



Διδακτορική Διατριβή

**Διερεύνηση της δυναμικής απόκρισης μη-γραμμικών συστημάτων
σε σεισμικούς παλμούς κοντινού πεδίου**

Ph.D. Dissertation

**Contributions to dynamic analysis of yielding systems
to near-fault earthquake motions**

Έλια Βογιατζάκη Elia Voyagaki

Πολιτικός Μηχανικός, M.Sc. Civil Engineer, M.Sc.



Διδακτορική Διατριβή

**Διερεύνηση της δυναμικής απόκρισης μη-γραμμικών συστημάτων
σε σεισμικούς παλμούς κοντινού πεδίου**

Ph.D. Dissertation

**Contributions to dynamic analysis of yielding systems
to near-fault earthquake motions**

Έλια Βογιατζάκη Elia Voyagaki

Πολιτικός Μηχανικός, M.Sc. Civil Engineer, M.Sc.

.....
Ι.Ν. ΨΥΧΑΡΗΣ
Αναπλ. Καθηγητής ΕΜΠ

.....
Χ. ΜΟΥΖΑΚΗΣ
Επικ. Καθηγητής ΕΜΠ

.....
Γ. ΜΥΛΩΝΑΚΗΣ
Αναπλ. Καθηγητής ΠΠ

.....
Χ. ΓΑΝΤΕΣ
Καθηγητής ΕΜΠ

.....
Δ. ΜΠΕΣΚΟΣ
Καθηγητής ΠΠ

.....
Γ. ΜΠΟΥΚΟΒΑΛΑΣ
Καθηγητής ΕΜΠ

.....
Κ. ΣΠΥΡΑΚΟΣ
Καθηγητής ΕΜΠ



ΕΘΝΙΚΟ ΜΕΤΣΟΒΙΟ ΠΟΛΥΤΕΧΝΕΙΟ
ΣΧΟΛΗ ΠΟΛΙΤΙΚΩΝ ΜΗΧΑΝΙΚΩΝ
ΤΟΜΕΑΣ ΔΟΜΟΣΤΑΤΙΚΗΣ

**ΔΙΕΡΕΥΝΗΣΗ ΤΗΣ ΔΥΝΑΜΙΚΗΣ ΑΠΟΚΡΙΣΗΣ ΜΗ-ΓΡΑΜΜΙΚΩΝ ΣΥΣΤΗΜΑΤΩΝ
ΣΕ ΣΕΙΣΜΙΚΟΥΣ ΠΑΛΜΟΥΣ ΚΟΝΤΙΝΟΥ ΠΕΔΙΟΥ**

ΔΙΔΑΚΤΟΡΙΚΗ ΔΙΑΤΡΙΒΗ

Έλιος Βογιατζάκης

Πολιτικού Μηχανικού Ε.Μ.Π., M.Sc.

Η διατριβή υποβλήθηκε στη Σχολή Πολιτικών Μηχανικών του Εθνικού Μετσόβιου Πολυτεχνείου προς εκπλήρωση των
προϋποθέσεων του τίτλου του Διδάκτορος Μηχανικού

ΤΡΙΜΕΛΗΣ ΣΥΜΒΟΥΛΕΥΤΙΚΗ ΕΠΙΤΡΟΠΗ:

1. Ι.Ν. Ψυχάρης, Αν. Καθηγητής Ε.Μ.Π.
(Επιβλέπων)

2. Χ. Μουζάκης, Επικ. Καθηγητής Ε.Μ.Π.

3. Γ. Μυλωνάκης, Αν. Καθηγητής Π.Π.

ΕΠΤΑΜΕΛΗΣ ΕΞΕΤΑΣΤΙΚΗ ΕΠΙΤΡΟΠΗ:

1. Ι.Ν. Ψυχάρης, Αν. Καθηγητής Ε.Μ.Π.
(Επιβλέπων)

2. Χ. Μουζάκης, Επικ. Καθηγητής Ε.Μ.Π.

3. Γ. Μυλωνάκης, Αν. Καθηγητής Π.Π.

4. Χ. Γαντές, Καθηγητής Ε.Μ.Π.

5. Δ. Μπέσκος, Καθηγητής Π.Π.

6. Γ. Μπουκοβάλας, Καθηγητής Ε.Μ.Π.

7. Κ. Σπυράκος, Καθηγητής Ε.Μ.Π.



NATIONAL TECHNICAL UNIVERSITY OF ATHENS
SCHOOL OF CIVIL ENGINEERING
DEPARTMENT OF STRUCTURAL ENGINEERING

**CONTRIBUTIONS TO DYNAMIC ANALYSIS OF YIELDING SYSTEMS
TO NEAR-FAULT EARTHQUAKE MOTIONS**

DOCTORAL THESIS

Elia Voyagaki

Civil Engineer, M.Sc.

The thesis is submitted to the School of Civil Engineering of the National Technical University of Athens in fulfillment of the requirements of the Degree of Doctor of Philosophy

ADVISORY COMMITTEE:

1. I.N. Psycharis, Assoc. Professor N.T.U.A.
(Supervisor)

2. H. Mouzakis, Assist. Professor N.T.U.A.

3. G. Mylonakis, Assoc. Professor U. Patras

EXAMINATION COMMITTEE:

1. I.N. Psycharis, Assoc. Professor N.T.U.A.
(Supervisor)

2. H. Mouzakis, Assist. Professor N.T.U.A.

3. G. Mylonakis, Assoc. Professor U. Patras

4. Ch. Gantes, Professor N.T.U.A.

5. D. Beskos, Professor U. Patras

6. G. Bouckovalas, Professor N.T.U.A.

7. C. Spyrakos, Professor N.T.U.A.

Copyright © Έλια Βογιατζάκη, 2013

Με επιφύλαξη παντός δικαιώματος

Απαγορεύεται η αντιγραφή, η αποθήκευση σε αρχείο πληροφοριών, η διανομή, η αναπαραγωγή, η μετάφραση ή μετάδοση της παρούσας εργασίας, εξ ολοκλήρου ή τμήματος αυτής, για εμπορικό σκοπό, υπό οποιαδήποτε μορφή και με οποιοδήποτε μέσο επικοινωνίας, ηλεκτρονικό ή μηχανικό, χωρίς την προηγούμενη έγγραφη άδεια του συγγραφέα. Επιτρέπεται η αναπαραγωγή, αποθήκευση και διανομή για σκοπό μη κερδοσκοπικό, εκπαιδευτικής ή ερευνητικής φύσης, υπό την προϋπόθεση να αναφέρεται η πηγή προέλευσης και να διατηρείται το παρόν μήνυμα. Ερωτήματα που αφορούν στη χρήση της εργασίας για κερδοσκοπικό σκοπό πρέπει να απευθύνονται προς το συγγραφέα.

Η έγκριση της διδακτορικής διατριβής από την Ανώτατη Σχολή Πολιτικών Μηχανικών του Εθνικού Μετσόβιου Πολυτεχνείου δεν υποδηλώνει αποδοχή των απόψεων του συγγραφέα (Ν. 5343/1932, Άρθρο 202).

Copyright © Elia Voyagaki, 2013

All rights reserved

Neither the whole nor any part of this doctoral thesis may be copied, stored in a retrieval system, distributed, reproduced, translated, or transmitted for commercial purposes, in any form or by any means now or hereafter known, electronic or mechanical, without the written permission from the author. Reproducing, storing and distributing this doctoral thesis for non-profitable, educational or research purposes is allowed, without prejudice to reference to its source and to inclusion of the present text. Any queries in relation to the use of the present doctoral thesis for commercial purposes must be addressed to its author.

Approval of this doctoral thesis by the School of Civil Engineering of the National Technical University of Athens (NTUA) does not constitute in any way an acceptance of the views of the author contained herein by the said academic organization (L. 5343/1932, art 202).

For
Villa, Max, Osama & Nadal

For my Friend
Demetri Elias

&

For my Mentor & Love
George

ACKNOWLEDGMENTS

This work would not have been possible without the help of a number of individuals to whom I feel deeply indebted. I would like to thank Professor Ioannis N. Psycharis for serving as my Committee Chair, for the ideas he contributed and the time he spent in working on the manuscript, especially the part related to rocking.

I would like to thank Professor George Mylonakis, for his advice, guidance, and encouragement during the course of this research. He has contributed to this work by providing innumerable ideas and exciting intellectual challenges. I sincerely and greatly appreciate his mentorship and support throughout my doctoral studies.

Thanks are due to Professor Harris Mouzakis for his time during the preparation of my presentation.

Professor and Researcher Extraordinaire Dimitri Beskos, I have been so privileged to have met you. I cannot express my gratitude for your advice and support.

Most valuable and witty comments on the material of this dissertation were provided by Professor George Bouckovalas, who thoroughly discussed the physical meaning of several of my solutions and looked for practical applications. Professor I am truly grateful.

Professor Charis Gantes provided constructive comments and suggestions during the preparation of the presentation. He also set the initial conditions for this task by serving as my undergraduate Diploma Thesis Advisor, getting me in touch with symbolic mathematics software, which I found immensely useful during the course of this research. Thank you for supporting me throughout the years.

Thanks are also due to Professor Constantin Spyrakos, for agreeing to serve on my Examination Committee.

Demetri Elias, thank you so much for being such a good friend and unique human being.

Dr. Yannis Taflampas for his kindness, support, advice and friendship all these years.

My dear friend Professor Costas Papantonopoulos, one of the wisest individuals I ever met for his generosity and teachings.

Thanks are due to all the friends I made in LEE and especially to Drs-to-be Charis Maniatakis & Vicky Dimakopoulou. It's your turn now guys!

Many thanks go to my dear friend, Dr-to-be, Christos Giarlelis for his belief in me and for always encouraging me to move forward.

Professor Anil Chopra for his valuable editorial comments regarding the Shift Theorem Approach. His experience and advice added a great deal in Chapters 6 & 7 of this Thesis.

Professor Elias Dimitrakopoulos for our discussions and for raising the bar by solving so elegantly the transcendental equations of the rocking block under a sinusoidal pulse.

Professor Dimitris Vamvatsikos for approaching my work from a different and exciting point of view.

Special thanks are due to Professor Jian Zhang of the University of California at Los Angeles for many interesting discussions and valuable suggestions.

Last, but not least, I would like to thank Professor Jonathan Stewart for his friendship, support and for always giving me opportunities to expand my horizons. The time I've spent in UCLA has been the most efficient and productive; I owe this to him.

Part of the research described in this paper was supported by National Technical University of Athens, Basic Research Program Grant #65/1720. This support is gratefully acknowledged.

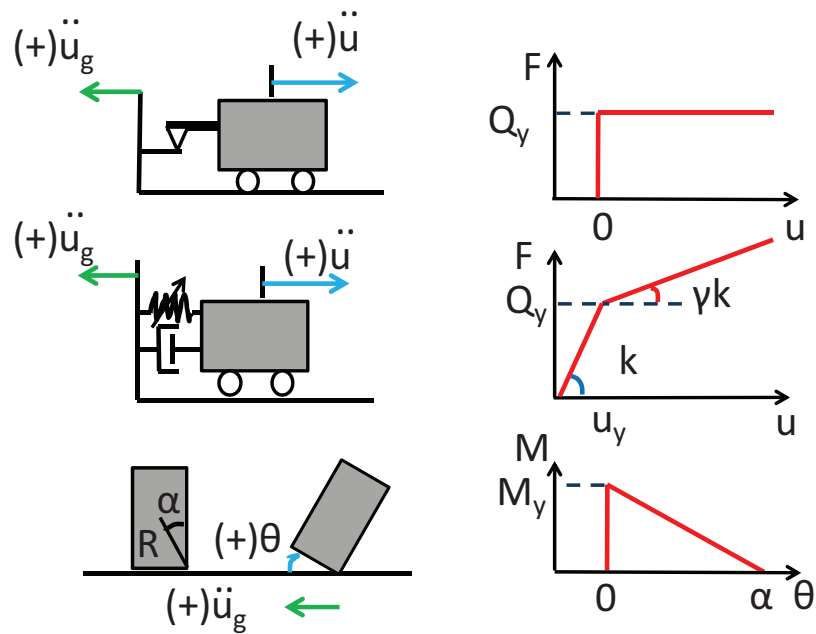
This work is dedicated to my parents for their endless love and support, to my childhood heroes Spyros & George Psimoulis, and to all those who dream of and fight for a better world.

Εκτενής Περίληψη

Σκοπός της διδακτορικής διατριβής είναι η διερεύνηση της απόκρισης συγκεκριμένων τύπων μη-γραμμικών συστημάτων σε ισχυρές παλμικές διεγέρσεις. Το παλμικό περιεχόμενο είναι τυπικό (όχι όμως αποκλειστικό) χαρακτηριστικό σεισμικών κινήσεων εγγύς πεδίου, οι οποίες συνδέονται εγγενώς με φαινόμενα πρόσω κατευθυντικότητας της σεισμικής πηγής και τείνουν να επιβάλλουν μεγάλες μόνιμες μετακινήσεις απουσία ικανής εδαφικής ή δομικής αντοχής. Η επίδραση του συγκεκριμένου τύπου διέγερσης σε συστήματα με πεπερασμένη αντοχή διαρροής ή δύναμη επαναφοράς δεν έχει μελετηθεί ούτε κατανοηθεί επαρκώς. Κύριοι άγνωστοι παραμένουν οι νόμοι ανακλιμάκωσης για το συσχετισμό της μέγιστης απόκρισης με τα κύρια χαρακτηριστικά της διεγερσης (εύρος, διάρκεια), και του δυναμικού συστήματος (οριακή αντοχή, υστέρηση). Στην περίπτωση πραγματικών σεισμικών καταγραφών, εμπλέκονται πρόσθετοι άγνωστοι οι οποίοι καθορίζονται από σεισμολογικούς παράγοντες που επηρεάζουν τα χαρακτηριστικά του σεισμικού παλμού όπως το εύρος, το σχήμα και το συχνοτικό περιεχόμενο. Παρά την μεγάλη σημασία αυτών των φαινομένων σε εφαρμογές της Επιστήμης του Πολιτικού Μηχανικού, η μελέτη τους βρίσκεται ακόμη σε στάδιο βασικής έρευνας.

Στόχος της διατριβής είναι η διεξοδική αναλυτική διερεύνηση μέσω εξαγωγής (όπου αυτό είναι εφικτό) κλειστών αναλυτικών λύσεων για διαφορετικούς τύπους ανελαστικών συστημάτων υπό εξιδανικευμένες παλμικές σεισμικές κινήσεις. Συγκεκριμένα, εξετάστηκαν τα παρακάτω μη-γραμμικά προβλήματα (Σχήμα 1):

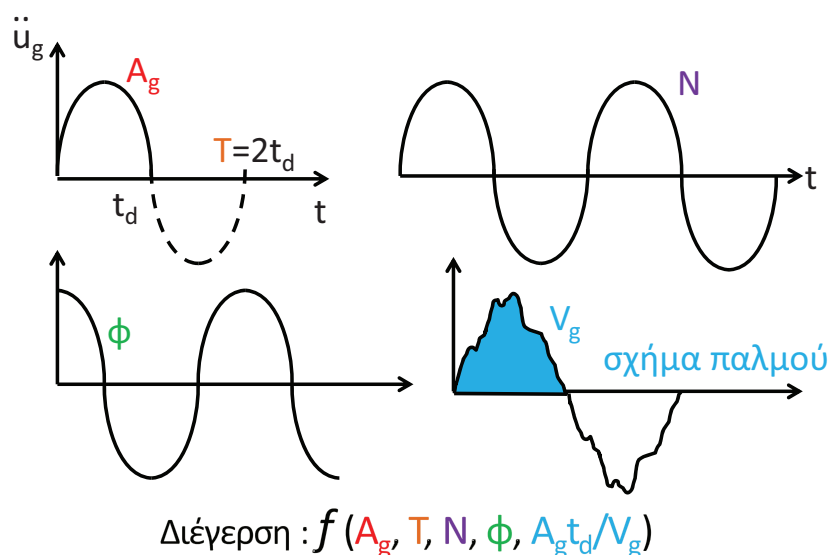
- (1) Ολίσθηση στερεού σώματος υπό συμμετρική ή ασύμμετρη τριβή.
- (2) Ανελαστική απόκριση ελαστικού-πλήρως πλαστικού και γενικευμένου διγραμμικού ταλαντωτή με απόσβεση.
- (3) Απόκριση λικνιζόμενου στερεού σώματος σε ανένδοτη βάση.



Σχήμα 1. Μη-γραμμικά συστήματα που εξετάζονται στη διατριβή

Έγινε θεώρηση των παρακάτω τύπων διεγέρσεων: (1) πραγματικές σεισμικές καταγραφές τύπου κοντινού πεδίου, (2) εξιδανικευμένοι παλμοί διάρκειας μισού και πλήρους κύκλου, διαφόρων σχημάτων. Η διέγερση περιγράφηκε μέσω απλών παραμέτρων όπως το εύρος εδαφικής επιτάχυνσης A_g , η περίοδος παλμού T , ο αριθμός κύκλων N , η γωνία φάσης φ και το σχήμα του παλμού, το οποίο εκφράστηκε με τη βοήθεια στοιχειώδους διαστατικής

ανάλυσης ως ο λόγος του γινομένου της εδαφικής επιτάχυνσης επί την ημιδιάρκεια του παλμού διά την μέγιστη αντίστοιχη ταχύτητα (Σχήμα 2).



Σχήμα 2. Παράμετροι της διέγερσης: μέγιστη επιτάχυνση A_g , περίοδος T , αριθμός κύκλων N , γωνία φάσης ϕ , παράμετρος σχήματος ($A_g t_d / V_g$).

Λόγω της εγγενούς μη-γραμμικότητας των υπό μελέτη φαινομένων, η επίλυση των εξισώσεων κίνησης επετεύχθη όχι μέσω γραμμικοποίησης της συνολικής απόκρισης, αλλά μέσω αναγνώρισης των σημείων διακλάδωσης, η οποία οδηγεί σε τμηματική, χρονικά, γραμμικοποίηση (μέθοδος διαδοχικών γεγονότων). Ο ρόλος των κυρίων χρονικών παραγώγων της διέγερσης (εδαφική επιτάχυνση, ταχύτητα, μετατόπιση) στην ανελαστική συμπεριφορά, διερευνήθηκε διεξοδικά με στόχο την εξαγωγή νέων νόμων ανακλιμάκωσης προσανατολισμένων σε πρακτικές εφαρμογές.

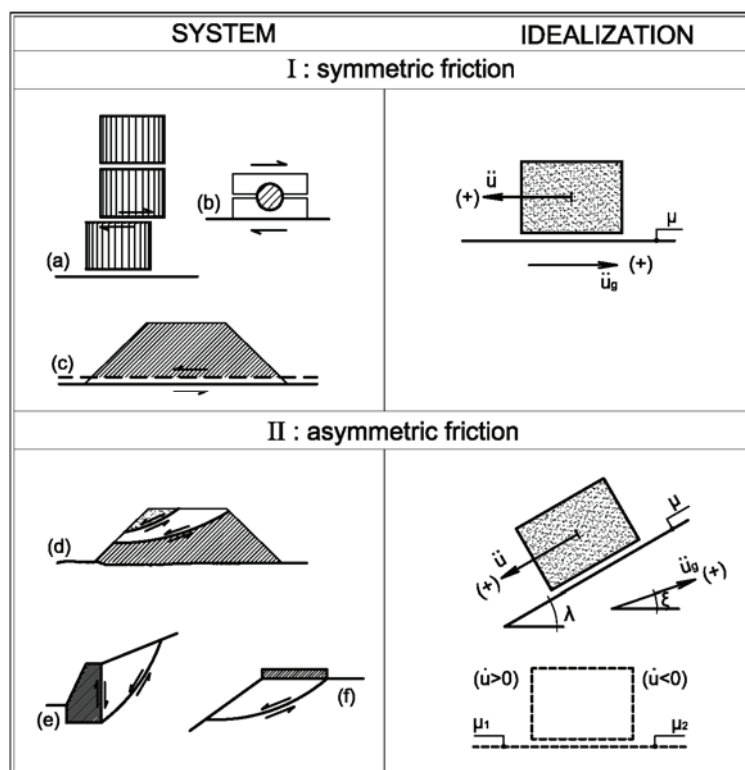
Η παρούσα εργασία είναι πρωτοποριακή καθώς για πρώτη φορά παρουσιάζονται ολοκληρωμένες αναλυτικές λύσεις για διαφορετικούς τύπους παλμών και δυναμικών συστημάτων. Είναι σημαντικό να αναφερθεί ότι η πλειονότητα των υφιστάμενων κλειστών

λύσεων εστιάζεται σε διεγέρσεις με μεγάλο αριθμό κύκλων φόρτισης (κατάλληλων π.χ. για βιομηχανικές εφαρμογές), οι οποίες συχνά λαμβάνονται με χρήση ειδικών τεχνικών γραμμικοποίησης (π.χ. απλοποίηση κατά Krylov και Bogoliubov 1943) ή βάσει της θεωρίας των διαταραχών (Nayfeh 2000). Η περίπτωση ωστικών φορτίων έχει βρει εφαρμογές στο αντικείμενο της βαλλιστικής. Οι λύσεις αυτές αφορούν μεγάλα εύρη διέγερσης και μικρούς χρόνους αποτόνωσης [π.χ. Timoshenko & Young 1948, Biggs 1964] και δεν είναι άμεσα εφαρμόσιμες στα προβλήματα που πραγματεύεται η παρούσα εργασία. Στο γνωστικό πεδίο της σεισμικής μηχανικής, οι διαθέσιμες λύσεις είναι εξαιρετικά περιορισμένες. Υπό αυτό το πρίσμα, η διατριβή συμβάλλει στην πρόοδο της επιστήμης.

Η διατριβή αποτελείται από οκτώ (8) Κεφάλαια και πέντε (5) Παραρτήματα. Μια ανασκόπηση του περιεχομένου παρέχεται στη συνέχεια:

Στο Κεφάλαιο 1 παρουσιάζεται ο σκοπός της διατριβής και γίνεται ανασκόπηση, της σχετικής με τις σεισμικές κινήσεις κοντινού πεδίου, βιβλιογραφίας. Επίσης γίνεται αναφορά στα μη-γραμμικά δυναμικά συστήματα και στις υφιστάμενες αναλυτικές λύσεις. Τέλος, παρουσιάζονται συνοπτικά τα περιεχόμενα των υπολοίπων κεφαλαίων.

Στο Κεφάλαιο 2, παρουσιάζονται νέες αναλυτικές λύσεις για την ολίσθηση στερεού σώματος υποκείμενου σε εξιδανικευμένους παλμούς εδαφικής επιτάχυνσης. Το προσομοίωμα αφορά στερεό σώμα επί (εν γένει) κεκλιμένου επιπέδου τριβής. Αν και εξιδανικευμένο, το συγκεκριμένο μοντέλο χρησιμοποιείται ευρέως για την ανάλυση ποικιλίας κατασκευών όπως τοίχων αντιστήριξης, κρηπιδότοιχων, φραγμάτων, πρανών, επιχωμάτων (Σχήμα 3).

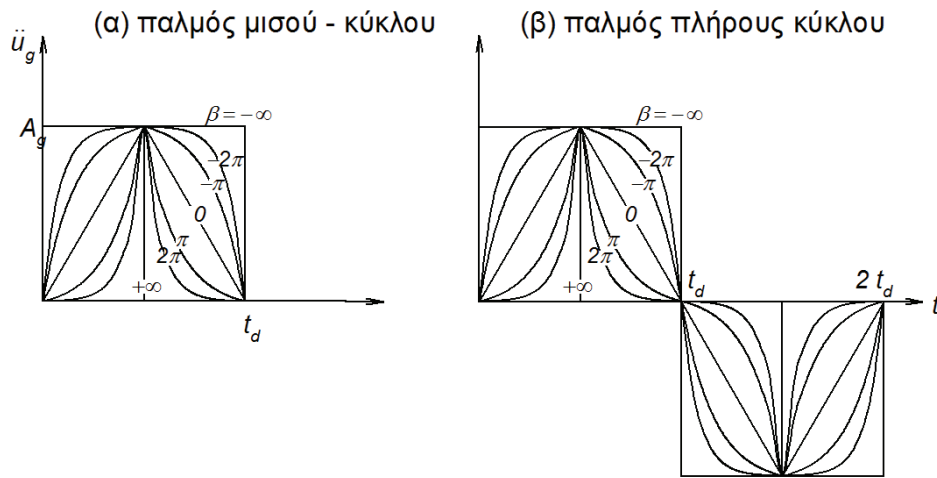


Σχήμα 3. Προβλήματα που αναλύονται μέσω προσομοιώματος ολισθαίνοντος στερεού σε οριζόντιο και κεκλιμένο επίπεδο.

Ο στόχος της παρούσας μελέτης είναι τριπλός: (1) η εξαγωγή αναλυτικών λύσεων για μεγάλο εύρος εξιδανικευμένων παλμών; (2) η διερεύνηση της ολίσθησης για συμμετρικές και ασύμμετρες συνθήκες τριβής υπό μονόπλευρη και αμφίπλευρη διέγερση; (3) η αναζήτηση εναλλακτικών τρόπων κανονικοποίησης της μέγιστης ολίσθησης συναρτήσει της μέγιστης παλμικής επιτάχυνσης και ταχύτητας, της διάρκειας και του σχήματος του παλμού. Για τον σκοπό αυτό γίνεται χρήση του γενικευμένου εκθετικού παλμού (Σχήμα 4), ο οποίος έχει την ιδιότητα να περιγράφει μέσω μίας μοναδικής παραμέτρου σχήματος β , άπειρο πλήθος παλμικών διεγέρσεων, από παλμούς ορθογωνικής μορφής για $\beta \rightarrow -\infty$ έως απόλυτα αιχμηρούς παλμούς για $\beta \rightarrow +\infty$ [Εξίσωση (1)].

$$\ddot{u}_g(t) = \underbrace{f(t)}_{0 \rightarrow t_d/2} + \underbrace{f(t_d-t)}_{t_d/2 \rightarrow t_d} - \underbrace{f(-t_d+t)}_{t_d \rightarrow 3t_d/2} - \underbrace{f(2t_d-t)}_{3t_d/2 \rightarrow 2t_d} \quad (1)$$

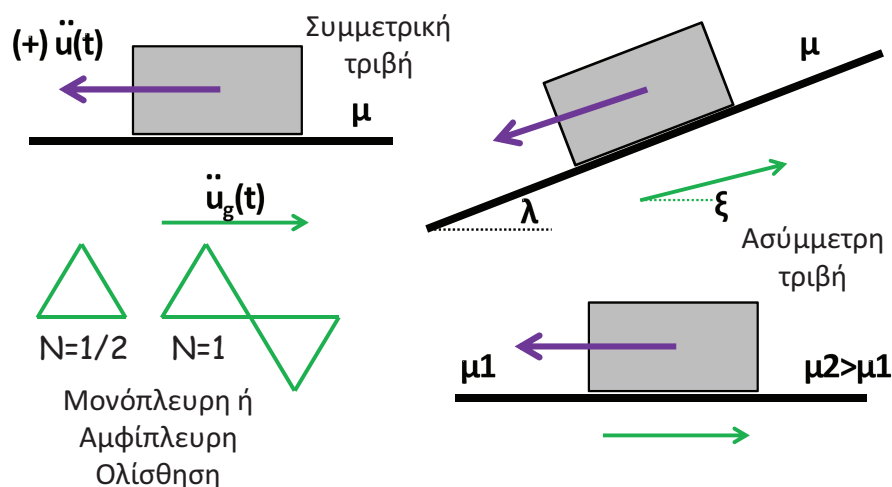
όπου η συνάρτηση $f(t) = A_g (1 - e^{2\beta t/t_d}) / (1 - e^\beta)$ εκφράζει τον πρώτο αύξοντα κλάδο της διέγερσης, ενώ το σύμβολο $\underbrace{\quad}_{t_1 \rightarrow t_2}$ αντιστοιχεί στο εύρος εφαρμογής κάθε όρου της εξίσωσης (π.χ., από t_1 έως t_2). Για παλμούς μισού κύκλου, ισχύουν φυσιολογικά μόνο οι δύο πρώτοι όροι της Εξ. (1).



Σχήμα 4. Χρονοϊστορία εδαφικής επιτάχυνσης γενικευμένου εκθετικού παλμού μισού και πλήρους κύκλου.

Οι μη-γραμμικές εξισώσεις κίνησης διατυπώθηκαν σε συμπαγή μορφή με χρήση κατάλληλων ασυνεχών ή ωστικών συναρτήσεων τύπου Heaviside και Dirac, αντίστοιχα. Η κανονικοποίηση πραγματοποιήθηκε βάσει τριών βασικών παραμέτρων (σχετικής αντοχής, ασυμμετρίας τριβής, χρονικής εξάρτησης τριβής). Αναλυτικές λύσεις προέκυψαν για τις παρακάτω τρεις γενικές περιπτώσεις (Σχήμα 5):

- (1) ολίσθηση σταθερής συμμετρικής τριβής
- (2) ολίσθηση σταθερής ασύμμετρης τριβής
- (3) ολίσθηση μεταβλητής ασύμμετρης τριβής



Σχήμα 5. Περιπτώσεις ολίσθησης στερεού σώματος που εξετάστηκαν.

Για την αναζήτηση κλειστών λύσεων, πέρα από τον γενικευμένο παλμό του Σχήματος 3 χρησιμοποιήθηκαν 10 ακόμη εξιδανικευμένοι παλμοί (box, sinusoidal, co-sinusoidal, versed-sine, triangular, ramp, inverse ramp, exponential) μισού και πλήρους κύκλου, αποτελέσματα για τους οποίους παρατίθενται στο Παράρτημα 2. Οι λύσεις παρουσιάστηκαν τόσο με τη μορφή εξισώσεων (σε μορφή πινάκων που αντιστοιχούν στις 3 γενικές περιπτώσεις για κάθε παλμό διάρκειας μισού και πλήρους κύκλου), όσο και με αντίστοιχα αδιάστατα γραφήματα. Εξήχθησαν και διατυπώθηκαν νέες σχέσεις ανακλιμάκωσης που συσχετίζουν τη μέγιστη απόκριση με τις βασικές παραμέτρους του προβλήματος (αντίσταση σε ολίσθηση, μέγεθος, διάρκεια διέγερσης).

Διερευνήθηκε η δυνατότητα επίλυσης του αντίστροφου προβλήματος και η αντικατάσταση των εξιδανικευμένων διεγέρσεων από ένα μοναδικό αντιπροσωπευτικό σχήμα για την

περίπτωση ολίσθησης υπό σταθερή τριβή. Ένα πρόσθετο σύνολο γραφημάτων και πινάκων κατασκευάστηκε για τον σκοπό αυτό.

Τα κυριότερα συμπεράσματα του Κεφαλαίου 2 είναι:

- (1) Η ταχύτητα του παλμού και η μέγιστη ολίσθηση λόγω τριβής είναι αλληλένδετες. Αυτό μπορεί να γίνει κατανοητό δεδομένου ότι όσο μεγαλύτερο είναι το εμβαδόν του παλμού επιτάχυνσης μετά την έναρξη της ολίσθησης (δηλαδή πάνω από τη στάθμη της επιτάχυνσης διαρροής), τόσο μεγαλύτερη είναι η διάρκεια της ολίσθησης, και συνεπώς τόσο μεγαλύτερη είναι η μετατόπιση του σώματος.
- (2) Καθώς η έναρξη της ολίσθησης προϋποθέτει υπέρβαση ενός συγκεκριμένου επιπέδου σεισμικής επιτάχυνσης ανεξάρτητα από την τιμή των άλλων παραμέτρων, η σεισμική ταχύτητα δεν πρέπει να θεωρείται από μόνη της ικανή να περιγράψει την καταστρεπτικότητα του παλμού. Αντίθετα, όπως καταδεικνύεται από την Εξίσωση (2.5), η ολίσθηση εξαρτάται άμεσα από την εδαφική μετακίνηση και μόνο έμμεσα από την ταχύτητα. Συμπεραίνουμε λοιπόν ότι η επίδραση της εδαφικής μετακίνησης στη μέγιστη ολίσθηση είναι σημαντικότερη από την επικρατούσα μέχρι σήμερα άποψη.
- (3) Οι χρόνοι έναρξης της ολίσθησης και της επίτευξης της μέγιστης ολίσθησης είναι αύξουσες και φθίνουσες συναρτήσεις, αντίστοιχα, της αντίστασης σε ολίσθηση η .
- (4) Για διεγέρσεις τύπου μισού κύκλου, μόνο το μισό της εδαφικής μετακίνησης κατά τη διάρκεια της απόκρισης (βήμα εδαφικής μετακίνησης) μπορεί να μεταφερθεί, ως σχετική ως προς τη βάση, ολίσθηση, στο στερεό σώμα. Η συμπεριφορά αυτή είναι παράδοξη, αν και ακριβής από μαθηματικής σκοπιάς, καθώς λογικά αναμένεται ότι

υπό συνθήκες μηδενικής τριβής η σχετική ως προς τη βάση ολίσθηση ισούται, κατ' απόλυτη τιμή, με την εδαφική μετακίνηση. Το φαινόμενο ερμηνεύεται πλήρως μέσω κατάλληλης ασυμπτωτικής ανάλυσης και οφείλεται στη φύση των παλμών μισού κύκλου. Ωστόσο, το παράδοξο αυτό δεν μπορεί να επαληθευτεί εύκολα πειραματικά, καθώς εκδηλώνεται σε χρόνο τάξεις μεγέθους μεγαλύτερο από την διάρκεια του παλμού.

- (5) Ιδιαίτερα περίπλοκη είναι η εξάρτηση της μέγιστης μετατόπισης από την αντίσταση σε ολίσθηση η όπως προκύπτει από τις Εξισώσεις (2.9) και (2.18). Έτσι, απλές σχέσεις ανακλιμάκωσης πολυωνυμικής ή εκθετικής μορφής μπορεί να μην είναι εφικτές ακόμη και στην περίπτωση εξιδανικευμένων παλμών όπως αυτών της παρούσας διατριβής.
- (6) Σε συνέχεια του προηγούμενου συμπεράσματος, η επιλογή του βήματος εδαφικής μετακίνησης Δu_{gm} , ως παραμέτρου κανονικοποίησης της μέγιστης ολίσθησης, υπερέχει άλλων συνδυασμών που προέκυψαν από διαστατική ανάλυση καθώς η συγκεκριμένη παράμετρος φράσσει τα αποτελέσματα τόσο για παλμούς μισού κύκλου όσο και για παλμούς πλήρους κύκλου μεταξύ $\frac{1}{2}$ -0 και 1-0, στα όρια $\eta = 0$ και 1 αντίστοιχα, και οδηγεί σε μικρότερη διασπορά συναρτήσεων των η και β (Σχήματα 2.13, 2.14). Συνεπώς συμπεραίνουμε ότι η κανονικοποίηση της ολίσθησης βάσει διαστατικής ανάλυσης δεν φαίνεται να υπερτερεί έναντι εναλλακτικών επιλογών που στηρίζονται στα φυσικά χαρακτηριστικά του προβλήματος.
- (7) Οι λύσεις για παλμούς μισού κύκλου μπορούν να περιγράψουν πιστά την απόκριση σε ολίσθηση για παλμούς πολλών κύκλων υπό συνθήκες ασύμμετρης τριβής (Σχήμα

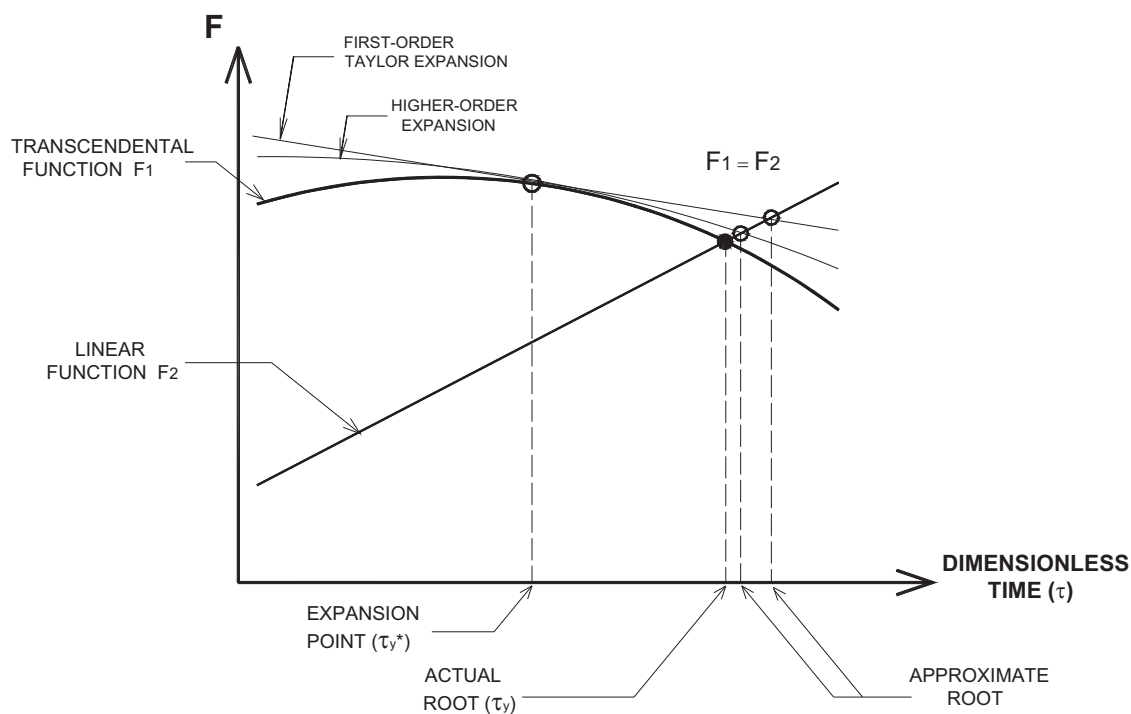
2.16, Πίνακας 2.4). Οι παλμοί μισού κύκλου δεν θα πρέπει λοιπόν να αντιμετωπίζονται ως διεγέρσεις «ακαδημαϊκού» τύπου χωρίς εφαρμοσιμότητα στην πράξη και με περιορισμένο φυσικό νόημα. Αντίστοιχα, η μη-μηδενική εδαφική ταχύτητα στο τέλος του παλμού μισού κύκλου δεν αποτελεί περιορισμό για την εφαρμογή των παλμών αυτών σε πραγματικά προβλήματα.

- (8) Οι παλμοί μισού κύκλου οδηγούν συστηματικά σε μεγαλύτερες μετατοπίσεις από παλμούς πλήρους κύκλου για ίδιο εύρος εδαφικής επιτάχυνσης και αντίστασης σε ολίσθηση. Αυτό έρχεται σε αντίθεση με την απόκριση σε συνήθεις εδαφικές κινήσεις πολλών κύκλων (ιδιαίτερα στην περίπτωση ασύμμετρης τριβής), για τις οποίες η μετατόπιση γενικώς αυξάνει με τον αριθμό των κύκλων.

Αξίζει να σημειωθεί ότι η καταστρεπτικότητα παλμών τύπου πρόσω κατευθυντικότητας όπως αυτοί που εξετάζονται στην παρούσα εργασία, δεν είναι απαραίτητα μεγαλύτερη από αυτή κινήσεων μικρότερης επιτάχυνσης και μεγαλύτερου αριθμού κύκλων φόρτισης (π.χ. καταγραφές με επιρροή όπισθεν κατευθυντικότητας), για γεωτεχνικές κατασκευές. Οι τελευταίες ενδέχεται να οδηγήσουν σε μεγαλύτερες μετατοπίσεις παρουσία στατικών δυνάμεων αστάθειας (π.χ., ενεργητικές ωθήσεις) και μεγάλου αριθμού κύκλων φόρτισης. Η μελέτη αυτών των φαινομένων ξεφεύγει από τους στόχους της παρούσας διατριβής.

Η διατύπωση σχέσεων ανακλιμάκωσης που συσχετίζουν τη μέγιστη απόκριση με τις βασικές παραμέτρους του προβλήματος (αντοχή, φυσική συχνότητα, διάρκεια διέγερσης, υστέρηση) αποτελεί το αντικείμενο του Κεφαλαίου 3. Συγκεκριμένα:

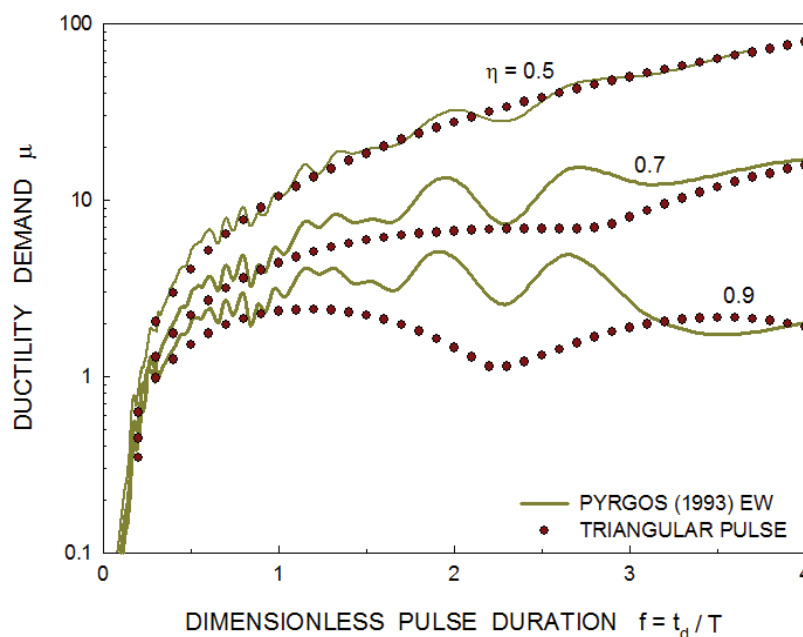
Στο Κεφάλαιο 3 διερευνάται η ελαστική και ανελαστική απόκριση μονοβάθμιων ταλαντωτών με διγραμμικό διάγραμμα δύναμης επαναφοράς-μετατόπισης σε εξιδανικευμένους παλμούς επιτάχυνσης. Εξετάζονται τα παρακάτω βασικά σχήματα παλμών: (1) συμμετρικός τριγωνικός, (2) ημιτονικός, (3) εκθετικός, (4) γενικευμένος εκθετικός, (5) ορθογωνικός, (6) γραμμικός ανιούσας επιτάχυνσης, (7) γραμμικός κατιούσας επιτάχυνσης. Αρχικά παρουσιάζεται αριθμητική μελέτη της επιρροής των χαρακτηριστικών του ταλαντωτή (αντοχή, δυσκαμψία, απόσβεση, κράτυνση) και της διέγερσης (αριθμός κύκλων, εύρος) στην ανελαστική απόκριση. Οι παράμετροι που μελετώνται περιλαμβάνουν τον χρόνο διαρροής, τον χρόνο μέγιστης απόκρισης και την απαίτηση πλαστιμότητας. Σχολιάζεται επίσης η επίδραση των ανώτερων αρμονικών της διέγερσης (φάση, εύρος, συχνότητα). Τα αποτελέσματα παρουσιάζονται με τη μορφή αδιάστατων γραφημάτων και σχέσεων παλινδρόμησης. Στη συνέχεια μελετώνται ταλαντωτές μηδενικής κράτυνσης και απόσβεσης. Για την συγκεκριμένη κατηγορία συστημάτων εξάγονται κλειστές αναλυτικές λύσεις που περιγράφουν τη συμπεριφορά του ταλαντωτή σε όλες τις φάσεις της απόκρισης (ανιούσα διέγερση, κατιούσα διέγερση, ελεύθερη ταλάντωση). Για τον καθορισμό του χρόνου διαρροής, οι σχετικές υπερβατικές εξισώσεις γραμμικοποιούνται μέσω δυναμοσειρών Taylor ο οποίες αναπτύσσονται γύρω από ένα χρονικό σημείο που προσεγγίζει τη λύση. Παρατηρούμε ότι όταν το κέντρο ανάπτυξης επιλέγεται κοντά στον πραγματικό χρόνο διαρροής, επιτυγχάνεται εξαιρετική ακρίβεια χωρίς ανάγκη χρήσης επαναληπτικής διαδικασίας ή δυναμοσειρών ανώτερης τάξης. Προτείνονται διάφορες τεχνικές για τον σκοπό αυτό (Σχήμα 6).



Σχήμα 6. Εύρεση ριζών υπερβατικών εξισώσεων, για τον υπολογισμό του χρόνου διαρροής, μέσω αναπτυγμάτων σε δυναμοσειρές Taylor.

Οι χρόνοι μέγιστης απόκρισης και η απαίτηση πλαστιμότητας είναι ευκολότερο να υπολογιστούν καθώς δεν εμπλέκονται υπερβατικές συναρτήσεις στους υπολογισμούς. Σε αντίθεση με τους σχεδόν καθολικώς εφαρμοζόμενους αριθμητικούς αλγορίθμους, οι προκύπτουσες κλειστές αναλυτικές εκφράσεις παρέχουν ισχυρή εποπτεία της φυσικής του προβλήματος. Οι προτεινόμενες λύσεις συγκρίνονται με αριθμητικά αποτελέσματα από πραγματικές καταγραφές εγγύς πεδίου (Σχήμα 7).

Τέλος, παρουσιάζονται δύο απλές εφαρμογές σε ιστορικά περιστατικά.



Σχήμα 7. Απαίτηση πλαστιμότητας ελαστοπλαστικού μονοβάθμιου ταλαντωτή. Σύγκριση αναλυτικής λύσης για τριγωνικό παλμό μισού κύκλου με αριθμητική επίλυση για πραγματική σεισμική καταγραφή.

Τα σημαντικότερα ευρήματα του Κεφαλαίου 3 περιλαμβάνουν τα παρακάτω:

- (1) Η απαίτηση πλαστιμότητας ελαστοπλαστικών συστημάτων με αντίσταση διαρροής μικρότερη από το εύρος του παλμού μπορεί να είναι πολύ υψηλή (μεγαλύτερη του 10). Είναι δε αύξουσα συνάρτηση της διάρκειας του παλμού, αντίθετα με την περίπτωση ελαστικών συστημάτων όπου η μέγιστη απόκριση επιτυγχάνεται κοντά στον συντονισμό ($f=1$).
- (2) Ο χρόνος διαρροής προβλέπεται με μεγάλη ακρίβεια συναρτήσει της αντοχής σε διαρροή η και της κανονικοποιημένης διάρκειας του παλμού f (Εξισώσεις 3.27, 3.32, 3.33).

- (3) Η μέγιστη απόκριση λαμβάνει χώρα μετά τη μέγιστη παλμική επιτάχυνση τόσο για ελαστικούς όσο και για ανελαστικούς ταλαντωτές. Ο χρόνος μέγιστης απόκρισης υπολογίζεται από την Εξίσωση (3.19).
- (4) Η απόκριση επηρεάζεται σημαντικά από την μετελαστική κράτυνση. Η συγκεκριμένη εξάρτηση είναι μη γραμμική και το φαινόμενο εξασθενεί για περιόδους μεγαλύτερες του $(1.5xT_d)$. Για μικρότερες περιόδους, ωστόσο, ταλαντωτές με κράτυνση περί το 5% παρουσιάζουν μέχρι και 100% μικρότερη παραμόρφωση συγκριτικά με αντίστοιχους ελαστικούς-πλήρως πλαστικούς ταλαντωτές. Η παρατήρηση αυτή ενισχύει την άποψη ότι ο σχεδιασμός κατασκευών στο εγγύς πεδίο σεισμογόνων ρηγμάτων θα πρέπει να εξασφαλίζει ικανή κράτυνση, πέρα από ικανή πλαστιμότητα.
- (5) Η επίδραση της απόσβεσης στη συμπεριφορά του ταλαντωτή εξετάζεται διεξοδικά. Προκύπτει ότι αύξηση του συντελεστή ζ οδηγεί πάντοτε σε μείωση της απόκρισης. Το φαινόμενο είναι ως επί το πλείστον ανεξάρτητο της περιόδου, της αντοχής σε διαρροή και της μορφής του παλμού.
- (6) Οι ανώτερες αρμονικές που συχνά εμπεριέχονται σε παλμούς σεισμικών καταγραφών μεταβάλλουν (συνήθως μειώνουν) την απαίτηση πλαστιμότητας, ιδιαίτερα για περιόδους μεγαλύτερες από $0.6xT_p$ και χαμηλές τιμές του λόγου (A_s/A_p) .
- (7) Ο συντελεστής συμπεριφοράς $R (= m SA / Q_y)$ φαίνεται ως λιγότερο κατάλληλος από την αδιάστατη αντοχή σε διαρροή $\eta (= Q_y / m A_g)$ για την κανονικοποίηση της ανελαστικής απόκρισης. Αυτό μπορεί να κατανοηθεί αν ληφθούν υπόψη οι θεμελιώδεις διαφορές μεταξύ ελαστικής και ανελαστικής συμπεριφοράς υπό παλμικές κινήσεις, και ότι η εμπλοκή της ελαστικής φασματικής επιτάχυνσης SA μέσω του

συντελεστή R στη λύση δεν σχετίζεται με την φυσική του προβλήματος (και συνεπώς περιπλέκει τα αποτελέσματα).

- (8) Οι παλμοί πλήρους κύκλου ($N = 1$) ενδέχεται να οδηγήσουν σε μεγαλύτερη απαίτηση πλαστιμότητας, με την αύξηση της κράτυνσης. Μια ερμηνεία για την συγκεκριμένη συμπεριφορά είναι ότι η μέγιστη απόκριση λαμβάνει χώρα μετά την αλλαγή της φοράς της κίνησης, δηλαδή προς την αντίθετη κατεύθυνση από την αρχική κίνηση του ταλαντωτή.
- (9) Για ελαστοπλαστικούς ταλαντωτές χωρίς απόσβεση υπό παλμούς μισού κύκλου, υπολογίστηκαν αναλυτικά οι χρόνοι διαρροής και μέγιστης απόκρισης, η μέγιστη απόκριση και η αντίστοιχη απαίτηση πλαστιμότητας. Οι αντίστοιχες εξισώσεις (3.19–3.34) συμφωνούν με τα δεδομένα από τις αριθμητικές επιλύσεις. Όπως και στην περίπτωση της ολίσθησης στερεού σώματος (Κεφάλαιο 2), η ανελαστική απόκριση, που τώρα περιγράφεται μέσω της απαίτησης πλαστιμότητας, εξαρτάται και από τις τρεις παραγώγους της εδαφικής κίνησης (επιτάχυνση, ταχύτητα και μετατόπιση), και άρα η εδαφική ταχύτητα δεν πρέπει να θεωρείται από μόνη της ικανή να περιγράψει την καταστρεπτικότητα του παλμού.
- (10) Μελετήθηκαν δύο ιστορικά περιστατικά αστοχιών λόγω παλμών εγγύς πεδίου: η κατάρρευση του νοσοκομείου Olive View το 1971 κατά τον σεισμό του San Fernando, και του κτηρίου διοίκησης της EBO (Ελληνική Βιομηχανία Όπλων) κατά τον σεισμό του Αιγίου το 1995. Χρησιμοποιήθηκαν οι καταγραφές Pacoima Dam και Aegion αντίστοιχα, και προέκυψε ότι οι κατασκευές δεν μπορούσαν να αναλάβουν την μεγάλη απαίτηση πλαστιμότητας, λόγω της εγγύτητας στο ρήγμα (επαυξημένη

λόγω φαινομένων πρόσω κατευθυντικότητας). Αν και οι παραμορφώσεις που υπολογίστηκαν από τις αναλυτικές σχέσεις βρίσκονται σε συμφωνία με τα αποτελέσματα ακριβέστερων αριθμητικών αναλύσεων μέσω θεώρησης συστημάτων πολλών βαθμών ελευθερίας, οι αντίστοιχες μέγιστες μετατοπίσεις προέκυψαν σημαντικά μικρότερες από τις πραγματικές. Οι λόγοι για την συγκεκριμένη απόκλιση είναι γνωστοί [Mahin et al. 1976, Bertero et al. 1978, Gazetas et al. 1995] και η ανάλυση τους ξεφεύγει από τους στόχους της παρούσας διατριβής.

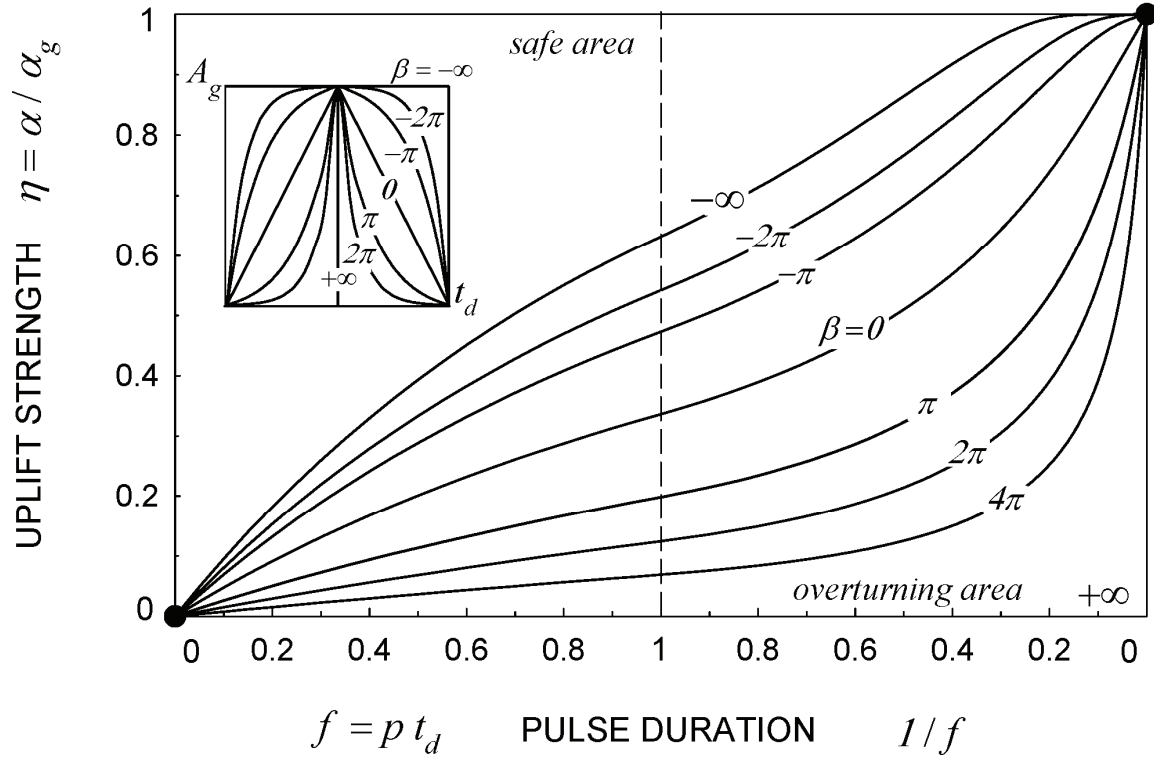
Στην περίπτωση του λικνισμού στερεών σωμάτων διατυπώθηκαν οι εξισώσεις κίνησης για σώμα εδραζόμενο σε ανένδοτη βάση. Εξήχθησαν αναλυτικές λύσεις των γραμμικοποιημένων εξισώσεων κίνησης για εξιδανικευμένους παλμούς τύπου μισού και πλήρους κύκλου. Μελετήθηκε η επίδραση στην απόκριση της γεωμετρίας του σώματος και των χαρακτηριστικών της διέγερσης, και έγινε σύγκριση των αποτελεσμάτων με διαθέσιμες λύσεις από την βιβλιογραφία. Πιο αναλυτικά, τα Κεφάλαια 4 και 5 έχουν ως εξής:

Στο Κεφάλαιο 4 επανεξετάζεται το κλασικό πρόβλημα του λικνισμού στερεού σώματος υποκείμενου σε παλμικές σεισμικές διεγέρσεις. Το σώμα θεωρείται ορθογωνικό, εδραζόμενο σε ανένδοτη βάση και υποβαλλόμενο σε ευρεία γκάμα εξιδανικευμένων παλμών εδαφικής επιτάχυνσης που περιγράφονται από την γενικευμένη εκθετική συνάρτηση που παρουσιάστηκε στο Κεφάλαιο 2. Το πρόβλημα αντιμετωπίζεται αναλυτικά μέσω των γραμμικοποιημένων εξισώσεων λικνισμού υπό την παραδοχή ραδινής γεωμετρίας και λικνισμού χωρίς ολίσθηση [Εξίσωση (2)]

$$\ddot{\theta} - p^2 \theta = +p^2 \ddot{u}_g / g - p^2 \alpha \operatorname{sgn}(\theta) \quad (2)$$

Η μέγιστη λικνιστική απόκριση και τα όρια ανατροπής για διαφορετικές κυματομορφές παρουσιάζονται υπό τη μορφή κανονικοποιημένων κλειστών εκφράσεων και γραφημάτων. Για τον σκοπό αυτό εισάγονται δύο κύριες παράμετροι: η αδιάστατη διάρκεια του παλμού f (που ορίζεται ως η απόλυτη ημιδιάρκεια του παλμού επί τη χαρακτηριστική συχνότητα του σώματος), και η αδιάστατη αντίσταση σε λικνισμό η (δηλαδή ο λόγος της απαιτούμενης εδαφικής επιτάχυνσης για ανύψωση-αποκόλληση διά τη μέγιστη επιτάχυνση του παλμού). Η κλειστή λύση στην Εξίσωση (3) περιγράφει το κριτήριο ανατροπής για τον γενικευμένο παλμό μισού κύκλου. Η κρίσιμη τιμή της παραμέτρου αντοχής, η_w , ερμηνεύεται ως η οριακή αντοχή που διαχωρίζει τα συστήματα που ανατρέπονται ($\eta < \eta_w$) από αυτά που δεν ανατρέπονται ($\eta > \eta_w$), ενώ οι καμπύλες $\eta_w = \eta_w(f, \beta)$ του Σχήματος 8 περιγράφουν αυτό ακριβώς το σύνορο και ονομάζονται όρια ανατροπής.

$$\eta_w = \frac{1}{1 - e^\beta} \left\{ 1 - \left[\frac{e^f (f + 2\beta)}{2f e^{\frac{f}{2} + \beta} - (f - 2\beta)} \right]^{\frac{2\beta}{f - 2\beta}} \right\} \quad (3)$$



Σχήμα 8. Περιοχές ανατροπής και ευστάθειας λικνιζόμενου στερεού σώματος υπό παλμούς μισού κύκλου. Το σχήμα του παλμού κυμαίνεται από ορθογωνικό έως τελείως αιχμηρό, βάσει της τιμής της παραμέτρου β .

Η γραμμικοποιημένη απόκριση συγκρίνεται με την μη-γραμμικοποιημένη μέσω ειδικής ενεργειακής μεθόδου που οδηγεί στην προσεγγιστική κλειστή λύση

$$\frac{\eta_{w,non-lin}}{\eta_{w,lin}} \cong \frac{\alpha^2 \cos[\alpha(1-\tilde{\chi})]}{2(1-\cos \alpha)} \quad (4)$$

όπου α η γωνία ραδινότητας του σώματος και $\tilde{\chi}$ μια αδιάστατη παράμετρος που συμβολίζει τη μέση τιμή της γωνίας περιστροφής (θ/α) κατά τον λικνισμό και παίρνει τιμές στο διάστημα $(0, 1)$. Η Εξίσωση (4) συνδέει τα κριτήρια ανατροπής βάσει της γενικής μη-

γραμμικής εξίσωσης κίνησης με την κλειστή λύση της Εξ. (3), η οποία προέκυψε μετά από γραμμικοποίηση, βάσει αντικατάστασης των σχετικών τριγωνομετρικών όρων με μονώνυμα Taylor πρώτης τάξης.

Καταδεικνύεται ότι οι μη-γραμμικοποιημένες εξισώσεις κίνησης οδηγούν σε μεγαλύτερη ευστάθεια από τις αντίστοιχες γραμμικοποιημένες λόγω της εγγενούς ανύψωσης του κέντρου βάρους (η οποία απαιτεί δαπάνη έργου) κατά τη διάρκεια του λικνισμού. Τέλος σχολιάζονται εν συντομία θέματα σχέσεων ανακλιμάκωσης.

Τα κύρια συμπεράσματα του Κεφαλαίου 4 είναι:

- (1) Κατ' εφαρμογή της γενικευμένης εκθετικής κυματομορφής που περιγράφεται από την Εξίσωση (1) της παρούσας περίληψης, προέκυψαν νέες αναλυτικές λύσεις για τον λικνισμό στερεών σωμάτων οι οποίες εφαρμόζονται σε άπειρο πλήθος συμμετρικών παλμών που κυμαίνονται από τελείως αιχμηρό ($\beta \rightarrow +\infty$) έως απόλυτα ορθογωνικό ($\beta \rightarrow -\infty$).
- (2) Η λικνιστική απόκριση παρουσιάζει πολυπλοκότητα ακόμη και για απλούς εξιδανικευμένους παλμούς τύπου μισού κύκλου όπως οι παραπάνω.
- (3) Η ανατροπή υπό παλμούς μισού κύκλου μπορεί να περιγραφεί μέσω δύο μόνο αδιάστατων παραμέτρων: την αδιάστατη διάρκεια του παλμού f (διάρκεια παλμού μισού κύκλου επί χαρακτηριστική συχνότητα σώματος) και την αδιάστατη αντίσταση σε λικνισμό η (λόγος της απαιτούμενης εδαφικής επιτάχυνσης για την ανύψωση διά την μέγιστη επιτάχυνση του παλμού). Παράλληλα, στην περίπτωση της γραμμικοποιημένης εξίσωσης κίνησης, η ραδινότητα α κανονικοποιεί πλήρως την

εξαρτημένη μεταβλητή θ , κάτι που μπορεί να θεωρηθεί ως μια ειδικου τύπου ιδιότητα ομοιότητας. Η ιδιότητα αυτή, ωστόσο, γεννάται από τη δομή της εξίσωσης κίνησης και όχι από διαστατική ανάλυση.

(4) Προέκυψαν αναλυτικές σχέσεις για την διάκριση μεταξύ περιοχών ευστάθειας (S) και ανατροπής (O) βάσει των αδιάστατων μεταβλητών f και η . Επιπλέον, οι περιοχές O διαχωρίστηκαν περαιτέρω σε τρεις υπό-περιοχές (Σχήματα 4.4-4.7): η απόκριση των συστημάτων που ανήκουν στην περιοχή O_1 γίνεται υπερκρίσιμη (ασταθής) κατά τη διάρκεια του ανιόντα κλάδου του παλμού ($\tau_c < 1/2$), στην O_2 η απόκριση γίνεται υπερκρίσιμη κατά τον κατιόντα κλάδο ($1/2 < \tau_c < 1$), ενώ στην O_3 η απόκριση γίνεται υπερκρίσιμη μετά το πέρας του παλμού. Οι περιοχές S διαχωρίζονται στις S_1 όπου η μέγιστη γωνία λικνισμού εκδηλώνεται κατά τη διάρκεια του παλμού, και S_2 όπου η μέγιστη γωνία λικνισμού λαμβάνει χώρα μετά τον παλμό. Σημειώνεται ότι καθώς οι περιοχές O_1 και O_2 αφορούν αστάθεια κατά τη διάρκεια του παλμού, βρίσκουν εφαρμογή σε πιο σύνθετες διεγέρσεις με χρονοϊστορίες που συνεχίζουν μετά το πέρας του μισού κύκλου, t_d .

(5) Παρουσιάστηκαν απλές λύσεις για την μέγιστη απόκριση σε όρους μέγιστης γωνίας λικνισμού και του αντίστοιχου χρόνου. Λόγω της υπερβατικής φύσης των εξισώσεων απόκρισης δεν κατέστη δυνατό να παραχθούν κλειστές λύσεις για όλες τις περιπτώσεις δυναμικών συστημάτων.

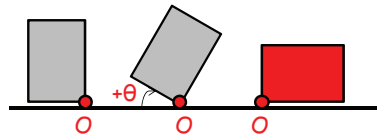
Τέλος, είναι σκόπιμο να αναφερθεί ότι λόγω θεώρησης διεγέρσεων μισού κύκλου, η λύση που παρουσιάστηκε στο συγκεκριμένο κεφάλαιο, δεν αφορά ανατροπή μετά από κρούση η οποία είναι κρίσιμη για μεγάλο αριθμό πραγματικών συστημάτων. Παρόλα αυτά, το

Κεφάλαιο 4 συμπληρώνει και επεκτείνει κλασικές μελέτες στο αντικείμενο παρέχοντας νέες λύσεις και φυσική εποπτεία του προβλήματος.

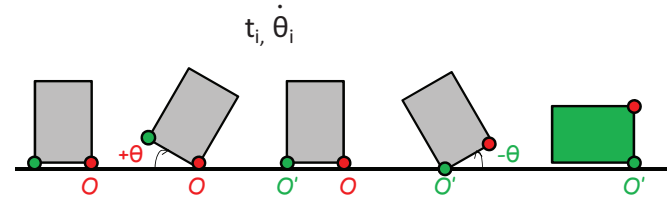
Η διερεύνηση της λικνιστικής συμπεριφοράς στερεού σώματος υπό παλμικές διεγέρσεις επεκτείνεται στο Κεφάλαιο 5 για την πιο ρεαλιστική περίπτωση παλμών διάρκειας πλήρους κύκλου. Γίνεται θεώρηση ορθογωνικού στερεού σώματος σε ανένδοτη βάση υποκειμένου σε εξιδανικευμένη κυματομορφή που αποτελείται από δύο συμμετρικούς ημι-παλμούς σταθερής επιτάχυνσης. Το πρόβλημα αντιμετωπίζεται αναλυτικά για ραδινή γεωμετρία σώματος και λικνισμό χωρίς ολίσθηση ή αναπήδηση, και αριθμητικά μέσω μη-γραμμικής προσομοίωσης που περιλαμβάνει κρουστικούς όρους και διαφορικούς τελεστές με τριγωνομετρικούς όρους.

Εξάγονται κλειστές εκφράσεις για: (1) τη γωνία λικνισμού, τη γωνιακή ταχύτητα και την επιτάχυνση του σώματος, (2) τα κριτήρια ανατροπής με ή χωρίς κρούση, (3) τη μέγιστη απόκριση. Οι λύσεις εκφράζονται σε όρους τριών αδιάστατων παραμέτρων: της αδιάστατης διάρκειας του παλμού (f), της αντίστασης σε λικνισμό (η) και του συντελεστή αποκατάστασης (ε). Παρά τη φαινομενική απλότητα του προβλήματος, ο υπολογισμός της απόκρισης είναι ιδιαίτερα απαιτητικός λόγω της δυνατότητας ανατροπής με δύο διαφορετικούς τρόπους (πρόσω και όπισθεν – Σχήμα 9), της μη-γραμμικής φύσης της κρούσης, του θετικού πόλου του διαφορικού τελεστή σε χώρο φάσης, και της παρουσίας πολλαπλών στάσιμων σημείων ανά κλάδο απόκρισης.

Τύπος 0 : Ανατροπή χωρίς κρούση ($\theta_{cr} > 0$)

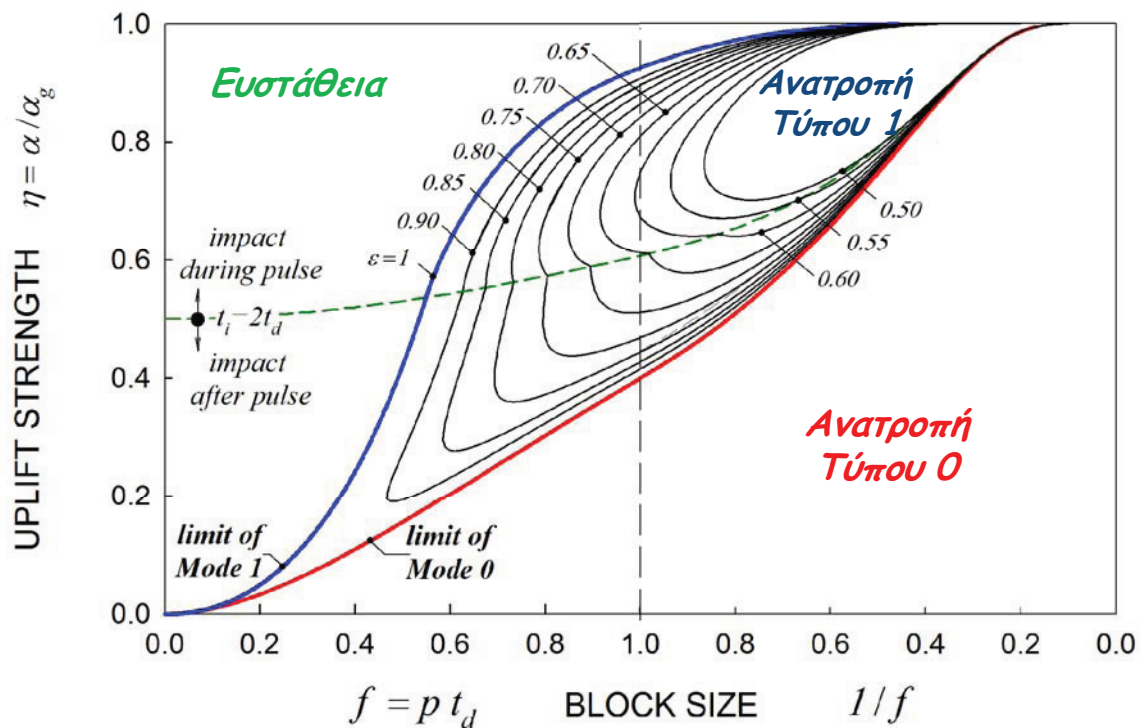


Τύπος 1 : Ανατροπή με 1 κρούση ($\theta_{cr} < 0$)



Σχήμα 9. Τύποι ανατροπής υπό παλμούς πλήρους κύκλου.

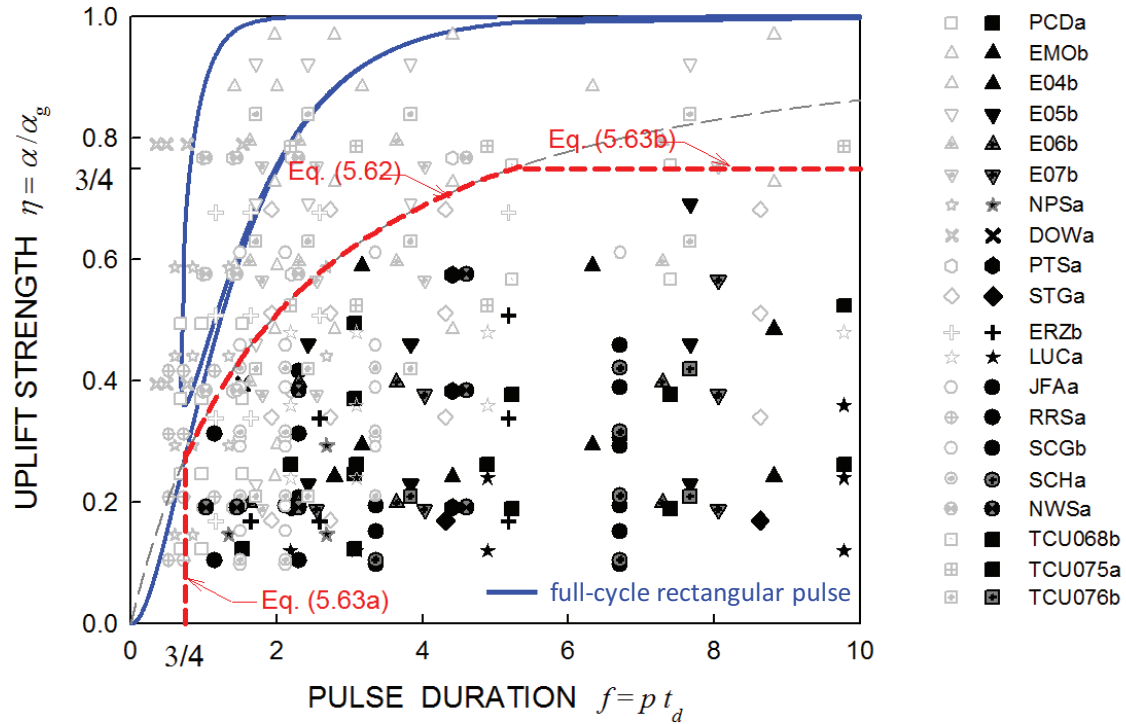
Τονίζεται η σχετιζόμενη με τους δύο τύπους ανατροπής συμπεριφορά διακλάδωσης. Τα αποτελέσματα δίνονται υπό τη μορφή κανονικοποιημένων γραφημάτων που βοηθούν στη φυσική κατανόηση της λικνιστικής απόκρισης. Οι εξισώσεις απόκρισης που διατυπώνονται στο Κεφάλαιο 5 αποτελούν (από όσο γνωρίζει η συγγραφέας) την πρώτη γνωστή ακριβή λύση κλειστής μορφής για την ανατροπή στερεού σώματος υπό παλμό πλήρους κύκλου. Στο Σχήμα 10, δίνονται τα όρια ανατροπής Τύπου 0 και Τύπου 1 για διαφορετικές τιμές του συντελεστή αποκατάστασης.



Σχήμα 10. Περιοχές ανατροπής χωρίς κρούση (Τύπου 0), μετά από μία κρούση (Τύπου 1) και ευστάθειας υπό ορθογωνικούς παλμούς πλήρους κύκλου.

Τέλος παρουσιάζονται και σχολιάζονται συγκρίσεις με άλλα σχήματα εξιδανικευμένων παλμών και πραγματικές καταγραφές κοντινού πεδίου. Τα αποτελέσματα των αριθμητικών αναλύσεων απεικονίζονται στο Σχήμα 11. Με άσπρο χρώμα συμβολίζονται τα συστήματα που δεν ανατρέπονται, ενώ με μαύρο χρώμα αυτά που ανατρέπονται. Παρατηρούμε ότι: (1) όπως στην περίπτωση των απλών παλμών τα συστήματα στο πάνω αριστερά τμήμα των γραφημάτων δεν ανατρέπονται, (2) όλα τα συστήματα που ανατρέπονται έχουν μεγάλες χαρακτηριστικές συχνότητες και μικρή αντοχή σε σχέση με τη διέγερση, (3) υπάρχουν συστήματα με τα συγκεκριμένα χαρακτηριστικά που δεν ανατρέπονται. Μια πιθανή εξήγηση είναι η παρουσία υψίσυχων παλμών μεγάλης επιτάχυνσης στην αρχή της χρονοιστορίας με αποτέλεσμα την έναρξη λικνισμού χαμηλού εύρους και μεγάλου αριθμού κρούσεων πριν την

άφιξη των κυρίως παλμών, (4) Το όριο ανατροπής για ορθογωνικούς παλμούς διάρκειας ενός κύκλου είναι πολύ συντηρητικό, (5) δεν φαίνεται να υπάρχει διαχωρισμός μεταξύ των τύπων ανατροπής πάλι λόγω μεγάλου αριθμού κύκλων και κρούσεων.



Σχήμα 11. Αριθμητική διερεύνηση ανατροπής λικνιζόμενου σώματος υπό πραγματικές καταγραφές κοντινού πεδίου. Με μαύρο χρώμα συμβολίζονται τα συστήματα που ανατρέπονται ($\varepsilon = 0.8$). Ο αριθμός των εξισώσεων αντιστοιχεί στο κείμενο της διατριβής (εξισώσεις 5, 6 στην Περίληψη).

Ένα λογικό άνω όριο των συστημάτων που ανατρέπονται δίνεται από την εξίσωση

$$\eta_w = 2 - \frac{2}{f} \ln[2e^{\frac{f}{2}} - 1] \quad (5)$$

η οποία περιγράφεται με στικτή γραμμή στο Σχήμα 11. Για μικρά f και μεγάλα η το κριτήριο γίνεται πολύ συντηρητικό και μπορεί να αντικατασταθεί από τις Εξισώσεις (6)

$$f_w \cong 3/4 \quad , \quad \eta_w \cong 3/4 \quad (6)$$

δεδομένου ότι δεν υπάρχουν συστήματα που ανατρέπονται αριστερά και πάνω από αυτές τις γραμμές.

Τα κύρια συμπεράσματα του Κεφαλαίου 5 είναι:

- (1) Ο λικνισμός στερεού σώματος σε ανένδοτη βάση αντιμετωπίστηκε αναλυτικά, περιγράφοντας όλα τα στάδια της απόκρισης μέσω ακριβών κλειστών λύσεων. Επισημαίνονται και αναλύονται δύο τύποι ανατροπής: *Τύπου 0*, που αφορά συστήματα που ανατρέπονται χωρίς κρούση δηλαδή με πρόσω λικνισμό, και *Τύπου 1* που αφορά συστήματα που ανατρέπονται κατόπιν μίας κρούσης και λικνισμό στην αντίθετη κατεύθυνση.
- (2) Πραγματοποιήθηκε διεξοδική μελέτη της συνάρτησης απόκρισης και προέκυψαν χρήσιμα συμπεράσματα για το φυσικό πρόβλημα. Όρια ανατροπής για λικνισμό *Τύπου 0* και *1* προέκυψαν αναλυτικά και σε κλειστή μορφή.
- (3) Ο λικνισμός των συστημάτων που δεν ανατρέπονται μελετήθηκε συστηματικά και προέκυψαν κλειστές λύσεις για την μέγιστη απόκριση (δηλαδή την μέγιστη γωνία περιστροφής) και τον αντίστοιχο χρόνο. Τα συγκεκριμένα φάσματα διαφέρουν ουσιαστικά από τα αντίστοιχα για απλούς ταλαντωτές.
- (4) Ο χρόνος και η ταχύτητα κατά την κρούση, υπολογίστηκαν επίσης σε κλειστή μορφή και μελετήθηκαν διεξοδικά.
- (5) Προέκυψε ένα γενικό κριτήριο ανατροπής για έναν συγκεκριμένο λικνιστικό κλάδο: περιττός αριθμός στάσιμων σημείων με αντιστροφή προσήμου γωνιακής ταχύτητας

σε χώρο $(\theta-t)$ οδηγεί σε ασφάλεια, ενώ άρτιος αριθμός στάσιμων σημείων με αντιστροφή προσήμου γωνιακής ταχύτητας οδηγεί σε ανατροπή.

(6) Η μελέτη των χαρακτηριστικών ανατροπής και ευστάθειας κατά την όπισθεν έννοια οδηγεί στις παρακάτω μη-αναμενόμενες συμπεριφορές: (α) αν τη στιγμή που τελειώνει ο παλμός η γωνία λικνισμού ισούται με τη γωνία ραδινότητας α , το σώμα δεν οδηγείται σε ανατροπή Τύπου θ , (β) αν κατά τη διάρκεια του παλμού δεν υπάρχουν στάσιμα σημεία, το σώμα οδηγείται πάντα σε ανατροπή Τύπου θ . Με άλλα λόγια, η ύπαρξη ενός στάσιμου σημείου μετά το τέλος του παλμού δεν εξασφαλίζει την ευστάθεια. Αντίθετα, αν ένα τέτοιο σημείο υπάρχει, θα είναι δεύτερο σημείο αλλαγής προσήμου της $\dot{\theta}$ που θα οδηγεί σε ανατροπή Τύπου θ (group O_4). Αυτό έρχεται σε αντίθεση με την περίπτωση παλμών μισού κύκλου, όπου μόνο ένα στάσιμο σημείο μπορεί να υπάρχει κατά τη διάρκεια ενός συγκεκριμένου κλάδου λικνισμού.

(7) Όλα τα συστήματα που εμφανίζουν τοπικά μέγιστα της γωνιακής ταχύτητας πριν το $3/2t_d$ δεν ανατρέπονται κατά τον Τύπο θ . Όλα τα συστήματα που εμφανίζουν τοπικά μέγιστα της ταχύτητας μετά το πέρας του παλμού ανατρέπονται. Σε όρους γωνιών λικνισμού, όλα τα συστήματα που λικνίζονται μέχρι γωνίες μικρότερες ή ίσες της γωνίας ραδινότητας α δεν ανατρέπονται με τον Τύπο θ , ενώ όσα φτάσουν ή ξεπεράσουν την γωνία 2α ανατρέπονται πάντα.

(8) Αντίθετα με την περίπτωση του πρόσω λικνισμού κατά τον οποίο ο χρόνος της μέγιστης απόκρισης είναι φραγμένος, κατά τον όπισθεν λικνισμό οι αντίστοιχοι

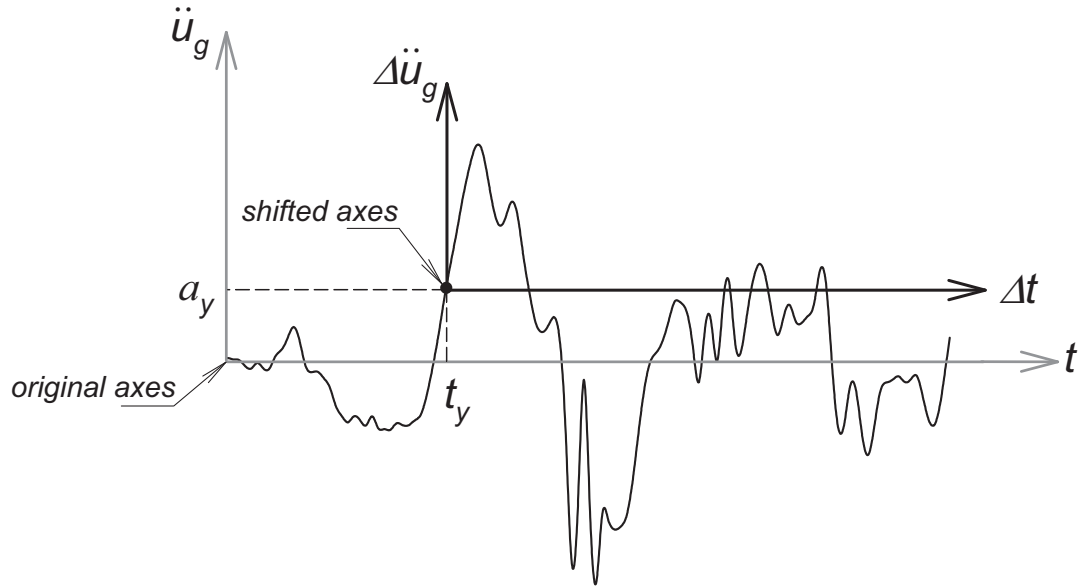
χρόνοι τείνουν στο άπειρο για συστήματα που βρίσκονται κοντά στο όριο της ανατροπής.

- (9) Τα όρια ανατροπής υπολογίστηκαν αριθμητικά για διαφορετικούς εξιδανικευμένους παλμούς διάρκειας πλήρους κύκλου: (α) ίδιου εύρους επιτάχυνσης A_g και ίδιας ημιδιάρκειας t_d , και (β) ίδιου εύρους ταχύτητας V_g και ίδιας διάρκειας t_d . Στην πρώτη περίπτωση παρατηρήθηκε ότι όσο μικρότερες είναι οι ταχύτητες της διέγερσης, τόσο μικρότερες είναι οι περιοχές ανατροπής και αντίστροφα. Στη δεύτερη περίπτωση, τα συστήματα που ανατρέπονται υπό ορθογωνικό παλμό, ανατρέπονται και υπό πιο αιχμηρές κυματομορφές, ενώ συστήματα που δεν ανατρέπονται υπό απόλυτα αιχμηρό παλμό δεν ανατρέπονται υπό λιγότερο αιχμηρές κυματομορφές.
- (10) Από τις αριθμητικές αναλύσεις για πραγματικές καταγραφές εγγύς πεδίου προέκυψαν απλά κριτήρια για τα συστήματα που ανατρέπονται. Η ύπαρξη υψηλών εδαφικών επιταχύνσεων στην αρχή της σεισμικής καταγραφής, οδηγεί σε πολλαπλές κρούσεις και γωνιακή ταχύτητα με τυχαίο πρόσημο πριν την άφιξη του κυρίως παλμού. Αυτό μπορεί να είναι ευεργετικό και να οδηγήσει σε ευστάθεια ακόμη και για συστήματα χαμηλής αντοχής και υψηλής συχνότητας. Τα κριτήρια των Εξισώσεων (5.62) και (5.63) εξασφαλίζουν την ευστάθεια καθώς δεν βρέθηκαν συστήματα που ανατρέπονται πέρα από αυτά τα όρια. Σε αυτό το πλαίσιο οδηγούμαστε στο συμπέρασμα ότι κινήσεις εγγύς πεδίου με PGA/g μικρότερα από το 75% της γωνίας ραδινότητας α δεν οδηγούν σε ανατροπή (Εξίσωση 5.63b). Επίσης, κινήσεις εγγύς πεδίου με δεσπόζουσα περίοδο μικρότερη του 75% της χαρακτηριστικής περιόδου του λικνιζόμενου σώματος δεν προκαλούν ανατροπή

(Εξίσωση 5.63a). Για όλες τις υπόλοιπες περιπτώσεις, τα συστήματα που ανατρέπονται φράσσονται από μια απλή καμπύλη (Εξίσωση 5.62). Ο ορθογωνικός παλμός πλήρους κύκλου οδηγεί πάντα σε μεγαλύτερη τάση ανατροπής από μια πραγματική καταγραφή ίσου εύρους επιτάχυνσης και ίδιας δεσπόζουσας περιόδου.

Η εύρεση κατάλληλων σχέσεων ανακλιμάκωσης που να λαμβάνουν υπόψη τις εγγενείς μη-γραμμικότητες των υπό μελέτη συστημάτων αποτελεί το αντικείμενο των Κεφαλαίων 6 και 7. Συγκεκριμένα:

Στο Κεφάλαιο 6 παρουσιάζεται μια νέα τεχνική μετάθεσης για τον υπολογισμό και την ερμηνεία της ολίσθησης στερεού σώματος υπό δυναμική φόρτιση. Οι νόμοι ανακλιμάκωσης για πλήρως πλαστικά συστήματα είναι, όπως δηλώνει η ονομασία τους, εγγενώς πολλαπλασιαστικής φύσης και συνδέουν τη μέγιστη δυναμική απόκριση με γινόμενα βασικών παραμέτρων του προβλήματος όπως είναι οι ελαστικές φασματικές συντεταγμένες, ο σεισμικός συντελεστής και οι μέγιστες τιμές της διέγερσης και των χρονικών της παραγώγων. Αντίθετα από τους παραπάνω κλασικούς νόμους, η προτεινόμενη μέθοδος είναι αθροιστικής φύσης και επιβάλλει μετάθεση των τετμημένων και τεταγμένων της συνάρτησης διέγερσης βάσει μιας ομάδας παραμέτρων που σχετίζονται με την αντοχή διαρροής του συστήματος [Σχήμα 12, Εξίσωση (7)].



Σχήμα 12. Τεχνική μετάθεσης των αξόνων για πλήρως πλαστικό σύστημα.

$$\Delta t \equiv t - t_y \quad (7\alpha)$$

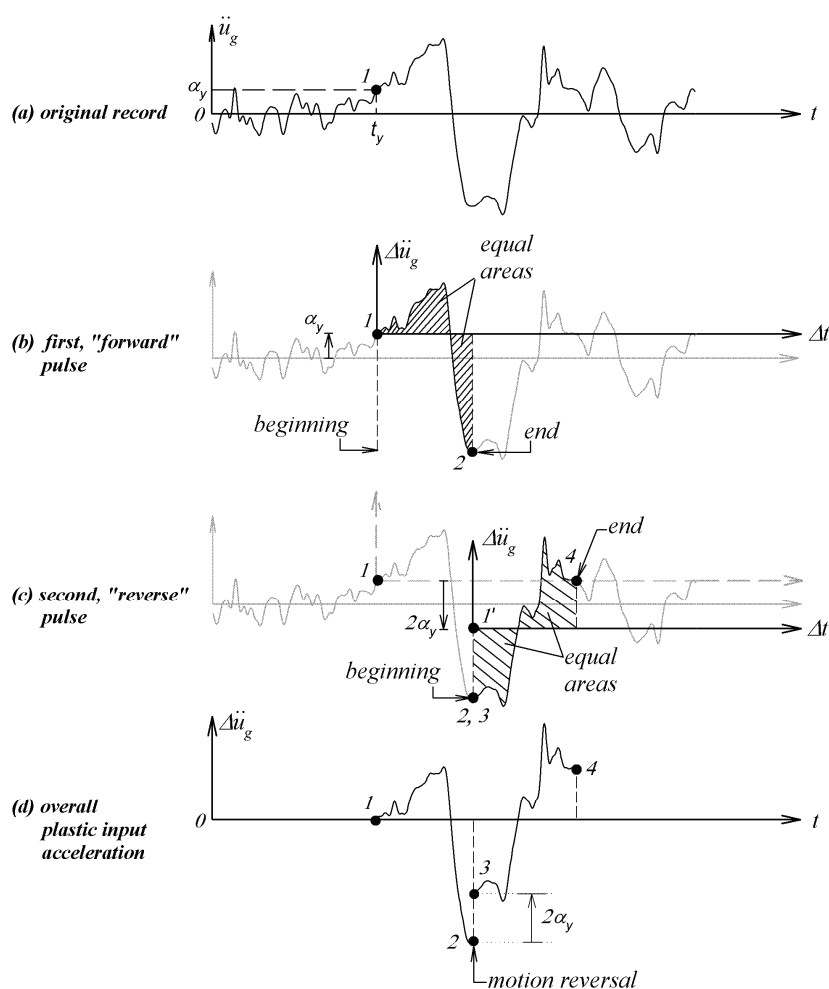
$$\Delta\ddot{u}_g(t) \equiv \ddot{u}_g(t) - a_y \quad (7\beta)$$

όπου a_y , t_y , η επιτάχυνση διαρροής και ο αντίστοιχος χρόνος. Η δυναμική απόκριση προκύπτει μέσω ολοκλήρωσης της τροποποιημένης σεισμικής επιτάχυνσης με γραμμικό τρόπο σε κάθε ανελαστικό κλάδο απόκρισης, καθώς η μη-γραμμικότητα περιλαμβάνεται πλέον στον όρο της διέγερσης [Εξίσωση (8)].

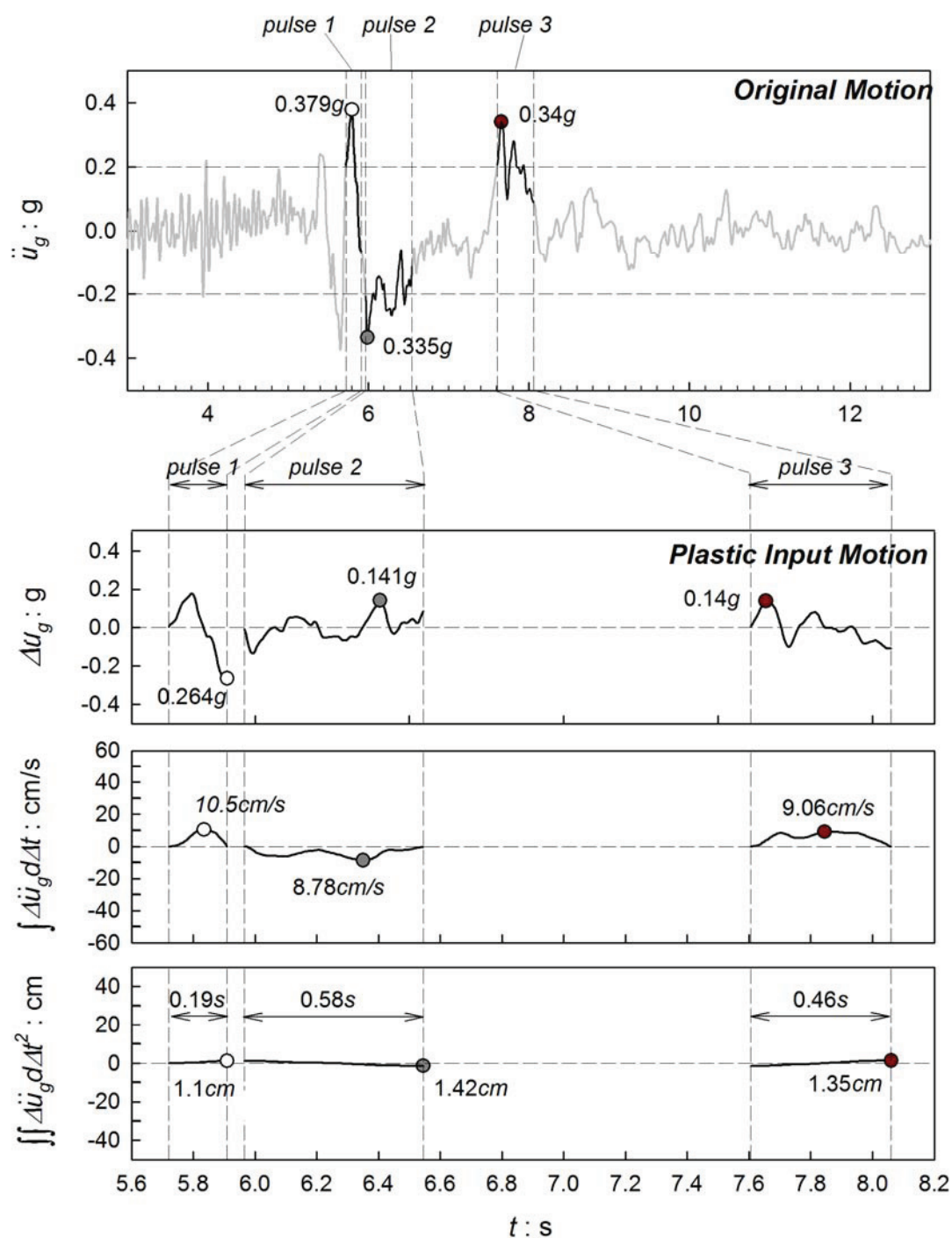
$$u = \int_0^{\Delta t} \int_0^{\Delta t} \Delta\ddot{u}_g dt^2 \quad (8)$$

Η μαθηματική ορθότητα της μεθόδου παρουσιάζεται αναλυτικά και τονίζεται η σημασία της για την αντιμετώπιση συστημάτων με συμμετρική αντοχή σε διαρροή υπό εδαφικές κινήσεις

εγγύς πεδίου. Η τροποποιημένη χρονοϊστορία μπορεί να είναι ασυνεχής στα σημεία αλλαγής του κλάδου διαρροής και σχετίζεται μονοσήμαντα με την πλαστική παραμόρφωση του συστήματος (π.χ. την παραμένουσα μετατόπιση – Σχήμα 13). Για τον λόγο αυτό η τροποποιημένη συνάρτηση ορίζεται ως *Πλαστική Σεισμική Διέγερση* [*Plastic Input Motion (PIM)*]. Οι τεταγμένες και η διάρκεια της *PIM* μπορεί να είναι σημαντικά (αν και όχι απαραίτητα) μικρότερες από τις αντίστοιχες της αρχικής καταγραφής ανάλογα με την αντοχή σε διαρροή (Σχήμα 14, Πίνακας 1).



Σχήμα 13. Εύρεση των ορίων ολοκλήρωσης και Πλαστική Σεισμική Επιτάχυνση σύμφωνα με την προτεινόμενη μεθοδολογία.



Σχήμα 14. Εφαρμογή τεχνικής μετάθεσης στο σεισμό Imperial Valley, USA, 1979, E05-SN record. $a_y = 0.20g$.

Πίνακας 1. Σύγκριση παραμέτρων μέγιστης απόκρισης λαμβάνοντας υπόψη μόνο τους κύριους παλμούς βάσει της προτεινόμενης μεθοδολογίας (method A) και μέσω αριθμητικής ολοκλήρωσης όλης της σεισμικής καταγραφής (method B); $a_y=0.1$.

Ground Motion	(g) (cm/s) (cm)	Method A					Method B	Deviation
		No. of effective pulses	Pulse 1	Pulse 2	Pulse 3	Peak value [†] (A)	Peak value (B)	(B-A) / B %
Pacoima Dam	A_{gp}	1	0.61			0.61	-1.13	46
	V_{gp}		95			95	95.6	0.63
	D_{gp}		52.7			52.7	45	17.1
Imperial Valley	A_{gp}	3	-0.435	-0.235	0.24	-0.435	-0.435	0
	V_{gp}		24.5	-52.6	47.7	-52.6	-56.7	7.2
	D_{gp}		3.95	-44.6	24	-40.6	-49.3	17.6
Erzican	A_{gp}	1	0.415			0.415	0.415	0
	V_{gp}		90			90	90	0
	D_{gp}		48			48	48	0
Rinaldi	A_{gp}	2	-0.867	-0.738		-0.867	-0.867	0
	V_{gp}		127	-102		127	127	0
	D_{gp}		36.1	-34.8		36.1	36.3	0.55
Aegion	A_{gp}	2	0.56	-0.543		0.56	0.56	0
	V_{gp}		-37.4	36		-37.4	-37.6	0.53
	D_{gp}		-8.5	4.76		-8.5	-8.5	0
Chi-Chi	A_{gp}	2	-0.362	0.24		-0.362	-0.362	0
	V_{gp}		-66.5	-109		-109	-109	0
	D_{gp}		-29.5	-257		-287	-265	8.3

Η προτεινόμενη μέθοδος συγκρίνεται επιτυχώς με υφιστάμενες μεθόδους και παραμέτρους της σεισμικής μηχανικής όπως την ευρύτητα γνωστή μέθοδο του ολισθαίνοντος σώματος κατά Newmark και την λίγομετρο διερευνημένη παράμετρο της σχετικής εδαφικής επιτάχυνσης.

Τα κύρια συμπεράσματα του Κεφαλαίου 6 είναι:

- (1) Αναπτύχθηκε μια μέθοδος μετάθεσης για τον υπολογισμό και την ερμηνεία της ολίσθησης στερεού σώματος υπό δυναμική φόρτιση. Αντίθετα με τους κλασικούς νόμους ανακλιμάκωσης οι οποίοι είναι πάντοτε πολλαπλασιαστικοί, η προτεινόμενη

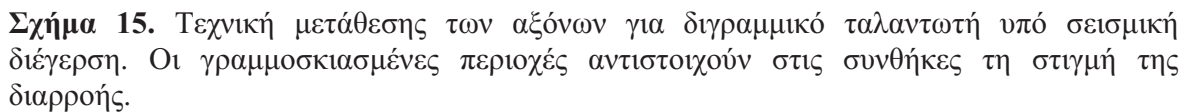
μέθοδος είναι αθροιστικής φύσης και αφορά μετάθεση του συστήματος συντεταγμένων τόσο ως προς τις τετμημένες (κατά ποσότητα ίση με τον χρόνο διαρροής) όσο και τις τεταγμένες (κατά ποσότητα ίση με την αντοχή διαρροής) της χρονοϊστορίας της διέγερσης.

- (2) Επίσης, αντίθετα από τις κλασικές αναλυτικές και αριθμητικές μεθόδους, η προτεινόμενη μέθοδος δεν απαιτεί γνώση της εδαφικής μετακίνησης και ταχύτητας κατά τη στιγμή της διαρροής. Αν και για την περίπτωση του ολισθαίνοντος σώματος η ιδιότητα αυτή φαίνεται αναμενόμενη, δεν ισχύει το ίδιο για γενικότερες κατηγορίες μη-γραμμικών συστημάτων όπως ο ελαστοπλαστικός ταλαντωτής.
- (3) Με εισαγωγή κατάλληλων παραμέτρων τροποποιημένης διέγερσης, η απόκριση προκύπτει μέσω ολοκλήρωσης της χρονοϊστορίας της πλαστικής σεισμικής επιτάχυνσης με γραμμικό τρόπο μέσω συμβατικών μεθοδολογιών ανάλυσης (Εξίσωση 6.19). Αυτό βέβαια αποτελεί απλώς μια ιδιότητα μαθηματικής φύσης της μεθόδου, καθώς η μη-γραμμικότητα είναι εγγενές χαρακτηριστικό της ολίσθησης.
- (4) Η τροποποιημένη χρονοϊστορία της εδαφικής επιτάχυνσης σχετίζεται μονοσήμαντα με την πλαστική παραμόρφωση του συστήματος. Για τον λόγο αυτό, αναφέρεται ως πλαστική σεισμική επιτάχυνση (*PIA*), ενώ οι χρονικές της παράγωγοι, αναφέρονται ως πλαστική σεισμική ταχύτητα (*PIV*) και πλαστική σεισμική μετατόπιση (*PID*), αντίστοιχα.
- (5) Υπολογίστηκαν οι μέγιστες τιμές της πλαστικής σεισμικής διέγερσης, οι οποίες περιλαμβάνουν τόσο τα χαρακτηριστικά της εδαφικής κίνησης όσο και του μη-γραμμικού συστήματος. Τα συγκεκριμένα μέγιστα μπορεί να είναι μικρότερα από τα

αντίστοιχα της αρχικής καταγραφής. Η αύξηση σε κάποιες περιπτώσεις, κυρίως της επιτάχυνσης, οφείλεται στην αλλαγή προσήμου και τους αρνητικούς κύκλους της χρονοϊστορίας.

- (6) Αν και στοιχεία της προτεινόμενης μεθοδολογίας υπάρχουν εδώ και καιρό στη βιβλιογραφία (ειδικά στη θεωρία του ολισθαίνοντος σώματος κατά Newmark), δεν έχουν μελετηθεί και γενικευθεί όπως γίνεται στην παρούσα εργασία. Η μέθοδος μπορεί να εφαρμοσθεί σε γενικότερες κατηγορίες δυναμικών συστημάτων όπως ο ελαστοπλαστικός και ο διγραμμικός ταλαντωτής. Τα συστήματα αυτά εξετάζονται διεξοδικά στο Κεφάλαιο 7.
- (7) Στην περίπτωση του ολισθαίνοντος σώματος, η πλαστική σεισμική επιτάχυνση ισούται με τη σχετική ως προς την βάση επιτάχυνση (Εξίσωση 6.21), η οποία δεν έχει λάβει την πρέπουσα προσοχή στη σεισμική μηχανική και την τεχνική σεισμολογία (αντίθετα από την αντίστοιχη απόλυτη επιτάχυνση). Φυσικά αυτό ισχύει για το συγκεκριμένο σύστημα και δεν πρέπει να γενικεύεται σε όλα τα μη-γραμμικά συστήματα.
- (8) Ο αδιάστατος συντελεστής αντοχής (a_y/A_g) δεν αποτελεί συντελεστή ανακλιμάκωσης αφού δεν πολλαπλασιάζει τον όρο της διέγερσης (Εξίσωση 6.32). Κάτι τέτοιο ισχύει μόνο για την εξιδανικευμένη περίπτωση σταθερής εδαφικής επιτάχυνσης (Εξίσωση 6.33). Παρόλα αυτά αποτελεί χρήσιμη παράμετρο για την οριοθέτηση της απόκρισης, καθώς $u \rightarrow u_g$ για $(a_y/A_g) = 0$ και $u \rightarrow 0$ για $(a_y/A_g) = 1$. Επίσης, αντιστοιχεί στην μέση εδαφική επιτάχυνση ενός κλάδου διαρροής (ολίσθησης) (Εξίσωση 6.34).

Η παραπάνω μέθοδος επεκτείνεται στο Κεφάλαιο 7 σε πιο γενικές περιπτώσεις ανελαστικών ταλαντωτών (Σχήμα 15).



Μέσω της συγκεκριμένης μεθόδου: (1) ο διαφορικός τελεστής γραμμικοποιείται, καθώς η μη-γραμμικότητα μεταφέρεται στο δεξί μέρος της εξίσωσης, (2) οι αρχικές συνθήκες απλοποιούνται, (3) η τροποποιημένη διέγερση σχετίζεται μονοσήμαντα με την ανάπτυξη πλαστικών παραμορφώσεων.

Η θεωρία εφαρμόζεται σε διάφορους ανελαστικούς ταλαντωτές υπό εξιδανικευμένους σεισμικούς παλμούς, και η τροποποιημένη χρονοϊστορία ορίζεται ως “Πλαστική Σεισμική Διέγερση” (*PIM*). Παρουσιάζεται αναλυτικά η μεθοδολογία εφαρμογής της μεθόδου σε χρονοϊστορίες γενικού τύπου. Οι τετμημένες των *PIM* είναι συχνά ασυνεχείς και αρκετά μικρότερες από αυτές της αρχικής καταγραφής, δεδομένου ότι σημαντικό μέρος της εδαφικής επιτάχυνσης καταναλώνεται για την υπέρβαση της ελαστικής αντίστασης του συστήματος. Η θεωρία είναι χρήσιμη στην σεισμική μηχανική, καθώς προσφέρει μια εναλλακτική προσέγγιση έναντι πραγματικών καταγραφών, μέσω των *Πλαστικών Σεισμικών Διεγέρσεων* που είναι συνυφασμένες με το εκάστοτε μη-γραμμικό σύστημα και αποτελούν από μόνες τους δείκτες σεισμικής απαίτησης.

Η ανελαστική απόκριση ενός μονοβάθμιου διγραμμικού ταλαντωτή με απόσβεση, περιγράφεται από την Εξίσωση (9)

$$\ddot{u} + 2\omega_n^* \zeta^* \dot{u} + \omega_n^{*2} u = \ddot{u}_g(t) - (\alpha_y - u_y \omega_n^{*2}) \text{sgn}(\dot{u}) + \delta(t - t_y) \dot{u}_y + [\dot{\delta}(t - t_y) + 2\zeta^* \omega_n^* \delta(t - t_y)] u_y \quad (9)$$

όπου $\delta()$ και $\dot{\delta}()$ η συνάρτηση δέλτα και η πρώτη παράγωγός της (συνάρτηση doublet), σε μονάδες Χρόνου⁻¹ και Χρόνου⁻², αντίστοιχα, ενώ $\gamma = K_{py} / K$, $\omega_n^* = \sqrt{\gamma K / m}$, $\zeta^* = c_{py} / 2\omega_n^*$ ο συντελεστής κράτυνσης, η μετελαστική ιδιοσυχνότητα, και η αντίστοιχη απόσβεση [π.χ. c_{py}

= $(\gamma \ c)$] του συστήματος, u_y , a_y είναι η μετακίνηση και η επιτάχυνση διαρροής στον συγκεκριμένο κλάδο διαρροής. Αλλάζοντας τις μεταβλητές του προβλήματος σύμφωνα με τη μεθοδολογία μετάθεσης του Κεφαλαίου 6, η Εξίσωση (9) μπορεί να γραφεί στη μορφή

$$\Delta \ddot{u} + 2\omega_n^* \zeta^* \Delta \dot{u} + \omega_n^{*2} \Delta u = \Delta \ddot{u}_g + \delta(t^*) \dot{u}_y + [\dot{\delta}(t^*) + 2\zeta^* \omega_n^* \delta(t^*)] u_y \quad (10)$$

η λύση της οποίας μπορεί να εκφραστεί ως

$$\Delta u = \int_0^{t^*} \left\{ \Delta \ddot{u}_g(\tau) + \delta(\tau) \dot{u}_y + [\dot{\delta}(\tau) + 2\zeta^* \omega_n^* \delta(\tau)] u_y \right\} h(\tau) d\tau \quad (11\alpha)$$

$$h = \frac{1}{\omega_D^*} \exp[-\zeta^* \omega_n^* (t^* - \tau)] \sin[\omega_D^* (t^* - \tau)] \quad (11\beta)$$

Προφανώς, στη γενική αυτή περίπτωση, η Πλαστική Σεισμική Επιτάχυνση (*PIA*) ισούται με την τροποποιημένη εδαφική επιτάχυνση $\Delta \ddot{u}_g$, συν ένα σύνολο όρων που σχετίζονται με τις αρχικές συνθήκες την στιγμή της διαρροής.

Τα παραπάνω μπορούν να ερμηνευθούν μέσω της γενικευμένης έκφρασης:

$$L(u) + \text{sgn}(\dot{u}) F_y = P(t) \quad (12)$$

όπου $L(\)$ ένας διαφορικός τελεστής, $P(t)$ η συνάρτηση της διέγερσης (η οποία περιλαμβάνει τις αρχικές συνθήκες) και F_y η αντοχή διαρροής του συστήματος. Αφού η δύναμη F_y είναι σταθερή, μπορεί να μεταφερθεί στο δεξί μέλος της εξίσωσης μαζί με τα εξωτερικά φορτία

$$L(u) = P(t) - \text{sgn}(\dot{u}) F_y \quad (13)$$

Με τον τρόπο αυτό η λύση απλοποιείται σημαντικά, αφού απαλλάσσεται από τις αρχικές συνθήκες των συναρτήσεων $u(t)$ και $P(t)$. Επίσης, αν $L(\cdot)$ είναι γραμμικός τελεστής όπως στη συγκεκριμένη περίπτωση, και γνωρίζοντας το δεξί μέλος της Εξ. (13), το πρόβλημα γραμμικοποιείται αφού η μη-γραμμικότητα περιλαμβάνεται πλέον στον όρο της διέγερσης.

Για την ειδική περίπτωση πλήρως πλαστικών συστημάτων η πλαστική σεισμική επιτάχυνση ισούται με την σχετική ως προς την βάση επιτάχυνση του συστήματος.

$$\ddot{u} = \ddot{u}_g(t) - \text{sgn}(\dot{u}) a_y \quad (14)$$

Ο όρος $\Delta \ddot{u}_g$ συν τους όρους των αρχικών συνθηκών στο δεξί μέλος της Εξ. (11) ονομάζεται Πλαστική Σεισμική Επιτάχυνση (*PIA*) ενώ το πρώτο και δεύτερο ολοκλήρωμα της *PIA*, ονομάζεται Πλαστική Σεισμική Ταχύτητα (*PIV*) και Πλαστική Σεισμική Επιτάχυνση (*PID*), αντίστοιχα. Αυτές οι συναρτήσεις αποτελούν τις συνιστώσες των Πλαστικών Σεισμικών Κινήσεων *PIM* για ένα δεδομένο σύστημα. Συγκεκριμένα,

$$PIA = \Delta \ddot{u}_g + Z \quad (15\alpha)$$

$$PIV = \int (\Delta \ddot{u}_g + Z) dt \quad (15\beta)$$

$$PID = \iint (\Delta \ddot{u}_g + Z) dt^2 \quad (15\gamma)$$

$$Z = \delta(t^*) \dot{u}_y + [\dot{\delta}(t^*) + 2\zeta^* \omega_n^* \delta(t^*)] u_y \quad (15\delta)$$

Η παράμετρος Z εισάγει την επιρροή των αρχικών συνθηκών και είναι μηδενική για πλήρως πλαστικά συστήματα.

Οι μέγιστες τιμές των PIM συμβολίζονται ως

$$A_{gp} \equiv \max |PIA| = \max |\Delta \ddot{u}_g + Z| \quad (16\alpha)$$

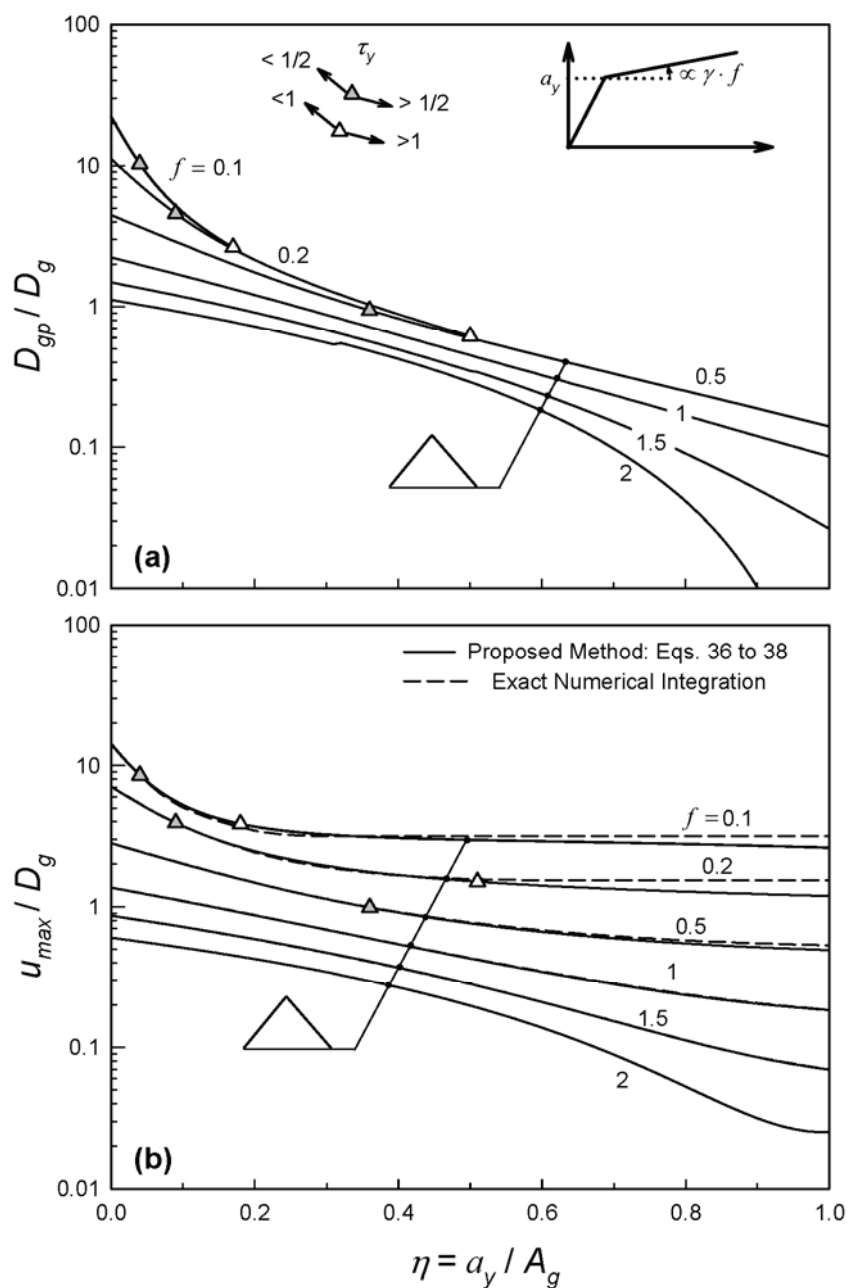
$$V_{gp} \equiv \max |PIV| = \max \left| \int (\Delta \ddot{u}_g + Z) dt \right| \quad (16\beta)$$

$$D_{gp} \equiv \max |PID| = \max \left| \iint (\Delta \ddot{u}_g + Z) dt^2 \right| \quad (16\gamma)$$

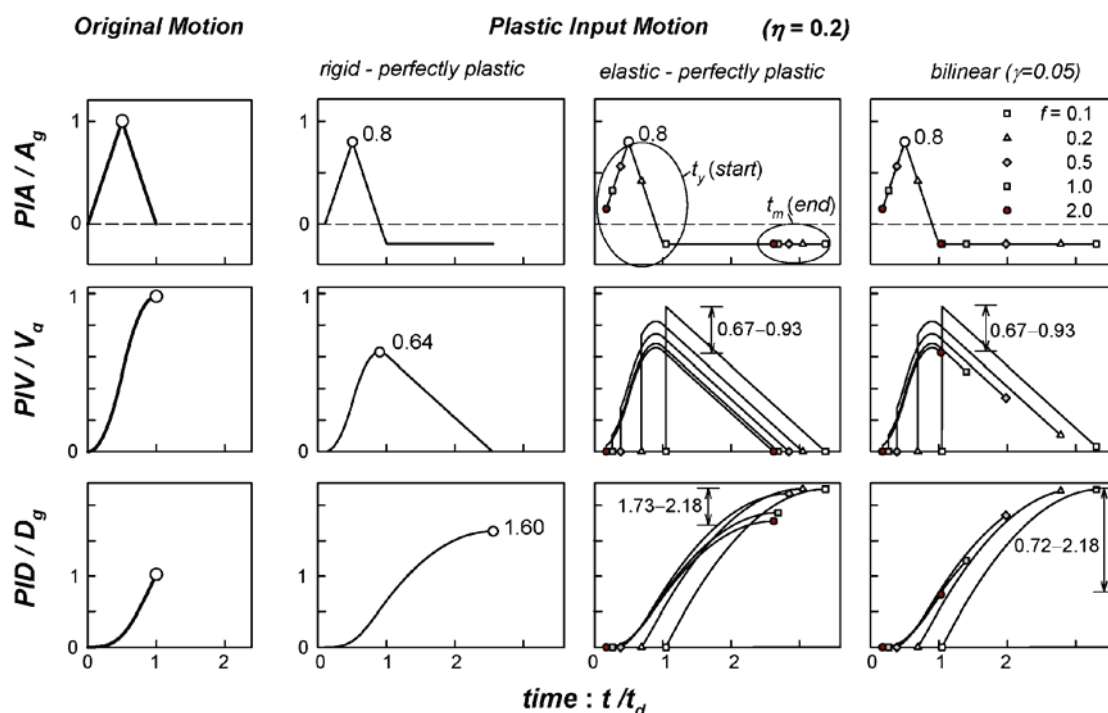
όπου ο δείκτης p δηλώνει “πλαστικό” (“plastic”) για να ξεχωρίζει τα μέγιστα των πλαστικών κινήσεων από αυτά των αρχικών καταγραφών (τα οποία συμβολίζονται A_g , V_g , D_g). Αξίζει να σχολιάσουμε το φυσικό νόημα των Εξισώσεων (16): οι παραπάνω ποσότητες εκφράζουν την καταστρεπτικότητα του τμήματος της χρονοϊστορίας που ευθύνεται αποκλειστικά για τις πλαστικές παραμορφώσεις. Με αυτήν τη λογική οι δείκτες A_{gp} , V_{gp} , D_{gp} μπορούν να αντικαταστήσουν τις συμβατικές παραμέτρους A_g , V_g , D_g για τον υπολογισμό της σεισμικής απαίτησης μη-γραμμικών συστημάτων.

Στο Σχήμα 16 παρουσιάζεται η μεταβολή της μέγιστης PID ως προς την αδιάστατη αντοχή σε διαρροή η , για την γενική περίπτωση του διγραμμικού ταλαντωτή. Στο Σχήμα 17 δίνονται οι πλαστικές σεισμικές κινήσεις για πλήρως πλαστικά, ελαστοπλαστικά και διγραμμικά συστήματα υπό τριγωνικό παλμό μισού κύκλου για αδιάστατη αντοχή σε διαρροή $\eta=0.2$. Παρατηρούμε τρεις κύριες τάσεις που αξίζει να σχολιαστούν: (1) Η χρονοϊστορία της πλαστικής σεισμικής επιτάχυνσης μοιάζει με την αρχική καταγραφή με την προσθήκη ενός σταθερού κλάδου. (2) Οι PIV και PID έχουν σημαντικές διαφορές με τις αντίστοιχες της αρχικής χρονοϊστορίας. (3) Οι μέγιστες τιμές των PIA και PIV είναι μικρότερες από τις

μέγιστες εδαφικές A_g και V_g , ενώ η μέγιστη τιμή των PID μπορεί να είναι μεγαλύτερη της μέγιστης μετακίνησης του εδάφους D_g .



Σχήμα 16. Σύγκριση αποτελεσμάτων μέσω της προτεινόμενης μεθοδολογίας και αριθμητικών μεθόδων για την απόκριση διγραμμικού ταλαντωτή υπό τριγωνικό παλμό μισού κύκλου ($\gamma=0.05$).



Σχήμα 17. Τριγωνικός παλμός μισού κύκλου: χρονοϊστορίες PIM για πλήρως πλαστικά, ελαστοπλαστικά και διγραμμικά συστήματα.

Τα κύρια συμπεράσματα του Κεφαλαίου 7 είναι:

- (1) Αντίθετα από τις συνήθεις αριθμητικές λύσεις η μέθοδος δεν απαιτεί πληροφορία, με τη μορφή αρχικών συνθηκών, για τις τιμές της εδαφικής κίνησης τη στιγμή της διαρροής, καθώς οι μόνες απαιτούμενες παράμετροι είναι ο χρόνος του γεγονότος και η αντίστοιχη ταχύτητα και μετατόπιση του ταλαντωτή. Σημειώνεται ότι για τον υπολογισμό αυτών των παραμέτρων, απαιτείται γενικώς ολοκλήρωση όλης της χρονοϊστορίας της εδαφικής κίνησης μέχρι τη συγκεκριμένη στιγμή.
- (2) Τα μέγιστα της πλαστικής σεισμικής επιτάχυνσης, ταχύτητας και μετατόπισης είναι συνήθως αρκετά μικρότερα από αυτά της αρχικής καταγραφής. Στην περίπτωση

σεισμικής διέγερσης, η μεγαλύτερη μείωση αφορά την ταχύτητα και την μετατόπιση, ενώ η πλαστική σεισμική επιτάχυνση μπορεί να είναι μεγαλύτερη από την αρχική – ειδικά για μικρές τιμές της αντοχής διαρροής.

- (3) Για τις περιπτώσεις πλήρως πλαστικών και ελαστοπλαστικών συστημάτων, οι πλαστικές σεισμικές διεγέρσεις συμπίπτουν με τη σχετική ως προς τη βάση πλαστική απόκριση. Αυτή η ιδιότητα είναι ιδιαίτερα χρήσιμη καθώς οι *PIM*'s παρέχουν άμεσα την ανελαστική απόκριση των συγκεκριμένων συστημάτων.
- (4) Στην περίπτωση των παλμών μισού κύκλου, οι μέγιστες πλαστικές σεισμικές μετατοπίσεις ενός ελαστοπλαστικού συστήματος είναι γενικώς μεγαλύτερες από αυτές του αντίστοιχου πλήρως πλαστικού. Εξαιρούνται τα πολύ εύκαμπτα συστήματα για τα οποία ισχύει το αντίστροφο. Αντίθετα, οι μέγιστες μετατοπίσεις (δηλαδή το άθροισμα ελαστικών και πλαστικών) είναι πάντοτε μεγαλύτερες από αυτές του πλήρως πλαστικού συστήματος. Αυτό σημαίνει ότι στην περίπτωση των εύκαμπτων ταλαντωτών, η μικρή απαίτηση πλαστιμότητας αναπληρώνεται από την μεγαλύτερη μετατόπιση διαρροής, το οποίο με τη σειρά του οδηγεί σε αύξηση της συνολικής απόκρισης.
- (5) Το Συμπέρασμα (4) ισχύει και στην περίπτωση διγραμμικών συστημάτων. Ωστόσο αυτά συνήθως έχουν μικρότερες πλαστικές σεισμικές μετατοπίσεις από την εδαφική μετατόπιση στο τέλος του παλμού.

Η μέθοδος έχει πρακτικό ενδιαφέρον διότι: (i) εντοπίζει το τμήμα της εδαφικής διέγερσης που ευθύνεται για την ανάπτυξη πλαστικής παραμόρφωσης, (ii) μπορεί να υπολογίσει με ακρίβεια τη διάρκεια κάθε κλάδου διαρροής. Επιπλέον, η τροποποιημένη χρονοϊστορία

διέγερσης αποτελεί δείκτη καταστρεπτικότητας με εφαρμογές στη σεισμική μηχανική και τη σεισμολογία. Βρίσκει δε ιδιαίτερη εφαρμογή σε παλμικές διεγέρσεις μικρού αριθμού κύκλων όπως κρουστικά φορτία και σεισμούς εγγύς πεδίου.

Σημειώνεται ότι η προτεινόμενη μεθοδολογία μετάθεσης δεν είναι υπολογιστικώς οικονομικότερη έναντι των καθιερωμένων μη-γραμμικών αριθμητικών αναλύσεων, αφού οι περιοχές πλαστικής απόκρισης πρέπει να εντοπιστούν με ολοκλήρωση της χρονοιστορίας της διέγερσης καθ'όλη τη διάρκειά της για τον εντοπισμό των συμβάντων διαρροής και στιγμιαίας ακινησίας. Ωστόσο, η συγκεκριμένη θεωρία προσφέρει τα μέσα για τον υπολογισμό των μέγιστων *PIM* που συνδέονται μονοσήμαντα με το μη-γραμμικό σύστημα και την ανάπτυξη πλαστικής παραμόρφωσης, και μπορούν δυνητικά να αντικαταστήσουν κλασικές παραμέτρους σεισμικής έντασης όπως την μέγιστη εδαφική επιτάχυνση, την ταχύτητα και την μετατόπιση, ως δείκτες πλαστικής συμπεριφοράς.

Το Κεφάλαιο 8 συνοψίζει τα συμπεράσματα και παρουσιάζει προτάσεις για περαιτέρω έρευνα.

Table of Contents

1.	Introduction	1
1.1	Scope of Work	1
1.2	Literature Review	3
1.2.1	Near – Fault Ground Motions	3
1.2.2	Existing Solutions	4
1.3	Treatment of Nonlinear Oscillators	5
1.4	Preview of Thesis Contents	7
2.	Rigid Block Sliding to Idealized Acceleration Pulses	11
2.1	Synopsis	11
2.2	Preface	12
2.2.1	Pulses in Earthquake Engineering	12
2.2.2	Sliding Block Model	12
2.3	Sliding Block Problem Presentation	14
2.4	Closed-Form Solutions	21
2.4.1	Half – Cycle Excitation	21
2.4.1.1	Sliding on a nearly smooth plane	29
2.4.2	Full-Cycle Excitation	33
2.4.2.1	Stick – slip effects: residual response	36
2.5	Displacement Normalization by Dimensional Analysis	47
2.6	Asymmetric Friction	63
2.7	Static vs Dynamic Friction	68
2.8	Comparison with Solutions from the Literature	79
2.9	Application Examples	81

2.9.1	Bilateral sliding on horizontal plane	81
2.9.2	Unilateral sliding on inclined plane	82
2.10	Conclusions	83
3.	Simple Yielding Oscillator	87
3.1	Synopsis	87
3.2	Preface	88
3.3	Problem and Parameter Definition	92
3.4	Elastic Oscillator	95
3.4.1	Effect of Damping and Number of Cycles	102
3.5	Inelastic Oscillator: Parameter Study	107
3.5.1	Effect of Higher Harmonics	144
3.6	Elastic Perfectly Plastic Oscillator: Closed-Form Solutions	153
3.6.1	Determination of Time of Yielding	156
3.7	Results	162
3.8	Comparison with Actual Near-Field Recordings	167
3.9	Case Studies	174
3.9.1	The Collapse of Olive View Hospital in the 1971 San Fernando Earthquake	174
3.9.2	The Collapse of EBO Building in the 1995 Aegion, Greece, Earthquake	179
3.10	Conclusions	184
4.	Rocking Response and Overturning Criteria for Free Standing Rigid Blocks to Single-Lobe Pulses	187
4.1	Synopsis	187
4.2	Preface	188
4.3	Problem and Parameter Definition	189
4.3.1	Rocking Initiation	193
4.3.2	Equations of Motion	196

4.4	Solutions	197
4.4.1	Rocking Response	198
4.4.2	Overturning Criterion	199
4.4.3	Peak Rocking Response	207
4.4.4	Time of Impact	216
4.5	Discussion	221
4.6	Linear vs Non-Linear Equations of Motion: Energy Formulation	223
4.7	Conclusions	231
5.	Complex Response of a Rocking Block to a Full-Cycle Pulse	233
5.1	Synopsis	233
5.2	Preface	234
5.3	Problem and Parameter Definition	236
5.3.1	Rocking Initiation	239
5.3.2	Equations of Motion	239
5.3.3	Linearized Equations of Motion	240
5.4	Solutions	241
5.4.1	Half-Cycle Rectangular Pulse	241
5.4.2	Full-Cycle Rectangular Pulse	245
5.4.2.1	Mode 0	246
5.4.2.2	Mode 1	261
5.5	Peak Rocking Response	279
5.5.1	Peak Rocking Response for Positive Rocking Angles	279
5.5.2	Peak Rocking Response for Negative Angles	283
5.6	Response to other pulse shapes	291
5.7	Asymmetric Rectangular Pulse	295
5.8	Actual near-fault ground motions	304
5.9	Conclusions	310

6.	Shift Approach for the Dynamic Response of Rigid-Plastic Systems	313
6.1	Synopsis	313
6.2	Preface	314
6.3	Shift-Approach for Rigid-Plastic Systems	318
6.3.1	Interpretation	323
6.4	Methodology and Applications	324
6.4.1	Methodology	328
6.4.2	Results	331
6.5	Discussion	362
6.6	Conclusions	366
7.	Plastic Input Motion: a Transformation for the Response of Yielding Oscillators	369
7.1	Synopsis	369
7.2	Preface	370
7.3	Shift Property for Elastic-Perfectly Plastic Systems	374
7.4	Special Case	379
7.4.1	Rigid-Perfectly Plastic System	379
7.4.2	Shift Property for Bilinear Yielding Systems	381
7.5	Interpretation	384
7.6	Implementation	385
7.7	Applications	389
7.8	Conclusions	409
8.	Conclusions	413
8.1	Summary and Main Points	413
8.2	Recommendations for Future Research	426
	List of Symbols	429

References

437

A.	Appendix A: Various Pulse Shapes	A1
A.1	Half-Cycle	A2
A.1.1	Rectangular Pulse	A2
A.1.2	Sinusoidal Pulse	A3
A.1.3	Triangular Pulse	A4
A.1.4	Exponential Pulse	A5
A.1.5	Generalized Exponential Pulse	A6
A.1.6	Versed Sine Pulse	A7
A.1.7	Co-sinusoidal Pulse	A8
A.2	Full-Cycle	A9
A.2.1	Rectangular Pulse	A9
A.2.2	Sinusoidal Pulse	A10
A.2.3	Triangular Pulse	A11
A.2.4	Exponential Pulse	A12
A.2.5	Generalized Exponential Pulse	A13
A.2.6	Versed Sine Pulse	A14
A.2.7	Co-sinusoidal Pulse	A15
B.	Appendix B: Rigid Block Tables	B1
B.1	Half-Cycle	B2
B.1.1	Rectangular Pulse	B2
B.1.2	Sinusoidal Pulse	B3
B.1.3	Triangular Pulse	B4
B.1.4	Exponential Pulse	B5
B.1.5	Generalized Exponential Pulse	B7
B.1.6	Versed Sine Pulse	B9

B.1.7	Co-sinusoidal Pulse	B10
B.2	Full-Cycle	B11
B.2.1	Rectangular Pulse	B11
B.2.2	Sinusoidal Pulse	B12
B.2.3	Triangular Pulse	B13
B.2.4	Exponential Pulse	B14
B.2.5	Generalized Exponential Pulse	B16
B.2.6	Versed Sine Pulse	B18
B.2.7	Co-sinusoidal Pulse	B20
C.	Appendix C: Analytical & Graphical Results for Yielding Oscillator	C1
C.1	Rectangular Pulse	C3
C.2	Linear Pulse	C9
C.3	Suddenly Applied Triangular Pulse	C15
D.	Appendix D: Additional Material on Rocking Response	D1
D.1	Closed – Form Solution for the Limit of Mode 1 for $t_i < 2t_d$	D2
D.2	Function Study of Rocking Response	D4
D.3	Impulsive Forces during Impact	D8
E.	Appendix E: Shift Theorem Derivations	E1
E.1	Proof of Equations (6.16) and (6.17)	E2
E.2	Proof of Equation (7.2b)	E3
E.3	Tables	E4

List of Figures

- Figure 2.1** Problems analyzed by frictional block models on horizontal and inclined planes: (I) a) classical column drums, b) seismic isolation device, c) geo-structure on geo-synthetic liner; (II) d) failures of dam/embankment, e) retaining wall, f) footing near slope.
- Figure 2.2** Ground acceleration, velocity and displacement time histories for the generalized pulse in Eqs (2.1) – (2.2).
- Figure 2.3** Schematic representation of times of first slippage, peak sliding velocity and peak sliding displacement.
- Figure 2.4** Normalized peak sliding velocity and time of yielding (slippage) for a rigid block on a frictional plane subjected to a generalized exponential pulse.
- Figure 2.5** Normalized peak sliding displacement and corresponding time for a rigid block subjected to a half-cycle acceleration pulse.
- Figure 2.6** Time variation of sliding displacement for a rectangular pulse ($\beta \rightarrow -\infty$) of half-cycle duration, as function of yield strength, in terms of normalized time τ/τ_m .
- Figure 2.7** Time variation of sliding displacement for a rectangular pulse ($\beta \rightarrow -\infty$) of half-cycle duration, as function of yield strength, in terms of dimensionless time τ .
- Figure 2.8** Normalized sliding displacement ($u_m/\Delta u_{gm}$) and time of peak sliding τ_m for a rigid block subjected to a full-cycle acceleration pulse.
- Figure 2.9** Graphical representation of full-cycle pulse excitation showing times of initiation of slippage (τ_{y1}, τ_{y2}) and times of zero velocity (τ_{m1}, τ_{m2})
- Figure 2.10** (a) Criterion for stick-slip behavior after first stop for a full-cycle pulse; (b) critical strength required to induce stick behavior
- Figure 2.11** Normalized maximum response, time of maximum sliding response and time of yield during second slide (motion reversal).

- Figure 2.12** Comparison of blocks maximum displacement during second and first slide.
- Figure 2.13** Residual (permanent) displacement for a full-cycle pulse as a fraction of maximum displacement. Note the progressively smaller residual displacements with increasing yield strength leading to negative and eventually zero values.
- Figure 2.14** Comparison of peak sliding displacement for half- and full-cycle excitation.
- Figure 2.15** Alternative normalizations of sliding displacement for half-cycle excitation.
- Figure 2.16** Alternative normalizations of sliding displacement for full-cycle excitation.
- Figure 2.17** Comparison of peak sliding displacement predicted by the regression formula in Eq (2.30) against the exact solution for a full – cycle excitation pulse.
- Figure 2.18** Schematic representation of sliding to full cycle excitation under asymmetric friction.
- Figure 2.19** Variation of normalized frictional coefficient in terms of (a) displacement, (b) time and (c) time of yielding and maximum sliding velocity of a sliding block under an exponential pulse considering this variation.
- Figure 2.20.** Normalized peak sliding velocity and time of yield with time of yielding for a rigid block on a frictional plane under a generalized exponential pulse for varying friction conditions ($\eta^*=a\eta$, $a=1, 1.05, 1.1, 1.15, 1.2$).
- Figure 2.21a** Normalized displacement response with yield strength for a rigid block subject to a rectangular acceleration pulse ($\beta = -\infty$) under varying friction conditions ($\eta^*=a\eta$, $a=1, 1.05, 1.1, 1.15, 1.2$).
- Figure 2.21b.** Normalized displacement response with yield strength for a rigid block subject to a half-cycle generalized exponential acceleration pulse ($\beta = -\pi$) under varying friction conditions ($\eta^*=a\eta$, $a=1, 1.05, 1.1, 1.15, 1.2$).
- Figure 2.21c.** Normalized displacement response with yield strength for a rigid block subject to a half-cycle exponential acceleration pulse ($\beta = \pi$) under varying friction conditions ($\eta^*=a\eta$, $a=1, 1.05, 1.1, 1.15, 1.2$).

-
- Figure 2.21d.** Normalized displacement response with yield strength for a rigid block subject to a half - cycle triangular acceleration pulse ($\beta = 0$) under varying friction conditions ($\eta^* = \alpha\eta$, $\alpha = 1, 1.05, 1.1, 1.15, 1.2$)
- Figure 2.22a** Normalized displacement response with yield strength for a rigid block subject to a full-cycle rectangular acceleration pulse ($\beta = -\infty$) under varying friction conditions ($\eta^* = \alpha\eta$, $\alpha = 1, 1.05, 1.1, 1.15, 1.2$)
- Figure 2.22b** Normalized displacement response with yield strength for a rigid block subject to a full-cycle triangular acceleration pulse ($\beta = 0$) under varying friction conditions ($\eta^* = \alpha\eta$, $\alpha = 1, 1.05, 1.1, 1.15, 1.2$)
- Figure 2.22c** Normalized displacement response with yield strength for a rigid block subject to a full-cycle exponential acceleration pulse ($\beta = \pi$) under varying friction conditions ($\eta^* = \alpha\eta$, $\alpha = 1, 1.05, 1.1, 1.15, 1.2$)
- Figure 2.23** Comparison of permanent displacement due to a half-cycle pulse obtained by the present methodology, with established solutions from the literature.
-
- Figure 3.1** Four selected near-field ground motions containing long-period, high- acceleration pulses: Pacoima-N164W (1971), Rinaldi-N228W (1994), Erzican-NS (1992), and Pyrgos-EW (1993). The corresponding peak ground velocities are, respectively, 113, 166, 84, and 25 cm/s (Table 3.1). (modified after Mylonakis & Reinhorn, 2001)
- Figure 3.2** System considered.
- Figure 3.3** Dimensionless ground acceleration, velocity and displacement time histories of four idealized earthquake pulses considered in this Chapter: (a) triangular; (b) sinusoidal; (c) exponential and (d) rectangular.
- Figure 3.4** Normalized peak displacement response (a) and corresponding time (b) of an undamped elastic oscillator subjected to a symmetric triangular acceleration pulse.
- Figure 3.5** Normalized peak displacement response (a) and corresponding time (b) of an undamped elastic oscillator subjected to a sinusoidal acceleration pulse.

- Figure 3.6** Normalized peak displacement response (a) and corresponding time (b) of an undamped elastic oscillator subjected to a rectangular acceleration pulse.
- Figure 3.7** Elastic spectral amplification for four idealized half-cycle pulses. $\zeta=5\%$
- Figure 3.8** Effect of damping on elastic spectral amplification for a sinusoidal pulse.
- Figure 3.9** Effect of number of cycles on elastic spectral amplification for a sinusoidal pulse.
- Figure 3.10** Effect of number of cycles and oscillator damping on peak spectral amplification for harmonic excitation.
- Figure 3.11** Ductility demand of a simple oscillator subjected to triangular pulse excitation; $\gamma=0.10$; $\zeta=5\%$
- Figure 3.12** Ductility demand of a simple oscillator subjected to sinusoidal pulse excitation; $\gamma=0.10$; $\zeta=5\%$
- Figure 3.13** Ductility demand of a simple oscillator subjected to exponential pulse excitation; $\gamma=0.10$; $\zeta=5\%$
- Figure 3.14** Ductility demand of a simple oscillator subjected to rectangular pulse excitation; $\gamma=0.10$; $\zeta=5\%$
- Figure 3.15** Effect of number of cycles on ductility demand due to triangular pulse excitation; $\gamma=0.10$; $\zeta=5\%$
- Figure 3.16** Effect of number of cycles on ductility demand due to sinusoidal pulse excitation; $\gamma=0.10$; $\zeta=5\%$
- Figure 3.17** Effect of number of cycles on ductility demand due to exponential pulse excitation; $\gamma=0.10$; $\zeta=5\%$
- Figure 3.18** Effect of number of cycles on ductility demand due to rectangular pulse excitation; $\gamma=0.10$; $\zeta=5\%$
- Figure 3.19** Effect of number of cycles on ductility demand due to triangular pulse excitation for different values of the force reduction factor R ; $\gamma=0.10$; $\zeta=5\%$
- Figure 3.20** Effect of number of cycles on ductility demand due to sinusoidal pulse excitation for different values of the force reduction factor R ; $\gamma=0.10$; $\zeta=5\%$
- Figure 3.21** Effect of number of cycles on ductility demand due to exponential pulse excitation for different values of the force reduction factor R ; $\gamma=0.10$; $\zeta=5\%$

Figure 3.22	Effect of number of cycles on ductility demand due to rectangular pulse excitation for different values of the force reduction factor R ; $\gamma=0.10$; $\zeta=5\%$
Figure 3.23	Effect of damping on ductility demand due to a triangular pulse; $\gamma=0.10$
Figure 3.24	Effect of damping on ductility demand due to a sinusoidal pulse; $\gamma=0.10$
Figure 3.25	Effect of damping on ductility demand due to an exponential pulse; $\gamma=0.10$
Figure 3.26	Effect of damping on ductility demand due to a rectangular pulse; $\gamma=0.10$
Figure 3.27	Effect of damping on ductility demand due to a triangular pulse; $\gamma=0.10$
Figure 3.28	Effect of damping on ductility demand due to a sinusoidal pulse; $\gamma=0.10$
Figure 3.29	Effect of damping on ductility demand due to an exponential pulse; $\gamma=0.10$
Figure 3.30	Effect of damping on ductility demand due to a rectangular pulse; $\gamma=0.10$
Figure 3.31	Effect of post-yielding stiffness on ductility demand due to a triangular pulse; $\zeta=5\%$
Figure 3.32	Effect of post-yielding stiffness on ductility demand due to a sinusoidal pulse; $\zeta=5\%$
Figure 3.33	Effect of post-yielding stiffness on ductility demand due to an exponential pulse; $\zeta=5\%$
Figure 3.34	Effect of post-yielding stiffness on ductility demand due to a rectangular pulse; $\zeta=5\%$
Figure 3.35	Effect of post-yielding stiffness on ductility demand due to a triangular pulse; $\zeta=5\%$
Figure 3.36	Effect of post-yielding stiffness on ductility demand due to a sinusoidal pulse; $\zeta=5\%$
Figure 3.37	Effect of post-yielding stiffness on ductility demand due to an exponential pulse; $\zeta=5\%$
Figure 3.38	Effect of post-yielding stiffness on ductility demand due to a rectangular pulse; $\zeta=5\%$

- Figure 3.39** Comparison of numerical data with an approximate regression formula for the effect of post-yielding stiffness on ductility demand due to a half-cycle pulse. ($N=1/2$, $\zeta=5\%$)
- Figure 3.40** Comparison of response due to a half- vs a full – cycle sinusoidal pulse; $\gamma=0.10$, $\zeta=5\%$, $\eta = 0.70$
- Figure 3.41** Effect of post-yielding stiffness on ductility demand, time of peak response and sign of peak displacement for a sinusoidal ground acceleration pulse; $N=1$, $\zeta=5\%$, $\eta = 0.70$
- Figure 3.42** Effect of post-yielding stiffness on ductility demand, time of peak response and sign of peak displacement for a sinusoidal ground acceleration pulse; $N=1/2$, $\zeta=5\%$, $\eta = 0.70$
- Figure 3.43** Effect of higher harmonics of a triangular pulse on ductility demand; $N=1$, $\gamma = 0.10$, $\zeta=5\%$, $\eta = 0.30$
- Figure 3.44** Effect of higher harmonics of a sinusoidal pulse on ductility demand; $N=1$, $\gamma = 0.10$, $\zeta=5\%$, $\eta = 0.30$
- Figure 3.45** Effect of phase angle of a higher harmonic on ductility demand for a sinusoidal pulse; $N=1$, $\gamma = 0.10$, $\zeta=5\%$, $\eta = 0.30$
- Figure 3.46** Effect of higher harmonics of a triangular pulse on spectral amplification; $N=1$, $\zeta=5\%$
- Figure 3.47** Effect of higher harmonics of a sinusoidal pulse on spectral amplification; $N=1$, $\zeta=5\%$
- Figure 3.48** Effect of higher harmonics of a triangular pulse on spectral amplification; $N=1$, $\zeta=5\%$
- Figure 3.49** Effect of higher harmonics of a sinusoidal pulse on spectral amplification; $N=1$, $\zeta=5\%$
- Figure 3.50** Procedure for determining the time of yielding by finding roots to transcendental equations via Taylor expansions.
- Figure 3.51** Ductility demand for an elastic-perfectly plastic oscillator subjected to four idealized ground acceleration pulses. Comparison of the exact numerical solution with the approximate solution in Eqs 3.20 to 3.23.

Figure 3.52	Dimensionless time of yielding for an undamped elastic-perfectly plastic oscillator subjected to four idealized ground acceleration pulses. Comparison of the exact numerical solution with the approximate solution in Eqs 3.24 to 3.35.
Figure 3.53	Dimensionless time of peak response for an undamped elastic-perfectly plastic oscillator subjected to four idealized ground acceleration pulses. Comparison of the exact numerical solution with the approximate solution in Eqs 3.19.
Figure 3.54	Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.
Figure 3.55	Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.
Figure 3.56	Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.
Figure 3.57	Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.
Figure 3.58	Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.
Figure 3.59	Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.
Figure 3.60	Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.
Figure 3.61	Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.
Figure 3.62	Olive View Hospital, Wing D: Inferred geometry and structural properties (modified after Mahin <i>et al</i> 1976).
Figure 3.63	EBO Building: approximate geometry and structural properties. [Data after Gazetas <i>et al</i> 1995.]

- Figure 4.1** Problem considered: rocking motion of a free-standing block resting on a rigid base.
- Figure 4.2** Ground acceleration time history for a generalized exponential pulse of half-cycle duration.
- Figure 4.3** Time of uplift with uplift strength due to a generalized exponential pulse of half-cycle duration.
- Figure 4.4** Areas of Safety (S) and Overturning (O_1 to O_3) for a rocking block under an exponential pulse excitation of half-cycle duration ($\beta=\pi$).
- Figure 4.5** Areas of Safety (S) and Overturning (O_1 to O_3) for a rocking block under a rectangular pulse excitation of half-cycle duration ($\beta\rightarrow-\infty$).
- Figure 4.6** Areas of Safety (S) and Overturning (O_1 to O_3) for a rocking block under a triangular pulse excitation of half-cycle duration ($\beta\rightarrow 0$).
- Figure 4.7** Areas of Safety (S) and Overturning (O_1 to O_3) for a rocking block under a sinusoidal – like pulse excitation of half-cycle duration ($\beta\rightarrow-\pi$).
- Figure 4.8** Areas of Safety (S) and Overturning (O) for a rocking block under an exponential pulse excitation of half-cycle duration for different values of shape factor β .
- Figure 4.9** Time of peak rocking response as function of uplift strength and pulse duration (region S) due to a half-cycle generalized exponential pulse of : (a) exponential ($\beta=\pi$), (b) rectangular ($\beta\rightarrow-\infty$), (c) triangular ($\beta\rightarrow 0$) and (d) sinusoidal-like shape ($\beta=-\pi$).
- Figure 4.10** Double asymptotic matching of the solution for time of peak response (rectangular pulse shape $\beta=-\infty$).
- Figure 4.11** Peak rocking response (“rocking spectra”) as function of uplift strength and pulse duration (region S) due to a single-lobe generalized pulse of : (a) exponential ($\beta=\pi$), (b) rectangular ($\beta\rightarrow-\infty$), (c) triangular ($\beta\rightarrow 0$) and (d) sinusoidal-like shape ($\beta=-\pi$).
- Figure 4.12** Double asymptotic matching of the solution for peak response (rectangular pulse shape $\beta=-\infty$).

-
- Figure 4.13** Time of impact as function of uplift strength and pulse duration (region S) due to a half-cycle generalized exponential pulse of (a) exponential ($\beta=\pi$), (b) rectangular ($\beta\rightarrow-\infty$), (c) triangular ($\beta\rightarrow 0$) and (d) sinusoidal-like shape ($\beta=-\pi$).
- Figure 4.14** Double asymptotic matching of the solution for time of impact (rectangular pulse shape $\beta=-\infty$).
- Figure 4.15** Normalization of time of impact with peak rocking response as function of uplift strength and pulse duration (region S).
- Figure 4.16** Examples of response for different regimes of a rocking block under an exponential pulse of half-cycle duration pulse ($\beta=\pi$).
- Figure 4.17** Examples of response for different regimes of a rocking block under an exponential pulse of half-cycle duration pulse ($\beta=\pi$).
- Figure 4.18** Ratio of the critical strength, according to the non – linear equations of motion, over the one from the linearized solution as function of geometric parameter α and averaging parameter $\tilde{\chi}$ (Eq. 4.46).
- Figure 4.19** Geometric effect in non-slender blocks as function of parameters α and f due to half – cycle pulse excitations of (a) rectangular (b) triangular and (c) exponential shape.
-
- Figure 5.1** The problem considered: rocking motion of a free-standing block on a rigid-base.
- Figure 5.2** Rectangular ground acceleration pulse of full-cycle duration.
- Figure 5.3** Areas of Safety (S) and Overturning (O_1 to O_3) for a rocking block under a rectangular pulse excitation of half-cycle duration.
- Figure 5.4** Conditions for instantaneous equilibrium under the full-cycle symmetric rectangular pulse of Fig. 5.2.

- Figure 5.5** Classification of overturning (O) and surviving (S) systems under *Mode 0* by means of immobility and equilibrium points. Note that systems in group O experience an even number of immobility points (zero or two) whereas systems in group S experience an odd number of immobility points (one). No other pattern is possible for the specific problem.
- Figure 5.6** Areas of Safety (S_1 to S_3) and Overturning (O_1 to O_4) during forward response without impact (*Mode 0*), for a rocking block under a symmetric, full-cycle rectangular pulse.
- Figure 5.7** Examples of response for different regimes of a rocking block under a symmetric rectangular pulse of full-cycle duration.
- Figure 5.8** Times of impact for a rocking block under a symmetric rectangular pulse of full-cycle duration.
- Figure 5.9** Velocity at impact as function of uplift strength and pulse duration (Region S) due to a symmetric rectangular pulse of full-cycle duration. Note that both numerator and denominator in the ordinate axis are dimensionless.
- Figure 5.10** Classification of overturning and surviving rocking systems against *Mode 1* by means of immobility and equilibrium points for a symmetric rectangular pulse of full – cycle duration.
- Figure 5.11** Areas of overturning with one impact (*Mode 1*) for a rocking block under a symmetric rectangular pulse excitation of full – cycle duration; $\varepsilon = 0.8$.
- Figure 5.12** Limits of *Mode 1* due to a symmetric rectangular pulse excitation of full – cycle duration for different values of coefficient of restitution ε .
- Figure 5.13** Areas of overturning with impact (*Mode 1*) for a rocking block under a symmetric rectangular pulse of full – cycle duration.
- Figure 5.14** Examples of response for different regimes of a rocking block under a symmetric rectangular pulse of full-cycle duration (systems overturning or surviving *Mode 1*). Times of impact, denoted by a square, correspond to singularities in block acceleration. Note the jump in velocity at impact.

-
- Figure 5.15** Time of peak *forward* ($\theta > 0$) rocking response as function of uplift strength and pulse duration (Region S) due to a symmetric full-cycle rectangular pulse.
- Figure 5.16** Peak rocking response in the forward mode ($\theta > 0$) as function of uplift strength and pulse duration (Region S) for a full-cycle symmetric rectangular pulse. Note the double asymptotic matching of the results for small and large angles in the right graph.
- Figure 5.17** Time of peak rocking response in the *backward* mode ($\theta < 0$) as function of uplift strength and pulse duration (Regions S) due to a full-cycle symmetric rectangular pulse.
- Figure 5.18** Peak rocking response in the *backward* mode ($\theta < 0$) as function of uplift strength and pulse duration (Regions S) due to a full-cycle symmetric rectangular pulse ($\varepsilon = 0.8$).
- Figure 5.19** Time of peak rocking response in the *backward* mode ($\theta < 0$) as function of pulse duration (Regions S) due to a full-cycle symmetric rectangular pulse for various uplift strengths. Dots indicate limit stability points referring to the Limits of Modes 0 and 1 as shown in the insets; $\varepsilon = 0.8$
- Figure 5.20** Peak rocking response in the *backward* mode ($\theta < 0$) as function of pulse duration (Regions S) due to a full-cycle symmetric rectangular pulse for various uplift strengths. Dots indicate limit stability points referring to the Limits of Modes 0 and 1 as shown in the insets; $\varepsilon = 0.8$
- Figure 5.21** Comparison of peak response during forward and backward rocking; combination of the two *Modes* for the overall (absolute) value.
- Figure 5.22** Limits of areas of overturning without impact (*Mode 0* – solid lines) and with one impact (*Mode 1* – bulleted lines) for symmetric full – cycle pulses of rectangular, sinusoidal, triangular and exponential shape; $\varepsilon = 0.8$
- Figure 5.23** Types of pulse asymmetry considered in this investigation: (a) Type 1; (b) Type 2; (c) Type (3) and (d) Type 4.
- Figure 5.24a** Areas of overturning without impact (*Mode 0*) and criteria of impact at the end of the pulse due to Type 1 and Type 2 asymmetric rectangular excitations as function of shape parameter z . ($\varepsilon = 0.8$)

- Figure 5.24b** Areas of overturning without impact (*Mode 0*) and with one impact (*Mode 1*) due to Type 1 and Type 2 asymmetric rectangular pulse excitations as function of shape parameter z . ($\varepsilon = 0.8$)
- Figure 5.25a** Areas of overturning without impact (*Mode 0*) and criteria of impact at the end of the pulse due to a Type 3 asymmetric rectangular excitation as function of shape parameter z . ($\varepsilon = 0.8$)
- Figure 5.25b** Areas of overturning without impact (*Mode 0*) and with one impact (*Mode 1*) due to a Type 3 asymmetric rectangular pulse as function of shape parameter z . ($\varepsilon = 0.8$)
- Figure 5.26a** Areas of overturning without impact (*Mode 0*) and criteria of impact at the end of the pulse due to a Type 4 asymmetric rectangular excitation as function of shape parameter z . ($\varepsilon = 0.8$)
- Figure 5.26b** Areas of overturning without impact (*Mode 0*) and with one impact (*Mode 1*) due to a Type 4 asymmetric rectangular pulse as function of shape parameter z . ($\varepsilon = 0.8$)
- Figure 5.27** Selected near – fault acceleration records containing distinct pulses.
- Figure 5.28** Numerical investigation of overturning to actual earthquake recordings. Black bullets denote overturning; uplift strength normalized with (a) PGA ; (b) $2PGV/T_p$; $\varepsilon = 0.8$
- Figure 6.1** Idealized rigid-perfectly-plastic system considered.
- Figure 6.2** Translation of axes for a rigid–perfectly plastic system according to Eqs (6.12) and (6.15).
- Figure 6.3** Selected near-field recorded and idealized ground motions containing long-period, high-acceleration pulses. Properties are summarized in Table 6.1.
- Figure 6.4** Definition of integration limits and Plastic Input Acceleration (PIA) according to the proposed approach.

Figure 6.5a	Chi-Chi, Taiwan, 1999, TCU068N record. Plastic input motions for $a_y=0.10g$.
Figure 6.5b	Chi-Chi, Taiwan, 1999, TCU068N record. Plastic input motions for $a_y=0.20g$.
Figure 6.5c	Chi-Chi, Taiwan, 1999, TCU068N record. Plastic input motions for $a_y=0.30g$.
Figure 6.5d	Chi-Chi, Taiwan, 1999, TCU068N record. Plastic input motions for $a_y=0.40g$.
Figure 6.6a	Aegion, Greece, 1995, OTE-T record. Plastic input motions for $a_y=0.10g$.
Figure 6.6b	Aegion, Greece, 1995, OTE-T record. Plastic input motions for $a_y=0.30g$.
Figure 6.6c	Aegion, Greece, 1995, OTE-T record. Plastic input motions for $a_y=0.30g$.
Figure 6.6d	Aegion, Greece, 1995, OTE-T record. Plastic input motions for $a_y=0.40g$.
Figure 6.7a	Erzican, Turkey, 1992, ERZ-N record. Plastic input motions for $a_y=0.10g$.
Figure 6.7b	Erzican, Turkey, 1992, ERZ-N record. Plastic input motions for $a_y=0.20g$.
Figure 6.7c	Erzican, Turkey, 1992, ERZ-N record. Plastic input motions for $a_y=0.30g$.
Figure 6.7d	Erzican, Turkey, 1992, ERZ-N record. Plastic input motions for $a_y=0.40g$.
Figure 6.8a	Northridge, USA, 1994, Rinaldi-L record. Plastic input motions for $a_y=0.10g$.
Figure 6.8b	Northridge, USA, 1994, Rinaldi-L record. Plastic input motions for $a_y=0.20g$.
Figure 6.8c	Northridge, USA, 1994, Rinaldi-L record. Plastic input motions for $a_y=0.30g$.
Figure 6.8d	Northridge, USA, 1994, Rinaldi-L record. Plastic input motions for $a_y=0.40g$.
Figure 6.9a	San Fernando, USA, 1971, Pacoima Dam-L record. Plastic input motions for $a_y=0.10g$.
Figure 6.9b	San Fernando, USA, 1971, Pacoima Dam-L record. Plastic input motions for $a_y=0.20g$.

- Figure 6.9c** San Fernando, USA, 1971, Pacoima Dam-L record. Plastic input motions for $a_y=0.30g$.
- Figure 6.9d** San Fernando, USA, 1971, Pacoima Dam-L record. Plastic input motions for $a_y=0.40g$.
- Figure 6.10a** Imperial Valley, USA, 1979, E05-SN record. Plastic input motions for $a_y=0.10g$.
- Figure 6.10b** Imperial Valley, USA, 1979, E05-SN record. Plastic input motions for $a_y=0.20g$.
- Figure 6.10c** Imperial Valley, USA, 1979, E05-SN record. Plastic input motions for $a_y=0.30g$.
- Figure 6.11** Peak plastic input motion parameters for the waveforms of Figures 6.3 and 6.12.
- Figure 6.12** Graphical representation of the shift – theorem on a full – cycle sinusoidal pulse.
- Figure 6.13** Comparison of the predictions of the proposed method against rigorous numerical integration and established approximate methods for the near-fault motions of Figure 6.3.
-
- Figure 7.1** Idealized yielding oscillators considered in this chapter. From left to right: (a) Bilinear, (b) Elastic – perfectly plastic, (c) Rigid – perfectly plastic.
- Figure 7.2** Translation of axes for an elastic – perfectly plastic system subjected to earthquake loading. Shaded areas correspond to conditions at yielding.
- Figure 7.3** Translation of axes for a rigid – perfectly plastic system subjected to earthquake loading.
- Figure 7.4** Translation of axes for a bilinear yielding system subjected to earthquake loading. Shaded areas correspond to conditions at yielding.
- Figure 7.5** Definition of integration limits according to the proposed approach.
- Figure 7.6** Time histories of four idealized ground acceleration pulses of half- and full-cycle duration: (a) box, (b) sinusoidal, (c) symmetric triangular, (d) exponential.

Figure 7.7	Dimensionless representation of peak plastic input motion parameters for a rigid plastic system subjected to four idealized pulse excitations of full-cycle duration.
Figure 7.8	Dimensionless representation of peak plastic input motion parameters for a rigid plastic system subjected to four idealized pulse excitations of half-cycle duration.
Figure 7.9	Time histories of plastic input motions for a rigid-perfectly plastic system of different strengths subjected to four full-cycle ground motion pulses. Peak values in the original motions are scaled to unity.
Figure 7.10	Comparison of the results of the proposed theory against rigorous numerical results for an elastic – perfectly plastic oscillator subjected to a triangular pulse of half- cycle duration.
Figure 7.11	Comparison of the results of the proposed theory against rigorous numerical results for a bilinear oscillator subjected to a triangular pulse of half- cycle duration ($\gamma=0.05$).
Figure 7.12	Normalized velocity at yielding and time of yielding for elastic – perfectly plastic and bilinear oscillators subjected to a triangular pulse of half-cycle duration.
Figure 7.13	Half-cycle triangular pulse: time histories of Plastic Input Motions for perfectly plastic, elastic perfectly plastic and bilinear oscillators.

List of Tables

Table 2.1	Solution for residual displacement under bilateral sliding to full-cycle excitation
Table 2.2a	Characteristic displacement D_c for different integer values of constants a, b . (Eq. 2.27a).
Table 2.2b	Characteristic displacement D_c for different integer values of constants a, c . (Eq. 2.27b).
Table 2.2c	Characteristic displacement D_c for different integer values of constants a, c . (Eq. 2.27c)
Table 2.2d	Characteristic displacement D_c for different integer values of constants a, c . (Eq. 2.27d).
Table 2.2e	Characteristic displacement D_c for different integer values of constants a, c . (Eq. 2.27e).
Table 2.2f	Characteristic displacement D_c for different integer values of constants a, c . (Eq. 2.27f).
Table 2.3	Six basic normalization schemes of displacement for the generalized pulse of half- and full-cycle duration in Eqs (2.1) and (2.2).
Table 2.4	Determination of sliding response to full-cycle excitation under asymmetric friction.
Table 3.1	Selected near-field motions with long-period, high-acceleration pulses.
Table 5.1	Closed-form solutions for the rocking response to the asymmetric pulses in Figs 5.23a,b.
Table 5.2	Closed-form solutions for the rocking response due to the asymmetric pulse of Fig 5.23c.
Table 5.3	Closed-form solutions for the rocking response due to the asymmetric pulse of Figs 5.23d.

Table 5.4	Data for the near-fault acceleration records with distinct pulses of Fig 5.27.
Table 6.1	Selected near-fault ground motions employed in the analyses. Waveforms are provided in Fig 6.3.
Table 6.2	Peak plastic input motion parameters for different levels of yield acceleration.
Table 6.3	Comparison of peak plastic input motion parameters considering only major pulses (method A) and by rigorous integration of the whole earthquake record (method B); $a_y=0.1g$.
Table 7.1.	Functional form of the ground acceleration pulses considered.
Table 7.2	Normalized peak <i>PIM</i> values for a rigid perfectly plastic system under four full-cycle acceleration pulses.
Table 7.3	Normalized peak <i>PIM</i> values for a rigid perfectly plastic system under four half-cycle acceleration pulses.
Table 7.4	Dimensionless time of yielding and time of peak plastic input motion for a rigid perfectly plastic system.

1

Introduction

1.1 Scope of Work

The herein reported work aims at investigating the dynamic response of certain classes of dynamic yielding systems to impulsive ground excitations. Pulse content is typical (though not limited to) of near-fault earthquake motions generated by forward fault-rupture directivity and may inflict damage in the absence of substantial structural strength, post-yielding resistance and ductility capacity. The influence of such excitations in systems possessing bounded restoring force and hysteresis has not been adequately understood. Key unknowns include the scaling laws relating maximum response, limit capacity, excitation amplitude, duration, hysteresis and natural vibrational characteristics of the system. In the case of earthquake loading, additional unknowns stem from seismological factors affecting the characteristics of the pulse such as amplitude, shape and frequency content. Despite their importance in Civil Engineering science and practice, investigation of these phenomena is still at a state of basic research.

The goal of the present work is to produce numerical and analytical solutions (the latter mainly in explicit form) for certain types of yielding systems to actual and idealized

pulse-like earthquake excitations. More specifically, the following non-linear problems are addressed:

- (1) Sliding of rigid blocks obeying symmetric and asymmetric friction
- (2) Inelastic response of elastic-perfectly plastic and bilinear simple oscillators
- (3) Rocking response of rigid free-standing blocks on a perfectly rigid base

The suite of excitations utilized here are in the form of: (1) actual recorded ground motions containing forward fault rupture directivity pulses; (2) idealized pulses of half-cycle, full-cycle and multiple cycle duration, of various shapes.

Due to the inherent non-linearity of the phenomena at hand, bifurcation branches are identified in the solutions of the governing equations to reduce the problems into a sequence of piece-wise linear ones in time. The role of the main time derivatives of the excitation function (i.e. ground acceleration, velocity and displacement) on inelastic behavior are studied and a set of new scaling laws is derived and discussed.

The present work is deemed original as for the first time an extensive set of integrated numerical and analytical closed-form solutions for nonlinear systems is presented covering a wide-range of pulse excitations. The majority of the existing solutions focuses on excitations involving a large number of cycles (i.e for mechanical engineering applications), obtained mainly via linearization techniques (Krylov & Bogoliubov 1943) or perturbation theory (Nayfeh 2000), all referring to steady-state response. Likewise, studies of blast loads have been conducted for ballistics (Timoshenko & Young 1948, Biggs 1964) and various other applications (Gantes & Pnevmatikos 2004, Kouretzis *et al.* 2007). Such solutions are usually restricted to large excitation amplitudes and very short

durations (shorter than those of interest in earthquake engineering and engineering seismology). In the latter fields, solutions of transient nonlinear dynamic problems are very limited.

1.2 Literature Review

1.2.1 Near – Fault Ground Motions

A number of (often spectacular) seismic failures has been attributed to near-fault pulses. These include the damage beyond repair of the Olive View Hospital in Los Angeles during the 1971 San Fernando earthquake (Bertero *et al.* 1978), the collapse and overturning of the elevated Hanshin Expressway in Kobe during the destructive 1995 earthquake (Mylonakis *et al.* 2006), collapses of several building structures during the Northridge, California, 1994 earthquake, as well as that of a weapon industry in Aegion, Greece, in 1995 (Gazetas 1996).

Over the past decade the destructiveness of near-field earthquake motions has become a major research topic in earthquake engineering. These motions are strongly affected by *forward* fault-rupture directivity – in addition to proximity to fault – as the fault rupture propagates towards a site (and slip vector points toward the site) located within approximately 15km from the ruptured zone. The resulting ground motions attain the form of high-amplitude shock-like pulses polarized in the strike-normal direction, concentrated in the early part of the earthquake record (Somerville & Graves 1993, Somerville *et al.* 1997, Somerville 2000, Mavroeidis & Papageorgiou 2003, Spyrakos *et al.* 2008, Taflampas *et al.* 2009).

The majority of research in the area is based on actual ground recordings in combination with numerical simulations for analyzing the response of Single-Degree-Of-Freedom (SDOF) systems. The use of idealized pulses that can cast light into the physics of the problem in a more fundamental manner than numerical data, has not received the proper attention by researchers (Cuesta & Aschheim 2001, Menun & Fu 2002, Alavi & Krawinkler 2004). Studying such pulses appears advantageous as there is ample evidence that only a small portion of the earthquake motion associated with the pulse controls inelastic response (Bertero *et al.* 1978, Rodriguez-Marek 2000, Alavi & Krawinkler 2004). Moreover, simple waveforms often allow closed-form solutions to be obtained, which have obvious advantages over numerical evaluations of the response.

1.2.2 Existing Solutions

a. Sliding Blocks

Sliding of a wide class of engineering systems, such as rigid blocks, to near-fault pulses has scarcely been investigated by analytical techniques. The few available solutions (Newmark 1965, Sarma 1975, Yegian *et al.* 1988, Conte & Dente 1989) pertain to half – or full – cycle excitations and constant friction conditions.

b. Simple Oscillators

Single-Degree-Of-Freedom (SDOF) systems with bounded restoring force, such as the elastic-plastic oscillators, are among the most commonly-used models in structural dynamics and earthquake engineering. Despite extensive research on the dynamics of such systems (Bertero *et al.* 1978, Rodriguez-Marek 2000, Alavi & Krawinkler 2004, Makris & Chang 2000, Cuesta & Aschheim 2001, Mylonakis & Reinhorn 2001, Mavroeidis *et al.* 2004, Gantes & Pnevmatikos 2004, Dimakopoulou *et al.* 2011) and the availability of a substantial number of near-fault earthquake recordings (Mavroeidis &

Papageorgiou 2003, Baker 2007), the inelastic response of structures to such excitations has not been adequately explored or even understood. Key unknowns include scaling laws relating peak response, yield strength, hysteresis and vibrational characteristics of the dynamic system.

c. Rocking of Inverted Pendula

Current research on rocking response of rigid bodies has elucidated the role of near-fault pulses on uplift at tensionless horizontal interfaces. Such are building foundations or supports of rigid-bodies (i.e., machinery, museum exhibits) whose uplift is controlled by the amplitude and frequency content of the ground motion. Despite the apparent simplicity of the problem, tackling analytically the equations of motion has proven demanding due to non-linear geometric effects that often lead to instabilities. As a result, the majority of existing studies are either numerical or experimental in nature (Ishiyama 1982, Psycharis 1982, 1990, 1991, Psycharis & Jennings 1983, 1985, Psycharis *et al.* 2002, 2003, Sinopoli 1989, Spanos & Koh 1984, Tso & Wong 1989a,b, Yim *et al.* 1980, Mouzakis *et al.* 2002, Papandonopoulos *et al.* 2002). Whereas certain mixed analytical – numerical formulations (Anooshehpour *et al.* 1999, Zhang & Makris 2001, Dimitrakopoulos & DeJong 2012a,b, Kounadis 2013) are available, exact analytical solutions are, remarkably, limited to only two cases: rectangular and sinusoidal pulses of half-cycle duration (Housner 1963, Shi *et al.* 1996).

1.3 Treatment of Nonlinear Oscillators

It is rather common in dynamics to handle complex governing equations that require introducing assumptions and approximations, which are consistent with the physics of the

problem, to simplify the equations of motion to forms more tractable by analytical means. The process of linearization (Newmark & Rosenblueth 1971, Harris 1996) has become an intrinsic part of the analysis of engineering problems. There are instances, however, when linear analysis fails to describe the behavior of the dynamic problem adequately. In fact, one of the most fascinating features of the study of nonlinear problems is the occurrence of new and totally unsuspected phenomena: i.e., new in the sense that the phenomena are not predicted, or even hinted at, by the linear theory. On the other hand, certain phenomena observed physically are unexplainable except by giving due consideration to nonlinearities present in the system.

Following these lines, the problems treated in this Dissertation are mainly forced oscillations of yielding single-degree-of-freedom systems described by a nonlinear ordinary differential equation. Exact solutions for these systems are notoriously hard to obtain, except if the systems are linearized or can be represented in a stepwise linear manner. The latter approach was first described by Timoshenko and Young (1948) and is the one adopted in this Dissertation. The specific systems treated in the ensuing are:

Simple inelastic oscillator

A simple spring – mass – damper system is characterized by the equation

$$m\ddot{u} + f_d + f_s = P(t) \tag{1.1}$$

in which m is the system mass, $P(t)$ the excitation function, $\ddot{u}$ the oscillatory acceleration and f_s and f_d are the bilinear restoring force and damping force, respectively. It is assumed that f_s initially obeys Hooke's law until a specific displacement (yield point) when the stiffness of the system suddenly changes. Up to that point Eq. (1.1) is linear; after it is again linear, but the transition is a nonlinear event.

Frictional Sliding of a Rigid-Block

The governing equation of motion of a rigid-block sliding on a frictional plane is

$$m\ddot{u} + Q_y \operatorname{sgn}(\dot{u}) = P(t) \quad (1.2)$$

where Q_y is the frictional resistance multiplied by the signum of sliding velocity. The nonlinearity in this case is twofold: First, due to the elastic-perfectly plastic nature of the constitutive relation; Second, due to the existence of the signum function specifying that frictional resistance always opposes velocity.

Rocking of a Free-Standing Block

The non-linear equation governing rocking response of a rigid – block is

$$I_o \ddot{\theta} + mgR \sin(\alpha - \theta \operatorname{sgn}(\dot{\theta})) \operatorname{sgn}(\dot{\theta}) = P(t)R \cos(\alpha - \theta \operatorname{sgn}(\dot{\theta})) \quad (1.3)$$

where θ is the rocking angle and I_o is the moment of inertia of the block with respect to the pivot point (not the center of mass). The nonlinearity is now triple: First, due to the trigonometric terms containing the unknown function θ ; Second, due to the existence of the signum function on the left and right-hand side; Third, due to the impacts that force angular velocity to drop over an infinitesimal time interval. It will be shown that for slender blocks the first type of nonlinearity can be eliminated through the first-order approximation

$$I_o \ddot{\theta} - mgR\theta = P(t)R - mgR\alpha \operatorname{sgn}(\dot{\theta}) \quad (1.4)$$

Approximate methods for nonlinear vibration analysis provide a large number of alternatives, each of which may or may not have advantages for certain problems. To name a few: *Duffing's method*, *Rauscher's method*, *perturbation method*, the method of

Krylov and *Bogoliubov*, the *Ritz* method. Another approach to handle nonlinear equations is by graphical methods of integration such as the *phase-plane* representation. Comprehensive reviews of these methods can be found in Jacobsen & Ayre (1958), Harris (1996) and Andronov (2011). Nevertheless, handling Eqs (1.1) to (1.4) analytically is a formidable task even for highly idealized excitation functions such as sinusoidal or rectangular pulses to be considered in the ensuing.

1.4 Preview of Dissertation Contents

In Chapter 2 of the Thesis the problem of frictional sliding of rigid blocks to idealized ground acceleration pulses is considered. Specifically: (1) analytical solutions for a wide set of idealized pulses are derived; (2) the effects of symmetric and asymmetric sliding under both unilateral and bilateral excitation conditions are investigated; (3) alternative normalization schemes of peak sliding with reference to peak pulse acceleration, velocity, duration and shape are proposed. A generalized exponential function, capable of simulating an infinite number of pulse waveforms based on a single shape parameter, is employed.

Chapter 3 deals with the elastic-perfectly plastic oscillator. Numerical and analytical solutions are presented for the elastic and inelastic response of single-degree-of freedom yielding oscillators to idealized ground acceleration pulses. A numerical study is presented of the effect of oscillator period, strength, damping, post-yielding stiffness and number of excitation cycles, on inelastic response. Results are presented in the form of dimensionless graphs and regression formulas that elucidate the salient features of the problem. Closed-form solutions are derived for post-yielding response and associated ductility demand. The derived solutions provide insight on the physics of inelastic

response, which is often obscured by the complexity of numerical algorithms and actual earthquake motions. The model is evaluated against numerical results from near-field recordings. Two case studies are presented.

The classical problem of rocking of a rigid, free-standing block to earthquake ground shaking containing strong pulses is presented in [Chapter 4](#). A rectangular block resting on a perfectly rigid base is considered, subjected to a suite of idealized ground acceleration pulses expressed by a generalized function. The problem is treated analytically in the realm of the linearized equations of motion under the assumption of slender block geometry and rocking without slippage. Peak rocking response and overturning criteria for different waveforms are presented in terms of dimensionless closed-form expressions and graphs. The linearized response is compared with the fully non-linear one using an energy formulation leading to a novel approximate closed-form solution.

The above investigation is extended in [Chapter 5](#) for the case of a full-cycle pulse. A free-standing rigid block is considered, subjected to an idealized ground pulse of constant acceleration composed of two half-cycles of opposite sign. Closed-form expressions are obtained for: (1) peak rotation, peak angular velocity and peak angular acceleration; (2) overturning criteria for conditions with and without impact. Despite the apparent simplicity of the problem, it is shown that the response can exhibit complex patterns, a trait attributed to the two excitation lobes of opposite sign, the non-linear nature of the impact and the real-valued (positive) pole of the differential operator. The bifurcation behaviour of overturning with and without impact are highlighted.

In [Chapter 6](#) a novel shift approach is presented for evaluating and interpreting the response of rigid-perfectly plastic single-degree-of-freedom systems to dynamic loading.

Contrary to classical scaling laws, the proposed approach is *additive*, imposing a shift in the ordinates and the abscissa of the excitation function by means of a set of parameters uniquely related to the yielding resistance of the system. The dynamic response is then obtained by integrating the modified excitation function in a linear-like manner within a particular yielding branch, for the nonlinearity is incorporated into the forcing term. The mathematical validity of the approach is demonstrated analytically and its importance is highlighted for systems with symmetric yielding resistance subjected to near-fault earthquake motions. The relation of the proposed approach to existing methods and parameters of earthquake engineering such as Newmark's sliding block and relative ground acceleration, is discussed.

Finally, Chapter 7 extends the shift approach to bilinear systems. By this approach: (1) the differential operator becomes linear like, with the nonlinearity transferred to the right-hand side; (2) the initial conditions are simplified; (3) the modified forcing term becomes uniquely associated with development of plastic deformation. The theory is applied to various yielding oscillators subjected to idealized earthquake pulses, with the modified excitation function termed "*Plastic Input Motion*" (*PIM*). A procedure for applying the method to general waveforms is provided. The theory can be useful in earthquake engineering, by offering a replacement to physical ground motions with system-dependent *PIM*'s for establishing demand indices.

2

Rigid Block Sliding to Idealized Acceleration Pulses

2.1 Synopsis

New analytical solutions are derived for the frictional sliding of rigid blocks to idealized ground acceleration pulses. These excitations are indicative of near-fault earthquake motions affected by forward fault-rupture directivity, which may inflict large permanent displacements in the absence of substantial frictional resistance at the sliding interface. The scope of the study reported in this chapter is threefold: (1) to derive analytical solutions for a wide set of idealized pulses; (2) to investigate the effects of symmetric and asymmetric sliding under both unilateral and bilateral excitation conditions; (3) to explore alternative normalization schemes of peak sliding with reference to peak pulse acceleration, velocity, duration and shape; A generalized exponential function, capable of simulating an infinite number of pulse waveforms based on a single shape parameter, is employed to this end. Results are presented in the form of dimensionless closed-form expressions and graphs that provide insight into the physics of the non-linear problem.

2.2 Preface

2.2.1 Pulses in Earthquake Engineering

Following the unprecedented ground motions recorded in the Northridge (1994), Kobe (1995) and Chi-Chi (1999) earthquakes, the destructiveness of near-fault earthquake shaking has evolved into a major research area in earthquake engineering and seismology. These motions are known to be the result of forward fault-rupture directivity, as the rupture propagates towards a site (and the slip vector points towards the site) located within approximately 15-20 kilometers from fault (Stewart *et al.* 2001). The resulting ground motions attain the form of high-amplitude, long duration pulses, polarized in the strike-normal direction, concentrated in the early part of the earthquake record (Mavroeidis *et al.* 2004, Kempton & Stewart 2006). Studying such excitations appears desirable, as there is ample evidence that only a short time interval in the time history, associated with the pulse, contributes to most of the inelastic seismic demand. Comprehensive catalogues of pulse-like motions are available (Mavroeidis *et al.* 2004). Efforts to model pulse content by idealized signals have likewise been reported (Rodriguez-Marek 2000, Mylonakis & Reinhorn 2001, Mavroeidis *et al.* 2004, Cuesta & Aschheim 2004, Makris & Black 2004, Tang & Zhang 2010).

2.2.2 Sliding Block Model

Contrary to the response of yielding structures to actual time histories containing a large number of irregular cycles, which has been extensively investigated since the 1960's for a variety of structural and geotechnical systems obeying different force-displacement relationships (Veletsos & Newmark 1964, Newmark & Rosenblueth 1971, Makdisi & Seed 1978, Whitman & Liao 1985, Ambraseys & Menu 1988, Miranda & Bertero 1994, Wartman *et al.* 2003), the apparently simpler problem of frictional response to simple

pulses has been explored to a much lesser degree. Although idealized, sliding block models are widely accepted for simulating a variety of engineering systems including retaining walls, slopes, footings, masonry, equipment and seismic isolation devices (Fig 2.1). Studying simple waveforms appears desirable as these motions often allow closed-form solutions to be obtained, which have obvious advantages over numerical solutions. The few available studies (Biggs 1964, Newmark 1965, Sarma 1975, Conte & Dente 1989, Yegian *et al.* 1991) are restricted to a small set of waveforms (mostly of rectangular, sinusoidal, and triangular shape), half-cycle durations and unilateral sliding under constant frictional resistance. Findings of this research have indicated the possibility of deriving closed-form solutions for a broader set of pulse shapes and excitation cycles, accounting for factors such as system flexibility, stick-slip effects and time-varying friction. Moreover, these studies have demonstrated the potential of normalizing sliding response by a wider set of schemes, obtained by means of both dimensional analysis and physically-motivated considerations, which were not previously explored.

With reference to frictional sliding of rigid bodies, the scope of this chapter is threefold: (1) to derive an analytical closed-form solution for a generalized waveform that can simulate an infinite set of symmetric pulses ranging from a perfect rectangle to a perfect impulse; (2) to obtain solutions for both unilateral and bilateral sliding under half- and full-cycle excitation acting at an angle with respect to the sliding plane; (3) to investigate a novel set of normalization schemes for frictional response established by means of dimensional analysis; It will be shown that new solutions and insight into the physics of the problem can be gained from these investigations.

2.3 Sliding Block Problem Presentation

The problem considered involves a rigid block resting on a frictional plane having a Coulomb coefficient μ , subjected to an idealized ground acceleration pulse acting at a constant angle ξ with respect to the horizontal plane, as depicted in Fig 2.1. The pulse, shown in Fig 2.2, is described by a shape parameter β , amplitude A_g , half-cycle duration t_d and number of cycles (half-cycle, full-cycle). This excitation was apparently first employed by Jacobsen & Ayre (1958) for the investigation of linear shock spectra. Its use will be extended here for the case of frictional systems, as it can simulate by means of a single parameter an infinite set of symmetric pulses, (Fig 2.2). This appears desirable as a single solution may suffice for describing dynamic response to vastly different waveforms.

The use of ground acceleration, instead of velocity, for defining a rupture-directivity affected pulse is worth noting. From a seismological viewpoint, velocity pulses possess certain advantages over acceleration pulses for describing near fault motions, as they tend to scale better with source parameters such as moment magnitude and stress drop (Mavroeidis *et al.* 2004). On the other hand, acceleration pulses appear to correlate better with engineering parameters such as yield acceleration and plastic response. This can be understood given that: (1) triggering of plastic deformations (slippage) is inherently associated with acceleration – not velocity. (2) Structural systems, including rigid blocks, possess characteristic accelerations (e.g. yield acceleration), but not characteristic velocities. Thereby, use of acceleration pulses may be equally suitable (or perhaps advantageous) over velocity pulses when plastic response is of main concern. With reference to Fig 2.2, the pulses at hand are described as:

$$\ddot{u}_g(t) = \frac{A_g}{1-e^\beta} \begin{cases} 1-e^{2\beta t/t_d} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ 1-e^{2\beta(1-t/t_d)} & , \quad \frac{t_d}{2} \leq t \leq t_d \\ 0 & , \quad t > t_d \end{cases} \quad \text{half-cycle} \quad (2.1a)$$

$$\ddot{u}_g(t) = \frac{A_g}{1-e^\beta} \begin{cases} \text{same as in Eq. (1a)} & , \quad t \leq t_d \\ -1+e^{2\beta(t/t_d-1)} & , \quad t_d \leq t \leq \frac{3t_d}{2} \\ -1+e^{2\beta(2-t/t_d)} & , \quad \frac{3t_d}{2} \leq t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases} \quad \text{full-cycle} \quad (2.2a)$$

Employing the Heaviside function (i.e. $H(x) = +1, x > 0$; $H(x) = 0, x < 0$) Eqs (2.1a) and (2.2a) may be rearranged in the more compact form

$$\ddot{u}_g(t) = \frac{A_g}{1-e^\beta} \left[1-e^{2\beta t/t_d} + \left(e^{2\beta t/t_d} - e^{2\beta(1-t/t_d)} \right) H\left(t - \frac{t_d}{2}\right) + \left(e^{2\beta(1-t/t_d)} - 1 \right) H(t - t_d) \right] \quad (2.1b)$$

$$\ddot{u}_g(t) = \frac{A_g}{1-e^\beta} \left[\text{same as in Eq(1)} + (e^{2\beta(1-t/t_d)} + e^{-2\beta(1-t/t_d)} - 2)H(t - t_d) + (e^{2\beta(2-t/t_d)} - e^{-2\beta(1-t/t_d)})H\left(t - \frac{3t_d}{2}\right) + (1 - e^{2\beta(2-t/t_d)})H(t - 2t_d) \right] \quad (2.2b)$$

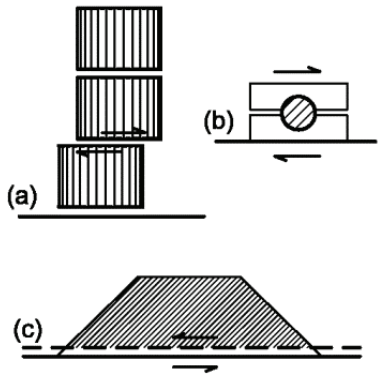
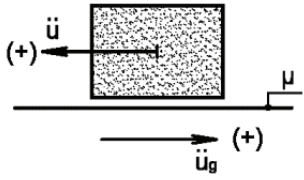
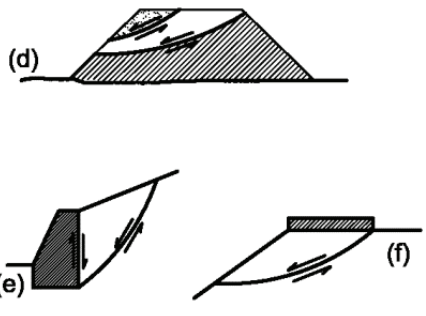
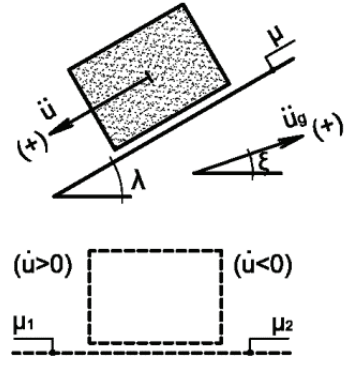
SYSTEM	IDEALIZATION
I : symmetric friction	
 (a) (b) (c)	
II : asymmetric friction	
 (d) (e) (f)	

Figure 2.1 Problems analyzed by frictional block models on horizontal and inclined planes: (I) a) classical column drums, b) seismic isolation device, c) geo-structure on geosynthetic liner; (II) d) failures of dam/embankment, e) retaining wall, f) footing near slope.

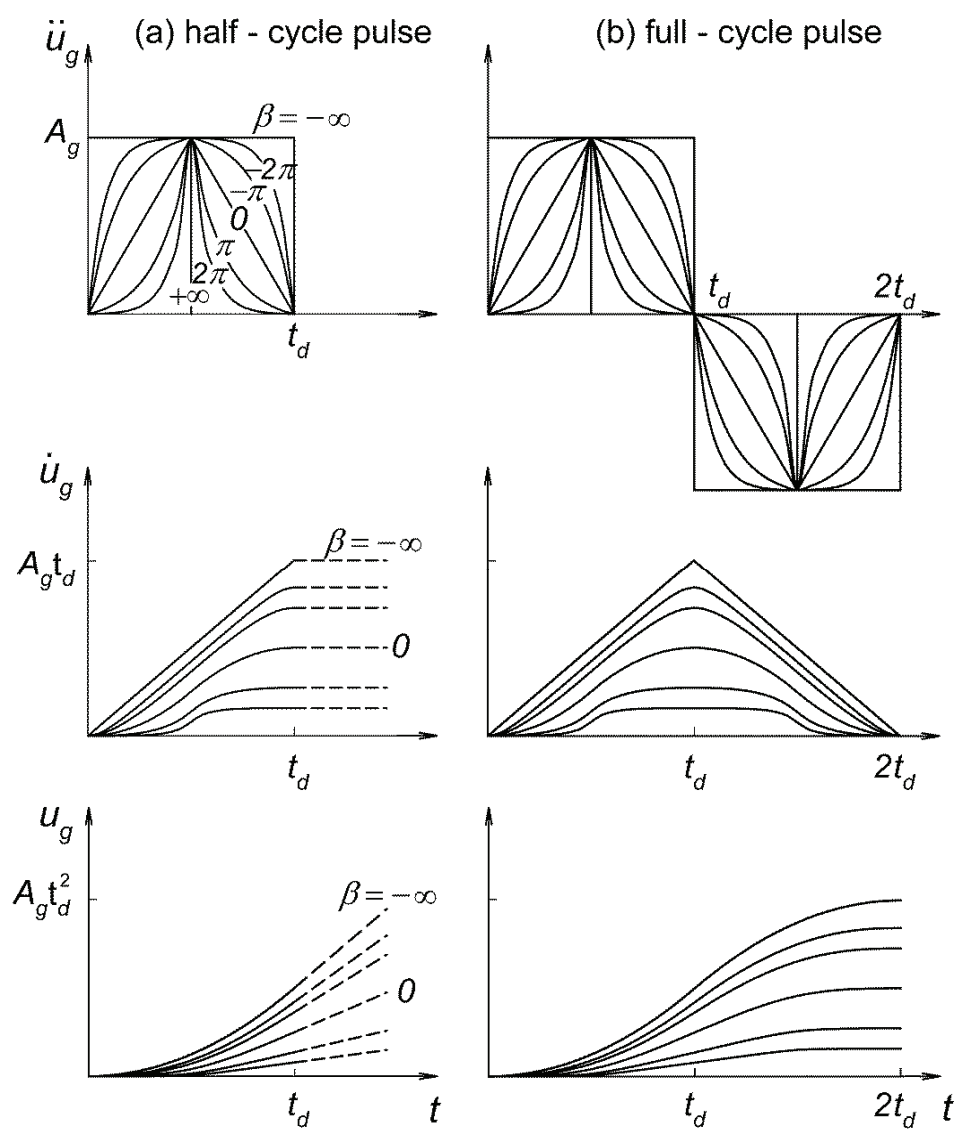


Figure 2.2 Ground acceleration, velocity and displacement time histories for the generalized pulse in Eqs (2.1) – (2.2).

Depending on the value of parameter β , different waveforms can be simulated by the above pair of equations. For instance, for $\beta=0$ a perfect triangular shape is reproduced, as shown in Fig 2.2. For smaller values of β the pulse shape gradually changes to sinusoidal like ($\beta=-\pi$) and box like ($\beta=-2\pi$); for $\beta \rightarrow -\infty$ the shape converges to that of a perfect rectangle. On the other hand, for increasing positive values of β the waveform becomes progressively more spiky ($\beta=\pi, 2\pi$), a shape referred by Jacobsen & Ayre (1958) to as *exponential*. For $\beta \rightarrow +\infty$ the pulse tends to an ideal spike of amplitude A_g and zero velocity*.

Strictly speaking, half-cycle pulses are not physically realizable, as ground velocity at the end of the excitation ($t = t_d$) is not zero. Nevertheless, these pulses incorporate the large acceleration and velocity excursions observed in actual near-fault recordings and, thereby, are well suited for investigating the salient features of seismic demand imposed by such motions. In addition, it will be shown that a wide class of sliding systems attains its peak displacement response *before* the end of the excitation and, thereby, whether ground velocity tends to zero or not at a later time is of less importance (Mylonakis & Voyagaki 2006). Finally, half-cycle pulses may be important for simulating conditions involving asymmetric friction, as explained later in this chapter.

The following dimensionless quantities are employed to describe the results presented below: yield strength $\eta = a_y/A_g$, time of first slippage $\tau_y = t_y/t_d$, time of peak sliding $\tau_m = t_m/t_d$, time of peak sliding velocity $\tau_{vm} = t_{vm}/t_d$. In addition, a_y = yield acceleration, $\dot{u}_{gy}$ = ground velocity at yielding, $\dot{u}_{gm}$ = ground velocity at peak sliding, t_y = time of yielding, t_m =

* A unitary delta function can be simulated by Eq(2.1) if the waveform is normalized by $A_g t_d [1/(1-e^\beta) + 1/\beta]$.

time of peak sliding, t_{vm} = time of peak sliding velocity, V_m = peak sliding velocity, u_m = peak sliding displacement, u_{res} = residual (permanent) displacement, V_g = peak pulse velocity, D_g = peak pulse displacement. Finally, the notions of incremental ground displacement, $\Delta u_{gm} = u_g(t_m) - u_g(t_y)$, and incremental ground velocity, $\Delta \dot{u}_{gm} = \dot{u}_{gm} - \dot{u}_{gy}$, during sliding are used as reference quantities. Some key parameters are illustrated in Fig. 2.3.

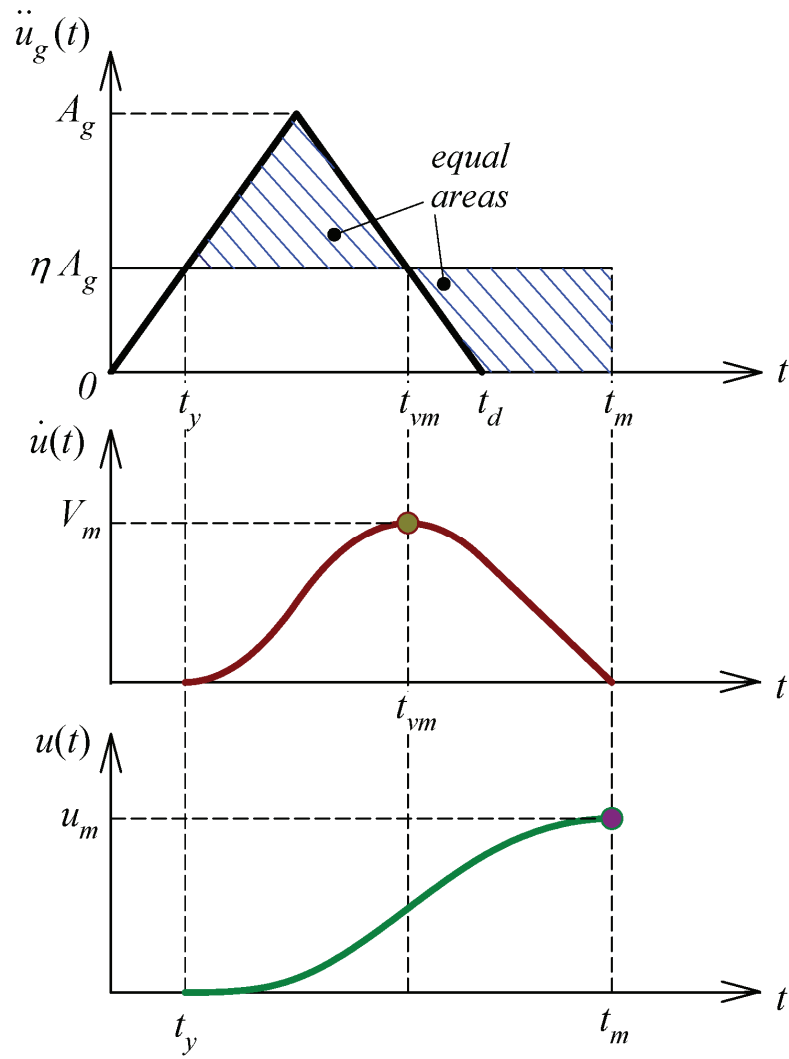


Figure 2.3 Schematic representation of times of first slippage, peak sliding velocity and peak sliding displacement.

2.4 Closed-Form Solutions

2.4.1 Half – Cycle Excitation

On the basis of the reference system of Fig 2.1, the governing equation of motion of a rigid block sliding on a frictional plane under seismic excitation is

$$\ddot{u} = +\ddot{u}_g - \frac{Q_y}{m} \text{sgn}(\dot{u}) \quad (2.3)$$

where $\ddot{u}$ denotes sliding acceleration relative to the frictional plane, m block mass and $Q_y = \mu mg$ ($= m a_y$) frictional resistance; $\text{sgn}()$ is the signum function. The positive sign in $\ddot{u}_g$ is to ensure positive block displacement for positive ground acceleration as evident from the reference system of Fig 2.1. Due to the unilateral nature of the response, Eq. (2.3) may refer to sliding either on a horizontal or on an inclined plane, provided that a suitable frictional force Q_y is employed. Details are provided later on.

Integrating Eq. (2.3) twice with respect to time, enforcing the conditions of zero relative velocity and displacement at first slippage ($t=t_y$) and considering unidirectional sliding (the only possibility for half-cycle excitation), block velocity and displacement are obtained from the expressions:

$$\dot{u}(t) = \dot{u}_g(t) - \dot{u}_{gy} - \frac{Q_y}{m}(t - t_y) \quad (2.4)$$

$$u(t) = u_g(t) - u_{gy} - (t - t_y) \dot{u}_{gy} - \frac{Q_y}{2m}(t - t_y)^2 \quad (2.5)$$

where u_{gy} and $\dot{u}_{gy}$ denote ground displacement and velocity at initiation of slippage (obtained by integrating Eq. (2.1)), respectively.

Time of peak sliding is determined from Eq. (2.4), by setting block velocity equal to zero

$$t_m = t_y + \frac{m}{Q_y}(\dot{u}_{gm} - \dot{u}_{gy}) = t_y + \frac{1}{a_y} \Delta \dot{u}_{gm} \quad (2.6)$$

where $\Delta \dot{u}_{gm}$ expresses the pulse area after initiation of sliding. The above time can be interpreted either as an instantaneous stop, with continuation of motion in the same direction, or a motion reversal, or a full stop. For the half-cycle pulses at hand, it is evident that t_m corresponds to a full stop and defines the time of peak sliding.

Equation (2.6) elucidates the intimate interconnection between pulse velocity and peak sliding displacement. Indeed, the higher the pulse area after initiation of slippage, the longer the block will stay in motion, the higher the ensuing sliding displacement. It should be kept in mind, however, that Eqs (2.3-2.6) tacitly assume that sliding has already initiated. As this requires exceedance of a certain level of acceleration ($|\ddot{u}_{gy}| > a_y$), ground velocity should not be viewed as the sole descriptor of pulse intensity (Rodriguez-Marek 2000, Mylonakis & Voyagaki 2006).

Additional support to the above suggestion comes from the observation that block displacement in Eq. (2.5) depends on time history of ground displacement u_g , not on that of ground velocity [term $\dot{u}_{gy}$ stands for a single value of ground velocity (at yielding), not a time history]. Evidently, a change in ground displacement during sliding may increase or decrease block displacement without altering the dynamic equilibrium in Eq. (2.3). This indicates that the time history of ground displacement might be more important in affecting dynamic response than presently thought. A similar discussion pertaining to the case of a yielding oscillator is provided in Mylonakis & Voyagaki (2006).

Expressed in terms of dimensionless time, Eq. (2.6) yields for the half-cycle pulses of Fig 2.2:

$$\tau_m = \tau_y + \frac{1}{\eta(1-e^\beta)} \left[1 + \frac{1}{2\beta} (1 + e^{2\beta\tau_y}) - \frac{e^\beta}{\beta} - \tau_y \right] \quad (2.7)$$

referring to the case where $\tau_m \geq 1$.

When maximum response occurs during forced sliding ($\tau_m \leq 1$), the corresponding equation is:

$$(\tau_m - \tau_y)[1 - \eta(1 - e^\beta)] + \frac{1}{2\beta} [e^{2\beta(1-\tau_m)} + e^{2\beta\tau_y} - 2e^\beta] = 0 \quad (2.8)$$

which can be solved numerically for a given pair of parameters β , η . Note that Eq. (2.8) never holds for a rectangular pulse ($\beta \rightarrow -\infty$), for which peak displacement always occurs during free sliding.

Setting τ equal to τ_m in Eq. (2.5) and after some straightforward algebra, the following explicit solution is obtained for the maximum displacement of the block:

$$\frac{u_m}{A_g t_d^2} = \frac{1}{4\beta^2(1-e^\beta)} \{X_1 + [-2\beta(1-\tau_m)[1 + \beta(1-\tau_m)] - 1 + e^{2\beta(1-\tau_m)}]H(\tau_m - 1)\} \quad (2.9a)$$

where

$$X_1 = -e^{2\beta(1-\tau_m)} + e^{2\beta\tau_y} + 2\beta[e^\beta(1-2\tau_m) + e^{2\beta\tau_y}(\tau_m - \tau_y)] + 2\beta^2(\tau_m - \tau_y)^2[1 - \eta(1 - e^\beta)] \quad (2.9b)$$

$H(\cdot)$ being the Heaviside (step) function. As the right side of Eq. (2.9) is dimensionless, peak sliding displacement is proportional to the product $(A_g t_d^2)$ for a given pulse shape and frictional resistance. For the special cases where $\beta \rightarrow -\infty$, $\beta \rightarrow 0$ and $\beta \rightarrow +\infty$, the generalized pulse attains the shape of a rectangle, a symmetric triangle and a spike, respectively (Fig 2.2). For the first two cases Eq. (2.9a) simplifies, respectively, to

$$\frac{u_m}{A_g t_d^2} = -\frac{\eta}{2} \tau_m^2 + \tau_m - \frac{1}{2} \quad (2.10a)$$

$$\frac{u_m}{A_g t_d^2} = -\frac{\eta}{2} (\tau_m - \tau_y)^2 + \tau_m^2 \left(1 - \frac{\tau_m}{3}\right) + \frac{1}{12} (1 - 6\tau_m) - \tau_y^2 \left(\tau_m - \frac{2}{3} \tau_y\right) + \frac{1}{3} (1 - \tau_m)^3 H(\tau_m - 1) \quad (2.10b)$$

which coincide with the solutions by Newmark (1965), Newmark & Rosenblueth (1971), Sarma (1975) and Conte & Dente (1989). The last case ($\beta \rightarrow +\infty$) yields $u_m = 0$, an anticipated result given the zero velocity content of the spike pulse (Fig 2.2). Note that the above peak displacements are also residual quantities due to the unilateral nature of the response.

The time of slippage can be obtained from the characteristic equation:

$$\ddot{u}_{gy} = \frac{Q_y}{m} = a_y = \eta A_g = \mu g \quad (2.11)$$

which, upon substitution of Eq. (2.1), yields the following solution for τ_y

$$\tau_y = \frac{1}{2\beta} \ln[1 - \eta(1 - e^\beta)] \quad (2.12)$$

The peak sliding velocity can be determined by setting $t = t_{vm} = t_d - t_y$ in Eq. (2.4); the solution can be expressed in normalized form as:

$$\frac{V_m}{V_g} = \frac{1}{e^\beta - \beta - 1} \left\{ (1 - \eta)(e^\beta - 1) - \beta[1 + \eta(e^\beta - 1)](1 - 2\tau_y) \right\} \quad (2.13)$$

where $V_g = A_g t_d (e^\beta - \beta - 1) / [\beta(e^\beta - 1)]$ is peak pulse velocity.

Numerical results from the above equations are shown in Figs 2.4 and 2.5, plotted as functions of yield strength. Time of yielding in Fig 2.4 is understandably an increasing function of η for all pulse shapes. Interestingly, the curves defining τ_y and τ_{vm} reproduce the shape of the generalized pulse (Fig 2.4b). This can be explained as τ_y in Eq. (2.12), the inverse function of Eq. (2.1), and $\tau_{vm} = 1 - \tau_y$ are symmetric with respect to $\tau = 1/2$. This attribute is useful as it holds for all symmetric pulses. For large negative and positive values of β , times τ_y and τ_{vm} tend to 0 and $1/2$, respectively, and vice versa. For small values of β , the above solutions reduce to linear functions $\tau_y = \eta/2$ and $\tau_{vm} = 1 - \eta/2$.

V_m/V_g in Fig 2.4a is naturally a decreasing function of η , starting from 1 at $\eta = 0$ and dropping to 0 at $\eta = 1$. For a box pulse ($\beta \rightarrow -\infty$), a linear variation $V_m/V_g = (1 - \eta)$ is obtained. For a spiky pulse ($\beta > 2\pi$) the dependence can be shown to converge to function $V_m/V_g = [1 - \eta(1 - \ln \eta)]$ which defines the lower bound of the results. All other cases lie between these two extremes.

With reference to displacement response, a rational normalizing parameter can be established on the basis of incremental base displacement $\Delta u_{gm} = u_{gm} - u_{gy}$, which stands for the amount of ground movement developing during sliding. For the pulses at hand Δu_{gm} is related to $(A_g t_d^2)$ through:

$$\frac{\Delta u_{gm}}{A_g t_d^2} = \frac{1}{4\beta^2(1-e^\beta)} \{X_2 + [e^{2\beta(1-\tau_m)} - 1 - 2\beta(\beta(\tau_m-1)^2 + 1 - \tau_m)]H(\tau_m-1)\} \quad (2.14a)$$

where τ_y and τ_m are given by Eqs (12) and (6–7), respectively, and

$$X_2 = -e^{2\beta(1-\tau_m)} + e^{2\beta\tau_y} + 2\beta[e^\beta(1-2\tau_m) + (\tau_m - \tau_y) + \beta(\tau_m^2 - \tau_y^2)] \quad (2.14b)$$

On the basis of this normalization, peak sliding displacements are plotted in Fig 2.5a. Contrary to dimensionless displacement ($u_m/A_g t_d^2$) in Eq. (2.9) which is unbounded[†] at small η 's, displacements normalized by means of Δu_{gm} are always finite. They are shown to decrease monotonically with increasing η , starting from $1/2$ at $\eta = 0$ and dropping to zero at $\eta = 1$. This suggests that up to 50% of incremental ground displacement can be transferred, as sliding displacement, to the block. This is a surprising behavior as intuition suggests that relative-to-ground displacement must equal ground displacement under zero friction (i.e., $u_m/\Delta u_{gm}=1$). An explanation is provided below. As regards the influence of pulse shape, Fig 2.5 indicates that for spiky pulses ($\beta > 2\pi$) the influence of shape on the response is negligible; for box-like pulses a stronger dependence is noticeable.

Time of maximum displacement in Fig 2.5b is, naturally, a decreasing function of sliding strength. In most cases peak displacement occurs during free sliding, corresponding to points located above the dashed horizontal line in the graph. Peak response during forced sliding occurs only above a certain level of frictional resistance which varies with pulse shape (e.g., $\eta > 0.25$ for $\beta = 2\pi$; $\eta > 0.88$ for $\beta = -2\pi$). The dashed line in both graphs defines the special case where $\tau_m=1$.

[†] This is due to the finite ground velocity at the end of the pulse that makes ground displacement to increase without limit

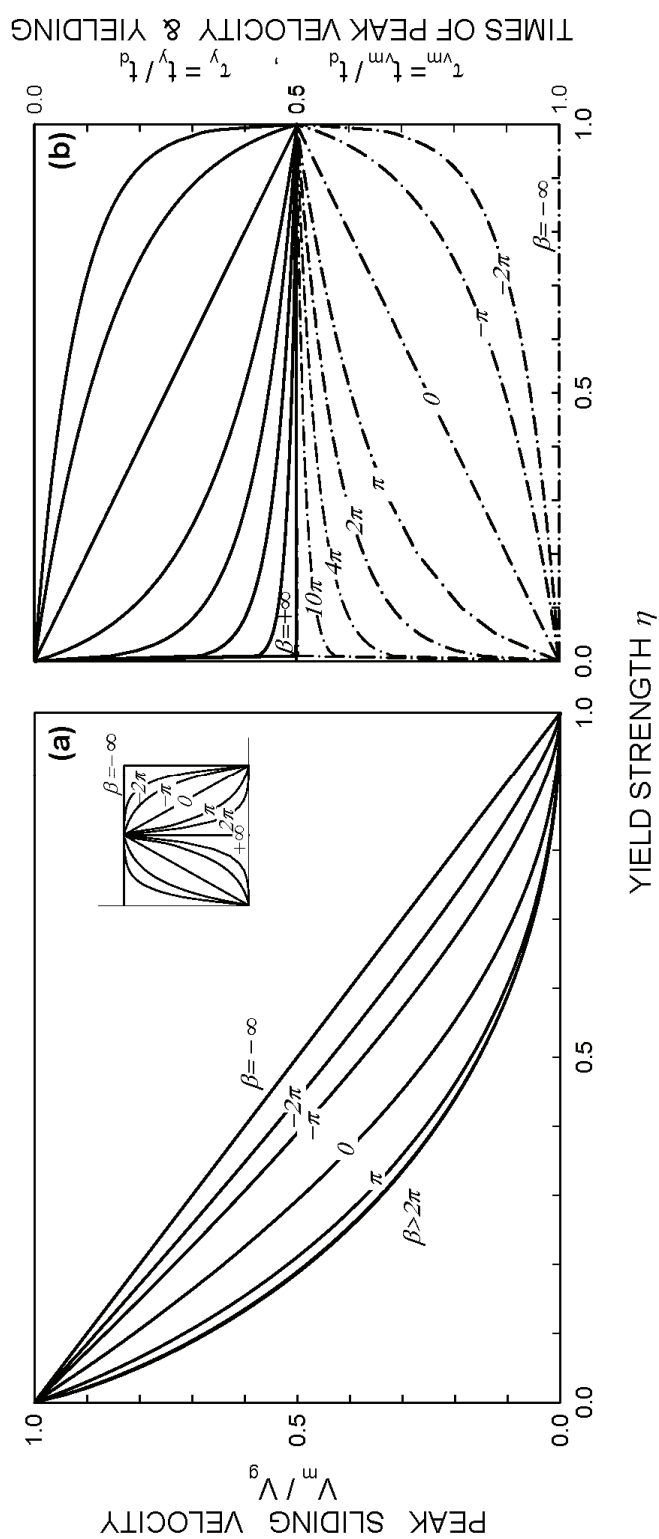


Figure 2.4 Normalized peak sliding velocity and time of yielding (slippage) for a rigid block on a frictional plane subjected to a generalized exponential pulse.

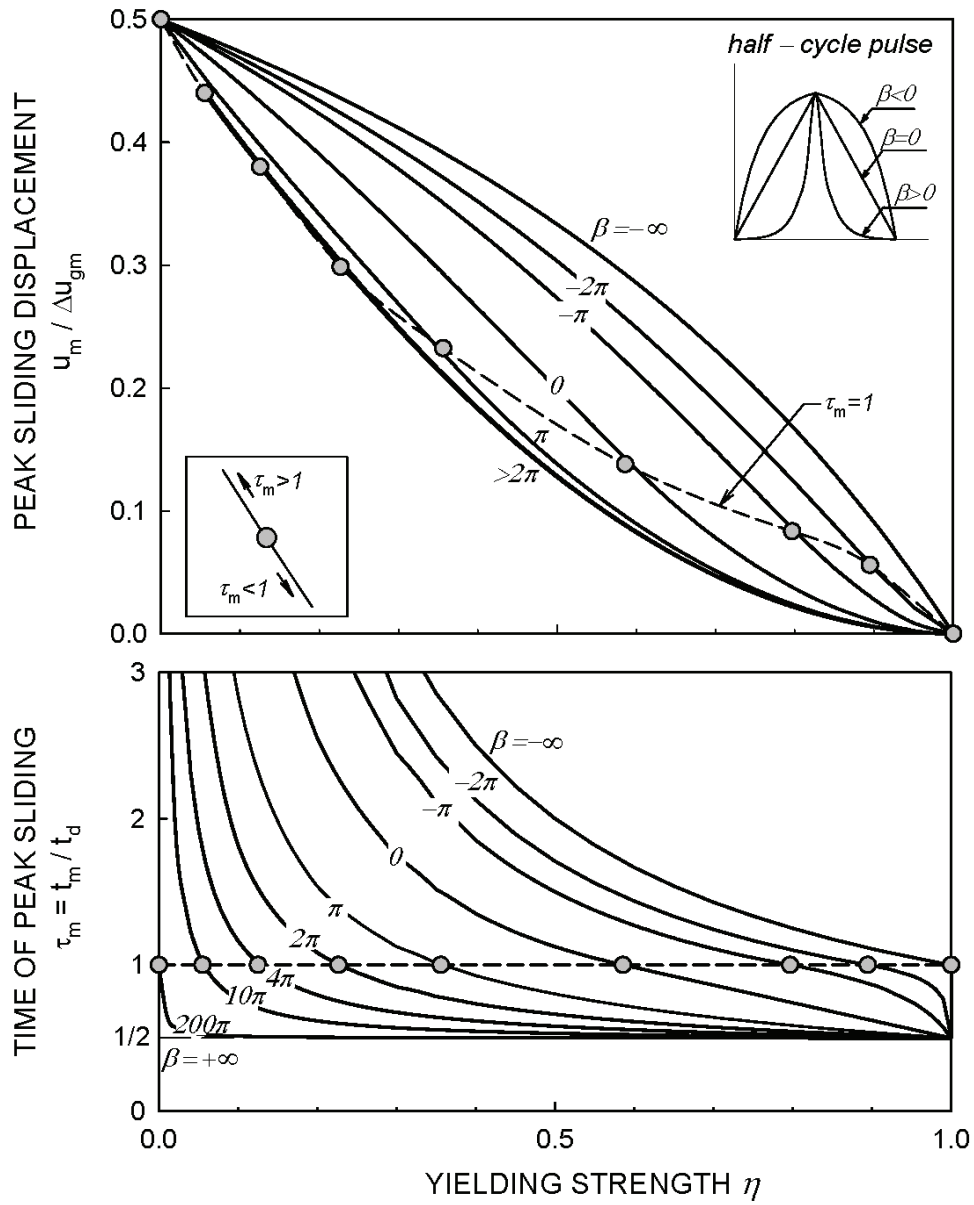


Figure 2.5 Normalized peak sliding displacement and corresponding time for a rigid block subjected to a half-cycle acceleration pulse.

2.4.1.1 Sliding on a nearly smooth plane

Reference has already been made to the anomalous behavior at $\eta \rightarrow 0$ resulting in a $u_m/\Delta u_{gm}$ ratio of 1/2 instead of an anticipated 1. This unexpected trend is due to the infinite sliding time associated with the finite velocity at the end of a half-cycle pulse in conjunction with the lack of sufficient friction to bring the block to a stop. By employing a pertinent asymptotic analysis, it will be shown that this behavior is of little interest in applications.

For instance, considering the simple case of a rectangular pulse ($\beta \rightarrow -\infty$), function $u/\Delta u_g$ during free sliding can be shown to be

$$\frac{u(\tau)}{\Delta u_g(\tau)} = \frac{2\tau - 1 - \eta\tau^2}{2\tau - 1} \quad (2.15)$$

where τ varies between $\tau_d = 1$ (end of pulse) and $\tau_m = 1/\eta$ (end of sliding). Note that the latter time is unbounded at $\eta \rightarrow 0$, due to the lack of sufficient friction to bring the block to a stop.

During forced sliding ($0 < \tau < 1$), the corresponding expression is

$$\frac{u(\tau)}{\Delta u_g(\tau)} = 1 - \eta \quad (2.16)$$

which, remarkably, is independent of time.

For the limit case of a nearly smooth interface ($\eta \rightarrow 0$), a naïve approach would be to drop, by dominant balance, the term ($\eta\tau^2$) from Eq. (2.15), which would lead to the spurious result $u/\Delta u_g = 1$. In reality this term does not vanish at $\tau = \tau_m$ but, instead, tends to infinity

as $(\eta \tau_m^2)=1/\eta$. Accordingly, instead of dropping $(\eta \tau^2)$, the numerator and denominator of Eq. (2.15) are divided by τ_m to give

$$\frac{u(\tau)}{\Delta u_g(\tau)} = \frac{2(\tau/\tau_m) - \eta - (\tau/\tau_m)^2}{2(\tau/\tau_m) - \eta} \quad (2.17)$$

Where, now, the new time variable (τ/τ_m) varies between η (end of pulse) and 1 (end of sliding).

An asymptotic analysis of Eq. (2.17) reveals that for small (τ/τ_m) 's and finite η 's, all terms in the numerator and the denominator are significant; setting $(\tau/\tau_m) = \eta$, corresponding to the end of pulse, yields the classical result $u/\Delta u_g = 1 - \eta$ (see Eq. 2.16) which, naturally, tends to 1 as η tends to 0. On the other hand, for (τ/τ_m) 's close to 1, corresponding to end of sliding, and for small η 's, the second term in the numerator and denominator vanishes leading to the asymptotic expression $u(\tau)/\Delta u_g(\tau) \sim 1 - (\tau/\tau_m)/2$ which tends to $1/2$ as (τ/τ_m) tends to 1. This is depicted in Fig 2.6, where Eq. (2.15) is plotted as function of time τ . Remarkably, the limit value $1/2$ cannot be attained physically, as τ_m is infinite for $\eta = 0$. Only a value of 1 would be experimentally observable at sufficiently small times for $\eta=0$ and, thereby, the conventional behavior $u/\Delta u_g = 1$ will not be violated in practice. Indeed, whereas the curve for $\eta = 0$ in Fig 2.6 cannot be reproduced experimentally as (τ/τ_m) is always zero for finite times, the rest of the results in the graph can be of importance in practice applications. Results expressed in terms of dimensionless time τ are provided in Fig. 2.7. The slow deviation from $u/\Delta u_g = 1$ at low η 's is evident in the graphs.

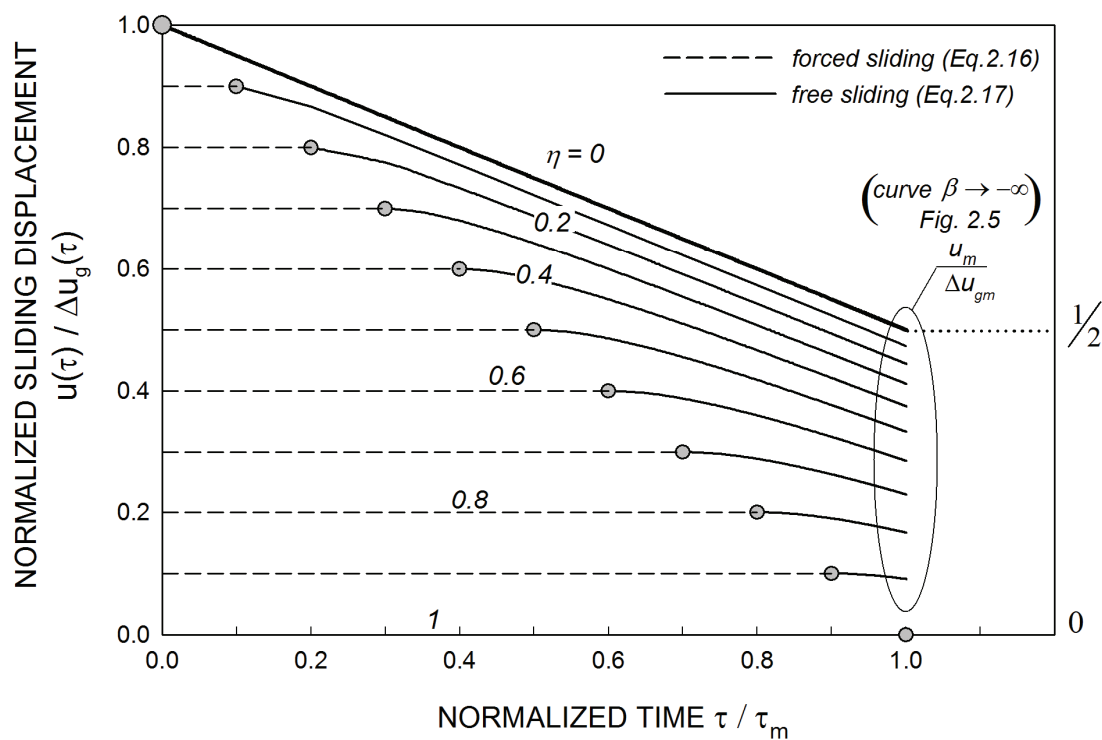


Figure 2.6 Time variation of sliding displacement for a rectangular pulse ($\beta \rightarrow -\infty$) of half-cycle duration, as function of yield strength, in terms of normalized time τ/τ_m .

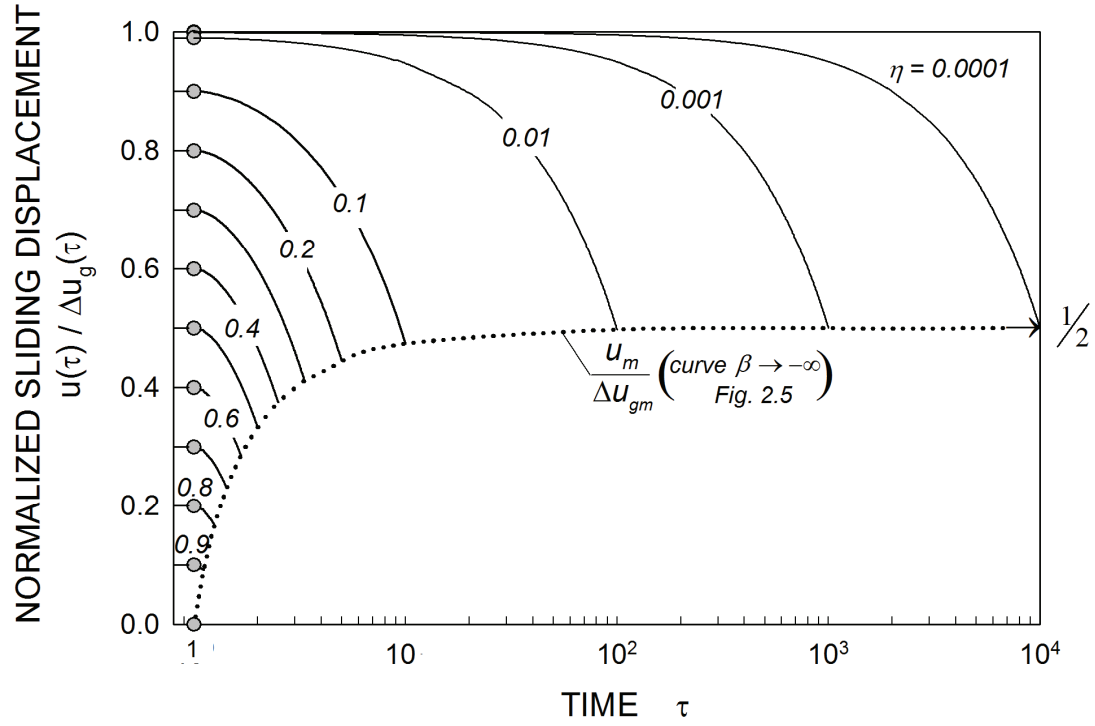


Figure 2.7 Time variation of sliding displacement for a rectangular pulse ($\beta \rightarrow -\infty$) of half-cycle duration, as function of yield strength, in terms of dimensionless time τ .

2.4.2 Full-Cycle Excitation

Considering initiation of sliding during the first excitation half-cycle (Fig 2.2), maximum sliding response is given by

$$\frac{u_m}{A_g t_d^2} = \frac{1}{4\beta^2(1-e^\beta)} \{X_1 + [e^{2\beta(1-\tau_m)} + e^{-2\beta(1-\tau_m)} - 2 - 4\beta^2(1-\tau_m)^2]H(\tau_m - 1) + [e^{2\beta(2-\tau_m)} - e^{-2\beta(1-\tau_m)} + 2\beta e^\beta(2\tau_m - 3)]H(\tau_m - 3/2)\} \quad (2.18)$$

where X_1 is given by Eq. (2.9b); τ_m is obtained from the solution of the following equation:

$$X_3 - [2\tau_m + \frac{1}{2\beta}[e^{2\beta(1-\tau_m)} - e^{-2\beta(1-\tau_m)}]]H(\tau_m - 1) + \frac{1}{2\beta}[-e^{-2\beta(1-\tau_m)} - e^{2\beta(2-\tau_m)} + 4\beta + 2e^\beta]H(\tau_m - 3/2) = 0 \quad (2.19a)$$

where

$$X_3 = (\tau_m - \tau_y)[1 - \eta(1 - e^\beta)] + \frac{1}{2\beta}[e^{2\beta(1-\tau_m)} + e^{2\beta\tau_y} - 2e^\beta] \quad (2.19b)$$

that coincides with the left-hand side of Eq. (2.8). The presence of Eqs (2.8)–(2.9) in the above expressions is anticipated, as they represent the effect of first excitation lobe on the response. Time of slippage, τ_y , and peak velocity response, V_m , are as for the half-cycle pulse (Eqs 2.12–2.13).

In the same spirit as before, peak sliding response can be normalized by incremental base displacement Δu_{gm} . For a full cycle pulse, this parameter is related to $A_g t_d^2$ as

$$\frac{\Delta u_{gm}}{A_g t_d^2} = \frac{1}{4\beta^2(1-e^\beta)} \{X_2 + [e^{-2\beta(1-\tau_m)}(1 - e^{2\beta(1-\tau_m)})^2 - 4\beta^2(\tau_m - 1)^2]H(\tau_m - 1) + [e^{2\beta(2-\tau_m)} - e^{-2\beta(1-\tau_m)} + 2\beta e^\beta(2\tau_m - 3)]H(\tau_m - 3/2)\} \quad (2.20)$$

where X_2 is given by Eq. (2.14b). Tabulated closed-form solutions for both half- and full-cycle generalized exponential, and cycloidal pulses are available in Appendices A & B.

Numerical results from the above equations are provided in Fig 2.8, plotted as functions of η . Similarly to the half-cycle pulse, time of maximum displacement is a decreasing function of η (Fig 2.8b). Contrary to the previous case, however, τ_m is always finite and limited within the duration of forced sliding ($1/2 \leq \tau_m \leq 2$). Peak response during first half cycle ($\tau_m \leq 1$) occurs only for sufficiently strong friction (e.g. $\eta > 0.60$ for $\beta=0$; $\eta > 0.90$ for $\beta=-2\pi$). For a rectangular pulse ($\beta \rightarrow -\infty$), peak response always occurs during the second half cycle ($1 \leq \tau_m \leq 2$). The functional dependence of τ_m on η is, understandably, more complex than that of a half-cycle pulse.

Peak sliding displacement is depicted in Fig 2.8a, normalized by incremental base displacement Δu_{gm} . This ratio is, naturally, smaller than 1 and decreases with increasing η , starting from 1 at $\eta = 0$ and reaching zero at $\eta = 1$. As in the previous case, the influence of pulse shape on the response becomes negligible beyond $\beta = 2\pi$. For smaller values of β , however, the influence of pulse shape becomes significant, with box-like pulses exhibiting stronger response than spiky ones. The anomalous behaviour encountered in the half-cycle excitation at $\eta \rightarrow 0$ is not observed in this case, as time of peak sliding is always finite, leading to $u_m / \Delta u_{gm} = 1$ for a smooth interface. White and gray dots in Fig 2.8 denote systems attaining peak response at dimensionless times equal to $3/2$ and 1 , respectively (they do not indicate a change in slope or curvature of the lines).

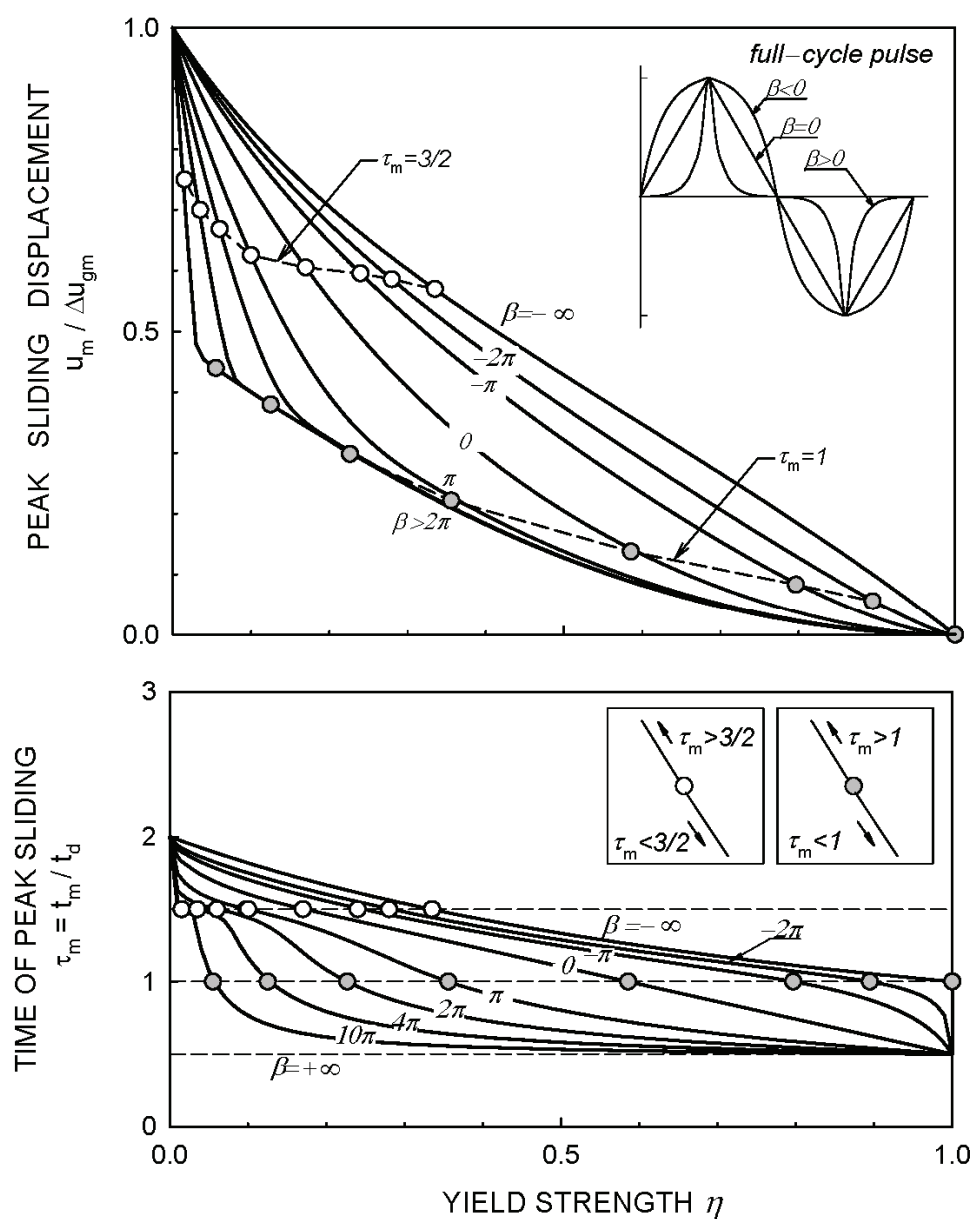


Figure 2.8 Normalized sliding displacement ($u_m/\Delta u_{gm}$) and time of peak sliding τ_m for a rigid block subjected to a full-cycle acceleration pulse.

2.4.2.1 Stick – slip effects: residual response

The foregoing invariably refer to a first stop ($\tau = \tau_{m1}$, Fig 2.9), which defines the maximum response of the block. Following first stop the block will either: (1) slip in the opposite direction, if base acceleration at τ_{m1} is greater than the frictional resistance ($|\ddot{u}_g(\tau_{m1})|/A_g > \eta$, Fig 2.9), a behavior that will be referred hereafter to as “slip”; (2) stick if base acceleration at τ_{m1} is smaller than frictional resistance (ηA_g). For the latter case, stick will last until a new exceedance of yield acceleration is encountered. This kind of behavior will be referred hereafter to as “stick”. For the particular family of pulses and on the basis of symmetry, time of second yield would always be $\tau_{y2} = 1 + \tau_{y1}$.

The criterion for stick-slip behavior can, accordingly, be reformulated in terms of the equations:

$$\text{stick } (\tau_{y2} = \tau_{y1} + 1), \text{ when } 1 - \tau_{y1} \leq \tau_{m1} \leq 1 + \tau_{y1} \quad (2.21a)$$

$$\text{slip } (\tau_{y2} = \tau_{m1}), \text{ when } 1 + \tau_{y1} < \tau_{m1} < 2 - \tau_{y1} \quad (2.21b)$$

which are demonstrated graphically in Fig 2.10a, plotted as functions of parameters η and β . [Note that η is linked to the physical frictional coefficient as $\mu = a_y/g = \eta A_g/g$.] The critical point separating stick from slip behavior corresponds to the intersection of curves τ_{m1} and $(1 + \tau_{y1})$ and is denoted by a bullet in the graphs. The portion of curve τ_{m1} above the intersection point refers to slip conditions, whereas the one below the intersection point to stick. Evidently, systems with small yield strength tend to exhibit slip behavior and vice versa. The critical yield strength separating the two regions tends to increase as

pulse shape changes from spiky ($\beta \geq 2\pi$) to box like ($\beta \rightarrow -\infty$). The hatched areas defined between curves $1 + \tau_{y1}$ and $2 - \tau_{y1}$ portray the shape of the pulse.

Based on these results the following regression formula was derived to approximate the critical strength required to induce stick behavior:

$$\eta_{cr} = 0.1 + 0.8 e^{-\exp(0.25\beta - 0.15)} \quad (2.22)$$

which is valid in the region $-4\pi \leq \beta \leq 4\pi$ having an R^2 coefficient of 0.988 (Fig 2.10b).

To describe the second (reverse) sliding branch under either slip or stick behavior, the solutions obtained for a half-cycle excitation can be extended to cover the case where τ_y occurs after the end of the ascending branch of the pulse ($\tau_y > 1/2$). Although this is never the case for first yielding, it is possible for motion reversal under *slip* conditions when $\tau_{m1} > 3/2$ (Fig 2.9d). Moreover, while τ_m for a half-cycle pulse can occur during either forced or free sliding, final stop for a full-cycle pulse (τ_{m2}) and slip conditions always occurs during free sliding (Fig 2.9c,d). Moreover, if τ_{m2} occurs during forced sliding, the particular response branch will induce a displacement u_{m2} which is equal and opposite to that of the first branch, u_{m1} , thereby residual displacement will be zero.

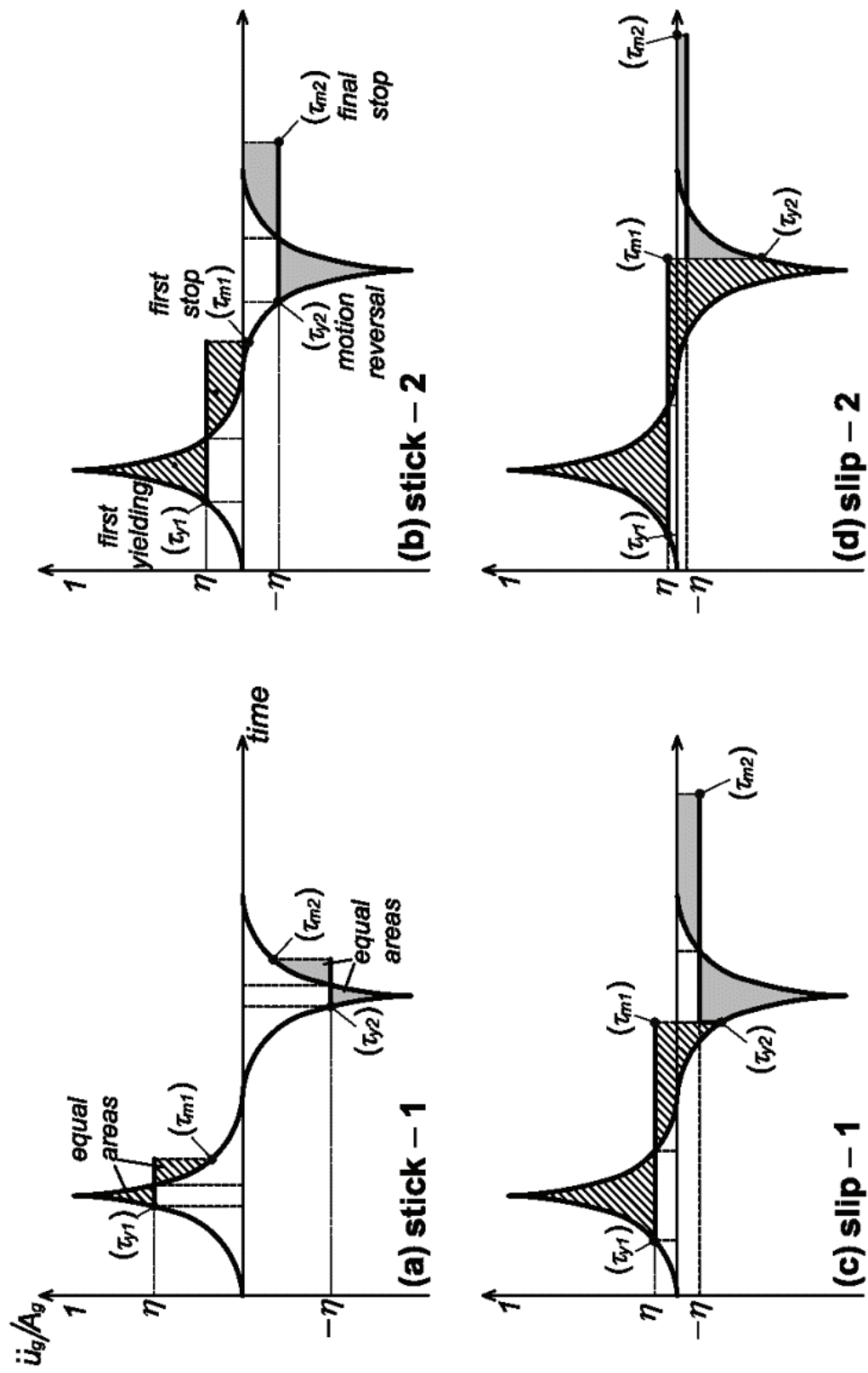


Figure 2.9 Graphical representation of full-cycle pulse excitation showing times of initiation of slippage (τ_{y1} , τ_{y2}) and times of zero velocity (τ_{m1} , τ_{m2})

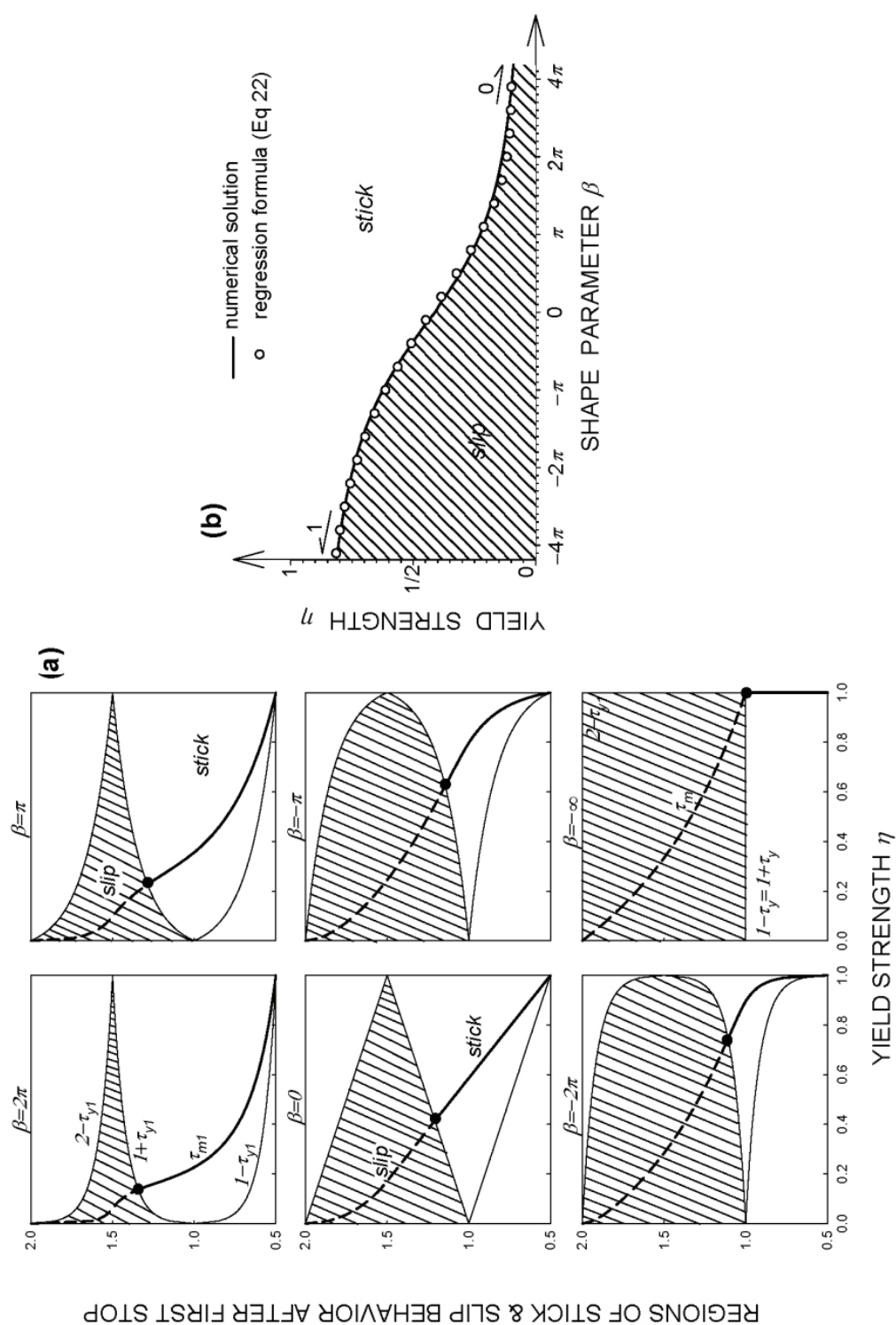


Figure 2.10 (a) Criterion for stick-slip behavior after first stop for a full-cycle pulse; (b) critical strength required to induce stick behavior.

Table 2.1 Solution for residual displacement under bilateral sliding to full-cycle excitation

sliding branch	behavior	τ_y	τ_m	u_m
1 st (<i>forward</i>)		τ_{y1} from Eq.(2.12)	τ_{m1} from Eq. (2.19)	u_{m1} from Eq. (2.18)
	stick	$\tau_{y2} = \tau_{y1} + 1$	$\bar{\tau}_{m2}$ from Eqs (2.7-2.8)	u_{m2} from Eq. (2.9)
			setting $\tau_y = \bar{\tau}_{y2} = \tau_{y2} - 1$	using $\bar{\tau}_{y2}$, $\bar{\tau}_{m2}$
2 nd (<i>reverse</i>)	slip $\bar{\tau}_{y2} \leq 1/2$	$\tau_{y2} = \tau_{m1}$	$\bar{\tau}_{m2}$ from Eqs (2.7-2.8)	u_{m2} from Eq. (2.9)
			setting $\tau_y = \bar{\tau}_{y2} = \tau_{y2} - 1$	using $\bar{\tau}_{y2}$, $\bar{\tau}_{m2}$
	slip $\bar{\tau}_{y2} > 1/2$	$\tau_{y2} = \tau_{m1}$	$\bar{\tau}_{m2}$ from Eq. (2.23)	u_{m2} from Eq. (2.24)
			setting $\tau_y = \bar{\tau}_{y2} = \tau_{y2} - 1$	using $\bar{\tau}_{y2}$, $\bar{\tau}_{m2}$
peak displacement = $u_m = u_{m1}$; residual displacement, $u_{res} = u_{m1} - u_{m2}$				

In light of the above, introducing the transformations $\bar{\tau}_{y2} \equiv \tau_{y2} - 1$ and $\bar{\tau}_{m2} \equiv \tau_{m2} - 1$ into the solutions for a half-cycle pulse, a simple procedure for obtaining reverse (u_{m2}) and residual (u_{res}) sliding displacements can be formulated as shown in Table 2.1. Note that according to this procedure u_{m2} is always calculated as a positive quantity.

In the special case of slip condition and when $\bar{\tau}_{y2} > 1/2$, the following explicit solution applies for time and displacement at second stop:

$$\bar{\tau}_{m2} = \bar{\tau}_{y2} + \frac{1}{\eta(1-e^\beta)} \left[1 + \frac{1}{2\beta} (1 - e^{2\beta(1-\bar{\tau}_{y2})}) - \bar{\tau}_{y2} \right] \quad (2.23)$$

$$\begin{aligned} \frac{u_{m2}}{A_g t_d^2} = & \frac{1}{4\beta^2(1-e^\beta)} \{ -2\beta^2[(1-\bar{\tau}_{y2})(1+\bar{\tau}_{y2}-2\bar{\tau}_{m2}) + \eta(\bar{\tau}_{m2}-\bar{\tau}_{y2})^2] + \\ & + 2\beta^2 e^{\beta(1+2\bar{\tau}_{y2})} \eta(\bar{\tau}_{m2}-\bar{\tau}_{y2})^2 + e^{2\beta(1-\bar{\tau}_{y2})} [1-2\beta(\bar{\tau}_{m2}-\bar{\tau}_{y2})] - 2\beta(1-\bar{\tau}_{m2}) - 1 \} \end{aligned} \quad (2.24)$$

It should be noticed that the above solutions are applicable to the case of asymmetric friction if different frictional strengths (η_1, η_2) are employed in forward and reverse sliding. These will also alter the criteria for “stick” and “slip” classification as will be explained in Section 2.6.

Figure 2.11 illustrates sliding response during motion reversal as described by Table 2.1. Dashed lines represent stick conditions.

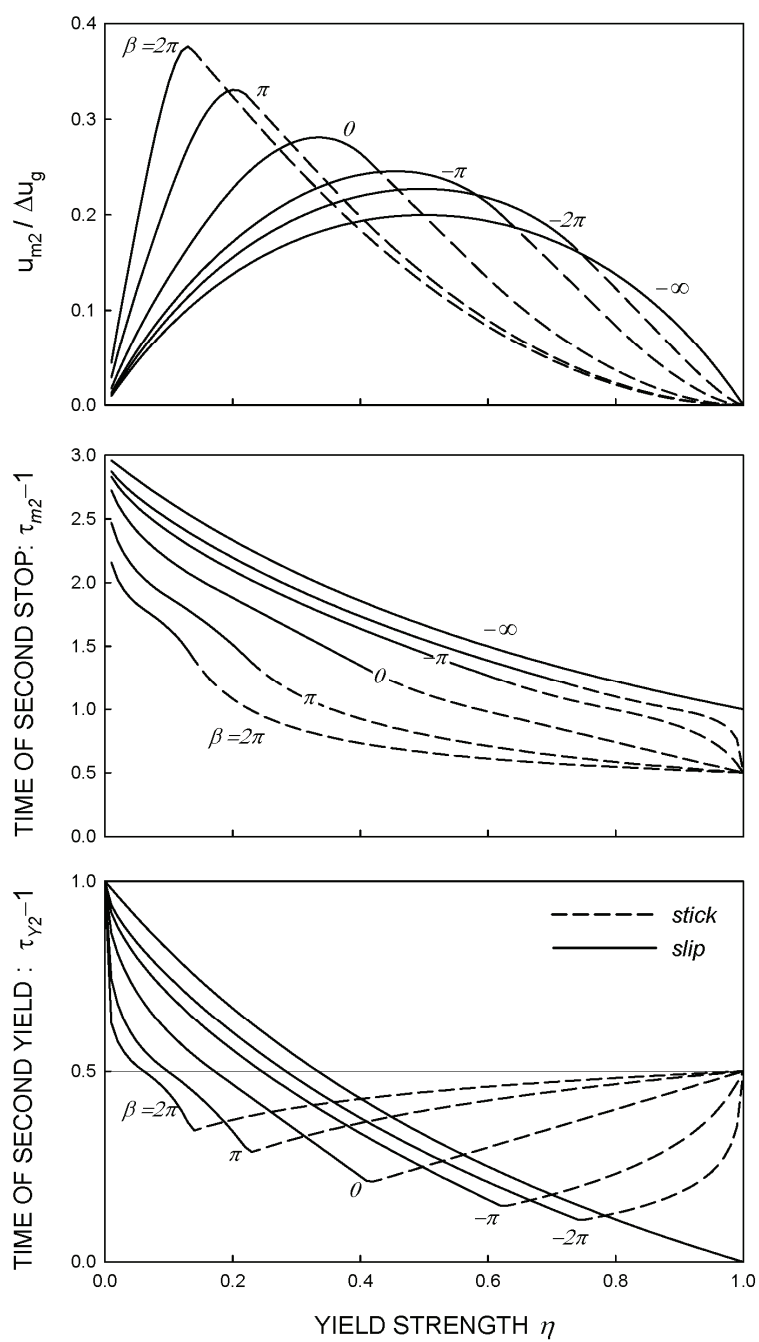


Figure 2.11 Normalized maximum response, time of maximum sliding response and time of yield during second slide (motion reversal).

Results for displacement at second stop u_{m2} , and residual displacement, $u_{res}=u_{m1}-u_{m2}$, evaluated according to the exact procedure of Table 2.1 are illustrated in Figs 2.12 and 2.13, normalized with peak displacement $u_m=u_{m1}$. As expected, the sum of displacement at second stop and residual displacement normalized with u_{m1} equals 1. Residual displacement is naturally smaller than u_m , thereby the plotted values are smaller than 1 and tend to decrease with increasing frictional strength. This is evident for slip conditions and relatively low friction, as shown in Fig 2.9d. An exception is the case of frictional strength close to the critical value in Eq.(2.22), where reverse sliding may be larger than forward, leading to negative residual displacements (Fig 2.9c). For stick conditions, u_{m1} is smaller than u_{m2} which generally leads to negative values in the graph. This is the case when τ_{m2} occurs during free sliding, as shown in Fig 2.9b. On the other hand, when τ_{m2} occurs during forced sliding, residual displacement is always zero as $u_{m1}=u_{m2}$ (Fig 2.9a).

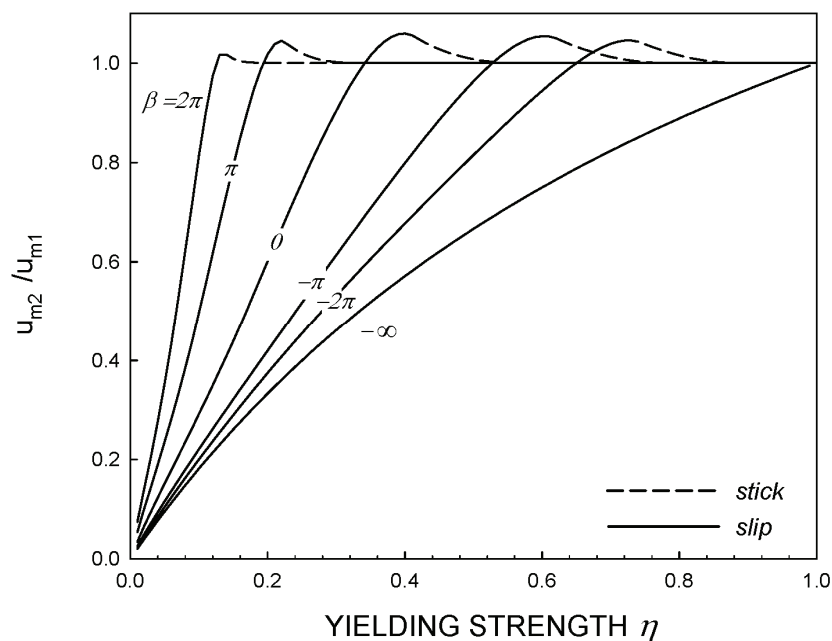


Figure 2.12 Comparison of blocks maximum displacement during second and first slide.

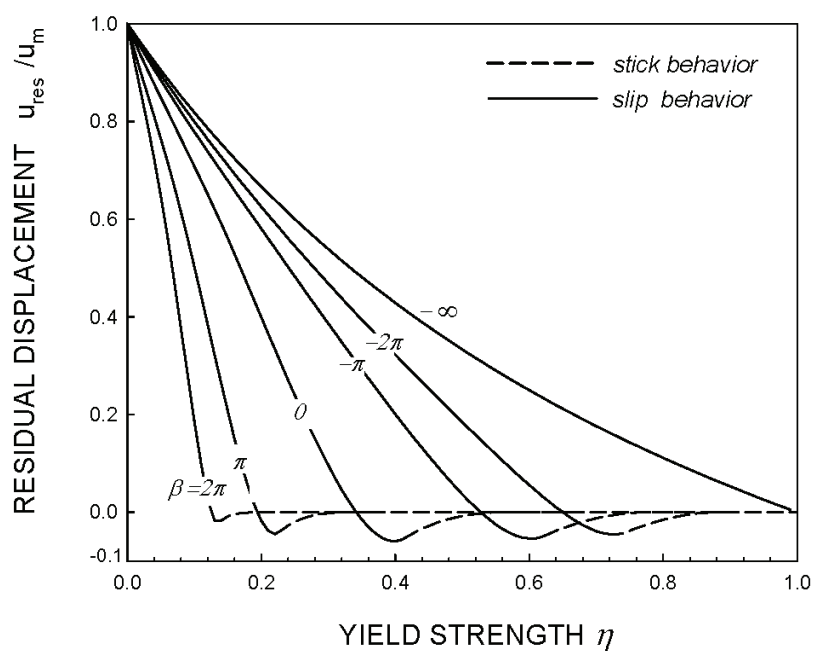


Figure 2.13 Residual (permanent) displacement for a full-cycle pulse as a fraction of maximum displacement. Note the progressively smaller residual displacements with increasing yield strength leading to negative and eventually zero values.

Additional explanation for the aforementioned trends for stick behaviour is provided in Fig 2.14, where peak sliding displacements for half- and full-cycle excitations are compared. Evidently, half-cycle pulses generate larger displacement than their full-cycle counterparts. The solutions converge with increasing yield strength and become identical beyond approximately $\eta = 0.36$ and 0.59 for $\beta = \pi$ and 0 , respectively. These limiting values are identical to those shown in Fig 2.5 and confirm that maximum displacement is obtained during the first half of the pulse.

As under moderately strong friction stick behaviour is compatible with that shown Fig 2.9b, u_{m2} is equal to peak sliding response to half-cycle excitation which is generally higher than u_{m1} to full-cycle excitation (i.e., points above white dot, Fig 2.14). The negative values observed in Fig 2.13 are, likewise, anticipated. As for larger frictional strengths stick behaviour is compatible to that in Fig 2.9a, the two modular responses are equal (i.e., points below white dot, Fig 2.14). Permanent displacement u_{res} is, accordingly, zero.

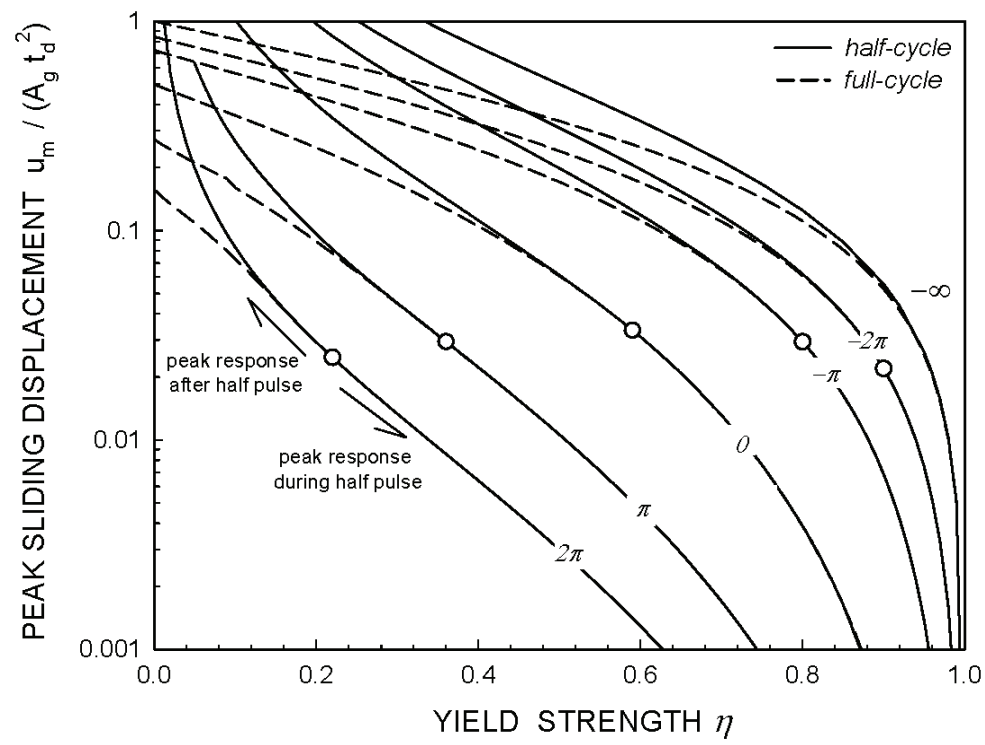


Figure 2.14 Comparison of peak sliding displacement for half- and full-cycle excitation.

2.5 Displacement Normalization by Dimensional Analysis

In addition to expressing results in a dimensionless form, a proper normalization scheme should be chosen to minimize scattering of the results (ideally leading to a single independent variable capturing the salient features of the solution). This is often achieved by means of dimensional analysis as explained in Barenblatt (1996) and Butterfield (1999). The approach is explored here for normalizing sliding displacement as an alternative to incremental ground displacement Δu_{gm} .

For the problem at hand a set of characteristic lengths D_c can be defined according to the expression:

$$D_c \propto A_g^a V_g^b D_g^c t_d^d \quad (2.25)$$

where A_g , V_g , D_g , t_d are the fundamental physical parameters of the pulse and a , b , c , d are pertinent dimensionless exponents. An advantage of this normalization over Δu_{gm} , is that D_c can be established *a priori*, for it is independent of the response and, thereby, does not vary with frictional strength.

Equating units of length and time between the left and right-hand sides of Eq. (2.25) yields the following pair of linear equations in four unknowns:

$$\begin{aligned} a + b + c &= 1 \\ 2a + b - d &= 0 \end{aligned} \quad (2.26a, b)$$

There are six possible formulations that can be produced from the solution of these equations depending on the parameters to be eliminated. For instance, eliminating parameters c and d

yields

$$\frac{D_c}{D_g} \propto \left(\frac{A_g t_d^2}{D_g} \right)^a \left(\frac{V_g t_d}{D_g} \right)^b \quad (2.27a)$$

which reveals the existence of three dimensionless groups associated with displacement response.

In the same spirit, eliminating constants b and d from Eqs (2.25–2.26), one obtains

$$\frac{D_c}{V_g t_d} \propto \left(\frac{A_g t_d}{V_g} \right)^a \left(\frac{D_g}{V_g t_d} \right)^c \quad (2.27b)$$

which reveals two additional dimensionless groups in the left- and right-hand side.

Repeating this procedure four times to eliminate the remaining pairs (a, b) , (a, c) , (a, d) , (b, c) , the following set of characteristic displacements is obtained:

$$D_c = A_g^a V_g^{d-2a} D_g^{1+a-d} t^d = D_g \left(\frac{A_g D_g}{V_g^2} \right)^a \left(\frac{V_g t}{D_g} \right)^d \quad (2.27c)$$

$$D_c = A_g^{1-b-c} V_g^b D_g^c t^{2-b-2c} = A_g t^2 \left(\frac{V_g}{A_g t} \right)^b \left(\frac{D_g}{A_g t^2} \right)^c \quad (2.27d)$$

$$D_c = A_g^{(d-b)/2} V_g^b D_g^{1-(d+b)/2} t^d = D_g \left(\frac{V_g}{A_g D_g} \right)^{b/2} \left(\frac{A_g t^2}{D_g} \right)^{d/2} \quad (2.27e)$$

$$D_c = A_g^{d+c-1} V_g^{2-d-2c} D_g^c t^d = \frac{V_g^2}{A_g} \left(\frac{A_g D_g}{V_g^2} \right)^c \left(\frac{A_g t}{V_g} \right)^d \quad (2.27f)$$

Evidently, depending on the values of constants a , b , c , d , an infinite number of normalization schemes is possible. Examples of the schemes generated by Eqs (2.27) are provided in Tables 2.2a to 2.2f for integer values of a , b , c , d in the range -4 to 4 . Highlighted cells refer to zero values of one (or more) constant.

The simplest set of characteristic displacements is obtained by setting both active powers in each of the above equations equal to zero or one, to get:

$$D_c = \left\{ A_g t_d^2 ; \frac{V_g^2}{A_g} ; D_g ; V_g t_d ; \frac{V_g D_g}{A_g t_d} ; \frac{A_g D_g t_d}{V_g} \right\} \quad (2.28a-f)$$

Of these normalization schemes, the first, $D_c = A_g t_d^2$, has been used by a number of researchers (Sarma 1975, Yegian *et al.* 1991), while the second, $D_c = V_g^2/A_g$ has been employed in the classical studies by Newmark (1965, 1971). The last four schemes have not been investigated in the past with reference to the particular problem. Note that the above normalizations are independent of waveform and, therefore, are applicable to any earthquake motion.

Table 2.2a Characteristic displacement D_c for different integer values of constants a , b . (Eq. 2.27a).

$$u = A^a V^b D^{1-a-b} t^{2a+b} = \left(\frac{At^2}{D} \right)^a \left(\frac{Vt}{D} \right)^b D \quad (\text{a})$$

$b \backslash a$	-4	-3	-2	-1	0	1	2	3	4
-4	$\frac{D^9}{A^4 V^4 t^{12}}$	$\frac{D^8}{A^3 V^4 t^{10}}$	$\frac{D^7}{A^2 V^4 t^8}$	$\frac{D^6}{A V^4 t^6}$	$\frac{D^5}{V^4 t^4}$	$\frac{AD^4}{V^4 t^2}$	$\frac{A^2 D^3}{V^4}$	$\frac{A^3 D^2 t^2}{V^4}$	$\frac{A^4 D t^4}{V^4}$
-3	$\frac{D^8}{A^4 V^3 t^{11}}$	$\frac{D^7}{A^3 V^3 t^9}$	$\frac{D^6}{A^2 V^3 t^7}$	$\frac{D^5}{A V^3 t^5}$	$\frac{D^4}{V^3 t^3}$	$\frac{AD^3}{V^3 t}$	$\frac{A^2 D^2 t}{V^3}$	$\frac{A^3 D t^3}{V^3}$	$\frac{A^4 t^5}{V^3}$
-2	$\frac{D^7}{A^4 V^2 t^{10}}$	$\frac{D^6}{A^3 V^2 t^8}$	$\frac{D^5}{A^2 V^2 t^6}$	$\frac{D^4}{A V^2 t^4}$	$\frac{D^3}{V^2 t^2}$	$\frac{AD^2}{V^2}$	$\frac{A^2 D t^2}{V^2}$	$\frac{A^3 t^4}{V^2}$	$\frac{A^4 t^6}{V^2 D}$
-1	$\frac{D^6}{A^4 V t^9}$	$\frac{D^5}{A^3 V t^7}$	$\frac{D^4}{A^2 V t^5}$	$\frac{D^3}{A V t^3}$	$\frac{D^2}{V t}$	$\frac{AD t}{V}$	$\frac{A^2 t^3}{V}$	$\frac{A^3 t^5}{VD}$	$\frac{A^4 t^7}{VD^2}$
0	$\frac{D^5}{A^4 t^8}$	$\frac{D^4}{A^3 t^6}$	$\frac{D^3}{A^2 t^4}$	$\frac{D^2}{A t^2}$	D	$A t^2$	$\frac{A^2 t^4}{D}$	$\frac{A^3 t^6}{D^2}$	$\frac{A^4 t^8}{D^3}$
1	$\frac{VD^4}{A^4 t^7}$	$\frac{VD^3}{A^3 t^5}$	$\frac{VD^2}{A^2 t^3}$	$\frac{VD}{A t}$	$V t$	$\frac{AV t^3}{D}$	$\frac{A^2 V t^5}{D^2}$	$\frac{A^3 V t^7}{D^3}$	$\frac{A^4 V t^9}{D^4}$
2	$\frac{V^2 D^3}{A^4 t^6}$	$\frac{V^2 D^2}{A^3 t^4}$	$\frac{V^2 D}{A^2 t^2}$	$\frac{V^2}{A}$	$\frac{V^2 t^2}{D}$	$\frac{AV^2 t^4}{D^2}$	$\frac{A^2 V^2 t^6}{D^3}$	$\frac{A^3 V^2 t^8}{D^4}$	$\frac{A^4 V^2 t^{10}}{D^5}$
3	$\frac{V^3 D^2}{A^4 t^5}$	$\frac{V^3 D}{A^3 t^3}$	$\frac{V^3}{A^2 t}$	$\frac{V^3 t}{AD}$	$\frac{V^3 t^3}{D^2}$	$\frac{AV^3 t^5}{D^3}$	$\frac{A^2 V^3 t^7}{D^4}$	$\frac{A^3 V^3 t^9}{D^5}$	$\frac{A^4 V^3 t^{11}}{D^6}$
4	$\frac{V^4 D}{A^4 t^4}$	$\frac{V^4}{A^3 t^2}$	$\frac{V^4}{A^2 D}$	$\frac{V^4 t^2}{AD^2}$	$\frac{V^4 t^4}{D^3}$	$\frac{AV^4 t^6}{D^4}$	$\frac{A^2 V^4 t^8}{D^5}$	$\frac{A^3 V^4 t^{10}}{D^6}$	$\frac{A^4 V^4 t^{12}}{D^7}$

Table 2.2b Characteristic displacement D_c for different integer values of constants a, c . (Eq. 2.27b).

$$u = A^a V^{1-a-c} D^c t^{1+a-c} = Vt \left(\frac{At}{V} \right)^a \left(\frac{D}{Vt} \right)^c \quad (\text{b})$$

$c \backslash a$	-4	-3	-2	-1	0	1	2	3	4
-4						$\frac{AV^4t^6}{D^4}$	$\frac{A^2V^3t^7}{D^4}$	$\frac{A^3V^2t^8}{D^4}$	$\frac{A^4Vt^9}{D^4}$
-3					$\frac{V^4t^4}{D^3}$	$\frac{AV^3t^5}{D^3}$	$\frac{A^2V^2t^6}{D^3}$	$\frac{A^3Vt^7}{D^3}$	$\frac{A^4t^8}{D^3}$
-2				$\frac{V^4t^2}{AD^2}$	$\frac{V^3t^3}{D^2}$	$\frac{AV^2t^4}{D^2}$	$\frac{A^2Vt^5}{D^2}$	$\frac{A^3t^6}{D^2}$	$\frac{A^4t^7}{VD^2}$
-1			$\frac{V^4}{A^2D}$	$\frac{V^3t}{AD}$	$\frac{V^2t^2}{D}$	$\frac{AVt^3}{D}$	$\frac{A^2t^4}{D}$	$\frac{A^3t^5}{VD}$	$\frac{A^4t^6}{V^2D}$
0		$\frac{V^4}{A^3t^2}$	$\frac{V^3}{A^2t}$	$\frac{V^2}{A}$	Vt	At^2	$\frac{A^2t^3}{V}$	$\frac{A^3t^4}{V^2}$	$\frac{A^4t^5}{V^3}$
1	$\frac{V^4D}{A^4t^4}$	$\frac{V^3D}{A^3t^3}$	$\frac{V^2D}{A^2t^2}$	$\frac{VD}{At}$	D	$\frac{ADt}{V}$	$\frac{A^2Dt^2}{V^2}$	$\frac{A^3Dt^3}{V^3}$	$\frac{A^4Dt^4}{V^4}$
2	$\frac{V^3D^2}{A^4t^5}$	$\frac{V^2D^2}{A^3t^4}$	$\frac{VD^2}{A^2t^3}$	$\frac{D^2}{At^2}$	$\frac{D^2}{Vt}$	$\frac{AD^2}{V^2}$	$\frac{A^2D^2t}{V^3}$	$\frac{A^3D^2t^2}{V^4}$	
3	$\frac{V^2D^3}{A^4t^6}$	$\frac{VD^3}{A^3t^5}$	$\frac{D^3}{A^2t^4}$	$\frac{D^3}{AVt^3}$	$\frac{D^3}{V^2t^2}$	$\frac{AD^3}{V^3t}$	$\frac{A^2D^3}{V^4}$		
4	$\frac{VD^4}{A^4t^7}$	$\frac{D^4}{A^3t^6}$	$\frac{D^4}{A^2Vt^5}$	$\frac{D^4}{AV^2t^4}$	$\frac{D^4}{V^3t^3}$	$\frac{AD^4}{V^4t^2}$			

Table 2.2c Characteristic displacement D_c for different integer values of constants a, c . (Eq. 2.27c).

$$u = A^a V^{d-2a} D^{1+a-d} t^d = D \left(\frac{AD}{V^2} \right)^a \left(\frac{Vt}{D} \right)^d \quad (c)$$

$d \backslash a$	-4	-3	-2	-1	0	1	2	3	4
-4	$\frac{V^4 D}{A^4 t^4}$	$\frac{V^2 D^2}{A^3 t^4}$	$\frac{D^3}{A^2 t^4}$	$\frac{D^4}{AV^2 t^4}$					
-3		$\frac{V^3 D}{A^3 t^3}$	$\frac{VD^2}{A^2 t^3}$	$\frac{D^3}{AV t^3}$	$\frac{D^4}{V^3 t^3}$				
-2		$\frac{V^4}{A^3 t^2}$	$\frac{V^2 D}{A^2 t^2}$	$\frac{D^2}{At^2}$	$\frac{D^3}{V^2 t^2}$	$\frac{AD^4}{V^4 t^2}$			
-1			$\frac{V^3}{A^2 t}$	$\frac{VD}{At}$	$\frac{D^2}{Vt}$	$\frac{AD^3}{V^3 t}$			
0			$\frac{V^4}{A^2 D}$	$\frac{V^2}{A}$	D	$\frac{AD^2}{V^2}$	$\frac{A^2 D^3}{V^4}$		
1				$\frac{V^3 t}{AD}$	Vt	$\frac{ADt}{V}$	$\frac{A^2 D^2 t}{V^3}$		
2				$\frac{V^4 t^2}{AD^2}$	$\frac{V^2 t^2}{D}$	At^2	$\frac{A^2 Dt^2}{V^2}$	$\frac{A^3 D^2 t^2}{V^4}$	
3					$\frac{V^3 t^3}{D^2}$	$\frac{AV t^3}{D}$	$\frac{A^2 t^3}{V}$	$\frac{A^3 Dt^3}{V^3}$	
4					$\frac{V^4 t^4}{D^3}$	$\frac{AV^2 t^4}{D^2}$	$\frac{A^2 t^4}{D}$	$\frac{A^3 t^4}{V^2}$	$\frac{A^4 Dt^4}{V^4}$

Table 2.2d Characteristic displacement D_c for different integer values of constants a, c . (Eq. 2.27d).

$$u = A^{1-b-c} V^b D^c t^{2-b-2c} = At^2 \left(\frac{V}{At} \right)^b \left(\frac{D}{At^2} \right)^c$$

(d)

$c \backslash b$	-4	-3	-2	-1	0	1	2	3	4
-4						$\frac{A^4 V t^9}{D^4}$	$\frac{A^3 V^2 t^8}{D^4}$	$\frac{A^2 V^3 t^7}{D^4}$	$\frac{A V^4 t^6}{D^4}$
-3					$\frac{A^4 t^8}{D^3}$	$\frac{A^3 V t^7}{D^3}$	$\frac{A^2 V^2 t^6}{D^3}$	$\frac{A V^3 t^5}{D^3}$	$\frac{V^4 t^4}{D^3}$
-2				$\frac{A^4 t^7}{V D^2}$	$\frac{A^3 t^6}{D^2}$	$\frac{A^2 V t^5}{D^2}$	$\frac{A V^2 t^4}{D^2}$	$\frac{V^3 t^3}{D^2}$	$\frac{V^4 t^2}{A D^2}$
-1			$\frac{A^4 t^6}{V^2 D}$	$\frac{A^3 t^5}{V D}$	$\frac{A^2 t^4}{D}$	$\frac{A V t^3}{D}$	$\frac{V^2 t^2}{D}$	$\frac{V^3 t}{A D}$	$\frac{V^4}{A^2 D}$
0		$\frac{A^4 t^5}{V^3}$	$\frac{A^3 t^4}{V^2}$	$\frac{A^2 t^3}{V}$	$A t^2$	$V t$	$\frac{V^2}{A}$	$\frac{V^3}{A^2 t}$	$\frac{V^4}{A^3 t^2}$
1	$\frac{A^4 D t^4}{V^4}$	$\frac{A^3 D t^3}{V^3}$	$\frac{A^2 D t^2}{V^2}$	$\frac{A D t}{V}$	D	$\frac{V D}{A t}$	$\frac{V^2 D}{A^2 t^2}$	$\frac{V^3 D}{A^3 t^3}$	$\frac{V^4 D}{A^4 t^4}$
2	$\frac{A^3 D^2 t^2}{V^4}$	$\frac{A^2 D^2 t}{V^3}$	$\frac{A D^2}{V^2}$	$\frac{D^2}{V t}$	$\frac{D^2}{A t^2}$	$\frac{V D^2}{A^2 t^3}$	$\frac{V^2 D^2}{A^3 t^4}$	$\frac{V^3 D^2}{A^4 t^5}$	
3	$\frac{A^2 D^3}{V^4}$	$\frac{A D^3}{V^3 t}$	$\frac{D^3}{V^2 t^2}$	$\frac{D^3}{A V t^3}$	$\frac{D^3}{A^2 t^4}$	$\frac{V D^3}{A^3 t^5}$	$\frac{V^2 D^3}{A^4 t^6}$		
4	$\frac{A D^4}{V^4 t^2}$	$\frac{D^4}{V^3 t^3}$	$\frac{D^4}{A V^2 t^4}$	$\frac{D^4}{A^2 V t^5}$	$\frac{D^4}{A^3 t^6}$	$\frac{V D^4}{A^4 t^7}$			

Table 2.2e Characteristic displacement D_c for different integer values of constants a, c . (Eq. 2.27e).

$$u = A^{(d-b)/2} V^b D^{1-(d+b)/2} t^d = D \left(\frac{V^2}{AD} \right)^{b/2} \left(\frac{At^2}{D} \right)^{d/2} \quad (e)$$

$d \backslash b$	-4	-3	-2	-1	0	1	2	3	4
-4	$\frac{D^5}{V^4 t^4}$		$\frac{D^4}{AV^2 t^4}$		$\frac{D^3}{A^2 t^4}$		$\frac{V^2 D^2}{A^3 t^4}$		$\frac{V^4 D}{A^4 t^4}$
-3		$\frac{D^4}{V^3 t^3}$		$\frac{D^3}{AV t^3}$		$\frac{VD^2}{A^2 t^3}$		$\frac{V^3 D}{A^3 t^3}$	
-2	$\frac{AD^4}{V^4 t^2}$		$\frac{D^3}{V^2 t^2}$		$\frac{D^2}{At^2}$		$\frac{V^2 D}{A^2 t^2}$		$\frac{V^4}{A^3 t^2}$
-1		$\frac{AD^3}{V^3 t}$		$\frac{D^2}{Vt}$		$\frac{VD}{At}$		$\frac{V^3}{A^2 t}$	
0	$\frac{A^2 D^3}{V^4}$		$\frac{AD^2}{V^2}$		D		$\frac{V^2}{A}$		$\frac{V^4}{A^2 D}$
1		$\frac{A^2 D^2 t}{V^3}$		$\frac{ADt}{V}$		Vt		$\frac{V^3 t}{AD}$	
2	$\frac{A^3 D^2 t^2}{V^4}$		$\frac{A^2 D t^2}{V^2}$		At^2		$\frac{V^2 t^2}{D}$		$\frac{V^4 t^2}{AD^2}$
3		$\frac{A^3 D t^3}{V^3}$		$\frac{A^2 t^3}{V}$		$\frac{AV t^3}{D}$		$\frac{V^3 t^3}{D^2}$	
4	$\frac{A^4 D t^4}{V^4}$		$\frac{A^3 t^4}{V^2}$		$\frac{A^2 t^4}{D}$		$\frac{AV^2 t^4}{D^2}$		$\frac{V^4 t^4}{D^3}$

Table 2.2f Characteristic displacement D_c for different integer values of constants a, c . (Eq. 2.27f).

$$u = A^{d+c-1} V^{2-d-2c} D^c t^d = \frac{V^2}{A} \left(\frac{AD}{V^2} \right)^c \left(\frac{At}{V} \right)^d \quad (f)$$

$d \backslash c$	-4	-3	-2	-1	0	1	2	3	4
-4						$\frac{V^4 D}{A^4 t^4}$	$\frac{V^2 D^2}{A^3 t^4}$	$\frac{D^3}{A^2 t^4}$	$\frac{D^4}{AV^2 t^4}$
-3						$\frac{V^3 D}{A^3 t^3}$	$\frac{VD^2}{A^2 t^3}$	$\frac{D^3}{AV t^3}$	$\frac{D^4}{V^3 t^3}$
-2					$\frac{V^4}{A^3 t^2}$	$\frac{V^2 D}{A^2 t^2}$	$\frac{D^2}{At^2}$	$\frac{D^3}{V^2 t^2}$	$\frac{AD^4}{V^4 t^2}$
-1					$\frac{V^3}{A^2 t}$	$\frac{VD}{At}$	$\frac{D^2}{Vt}$	$\frac{AD^3}{V^3 t}$	
0				$\frac{V^4}{A^2 D}$	$\frac{V^2}{A}$	D	$\frac{AD^2}{V^2}$	$\frac{A^2 D^3}{V^4}$	
1				$\frac{V^3 t}{AD}$	$\frac{Vt}{D}$	$\frac{ADt}{V}$	$\frac{A^2 D^2 t}{V^3}$		
2			$\frac{V^4 t^2}{AD^2}$	$\frac{V^2 t^2}{D}$	At^2	$\frac{A^2 D t^2}{V^2}$	$\frac{A^3 D^2 t^2}{V^4}$		
3			$\frac{V^3 t^3}{D^2}$	$\frac{AV t^3}{D}$	$\frac{A^2 t^3}{V}$	$\frac{A^3 D t^3}{V^3}$			
4		$\frac{V^4 t^4}{D^3}$	$\frac{AV^2 t^4}{D^2}$	$\frac{A^2 t^4}{D}$	$\frac{A^3 t^4}{V^2}$	$\frac{A^4 D t^4}{V^4}$			

Focusing on the pulses at hand, it can be shown by integration of Eqs (2.1) and (2.2) that the six characteristic displacements are interconnected through the relations:

$$V_g t_d = n D_g \quad , \quad A_g t_d^2 = n \frac{A_g D_g t_d}{V_g} \quad , \quad \frac{V_g^2}{A_g} = n \frac{V_g D_g}{A_g t_d} \quad (2.29\text{a-c})$$

where $n = 1$ for full-cycle and $n = 2$ for half-cycle conditions. Evidently, the six normalization schemes in Eq. (2.28) can be reduced to three through Eqs (2.29). These properties are highlighted in Table 2.3.

Table 2.3 Six basic normalization schemes of displacement for the generalized pulse of half- and full-cycle duration in Eqs (2.1) and(2.2).

D_c	<i>Half – Cycle</i>	<i>Full – Cycle</i>
D_g	$\frac{A_g t_d^2}{2} \left[\frac{1 - e^\beta + \beta}{\beta(1 - e^\beta)} \right]$	$A_g t_d^2 \left[\frac{1 - e^\beta + \beta}{\beta(1 - e^\beta)} \right]$
$V_g t_d$	$A_g t_d^2 \left[\frac{1 - e^\beta + \beta}{\beta(1 - e^\beta)} \right]$	$A_g t_d^2 \left[\frac{1 - e^\beta + \beta}{\beta(1 - e^\beta)} \right]$
$A_g t_d^2$	$A_g t_d^2$	$A_g t_d^2$
$\frac{V_g^2}{A_g}$	$A_g t_d^2 \left[\frac{1 - e^\beta + \beta}{\beta(1 - e^\beta)} \right]^2$	$A_g t_d^2 \left[\frac{1 - e^\beta + \beta}{\beta(1 - e^\beta)} \right]^2$
$\frac{V_g D_g}{A_g t_d}$	$\frac{A_g t_d^2}{2} \left[\frac{1 - e^\beta + \beta}{\beta(1 - e^\beta)} \right]^2$	$A_g t_d^2 \left[\frac{1 - e^\beta + \beta}{\beta(1 - e^\beta)} \right]^2$
$\frac{A_g D_g t_d}{V_g}$	$\frac{A_g t_d^2}{2}$	$A_g t_d^2$

Displacement response normalized according to the above parameters is presented in Figs 2.15 and 2.16, in comparison to that obtained based on incremental displacement Δu_{gm} . The advantage of using Δu_{gm} over the other schemes becomes apparent in the graphs, as the specific parameter yields bounded displacements for both half- and full-cycle excitations (anchored at the simple values $\frac{1}{2}$ and 1 in the limit $\eta = 0$) and provides the smallest dispersion in the results. Evidently, naïve dimensional analysis such as the one discussed above does not appear superior to physically motivated choices for establishing normalization schemes for the problem at hand.

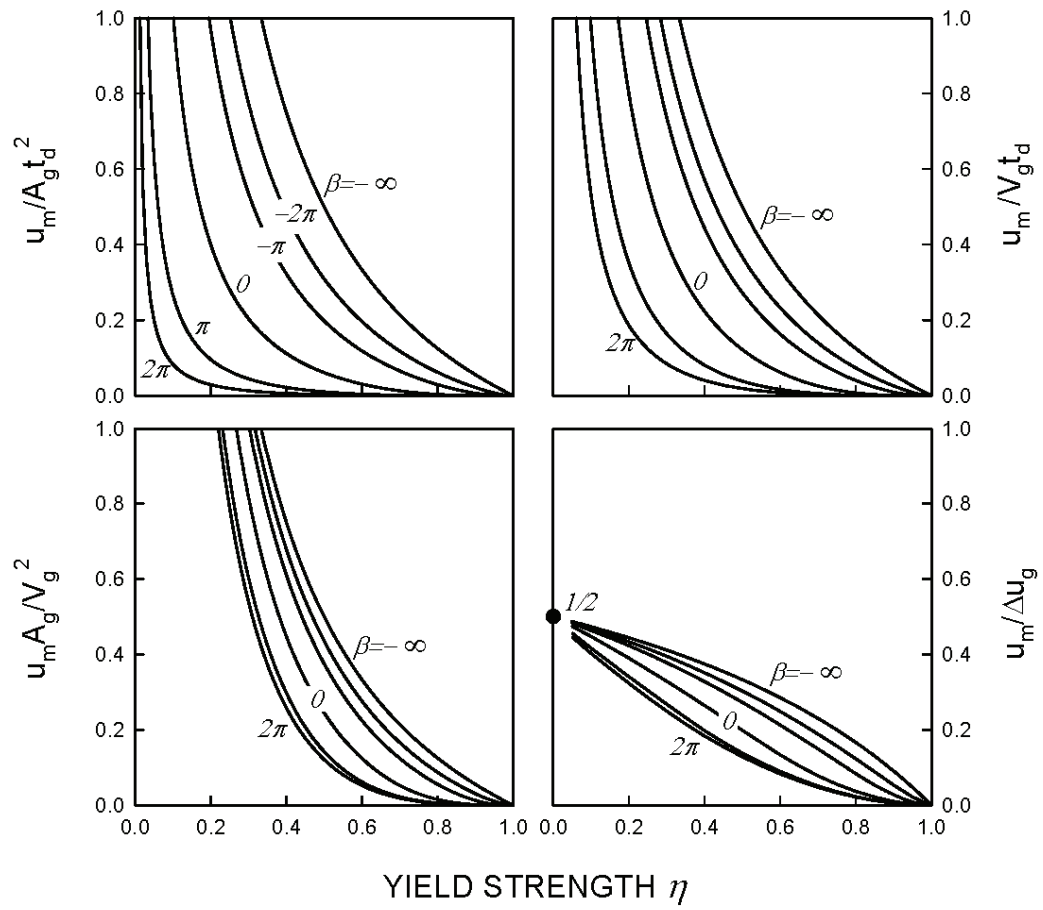


Figure 2.15 Alternative normalizations of sliding displacement for half-cycle excitation.

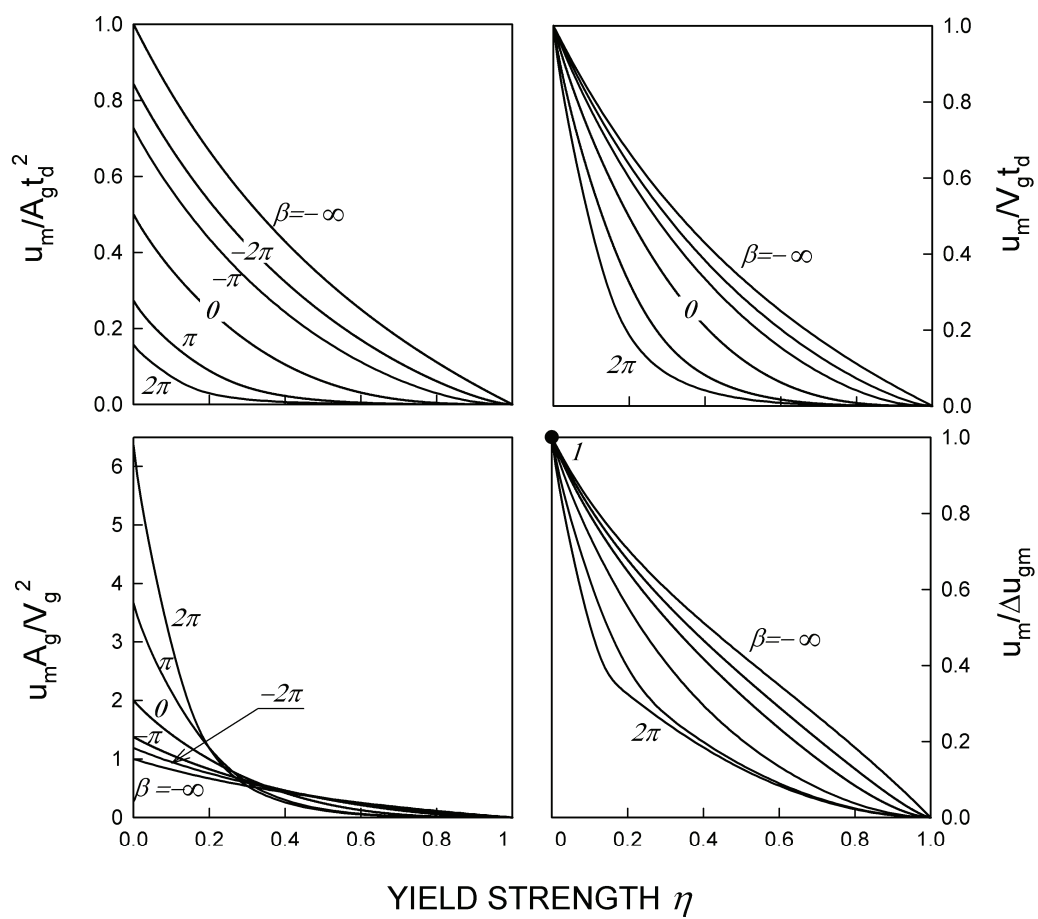


Figure 2.16 Alternative normalizations of sliding displacement for full-cycle excitation.

Using non-linear regression analysis, the following simplified expression was developed to approximate peak sliding response for a full-cycle pulse

$$u_m / D_g = u_m / V_g t_d = (1 - \eta)^f \quad (2.30)$$

where $f = -9 + 12e^{\beta/20}$, for $\beta > 0$ and $f = (3 - 0.9\beta)/(1 - 0.6\beta)$, for $\beta < 0$. Both functions are in excellent agreement with the exact solution (Fig 2.17) exhibiting R^2 values of 0.99 or higher.

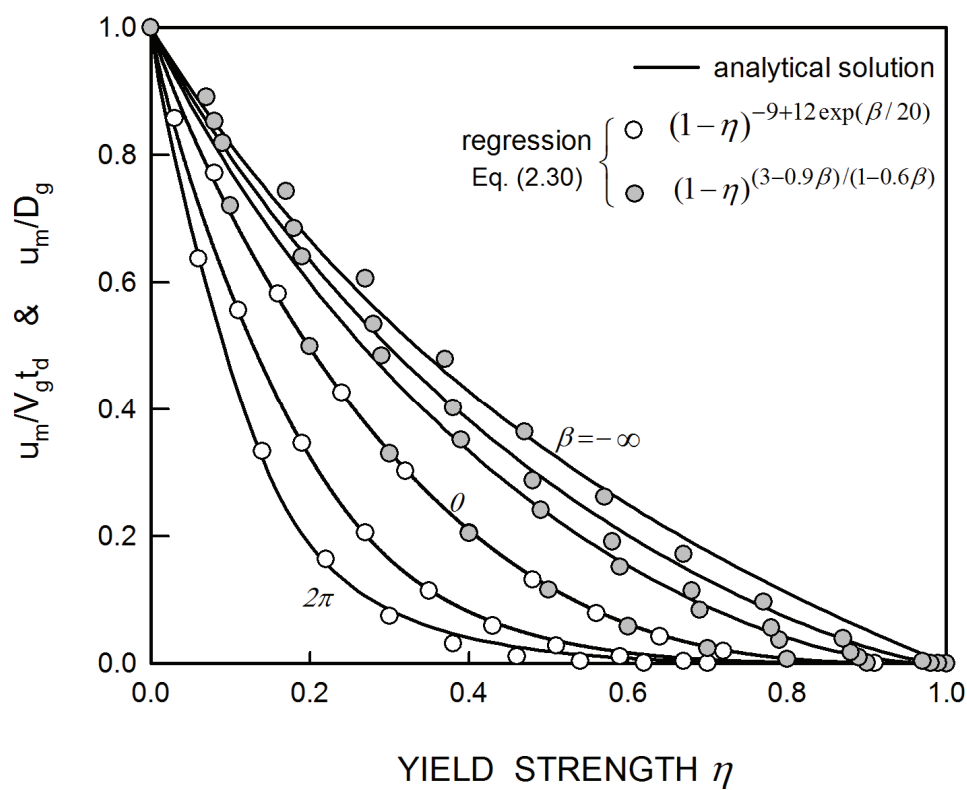


Figure 2.17 Comparison of peak sliding displacement predicted by the regression formula in Eq. (2.30) against the exact solution for a full – cycle excitation pulse.

2.6 Asymmetric Friction

Asymmetric friction provides a convenient way to model sliding on an inclined plane under the combined action of gravitational and inertial acceleration, as shown in Fig 2.1 (lower graph). For plane inclination λ , frictional coefficient μ , and pulse inclination ξ with respect to the horizontal plane (Fig 2.1), the frictional strength for sliding in the left and right direction is, respectively

$$\eta_{1,2} = \eta \frac{\cos \lambda}{\cos(\lambda - \xi)} \left[\frac{1 \mp \mu^{-1} \tan \lambda}{1 \pm \mu \tan(\lambda - \xi)} \right] \quad (2.31)$$

η being the frictional resistance on horizontal plane. In this equation, the upper sign corresponds to downward sliding and vice versa. Positive slope inclination λ is indicated in Fig 2.18. In the special case of a pulse acting parallel to the inclined plane ($\xi = \lambda$), Eq. (2.31) simplifies to $\eta_{1,2} = \eta (\cos \lambda \mp \mu^{-1} \sin \lambda)$ (Garini *et al.* 2011, Ray & Hutchinson 2006); when excitation is acting horizontally ($\xi = 0$), $\eta_{1,2} = \eta (1 \mp \mu^{-1} \tan \lambda) / (1 \pm \mu \tan \lambda)$. Finally, for $\xi = \lambda = 0$ both expressions yield $\eta_{1,2} = \eta$.

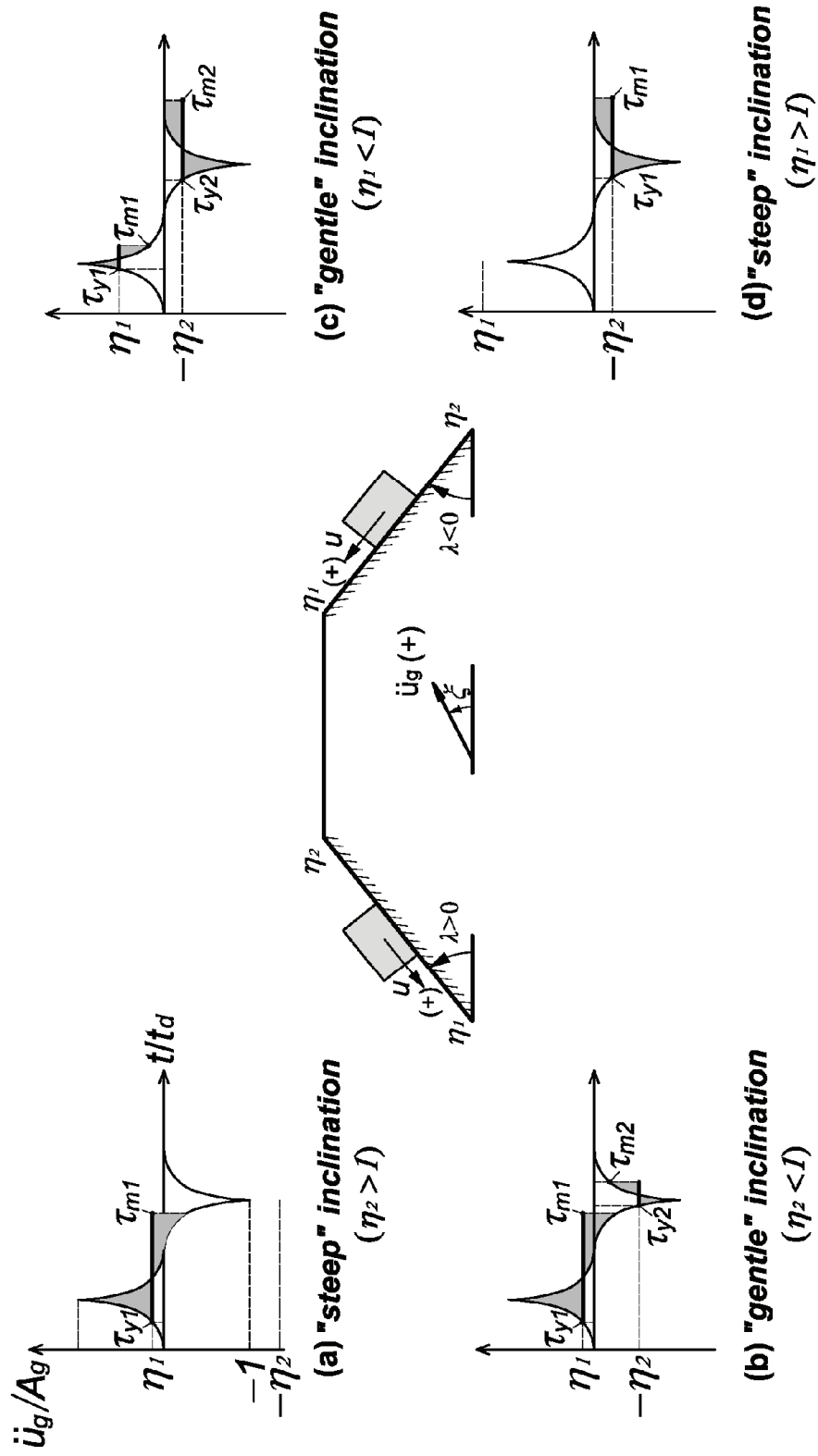


Figure 2.18 Schematic representation of sliding to full cycle excitation under asymmetric friction.

For the pair of sliding blocks subjected to full-cycle excitation under asymmetric friction, four distinct cases are possible, as shown in Fig 2.18:

(A) when frictional resistance in one direction exceeds peak ground acceleration (i.e. when either η_1 or η_2 is greater than 1, sliding can only be unilateral. This corresponds to the case of steep inclinations in Figs 2.18a and 2.18d

(B) when frictional resistance in both directions is smaller than 1, sliding will be bilateral. This corresponds to the case of gentle inclinations in Figs 2.18b and 2.18c.

For the last two cases, sliding conditions may be either stick or slip depending on pulse shape and frictional strength. The modified stick-slip criterion is:

$$\text{stick } (\tau_{y2} = 1 + \tau_y(\eta_2)), \text{ when } 1 - \tau_{y1} \leq \tau_{m1} \leq \tau_{y2} \quad (2.32a)$$

$$\text{slip } (\tau_{y2} = \tau_{m1}), \text{ when } \tau_{y2} < \tau_{m1} < 1 - \tau_{y2} \quad (2.32b)$$

In light of the above, the analysis described earlier can be adjusted for conditions of asymmetric friction. The modified procedure is summarized in Table 2.4 for both peak and residual displacements.

Table 2.4 Determination of sliding response to full-cycle excitation under asymmetric friction.

sliding branch	behavior	η	τ_y	τ_m	u_m
1st (<i>forward</i>)		η_1	τ_{y1} from Eq. (2.12)	τ_{m1} from Eq. (2.19)	u_{m1} from Eq. (2.18)
	Stick	η_2	$\tau_{y2} = 1 + \frac{1}{2\beta} \ln[1 - \eta_2(1 - e^\beta)]$	$\bar{\tau}_{m2}$ from Eqs (2.7-2.8) setting $\tau_y = \bar{\tau}_{y2} = \tau_{y2} - 1$	u_{m2} from Eq. (2.9) using $\bar{\tau}_{y2}, \bar{\tau}_{m2}$
2nd (<i>reverse</i>)	Slip $\bar{\tau}_{y2} \leq 1/2$	η_2	$\tau_{y2} = \tau_{m1}$	$\bar{\tau}_{m2}$ from Eqs (2.7-2.8) setting $\tau_y = \bar{\tau}_{y2} = \tau_{y2} - 1$	u_{m2} from Eq. (2.9) using $\bar{\tau}_{y2}, \bar{\tau}_{m2}$
	slip $\bar{\tau}_{y2} > 1/2$	η_2	$\tau_{y2} = \tau_{m1}$	$\bar{\tau}_{m2}$ from Eq. (2.23) setting $\tau_y = \bar{\tau}_{y2} = \tau_{y2} - 1$	u_{m2} from Eq. (2.24) using $\bar{\tau}_{y2}, \bar{\tau}_{m2}$
in all cases: peak displacement $u_m = u_{m1}$; residual displacement $u_{res} = u_{m1} - u_{m2}$					

* $\bar{\tau} = \tau - 1$ for all subscripts.

With reference to the effect of number of cycles, it should be noticed that whereas sliding to half-cycle excitation can only be unilateral, response to full-cycle excitation can be either unilateral or bilateral depending on the nature of frictional resistance (symmetric or asymmetric; Fig 2.18). In case of asymmetric friction, if sliding does not initiate during the first ascending branch of the pulse, the solution to a half-cycle pulse is applicable to a full-cycle pulse. This suggests that half-cycle pulses should be viewed as being more than fictitious excitations adopted for the sake of analytical simplicity. Accordingly, the finite ground velocity at the end of a half-cycle excitation should not be viewed as a physical limitation for using the pulse in engineering applications.

2.7 Static vs Dynamic Friction

The behavior of friction has been extensively examined during the past century (Olsson *et al.* 1998). A number of frictional models has been proposed by engineers and physicists describing the variation of friction with time, displacement (Rabinowicz 1951, Dahl 1968) or velocity (Stribeck 1902, Hess & Soom 1990), considering the physical mechanisms driving this highly nonlinear phenomenon. The use of these models lies beyond the scope of this chapter.

In the foregoing, the frictional coefficient was assumed to be of the Coulomb type that is, constant dry friction. A more realistic assumption would be to distinguish static from kinetic (dynamic) friction (Bowden & Tabor, 1954) which occurs when two objects are moving relative to each other and rub together. Static friction, that is friction between two solid objects that are not moving relative to each other, prevents the block from sliding and controls the initiation of slippage. After yielding, the block slides relative to the base under kinetic friction. The coefficient of static friction (η^*) is typically higher than the coefficient of kinetic friction (η), even if by a small amount. A simple way to incorporate this variation in the present analysis is shown in Fig 2.19: the effect of static friction $\eta^* > \eta$ on the response manifests itself in the removal of a triangular like area under the acceleration diagram, which leads to a drop in the peak velocity of the block.

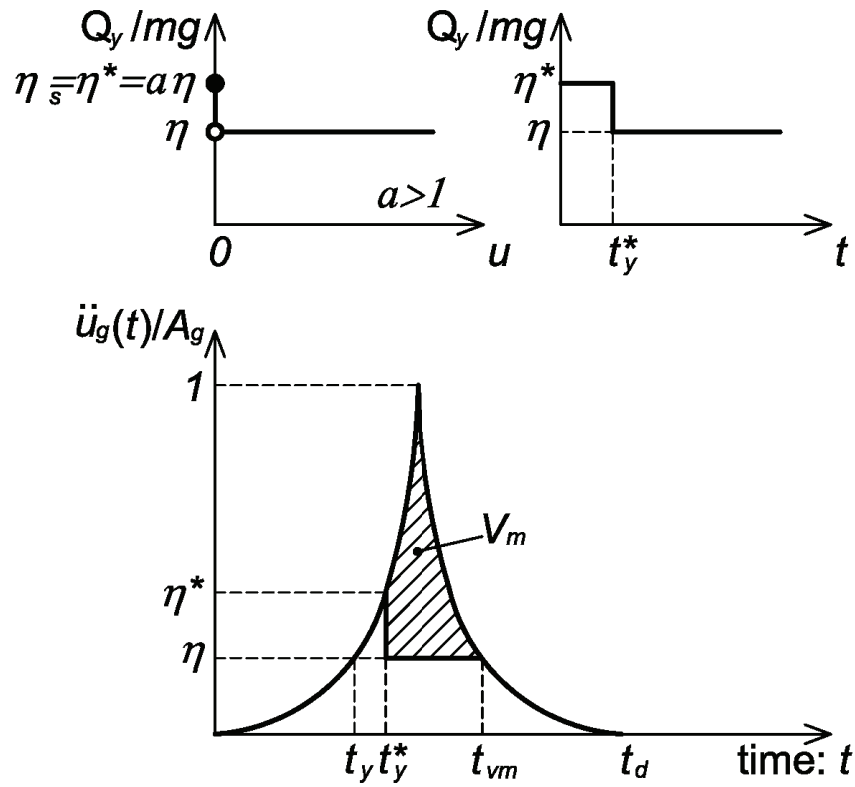


Figure 2.19 Variation of normalized frictional coefficient in terms of (a) displacement, (b) time and (c) time of yielding and maximum sliding velocity of a sliding block under an exponential pulse considering this variation.

In light of the above, normalized frictional coefficient can be expressed as.

$$\eta(t) = \begin{cases} a \cdot \eta, & t \leq t_y \\ \eta, & t_y \leq t \leq t_m \end{cases} \quad (2.33)$$

where a is dimensional coefficient having values greater than one.

The modified time of yielding then becomes

$$\tau_y^* = \frac{1}{2\beta} \ln[1 - a \cdot \eta(1 - e^\beta)] \quad (2.34)$$

which differs from the one in Eq (2.12) in the presence of the a multiplier in η .

Accordingly the maximum velocity response is now:

$$\frac{V_m}{V_g} = \frac{1}{e^\beta - \beta - 1} \left[\frac{-e^{2\beta\tau_y^*} - e^{2\beta\tau_y}}{2} + \beta(\eta - 1)(1 - \tau_y^* - \tau_y) + e^\beta(1 - \beta\eta + \beta\eta\tau_y^* + \beta\eta\tau_y) \right] \quad (2.35)$$

Eqs (2.34) and (2.35) are graphically presented in Fig 2.20.

Simply by substituting τ_y^* in the equations for constant friction we can determine the time of maximum sliding, or first stop, and the corresponding block displacement. For the case of a half – cycle generalized exponential pulse and values of the shape factor $\beta = -\infty, -\pi, 0, \pi$, the variation of maximum response with yield strength is graphically shown in Figs 2.21a-d. Equivalent illustrations for the full-cycle case are provided in Figs 2.22a-c. Evidently the effect of kinetic friction doesn't dramatically affect sliding behavior and can be neglected in engineering analysis of this kind.

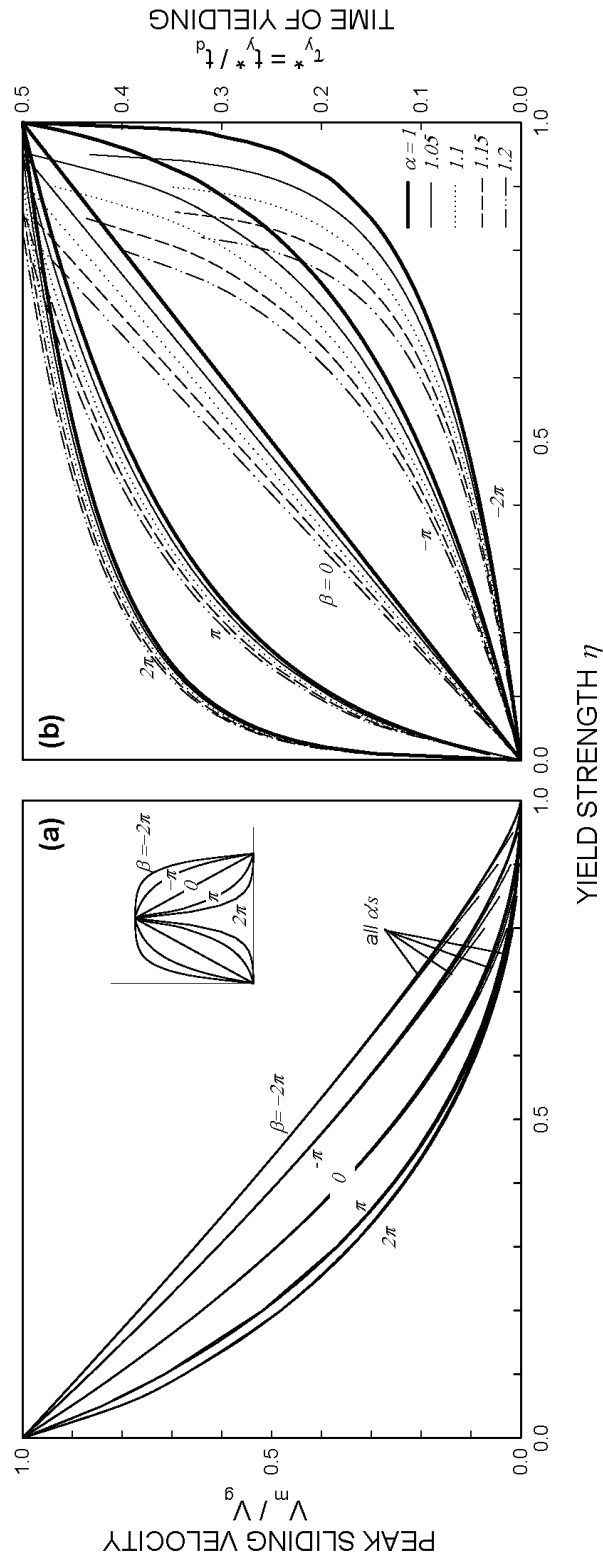


Figure 2.20. Normalized peak sliding velocity and time of yield with time of yield for a rigid block on a frictional plane under a generalized exponential pulse for varying friction conditions ($\eta^* = a\eta$, $a=1, 1.05, 1.1, 1.15, 1.2$).

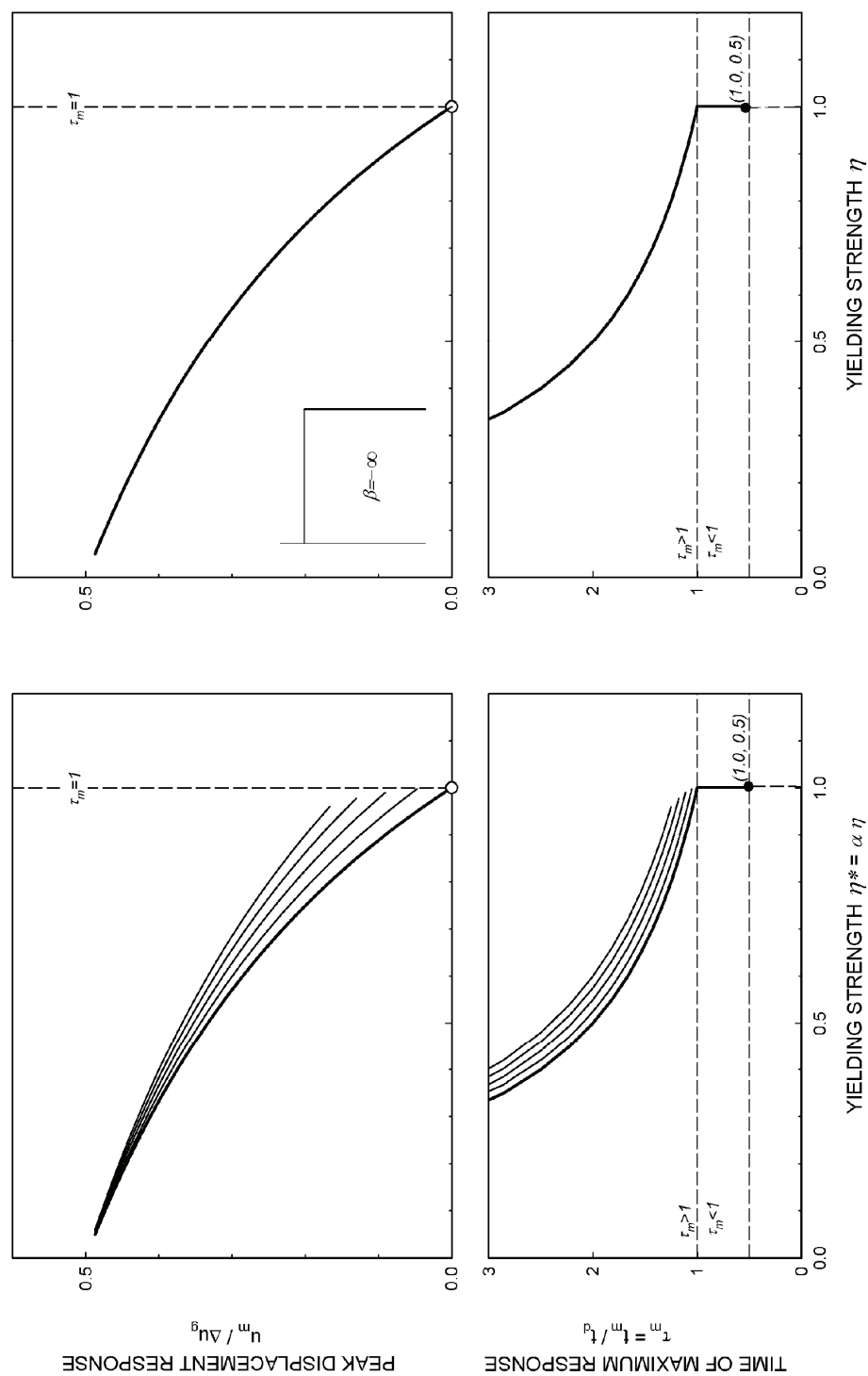


Figure 2.21a Normalized displacement response with yield strength for a rigid block subject to a rectangular acceleration pulse ($\beta = -\infty$) under varying friction conditions ($\eta^* = \alpha \eta$, $\alpha = 1, 1.05, 1.1, 1.15, 1.2$)

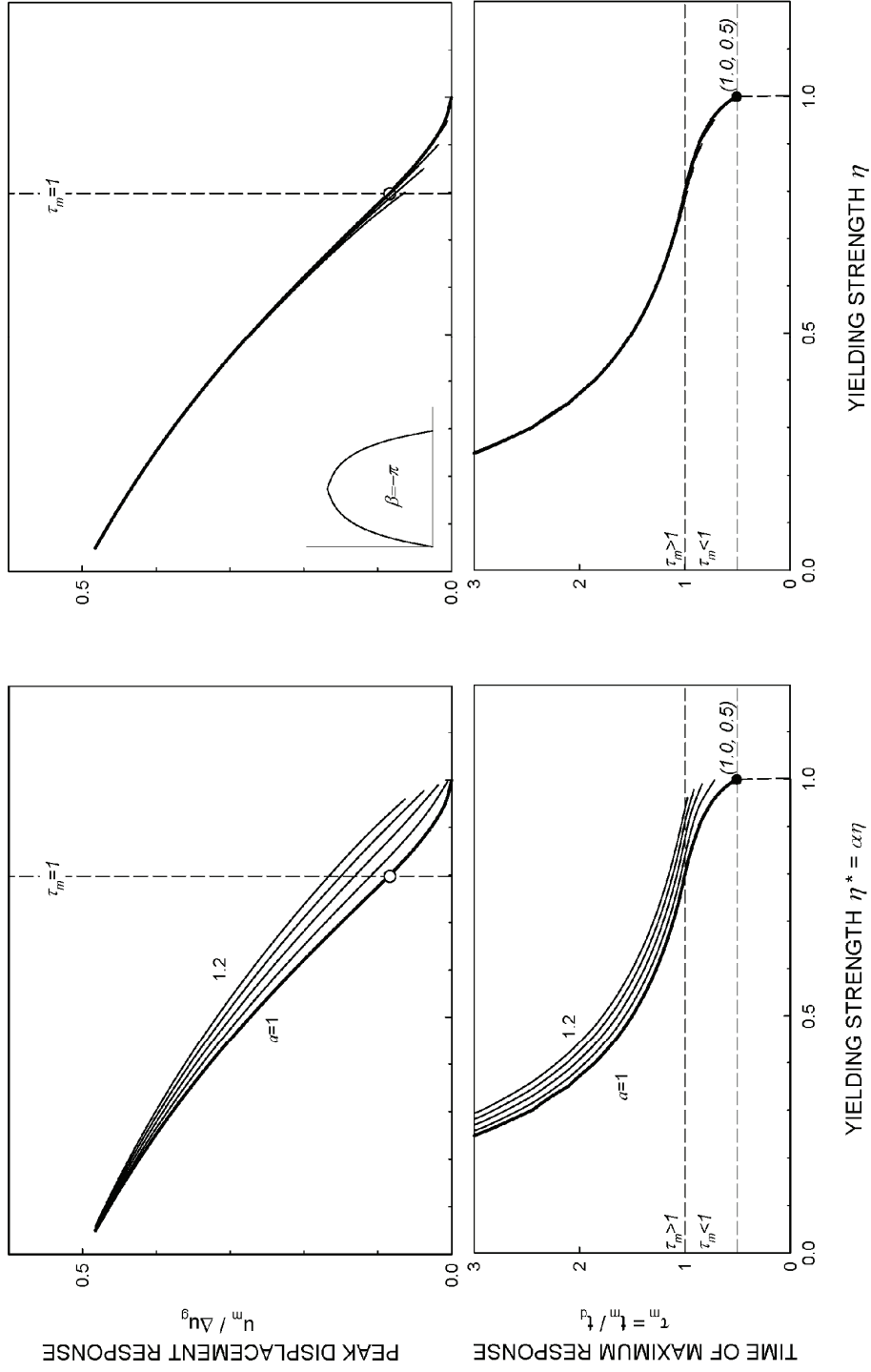


Figure 2.21b. Normalized displacement response with yield strength for a rigid block subject to a half-cycle generalized exponential acceleration pulse ($\beta = -\pi$) under varying friction conditions ($\eta^* = \alpha\eta$, $\alpha = 1, 1.05, 1.1, 1.15, 1.2$)

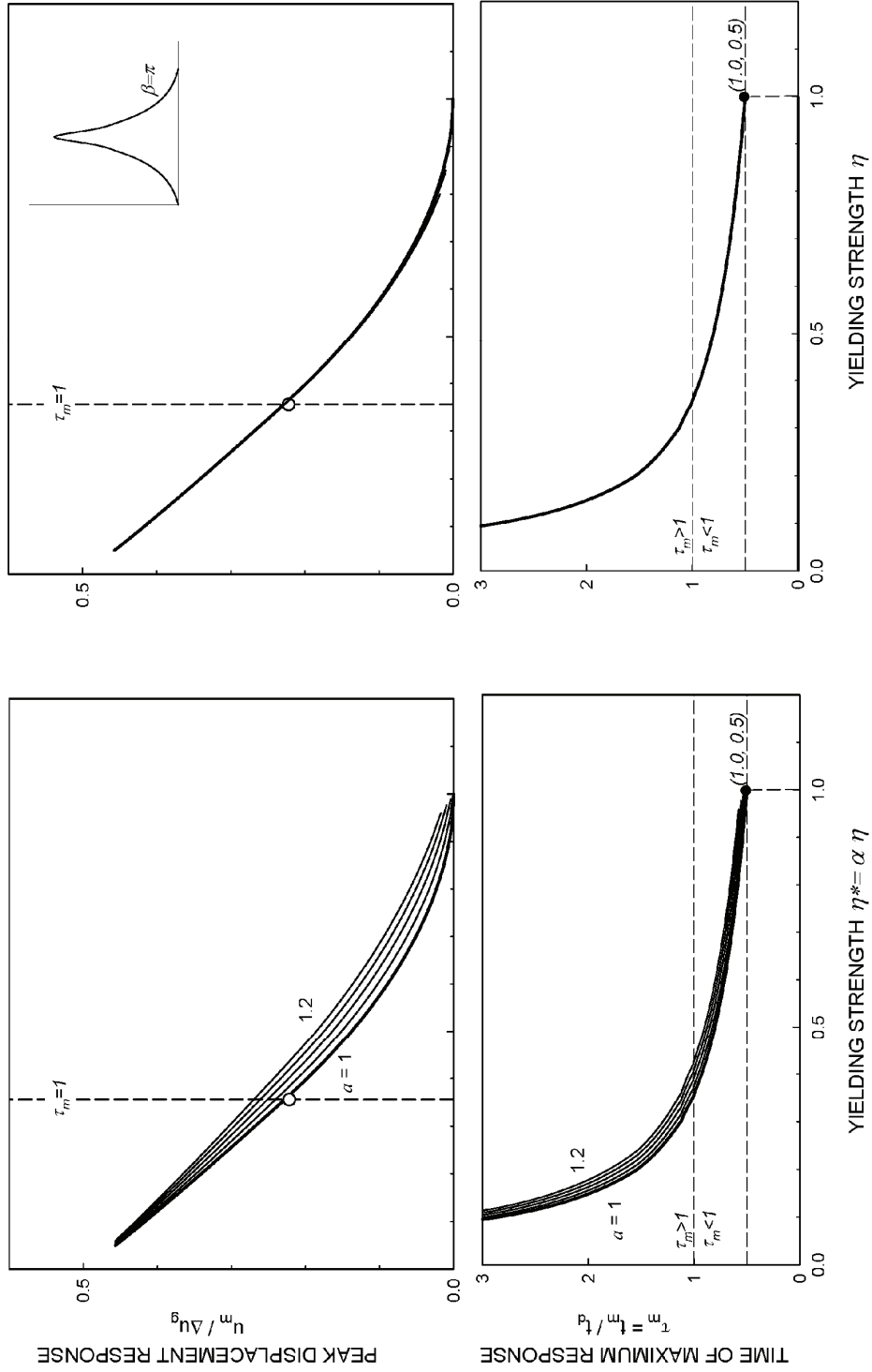


Figure 2.21c. Normalized displacement response with yield strength for a rigid block subject to a half-cycle exponential acceleration pulse ($\beta = \pi$) under varying friction conditions ($\eta^* = a\eta$, $a = 1, 1.05, 1.1, 1.15, 1.2$)

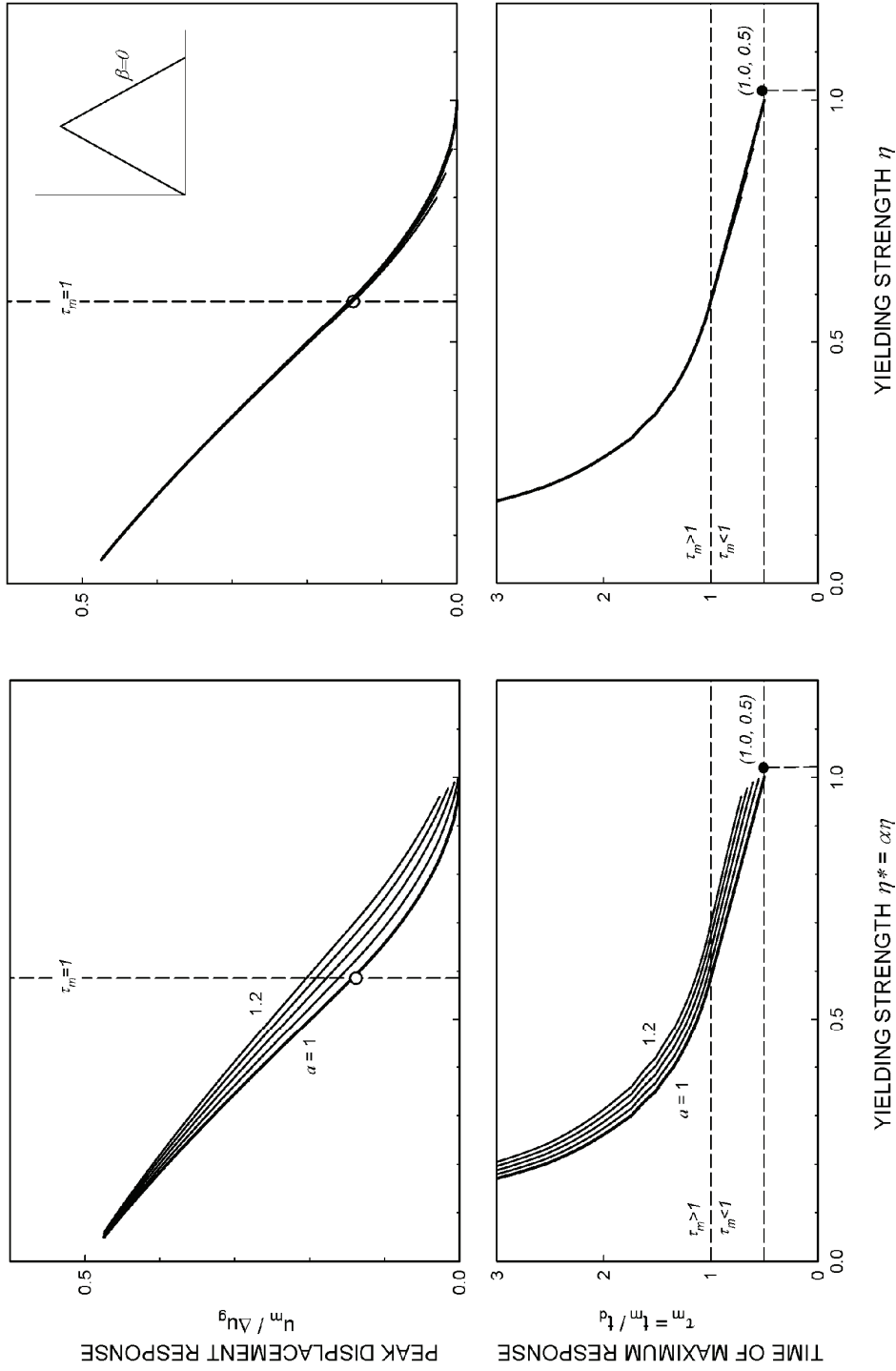


Figure 2.21d. Normalized displacement response with yield strength for a rigid block subject to a half - cycle triangular acceleration pulse ($\beta = 0$) under varying friction conditions ($\eta^* = a\eta$, $a=1, 1.05, 1.1, 1.15, 1.2$)

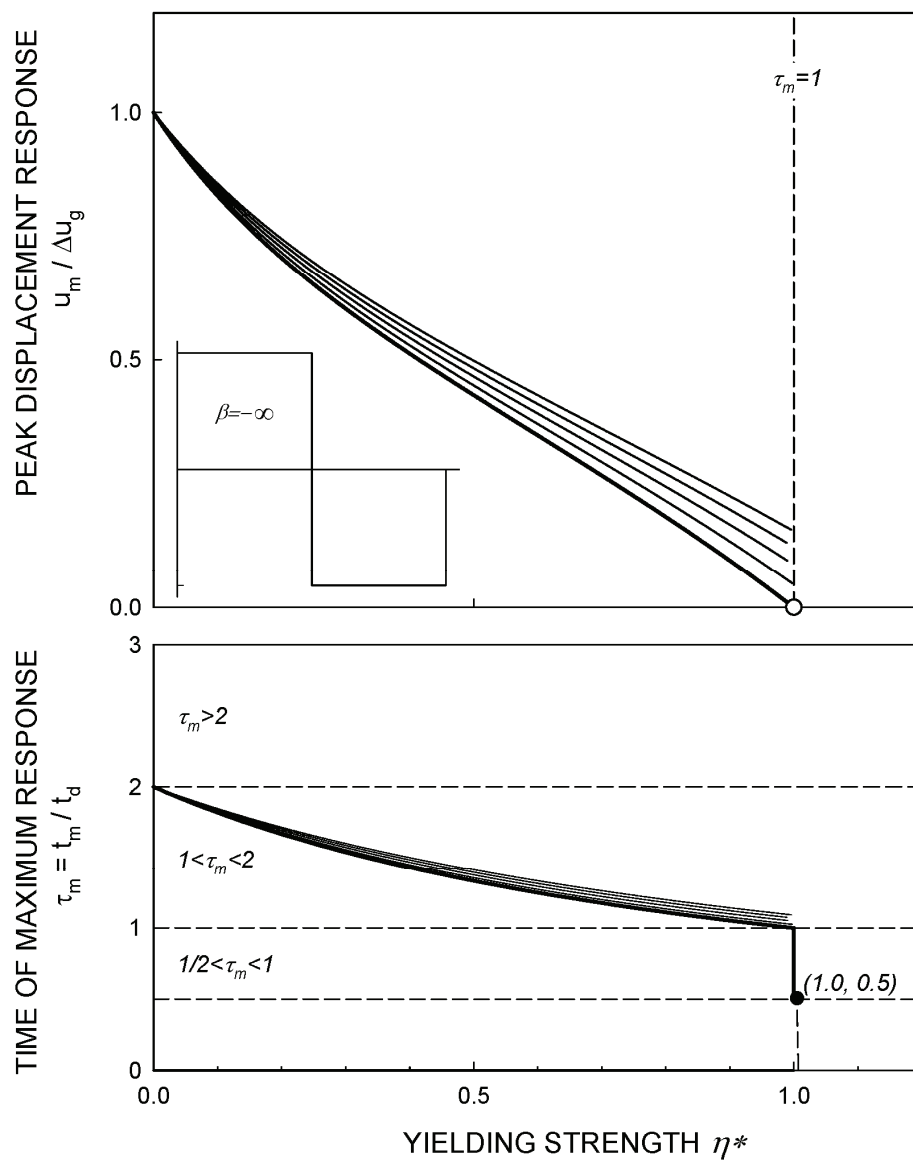


Figure 2.22a Normalized displacement response with yield strength for a rigid block subject to a full-cycle rectangular acceleration pulse ($\beta = -\infty$) under varying friction conditions ($\eta^* = a\eta$, $a = 1, 1.05, 1.1, 1.15, 1.2$)

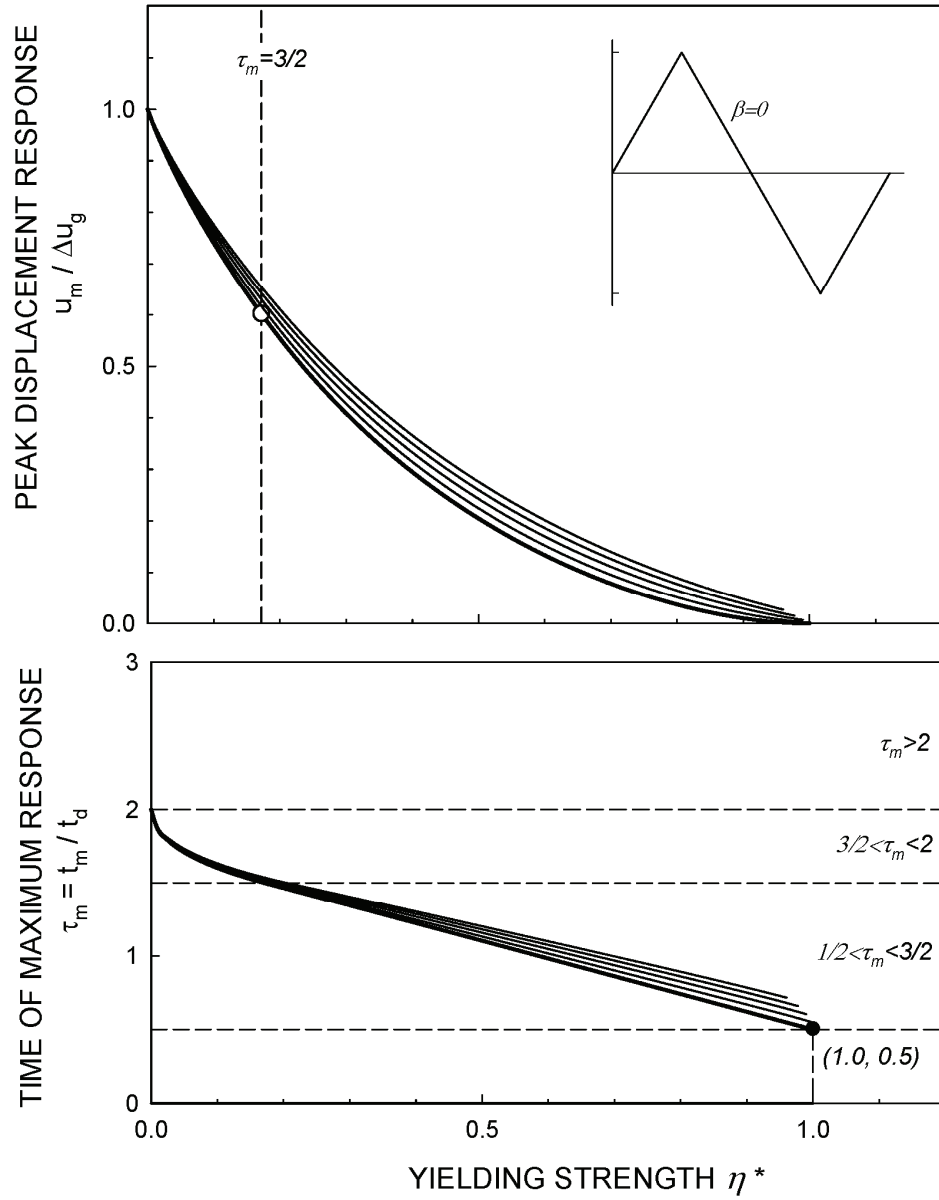


Figure 2.22b Normalized displacement response with yield strength for a rigid block subject to a full-cycle triangular acceleration pulse ($\beta=0$) under varying friction conditions ($\eta^*=a\eta$, $a=1, 1.05, 1.1, 1.15, 1.2$)

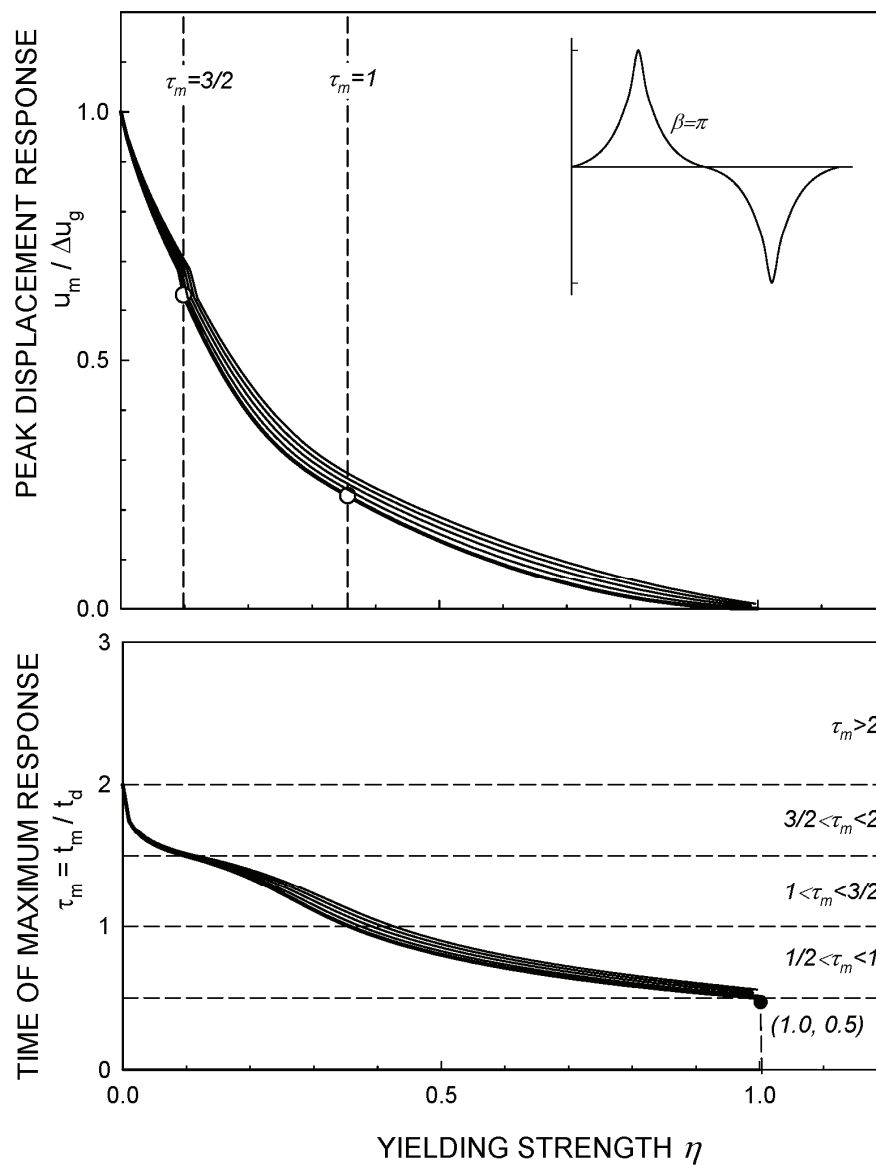


Figure 2.22c Normalized displacement response with yield strength for a rigid block subject to a full-cycle exponential acceleration pulse ($\beta = \pi$) under varying friction conditions ($\eta^* = a\eta$, $a = 1, 1.05, 1.1, 1.15, 1.2$)

2.8 Comparison with Solutions from the Literature

A comparison of permanent displacements obtained for half-cycle pulses of different shape against established solution by Newmark (1965), Richards & Elms (1979), Whitman & Liao (1985) and Yegian *et al.* (1991) is provided in Fig 2.23.

Except for the solution by Newmark, all other schemes are derived on the basis of conventional (non-impulsive) motions containing multiple irregular cycles. The solutions by Newmark and Richards-Elms refer to maximum expected values, whereas those by Whitman & Liao and Yegian *et al.* to average or median values. The proposed solution provides results which are comparable to the median predictions – a remarkable behavior given the half-cycle nature of the excitation. Of particular interest are: (1) the case $\beta = \pi$ which is in very good agreement with the solution by Yegian *et al.*; (2) the case $\beta \rightarrow -\infty$ which is in excellent agreement with the solution by Whitman & Liao for low-strength systems ($\eta < 0.3$) and the solution of Richards-Elms for a set of high-strength systems ($0.5 < \eta < 0.9$).

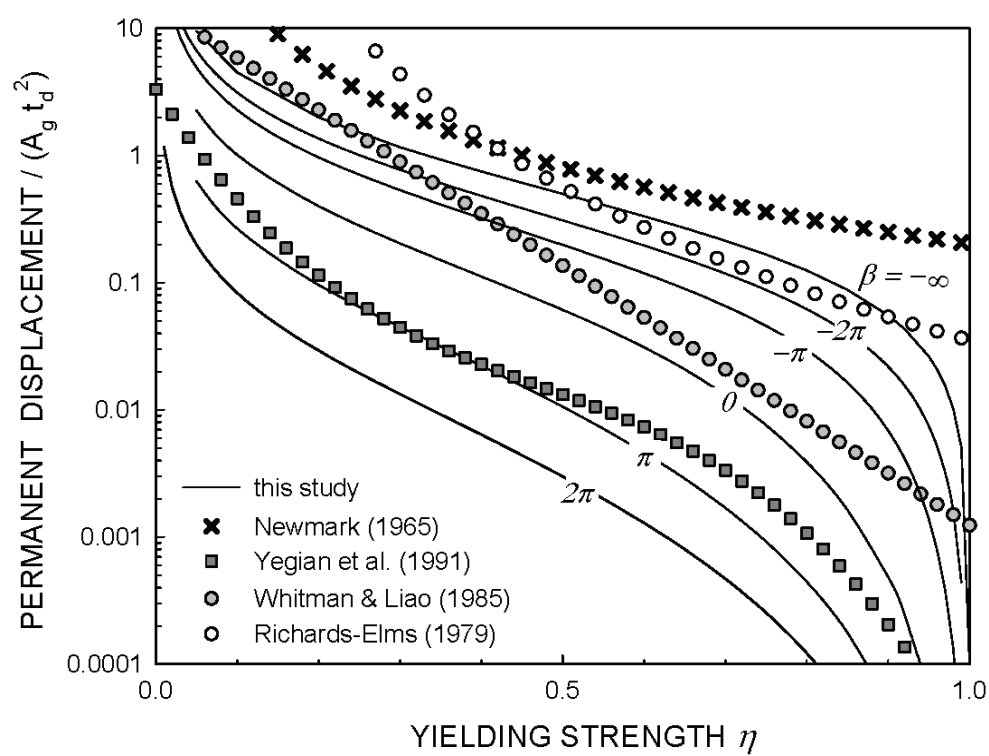


Figure 2.23 Comparison of permanent displacement due to a half-cycle pulse obtained by the present methodology, with established solutions from the literature.

2.9 Application Examples

With reference to the Pacoima Dam (1971) record, a full-cycle sinusoidal like pulse can be identified in the early portion of the time history (not shown here in the interest of space) having amplitude $A_g = 0.66g$ and full-cycle duration $2 \times t_d = 1.2s$. Application of the proposed theory is demonstrated below using a rigid block on a frictional plane with $\mu = 0.33$, for the following two cases: (1) bilateral sliding on horizontal plane; (2) unilateral sliding on inclined plane.

2.9.1 Bilateral sliding on horizontal plane

For the purposes of this demonstration the pulse is modeled using Eq. (2.1a) with $\beta = -\pi$. The yield strength is (Eq. 2.11): $\eta = 0.33/0.66 = 0.5$. Accordingly, τ_y is evaluated from Eq. (2.12) as:

$$\tau_y = \ln[1 - 0.5(1 - e^{-\pi})]/(-2\pi) = 0.104 \quad \text{or} \quad t_y = 0.6 \times 0.104 = 0.062s \quad (2.36)$$

Time of maximum response is computed from Eq. (2.19) as $\tau_{m1} = 1.25$ or $t_{m1} = 0.75s$.

Maximum sliding displacement is then obtained from Eq. (2.18) as: $u_{m1} / A_g t_d^2 = 0.17$ or $u_{m1} = 0.17 \times 0.66g \times 0.6^2 = 0.40m$.

With reference to reverse sliding, Eq. (2.21b) gives $1 + \tau_{y1} < \tau_{m1} < 2 - \tau_{y1}$ or $1.104 < 1.25 < 1.896$, which corresponds to “slip” condition. Accordingly time of second slippage is (Eq. 2.21b): $\tau_{y2} = \tau_{m1} = 1.25$ and $\bar{\tau}_{y2} = \tau_{y2} - 1 = 1/4$. Time of final stop is obtained from Eq. (2.7):

$$\bar{\tau}_{m2} = \frac{1}{4} + \frac{2}{(1 - e^{-\pi})} \left[1 - \frac{1}{2\pi} (1 + e^{-\pi/2}) + \frac{e^{-\pi}}{\pi} - \frac{1}{4} \right] = 1.44 \quad \text{or} \quad \bar{t}_{m2} = 0.6 \times 1.44 = 0.86s \quad (2.37)$$

Maximum reverse displacement is then determined from Eq. (2.9) as the positive quantity:

$u_{m2} / A_g t_d^2 = 0.163$ or $u_{m2} = 0.163 \times 0.66g \times 0.6^2 = 0.38m$. Combining the above displacements yields the residual value $u_{res} = u_{m1} - u_{m2} = 0.40 - 0.38 = 0.02m$ (about 5% of peak displacement, in agreement with Fig. 2.13).

2.9.2 Unilateral sliding on inclined plane

Considering an inclined plane with $\lambda = 30^\circ = \pi/6$ and horizontal seismic action ($\xi=0$), Eq.(2.31)

provides the pair of frictional resistances

$$\eta_{1,2} = 0.33/0.66 (1 \mp 0.33^{-1} \tan \pi/6) / (1 \pm 0.33 \tan \pi/6) \quad \text{or} \quad \eta_1 = 0.315, \eta_2 = 1.70 \quad (2.38)$$

corresponding to downward and upward sliding, respectively.

Considering the first half-cycle of the pulse pushing the block downhill, time of first slippage

is given from Eq. (2.12) as: $\tau_{y1} = \ln[1 - 0.315(1 - e^{-\pi})] / (-2\pi) = 0.034$ or

$t_{y1} = 0.6 \times 0.034 = 0.02s$. Time of peak sliding is obtained in the same spirit as before (Eq.

2.19): $\tau_{m1} = 1.42$ or $t_{m1} = 0.85s$.

Accordingly, peak displacement is (Eq. 2.18):

$$u_{m1} / A_g t_d^2 = 0.316 \quad \text{or} \quad u_{m1} = 0.316 \times 0.66g \times 0.6^2 = 0.74m \quad (2.39)$$

As $\eta_2 > 1$ sliding in the upward direction is not possible; therefore the above quantity also stands for the residual displacement of the block.

2.10 Conclusions

Exact analytical solutions were derived for the frictional sliding of rigid blocks on a frictional plane subjected to a family of idealized acceleration pulses of half- and full-cycle duration, acting at an arbitrary inclination with respect to the frictional interface. The main conclusions this chapter are:

- (1) Pulse velocity and peak sliding displacement appear to be intimately interconnected (Eq. 2.6). This is understood given that the higher the pulse area after initiation of slippage, the longer the block will stay in motion, the higher the ensuing sliding displacement.
- (2) As initiation of slippage requires exceedance of a certain level of ground acceleration regardless of other parameters, ground velocity alone should not be viewed as the main descriptor of pulse intensity. Support to this finding comes from Eq. (2.5), which shows that sliding displacement depends on the time history of ground displacement – not of ground velocity. Accordingly, ground displacement might be more important in affecting peak dynamic response than presently thought.
- (3) Times of first slippage and peak response are, naturally, increasing and decreasing functions of yield strength, respectively. Peak sliding response is a decreasing function of yield strength.

(4) For half-cycle excitation, only up to 1/2 of incremental ground displacement can be transferred, as sliding displacement, to the block. This behavior is paradoxical, yet rigorous from a mathematical viewpoint, as common sense suggests that relative-to-ground-displacement must equal ground displacement under zero friction. Nevertheless, the above value cannot be reproduced experimentally, as time of peak sliding τ_m is infinite and, thereby, dimensionless time (τ/τ_m) would never be sufficiently large to make the trend observable in a real test. Instead, the value $u_m/\Delta u_{gm}=1$ will be obtained, that is, the conventional behavior would not be violated in practice.

(5) Half-cycle excitation tends to generate higher sliding displacement than full-cycle excitation under equal ground acceleration and frictional strength. This is in contrast to ordinary motions containing multiple cycles, for which displacement generally increases with number of cycles.

(6) The complex dependence of peak displacement on frictional strength η is evident in Eqs (2.9) and (2.18). This suggests that simple scaling expressions of the power- or exponential-law type might not be possible for the problem at hand, despite the idealized nature of friction and pulse shape.

(7) Following up on the previous conclusion, incremental ground displacement Δu_{gm} appears superior to schemes obtained by dimensional analysis for normalizing sliding displacement, as the specific parameter yields bounded results for both half- and full-cycle excitations (anchored at the simple values 1/2, 1 and 0 in the limits $\eta=0$ and 1) and provides the smallest dispersion of the results as function of η and β (Figs 2.13, 2.14). Accordingly, dimensional

analysis does not seem superior to physically motivated choices for deriving optimal normalization schemes for the problem at hand.

(8) Solutions to half-cycle excitations may faithfully simulate response to full- or multiple-cycle loading under conditions of asymmetric friction (Fig 2.16, Table 2.4). Thereby, half-cycle pulses should be regarded as being more than highly idealized excitations employed for the sake of analytical simplicity. In this light, the finite ground velocity at the end of a half-cycle pulse should not be viewed as a physical limitation for employing these waveforms in real-life problems.

3

Simple Yielding Oscillator

3.1 Synopsis

Analytical investigations are presented for the elastic and inelastic response of bilinear simple oscillators to idealized ground acceleration pulses. These motions are typical of near-field recordings generated by forward-rupture directivity and may generate damage in the absence of substantial structural strength and ductility. Seven basic pulse types are examined: (1) symmetric triangular; (2) sinusoidal; (3) exponential; (4) rectangular; (5) generalized exponential; (6) linear ramp; (7) inverse ramp. First, a numerical study is presented on the effect of oscillator strength, stiffness, damping, post-yielding stiffness and number of excitation cycles on inelastic response. Parameters investigated include time of yielding, time of peak response and ductility demand. The effect of high-frequency motions (higher harmonics) that override actual recorded pulses is investigated. Results are presented in the form of dimensionless graphs, charts and regression formulas involving the salient parameters of the problem. Second, oscillators having zero post-yielding stiffness and damping are investigated analytically. For these systems, closed-form solutions are derived for the inelastic response during the rising and decaying phases

of the pulse as well as the ensuing free oscillations. To determine the time of yielding, the associated transcendental equations are linearized using pertinent Taylor expansions over the time variable. It is shown that when the expansion point is located close to the actual time of yielding, excellent accuracy is achieved by means of first-order expansions. Different techniques are proposed to this end. Times of peak response and ductility demand are easier to obtain, as no transcendental functions are typically involved. Contrary to presently – used numerical algorithms, the derived analytical expressions provide useful insight on the response of yielding structures to pulse-like motions. The model is evaluated against numerical results from actual near-field recordings. Finally, two simple case studies are presented.

3.2 Preface

As already discussed in Chapters 1 and 2, significant interest has emerged in recent years in exploring the destructiveness of strong near-source earthquake motions. According to seismologists, these motions are the result of superimposed shear waves propagating in the same direction as the fault rupture, generating a long-duration high-amplitude directivity pulse. The phenomenon had already been described (Boore & Zoback 1974; Bertero *et al.* 1978; Singh 1985; Somerville & Graves 1993) when Northridge and Kobe earthquakes occurred and the interest in the subject skyrocketed. Several structural failures have been attributed to near-field pulses including those of Olive View Hospital in the 1971 San Fernando Earthquake (Mahin *et al.* 1976) and Elevated Hanshin Expressway in Kobe (Park 1996). Evidence of the role of near-fault pulses on the collapse of a large industrial building in Greece during the 1995 Aegion earthquake, has been presented by Gazetas *et al.* (1995). Examples of near-field recorded motions from four

destructive earthquakes are provided in Figure 3.1. Comprehensive catalogues of near – fault earthquake motions are available (Somerville *et al.* 1997, Somerville 2000, Rodriguez-Marek 2000, Stewart *et al.* 2001).

Despite an ever-increasing database of near-field recorded accelerograms and the intense research in the subject (Hall *et al.* 1995; Iwan 1997; Malhotra 1999; Baez & Miranda 2000), the inelastic response of structures to such motions has not been sufficiently explored (Karavasilis *et al.* 2010, Dimakopoulou *et al.* 2011). It is noted that most of the foregoing studies utilize actual time histories in conjunction with numerical algorithms to analyze the response of inelastic single- or multi-degree structures to a single or multiple records (Bertero *et al.* 1978; Iwan 1997; Baez & Miranda 2000). In contrast, idealized pulses and analytical techniques that can provide insight into the physics of the problem have received less attention (Walker 1965; Bozorgnia & Mahin 1998; Cuesta & Aschheim 2001; Mylonakis & Reinhorn 2001, Gantes & Pnevmatikos 2004, Kouretzis *et al.* 2007). This is desirable since there is evidence that only a short interval in the ground motion associated with the pulse contributes to most of the seismic demand (Bertero *et al.* 1978; Cuesta & Aschheim 2000; Bozorgnia & Mahin 1998).

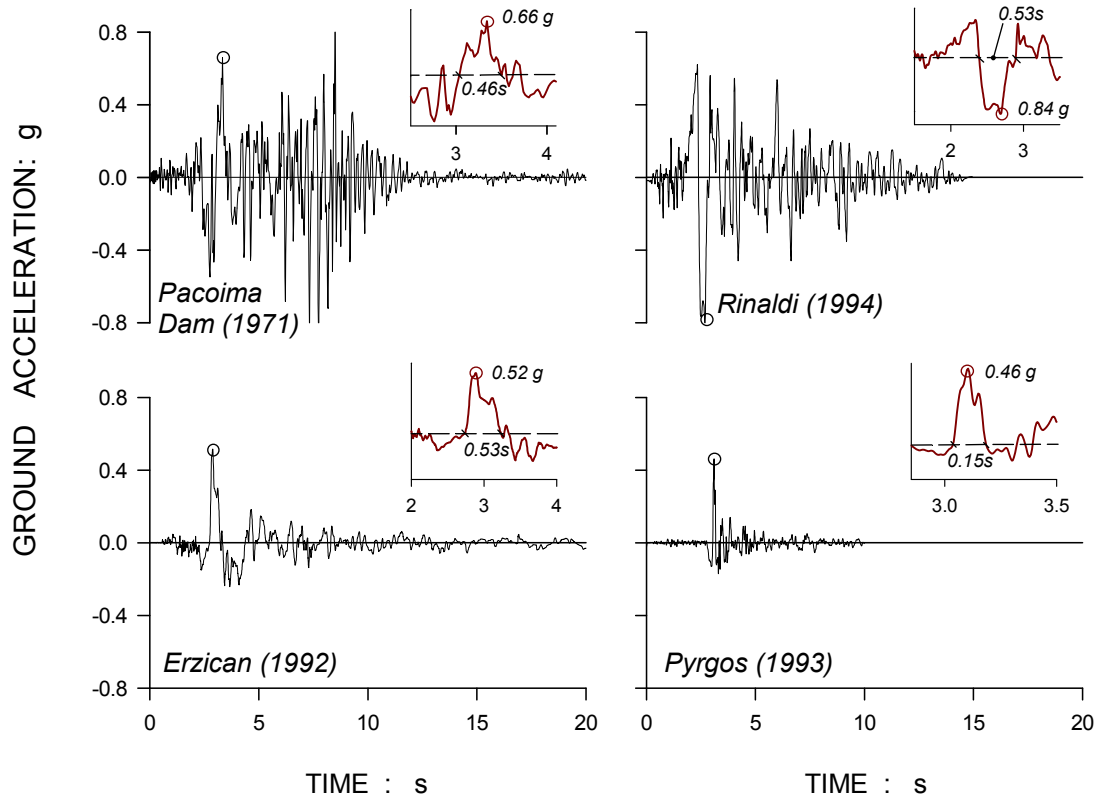


Figure 3.1 Four selected near-field ground motions containing long-period, high-acceleration pulses (Table 3.1): Pacoima-N164W (1971), Rinaldi-N228W (1994), Erzican-NS (1992), and Pyrgos-EW (1993). The corresponding peak ground velocities are, respectively, 113, 166, 84, and 25 cm/s. (modified after Mylonakis & Reinhorn, 2001).

The scope of this Chapter is to explore the fundamentals of the problem by studying the response of bilinear yielding oscillators to idealized acceleration pulses. The specific objectives are: (1) to compare the inelastic seismic demands resulting from different pulses; (2) examine the sensitivity of the response to the oscillator strength, natural period, damping, and post-yielding stiffness; (3) examine the sensitivity of the response to the number of excitation cycles; (4) study the effect of high-frequency motions that override actual recorded pulses; (5) derive explicit solutions for the response of elastic-perfectly plastic oscillators to certain idealized pulses. It is noted that, while perfectly elastoplastic systems subjected to pulse-like excitations have been studied as early as the 1960's (Biggs 1964; Walker 1965), the response of oscillators with hardening (non-zero post yielding stiffness) has not been sufficiently explored. In addition, contrary to the elastic problem which has been studied extensively in the past (Jacobsen & Ayre 1958; Ayre 1995; Chopra 2001), little effort has been put in deriving analytical closed-form solutions for the inelastic problem (Newmark & Rosenblueth 1971; Mylonakis & Reinhorn 2001). Such explicit solutions can provide valuable insight into the physics of inelastic response, which is often obscured by numerical data. The results presented in this Chapter complement and extend foregoing studies on inelastic shock spectra by Jacobsen & Ayre (1958), Biggs (1964), Cuesta & Aschheim (2001), Mylonakis & Reinhorn (2001), among others. Because of their simplicity, the proposed solutions can advance the development of engineering provisions for structural design against near-fault motions.

3.3 Problem and Parameter Definition

The problem considered is depicted in Figure 3.2: a Single-Degree-of-Freedom (SDOF) oscillator having mass m , elastic stiffness K (leading to an elastic period $T = 2\pi\sqrt{m/K}$), yield strength Q_y (corresponding to a yielding displacement $u_y = Q_y/K$), post-yielding stiffness K_{py} (expressed through the dimensionless factor $\gamma = K_{py} / K$), and viscous damping ratio ζ , is subjected to four different ground acceleration pulses, as shown in Figure 3.3. The pulses are described by their shape (triangular, sinusoidal, exponential, rectangular), amplitude A_g , half-cycle duration t_d (or, equivalently, period $T_d = 2 \times t_d$), and number of excitation cycles N . It is noted that for non-integer values of N the pulses are not physically realizable, since ground velocity at the end of the pulse is not zero. Nevertheless, these motions incorporate the large acceleration and velocity excursions observed in actual near-field recordings and, thereby, are well suited for exploring the salient features of structural response to such motions. In addition, it will be shown that a large set of yielding oscillators attain their peak response before the end of the excitation and, thereby, whether ground velocity goes to zero after the end of the pulse is of less importance.

The following dimensionless quantities are used to describe the results presented below:

pulse duration (oscillator frequency) $f = t_d / T$, *oscillator period* T / T_d , *yield strength* $\eta = Q_y / m A_g$, *force reduction factor* $R = S A_g / Q_y$, *time of yielding* $\tau_y = t_y / t_d$, *time of peak response* $\tau_m = t_m / t_d$, *displacement (“spectral”) amplification* $D = u / u_s$, *ductility demand* $\mu = u_{\max} / u_y$, where $u_{\max}$ = maximum oscillator displacement relative to the base, t_y = time of yielding, t_m = time of $u_{\max}$, $u_s = m A_g / K$ = static oscillator displacement. A complete notation is provided in the List of Symbols at the end of the volume.

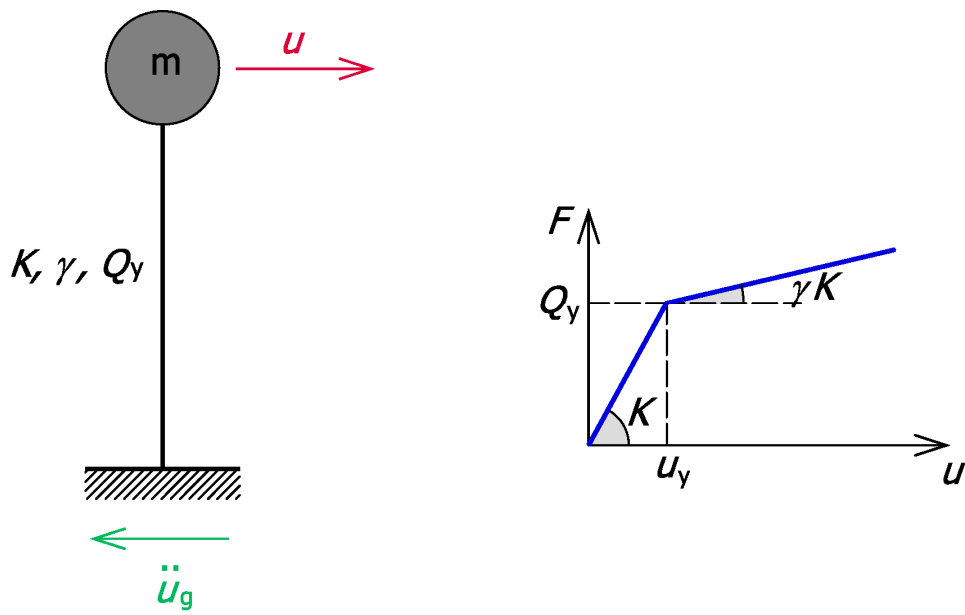


Figure 3.2 System considered.

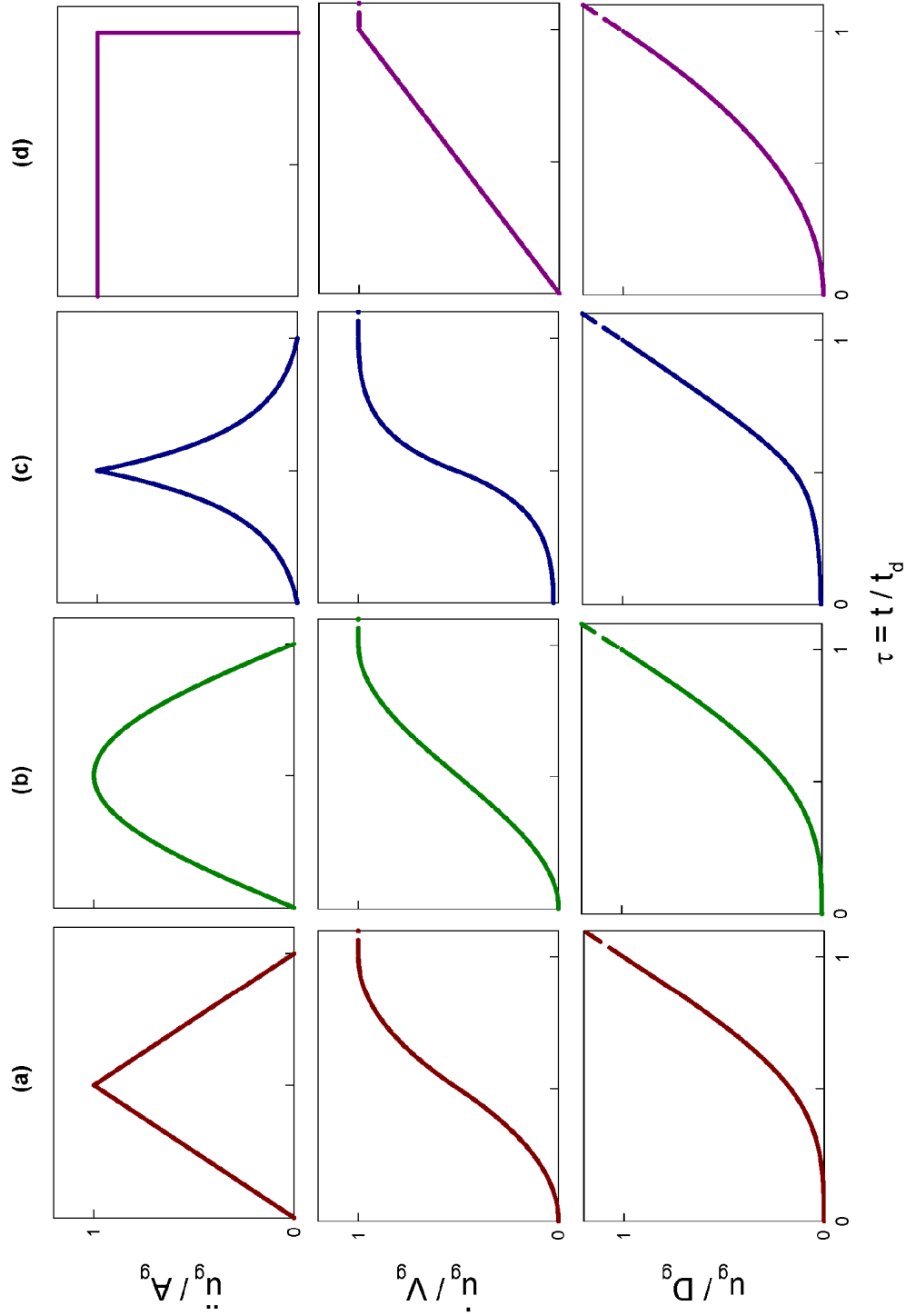


Figure 3.3 Dimensionless ground acceleration, velocity and displacement time histories of four idealized earthquake pulses considered in this Chapter: (a) triangular; (b) sinusoidal; (c) exponential and (d) rectangular.

3.4 Elastic Oscillator

The response of an elastic undamped oscillator can be conveniently established using the method of undetermined coefficients. The method is known (Jacobsen & Ayre 1958, Biggs 1962; Ayre 1995; Chopra 2001; Katsikadelis 2002) and will not be repeated here. Only results are discussed below.

The differential equation of dynamic equilibrium of the elastic oscillator is written as:

$$\ddot{u} + \omega^2 u = \ddot{u}_g [H(t) - H(t - t_d)] \quad (3.1)$$

where $H()$ denotes the Heaviside (step) function. The second factor on the right-hand side implies that the system is subjected to forced oscillations in the interval $0 < t < t_d$. The negative sign on the right-hand side has been omitted for simplicity.

Ground acceleration $\ddot{u}_g$ is written, for each different pulse, as

$$\ddot{u}_g = \begin{cases} \frac{2A_g}{t_d} [t + H(t - \frac{t_d}{2})(t_d - 2t)] & \text{triangular} \\ A_g \sin \pi \frac{t}{t_d} & \text{sinusoidal} \\ \frac{A_g}{1 - e^{-\pi}} [1 - e^{-2\pi \frac{t}{t_d}} + H(t - \frac{t_d}{2})(e^{-2\pi \frac{t}{t_d}} - e^{-2\pi(1 - \frac{t}{t_d})})] & \text{exponential} \\ A_g & \text{rectangular} \end{cases} \quad (3.2)$$

The solution to the governing equation (3.1) with at rest initial conditions is presented for the general case $f = t_d / T \neq 1/2$. Resonant conditions ($f = 1/2$) can be obtained from the

general case using a limit process. Presentation of results is organized in two phases: forced vibrations and free vibrations.

Using the normalizations $D = u / u_s$, $f = t_d / T$, $\tau = t / t_d$, $\tau_m = t_m / t_d$, one obtains for each response interval (Biggs 1964, Chopra 2001, Jacobsen & Ayre 1958, Mylonakis & Reinhorn 2001):

Forced vibration phase ($\tau \leq 1$)

$$D(\tau) = \begin{cases} 2\tau - \frac{1}{\pi f} \sin[2\pi f \tau] + H(\tau - 1/2) \left\{ -4\tau + 2 + \frac{1}{\pi f} [\sin \pi f \cos 2\pi f(\tau - \frac{1}{2}) - (2 - \cos \pi f) \sin 2\pi f(\tau - \frac{1}{2}) + \sin(2\pi f \tau)] \right\} & \text{triangular} \\ \frac{4f^2}{4f^2 - 1} \left(\sin \pi \tau - \frac{1}{2f} \sin 2\pi f \tau \right) & \text{sinusoidal} \\ \frac{1}{(1 - e^\pi)(1 + f^2)} \left(\begin{aligned} &1 + f^2(1 - e^{2\pi\tau}) - \cos 2\pi f \tau + \\ &+ f \sin 2\pi f \tau + H(\tau - 1/2) \times 2fe^\pi \times \\ &\times \{ \sin[f\pi(1 - 2\tau)] - f \sinh[\pi(1 - 2\tau)] \} \end{aligned} \right) & \text{exponential} \\ 1 - \cos 2\pi f \tau & \text{rectangular} \end{cases} \quad (3.3a-d)$$

Free vibration phase ($\tau \geq 1$)

$$D(\tau) = \begin{cases} \frac{1}{\pi f} [2 \sin \pi f(2\tau - 1) - \sin 2\pi f(\tau - 1) - \sin 2\pi f \tau] & \text{triangular} \\ \frac{4f^2}{4f^2 - 1} \cos \pi f \cdot \sin \pi f(1 - 2\tau) & \text{sinusoidal} \\ \frac{2 \sin[\pi f(1 - 2\tau)]}{(1 - e^\pi)(1 + f^2)} (fe^\pi - f \cos \pi f - \sin \pi f) & \text{exponential} \\ 2 \sin \pi f \cdot \sin \pi f(2\tau - 1) & \text{rectangular} \end{cases} \quad (3.4a-d)$$

Differentiating Eqs (3.3) and (3.4) one can determine the critical time at which velocity changes sign and the system reaches its peak response. After some tedious yet straightforward algebra, the following expressions are obtained

$$\tau_{m,el} = \begin{cases} \frac{1}{2} + \frac{1}{2\pi f} \left[\arctan \left(\frac{\sin \pi f}{2 - \cos \pi f} \right) + \arccos \left(\frac{1}{\sqrt{5 - 4 \cos \pi f}} \right) \right] & \text{triangular pulse} \\ \frac{2 \operatorname{Int} \left[\frac{1}{2} f + \frac{3}{4} \right]}{1 + 2f} & \text{sinusoidal pulse} \\ \frac{1}{2f} & \text{rectangular pulse} \end{cases} \quad (3.5)$$

which pertain to forced oscillations and are applicable for $f > 1/2$. In the above equations, $\operatorname{Int}[]$ denotes integer part and corresponds to different peaks (oscillation cycles) for the sinusoidal pulse (see for instance Chopra 2001, pp 146-147). For $f \leq 1/2$, it can be easily shown that no peak occurs during forced vibrations. The following simple expression applies to all pulses for time of peak response during free vibrations:

$$\tau_{m,el} = \frac{1}{4f} + \frac{1}{2} \quad (3.6)$$

Obviously, only dimensionless times greater than 1 are admissible from Eq. (3.6), and times of less than 1 are admissible from Eqs (3.5).

In Figures 3.4 to 3.6, the maximum response of the system, D_m , is plotted as function of dimensionless pulse duration f for all three pulses. The response at the peak of the pulse, $D_{1/2}$, end of pulse D_l , and free vibration amplitude D_f , are shown for comparison. The

following are worthy note: *Firstly*, the overall peak value D_m ($= 1.52, 1.77, 2$ for the triangular, sinusoidal, and rectangular pulse, respectively) is quite small which indicates that resonant effects are of minor importance for this type of loading. *Secondly*, for small pulse durations ($f < 0.5$) the response at the end of the pulse is very close to maximum. In contrast, for long pulse durations ($f > 1.3$), maximum response can be well approximated by the response at pulse peak. *Thirdly*, the response amplitude during free oscillations is not accurate for predicting peak response for values of f greater than approximately 0.5.

The peak value $D_m = 2$ obtained for the rectangular pulse can be easily verified from the energy equation of a spring-mass system in a gravitational field

$$m \cdot g \cdot u_m = \frac{1}{2} K u_m^2 \quad (3.7)$$

which expresses the conservation of energy between the compressed and the fully stretched position under the weight of the mass.

Solving Eq. (3.7) for u_m yields the expression

$$u_m = 2 \cdot m \cdot g / K = 2 \cdot u_s \quad (3.8)$$

which confirms the amplification value $D_m = 2$.

In the same figures (3.4 to 3.6), the time of peak response is plotted as function of f . As mentioned earlier, the maximum response occurs before the end of the pulse ($\tau_m < 1$) if f is greater than 0.5. For shorter pulse durations ($f < 0.5$), peak response takes place during free vibrations (Eq. 3.6). It is noted that the derivative of the curves is discontinuous at $f = 0.5$.

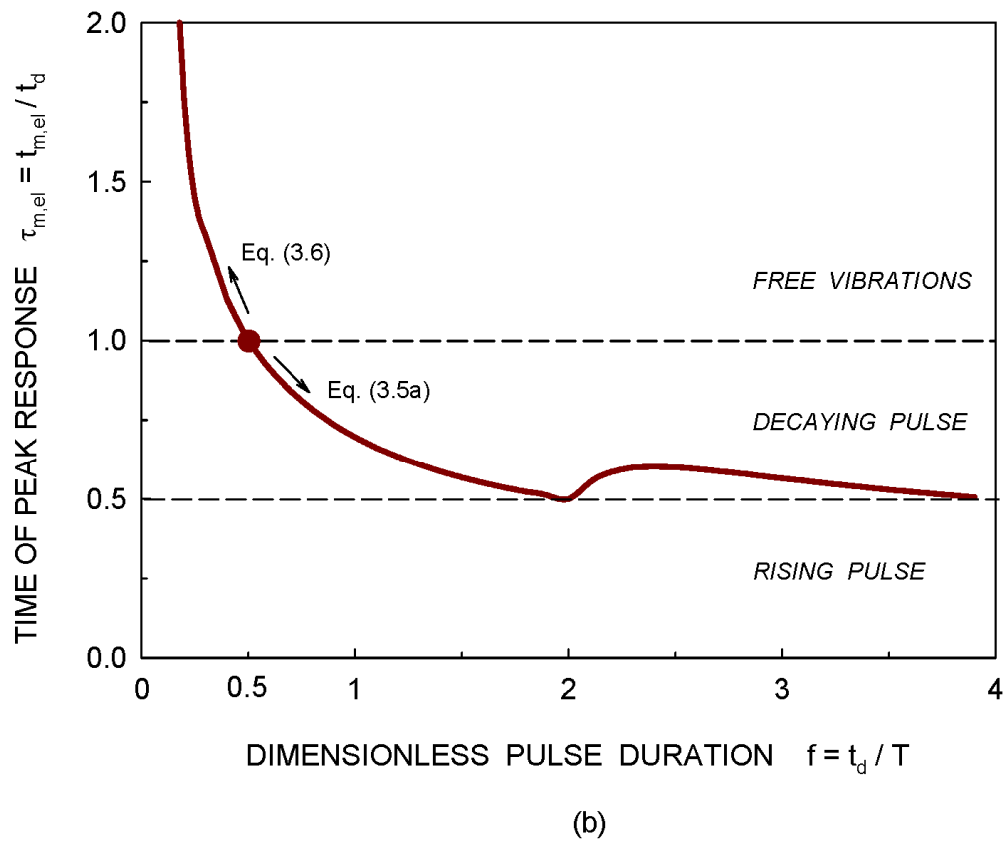
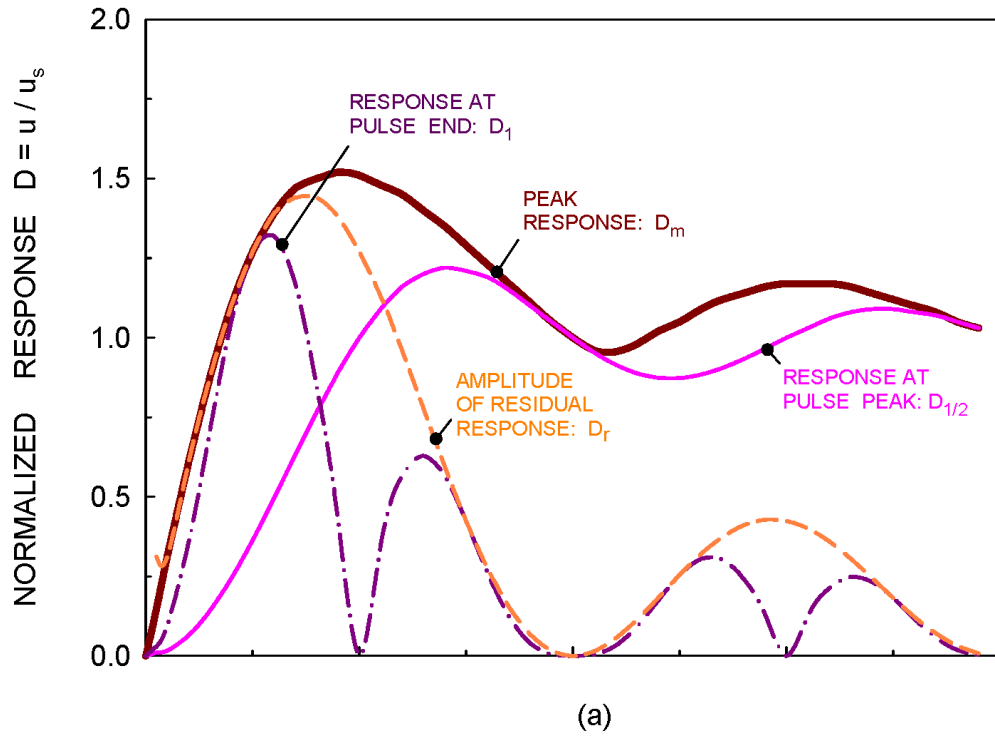


Figure 3.4 Normalized peak displacement response (a) and corresponding time (b) of an undamped elastic oscillator subjected to a symmetric triangular acceleration pulse.

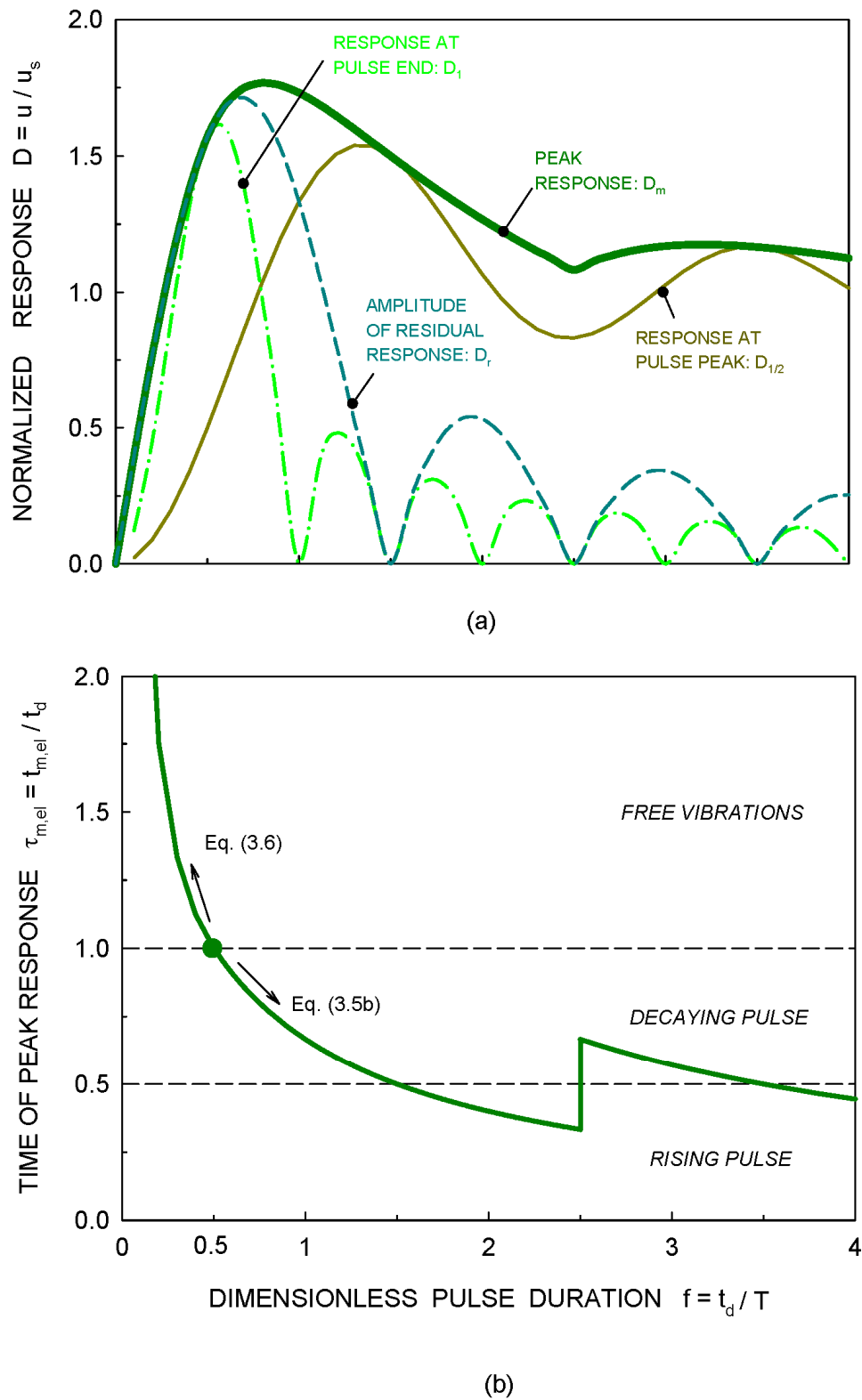
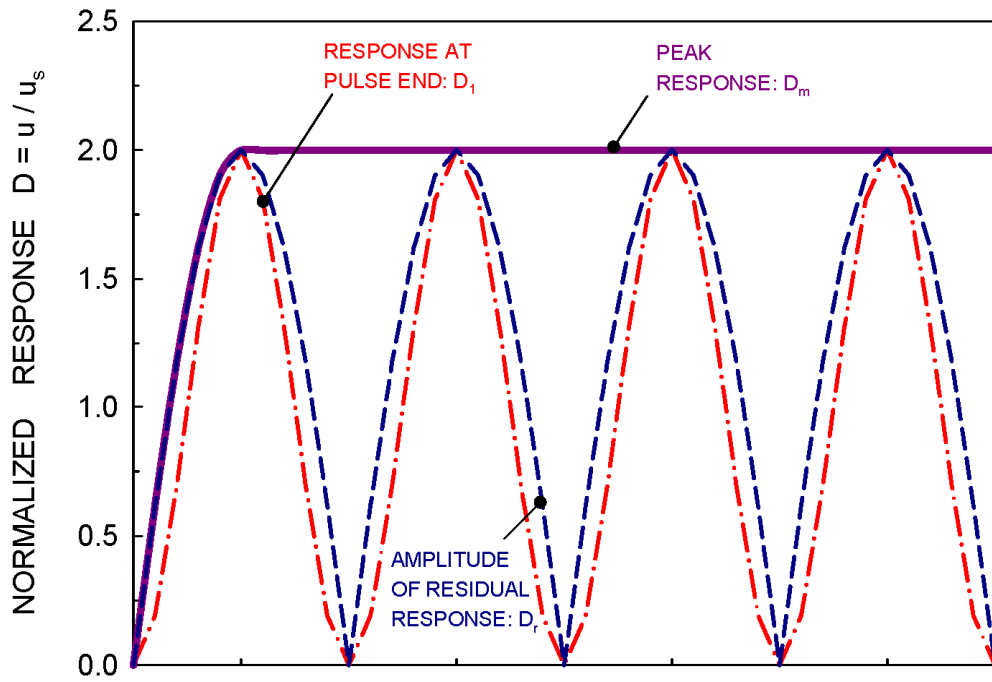
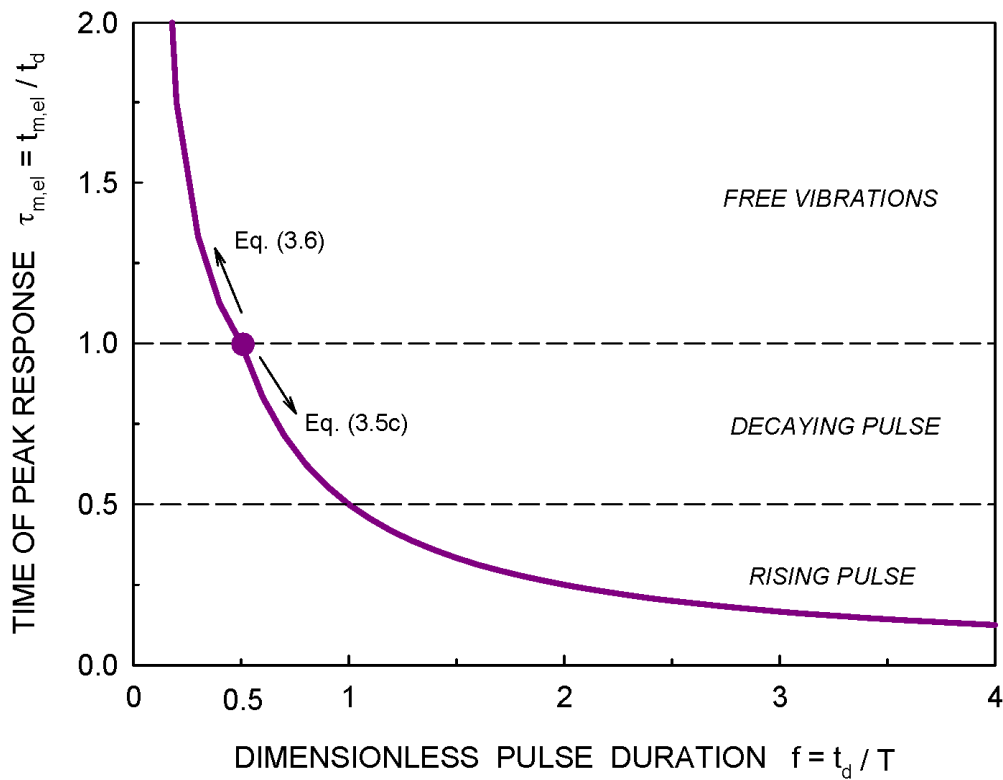


Figure 3.5 Normalized peak displacement response (a) and corresponding time (b) of an undamped elastic oscillator subjected to a sinusoidal acceleration pulse.



(a)



(b)

Figure 3.6 Normalized peak displacement response (a) and corresponding time (b) of an undamped elastic oscillator subjected to a rectangular acceleration pulse.

3.4.1 Effect of Damping and Number of Cycles

An alternative representation of the above results is depicted in Figure 3.7, where D_m is plotted as function of dimensionless oscillator *period* T/T_d for 5% viscous damping. D is termed here “spectral amplification” in accordance with the analogous $(SA(T)/PGA)$ normalized earthquake response spectra. Due to analytical difficulties associated with deriving the solution in presence of damping, the results presented in this paragraph have been obtained numerically. Evidently, the maximum amplification is obtained for the rectangular shape followed by the sinusoidal and the triangular shape, in accordance with Figures 3.4 to 3.6. Due to the presence of damping, the peak values are somewhat smaller than for the undamped oscillator. The effect of damping is further examined in Figure 3.8 which shows spectral amplification as function of T/T_d for a sinusoidal pulse of two different durations. The minor importance of damping for low-cycle excitation is visible in the graph. As anticipated, the opposite holds for a larger number of cycles, especially at resonance.

Figure 3.9 depicts the effect of number of cycles, N , on spectral amplification for a sinusoidal pulse and two different damping ratios. Evidently the response close to resonance increases substantially with increasing N . The reduction in response with increasing N beyond approximately $T/T_d = 1.5$ is worth noting.

Figure 3.10 presents the effect of N on the peak spectral amplification $(D_m)_{max}$ for different values of viscous damping. For small number of cycles, peak response increases linearly with N . For damped systems, the dependence of $(D_m)_{max}$ to N becomes less than linear beyond $N = 3$, and diminishes altogether when N exceeds approximately 10. For undamped systems, the dependence of $(D_m)_{max}$ on N is always linear.

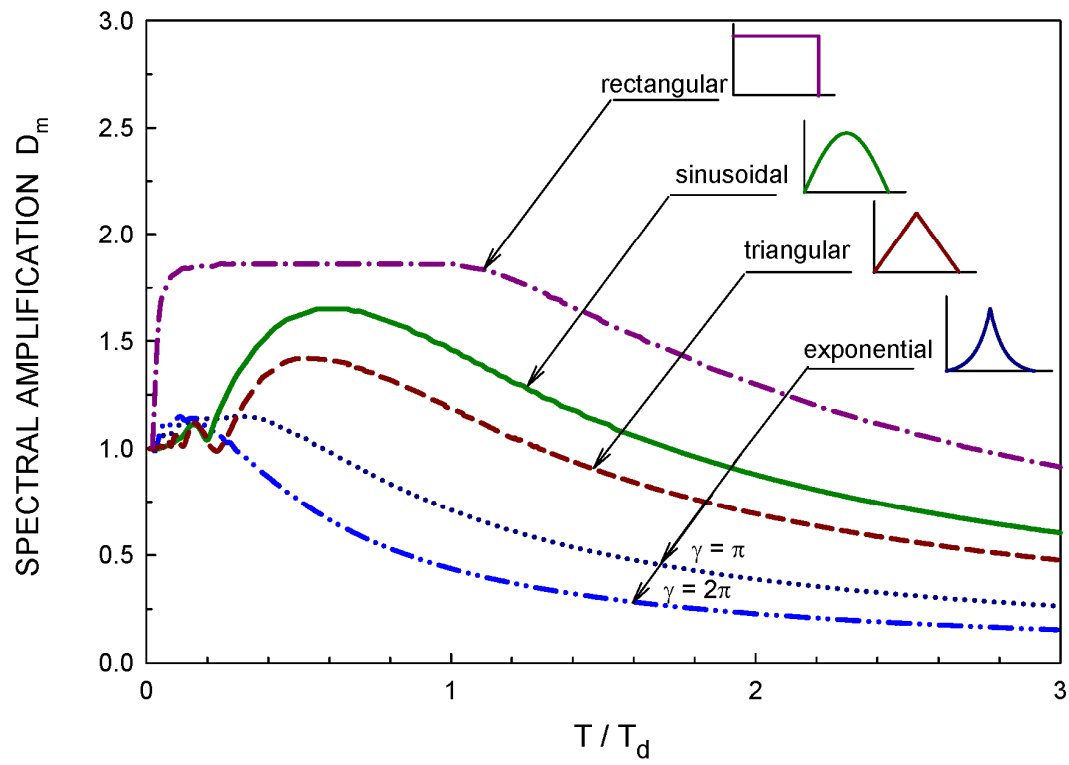


Figure 3.7 Elastic spectral amplification for four idealized half-cycle pulses. $\zeta=5\%$

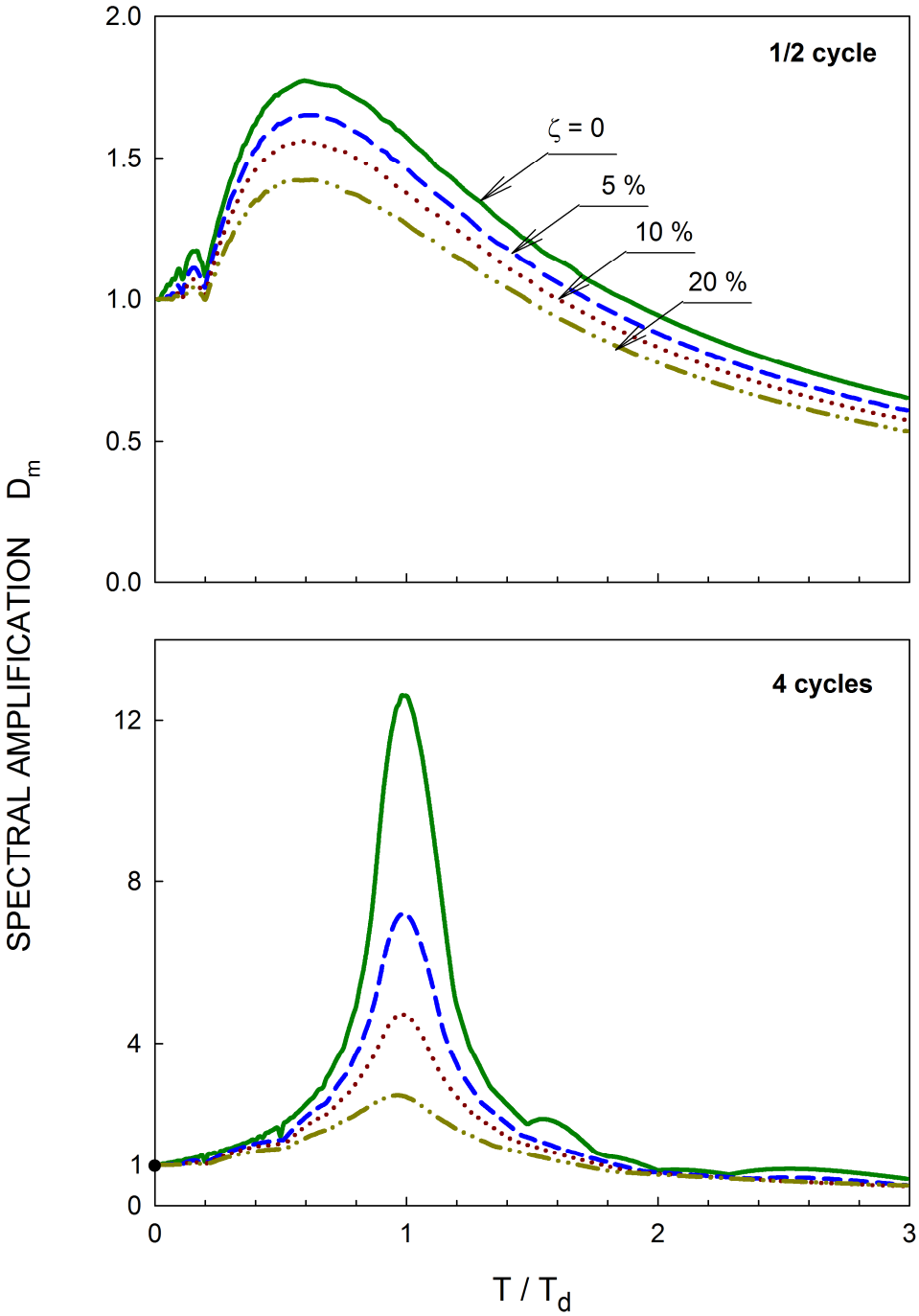


Figure 3.8 Effect of damping on elastic spectral amplification for a sinusoidal pulse.

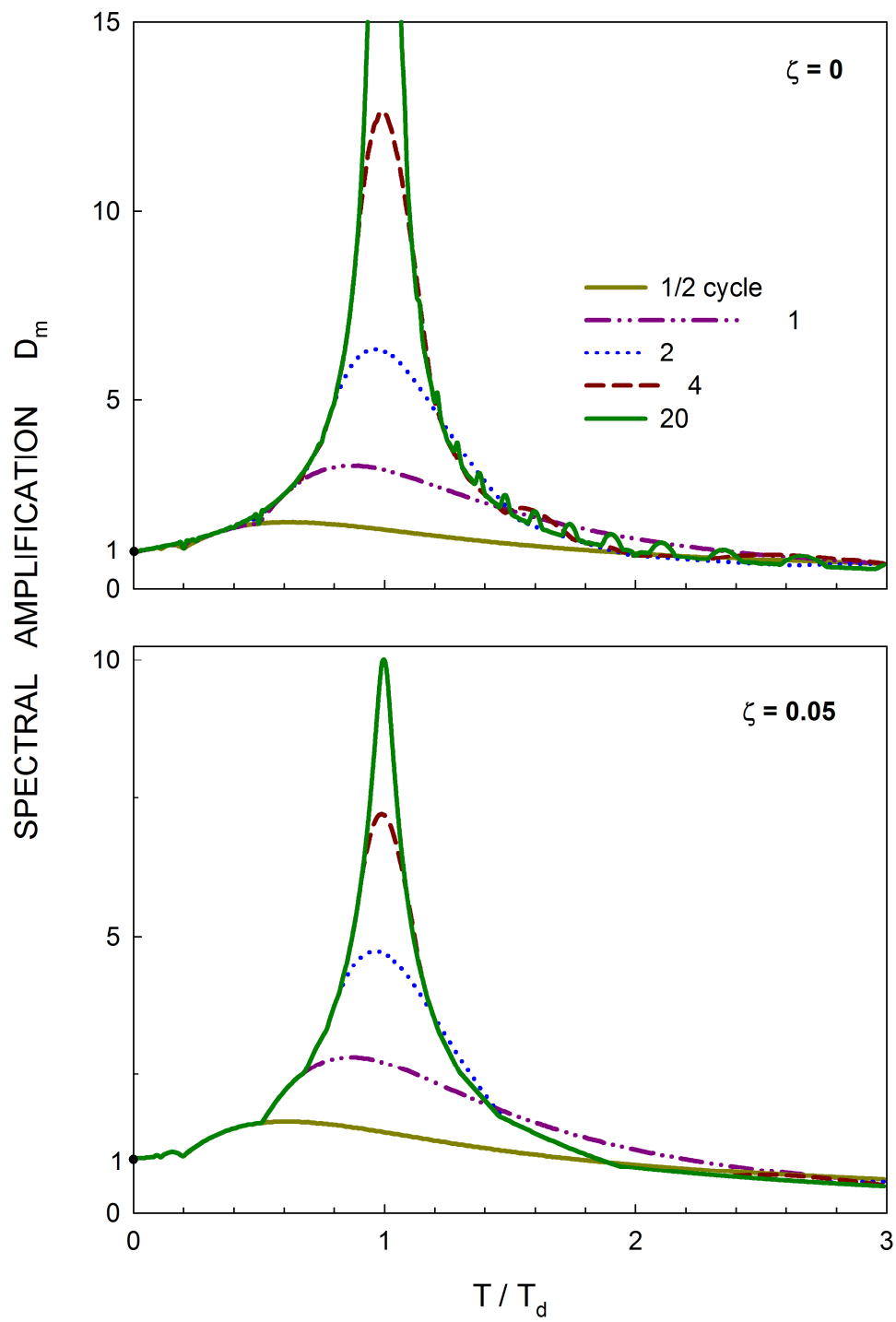


Figure 3.9 Effect of number of cycles on elastic spectral amplification for a sinusoidal pulse.

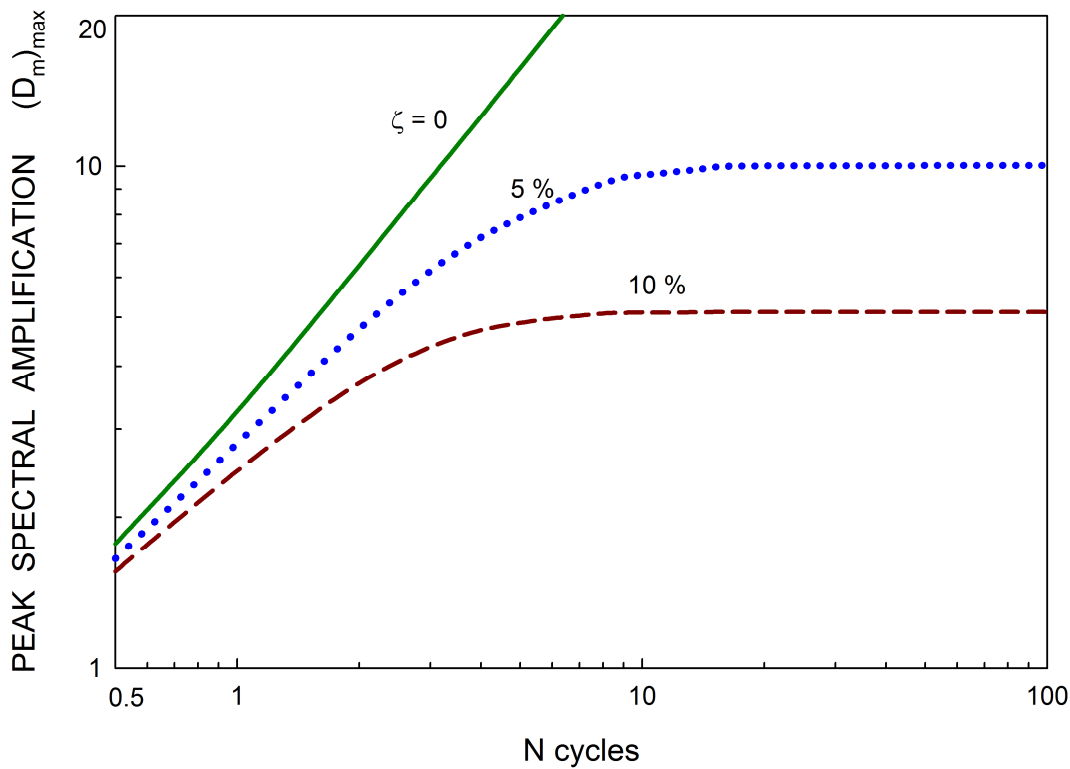


Figure 3.10 Effect of number of cycles and oscillator damping on peak spectral amplification for harmonic excitation.

3.5 Inelastic Oscillator: Parameter Study

The differential equation of dynamic equilibrium of a yielding oscillator under seismic excitation is

$$m\ddot{u} + f_d + f_s = -m\ddot{u}_g \quad (3.9)$$

in which f_s and f_d denote the bilinear restoring force and damping force, respectively.

A Newmark – beta computational scheme was employed for numerically solving Eq. (3.9). Figures 3.11 to 3.14 present seismic ductility demand, μ , as function of normalized oscillator period and strength, for two different pulse durations. The following trends are worthy of note: *Firstly*, ductility demand is a decreasing function of oscillator period (or, equivalently, an increasing function of pulse duration). *Secondly*, μ is a decreasing function of oscillator strength. *Thirdly*, for relatively weak oscillators ($\eta < 0.7$) and oscillator periods of less than 1, μ may attain extremely high values (above 10), which far exceed the ductility capacity of conventionally designed structures. *Fourthly*, the ductility demand generated by a rectangular pulse exceeds that of the other pulses for the same oscillator strength. This is in contrast to the results for elastic systems presented earlier. *Fifthly*, the peak ductility generally occurs at periods smaller than the resonant period $T/T_d = 1$. This is an anticipated trend as a stiff oscillator ($T/T_d \ll 1$) is expected to respond more strongly after yielding, as its period lengthens approaching the resonant value T_d . On the contrary, a resonant oscillator by elastic considerations ($T/T_d = 1$) is expected to respond less strongly, as its period moves away from resonance after yielding.

The effect of number of cycles on ductility demand is presented in Figures 3.15 to 3.18 for two different oscillator strengths. For “weak” ($\eta = 0.3$) oscillators, three distinct

period ranges are observed: For periods greater than approximately 0.8, ductility demand is essentially *independent* of number of cycles (an exception is the case of the half-cycle pulse which is discussed below). For somewhat stiffer systems ($0.2 < T/T_d < 0.8$), ductility demand increases substantially with number of cycles (*inelastic resonance*). For very stiff oscillators ($T/T_d < 0.2$), the response becomes again independent of N . The peak ductility occurs at around $T/T_d = 0.3$ for all but the exponential pulse. Similar (yet not identical) trends are observed for strong ($\eta = 0.7$) oscillators.

An interesting special case is observed that for $N = 1/2$. Indeed, a half-cycle pulse generates higher ductility demand than pulses with greater number of cycles for weak oscillators and $T/T_d > 0.8$. With strong oscillators this trend is visible only in the rectangular pulse (Fig 3.18).

An alternative representation of the above results is provided in Figures 3.19 to 3.22. In these, force reduction factor $R (= SA g / Q_y)$ is used instead of η . The variation in ductility demand with both period and oscillator strength is significantly more complex than with the previous representation, suggesting that R is not well suited for expressing oscillator strength for this type of loading. A possible explanation is that the response of a yielding oscillator to impulse loading is very different than the response of an elastic oscillator of similar vibrational characteristics and, thereby, using R in the presentation of results essentially introduces an uncorrelated parameter, which complicates the interpretation of the data.

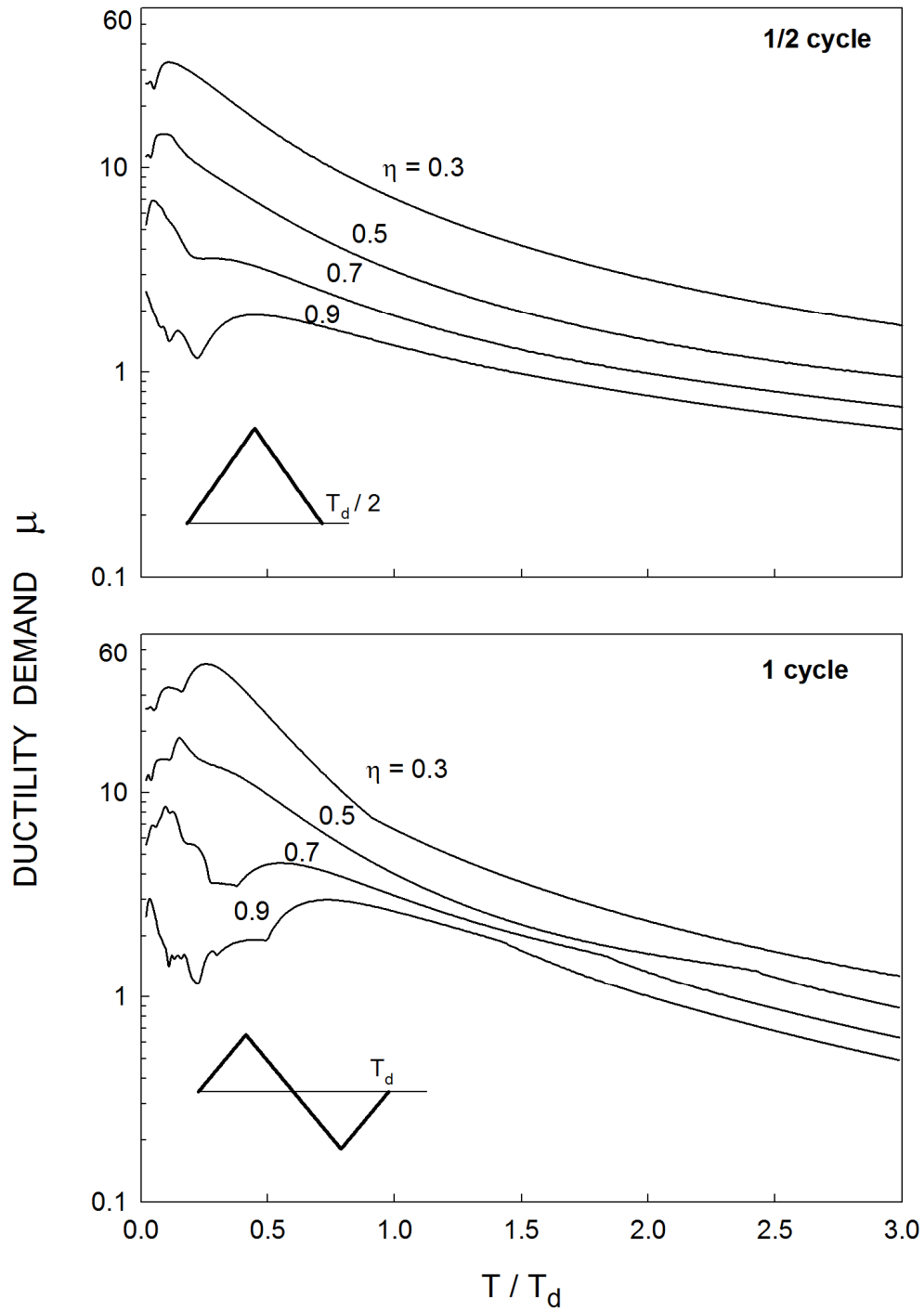


Figure 3.11 Ductility demand of a simple oscillator subjected to triangular pulse excitation; $\gamma=0.10$; $\zeta=5\%$

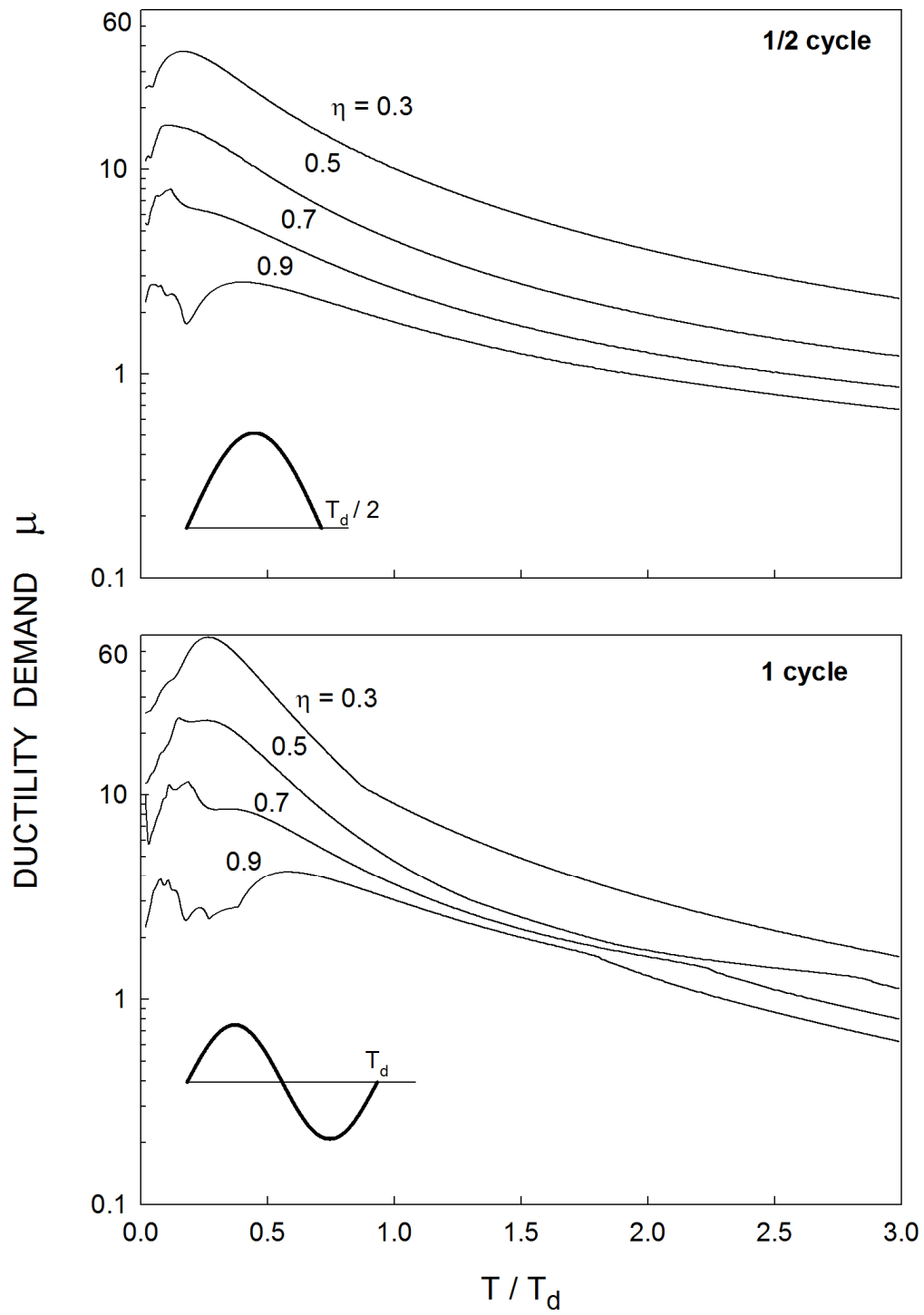


Figure 3.12 Ductility demand of a simple oscillator subjected to sinusoidal pulse excitation; $\gamma=0.10$; $\zeta=5\%$

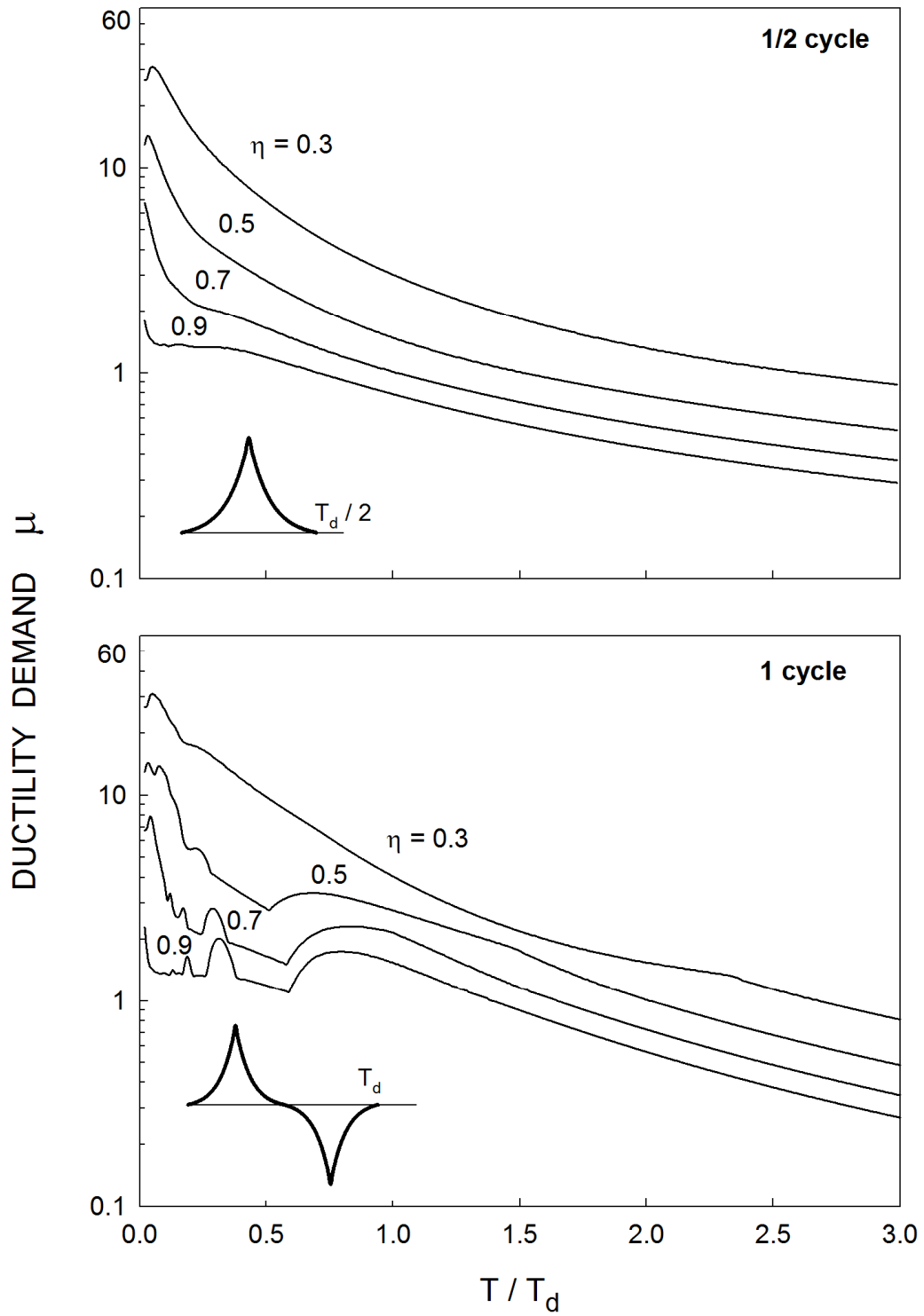


Figure 3.13 Ductility demand of a simple oscillator subjected to exponential pulse excitation; $\gamma=0.10$; $\zeta=5\%$

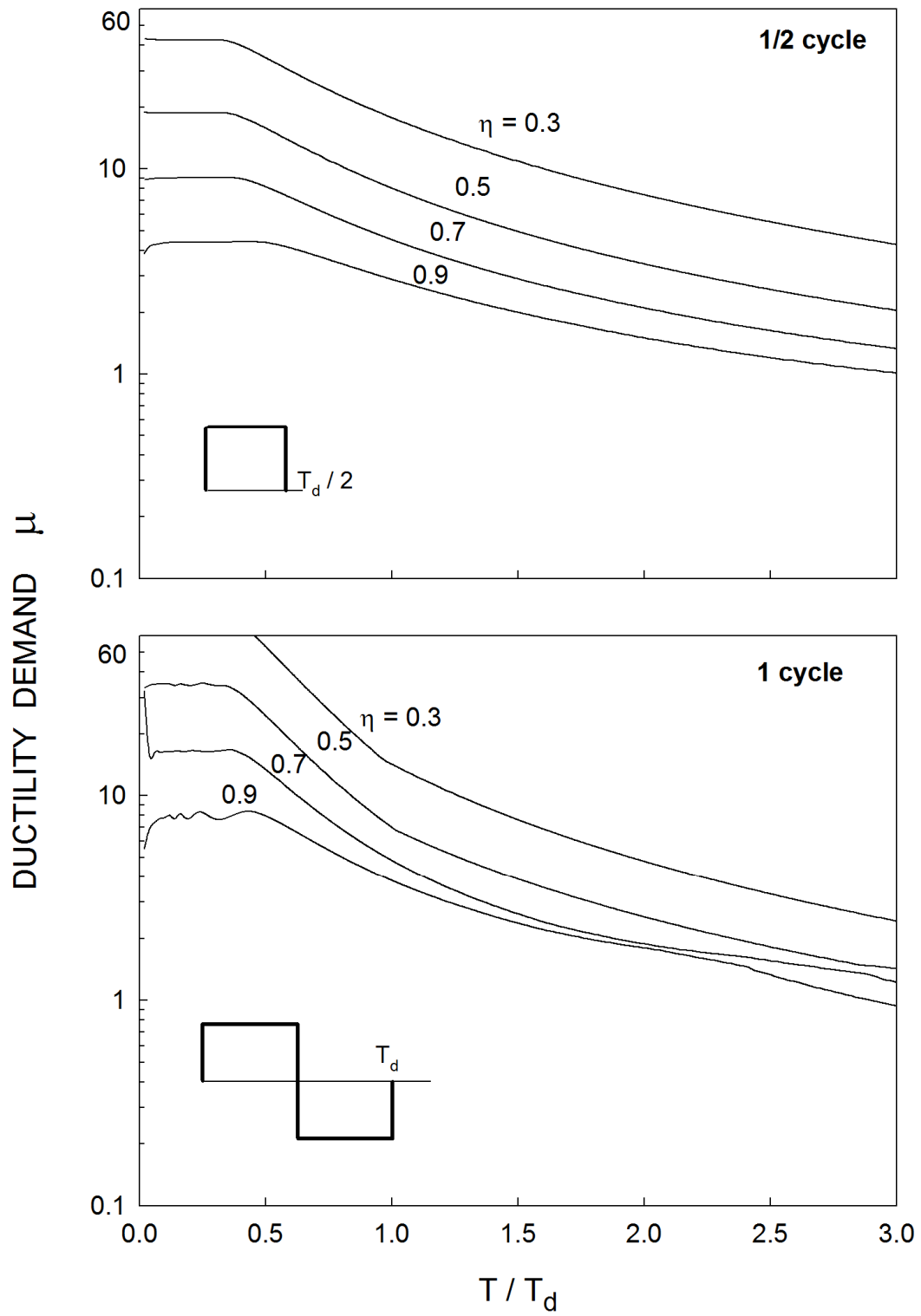


Figure 3.14 Ductility demand of a simple oscillator subjected to rectangular pulse excitation; $\gamma=0.10$; $\zeta=5\%$

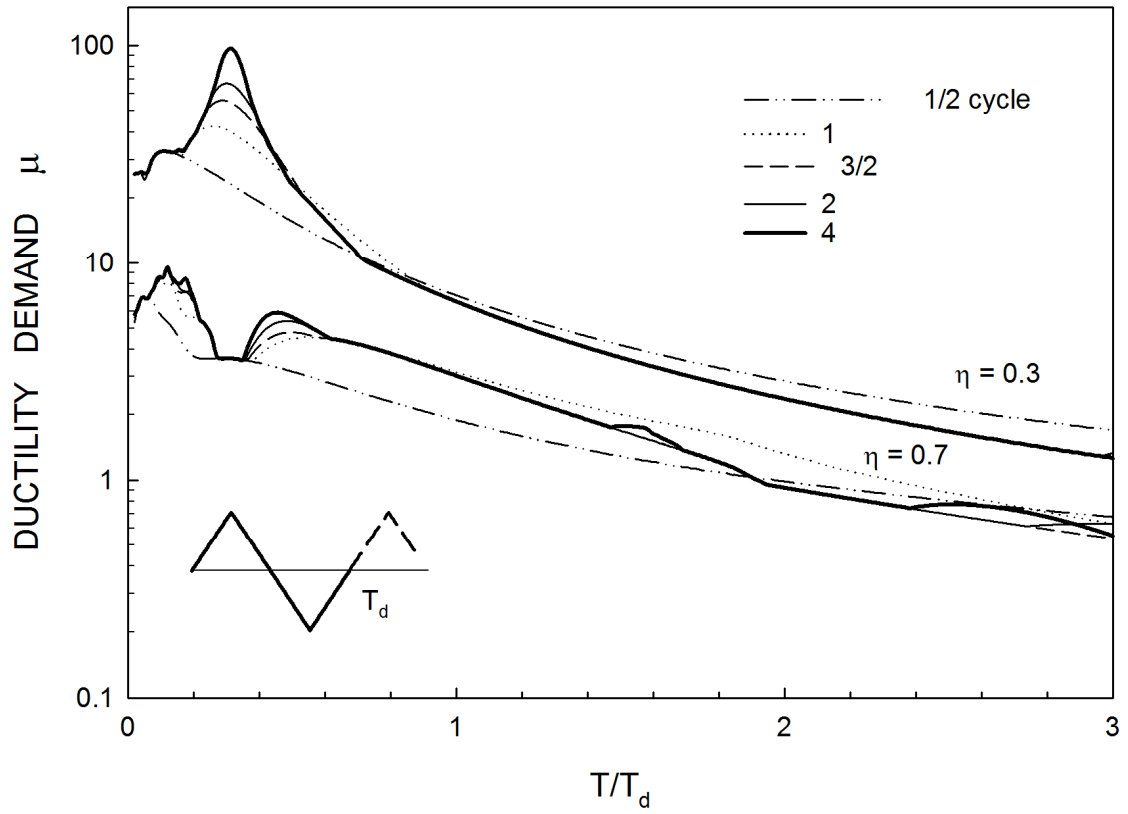


Figure 3.15 Effect of number of cycles on ductility demand due to triangular pulse excitation; $\gamma=0.10$; $\zeta=5\%$

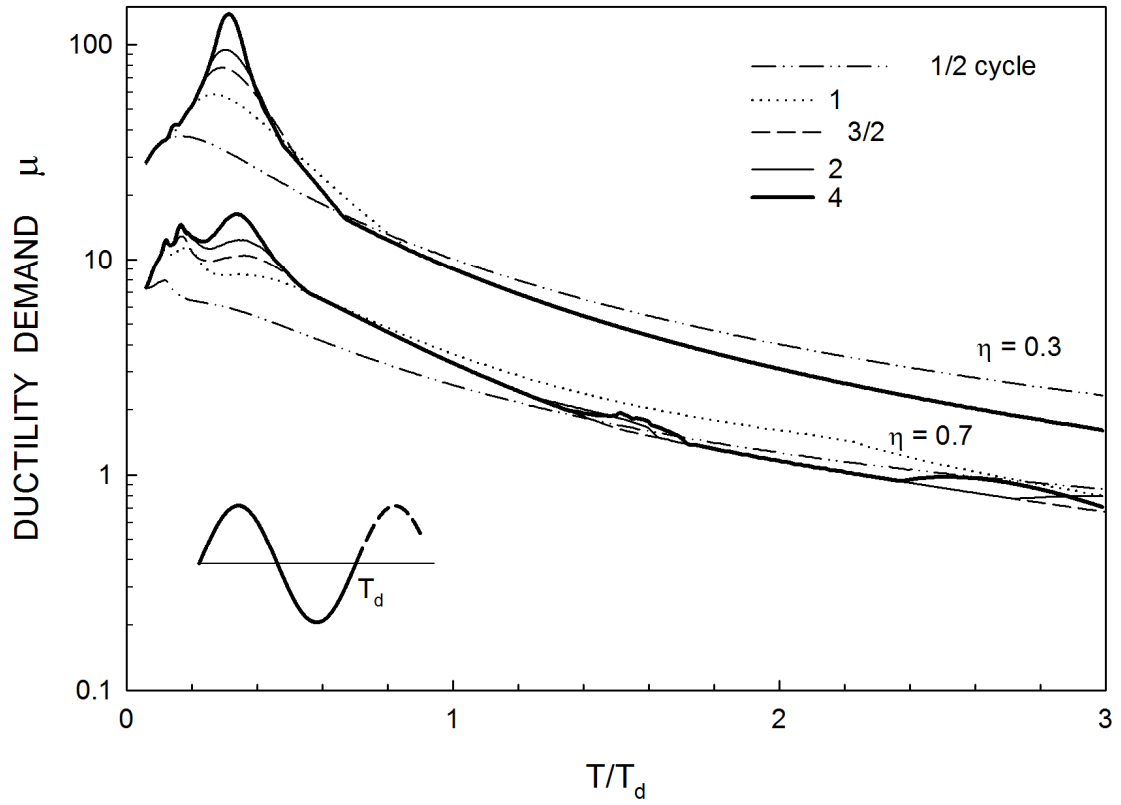


Figure 3.16 Effect of number of cycles on ductility demand due to sinusoidal pulse excitation; $\gamma=0.10$; $\zeta=5\%$

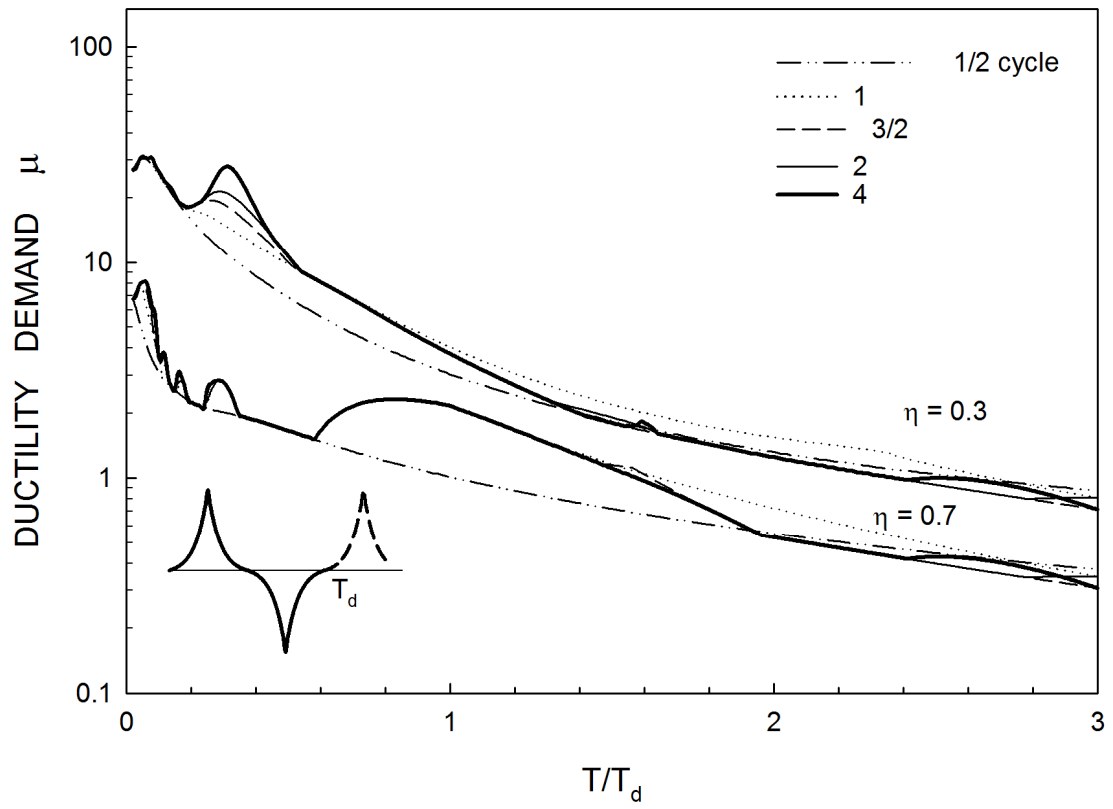


Figure 3.17 Effect of number of cycles on ductility demand due to exponential pulse excitation; $\gamma=0.10$; $\zeta=5\%$

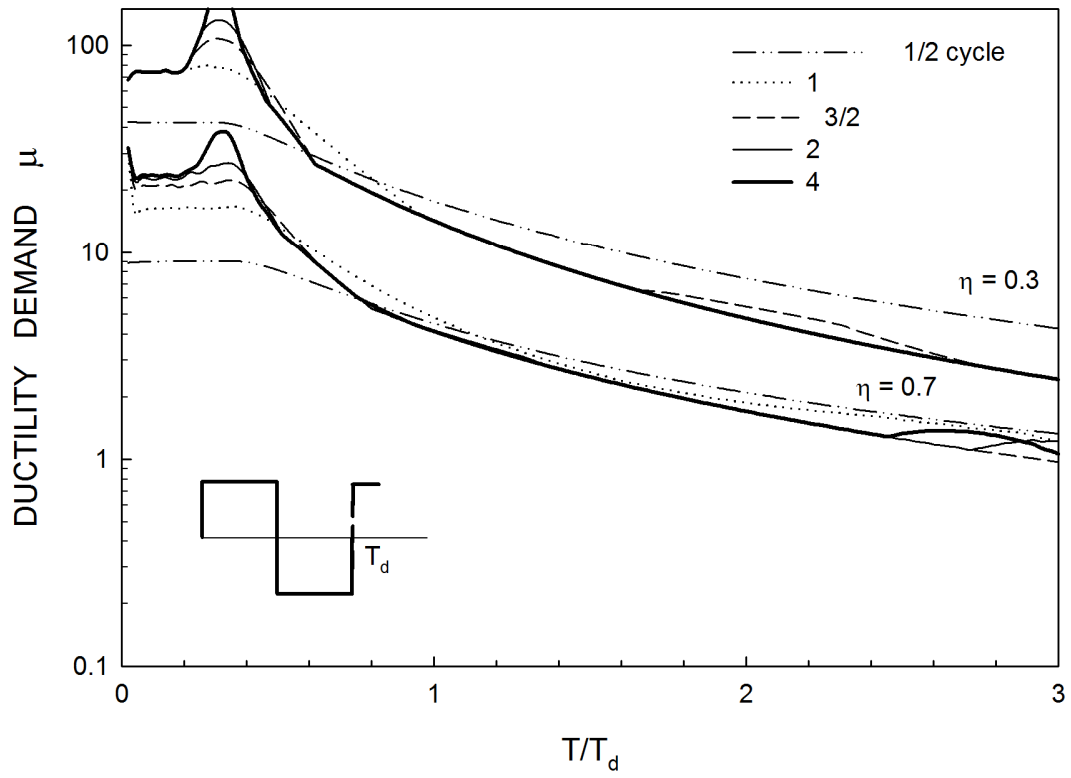


Figure 3.18 Effect of number of cycles on ductility demand due to rectangular pulse excitation; $\gamma=0.10$; $\zeta=5\%$

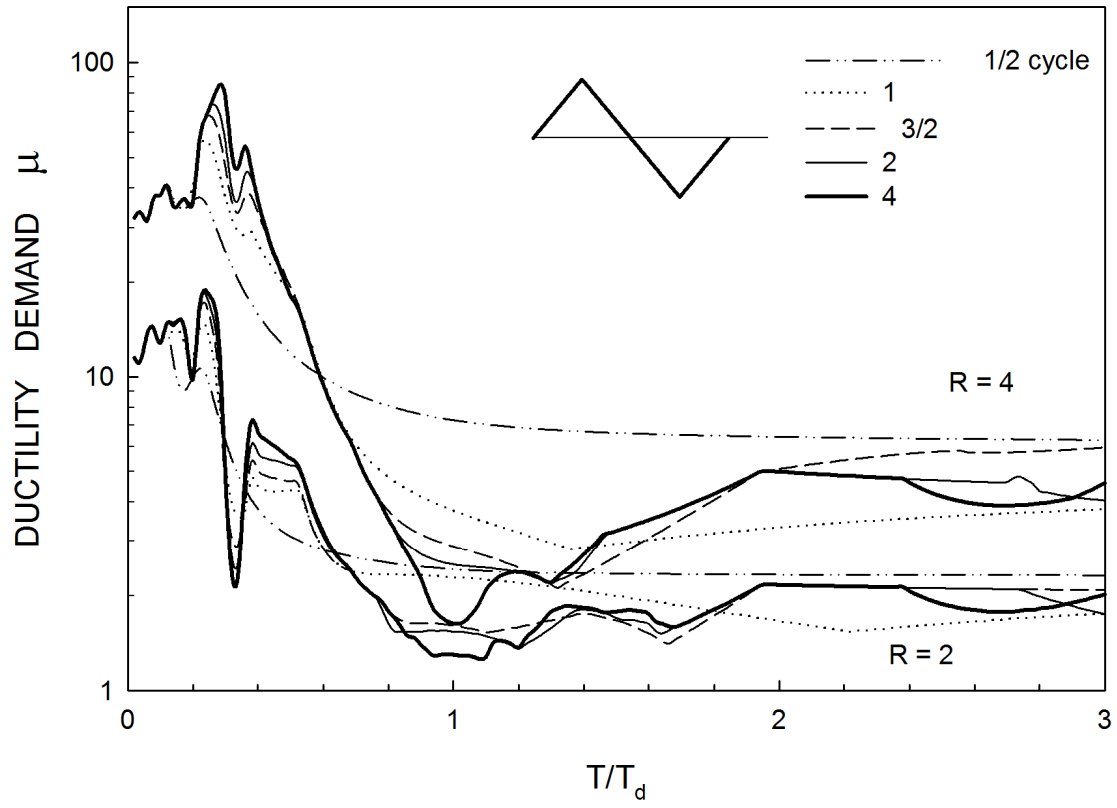


Figure 3.19 Effect of number of cycles on ductility demand due to triangular pulse excitation for different values of the force reduction factor R ; $\gamma=0.10$; $\zeta=5\%$

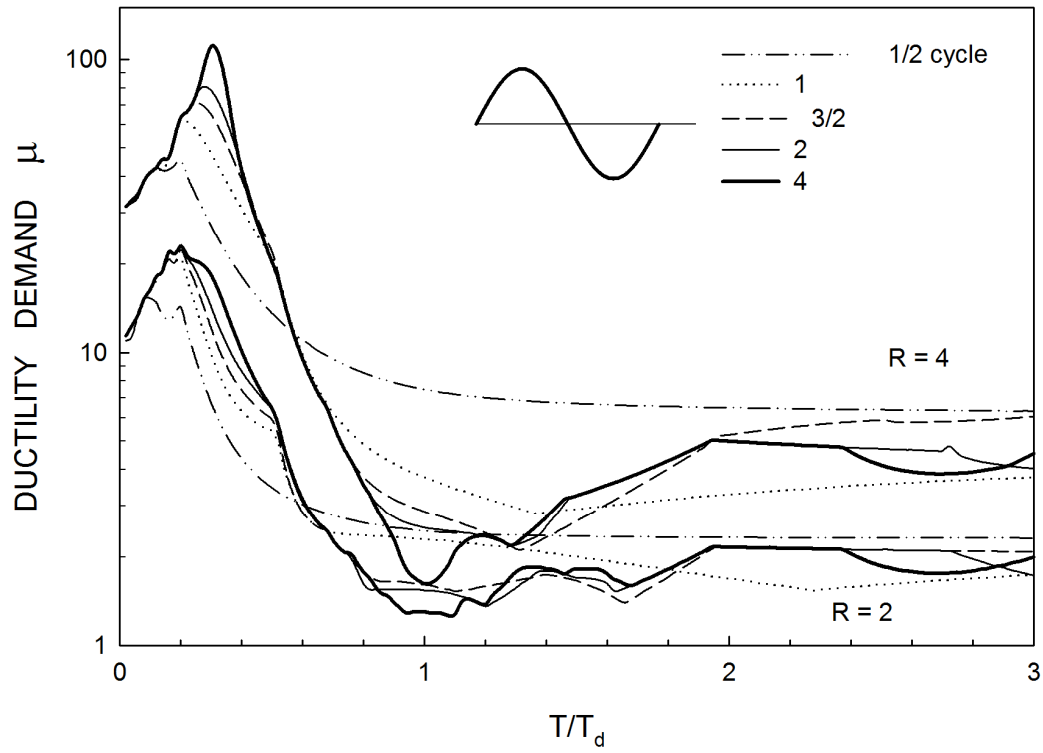


Figure 3.20 Effect of number of cycles on ductility demand due to sinusoidal pulse excitation for different values of the force reduction factor R ; $\gamma=0.10$; $\zeta=5\%$

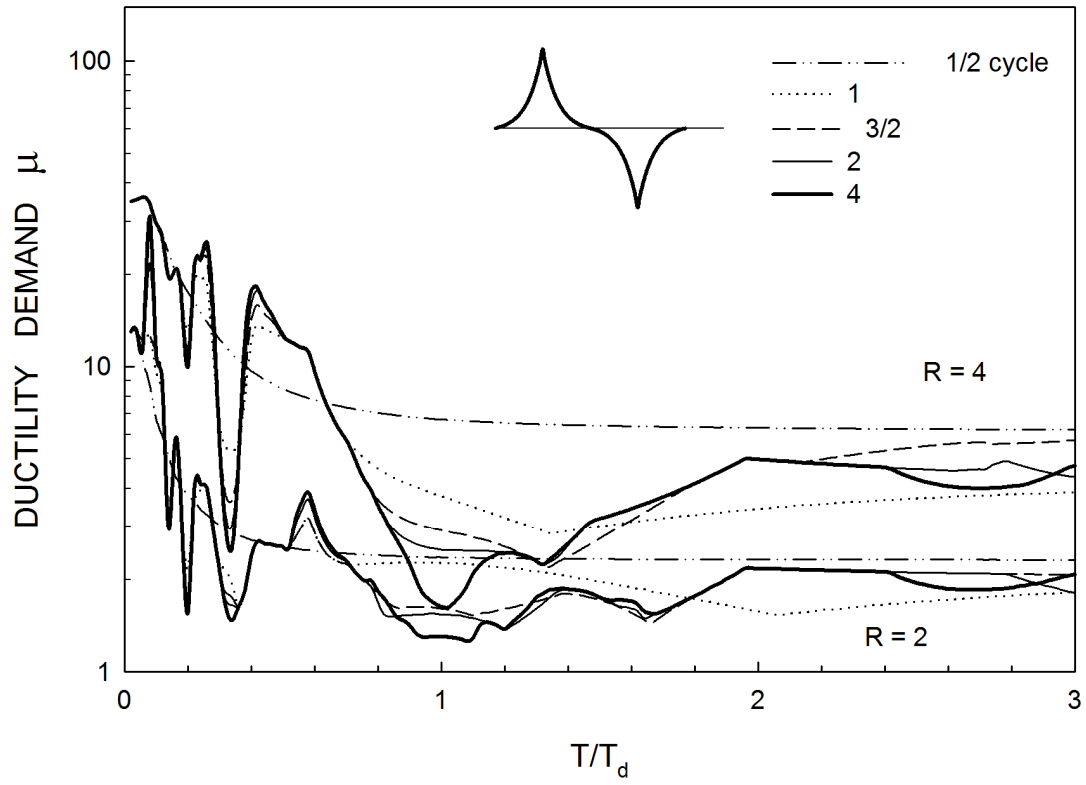


Figure 3.21 Effect of number of cycles on ductility demand due to exponential pulse excitation for different values of the force reduction factor R ; $\gamma=0.10$; $\zeta=5\%$

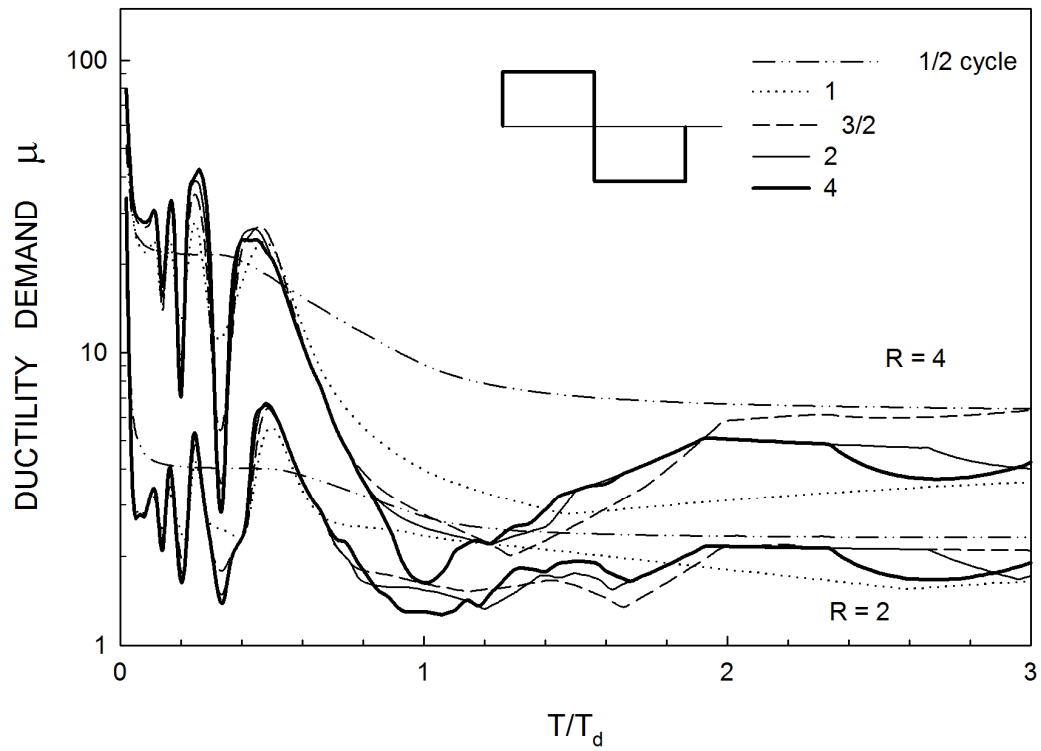


Figure 3.22 Effect of number of cycles on ductility demand due to rectangular pulse excitation for different values of the force reduction factor R ; $\gamma=0.10$; $\zeta=5\%$

Figures 3.23 to 3.26 depict the effect of damping on inelastic response. Increasing ζ reduces response in accordance with the trends observed for the elastic oscillator. The reduction is more or less the same in all frequency ranges, oscillator strengths, and pulse types. This is reflected in the normalized results presented in Figures 3.27 to 3.30.

Based on these results, the following expression was developed to account for the variation in response with damping, which pertains to all pulse types, oscillator periods, and strengths.

$$\frac{\mu(\zeta)}{\mu(5\%)} = \left[\frac{100}{95 + \zeta(\%)} \right]^{\frac{11}{4}} \quad (3.10)$$

The coefficient of correlation, R^2 , of the above expression is equal to 0.94.

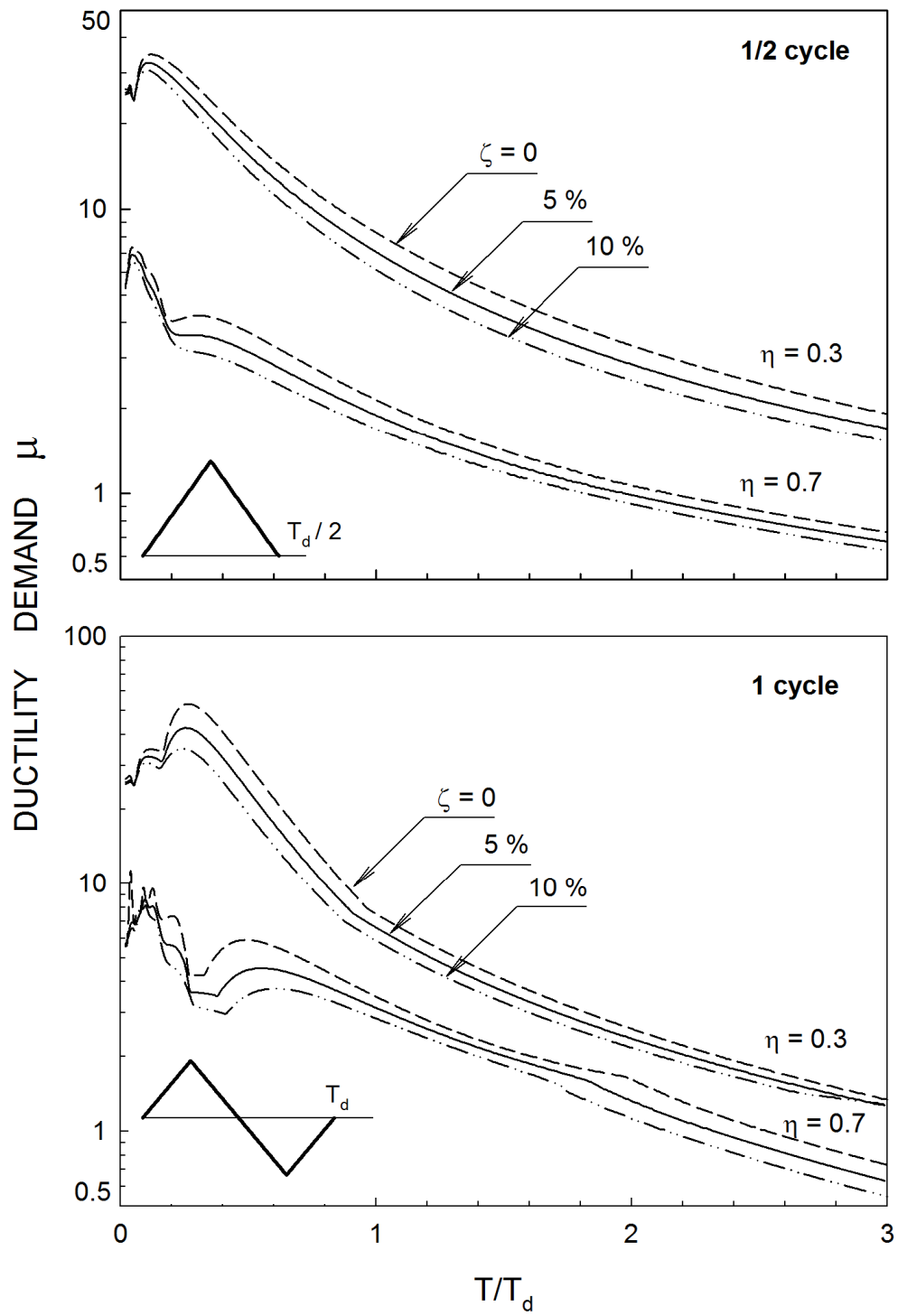


Figure 3.23 Effect of damping on ductility demand due to a triangular pulse; $\gamma=0.10$

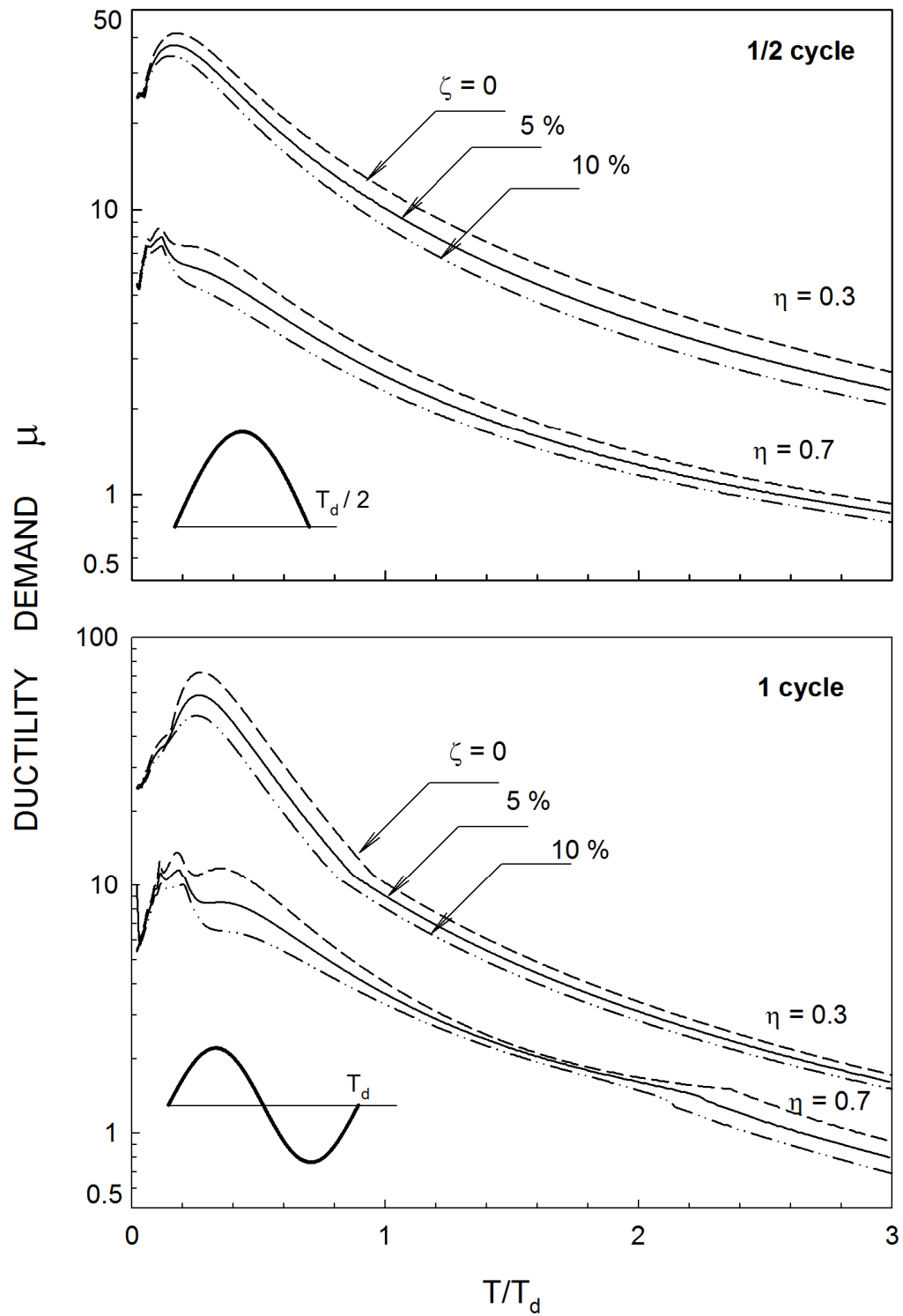


Figure 3.24 Effect of damping on ductility demand due to a sinusoidal pulse; $\gamma=0.10$

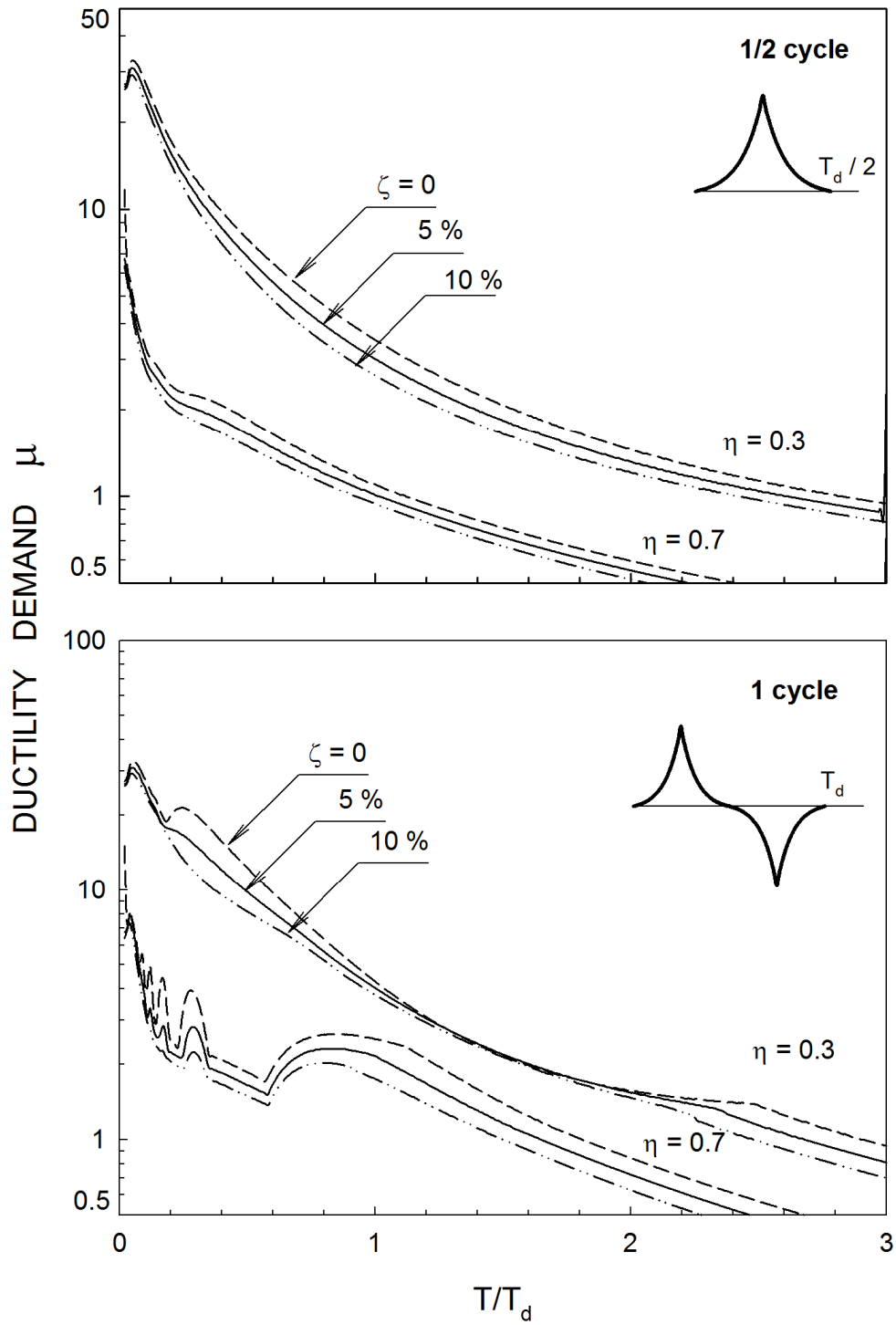


Figure 3.25 Effect of damping on ductility demand due to an exponential pulse; $\gamma=0.10$

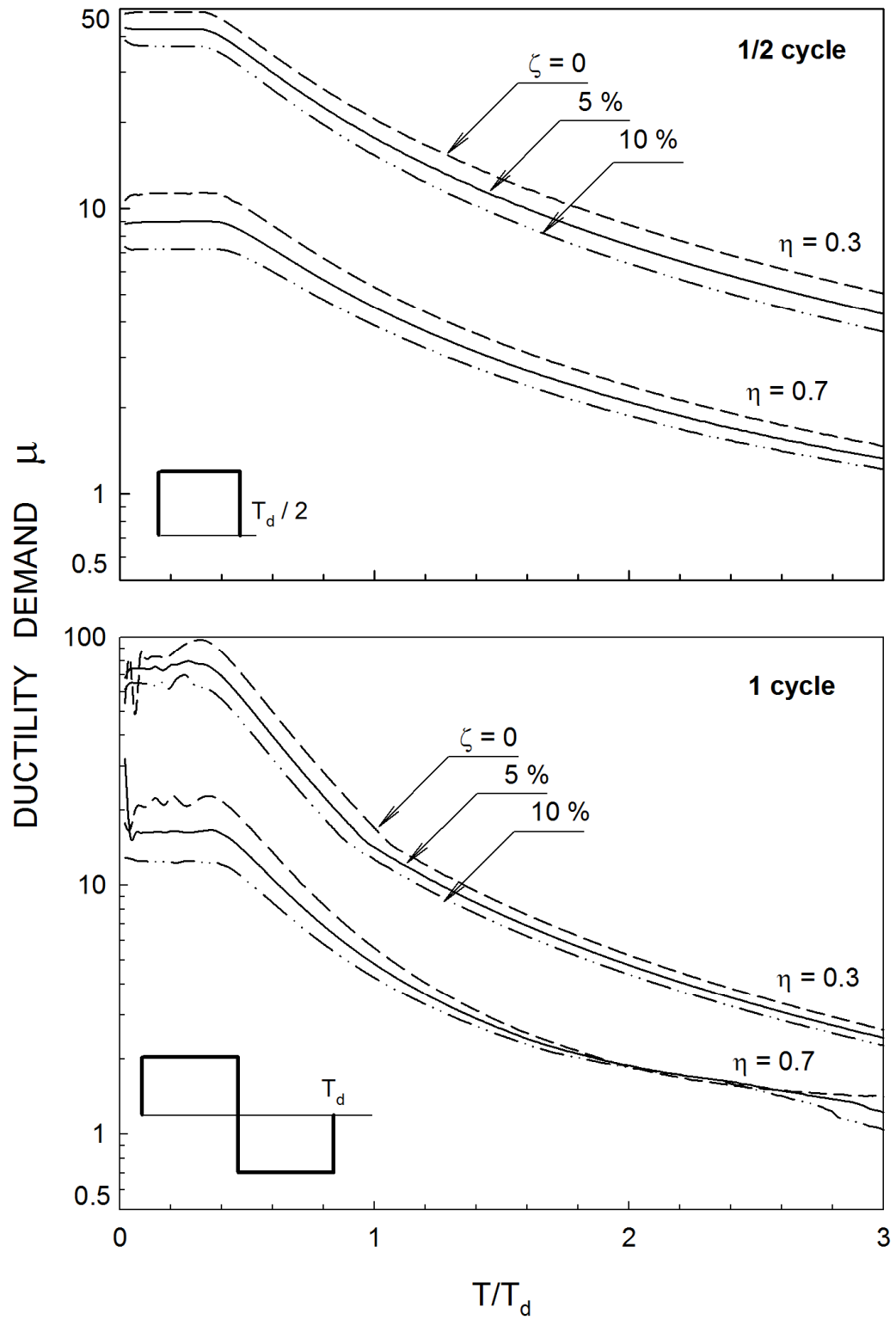


Figure 3.26 Effect of damping on ductility demand due to a rectangular pulse; $\gamma=0.10$

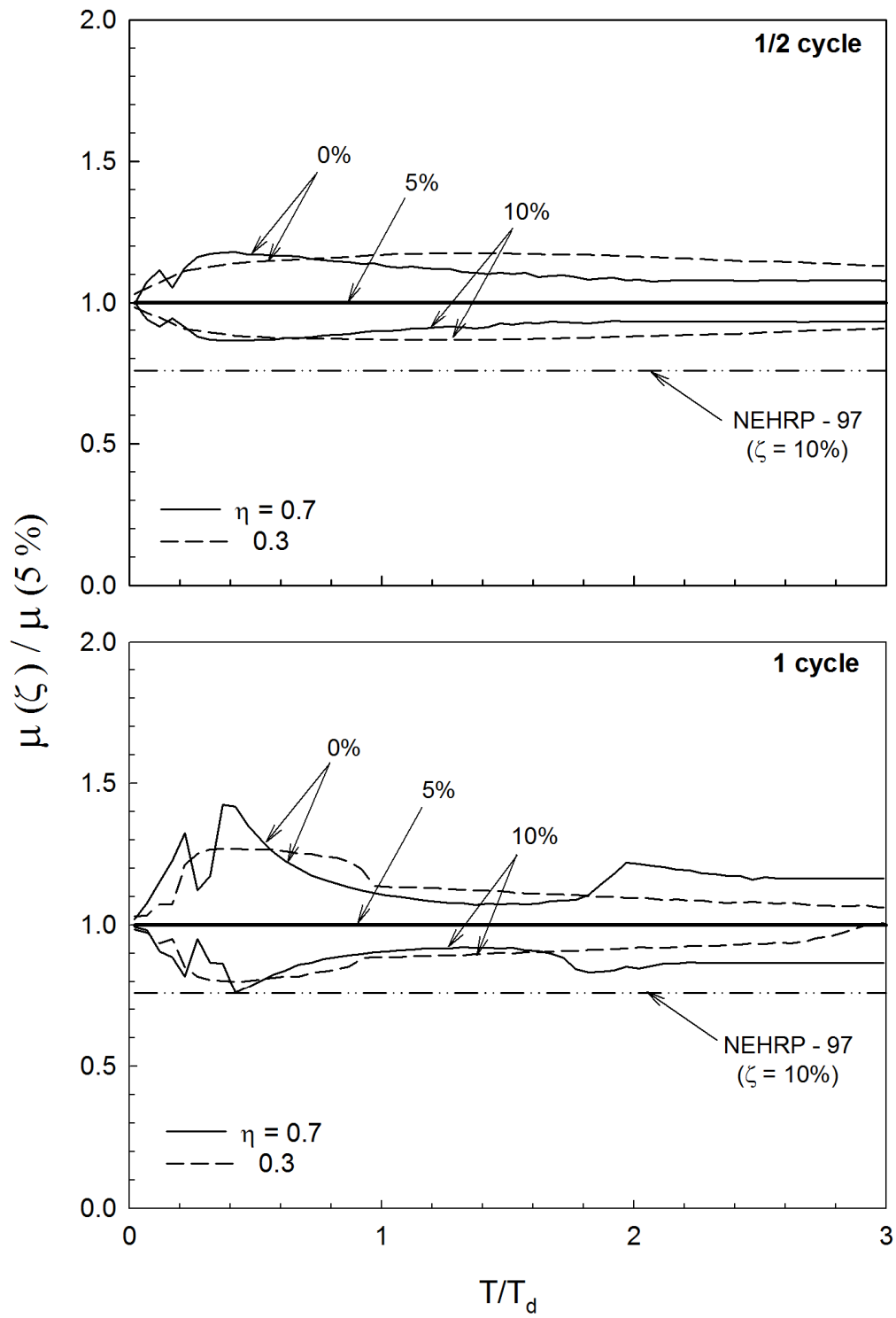


Figure 3.27 Effect of damping on ductility demand due to a triangular pulse; $\gamma=0.10$

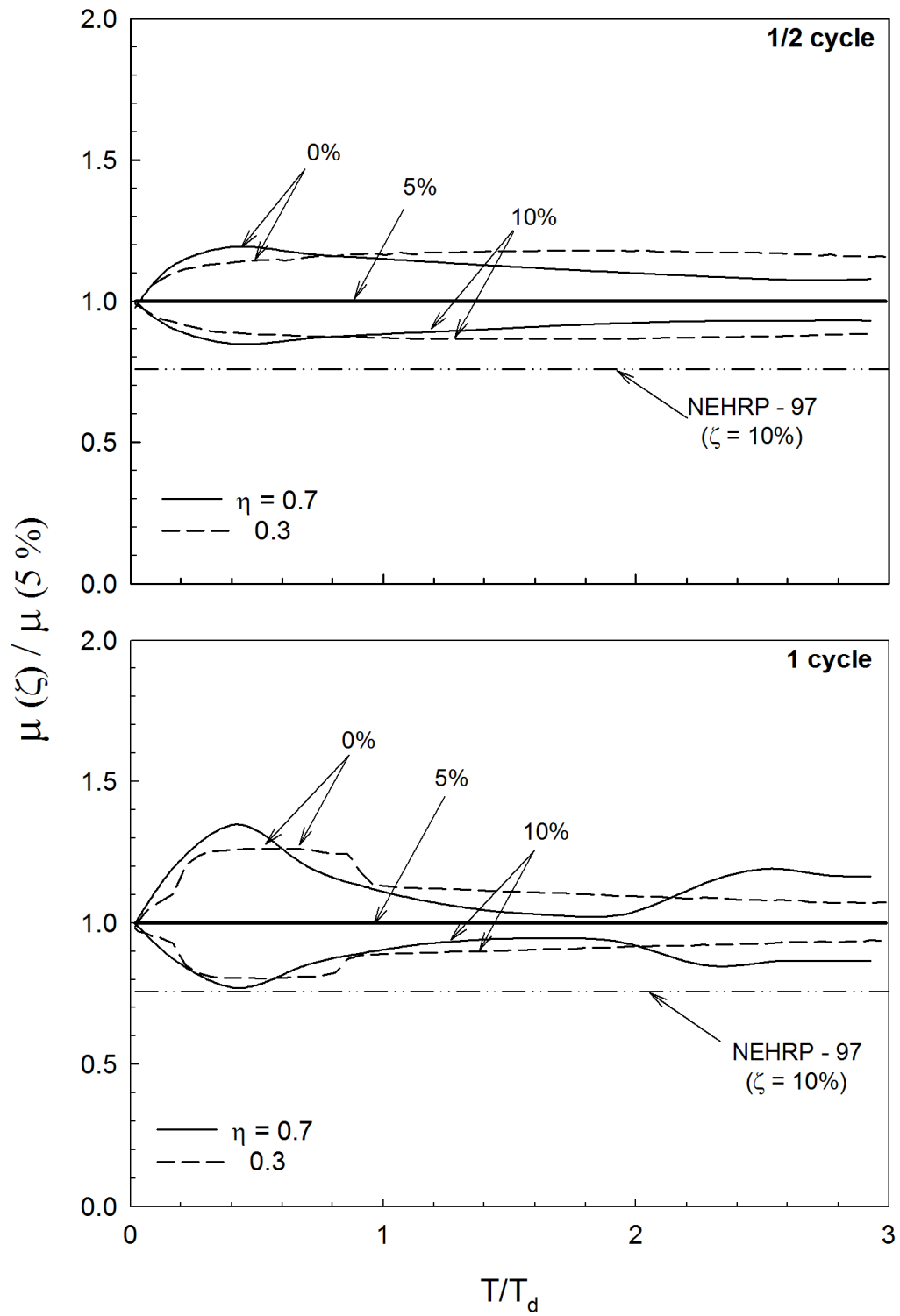


Figure 3.28 Effect of damping on ductility demand due to a sinusoidal pulse; $\gamma=0.10$

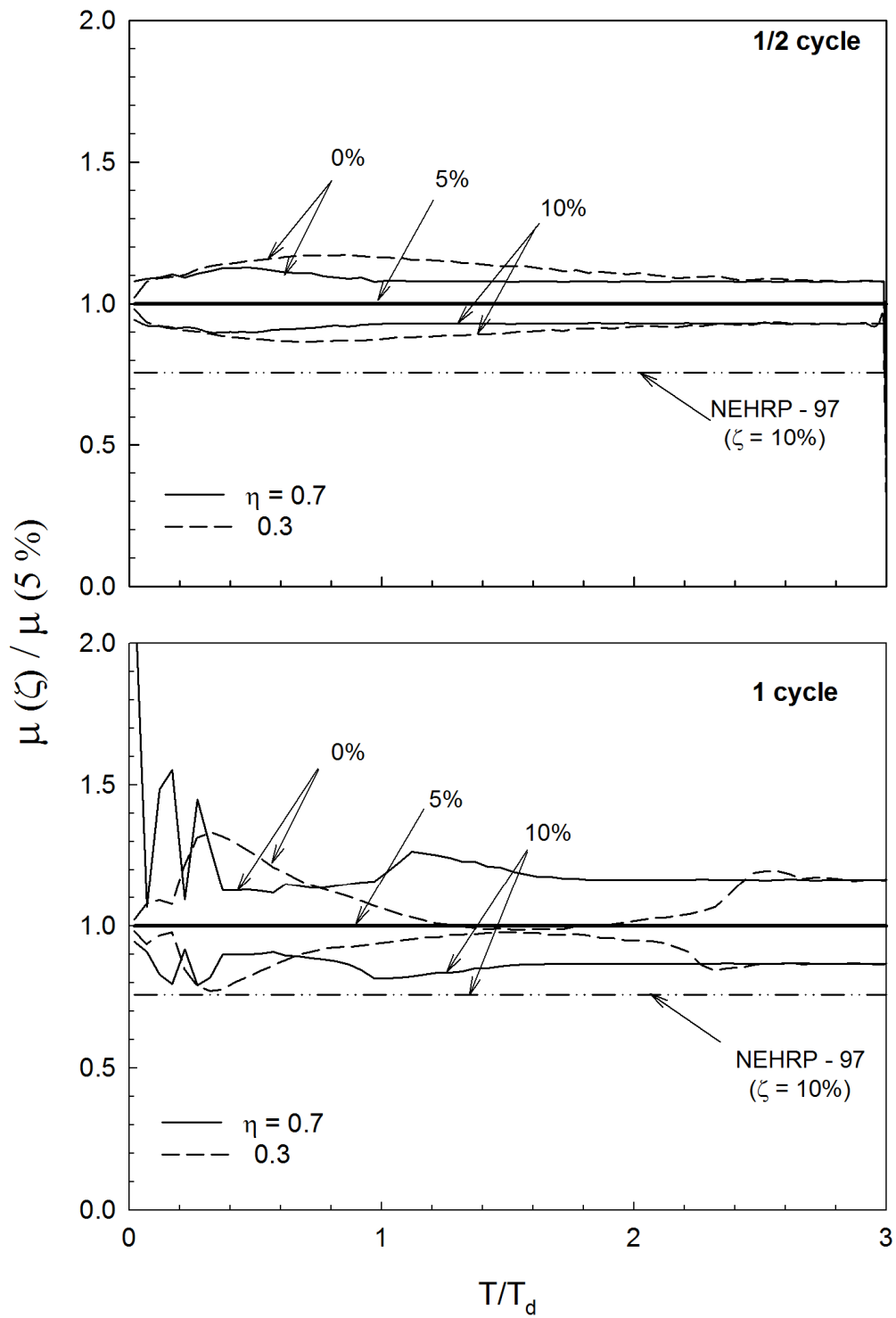


Figure 3.29 Effect of damping on ductility demand due to an exponential pulse; $\gamma=0.10$

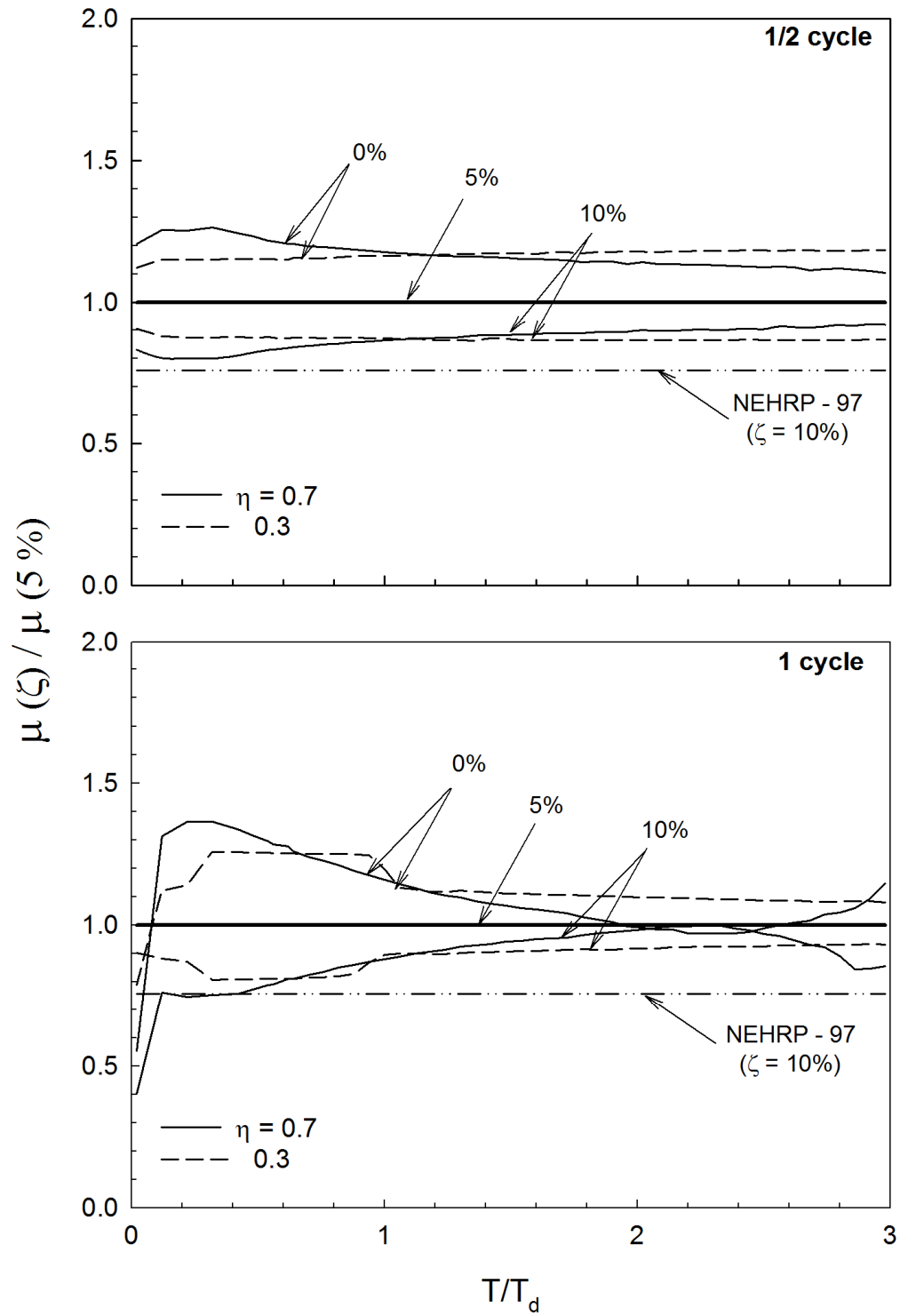


Figure 3.30 Effect of damping on ductility demand due to a rectangular pulse; $\gamma=0.10$

The effect of post-yielding stiffness is examined in Figures 3.31 to 3.34 for two different pulse durations and oscillator strengths. Two distinct patterns are observed: *First*, for a half-cycle pulse ($N = 1/2$) ductility demand is a *decreasing* function of γ . The trend is understandably stronger for weak ($\eta = 0.3$) oscillators. *Second*, for a full-cycle pulse ($N = 1$) the response may increase or decrease with increasing γ . A possible explanation for this counterintuitive trend is that peak response is attained after a motion reversal (i.e., in the direction opposite to the initial oscillator movement). This implies that the higher the value of γ , the more energy is stored in the initial “forward” movement, the higher the displacement will be after the reversal. Evidence for this behavior is presented in the sequel.

The effect of post-yielding stiffness is further examined in Figures 3.35 to 3.38 in the form of the dimensionless ratio $\mu(\gamma)/\mu(0)$. Based on these graphs, the following regression formula was fitted to the results for a half-cycle pulse ($N = 1/2$):

$$\frac{\mu(\gamma)}{\mu(0)} = \left[\frac{(T/T_d)^2 + (T/T_d)}{(T/T_d)^2 + 3/4(T/T_d) + 3/2} \right]^{\frac{3(\gamma)}{2(\eta)}} \quad (3.11)$$

The coefficient of correlation, R^2 , of Eq. (3.11) is 0.90. The fitted formula is graphically presented in Figure 3.39.

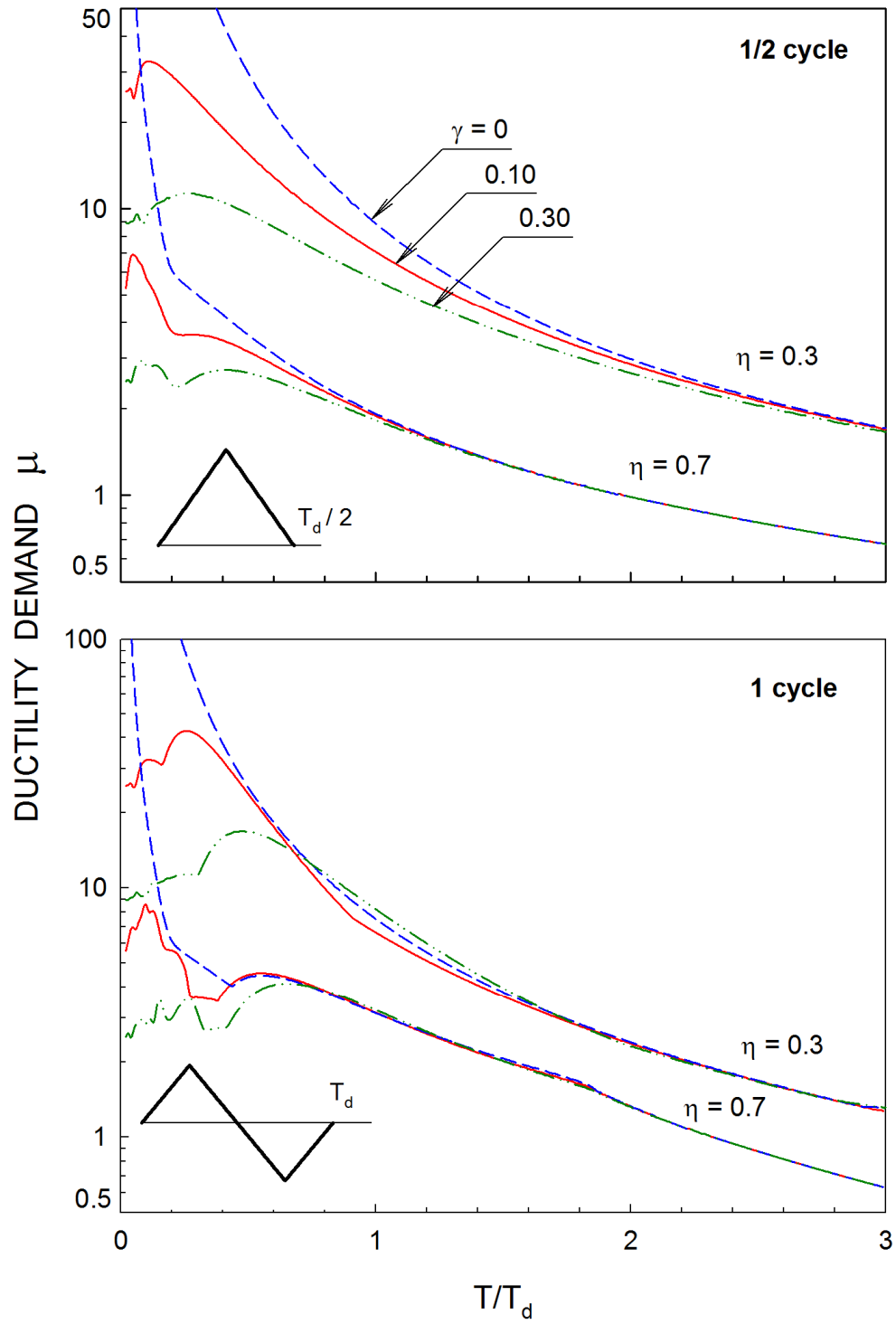


Figure 3.31 Effect of post-yielding stiffness on ductility demand due to a triangular pulse; $\zeta=5\%$

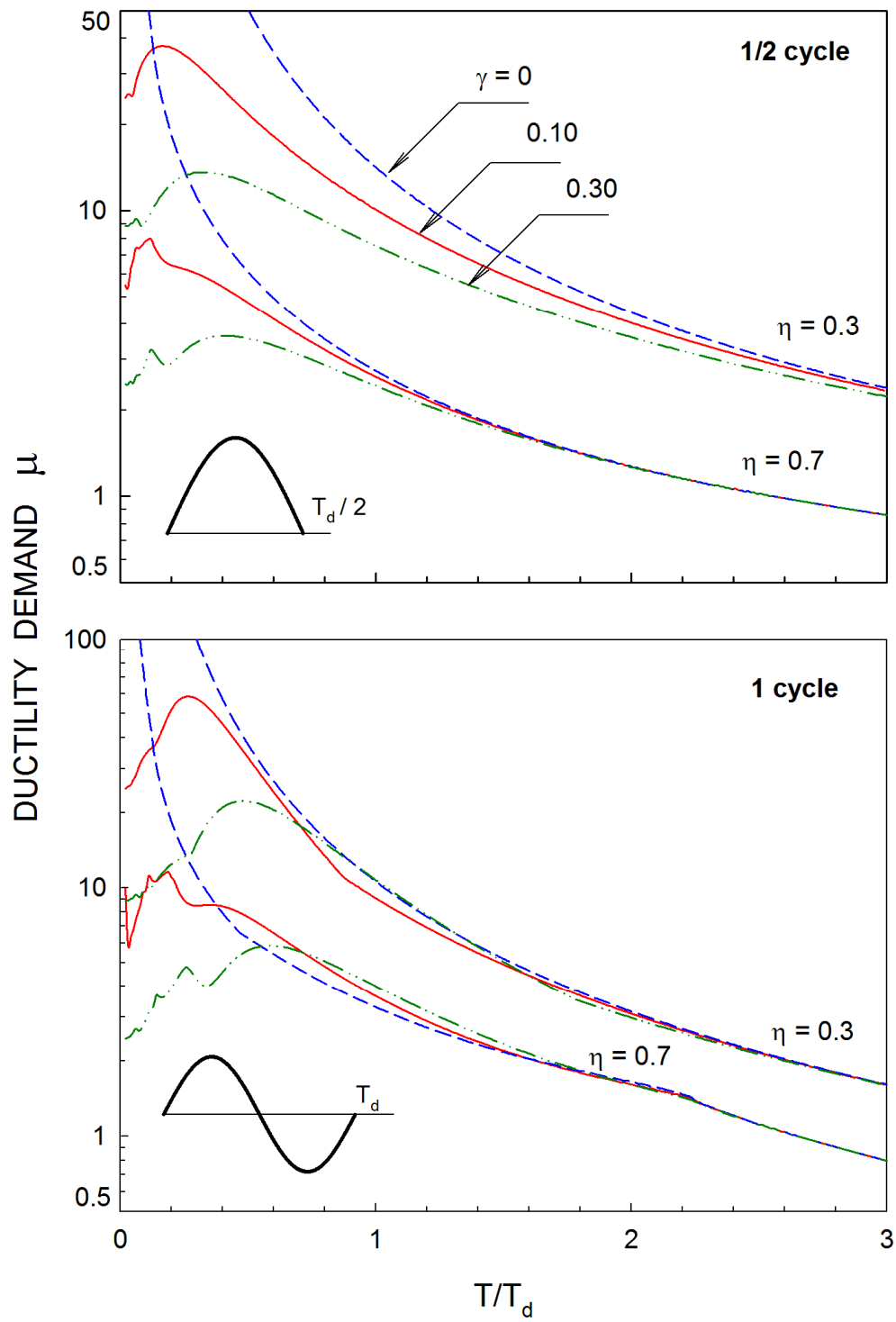


Figure 3.32 Effect of post-yielding stiffness on ductility demand due to a sinusoidal pulse; $\zeta=5\%$

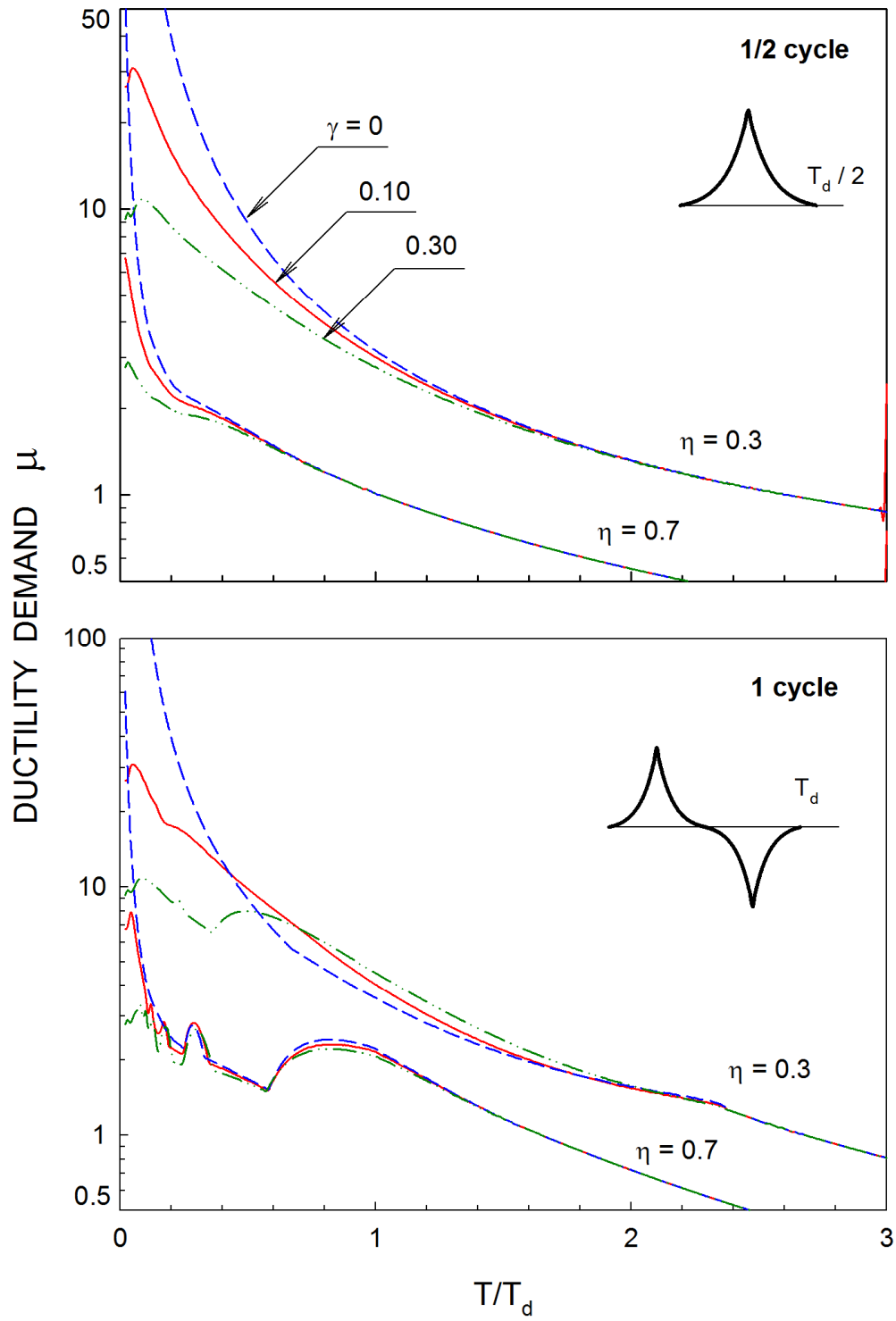


Figure 3.33 Effect of post-yielding stiffness on ductility demand due to an exponential pulse; $\zeta=5\%$

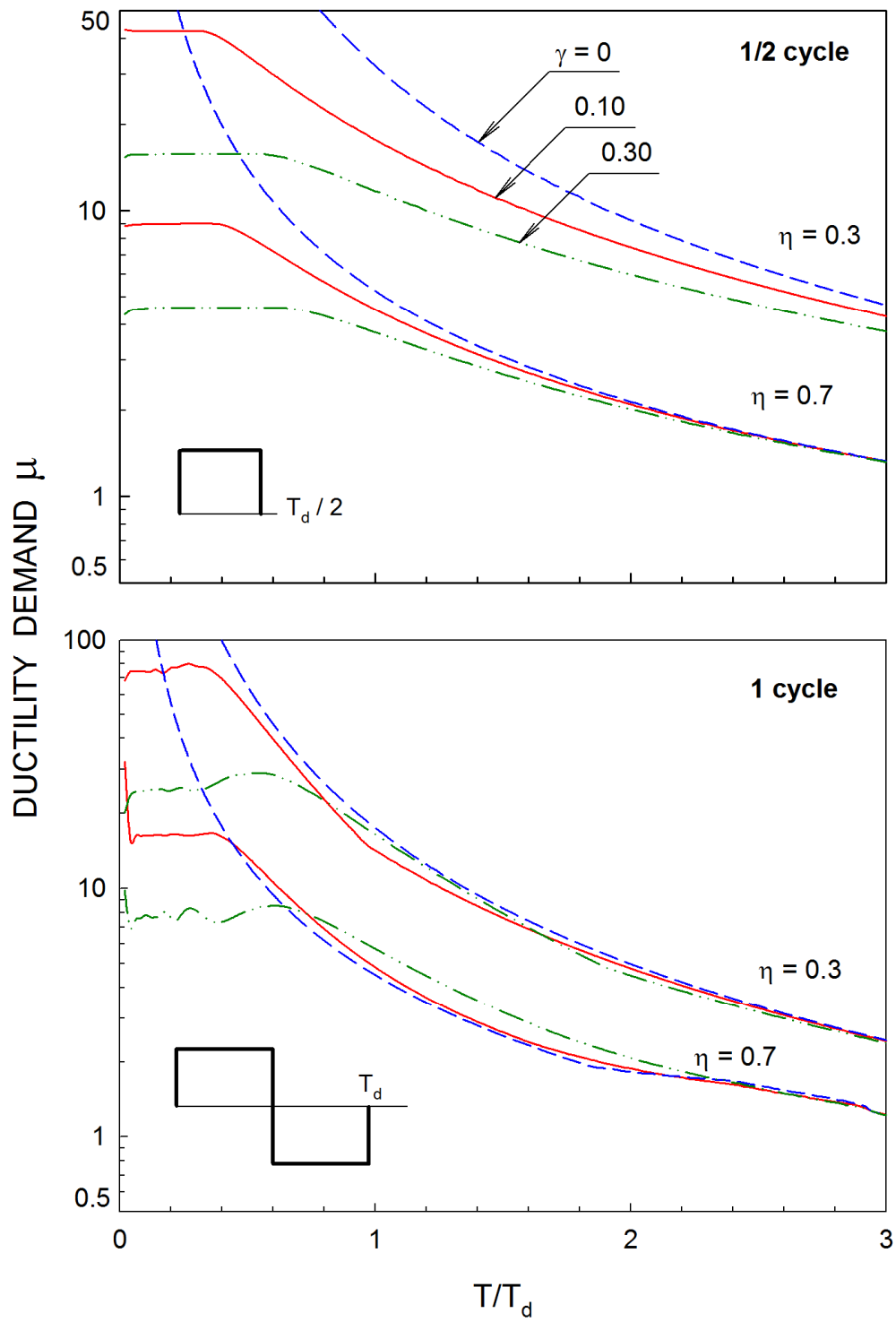


Figure 3.34 Effect of post-yielding stiffness on ductility demand due to a rectangular pulse; $\zeta=5\%$

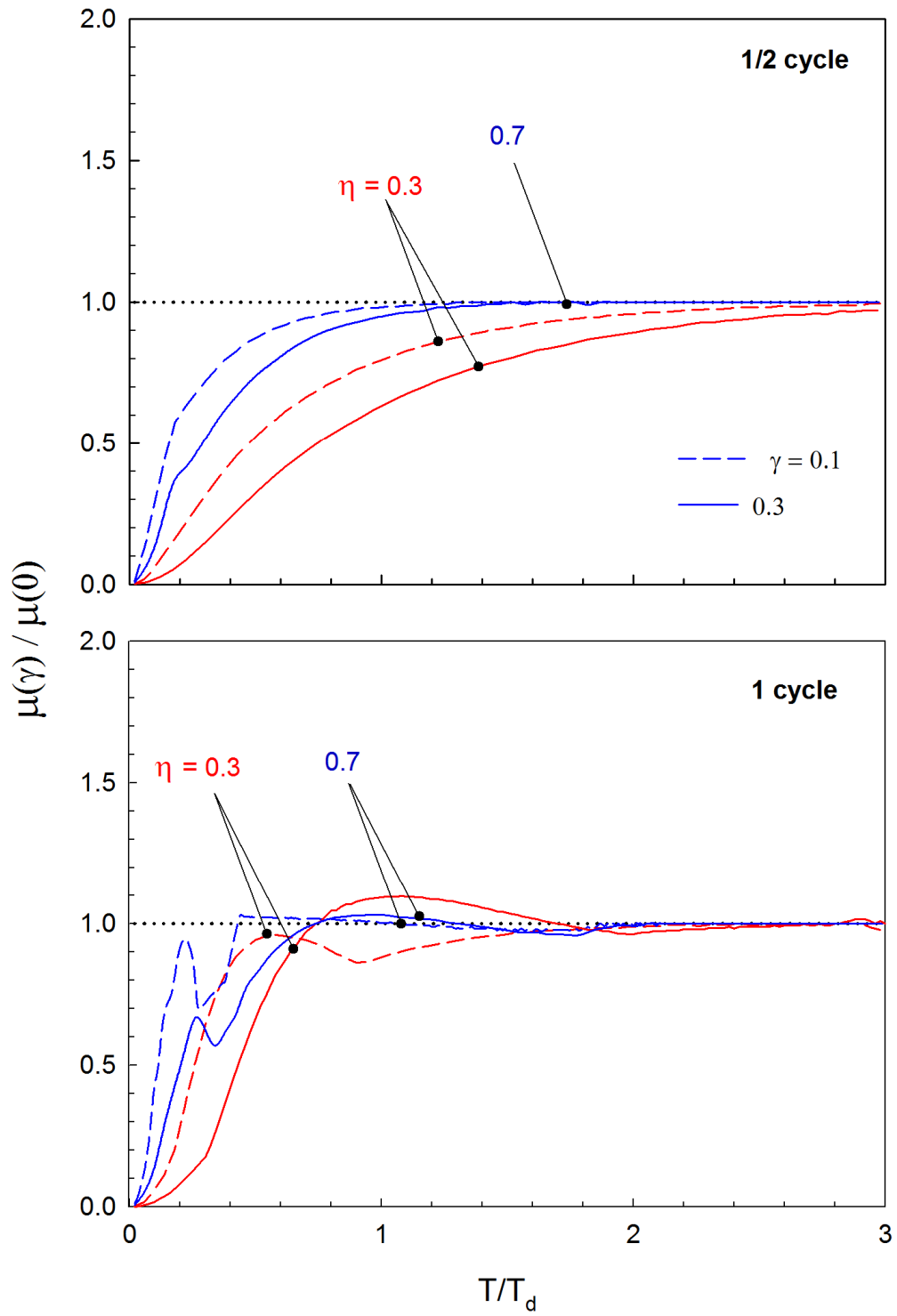


Figure 3.35 Effect of post-yielding stiffness on ductility demand due to a triangular pulse; $\zeta=5\%$

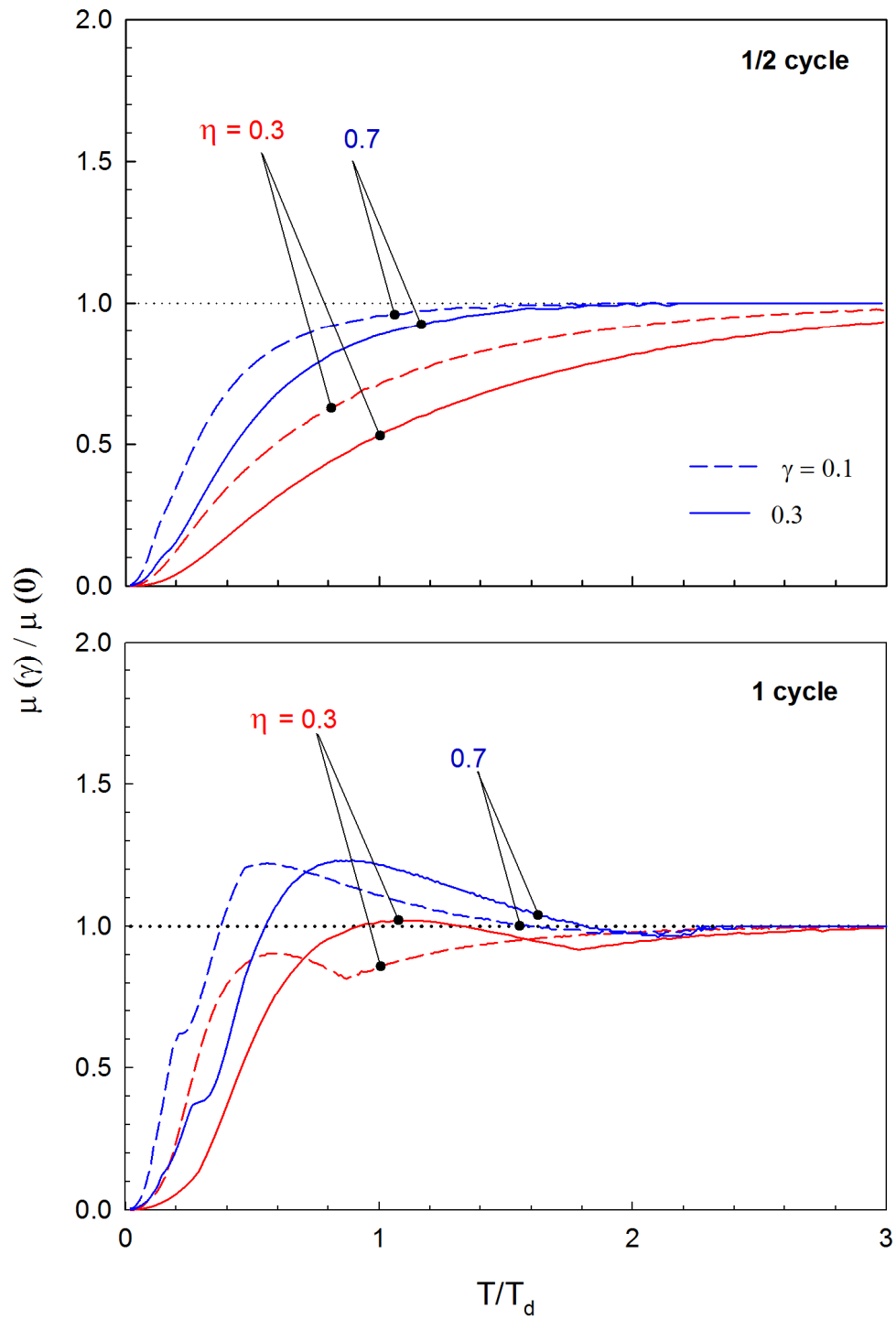


Figure 3.36 Effect of post-yielding stiffness on ductility demand due to a sinusoidal pulse; $\zeta=5\%$

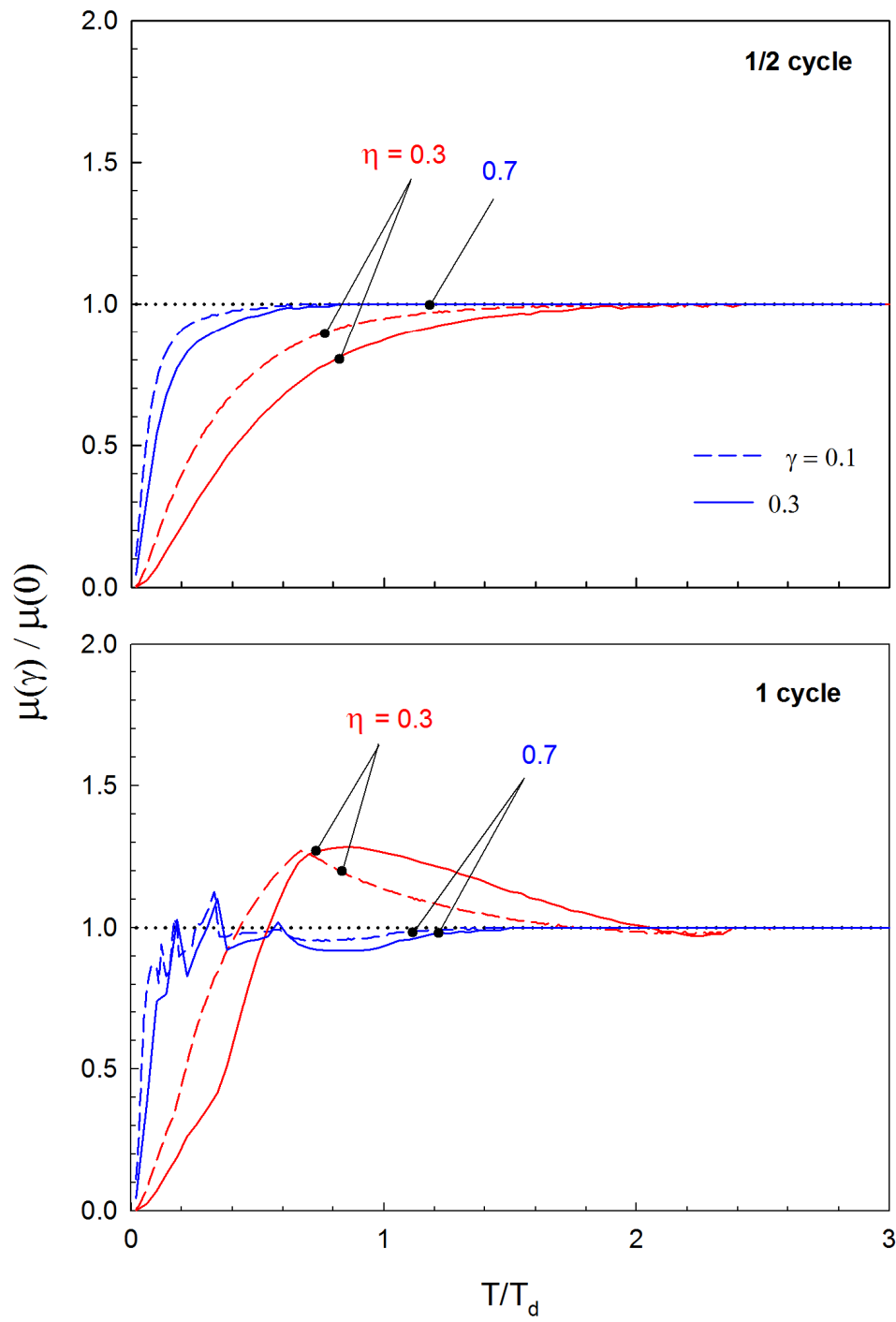


Figure 3.37 Effect of post-yielding stiffness on ductility demand due to an exponential pulse; $\zeta=5\%$

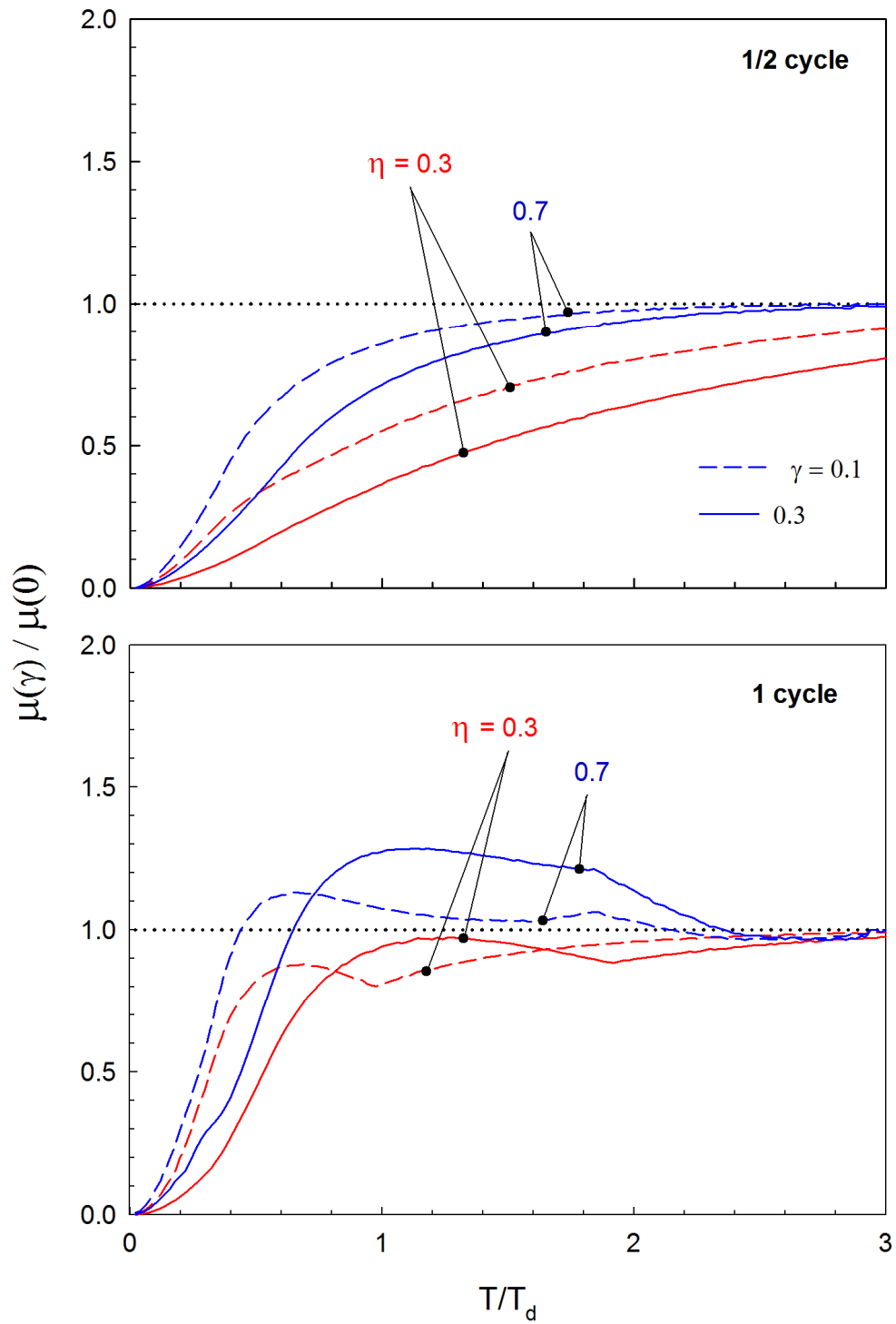


Figure 3.38 Effect of post-yielding stiffness on ductility demand due to a rectangular pulse; $\zeta=5\%$

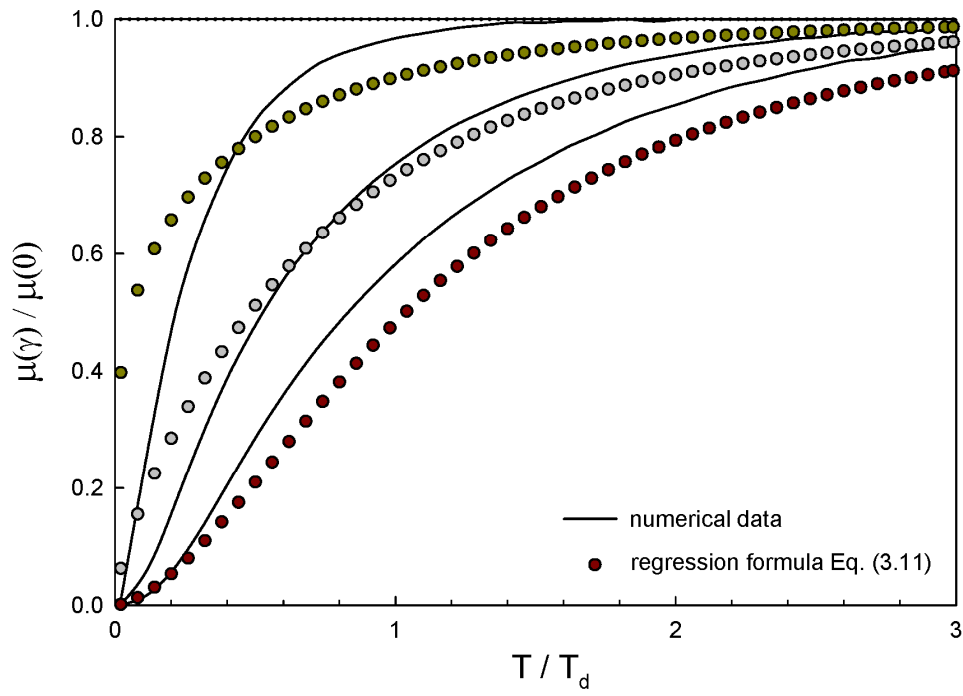


Figure 3.39 Comparison of numerical data with an approximate regression formula for the effect of post-yielding stiffness on ductility demand due to a half-cycle pulse. ($N=1/2$, $\zeta=5\%$)

Additional results are presented in Figures 3.40 to 3.42. Figure 3.41 depicts the time of peak response and corresponding sign of maximum displacement for a sinusoidal pulse of two different durations and 10% post-yielding stiffness. Peak displacement for a half-cycle pulse is positive, which implies that peak response is attained during the first (forward) oscillator movement. In contrast, peak response for a full-cycle pulse is negative for $T/T_d > 0.1$, implying that most oscillators attain their peak after reversal. Also, time of peak response t_m is higher than the duration of the pulse for T/T_d greater than approximately 0.4 and 0.6 for the full sine and the half sine pulse, respectively.

Insight on the effect of post-yielding stiffness on inelastic response is provided in Figures 3.41 and 3.42, for a full-cycle and a half-cycle pulse, respectively. For the full-cycle pulse, it is evident that when ductility demand increases with γ (i.e., when $T/T_d > 0.5$), the maximum response occurs in the negative direction (after the motion reverses). In contrast, for the half-cycle pulse, peak response always occurs in the positive direction.

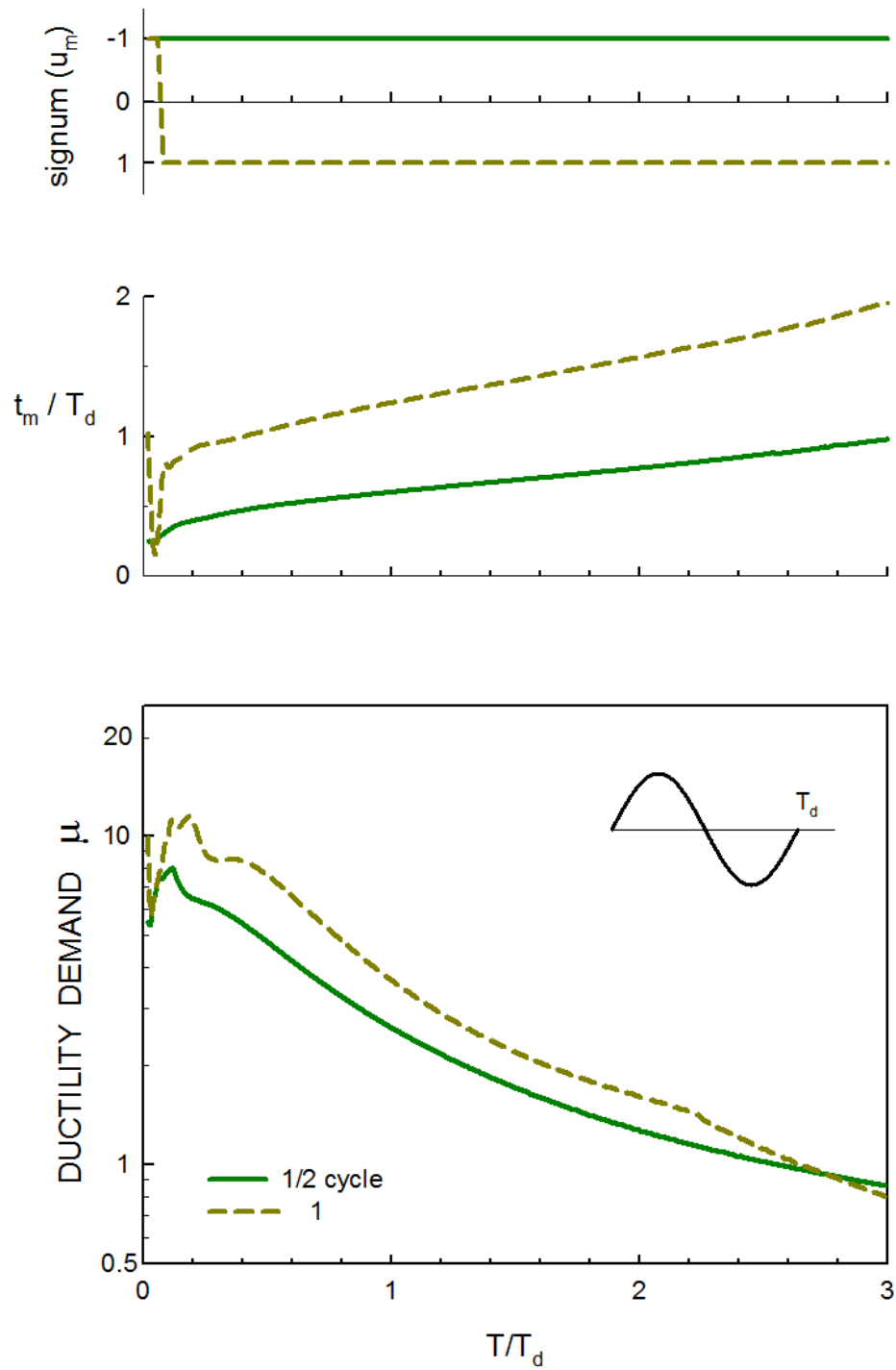


Figure 3.40 Comparison of response due to a half- vs a full – cycle sinusoidal pulse;
 $\gamma=0.10$, $\zeta=5\%$, $\eta=0.70$

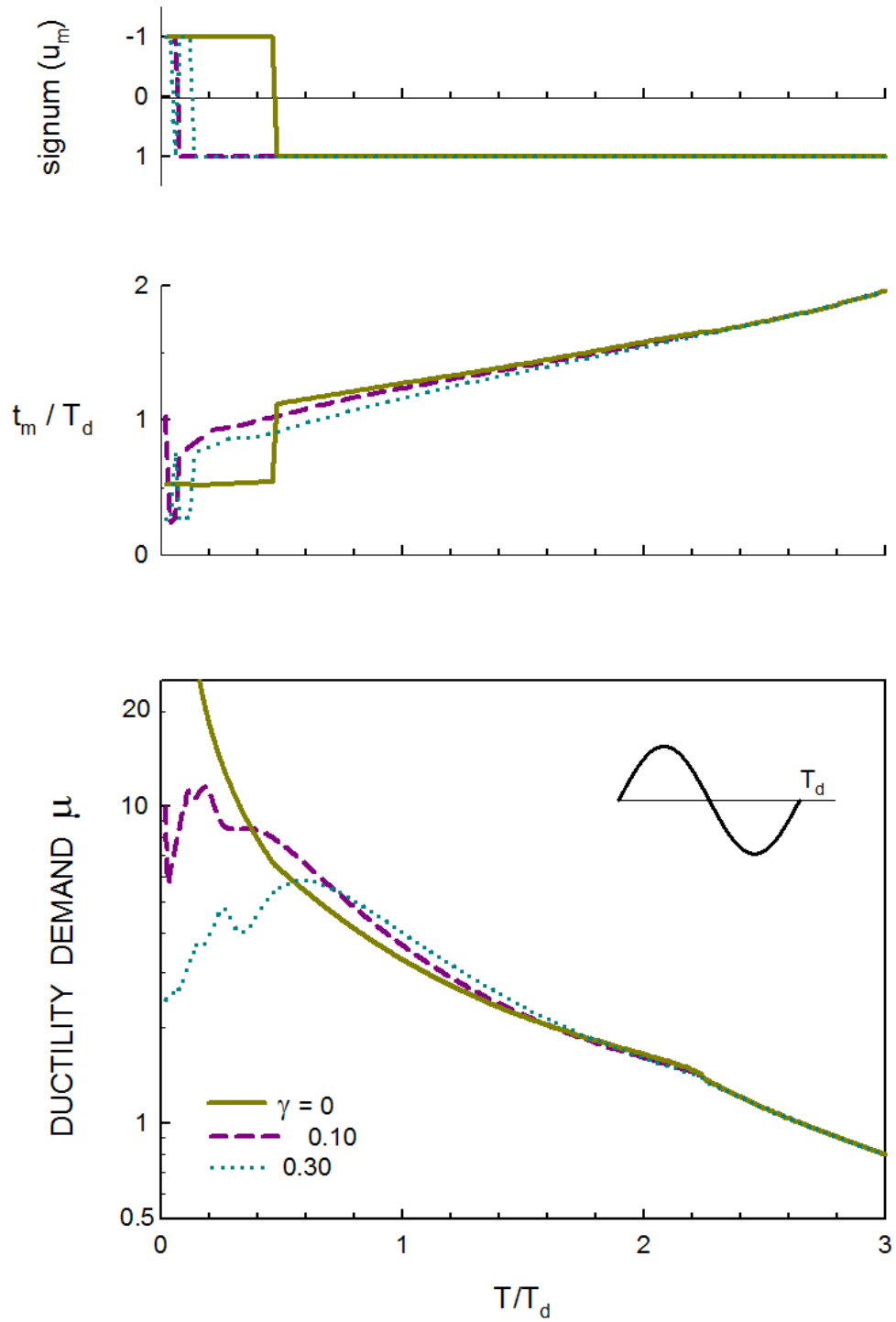


Figure 3.41 Effect of post-yielding stiffness on ductility demand, time of peak response and sign of peak displacement for a sinusoidal ground acceleration pulse; $N=1$, $\zeta=5\%$, $\eta=0.70$

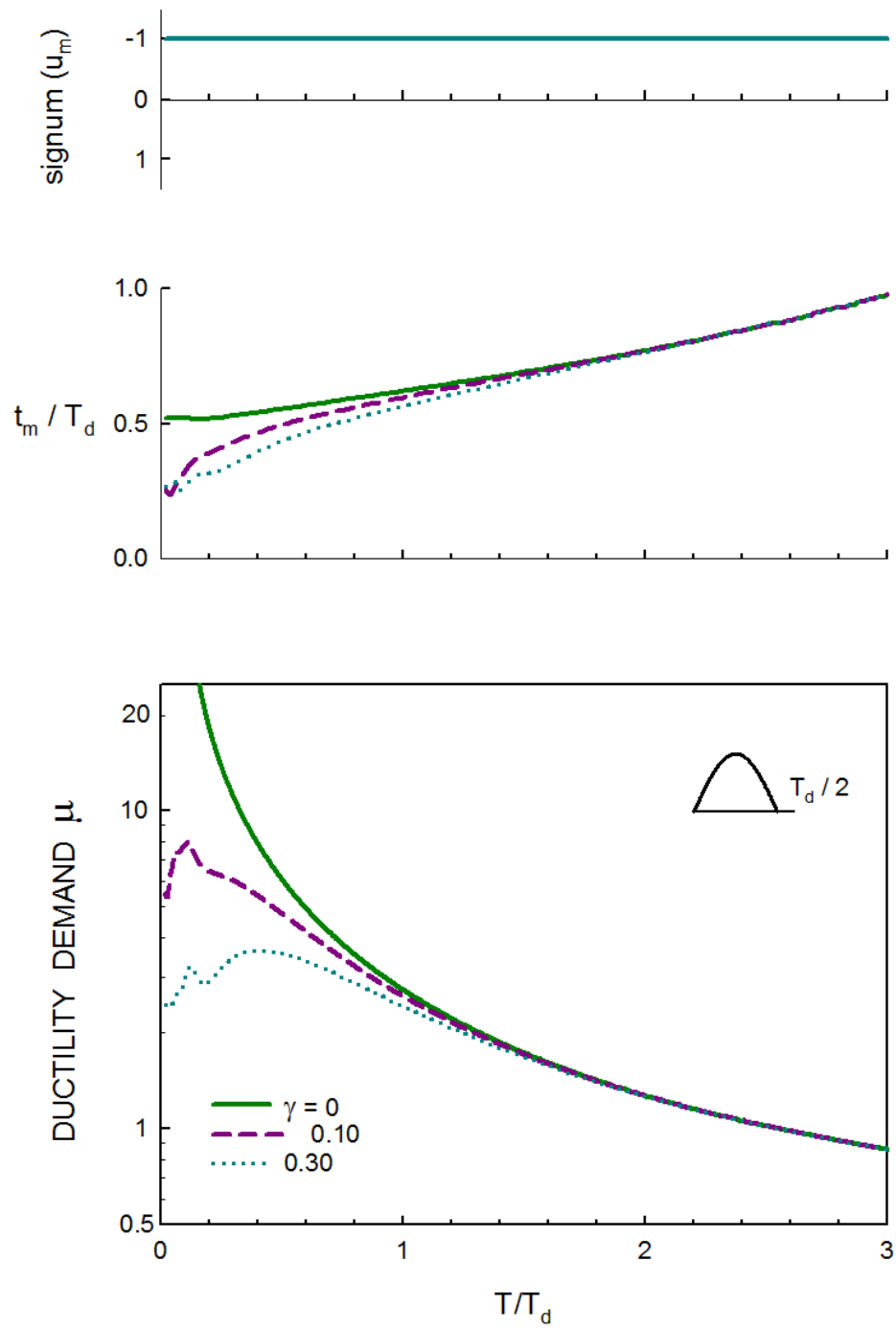


Figure 3.42 Effect of post-yielding stiffness on ductility demand, time of peak response and sign of peak displacement for a sinusoidal ground acceleration pulse; $N=1/2$, $\zeta=5\%$, $\eta=0.70$

3.5.1 Effect of Higher Harmonics

Earthquake pulses contain higher harmonics, which override the main pulse and alter (usually reduce) ductility demand. The following simple function was adopted to elucidate this effect:

$$\ddot{u}_g = A_p \sin\left(\frac{2\pi}{T_p}t\right) + A_s \sin\left(\frac{2\pi}{T_s}t - \phi_s\right) \quad (3.12)$$

where A_p , A_s denote the amplitude of the main pulse and the higher harmonic, respectively. T_p and T_s denote the corresponding periods while ϕ_s is a phase angle. For the sake of simplicity only the values $\phi_s = 0$ (same phase) and π (opposite phase) were considered.

Based on equation (3.12), the following dimensionless parameters are used to describe the results: *amplitude ratio* A_s/A_p ; *period ratio* T_s/T_p and *phase angle* ϕ_s .

Characteristic results are presented in Figures 3.43 to 3.45. Figure 3.44 shows the ratio $\mu/\mu_{\text{basic harmonic}}$ for a sinusoidal pulse ($\eta = 0.3$) and two values of the period ratio T_s/T_p . The reduction in ductility demand for all periods above $0.2 T_p$ is evident in the graph. Naturally, the reduction is stronger as the ratio A_s/A_p increases, while the effect of period ratio T_s/T_p is of secondary importance. Similar trends are shown in Figure 3.43 for a triangular pulse.

The effect of phase angle ϕ_s on ductility demand is depicted in Figure 3.45 for a sinusoidal pulse. It is seen that a change in ϕ_s may increase or decrease ductility demand depending on the amplitude and period ratio. The effect of phase angle on ductility demand is rather minor for periods up to $0.6 T_p$. For higher periods the effect is more significant particularly for low values of the amplitude ratio A_s/A_p .

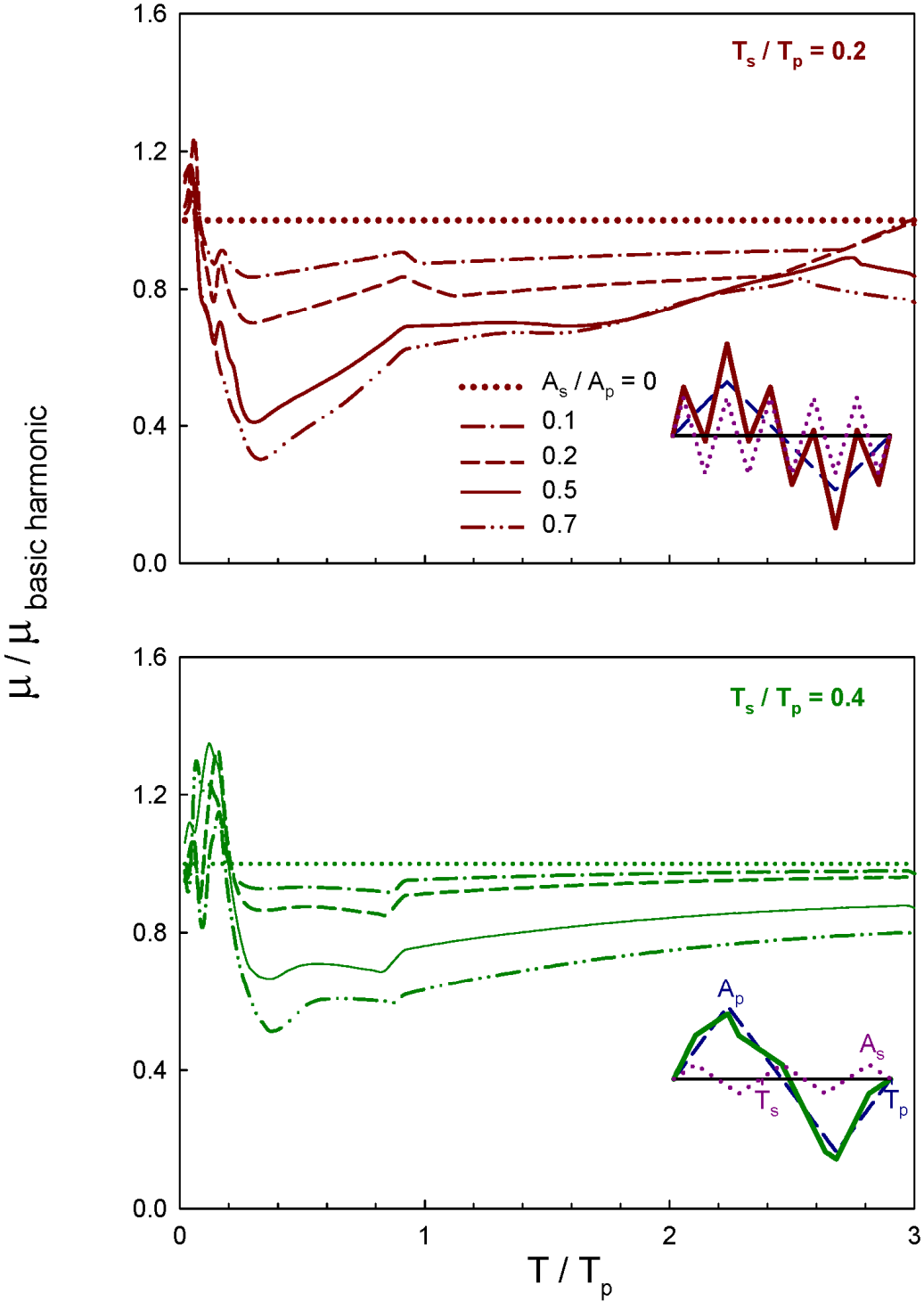


Figure 3.43 Effect of higher harmonics of a triangular pulse on ductility demand; $N=1$, $\gamma = 0.10$, $\zeta=5\%$, $\eta = 0.30$

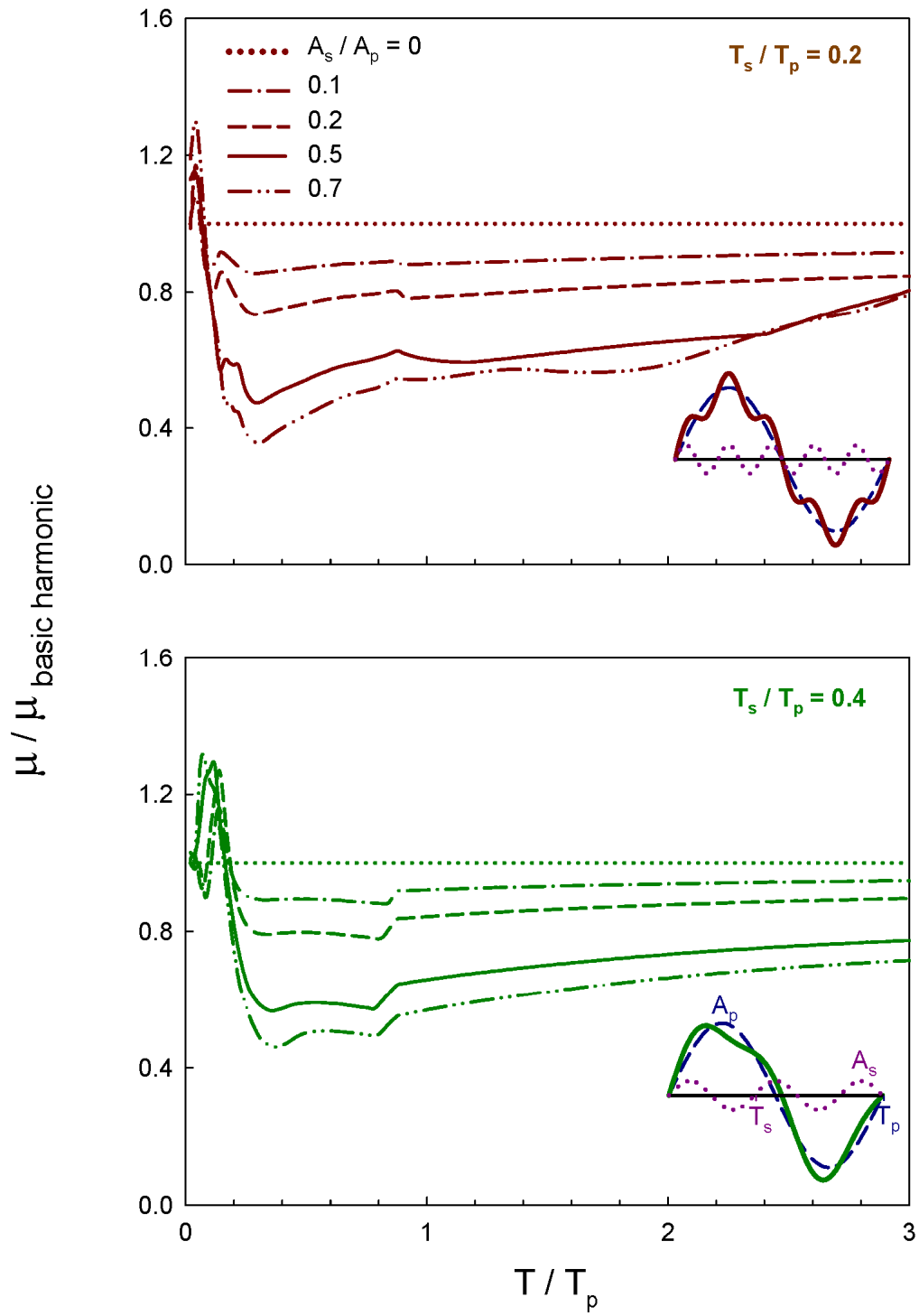


Figure 3.44 Effect of higher harmonics of a sinusoidal pulse on ductility demand; $N=1$, $\gamma = 0.10$, $\zeta=5\%$, $\eta = 0.30$

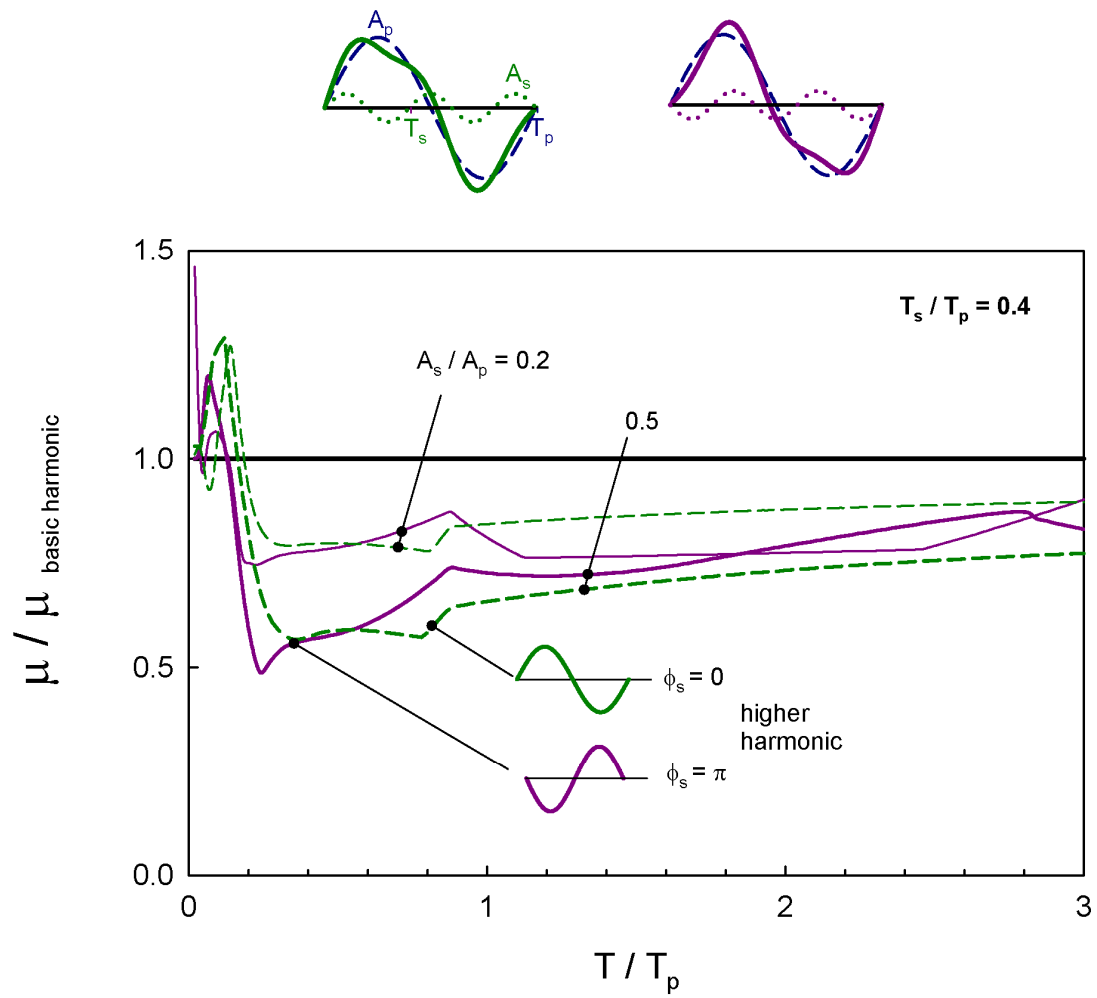


Figure 3.45 Effect of phase angle of a higher harmonic on ductility demand for a sinusoidal pulse; $N=1$, $\gamma = 0.10$, $\zeta=5\%$, $\eta = 0.30$

The effect of higher harmonics, that override the main pulse, on elastic response, is illustrated in Figures 3.46-3.49 for triangular and sinusoidal shapes and two alternative normalizations (i.e., with the response of the main pulse and with peak ground acceleration). The trend is straightforward as higher harmonics tend to invariably increase elastic response. Evidently, for larger periods (T_s) and for larger amplitudes (A_s) of the higher harmonic the effect of the higher harmonics is amplified.

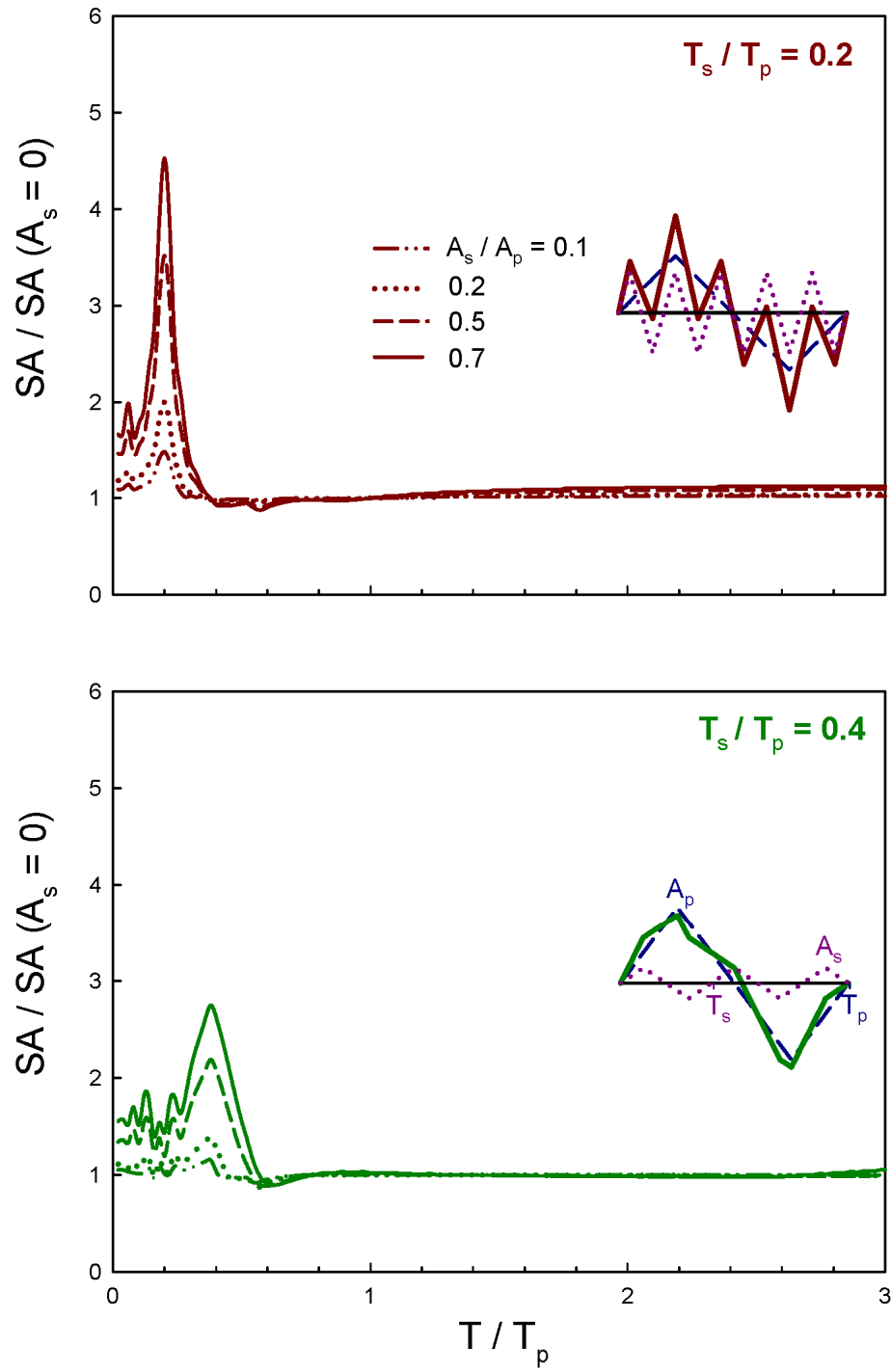


Figure 3.46 Effect of higher harmonics of a triangular pulse on spectral amplification;
 $N=1$, $\zeta=5\%$

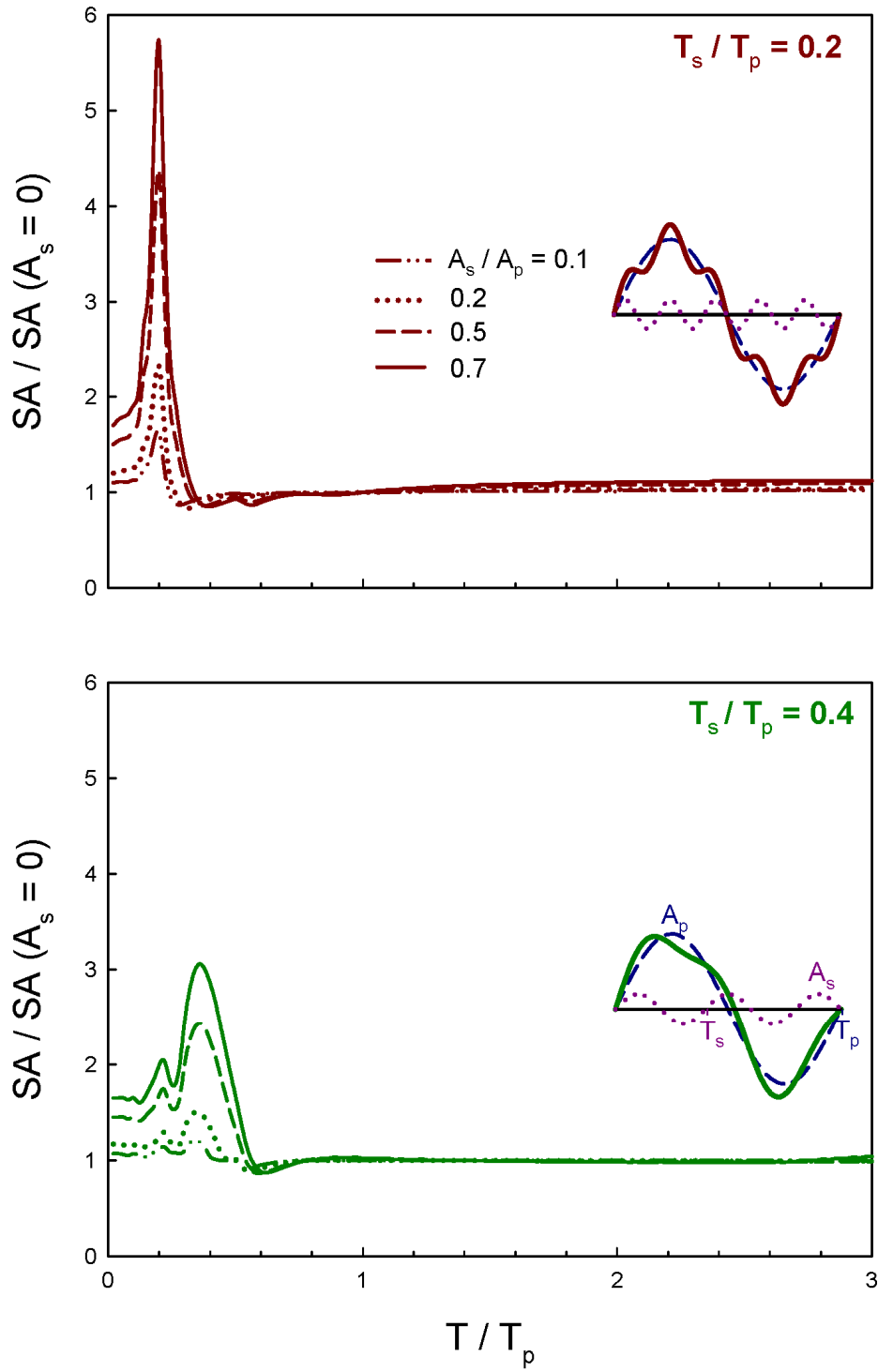


Figure 3.47 Effect of higher harmonics of a sinusoidal pulse on spectral amplification;
 $N=1$, $\zeta=5\%$

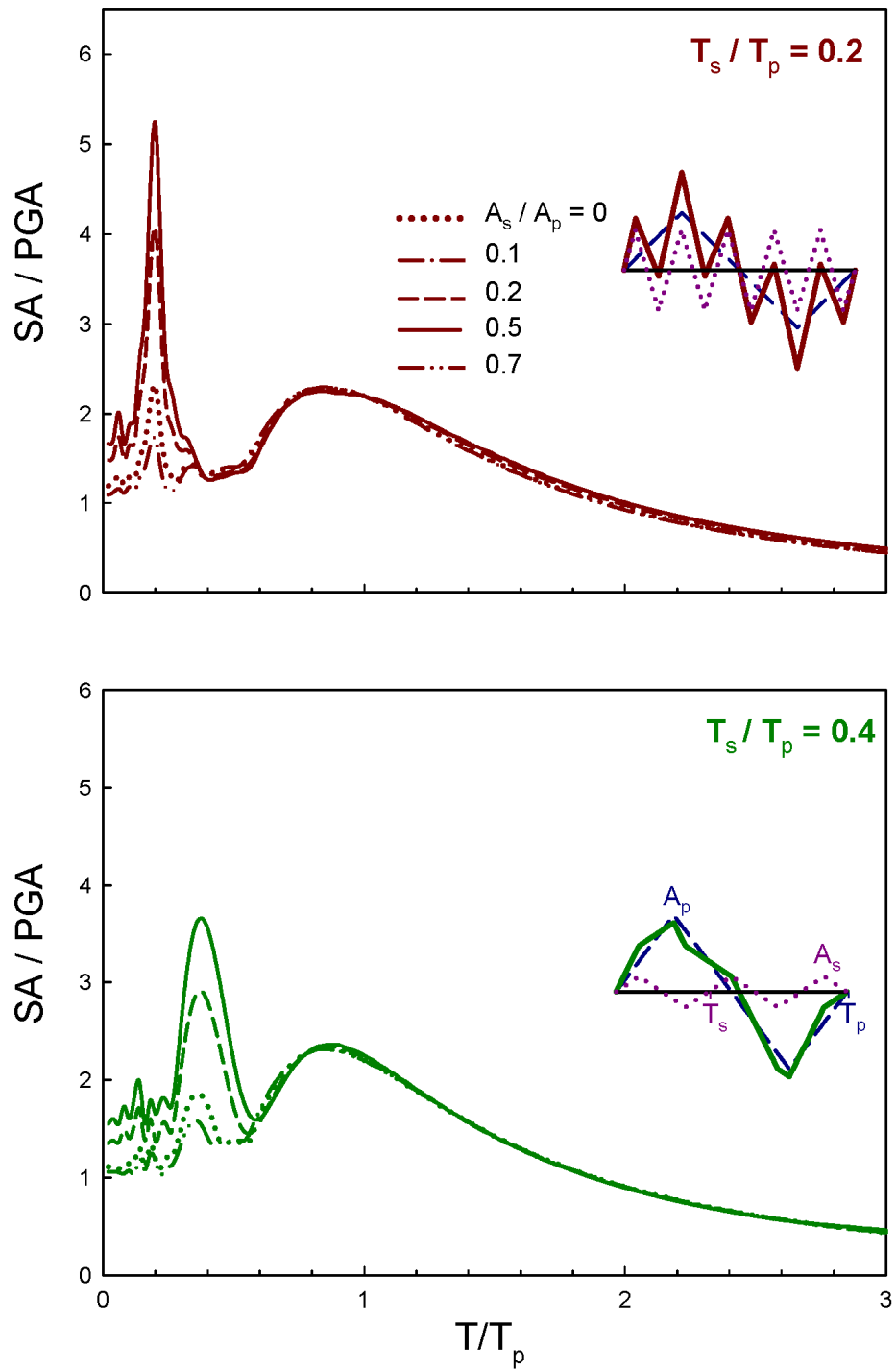


Figure 3.48 Effect of higher harmonics of a triangular pulse on spectral amplification;
 $N=1$, $\zeta=5\%$

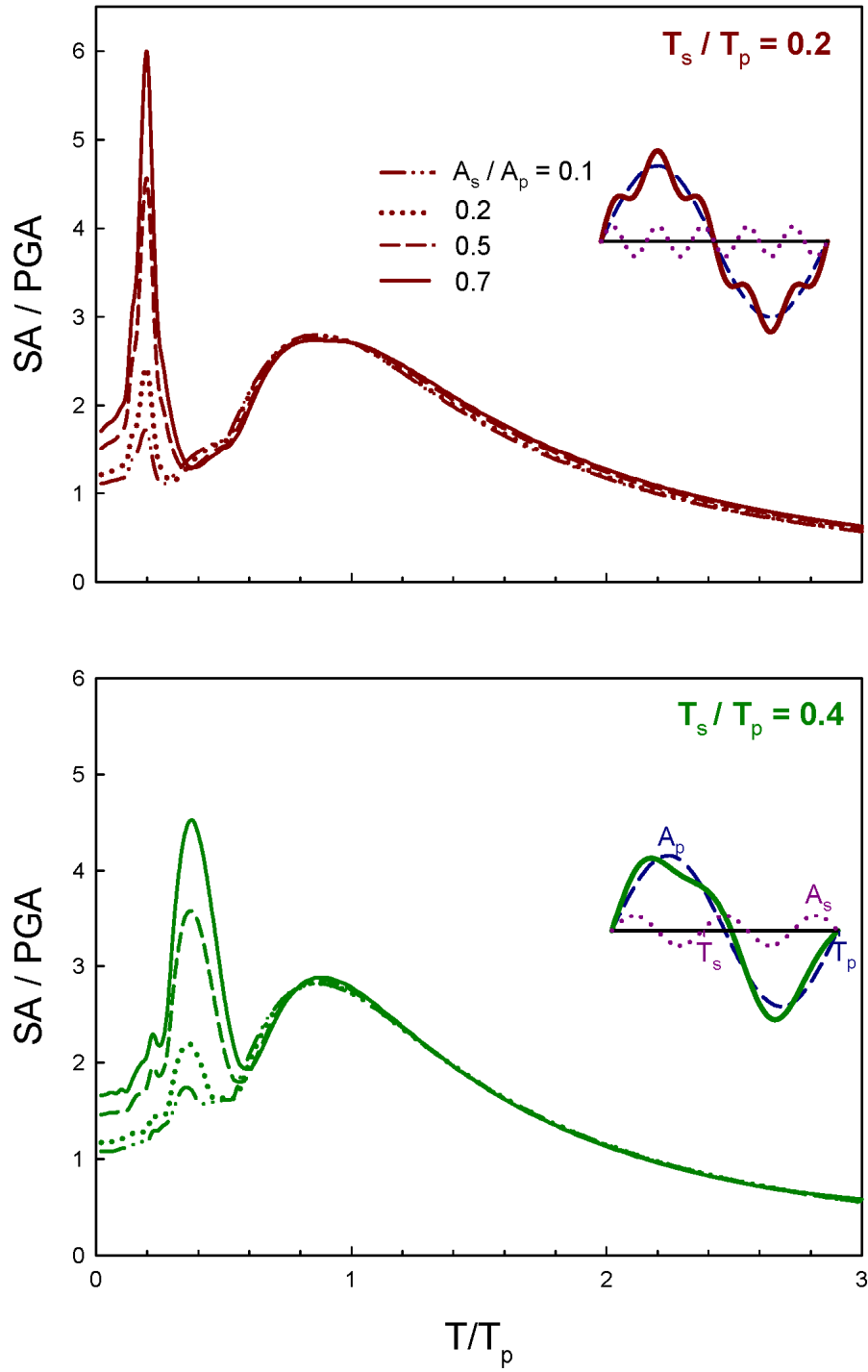


Figure 3.49 Effect of higher harmonics of a sinusoidal pulse on spectral amplification;
 $N=1$, $\zeta=5\%$

3.6 Elastic Perfectly – Plastic Oscillator: Closed-Form Solutions

An important special case is that of an elastic-perfectly plastic un-damped oscillator ($\gamma = 0$; $\zeta = 0$). For this system, the equation of dynamic equilibrium is

$$\ddot{u} = \ddot{u}_g - \frac{Q_y}{m} \quad (3.13)$$

The above case is particularly useful for practical applications as: (1) it represents a conservative assumption; (2) the effect of damping on the response is usually minor and can be assessed at a later stage (see Eq. 3.10).

Integrating the above equation twice with respect to time and enforcing the initial conditions at time of yielding, the relative velocity and displacement of the system are obtained as

$$\dot{u} = \dot{u}_y + [\dot{u}_g - \dot{u}_g(t_y)] - \frac{Q_y}{m}(t - t_y) \quad (3.14)$$

$$u = u_y + [u_g - u_g(t_y)] + [\dot{u}_y - \dot{u}_g(t_y)](t - t_y) - \frac{Q_y}{2m}(t - t_y)^2 \quad (3.15)$$

The time of peak response, t_m , is obtained from Eq. (3.14) by setting the oscillator velocity equal to zero

$$t_m = t_y + \frac{m}{Q_y}[\dot{u}_y + \dot{u}_g(t_m) - \dot{u}_g(t_y)] \quad (3.16)$$

Considering a half-cycle pulse, it is reasonable to assume (see discussion in previous Section) that the peak response occurs after (or slightly before) the end of the pulse. Accordingly, ground velocity at $t = t_m$ can be well approximated by the value at the end of the pulse i.e.,

$$\dot{u}_g(t_m) \cong \dot{u}_g(t_d) \quad (3.17)$$

Under this realistic assumption, the critical time t_m can be easily determined as^{*},

$$t_m = t_y + \frac{m}{Q_y} [\dot{u}_y + (Pulse\ Area) - \dot{u}_g(t_y)] \quad (3.18)$$

In terms of dimensionless time, equation (3.16) yields, for each pulse type,

$$\tau_m = \tau_y + \begin{cases} \frac{1}{\eta} \left[\frac{1}{2} + \frac{1}{2\pi^2 f^2} (1 - \cos 2\pi f \tau_y) - \tau_y^2 \right] & \text{triangular} \\ \frac{1}{\eta \pi (1 - 4f^2)} [1 - 4f^2 - 4f^2 \cos \pi \tau_y + \cos 2\pi f \tau_y] & \text{sinusoidal} \\ \frac{e^{2\pi \tau_y} f^2 + (1 + f^2) [1 - 2e^\pi + 2\pi(1 - \tau_y)] + \cos 2\pi f \tau_y + \frac{1}{f} \sin 2\pi f \tau_y}{2\eta \pi (1 - e^\pi) (f^2 + 1)} & \text{exponential} \\ \frac{1}{\eta} \left[\frac{1}{2\pi f} \sin 2\pi f \tau_y + 1 - \tau_y \right] & \text{rectangular} \end{cases} \quad (3.19a-d)$$

Equation (3.19a) was first derived by Mylonakis & Reinhorn (2001). It is noted that the above equations are exact for $\tau_m \geq 1$ and only approximate for $\tau_m < 1$. As will be shown later on, the approximation is satisfactory in the range ($f \leq 2$, $\eta \leq 0.8$), which suffices for most cases of practical interest.

Substituting Eq. (3.19) into Eq. (3.15) and after some lengthy yet straightforward algebraic manipulations, the following explicit solutions were obtained for the imposed ductility demand:

^{*} The assumption in Eq. (3.17) is introduced for mathematical convenience, to avoid solving transcendental equations as in the time of yielding.

- *Triangular Pulse*

$$\mu = 1 + \frac{\pi^2 f^2}{\eta} \left(E_t - \frac{4}{3} \tau_y^3 \right) - \frac{2}{\eta} \left[2\pi^2 f^2 \tau_y^2 + \cos 2\pi f \tau_y - 1 \right] (\tau_m - \tau_y) - 2\pi^2 f^2 (\tau_m - \tau_y)^2 \quad (3.20a)$$

where

$$E_t = \begin{cases} -\frac{4}{3} \tau_m^3 + 4\tau_m^2 - 2\tau_m + \frac{1}{3} & , \quad 0.5 \leq \tau_m \leq 1 \\ 2\tau_m - 1 & , \quad \tau_m \geq 1 \end{cases} \quad (3.20b)$$

- *Sinusoidal Pulse*

$$\mu = (8f^3 \pi (\tau_m - \tau_y) \cos \pi \tau_y + 8f^3 \sin \pi \tau_y - f(4f^2 - 1) [\eta \pi^2 (\tau_m - \tau_y)^2 + E_s] - \sin 2\pi f \tau_y - 2f \pi (\tau_m - \tau_y) \cos 2\pi f \tau_y) / (2f \sin \pi \tau_y - \sin 2\pi f \tau_y) \quad (3.21a)$$

where

$$E_s = \begin{cases} 2 \sin \pi \tau_m & , \quad \tau_m < 1 \\ 2\pi(1 - \tau_m) & , \quad \tau_m \geq 1 \end{cases} \quad (3.21b)$$

- *Exponential Pulse*

$$\mu = \frac{1}{\eta(e^\pi - 1)} \left[\frac{f}{f^2 + 1} (-f e^{2\pi \tau_y} (1 + f^2 (1 + 2\pi(\tau_m - \tau_y))) - 2\pi(\tau_m - \tau_y)(f \cos 2\pi f \tau_y + \sin 2\pi f \tau_y)) + E_e \right] \quad (3.22a)$$

where

$$E_e = \begin{cases} f^2 e^{2\pi(1-\tau_m)} + \chi_1 - \chi_2(1 + \chi_1) + \chi_3 e^\pi (2\tau_m - 1) & \tau_m \leq 1 \\ \eta(e^\pi - 1)(1 - \chi_2) - f^2 + \chi_3 \left[2\tau_m(e^\pi + \pi\tau_y - \pi - 1/2) + \pi(1 - \tau_y^2) + (1 - e^\pi) \right] & \tau_m \geq 1 \end{cases} \quad (3.22b)$$

in which $\chi_1 = 2e^{\pi/2}\eta \sinh \frac{\pi}{2}$, $\chi_2 = 2\pi^2 f^2 (\tau_m - \tau_y)^2$, $\chi_3 = 2\pi f^2$

- *Rectangular Pulse*

$$\mu = 1 + \frac{4\pi^2 f^2}{1 - \cos 2\pi f \tau_y} \left[E_r - \frac{1}{2} \tau_y^2 + (\tau_m - \tau_y) \left(\frac{\sin 2\pi f \tau_y}{(2\pi f)} - \tau_y \right) - \frac{1}{2} \eta (\tau_m - \tau_y)^2 \right] \quad (3.23a)$$

where:

$$E_r = \begin{cases} 0.5\tau_m^2, & \tau_m \leq 1 \\ \tau_m - 0.5, & \tau_m > 1 \end{cases} \quad (3.23b)$$

It is noted that the above equations are exact solutions to Eq. (3.13). Equation (3.20a) was first derived for the special case $\tau_m > 1$ by Mylonakis & Reinhorn (2001).

3.6.1 Determination of time of yielding

Determining t_y is not straightforward, since the response function $u(t)$ typically involves transcendental functions of time. A useful property to this end, is that the normalized yielding strength η and the displacement amplification function D at yielding are numerically equal (i.e., $D_y = \eta$).

With reference to a triangular pulse, determining t_y requires solving the equation (Mylonakis & Reinhorn 2001)

$$f\tau_y - \frac{1}{2\pi} \sin[2\pi f\tau_y] = \frac{1}{2} f\eta \quad (3.24)$$

In the case of a sinusoidal pulse, the corresponding equation is

$$\sin[\pi\tau_y] - \frac{1}{2f} \sin[2\pi f\tau_y] = \frac{4f^2 - 1}{4f^2} \eta \quad (3.25)$$

For a rectangular pulse,

$$1 - \cos[2\pi f\tau_y] = \eta \quad (3.26)$$

The last of the above equations has the elementary solution

$$\tau_y = \frac{1}{2\pi f} \arccos[1 - \eta] \quad (3.27)$$

which elucidates the importance of pulse duration, f , on time of yielding.

A simple way to solving Eqs (3.25) and (3.27) is to transform them into algebraic equations by expanding the transcendental functions in power series. As will be shown below, if the expansion point is located in the vicinity of the root, good accuracy can be achieved by means of first-order analysis.

A general form of the above equations is

$$\sum_{n=1}^N a_n \sin b_n t_y + \sum_{n=1}^N c_n \cos d_n t_y = S t_y + T \quad (3.28)$$

where $N, a_n, b_n, c_n, d_n, S, T$ are problem parameters.

Expanding the trigonometric functions in power series up to first-order terms yields

$$\sin bt_y \cong \sin bt_y^* + b \cos bt_y^* (t_y - t_y^*) \quad (3.29)$$

$$\cos dt_y \cong \cos dt_y^* - d \sin dt_y^* (t_y - t_y^*) \quad (3.30)$$

in which t_y^* denotes the expansion point.

Substituting Eqs (3.29) and (3.30) into Eq. (3.28), the following explicit solution is obtained for the time of yielding

$$t_y = \frac{T - \sum_{n=1}^N (a_n \sin b_n t_y^* + c_n \cos d_n t_y^*)}{\sum_{n=1}^N (a_n b_n \cos b_n t_y^* - c_n d_n \sin d_n t_y^*) - S} \quad (3.31)$$

The idea is illustrated in Figure 3.50.

Applying the above procedure to Eqs (3.25) and (3.26) yields the solutions,

$$\tau_y = \frac{\frac{1}{2}\eta + \frac{1}{2\pi f} \sin[2\pi f \tau_{yt}^*] - \cos[2\pi f \tau_{yt}^*] \tau_{yt}^*}{1 - \cos[2\pi f \tau_{yt}^*]} \quad (3.32)$$

$$\tau_y = \tau_{ys}^* + \frac{\frac{4f^2 - 1}{4f^2} \eta + \frac{1}{2f} \sin[2\pi f \tau_{ys}^*] - \sin[\pi \tau_{ys}^*]}{\pi (\cos[\pi \tau_{ys}^*] - \cos[2\pi f \tau_{ys}^*])} \quad (3.33)$$

pertaining to a triangular and a sinusoidal pulse, respectively.

In the above equations, the expansion points τ_{yt}^* and τ_{ys}^* can be viewed as gross approximations of the time of yielding (hence the symbols) and can be obtained from high-order Taylor expansions of the corresponding equations around $t = 0$. For instance,

for the sinusoidal pulse, expanding the sine functions in Eq. (3.23) up to third-order terms around $\tau = 0$ yields

$$\tau_{ys}^* = \sqrt[3]{\left(\frac{2}{\pi}\right) \frac{3\eta}{4\pi^2 f^2}} \quad (3.34)$$

For a triangular pulse,

$$\tau_{yt}^* = \sqrt[3]{\frac{3\eta}{4\pi^2 f^2}} \quad (3.35)$$

In principle, the predictions of Eqs (3.32) and (3.33) can be improved by iterating the values of t_y and t_y^* , starting from the initial estimates in Eqs (3.34) and (3.35). The improvement in accuracy, however, hardly compensates the extra effort required.

In the results presented below, no iterations have been employed. It is worth mentioning that the predictions of Eq. (3.35) become identical to those of Eq. (3.34) if the dimensionless strength factor η is multiplied by $2/\pi$ i.e.,

$$\eta_t \rightarrow \frac{2}{\pi} \eta_s \quad (3.36)$$

Interestingly, the above factor is equal to the ratio of the peak velocities of the two pulses

$$\frac{\dot{u}_{gm,s}}{\dot{u}_{gm,t}} = \frac{\frac{2}{\pi} A_g t_d}{\frac{1}{2} A_g t_d} = 2 \times \left(\frac{2}{\pi}\right) \quad (3.37)$$

The above ratio can be used to define a sinusoidal pulse from an “equivalent” triangular pulse and vice versa. An application is presented later on.

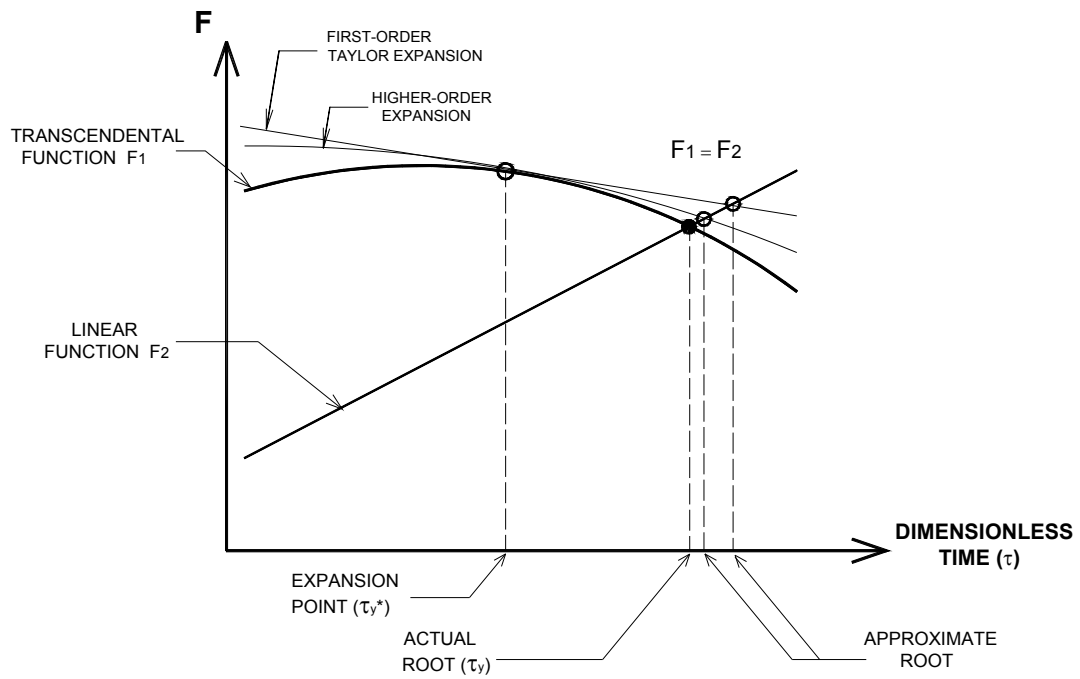


Figure 3.50 Procedure for determining the time of yielding by finding roots to transcendental equations via Taylor expansions.

3.7 Results

In Figures 3.51, ductility demand $\mu (= D_m / D_y)$ is plotted as function of pulse duration f for a triangular, sinusoidal and rectangular pulse. Both numerical results and results from Eqs (3.20), (3.21), and (3.22) are provided. Many noteworthy features are apparent in the graphs: *First*, μ is generally an increasing function of f . For instance, when $f = 1$ (i.e., pulse duration is equal to the elastic structural period) and $\eta = 0.5$ (i.e., yield structural acceleration is equal to 50% of peak pulse acceleration), μ may exceed the astonishing value of 10. The trend is understandably stronger with low-strength systems. It is noted that the increased response with increasing f is in contrast to the trends observed for the elastic oscillator in which maximum response is obtained near $f = 1$ (Fig 3.47). *Second*, the response is sensitive to the yielding resistance of the structure. For example, for $\eta = 0.8$ and $f = 2$, a reduction in strength by just 15% ($\eta = 0.7$) leads to an increase in ductility demand by 150% or more. The agreement of the closed-form solutions with the numerical solutions is meaningful.

Figure 3.52 presents results for time of yielding as function of pulse duration, for the three aforementioned pulses. It is observed that τ_y is a decreasing function of η , which implies that the smaller the yield strength, the sooner the system will yield. The accuracy of the approximate solution (Eqs 3.27, 3.32 and 3.33) is indeed very satisfactory.

In Figure 3.53, the critical time τ_m associated with peak displacement response is presented. For $f < 1$ peak response occurs after the end of the pulse, regardless of yield strength and pulse type. For longer pulse durations two different types of behavior are observed: relatively strong systems ($\eta > 0.7$) attain their maximum response before or shortly after the end of the pulse ($\tau_m < 1$), while weak systems ($\eta \leq 0.5$) attain their

maximum response during free oscillations ($\tau_m > 1$). Also, for $f > 1$ time of peak response depends solely on η , being practically independent of pulse duration. However, for sinusoidal and rectangular pulse excitations the system reaches its peak response after the end of the pulse ($\tau_m > 1$) for all η 's. The analytical results are in excellent agreement with the numerical solution.

Results (in tabulated and graphical form) for some additional pulse shapes are available in Appendix C.

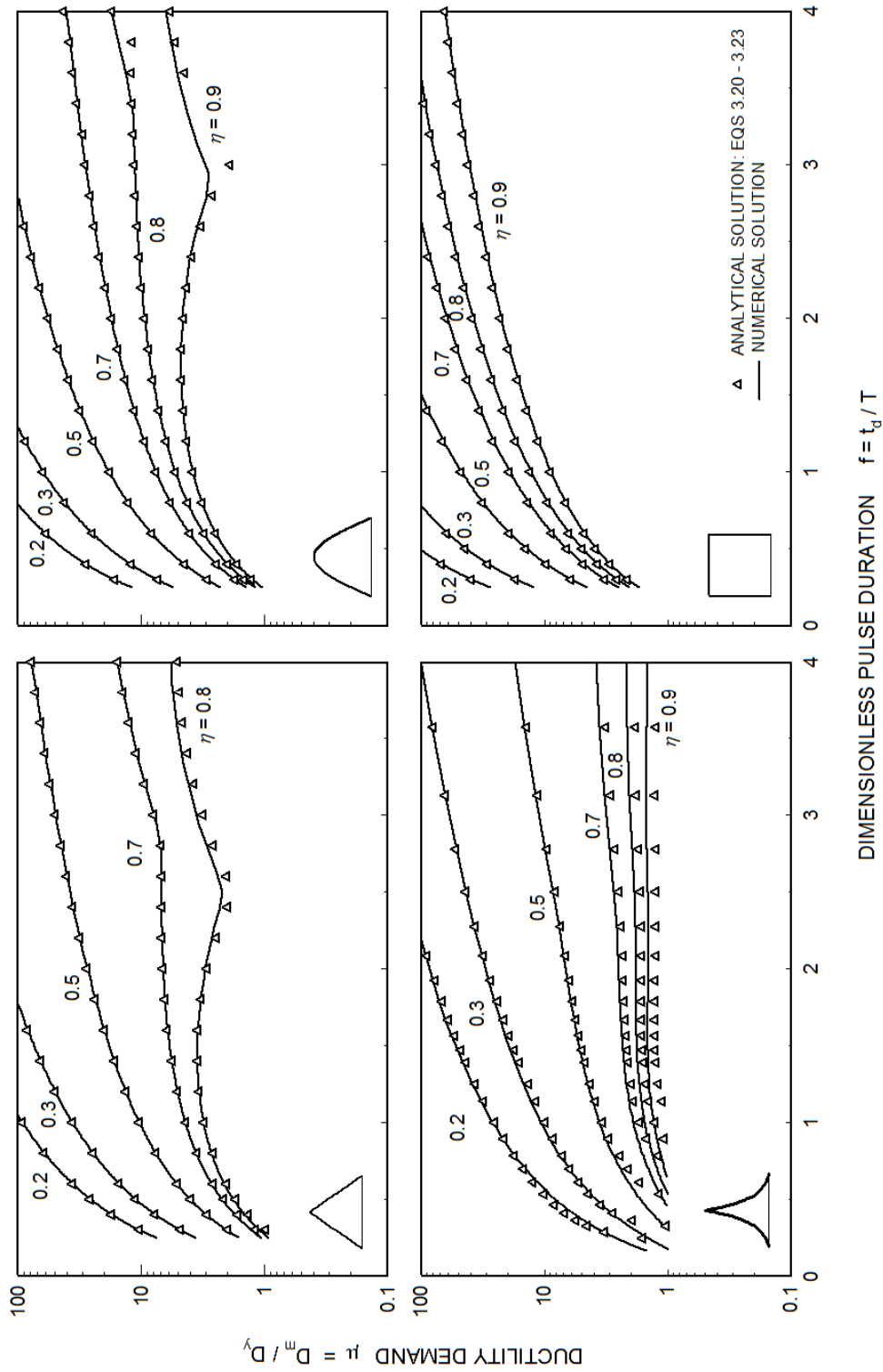


Figure 3.51 Ductility demand for an elastic-perfectly plastic oscillator subjected to four idealized ground acceleration pulses. Comparison of the exact numerical solution with the approximate solution in Eqs 3.20 to 3.23.

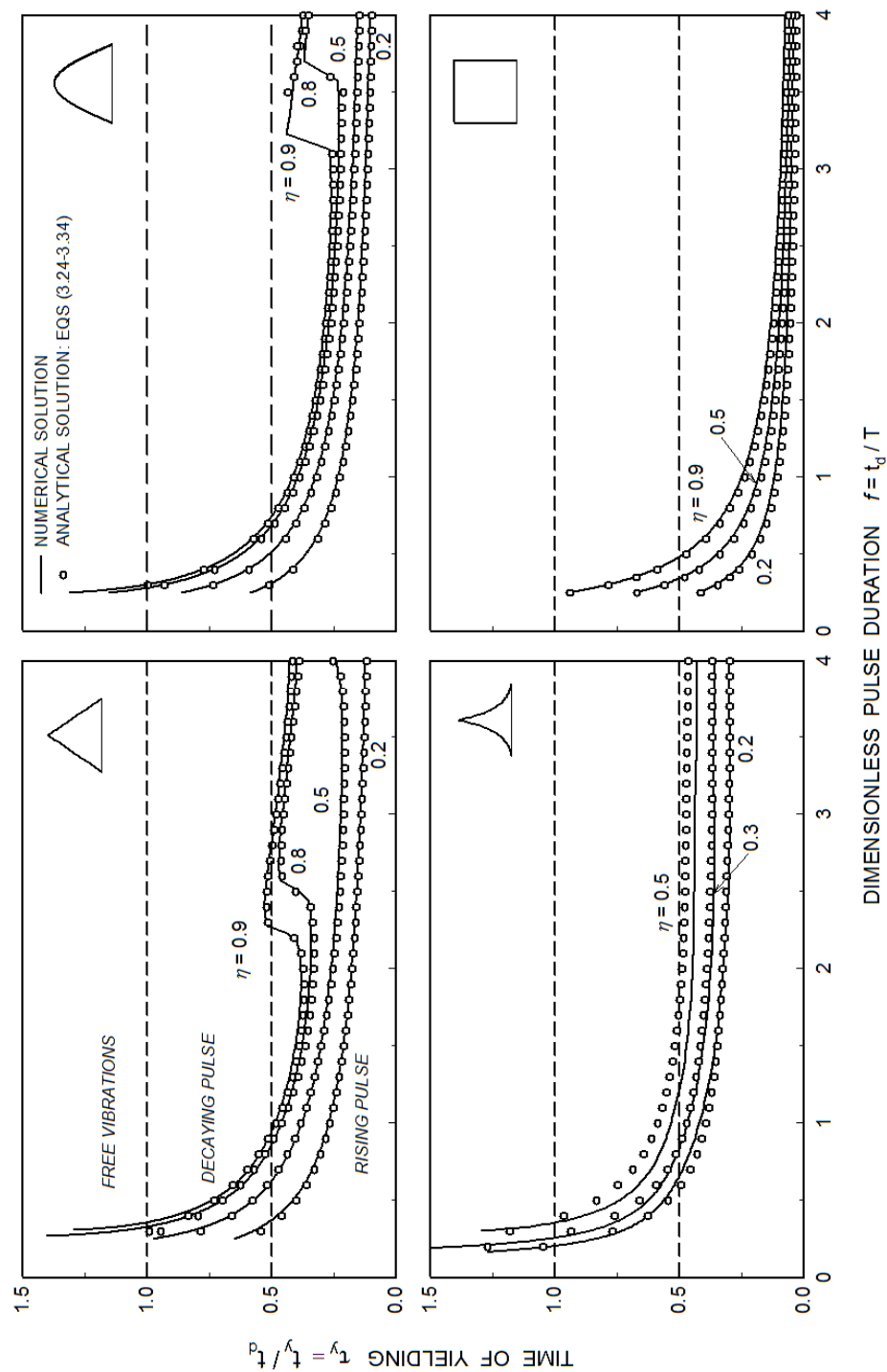


Figure 3.52 Dimensionless time of yielding for an undamped elastic-perfectly plastic oscillator subjected to four idealized ground acceleration pulses. Comparison of the exact numerical solution with the approximate solution in Eqs 3.24 to 3.35.

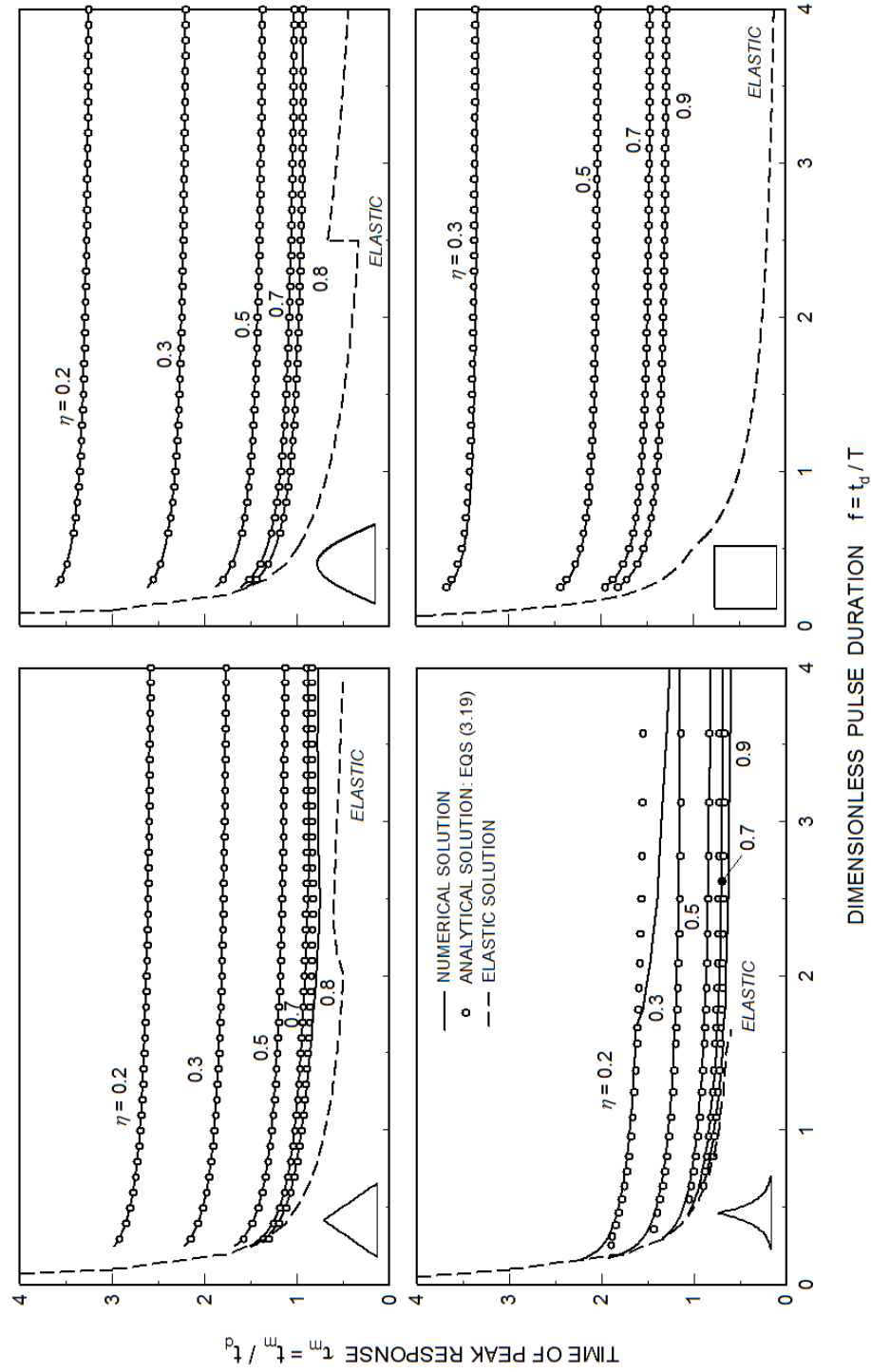


Figure 3.53 Dimensionless time of peak response for an undamped elastic-perfectly plastic oscillator subjected to four idealized ground acceleration pulses. Comparison of the exact numerical solution with the approximate solution in Eqs 3.19.

3.8 Comparison with Actual Near-Field Recordings

To evaluate the applicability of the model to earthquake engineering, its predictions are compared against inelastic spectra from four actual near-field motions. The accelerograms shown in Figure 3.1 were recorded during four destructive earthquakes of different magnitudes around the world. The pulses contained in the records exhibit various degrees of similarity to the sinusoidal and rectangular pulses of the model. For instance, the 0.46g pulse of Pyrgos (1993) record and the 0.84g pulse of Rinaldi (1994) record resembles closely the triangular and sinusoidal pulses of the model, respectively. In contrast, the 0.66g pulse of Pacoima (1971) record does not possess such an obvious similarity. Erzican (1992) motion represents an intermediate case. Key engineering characteristics of the ground motions are summarized in Table 3.1.

To determine the duration of the pulse in an unambiguous manner, the following definition was adopted (Cuesta & Aschheim 2000):

$$t_d \approx \frac{T_g}{2} \quad (3.38)$$

where T_g is the period at which the 5%-damped velocity spectrum of the motion becomes maximum (Miranda & Bertero 1994). Notice the good agreement between the predictions based on the above equation (Table 3.1) and the pulse durations indicated in Figure 3.1.

For simplicity, the amplitude of the pulse in the model was considered equal to the peak pulse acceleration, PPA , (Table 3.1) i.e.,

$$A_g \approx PPA \quad (3.39)$$

Table 3.1 Selected near-field motions with long-period, high-acceleration pulses.

Station Name	Event /Date	Magnitude (M_s)	Distance from Fault (km)	Direction	Peak Pulse Acceleration (g)	PGV (m/s)	T_a (s)	T_g (s)	$t_d^\dagger$ (s)
Pyrgos	Pyrgos, Greece 3/26/93	5.2	<5	EW	0.46	0.25	0.27	0.37	0.19
Erzican	Erzican, Turkey 3/13/92	6.9	2	NS	0.52	0.84	0.30	2.03	1.01
Rinaldi	Northridge CA, USA 1/17/94	6.7	7.1	N228W	0.84	1.66	0.72	1.06	0.58
Pacoima Dam	San Fernando CA, USA 2/9/71	6.6	2.8	N164W	0.66	1.13	0.39	1.20	0.60

† Computed as $t_d = T_g / 2$ (Eq. 3.38)

In Figures 3.54 and 3.55, ductility demands from the Pyrgos (1993) record are presented for three different oscillator strengths. The good agreement between the numerical results and the predictions of the model for the triangular pulse is anticipated given the similarity of the waveforms. The comparison with the predictions of the sinusoidal pulse is, evidently, less favorable.

Corresponding results obtained with the Erzican (1992) record are shown in Figures 3.56 and 3.57. The agreement between the triangular pulse model and the earthquake results is again satisfactory. The largest discrepancies are observed with the “weak” $\eta = 0.5$ oscillator and can be possibly attributed to a pre-pulse contained in the record. The comparison with the sinusoidal record is satisfactory from a qualitative viewpoint.

The Pacoima (1971) record is examined in Figures 3.58 and 3.59. In this case the two solutions do not compare well, particularly for normalized pulse durations higher than about 1. This is understood in view of the different shapes of the pulses in the two motions. Nevertheless, in the frequency range of the most practical importance (i.e., $f < 1$), the predictions of the two models are quite similar. Additional discussion on the Pacoima record is provided in the following section.

Results for the Rinaldi record are provided in Figures 3.60 and 3.61. The predictions of the model are excellent (except perhaps for the case where $\eta = 0.9$), particularly for the sinusoidal pulse, which is expected in view of the similarity in the shape of the pulses. The comparison with the triangular pulse is less favorable.

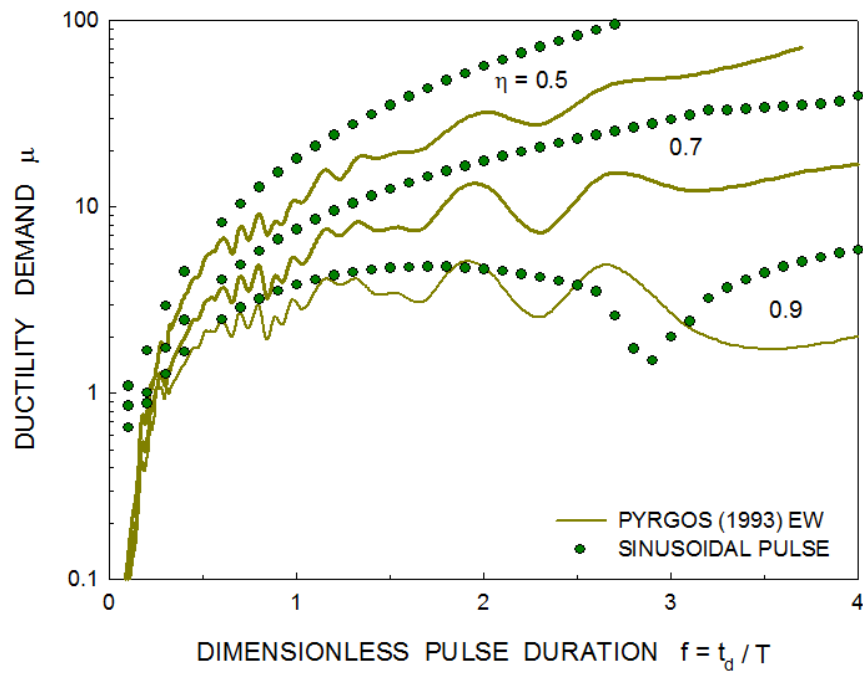


Figure 3.54 Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.

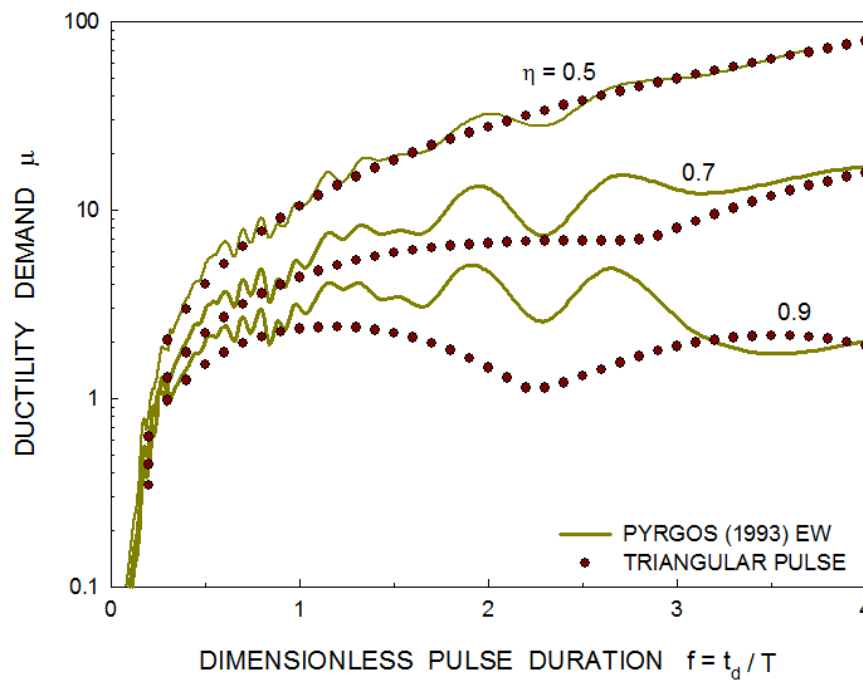


Figure 3.55 Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.

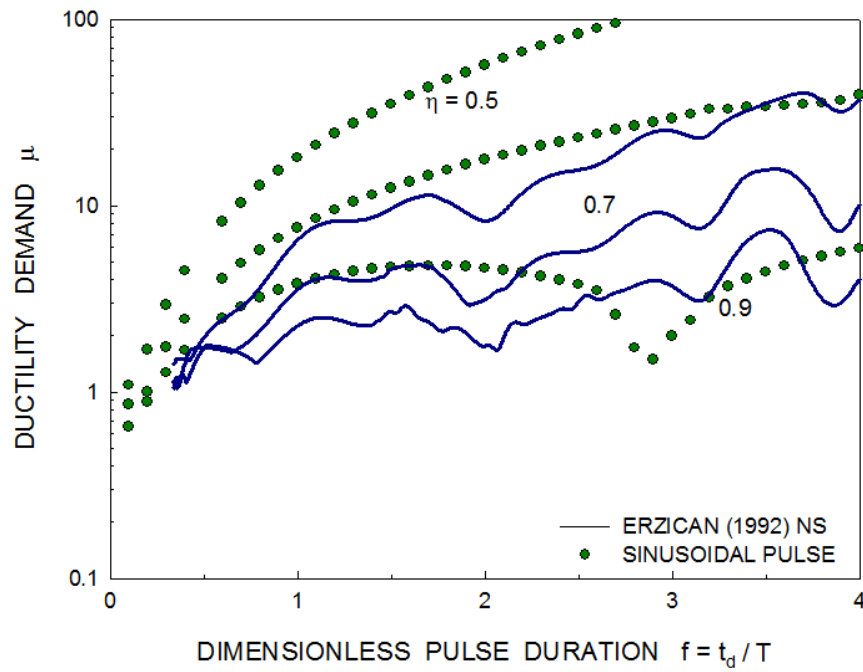


Figure 3.56 Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.

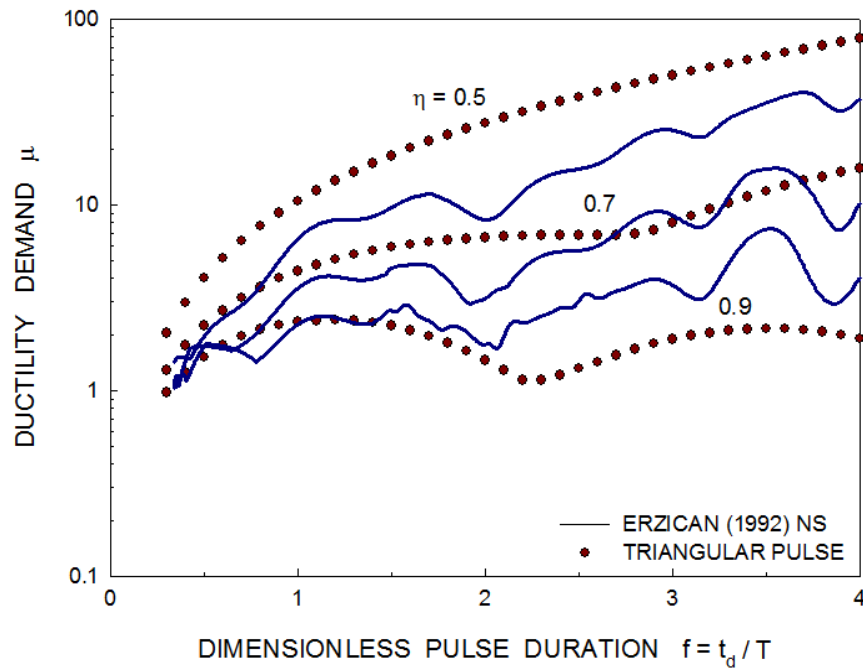


Figure 3.57 Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.

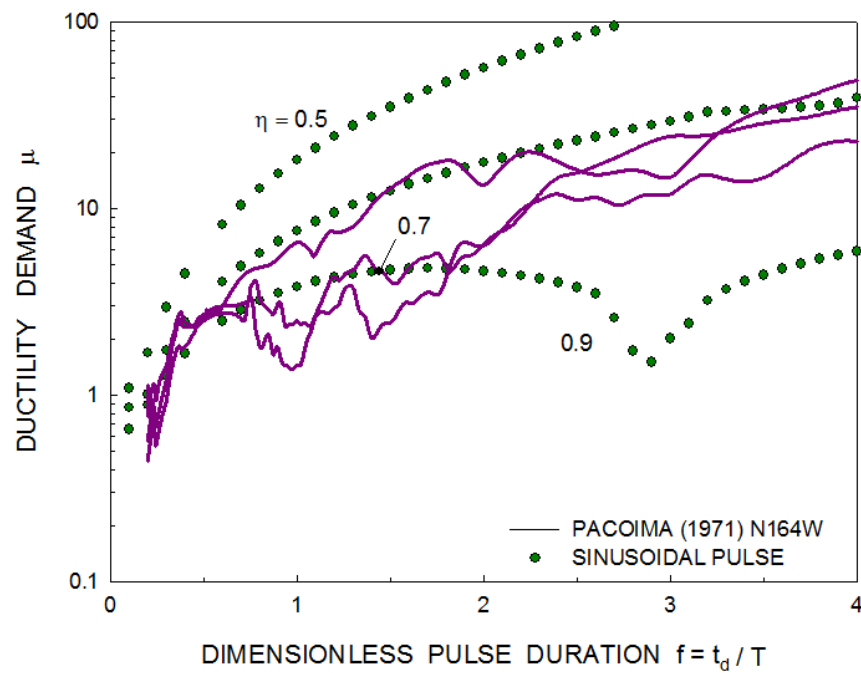


Figure 3.58 Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.

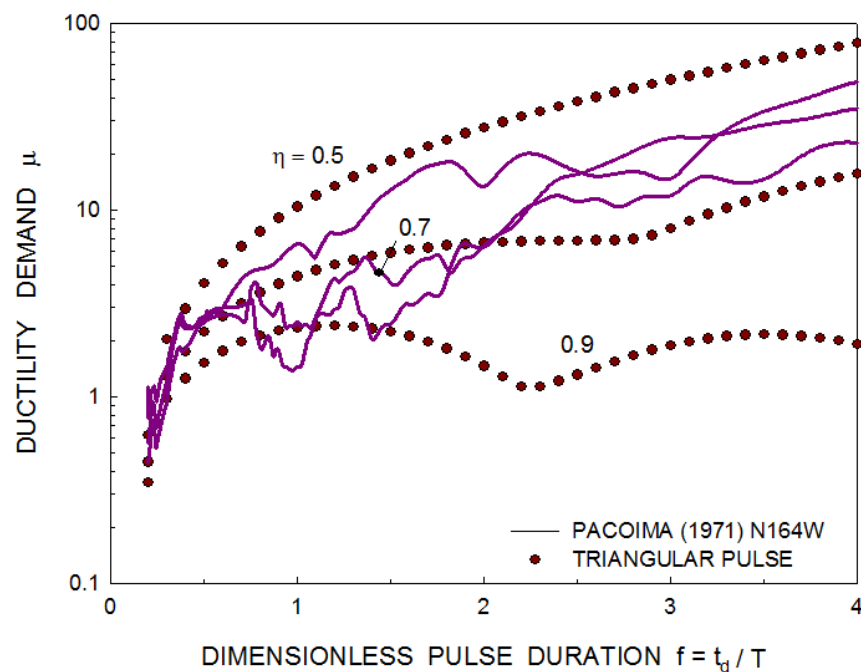


Figure 3.59 Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.

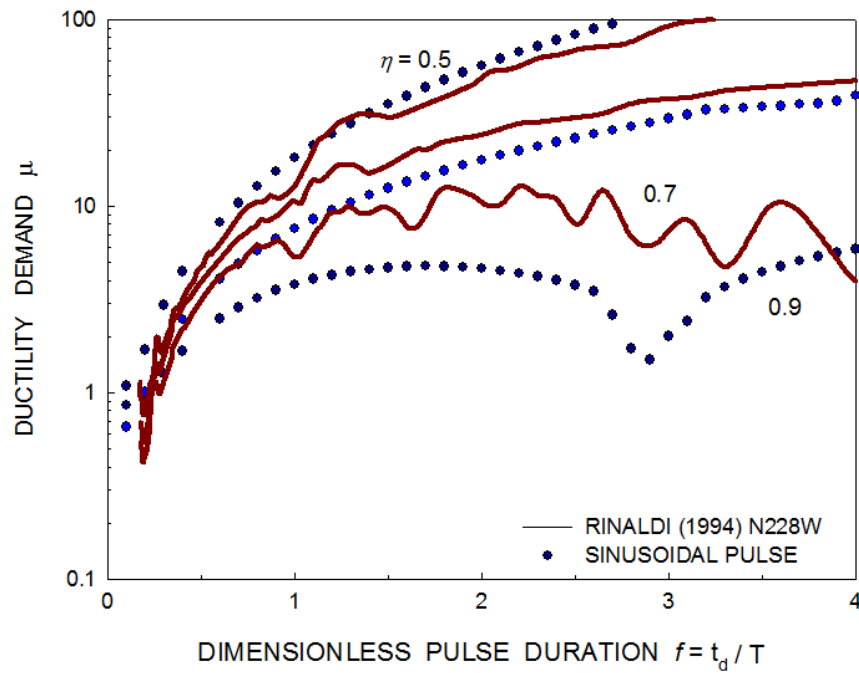


Figure 3.60 Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.

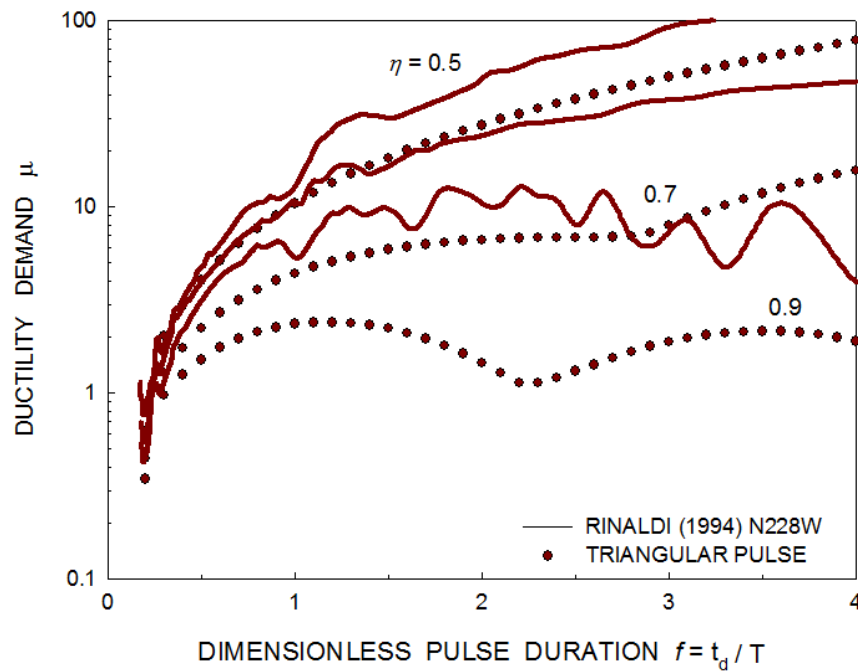


Figure 3.61 Ductility demand for an elastic – perfectly plastic simple oscillator. Comparison of the predictions of the model with an actual record.

3.9 Case Studies

3.9.1 The Collapse of Olive View Hospital in the 1971 San Fernando Earthquake

The Olive View Hospital was a six-story reinforced concrete building (Figure 3.62) located in the meizoseismal area of the 1971 San Fernando Earthquake. The structural system included shear walls that did not extend to the lower two stories. The discontinuity in stiffness made the upper four floors essentially a rigid box supported on soft columns. During the earthquake, the columns were displaced horizontally with respect to their bases by as much as 30 inches (Mahin *et al.* 1976). Because of the extensive damage, the building, which had been completed only a few months before the earthquake, had to be demolished. While no records were obtained at the site, there is evidence that the imposed excitation was similar to that recorded at the nearby Pacoima Dam (Mahin *et al.* 1976; Bertero *et al.* 1978). The performance of the building is analyzed here in conjunction with the Pacoima record to demonstrate the destructiveness of strong pulse-like motions and evaluate the predictions of the model.

Based on earlier investigations (Mahin *et al.* 1976), the estimated fundamental natural period and yielding seismic coefficient of the building are, respectively, $T = 0.58\text{s}$ and $C_y = 0.35$. From Table 3.1, using the values $t_d = 0.6\text{s}$, $PPA = 0.66g$ corresponding to Pacoima record[†], the dimensionless pulse duration and oscillator strength are, respectively

$$f = \frac{t_d}{T} = \frac{0.60}{0.58} \approx 1.03 \quad (3.40)$$

$$\eta = \frac{C_y g}{PPA} = \frac{0.35}{0.66\pi/4} \approx 0.67 \quad (3.41)$$

[†] Note that the modified Pacoima motion used by Mahin *et al.* (1976) and Bertero *et al.* (1978) has the same PPA and t_d .

which pertain to a sinusoidal pulse. To compare the results with those due to a sinusoidal pulse, the amplitude of the pulse in the above equation has been adjusted according to Eq. (3.37).

The "exact" time of yielding is obtained from Eq. (3.33):

$$\tau_y = 0.31 + \frac{\frac{4 \times 1.03^2 - 1}{4 \times 1.03^2} 0.67 + \frac{1}{2 \times 1.03} \sin[2\pi \times 1.03 \times 0.31] - \sin[\pi \times 0.31]}{\pi(\cos[\pi \times 0.31] - \cos[2\pi \times 1.03 \times 0.31])} \approx 0.35 \quad (3.42)$$

which is equivalent to $(0.35) \times (0.6s) = 0.21s$ after the beginning of the pulse. Further, considering zero post-yield hardening (a reasonable approximation in view of the inadequate ductility capacity of the columns), time of peak deformation is computed by means of Eq. (3.19):

$$\tau_m = 0.35 + \frac{(1 - 4 \times 1.03^2 - 4 \times 1.03^2 \cos[\pi \times 0.35] + \cos[2\pi \times 1.03 \times 0.35])}{0.67 \times \pi(1 - 4 \times 1.03^2)} \approx 1.19 \quad (3.43)$$

which corresponds to $(1.19) \times (0.6s) = 0.71s$ after the beginning of the pulse.

The ductility demand on the structure is computed from Eq. (3.21a):

$$\mu = (8 \times 1.03^3 \pi(1.19 - 0.35)C_1 + 8 \times 1.03^3 S_1 - 1.03(4 \times 1.03^2 - 1)[0.67\pi^2(1.19 - 0.35)^2 - 1.19] - S_2 - 2 \times 1.03 \times \pi(1.19 - 0.35)C_2) / (2 \times 1.03 S_1 - S_2) \approx 8.6 \quad (3.44)$$

where $S_1 = \sin 0.35\pi$, $S_2 = \sin 2\pi \times 1.03 \times 0.35$, $C_1 = \cos 0.35\pi$, $C_2 = \cos 2\pi \times 1.03 \times 0.35$.

Assuming $\zeta=5\%$, the effect of damping is assessed from Eq. (3.10) as $\mu(\zeta=5\%)/\mu(0)=0.87$. In addition, for post yielding stiffness equal to 3% of elastic stiffness

($\alpha=0.03$), The modification in ductility demand is obtained via Eq. (3.11) as $\mu(\alpha=3\%)/\mu(0)=0.93$.

Overall,

$$\mu = 0.93 \times 0.87 \times 8.6 \approx 6.9 \quad (3.45)$$

The above value is obviously higher than the ductility capacity of the inadequately designed columns. It is worth mentioning that according to traditional design procedures (Newmark & Hall 1982), the ductility demand would have been estimated from the ratio of the elastic spectral acceleration and the yielding acceleration of the structure. From the response spectrum of the motion (not shown): $SA(T=0.58s) \approx 0.8g$; thus,

$$\mu = \frac{0.8g}{0.35g} \approx 2.3 \quad (3.46)$$

which is about 3 times smaller than the value in Eq. (3.45). Alternatively, with the more conservative equal energy rule one would have obtained the value $(2.3^2 + 1)/2 = 3.1$ which is 2.2 times smaller than the above estimate. A numerical non-linear dynamic analysis of the structure modeled as a single-degree-of-freedom oscillator predicted a ductility demand of about 7 to 8 for periods between 0.5 and 0.6 seconds, which is in agreement with the predictions of the simplified model.

The maximum imposed deformation is computed from the yielding displacement of the system

$$u_y = \frac{C_y}{4\pi^2} g T^2 \approx 0.03 m \quad (3.47)$$

which leads to

$$u_{\max} = \mu u_y = 6.9 \times 0.03 \approx 0.21m \quad (3.48)$$

The above value, although substantially large, is much smaller than the observed residual offset of about 0.50 to 0.75 *meters*, in agreement with earlier analytical investigations (Mahin *et al.* 1976; Bertero *et al.* 1978). Discussing the reasons for this discrepancy lies beyond the scope of this work.

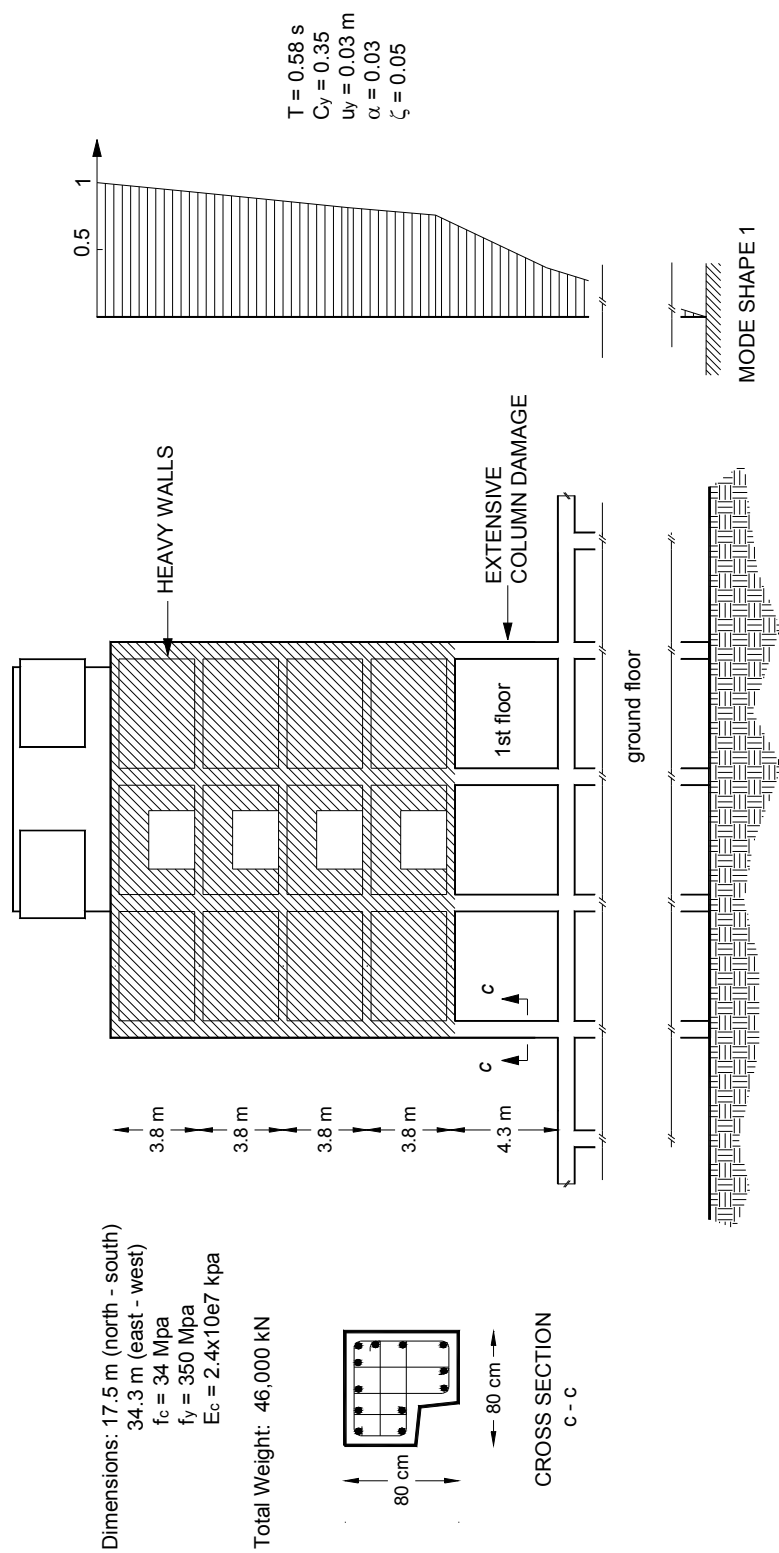


Figure 3.62 Olive View Hospital, Wing D: Inferred geometry and structural properties (modified after Mahin *et al.* 1976).

3.9.2 The Collapse of EBO Building in the 1995 Aegion, Greece, Earthquake

The collapsed portion of the three-storey administration building of the army factory (EBO) consists of frames with square columns, 0.60 m by 0.60 m, reinforced with 12 steel bars of 20 mm diameter. Lateral reinforcement consisted of 8mm stirrups at 200mm interval, which would be judged unacceptably low with today's understanding of the need for ductility. The soft first storey is flexible, consisting exclusively of the square columns, whereas the other two storeys constitute a relatively stiff system. A rather simple but realistic model of EBO building considers the columns of the first storey as the deformable portion of the system and the upper stories as a rigid body (Figure 3.63).

During the earthquake, the lateral displacement of columns was so large that the building collapsed and the ground floor disappeared. The performance of the building is analyzed here in conjunction with the Aegion record, to demonstrate the destructiveness of strong pulse-like motions and evaluate the predictions of the model.

Based on earlier investigations (Gazetas *et al.* 1995), the estimated fundamental natural period and yielding seismic coefficient of the building are, respectively, $T = 0.26\text{s}$ and $C_y = 0.30$. From Table 3.1, using the values $t_d = 0.295\text{s}$, $PPA = 0.54g$ corresponding to Aegion record, the dimensionless pulse duration and oscillator strength are, respectively

$$f = \frac{t_d}{T} = \frac{0.295}{0.26} \approx 1.13 \quad (3.49)$$

$$\eta = \frac{C_y g}{PPA} = \frac{0.30}{0.54\pi/4} \approx 0.71 \quad (3.50)$$

which pertain to a sinusoidal pulse.

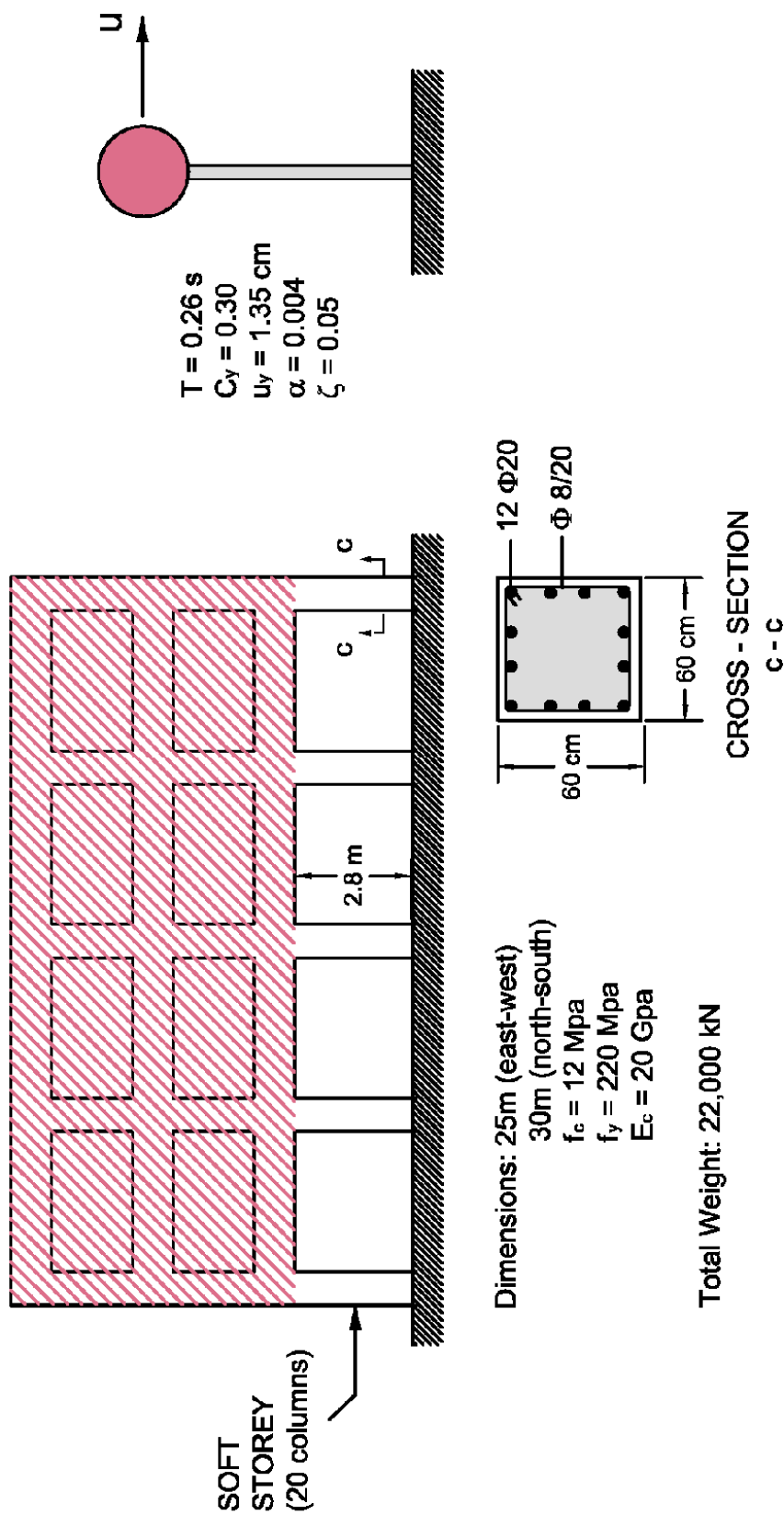


Figure 3.63 EBO Building: approximate geometry and structural properties. [Data after Gazetas *et al.* 1995.]

The "exact" yielding time is obtained from Eq. (3.33):

$$\tau_y = 0.30 + \frac{\frac{4 \times 1.13^2 - 1}{4 \times 1.13^2} 0.71 + \frac{1}{2 \times 1.13} \sin[2\pi \times 1.13 \times 0.30] - \sin[\pi \times 0.30]}{\pi(\cos[\pi \times 0.30] - \cos[2\pi \times 1.13 \times 0.30])} \approx 0.34 \quad (3.51)$$

which is equivalent to $(0.34) \times (0.295s) = 0.10s$ after the beginning of the pulse. Further, considering zero post-yield hardening (a reasonable approximation in view of the inadequate column ductility), the time of peak deformation is computed by means of Eq. (3.19):

$$\tau_m = 0.34 + \frac{(1 - 4 \times 1.13^2 - 4 \times 1.13^2 \cos[\pi \times 0.34] + \cos[2\pi \times 1.13 \times 0.34])}{0.71 \times \pi(1 - 4 \times 1.13^2)} \approx 1.14 \quad (3.52)$$

which corresponds to $(1.14) \times (0.295s) = 0.34s$ after the beginning of the pulse.

The ductility demand on the structure is computed from Eq. (3.21a):

$$\mu = (8 \times 1.13^3 \pi (1.14 - 0.34) C_1 + 8 \times 1.13^3 S_1 - 1.13 (4 \times 1.13^2 - 1) [0.71 \pi^2 (1.14 - 0.34)^2 - 0.88] - S_2 - 2 \times 1.13 \times \pi (1.14 - 0.34) C_2) / (2 \times 1.13 S_1 - S_2) \approx 8.5 \quad (3.53)$$

where $S_1 = \sin 0.34\pi$, $S_2 = \sin 2\pi \times 1.13 \times 0.34$, $C_1 = \cos 0.34\pi$, $C_2 = \cos 2\pi \times 1.13 \times 0.34$.

The above value is obviously higher than the ductility capacity of the inadequately designed columns. It is worth mentioning that according to traditional design procedures (Newmark & Hall 1982), the ductility demand would have been estimated from the ratio of the elastic spectral acceleration and the yielding acceleration of the structure. From the response spectrum of the motion (not shown): $SA(T = 0.26s) \approx 1.07g$; thus,

$$\mu = \frac{1.07g}{0.30g} \approx 3.57 \quad (3.54)$$

which is about 4 times smaller than the value in Eq. (3.53). Alternatively, with the more conservative equal energy rule one would obtain the value $(3.57^2 + 1)/2 = 6.86$ which is 1.25 times smaller than the above estimate. A numerical non-linear dynamic analysis, using Aegion record, of the structure modeled as a single-degree-of-freedom oscillator predicted a ductility demand of about 8 to 10, for periods between 0.2 and 0.3 seconds, which is in a good agreement with the predictions of the simplified model. Additional numerical analysis considering the effects of damping (taken equal to 5%) and post – yielding stiffness (0.4%) indicated minor reductions in ductility demand of the order of 10 percent.

Incidentally, considering a triangular pulse, one gets (Mylonakis & Reinhorn 2001; Eqs 3.19, 3.20a):

$$\tau_y = \frac{1}{2} (0.56)^{0.4} (1.13)^{-0.6} \approx 0.37 \quad (3.55a)$$

or using Eqs (3.32) and (3.35)

$$\tau_y = \frac{0.5 \cdot 0.56 + \frac{1}{2\pi \cdot 1.13} S_2^* - C_2^* \tau_{yt}^*}{1 - C_2^*} \approx 0.36 \quad (3.55b)$$

$$\text{where } \tau_{yt}^* = \sqrt[3]{\frac{3 \cdot 0.56}{4\pi^2 \cdot 1.13^2}} = 0.32, \quad S_2^* = \sin[2\pi \cdot 1.13 \cdot 0.32], \quad C_2^* = \cos[2\pi \cdot 1.13 \cdot 0.32]$$

which is equivalent to $(0.37) \times (0.295s) = 0.11s$ after the beginning of the pulse.

$$\tau_m = 0.37 - \frac{1}{0.56} \left(0.37^2 - \frac{1}{2} - \frac{1 - \cos[2\pi \times 1.13 \times 0.37]}{2\pi^2 1.13^2} \right) \approx 1.16 \quad (3.56)$$

$$\begin{aligned} \mu = & 1 + \frac{\pi^2 1.13^2}{0.56} \left(2 \times 1.16 - 1 - \frac{4}{3} \times 0.37^3 \right) - \\ & - \frac{2}{0.56} \left[2\pi^2 \times 1.13^2 \times 0.37^2 + \cos(2\pi \times 1.13 \times 0.37) - 1 \right] (1.16 - 0.37) - \\ & - 2\pi^2 \times 1.13^2 (1.16 - 0.37)^2 \approx 9.3 \end{aligned} \quad (3.57)$$

which compares well with the value obtained above.

3.10 Conclusions

A simple analytical procedure was developed for determining the response of a simple oscillator subjected to four idealized ground acceleration pulses. Pulse types under discussion were: (a) symmetric triangular; (b) sinusoidal; (c) exponential and (d) rectangular. Both elastic and inelastic response were evaluated and presented numerically as well as analytically. The analytical method allows several key response parameters to be evaluated in closed-form and valuable insight to be gained on the physics of the problem. The most important findings gleaned from the study are:

- (1) Ductility demand on elastic-perfectly-plastic structures with yield resistance smaller than the amplitude of the pulse may attain considerably high values (i.e., larger than 10). Ductility demand is generally an increasing function of pulse duration. This is in contrast with elastic systems where the maximum response is obtained near resonance ($f = 1$).
- (2) Time of yielding can be predicted accurately based on normalized yield strength η and pulse duration f (Eqs 3.27, 3.32, 3.33).
- (3) Peak response always takes place after pulse peak for both elastic and yielding oscillators. Time of peak response can be determined from Eq. (3.19).
- (4) Structural response is sensitive to post-yielding stiffness. The relationship is non linear as this effect is less significant for periods greater than $1.5 T_d$. For smaller periods, however, oscillators with post-yield hardening of approximately 5% may experience 100% less deformation as compared to corresponding perfectly plastic

systems. This emphasizes the need for providing sufficient post-yielding stiffness (ductility) to structures located in the vicinity of active faults.

- (5) The effect of damping is thoroughly examined and is shown that increasing ζ always reduces response. The reduction is more or less the same in all period ranges, oscillator strengths and pulse types while effect on inelastic response is in accordance with the trends observed with the elastic oscillator.
- (6) Higher harmonics often contained in earthquake pulses alter (usually reduce) ductility demand especially for periods greater than $0.6 T_p$ and low values of the amplitude ratio A_s/A_p .
- (7) Force-reduction factor $R (= m SA / Q_y)$ appears inferior to normalized oscillator strength $\eta (= Q_y / m A_g)$ for quantifying inelastic response to impulsive loading. This can be understood given that elastic and inelastic response to strong pulses are fundamentally *different* and, thereby, using R introduces an elastic parameter (spectral acceleration SA) to the solution, which complicates the results, as it is not related to the physical problem.
- (8) Full-cycle pulses ($N = 1$) may generate higher ductility demand with increasing post-yielding stiffness. An explanation for this counter-intuitive effect is that peak response is attained after displacement reversal i.e., in the opposite direction to the initial oscillator movement.
- (9) For elastic–perfectly plastic undamped oscillators subjected to half–cycle pulses, time of yielding, peak deformation, and associated ductility demand can be predicted analytically. The predictive equations (3.19–3.34) are in good agreement with

rigorous numerical solutions. These solutions indicate that ductility demand is controlled by all three time histories of ground movement (i.e., acceleration, velocity, displacement) and, thereby, ground velocity should not be viewed as the sole descriptor of pulse destructiveness.

- (10) Studies of the failure of Olive View Hospital in the 1971 San Fernando earthquake as well as that of EBO building during the 1995 Aegion earthquake were presented. Using the Pacoima Dam and Aegion records respectively it was shown that the structures could not sustain the imposed ductility demand due to the proximity to fault and the associated directivity effects. While the deformations predicted by the models were found to be in reasonable agreement with results from inelastic numerical analyses, the estimated peak displacements were significantly smaller than the observed offset. This is in accord with earlier investigations of the failures using classical numerical approaches.

4

Rocking Response and Overturning Criteria for Free Standing Rigid

Blocks to Single – Lobe Pulses

4.1 Synopsis

The classical problem of rocking of a rigid, free-standing block to earthquake ground shaking containing distinct pulses, as is the case of near-fault earthquake motions is revisited. A rectangular block resting on a rigid base is considered, subjected to a range of idealized ground acceleration pulses expressed by a generalized function controlled by a single shape parameter. The problem is treated analytically in the realm of linearized equations of motion under the assumption of slender block geometry and rocking without slipping. Peak rocking response and overturning criteria for different waveforms are presented in terms of dimensionless closed-form expressions and graphs. Two parameters are employed to this end: dimensionless pulse duration f (actual pulse duration times characteristic block frequency) and dimensionless uplift strength η (ratio of minimum required acceleration for initiation of uplift over peak pulse acceleration). The linearized response is compared with the fully non-linear one using an ad hoc energy formulation leading to a novel approximate closed-form solution. It is shown that the non-linear equations of motion lead to more stable response than their linearized counterparts. A brief discussion on scaling laws is provided.

4.2 Preface

Following the pioneering studies by Milne (1885, 1893), Kirkpatrick (1927) and Housner (1963), the problem of determining the rocking response and overturning of a rigid, free-standing block to ground pulses has been a subject of intense research for over half a century. A basic aim of these investigations is the estimation of a minimum and a maximum level of ground motion intensity given the overturning or survival of slender bodies during strong earthquakes. Despite the apparent simplicity of the problem, tackling analytically the equations of motion has proven to be a formidable task – even for very simple waveforms – due to the non-linear geometric effects, the non-conservative nature of the impact and the exponential character of the response functions. As a result, the majority of available studies are either numerical (Yim *et al.* 1980; Ishiyama 1982; Psycharis & Jennings 1983, 1985; Spanos & Koh 1984; Tso & Wong 1989a; Cai *et al.* 1995; Spanos *et al.* 2001; Prieto *et al.* 2004) or experimental in nature (Muto *et al.* 1960; Aslam *et al.* 1980; Priestley *et al.* 1983; Tso & Wong 1989a; Peña *et al.* 2007; Purvance *et al.* 2008). Whereas certain mixed analytical-numerical formulations (Anooshehpour *et al.* 1999; Zhang & Makris 2001, Kounadis 2013) including approximate closed-form solutions (Dimitrakopoulos & DeJong 2012), are available – exact solutions are limited to rectangular and sinusoidal pulses of half-cycle duration (Housner 1963; Shi *et al.* 1996).

Despite the aforementioned difficulties, analytical investigations are known to be exceptionally valuable as they can cast light on the complex physics of the response, including certain counterintuitive trends observed in numerical and experimental studies. For instance: (a) the stability of a block subjected to a particular ground motion does not decrease monotonically with decreasing size or increasing slenderness ratio (Yim *et al.* 1980), (b) overturning of a block does not imply overturning for an increase in ground

motion amplitude (Yim *et al.* 1980, Psycharis & Jennings 1983) and (c) reduction in restitution coefficient, and thus an increase in system damping, does not imply a decrease in response (Aslam *et al.* 1980). The limited number of pure analytical solutions to the problem provided the initial motivation for the herein reported work.

With reference to a free-standing rigid block subjected to the generalized waveform presented in Chapter 2, that can simulate an infinite set of symmetric pulses of half-cycle duration ranging from a perfect rectangle (“box”) to a perfect impulse (“spike”), an exact analytical solution is presented in this chapter for the rocking response of the block and the conditions of overturning. It will be shown that despite the simplicity of the system and the idealized nature of the excitation, intriguing response patterns can be identified.

4.3 Problem and Parameter Definition

The problem considered involves a rigid block resting on a perfectly stiff horizontal plane, subjected to an idealized ground acceleration pulse acting parallel to the plane. The block, shown in Fig 4.1, is of rectangular shape having uniformly distributed mass m , dimensions $(2b)$ by $(2h)$ – leading to a radial distance from the centre of rotation to the centre of gravity $R = (b^2 + h^2)^{1/2}$ and a dimensionless slenderness angle $\alpha = \tan^{-1}(b/h)^*$. The interface between the block and the rigid base is both tensionless and cohesionless, obeying a linear friction law characterized by a Coulomb coefficient μ which is assumed to be sufficiently large to prevent sliding at all times. As shown by Housner (1963), the dynamic characteristics of the block can be expressed by its characteristic frequency $p = (mgR/I_0)^{1/2}$ (measured in radians $\times$ time $^{-1}$), g being the gravitational acceleration and I_0 the moment of

* Note that block slenderness is actually represented by angle $(\pi/2 - \alpha)$; to conform to the rest of the literature, the complementary angle α will be referred hereafter to as “slenderness”.

inertia of the block with respect to the edge of the base. For a uniform rectangular block, $p = (3g/4R)^{1/2}$. The proposed analysis is also valid for blocks of any symmetric shape about the vertical axis through their centre of mass; in that case, h is the height of the centre of mass from the base and b is the half-width of the base. The dependence of the characteristic frequency solely on block size and gravitational action (not mass) is analogous to the behaviour of a linear pendulum – hence the term “inverted pendulum” coined by Housner (1963).

Ground excitation consists of an idealized horizontal ground acceleration pulse described by a shape parameter β (to be discussed below), amplitude A_g (normalized amplitude $\alpha_g = A_g/g$) and half-cycle duration t_d . With reference to Fig 4.2, the pulse at hand is described as:

$$\ddot{u}_g(t) = \frac{A_g}{1-e^\beta} [1 - e^{2\beta t/t_d} + (e^{2\beta t/t_d} - e^{2\beta(1-t/t_d)})H(t-t_d/2) - (1 - e^{2\beta(1-t/t_d)})H(t-t_d)] \quad (4.1)^\dagger$$

where $H(\cdot)$ denotes the Heaviside (step) function.

The above excitation function was apparently first employed by Jacobsen & Ayre (1958) for the investigation of linear shock spectra. Its use is extended here to the case of rocking blocks, since it can simulate by means of a single parameter (β) an infinite set of symmetric pulses (Fig 4.2). This appears desirable as a single solution suffices for describing dynamic response to vastly different waveforms. Other idealized pulse shapes employed in related research are available in Jacobsen & Ayre (1958), Mavroeidis & Papageorgiou (2003) and Ricker (1944).

[†] Note that Eq. (4.1) is the same with Eq. (2.1.a) written in a more compact way.

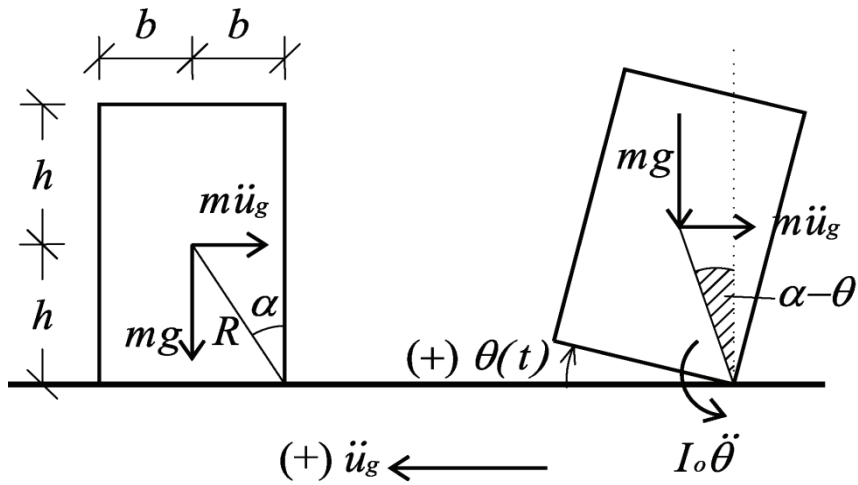


Figure 4.1 Problem considered: rocking motion of a free-standing block resting on a rigid base.[‡]

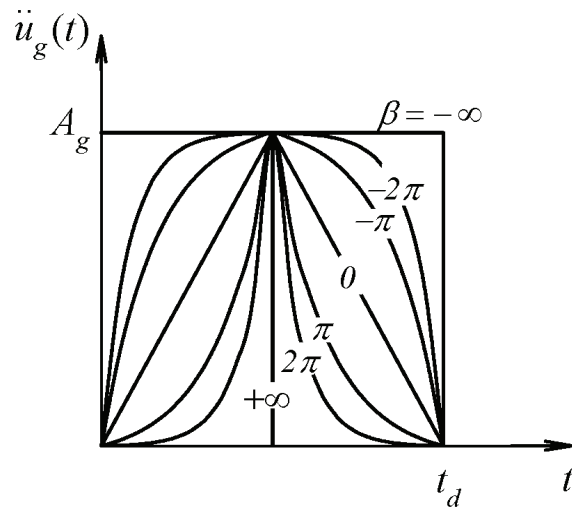


Figure 4.2 Ground acceleration time history for a generalized exponential pulse of half-cycle duration.

[‡] The inertial force associated with the translation of the center of mass of the block ($mh\ddot{\theta}$) is not indicated in the graph, as moment of inertia I_o is taken around the edge of the base. This is a simpler approach as it leads to three equilibrating actions instead of four.

Strictly speaking, half-cycle pulses are not physically realizable, as ground velocity at the end of the excitation ($t = t_d$) is not zero. However, a half-cycle pulse can be thought as the first lobe of more complicated waveforms that incorporates the large acceleration and velocity excursions observed in actual near-fault earthquake recordings and, thereby, is well suited for investigating the salient features of seismic demand imposed by such motions. In addition it will be shown that certain rocking systems attain their peak response before the end of excitation and, thereby, whether or not ground velocity tends to zero or not at a later time is of less importance. Note that the above attributes apply to other dynamic systems as well, including simple oscillators (Mylonakis & Voyagaki 2006) and sliding blocks (Chapter 2).

The following dimensionless quantities are employed to describe the results presented below: rocking angle θ , peak rocking angle θ_{max} , uplift strength $\eta = \alpha/\alpha_g$, dimensionless pulse duration $f = p t_d$, normalized time of rocking initiation $\tau_{up} = t_{up}/t_d$, normalized time of maximum response $\tau_m = t_m/t_d$, normalized time of impact $\tau_i = t_i/t_d$, and normalized critical (“overturning”) time $\tau_c = t_c/t_d$ when $\theta(\tau_c) = \alpha$.

From a dimensional analysis viewpoint, given that the problem is described by 6 dimensional variables (i.e. m, R, b, g, A_g, t_d) and the rank of the dimensional matrix is 3 (i.e. Mass, Time, Length), Buckingham’s theorem suggests that 3 (=6–3) dimensionless parameters (in addition to shape parameter β) suffice for describing peak rocking response. In this chapter parameters α, η and f are selected for this purpose. Accordingly one may write

$$\theta_{max} = F(f, \eta, \alpha) \tag{4.2}$$

which can be established *a priori* i.e., without knowing the solution or even formulating the governing equations. Note that this analysis does not take advantage of (nor can it benefit from) the observation that the response of the block is independent of its mass. It should also be noticed that due to the very nature of the problem, dimensional analysis alone cannot contribute much more than Eq. (4.2), so tackling the equations of motion is essential. Finally, it is noteworthy that the coefficient of restitution has been omitted from Eq. (4.2), as it is irrelevant for peak response to single-lobe pulses such as those examined here. Issues related to the complementary concept of orientational homogeneity are discussed by Dimitrakopoulos & DeJong (2012).

4.3.1 Rocking Initiation

Depending on the characteristics of the excitation and the properties of the frictional interface, the block can either: stay still, slide (without rocking), rock (without sliding), slide and rock at the same time, or upthrow following an impact (Ishiyama 1982; Shendon 1996; Pompei *et al.* 1998). Our analysis focuses solely on rocking behaviour and thus assumes a sufficiently large frictional coefficient and a perfectly rigid base so that sliding and upthrow are prevented.

To trigger rocking motion, the overturning moment due to inertial action should exceed the restoring moment due to gravity, which can be written as $(\ddot{u}_g / g) \geq \tan \alpha$; for slenderness angles α of less than approximately 20 degrees (Housner 1963; Yim *et al.* 1980), the trigonometric argument on the right side of the inequality can be dropped to give

$$\ddot{u}_g / g \geq \alpha \quad (4.3)$$

This criterion can be expressed in terms of uplift strength η as follows: if $\eta > 1$ no rocking occurs, if $\eta < 1$ rocking initiates at $\tau = \tau_{up}$, when $\ddot{u}_g(\tau_{up}) = \alpha g$.

Substituting the above critical acceleration into Eq. (4.1), the time of uplift can be readily determined as

$$\tau_{up} = \frac{1}{2\beta} \ln[1 - \eta(1 - e^\beta)] \quad (4.4)$$

Numerical results obtained from Eq. (4.4) are shown in Fig 4.3, plotted as function of uplift strength. Evidently if uplift takes place ($\eta < 1$), corresponding times for the waveforms at hand occur during rising acceleration ($0 \leq \tau_{up} \leq 1/2$). For a rectangular pulse ($\beta \rightarrow -\infty$) rocking initiates at $\tau_{up} = 0$, whereas for a perfect spike pulse ($\beta \rightarrow +\infty$) at $\tau_{up} = 1/2$. Note that since uplift depends solely on ground acceleration, Eq. (4.4) also describes initiation of sliding of a rigid body on a frictional interface (Chapter 2).

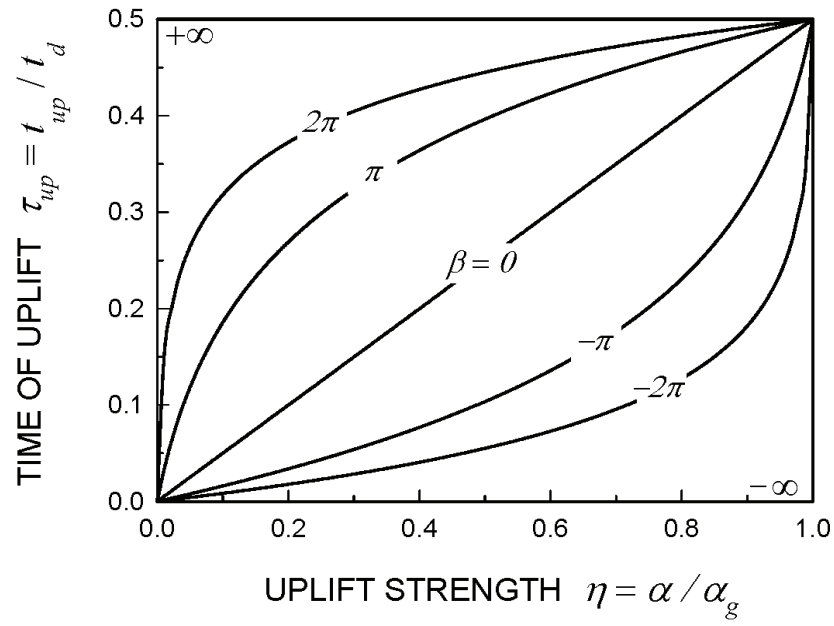


Figure 4.3 Time of uplift with uplift strength due to a generalized exponential pulse of half-cycle duration.

4.3.2 Equations of Motion

The non-linear equations governing rocking response of the system at hand are (Housner 1963)

$$I_o \ddot{\theta} + mgR \sin(\alpha - \theta) = +m\ddot{u}_g R \cos(\alpha - \theta), \quad \theta(t) > 0 \quad (4.5)$$

$$I_o \ddot{\theta} - mgR \sin(\alpha + \theta) = +m\ddot{u}_g R \cos(\alpha + \theta), \quad \theta(t) < 0 \quad (4.6)$$

where θ is the rocking angle and I_o is the aforementioned moment of inertia of the block with respect to the edge of the base (pivot point). For rectangular geometry, $I_o = (4/3) mR^2$. The positive sign on the right-hand side of Eqs (4.5) and (4.6) is to ensure positive response for positive ground acceleration, as evident from the reference system of Fig 4.1.

For slender blocks, angles θ and α are small; the equations of motion (4.5) & (4.6) can be linearized using the first-order approximations $\sin(\alpha \pm \theta) \cong \alpha \pm \theta$ and $\cos(\alpha \pm \theta) \cong 1$ to give

$$\ddot{\theta} - p^2 \theta = +p^2 \ddot{u}_g / g - p^2 \alpha \operatorname{sgn}(\theta) \quad (4.7)$$

where $\operatorname{sgn}(\)$ denotes the signum function. It should be noticed that the second term on the right-hand side refers to a constant, in each response branch, restoring moment associated with the weight of the block, whereas the second term on the left-hand side can be interpreted as a negative (geometric) stiffness.

Inspecting Eq. (4.7) and recalling that $\eta = \alpha / \alpha_g$ one can notice that in the linearized equation of motion, slenderness α perfectly normalizes the dependent variable θ i.e.,

$$\theta_{max} = \alpha F(f, \eta) \quad (4.8)$$

The monomial dependence of θ on α effectively reduces the number of independent variables of the problem by one and thus can be interpreted as a similarity attribute. Interestingly, this property arises from the structure of the governing equation, not from dimensional considerations as commonly thought (Barenblatt 1996).

As mentioned in Section 4.2, the above equation has been solved in an exact manner only for two cases: (1) a half-cycle rectangular pulse (Housner 1963); (2) a half-cycle sinusoidal pulse (Anooshehpour *et al.* 1999). Note that an elegant, yet still approximate, closed-form solution pertaining to a full-cycle trigonometric pulse has been recently derived by Dimitrakopoulos and DeJong (2012). Analytical solutions for the more general waveforms in Eq. (4.1) are presented in the ensuing sections.

4.4 Solutions

4.4.1 Rocking Response

For the generalized exponential pulse in Eq. (4.1), upon introducing dimensionless peak ground acceleration $\alpha_g = A_g/g$ and considering positive rocking angles (the only case of interest for single-lobe excitation), Eq. (4.7) can be cast in the form

$$\ddot{\theta} - p^2\theta = +p^2\alpha_g(1 - \exp[2\beta[\tau + (1-2\tau)H(\tau-1/2) - (1-\tau)H(\tau-1)])/(1-e^\beta) - p^2\alpha \quad (4.9)$$

which holds under uplift conditions i.e., when $\alpha_g > \alpha$ or, equivalently, when $\alpha/\alpha_g = \eta < 1$.

Enforcing the initial conditions of zero angular velocity $\dot{\theta}(\tau_{up}) = 0$ and rotation $\theta(\tau_{up}) = 0$, the response functions are obtained as:

$$\frac{\theta(\tau)}{\alpha} = 1 - \frac{x_1 + x_2 H(\tau-1/2) + x_3 H(\tau-1)}{\eta(1-e^\beta)(f^2 - 4\beta^2)} \quad (4.10a)$$

$$\frac{\dot{\theta}(\tau)}{\alpha f} = -\frac{\dot{x}_1 + \dot{x}_2 H(\tau - 1/2) + \dot{x}_3 H(\tau - 1)}{\eta f (1 - e^\beta) (f^2 - 4\beta^2)} \quad (4.10b)$$

where

$$x_1 = \beta(f + 2\beta)[1 - \eta(1 - e^\beta)]^{1 - \frac{f}{2\beta}} e^{f\tau} - \beta(f - 2\beta)[1 - \eta(1 - e^\beta)]^{1 + \frac{f}{2\beta}} e^{-f\tau} + f^2(1 - e^{2\beta\tau}) - 4\beta^2 \quad (4.11a)$$

$$x_2 = f^2(e^{2\beta\tau} - e^{2\beta(1-\tau)}) - 2\beta f e^\beta (e^{f(\tau-1/2)} - e^{-f(\tau-1/2)}) \quad (4.11b)$$

$$x_3 = f^2 e^{2\beta(1-\tau)} - (f^2 - 4\beta^2) + \beta[(f - 2\beta)e^{f(\tau-1)} - (f + 2\beta)e^{-f(\tau-1)}] \quad (4.11c)$$

in which dot denotes differentiation with respect to normalized time (i.e., $\dot{x}_i = dx_i / d\tau$).

In the limit case of a rectangular pulse, $\beta \rightarrow -\infty$; Eqs (4.10) and (4.11) simplify to

$$\frac{\theta(\tau)}{\alpha} = \left(\frac{1}{\eta} - 1\right)(\cosh(f\tau) - 1) - \frac{1}{\eta}(\cosh[f(\tau - 1)] - 1)H(\tau - 1) \quad (4.12)$$

$$\frac{\dot{\theta}(\tau)}{\alpha f} = \left(\frac{1}{\eta} - 1\right)\sinh(f\tau) - \frac{1}{\eta}\sinh[f(\tau - 1)]H(\tau - 1) \quad (4.13)$$

Likewise, for a triangular pulse $\beta \rightarrow 0$; Eqs (4.10) and (4.11) yield

$$\begin{aligned} \frac{\theta(\tau)}{\alpha} = & 1 - \frac{1}{\eta}\left(2\tau - \frac{2}{f}\sinh\left[f\left(\tau - \frac{\eta}{2}\right)\right]\right) - \frac{1}{\eta}\left(2 - 4\tau + \frac{4}{f}\sinh\left[f\left(\tau - \frac{1}{2}\right)\right]\right)H\left(\tau - \frac{1}{2}\right) - \\ & - \frac{1}{\eta}\left(2(\tau - 1) - \frac{2}{f}\sinh[f(\tau - 1)]\right)H(\tau - 1) \end{aligned} \quad (4.14)$$

$$\begin{aligned} \frac{\dot{\theta}(\tau)}{\alpha f} = & -\frac{2}{\eta f}(1 - \cosh[f(\tau - \frac{\eta}{2})]) - \frac{4}{\eta f}(-1 + \cosh[f(\tau - \frac{1}{2})])H(\tau - \frac{1}{2}) - \\ & - \frac{2}{\eta f}(1 - \cosh[f(\tau - 1)])H(\tau - 1) \end{aligned} \quad (4.15)$$

Note that in the limit $\beta \rightarrow +\infty$ the pulse degenerates to a perfect spike pulse of zero area (i.e., not a delta function); both θ and $\dot{\theta}$ are zero regardless of pulse amplitude. Note, however, that this is a trivial case with little practical significance, as, evidently, for large but finite values of β any impulsive excitation with $\alpha_g > \alpha$ will induce rocking of finite amplitude.

4.4.2 Overturning Criterion

A primary objective of the analysis at hand is to determine under which conditions the block will overturn. Defining “overturning”, however, is not straightforward, as the obvious condition $\theta = \pi/2$ is inapplicable in the realm of the linearized equations of motion (Eq. 4.6) and, thereby, alternative definitions must be sought.

For unilateral excitations like the half-cycle pulse in Eq. (4.1), if rocking angle θ exceeds slenderness α , the restoring moment due the weight of the block will switch into a driving moment response will become unstable (to be referred in the ensuing as “supercritical”), and the block will overturn. Accordingly, a sufficient acceleration amplitude to overturn the block can be defined – as a sufficient condition – by setting the angle θ in Eq. (4.10) equal to α .

Enforcing this simple criterion at the end of pulse ($\tau_c = I$), corresponding to the end of the decreasing acceleration branch in Fig 4.2, the following discriminant equation is obtained

$$\beta[(f + 2\beta)[1 - \eta(1 - e^\beta)]^{1 - \frac{f}{2\beta}} e^f - (f - 2\beta)[1 - \eta(1 - e^\beta)]^{1 + \frac{f}{2\beta}} e^{-f}] = 4\beta^2 - f^2 + 2\beta f e^\beta (e^{\frac{f}{2}} - e^{-\frac{f}{2}}) \quad (4.16a)$$

which can be solved numerically in the form $\eta = \eta(f)$ or $f = f(\eta)$.

Repeating the analysis for $\tau_c = 1/2$ (end of rising acceleration branch), yields the analogous expression

$$\beta[(f + 2\beta)[1 - \eta(1 - e^\beta)]^{1 - \frac{f}{2\beta}} e^{\frac{f}{2}} - (f - 2\beta)[1 - \eta(1 - e^\beta)]^{1 + \frac{f}{2\beta}} e^{-\frac{f}{2}}] = 4\beta^2 - f^2 + f^2 e^\beta \quad (4.16b)$$

For $\beta \rightarrow -\infty$, corresponding to a rectangular pulse, Eqs (4.16a, b) provide, respectively, the exact solutions

$$\eta = 1 - 1/\cosh(f) \quad (4.17a)$$

$$\eta = 1 - 1/\cosh(f/2) \quad (4.17b)$$

In the case of a triangular pulse ($\beta \rightarrow 0$), the corresponding solutions are

$$\eta = 2 + \frac{1}{f} \ln[1 + 2f(f - \sqrt{f^2 + 1})] \quad (4.18a)$$

$$\eta = 1 + \frac{1}{f} \ln[1 + \frac{f}{2}(f - \sqrt{f^2 + 4})] \quad (4.18b)$$

The above equations are evidently applicable even to more complicated base excitations in which the ground acceleration time history continues after t_d . However, overturning during pulse does not represent the most critical condition for the particular system. The exact overturning criterion can be developed using the notion of simultaneous immobility ($\dot{\theta} = 0$) and equilibrium ($\ddot{\theta} = 0$) of the block i.e.,

$$\dot{\theta}(\tau_c) = 0 \quad (4.19a)$$

$$\ddot{\theta}(\tau_c) = 0 \text{ or } \theta(\tau_c) = \alpha \quad (4.19b)$$

that define a *saddle point* in the response [provided that $\ddot{\theta}(\tau_c) \neq 0$], at critical time $\tau = \tau_c$.

As for any system that overturns at a finite time, there is always a stronger system that overturns at a later time, it is straightforward to show (Zhang & Makris 2001; Dimitrakopoulos & DeJong 2012) that the saddle point is always located at infinity (i.e., $\tau_c \rightarrow \infty$).

By virtue of Eqs. (4.10) and (4.11), the above conditions yield the solution

$$\eta_w = \frac{1}{1-e^\beta} \left\{ 1 - \left[\frac{e^f (f+2\beta)}{2f e^{\frac{f}{2}+\beta} - (f-2\beta)} \right]^{\frac{2\beta}{f-2\beta}} \right\} \quad (4.20)$$

which expresses the overturning criterion for the suite of pulses at hand. In the above equation η_w is interpreted as a limit strength separating systems that overturn ($\eta < \eta_w$) from those that do not ($\eta > \eta_w$). In this light, the curve $\eta_w = \eta_w(f, \beta)$ will be referred in the ensuing to as “*safety wall*”. Unlike the results in Eqs (4.16) to (4.18), the above overturning criterion is complete as it describes systems reaching $\theta = \alpha$ at $t \rightarrow +\infty$ under zero angular velocity and acceleration, yet with a tendency for increasing response (i.e., $d(\ddot{\theta})/dt > 0$ - a strictly increasing point of inflection). This extra “push” guarantees overturning in the limit sense.

Considering the simple case of a rectangular pulse ($\beta \rightarrow -\infty$), Eq. (4.20) simplifies to

$$\eta_w = 1 - e^{-f} \quad (4.21)$$

which coincides with the solution by Housner (1963). For a triangular pulse ($\beta \rightarrow 0$) one obtains

$$\eta_w = 2 - \frac{2}{f} \ln[2e^{\frac{f}{2}} - 1] \quad (4.22)$$

The above solutions are presented graphically in Figure 4.4 which depicts the areas of safety (denoted by letter S) and overturning (denoted by letter O) as function of uplift strength η and pulse duration f for an exponential-like pulse obtained for a shape factor $\beta = \pi$. Evidently, for large values of uplift strength overturning requires large pulse duration and vice versa. Regions O_1 and O_2 correspond to supercritical response during pulse (the former taking place before $t_d/2$ and the latter after $t_d/2$), whereas region O_3 corresponds to supercritical response after pulse. The line separating area O_2 and O_3 corresponds to the solution in Eq. (4.16a), whereas the line separating area O_3 from the area of safety S corresponds to the safety wall and is given by Eq. (4.20). Finally the line separating areas O_1 and O_2 is defined by the solution of Eq. (4.16b). Note that the two dimensionless parameters η and f suffice for describing rocking response, a property associated with the similarity attribute of the linearized problem in Eq. (4.8). Corresponding results for the cases of a rectangular ($\beta \rightarrow -\infty$) and a triangular ($\beta = 0$) and a sinusoidal – like ($\beta = -\pi$) pulse are provided in Figs 4.5, 4.6 and 4.7, respectively.

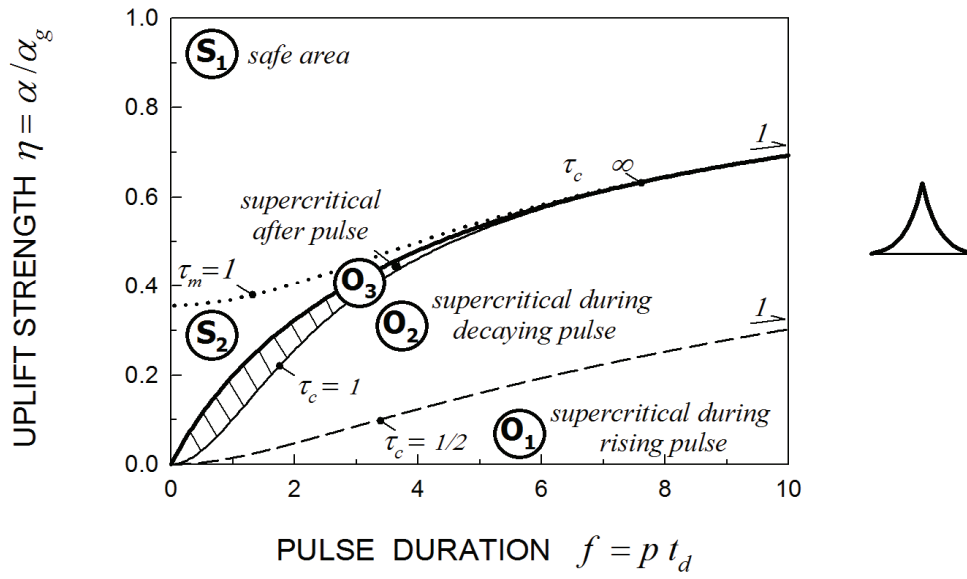


Figure 4.4 Areas of Safety (S) and Overturning (O₁ to O₃) for a rocking block under an exponential pulse excitation of half-cycle duration ($\beta = \pi$).

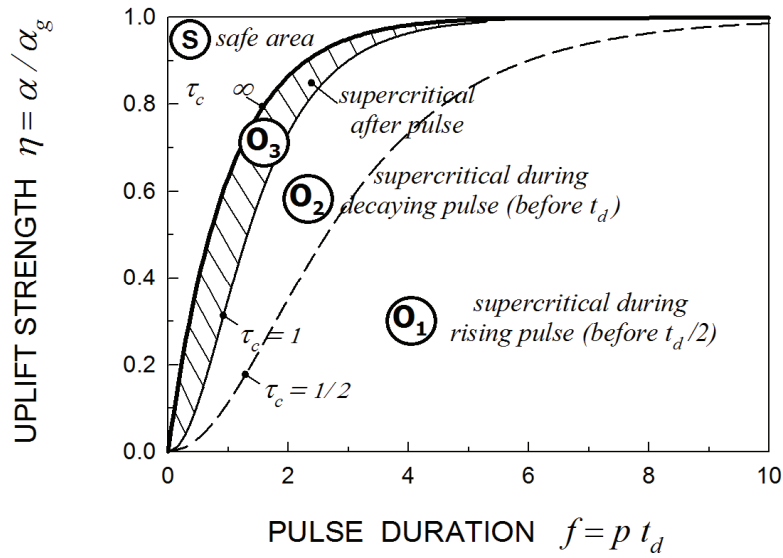


Figure 4.5 Areas of Safety (S) and Overturning (O₁ to O₃) for a rocking block under a rectangular pulse excitation of half-cycle duration ($\beta \rightarrow \infty$).

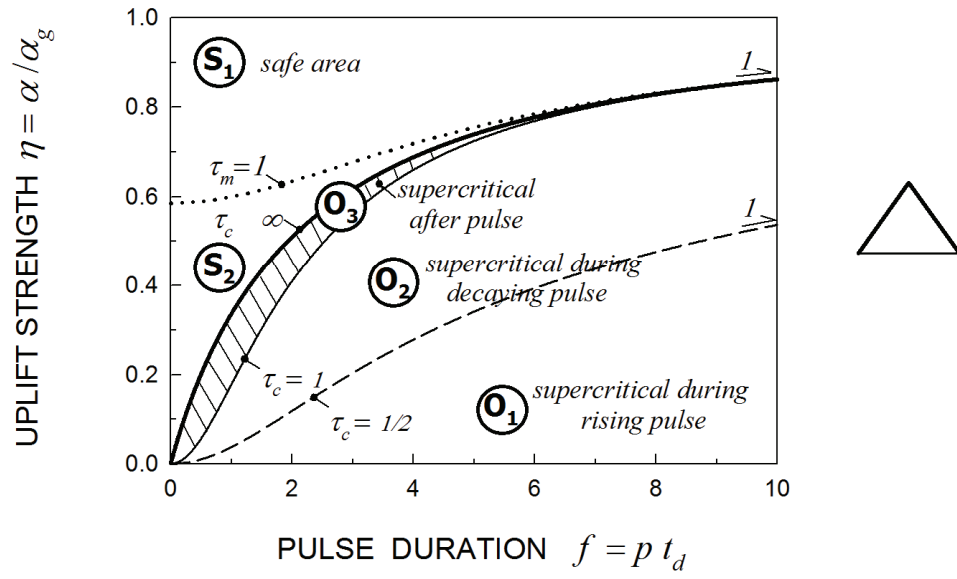


Figure 4.6 Areas of Safety (S) and Overturning (O₁ to O₃) for a rocking block under a triangular pulse excitation of half-cycle duration ($\beta \rightarrow 0$).

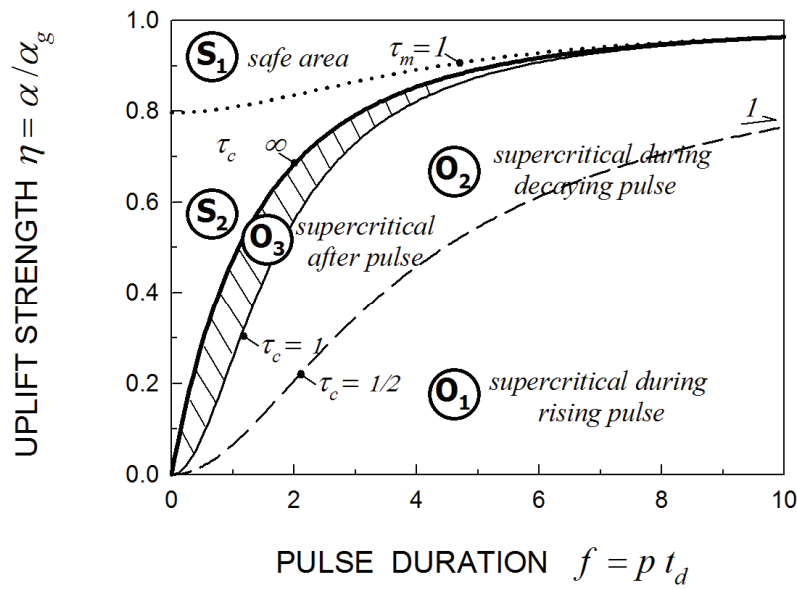


Figure 4.7 Areas of Safety (S) and Overturning (O₁ to O₃) for a rocking block under a sinusoidal-like pulse excitation of half-cycle duration ($\beta \rightarrow -\pi$).

Figure 4.8 depicts the overturning criteria in Eq. (4.20) (*safety walls*) as functions of uplift strength η and pulse duration f for a rocking block under a generalized exponential pulse of half-cycle duration, for different values of shape factor β , ranging from $-\infty$ (perfect rectangle) to $+\infty$ (perfect spike). Evidently, the *safety wall* is anchored at $(f, \eta) = (0, 0)$ and $(+\infty, 1)$, regardless of pulse shape. Naturally, the higher the value of shape factor β , the smaller the region of overturning. For high values of uplift strength overturning requires large pulse durations and vice versa.

It must be emphasized that the above comparisons consider pulses of equal acceleration and duration, but different velocities (depending on β). Accordingly these results cannot be interpreted in the context of “equal energy” principles.

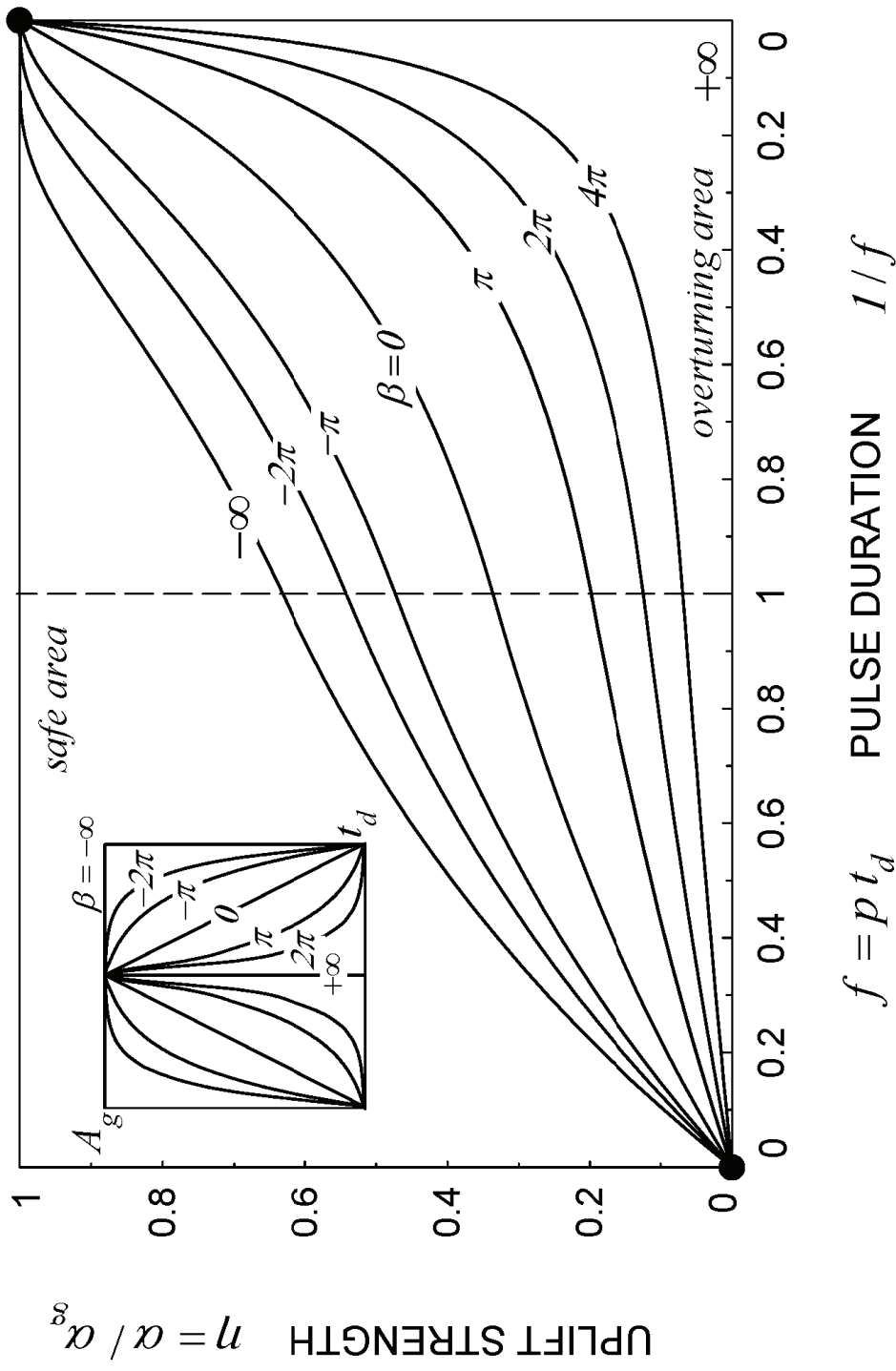


Figure 4.8 Areas of Safety (S) and Overturning (O) for a rocking block under an exponential pulse excitation of half-cycle duration for different values of shape factor β .

4.4.3 Peak Rocking Response

With reference to systems that do not overturn (i.e., blocks in region S having strength higher than critical), there exists a time $\tau_m = t_m / t_d$ when rocking velocity becomes zero and the motion reverses leading to an impact. Naturally, rocking angle $\theta(\tau_m)$ defines a maximum (θ_{max}).

For the family of pulses examined here, peak response may take place during decaying pulse ($1/2 \leq \tau_m \leq 1$), or after pulse ($\tau_m \geq 1$). It can be shown that for symmetric half-cycle excitations like the ones in Fig 4.2,

$$\tau_m \geq 1 - \tau_{up} \quad (4.23)$$

which can be derived by setting the angular velocity $\dot{\theta}(\tau_m)$ equal to zero.

For systems in region S_I , (i.e., $1/2 \leq \tau_m \leq 1$), τ_m is obtained by solving the transcendental equation

$$\begin{aligned} (f + 2\beta)[1 - \eta(1 - e^\beta)]^{1 - \frac{f}{2\beta}} e^{f\tau_m} + (f - 2\beta)[1 - \eta(1 - e^\beta)]^{1 + \frac{f}{2\beta}} e^{-f\tau_m} = \\ = -2fe^{2\beta(1-\tau_m)} + 2fe^\beta (e^{f(\tau_m-1/2)} + e^{-f(\tau_m-1/2)}) \end{aligned} \quad (4.24a)$$

which should be treated numerically except for certain special cases, to be discussed below. Note that approximate solutions to Eq. (4.24a) can be obtained in closed-form using the technique described in Dimitrakopoulos & DeJong (2012).

For systems in region S_2 (i.e., $\tau_m \geq 1$) the following closed-form solution is possible

$$\tau_m = \frac{1}{2f} \ln \left[\frac{\left(f - 2\beta + 2fe^{\frac{f}{2} + \beta} \right) \left[e^f (f + 2\beta) - 2fe^{\frac{f}{2} + \beta} + (f - 2\beta)[1 - \eta(1 - e^\beta)]^{1 + \frac{f}{2\beta}} \right]}{4f^2 e^{2\beta} - e^{-f} (f - 2\beta)^2 - (f + 2\beta)(f - 2\beta + 2fe^{\frac{f}{2} + \beta})[1 - \eta(1 - e^\beta)]^{1 - \frac{f}{2\beta}}} \right] \quad (4.24b)$$

For the limit cases of a rectangular ($\beta \rightarrow -\infty$) and a triangular pulse ($\beta \rightarrow 0$) the times of peak rocking response are provided, respectively, by the expressions

$$\tau_m = \frac{1}{2f} \ln \left[\frac{1 - \eta - e^f}{1 - \eta - e^{-f}} \right] \quad (4.25)$$

$$\tau_m = \frac{1}{f} \ln \left[\frac{1 + 2\sqrt{\cosh\left[\frac{f}{2}(1 - \eta)\right] - 1}}{2e^{-\frac{f}{2}} - e^{-\frac{f\eta}{2}}} \right], \quad 1/2 \leq \tau_m \leq 1 \quad (4.26a)$$

$$\tau_m = \frac{1}{2f} \ln \left[-\frac{2e^{\frac{f}{2}} - e^{\frac{f\eta}{2}} - e^f}{2e^{-\frac{f}{2}} - e^{-\frac{f\eta}{2}} - e^{-f}} \right], \quad \tau_m > 1 \quad (4.26b)$$

Note that for a rectangular pulse, peak response always occurs after pulse ($\tau_m > 1$), whereas for a triangular pulse peak response may occur either during pulse or after pulse depending on the values of f and η . Numerical results for τ_m based on the above solutions are provided in Fig. 4.9.

The special case of systems reaching their peak response at the end of the pulse ($\tau_m = 1$) is represented in Figs 4.4, 4.6 and 4.7 by the line separating regions S_1 ($1/2 \leq \tau_m \leq 1$) and S_2 (i.e., $\tau_m \geq 1$). This boundary can be determined numerically by setting $\dot{\theta} = 0$ in Eq. (4.11) at $\tau = \tau_m = 1$.

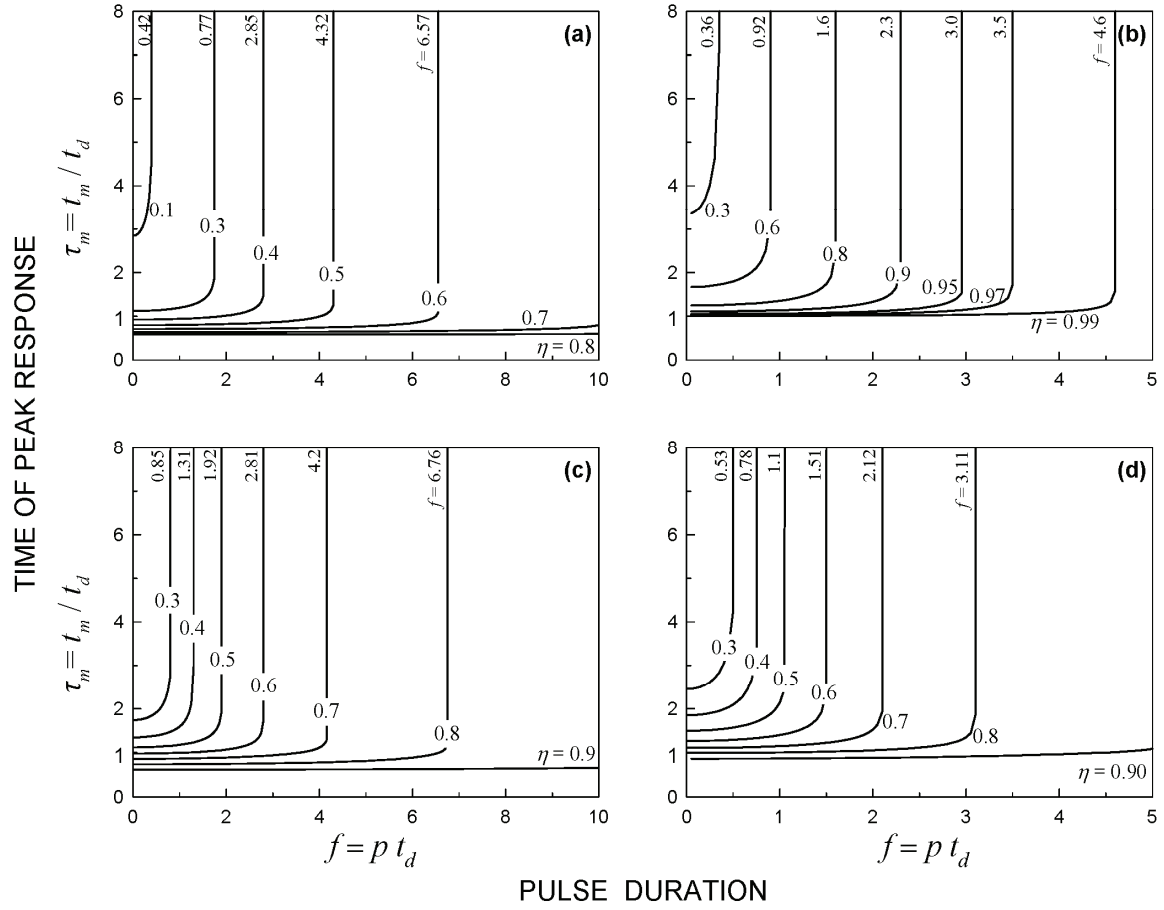


Figure 4.9 Time of peak rocking response as function of uplift strength and pulse duration (region S) due to a half-cycle generalized exponential pulse of : (a) exponential ($\beta = \pi$), (b) rectangular ($\beta \rightarrow \infty$), (c) triangular ($\beta \rightarrow 0$) and (d) sinusoidal-like shape ($\beta = -\pi$).

Normalizing τ_m with $1/\eta$, which corresponds to the value of τ_m for zero pulse duration (i.e. for the rectangular pulse Eq. (4.25) for $f \rightarrow 0$), and pulse duration with $\ln[1/(1-\eta)]$, which pertains to the safety wall conditions described by Eq. (4.21), Equation (4.25) can be further manipulated to give

$$\eta\tau_m = -\frac{1}{2\bar{f}} \frac{\eta}{\ln[1-\eta]} \ln \left[\frac{(1-\eta)^{-\bar{f}} - 1 + \eta}{(1-\eta)^{\bar{f}} - 1 + \eta} \right] \quad (4.27)$$

where $\bar{f} = f / \ln[1/(1-\eta)]$. The limit behavior of Eq. (15) at $\eta \rightarrow 0$ and $\eta \rightarrow 1$ is given by Eqs (16) & (17) respectively providing upper and lower bounds for the response and a double asymptotic matching as illustrated in Figure 4.10.

$$\eta\tau_m \rightarrow \frac{1}{2\bar{f}} \ln \left[\frac{1+\bar{f}}{1-\bar{f}} \right] \quad (4.28a)$$

$$\eta\tau_m \rightarrow 1 \quad (4.28b)$$

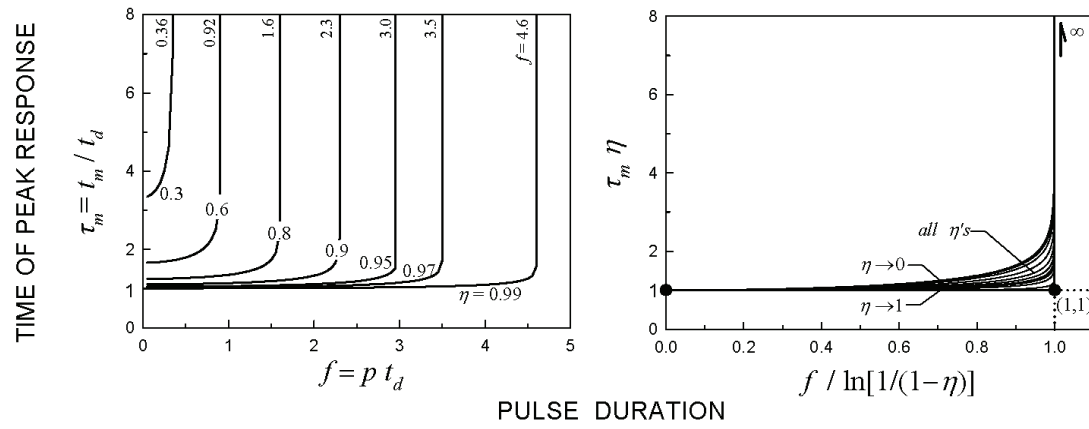


Figure 4.10 Double asymptotic matching of the solution for time of peak response (rectangular pulse shape $\beta = -\infty$).

Maximum rocking response can be obtained from Eqs (4.10) and (4.24), as a fraction of slenderness angle α . For example, in the simple case of a rectangular pulse, maximum rocking angle is given by

$$\theta_{max} = \frac{\alpha}{\eta} [\eta + (1 - \eta) \cosh(f\tau_m) - \cosh[f(\tau_m - 1)]] \quad (4.29)$$

Figure 4.11 illustrates Eq. (4.29) plotted as function of pulse duration and uplift strength. Evidently, systems in region S rock to angles which are smaller than α . Corresponding dimensionless times naturally exceed 1 in all cases. Note that when θ_{max} reaches α , the system lies on the critical limit (safety wall), as defined by Eq. (4.20) and τ_m tends to infinity.

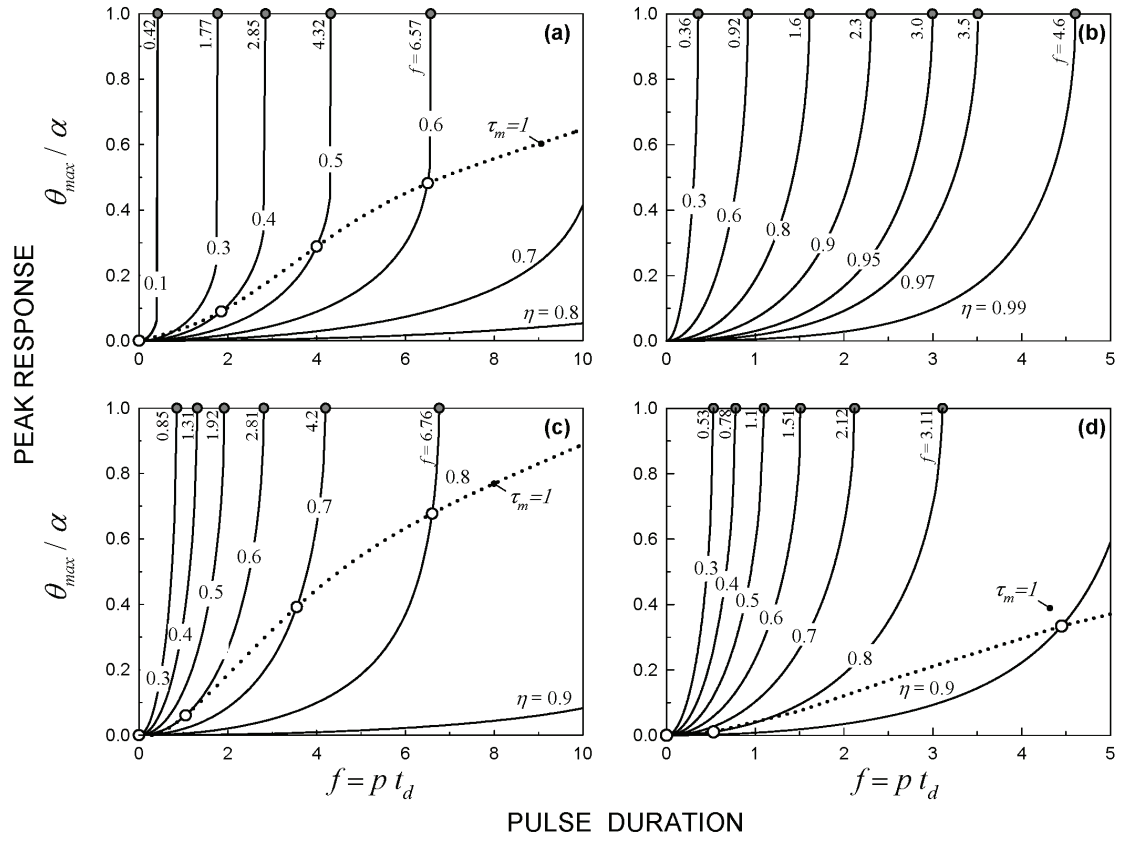


Figure 4.11 Peak rocking response (“rocking spectra”) as function of uplift strength and pulse duration (region S) due to a single-lobe generalized pulse of: (a) exponential ($\beta = \pi$), (b) rectangular ($\beta \rightarrow \infty$), (c) triangular ($\beta \rightarrow 0$) and (d) sinusoidal-like shape ($\beta = -\pi$).

In the same spirit as before Eq. (4.29) may be further normalized with $\ln[1/(1-\eta)]$ to give

$$\frac{\theta_{max}}{\alpha} = \frac{1}{\eta} \left[\eta + (1-\eta) \cosh\left(\ln \left[\frac{(1-\eta)^{-\bar{f}} - 1 + \eta}{(1-\eta)^{\bar{f}} - 1 + \eta} \right] \right) - \cosh\left(\ln \left[\frac{(1-\eta)^{-\bar{f}} - 1 + \eta}{(1-\eta)^{\bar{f}} - 1 + \eta} \right] - \bar{f} \ln \left[\frac{1}{1-\eta} \right] \right) \right] \quad (4.30)$$

The limit behavior of Eq. (4.30) at $\eta \rightarrow 0$ and $\eta \rightarrow 1$ is given by Eqs (4.31a) & (4.31b) respectively providing upper and lower bounds for the response and a double asymptotic matching (Figure 4.12).

$$\frac{\theta_{max}}{\alpha} \rightarrow 1 - \cosh\left(\ln \sqrt{\frac{1+\bar{f}}{1-\bar{f}}}\right) + \bar{f} \sinh\left(\ln \sqrt{\frac{1+\bar{f}}{1-\bar{f}}}\right) \quad (4.31a)$$

$$\frac{\theta_{max}}{\alpha} \rightarrow 0 \quad (4.31b)$$

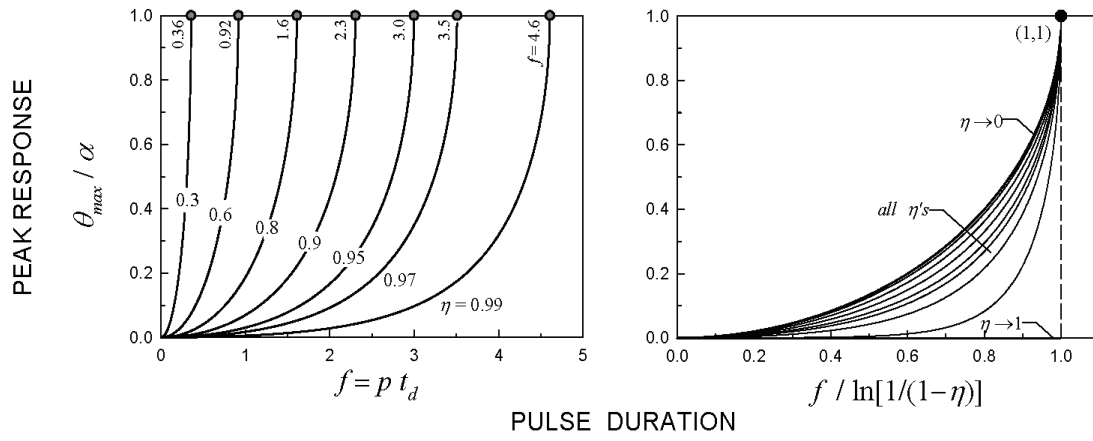


Figure 4.12 Double asymptotic matching of the solution for peak response (rectangular pulse shape $\beta = -\infty$).

4.4.4 Time of Impact

Following peak response and for the half-cycle pulses studied here, angular velocity and acceleration are negative and rocking angle gradually decreases until impact. After impact, rocking will switch from one edge of the block to the other leading to negative uplift angles (Fig 4.1). The transition will always be accompanied by energy loss described by a restitution coefficient. Given the half-cycle nature of the excitation, the block will rock to gradually smaller angles until coming to rest. Only times of first impact are examined below.

For $\tau_m \leq \tau_i \leq 1$ time of impact τ_i is obtained by setting rocking angle in Eq. (4.10) equal to zero, to get

$$\eta(1-e^\beta)(f^2-4\beta^2) = \beta[(f+2\beta)[1-\eta(1-e^\beta)]^{1-\frac{f}{2\beta}} e^{f\tau_i} - (f-2\beta)[1-\eta(1-e^\beta)]^{1+\frac{f}{2\beta}} e^{-f\tau_i}] + f^2 - 4\beta^2 - f^2 e^{2\beta(1-\tau_i)} - 2\beta f e^\beta (e^{f(\tau_i-1/2)} - e^{-f(\tau_i-1/2)}) \quad (4.32a)$$

which, for arbitrary values of β , should be treated numerically.

For impact after the end of the pulse (i.e., $\tau_i \geq 1$), the following closed-form solution is possible

$$\tau_i = \frac{1}{f} \ln \left[\frac{\sqrt{y_2 y_3 + y_1^2} - y_1}{y_3} \right] \quad (4.32b)$$

where

$$y_1 = -\frac{\eta}{2\beta}(1-e^\beta)(f^2-4\beta^2) \quad (4.32c)$$

$$y_2 = -2fe^{\frac{f}{2}+\beta} + e^f(f+2\beta) + (f-2\beta)[1-\eta(1-e^\beta)]^{1+\frac{f}{2\beta}} \quad (4.32d)$$

$$y_3 = -2fe^{-\frac{f}{2}+\beta} + e^{-f}(f-2\beta) + (f+2\beta)[1-\eta(1-e^\beta)]^{1-\frac{f}{2\beta}} \quad (4.32e)$$

Time of impact for the limit cases of rectangular ($\beta \rightarrow -\infty$) and triangular ($\beta \rightarrow 0$) pulses are provided, respectively, by Eqs. (4.33) and (4.34a,b) as listed below.

$$\tau_i = \frac{1}{f} \ln \left[\frac{(e^{-f}-1)\sqrt{e^f(1-\eta)}-\eta}{1-\eta-e^{-f}} \right] \quad (4.33)$$

$$\sinh f(\tau_i - \frac{\eta}{2}) - 2 \sinh f(\tau_i - \frac{1}{2}) + f(\tau_i + \frac{\eta}{2} - 1) = 0 \quad (4.34a)$$

$$\tau_i = \frac{1}{f} \ln \left[\frac{-\sqrt{(e^f - 2e^{\frac{f}{2}} + e^{\frac{f\eta}{2}})(e^{-f} - 2e^{-\frac{f}{2}} + e^{-\frac{f\eta}{2}}) + \left(\frac{f\eta}{2}\right)^2} - \frac{f\eta}{2}}{e^{-f} - 2e^{-\frac{f}{2}} + e^{-\frac{f\eta}{2}}} \right] \quad (4.34b)$$

Naturally, for a rectangular pulse peak response always occurs during free rocking ($\tau_i > 1$), whereas for a triangular pulse Eq. (4.34a) refers to $1/2 \leq \tau_i \leq 1$ and Eq. (4.34b) to $\tau_i > 1$.

Numerical results obtained from these expressions are provided in Fig 4.13.

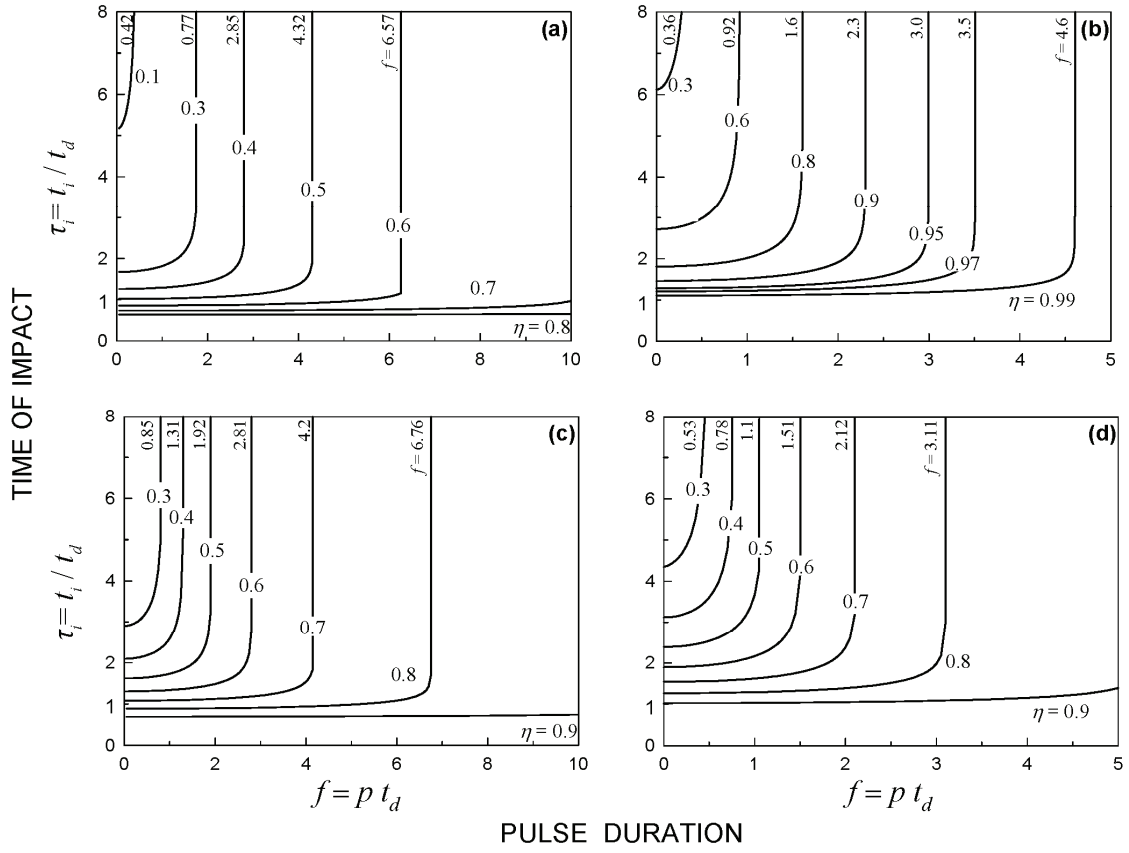


Figure 4.13 Time of impact as function of uplift strength and pulse duration (region S) due to a half-cycle generalized exponential pulse of (a) exponential ($\beta = \pi$), (b) rectangular ($\beta \rightarrow \infty$), (c) triangular ($\beta \rightarrow 0$) and (d) sinusoidal-like shape ($\beta = -\pi$).

Like time of maximum response, time of impact may be further normalized by $(1 + \sqrt{1 - \eta}) / \eta$, which corresponds to the value of τ_i for zero pulse duration (Eq. (4.33) for $f \rightarrow 0$), and pulse duration by $\ln[1/(1 - \eta)]$ to give

$$\frac{\eta \tau_i}{1 + \sqrt{1 - \eta}} = \frac{1}{\bar{f}} \frac{\eta}{(1 + \sqrt{1 - \eta}) \ln[1/(1 - \eta)]} \ln \left[\frac{[1 - (1 - \eta)^{-\bar{f}}] \sqrt{(1 - \eta)^{-\bar{f} + 1}} + \eta}{(1 - \eta)^{\bar{f}} - 1 + \eta} \right] \quad (4.35)$$

The limit behavior of Eq. (4.35) at $\eta \rightarrow 0$ and $\eta \rightarrow 1$ is given by Eqs (4.36a) & (4.36b) respectively providing upper and lower bounds for the response and a double asymptotic matching (Figure 4.14). The results suggest that the time of impact is practically independent of pulse duration as long as $f / \ln[1/(1 - \eta)]$ is smaller than 0.8 or so.

$$\frac{\eta \tau_i}{1 + \sqrt{1 - \eta}} \rightarrow \frac{1}{2\bar{f}} \ln \left[\frac{1 + \bar{f}}{1 - \bar{f}} \right] \quad (4.36a)$$

$$\frac{\eta \tau_i}{1 + \sqrt{1 - \eta}} \rightarrow 1 \quad (4.36b)$$

An alternative view is that of Figure 4.15 which depicts the ratio of time of impact over time of maximum response. Although normalizing with $\ln[1/(1 - \eta)]$ matches results at $\eta \rightarrow 1$, it does not behave evenly well for $\eta \rightarrow 0$.

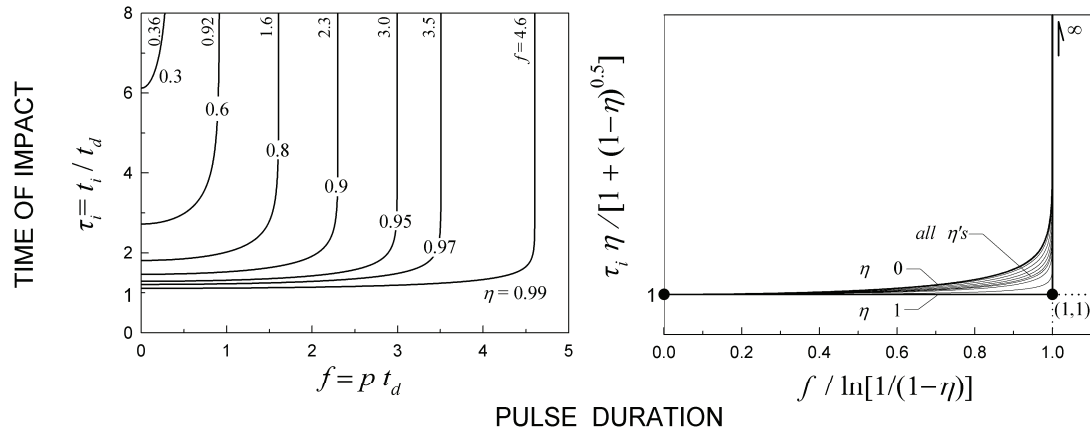


Figure 4.14. Double asymptotic matching of the solution for time of impact (rectangular pulse shape $\beta = -\infty$).

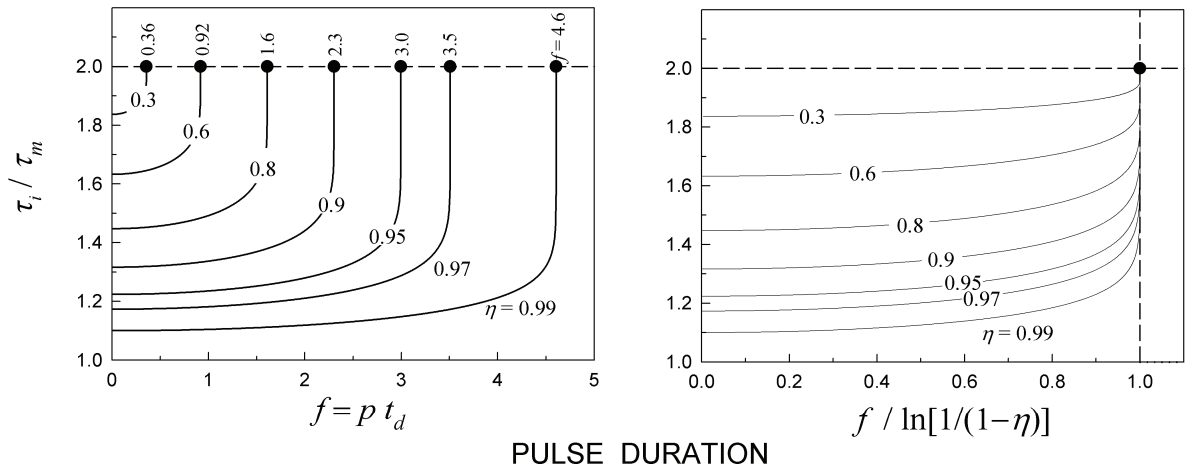


Figure 4.15. Normalization of time of impact with peak rocking response as function of uplift strength and pulse duration (region S).

4.5 Discussion

With reference to the systems in Figure 4.16, examples of rocking behaviour in each of the aforementioned regimes are given in Figure 4.17. Evidently, region O_1 is associated with a monotonic increase in angular displacement, velocity and acceleration, leading to an “instability” (rocking past angle α) in response during rising pulse. Region O_2 is associated with analogous trends except for a drop in acceleration at the end of the pulse. For systems in region O_3 acceleration becomes negative after the end of the pulse (since there is sufficient restoring moment at $t = t_d$), yet this cannot prevent overturning as velocity never drops to zero. Finally, in region S negative acceleration at the end of the pulse leads to a reversal in motion and, thereby, safety as the block cannot overturn after impact. Note that for the limit case of a rectangular pulse, all three response parameters (i.e. angular displacement, velocity and acceleration) are positive and tend to increase during pulse ($0 \leq \tau \leq 1$). This implies that peak rocking response always occurs after the end of the excitation. However, as shown in Fig 4.17, this is not true for higher values of β .

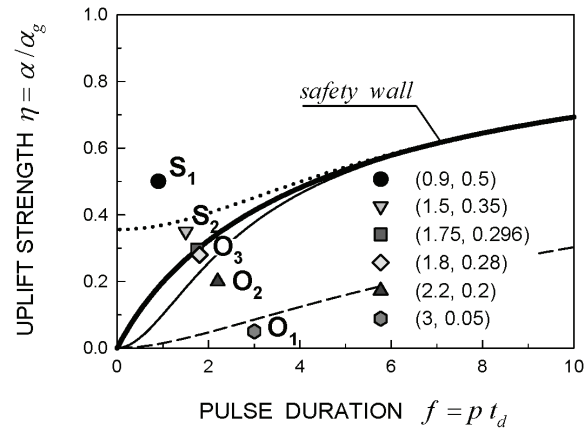


Figure 4.16 Examples of response for different regimes of a rocking block under an exponential pulse of half-cycle duration pulse ($\beta = \pi$).

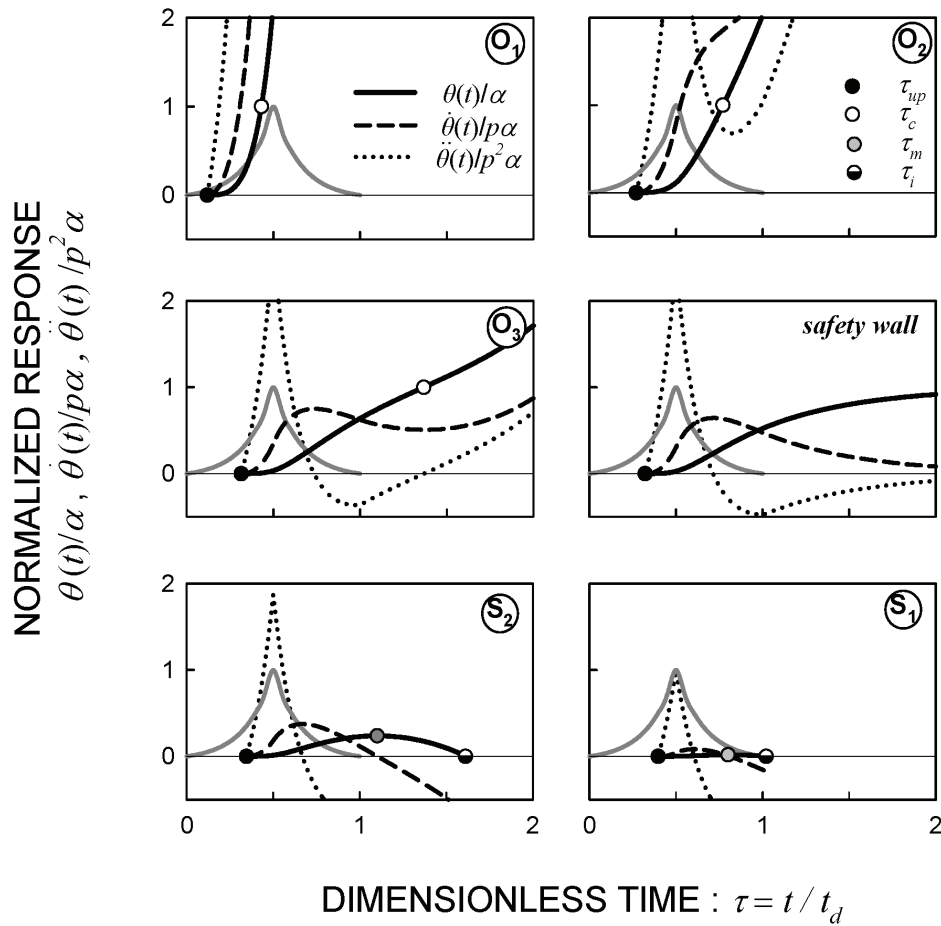


Figure 4.17 Examples of response for different regimes of a rocking block under an exponential pulse of half-cycle duration pulse ($\beta = \pi$).

4.6 Linear vs Non-Linear Equations of Motion: Energy Formulation

Due to the inherent non-linear nature of the problem, no exact analytical solutions to Eqs (4.5) and (4.6) are available. As already mentioned, for slender blocks these equations can be linearized leading to Eq. (4.7). An ad hoc investigation on the accuracy of this approach based on energy considerations is explored in this section, aiming at quantifying the response of critical systems as function of slenderness parameter α .

Integrating the governing differential equation (4.5) over the rocking angle and switching independent variable from θ to t using the chain rule of differentiation, yields

$$\underbrace{I_o \int_{t_o}^t \ddot{\theta} \dot{\theta} dt}_{\text{change in kinetic energy}} + \underbrace{mgR \int_{\theta_o}^{\theta} \sin(\alpha - \theta) d\theta}_{\text{change in potential energy}} = \underbrace{mR \int_{t_o}^t \ddot{u}_g \cos(\alpha - \theta) \dot{\theta} dt}_{\text{work of base acceleration}} \quad (4.37)$$

which expresses the work of the inertial and gravitational forces acting on the block between the arbitrary response angles $\theta_o = \theta(t_o)$ and $\theta = \theta(t)$, referring to times t_o and t (not separated by an impact).

Carrying out the integrations on the left side of Eq. (4.37) yields

$$\frac{1}{2} I_o \left[\dot{\theta}^2 \right]_{t_o}^t + mgR \left[\cos(\alpha - \theta) \right]_{\theta_o}^{\theta} = mR \int_{t_o}^t \ddot{u}_g \cos(\alpha - \theta) \dot{\theta} dt \quad (4.38)$$

which elucidates the conservative nature of the above forces.

Concentrating on a critical system ($\eta = \eta_w$) that lies on the safety wall in Figs 4.4–4.8) and upon setting $t_o = t_{up}$, $t \rightarrow \infty$ (recall that for a critical system instability is reached at an infinite time), $\theta_o = 0$, $\theta = \alpha$, $\dot{\theta}_o = \dot{\theta} = 0$, one obtains the simple expression

$$mgR(1 - \cos \alpha) = mR \int_{t_{up}}^{\infty} \ddot{u}_g(t) \cos(\alpha - \theta) \dot{\theta} dt \quad (4.39)$$

which merely suggests that for the conditions at hand, the work done by the inertial earthquake force equals the change in potential energy resulting from lifting the centre of mass of the block by an amount equal to $R(1 - \cos \alpha)$. For slender blocks and using the power series expansion $1 - \cos \alpha \cong \alpha^2/2$, this change in elevation can be written as $R\alpha^2/2$, showing that it tends to zero as the slenderness angle α decreases, hence suggesting that an infinitesimal amount of work suffices for overturning the block. Nevertheless, the ratio of the two terms in Eq. (4.39) is always finite and allows determining the strength of the system as demonstrated below.

Eliminating mass and radius from both sides of the above equation introducing dimensionless time $\tau = t/t_d$, dimensionless strength $\eta = \alpha / \alpha_g$ and calling

$$\ddot{\psi}_g(\beta, \tau) = \ddot{u}_g(\tau) / \alpha_g g \quad (4.40a)$$

$$\chi(\beta, \tau, \eta_w, f, \alpha) = \theta(\tau) / \alpha \quad (4.40b)$$

a normalized pulse acceleration function and a dimensionless response function (both bounded between 0 and 1) respectively, Eq. (4.39) yields the non-linear integral equation

$$\eta_w = \frac{\alpha^2}{1 - \cos \alpha} \int_{\tau_{up}}^{\infty} \ddot{\psi}_g(\beta, \tau) \cos[\alpha(1 - \chi)] \dot{\chi} d\tau \quad (4.41)$$

which can be viewed as an alternative to Eq. (4.5) for a critical system.

For the special case of a slender block, $\alpha^2 \ll \alpha$; $\cos[\alpha(1 - \chi)] \rightarrow 1$, employing again the power series expansion of $\cos \alpha$, Eq. (4.41) can be linearized to

$$\eta_{w,lin} = 2 \int_{\tau_{up}}^{\infty} \ddot{\psi}_g(\beta, \tau) \dot{\chi} d\tau \quad (4.42)$$

an alternative to Eqs (4.7) and (4.20) for the systems at hand.

For the more general case in which no linearization is applied, employing the mean-value theorem of integral calculus, the term $\cos[\alpha(1-\chi)]$ can be moved outside the integral sign to give

$$\eta_{w,non-lin} = \frac{\alpha^2 \cos[\alpha(1-\tilde{\chi})]}{1-\cos\alpha} \int_{\tau_{up}}^{\infty} \ddot{\psi}_g(\beta, \tau) \dot{\chi} d\tau \quad (4.43)$$

where $\tilde{\chi}$ is a pertinent mean normalized angular displacement (θ/α) in the interval (0, 1).

Further, if function $\ddot{\psi}_g(\beta, \tau)$ is moved outside the above integrals in terms of an average (“effective”) ground acceleration $\tilde{\ddot{\psi}}_g(\beta)$ acting on the block over the whole response

interval (τ_{up}, ∞) , and recalling that $\int_{\tau_{up}}^{\infty} \dot{\chi} d\tau = 1$, Eqs (4.42) and (4.43) become

$$\eta_{w,lin} = 2 \tilde{\ddot{\psi}}_g(\beta) \quad (4.44)$$

$$\eta_{w,non-lin} = \frac{\alpha^2 \cos[\alpha(1-\tilde{\chi})]}{1-\cos\alpha} \tilde{\ddot{\psi}}_g(\beta) \quad (4.45)$$

Eq. (4.44) reveals that the “effective” ground acceleration acting on a slender block is half its dimensionless critical strength.

Taking the ratio of Eqs (4.45) and (4.44), yields the simple expression

$$\frac{\eta_{w,non-lin}}{\eta_{w,lin}} \cong \frac{\alpha^2 \cos[\alpha(1-\tilde{\chi})]}{2(1-\cos \alpha)} \quad (4.46)$$

which relates the critical strength according to the linearized equations of motion with the ‘exact’ value for the non-linear equations. Note that Eq. (4.46) does not depend explicitly on η and f , being a function of slenderness α and the averaging parameter $\tilde{\chi}$. The above equation is plotted in Fig 4.18.

For blocks with angles α up to approximately 20 degrees, for which the power series expansion $1 - \cos x \cong x^2/2$ can be applied, Eq. (4.46) simplifies to

$$\frac{\eta_{w,non-lin}}{\eta_{w,lin}} \cong 1 - \frac{1}{2}[(1-\tilde{\chi})\alpha]^2 \quad (4.47)$$

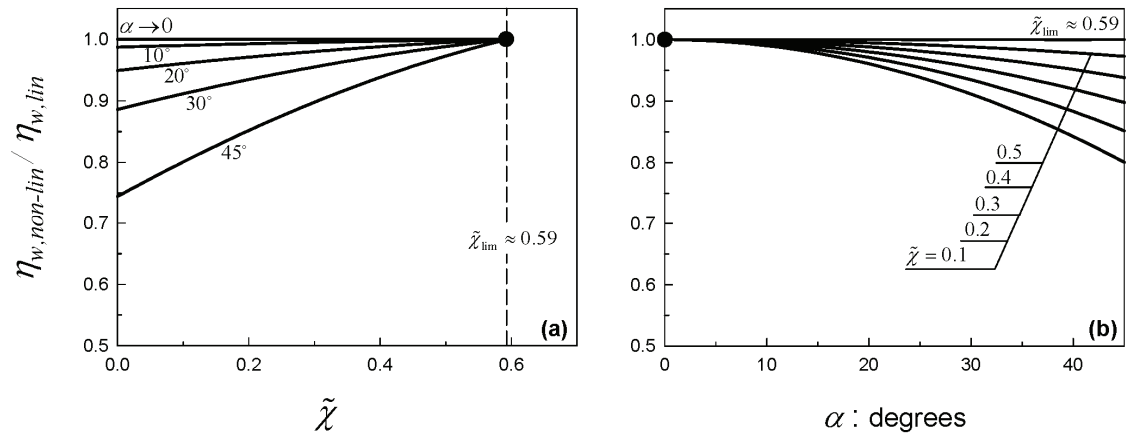


Figure 4.18 Ratio of the critical strength, according to the non – linear equations of motion, over the one from the linearized solution as function of geometric parameter α and averaging parameter $\tilde{\chi}$ (Eq. 4.46).

Results from the above analysis are provided in Figs 4.18 and 4.19. Evidently, Eqs (4.46) and (4.47) attain values always smaller than 1 (typically ranging between 0.95 and 0.99 for geometric angles α less than 20 degrees), which suggests that the critical strength predicted by the linearized equations of motion is higher than the one for the non-linear equations. Recalling that $\eta = \alpha / \alpha_g$, this observation implies that the linearized equations predict overturning of the block for a smaller value of α_g i.e. they lead to a less stable system. Naturally, in the extreme case where $\alpha \rightarrow 0$ the right side of the above expressions tends to one and the two limit strengths coincide. It should be kept in mind that this property applies only to overturning in the forward sense and should not be generalized to more complex cases such as those involving overturning after impact. Results obtained by means of non-linear numerical analysis using a Runge-Kutta algorithm in Mathematica platform (Wolfram 2010), shown in Fig 4.19, confirm these findings. It is also observed in Fig 4.19 that the ratio $(\eta_{w,non-lin} / \eta_{w,lin})$ is generally higher than 0.95 for values of α up to 20 degrees and does not drop below 0.8 even for squat blocks having α angles of the order of 40 degrees. For squatter blocks uplift requires peak pulse acceleration higher than $1g$ suggesting that the block is more likely to slide than rock. Studying such phenomena lies beyond the scope of this work.

It is noted in passing that an upper bound to the mean angular displacement $\tilde{\chi}$ can be obtained from Eq. (4.46), by setting the left hand side equal to one for a finite angle α . Then,

$$\tilde{\chi}_{lim} = 1 - \frac{1}{\alpha} \cos^{-1} \left[\frac{2(1 - \cos \alpha)}{\alpha^2} \right] \quad (4.48)$$

which, for all systems of interest, attains values close to 0.59 providing an extreme upper bound for the factor at hand. This trend is also visible in Figure 4.18.

In light of the numerical results in Fig 4.19, it was found that the average value $\tilde{\chi} = 0.1$ can be selected to give via Eqs (4.46) and (4.47)

$$\frac{\eta_{w,non-lin}}{\eta_{w,lin}} \cong \frac{\alpha^2 \cos[9\alpha/10]}{2(1 - \cos \alpha)} \quad (4.49)$$

$$\frac{\eta_{w,non-lin}}{\eta_{w,lin}} \cong 1 - \frac{2}{5}\alpha^2 \quad (4.50)$$

which provide, respectively, good upper- and lower-bound estimates of the non-linear geometric effect in non-slender blocks subjected to different half-cycle pulses. The average of these predictions may be used in a best estimate sense.

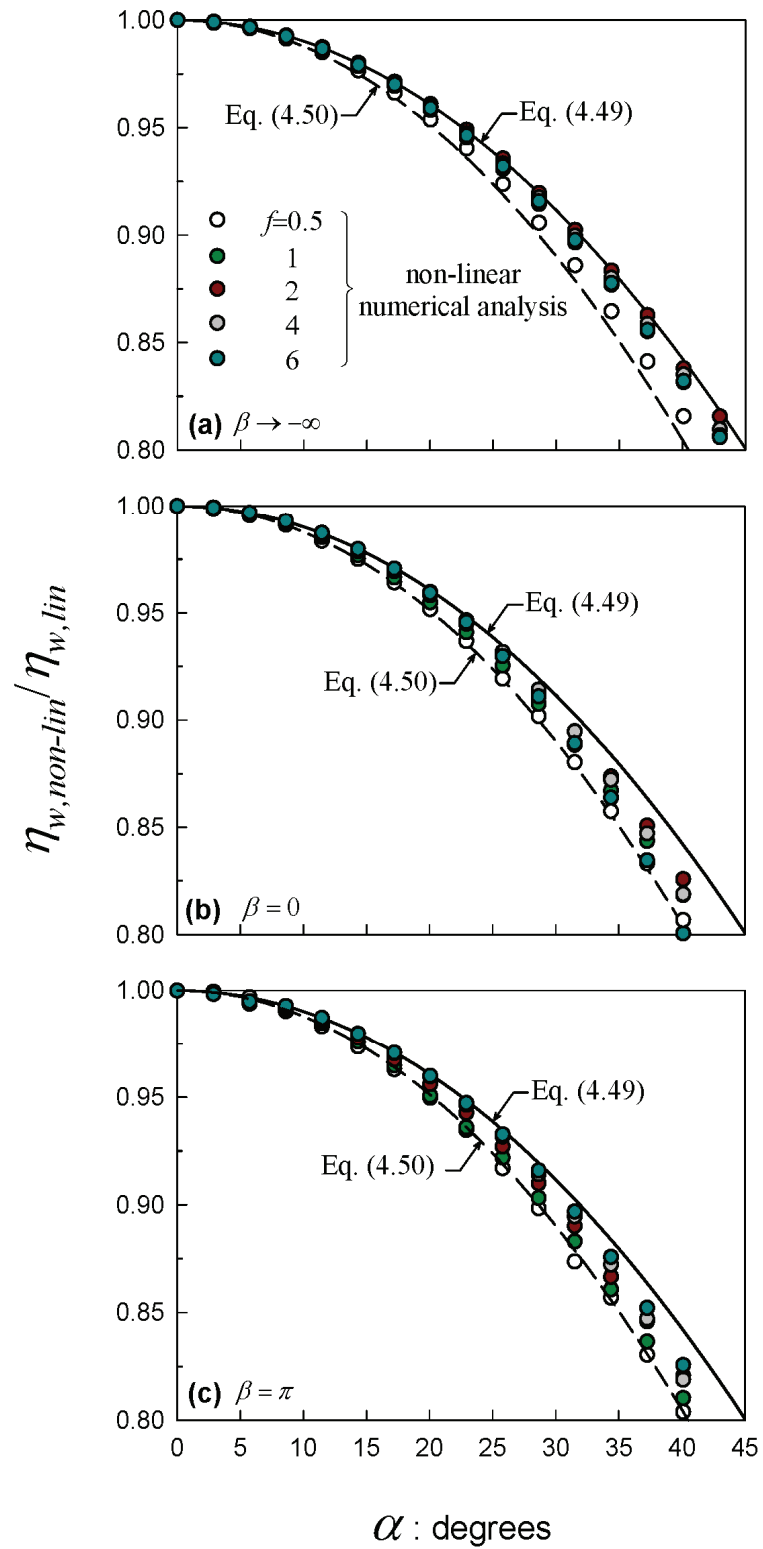


Figure 4.19 Geometric effect in non-slender blocks as function of parameters α and f due to half – cycle pulse excitations of (a) rectangular (b) triangular and (c) exponential shape.

4.7 Conclusions

The main conclusions of the chapter are:

1. By virtue of a generalized pulse shape in Eq. (4.1) novel analytical solutions for rocking of rigid blocks were presented covering an infinite number of pulse shapes ranging from a perfect spike ($\beta \rightarrow +\infty$), to a perfect triangle ($\beta \rightarrow 0$), to a perfect rectangle ($\beta \rightarrow -\infty$).
2. Rocking response was found to exhibit complex patterns even for highly idealized pulses like the ones at hand.
3. It was shown that overturning is controlled by only two dimensionless parameters: dimensionless pulse duration f (i.e., actual pulse duration times characteristic block frequency) and dimensionless uplift strength η (i.e., ratio of minimum required acceleration for initiation of uplift over peak pulse acceleration). On the other hand in the realm of the linearized equation of motion, block slenderness α was found to perfectly normalise the dependent variable θ , which can be viewed as a similarity property. This attribute however, stems from the very structure of the governing equation, not from dimensional considerations as commonly thought.
4. Analytical formulae were derived for distinguishing between Safe (S) and Overturning (O) regions as function of f and η . In turn, regions O may be divided into three sub-areas (Figs 4.4-4.7): Region O_1 corresponding to instability in response during rising pulse, O_2 corresponding to instability in response during decaying pulse, and O_3 where critical angle is attained after the end of the pulse. Regions S may be subdivided into subgroups S_1 for peak rocking response attained during pulse and S_2 for peak response attained after pulse. It is noted that, as regions O_1

and O_2 correspond to overturning during pulse, they are applicable even to ground motions with time histories that extend beyond the end of the pulse, t_d .

5. A critical line separating safe from overturning response was established by means of a simple closed-form expression (Eq. 4.20).
6. Simple solutions for peak rocking response in terms of peak angular displacement and corresponding time were presented. Due to the transcendental nature of the response functions however, not all solutions could be obtained in closed form.
7. It was shown that the full non-linear equations of motion lead to more stable systems than the linearized equations (Eq. 4.44). This property, however, strictly applies to overturning in the forward sense and should not be generalized to more complex cases such as those involving overturning after impact.

Finally, it is fair to reiterate that, due to the unilateral excitation considered, the work at hand does not address overturning after an impact, which represents the most critical case for a wide class of real-life systems. Nevertheless, Chapter 4 complements and extends classical investigations in the subject by providing novel solutions and physical insight into the nature of the problem. Moreover, it provides conservative criteria for overturning under full-cycle pulses when f is less than approximately 0.75.

5

The Complex Response of a Rocking Block to a Full-Cycle Pulse

5.1 Synopsis

The investigation of rocking behavior of a rigid, free-standing block to impulsive earthquake shaking discussed in the previous Chapter is extended here to the more realistic case of a full-cycle pulse. A rectangular block resting on a rigid base is considered, subjected to an idealized ground acceleration pulse composed of two constant half-cycles of equal amplitude, duration and opposite sign. The problem is treated both analytically, in the realm of slender block geometry and rocking without slippage or upthrow, and numerically by means of a more elaborate nonlinear formulation encompassing impulsive and transcendental terms. Closed-form expressions are obtained for: (1) block rotation, angular velocity and acceleration; (2) overturning criteria for conditions with and without impact; (3) peak rocking response. The solutions are expressed in terms of three dimensionless parameters, namely pulse duration, uplift strength and restitution coefficient. Despite the apparent simplicity of the problem, the response can exhibit complex – even counterintuitive – patterns, a trait attributed to the possibility of overturning in two distinct modes (forward and backward), the non-linear nature of the impact, the real-valued (positive) pole of the differential operator and the presence of multiple immobility points in a particular response branch. The bifurcation behaviour associated with the two overturning modes is highlighted. Results are presented

in the form of dimensionless graphs that provide insight into the physics of the response. The derivations presented in the chapter are, to the best of the author's knowledge, the first exact solutions for overturning of a rigid block to a full-cycle pulse. Comparisons against idealized pulses of other shapes and actual near fault recorded ground motions are presented and commented.

5.2 Preface

As discussed in Chapter 4, despite the apparent simplicity in describing the rocking problem and formulating the governing equations, determining analytically the response is challenging even for the most basic waveforms, due to the non-linear nature of the differential operator due to both impact and geometric effects, and the exponential character of the response functions. As a result, the majority of available studies are either purely numerical (Yim *et al.* 1980, Ishiyama 1982, Psycharis & Jennings 1983, Spanos & Koh 1984, Tso & Wong 1989a, b) or experimental in nature (Muto *et al.* 1960, Priestley *et al.* 1983, Aslam *et al.* 1980). Whereas a number of valuable hybrid analytical-numerical solutions is available (Anooshehpour *et al.* 1996, Zhang & Makris 2001, Kounadis 2013) pure analytical investigations leading to exact closed-form solutions (Housner 1963, Shi *et al.* 1996) are limited to pulses of half-cycle duration, which can induce overturning only in the forward sense (i.e., without impact). Exact analytical solutions for overturning to motions containing more than a half-cycle pulse (and, thereby, capable of inducing overturning after impact) have not been developed to date.

The importance of analytical investigations to the rocking problem can hardly be overstated, as they provide the fundamental means for casting light into counterintuitive trends observed in numerical and experimental studies. For example: (a) the risk of

overturning of a block subjected to a particular ground motion does not vary monotonically with size or slenderness ratio (Yim *et al.* 1980), (b) overturning of a block does not imply overturning for an increase in ground motion amplitude (Psycharis and Jennings 1983) and (c) reduction in restitution coefficient, and thus increase in system damping, does not imply a decrease in response (Aslam *et al.* 1980). As will be shown in the ensuing, these trends are associated with changes in the mode of overturning i.e., from overturning in the forward manner to overturning in the backward following an impact and vice versa.

Exact analytical solutions have been developed for response to half-cycle pulses of rectangular (Housner 1963) and sinusoidal shape (Shi *et al.* 1996). Attempts to handle full-cycle sinusoidal pulses have been presented by Anooshehpour *et al.* (1999, 2000), Zhang and Makris (2001) and Kounadis 2013. Notwithstanding the theoretical significance of these studies, they can all be viewed as quasi-analytical, for the overturning limits are determined numerically due to the complex (transcendental) nature of the governing equations and the response functions. It is worth mentioning that an analytical – yet still approximate – closed-form solution for a sinusoidal full-cycle pulse has recently been presented by Dimitrakopoulos & DeJong (2012). The lack of exact analytical solutions to the problem provided the initial motivation for the herein-reported work.

With reference to a free-standing rigid block subjected to a full-cycle ground acceleration pulse, an exact analytical solution – accompanied by numerical investigations – is presented in this Chapter for the rocking response of the block and the conditions of overturning. It will be shown that complex response patterns develop which can be

described analytically by transcendental functions. Other pulse shapes, ranging from a perfect rectangle to a perfect impulse are also examined through numerical analyses. It is shown that for the same peak ground velocity and pulse duration, blocks that topple under a square pulse, overturn for pulses of more spiky shape, and blocks that survive a perfect impulse, also survive pulses of less spiky shape. Finally, the stability curves are checked against numerical results for actual near-fault recorded ground motions containing directivity pulses.

5.3 Problem and Parameter Definition

The problem considered involves a rigid block resting on a horizontal, perfectly rigid plane, subjected to an idealized ground acceleration pulse acting parallel to the plane. The block, shown in Fig 5.1, is of rectangular shape^{*} having mass m , dimensions $(2 \times b)$ by $(2 \times h)$ – leading to a radial distance from the center of rotation to the center of gravity $R = (b^2 + h^2)^{1/2}$ and a dimensionless slenderness angle[†] $\alpha = \tan^{-1}(b/h)$. The interface between the block and the rigid base is assumed to be cohesionless, obeying a linear friction law characterized by a Coulomb coefficient μ . As shown by Housner (1963), the dynamic characteristics of the block can be expressed by its pseudo natural frequency $p = (3g/4R)^{1/2}$ (measured in radians per time), g being the gravitational acceleration. As the restoring force is exclusively due to gravity (as in the classical pendulum), the above intrinsic frequency is independent of the block mass.

^{*} As mentioned in Chapter 4, the proposed analysis is also valid for blocks of any symmetric shape about the vertical axis through their center of mass; in that case, h is the height of the center of mass from the base and b is the half-width of the base.

[†] Block slenderness is actually represented by the complementary angle $(\pi/2 - \alpha)$. In the remainder of this work, angle α will be referred to as “slenderness”, to conform to the rest of the literature.

Ground excitation consists of an acceleration pulse of rectangular shape composed of two symmetric cycles of opposite sign. The waveform is described by its amplitude A_g (normalized amplitude $\alpha_g = A_g/g$) and half-cycle duration t_d (Fig 5.2). It is worth stressing that this function, although idealized, suffices for revealing the main aspects of rocking response to impulsive ground motions.

The following dimensionless quantities are employed to express the results presented in the ensuing sections: uplift strength $\eta = \alpha_g / A_g = \alpha / \alpha_g$, pulse duration $f = p t_d$, time of rocking initiation $\tau_{up} = t_{up} / t_d$, time of impact $\tau_i = t_i / t_d$, time of instantaneous equilibrium $\tau_e = t_e / t_d$, time of instantaneous immobility $\tau_s = t_s / t_d$, time of peak rocking response $\tau_m = t_m / t_d$, rocking angle θ and peak rocking angle θ_{max} . Moreover, r = kinetic energy reduction during impact and $\varepsilon = \sqrt{r}$ coefficient of restitution.

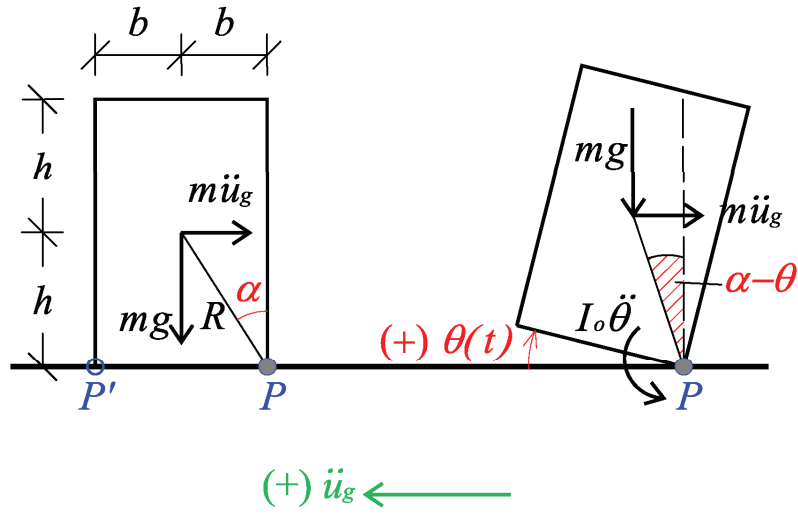


Figure 5.1 The problem considered: rocking motion of a free-standing block on a rigid-base. Note the positive direction of ground acceleration to induce a positive rocking angle.[‡]

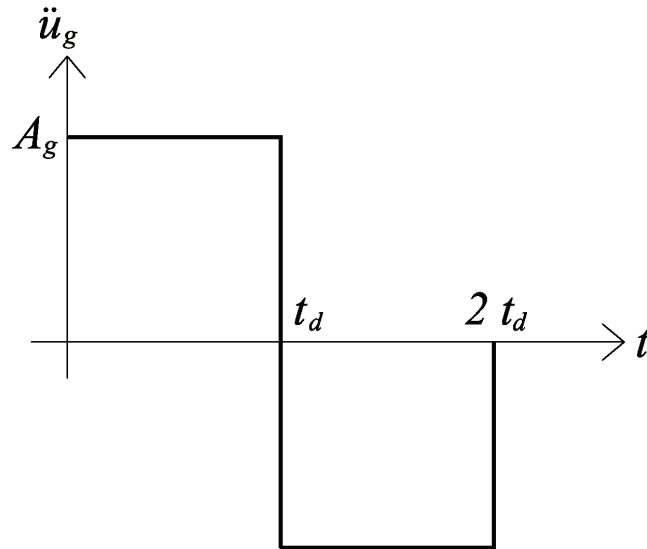


Figure 5.2 Rectangular ground acceleration pulse of full-cycle duration.

[‡] For a discussion on notation, see footnote on page 191.

5.3.1 Rocking Initiation

Depending on the characteristics of the excitation and the properties of the frictional interface, the block in Fig. 5.1 will either: slide (without rocking), rock (without sliding), slide and rock at the same time, or upthrow following an impact (Psycharis & Jennings 1985, Shenton 1996, Pompey al 1998). As shown in Chapter 4, the initiation of rocking requires exceedance of the restoring moment due to self-weight, by the overturning moment due to inertia i.e., $m \ddot{u}_g h \geq m g b$. This can be written as $\ddot{u}_g / g \geq \tan \alpha$; for slender blocks ($\alpha^2 \ll \alpha$) the trigonometric argument can be dropped for the simplified form $\ddot{u}_g / g \geq \alpha$ (Eq. 4.3) which holds for angles α less than about 20 degrees (Yim *et al.* 1980).

On the other hand, sliding occurs for $\ddot{u}_g / g \geq \mu$, where μ is the aforementioned frictional coefficient. Conditions for simultaneous sliding and rocking are provided in the above references. Our analysis focuses solely on rocking behavior and, thus, assumes a sufficiently large frictional coefficient so that sliding is prevented.

The ratio of slenderness over dimensionless peak pulse acceleration ($\alpha g/A_g$), to be referred hereafter to as *uplift strength* η , has the following properties: if $\eta > 1$ no rocking occurs, if $\eta < 1$ rocking initiates at $t = t_{up}$, when $\ddot{u}_g(t_{up}) \cong \alpha g$. Naturally, for a rectangular pulse and $\eta < 1$ rocking initiates at $t_{up} = 0$.

5.3.2 Equations of Motion

The equations governing planar rocking response of the block (Housner 1963) have been presented in Chapter 4 (Eqs. 4.5 and 4.6); they are re-written below for completeness

$$I_o \ddot{\theta} + mgR \sin(\alpha - \theta) = +m\ddot{u}_g R \cos(\alpha - \theta) \quad , \quad \theta(t) > 0 \quad (5.1a)$$

$$I_o \ddot{\theta} - mgR \sin(\alpha + \theta) = +m\ddot{u}_g R \cos(\alpha + \theta) \quad , \quad \theta(t) < 0 \quad (5.1b)$$

where I_o is the moment of inertia of the block with respect to its corner P or P' (for rectangular geometry, $I_o = 4/3 mR^2$). The positive sign in the right-hand side of the equations is to ensure positive rocking response for positive ground acceleration, as evident from the reference system of Fig 5.1.

5.3.3 Linearized Equations of Motion

For slender blocks angle α is small; the above equations can be linearized using the first-order approximations $\sin(\alpha \pm \theta) \cong \alpha \pm \theta$, $\cos(\alpha \pm \theta) \cong 1$ and re-written in the compact form:

$$\ddot{\theta} - p^2 \theta = +p^2 \ddot{u}_g / g - p^2 \alpha \operatorname{sgn}(\theta) \quad (5.2)$$

where $\operatorname{sgn}(\)$ denotes the signum function and $p = (3g/4R)^{1/2}$ a measure of the frequency characteristics of the block (Housner 1963). It should be noticed that the second term on the right side of the equation refers to a constant, in each response branch, restoring force, whereas the second term on the left side can be interpreted as a negative (geometric) stiffness. As mentioned in Section 5.2, the above equation has been solved in an exact manner only for two cases: (1) a half-cycle rectangular pulse (Housner 1963); (2) a half-cycle sinusoidal pulse (Shi *et al.* 1996). An exact analytical solution for the aforementioned full-cycle waveform is presented below.

5.4 Solutions

5.4.1 Half-cycle rectangular pulse

For completeness and as a first step towards addressing more complex cases, the problem of a pulse of half-cycle duration will be briefly revisited. With reference to Fig 5.1, the half pulse is described as

$$\ddot{u}_g(t) = A_g[1 - H(t - t_d)] \quad (5.3)$$

where $H(\cdot)$ denotes the Heaviside (step) function. Upon introducing dimensionless peak ground acceleration $\alpha_g = A_g/g$ and considering positive rocking angles (the only case of interest for half-cycle excitation), Eq. (5.2) can be cast in the form

$$\ddot{\theta} - p^2\theta = +p^2\alpha_g[1 - H(t - t_d)] - p^2\alpha \quad (5.4)$$

which holds under the assumption $\alpha_g > \alpha$ or, equivalently, $\alpha/\alpha_g = \eta < 1$. Enforcing the initial conditions of zero angular velocity and displacement at time of uplift ($t_{up} = 0$), the time histories of angular velocity and displacement during pulse ($t_{up} \leq t \leq t_d$) are obtained as:

$$\dot{\theta}(t) = (\alpha_g - \alpha)p \sinh(pt) \quad (5.5)$$

$$\theta(t) = (\alpha_g - \alpha)[\cosh(pt) - 1] \quad (5.6)$$

Likewise, rocking response after the end of the pulse ($t \geq t_d$) is obtained from the homogeneous part of Eq. (5.4) in conjunction with the initial conditions $\dot{\theta}(t_d)$ and $\theta(t_d)$ to give:

$$\dot{\theta}(t) = (\alpha_g - \alpha)p \sinh(pt) - \alpha_g p \sinh[p(t - t_d)] \quad (5.7)$$

$$\theta(t) = \alpha + (\alpha_g - \alpha) \cosh(pt) - \alpha_g \cosh[p(t - t_d)] \quad (5.8)$$

As discussed earlier, a primary objective of the analysis is to determine under which conditions the block will overturn. Defining overturning, however, is not straightforward, as the obvious condition $\theta(t) = \pi/2$ cannot be employed in the realm of the linearized Eq. (5.2), thereby alternative definitions must be sought.

For unilateral excitations like the one at hand, if response angle θ exceeds slenderness α the restoring moment due to self-weight will turn into a driving moment, response will become unstable (to be referred in the ensuing as “supercritical”), and the block will overturn. Accordingly, a sufficient acceleration amplitude to overturn the block can be defined by setting $\theta(t)$ in Eqs (5.6) or (5.8) equal to $\pm\alpha$.

Enforcing this condition at $t = t_d/2$ and the end of the pulse ($t = t_d$), the following simple expressions are obtained, respectively

$$\eta = 1 - 1/\cosh(f/2) \quad (5.9a)$$

$$\eta = 1 - 1/\cosh(f) \quad (5.9b)$$

where $f = p t_d$ is the dimensionless duration of the pulse (or, equivalently, the normalized pseudo - frequency of the block).

As overturning during pulse clearly does not represent the most critical condition for the particular system, the limit criterion (*safety wall*) against overturning can be defined by the condition of an uplift angle equal to α under zero rocking velocity, after the end of the

pulse. This criterion corresponds to simultaneous immobility and equilibrium, i.e., $\dot{\theta}(t_c) = 0$ and $\ddot{\theta}(t_c) = 0$ (Eqs 4.19a and 4.19b) that define a *saddle point* in $\theta - t$ space, at critical time $t = t_c$.

By virtue of Eqs. (5.4), (5.7) and (5.8), the above conditions yield the simple expression

$$\eta = 1 - e^{-f} \quad (5.10)$$

referring to $t_c \rightarrow \infty$. This result coincides with the solution of Housner (1963).

Figure 5.3 shows the areas of safety (denoted by letter S) and overturning (denoted by letter O) in $f - \eta$ space for the specific pulse. Evidently, for large values of uplift strength overturning requires large pulse durations and vice versa. Regions O_1 and O_2 correspond to systems exceeding critical angle α during pulse (the former taking place before $t_d/2$; the latter after $t_d/2$), whereas systems in region O_3 become supercritical after pulse. The curve separating area O_2 and O_3 corresponds to Eq. (5.9b) while the line separating area O_3 from the area of safety S corresponds to Eq. (5.10). Finally, the line separating areas O_1 and O_2 is defined by Eq. (5.9a).

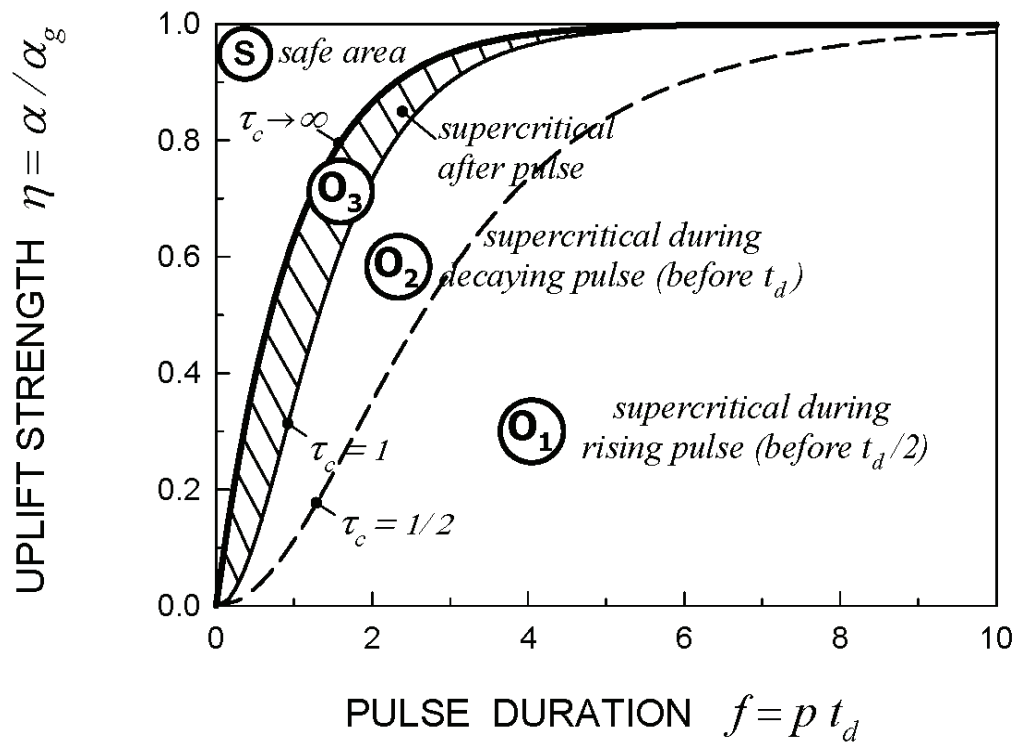


Figure 5.3 Areas of Safety (S) and Overturning (O₁ to O₃) for a rocking block under a rectangular pulse excitation of half-cycle duration.

With reference to systems that do not overturn (i.e., blocks in region S), time of peak response, $\tau_m = t_m/t_d$, is determined from Eq. (5.7) by setting angular velocity equal to zero, to get

$$\tau_m = \frac{1}{2f} \ln \left[\frac{1-\eta-e^f}{1-\eta-e^{-f}} \right] \quad (5.11)$$

Based on this result, maximum rocking response is obtained from Eqs (5.8) as fraction of slenderness angle α

$$\frac{\theta_{max}}{\alpha} = \frac{1}{\eta} \left[\eta + (1-\eta) \cosh(f\tau_m) - \cosh[f(\tau_m - 1)] \right] \quad (5.12)$$

Finally, setting $\theta = 0$ in Eq. (5.8) one obtains the time of impact in closed-form

$$\tau_i = \frac{1}{f} \ln \left[\frac{(1-e^{-f})\sqrt{e^f(1-\eta)} + \eta}{e^{-f} - 1 + \eta} \right] \quad (5.13)$$

More details on the derivation of the above equations are provided in Chapter 4 [Eqs (4.25), (4.29) & (4.33) and Figures 4.9 to 4.15].

5.4.2 Full-cycle rectangular pulse

Response to a full-cycle pulse is far more complicated than that to its half-cycle counterpart, as the existence of the second, negative, excitation lobe allows overturning of the block under two distinct, mutually exclusive modes: *Mode 0*, corresponding to overturning in the forward sense without impact; and *Mode 1*, that is overturning in the backward sense following a single impact and rocking reversal. It will be shown that complexity arises due to the bifurcation behavior associated with these two modes.

5.4.2.1 Mode 0

Upon initiation of rocking ($\eta < 1$) there is a class of systems that may overturn without an impact i.e., under an always positive angle $\theta(t)$ from initiation of rocking to supercritical response.

On the basis of this hypothesis (to be referred below to as *Mode 0*), equation of motion (5.2) simplifies to

$$\ddot{\theta}(t) - p^2 \theta(t) = (+) p^2 \ddot{u}_g / g - p^2 \alpha \quad (5.14)$$

where now

$$\ddot{u}_g(t) = A_g [1 - 2H(t - t_d) + H(t - 2t_d)] \quad (5.15)$$

The time histories of angular velocity and displacement during the first excitation lobe ($t_y \leq t \leq t_d$) are as for the half-cycle pulse in Eqs (5.5) and (5.6).

During the second pulse lobe ($t_d \leq t \leq 2t_d$), rocking response is obtained from the expressions

$$\dot{\theta}(t) = (\alpha_g - \alpha) p \sinh(pt) - 2\alpha_g p \sinh[p(t - t_d)] \quad (5.16)$$

$$\theta(t) = \alpha_g + \alpha + (\alpha_g - \alpha) \cosh(pt) - 2\alpha_g \cosh[p(t - t_d)] \quad (5.17)$$

which satisfy the governing equation (5.14) and the initial conditions $\dot{\theta}(t_d)$ and $\theta(t_d)$ obtained from Eqs (5.5) and (5.6).

Free rocking response after the end of pulse ($t \geq 2t_d$) is, likewise, obtained from

$$\dot{\theta}(t) = (\alpha_g - \alpha)p \sinh(pt) - 2\alpha_g p \sinh[p(t - t_d)] + \alpha_g p \sinh[p(t - 2t_d)] \quad (5.18)$$

$$\theta(t) = \alpha + (\alpha_g - \alpha) \cosh(pt) - 2\alpha_g \cosh[p(t - t_d)] + \alpha_g \cosh[p(t - 2t_d)] \quad (5.19)$$

which, in turn, satisfy the homogeneous part of Eq. (5.2) and the conditions $\dot{\theta}(2t_d)$ and $\theta(2t_d)$ obtained from Eqs (5.16) and (5.17).

For unilateral excitations like the half-cycle pulses discussed in Chapter 4, if response angle θ exceeds slenderness α , the restoring moment due to the self-weight of the block will turn into a driving moment and the block will overturn. Accordingly, a sufficient acceleration amplitude to overturn the block can be determined by setting the response angle $\theta(t)$ equal to α . For bilateral excitations, however, developing overturning criteria is much more demanding. To this end, one needs to revisit two fundamental aspects of dynamic response, namely *equilibrium* and *immobility*. These notions are of key importance to the problem, as they provide the means by detecting pertinent inflection points in the response.

Equilibrium naturally occurs when the overall moment with respect to the pivot point P at the base of the block becomes zero. i.e.,

$$\ddot{\theta}(t_e) = 0 \quad (5.20)$$

where t_e is the corresponding time (subscript e standing for “equilibrium”).

For a symmetric rectangular pulse and for positive rocking angles, associated with the notation of Figure 5.1, the equilibrium condition is obtained from Eq. (5.1a) as

$$g \sin[\alpha - \theta(t_e)] = (+)\ddot{u}_g \cos[\alpha - \theta(t_e)] \quad (5.21a)$$

or, in linearized form,

$$\theta(t_e) = \alpha - \ddot{u}_g(t_e)/g \quad (5.21b)$$

Equation (5.21b) implies that instantaneous equilibrium may occur during the first lobe when $\theta(t_e) = \alpha - \alpha_g$, during second lobe when $\theta(t_e) = \alpha + \alpha_g$ and during free rocking when $\theta(t_e) = \alpha$. These conditions are presented graphically in Figure 5.4. The equilibrium points clearly provide only necessary conditions for overturning. For instance, if equilibrium is achieved during the first pulse lobe (i.e., when $\theta(t_e) = \alpha - \alpha_g$) the block will not necessarily overturn. Actually, only if the uplift angle becomes equal to the maximum of the equilibrium conditions in Fig. 5.4 overturning will be certain.

This sufficient condition is written as

$$\theta = \alpha + \alpha_g \quad (5.21c)$$

Immobility defines a second family of points useful in exploring the physics of the response. Naturally, immobility occurs when angular velocity $\dot{\theta}$ becomes zero i.e.,

$$\dot{\theta}(t_s) = 0 \quad (5.22)$$

where t_s is the corresponding time (subscript s standing for “stationary”).[§]

Under unilateral excitation, existence of an immobility point indicates a system that does not overturn and vice versa. Under bilateral excitation, however, this is not always true as multiple immobility points as possible. To this end, the following alternative definition applies: Under bilateral excitation, a system survives rocking if subjected to an *odd*

[§] symbol i is retained to denote impact in the ensuing.

number of immobility points in each response branch following uplift or impact. Conversely, a rocking system overturns if subjected to an *even* number of immobility points in each response branch. These definitions generalize those presented in Chapter 4 for half-cycle excitations. To elucidate the point the following trends (to be discussed in detail later on) are worth noting:

1. If uplift angle at the end of the pulse equals block slenderness (i.e., $\theta(2t_d) = \alpha$ – a sufficient condition for overturning under half-cycle excitation) the block does not overturn in the forward mode. In other words, although the system has reached a point of unstable equilibrium, remarkably survives *Mode 0*.
2. In order for the block to survive overturning under *Mode 0* there must be an immobility point before the end of the pulse. On the other hand, if angular velocity at the end of pulse is positive, it will always remain positive, regardless of uplift angle θ , during free rocking and the block will overturn.

These patterns are discussed in more detail in the ensuing.

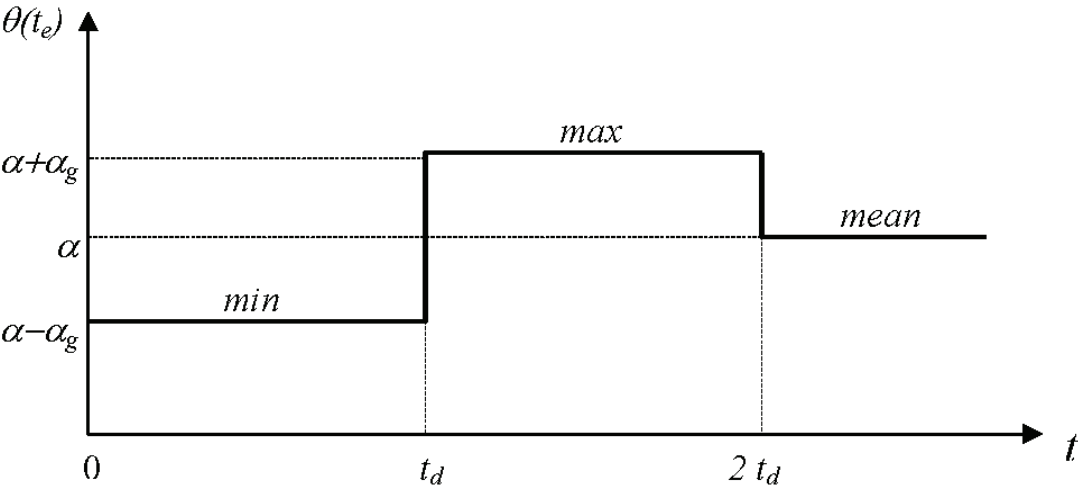


Figure 5.4 Conditions for instantaneous equilibrium under the full-cycle symmetric rectangular pulse of Fig. 5.2.

In light of the foregoing, a complete classification of rocking systems with respect to *Mode 0* in surviving (*S*) and overturning (*O*) groups is feasible. Specifically:

(a) Systems Overturning with Mode 0:

- Area O_1 : systems belonging this group attain supercritical response during the first excitation lobe i.e. $\theta(t_d) > \alpha + \alpha_g$.
- Area O_2 : Supercritical response for this group is achieved during the second excitation lobe, i.e. $\theta(t_d) < \alpha + \alpha_g$ and $t_d < t_e < 2t_d$.
- Area O_3 : For systems in O_3 limit uplift angle is not attained during the course of the pulse. Instead, the block experiences instantaneous equilibrium as response changes from forced to free and the mean equilibrium angle is surpassed at the end of the pulse (i.e., $\alpha < \theta(2t_d) < \alpha + \alpha_g$). For this group, like those in O_1 and O_2 , angular velocity is always positive which implies the absence of an immobility point.
- Area O_4 : Rocking behavior of systems in this group is similar to those in O_3 (again $\alpha < \theta(2t_d) < \alpha + \alpha_g$), except for angular velocity which changes sign twice after uplift: once during second lobe ($t_d < t_{s1} < 2t_d$) and then after the end of the pulse ($t_{s2} > 2t_d$).

Details on the response systems in these groups and the boundaries between them are provided below.

Mode 0

- instantaneous equilibrium ▲ immobility ■ impact ● limit uplift angle

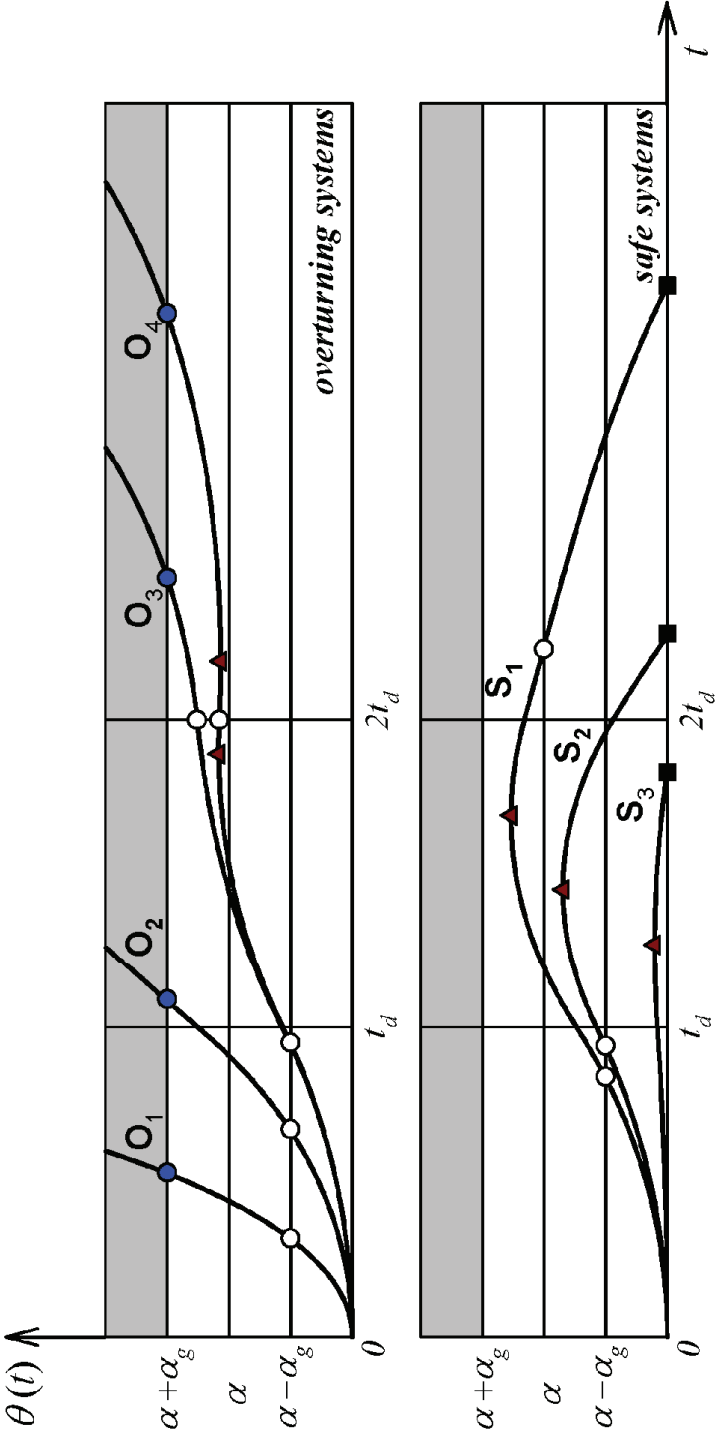


Figure 5.5 Classification of overturning (O) and surviving (S) systems under *Mode 0* by means of immobility and equilibrium points. Note that systems in group O experience an even number of immobility points (zero or two) whereas systems in group S experience an odd number of immobility points (one). No other pattern is possible for the specific problem.

(b) Systems Surviving Mode 0:

- Area S_I defines systems that narrowly survive *Mode 0*. In this group mean equilibrium angle is surpassed at the end of the pulse (i.e., $\alpha < \theta(2t_d) < \alpha + \alpha_g$), yet under negative velocity ($t_d < t_s < 2t_d$) that remains negative until impact and beyond. The difference between systems in groups O_4 and S_I (both of which attain rocking angles between α and $\alpha + \alpha_g$ at the end of the pulse) is noteworthy: safety in the latter case is achieved by the lack of a second immobility point, to switch the sign of velocity from negative to positive after the end of the pulse. In other words systems in O_4 experience an even number of immobility points (two) whereas systems in S_I an odd number (one).
- Area S_2 refers to systems experiencing uplift angles smaller than the mean equilibrium angle ($\theta < \alpha$), leading to a single immobility point before the end of the pulse ($t_d < t_s < 2t_d$) and impact after the end of pulse.
- Area S_3 represents the strongest group of surviving systems which never attain equilibrium, while impact occurs during the second lobe of pulse. Like all systems in group S, a single immobility point is experienced during forced excitation ($t_d < t_s < 2t_d$). It will be shown that these systems behave in a different manner after impact.

Examples of the above classification are presented in Figure 5.5. Criteria separating systems in the various regimes are provided below:

- Limit between Areas O_1 & O_2

In light of the foregoing the criterion separating zones O_1 and O_2 is

$$\theta(t_d) = \alpha + \alpha_g \quad (5.23)$$

which corresponds to the upper equilibrium condition.

Combining Eqs (5.6) and (5.24) the limit between the above areas, formulated on f - η space is

$$\eta = 1 - \frac{2}{\cosh f} \quad (5.24)$$

- Limit between Areas O_2 & O_3

The corresponding criterion for the boundary of the above zones is

$$\theta(2t_d) = \alpha + \alpha_g \quad (5.25)$$

which is identical to that in Eq. (5.23) except for a translation in time ($2t_d$ instead of t_d).

From Eqs (5.17) and (5.25) one obtains the expression

$$\eta = 1 - \frac{2 \cosh f}{\cosh 2f} \quad (5.26)$$

- Limit between Areas O_3 & O_4

Setting angular velocity at the end of the pulse equal to zero, or the uplift angle at t_d equal to α , yields the criterion separating the above zones. Note in this regard that

this immobility point is not unique as overturning requires an even number of immobility points in each response mode.

$$\dot{\theta}(2t_d) = 0 \text{ or } \theta(t_d) = \alpha \quad (5.27)$$

From Eqs (5.16) and (5.27) this condition is written as

$$\eta = 1 - \frac{2 \sinh f}{\sinh 2f} \quad (5.28)$$

which, remarkably, is identical to Eq. (5.26) except for the presence of hyperbolic sines instead of cosines.

- Limit between Areas O_4 & S_1 : Limit of Mode 0

The limit between areas O_4 & S_1 defines the limit between systems overturning and systems surviving under *Mode 0*, and thus the limit criterion (*safety wall*) against the specific mode. As for the case of half-cycle excitation, the limit corresponds to an uplift angle of α (equilibrium condition) under zero rocking velocity (immobility condition), at an infinite time after the end of the pulse i.e.,

$$\ddot{\theta}(t_\infty) = 0 \quad (5.29a)$$

$$\dot{\theta}(t_\infty) = 0 \quad (5.29b)$$

Combining Eq. (5.18) with Eqs (5.29) we obtain the pair of equations

$$(1 - \eta) \sinh(f\tau_\infty) - 2 \sinh[f(\tau_\infty - 1)] + \sinh[f(\tau_\infty - 2)] = 0 \quad (5.30a)$$

$$(1 - \eta) \cosh(f\tau_\infty) - 2 \cosh[f(\tau_\infty - 1)] + \cosh[f(\tau_\infty - 2)] = 0 \quad (5.30b)$$

Equations (5.30a) and (5.30b) are identical and give

$$\eta = (1 - e^{-f})^2 \quad (5.30c)$$

or

$$f = \ln[1/(1 - \eta^{1/2})] \quad (5.30d)$$

which defines the overturning criterion for *Mode 0* in f - η space. Note that the above criterion is very similar to the one for the half-cycle pulse in Eq. (5.10), except for the presence of the second power in the right side. This suggests that a block subjected to a full-cycle pulse is less vulnerable to overturning under *Mode 0* than an identical block subjected to a half-cycle pulse.

- Limit between Areas S_1 & S_2

The condition separating the two zones in Fig 5.6 is

$$\theta(2t_d) = \alpha \quad (5.31)$$

which corresponds to the mean equilibrium point at the end of the pulse.

Using Eqs (5.17) and (5.31) yields the solution

$$\eta = 1 - \frac{2 \cosh f - 1}{\cosh 2f} \quad (5.32)$$

which is identical to Eq. (5.26) except for the presence of the minus one term in the numerator.

- Limit between Areas S_2 & S_3

By definition, the criterion distinguishing between the two zones is

$$\theta(2t_d) = 0 \quad (5.33)$$

Accordingly, using Eqs (5.17) and (5.33) yields the condition

$$\eta = 1 - \frac{2 \cosh f - 2}{1 - \cosh 2f} \quad (5.34)$$

Results from the above analysis are summarized in Figure 5.6. As explained earlier, systems in areas O overturn under *Mode 0*, while systems in areas S survive the specific failure pattern. Evidently, more prone to overturning are systems in region O_1 which experience instability ($\theta = \alpha + \alpha_g$) during the first excitation lobe (Fig 5.4). Systems in area O_2 follow, whose response becomes supercritical during the second lobe. Systems in group O_3 reach *mean* equilibrium conditions (i.e. $\theta = \alpha$) during the second pulse lobe and experience rocking angles equal or higher than slenderness angle α at the end of the pulse. Area O_4 is a special case of O_3 in which two immobility points are encountered.

As far as surviving (S) systems are concerned, all such blocks attain a single immobility point during forward rocking. Area S_3 involves systems that experience impact during pulse, while systems S_1 and S_2 experience impact during free rocking. Understandably, the limit between S_2 and S_3 (Eq. 5.34) indicates systems that impact at pulse end ($t_i = 2t_d$).

The narrow areas corresponding to groups S_1 and O_4 are worth noting.

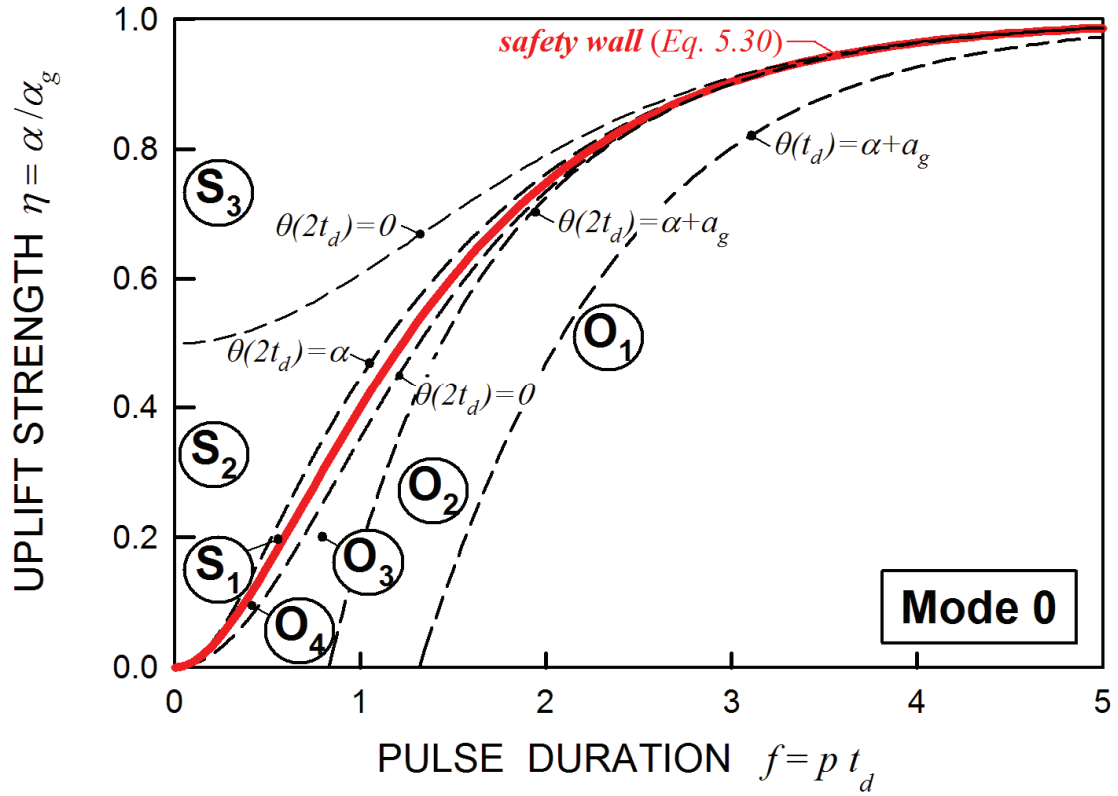


Figure 5.6 Areas of Safety (S_1 to S_3) and Overturning (O_1 to O_4) during forward response without impact (*Mode 0*), for a rocking block under a symmetric, full-cycle rectangular pulse.

Examples of the above classification are provided in Figure 5.7, where acceleration, velocity and displacement response for eight systems is depicted belonging to different groups. Evidently, region O_1 corresponds to a monotonic increase in block angular displacement, velocity and acceleration leading to an instability during the first half of the pulse. For systems in region O_2 acceleration becomes negative at the end of the first lobe and then switches back to positive before t_d , when $\theta = \alpha + \alpha_g$. Region O_3 exhibits negative rocking acceleration after the first lobe which, in this case, lasts up to the end of the pulse and then it goes positive again; yet this is not sufficient to prevent overturning, as velocity does not drop to zero. Region O_4 is analogous to O_3 , except for the sign of velocity which switches from positive to negative during pulse, and back to positive after pulse (i.e., two immobility points). In region S_1 acceleration is positive at the end of the pulse, but under negative velocity which leads to decrease in response and, thereby, to safety. Likewise, systems in region S_2 exhibits negative acceleration and velocity at the end of the pulse. Finally, systems in Region S_3 attain motion reversal and impact before the end of the pulse. No other response patterns are possible for the particular excitation.

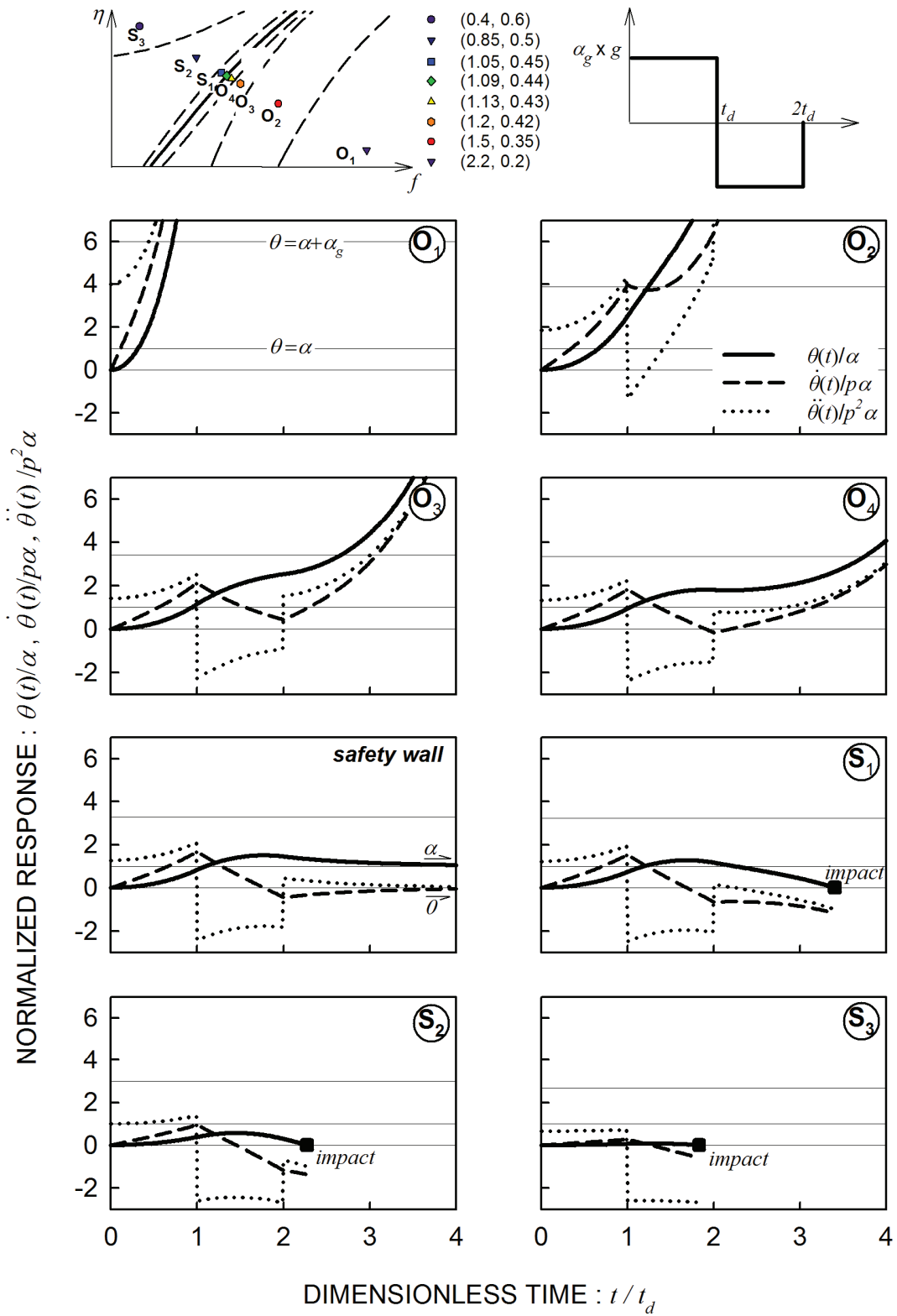


Figure 5.7 Examples of response for different regimes of a rocking block under a symmetric rectangular pulse of full-cycle duration.

5.4.2.2 Mode 1

By definition, systems in *Mode 1* overturn in the direction opposite to first movement after experiencing one impact. For this to be possible, rocking velocity should change direction during forced response so that the block can accelerate in the negative direction. This requires an odd number of immobility points before the end of the pulse ($t_s \leq 2t_d$). [It will be shown that in all cases relevant to the specific pulse there will be just a single immobility point.] After t_s , rocking angle will naturally decrease with time until impact, ($\theta(t_i)=0$), beyond which it will become negative. Impact may occur during pulse ($t_s \leq t_i \leq 2t_d$) for systems in area S_3 , or after pulse ($t_i \geq 2t_d$) for systems in areas S_1 & S_2 .

Setting the rocking angle in Eqs (5.17) and (5.19) equal to zero and solving for t_i we obtain the closed-form expressions

$$\tau_i = \frac{1}{f} \ln \left[\frac{(1-e^f) \sqrt{2e^{-f}(1-\eta)} - 1 - \eta}{1 - 2e^{-f} - \eta} \right] \quad (5.35a)$$

$$\tau_i = \frac{1}{f} \ln \left[\frac{-2 \sinh \frac{f}{2} \sqrt{2 - 2(1-\eta) \cosh f} - \eta}{1 - 2e^{-f} + e^{-2f} - \eta} \right] \quad (5.35b)$$

corresponding to the time of impact during and after pulse, respectively.

The above equations are plotted for different uplift strengths, as functions of pulse duration in Figure 5.8. τ_i naturally increases with pulse duration and decreases with system strength. The singularities observed at specific durations correspond to a root in the denominator of the logarithmic arguments and pertain to systems located on the safety

wall for overturning in *Mode 0*. Beyond the specific value of pulse duration f , no times of impact are possible due to overturning of the blocks.

Corresponding velocities can be readily determined by combining Eqs (5.16) and (5.18) with Eqs (5.35) for impact during and after the pulse, respectively, to get

$$\frac{\dot{\theta}(\tau_i^-)}{\alpha_g f} = (1 - e^f) \sqrt{2e^{-f}(1 - \eta)} \quad (5.36a)^{**}$$

$$\frac{\dot{\theta}(\tau_i^-)}{\alpha_g f} = (1 - e^f) \sqrt{2e^{-f} - (1 + e^{-2f})(1 - \eta)} \quad (5.36b)$$

which pertain to times shortly before impact ($\tau = \tau_i^-$).

Equating the right-hand sides of Eqs (5.36a) and (5.36b) and solving for η one gets

$$\eta(\tau_i = 2) = \frac{1 + e^{2f}}{(1 + e^f)^2} \quad (5.37)$$

which defines the uplift strength of systems experiencing impact right at the end of the pulse.

Substituting η in Eq. (5.36b) with that in Eq. (5.37) one gets the following expression for the velocity at impact at pulse end ($\tau_i=2$)

$$\frac{\dot{\theta}(\tau_i^- = 2)}{\alpha_g f} = -2 \tanh \frac{f}{2} \quad (5.38)$$

which, naturally, is always negative like those in Eqs (5.36).

^{**} In this case the dot symbol denotes derivative with respect to dimensionless time τ ; this differs from the classical derivative with respect to t , since $\dot{\theta}(\tau_i) = \frac{d\theta(\tau_i)}{d\tau} = \frac{d\theta(t_i)}{dt} \cdot \frac{dt}{d\tau} = \dot{\theta}(t_i) \cdot t_d$

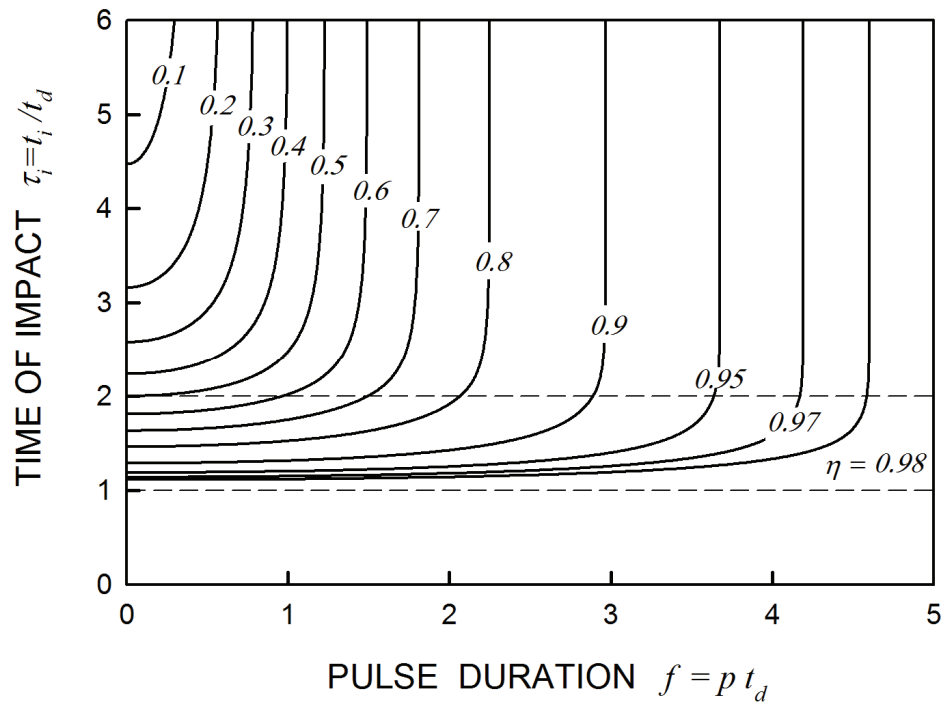


Figure 5.8 Times of impact for a rocking block under a symmetric rectangular pulse of full-cycle duration.

Results from the above equations are depicted in Figure 5.9. Equations (5.36a) and (5.36b) are depicted by light and dark green lines, respectively. The dashed line (Eq. 5.38), separates the solutions for impact before and after the pulse, and provides a lower bound for the velocity at impact. The dotted line corresponds to the safety wall of *Mode 0* beyond which the block overturns before impact. Evidently, the maximum, in absolute terms, velocity at impact, realizes for systems experiencing impact at the end of the pulse. A closed-form expression for this limit is possible by substituting uplift strength in Eq. (5.36b) by critical strength from Eq. (5.30c), to get

$$\frac{\dot{\theta}(\tau_i^- \rightarrow \infty)}{\alpha_g f} = -e^{-2f} (1 - e^f)^2 \quad (5.39)$$

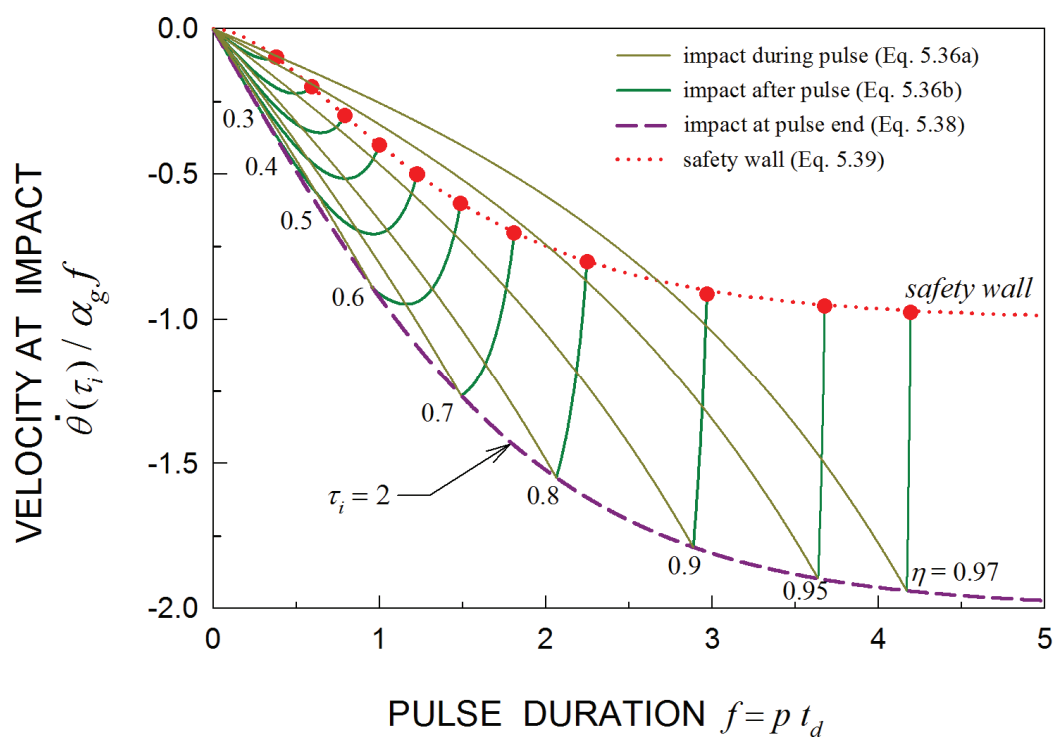


Figure 5.9 Velocity at impact as function of uplift strength and pulse duration (Region S) due to a symmetric rectangular pulse of full-cycle duration. Note that both numerator and denominator in the ordinate axis are dimensionless.

According to the adopted notation (Fig 5.1), up to the point of impact the block rocks around the right edge (pole P) with positive uplift angles; after impact rocking continues around the left edge (pole P') with negative uplift angles. The transition from one pole to the other is accompanied by energy loss during impact even for non-dissipative block and base materials. Assuming perfectly inelastic impact (no bouncing), the reduction in kinetic energy during transition is

$$r = \left(\frac{\dot{\theta}_i^+}{\dot{\theta}_i^-} \right)^2 \quad (5.40)$$

Using conservation of angular momentum, Housner (1963) evaluated r for rectangular blocks to be

$$r = \left(1 - \frac{3}{2} \sin^2 \alpha \right)^2 \quad (5.41a)$$

Restricting angle α to small values pertaining to slender blocks the above result simplifies to

$$\varepsilon = \sqrt{r} = 1 - \frac{3}{2} \alpha^2 \quad (5.41b)$$

which defines the coefficient of restitution. This is an upper bound estimate, as the true coefficient of restitution is always smaller than the one disregarding bouncing. A formulation of impact in terms of an impulsive forcing function is provided in Appendix D.3.

Rocking response in the realm of *Mode 1* is governed by Eq. (5.2) where now the sign of uplift angle is negative. For systems experiencing impact during pulse ($t_s \leq t_i \leq 2t_d$), Eq. (5.2) becomes

$$\ddot{\theta}(t) - p^2\theta(t) = -p^2\alpha_g[1 - H(2t_d - t)] + p^2\alpha \quad (5.42)$$

Applying the initial conditions $\theta(t_i) = 0$ and $\dot{\theta}(t_i^+) = \varepsilon \dot{\theta}(t_i^-)$, ε being the coefficient of restitution, one obtains the response during pulse ($t_i \leq t \leq 2t_d$)

$$\dot{\theta}(t) = -(\alpha_g - \alpha)p \sinh[p(t - t_i)] + \varepsilon \cosh[p(t - t_i)]\dot{\theta}(t_i^-) \quad (5.43a)$$

$$\theta(t) = (\alpha_g - \alpha)[1 - \cosh[p(t - t_i)]] + \frac{\varepsilon}{p} \sinh[p(t - t_i)]\dot{\theta}(t_i^-) \quad (5.43b)$$

For the response after pulse ($t \geq 2t_d$), the corresponding solution is

$$\dot{\theta}(t) = -(\alpha_g - \alpha)p \sinh[p(t - t_i)] + \alpha_g p \sinh[p(t - 2t_d)] + \varepsilon \cosh[p(t - t_i)]\dot{\theta}(t_i^-) \quad (5.44a)$$

$$\theta(t) = (\alpha_g - \alpha)[1 - \cosh[p(t - t_i)]] - \alpha_g[1 - \cosh[p(t - 2t_d)]] + \frac{\varepsilon}{p} \sinh[p(t - t_i)]\dot{\theta}(t_i^-) \quad (5.44b)$$

where $\dot{\theta}(t_i^-)$ is given by Eq. (5.16).

The predictions of the above equations match at $t=t_i$ to satisfy continuity of motion and energy loss during impact.

For systems experiencing impact after pulse ($t_i \geq 2t_d$), Eq. (5.2) becomes

$$\ddot{\theta}(t) - p^2\theta(t) = p^2\alpha \quad (5.45)$$

In the same spirit as before, applying the initial conditions $\theta(t_i) = 0$ and $\dot{\theta}(t_i^+) = \varepsilon \dot{\theta}(t_i^-)$ one obtains

$$\dot{\theta}(t) = \alpha p \sinh[p(t - t_i)] + \varepsilon \cosh[p(t - t_i)] \dot{\theta}(t_i^-) \quad (5.46a)$$

$$\theta(t) = -\alpha[1 - \cosh[p(t - t_i)]] + \frac{\varepsilon}{p} \sinh[p(t - t_i)] \dot{\theta}(t_i^-) \quad (5.46b)$$

Equilibrium and immobility points during backward rocking provide the means of classifying systems in an analogous manner to that in *Mode 0*. However, in this case the procedure is less complicated due to the absence of an acceleration reversal which precludes multiple immobility points. Indeed, there is only one type of equilibrium (when $\theta = -\alpha$) that may occur during free rocking and provides a sufficient condition for overturning. In this context, systems in the context of *Mode 1* are divided in two main groups: overturning systems (denoted by E) and surviving systems (denoted by S). Each class is divided into subgroups as shown below.

(a) Systems Overturning with Mode 1:

- Group E_1 consists of systems that classify as S_1 during forward rocking, thus, narrowly surviving overturning under *Mode 0*, with impact taking place after pulse. During backward motion, angular displacement exceeds critical angle ($\theta = -\alpha$) and the block overturns.
- Group E_2 consists of systems that belong to group S_2 during forward rocking. As with the previous group impact for these systems occurs after pulse and the condition for overturning ($\theta = -\alpha$) is met during free rocking.

- Group E_3 consists of systems that in the realm of *Mode 0* belong to area S_3 and are the ones that respond the least during forward rocking. For these systems impact occurs during second pulse lobe thus forcing them to rock with negative angles before the end of the pulse. The overturning angle ($\theta = -\alpha$) is again met during free rocking.

(b) Systems Surviving Mode 1: The *safe* areas against *Mode 1* naturally coincide with area S of *Mode 0*, if one extracts the overturning areas E against *Mode 1*. During *Mode 1* rocking systems classifying under areas S_1 and S_2 exhibit identical behaviors [i.e., experience motion reversal, a single immobility point, and a second impact after pulse (Fig. 5.10)]. Systems in S_3 experience impact during second pulse lobe.

In the realm of this specific *Mode*, like for the previous one, an even number of immobility points leads to overturning, while an odd number imply safety. Figure 5.10 illustrates the behaviour of systems belonging to each of these groups.

Mode 1

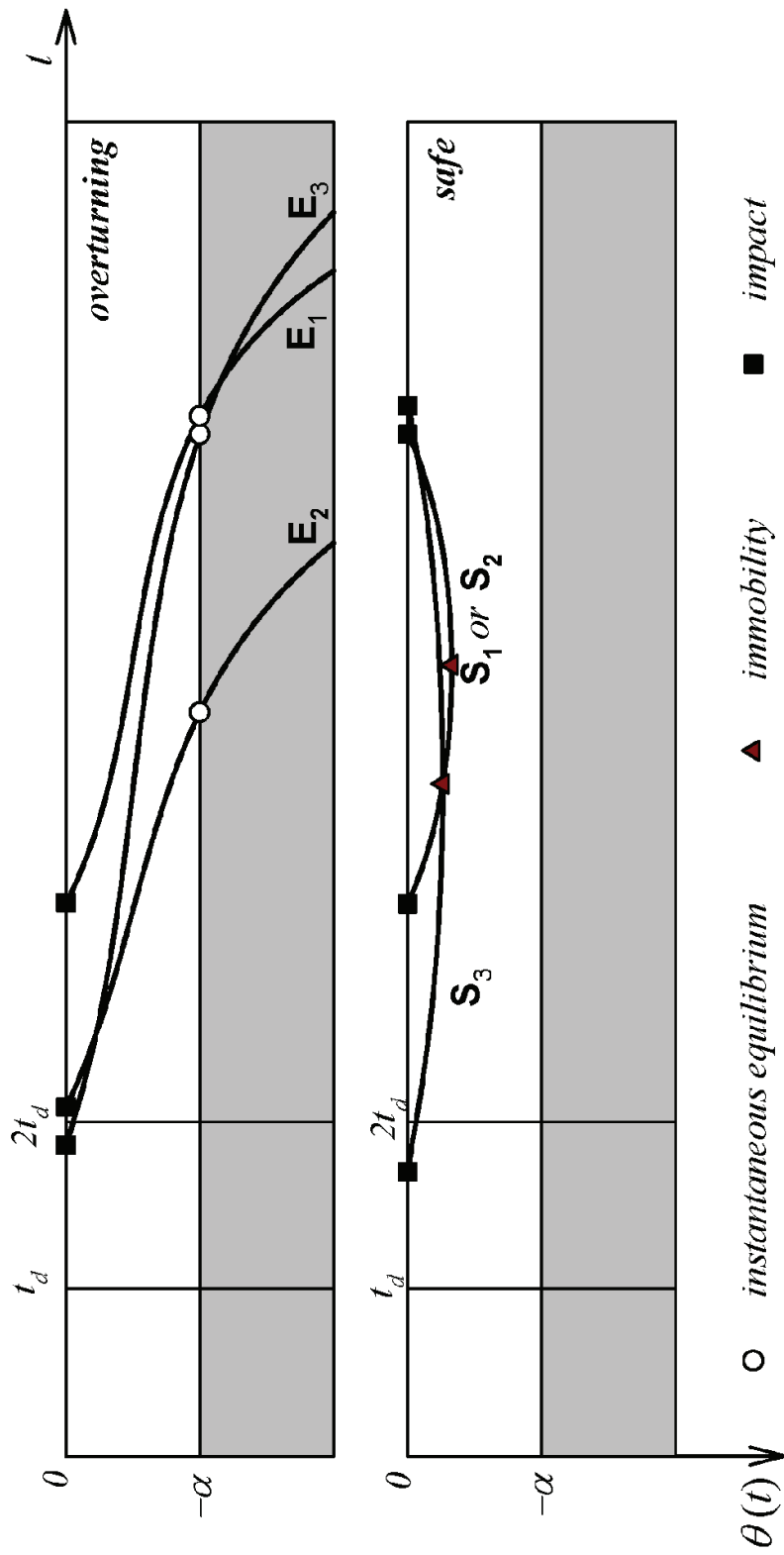


Figure 5.10 Classification of overturning and surviving rocking systems against *Mode 1* by means of immobility and equilibrium points for a symmetric rectangular pulse of full – cycle duration.

Classification of systems based on *Mode 1* is depicted in Figure 5.11 on f - η space. The boundaries separating groups E_1 , E_2 and E_3 coincide with those separating areas S_1 , S_2 and S_3 in *Mode 0* which, naturally, are the same for the two Modes and are described by Eqs (5.32) and (5.34). Interestingly, the boundaries separating E_1 from S_1 , E_2 from S_2 and E_3 from S_3 define the *Limit of Mode 1* as explained below.

Boundary between Areas E & S: Limit of Mode 1

The boundary between areas E & S defines the limit of systems overturning under *Mode 1* and, therefore, the corresponding *safety wall*. Like in *Mode 0* it can be determined as an equilibrium and an immobility point reached simultaneously at an infinite time (Eqs 5.29). The difference from *Mode 0* is that now the limit can be reached starting from two different initial conditions i.e., when impact occurs during pulse corresponding to areas $E_3 - S_3$ and when impact occurs after pulse corresponding to areas $E_1 - S_1$ and $E_2 - S_2$. Each boundary is examined separately below.

- Impact during pulse ($t_d \leq t_i \leq 2t_d$)

Combining Eqs (5.29) with the first derivative of Eq. (5.44a) we obtain

$$-1 + \eta + \frac{\cosh[f(\tau_c - 2)]}{\cosh[f(\tau_c - \tau_i)]} + \frac{\varepsilon \dot{\theta}(\tau_i^-)}{a_g f} \tanh[f(\tau_c - \tau_i)] = 0 \quad (5.47)$$

Letting $\tau_c \rightarrow +\infty$, $\tanh[f(\tau_c - \tau_i)] \rightarrow 1$, $\cosh[f(\tau_c - 2)] / \cosh[f(\tau_c - \tau_i)] \rightarrow e^{f(\tau_i - 2)}$,

and employing Eqs (5.35a) and (5.36a), the above result can be cast in the form

$$-1 + \eta + e^{-2f} \frac{(1 - e^f) \sqrt{2e^{-f}(1 - \eta)} - 1 - \eta}{1 - 2e^{-f} - \eta} + \varepsilon(1 - e^f) \sqrt{2e^{-f}(1 - \eta)} = 0 \quad (5.48)$$

The above equation can be solved analytically to provide the *Limit of Mode 1* between regions E_3 and S_3 . As the solution is rather complicated, the results are provided in Appendix D.

- Impact after pulse ($t_i > 2t_d$)

Combining Eqs (5.29) with the first derivative of Eq. (5.46a) and letting $\tau_c \rightarrow +\infty$, $\tanh[f(\tau_c - \tau_i)] \rightarrow 1$, we get

$$\frac{\varepsilon \dot{\theta}(\tau_i^-)}{a_g f} = -\eta \quad (5.49)$$

which, combined with Eq (5.36b), yields

$$\varepsilon(1 - e^f) \sqrt{2e^{-f} - (1 + e^{-2f})(1 - \eta)} = -\eta \quad (5.50a)$$

Equation (5.50a) admits the simple solution

$$\eta = 2\varepsilon^2 \sinh^2 \frac{f}{2} \left[2 \cosh f \pm \sqrt{2 + 2 \cosh 2f - \frac{4}{\varepsilon^2}} \right] \quad (5.50b)$$

which provide the *Limit of Mode 1* between groups E_1 - S_1 and E_2 - S_2 .

Note that for the limit case where $\varepsilon=1$, a perfectly linear impact, the specific boundary coincides with the *Limit of Mode 0*.

The double sign in Eq. (5.50b) expresses the two roots of the solution. It is straightforward to determine the limit between the two roots by setting the term under the square root equal to zero, to get

$$\varepsilon^2 = \frac{2}{1 + \cosh 2f} \quad (5.51)$$

Substituting Eq. (5.51) back into Eq. (5.50b) one obtains the following expression which represents the line separating the two roots of the solution – shown with a dotted line in Figure 5.11.

$$\eta = \frac{8 \cosh f \sinh^2 \frac{f}{2}}{1 + \cosh 2f} \quad (5.52)$$

The above equations form the *Limits of Mode 1* and are depicted for different values of the restitution coefficient in Figure 5.12. An alternative way to present the *Limits of Mode 1* for values of f spanning from 0 to infinity is illustrated in Figure 5.13. The patterns shown in these graphs are self explanatory.

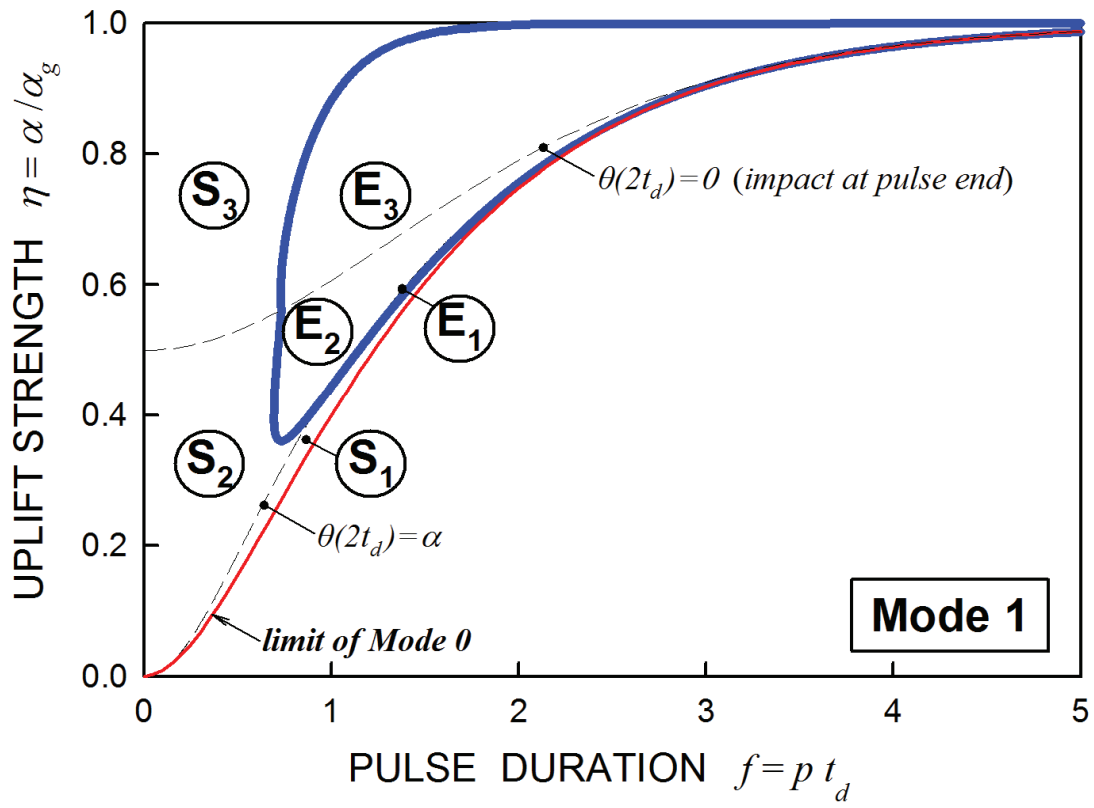


Figure 5.11 Areas of overturning with one impact (*Mode 1*) for a rocking block under a symmetric rectangular pulse excitation of full – cycle duration; $\varepsilon = 0.8$.

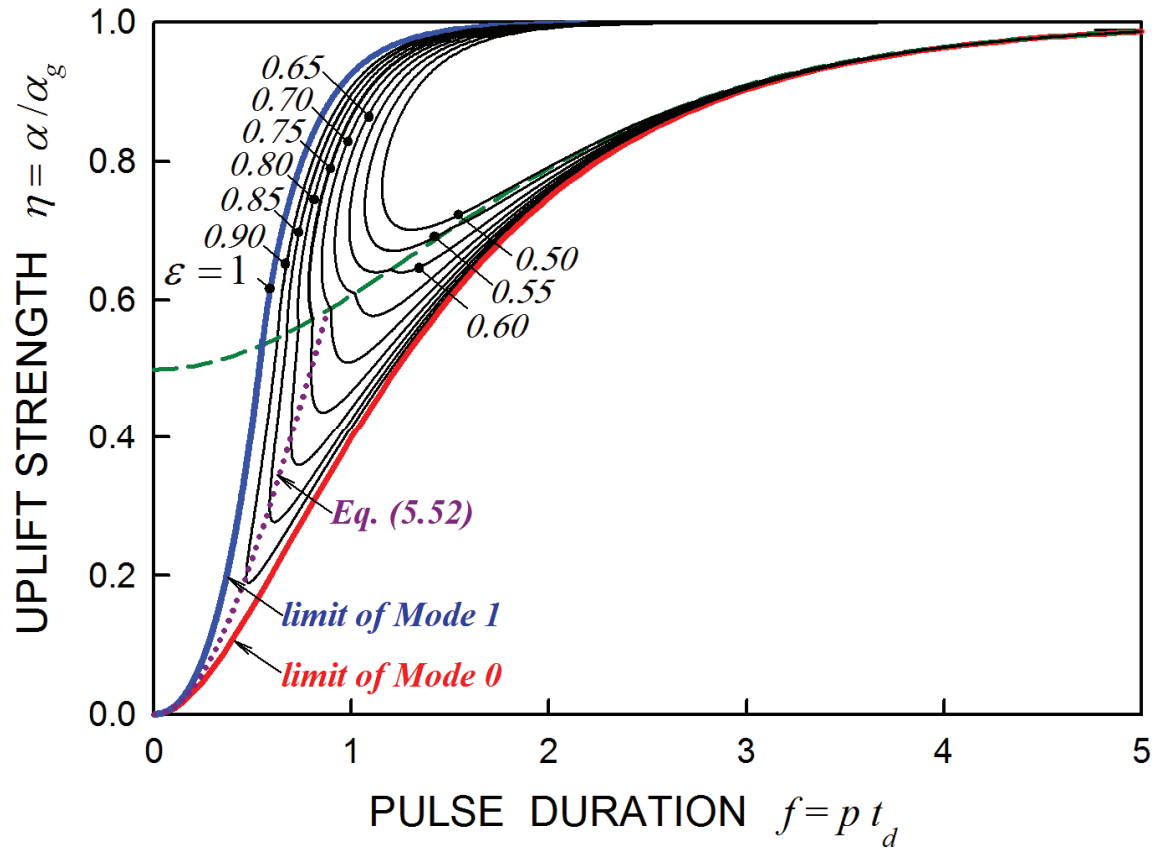


Figure 5.12 Limits of *Mode 1* due to a symmetric rectangular pulse excitation of full – cycle duration for different values of coefficient of restitution ε .

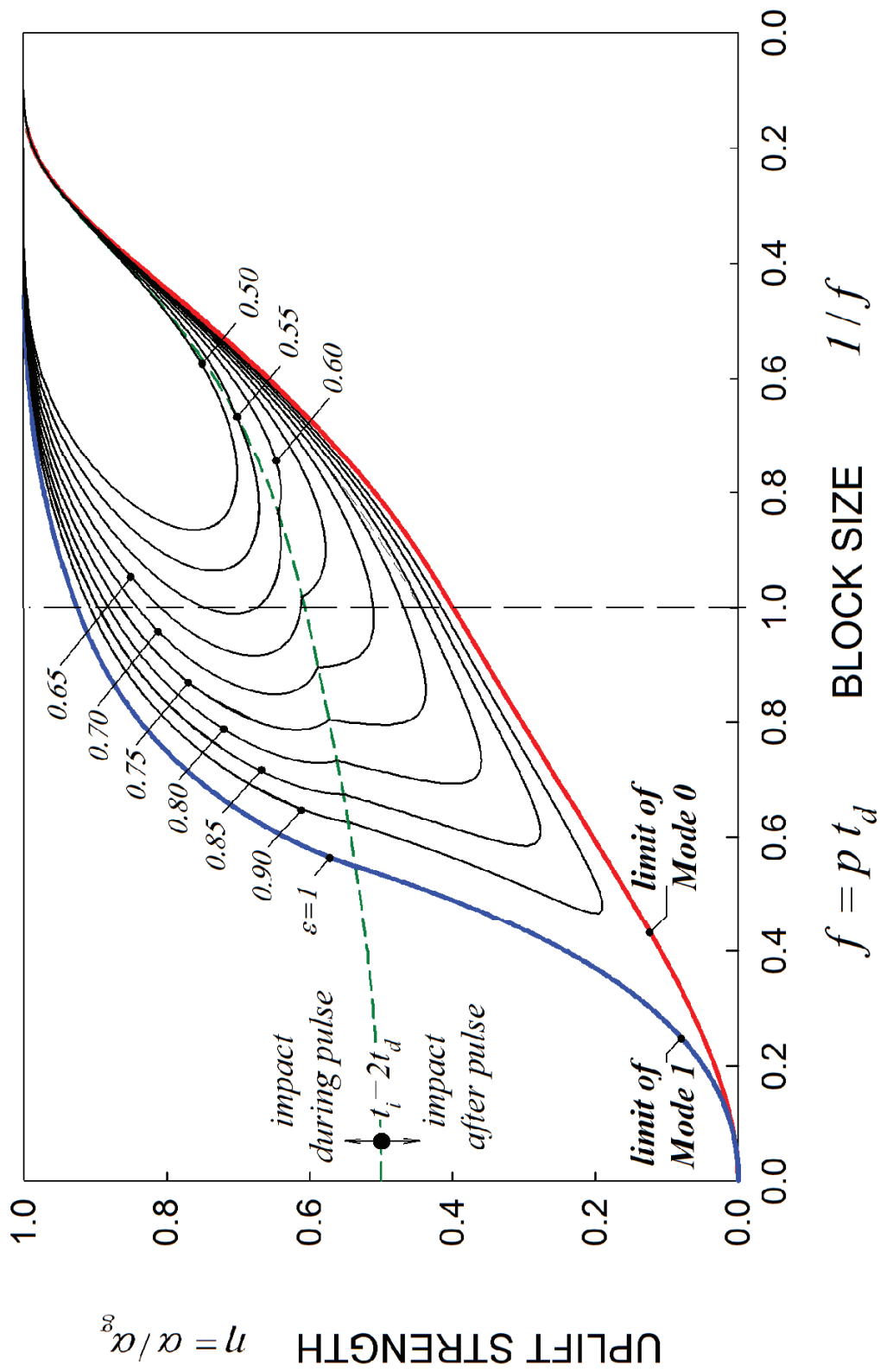


Figure 5.13 Areas of overturning with impact (Mode 1) for a rocking block under a symmetric rectangular pulse of full cycle duration.

Examples of rocking behavior for blocks in each of the above regimes are provided in Fig. 5.14. Regions E correspond to a monotonic decrease (increase in absolute terms) in angular displacement, velocity and acceleration leading to an instability in response ($\theta < -\alpha$) during the second half pulse. Regions S exhibit a change in sign of velocity after the end of the pulse and, therefore, to a second impact leading to stability. Overall, what distinguishes the two regions is the presence of a single immobility point after impact for systems in S , as opposed to none for systems in E .

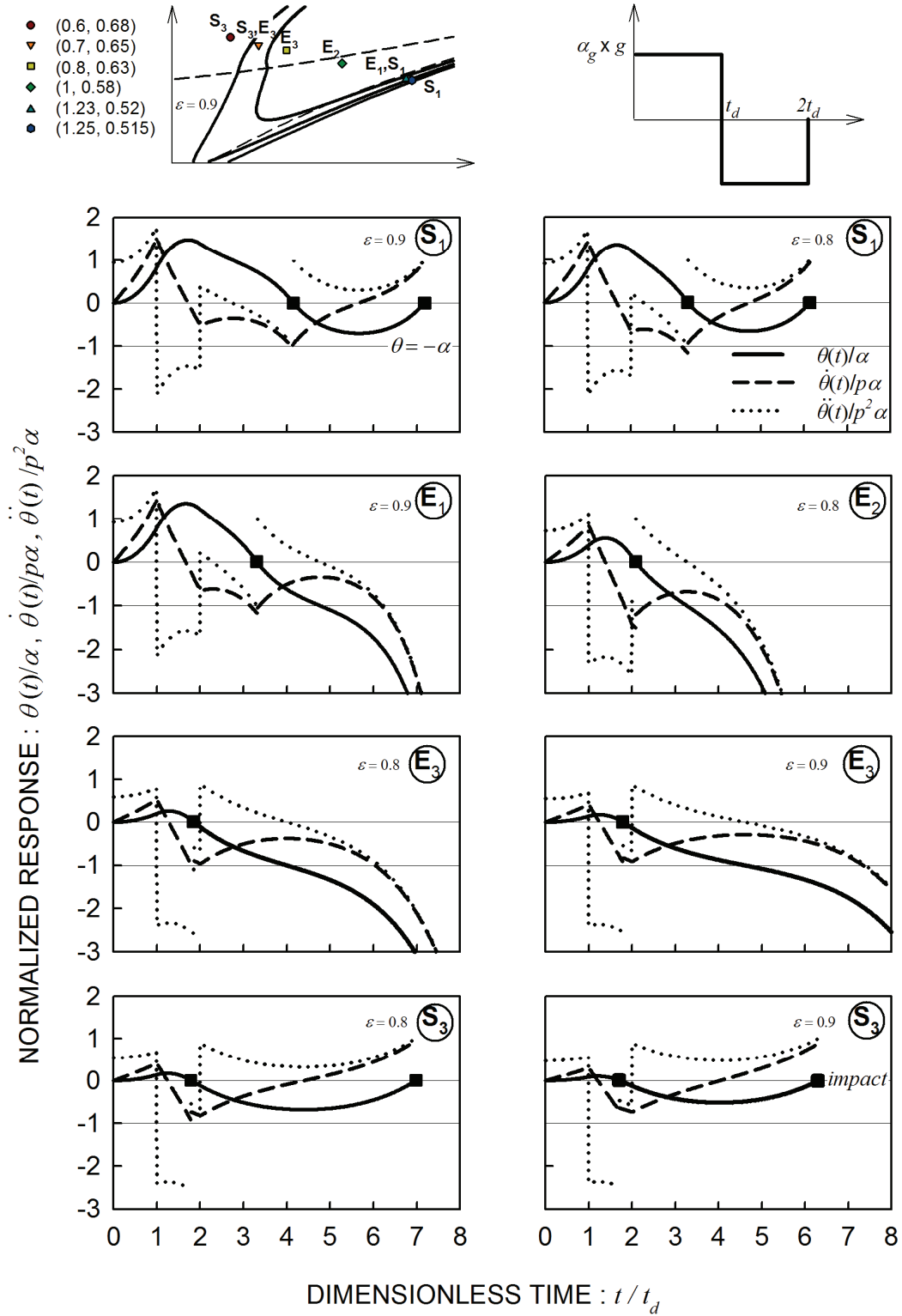


Figure 5.14 Examples of response for different regimes of a rocking block under a symmetric rectangular pulse of full-cycle duration (systems overturning or surviving *Mode 1*). Times of impact, denoted by a square, correspond to singularities in block acceleration and discontinuities in velocity.

5.5 Peak Rocking Response

In this section analytical solutions for peak rocking response and corresponding times are discussed. Whereas for a half-cycle pulse this would take place exclusively under positive rocking angles, peak response may now occur under either positive or negative angles depending on the circumstances.

5.5.1 Peak response for positive rocking angles ($t_{m0} \leq t_i$)

As already discussed in the foregoing, all systems surviving *Mode 0* attain an immobility point during forward rocking (second pulse lobe). These stationary points define the time of peak response during forward rocking (t_{m0}) i.e.,

$$\dot{\theta}(t_{m0}) = 0, \quad t_d \leq t_{m0} \leq 2t_d \quad \text{and} \quad \theta \geq 0 \quad (5.53)$$

Employing Eq. (5.16), Eq. (5.53) yields the following expression for dimensionless time of peak response during forward rocking

$$\tau_{m0} = \frac{1}{2f} \ln \left[\frac{1 - 2e^f - \eta}{1 - 2e^{-f} - \eta} \right] \quad (5.54)$$

If the limit condition for *Mode 0* in Eq. (5.30c) is enforced into Eq. (5.54), the time of peak response for systems on the *Limit of Mode 0* is

$$(\tau_{m0})_{cr} = \frac{1}{2f} \ln \left[1 - 2e^f + 2e^{3f} \right] \quad (5.55)$$

which is bounded between 2 and 1.5, as $\lim_{f \rightarrow 0} (\tau_{m0})_{cr} = 2$ and $\lim_{f \rightarrow +\infty} (\tau_{m0})_{cr} = 3/2$. This

implies that systems attaining local maxima before $3/2t_d$ survive *Mode 0*, while systems reaching local maxima after $2t_d$ overturn, as they exhibit one additional stationary point

and their velocity switches back to positive. The second observation is evident by the study of function in Section 5.4.

Figure 5.15 plots the times of peak response as function of pulse duration f for different values of uplift strength η . Equation (5.55) is shown by a dashed line in this graph.

Using the solutions for τ_{m0} it is straightforward to evaluate the corresponding rocking angles by setting $\tau = \tau_{m0}$ in Eq. (5.17) and normalizing with respect to α

$$\frac{\theta_{max0}}{\alpha} = \left(1 + \frac{1}{\eta}\right) - \frac{1}{\eta} \sqrt{(1 - 2e^f - \eta)(1 - 2e^{-f} - \eta)} \quad (5.56)$$

If the limit of *Mode 0* (Eq. 5.30c) is enforced into Eq. (5.56) yields the peak response for systems in the limit of *Mode 0*

$$\left(\frac{\theta_{max0}}{\alpha}\right)_{cr} = 1 + \frac{e^{2f} - \sqrt{1 - 2e^f + 2e^{3f}}}{(e^f - 1)^2} \quad (5.57)$$

which for $f \rightarrow 0$ tends to 1 and for $f \rightarrow +\infty$ to 2, implying that all systems rocking to angles smaller or equal to α survive *Mode 0*, while all systems rocking to angles beyond 2α overturn with *Mode 0*. Figure 5.16a plots peak response as function of pulse duration f for different values of uplift strength η . Equation (5.57) is shown by the dashed line. Figure 5.16b illustrates an alternative normalization by dividing Eq. (5.56) with (5.57) and dimensionless pulse duration f with Eq. (5.30d). A double asymptotic matching at both small and large rocking angles is thus achieved.

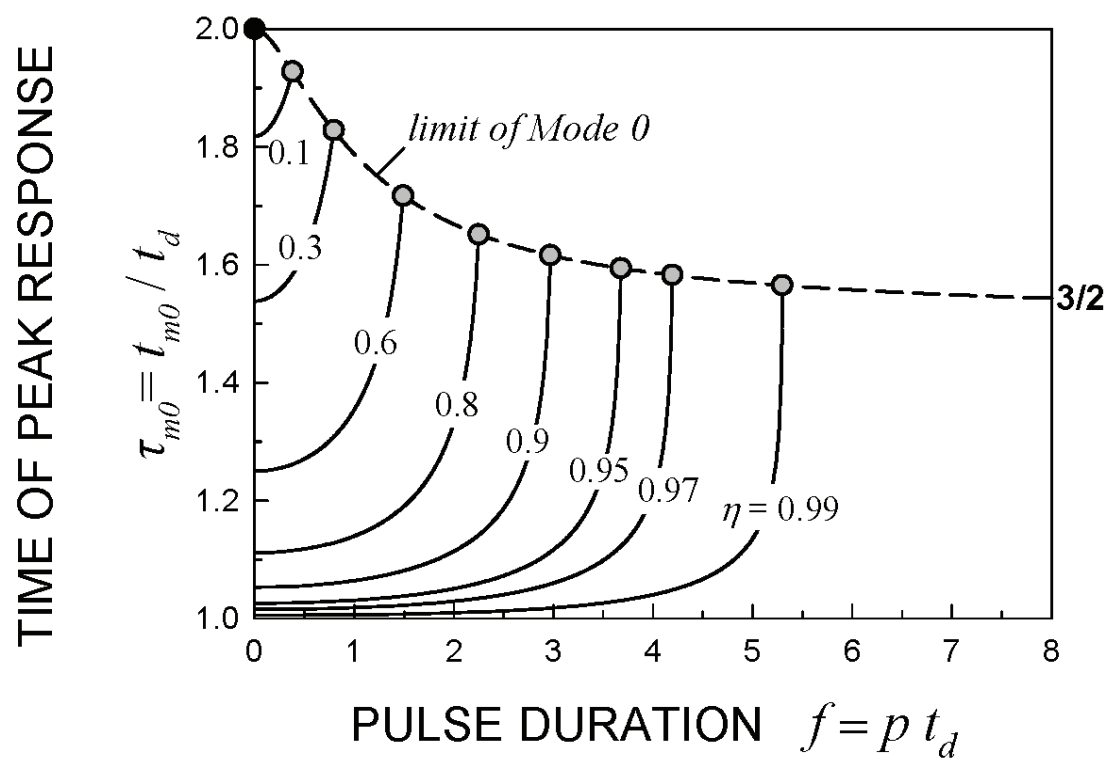


Figure 5.15 Time of peak *forward* ($\theta > 0$) rocking response as function of uplift strength and pulse duration (Region S) due to a symmetric full-cycle rectangular pulse.

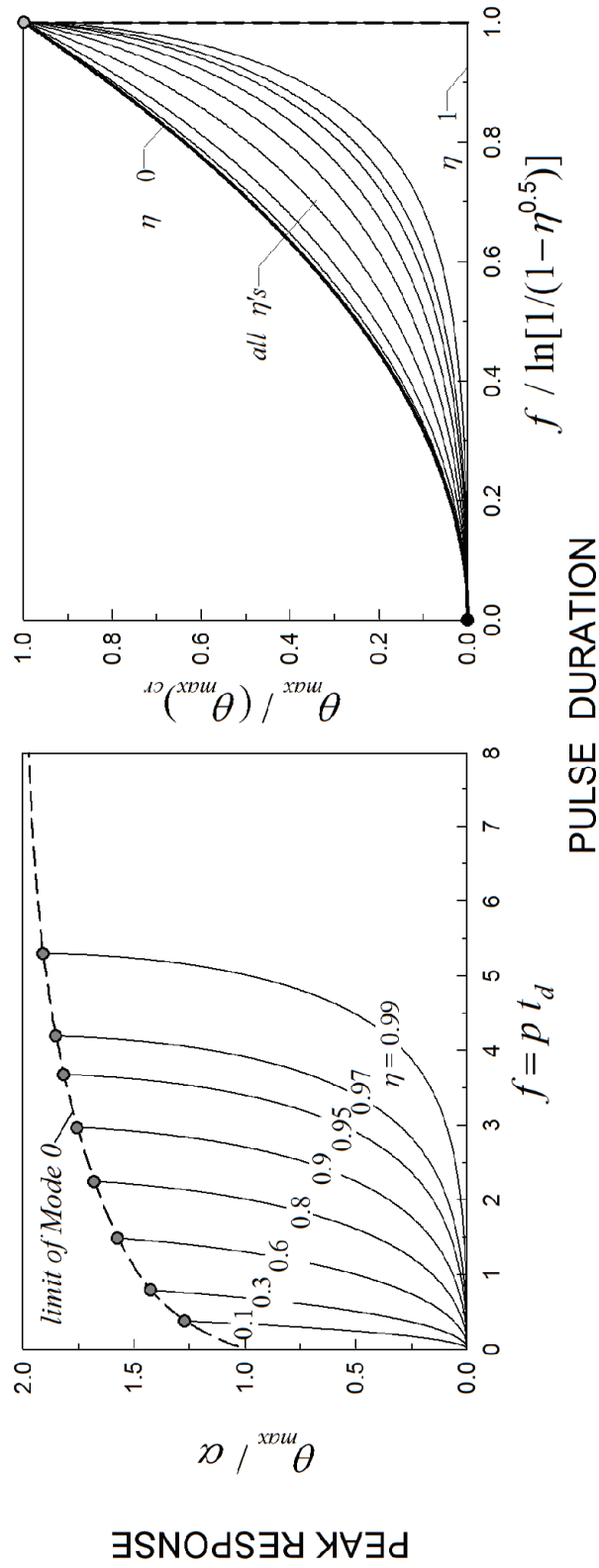


Figure 5.16 Peak rocking response in forward mode ($\theta > 0$) as function of uplift strength and pulse duration (Region S) for a full-cycle symmetric rectangular pulse. Note the double asymptotic matching of the results for small and large angles in the right graph.

5.5.2 Peak response for negative rocking angles ($t_{m1} \geq t_i$)

As already demonstrated all systems surviving *Mode 1* attain a single immobility point and experience a motion reversal after the end of the pulse. This can be expressed as

$$\dot{\theta}(t_{m1}) = 0, \quad t_{m1} \geq 2t_d \quad \text{and} \quad \theta \leq 0 \quad (5.58)$$

To evaluate t_{m1} using the rocking equations for negative angles we consider two distinct cases:

(a) Systems for which impact occurs during pulse (S_3)

(b) Systems for which impact occurs after pulse (S_1, S_2)

For systems in (a), $t_d \leq t_i \leq 2t_d$; employing Eqs (5.36a) and (5.44a), Eq. (5.58) yields the following expression for time of peak response during backward rocking

$$\tau_{m1a} = \frac{1}{2f} \ln \left[\frac{2e^f [e^{2f} - (1-\eta)e^{f\tau_i}] + \varepsilon [e^{2f\tau_i} [2 - (1-\eta)e^f] - e^f (2e^f - 1 + \eta)]}{2e^f [e^{-2f} - (1-\eta)e^{-f\tau_i}] - \varepsilon e^{-2f\tau_i} [e^{2f\tau_i} [2 - e^f (1-\eta)] - e^f (2e^f - 1 + \eta)]} \right] \quad (5.59a)$$

where τ_i is given by Eq. (5.35a).

For systems in (b) $t_i \geq 2t_d$; employing Eqs (5.36b) and (5.46a), Eq. (5.58) yields the following expression for the dimensionless time of peak response during backward rocking

$$\tau_{m1b} = \tau_i + \frac{1}{2f} \ln \left[\frac{-2\eta e^{f(\tau_i+2)} + \varepsilon [-e^{4f} + 2e^{3f} - e^{2f}(1-\eta) + e^{2f\tau_i} + e^{2f(\tau_i+1)}(1-\eta) - 2e^{f(2\tau_i+1)}]}{-2\eta e^{f(\tau_i+2)} - \varepsilon [-e^{4f} + 2e^{3f} - e^{2f}(1-\eta) + e^{2f\tau_i} + e^{2f(\tau_i+1)}(1-\eta) - 2e^{f(2\tau_i+1)}]} \right] \quad (5.59b)$$

where τ_i is given from Eq. (5.35b).

For $\tau_f=2$ the above equations yield the common result

$$\tau_{m1} = 2 + \frac{1}{2f} \ln \left[\frac{e^f + e^{-f} + 2\varepsilon(e^f - e^{-f})}{e^f + e^{-f} - 2\varepsilon(e^f - e^{-f})} \right] \quad (5.59c)$$

Contrary to forward rocking for which time of peak response is always bounded by the end of the pulse, time of peak response for backward rocking is unbounded and tends to infinity for systems approaching the limit of Mode 1. The above trends are elucidated in Fig. 5.17 where Eq. (5.59b), corresponding to systems in areas S_1 and S_2 (as indicated by the insets), is plotted in the left graph; Eq. (5.59a), corresponding to systems in S_3 , is plotted in the right graph.

Corresponding peak rocking angles are evaluated for systems in (a) from Eq. (5.44b) using Eqs (5.35a), (5.36a) and (5.59a); for systems in (b) peak rocking angles are evaluated from Eq. (5.46b) using Eqs (4.35b), (5.36b) and (5.59b). In accordance with Figure 5.17, Figure 5.18 demonstrates peak response under negative rocking angles.

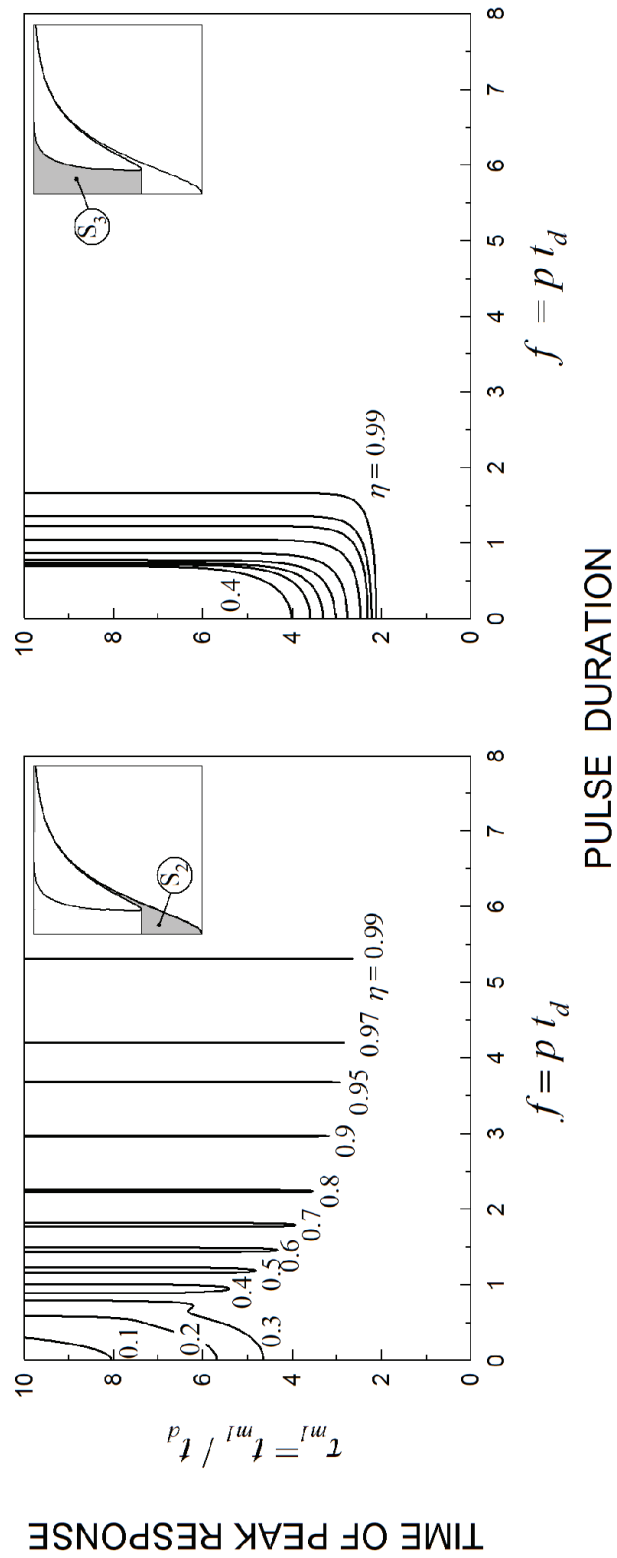


Figure 5.17 Time of peak rocking in the *backward* mode ($\theta < 0$) as function of uplift strength and pulse duration (Regions S) due to a symmetric rectangular pulse of full-cycle duration.

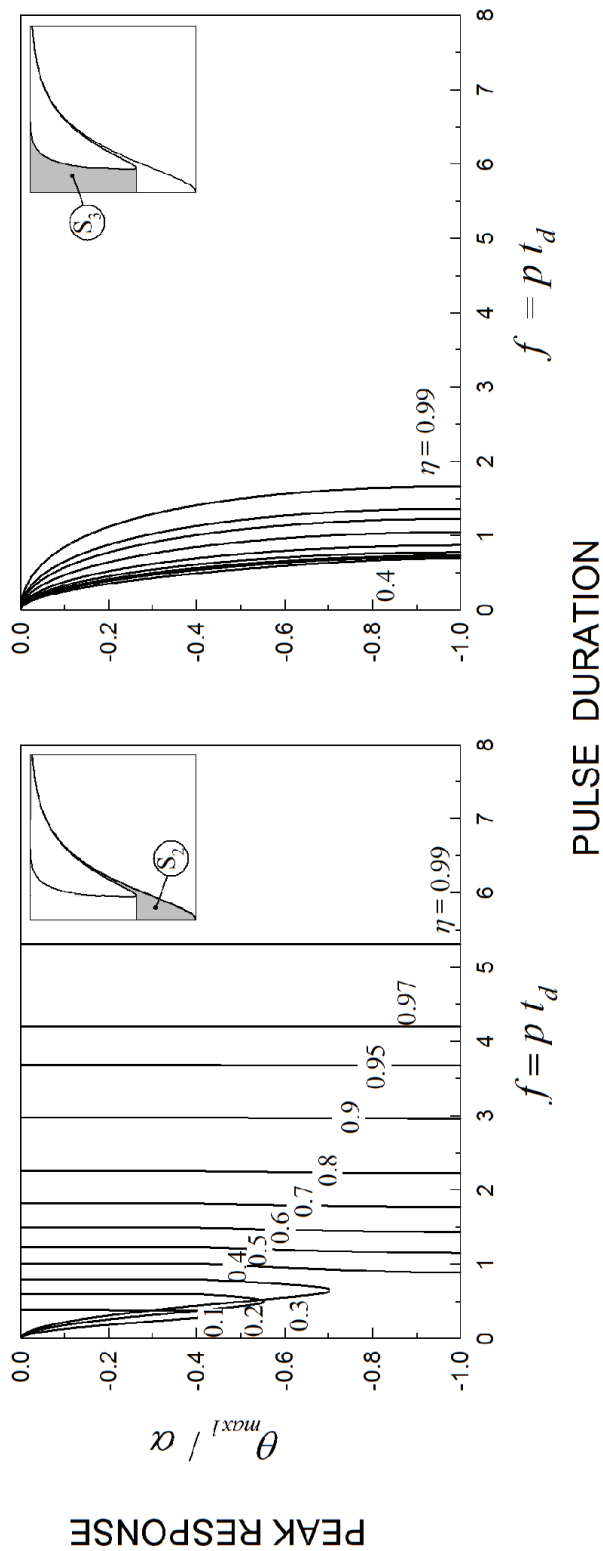


Figure 5.18 Peak rocking response in the *backward* mode ($\theta < 0$) as function of uplift strength and pulse duration (Regions S) due to a full-cycle symmetric rectangular pulse ($\varepsilon = 0.8$).

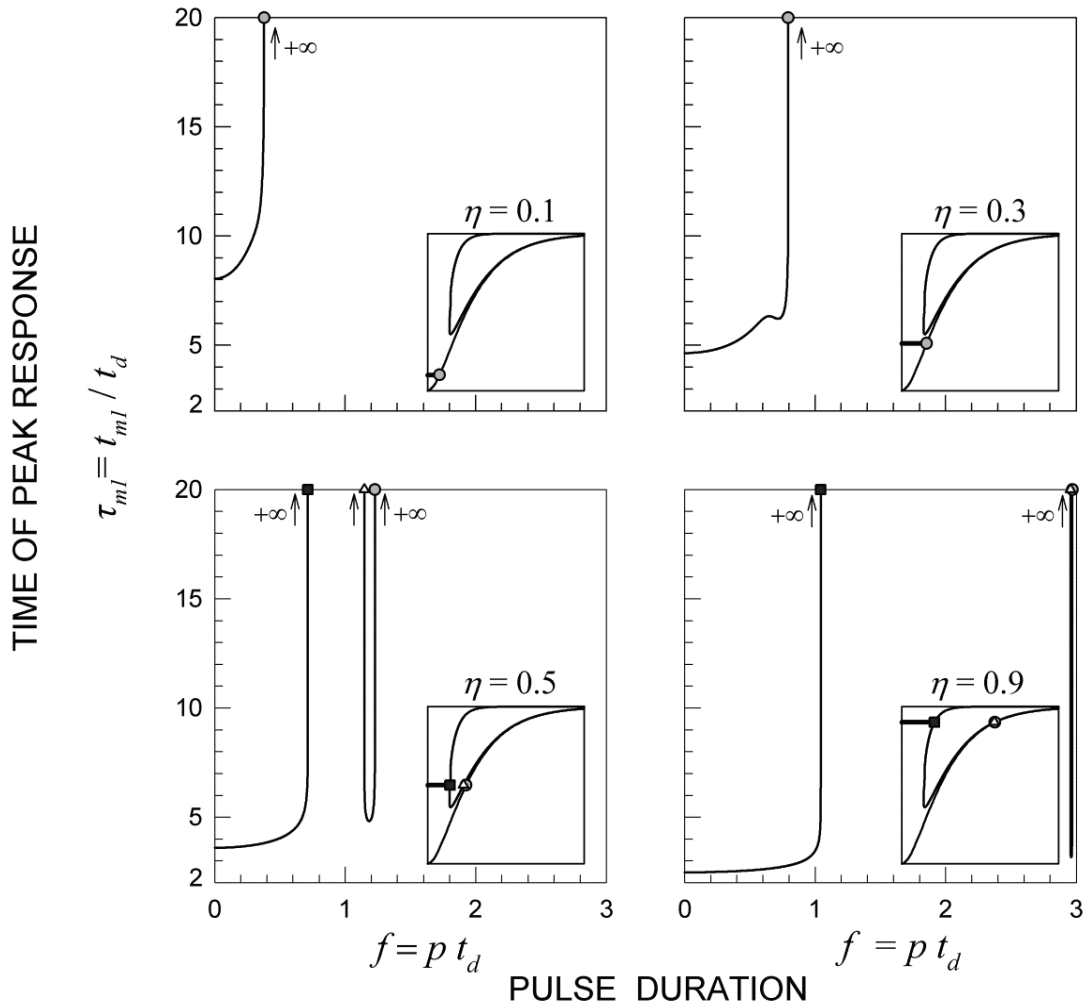


Figure 5.19 Time of peak rocking response in the *backward* mode ($\theta < 0$) as function of pulse duration (Regions S) due to a full-cycle symmetric rectangular pulse for various uplift strengths. Dots indicate limit stability points referring to the Limits of Modes 0 and 1 as shown in the insets; $\varepsilon = 0.8$

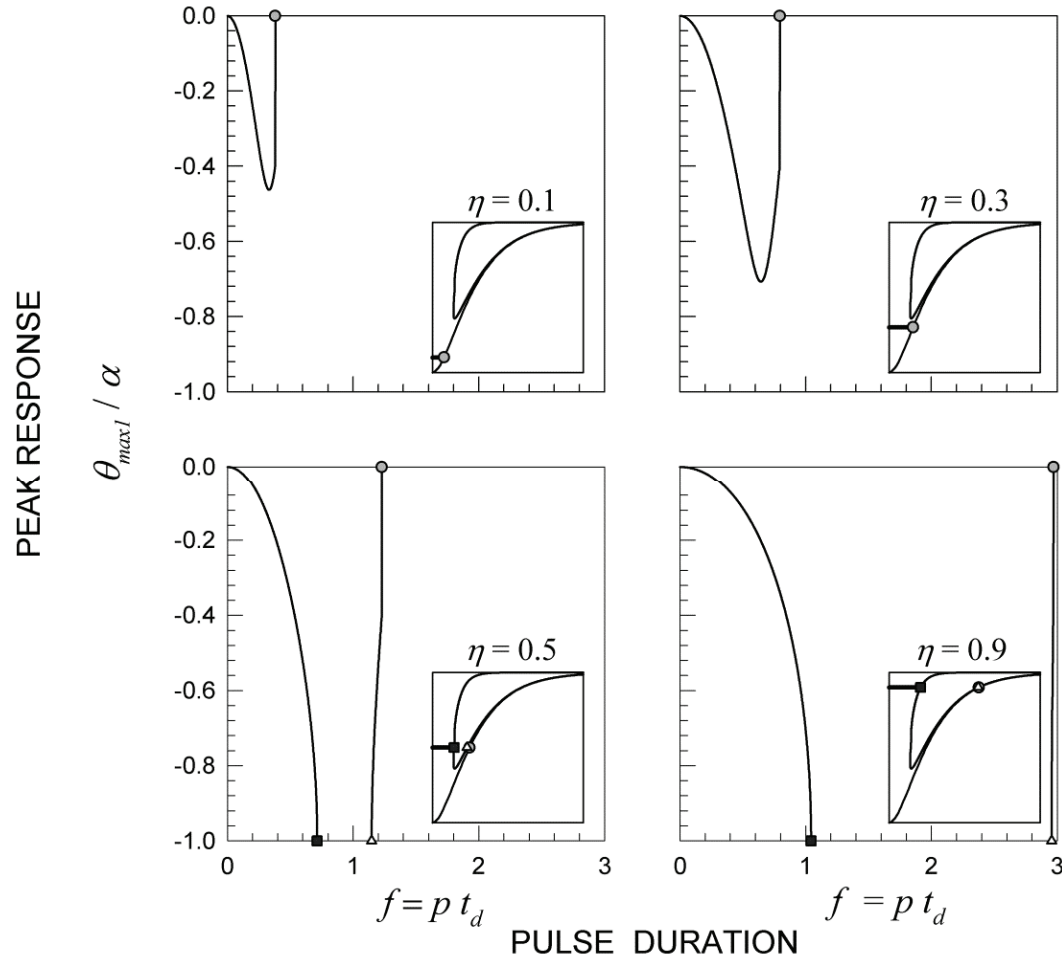


Figure 5.20 Peak rocking response in the *backward* mode ($\theta < 0$) as function of pulse duration (Regions S) due to a full-cycle symmetric rectangular pulse for various uplift strengths. Dots indicate limit stability points referring to the Limits of Modes 0 and 1 as shown in the insets; $\varepsilon = 0.8$

An alternative representation of the above results is provided in Figs 5.19 and 5.20, where time of peak response during backward rocking and corresponding angles are plotted for different uplift strengths.

Summarizing, peak response may occur either before or after impact. A comparison of the two cases is depicted in Figure 5.21, where maximum rocking response is presented in terms of forward, backward and overall (absolute) maximum, that is maximum regardless of rocking path and sign. It turns out that peak rocking response for the pulse at hand mostly occurs during forward motion. For short pulse durations ($f < 2$) and values of uplift strength higher than 0.3 the trend reverses and peak rocking response is met during backward rocking.

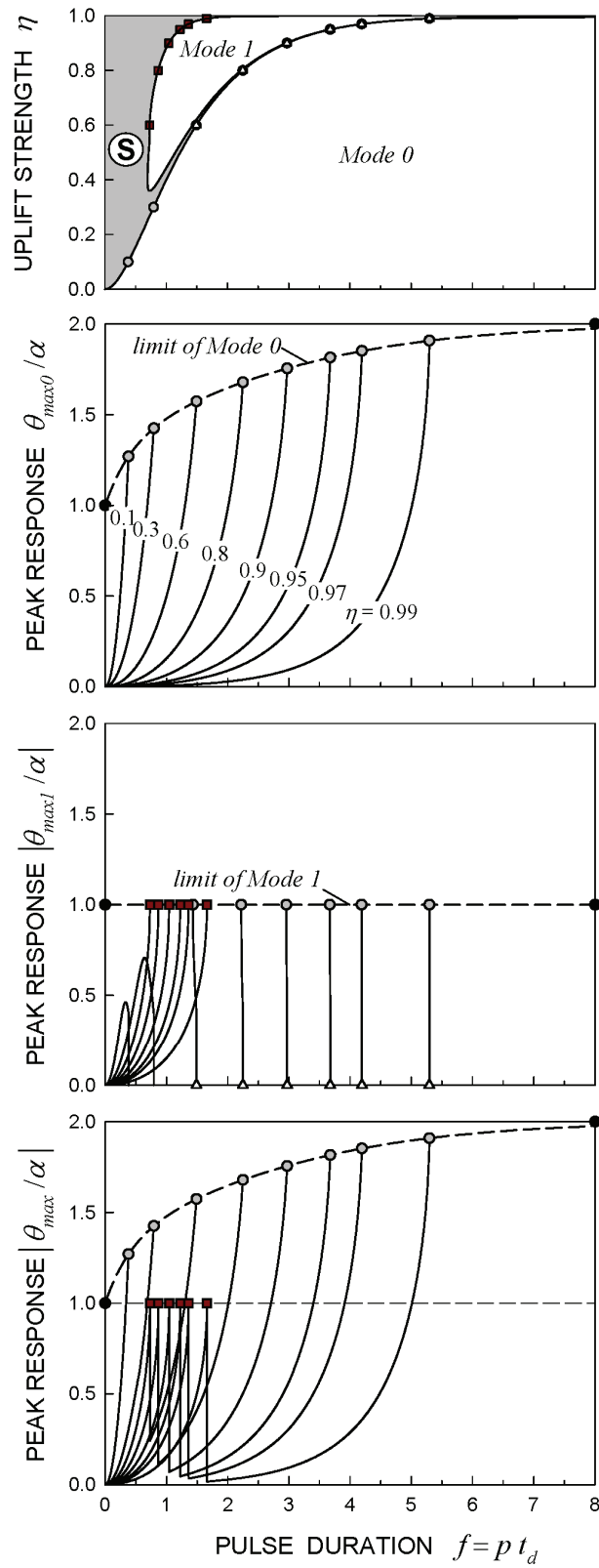


Figure 5.21 Comparison of peak response during forward and backward rocking; combination of the two *Modes* for the overall (absolute) value.

5.6 Response to other pulse shapes

Rocking response to symmetric rectangular pulses of full-cycle duration has been investigated numerically to account for: (a) pulse-shapes different from rectangular; (b) geometric non-linearities of the equation of motion.

To this end, a simple code in Mathematica platform was developed, capable of solving the nonlinear equations of rocking motion in a continuous manner for any pulse waveform. Following the elegant solution by Prieto *et al.* (2004), a compact formulation was derived to incorporate nonlinearities stemming from: (1) the transition from one rotation pole to the other; (2) the energy dissipation during impact and the associated discontinuity in angular velocity; and (3) the geometric nonlinearities expressed by the trigonometric terms. The compact equation in dimensionless form is provided below. A derivation about handling of impulsive loads is provided in Appendix D.3.

$$\ddot{x} + \frac{f^2}{\alpha} \operatorname{sgn}(x) \sin[\alpha(1 - x \operatorname{sgn}(x))] = \frac{f^2}{\eta} \cos[\alpha(1 - x \operatorname{sgn}(x))] \ddot{\psi}_g(\tau) + \ln(\varepsilon) \dot{x}^2 \delta(x) \operatorname{sgn}(\dot{x}) \quad (5.60)$$

where $x = \theta/\alpha$ the normalized rocking angle and $\ddot{\psi}_g(\tau) = \ddot{u}_g/(a_g \times g)$ the dimensionless ground excitation, bounded between +1 and -1.

In the special case of a slender block, Eq. (5.60) simplifies to

$$\ddot{x} - f^2 x + f^2 \operatorname{sgn}(x) = f^2 \frac{1}{\eta} \ddot{\psi}_g(\tau) + \ln(\varepsilon) \dot{x}^2 \delta(x) \operatorname{sgn}(\dot{x}) \quad (5.61)$$

It should be noticed that the presence of the impulsive term in Eq. (5.61) obliterates the similarity property (linear dependence) between θ on α identified on Chapter 4. This is

hardly surprising however as it has already been shown that the restitution coefficient ε is a nonlinear function of α .

Starting from the linearized equation (5.61) and considering a small slenderness angle $\alpha = 0.05\text{rad}$ and a coefficient of restitution $\varepsilon = 0.8$, the limits of *Mode 0* and *Mode 1* were identified numerically for four idealized full-cycle pulse shapes: (1) rectangular; (2) sinusoidal; (3) triangular; and (4) exponential, as shown Figure 5.22. These pulses are the same as those studied in Chapters 2 and 3. Results are presented in Figure 5.22 for two cases: (a) pulses of the above-mentioned shapes having equal acceleration amplitudes A_g and duration t_d ; and (b) same pulse shapes having equal velocity amplitude V_g and duration t_d . In the first case (Figure 5.22a), the amplitude of ground velocity varies among different pulses, attaining its maximum for the rectangular pulse and its minimum for the exponential. In the second case (Figure 5.22b), the amplitude of ground acceleration is varying, attaining its maximum value for the exponential pulse and its minimum for the rectangular. To account for this, the vertical axis of the plot in Figure 10b shows the normalized parameter $\eta^* = \alpha/\alpha_{g,rect}$, where $\alpha_{g,rect}$ is the amplitude of the acceleration of the rectangular pulse. Note that η^* does not denote actual uplift strength anymore (except for the rectangular pulse) and that $\eta^* \geq \eta$, thus η^* can attain values larger than unity.

In Figure 5.22 the numerical results for the rectangular pulse are in excellent agreement (not shown) with the analytical solution. The trend observed in Figure 5.22a, showing that the smaller the velocity content of the excitation, the smaller the areas of overturning and vice versa, is anticipated. Interesting results can be drawn from Figure 5.22b: *First* the rectangular pulse provides a lower bound for the overturning of a block, meaning that if the block overturns under a rectangular pulse, it will also overturn under any other pulse

of same peak velocity and duration; *Second* extrapolating the trend shown by these results to the extreme case of a perfect impulse of magnitude V_g/t_d (actually two opposite impulses, one applied at $t = t_d/2$ and the other at $t = 3t_d/2$) it can be concluded that the impulse excitation provides, in general, the upper bound for the stability of a block, meaning that if the block is stable under an impulse excitation, it will also be stable under any other pulse of peak velocity V_g and duration $2t_d$. This conclusion implies that for a given velocity and duration, the more spiky the pulse, the higher the overturning potential of the block. The opposite trend holds for a given acceleration and duration, for which the more spiky the pulse, the lower the overturning potential of the block.

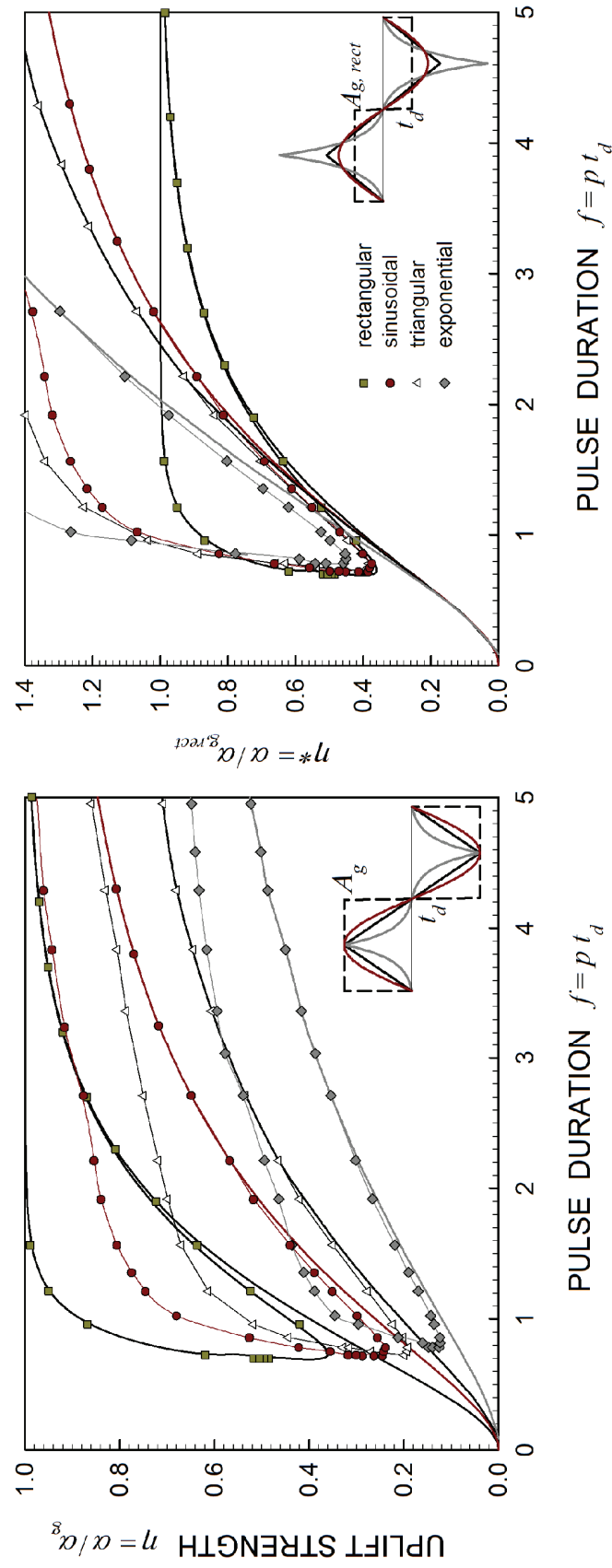


Figure 5.22 Limits of areas of overturning without impact (*Mode 0* – solid lines) and with one impact (*Mode 1* – bulleted lines) for symmetric full – cycle pulses of rectangular, sinusoidal, triangular and exponential shape; $\varepsilon = 0.8$

5.7 Asymmetric Rectangular Pulse

An extension of the waveform studied in the foregoing is that involving two asymmetric rectangular lobes of equal area. One of the challenges involved is the definition of asymmetry, as one could choose among different options. To this end, it is useful to reiterate that the pulse can be described by four principal interrelated variables: (a) peak pulse acceleration A , (b) peak pulse velocity V , (c) peak pulse displacement D , and (d) pulse period T . For a symmetric pulse: $A=A_g$, $V=A_g t_d$, $D=A_g t_d^2$ and $T=2t_d$. Different asymmetric pulses can be formed by varying the above parameters as shown below.

- Asymmetry Type 1

Maintaining peak pulse velocity, displacement and period, and allowing peak acceleration to vary, leads to a pulse with properties: $A=A_g/z$, $V=A_g t_d$, $D=A_g t_d^2$ and $T=2t_d$, z being an asymmetry parameter standing for the duration of the first lobe and taking values between 0 and 1 for pulses with peak acceleration during first lobe and larger than 1 for peak acceleration during second lobe (Figure 5.23a). The case $z=1$ refers to a symmetric pulse.

- Asymmetry Type 2

Maintaining peak pulse acceleration and period, and allowing peak velocity and displacement to vary, a pulse can be formed with properties: $A=A_g$, $V=zA_g t_d$, $D=zA_g t_d^2$ and $T=2t_d$. Asymmetry parameter z takes again values from 0 to 1 if the duration of the first lobe is smaller than that of the second lobe and vice versa (Figure 5.23b). Likewise, $z=1$ refers to a symmetric pulse.

- Asymmetry Type 3

Maintaining peak pulse acceleration and velocity, and allowing peak displacement and period to vary, a pulse is formed with properties: $A=A_g$, $V=A_g t_d$, $D=z A_g t_d^2/2$ and $T=z t_d$. z in this case takes values from 1 to 2 (Figure 5.23c) with $z=1$ corresponding to a single lobe pulse and $z=2$ to a symmetric full-cycle pulse.

- Asymmetry Type 4

The last type of asymmetry considered in this investigation involves constant forward peak pulse acceleration and displacement, and allows peak velocity and period to vary i.e., $A=A_g$, $V=z A_g t_d$, $D=A_g t_d^2$ and $T=2t_d/z$, where now z ranges from 0 to 1 (Figure 5.23d) with $z=1$ referring to the symmetric case.

Rocking response can be evaluated analytically for each of the above pulses. The results are summarized in Tables 5.1 – 5.3 and Figures 5.24 – 5.26.

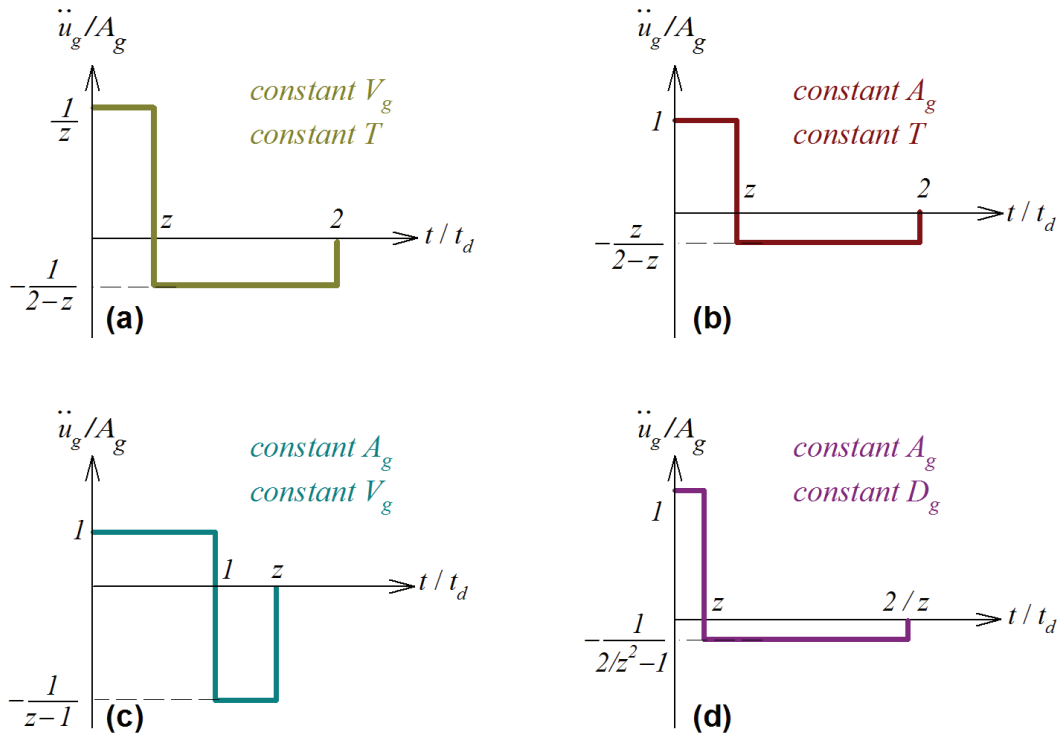


Figure 5.23 Types of pulse asymmetry considered in this investigation: (a) Type 1; (b) Type 2; (c) Type (3) and (d) Type 4.

Table 5.1 Closed-form solutions for the rocking response to the asymmetric pulses in Figs 5.23a,b

	Type 1	Type 2
<i>Uplift Strength</i>	$\eta = \frac{az}{a_g}$	$\eta = \frac{a}{a_g}$
<i>Limit of Mode 0</i>		$\eta = 1 - \frac{2e^{-\beta} - ze^{-2f}}{2-z}$
<i>Limit of Mode 1</i>	$\frac{z}{2-z} - \varepsilon(1-e^{\beta})\sqrt{\frac{2e^{-\beta}(1-\eta)}{2-z}} - \eta - \frac{ze^{-2f}\left[-(1-e^{\beta})\sqrt{2e^{-\beta}(2-z)(1-\eta)} + z + \eta(2-z)\right]}{(2-z)[-2e^{-\beta} + (2-z)(1-\eta)]} = 0$	
$\tau_i \geq 2$	(solved numerically due to complexity)	
<i>Time of Impact</i>	$\tau_i = \frac{1}{f} \ln \left[\frac{(1-e^{\beta})\sqrt{2e^{-\beta}(2-z)(1-\eta)} - z - (2-z)\eta}{-2e^{-\beta} + (2-z)(1-\eta)} \right]$	
$\tau_i \geq 2$	$\tau_i = \frac{1}{f} \ln \left[\frac{\eta(2-z) - \sqrt{\eta^2(2-z)^2 + \{-2e^{-\beta} + ze^{-2f} + (2-z)(1-\eta)\}\{2e^{\beta} - ze^{2f} - (2-z)(1-\eta)\}}}{-2e^{-\beta} + ze^{-2f} + (2-z)(1-\eta)} \right]$	
	$\dot{\theta}(\tau_i^-)/(a_g f/z)$	$\dot{\theta}(\tau_i^-)/(a_g f)$
<i>Velocity at the Time of Impact</i>		$\frac{(1-e^{\beta})\sqrt{2e^{-\beta}(2-z)(1-\eta)}}{2-z}$
$\tau_i \geq 2$	$\frac{z \sinh[f(\tau_i - 2)] + (2-z)(1-\eta) \sinh f\tau_i - 2 \sinh[f(\tau_i - z)]}{2-z}$	

Table 5.2 Closed-form solutions for the rocking response due to the asymmetric pulse of Fig 5.23c

<i>Type 3</i>		
<i>Uplift Strength</i>		$\eta = \frac{a}{a_g}$
<i>Limit of Mode 0</i>		$\eta = 1 - \frac{ze^{-f} - e^{-zf}}{z-1}$
<i>Limit of Mode 1</i>	$1 \leq \tau_i \leq z$	$\eta - \frac{1 - e^{f(\tau_i - z)}}{z-1} + \varepsilon(1 - e^f) \sqrt{\frac{ze^{-f}(1-\eta)}{z-1}} = 0$
	$\tau_i \geq z$	(solved numerically due to complexity)
<i>Time of Impact</i>	$1 \leq \tau_i \leq z$	$\tau_i = \frac{1}{f} \ln \left[\frac{(1 - e^f) \sqrt{ze^{-f}(z-1)(1-\eta)} - 1 + (1-z)\eta}{-ze^{-f} - (1-z)(1-\eta)} \right]$
	$\tau_i \geq z$	$\tau_i = \frac{1}{f} \ln \left[\frac{-(1-z)\eta + \sqrt{2 \left[-z - (1-z)^2 (1-\eta) - z(1-z)(1-\eta) \cosh f + (1-z)(1-\eta) \cosh fz + z \cosh f \cosh fz - z \sinh f \sinh fz \right]}}{-e^{-fz} + ze^{-f} + (1-z)(1-\eta)} \right]$
<i>Velocity at the Time of Impact</i>	$1 \leq \tau_i \leq z$	$\frac{\dot{\theta}(\tau_i^-)}{a_g f} = -(1 - e^f) \sqrt{\frac{-ze^{-f}(1-\eta)}{1-z}}$
	$\tau_i \geq z$	$\frac{\dot{\theta}(\tau_i^-)}{a_g f} = \frac{-z \sinh[f(1-\tau_i)] + (1-z)(1-\eta) \sinh[f\tau_i] - \sinh[f(\tau_i - z)]}{1-z}$

Table 5.3 Closed-form solutions for the rocking response due to the asymmetric pulse of Figs 5.23d

<i>Type 4</i>	
<i>Uplift Strength</i>	$\eta = \frac{a}{a_g}$
<i>Limit of Mode 0</i>	$\eta = 1 - \frac{2e^{-fz} - z^2 e^{-2f/z}}{2 - z^2}$
<i>Limit of Mode 1</i>	<i>obtained numerically due to complexity</i>
<i>Time of Impact</i>	$z \leq \tau_i \leq 2/z \quad \tau_i = \frac{1}{f} \ln \left[\frac{(1 - e^{fz}) \sqrt{2e^{-fz}(2 - z^2)(1 - \eta)} - 2\eta - z^2(1 - \eta)}{-2e^{-fz} + (2 - z^2)(1 - \eta)} \right]$ $\tau_i \geq 2/z \quad \tau_i = \frac{1}{f} \ln \left[\frac{-\eta(2 - z^2)e^{2f/z + fz} - \sqrt{2e^{2f/z(2 + z^2)} \left[(2 - z^2)^2(\eta - 2z^2) + 2z^2 \cosh[f(2/z - z)] + (2 - z^2)(1 - \eta)(2 \cosh fz - z^2 \cosh \frac{2f}{z}) \right]}}{-2e^{f/z} + e^{fz}(z^2 + e^{2f/z}(2 - z^2)(1 - \eta))} \right]$
<i>Velocity at the Time of Impact</i>	$z \leq \tau_i \leq 2/z \quad \frac{\dot{\theta}(\tau_i^-)}{a_g f} = (1 - e^{fz}) \sqrt{\frac{2e^{-fz}(1 - \eta)}{2 - z^2}}$ $\tau_i \geq 2/z \quad \frac{\dot{\theta}(\tau_i^-)}{a_g f} = \frac{(2 - z^2)(1 - \eta) \sinh f\tau_i + z^2 \sinh[f(\tau_i - 2/z)] - 2 \sinh[f(\tau_i - z)]}{2 - z^2}$

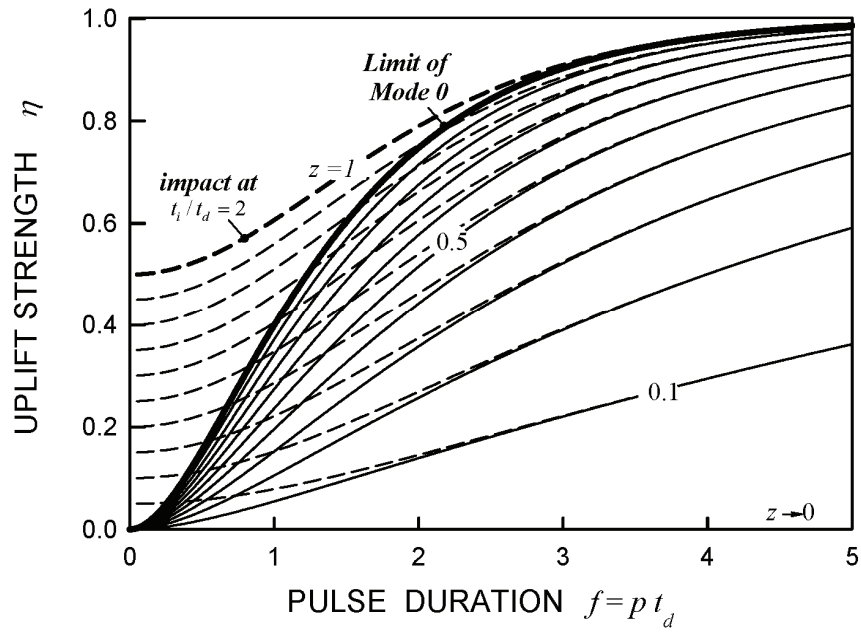


Figure 5.24a Areas of overturning without impact (*Mode 0*) and criteria of impact at the end of the pulse due to Type 1 and Type 2 asymmetric rectangular excitations as function of shape parameter z . ($\varepsilon = 0.8$)

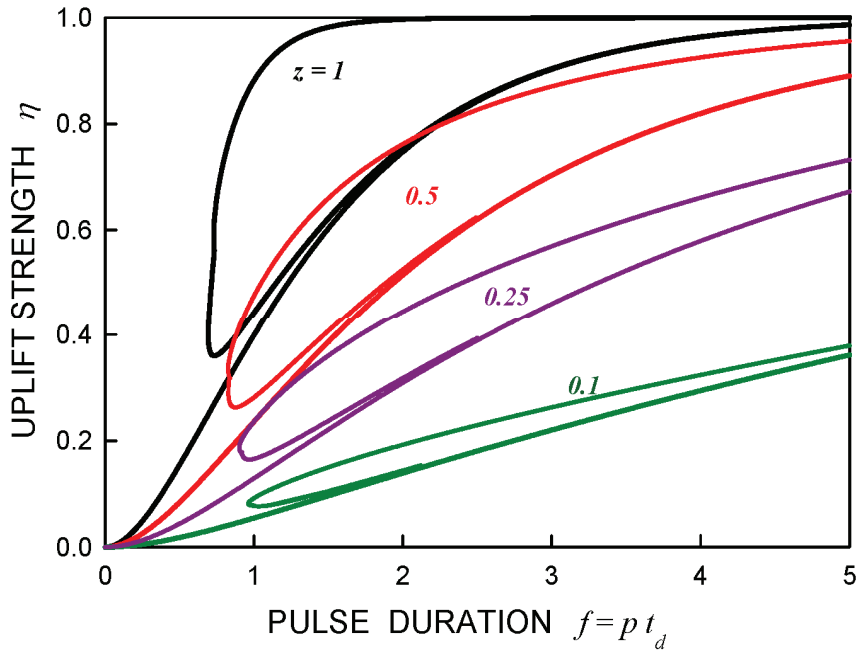


Figure 5.24b Areas of overturning without impact (*Mode 0*) and with one impact (*Mode I*) due to Type 1 and Type 2 asymmetric rectangular pulse excitations as function of shape parameter z . ($\varepsilon = 0.8$)

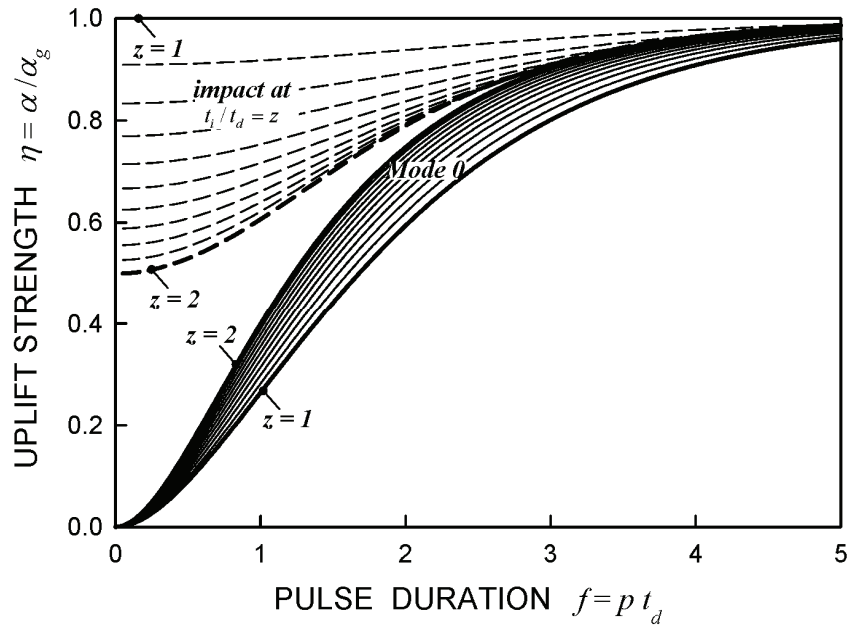


Figure 5.25a Areas of overturning without impact (*Mode 0*) and criteria of impact at the end of the pulse due to a Type 3 asymmetric rectangular excitation as function of shape parameter z . ($\varepsilon = 0.8$)

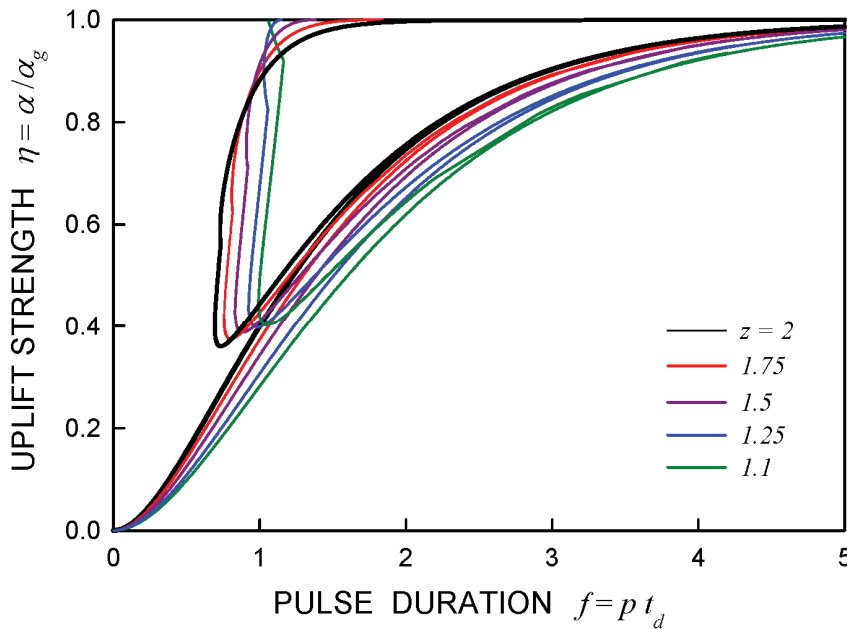


Figure 5.25b Areas of overturning without impact (*Mode 0*) and with one impact (*Mode I*) due to a Type 3 asymmetric rectangular pulse as function of shape parameter z . ($\varepsilon = 0.8$)

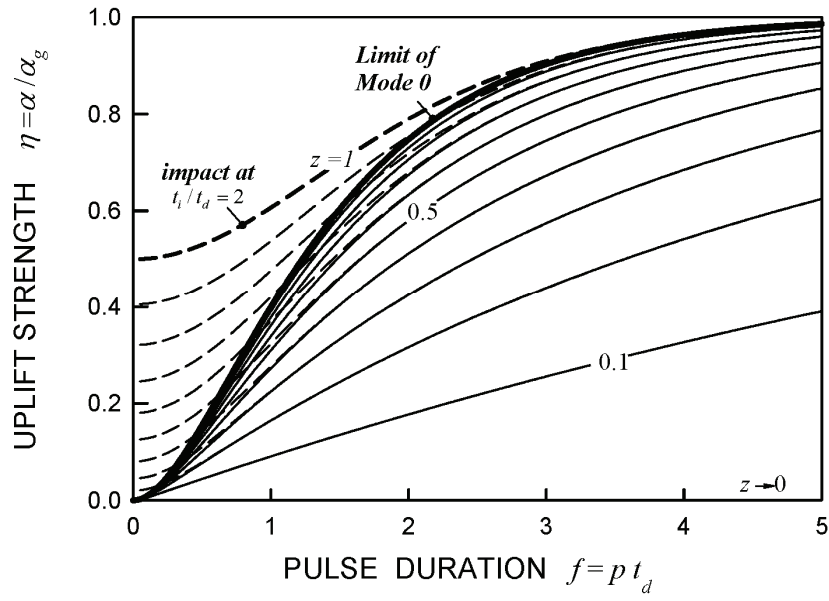


Figure 5.26a Areas of overturning without impact (*Mode 0*) and criteria of impact at the end of the pulse due to a Type 4 asymmetric rectangular excitation as function of shape parameter z . ($\varepsilon = 0.8$)

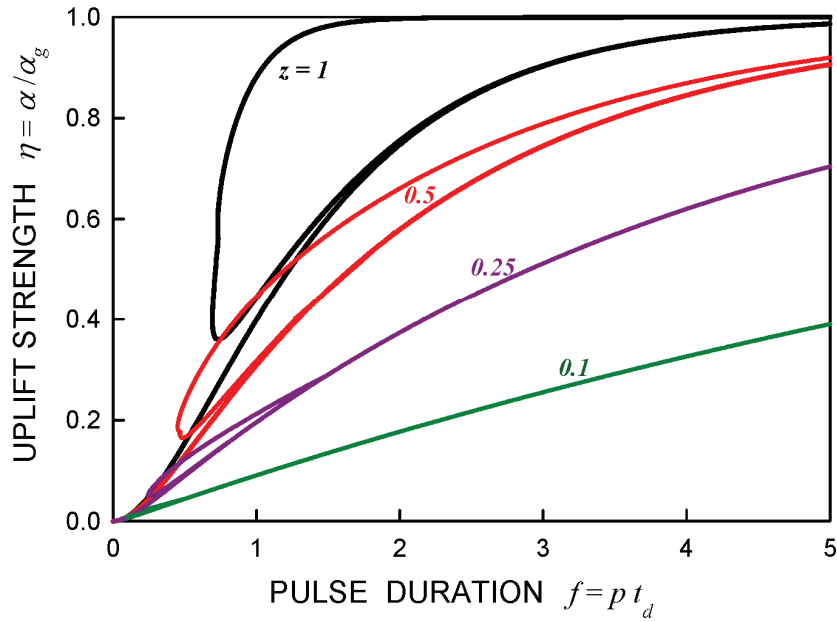


Figure 5.26b Areas of overturning without impact (*Mode 0*) and with one impact (*Mode I*) due to a Type 4 asymmetric rectangular pulse as function of shape parameter z . ($\varepsilon = 0.8$)

5.8 Actual near-fault ground motions

The information provided in the foregoing focused on analytical investigation of rocking response to full-cycle pulses and elucidated the difficulties associated with solving the equation of motion even for the highly idealized waveforms such as the rectangular pulse at hand. Determining rocking response to actual ground motions is a task feasible only by numerical means. Nevertheless, it will be proven useful to examine and compare response to impulsive near-fault ground motions with the proposed analytical solutions. It will be shown that simple analytical solutions for idealized excitations can provide good approximations, mostly at an upper bound sense, of the response to actual earthquakes motions.

With reference to the system at hand, a set of twenty impulsive ground motions affected by forward fault-rupture directivity is employed, as shown in Fig. 5.27 and Table 5.4. The set encompasses different recorded earthquake ground motions with peak accelerations (PGA) varying between 0.22 to 1.29g and peak velocities (PGV) varying between 31 and 137cm/s. Corresponding moment magnitudes (M_w) vary between 6.1 to 7.6. The impulsive excitation periods were taken from Mavroeidis *et al.* (2003) and Baker (2007) (both approaches were explored). The record components and corresponding acceleration amplitudes considered in the analysis are provided on Table 5.4. Four block slenderness ratios ($\alpha = 5, 10, 15$ and 20 degrees) and four block radii ($R=0.5, 2, 5, 10$ m) were considered, covering a wide range of system periods ($0.34 < f < 9.8$) and strengths ($0.1 < \eta < 0.97$) leading to 320 parameter combinations for the whole suite of 20 records. The analysis is performed via the in house computer code BLOCK.

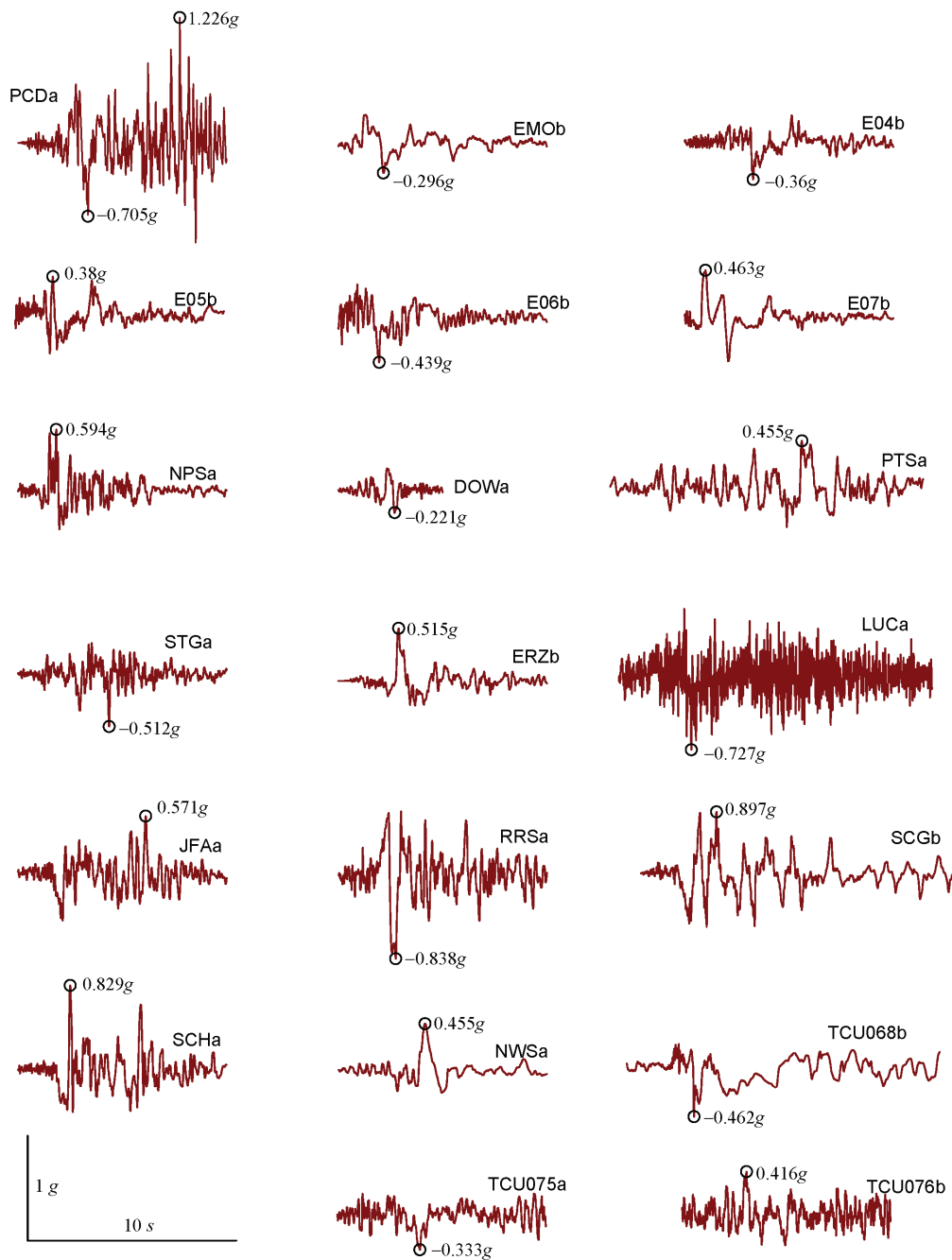


Figure 5.27 Selected near – fault acceleration records containing distinct pulses.

Table 5.4 Data for the near-fault acceleration records with distinct pulses of Fig 5.27.

No.	Event	Year	M_w	Station	Instrument Orientation	PGA (g)	PGV (cm/s)	$T_p^{b, \ddagger}$ (s)	$T_p^{imp, \dagger}$ (s)
1	San Fernando	1971	6.6	PCD: Pacoima Dam	164	1.29	120	1.6	2.6
2	Imperial Valley	1979	6.5	E04: El Centro Array #4	230	0.36	78.3	4.6	2.5
3				E05: El Centro Array #5	230	0.38	91.8	4.0	2.5
4				E06: El Centro Array #6	230	0.439	112	3.8	2.5
5				E07: El Centro Array #7	230	0.463	109	4.2	2.5
6				EMO: EC Meloland Overpass FF	270	0.296	115	3.3	2.5
7	Palm Springs	1986	6.1	NPS: North Palm Springs	210	0.594	73.6	1.4	1.7
8	Whittier Narrows	1987	5.9	DOW: Downey-co maintenance bldg	180	0.221	30.7	0.8	1.5
9	Superstition Hills	1987	6.4	PTS: Parachute Test Site	225	0.455	109	2.3	2.3
10	Loma Prieta	1989	6.9	STG: Saratoga – Aloha Ave.	000	0.512	56.4	4.5	3.6
11	Erzican, Turkey	1992	6.6	ERZ: Erzican	NS	0.515	95.2	2.7	2.8
12	Landers	1992	7.2	LUC: Lucerne	260	0.727	114	5.1	4.8
13	Northridge	1994	6.7	JFA: Jensen Filter Plant	022	0.571	105	3.5	3
14				RRS: Rinaldi Receiving Station	228	0.838	173	1.2	3
15				SCG: Sylmar-Converter Station	142	0.897	134	3.5	3
16				SCII: Sylmar-Converter Station East	018	0.829	122	3.5	3
17				NWS: Newhall-West Pico Canyon Rd.	046	0.455	117	2.4	3
18	Chi-Chi, Taiwan	1999	7.6	TCU068	N	0.462	380	12.2	6.9
19				TCU075	E	0.333	115	5.1	6.9
20				TCU076	N	0.416	88	4	6.9

$\ddagger$ after Baker 2007, $\dagger$ after Mavroeidis, Dong & Papageorgiou 2004

The results are presented in dimensionless form in Figures 5.28. f values were evaluated by employing the pulse durations t_d reported in Baker (2007). Uplift strengths in Figure 5.28a were normalized with PGA , while an alternative normalization with $2PGV/T_p$ is presented in Figure 5.28b. Black bullets in the graphs denote overturning while white bullets denote safety. The following observations can be made from these results: (1) as in the case of idealised excitations, systems plotting in the upper left part of the graph, having low characteristic frequencies and high strengths relative to the earthquake signal, survive rocking; (2) likewise, blocks most vulnerable to overturning are concentrated at the lower right part of the graph, having high characteristic frequencies and low strengths relative to the earthquake signal; (3) on the other hand, there are several low-strength systems that survive rocking, plotting on the lower part of the graph. A possible explanation for this behavior is the high acceleration content in the early parts of the earthquake signal that leads to multiple impacts and development of rocking velocities of arbitrary sign before the arrival of the main earthquake pulses; (4) evidently, the safety wall of the full-cycle rectangular pulse, shown by a solid line, represents a very conservative assumption plotting above all overturning systems; (5) there is no clear meaning or distinction between the aforementioned overturning Modes for multiple cycle – excitations.

A reasonable approximation of a safety wall for near-fault earthquake induced rocking, bounding overturning systems from the top, is provided by the equation

$$\eta_w = 2 - \frac{2}{f} \ln[2e^{\frac{f}{2}} - 1] \quad (5.62)$$

which is shown by a broken line in Figure 5.28a.

Interestingly, this expression describes the safety wall against overturning without an impact for a triangular pulse of half-cycle duration (Eq. 4.22). At both small ($f < 1$) and large ($f > 4$) pulse durations criterion gradually becomes overconservative.

Accordingly, two auxiliary, less conservative, safety criteria can be established in the form

$$f_w \cong 3/4 \quad , \quad \eta_w \cong 3/4 \quad (5.63a,b)$$

since no overturning systems plot to the left or above these lines.

The above criteria provide a limit for systems that overturn. In other words, they can be used to exclude the possibility of overturning for sufficiently “large” or “strong” systems. However the opposite does not hold as there are numerous surviving systems (for the reasons explained above) plotting below these limits.

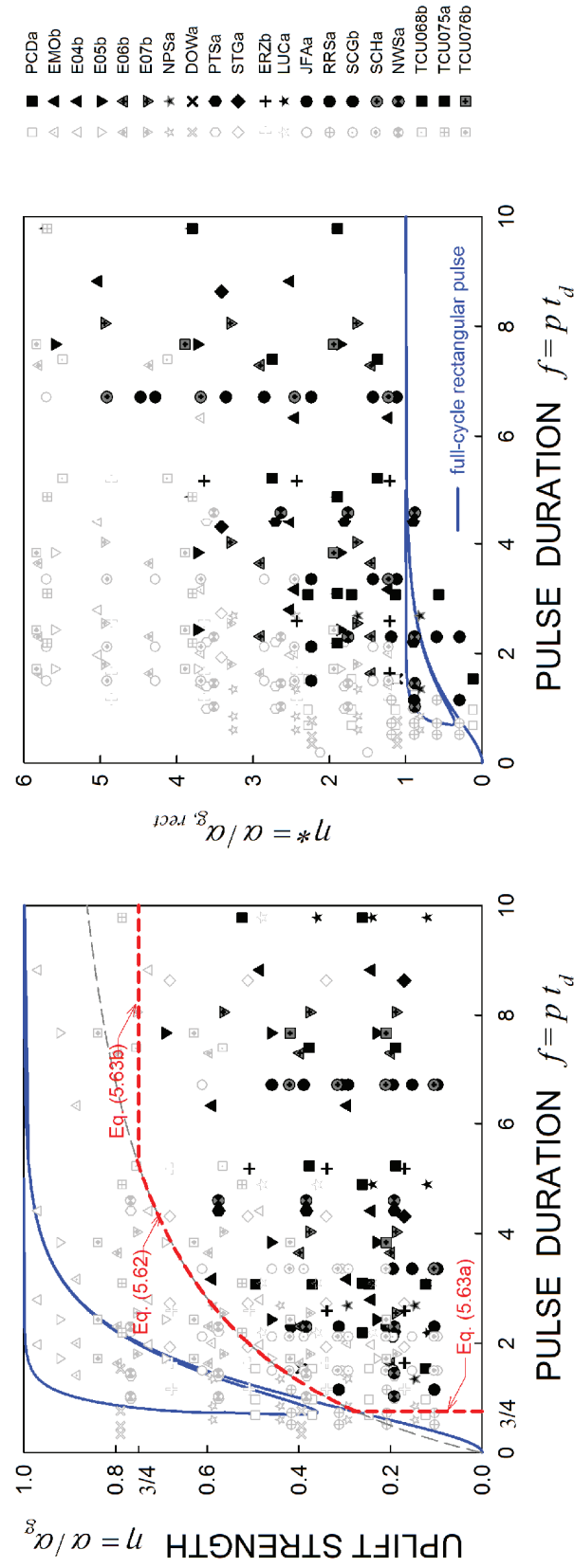


Figure 5.28 Numerical investigation of overturning to actual earthquake recordings. Black bullets denote overturning; uplift strength normalized with (a) PGA ; (b) $2PGV/T_p$; $\varepsilon=0.8$

5.9 Conclusions

The main conclusions of the study are:

1. Rocking behavior of a free-standing block on a rigid base due to a full-cycle rectangular pulse has been treated analytically, leading to closed-form solutions describing all phases of the response. Two overturning *Modes* were identified and rigorously analyzed: *Mode 0*, referring to systems overturning without an impact during forward rocking; *Mode 1*, referring to systems overturning after one impact during backward rocking.
2. A function study of the response was conducted providing useful insight into the physics of rocking behavior. Limit conditions for overturning in *Modes 0* and *1* were derived.
3. The behavior of blocks surviving rocking response was investigated and closed-form solutions for peak rocking parameters (angular displacement and corresponding time) were produced.
4. Time of impact and velocity at impact, that play a crucial role in the analysis of backward rocking response were also determined in closed-form.
5. In general, an odd number of immobility points during a specific rocking branch implies stability. On the contrary an even number of immobility points during a specific rocking branch implies overturning.
6. Observing the criteria for surviving and overturning due to *Mode 1* under a full-cycle pulse excitation, two counterintuitive behaviors arise: (a) if at the end of the

pulse uplift angle equals slenderness α , the block always survives *Mode 0*; (b) if an immobility point does not occur before the end of the pulse, the block always overturns in *Mode 0*. In other words it is impossible to have an immobility point leading to safety after the end of the pulse. If such a point exists, it shall be a second immobility point leading to overturning with *Mode 0* (group O_4). This is different from the case of single-lobe pulses for which no more than a single immobility point can exist during a particular rocking branch.

7. All systems attaining local maxima in angular velocity before $3/2t_d$ survive *Mode 0*. Conversely all systems attaining local maxima after $2t_d$ overturn. In terms of peak rocking angles, all systems rocking to angles smaller or equal to α survive *Mode 0*, whereas all systems rocking to angles equal or higher than 2α overturn with *Mode 0*.
8. In contrast with forward rocking for which blocks in region S possess bounded times of peak response, in backward rocking times of peak response are unbounded and tend to infinity for systems approaching the limit of the two rocking *Modes*.
9. The two rocking modes were numerically evaluated for a range of idealized pulses, of (a) equal acceleration amplitude A_g and duration t_d ; and (b) equal velocity amplitude V_g and duration t_d . In the first case it was shown that the smaller the velocity content of the excitation the smaller the areas of overturning and vice versa; in the second case it was shown that blocks that overturn for a square pulse, overturn for pulses of a more spiky shape, and blocks that survive a perfect impulse, also survive pulses of a less spiky shape.

10. Numerical results for response to actual impulsive ground motions provided simple limit criteria for systems that overturn. The high acceleration content early in the earthquake signal that leads to multiple impacts and development of angular velocity of arbitrary sign before the arrival of the main earthquake pulse may be beneficiary, leading to safety even for low strength, high frequency systems. Accordingly, the criteria in Eqs (5.62) and (5.63) can be used to exclude the possibility of overturning as no toppling systems plot beyond these limits. In this light, a near-fault motion with PGA/g less than $3/4$ times the slenderness angle of the block cannot induce overturning (strength criterion in Eq. 5.63b); a near-fault motion with predominant period less than $3/4$ of the characteristic period of the block cannot induce overturning either (frequency criterion in Eq. 5.63a); for all other cases, a simple curve bounds all toppling systems from the safe side (Eq. 5.62). Finally, a full-cycle rectangular pulse has always higher toppling potential than an actual record of equal PGA and predominant period.

6

A shift approach for the dynamic response of rigid-plastic systems

6.1 Synopsis

In this chapter a novel shift approach is presented for evaluating and interpreting the response of rigid-perfectly plastic single-degree-of-freedom systems to dynamic loading. Scaling laws for such systems are, as the term suggests, *multiplicative* in nature, relating peak dynamic response to products of key problem parameters such as linear spectral coordinates, force reduction coefficient and peak values of the excitation and its time derivatives. Contrary to classical laws, the proposed approach is *additive*, imposing a shift in the ordinates and the abscissa of the excitation function by means of a set of parameters uniquely related to the yielding resistance of the system. The dynamic response is then obtained by integrating the modified excitation function in a linear-like manner within a particular yielding branch, for the nonlinearity is incorporated into the forcing term. The mathematical validity of the approach is demonstrated analytically and its importance is highlighted for systems with symmetric yielding resistance subjected to near-fault earthquake motions. The modified excitation function may be discontinuous between different yielding branches and relates uniquely to the development of plastic deformation (permanent displacement). It is thereby referred to as *Plastic Input Motion (PIM)*. It is shown that the ordinates and the duration of this excitation function may be significantly (yet not necessarily) smaller than those of the original ground motion depending on

yielding strength. The relation of the proposed approach to existing methods and parameters of earthquake engineering such as Newmark's sliding block and relative ground acceleration, is discussed.

6.2 Preface

A fundamental scaling relationship in structural dynamics lies on the proportionality between excitation, $P(t)$, and response, $u(t)$, of a linear simple oscillator of natural frequency ω_n , viscous damping ζ and accelerating mass m i.e.,

$$\frac{d^2}{dt^2}(\lambda \times u) + 2\zeta \omega_n \frac{d}{dt}(\lambda \times u) + \omega_n^2 (\lambda \times u) = \frac{1}{m} (\lambda \times P) \quad (6.1)$$

λ being an arbitrary scaling parameter. This property is a direct consequence of the linearity of the differential operator in Eq. (6.1).

In the particular case of earthquake loading, a fundamental property is often employed for yielding oscillators, which is described by (Chopra 2006)

$$\ddot{\mu} + 2\zeta \omega_n \dot{\mu} + \omega_n^2 \bar{f}_s(\dot{\mu}, \mu) = -\omega_n^2 \frac{\ddot{u}_g(t)}{a_y} \quad (6.2)$$

where $\bar{f}_s(\dot{\mu}, \mu)$ is a dimensionless force-displacement function defined as the restoring force f_s over the restoring force at yielding f_y . $\ddot{u}_g(t)$ is the ground acceleration, $a_y = f_y/m$ is the yielding acceleration, and μ is the well-known *ductility demand factor* (Veletsos & Newmark 1960)

$$\mu \equiv \frac{u}{u_y} \quad (6.3)$$

The above equations suggest that the response of a yielding system depends on elastic natural frequency, damping ratio, shape of dimensionless force-displacement loop $\bar{f}_s$ and ratio of ground acceleration to yield acceleration, as evident from the right-hand side of Eq. (6.2). In particular the dimensionless ratio $\ddot{u}_g(t)/a_y$ implies that the response of the system is invariant to a uniform scaling (i.e., by the same factor λ) in both ground acceleration $\ddot{u}_g(t)$ and yielding acceleration a_y . It also suggests that a λ -fold increase in seismic demand can be achieved either by a λ -fold increase in ground acceleration, or by an $(1/\lambda)$ -fold decrease in yielding resistance. This property allows an infinite set of oscillators of different yielding strengths, subjected to different levels of ground motion, to exhibit identical responses and is, thereby, referred to as *similarity* (Barenblatt 1996).

As shown by Veletsos & Newmark (1960) and Bertero *et al.* (1978), the above feature relates to the well-known *strength reduction factor* and *dimensionless yield strength* parameters

$$R \equiv \frac{PSA(\omega_n, \zeta)}{a_y} \quad (6.4a)$$

$$\eta \equiv \frac{a_y}{A_g} \quad (6.4b)$$

which, evidently, scale in an identical manner with the ratio $\ddot{u}_g(t)/a_y$ in Eq. (6.2). In these equations, $PSA(\omega_n, \zeta)$ denotes the pseudo-acceleration parameter and A_g the amplitude of ground acceleration.

A useful set of scaling laws, akin to the definitions in Eqs (6.2-6.4), exist in the form of approximate *R-μ relations* of the type

$$\mu / R = g_1(\omega_n, \zeta, R) \quad (6.5a)$$

$$R / \mu = g_2(\omega_n, \zeta, \mu) \quad (6.5b)$$

where g_1 and g_2 are pertinent dimensionless functions* (Veletsos & Newmark 1960, Newmark & Rosenblueth 1971, Makris & Black 2004, Riddell *et al.* 2002, Cuesta & Aschheim 2001).

In the important case of the equal displacement rule, $g = 1$; Eq. (6.5) can be written with the help of Eqs. (6.4a) and (6.4b) as

$$u_{\max} = \frac{u_y}{a_y} PSA(\omega_n, \zeta) \quad (6.6)$$

which expresses the property that the peak displacement of an inelastic system is proportional to the amplitude of ground excitation A_g , as $PSA(\omega_n, \zeta)$ is proportional to A_g and (u_y/a_y) is constant, proportional to the inverse of elastic stiffness.

In the particular case of rigid-plastic systems, Newmark (1965, 1971) and later Richards & Elms (1979) showed that the displacement response to an earthquake motion can be effectively bounded by

$$u_{\max} = \chi_1 \frac{V_g^2}{A_g} \eta^{-b_1} \quad (6.7)$$

where A_g and V_g are the peak values of ground acceleration and velocity, respectively; χ_1 and b_1 are dimensionless parameters depending mainly on the assumptions as to unilateral

* The need to normalize Eqs (6.5) by means of a characteristic frequency (or time) to ensure dimensional homogeneity of functions g lies beyond the scope of this chapter.

or bilateral response associated with symmetric or asymmetric yielding resistance, respectively.

With reference to unilateral response pertaining to earth structures and retaining walls, the following modified expression has been proposed by Whitman & Liao (1985)

$$u_{\max} = \chi_2 \frac{V_g^2}{A_g} \exp(-b_2 \eta) \quad (6.8)$$

where χ_2 and b_2 are dimensionless parameters analogous to χ_1 and b_1 in Eq. (6.7). Equation (6.8) differs from the previous one in the functional form of the term involving system strength. A more general empirical expression was developed by Yegian *et al.* (1991) in the form

$$u_{\max} = A_g T_g^2 N_{eq} \exp \left[\sum_{m=1,2,3} c_m \eta^m \right] \quad (6.9)$$

where T_g denotes the predominant period of ground motion and N_{eq} the number of significant excitation cycles. c_m ($m = 1, 2, 3$) are dimensionless constants established from regression analyses. Additional relationships can be found in the literature (Newmark & Rosenblueth 1971, Makris & Black 2004, Riddell *et al.* 2002, Cuesta & Aschheim 2001, Ambraseys & Menu 1988, Sidiropoulos 2012).

6.3 Shift Approach for Rigid – Plastic Systems

The aforementioned scaling laws invariably involve *multiplicative* relations between dimensional parameters for various dynamic systems. For the particular case of a rigid-plastic system, a different type of relationship, *additive* rather than multiplicative in nature, can be developed as explained in the ensuing. It will be shown that this approach is general and can be of practical importance for high-intensity, low-cycle loading such as that induced by near-fault earthquake motions.

For the reference system of Fig 6.1, the equation of motion after yielding of a rigid perfectly-plastic system can be written as[†]

$$\ddot{u} + a_y \operatorname{sgn}(\dot{u}) = +\ddot{u}_g(t) \quad (6.10)$$

in which $u = u(t)$ denotes displacement relative to ground and $\operatorname{sgn}(\)$ the signum function [i.e., $\operatorname{sgn}(x > 0) = +1$, $\operatorname{sgn}(x < 0) = -1$]. For simplicity and without loss of generality, one may consider positive system velocity during a particular response branch; the above equation can be integrated twice to read

$$u = u_g - \frac{1}{2}(t - t_y)^2 a_y - (t - t_y) \dot{u}_{gy} - u_{gy} \quad (6.11)$$

where u_{gy} , $\dot{u}_{gy}$ stand, respectively, for ground displacement and velocity at the time of yielding ($t = t_y$). Note that due to rigidity, the relative velocity and displacement of the system at yielding (i.e., u_y , $\dot{u}_y$) have been taken equal to zero.

[†] The positive sign in the right-hand side of Eq. (6.10) is to ensure positive response $u(t)$ for positive ground acceleration $\ddot{u}_g(t)$, as evident from the reference axes of Fig 6.1.

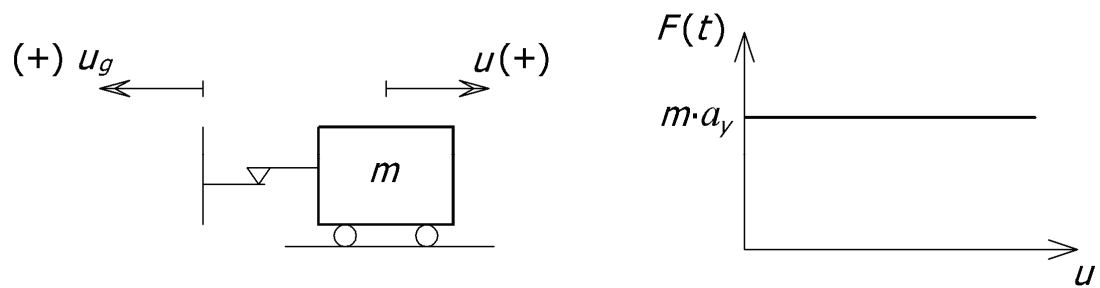


Figure 6.1. Idealized rigid-perfectly-plastic system considered.

A difficulty associated with the interpretation of Eq. (6.11) is that the right side depends on a rather large number of terms associated with different parameters of the problem. In addition, the response appears to depend on the “snapshot” ground motion parameters u_{gv} and $\dot{u}_{gv}$ which are difficult to incorporate in design-oriented relations such as those in Eqs (6.3-6.9). Moreover, the influence of the second of these parameters on the solution appears to be unreasonably strong, for it is multiplied by time, thereby its contribution tends to dominate in long signals. Evidently, a direct integration of Eq. (6.10), like the one in Eq. (6.11), although mathematically exact, is not optimal for revealing the underlying physics of the problem at hand. A simpler alternative can be obtained as shown below.

By virtue of the definitions of the shifted ground motion parameters,

$$\Delta t \equiv t - t_y \quad (6.12)$$

$$\Delta u_g(t) \equiv u_g(t) - u_{gv} \quad (6.13)$$

$$\Delta \dot{u}_g(t) \equiv \dot{u}_g(t) - \dot{u}_{gv} \quad (6.14)$$

$$\Delta \ddot{u}_g(t) \equiv \ddot{u}_g(t) - a_y \quad (6.15)$$

and the pair of the easy-to-derive[†] integral properties,

$$\Delta u_g = \int_0^{\Delta t} \Delta \dot{u}_g d\Delta t + \dot{u}_{gv} \Delta t \quad (6.16)$$

$$\Delta \dot{u}_g = \int_0^{\Delta t} \Delta \ddot{u}_g d\Delta t + a_y \Delta t \quad (6.17)$$

it is straightforward to show that Eq. (6.11) can be written in the simpler form

$$u = \int_0^{\Delta t} \Delta \dot{u}_g d\Delta t - \frac{1}{2} a_y \Delta t^2 \quad (6.18)$$

For a rigid-plastic system, $a_y = \ddot{u}_{gy}$; the above result can further simplify to

$$u = \int_0^{\Delta t} \int_0^{\Delta t} \Delta \ddot{u}_g dt^2 \quad (6.19)$$

in which the property $d(\Delta t) = d(t - t_y) = dt$ has been employed. This remarkably simple expression suggests that the displacement response of the system can be obtained by integrating twice ground acceleration shifted by an amount equal to yield acceleration without regard for initial conditions. In other words, the yielding resistance and the initial conditions can be incorporated into the excitation function by translating the origin of axes from point $(0, 0)$ to point (a_y, t_y) , thereby replacing co-ordinates $(\ddot{u}_g, t)$ by $(\Delta \ddot{u}_g, \Delta t)$, as shown in Fig 6.2. Evidently, Eq. (6.19) can be viewed as a simpler alternative to Eq. (6.11), for it is compact and does not involve the “snapshot” ground motion parameters u_g and $\dot{u}_{gy}$. Accordingly, the time history $\Delta \ddot{u}_g(t)$ in Eq. (6.15) will be referred hereafter to as *Plastic Input Acceleration (PIA)* or *Plastic Input Motion (PIM)*, for it is associated uniquely with the development of plastic deformation in the system. It must be emphasized that $\Delta \ddot{u}_g(t)$ is not the derivative of $\Delta \dot{u}_g(t)$ nor is $\Delta \dot{u}_g(t)$ the derivative of $\Delta u_g(t)$, as evident from Eqs (6.16) and (6.17).

[‡] A derivation is provided in Appendix E.

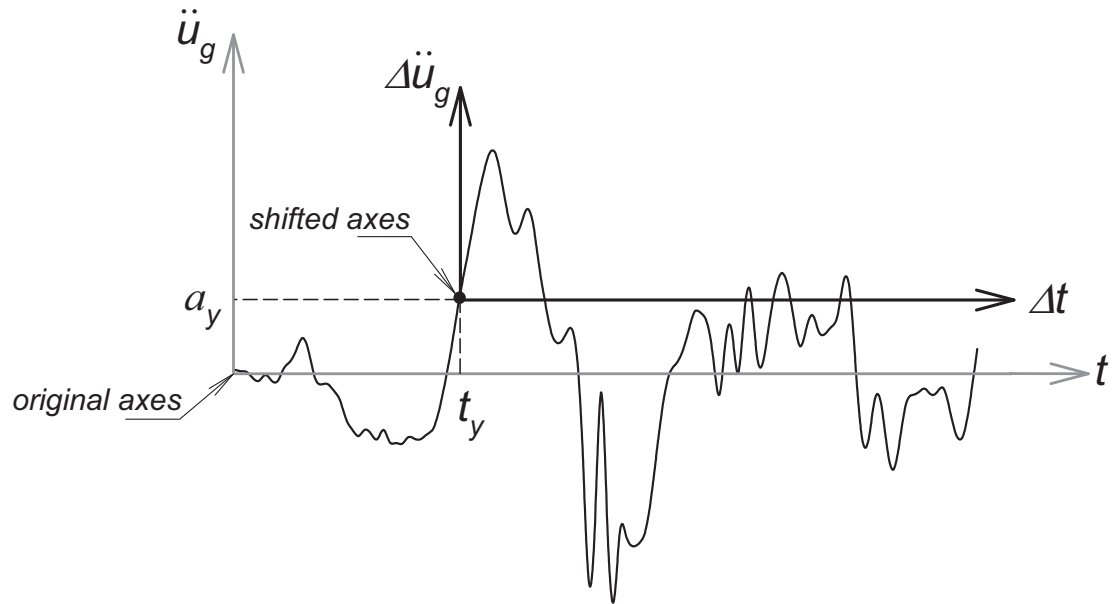


Figure 6.2 Translation of axes for a rigid-perfectly plastic system according to Eqs (6.12) and (6.15).

Incidentally it is noted that the above approach can be easily extended to more general classes of yielding systems, including elastic perfectly-plastic and bilinear oscillators.

Note that for the former type of systems, the following simple solution is possible

$$\Delta u = \int_0^{\Delta t} \int_0^{\Delta t} [\Delta \ddot{u}_g + \dot{u}_y \delta(t - t_y)] dt^2 \quad (6.20)$$

in which $\Delta u = u - u_y$ denotes the post-yielding (plastic) displacement of the system, $\dot{u}_y$ the relative to ground velocity at yielding and $\delta(t)$ the Dirac delta function. Note that setting u_y and $\dot{u}_y$ equal to zero, the above expression duly reduces to Eq. (4.19). These systems will be discussed in more detail in Chapter 7.

6.3.1 Interpretation

In summarizing the foregoing, it is instructive to note that they can be interpreted in light of the expression:

$$\ddot{u} = \ddot{u}_g(t) - \text{sgn}(\dot{u}) a_y \quad (6.21)$$

which is a re-arranged form of Eq. (6.10). Evidently, since a_y is constant, it is tantamount to an external load and, thereby, can be transferred to the right-hand side of the equation to induce the aforementioned shift in the excitation function. Note, however, that this operation leads to more than a mere “right-hand siding” of the yielding resistance, for it simplifies the solution and eliminates the need to account for a number of initial conditions associated with function $\ddot{u}_g(t)$ (see Eqs. 6.11 and 6.19). Moreover, knowledge of the right-hand side of Eq. (6.21) transforms the problem to a linear like, for the

nonlinearity is embedded into the forcing term, leaving a linear operator in the left hand side of the governing equation. This sense of linearity, however, should be viewed merely as a mathematical device, for the response of the system is inherently plastic.

Equation (6.21) also suggests that the *Plastic Input Acceleration (PIA)* coincides with the relative to ground acceleration of the system. Thereby, the integrals of $\Delta\ddot{u}_g$ describe the corresponding relative to ground velocity and displacement. This interpretation should, clearly, not be generalized for other types of yielding systems and is only valid for rigid-plastic ones. Accordingly, in the remainder of this chapter and following the definition of $\Delta\ddot{u}_g$, the integrals $\int \Delta\ddot{u}_g dt$ and $\int \int \Delta\ddot{u}_g dt^2$ will be referred to as *Plastic Input Velocity (PIV)* and *Plastic Input Displacement (PID)*, respectively. Note that *PIV* and *PID* are generally different from functions Δu_g and $\Delta\dot{u}_g(t)$ as suggested by Eqs. (6.16) and (6.17).

6.4 Methodology & Applications

In applying the above, it must be kept in mind that the approach pertains only to yielding response (i.e., where the resistance term a_y is constant), thereby identification of the corresponding time intervals is essential. When the excitation contains a large number of high intensity cycles, a numerical approach often provides the only practical means to carry out this task (Chopra 2006). On the other hand, when only a handful of severe loading cycles exist, the analysis can be implemented at a substantially smaller computational cost, possibly by means of hand calculations. This allows valuable insight to be gained on the nature of earthquake demand on yielding structures. An important application referring to impulsive near-fault earthquake motions is examined in the ensuing.

With reference to the system at hand, a set of six impulsive ground motions affected by forward fault-rupture directivity is employed, as shown in Fig. 6.3 and Table 6.1[§]. The set encompasses different recorded earthquake ground motions with peak accelerations (A_g) varying between 0.38 to 1.26g, peak velocities (V_g) ranging from 51 to 263cm/s and peak displacements (D_g) ranging from 9.6 to 430cm. Corresponding moment magnitudes (M_w) vary between 6.3 to 7.6. An idealized sinusoidal pulse of full-cycle duration, normalized to 0.5g acceleration, 156cm/s velocity and 78cm displacement (for an overall duration of 1s) is also examined. Four yielding acceleration levels ($a_y = 0.1, 0.2, 0.3$ and $0.4g$) are considered, covering a wide range of system strengths.

[§] A set of 40 near-fault ground motions was examined, corresponding results are provided in Appendix E.

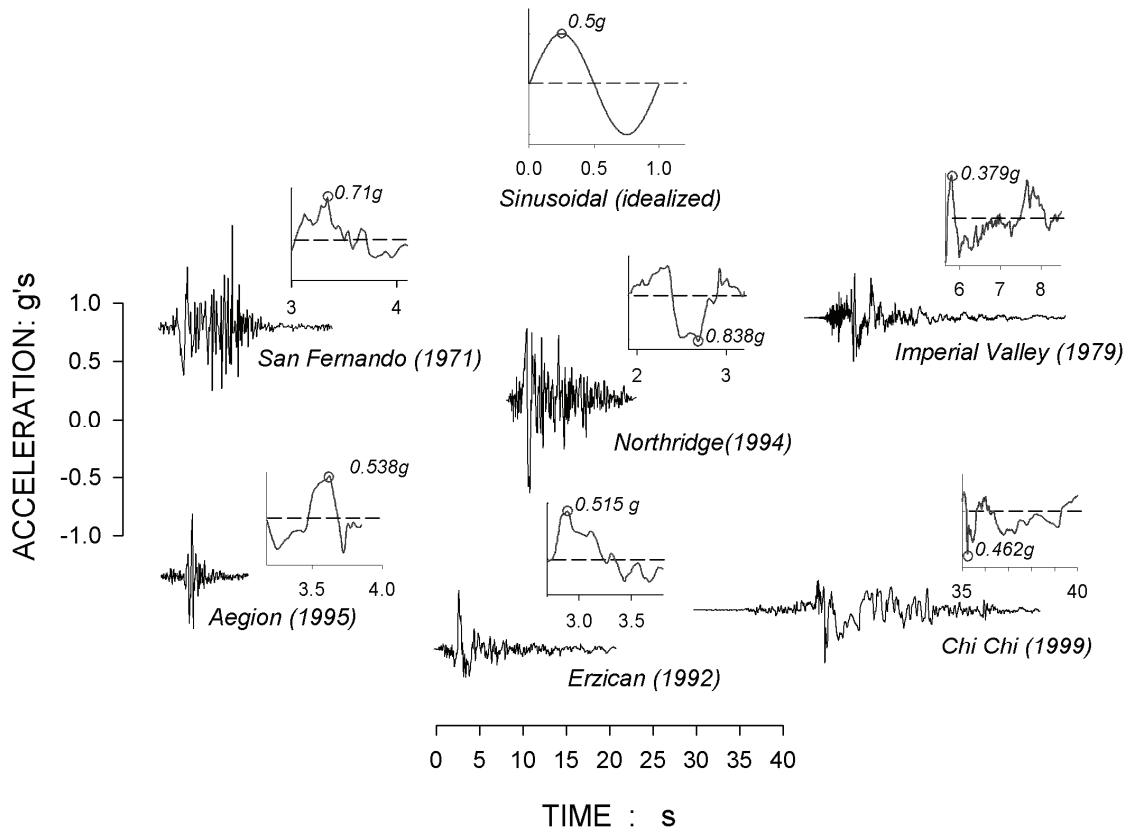


Figure 6.3 Selected near-field recorded and idealized ground motions containing long-period, high-acceleration pulses. Properties are summarized in Table 6.1.

Table 6.1 Selected near-fault ground motions employed in the analyses. Waveforms are provided in Fig 6.3.

No.	Earthquake	Date	Magnitude M_w	Fault Distance (km)	Fault Mechanism [†]	Station	Site Conditions [*]	Component [‡]	A_g (g)	V_g (cm/s)	D_g (cm)	$D_{5\%}$ (s)	Duration (s)
1	San Fernando, CA, U.S.A.	9/2/1971	6.6	3.0	R	Pacoima Dam	HR	L	1.23	113	35.4	7	40
2	Imperial Valley, CA, U.S.A.	15/10/1979	6.5	2.7	S	E05	SL	T	0.379	90.5	63.1	9.4	30
3	Erzican, Turkey	13/3/1992	6.6	2.0	S	ERZ	SL	N-S	0.515	84	27.7	7.5	21
4	Northridge, CA, U.S.A.	17/1/1994	6.7	6.0	R	Rinaldi	SL	L	0.838	166	28.2	7	15
5	Aegion, Greece	15/6/1995	6.3	6.0	N	OTE Building	SL or SR	T	0.538	51	9.6	2	30
6	Chi-Chi, Taiwan	20/9/1999	7.6	0.2	O	TCU068	SL	E-W	0.462	263	430	13.1	90

[†] N: Normal, S: Strike-Slip, R: Reverse, O: Oblique

[‡] L: Longitudinal, T: Transverse, N-S: North-South, E-W: East West

^{*} HR: Hard Rock, SR: Sedimentary and Conglomerate Rock, SL: Soil and Alluvium

6.4.1 Methodology

Identifying pulse content in an earthquake record is not a trivial task. Alternative proposals have been put forth over the years to determine the initiation and end of these excursions by visual and computational means (Rodriguez-Marek 2000, Mavroeidis & Papageorgiou A 2003, Mavroeidis *et al.* 2004, Baker 2007, Bray & Rodriguez-Marek 2004, Tang & Zhang 2011, Yang *et al.* 2009, Chioccarelli & Iervolino 2010). Due to a lack of a precise definition for an earthquake pulse and the complexity of the associated time histories, all available methods encompass some degree of ambiguity. By virtue of the physics of a rigid-perfectly plastic system, a simple unambiguous procedure is employed in this chapter, as depicted in Fig. 6.4: *First*, acceleration peaks associated with large velocity content are identified. The initiation of the pulse is set at the time where $\ddot{u}_g = |a_y|$, that coincides with the initiation of yielding and which can be readily established on the acceleration record (point 1, Fig. 6.4a). *Second*, the ordinates and the abscissa of the time history are shifted by a_y and t_y , respectively, to set the origin of the new axes at the beginning of the pulse (point 1, Fig 6.4b). *Third*, integration of the modified ground acceleration is carried out considering zero initial conditions, up to the point where modified ground velocity function becomes zero. This time instant (point 2, Fig 6.4b) corresponds to a possible end of yielding and manifests itself in the form of equal areas of opposite signs in the modified record. *Fourth*, if the acceleration level in the original record at the zero-velocity time instant (point 2) is smaller than a_y (this case does not correspond to the one shown in Fig 6.4), the yielding phase and the associated pulse end, and a new surpass of a_y is sought in the remainder of the time history. *Fifth*, if the acceleration level in the zero velocity point is larger, in absolute terms, than a_y (as is the case in Fig. 6.4), the motion of the system reverses and the yielding resistance changes

sign. Accordingly, the ordinates of the excitation function are shifted in the opposite direction leading to a jump in PIA by an amount equal to $2 \times a_y$ (point 1', Fig 6.4c). In the proposed procedure the new phase is treated as an independent pulse, thus integration of PIA is carried out from the new origin of axes considering zero initial conditions until a new zero velocity instant is detected (point 4, Fig. 6.4c).

Sixth, the procedure is repeated until the pulse content of the record is exhausted. *Seventh*, the largest, in absolute terms, maximum ground acceleration and velocity among all individual pulses defines the overall peak values of the plastic input motion. On the other hand, plastic input displacements are naturally cumulative, thereby the overall peak value is obtained by considering the succession of all individual pulse displacements as a single time history.

The identified pulses can be cast in the form of a single discontinuous excitation function using the ordinates of the first pulse and the abscissa of the original record (Fig. 6.4d). The observed discontinuity between points 2 and 3 is due to the axis shift following the initiation of the second pulse in Fig. 6.4c. Evidently, this is a different excitation function as compared to the original time history (Fig. 6.4a), yet it is the one that should be integrated over time for the plastic response of the system. More importantly, the latter function may be advantageous to the original acceleration record for describing the main features of post-yielding seismic demand, for it is uniquely related to the development of plastic deformation.

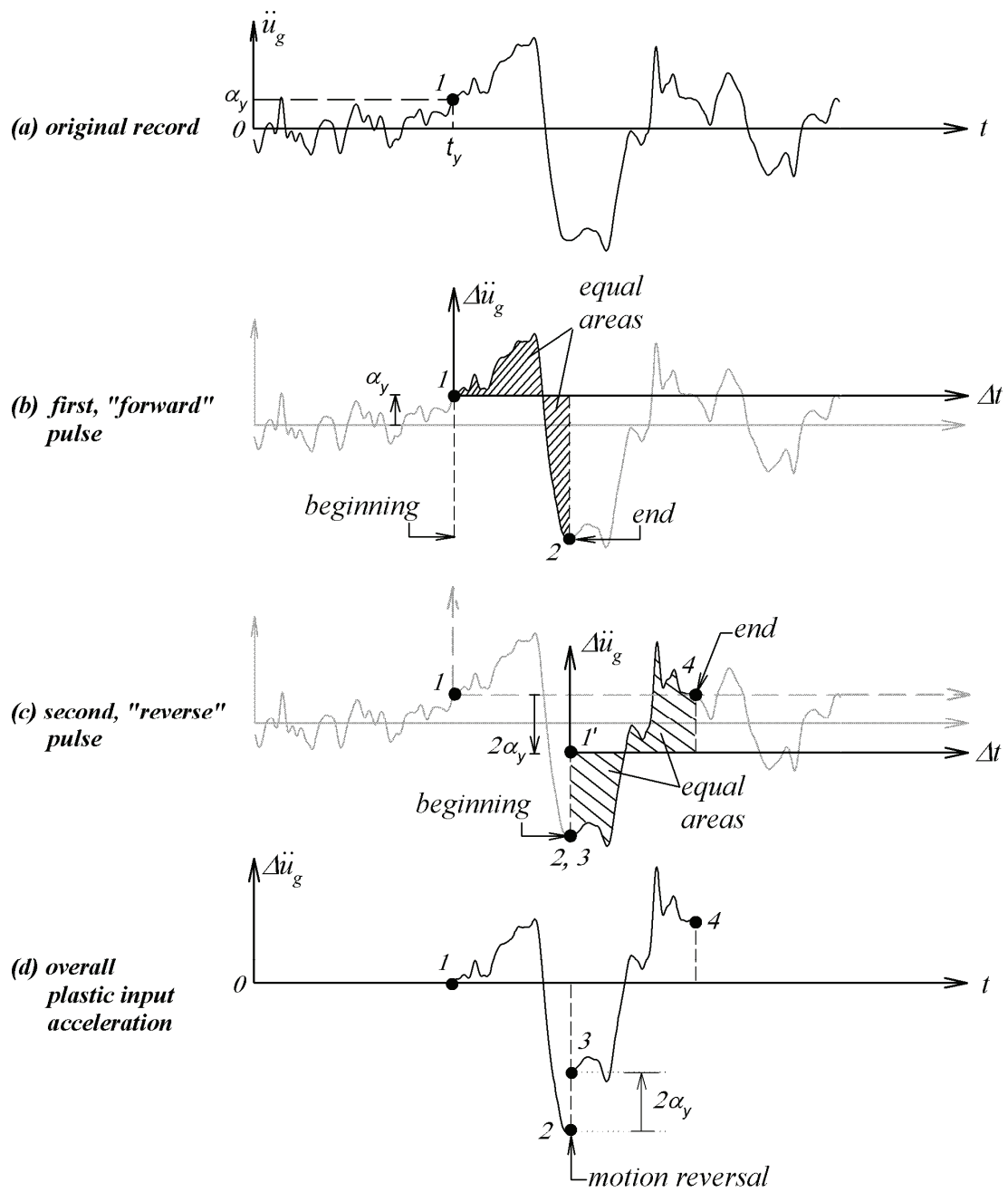


Figure 6.4 Definition of integration limits and Plastic Input Acceleration (PIA) according to the proposed approach.

6.4.2 Results

Results for the selected ground motions are provided in Figs. 6.5a-d to 6.10a-c, plotted in terms of $PIA = \Delta \ddot{u}_g(t)$ and its time integrals, for $a_y = 0.1g, 0.2g, 0.3g$ and $0.4g$. In all analyses presented here, a_y is considered to be the same in both directions (corresponding, for example, to the case of symmetric friction for a sliding system), which allows bilateral response to take place. Despite significant seismological differences as well as differences in shape, duration and intensity among the various waveforms (Table 6.1), the results exhibit several common features: *First*, the number of significant pulses is limited between 1 and 3, with most motions containing 2 pulses (Figs. 6.5a, 6.6a-c, 6.8a-d, 6.9c-d). *Second*, the duration of the plastic input motions is limited between about 0.07s (Fig 6.5d) and 4.2s (Fig 6.5a), in contrast with the significant duration $D_{5\%}$ of the original records (Table 6.1). *Third*, all motions containing more than one pulse exhibit characteristic jumps in PIA due to alternating directions of yielding (and, thereby, shifts in signs of yielding resistance) between successive pulses (Figs 6.5a, 6.6a-c, 6.8a-d, 6.9c-d, 6.10a). Also, when the conditions of Fig 6.4c are satisfied, the jumps equal exactly ($2 \times a_y$) as evident in Figs 6.6a-b, 6.8a-d, 6.9c and 6.10a. *Fourth*, peak values of plastic input motion may be significantly lower than those of the original ground motion.

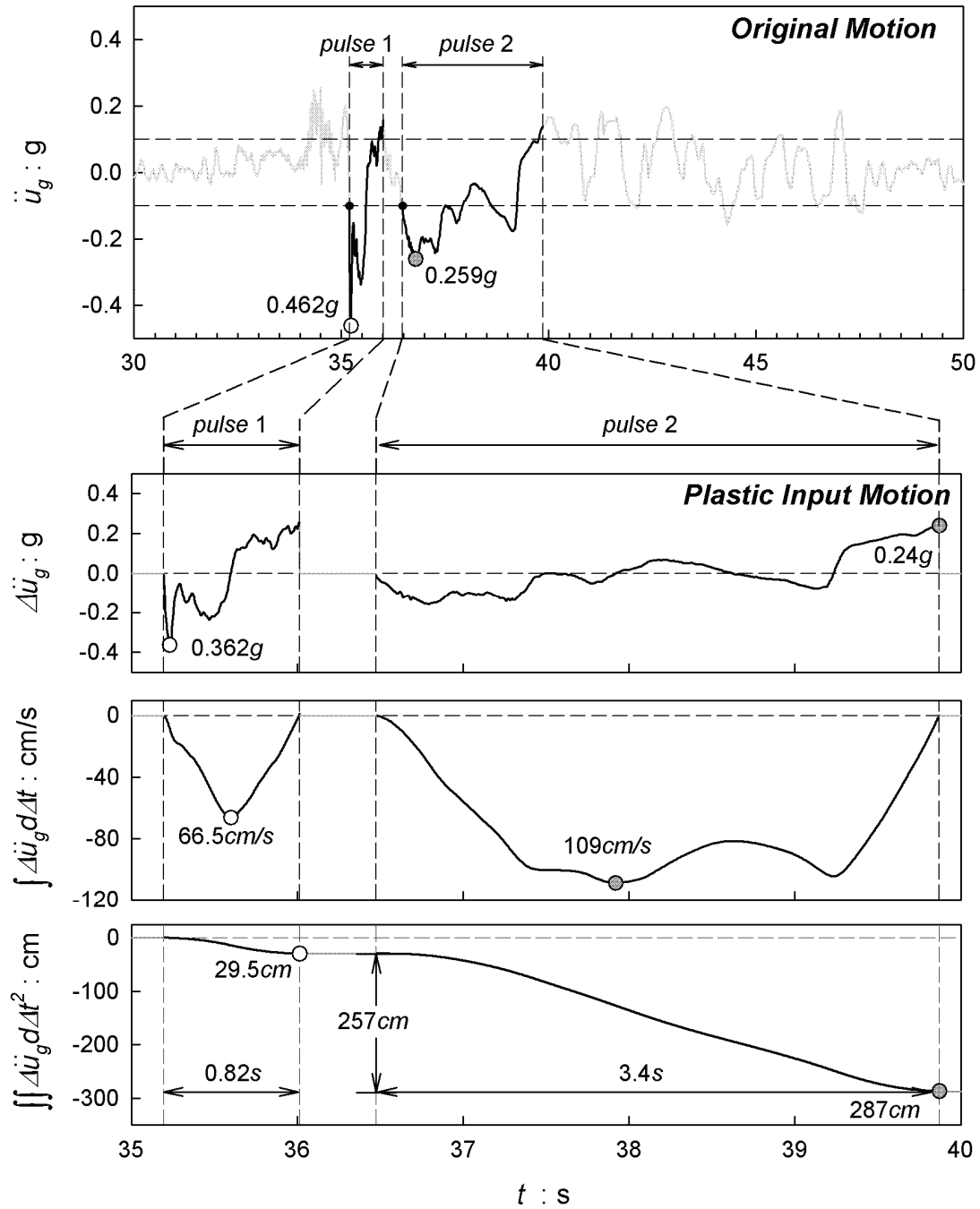


Figure 6.5a Chi-Chi, Taiwan, 1999, TCU068N record. Plastic input motions for $a_y=0.10g$.

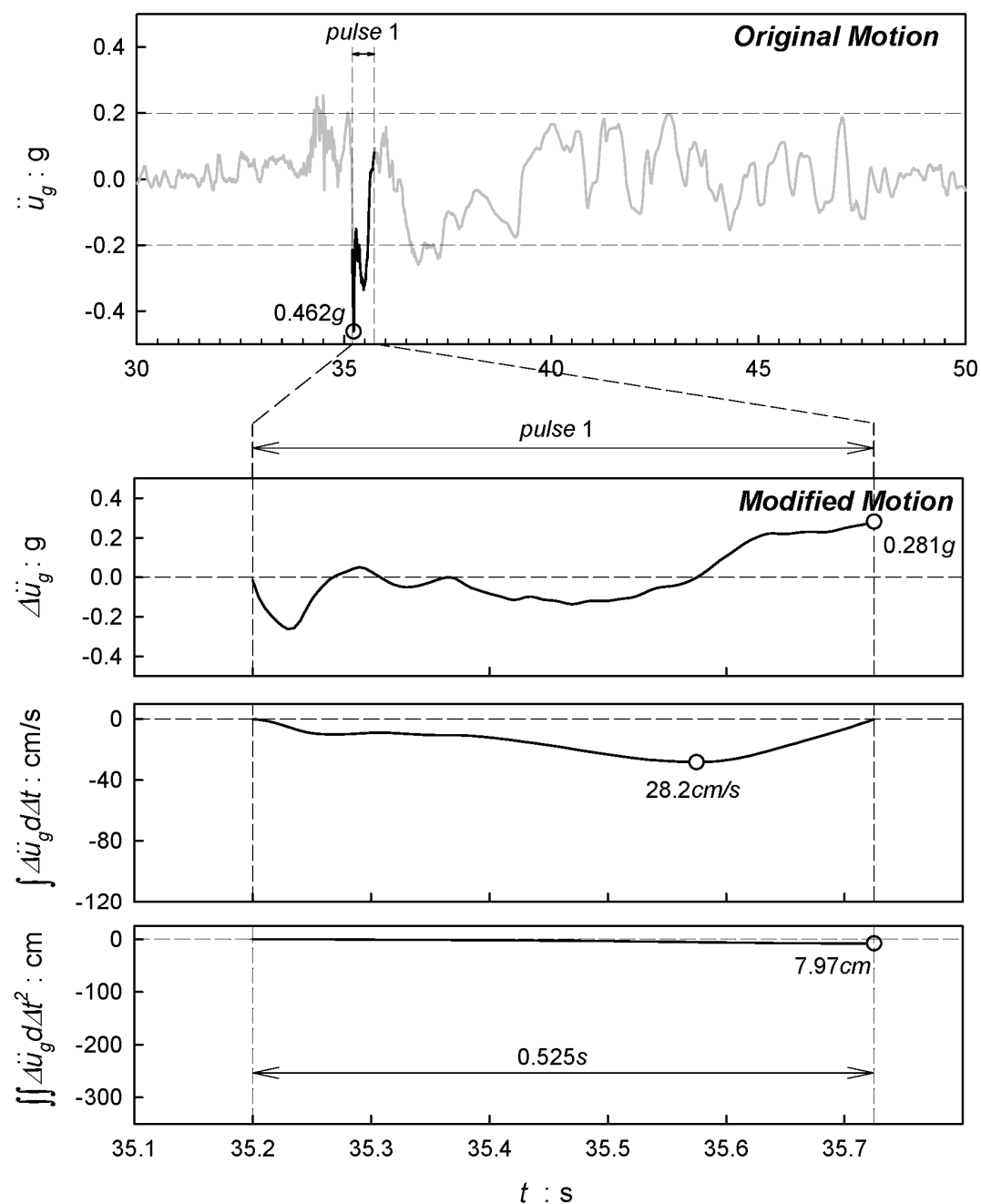


Figure 6.5b Chi-Chi, Taiwan, 1999, TCU068N record. Plastic input motions for $a_y=0.20g$.

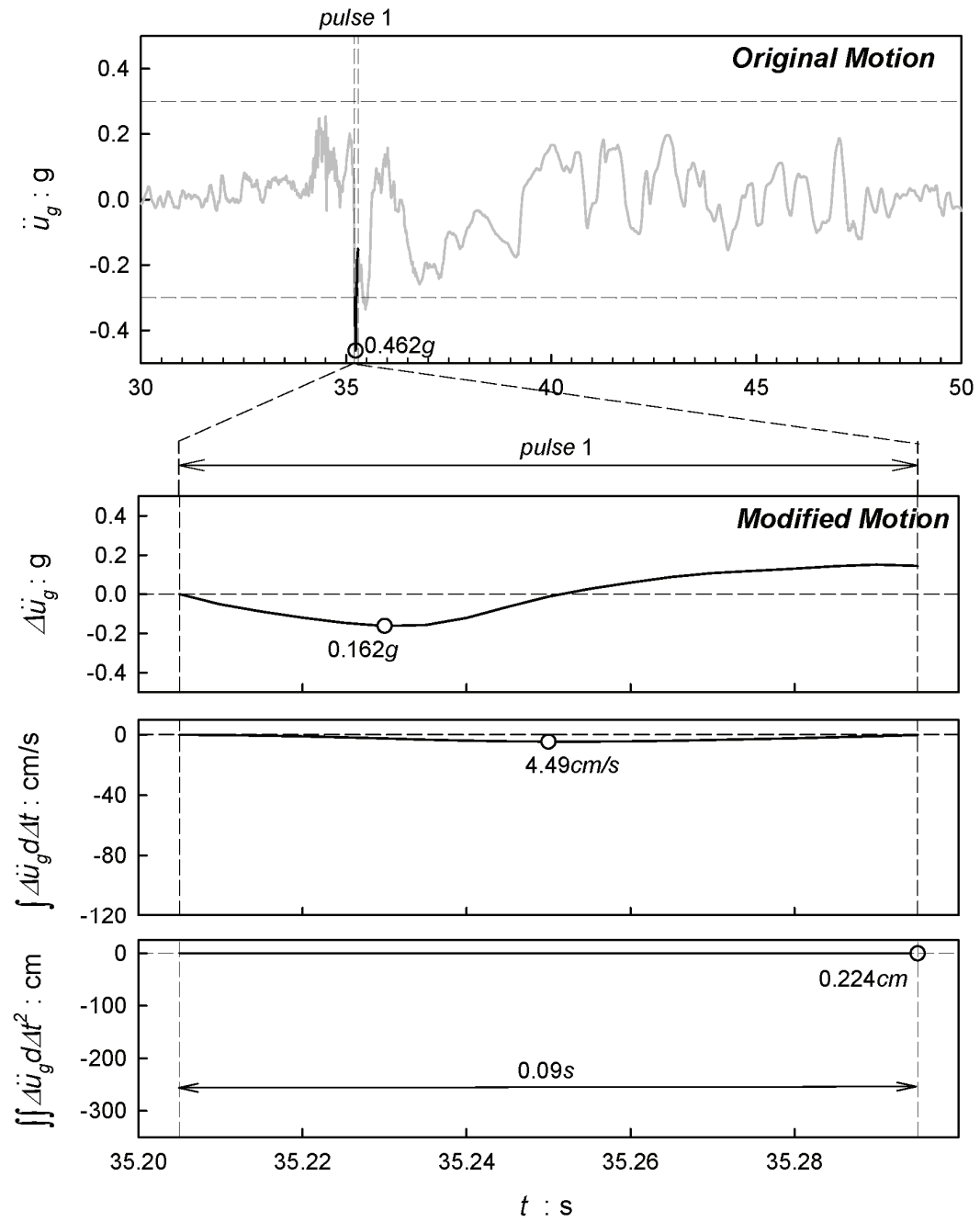


Figure 6.5c Chi-Chi, Taiwan, 1999, TCU068N record. Plastic input motions for $a_y=0.30g$.

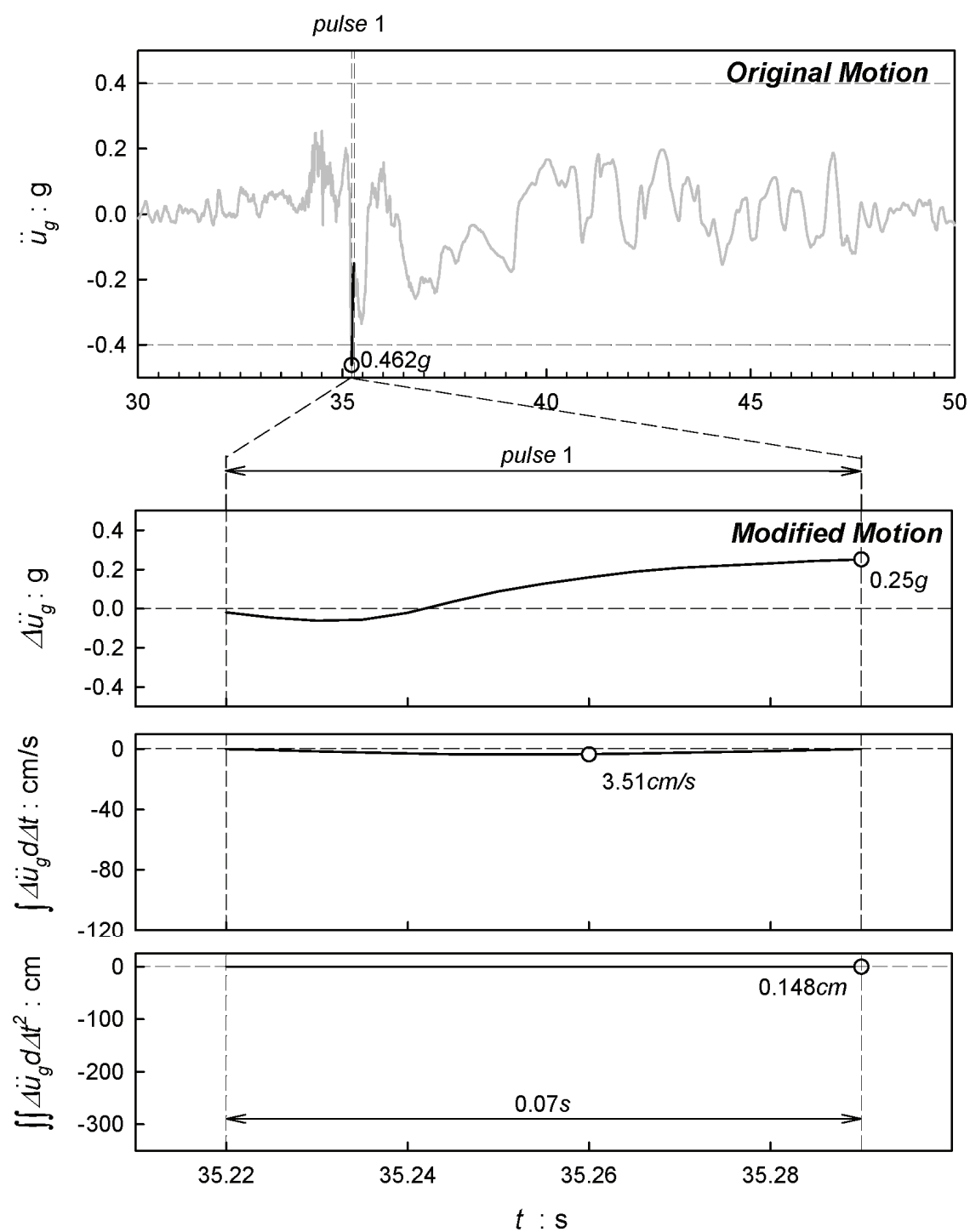


Figure 6.5d Chi-Chi, Taiwan, 1999, TCU068N record. Plastic input motions for $a_y=0.40g$.

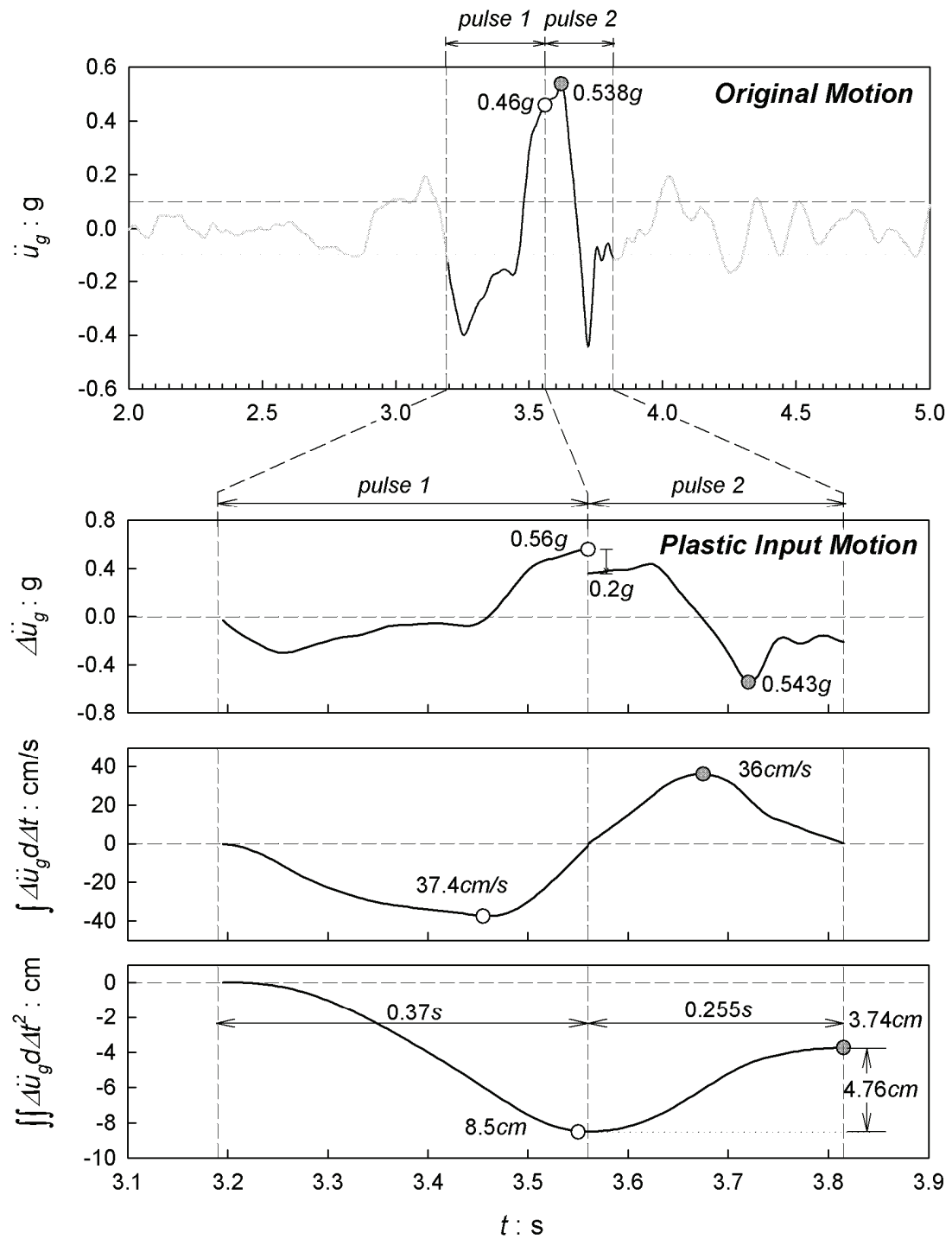


Figure 6.6a Aegion, Greece, 1995, OTE-T record. Plastic input motions for $a_y=0.10g$.

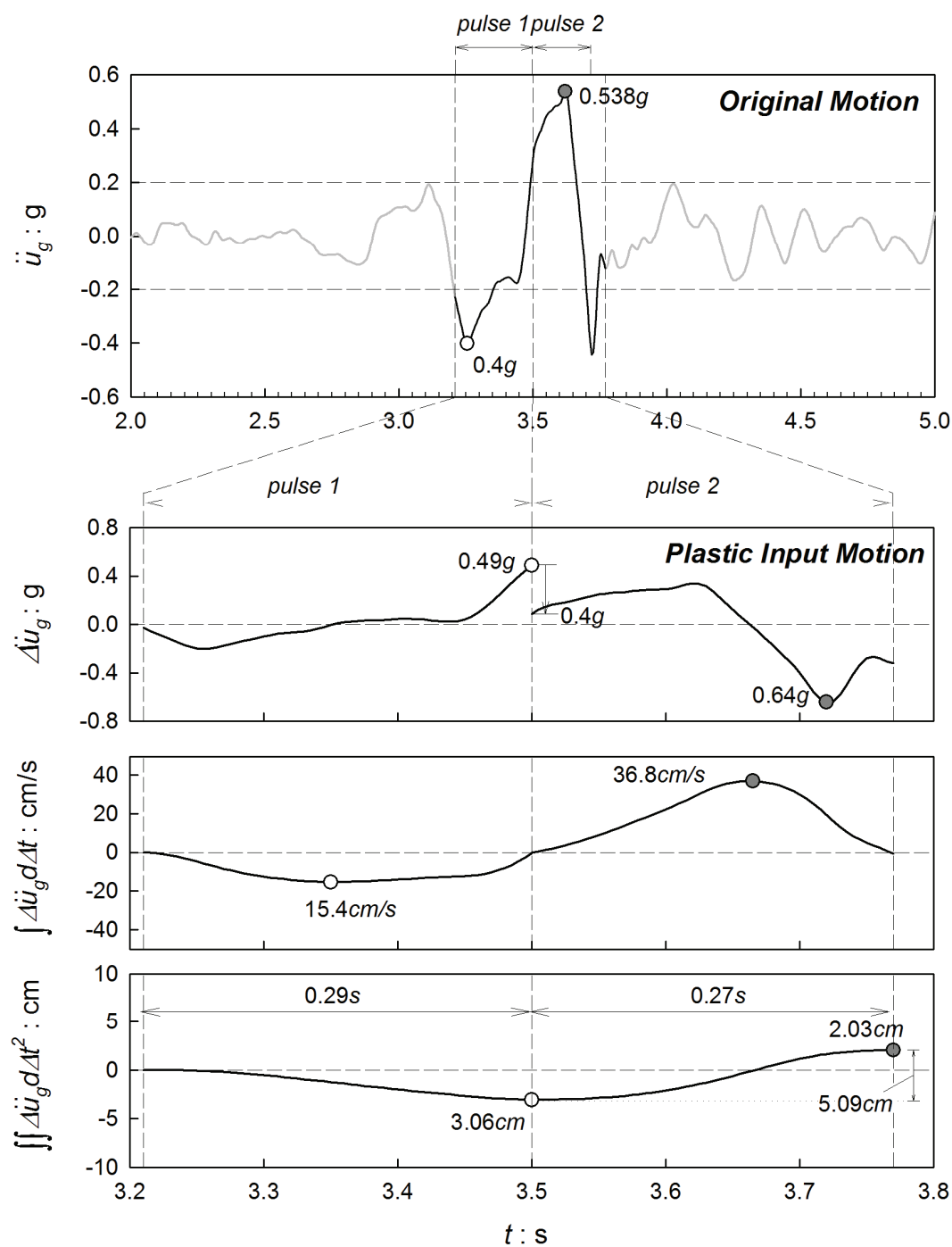


Figure 6.6b Aegion, Greece, 1995, OTE-T record. Plastic input motions for $a_y=0.20g$.

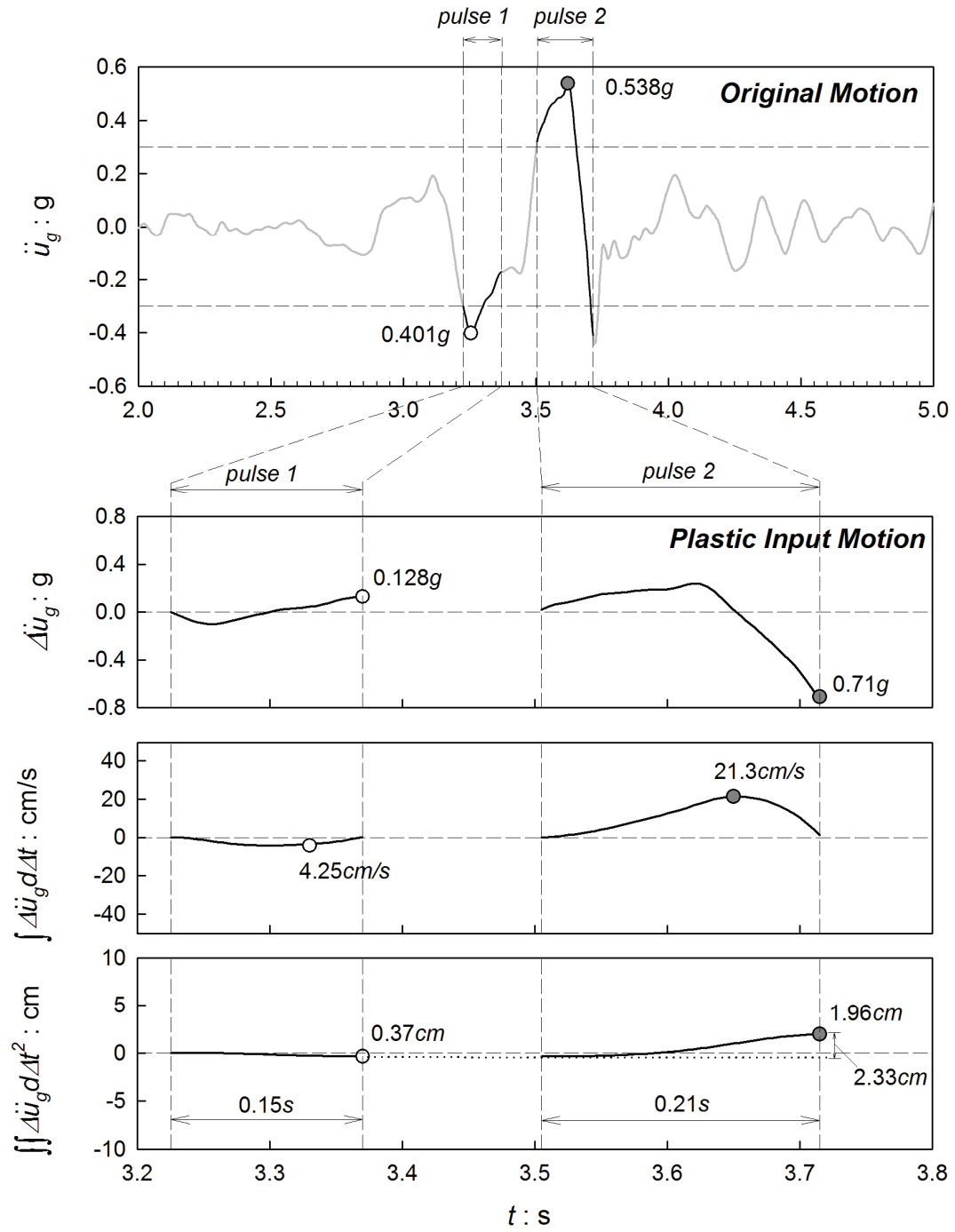


Figure 6.6c Aigion, Greece, 1995, OTE-T record. Plastic input motions for $a_y=0.30g$.

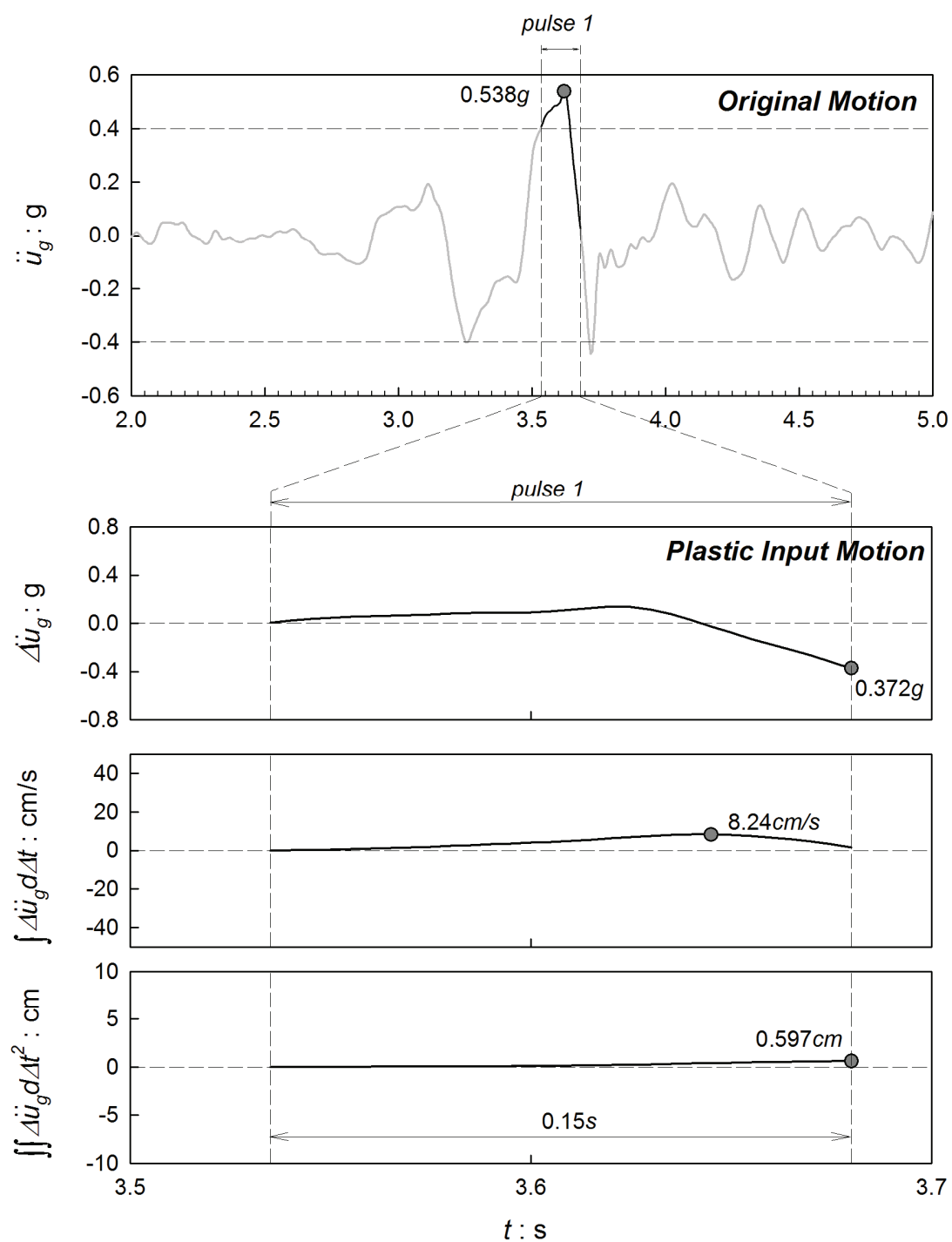


Figure 6.6d Aegion, Greece, 1995, OTE-T record. Plastic input motions for $a_y=0.40g$.

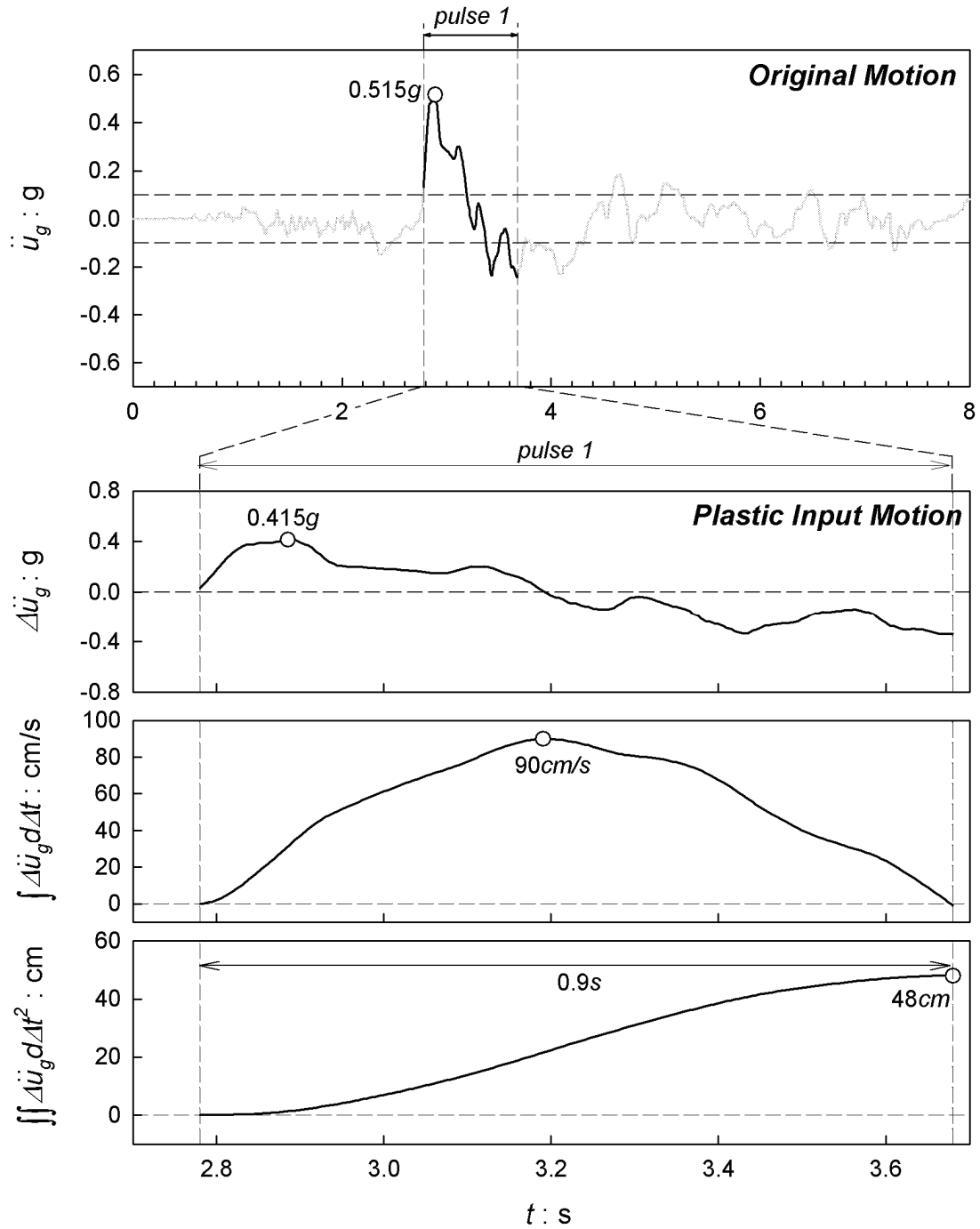


Figure 6.7a Erzican, Turkey, 1992, ERZ-N record. Plastic input motions for $a_y=0.10g$.

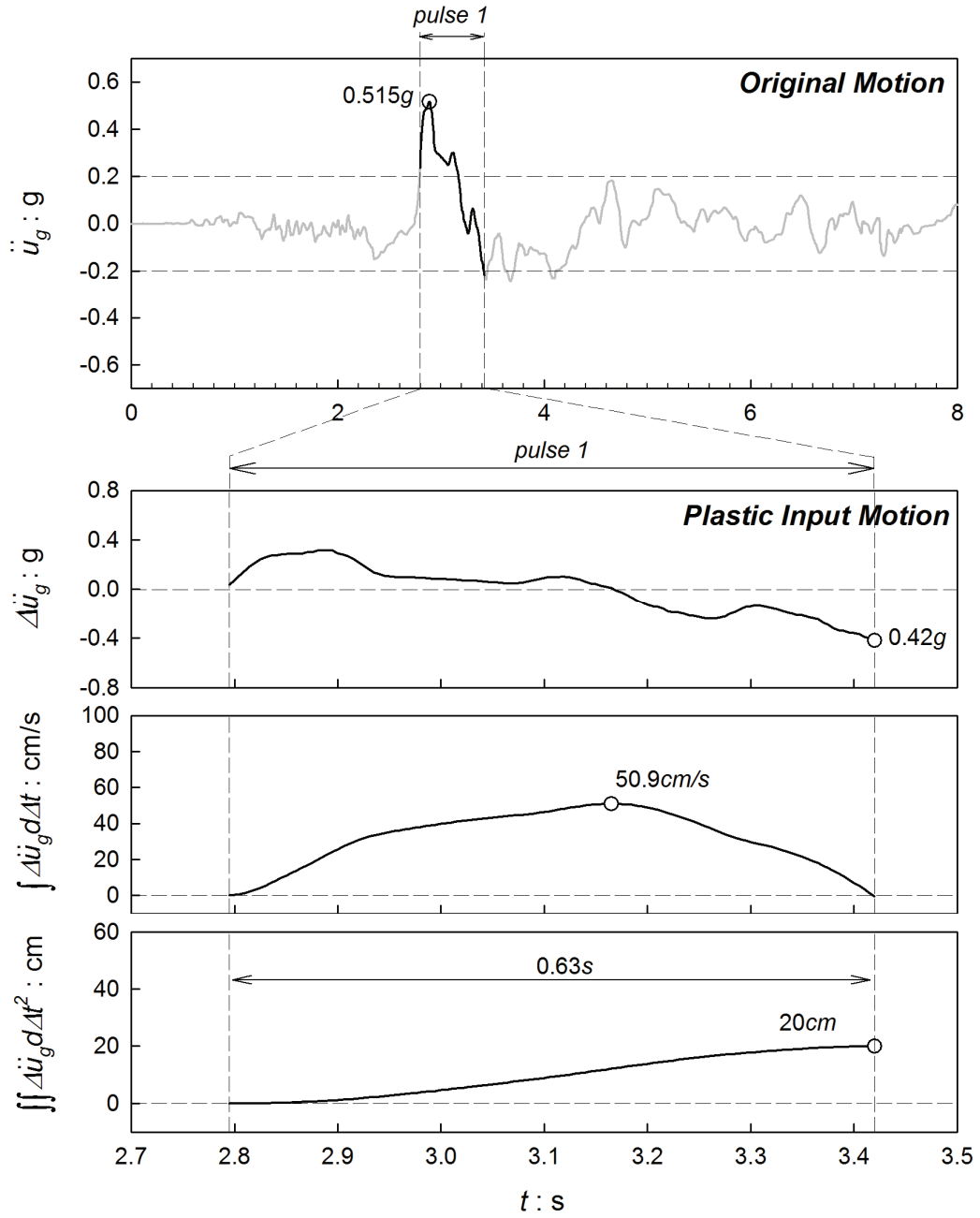


Figure 6.7b Erzican, Turkey, 1992, ERZ-N record. Plastic input motions for $a_y=0.20g$.

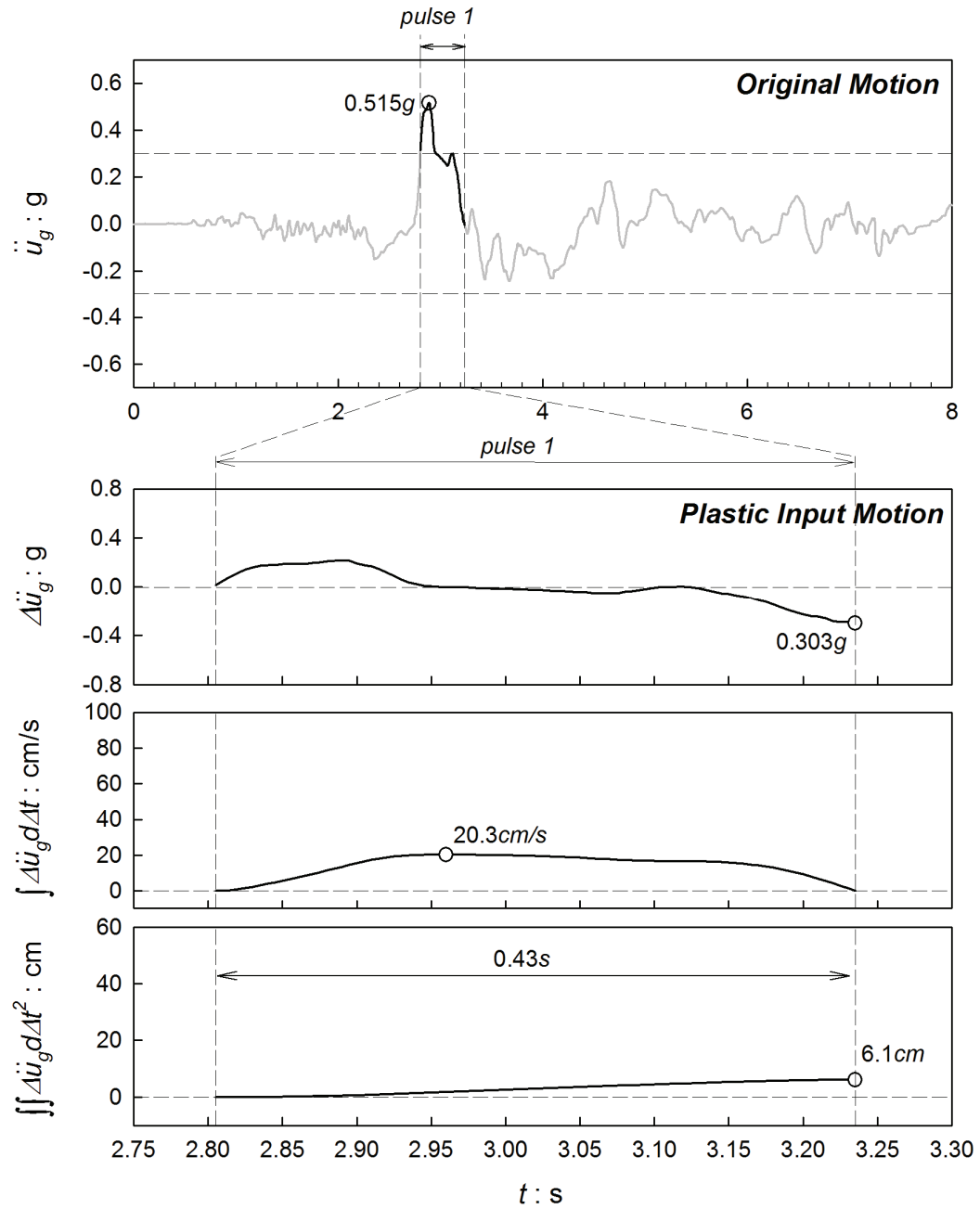


Figure 6.7c Erzican, Turkey, 1992, ERZ-N record. Plastic input motions for $a_y=0.30g$.

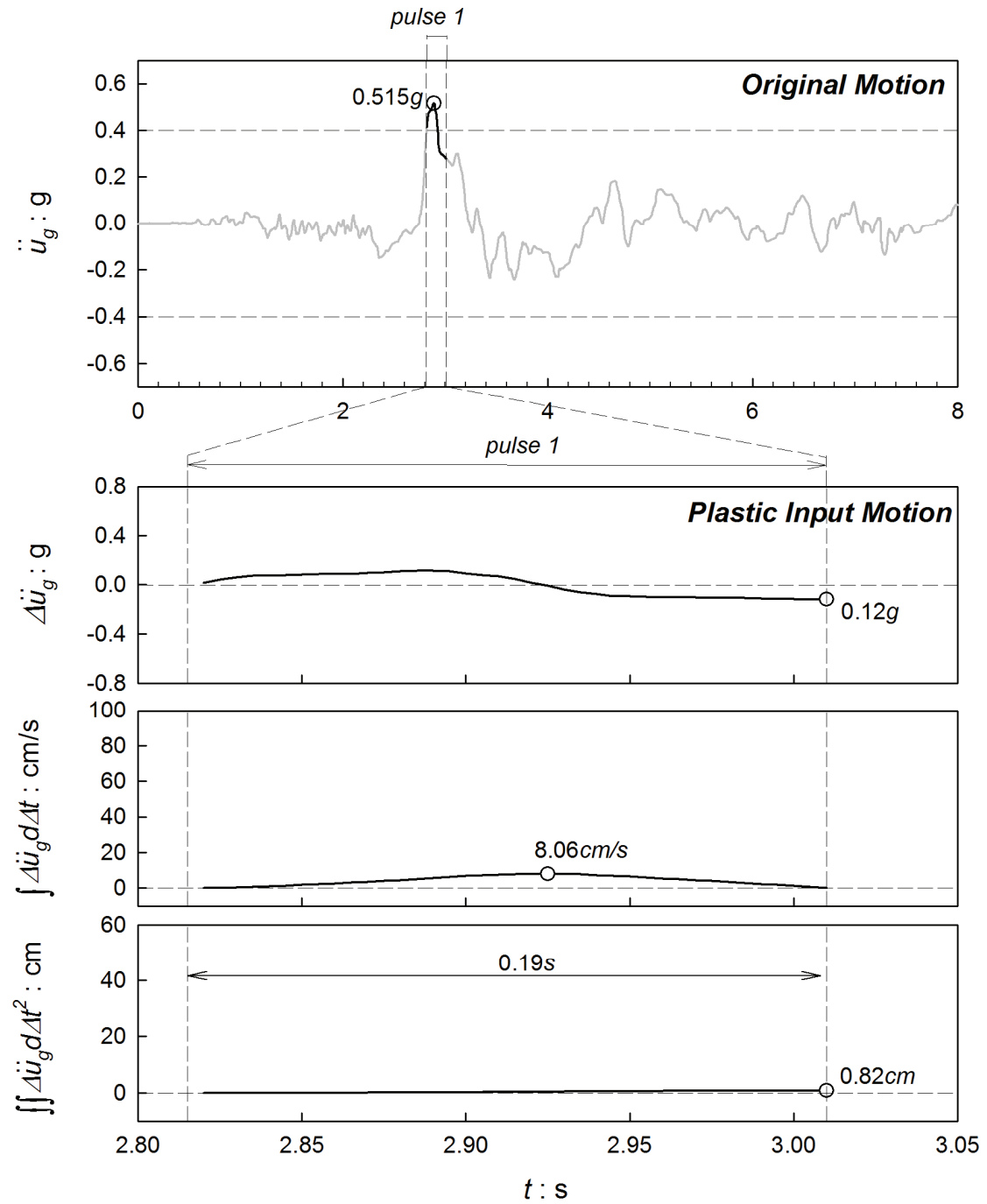


Figure 6.7d Erzican, Turkey, 1992, ERZ-N record. Plastic input motions for $a_y=0.40g$.

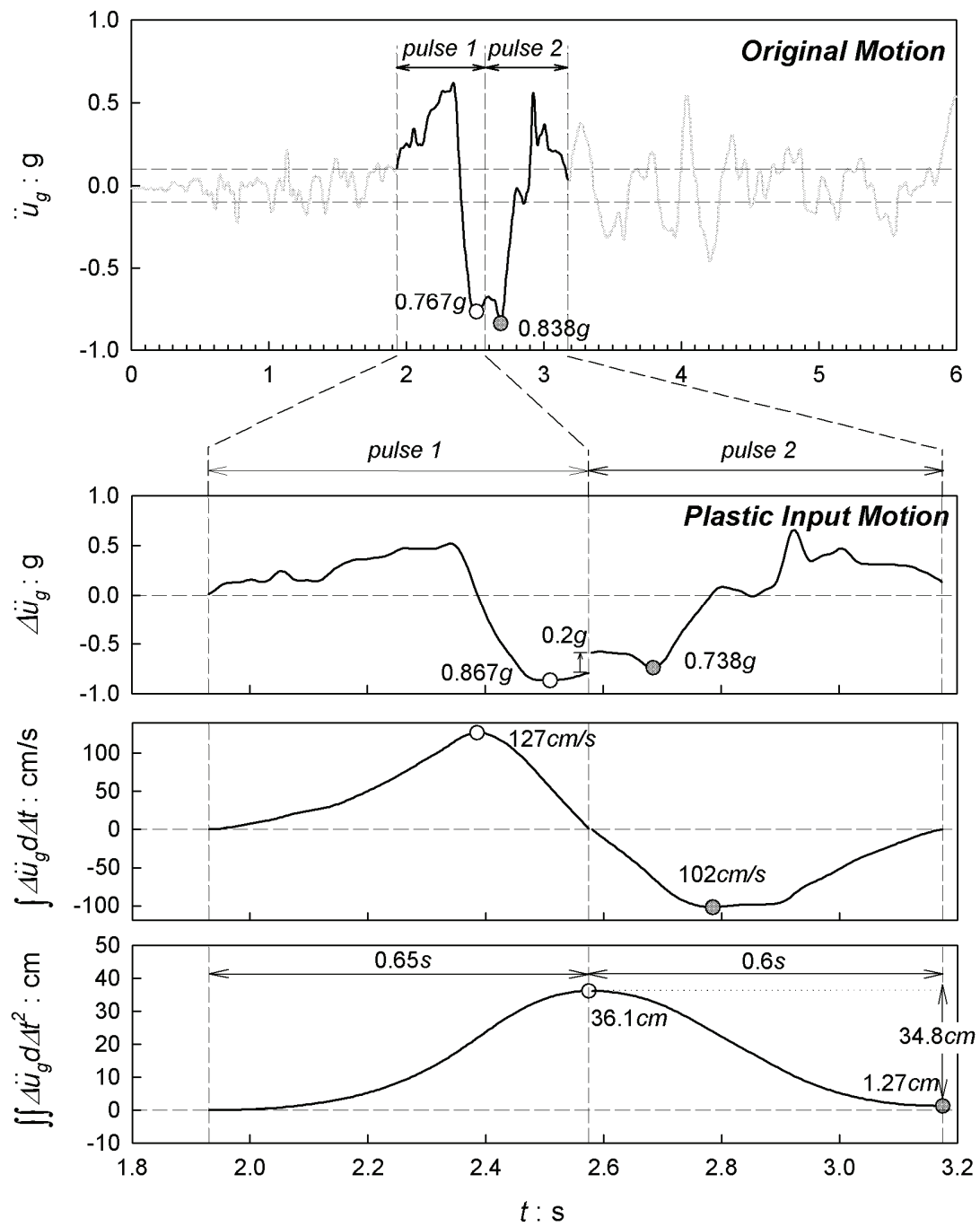


Figure 6.8a Northridge, USA, 1994, Rinaldi-L record. Plastic input motions for $a_y=0.10g$.

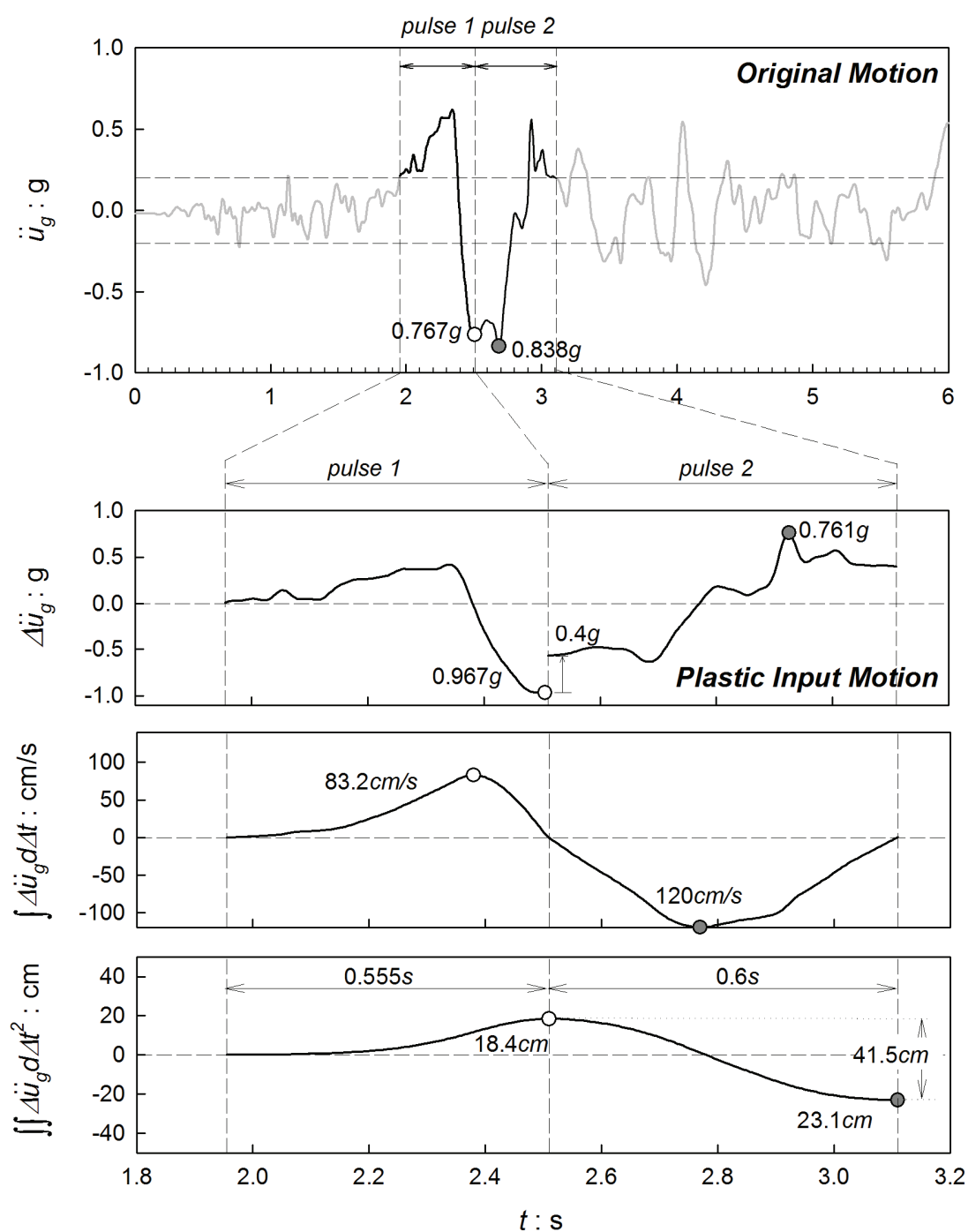


Figure 6.8b Northridge, USA, 1994, Rinaldi-L record. Plastic input motions for $a_y=0.20g$.

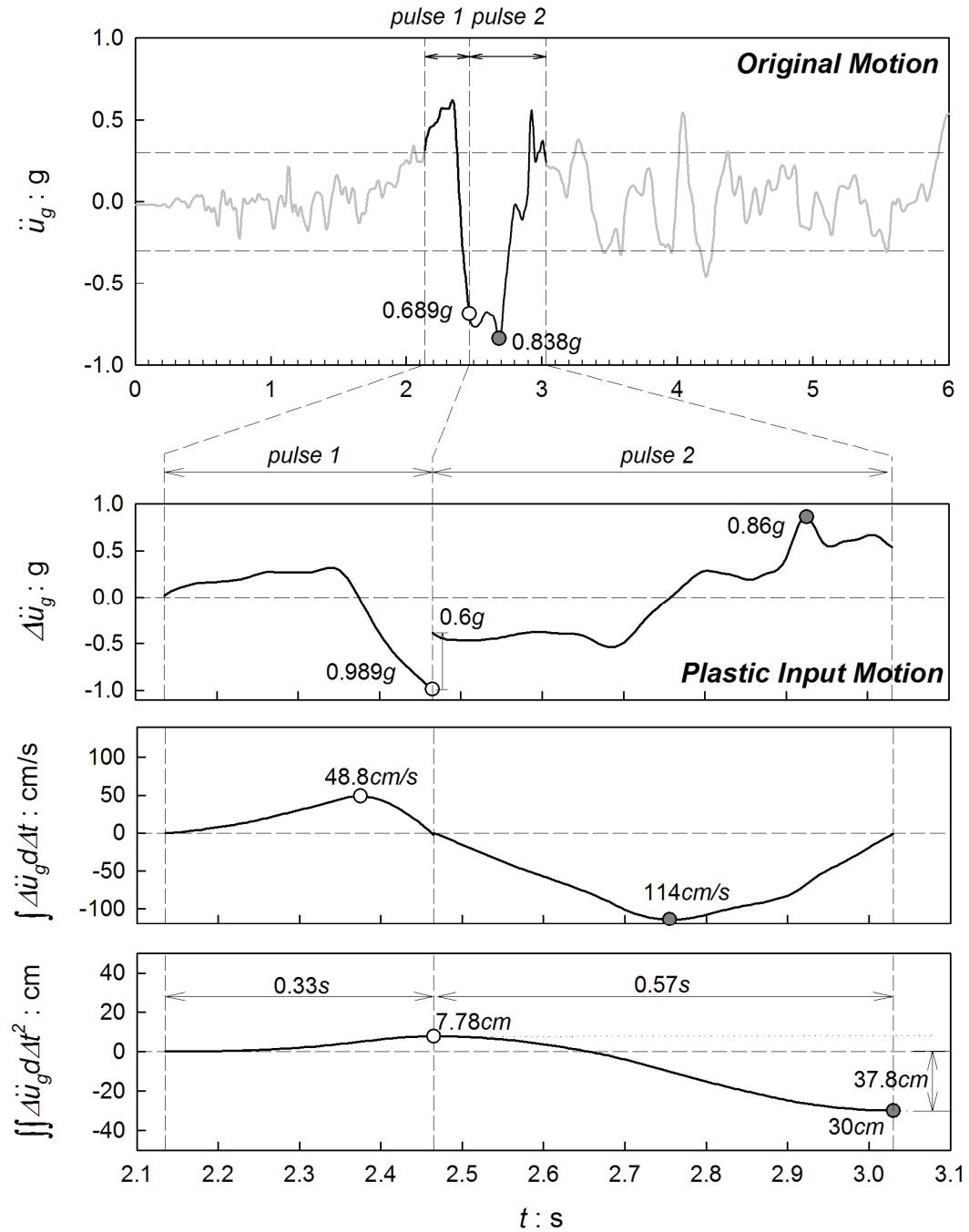


Figure 6.8c Northridge, USA, 1994, Rinaldi-L record. Plastic input motions for $a_y=0.30g$.

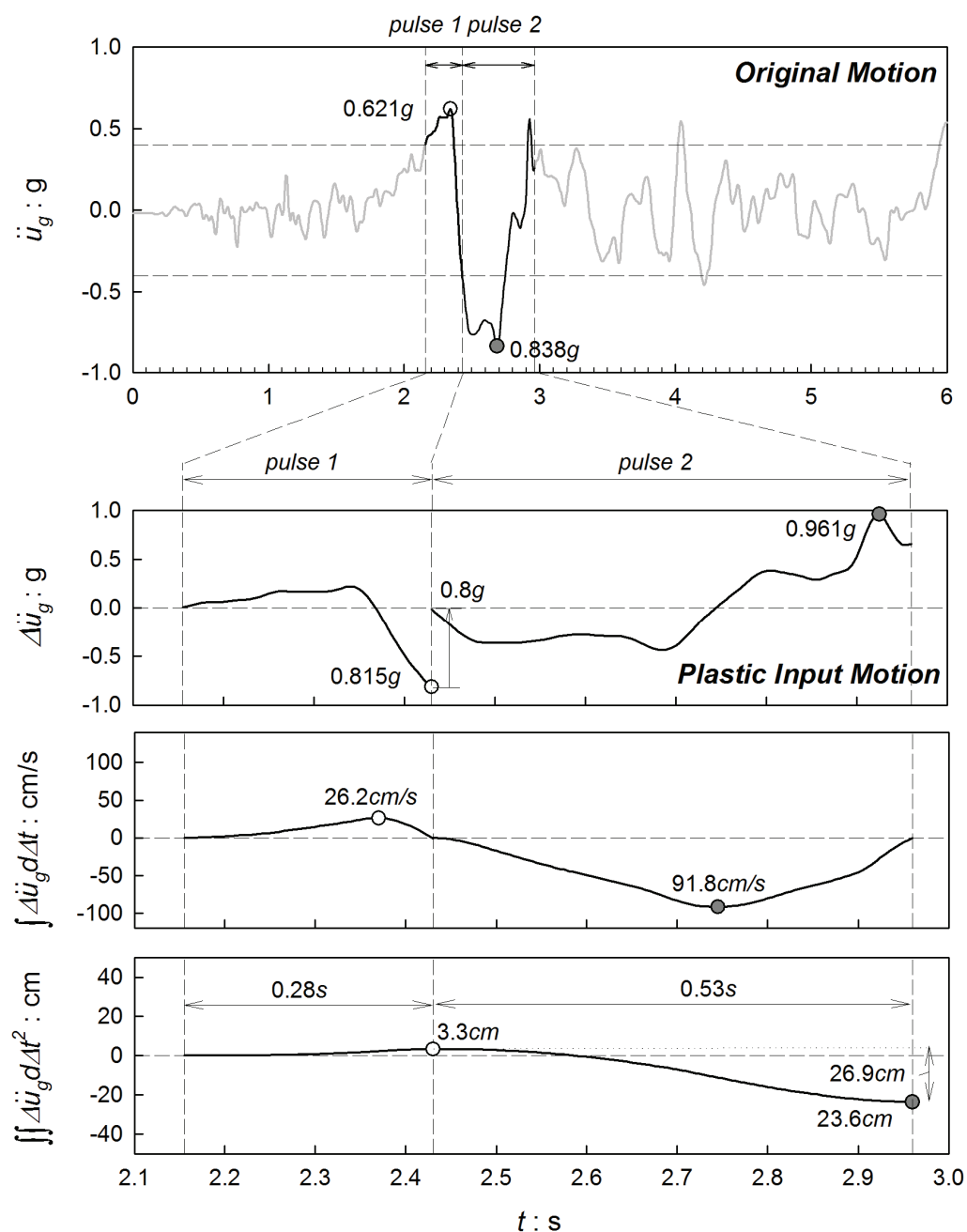


Figure 6.8d Northridge, USA, 1994, Rinaldi-L record. Plastic input motions for $a_y=0.40g$.

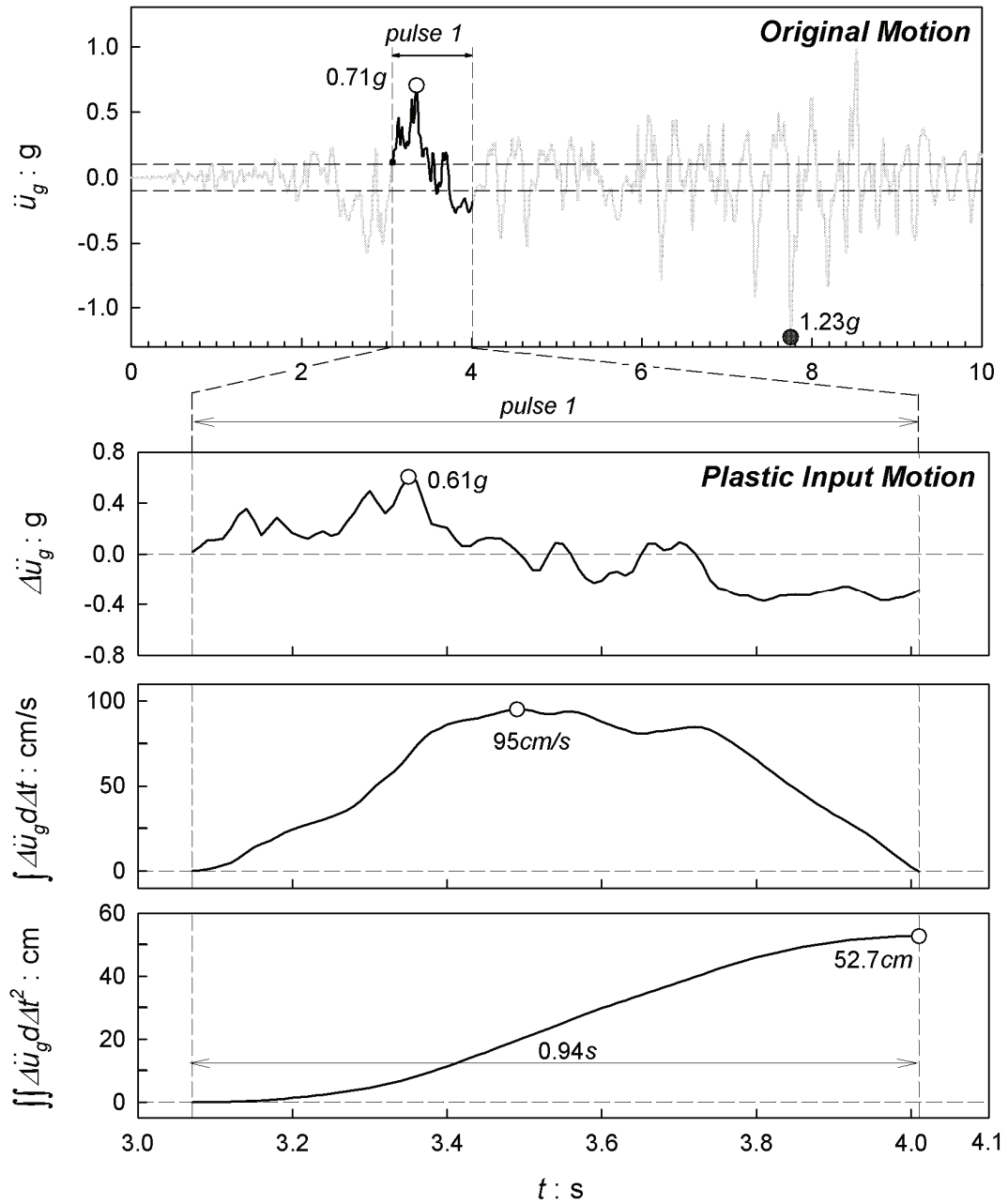


Figure 6.9a San Fernando, USA, 1971, Pacoima Dam-L record. Plastic input motions for $a_y=0.10g$.

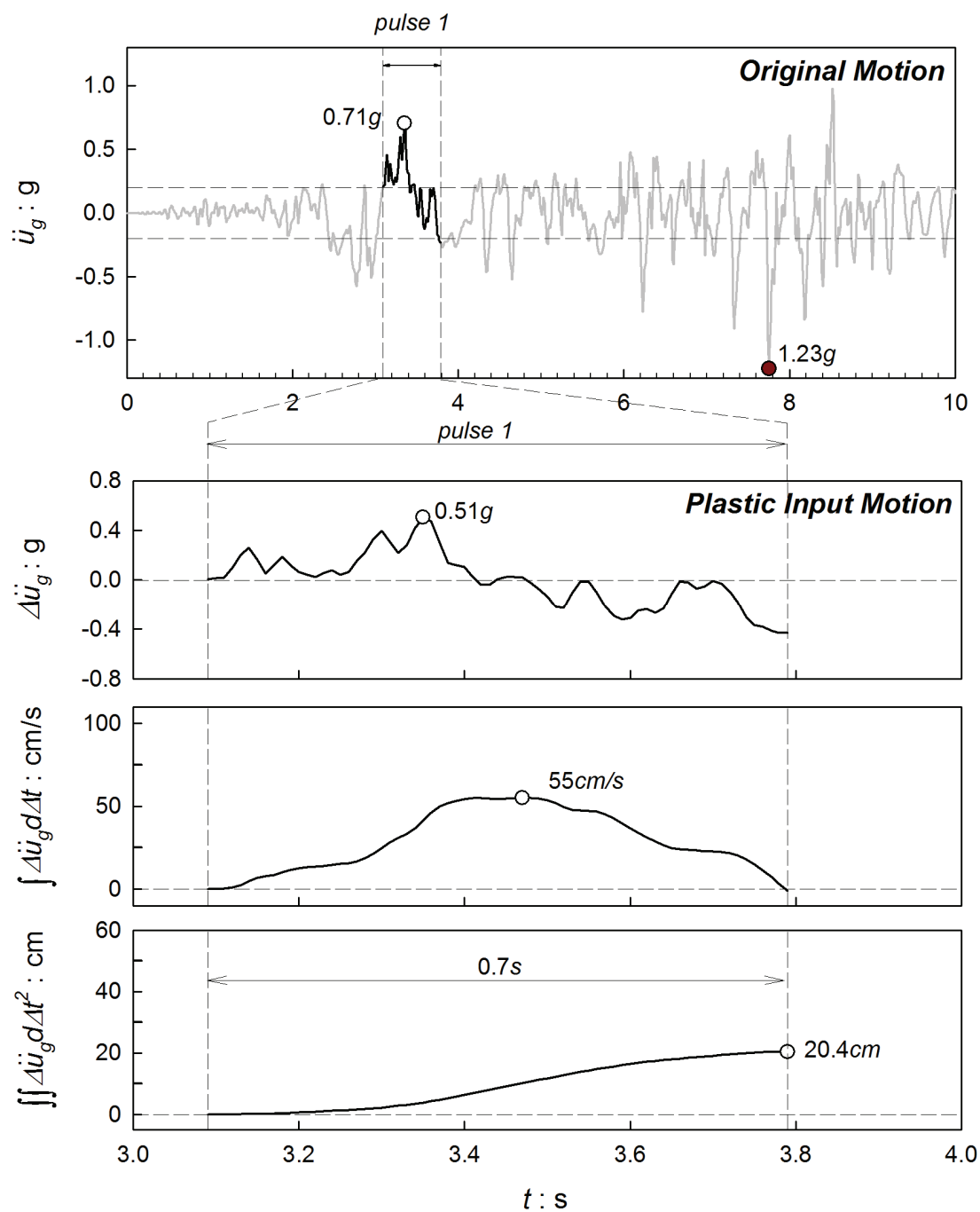


Figure 6.9b San Fernando, USA, 1971, Pacoima Dam-L record. Plastic input motions for $a_y=0.20g$.

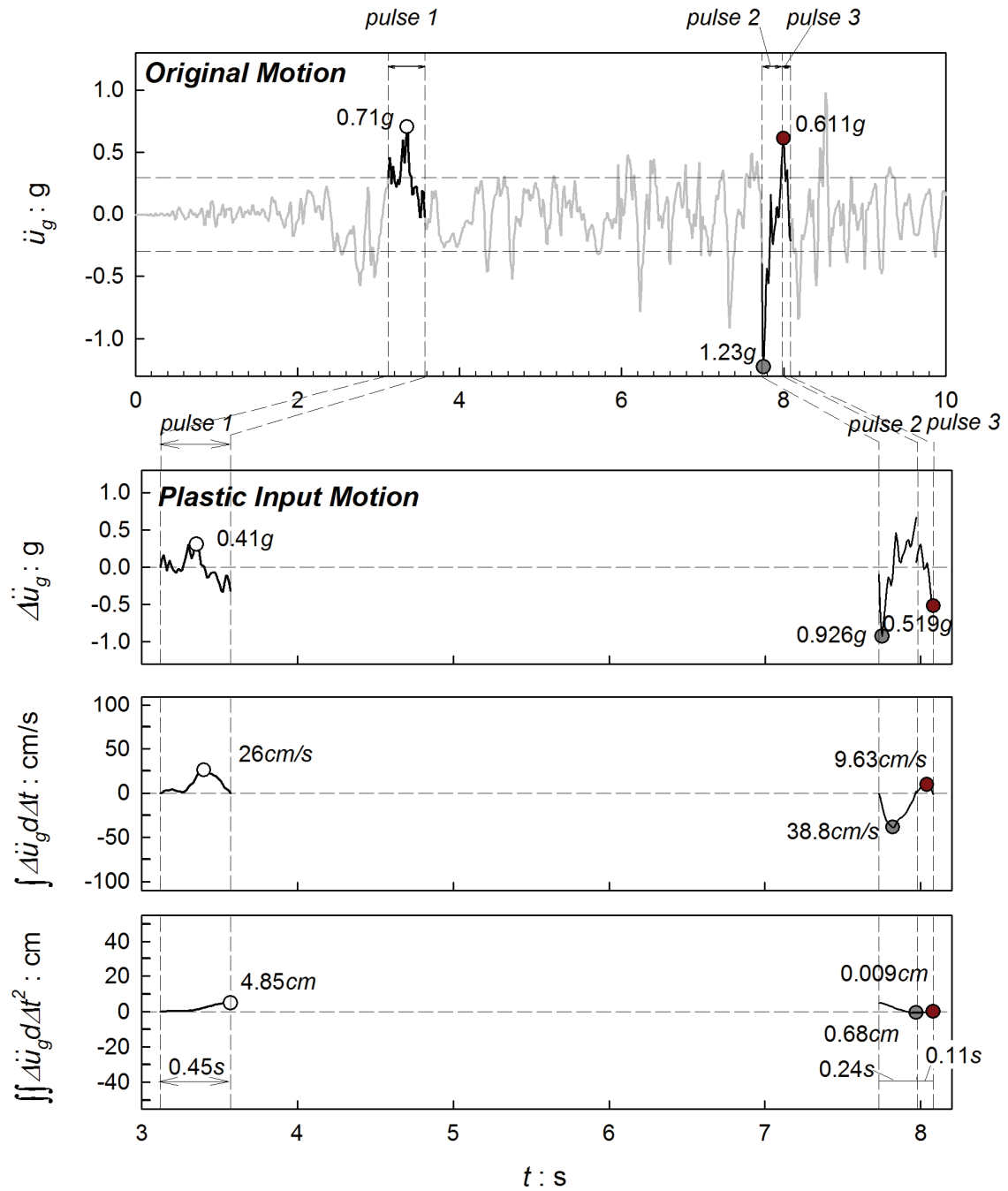


Figure 6.9c San Fernando, USA, 1971, Pacoima Dam-L record. Plastic input motions for $a_y=0.30g$.

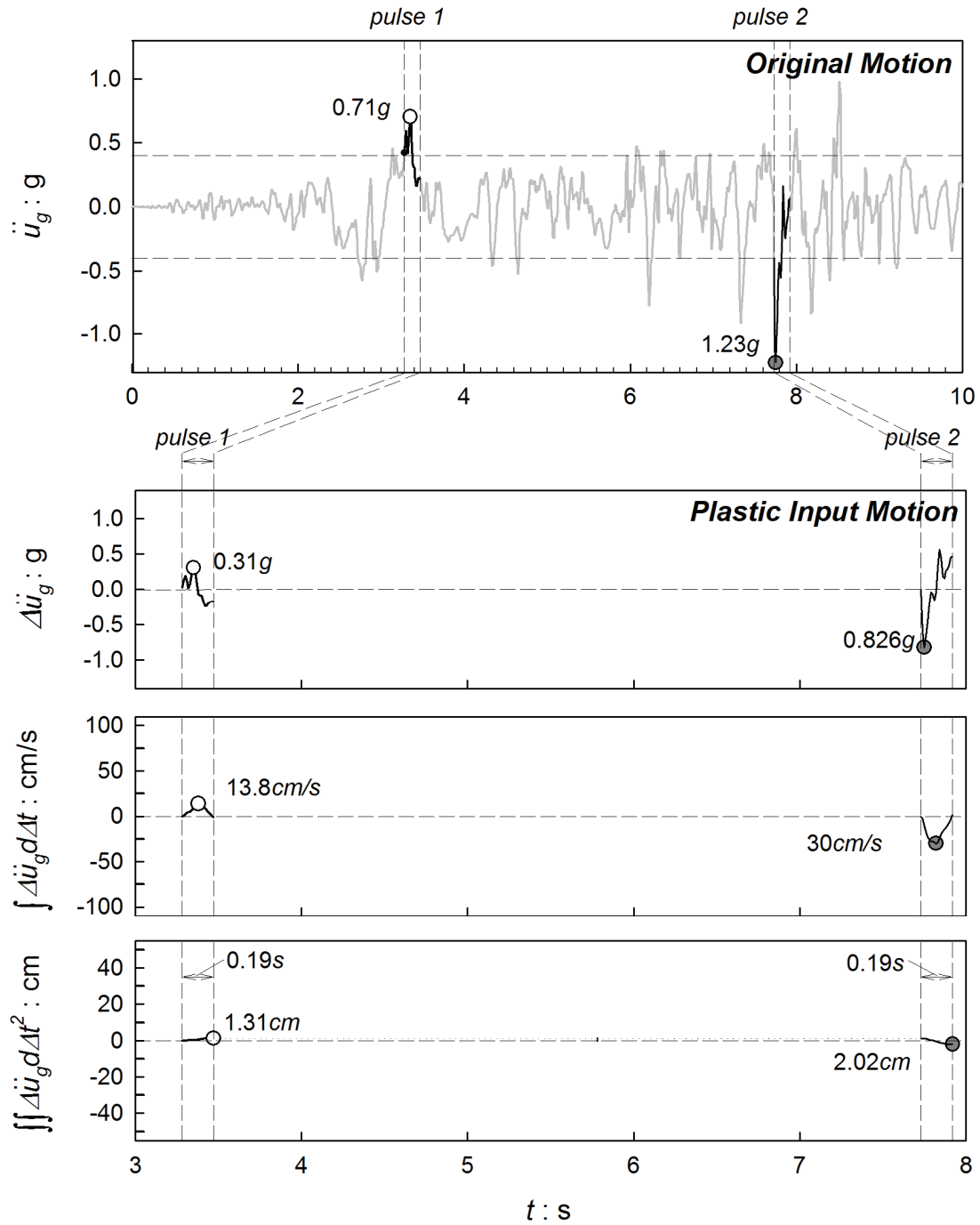


Figure 6.9d San Fernando, USA, 1971, Pacoima Dam-L record. Plastic input motions for $a_y=0.40g$.

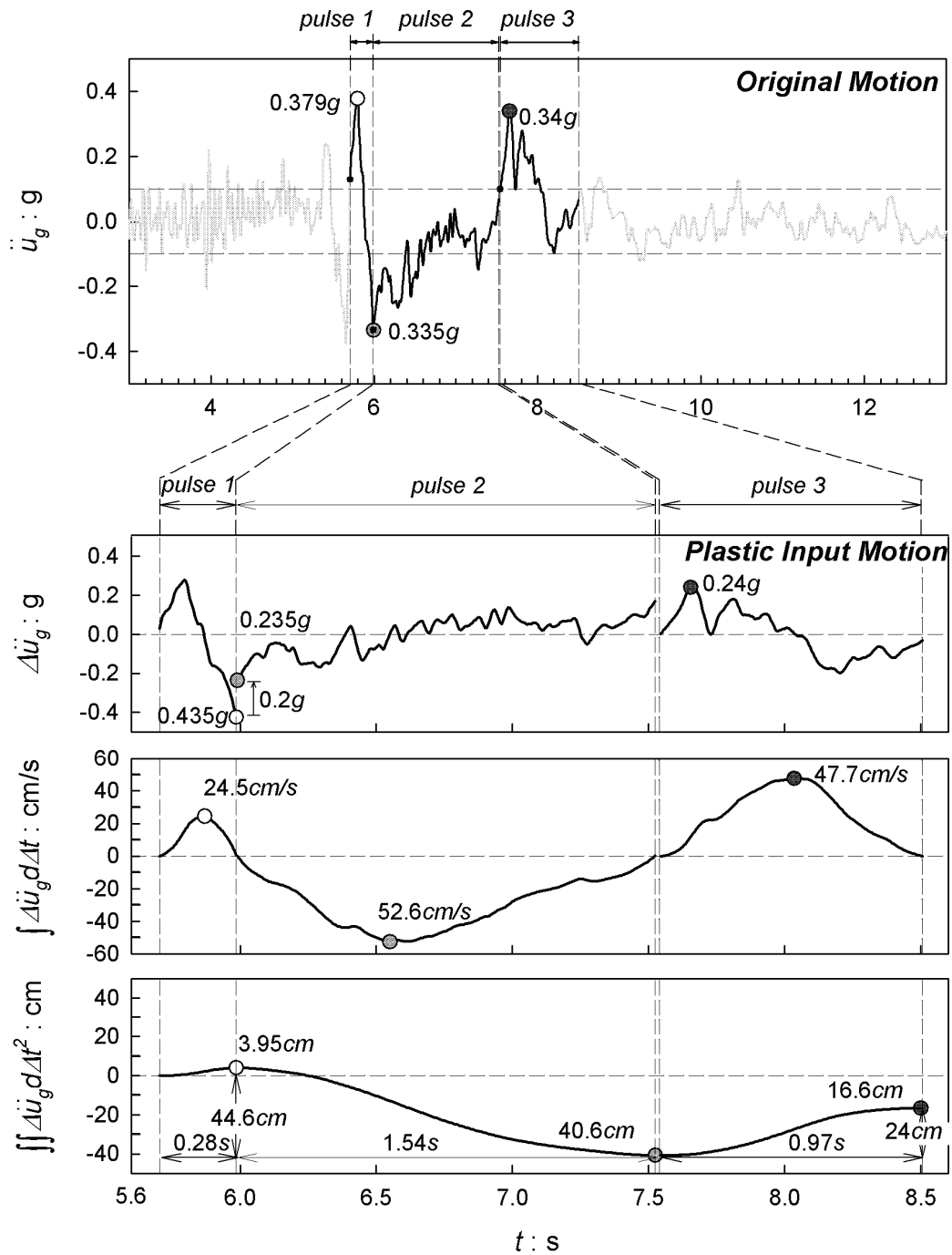


Figure 6.10a Imperial Valley, USA, 1979, E05-SN record. Plastic input motions for $a_y=0.10g$.

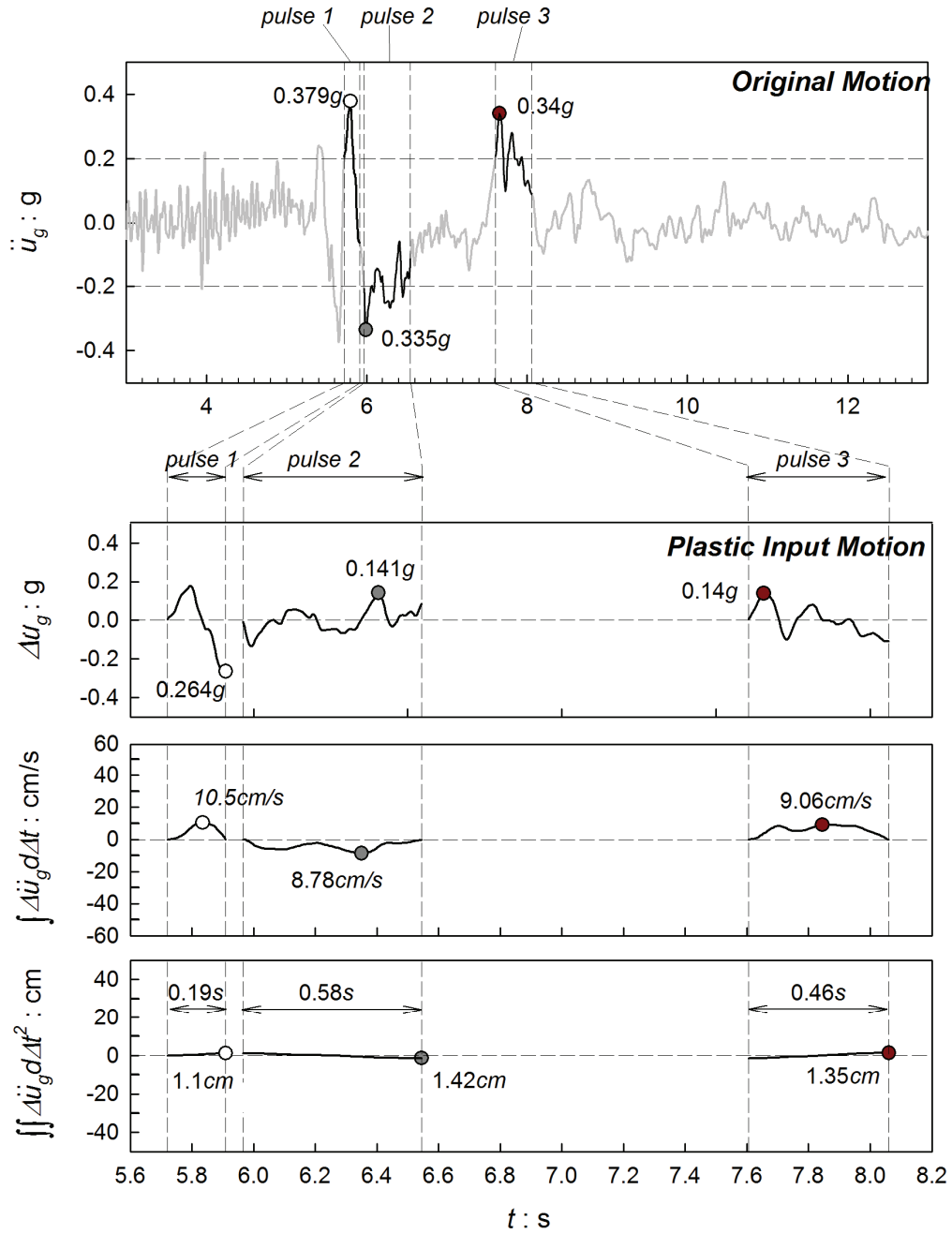


Figure 6.10b Imperial Valley, USA, 1979, E05-SN record. Plastic input motions for $a_y=0.20g$.

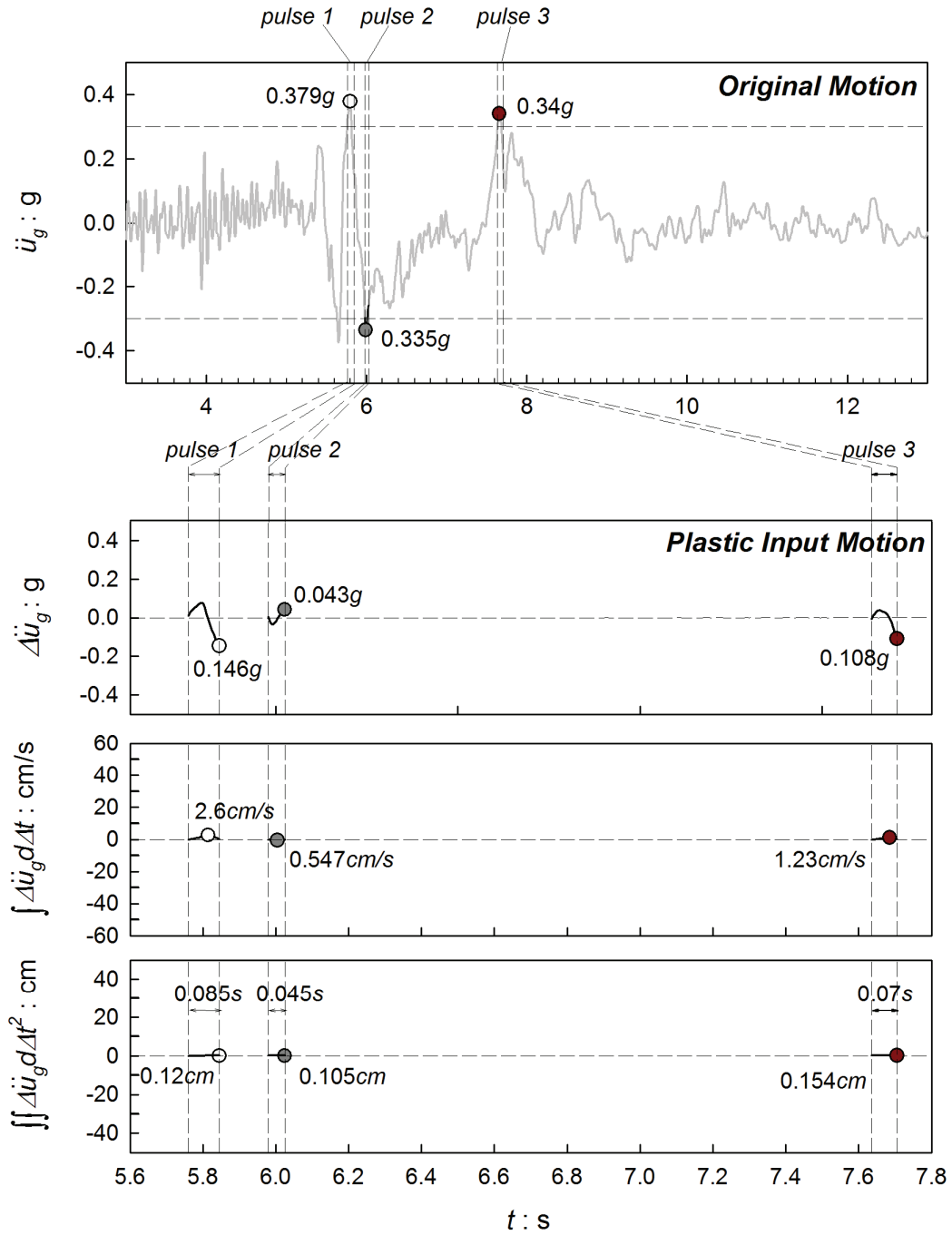


Figure 6.10c Imperial Valley, USA, 1979, E05-SN record. Plastic input motions for $a_y=0.30g$.

Peak values obtained from the above analyses are summarized in Table 6.2, presented by means of the following peak plastic input motion parameters

$$A_{gp} = \max |\Delta \ddot{u}_g| \quad (6.22)$$

$$V_{gp} = \max \left| \int \Delta \ddot{u}_g d\Delta t \right| \quad (6.23)$$

$$D_{gp} = \max \left| \iint \Delta \ddot{u}_g d\Delta t^2 \right| \quad (6.24)$$

which are defined over the duration of the pulses shown in Figs. 6.5 to 6.10. In the above equations, symbol $||$ denotes absolute value and subscript p stands for “*plastic*”. As mentioned earlier, for the system at hand the above equations also represent corresponding peak response parameters (i.e., relative acceleration, velocity and displacement, respectively). However, as this interpretation is limited to the specific system, it is not adopted in the ensuing.

Evidently, yielding resistance tends to impede plastic input motions leading to progressively smaller values of acceleration, velocity and displacement with increasing levels of a_y . Interestingly, in a few cases (e.g., Pacoima Dam and Erzican motions), peak PIM are higher than in the original motions. This counter-intuitive behavior should be attributed to several factors. For accelerations, it is clearly due to the axes shift, which tends to increase accelerations having opposite sign than the one originally exceeded at the initiation of the pulse (see for instance Figs. 6.6, 6.8 and 6.10). On the other hand, for velocities and displacements the increase should be sought in differences in the initial conditions (e.g., negative initial velocities and displacements in the original ground motion, which do not exist in the modified motions).

Table 6.2 Peak plastic input motion parameters for different levels of yielding acceleration considering only major pulses (Method A).

ground motion	(g) (cm/s) (cm)	original waveform	peak plastic motions (Eq.6.19)			
			$a_y = 0.1g$	$a_y = 0.2g$	$a_y = 0.3g$	$a_y = 0.4g$
full – cycle sinusoid	A_g	0.5	0.6	0.614	0.484	0.276
	V_g	156	110	70.6	38	13.3
	D_g	78	44.6	23.3	9.8	2.3
Pacoima Dam	A_g	1.23	0.61	0.51	0.926	0.826
	V_g	113	95	55	38.8	30
	D_g	35.4	52.7	20.4	4.85	2.02
Imperial Valley	A_g	0.379	0.435	0.264	0.146	-
	V_g	90.5	52.6	10.5	2.6	-
	D_g	63.1	40.6	1.42	0.154	-
Erzican	A_g	0.515	0.415	0.42	0.303	0.12
	V_g	84	90	50.9	20.3	8.06
	D_g	27.7	48	20	6.1	0.82
Rinaldi	A_g	0.838	0.867	0.967	0.989	0.961
	V_g	166	127	120	114	91.8
	D_g	28.2	36.3	23.1	30	23.6
Aegion	A_g	0.538	0.56	0.64	0.71	0.372
	V_g	51	37.4	36.8	21.3	8.24
	D_g	9.6	8.5	3.06	1.96	0.597
Chi-Chi	A_g	0.462	0.362	0.281	0.162	0.25
	V_g	263	109	28.2	4.49	3.51
	D_g	430	287	7.97	0.224	0.148

Normalized results for different levels of yielding resistance are provided in Fig. 6.11, plotted in terms of ratios A_{gp}/A_g , V_{gp}/V_g and D_{gp}/D_g as functions of normalized strength a_y/A_g . Naturally, for all records these ratios equal 1 for $a_y/A_g = 0$, and 0 for $a_y/A_g = 1$. As mentioned earlier, these ratios generally decrease with increasing levels of a_y/A_g , yet may exhibit values higher than 1 for small values of yielding resistance. The drop in their values reaches dramatic proportions for a_y/A_g exceeding approximately 0.75. Interestingly, the full – cycle sinusoidal pulse (Eq. 6.25, Fig 6.12) seems to represent quite well, at least on a qualitative basis, the behaviour of the more complex ground motions.

$$\ddot{u}_g(t) = \begin{cases} A_g \sin \pi \frac{t}{t_d} & , \quad 0 \leq t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases} \quad (6.25)$$

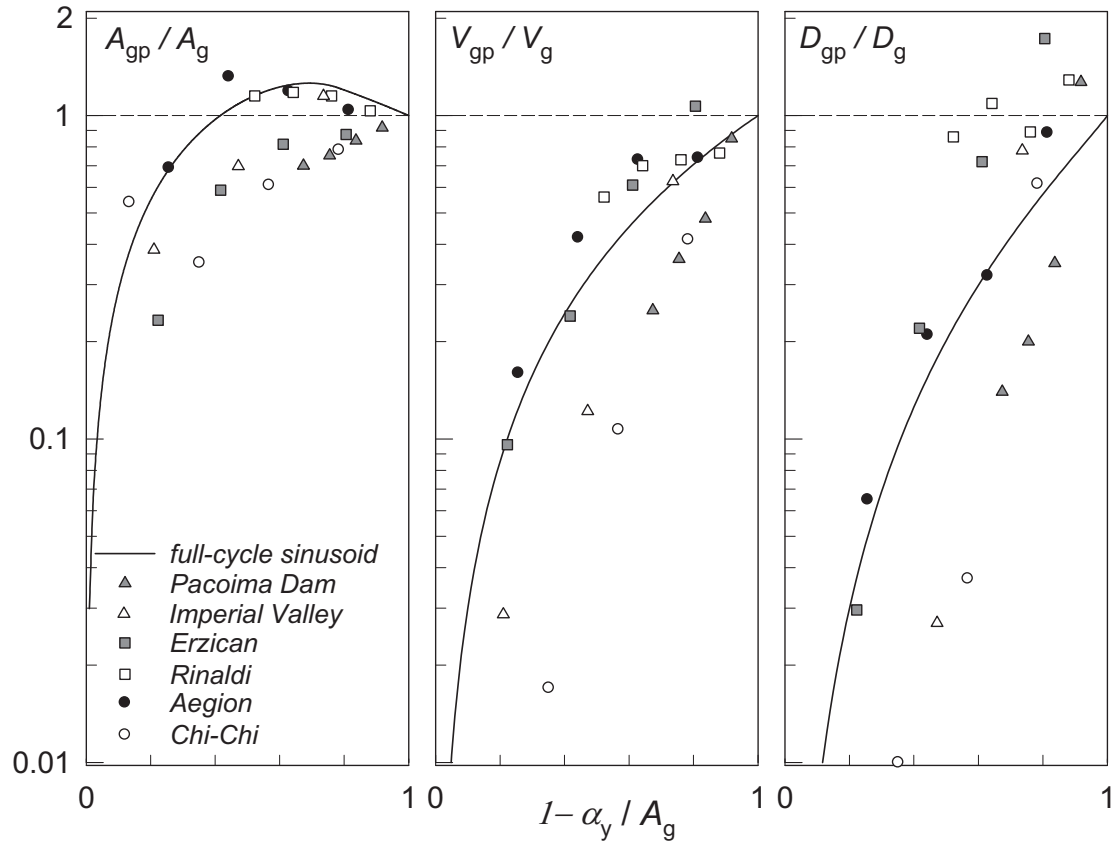


Figure 6.11 Peak plastic input motion parameters for the waveforms of Figures 6.3 and 6.12.

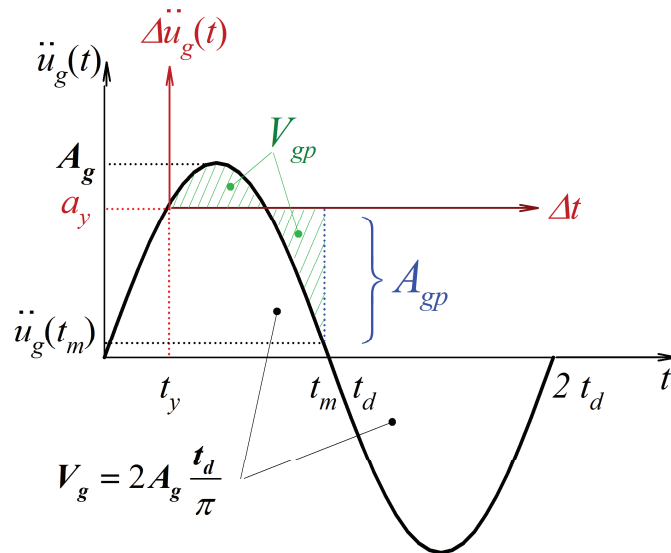


Figure 6.12 Graphical representation of the shift – theorem on a full – cycle sinusoidal pulse.

For this idealized waveform which is graphically illustrated in Figure 6.12, the following closed-form solution is possible (see also Table B2 in Appendix B).

$$\frac{V_{gp}}{V_g} = \left| \cos\left(\pi \frac{t_y}{t_d}\right) - \pi \frac{a_y}{A_g} \left(\frac{1}{2} - \frac{t_y}{t_d}\right) \right| \quad (6.26)$$

$$\frac{A_{gp}}{A_g} = \begin{cases} \max \left\{ \left(1 - \frac{a_y}{A_g}\right), \left| \frac{a_y}{A_g} - \sin\left(\frac{\pi t_m}{t_d}\right) \right| \right\} & , t_m < 3t_d/2 \\ \left(1 + \frac{a_y}{A_g}\right) & , t_m \geq 3t_d/2 \end{cases} \quad (6.27)$$

$$\frac{D_{gp}}{D_g} = \left| -\frac{a_y}{A_g} \frac{\pi}{4} \left(\frac{t_m}{t_d} - \frac{t_y}{t_d}\right)^2 + \frac{1}{2} \left(\frac{t_m}{t_d} - \frac{t_y}{t_d}\right) \cos\left(\pi \frac{t_y}{t_d}\right) - \frac{1}{2\pi} \left(\sin\left(\pi \frac{t_m}{t_d}\right) - \sin\left(\pi \frac{t_y}{t_d}\right)\right) \right| \quad (6.28)$$

In Eqs (6.26–6.28), t_y and t_m denote time of yielding and time of peak displacement, and are given by the solution of the following equations (see also Table B2 in Appendix B).

$$\frac{t_y}{t_d} = \frac{1}{\pi} \arcsin\left(\frac{a_y}{A_g}\right) \quad (6.29)$$

$$\cos\left(\pi \frac{t_y}{t_d}\right) - \cos\left(\pi \frac{t_m}{t_d}\right) = \pi \frac{a_y}{A_g} \left(\frac{t_m}{t_d} - \frac{t_y}{t_d}\right) \quad (6.30)$$

t_d being the half duration of the pulse. A somewhat different set of results, not referring to near-fault motions, has been presented by Newmark and Rosenblueth (1971).

An interesting set of results is presented in Table 6.3, referring to the case where $\alpha_y = 0.1g$. Evidently, this is a situation where yielding resistance may not be sufficiently strong to suppress a number of minor pulses present in the accelerograms (see for instance the Pacoima record in Fig. 6.9a). Two types of analyses are reported:

(A) the analysis presented earlier based only on the major pulses shown in Figs. 6.5a-6.10a

(B) a rigorous numerical integration of the whole earthquake record by means of Eq. (6.19).^{**}

The integration limits were specified as depicted in Fig. 6.4, yet the full length of the time history was utilized. With minor exception, the results from the two approaches compare very well, with maximum discrepancies in plastic input velocities and displacements not exceeding 7% and 18%, respectively. In most cases the error is negligible. This observation is in agreement with earlier finding suggesting that most of the seismic demand in near-fault motions is concentrated in a small part of the earthquake record (Bertero *et al.* 1978, Tang & Zhang 2011, Mylonakis & Voyagaki 2006, Mylonakis & Reinhorn 2001, Chopra & Chintanapakdee 2001, Malhotra 1999). This attribute could be potentially useful towards reducing computational effort in dynamic analyses of more complex yielding systems.

^{**} Rigorous results for $\alpha_y=0.2, 0.3$ and $0.4g$ are provided in Appendix E, Table B1.

Table 6.3 Comparison of peak plastic input motion parameters considering only major pulses (method A) and by rigorous integration of the whole earthquake record (method B); $a_y=0.1g$.

Ground Motion	(g) (cm/s) (cm)	Method A			Method B		Deviation (B-A) / B %
		No. of effective pulses	Pulse 1	Pulse 2	Pulse 3	Peak value [†] (A)	Peak value (B)
Pacoima Dam	A_{gp}	1	0.61			0.61	-1.13
	V_{gp}		95			95	95.6
	D_{gp}		52.7			52.7	45
Imperial Valley	A_{gp}	3	-0.435	-0.235	0.24	-0.435	-0.435
	V_{gp}		24.5	-52.6	47.7	-52.6	-56.7
	D_{gp}		3.95	-44.6	24	-40.6	-49.3
Erzican	A_{gp}	1	0.415			0.415	0.415
	V_{gp}		90			90	90
	D_{gp}		48			48	48
Rinaldi	A_{gp}	2	-0.867	-0.738		-0.867	-0.867
	V_{gp}		127	-102		127	127
	D_{gp}		36.1	-34.8		36.1	36.3
Aegion	A_{gp}	2	0.56	-0.543		0.56	0.56
	V_{gp}		-37.4	36		-37.4	-37.6
	D_{gp}		-8.5	4.76		-8.5	-8.5
Chi-Chi	A_{gp}	2	-0.362	0.24		-0.362	-0.362
	V_{gp}		-66.5	-109		-109	-109
	D_{gp}		-29.5	-257		-287	-265

[†] Overall peak displacement is defined as the maximum, in absolute terms, considering the succession of all individual pulse displacements as a single time history (Figs 5-10).

6.5 Discussion

It is fair to mention that truncated (over the dependent variable) time histories have long been employed in earthquake engineering, mostly in the realm of the so-called “Newmark sliding block” model and its variants (Newmark 1965, 1971, Ambraseys & Menu 1988, Sarma 1975). Although conceptually related, the proposed theory differs from the early model in a number of important aspects: First and foremost, Newmark’s model is limited to rigid-perfectly plastic systems (experiencing mainly unilateral response), whereas the present approach can be extended to general types of yielding systems (e.g., Eq. 6.21) subjected to both unilateral and bilateral response. Second, the truncated ground acceleration in the early model is used primarily as a graphical device, without the mathematical basis provided in the present work. Third, the concept of plastic input motion, through the “right-hand siding” of yielding resistance, is not part of the classical model. Fourth, the importance of relative-to-ground acceleration, as a pseudo-linear process for assessing earthquake demand in yielding systems is not part of the early work.

A comparison of the predictions of the proposed approach against results from established methods in the literature is presented in Fig. 6.13 for the six recorded motions in Fig. 6.3 and Table 6.1. Apart from the aforementioned rigorous numerical integration, three additional approaches are presented:

(A) Newmark’s “upper-bound” scheme in Eq. (6.7) evaluated for $\chi_I = 0.5$ and $b_I = 1$ pertaining to bilateral response (Newmark 1965, 1971)

(B) the spectral approach recommended by Eurocode 8 (2004) using an equivalent simple oscillator of natural period $T = 2\pi\sqrt{(D_{ge}/a_y)}$ and damping $\zeta = 0.637$

(C) the approximate formula of Newmark (1965, 1971) corresponding to the response of a rigid-perfectly plastic system to an acceleration pulse of constant amplitude and half-cycle duration. The solution to this problem is expressed by

$$u_{\max} = \frac{V_g^2}{2A_g} \frac{1-\eta}{\eta} \quad (6.31)$$

with the symbols in the right side having the same meaning as in Eq. (6.7). The comparison represents an extension of that presented in Table III covering a wider range of yield strengths. With minor exception, Newmark's formulas overestimate (often by a significant amount) the response and, thereby, can be viewed as conservative gross predictors, especially for small values of yielding resistance a_y/A_g . This is particularly true with the "upper bound" solution in Eq. (6.7). On the other hand, the spectral approach of EC-8 generally yields lower predictions than Newmark's formulas – especially at low values of dimensionless strength ratio a_y/A_g . The performance of the EC-8 method is judged as superior to the earlier methods. The proposed approach yields almost exact predictions of peak displacement for values of a_y/A_g ranging from approximately 0.1 to 0.85 for the six records, suggesting that use of only major pulses is realistic. An exception is the Pacoima Dam record (upper left graph in Fig. 6.13) for which the proposed method may underestimate the response at large a_y/A_g 's. This should not come as a surprise given the large number of high amplitude spikes present in this record (Fig. 6.9).

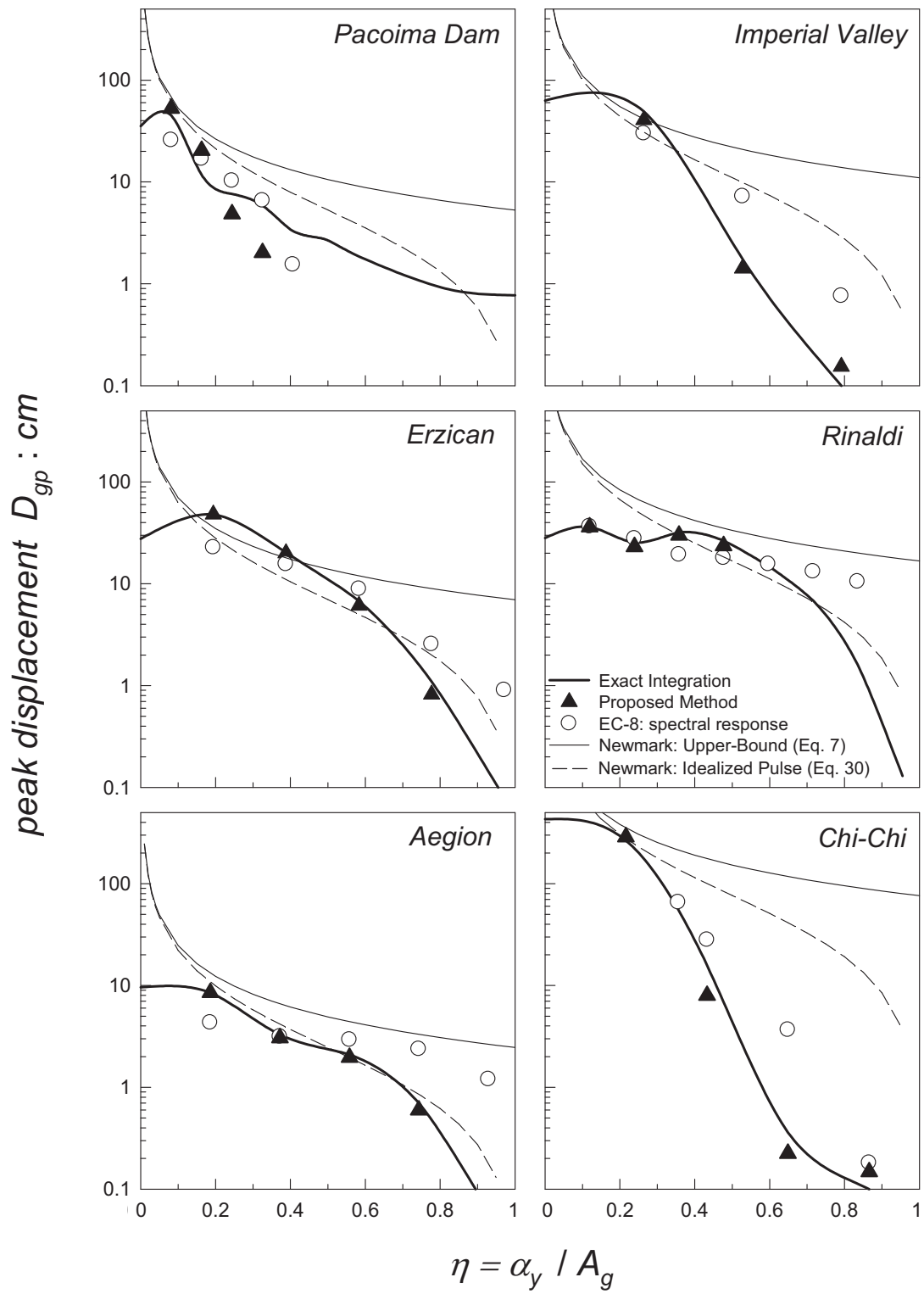


Figure 6.13 Comparison of the predictions of the proposed method against rigorous numerical integration and established approximate methods for the near-fault motions of Figure 6.3.

As a final remark, the importance of dimensionless ratio (a_y/A_g) for describing displacement response of sliding blocks is worth investigating. This can be done by operating on the right-hand side of Eq. (6.21) as follows:

$$\ddot{u}_g(t) - \text{sgn}(\dot{u}) a_y = A_g \left[\left(\frac{\ddot{u}_g(t)}{A_g} \right) \pm \left(\frac{a_y}{A_g} \right) \right] \quad (6.32)$$

Evidently, (a_y/A_g) cannot be separated from dimensionless ground acceleration $(\ddot{u}_g/A_g)$, thereby it merely serves as a shift (“up” or “down”) modifier of the forcing function depending on the value of the signum function. Accordingly, (a_y/A_g) is not a multiplier of the forcing function, thereby it cannot be viewed as a scaling factor of the response in the classical sense.

Nevertheless, (a_y/A_g) is useful in establishing limits for the response as, naturally, $u \rightarrow u_g$ for $(a_y/A_g) = 0$ and $u \rightarrow 0$ for $(a_y/A_g) = 1$ (Newmark 1965). The existence of these limits may explain the successful graphical representations of the response in terms of this parameter presented in Figs 6.11 and 6.13.

An exception to the above interpretation is the case when ground acceleration is constant with time. In this highly idealized situation, Eq. (6.32) simplifies to

$$\ddot{u}_g(t) - \text{sgn}(\dot{u}) a_y = A_g \left[1 - \left(\frac{a_y}{A_g} \right) \right] \quad (6.33)$$

in which, evidently, constant $[1 - a_y/A_g]$ is a multiplier of the forcing term and, thereby, a scaling factor of the response. [This explains the presence of factor $(1-\eta)$ in Eq. (6.31) derived by Newmark under the specific assumption.]

It should be noticed that a useful mathematical property involving a_y in the context of this analysis is

$$a_y = \frac{\pm 1}{t_m - t_y} \int_{t_y}^{t_m} \ddot{u}_g(t) dt \quad (6.34)$$

which suggests that yielding acceleration is, in absolute terms, equal to the mean value of the original ground acceleration over the time interval corresponding to a yielding branch. This can be viewed as a consequence of Eq. (6.10), i.e. to ensure zero relative to ground velocity at the beginning and end of yielding.

6.6 Conclusions

The main conclusions of this chapter are:

1. A transformation approach was derived for determining and interpreting the response of rigid-perfectly plastic systems to dynamic loading. Contrary to conventional scaling procedures which are invariably multiplicative, the derived relation is additive, leading to a shift in the ordinates and the abscissa of the excitation function by an amount equal to the yielding resistance and the time of yielding of the system, respectively. Accordingly, the proposed approach cannot be interpreted as a scaling law in the classical sense.
2. Contrary to classical analytical and numerical analysis schemes, the proposed theory does not require information as to the value of ground displacement and velocity at yielding. Whereas this property may appear anticipated for the particular system, it might not be so for more general classes of yielding systems such as those obeying elastic-perfectly plastic force-displacement rules.

3. By introducing pertinent modified excitation parameters, the response was shown to be derivable by integrating the modified acceleration time history in a linear fashion by conventional analysis procedures (Eq. 6.19). This, however, should be viewed merely as a mathematical device, for the nonlinearity is inherent in the response.

4. The transformed time history of ground acceleration is related uniquely to the development of plastic deformation. Accordingly, this time history and its time integrals are referred to as plastic input acceleration (*PIA*), velocity (*PIV*) and displacement (*PID*), respectively.

5. Peak values of the plastic input motion time histories were determined, which encompass the characteristics of the ground motion as well as those of the yielding system. These peak input motions can be substantially (yet not necessarily) smaller than those of the original ground motion. The increase in values may be particularly pronounced in the acceleration component (a discontinuous function) due to the presence of alternating positive and negative cycles of ground acceleration.

6. While elements of the proposed approach have long been utilized (notably in Newmark's sliding block theory), they have not been theoretically investigated and generalized as done in this work. The method was shown to be applicable to more general classes of dynamic systems such as those having elastic-perfectly plastic and bilinear yielding forced-displacements relationships. These systems will be dealt with in Chapter 7.

7. For the rigid-plastic system, plastic input acceleration can be interpreted as relative-to-ground acceleration (Eq. 6.21), a parameter that has received limited attention in earthquake engineering and seismology. However, this is a special case and should not be generalized for more general classes of yielding systems.

8. The dimensionless strength factor (a_y/A_g) cannot be viewed as a scaling factor for it is not a multiplier of the forcing term (Eq. 6.32). It was shown to be so only for the idealized case of constant ground acceleration (Eq. 6.33). It is nevertheless a useful parameter for providing limits to the response, as $u \rightarrow u_g$ for $(a_y/A_g) = 0$ and $u \rightarrow 0$ for $(a_y/A_g) = 1$. It was also shown to represent the average ground acceleration over a yielding response branch (Eq. 6.34).

9. It is fair to mention that in the absence of a small number of distinct pulses, the proposed approach does not guarantee a smaller number of operations in carrying out dynamic response analysis of a rigid-perfectly plastic system. This is because the suite of points of yielding and unloading should be determined in the classical way that is, by tracing the time history of ground acceleration over the whole length of the record. The strength of the proposed approach is in providing insight in the physics of the problem and should not be viewed as a shortcut for non-linear dynamic analysis of such systems.

Plastic Input Motion:

a Transformation for the Response of Yielding Oscillators

7.1 Synopsis

A transformation method is presented for the response of yielding oscillators to dynamic loading. The method employs a translation in the ordinates and the abscissa of the excitation function by means of a pair of parameters uniquely dependent on the yielding resistance and the vibrational characteristics of the system. By this approach: (1) the differential operator becomes linear like, with the nonlinearity transferred to the right-hand side; (2) the initial conditions are simplified; (3) the modified forcing term becomes uniquely associated with development of plastic deformation. The theory is applied to various yielding oscillators subjected to idealized earthquake pulses, with the modified excitation function termed “*Plastic Input Motion*” (*PIM*). A procedure for applying the method to general waveforms is provided. The co-ordinates of *PIM* may be discontinuous and significantly smaller than those of the original excitation function, as a considerable amount of ground acceleration is devoted to overcome the elastic resistance of the system. The theory can be useful in earthquake engineering, by offering a replacement to physical ground motions with system-dependent *PIM*’s for establishing demand indices.

7.2 Preface

As already discussed in Chapter 6, the majority of demand indices in earthquake engineering rely on the properties of recorded ground motions. These include maximum values of physical parameters such as peak ground acceleration and velocity, and integral properties such as peak spectral response and Arias intensity (Kramer 1995). Establishing the above quantities requires using the whole duration of the time history and does not consider yield strength, nor does it distinguish between intervals of elastic and inelastic response. A new kind of excitation time history is proposed in this chapter, derived from a ground motion record and the properties of a Single-Degree-of-Freedom (SDOF) yielding oscillator, to isolate the portion of the excitation function responsible for the development of plastic deformation. The properties of the modified excitation function can be used as a replacement to the ordinary ground motion parameters for deriving system-dependent demand indices. It will be shown that the proposed operation is not a mathematical artifact, as it is intrinsically related to the scaling properties of a yielding oscillator. To this end, the concept of excitation shift is introduced as an alternative to the classical scaling laws for dynamic systems. To address these issues, a brief review of some properties of inelastic systems is presented.

The post-yielding response of SDOF oscillators is commonly described by idealized force-displacement relationships such as the bilinear law shown in Fig. 7.1a. In many occasions, it is acceptable to simplify this relationship by assuming a horizontal post-yielding branch corresponding to an elastic-perfectly plastic system (Fig. 7.1b). In the special case of very stiff elastic behavior, such as that exhibited by frictional devices or certain geotechnical structures, the force-displacement relationship can further simplify to a rigid-perfectly plastic one (Fig. 7.1c.).

The differential equation (Eq. (6.2)) describing the response of the above systems can be cast in the alternative form

$$\ddot{\mu} + 2\zeta\omega_n\dot{\mu} + \omega_n^2\overline{F}_s(\dot{\mu}, \mu) = \omega_n^2 \frac{P(t)}{F_y} \quad (7.1)$$

where ω_n is the elastic (initial) natural frequency, ζ a viscous damping coefficient, $P(t)$ a time-varying forcing function, F_y the yielding restoring force and μ the well-known *ductility demand factor* (Veletsos & Newmark 1960). $\overline{F}_s(\dot{\mu}, \mu)$ is a dimensionless force-displacement function defined as the restoring force F_s over the restoring force at yielding F_y . As mentioned in Chapter 6, it is evident from the right-hand side of Eq. (7.1) that the response of the system is invariant to uniform scaling (i.e., by the same factor λ) in both forcing function $P(t)$ and yield resistance F_y . It is also suggested that a λ -fold increase in excitation intensity under constant yield resistance is equivalent to an $(1/\lambda)$ -fold decrease in yield resistance under constant excitation intensity. Accordingly, this property allows an infinite set of oscillators of different yield strengths subjected to different excitation levels, to exhibit identical responses and is, thereby, referred to as *similarity* (Barenblatt 1996).

In the important special case of earthquake loading, the ratio $P(t)/F_y$ is equivalent to the dimensionless ratio $\ddot{u}_g(t)/a_y$ where $\ddot{u}_g(t)$ denotes ground acceleration and a_y yield acceleration, i.e. $a_y \equiv F_y / m$, m being the mass of the system.

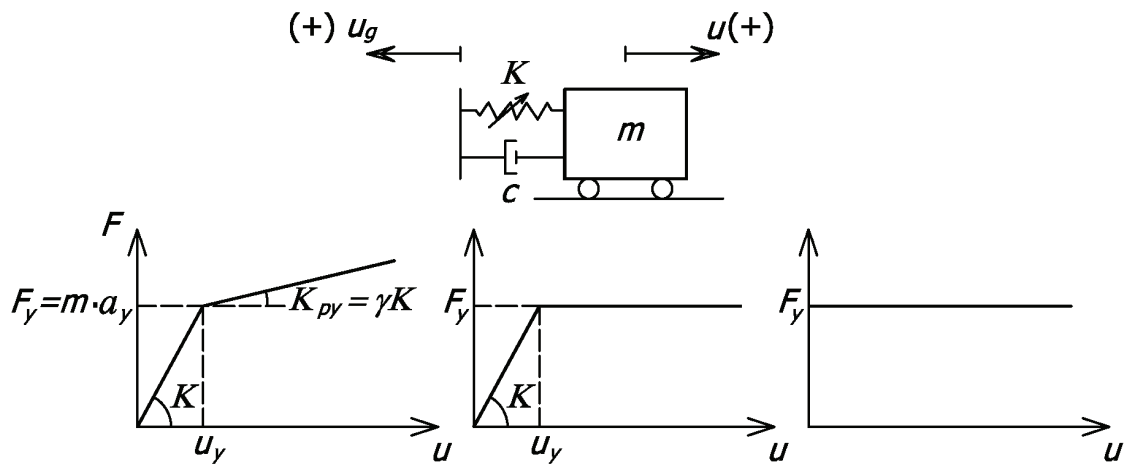


Figure 7.1 Idealized yielding oscillators considered in this chapter. From left to right: (a) Bilinear, (b) Elastic – perfectly plastic, (c) Rigid – perfectly plastic.

A common scaling law between excitation and response for the above systems can be expressed through *R-μ relations* of the type $\mu / R = g_1(\omega_n, \zeta, R)$ or $R / \mu = g_2(\omega_n, \zeta, \mu)$ (see Chapter 6), where $R = PSA(\omega_n, \zeta) / a_y$ is the familiar strength reduction factor defined by means of the elastic spectral pseudo-acceleration, while g_1 and g_2 are dimensionless functions dependent on the frequency content of ground motion. A variety of approximate expressions for g_1 and g_2 can be found in the literature (Veletsos & Newmark 1960, Newmark & Rosenblueth 1971, Miranda & Bertero 1994, Makris & Black 2004, Cuesta & Aschheim 2001). A set of exact solutions for g_1 , pertaining to pulse-like excitations, has been derived by Mylonakis & Voyagaki (2006).

The above scaling laws naturally involve *multiplicative* relations between excitation and response parameters for various yielding oscillators. Following the developments in Chapter 6, a different approach is investigated in this chapter, based on a shift in the coordinates of the excitation function. The outcome of the operation is the isolation of an excitation residue that controls the post-yielding response of the system. In Chapter 6 the method was applied to a rigid-plastic system (Fig. 7.1c). Here, the method is extended to yielding systems with bilinear characteristics, with or without hardening (Fig 7.1a, 7.1b). The method has practical implications as: (1) it can detect the portion of ground excitation responsible for the development of plastic deformation, (2) it can estimate the duration of each yielding branch, (3) the modified excitation function has distinct peak values (e.g., PGA , PGV), which can be calculated by standard procedures as demand indices. The above may be of particular interest for low-cycle impulsive loading such as that induced by blasts or near-fault earthquake motions.

7.3 Shift Property for Elastic-Perfectly Plastic Systems

Let's assume that the force-displacement relationship of the non-linear oscillator follows the elastic-perfectly plastic behavior shown in Fig. 7.1b. Under a base excitation that brings the system to yielding at $t = t_y$, zero viscous damping, and for the reference ordinates of Fig. 7.1, the post-yield response for $t > t_y$ is governed by the equation

$$\ddot{u} + a_y \operatorname{sgn}(\dot{u}) = +\ddot{u}_g(t) \quad (7.2a)$$

subject to the initial conditions u_y and $\dot{u}_y$, corresponding to relative-to-base system displacement and velocity at yielding. $u = u(t)$ denotes displacement relative to ground and $\operatorname{sgn}(\cdot)$ the signum function [i.e., $\operatorname{sgn}(x > 0) = +1$, $\operatorname{sgn}(x < 0) = -1$]. The positive sign in $\ddot{u}_g(t)$ is to ensure positive response for positive ground acceleration, as indicated from the reference axes of Fig 7.1. Equation (7.2a) can be written in the more compact form (Veletsos & Ventura 1984, Bracewell 1986)

$$\ddot{u} + a_y \operatorname{sgn}(\dot{u}) = +\ddot{u}_g(t) + \delta(t - t_y)\dot{u}_y + \dot{\delta}(t - t_y)u_y \quad (7.2b)$$

in which $\delta(\cdot)$ and $\dot{\delta}(\cdot)$ denote the delta function and its derivative (doublet function), measured in units of Time^{-1} and Time^{-2} , respectively. In the ensuing derivations it is assumed that the delta functions and the initial conditions are applied at time $t - t_y = 0^+$,

so that $\int_{t_y}^{\infty} \delta(t - t_y)\dot{u}_y dt = \dot{u}_y$ and $\int_{t_y}^{\infty} \int_{t_y}^{\infty} \dot{\delta}(t - t_y)u_y dt^2 = u_y$. The advantage of using Eq. (7.2b)

over the conventional formulation of Eq. (7.2a) is evident as the differential operator and the initial conditions are cast in a single line. A derivation of Eq. (7.2b) is provided in Appendix 5. Note that this can be viewed as a special form of Eq (7.1) for $\zeta = 0$, $\bar{F}_s = \operatorname{sgn}(\dot{u})$, $\omega_n^2 = (F_y/mu_y)$ and $P(t) = \text{mass} \times \text{right-hand side of Eq. (7.2b)}$.

For simplicity and without loss of generality, one may assume positive system velocity during a particular response branch; Eq. (7.2a) can be integrated twice for displacement to read

$$u = u_g - \frac{1}{2}(t - t_y)^2 a_y - (t - t_y) \dot{u}_{gy} - u_{gy} + (t - t_y) \dot{u}_y + u_y \quad (7.3)$$

where u_{gy} , $\dot{u}_{gy}$ stand, respectively, for ground displacement and velocity at yielding ($t = t_y$). A difficulty associated with the interpretation of Eq. (7.3) is that the right-hand side depends on a large number of terms which do not scale uniformly with ground acceleration and yielding resistance. In addition, the response depends on the “snapshot” ground motion parameters u_{gy} and $\dot{u}_{gy}$, which are difficult to incorporate in design-oriented relations such as those discussed in Section 6.2. Moreover, the influence of the second of these parameters, $\dot{u}_{gy}$, on the solution seems to be unreasonably strong, for the parameter is multiplied by time, thereby its contribution tends to dominate in long signals. Evidently, a direct integration of Eq. (7.2a), like the one in Eq. (7.3), although mathematically rigorous, might not be best suited for exploring the physics of the response at hand. An alternative approach can be developed to this end, as demonstrated in the ensuing.

In an analogous manner as in Chapter 6, by virtue of the definitions of the shifted ground motion parameters,

$$t^* \equiv t - t_y \quad (7.4)^*$$

$$\Delta u_g(t) \equiv u_g - u_{gy} \quad (7.5)$$

$$\Delta \dot{u}_g(t) \equiv \dot{u}_g - \dot{u}_{gy} \quad (7.6)$$

$$\Delta \ddot{u}_g(t) \equiv \ddot{u}_g - a_y \quad (7.7)$$

and the pair of integral properties (Chapter 6),

$$\Delta u_g = \int_0^{t^*} \Delta \dot{u}_g d\tau + \dot{u}_{gy} t^* \quad (7.8)$$

$$\Delta \dot{u}_g = \int_0^{t^*} \Delta \ddot{u}_g d\tau + a_y t^* \quad (7.9)$$

τ being a dummy time variable, Eq. (7.3) can be written in the simpler form

$$u(t^*) = \int_0^{t^*} \int_0^{t^*} \Delta \ddot{u}_g d\tau^2 + \dot{u}_{gy} t^* + u_y \quad (7.10)$$

where $\Delta \ddot{u}_g$ is interpreted as a shifted acceleration in light of Eq. (7.7).

Upon introducing the plastic displacement

$$\Delta u \equiv u - u_y \quad (7.11)$$

Eq. (7.10) can be written in the more compact form

$$\Delta u = \int_0^{t^*} \int_0^{t^*} \Delta \ddot{u}_g d\tau^2 + \dot{u}_{gy} t^* \quad (7.12a)$$

Moving the last term under the integral sign yields the alternative formulation

* Note that due to the more complex formulas presented in this chapter variable Δt has been changed to t^* .

$$\Delta u = \int_0^{t^*} \int_0^{t^*} [\Delta \ddot{u}_g + \dot{u}_y \delta(\tau)] d\tau^2 \quad (7.12b)$$

which suggests that the post-yielding displacement can be obtained by integrating twice the shifted ground acceleration $\Delta \ddot{u}_g$ plus a velocity impulse centred at time of yielding (Fig. 7.2). For this system, the integrand in Eq. (7.12b) defines the *Plastic Input Acceleration (PIA)*.

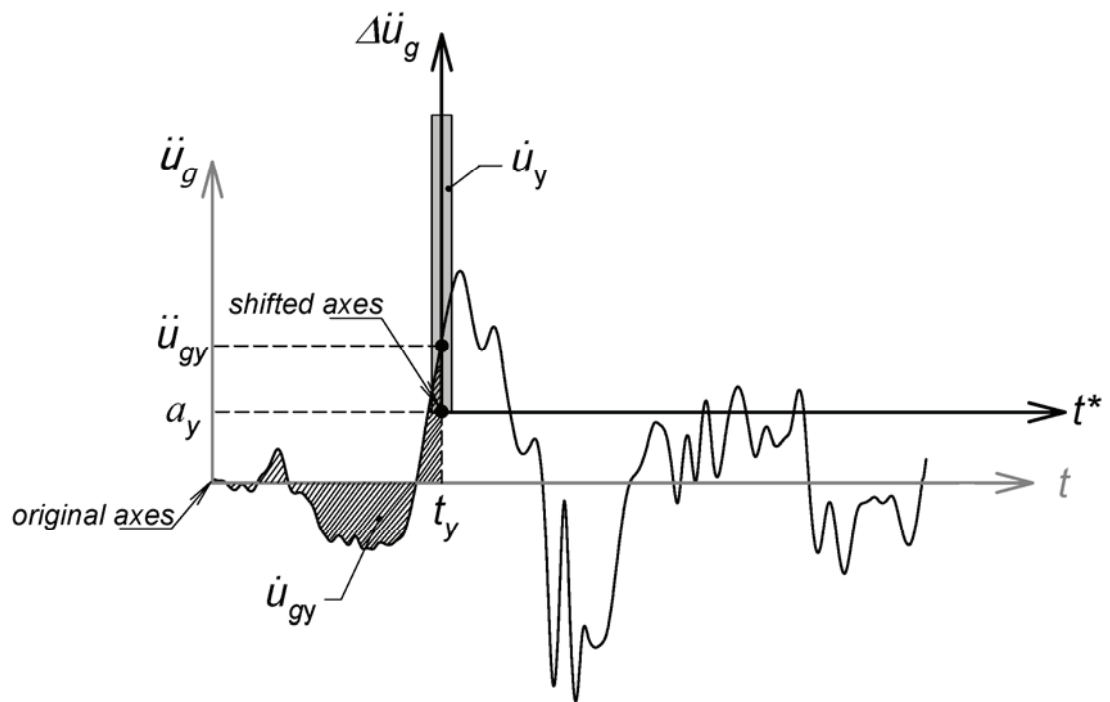


Figure 7.2 Translation of axes for an elastic – perfectly plastic system subjected to earthquake loading. Shaded areas correspond to conditions at yielding.

7.4 Special Cases

7.4.1 Rigid-Perfectly Plastic System

A rigorous analysis of the response of rigid-perfectly plastic systems by means of the proposed methodology can be found in Chapter 6. In this case, the relative velocity and displacement at yielding (i.e., u_y , $\dot{u}_y$) are zero, therefore $\Delta u = u$; Eq. (7.12a) yields

$$u = \int_0^{t^*} \int_0^{t^*} \Delta \ddot{u}_g d\tau^2 \quad (7.13)$$

This simple expression suggests that the displacement response of a rigid-perfectly plastic system can be obtained by integrating twice ground acceleration shifted by an amount equal to yield acceleration. In other words, the yielding resistance can be incorporated into the excitation function by translating the origin of axes from point $(0, 0)$ to (a_y, t_y) , thereby replacing co-ordinates $(\ddot{u}_g, t)$ by $(\Delta \ddot{u}_g, t^*)$, as shown in Fig. 7.3.

Evidently, Eq. (7.13) can be viewed as a more efficient solution to Eq. (7.3) for it is compact and does not involve the snapshot ground motion parameters u_{gy} and $\dot{u}_{gy}$. Moreover, the time history $\Delta \ddot{u}_g(t) = \ddot{u}_g - a_y$ in Eqs (7.7) and (7.13) refers exclusively to the development of plastic deformation and, thereby, defines the *Plastic Input Acceleration (PIA)*.

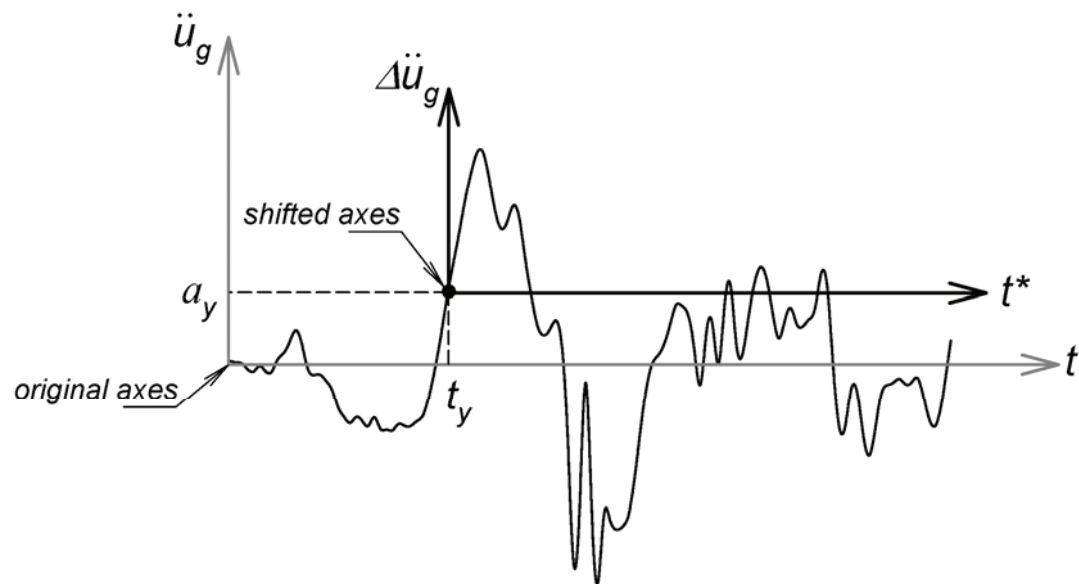


Figure 7.3 Translation of axes for a rigid – perfectly plastic system subjected to earthquake loading.

7.4.2 Shift Property for Bi-Linear Yielding Systems

In this more general case, the post-yielding response of a simple damped oscillator with the force-displacement characteristics in Fig. 7.1a is governed by the generalized equation

$$\ddot{u} + 2\omega_n^* \zeta^* \dot{u} + \omega_n^{*2} u = +\ddot{u}_g(t) - (\alpha_y - u_y \omega_n^{*2}) \text{sgn}(\dot{u}) + \delta(t - t_y) \dot{u}_y + [\dot{\delta}(t - t_y) + 2\zeta^* \omega_n^* \delta(t - t_y)] u_y \quad (7.14)$$

$\gamma = K_{py} / K$, $\omega_n^* = \sqrt{\gamma K / m}$, $\zeta^* = c_{py} / 2\omega_n^*$ denoting, respectively, the post-yielding hardening parameter, post-yielding natural frequency and post-yielding viscous damping [for stiffness-proportional damping, $c_{py} = (\gamma c)$] of the oscillator and u_y , a_y are the yield displacement and acceleration, respectively, at the particular post-yielding branch. Note that ω_n^* should not be viewed as a conventional natural frequency for it is not constant over a complete oscillation cycle of the non-linear system.

Using Eqs. (7.4) and (7.11), Eq. (7.14) takes the form

$$\Delta \ddot{u} + 2\omega_n^* \zeta^* \Delta \dot{u} + \omega_n^{*2} \Delta u = \Delta \ddot{u}_g + \delta(t^*) \dot{u}_y + [\dot{\delta}(t^*) + 2\zeta^* \omega_n^* \delta(t^*)] u_y \quad (7.15)$$

Considering, as before, positive velocity response, the solution to Eq. (7.15) can be expressed in terms of the convolution integral (Chopra 2006)

$$\Delta u = \int_0^{t^*} \left\{ \Delta \ddot{u}_g(\tau) + \delta(\tau) \dot{u}_y + [\dot{\delta}(\tau) + 2\zeta^* \omega_n^* \delta(\tau)] u_y \right\} h(\tau) d\tau \quad (7.16)$$

where $h(\tau)$ denotes the kernel

$$h = \frac{1}{\omega_D^*} \exp[-\zeta^* \omega_n^* (t^* - \tau)] \sin[\omega_D^* (t^* - \tau)] \quad (7.17)$$

in which $\omega_D^* = \omega_n^* \sqrt{1 - \zeta^{*2}}$ is the damped frequency corresponding to post-yielding frequency ω_n^* .

Evidently in this more general case, the *Plastic Input Acceleration (PIA)* of the bilinear system is equal to the shifted ground acceleration $\Delta \ddot{u}_g$, plus a set of impulsive terms related to the initial conditions at yielding (Fig. 7.4). Accordingly, Eq. (7.16) can be viewed as an extension of the previous two cases. The lack of a double integration in Eq. (7.16) should not come as a surprise, as the integration has been (implicitly) carried out in determining the kernel $h(\tau)$. Therefore, the integration in Eq. (7.16) has a different meaning than those in Eqs. (7.12b) and (7.13), yet the associated input accelerations are equivalent from a physical viewpoint.

Incidentally, it is noted that for $\gamma = 0$ and $\omega_n^* = 0$, corresponding to an elastic-perfectly plastic system, Eq. (7.16) duly reduces to

$$\Delta u = \int_0^{t^*} [\Delta \ddot{u}_g(\tau) + \delta(\tau) \dot{u}_y + \dot{\delta}(\tau) u_y] (t^* - \tau) d\tau \quad (7.18)$$

which can be readily integrated by parts for Eq. (7.3).

It is noted that Eq. (19) is not computationally efficient and should not be employed in cases where numerical performance is critical. Nevertheless, the scope of the derivation is to establish *PIA*— not a numerical treatment.

– 383 –

7.5 Interpretation

In summarizing the foregoing, it is instructive to point out that they can be interpreted by means of the generalized expression:

$$L(u) + \operatorname{sgn}(\dot{u}) F_y = P(t) \quad (7.19)$$

where $L(\)$ is a differential operator, $P(t)$ the excitation function (encompassing impulsive terms to account for initial conditions) and F_y the yielding resistance of the system. Since F_y is constant, it is tantamount to an external load, thereby, can be transferred to the right side of the equation to read

$$L(u) = P(t) - \operatorname{sgn}(\dot{u}) F_y \quad (7.20)$$

which introduces the aforementioned shift in $P(t)$ for a given value of the signum function. Note, however, that this property is more than a mere “right-hand siding” of yielding resistance, for it simplifies the solution and eliminates the need to account for certain initial conditions associated with functions $u(t)$ and $P(t)$. In addition, if $L(\)$ is a linear operator such as that in Eqs. (7.2) and (7.14), hence knowledge of the right-hand side of Eq. (7.20) will transform the problem to a linear one with the nonlinearity embedded into the forcing term. This sense of linearity, however, should be merely viewed as a mathematical device, for the response of the system is inherently non-linear.

In the special case of a rigid-plastic system (e.g. a rigid block on a frictional plane) subjected to earthquake loading, Eq. (7.20) simplifies to

$$\ddot{u} = \ddot{u}_g(t) - \operatorname{sgn}(\dot{u}) a_y \quad (7.21)$$

which suggests that the plastic input acceleration coincides with the relative-to-ground acceleration response of the block. This interpretation, however, should not be generalized to other forms of differential operator $L(\cdot)$ and function $P(t)$.

In the remainder of this chapter, $\Delta\ddot{u}_g$ and the associated impulsive terms in the right-hand side of Eq. (7.16) will be referred to as *Plastic Input Acceleration (PIA)* while the first and second integrals of *PIA* will be referred to as *Plastic Input Velocity (PIV)* and *Plastic Input Displacement (PID)*, respectively. These functions constitute the components of *PIM* for a given system. Specifically,

$$PIA = \Delta\ddot{u}_g + Z \quad (7.22)$$

$$PIV = \int (\Delta\ddot{u}_g + Z) dt \quad (7.23)$$

$$PID = \iint (\Delta\ddot{u}_g + Z) dt^2 \quad (7.24)$$

$$Z = \delta(t^*)\dot{u}_y + [\dot{\delta}(t^*) + 2\zeta^*\omega_n^*\delta(t^*)]u_y \quad (7.25)$$

Evidently, parameter Z stands for the effect of the impulsive terms and is zero for a rigid-perfectly plastic system.

7.6 Implementation

In applying the foregoing, it must be kept in mind that the theory pertains exclusively to yielding response, thereby identification of the corresponding time intervals is essential. For each post-yielding branch, the corresponding time interval can be derived as shown in Fig. 7.5 for elastic–perfectly plastic systems. Specifically:

First, acceleration peaks associated with large velocity content are identified. The beginning of the pulse is set at the time where $P=|F_y|$ that coincides with the initiation of yielding and can be determined through a conventional linear dynamic analysis (point 1, Fig. 7.5a).

Second, the ordinates and the abscissa of the ground acceleration time history are shifted by a_y and t_y , respectively, to set the origin of the new axes at the beginning of the pulse (point 1, Fig. 7.5b). The system velocity at yielding corresponds to an acceleration impulse having area $E_I = \dot{u}_y$ centered at $t^* = 0^+$ of the new time axis.

Third, integration of the modified ground acceleration is carried out according to Eq. (7.12), up to the point where PIV becomes zero. The specific time instant (point 2, Fig. 7.5b) corresponds to a possible end of yielding (i.e., a motion reversal), and manifests itself in the form of equal areas of opposite signs in the modified record (i.e., $E_I + E_2 = E_3$, Fig. 7.5b). Maximum PIV and PID values are traced over the whole time interval.

Fourth, a new exceedance of yielding resistance $|F_y|$ is sought in the time history. If this takes place under a negative restoring force, the ordinates of the excitation function would be shifted in the opposite direction leading to a jump in PIA (point 1', Fig. 7.5c). The smaller the time interval between points 2 and 3, the closer would be the jump to $2a_y$. In this case, the initial velocity will be negative, as $\dot{u}_y$ and F_y always possess the same sign. In the proposed procedure, the new phase is treated as an independent pulse, thus integration of PIA is carried out from the new origin of axes until a new zero velocity point is detected (point 4, Fig. 7.5c).

Fifth, the procedure is repeated until the pulse content of the record is exhausted.

Sixth, the largest, in absolute terms, maximum ground acceleration and velocity among all individual pulses, corresponds to the overall peak values of the *PIM*. On the other hand, *PID* are cumulative, thereby the overall peak value is obtained by superimposing all individual pulse displacements in a single time history. It should be noticed that for a rigid plastic system, the yield criterion is simpler i.e., $\ddot{u}_g = |a_y|$, which can be visually established on the acceleration record (Chapter 6). Also, the procedure can be easily extended to a bilinear system. In such a case, the end of pulse is not associated with equal areas under the modified acceleration record, but directly with a zero-velocity point ($\dot{u} = 0$). For piece-wise linear variation of ground acceleration this point can be determined analytically, as shown in Mylonakis & Reinhorn (2001).

The identified pulses can be cast in the form of a single discontinuous excitation function plus a suite of velocity impulses using the ordinates of the first pulse and the abscissa of the original record (Fig. 7.5d). The observed discontinuity between points 2 and 3 is due to the axis shift following the initiation of the second pulse in Fig. 7.5c. Evidently, this is a radically different excitation function as compared to the original one (Fig. 7.5a), yet it is the one that should be integrated over time for the response of the system. More importantly, the latter function may have certain advantages over the original acceleration record for describing the salient features of seismic demand, as it combines the properties of the ground acceleration time history with those of the yielding system. Evidently, this approach goes beyond the well-known numerical right-hand siding of non-linear terms employed in numerical dynamic analyses of yielding systems (Bathe & Gracewski 1981, Villaverde & Hanna 1992).

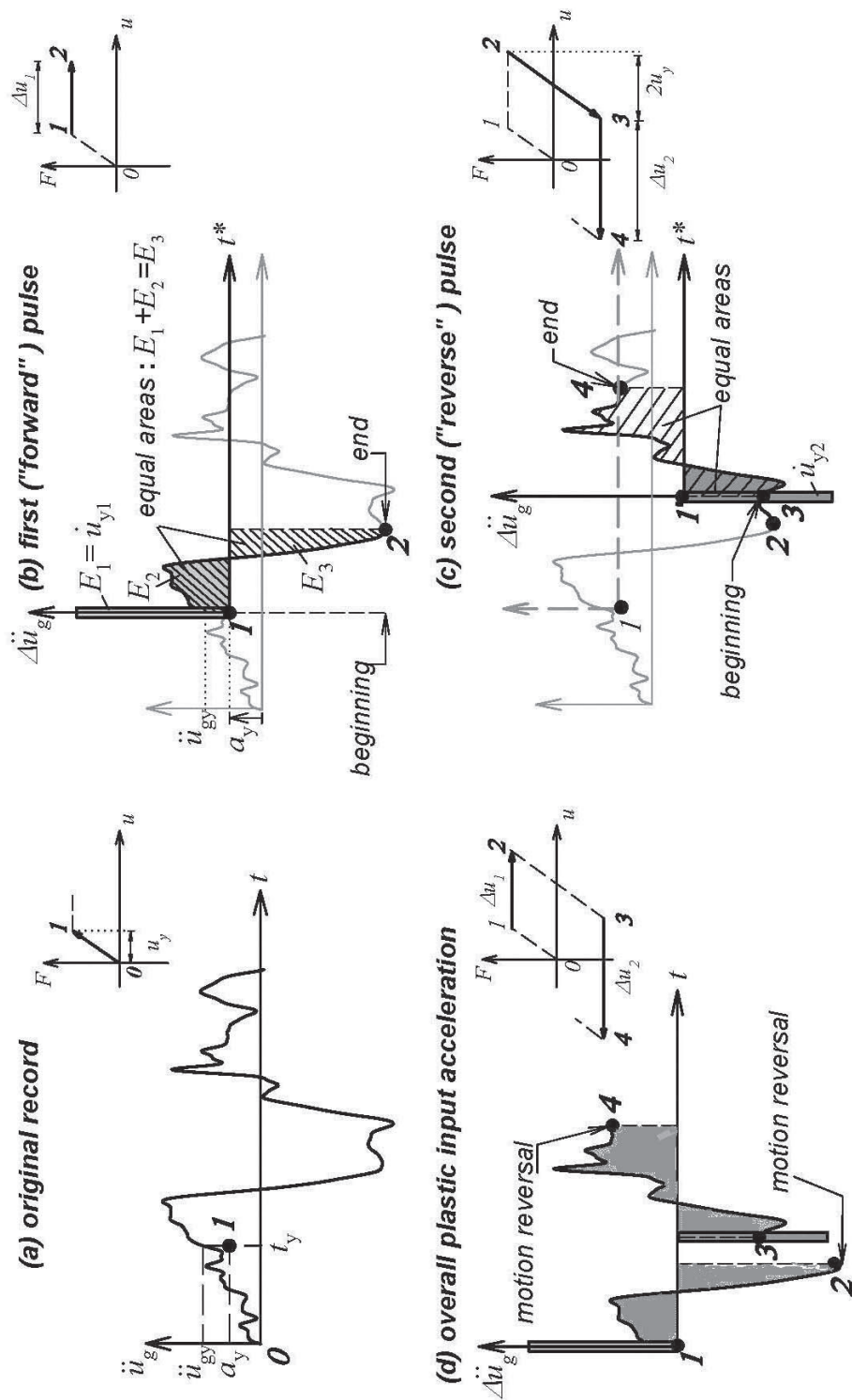


Figure 7.5 Definition of integration limits according to the proposed approach.

7.7 Applications

In what follows, results are presented by means of the following peak *PIM* parameters:

$$A_{gp} \equiv \max |PIA| = \max |\Delta \ddot{u}_g + Z| \quad (7.26)$$

$$V_{gp} \equiv \max |PIV| = \max \left| \int (\Delta \ddot{u}_g + Z) dt \right| \quad (7.27)$$

$$D_{gp} \equiv \max |PID| = \max \left| \iint (\Delta \ddot{u}_g + Z) dt^2 \right| \quad (7.28)$$

where subscript *p* stands for “plastic” to distinguish the above peak values from those of the original ground motion (to be referred hereafter to as A_g , V_g , D_g). The physical meaning of Eqs (7.26) to (7.28) is worth discussing: the above quantities are measures of the “effective” ground motion intensity that induces exclusively plastic deformations. This is achieved by: (i) removing the portion of ground motion associated with elastic response, (ii) subtracting from the excitation time history the yield resistance of the structure during plastic response, (iii) incorporating the initial conditions. Accordingly A_{gp} , V_{gp} , D_{gp} can be viewed as replacements to the conventional ground motions parameters A_g , V_g , D_g for quantifying earthquake demand in yielding structures.

An important application, referring to impulsive loading such as that induced by near-fault earthquake motions, is considered. With reference to the systems at hand, a suite of four idealized ground acceleration pulses representing near-fault motions affected by forward fault-rupture directivity (Bray & Rodriguez-Marek 2004, Tang & Zhang 2010) is employed, as shown in Fig 7.6 and Table 7.1. The motions are described by their peak acceleration A_g , peak velocity V_g , peak displacement D_g and half-cycle duration t_d . Both half- and full-cycle excitations are examined. A large set of normalized yield

accelerations $\eta = a_y/A_g$ varying from 0 to 1, and normalized natural frequencies $f = \omega_n t_d / 2\pi$, varying from 0.1 to 2 are considered, representing a wide range of yielding oscillators. In the analytical expressions provided in the ensuing, $H(\cdot)$ denotes the Heaviside function (i.e. $H(x>0) = +1$; $H(x<0) = 0$).

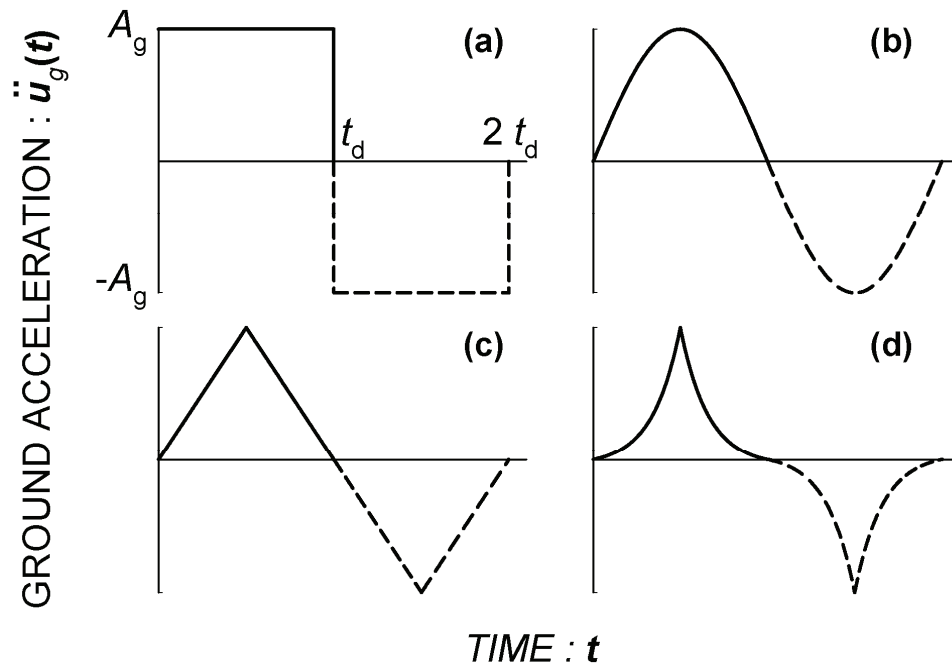


Figure 7.6 Time histories of four idealized ground acceleration pulses of half- and full-cycle duration: (a) box, (b) sinusoidal, (c) symmetric triangular, (d) exponential.

Table 7.1. Functional form of the ground acceleration pulses considered.

<i>Pulse Type</i>	<i>Ground Acceleration $\ddot{u}_g(t)$</i>
<i>Full – cycle</i>	
<i>Box</i>	$A_g[1 - 2H(t - t_d) + H(t - 2t_d)]$
<i>Sinusoidal</i>	$A_g \sin(\pi t/t_d)[1 - H(t - 2t_d)]$
<i>Triangular</i>	$2A_g t/t_d [1 + (t_d/t - 2)H(t - t_d/2) + (2 - 3t_d/t)H(t - 3t_d/2) + (2t_d/t - 1)H(t - 2t_d)]$
<i>Exponential</i>	$A_g [1 - \lambda_1 + (\lambda_1 - \lambda_2)H(t - t_d/2) + (\lambda_2 + \lambda_2^{-1} - 2)H(t - t_d) + (\lambda_3 - \lambda_2^{-1})H(t - 3t_d/2) + (1 - \lambda_3)H(t - 2t_d)] / (1 - e^\pi)$
<i>Half cycle</i>	
<i>Box</i>	$A_g [1 - H(t - t_d)]$
<i>Sinusoidal</i>	$A_g \sin(\pi t/t_d)[1 - H(t - t_d)]$
<i>Triangular</i>	$2A_g t/t_d [1 + (t_d/t - 2)H(t - t_d/2) + (1 - t_d/t)H(t - t_d)]$
<i>Exponential</i>	$A_g [1 - \lambda_1 + (\lambda_1 - \lambda_2)H(t - t_d/2) + (\lambda_2 - 1)H(t - t_d)] / (1 - e^\pi)$

where $\lambda_1 = e^{2\pi i/t_d}$, $\lambda_2 = e^{2\pi(1-i)/t_d}$, $\lambda_3 = e^{2\pi(2-i)/t_d}$; $H(\cdot)$ = Heaviside function

With reference to a rigid-perfectly plastic system, closed-form solutions for the above quantities were derived, as summarized in Tables 7.2, 7.3 and 7.4 (see also Appendix 2).

Normalized results for different levels of yielding resistance are provided in Fig 7.7 for the idealized pulses of Fig 7.6 considering a full excitation cycle. The results are plotted in terms of ratios A_{gp}/A_g , V_{gp}/V_g and D_{gp}/D_g , as functions of normalized yield strength $\eta = a_y/A_g$.

Naturally, for all pulse waveforms the above ratios equal one for $a_y/A_g = 0$ and zero for $a_y/A_g = 1$. With the exception of curve A_{gp}/A_g for the box pulse, all ratios decrease with increasing a_y/A_g . The drop reaches dramatic proportions for a_y/A_g exceeding approximately 0.75. Also, the ordinates of all ratios are lower for waveforms associated with lower velocity content such as the exponential pulse as opposed to the box pulse. Interestingly, the *PIA* function may exhibit values higher than 1 for all waveforms under small values of yielding resistance. This is a counterintuitive trend which can be attributed to the shift in the ordinates of ground acceleration that generates an increase in values associated with a negative acceleration branch.

Table 7.2 Normalized peak PIM values for a rigid perfectly plastic system under four full-cycle acceleration pulses.[†]

<i>Pulse Type</i>	A_{gp}/A_g	V_{gp}/V_g	D_{gp}/D_g
	$\max\{1-\eta, \bar{A}\}$ where $\bar{A}$ is:		
	$1/2 \leq \tau_m \leq 3/2$	$\tau_m \geq 3/2^{**}$	
<i>box</i>	—		$(1-\eta)/(1+\eta)$
<i>sinusoidal</i>	$ \eta - \sin \pi \tau_m $		$-\eta \frac{\pi}{4} (\tau_m - \tau_y)^2 + \frac{1}{2} (\tau_m - \tau_y) \cos \pi \tau_y - \frac{1}{2\pi} (\sin \pi \tau_m - \sin \pi \tau_y)$
<i>triangular</i>	$\eta - 2(1 - \tau_m)$	$1 + \eta$	$-\frac{1}{6} + \tau_m - 2\tau_m^2 + \frac{2}{3}\tau_m^3 + 2\tau_m\tau_y^2 -$ $-\frac{4}{3}\tau_y^3 + \eta(\tau_m - \tau_y)^2 - \frac{1}{6}(2\tau_m - 3)^3 H(\tau_m - 3/2)$
<i>exponential</i>	$\eta - \operatorname{sgn}(1 - \tau_m) \frac{1 - e^{2\alpha \tau_m}}{1 - e^\alpha}$	same as in Table 7.3	
			$\frac{1}{4\pi(e^\pi - \pi - 1)} \left[\begin{aligned} &-e^{2\pi(1-\tau_m)} + e^{2\pi\tau_y} + 2\pi[e^\pi(1-2\tau_m) + e^{2\pi\tau_y}(\tau_m - \tau_y)] + \\ &+ 2\pi^2(\tau_m - \tau_y)^2[1 + \eta(e^\pi - 1)] - \\ &- [2 + 4\pi^2(1 - \tau_m)^2 - e^{2\pi(1-\tau_m)} - e^{-2\pi(1-\tau_m)}]H(\tau_m - 1) \\ &+ [e^{2\pi(2-\tau_m)} - e^{-2\pi(1-\tau_m)} + 2\pi e^\pi(2\tau_m - 3)]H(\tau_m - 3/2) \end{aligned} \right]$

[†] τ_y , τ_m = dimensionless time of yielding and peak plastic input displacement, respectively (Table 7.4); $H()$ = Heaviside function, $\operatorname{sgn}()$ = signum function

**for box pulse conditions $\tau_m \geq 1$

Table 7.3 Normalized peak *PIM* values for a rigid perfectly plastic system under four half-cycle acceleration pulses.[†]

Pulse Type	A_{gp}/A_g	V_{gp}/V_g	D_{gp}/D_g
	$\max\{1-\eta, \overline{A}\}$ where $\overline{A} :$		
	$1/2 \leq \tau_m \leq 1$	$\tau_m \geq 1$	
<i>box</i>	—	$1-\eta$	$(1-\eta)/\eta$
<i>sinusoidal</i>	$ \eta - \sin \pi \tau_m $	$\cos \pi \tau_y + \eta \pi (\tau_y - 1/2)$	$-\eta \frac{\pi}{2} (\tau_m - \tau_y)^2 + (\tau_m - \tau_y) \cos \pi \tau_y - \frac{1}{\pi} (\sin \pi \tau_m - \sin \pi \tau_y) + \frac{1}{\pi} [\sin \pi \tau_m + \tau_m - 1] H(\tau_m - 1)$
<i>triangular</i>	$\eta - 2(1 - \tau_m)$	$(1-\eta)^2$	$-\frac{1}{3} + 2\tau_m - 4\tau_m^2 + \frac{4}{3}\tau_m^3 + 4\tau_m^2\tau_y - \frac{8}{3}\tau_y^3 + 2\eta(\tau_m - \tau_y)^2 - \frac{4}{3}(\tau_m - 1)^3 H(\tau_m - 1)$
<i>exponential</i>	$\eta - \frac{1 - e^{2\pi(1-\tau_m)}}{1 - e^\pi}$	$\frac{1}{e^\pi - \pi - 1} [-e^{2\pi\tau_y} + \pi(\eta - 1)(1 - 2\tau_y) + e^\pi(1 - \pi\eta + 2\pi\eta\tau_y)]$	$\frac{1}{2\pi(e^\pi - \pi - 1)} \left[\frac{-e^{2\pi(1-\tau_m)} + e^{2\pi\tau_y} + 2\pi[e^\pi(1 - 2\tau_m) + e^{2\pi\tau_y}(\tau_m - \tau_y)] + 2\pi^2(\tau_m - \tau_y)^2[1 + \eta(e^\pi - 1)] - [1 + 2\pi(1 - \tau_m) + 2\pi^2(1 - \tau_m)^2 - e^{2\pi(1-\tau_m)}]H(\tau_m - 1)}{2\pi(e^\pi - \pi - 1)} \right]$

[†] τ_y , τ_m = dimensionless time of yielding and peak plastic input displacement, respectively (see Table 7.4); $H(\cdot)$ = Heaviside function

Table 7.4 Dimensionless time of yielding and time of peak plastic input motion for a rigid perfectly plastic system.

<i>Pulse type</i>	$\tau_y \equiv t_y / t_d$	$\tau_m \equiv t_m / t_d$
<i>same for all durations</i>		
<i>box</i>	0	η^{-1}
<i>sinusoidal</i>	$\frac{1}{\pi} \sin^{-1} \eta$	$\cos \pi \tau_y - \cos \pi \tau_m - \eta \pi (\tau_m - \tau_y) = 0$
<i>triangular</i>	$\frac{\eta}{2}$	$\tau_m^2 + (\eta - 2)\tau_m + \tau_y^2 - \eta \tau_y + \frac{1}{2} -$ $- [2\tau_m^2 - 6\tau_m - \frac{7}{2}] H(\tau_m - \frac{3}{2}) = 0$
<i>exponential</i>	$\frac{1}{2\pi} \ln[1 - \eta(1 - e^\pi)]$	$[1 - \eta(1 - e^\pi)](\tau_m - \tau_y) + \frac{e^{2\pi(1-\tau_m)} + e^{2\pi\tau_y} - 2e^\pi}{2\pi} +$ $+ [2(1 - \tau_m) + \frac{e^{-2\pi(1-\tau_m)} - 2e^{2\pi(1-\tau_m)}}{2\pi}] H(\tau_m - 1) +$ $+ \frac{-e^{2\pi(2-\tau_m)} - e^{-2\pi(1-\tau_m)} + 2e^\pi}{2\pi} H(\tau_m - 3/2) = 0$

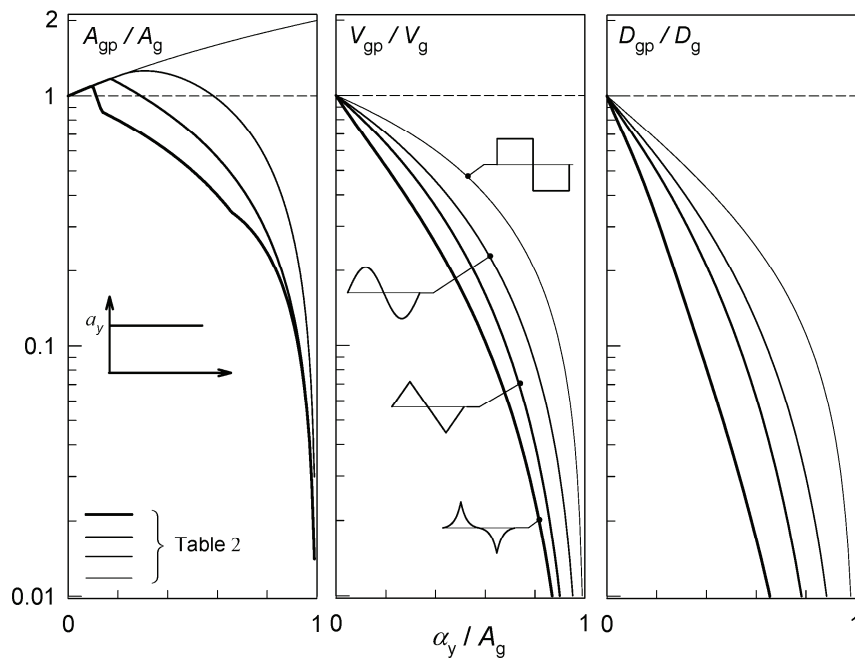


Figure 7.7 Dimensionless representation of peak plastic input motion parameters for a rigid plastic system subjected to four idealized pulse excitations of full-cycle duration.

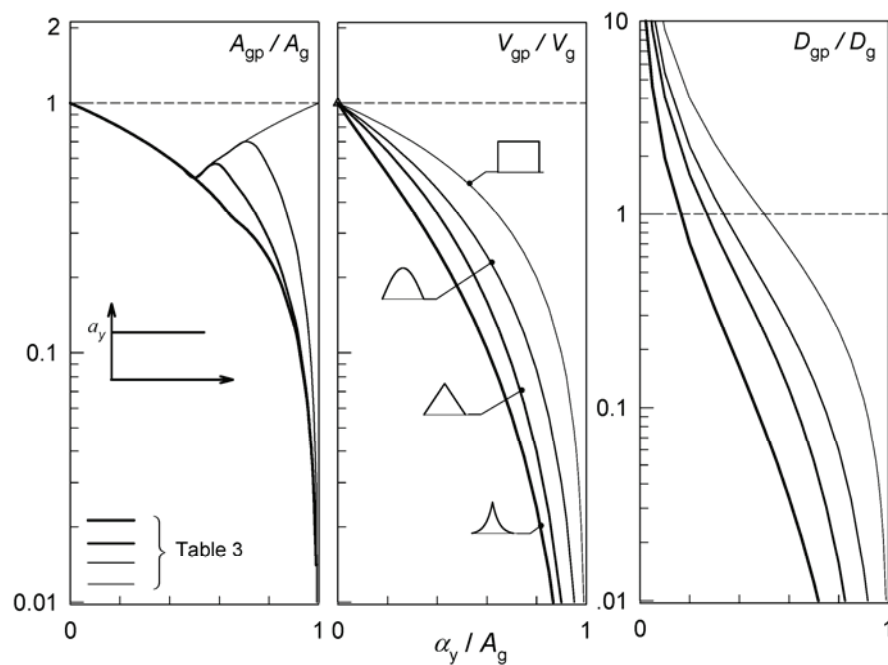


Figure 7.8 Dimensionless representation of peak plastic input motion parameters for a rigid plastic system subjected to four idealized pulse excitations of half-cycle duration.

The case of half-cycle pulses is examined in Fig. 7.8. The patterns are analogous to those in the previous figure. However, *PIA*'s do not attain the same kind of increase (i.e., values higher than 1) observed with the full-cycle pulses, apparently due to a lack of a negative acceleration branch. Indeed, accelerations tend to decrease in an identical manner with a_y/A_g , up to the value $a_y/A_g = 0.5$ beyond which they start to increase, by different amounts, and then decrease again reaching zero as a_y/A_g approaches 1. Peak velocities are identical to those for the full-cycle pulses (see Tables 7.2 and 7.3). A different kind of behaviour is observed with displacements as these are unbounded at $a_y/A_g = 0$. This is anticipated since half-cycle pulses possess finite residual velocity - thereby they tend to induce infinite displacements in systems with zero yielding resistance (Newmark 1965). Obviously, this behavior is not physically realizable.

A set of results for *PIM*'s in the time domain obtained by means of the proposed procedure for the four idealized pulses at hand is provided in Fig 7.9. The discontinuous nature of plastic acceleration time histories is evident in the graphs. This is in contrast with velocities and displacements which are invariably continuous functions of time. The drop in values of these functions with increasing yield strength η is noteworthy.

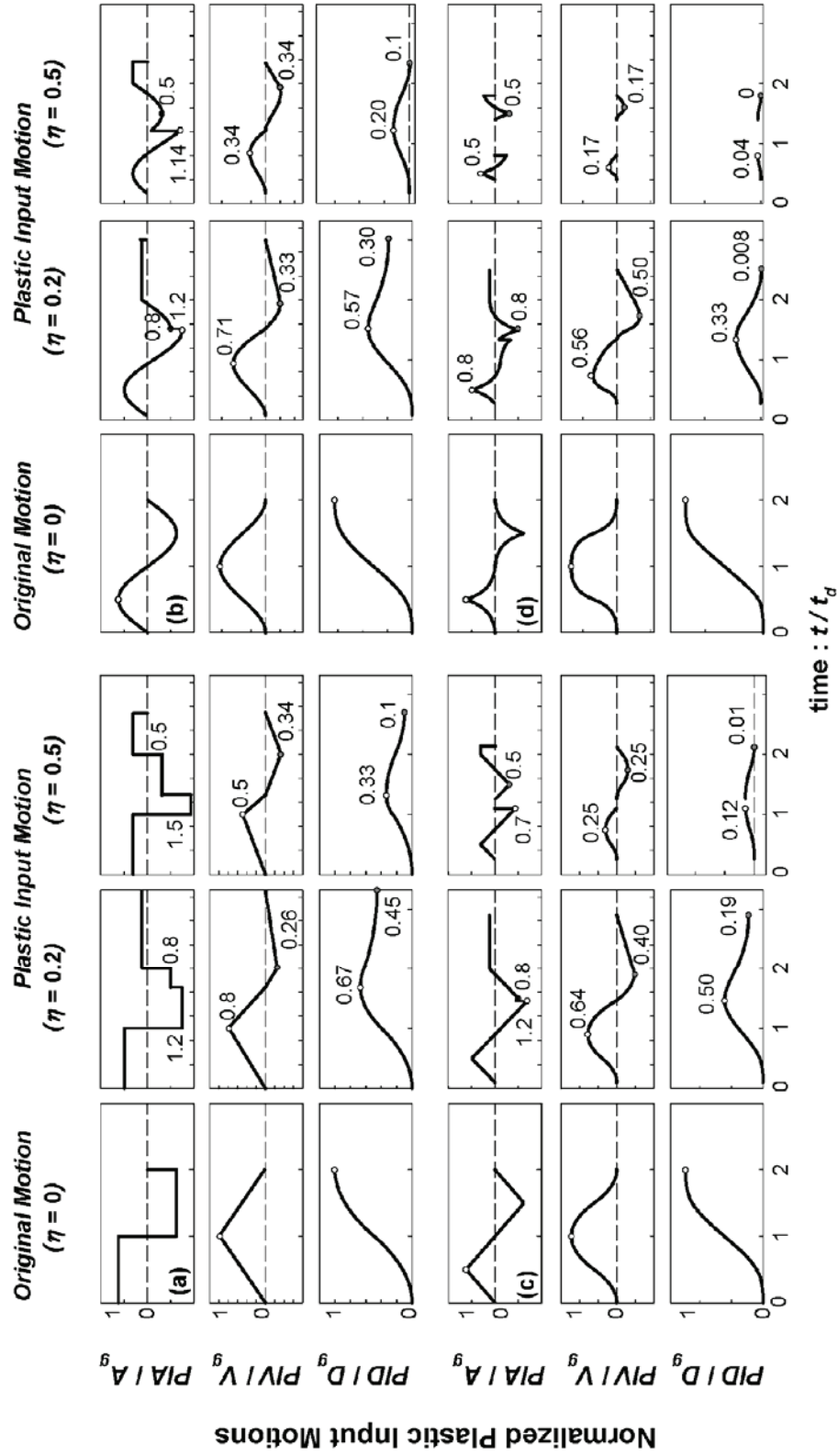


Figure 7.9 Time histories of plastic input motions for a rigid-perfectly plastic system of different strengths subjected to four full-cycle ground motion pulses. Peak values in the original motions are scaled to unity.

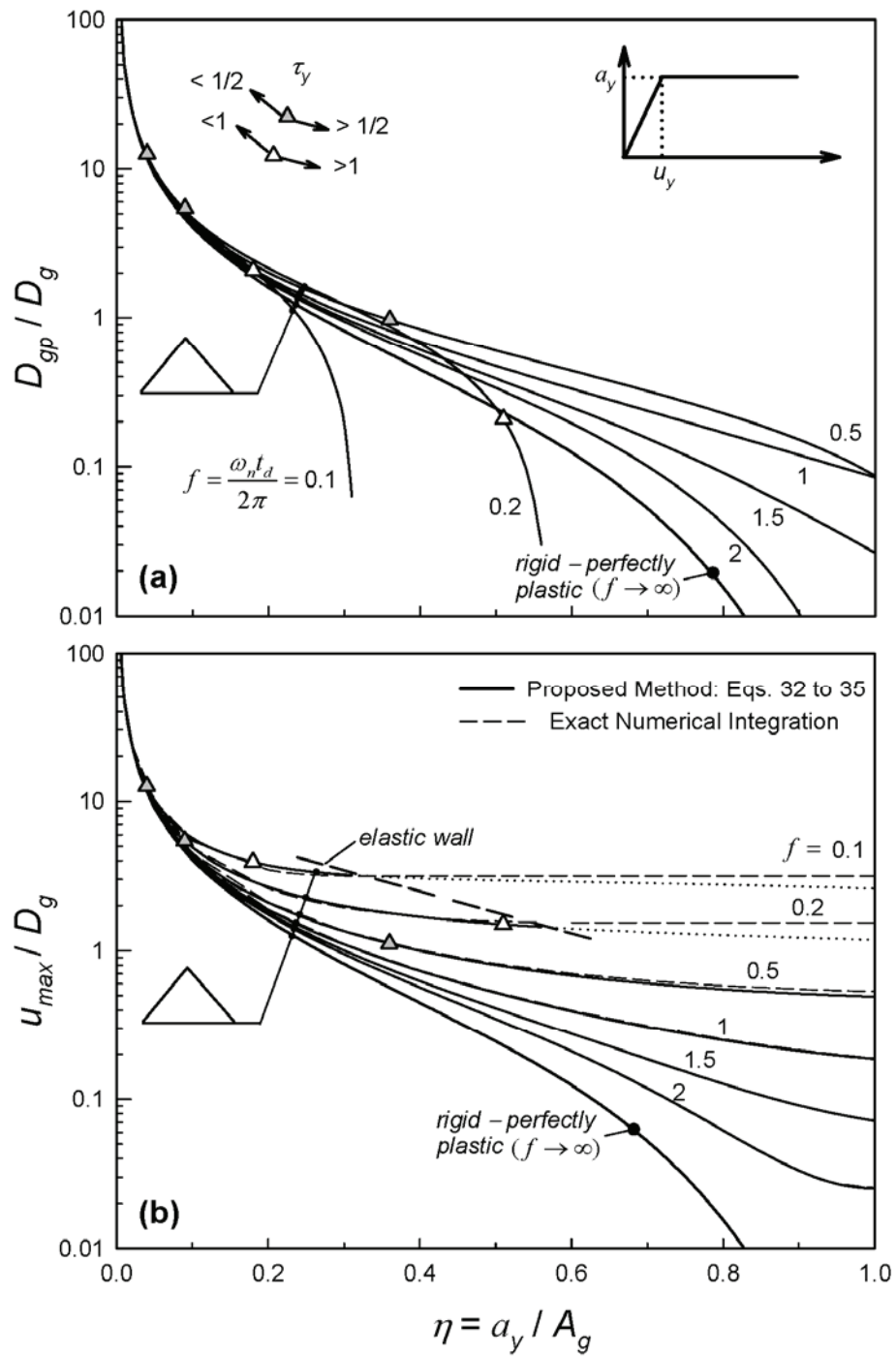


Figure 7.10 Comparison of the results of the proposed theory against rigorous numerical results for an elastic – perfectly plastic oscillator subjected to a triangular pulse of half-cycle duration.

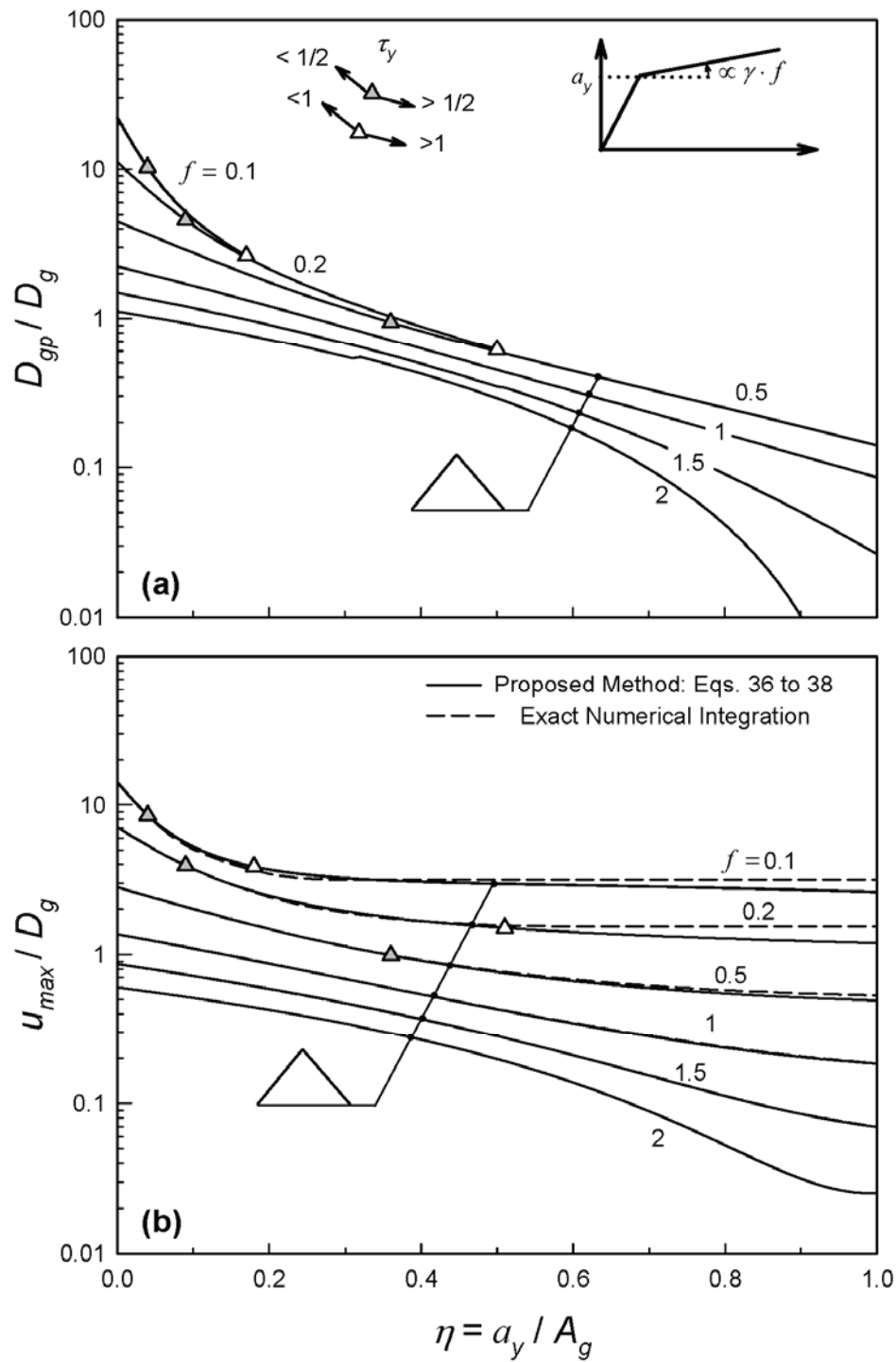


Figure 7.11 Comparison of the results of the proposed theory against rigorous numerical results for a bilinear oscillator subjected to a triangular pulse of half-cycle duration ($\gamma=0.05$).

With reference to an elastic-perfectly plastic and bilinear yielding oscillator subjected to a half-cycle triangular pulse, results are presented in Figs. 7.10 and 7.11.

For an elastic-perfectly plastic undamped oscillator subjected to this pulse, the following closed-form solution for plastic input displacement was derived:

$$\frac{D_{gp}}{D_g} = E_t - \frac{4}{3}\tau_y^3 - [4\tau_y^2 + \frac{2}{\pi^2 f^2}(\cos 2\pi f \tau_y - 1)](\tau_m - \tau_y) - 2\eta(\tau_m - \tau_y)^2 \quad (7.29)$$

where

$$E_t = \begin{cases} -\frac{4}{3}\tau_m^3 + 4\tau_m^2 - 2\tau_m + \frac{1}{3} & , \quad 1/2 \leq \tau_m \leq 1 \\ 2\tau_m - 1 & , \quad \tau_m \geq 1 \end{cases} \quad (7.30)$$

$$\tau_m = \begin{cases} 1 - \frac{\eta}{2} + \frac{1}{2\pi f} \sqrt{2 + \pi^2 f^2 (2 + \eta^2 + 4\eta(\tau_y - 1) - 4\tau_y^2) - 2\cos 2\pi f \tau_y} & , \quad 1/2 \leq \tau_m \leq 1 \\ \tau_y - \frac{1}{\eta} [\tau_y^2 - \frac{1}{2} - \frac{1}{2\pi^2 f^2} (1 - \cos 2\pi f \tau_y)] & , \quad \tau_m \geq 1 \end{cases} \quad (7.31)$$

$$\tau_y = \frac{\frac{1}{2}\eta + \frac{1}{2\pi f} \sin 2\pi f \tau_{yo} - \tau_{yo} \cos 2\pi f \tau_{yo}}{1 - \cos 2\pi f \tau_{yo}} \quad , \quad \tau_{yo} = \sqrt[3]{\frac{3\eta}{4\pi^2 f^2}} \quad (7.32a, b)$$

where $\tau_y = t_y/t_d$ and $\tau_m = t_m/t_d$ the normalized times of yielding and maximum plastic input displacement PID , and $f = \omega_n t_d / 2\pi$ stands for the dimensionless pulse duration (or natural frequency) of the system. This solution was first obtained, in a somewhat different form, by Mylonakis & Reinhorn (2001) and Mylonakis & Voyagaki (2006). It should be noticed that for this oscillator $D_{gp} = u_{max} - u_y$ (Eqs 7.10-7.12, 7.28) which suggests that plastic input displacement coincides with the plastic displacement response of the system.

The above solution is presented graphically in Fig. 7.10a. Evidently, in addition to yield strength displacement now depends on both natural frequency and pulse duration. Peak PID 's are generally higher than displacements for the corresponding rigid-plastic system, a behavior which is due to the finite velocity at yielding $\dot{u}_y$ which always points to the same direction as yield displacement u_y , thereby giving rise to stronger plastic response. Figure 7.12 illustrates the variation of velocity at yielding, normalized with characteristic velocity $u_s \times \omega_n$, where $u_s = A_g / \omega_n^2$ (Chapter 3) the static displacement of an elastic oscillator to peak pulse acceleration, and that of the corresponding time of yielding as function of yield strength η , for different values of the normalized frequency f . An opposing factor is time of yielding t_y which, due to inertia, tends to increase with increasing system flexibility, thereby limiting the duration of the post-yielding forced oscillations (Biggs 1964, Mylonakis & Reinhorn 2001).

As a result, a very flexible system may yield after the end of the pulse (or not yield at all), which results to a lower plastic input motion. This trend is reflected in the curves for $f = 0.1$ and 0.2 whose ordinates drop quickly with increasing yield strength and attain values lower than those for a rigid-perfectly plastic system. It is also noteworthy that displacements tend to infinity as yield strength approaches zero, regardless of natural period – in analogy to the trends observed for the rigid-plastic system.

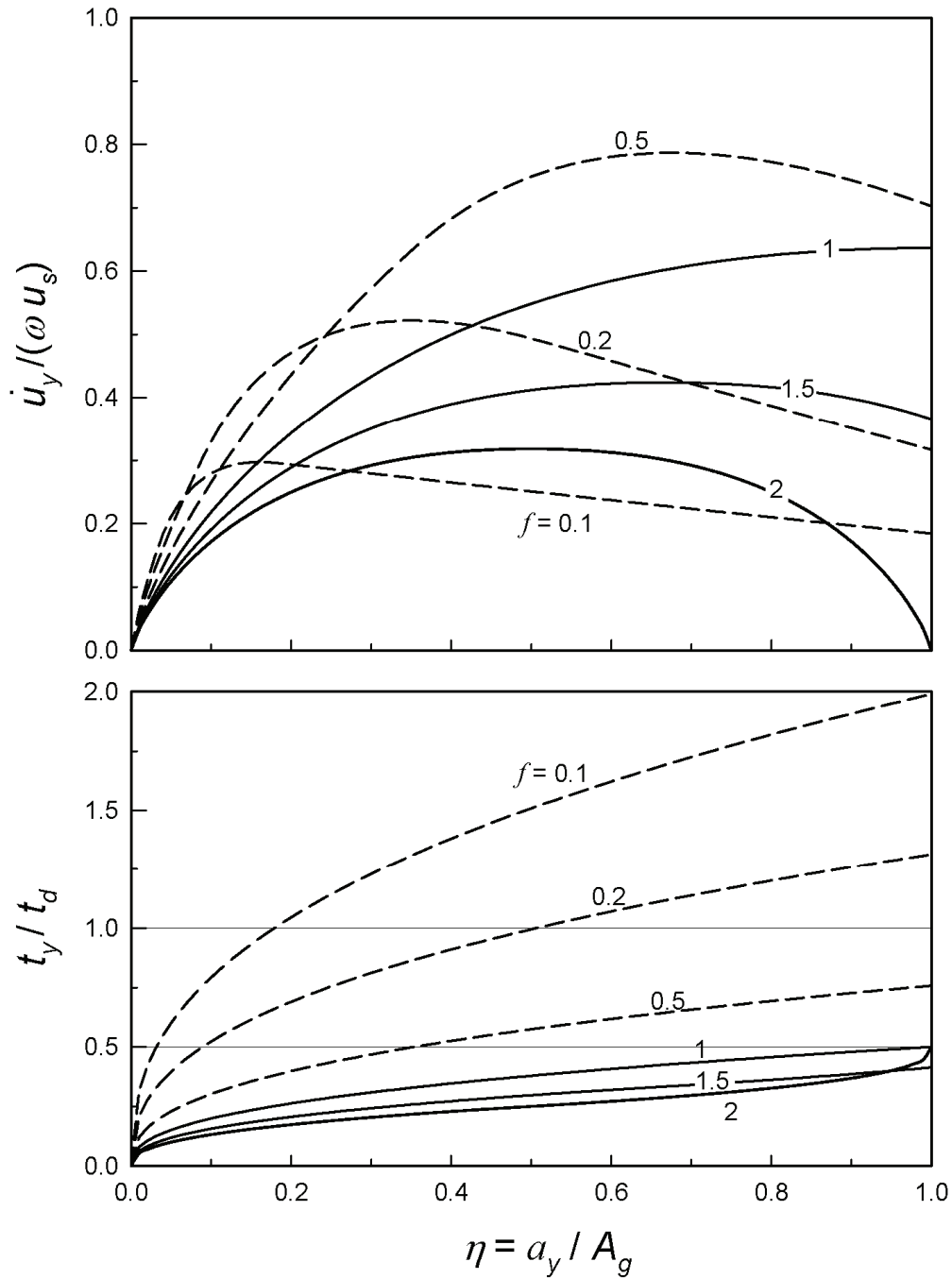


Figure 7.12 Normalized velocity at yielding and time of yielding for elastic – perfectly plastic and bilinear oscillators subjected to a triangular pulse of half-cycle duration.

For comparison purposes, the peak displacement response of the system at hand is presented graphically in Fig. 7.10b, determined by both numerical and analytical means. Evidently, peak displacement of the elastic-perfectly plastic system exhibits more regular behavior than plastic input displacement, for it is always higher than that of a corresponding rigid-perfectly plastic system. This indicates that for flexible oscillators, the smaller plastic demand observed Fig. 7.10a is compensated by a larger yielding displacement, which lifts the curves above that of the corresponding perfectly rigid system.

A somewhat different behavior is observed for an undamped bilinear yielding oscillator in Fig. 7.11.

Application of the theory leads to the following solution:

$$D_{gp} = u_g(t_m) - u_{gy} - (\dot{u}_{gy} - \dot{u}_y)(t_m - t_y) - \frac{a_y}{2}(t_m - t_y)^2 \quad (7.33)$$

where

$$\dot{u}_y = \frac{A_g t_d}{2\pi^2 f^2} \begin{cases} 1 - \cos 2\pi f \tau_y, & \tau_y \leq 1/2 \\ (2 - \cos \pi f) \cos[2\pi f(\tau_y - \frac{1}{2})] + \sin \pi f \sin[2\pi f(\tau_y - \frac{1}{2})] - 1, & 1/2 \leq \tau_y \leq 1 \\ (2 \cos \pi f - \cos 2\pi f - 1) \cos[2\pi f(\tau_y - 1)] - \\ - 2 \sin \pi f (1 - \cos \pi f) \sin[2\pi f(\tau_y - 1)] & , \quad \tau_y \geq 1 \end{cases} \quad (7.34)$$

$$\tau_m = \tau_y + \frac{1}{2\pi f \sqrt{\gamma}} \left\{ \tan^{-1} \frac{-\sqrt{\gamma} \left[D_0 - \frac{b_3}{\gamma \pi f} \right]}{V_0 - \frac{b_1}{\gamma \pi f}} + \cos^{-1} \frac{-b_1}{\gamma \pi f \sqrt{\left[V_0 - \frac{b_1}{\gamma \pi f} \right]^2 + \gamma \left[D_0 - \frac{b_3}{\gamma \pi f} \right]^2}} \right\} \quad (7.35)$$

in which γ stand for the post-yielding stiffness coefficient, while b_1 , b_3 , D_0 , V_0 are dimensionless parameters which are provided in Mylonakis & Reinhorn (2001) and τ_y is given by Eq. (7.32). Ground displacements u_{gy} and $\dot{u}_{gy}$ are obtained from Table 1 for times τ_y and τ_m . Note that τ_m , which defines the end of modified pulse according to the proposed procedure, is obtained in closed-form from Eq. (7.35). This is possible as a consequence of the piece-wise linear nature of the forcing function (Mylonakis & Reinhorn 2001). Note that for the particular system D_{gp} does not coincide with plastic displacement response, as the differential operator $L(u)$ in Eq. (7.20) is not of the elementary form (d^2/dt^2) as for the perfectly plastic system.

Evidently, the peak PID for a bilinear system in Fig 7.11a is generally smaller than ground displacement at the end of the pulse (D_g), and does not tend to infinity even at zero yield strength. This is anticipated due to the restraining action of the finite post-yielding stiffness. The maximum displacement response shown in Fig 7.11b compares with the plastic input displacement in more or less the same way as do the corresponding parameters in Fig 7.10.

Time histories of plastic input motions for the rigid-perfectly plastic, elastic-perfectly plastic and the bilinear system subjected to the half cycle triangular pulse are provided in Fig 7.13 for $\eta=0.2$. Three main trends are worthy of note: First, the plastic input

acceleration resembles the original function with the addition of a constant acceleration branch. Second, the plastic input velocity and displacement possess significant differences to the corresponding original functions. Third, the peak values of PIA and PIV are smaller than A_g and V_g , yet PID may be larger than D_g .

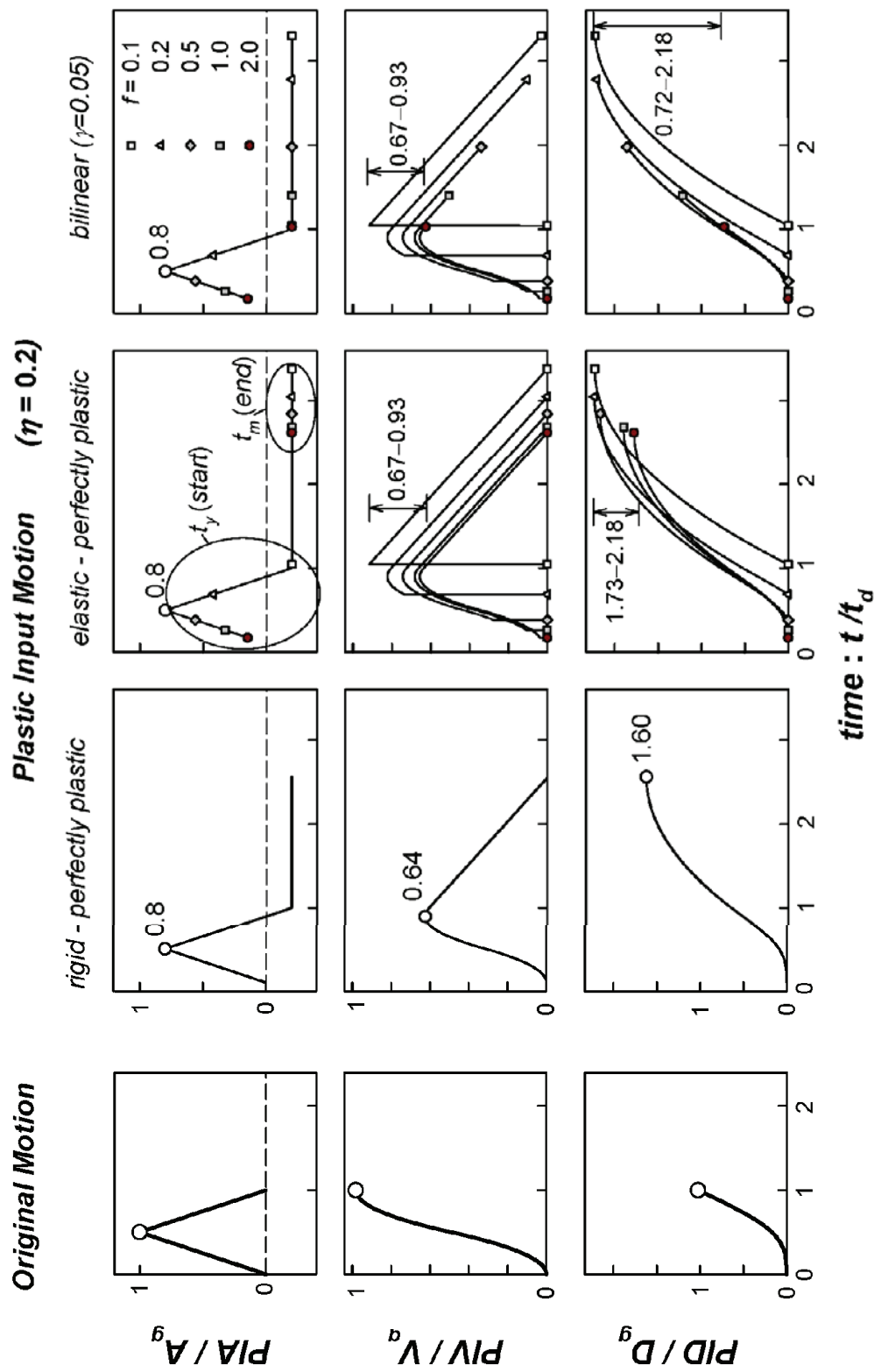


Figure 7.13 Half-cycle triangular pulse: time histories of Plastic Input Motions for perfectly plastic, elastic perfectly plastic and bilinear oscillators.

7.8 Conclusions

Analytical solutions and the novel concept of plastic input motion were presented for the response of single-degree-of-freedom yielding systems to dynamic loading. Contrary to conventional scaling laws, the derived relations are additive, thus imposing to a shift in the ordinates and the abscissa of the excitation function by an amount equal to yield strength and the time of yielding of the system, respectively. The response was shown to be derivable by integrating the modified forcing function in a linear fashion. While elements of this approach have been long utilized (notably in sliding block theory and in numerical algorithms for multi-degree-of-freedom systems), they have not been theoretically investigated and generalized as done in this work. Closed-form solutions were derived for a set of impulsive loads, akin to waveforms imposed by blasts and near-fault earthquake motions. The main conclusions of the study are:

1. Contrary to ordinary solutions, the proposed approach does not require information prior to time of yielding. In addition, for earthquake loading, knowledge of the (often difficult to establish) ground velocity and displacement time histories at yielding is not essential.
2. Peak values of the modified excitation function and its time integrals (Plastic Input Motion, *PIM*) can be substantially, yet not necessarily, smaller than those of the original excitation function. In the case of earthquake loading, the largest decrease occurs in the ground velocity and displacement time histories. On the other hand, Plastic Input Acceleration (*PIA*) may be higher than the original ground acceleration – especially for low values of yielding resistance.

3. In the special cases of rigid-plastic and elastic-perfectly plastic systems, plastic input motions coincide with the relative-to-ground plastic response of the system. This attribute is particularly valuable as the *PIM*'s can readily provide the response of such systems.
4. For an elastic-perfectly plastic system subjected to half-cycle earthquake loading, peak plastic input displacements are generally higher than those of a corresponding rigid-perfectly plastic system. An exception is the case of very flexible systems, where the opposite trend is observed. This is in contrast to peak displacement response (elastic+plastic), which is always higher than that of a rigid-perfectly plastic system. This suggests that for a flexible oscillator, the smaller plastic demand is compensated by a larger yield displacement, thus leading to an increasing overall response.
5. The finding in 4 is valid for bilinear systems as well. However, these systems typically experience smaller peak plastic input displacement than ground displacement at the end of the pulse.

The method has practical implications as: (i) it can detect the portion of ground excitation responsible for the development of plastic deformation, (ii) it can estimate the duration of each yielding branch. Also, the modified excitation function has distinct intensity characteristics which can be established by standard methods of engineering seismology and earthquake engineering. The above may be of particular interest for low-cycle impulsive loading such as that induced by blasts or near-fault earthquake motions.

It should be noticed that the proposed theory should not be viewed as a “shortcut” in the context of numerical analysis, as plastic response regimes should be identified by integrating the excitation record over its whole duration to detect times of yielding and unloading. Nevertheless, the theory offers the means of establishing modified peak ground motion parameters that are uniquely associated with the development of plastic deformation, which can replace ordinary parameters such as peak ground acceleration, velocity and displacement as demand indices for yielding systems.

It is fair to mention that extending the approach to multi-degree-of-freedom systems is not straightforward, as, typically, only a limited portion of the structure experiences yielding at a given time. This, however, should not be viewed as a limitation, as *PIM*'s, like other related notions in earthquake engineering, are based on the theory of simple oscillators.

8

Conclusions

8.1 Summary and Main Points

The work at hand investigated the dynamic response of certain classes of yielding systems to pulse ground excitations. Towards this direction three main objectives were pursued: (1) the derivation of new analytical solutions for a wide range of excitation waveforms; (2) the identification of pertinent dimensionless variables to express the response; (3) the investigation of the role of the time derivatives of the excitation on inelastic behavior and the introduction of new scaling laws. All objectives were achieved leading to a set of novel results. Specifically:

- (1) Frictional sliding of rigid blocks was quantified through exact analytical solutions considering a family of idealized acceleration pulses of half- and full-cycle duration, acting at an arbitrary inclination with respect to the frictional interface. With reference to the amplitude of response, pulse velocity and peak sliding displacement appear to be intimately interconnected (Eq. 2.6). This is anticipated given that the higher the area under the pulse after initiation of slippage, the longer the block will stay in motion, the higher ensuing sliding displacement. Nevertheless, as initiation of

slippage requires exceedance of a certain level of ground acceleration, ground velocity alone should not be viewed as the main descriptor of pulse destructiveness. Support to this finding comes from Eq. (2.5), which shows that sliding displacement depends on the time history of ground displacement – not of ground velocity. In this light, ground displacement might be more important in controlling peak dynamic response than presently thought.

- For half-cycle excitation, only up to 1/2 of incremental ground displacement can be transferred, as sliding displacement, to the block. This behavior is counterintuitive (yet rigorous from a mathematical viewpoint), as common sense suggests that relative-to-ground-displacement must equal ground displacement under zero friction. Nevertheless, the above value cannot be reproduced experimentally, as corresponding time of peak sliding τ_m is infinite and, thereby, dimensionless time (τ / τ_m) would never be sufficiently large to make the trend observable in a real test. Instead, the value $u_m / \Delta u_{gm} = 1$ is obtained, which means that the conventional behavior is not violated in practice.
- Half-cycle excitations tend to generate higher sliding displacement than corresponding full-cycle waveforms under equal ground acceleration, velocity and frictional strength. This is in contrast to ordinary motions containing multiple cycles, for which displacement generally increases with number of cycles.
- The complex dependence of peak displacement on frictional strength η is evident in Eqs (2.9) and (2.18). This suggests that simple scaling expressions of the power or exponential-law type might not be possible for the problem at hand, despite the idealized nature of the equation of motion and pulse.

- Incremental ground displacement $\Delta u_{gm} = u_{gm} - u_{gy}$ appears superior to schemes obtained by naïve dimensional analysis for normalizing sliding displacement, as the specific parameter bounds results for both half- and full-cycle excitations (anchored at the simple values $\frac{1}{2}$, 1 and 0 in the limits $\eta=0$ and 1) and provides the smallest dispersion of the results as function of η and β (Figs 2.13, 2.14). Accordingly, dimensional analysis alone does not seem superior to physically motivated choices for deriving optimal normalization schemes for the problem at hand.
 - Solutions to half-cycle excitations may faithfully simulate response to full- or multiple-cycle loading under conditions of asymmetric friction (Fig 2.16, Table 2.4). Thereby, half-cycle pulses should be regarded as being more than fictitious excitations employed for the sake of analytical simplicity. In this light, the finite ground velocity at the end of a half-cycle pulse should not be viewed as a physical limitation for employing these waveforms in real-life problems.
- (2) A simple analytical procedure was developed for determining the response of a simple oscillator subjected to four idealized ground acceleration pulses: (a) symmetric triangular; (b) sinusoidal; (c) exponential and (d) rectangular. Both elastic and inelastic response was evaluated and presented numerically and analytically. The most important findings gleaned from the study are:
- Ductility demand on elastic-perfectly-plastic structures with yield resistance smaller than the amplitude of the pulse may attain considerably high values (i.e., larger than 10), which elucidates the destructive nature of these motions. Ductility

demand is generally an increasing function of pulse duration. This is in contrast with elastic systems where maximum response is always obtained near resonance ($f = 1$).

- Time of yielding can be reliably (yet approximately) predicted by analytical means based on normalized yield strength η and pulse duration f (Eqs 3.27, 3.32, 3.33). Likewise, time of peak response can be determined analytically, in an exact manner, when it takes place after the end of the pulse. Peak response always takes place after pulse peak for both elastic and yielding oscillators.
- Structural response is sensitive to post-yielding stiffness. The relationship is non linear as this effect is less significant for periods greater than $1.5 T_d$. For smaller periods, however, oscillators with post-yield hardening of approximately 5% may experience 100% less deformation in comparison to corresponding perfectly plastic systems. This emphasizes the need for providing sufficient post-yielding stiffness (ductility) to structures located in the vicinity of active faults.
- The effect of damping was found to always reduce response. The reduction is more or less the same in all period ranges, oscillator strengths and pulse types while its effect on inelastic response is in accordance with the trends observed with the elastic oscillator.
- Higher harmonics contained in earthquake pulses alter (usually reduce) ductility demand especially at periods greater than $0.6 T_p$ and low values of amplitude ratio A_s/A_p .

- Force-reduction factor $R (= m SA / Q_y)$ appears inferior to normalized oscillator strength $\eta (= Q_y / m A_g)$ for quantifying inelastic response to impulsive loading. This can be understood given that elastic and inelastic response to strong pulses are fundamentally *different* and, thereby, using R introduces an elastic parameter (spectral acceleration SA) to the solution, which is not related to the physical problem.
- Full-cycle pulses ($N = 1$) may generate higher ductility demand with increasing post-yielding stiffness. An explanation for this counter-intuitive behavior is that peak response is attained after displacement reversal i.e., in the opposite direction to initial oscillator movement.
- For elastic–perfectly plastic undamped oscillators subjected to half–cycle pulses, time of yielding, peak deformation, and associated ductility demand can be evaluated analytically. The predictive equations (3.19–3.34) are in good agreement with rigorous numerical solutions. These solutions indicate that ductility demand is controlled by all three time histories of ground movement (i.e., acceleration, velocity, displacement) and, thereby, ground velocity alone should not be viewed as the sole descriptor of pulse destructiveness.
- Studies of the failure of Olive View Hospital in the 1971 San Fernando earthquake as well as that of EBO military factory headquarters during the 1995 Aegion earthquake were presented. Using the Pacoima Dam and Aegion records it was shown that the structures could not sustain the imposed ductility demand due to the proximity to fault and the associated forward rupture directivity effect. While the deformations predicted by the models were found to be in reasonable

agreement with results from inelastic numerical analyses, the estimated peak displacements were significantly smaller than the observed offset. This is in accord with earlier investigations of the failures using classical numerical approaches.

(3) By virtue of a half-cycle generalized pulse shape, novel analytical solutions for rocking of rigid blocks were presented covering an infinite number of pulse shapes ranging from a perfect spike ($\beta \rightarrow +\infty$), to a perfect rectangle ($\beta \rightarrow -\infty$). Rocking response was found to exhibit complex patterns even for highly idealized pulses like the ones at hand.

- It was shown that overturning is controlled by two dimensionless parameters: dimensionless pulse duration f (i.e., actual pulse duration times characteristic block frequency) and dimensionless uplift strength η (i.e., ratio of minimum required acceleration for initiation of uplift over peak pulse acceleration). On the other hand, in the realm of the linearized equation of motion block slenderness α was found to perfectly normalize the dependent variable θ , which can be interpreted as a similarity property for slender blocks. This attribute however, stems from the structure of the governing equation, not from dimensional considerations.
- Analytical formulae were derived for distinguishing between Safe (S) and Overturning (O) regions as function of f and η . Region O may be divided into three sub-areas (Figs 4.4-4.7): Region O_1 corresponding to instability in response during rising pulse, O_2 corresponding to instability in response

during decaying pulse, and O_3 where critical angle is attained after the end of the pulse. Region S is likewise subdivided into subgroups S_1 for which peak rocking response is attained during pulse and S_2 for which peak response is attained after pulse. As regions O_1 and O_2 correspond to overturning during pulse, they are applicable even to ground motions with time histories extending beyond the end of the pulse, t_d .

- A critical line separating safe from overturning response was established by means of a closed-form expression (Eq. 4.20). Likewise, simple solutions for peak rocking response in terms of peak angular displacement and corresponding time were presented (Eqs 4.24–4.31). Due to the transcendental nature of the response functions, however, not all solutions could be obtained in closed form.
- It was shown that the full non-linear equations of motion lead to more stable response than the linearized equations (Eq. 4.44). This property, however, strictly applies to overturning in the forward sense and should not be generalized to more complex cases such as those involving overturning after impact.

(4) As a follow up to the problem discussed in (3), rocking behavior of a free-standing block on a rigid base under a full-cycle rectangular pulse has been treated analytically, leading to closed-form solutions describing all phases of the response. To the best of the author's knowledge, this is the first fully analytical solution of rocking response to

a full-cycle pulse. The main conclusions gleaned from the analytical study are as follows:

- Two overturning *Modes* were identified and rigorously analyzed: *Mode 0*, referring to systems overturning without an impact during forward rocking; *Mode 1*, referring to systems overturning after one impact during backward rocking. A function study of the response was conducted providing useful insight into the physics of rocking behavior. Limit conditions for overturning in *Modes 0* and *1* were derived.
- The behavior of blocks surviving rocking response was investigated and closed-form solutions for peak rocking parameters (maximum angular displacement and corresponding time) were produced. Likewise, time of impact and velocity at impact (that play a crucial role in the analysis of backward rocking response), were determined in closed-form.
- Due to the full-cycle nature of the excitation, an odd number of immobility points during a specific response branch (i.e. rocking following an impact) implies stability, while an even number of immobility points implies overturning. This is different from half-cycle pulses for which stability and overturning are associated with a single immobility point and no immobility points, respectively.
- Observing the criteria for surviving and overturning due to *Mode 1* under full-cycle pulse excitation, two counterintuitive behaviors arise: (a) if at the end of pulse uplift angle equals slenderness α , the block will always survive *Mode 0*; (b) if an immobility point does not occur before the end of the pulse, the block

will always overturn in *Mode 0*. In other words it is impossible for a single immobility point to exist after the end of the pulse, leading to safety. If such a point exists, it shall be a second immobility point leading to overturning under *Mode 0* (group O_4). This is different from the case of single-lobe pulses for which no more than a single immobility point can take place during a particular rocking branch.

- All systems attaining local maxima in angular velocity before $3/2t_d$ survive *Mode 0*. Conversely, all systems attaining such local maxima after $2t_d$ overturn. In terms of peak rocking angles, all systems rocking to angles smaller or equal to α survive *Mode 0*, whereas all systems rocking to angles equal or higher than 2α overturn with *Mode 0*.
- In contrast with forward rocking for which blocks in region S possess bounded times of peak response, in backward rocking times of peak response are unbounded and tend to infinity for systems approaching the limit of the two rocking *Modes*.
- The two rocking modes were numerically evaluated for a range of idealized pulses, of (a) equal acceleration amplitude A_g and duration t_d ; and (b) equal velocity amplitude V_g and duration t_d . In the first case it was shown that the smaller the velocity content of the excitation the smaller the areas of overturning and vice versa; in the second case it was shown that blocks that overturn for a square pulse, overturn for pulses of a more spiky shape, and blocks that survive a perfect impulse, also survive pulses of a less spiky shape.

- Numerical results for response to actual impulsive ground motions provided simple limit criteria for systems that overturn. The high acceleration content early in the earthquake signal that leads to multiple impacts and development of angular velocity of arbitrary sign before the arrival of the main earthquake pulse may be beneficiary, leading to safety even for low strength, high frequency systems. Accordingly, the criteria in Eqs (5.62) and (5.63) can be used to exclude the possibility of overturning as no toppling systems plot beyond these limits. In this light, a near-fault motion with PGA/g less than $4/3$ times the slenderness angle of the block cannot induce overturning (strength criterion in Eq. 5.63b); a near-fault motion with predominant period less than $3/4$ of the characteristic period of the block cannot induce overturning either (frequency criterion in Eq. 5.63a); for all other cases, a simple curve bounds all toppling systems from the safe side (Eq. 5.62). Finally, a full-cycle rectangular pulse has always higher toppling potential than an actual record of equal PGA and predominant period.

(5) A transformation approach was derived for determining and interpreting the response of rigid-perfectly plastic, elastic-perfectly plastic and bilinear systems to dynamic loading. Contrary to conventional scaling procedures which are invariably multiplicative, the derived relation is additive, leading to a shift in the ordinates and the abscissa of the excitation function by an amount equal to the yield resistance and the time of yielding of the system, respectively. Accordingly, the proposed approach cannot be interpreted as a scaling law in the classical sense.

- Contrary to classical analytical and numerical schemes, the proposed theory does not require information as to the value of ground displacement and velocity at yielding. In addition, for earthquake loading, knowledge of the (often difficult to establish) ground velocity and displacement time histories at yielding is not essential.
- By introducing pertinent modified excitation parameters, the response was shown to be derivable by integrating the modified acceleration time history in a linear fashion by conventional analysis procedures (Eqs 6.19). This, however, should be viewed merely as a mathematical device, for the nonlinearity is inherent in the response.
- The transformed time history of ground acceleration is uniquely related to the development of plastic deformation. Accordingly, the particular time history and its time integrals are referred to as plastic input acceleration (*PIA*), velocity (*PIV*) and displacement (*PID*), respectively.
- Peak values of plastic input motion time histories were determined, encompassing the characteristics of ground motion as well as those of the yielding system. These peak input motions can be substantially (yet not necessarily) smaller than those in the original ground motion. The increase in values may be particularly pronounced in the acceleration component (a discontinuous function of time) due to the presence of alternating positive and negative cycles of ground acceleration.
- While elements of the proposed approach have long been utilized (notably in Newmark's sliding block theory), they have not been theoretically investigated

and generalized as done in this work. The method was shown to be applicable to more general classes of dynamic systems, such as those possessing elastic-perfectly plastic and bilinear yielding forced-displacements relationships.

- For a rigid-plastic system, plastic input acceleration can be interpreted as relative-to-ground acceleration (Eq. 6.21), a parameter that has received limited attention in earthquake engineering and seismology. However, this is a special case that should not be generalized for more general classes of yielding systems.
- The dimensionless strength factor (a_y/A_g) cannot be viewed as a classical scaling factor for it is not a multiplier of the forcing term (Eq. 6.32). It was shown to be so only for the idealized case of constant ground acceleration (Eq. 6.33). It is nevertheless a useful parameter for providing limits to the response, as $u \rightarrow u_g$ for $(a_y/A_g) = 0$ and $u \rightarrow 0$ for $(a_y/A_g) = 1$. It was also shown that this parameter represents the average ground acceleration over a particular yielding response branch (Eq. 6.34).
- It is fair to mention that in the absence of a small number of distinct pulses, the proposed approach does not guarantee a smaller number of mathematical operations in carrying out a dynamic response analysis of a rigid-perfectly plastic system. This is because the suite of points of yielding and unloading should be determined in the classical way that is, by tracing the time history of ground acceleration and block velocity over the whole length of the earthquake record. The strength of the proposed approach is in providing

insight into the physics of the problem and should not be viewed as a shortcut for the non-linear dynamic analysis of such systems.

- Peak values of the modified excitation function and its time integrals (Plastic Input Motion, *PIM*) can be substantially, yet not necessarily, smaller than those of the original excitation function. In the case of earthquake loading, the largest decrease occurs in the time histories of ground velocity and displacement. On the other hand, Plastic Input Acceleration (*PIA*) may be higher than the original ground acceleration – especially for low values of yield resistance.
- In the special cases of rigid-plastic and elastic-perfectly plastic systems, plastic input motions coincide with the relative-to-ground plastic response of the system. This attribute is particularly valuable as the *PIM*'s can readily provide the response of such systems.
- For an elastic-perfectly plastic system subjected to half-cycle earthquake loading, peak *PID*'s are generally higher than those of a corresponding rigid-perfectly plastic system. An exception is the case of very flexible oscillators, where the opposite trend is observed. This is in contrast to peak displacement response (elastic + plastic), which is always higher than that of a rigid-perfectly plastic system, and suggests that for a flexible oscillator, the smaller plastic demand is compensated by a larger yield displacement, thus leading to an increased overall response. This finding is valid for bilinear systems as well. However, these systems typically experience smaller peak *PID*'s than ground displacement at the end of the pulse.

- The method has practical implications as: (i) it can detect the portion of ground excitation responsible for the development of plastic deformation, (ii) it can estimate the duration of each yielding branch. Also, the modified excitation function has distinct intensity characteristics which can be established by standard methods of engineering seismology and earthquake engineering. The above may be of particular interest for low-cycle, high intensity impulsive loading such as that induced by blasts or near-fault earthquake motions.
- The theory provides the means of establishing modified peak ground motion parameters that are uniquely associated with the development of plastic deformation, which can replace ordinary parameters such as peak ground acceleration, velocity and displacement as demand indices for yielding systems.
- It is fair to mention that extending the approach to multi-degree-of-freedom systems is not straightforward, as, typically, only a limited portion of the structure experiences yielding at a given time. This, however, should not be viewed as a limitation, as *PIM*'s, like other related notions in earthquake engineering, are based on the theory of simple oscillators.

8.2 Recommendations for Future Research

The research reported above focused on an analytical investigation of non-linear single-degree-of-freedom dynamic systems exhibiting yielding behavior. Closed-form solutions were derived for the dynamic response of sliding blocks, rocking bodies and bilinear

oscillators to idealized ground pulses and actual recorded motions. A new framework for handling these systems in the realm of earthquake engineering analysis by truncating the excitation record, retaining only impulsive portions above yielding acceleration, was developed. As the subject is essentially open-ended, it would be worth continuing in the following directions:

- (1) The rocking response analysis presented in Chapter 5 can be extended to incorporate the non-linear equations of motion via energy solutions such as the one presented in Section 4.6 or perturbation theory.
- (2) Development of analytical solutions for the rocking behavior of multi-block assemblies such as two blocks, one placed on top of the other and two blocks connected by an epistyle.
- (3) The problem of the rocking block can be extended to account for an elastic foundation and/or a flexible superstructure.
- (4) An experimental verification (and extension) can be conducted of the results presented in Chapters 2, 4 and 5 considering rigid block sliding and rocking of rigid bodies.
- (5) The shift approaches presented in Chapters 6 and 7 can be extended to additional inelastic systems such as rocking blocks and various types of multi-degree-of-freedom systems.

List of Symbols

Latin symbols

<i>Symbol</i>	<i>Chapter</i> [*]	<i>Definition</i>
A		acceleration
A_g		peak pulse/ground acceleration
A_{gp}		peak <i>PIA</i>
A_p		amplitude of the main pulse
A_s		amplitude of the higher harmonic
a_y		yield acceleration
a, b, c, d	2	dimensionless exponents
b	4, 5	base half width
b_1, b_2		scaling parameters
c		damping
c_{py}		stiffness-proportional damping
C_y		ismic coefficient
D		dsplacement
D_c		characteristic displacement
D_g		peak pulse/ground displacement
D_{gp}		peak <i>PID</i>
D_m		maximum displacement amplification ($=u_m/u_s$)
f	3,7	dimensionless pulse duration ($=t_d/T$)

^{*} Indicated only in case of multiple use of a symbol.

	4,5	dimensionless pulse duration ($= pt_d$)
f_d		damping force
f_s, F_s		bilinear restoring force
F_y		restoring force at yielding
g		gravitational acceleration
g_1, g_2		scaling functions
h		half height of rigid block
I_o		block moment of inertia
K		elastic stiffness
K_{py}		post-yielding stiffness
L		differential operator
m		mass
N		number of cycles
p		block pseudo-frequency
$P(t)$		forcing function
PGA		peak ground acceleration
PGV		peak ground velocity
PIA		Plastic Input Acceleration
PID		Plastic Input Displacement
PIV		Plastic Input Velocity
PIM		Plastic Input Motion
PSA		spectral pseudo acceleration
	2	frictional resistance
Q_y	3	yield strength
r		ratio of kinetic energy
R	3,7	force reduction factor

	4,5	half block diameter
SA		elastic spectral acceleration
t		time
t^*		incremental time ($t-t_y$)
t_c		critical time
t_d		half-cycle pulse duration
T_d		pulse period ($=2 \times t_d$)
t_e		equilibrium point
t_{e1}		first equilibrium point
t_{e2}		second equilibrium point
t_i		time of impact
		time of u_{max}
	2	time of motion reversal
t_m	3	time of of peak deformation
	4, 5	time of maximum response
	6, 7	time of D_{gp}
t_{m0}		t_m for forward rocking
t_{m1}		t_m for backward rocking
$t_{m,el}$		time of peak elastic response
t_s		immobility point
t_{up}		time of uplift
t_{vm}		time of peak velocity
t_y		time of yielding
t_y^*		modified time of yielding
t_∞		infinite time after uplift
Δt		incremental time ($t-t_y$)

T		elastic period
T_d		excitation period
T_p		period of main pulse
T_s		period of the higher harmonic
u		displacement
u_m or u_{max}	2, 6	peak sliding displacement
	3, 7	maximum oscillator displacement
$u_{m1}=u_m$		sliding displacement at first stop
u_{m2}		sliding displacement at second stop
u_{res}		residual displacement
u_y		yield displacement
u_g		ground displacement
u_{gm}		ground displacement at peak response
u_{gy}		ground displacement at yielding
u_s		static displacement ($=A_g/\omega_n^2$)
$\dot{u}$		velocity
$\dot{u}_g$		ground velocity
$\dot{u}_{gm}$		ground velocity at peak response
$\dot{u}_{gy}$		ground velocity at yielding
$\dot{u}_y$		velocity at yielding
$\ddot{u}$		acceleration
$\ddot{u}_g$		ground acceleration
$\ddot{u}_{gy}$		ground acceleration at yielding
Δu		plastic displacement
Δu_g		shifted ground displacement
Δu_{gm}		incremental ground displacement

$\Delta \dot{u}_g$	shifted ground velocity
$\Delta \ddot{u}_g$	shifted ground acceleration
V	velocity
V_g	peak pulse/ground velocity
V_{gp}	peak <i>PIV</i>
V_m	peak sliding velocity
Z	impulsive terms parameter

Greek symbols

<i>Symbol</i>	<i>Chapter[†]</i>	<i>Definition</i>
α	2	static friction multiplier
α	4, 5	block slenderness
α_g		dimensionless peak ground acceleration
β		pulse shape parameter
γ		post-yielding hardening coefficient
ε		coefficient of restitution
ζ		viscous damping coefficient
ζ^*		post-yield viscous damping
η		dimensionless yield strength
η_1, η_2		dimensionless (asymmetric) yield strength
η_κ		kinetic dimensionless yield strength
$\eta^*=\eta_s$		static dimensionless yield strength
θ		uplift angle
$\dot{\theta}$		angular velocity
$\ddot{\theta}$		angular acceleration
θ_{max}		maximum uplift angle
θ_{max0}		peak response during forward rocking
θ_{max1}		peak response during backward rocking
$\dot{\theta}_i^-$		velocity before impact
$\dot{\theta}_i^+$		velocity after impact
λ_i		scaling parameter
μ	2, 4, 5	Coulomb frictional coefficient
μ	3, 6, 7	ductility demand factor
τ		dimensionless time

[†] Indicated only in case of multiple use of a symbol.

$\bar{\tau}$		shifted dimensionless time
τ_c		dimensionless critical time
τ_i		dimensionless time of impact
τ_i^-		before impact
τ_i^+		after impact
		time of u_{max}
	2	dimensionless time of motion reversal
τ_m	3	dimensionless time of peak deformation
	4, 5	dimensionless time of peak response
	6, 7	normalized time of D_{gp}
τ_{m0}		τ_m for forward rocking
	5	τ_m for backward rocking
τ_{m1}	2	dimensionless time of first stop ($=\tau_m$)
τ_{m2}		dimensionless time of second stop
$\tau_{m,el}$		dimensionless time of peak elastic response
τ_y		dimensionless time of yielding
$\tau_{y1}=\tau_y$		dimensionless time of first yield
τ_{y2}		dimensionless time of second yield
τ_y^*		modified time of yielding
τ_{y0}		expansion point for time of yielding
τ_{vm}		dimensionless time of peak velocity
τ_{up}		dimensionless time of uplift
τ_{∞}		infinite time after uplift
ϕ_s		phase angle
χ_i		scaling parameter
ω_D^*		damped post-yield frequency

 ω_n

natural frequency

 ω_n^*

post – yielding natural frequency

References

1. Alavi, B., Krawinkler, H. (2000). "Consideration of near-fault ground motion effects in seismic design." *Proceedings, 12th World Conference on Earthquake Engineering*, New Zealand, 8pp.
2. Alavi, B., Krawinkler, H. (2004). "Behavior of moment-resisting frame structures subjected to near-fault ground motions." *Earthquake Engineering & Structural Dynamics*, **33**: 687-706.
3. Ambraseys, N.N. & Menu, J.M. (1988). "Earthquake-induced ground displacements." *Earthquake Engineering & Structural Dynamics*, **16**: 985-1006.
4. Andronov, A.A., Vitt, A.A., Khaikin, S.E. (2011). "*Theory of oscillators*" Dover Publications
5. Anooshehpour, A., Heaton, T.H., Shi, B. & Brune, J.N. (1999). "Estimates of the ground accelerations at Point Reyes station during the 1906 San Francisco earthquake." *Bulletin of the Seismological Society of America*, **89** (4): 845-853.
6. Aslam, M., Godden, W.G. & Scalise, D.T. (1980). "Earthquake rocking response of rigid bodies." *Journal of the Structural Division, ASCE*, **106** (ST2): 377-392.
7. Baez, J.I., Miranda, E. (2000). "Amplification factors to estimate inelastic displacement demands for the design of structures in the near field." *12th World Conference on Earthquake Engineering*, New Zealand, Paper No. 1561.
8. Baker, J.W. (2007). "Quantitative classification of near-fault ground motions using wavelet analysis." *Bulletin of the Seismological Society of America*, **97** (5): 1486-1501.
9. Barenblatt, G.I. (1996). "*Scaling, self – similarity, and intermediate asymptotics*." Cambridge University Press.
10. Bathe, K.J., Gracewski, S. (1981). "On non-linear dynamic analysis using substructuring and mode superposition." *Comput. Struct.*, **13**: 699-707.
11. Bertero, V.V., Mahin, S.A., Herrera, R.A. (1978). "Aseismic design implications of near-fault San Fernando earthquake records." *Earthquake Engineering & Structural Dynamics*, **6**: 31-42.
12. Biggs, J.M. (1964). "*Introduction to structural dynamics*." Mc-Graw Hill, New York.
13. Boore, D.M., Zoback, M.D. (1974) "Near-field motions from kinematic models of propagating faults." *Bulletin of the Seismological Society of America*, **64**: 321-342.

-
14. Bowden F.P., Tabor, D. (1954). "*The friction and lubrication of solids.*" Oxford, Clarendon, UK
 15. Bozorgnia, Y., Mahin, S.A. (1998). "Ductility and strength demands of near-fault ground motions of the Northridge earthquake." *Proceedings of the 6th U.S. National Conference on Earthquake Engineering*, EERI, Seattle, Washington.
 16. Bracewell, R.N., (1986). "*The Fourier transform and its applications.*" (2nd ed.), Chapter 14, pp. 392-396, McGraw-Hill International Editions, New York.
 17. Bray, J.D., Rodriguez-Marek, A. (2004). "Characterization of forward-directivity ground motions in the near-fault region." *Soil Dynamics & Earthquake Engineering* **24**: 815-828.
 18. Butterfield, R. (1999). "Dimensional analysis for geotechnical engineers" *Geotechnique*, **49**(3), 357-366
 19. Cai, G.Q., Yu, J.S. & Lin, Y.K. (1995). "Toppling of rigid block under evolutionary random base excitations." *Journal of Engineering Mechanics, ASCE*. **121** (8): 924-929.
 20. Chioccarelli, E., Iervolino, I. (2010). "Near-source seismic demand and pulse-like records: A discussion for L'Aquila earthquake." *Earthquake Engineering & Structural Dynamics*, **39** (9): 1039–1062
 21. Chopra, A.K., Chintanapakdee, C. (2001). "Comparing response of SDF systems to near-fault and far-fault earthquake motions in the context of spectral regions." *Earthquake Engineering & Structural Dynamics*, **30** (12): 1769-1789.
 22. Chopra, A.K. (2006). "*Dynamics of structures.*" Prentice Hall: Upper Saddle River, NJ.
 23. Clough, R.W., Penzien, J. (1993). "*Dynamics of structures.*" McGraw-Hill: New York.
 24. Conte, E. & Dente, G. (1989). "An analytical solution for Newmark's sliding block." *Soils & Foundations*, **29** (3), 152-156.
 25. Cuesta, I., Aschheim, M.A. (2000). "*Waveform independence of R-factors.*" paper 1546, *12th Earthquake Conference on Earthquake Engineering*, New Zealand
 26. Cuesta, I., Aschheim, M.A. (2001). "Isoductile strengths and strength reduction factors of elasto-plastic SDOF systems subjected to simple waveforms." *Earthquake Engineering & Structural Dynamics*, **30** (7): 1043-1059.
 27. Cuesta I, Aschheim M.A. (2001) "Inelastic response spectra using conventional and pulse R-factors." *Journal of Structural Engineering*, **127** (9): 1013-1020.
 28. Cuesta, I., Aschheim, M.A. (2004). "The use of simple pulses to estimate inelastic response spectra." *Journal of Earthquake Engineering*, **8** (6): 865-893.
 29. Dahl, P. (1968). "*A solid friction model.*" Technical Report TOR-0158(3107-3118)-1, The Aerospace Corporation, El Segundo, CA.
 30. Dimakopoulou, V., Fragiadakis, M., Spyarakos, C.C., (2011). "Seismic response of degrading SDOF systems due to near-field ground motions", *COMPDYNN 2011 3rd ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics & Earthquake Engineering*, Corfu, Greece, May. 25-28.

31. Dimitrakopoulos, E. & DeJong, M. (2012). "Revisiting the rocking block: closed- form solutions and similarity laws." *Proceedings of the Royal Society*. **468** (2144): 2294-2318
32. Dimitrakopoulos, E. & DeJong, M. (2012). "Overturning of retrofitted rocking structures under pulse-type excitations" *J. Eng. Mech.*, **138**(8), 963–972.
33. European Committee for Standardization (CEN). (2004). "*Eurocode 8: Design of structures for earthquake resistance – Part 1: General rules, seismic actions and rules for buildings.*" EN 1998-1, Brussels.
34. Gantes, C.J., Pnevmatikos, N.G. (2004). "Elastic-plastic response spectra for exponential blast loading." *International Journal of Impact Engineering*, **30** (3): 323-343.
35. Garini, E., Gazetas, G., Anastasopoulos, I. (2011). "Asymmetric Newmark sliding caused by motions containing severe directivity and fling pulses" *Geotechnique* **61** (9): 733–756.
36. Gazetas, G. (1996). "*Soil dynamics & earthquake engineering: case histories*" Symeon Eds (in Greek)
37. Gazetas, G., Garini, E., Anastasopoulos, I. (2009) "Effects of Near–Fault Ground Shaking on Sliding Systems", *Journal of Geotechnical Engineering*, ASCE, **135**: 12: 1906 – 1921.
38. Gazetas, G., Garini, E., Berrill, J.B., Apostolou, M. (2012). "Sliding and overturning potential of Christchurch 2011 earthquake records" *Earthquake Engineering & Structural Dynamics* **41** (14): 1921–1944.
39. Hall, J.F., Heaton, T.H., Wald, D.J., Halling, M.W. (1995). "Near-source ground motion and its effects on flexible buildings." *Earthquake Spectra*, **11** (4): 569-605.
40. Harris, C.M. (1996). "*Shock and vibration handbook*" 4th Edition, McGraw-Hill
41. Hess, D.P., Soom, A. (1990). "Friction at a lubricated line contact operating at oscillating sliding velocities." *Journal of Tribology*, **112**: 147–152.
42. Hogan, S.J. (1994) "Slender rigid block motion." *Journal of Engineering Mechanics, ASCE*, **120** (1): 11-24.
43. Housner, G.W. (1963). "The behavior of inverted pendulum structures during earthquakes." *Bulletin of the Seismological Society of America*. **52** (2): 404-417.
44. Ishiyama, Y. (1982). "Motion of rigid bodies and criteria for overturning by earthquake excitations." *Earthquake Engng and Structural Dynamics*. **10**: 635-650.
45. Iwan, W.D. (1961). "*The dynamic response of bilinear hysteretic systems*" Ph.D. Dissertation, California Institute of Technology, Pasadena.
46. Iwan, W.D. (1997). "Drift spectrum: measure of demand for earthquake ground motions." *Journal of Structural Engineering, ASCE*, **123** (4): 397-404.
47. Jacobsen, L.S. & Ayre, S.M. (1958). "*Engineering vibrations.*" McGraw Hill: New York.
48. Jennings, P.C. (2000). "Ground motion pulses and structural response." *Proceedings of China – U.S. Millennium, Symposium of Earthquake Engineering: Earthquake Engineering Frontiers in the New Millennium*, 7 pages in electronic file, Beijing, China, November 8-10.

-
49. Karavasilis T.L., Seo C-Y., Makris N. (2011). "Dimensional response analysis of bilinear systems subjected to non-pulse-like earthquake ground motions." *Journal of Structural Engineering ASCE*, **137**(5):600-606
 50. Karavasilis T.L., Makris N., Bazeos N., Beskos D.E. (2010). "Dimensional response analysis of multi-storey regular steel MRF subjected to pulse-like earthquake ground motions." *Journal of Structural Engineering ASCE*, **136**(8): 921-932.
 51. Katsikadelis, J. (2002). "Dynamics of structures-Parts I & II" Symmetria Eds (in Greek)
 52. Kempton, J.J. & Stewart, J.P. (2006). "Prediction equations for significant duration of earthquake ground motions considering site and near-source effects." *Earthquake Spectra*, **22** (4): 985-1013.
 53. Kirkpatrick, P. (1927). "Seismic measurements by the overthrow of columns." *Bulletin of the Seismological Society of America*. **17**(2).
 54. Kounadis, A.N. (2013). "Parametric study in rocking instability of a rigid block under harmonic ground pulse: A unified approach." *Soil Dynamics & Earthquake Engineering*. **45**: 125–143
 55. Kouretzis, G.P., Bouckovalas, G.D. & Gantes, C.J., (2007). "Analytical calculation of blast-induced strains to buried pipelines.", *International Journal of Impact Engineering*, **34** (10): 1683-1704.
 56. Kramer, S.L. (1996). "Geotechnical earthquake engineering." Prentice Hall: Upper Saddle River, NJ.
 57. Krylov, N., Bogoliubov, N. (1943). "Introduction to nonlinear mechanics." Princeton University Press N.J.
 58. Mahin, S.A., Bertero, V.V., Chopra, A.K., Collins, R.G. (1976). "Response of the Olive View hospital main building during the San Fernando earthquake." *Report No. EERC 76-22, Earthquake Engineering Research Center, University of California, Berkeley*.
 59. Makdisi, F.I. & Seed, H.B. (1978). "Simplified procedure for estimating dam and embankment earthquake – induced deformations." *Journal of the Geotechnical Engineering Division*, **7**: 849-867.
 60. Makris, N., Chang, Sh.P. (2000) "Response of damped oscillators to cycloidal pulses." *Journal of Engineering Mechanics, ASCE*, **126** (02): 123-131.
 61. Makris, N., Konstantinidis, D. (2003) "The rocking spectrum and the limitations of practical design methodologies." *Earthquake Engineering & Structural Dynamics*, **32**: 265-289
 62. Makris, N. & Black, C.J. (2004). "Dimensional analysis of rigid-plastic & elastoplastic structures under pulse-type excitations." *Journal of Engineering Mechanics ASCE*, **130** (9), 1006-1018.
 63. Makris, N., Black, C.J. (2004). "Dimensional analysis of bilinear oscillators under pulse-type excitations." *Journal of Engineering Mechanics, ASCE*. **130**(9): 1019-1031.
 64. Malhotra, P.K. (1999). "Response of buildings to near-field pulse-like ground motions." *Earthquake Engineering & Structural Dynamics*, **28** (11): 1309-1326

65. Mavroeidis, G., Papageorgiou, A. (2003). "A mathematical representation of near-fault ground motions." *Bulletin of the Seismological Society of America*, **93** (3): 1099-1131.
66. Mavroeidis, G., Dong, Papageorgiou, A. (2004). "Near-fault ground motions, and the response of elastic and inelastic single-degree-of-freedom (SDOF) systems." *Earthquake Engineering & Structural Dynamics*, **33** (9), 1023-1049.
67. Menun, C., Fu, Qiang. (2002). "An analytical model for near-fault ground motions and the response of SDOF systems." *Seventh U S National Conference on Earthquake Engineering, Boston, Massachusetts*, 7 pages in CD ROM
68. Milne, J. (1885). "Seismic experiments." *Transactions of the Seismological Society of Japan*. **8**:1-82.
69. Milne, J. & Omori, F. (1893). "On the overturning and fracturing of brick and columns by horizontally applied motion." *Seismological Journal of Japan*. **17**: 59-86.
70. Miranda, E., Bertero, V. (1994). "Evaluation of strength reduction factors of earthquake – resistant design." *Earthquake Spectra*. **10** (2): 357-379.
71. Mouzakis, H., Psycharis, I.N., Papastamatiou, D.Y., Carydis, P.G., Papantonopoulos, C., Zambas, C. (2002). "Experimental investigation of the earthquake response of a marble classical column." *Earthquake Engineering and Structural Dynamics*. **31**:1681-1698.
72. Muto, K., Umemura, H. & Sonobe, Y. (1960). "Study of overturning vibration of slender structures." *Proc. 2nd World Conference on Earthquake Engng, Tokyo, Japan*. 2, 1293.
73. Mylonakis, G. & Reinhorn, A.M. (2001). "Yielding oscillator under triangular ground acceleration pulse." *Journal of Earthquake Engineering*, **5**(2), 225-251.
74. Mylonakis, G., Syngros, C., Gazetas, G., Tazoh, T. (2006). "The role of soil in the collapse of 18 piers of Hanshin Expressway in the Kobe earthquake." *Earthquake Engineering & Structural Dynamics*, **35** (5): 547-575.
75. Mylonakis, G., Voyagaki, E., Price, T. (2003). "Damage potential of the 1999 Athens, Greece, accelerograms." *Bulletin of Earthquake Engineering*, **1**: 205-240.
76. Mylonakis, G. & Voyagaki, E. (2006). "Yielding oscillator subjected to simple pulse waveforms: numerical analysis & closed-form solutions." *Earthquake Engineering & Structural Dynamics*, **35** (15): 1949-1974.
77. Nayfeh, A.H. (2000). "*Perturbation methods*" Wiley Classics Library Edition
78. Newmark, N.M. (1965). "Effects of earthquakes on dams and embankments. Fifth Rankine Lecture.", *Geotechnique*, **15** (2), 139-160.
79. Newmark, N.M., Rosenblueth, E. (1971). "*Fundamentals of earthquake engineering*." Prentice Hall
80. Newmark, N.M., Hall, W.J. (1982). "Earthquake spectra and design." *EERI Monograph Series*, Earthquake Engineering Research Institute, Oakland, CA.
81. Olsson, H., Åström, K.J., Canudas de Wit, C., Gäfvert, M., Lischinsky, P. (1998). "Frictional models and friction compensation." *Eur. J. Control*, **4** (3): 176-195.

-
82. Papantonopoulos, C., Psycharis, I. N., Papastamatiou, D. Y., Lemos J.V., H. P. Mouzakis, H. P. (2002). "Numerical prediction of the earthquake response of classical columns using the distinct element method." *Earthquake Engineering and Structural Dynamics*, **31**(9):1699-1717.
 83. Paret, T., Attala, M., Freeman, S., Borchardt, G.A. (1997). "Permanent directed deformations of buildings as evidence for a directed near-field pulse in the M6.7 Northridge earthquake of January 17, 1994." *Proceedings of the NEHRP Conference on Research on the Northridge California Earthquake of January 17, 1994, Los Angeles, California*, August 20-22.
 84. Peña, F., Prieto, F., Lourenço, P.B., Campos Costa, A. & Lemos, J.V. (2007). "On the dynamics of rocking motion of single rigid-block structures." *Earthquake Engineering and Structural Dynamics*. **36**: 2383-2399.
 85. Pompei, A., Scalia, A. & Sumbatyan, M.A. (1998). "Dynamics of rigid block due to horizontal ground motion." *Journal of Engng Mechanics ASCE*. **124** (7): 713-717.
 86. Priestley, M.J.N., Evison, R.J. & Carr, A.J. (1983). "Seismic response of structures free to rock on their foundations." *Bull. New Zealand Natl. Soc. Earthquake Engineering*. **11**: 141-150.
 87. Prieto, F., Lourenço, P.B. & Oliveira, C.S. (2004). Impulsive Dirac-delta forces in the rocking motion. *Earthquake Engineering & Structural Dynamics*. **33** (7): 839-857.
 88. Psycharis, I.N. (1982). "Dynamic behavior of rocking structures allowed to uplift." Ph.D. Dissertation, California Institute of Technology, Pasadena.
 89. Psycharis, I.N. & Jennings, P.C. (1983). "Rocking of slender bodies allowed to uplift." *Earthquake Engineering and Structural Dynamics*. **11**: 57-76.
 90. Psycharis, I.N. & Jennings, P.C. (1985). "Upthrow of objects due to horizontal impulse excitation." *Bulletin of the Seismological Society of America*. **75** (2): 543-561.
 91. Psycharis, I.N. (1990). "Dynamic behaviour of rocking two-block assemblies." *Earthquake Engineering and Structural Dynamics*, **19**: 555-575.
 92. Psycharis, I.N. (1991). "Effect of base uplift to the dynamic response of SDOF structures." *Journal of Structural Engineering ASCE*, **117**: 733-754.
 93. Psycharis, I.N., Syngros, C., Mimoglou, P., Taflampas, I. (2002) "Parametric investigation of the overturning of rigid blocks under dynamic loading." *12th European Conference on Earthquake Engineering*, London, September 9-13.
 94. Psycharis, I.N., Lemos, J.V., Papastamatiou, D.Y., Zambas, C., Papantonopoulos, C.. (2003). "Numerical study of the seismic behaviour of part of the Parthenon Pronaos." *Earthquake Engineering and Structural Dynamics*, **32**: 2063-2084.
 95. Purvance, M.D., Anooshehpour, A. & Brune, J.N. (2008). "Freestanding block overturning fragilities: Numerical simulation and experimental validation." *Earthquake Engineering and Structural Dynamics*. **37**: 791-808.
 96. Rabinowicz, E. (1951). "The nature of the static and kinetic coefficients of friction." *Journal of Applied Physics*, **22**(11):1373-1379.

97. Ray Chaudhuri, S. & Hutchinson, T.C. (2006). "Fragility of bench-mounted equipment considering uncertain parameters." *Journal of Structural Engineering*, ASCE, **132** (6): 884-898.
98. Richards, R.Jr, Elms, D.G. (1979). "Seismic behavior of gravity retaining walls." *Journal of the Geotechnical Engineering Division*, **4**: 449-464.
99. Riddell, R., Garcia, J.E., Garces, E. (2002). "Inelastic deformation response of SDOF systems subjected to earthquakes." *Earthquake Engineering and Structural Dynamics*, **31**: 515-538.
100. Rodriguez-Marek, A. (2000). "Near-fault seismic site response." Ph.D. Dissertation, Department of Civil Engineering, University of California, Berkeley.
101. Sarma, S.K. (1975). "Seismic Stability of earth dams & embankments" *Geotechnique*, **25** (4), 743-761
102. Sasani, M., Bertero, V.V. (2000). "Importance of severe pulse-type ground motion in performance – based engineering: historical and critical review." *Proceedings of the 12th World Conference on Earthquake Engineering*, New Zealand, 8pp.
103. Shendon, H.W. III. (1996). "Criteria for initiation of slide, rock, and slide-rock rigid-body modes." *Journal of Engineering Mechanics ASCE*. **122** (7): 690-693.
104. Shi, B., Anooshehpour, A., Zheng, Y. & Brune, J.N. (1996). "Rocking and overturning of precariously balanced rocks by earthquakes." *Bulletin of the Seismological Society of America*. **86** (5): 1364-1371.
105. Sidiropoulos, P. (2012). "Seismic sliding displacement on a dry sand surface" Master Thesis, Hellenic Open University (in Greek)
106. Singh, J.P. (1958). "Earthquake ground motions: implications for designing structures and reconciling structural damage." *Earthquake Spectra*, **1** (2): 239-270.
107. Sinopoli, A. (1989). "Dynamic analysis of a stone column excited by a sine wave ground motion." *Applied Mechanics Reviews*, **44**(11): S246-S255 (Part 2)
108. Somerville, P., Graves, R. (1993). "Conditions that give rise to unusually large long-period ground motions." *Structural Design of Tall Buildings*, **2**: 211-232.
109. Somerville, P.G., Smith, N.F., Graves, R.W., Abrahamson, N.A. (1997). "Modification of empirical strong motion attenuation relations to include the amplitude and duration effects of rupture directivity." *Seismological Research Letters*, **68**: 199 – 222.
110. Somerville, P.G. (2000). "Magnitude scaling of near fault ground motion." *Proc. Int. Workshop on Annual Commemoration of Chi – Chi earthquake*, **1**: 59 – 70.
111. Spanos, P.D. & Koh, A.S. (1984). "Rocking of rigid bodies due to harmonic shaking." *Journal of Engineering Mechanics, ASCE*. **110** (11): 1627-1642.
112. Spanos, P.D., Roussis, P.C. & Politis, N.P.A. (2001). "Dynamic analysis of stacked rigid blocks." *Soil Dynamics and Earthquake Engineering*. **21** (7): 559-578.

-
113. Spyrakos, C.C. (1996). *Finite element modeling in engineering practice*, Algor Inc, Pittsburgh, PA, USA.
 114. Spyrakos, C.C., Maniatakis, Ch.A., & Taflampas, J. (2008). "Evaluation of near-source seismic records based on damage potential parameters. Case Study: Greece" *Soil Dynamics and Earthquake Engineering*, **28**: 738-753
 115. Stewart, J.P. et al. (2001). "Ground motion evaluation procedures for performance-based design." *PEER-2001/09*, University of California, Berkeley.
 116. Stribeck, R., (1902). Die wesentlichen Eigenschaften der Gleit- und Rollenlager – The key qualities of sliding and roller bearings. *Zeitschrift des Vereines Deutscher Ingenieure*, 46(38,39):1342-1348, 1432-1437.
 117. Stronge WJ, Yu TX. (1995). *Dynamic models for structural plasticity* (corrected 2nd printing). Springer, London.
 118. Taflampas, I.M., Spyrakos, C.C, Koutromanos, I.A. (2009). "A new definition of strong motion duration and related parameters affecting the response of medium-long period structures." *Soil Dynamics & Earthquake Engineering*, **29**: 752-763.
 119. Tang, Y., Zhang, J. (2011). "Response spectrum-oriented pulse identification and magnitude scaling of forward directivity pulses in near-fault ground motions." *Soil Dynamics & Earthquake Engineering*, **31** (1): 59-76.
 120. Timoshenko, S., Young, D.H. (1948). "*Advanced Dynamics*." McGraw-Hill Book Company,
 121. Tso, W.K. & Wong, C.M. (1989a). "Steady state rocking response of rigid block. Part I: Analysis." *Earthquake Engineering and Structural Dynamics*. **18**: 89-106.
 122. Tso, W.K. & Wong, C.M. (1989b). "Steady state rocking response of rigid block. Part II: Experiments." *Earthquake Engineering and Structural Dynamics*. **18**: 107-120.
 123. Veletsos, A.S., Newmark, N.M. (1960). "Effect of inelastic behavior on the response of simple system to earthquake motion." *Proc. 2nd World Conference on Earthquake Engineering*, Tokyo, 895-912.
 124. Veletsos, A.S., Newmark, N.M. (1964). "Design Procedures for shock isolation systems of underground protective structures." *Technical Documentary Report No. RTD TDR-63-3096, vol. III, Air Force Laboratory*, New Mexico.
 125. Veletsos, A.S, Newmark, N.M., Chelapati, C.V. (1965). "Deformation spectra for elastic and elastoplastic systems subjected to ground shock and earthquake motions." *Proceedings of the Third World Conference on Earthquake Engng*, Wellington, New Zealand, **2**: 663 – 680.
 126. Veletsos, A.S. (1969). "Maximum deformations of certain nonlinear systems." *Proceedings of the 4th World Conference on Earthquake Engineering*, Santiago, Chile, **2**:155-170.
 127. Veletsos, A.S., and Ventura, C.E. (1984). "Efficient analysis of dynamic response of linear systems." *Earthquake Engineering and Structural Dynamics*, **12**: 521–536.

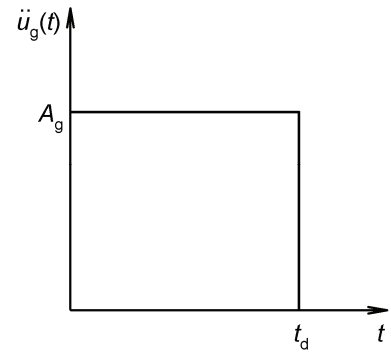
128. Villaverde, R., and Hanna, M.M. (1992). "Efficient mode superposition algorithm for seismic analysis of non-linear structures." *Earthquake Engineering and Structural Dynamics*, **21**: 849-858.
129. Voyagaki, E. (2003). "*Destructiveness of near-field earthquake pulses: applications in the 1999 Athens, Greece, earthquake.*" Master Thesis, City University of New York.
130. Walker, G.R. (1965). "Earthquake resistant design—the pulse method." *Proceedings of the Third World Conference on Earthquake Engineering*, National Committee on Earthquake Engineering, Wellington, New Zealand, Vol. II: 683-694.
131. Wartman, J., Bray, J.D., Seed, R.B. (2003). "Inclined plane studies of the Newmark sliding block procedure." *Journal of the Geotechnical & Geoenvironmental Engineering ASCE*, **129** (8), 673-684.
132. Whitman, R.V. and Liao, S. (1985). "Seismic design of gravity retaining walls" *Miscellaneous Paper GL-85-1*, U. S. Army Engineer Waterways Experiment Station, Vicksburg, MS 39180.(89).
133. Wolfram, S. (2010). "*Mathematica*" (version 8) [software]. Wolfram Research Inc.
134. Yang, D., Pan, J., Li, G. (2009). "Non-structure-specific intensity measure parameters and characteristic period of near-fault ground motions." *Earthquake Engineering and Structural Dynamics*, **38** (11): 1257-1280.
135. Yegian, M.K., Marciano, E.A., Gharaman, V.G. (1991). "Earthquake - induced permanent deformations: probabilistic approach." *Journal of Geotechnical Engineering, ASCE*, **117** (1): 35-50.
136. Yim, C.S., Chopra, A.K. & Penzien, J. (1980). "Rocking response of rigid bodies to earthquakes." *Earthquake Engineering and Structural Dynamics*. **8**: 565-587.
137. Yim, C.S, Lin, H. (1991). "Nonlinear impact and chaotic response of slender rocking objects." *Journal of Engineering Mechanics, ASCE* , **117** (9): 2079-2100.
138. Zhang, J. & Makris, N. (2001). "Rocking response of a free-standing blocks under cycloidal pulses." *Journal of Engineering Mechanics, ASCE*. **127** (5): 473-483.

Appendix A

Various Pulse Shapes

A.1 Half-Cycle

A1.1 Rectangular Pulse



Acceleration

$$\ddot{u}_g(t) = \begin{cases} A_g & , \quad t \leq t_d \\ 0 & , \quad t > t_d \end{cases}$$

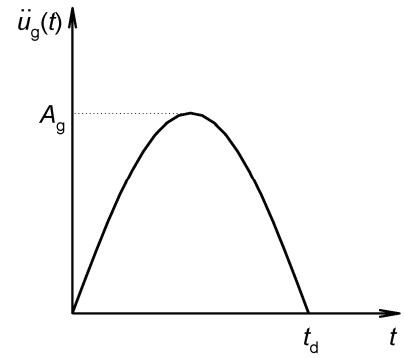
Velocity

$$\dot{u}_g(t) = \begin{cases} A_g t & , \quad t \leq t_d \\ A_g t_d & , \quad t > t_d \end{cases}$$

Displacement

$$u_g(t) = \begin{cases} \frac{1}{2} A_g t^2 & , \quad t \leq t_d \\ A_g t_d \left(t - \frac{t_d}{2} \right) & , \quad t > t_d \end{cases}$$

A1.2 Sinusoidal Pulse



Acceleration

$$\ddot{u}_g(t) = \begin{cases} A_g \sin \pi \frac{t}{t_d} & , \quad t \leq t_d \\ 0 & , \quad t > t_d \end{cases}$$

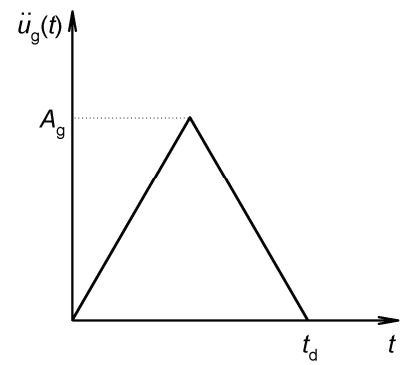
Velocity

$$\dot{u}_g(t) = \begin{cases} A_g \frac{t_d}{\pi} (1 - \cos \pi \frac{t}{t_d}) & , \quad t \leq t_d \\ A_g \frac{2 \cdot t_d}{\pi} & , \quad t > t_d \end{cases}$$

Displacement

$$u_g(t) = \begin{cases} A_g \frac{t_d^2}{\pi} \left(\frac{t}{t_d} - \frac{1}{\pi} \sin \pi \frac{t}{t_d} \right) & , \quad t \leq t_d \\ A_g \frac{t_d^2}{\pi} \left(2 \frac{t}{t_d} - 1 \right) & , \quad t > t_d \end{cases}$$

A1.3 Triangular Pulse



Acceleration

$$\ddot{u}_g(t) = \begin{cases} 2A_g \frac{t}{t_d} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ 2A_g(1 - \frac{t}{t_d}) & , \quad \frac{t_d}{2} \leq t \leq t_d \\ 0 & , \quad t > t_d \end{cases}$$

Velocity

$$\dot{u}_g(t) = \begin{cases} A_g \frac{t^2}{t_d} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ A_g t_d (2 \frac{t}{t_d} - (\frac{t}{t_d})^2 - \frac{1}{2}) & , \quad \frac{t_d}{2} \leq t \leq t_d \\ A_g \frac{t_d}{2} & , \quad t > t_d \end{cases}$$

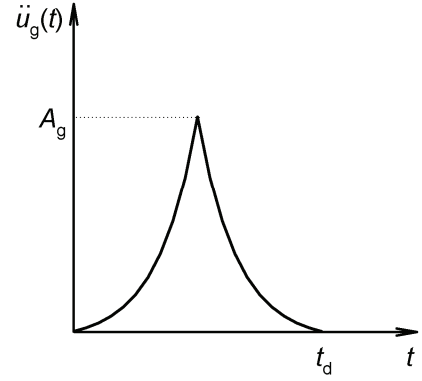
Displacement

$$u_g(t) = \begin{cases} A_g \frac{t^3}{3t_d} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ A_g (t^2 - \frac{t^3}{3t_d} - \frac{t \cdot t_d}{2} + \frac{t_d^2}{12}) & , \quad \frac{t_d}{2} \leq t \leq t_d \\ A_g t_d (\frac{t}{2} - \frac{t_d}{4}) & , \quad t > t_d \end{cases}$$

A1.4 Exponential Pulse

Acceleration

$$\ddot{u}_g(t) = \begin{cases} A_g \frac{1 - e^{2\pi t/t_d}}{1 - e^\pi} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ A_g \frac{1 - e^{2\pi(1-t/t_d)}}{1 - e^\pi} & , \quad \frac{t_d}{2} \leq t \leq t_d \\ 0 & , \quad t > t_d \end{cases}$$



Velocity

$$\dot{u}_g(t) = \begin{cases} A_g \frac{t + \frac{t_d}{2\pi}(1 - e^{2\pi t/t_d})}{1 - e^\pi} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ A_g \frac{t + \frac{t_d}{2\pi}(1 - 2e^\pi + e^{2\pi(1-t/t_d)})}{1 - e^\pi} & , \quad \frac{t_d}{2} \leq t \leq t_d \\ A_g \frac{1 - e^\pi + \pi}{1 - e^\pi} \cdot \frac{t_d}{\pi} & , \quad t > t_d \end{cases}$$

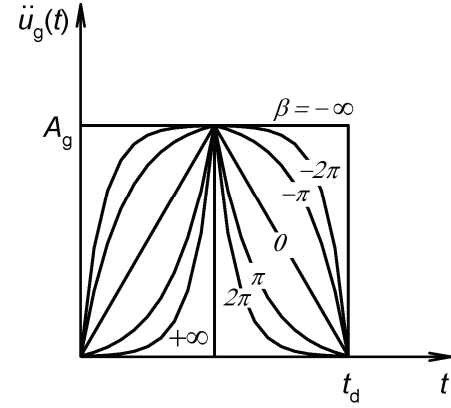
Displacement

$$u_g(t) = \begin{cases} A_g \frac{\frac{t^2}{2} + \frac{t \cdot t_d}{2\pi} + \frac{t_d^2}{4\pi^2}(1 - e^{2\pi t/t_d})}{1 - e^\pi} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ A_g \frac{\frac{t^2}{2} + \frac{t_d}{2\pi}(t - 2te^\pi + t_d e^\pi) - \frac{t_d^2}{4\pi^2}(1 - e^{2\pi(1-t/t_d)})}{1 - e^\pi} & , \quad \frac{t_d}{2} \leq t \leq t_d \\ A_g \frac{(1 - e^\pi + \pi)(2t - t_d)}{1 - e^\pi} \cdot \frac{t_d}{2\pi} & , \quad t > t_d \end{cases}$$

A1.5 Generalized Exponential Pulse

Acceleration

$$\ddot{u}_g(t) = \begin{cases} A_g \frac{1 - e^{2\beta t/t_d}}{1 - e^\beta} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ A_g \frac{1 - e^{2\beta(1-t/t_d)}}{1 - e^\beta} & , \quad \frac{t_d}{2} \leq t \leq t_d \\ 0 & , \quad t > t_d \end{cases}$$



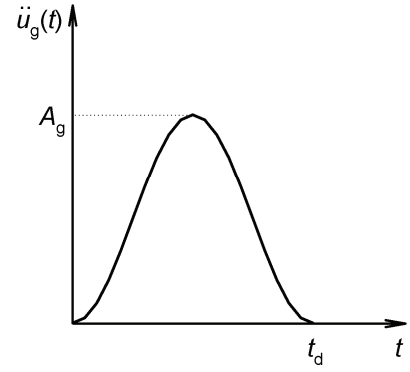
Velocity

$$\dot{u}_g(t) = \begin{cases} A_g \frac{t + \frac{t_d}{2\beta}(1 - e^{2\beta t/t_d})}{1 - e^\beta} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ A_g \frac{t + \frac{t_d}{2\beta}(1 - 2e^\beta + e^{2\beta(1-t/t_d)})}{1 - e^\beta} & , \quad \frac{t_d}{2} \leq t \leq t_d \\ A_g \frac{1 - e^\beta + \beta \cdot \frac{t_d}{\beta}}{1 - e^\beta} & , \quad t > t_d \end{cases}$$

Displacement

$$u_g(t) = \begin{cases} A_g \frac{\frac{t^2}{2} + \frac{t \cdot t_d}{2\beta} + \frac{t_d^2}{4\beta^2}(1 - e^{2\beta t/t_d})}{1 - e^\beta} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ A_g \frac{\frac{t^2}{2} + \frac{t_d}{2\beta}(t - 2te^\beta + t_d e^\beta) - \frac{t_d^2}{4\beta^2}(1 - e^{2\beta(1-t/t_d)})}{1 - e^\beta} & , \quad \frac{t_d}{2} \leq t \leq t_d \\ A_g \frac{(1 - e^\beta + \beta)(2t - t_d)}{1 - e^\beta} \cdot \frac{t_d}{2\beta} & , \quad t > t_d \end{cases}$$

A1.6 Versed Sine Pulse



Acceleration

$$\ddot{u}_g(t) = \begin{cases} \frac{A_g}{2} [1 - \cos 2\pi \frac{t}{t_d}] & , \quad 0 \leq t \leq t_d \\ 0 & , \quad t > t_d \end{cases}$$

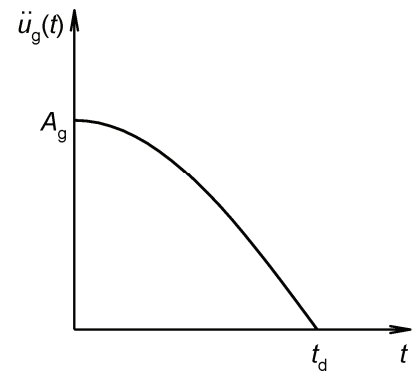
Velocity

$$\dot{u}_g(t) = \begin{cases} A_g \frac{t_d}{2} [\frac{t}{t_d} - \frac{1}{2\pi} \sin 2\pi \frac{t}{t_d}] & , \quad 0 \leq t \leq t_d \\ A_g \frac{t_d}{2} & , \quad t > t_d \end{cases}$$

Displacement

$$u_g(t) = \begin{cases} A_g \frac{t_d^2}{4} [\frac{t^2}{t_d^2} - \frac{1}{2\pi^2} (1 - \cos 2\pi \frac{t}{t_d})] & , \quad 0 \leq t \leq t_d \\ A_g \frac{t_d^2}{2} \left(\frac{t}{t_d} - \frac{1}{2} \right) & , \quad t > t_d \end{cases}$$

A1.7 Co-sinusoidal Pulse



Acceleration

$$\ddot{u}_g(t) = \begin{cases} A_g \cos \frac{\pi}{2} \frac{t}{t_d} & , \quad t \leq t_d \\ 0 & , \quad t > t_d \end{cases}$$

Velocity

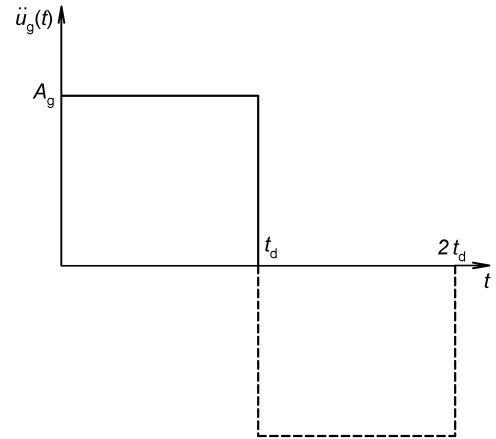
$$\dot{u}_g(t) = \begin{cases} 2A_g \frac{2t_d}{\pi} \cos \frac{\pi}{4} \frac{t}{t_d} \sin \frac{\pi}{4} \frac{t}{t_d} & , \quad t \leq t_d \\ A_g \frac{2 \cdot t_d}{\pi} & , \quad t > t_d \end{cases}$$

Displacement

$$u_g(t) = \begin{cases} 2A_g \left(\frac{2t_d}{\pi} \right)^2 \sin^2 \left(\frac{\pi}{4} \frac{t}{t_d} \right) & , \quad t \leq t_d \\ 2A_g \frac{t_d^2}{\pi} \left(\frac{t}{t_d} - 1 + \frac{2}{\pi} \right) & , \quad t > t_d \end{cases}$$

A.2 Full-Cycle

A2.1 Rectangular Pulse



Acceleration

$$\ddot{u}_g(t) = \begin{cases} A_g & , \quad 0 \leq t \leq t_d \\ -A_g & , \quad t_d \leq t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases}$$

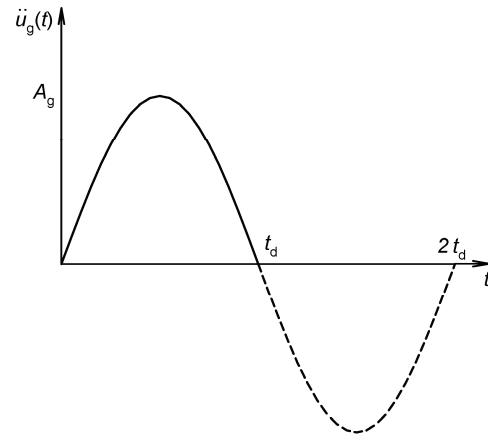
Velocity

$$\dot{u}_g(t) = \begin{cases} A_g t & , \quad 0 \leq t \leq t_d \\ A_g (2t_d - t) & , \quad t_d \leq t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases}$$

Displacement

$$u_g(t) = \begin{cases} \frac{1}{2} A_g t^2 & , \quad 0 \leq t \leq t_d \\ A_g (2t \cdot t_d - t_d^2 - \frac{t^2}{2}) & , \quad t_d \leq t \leq 2t_d \\ A_g t_d^2 & , \quad t > 2t_d \end{cases}$$

A2.2 Sinusoidal Pulse



Acceleration

$$\ddot{u}_g(t) = \begin{cases} A_g \sin \pi \frac{t}{t_d} & , \quad 0 \leq t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases}$$

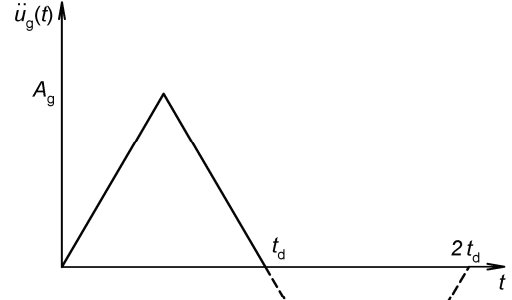
Velocity

$$\dot{u}_g(t) = \begin{cases} A_g \frac{t_d}{\pi} (1 - \cos \pi \frac{t}{t_d}) & , \quad 0 \leq t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases}$$

Displacement

$$u_g(t) = \begin{cases} A_g \frac{t_d^2}{\pi} \left(\frac{t}{t_d} - \frac{1}{\pi} \sin \pi \frac{t}{t_d} \right) & , \quad 0 \leq t \leq 2t_d \\ A_g \frac{2t_d^2}{\pi} & , \quad t > 2t_d \end{cases}$$

A2.3 Triangular Pulse



Acceleration

$$\ddot{u}_g(t) = \begin{cases} 2A_g \frac{t}{t_d} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ 2A_g(1 - \frac{t}{t_d}) & , \quad \frac{t_d}{2} \leq t \leq \frac{3t_d}{2} \\ 2A_g(\frac{t}{t_d} - 2) & , \quad \frac{3t_d}{2} \leq t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases}$$

Velocity

$$\dot{u}_g(t) = \begin{cases} A_g \frac{t^2}{t_d} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ A_g t_d (2 \frac{t}{t_d} - (\frac{t}{t_d})^2 - \frac{1}{2}) & , \quad \frac{t_d}{2} \leq t \leq \frac{3t_d}{2} \\ A_g t_d (-4 \frac{t}{t_d} + (\frac{t}{t_d})^2 + 4) & , \quad \frac{3t_d}{2} \leq t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases}$$

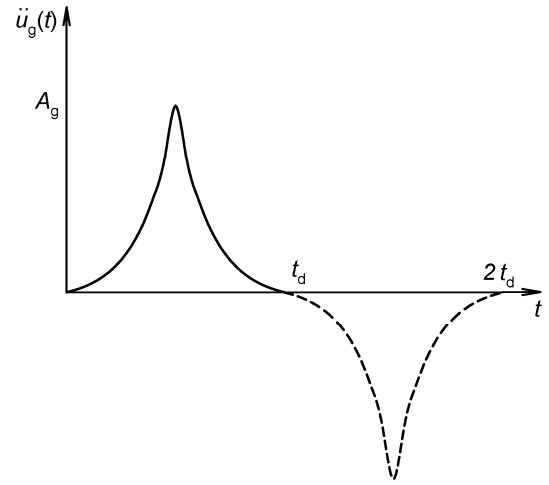
Displacement

$$u_g(t) = \begin{cases} A_g \frac{t^3}{3t_d} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ A_g (t^2 - \frac{t^3}{3t_d} - \frac{t \cdot t_d}{2} + \frac{t_d^2}{12}) & , \quad \frac{t_d}{2} \leq t \leq \frac{3t_d}{2} \\ A_g (-2t^2 + \frac{t^3}{3t_d} + 4t \cdot t_d - \frac{13t_d^2}{6}) & , \quad \frac{3t_d}{2} \leq t \leq 2t_d \\ A_g \frac{t_d^2}{2} & , \quad t > 2t_d \end{cases}$$

A2.4 Exponential Pulse

Acceleration

$$\ddot{u}_g(t) = \begin{cases} A_g \frac{1 - e^{2\pi t/t_d}}{1 - e^\pi} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ A_g \frac{1 - e^{2\pi(1-t/t_d)}}{1 - e^\pi} & , \quad \frac{t_d}{2} \leq t \leq t_d \\ -A_g \frac{1 - e^{2\pi(t/t_d - 1)}}{1 - e^\pi} & , \quad t_d \leq t \leq \frac{3t_d}{2} \\ -A_g \frac{1 - e^{2\pi(2-t/t_d)}}{1 - e^\pi} & , \quad \frac{3t_d}{2} \leq t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases}$$



Velocity

$$\dot{u}_g(t) = \frac{A_g}{1 - e^\pi} \begin{cases} t + \frac{t_d}{2\pi} (1 - e^{2\pi t/t_d}) & , \quad 0 \leq t \leq \frac{t_d}{2} \\ t + \frac{t_d}{2\pi} (1 - 2e^\pi + e^{2\pi(1-t/t_d)}) & , \quad \frac{t_d}{2} \leq t \leq t_d \\ -t + \frac{t_d}{2\pi} (1 - 2e^\pi + 4\pi + e^{-2\pi(1-t/t_d)}) & , \quad t_d \leq t \leq \frac{3t_d}{2} \\ -t + \frac{t_d}{2\pi} (1 + 4\pi - e^{2\pi(2-t/t_d)}) & , \quad \frac{3t_d}{2} \leq t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases}$$

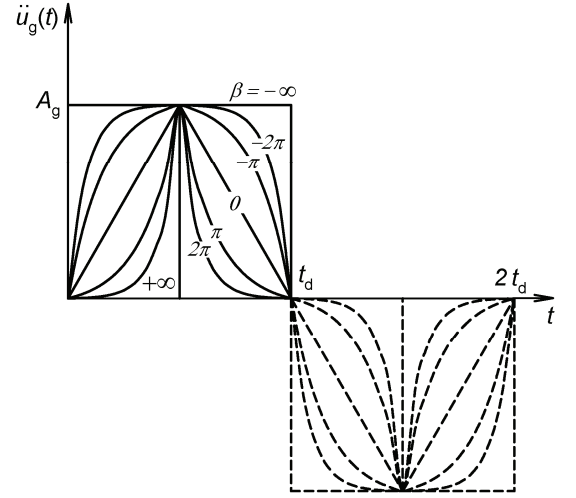
Displacement

$$u_g(t) = \frac{A_g}{1 - e^\pi} \begin{cases} \frac{t^2}{2} + \frac{t \cdot t_d}{2\pi} + \frac{t_d^2}{4\pi^2} (1 - e^{2\pi t/t_d}) & , \quad 0 \leq t \leq \frac{t_d}{2} \\ \frac{t^2}{2} + \frac{t_d}{2\pi} (t - 2te^\pi + t_d e^\pi) + \frac{t_d^2}{4\pi^2} (1 - e^{2\pi(1-t/t_d)}) & , \quad \frac{t_d}{2} \leq t \leq t_d \\ -\frac{t^2}{2} + 2t \cdot t_d - t_d^2 + \frac{t_d}{2\pi} (t - 2te^\pi + t_d e^\pi) - \frac{t_d^2}{4\pi^2} (1 - e^{-2\pi(1-t/t_d)}) & , \quad t_d \leq t \leq \frac{3t_d}{2} \\ -\frac{t^2}{2} + 2t \cdot t_d - t_d^2 + \frac{t_d}{2\pi} (t - 2t_d e^\pi) - \frac{t_d^2}{4\pi^2} (1 - e^{2\pi(2-t/t_d)}) & , \quad \frac{3t_d}{2} \leq t \leq 2t_d \\ \frac{t_d^2}{\pi} (1 + \pi - e^\pi) & , \quad t \geq 2t_d \end{cases}$$

A2.5 Generalized Exponential Pulse

Acceleration

$$\ddot{u}_g(t) = \begin{cases} A_g \frac{1-e^{2\beta t/t_d}}{1-e^\beta} & , \quad 0 \leq t \leq \frac{t_d}{2} \\ A_g \frac{1-e^{2\beta(1-t/t_d)}}{1-e^\beta} & , \quad \frac{t_d}{2} \leq t \leq t_d \\ -A_g \frac{1-e^{2\beta(t/t_d-1)}}{1-e^\beta} & , \quad t_d \leq t \leq \frac{3t_d}{2} \\ -A_g \frac{1-e^{2\beta(2-t/t_d)}}{1-e^\beta} & , \quad \frac{3t_d}{2} \leq t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases}$$



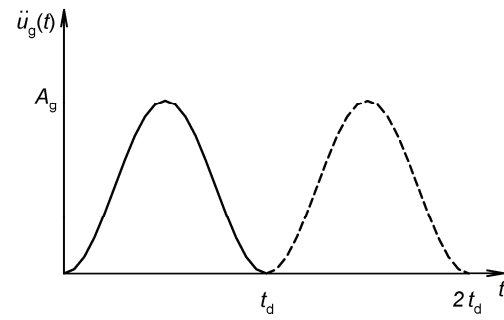
Velocity

$$\dot{u}_g(t) = \frac{A_g}{1-e^\beta} \begin{cases} t + \frac{t_d}{2\beta}(1-e^{2\beta t/t_d}) & , \quad 0 \leq t \leq \frac{t_d}{2} \\ t + \frac{t_d}{2\beta}(1-2e^\beta + e^{2\beta(1-t/t_d)}) & , \quad \frac{t_d}{2} \leq t \leq t_d \\ -t + \frac{t_d}{2\beta}(1-2e^\beta + 4\beta + e^{-2\beta(1-t/t_d)}) & , \quad t_d \leq t \leq \frac{3t_d}{2} \\ -t + \frac{t_d}{2\beta}(1+4\beta - e^{2\beta(2-t/t_d)}) & , \quad \frac{3t_d}{2} \leq t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases}$$

Displacement

$$u_g(t) = \frac{A_g}{1-e^\beta} \begin{cases} \frac{t^2}{2} + \frac{t \cdot t_d}{2\beta} + \frac{t_d^2}{4\beta^2}(1-e^{2\beta t/t_d}) & , \quad 0 \leq t \leq \frac{t_d}{2} \\ \frac{t^2}{2} + \frac{t_d}{2\beta}(t-2te^\beta + t_d e^\beta) + \frac{t_d^2}{4\beta^2}(1-e^{2\beta(1-t/t_d)}) & , \quad \frac{t_d}{2} \leq t \leq t_d \\ -\frac{t^2}{2} + 2t \cdot t_d - t_d^2 + \frac{t_d}{2\beta}(t-2te^\beta + t_d e^\beta) - \frac{t_d^2}{4\beta^2}(1-e^{-2\beta(1-t/t_d)}) & , \quad t_d \leq t \leq \frac{3t_d}{2} \\ -\frac{t^2}{2} + 2t \cdot t_d - t_d^2 + \frac{t_d}{2\beta}(t-2t_d e^\beta) - \frac{t_d^2}{4\beta^2}(1-e^{2\beta(2-t/t_d)}) & , \quad \frac{3t_d}{2} \leq t \leq 2t_d \\ \frac{t_d^2}{\beta}(1+\beta - e^\beta) & , \quad t \geq 2t_d \end{cases}$$

A2.6 Versed Sine Pulse



Acceleration

$$\ddot{u}_g(t) = \begin{cases} \frac{A_g}{2} \left[1 - \cos 2\pi \frac{t}{t_d} \right] & , \quad 0 \leq t \leq t_d \\ \frac{A_g}{2} \left[\cos 2\pi \frac{t}{t_d} - 1 \right] & , \quad t_d \leq t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases}$$

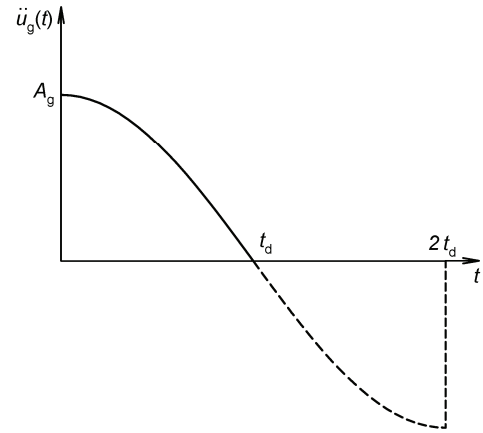
Velocity

$$\dot{u}_g(t) = \begin{cases} A_g \frac{t_d}{2} \left[\frac{t}{t_d} - \frac{1}{2\pi} \sin 2\pi \frac{t}{t_d} \right] & , \quad 0 \leq t \leq t_d \\ A_g \frac{t_d}{2} \left[\frac{1}{2\pi} \sin 2\pi \frac{t}{t_d} + 2 - \frac{t}{t_d} \right] & , \quad t_d \leq t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases}$$

Displacement

$$u_g(t) = \begin{cases} A_g \frac{t_d^2}{4} \left[\frac{t^2}{t_d^2} - \frac{1}{2\pi^2} (1 - \cos 2\pi \frac{t}{t_d}) \right] & , \quad 0 \leq t \leq t_d \\ A_g \frac{t_d^2}{4} \left[-\frac{t^2}{t_d^2} + 4 \frac{t}{t_d} - 2 + \frac{1}{2\pi^2} (1 - \cos 2\pi \frac{t}{t_d}) \right] & , \quad t_d \leq t \leq 2t_d \\ A_g \frac{t_d^2}{2} & , \quad t > 2t_d \end{cases}$$

A2.7 Co-sinusoidal Pulse



Acceleration

$$\ddot{u}_g(t) = \begin{cases} A_g \cos \frac{\pi}{2} \frac{t}{t_d} & , \quad t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases}$$

Velocity

$$\dot{u}_g(t) = \begin{cases} 2A_g \frac{2t_d}{\pi} \cos \frac{\pi}{4} \frac{t}{t_d} \sin \frac{\pi}{4} \frac{t}{t_d} & , \quad t \leq 2t_d \\ 0 & , \quad t > 2t_d \end{cases}$$

Displacement

$$u_g(t) = \begin{cases} 2A_g \sin^2 \left(\frac{\pi}{4} \frac{t}{t_d} \right) & , \quad t \leq 2t_d \\ 2A_g \left(\frac{2t_d}{\pi} \right)^2 & , \quad t > 2t_d \end{cases}$$

Appendix B

Rigid Block Tables

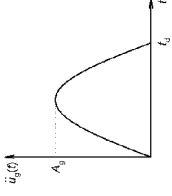
B.1 Half-Cycle

B1.1 Rectangular Pulse

<i>Time of Yielding</i>	$\tau_y = 0$	
<i>Peak Velocity Response</i>	$\frac{V_m}{PPV} = 1 - \eta$	$\tau_{vm} = 1$
<i>Time of Maximum Displacement</i>	$\tau_m \leq 1$	$\tau_m \geq 1$
	<i>Doesn't apply</i>	$\tau_m = \frac{1}{\eta}$
<i>Block Response</i>	$u(t) = A_g \frac{(\eta - 1)}{2} (t - t_y)^2$ $\dot{u}(t) = A_g (\eta - 1)(t - t_y)$ $\ddot{u}(t) = A_g (\eta - 1)$	$u(t) = A_g (t \cdot t_d - \frac{t_d^2}{2} \eta \cdot t^2)$ $\dot{u}(t) = A_g [t_d - \eta \cdot t]$ $\ddot{u}(t) = -A_g \eta$
<i>Normalized Peak Displacement Response</i> †	$\frac{u_{\max}}{\Delta u_g} = 1 - \eta \frac{\tau_m^2}{2\tau_m - 1}$	

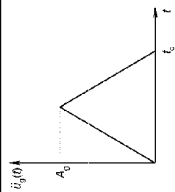
† $\Delta u_g = u_g(\tau_m) - u_g(\tau_y)$ defines incremental ground displacement between times of yielding and peak displacement

B1.2 Sinusoidal Pulse

Time of Yielding	$\tau_y = \frac{1}{\pi} \sin^{-1} \eta$	
Peak Velocity Response	$\frac{V_m}{ppV} = \cos \pi \tau_y + \eta \pi (\tau_y - 1/2)$	$\tau_{vm} = 1 - \tau_y$
Time of Maximum Displacement	$\tau_m \leq 1$	$\tau_m \geq 1$
	$\cos \pi \tau_y - \cos \pi \tau_m = \eta \pi (\tau_m - \tau_y)$	$\tau_m = \tau_y + \frac{1}{\eta \pi} (1 + \cos \pi \tau_y)$
Block Response	$u(t) = A_g \left[-\frac{\eta}{2} (t - t_y)^2 + \frac{t_d}{\pi} \left((t - t_y) \cos \frac{\pi t_y}{t_d} - \frac{t_d}{\pi^2} \left(\sin \frac{\pi t}{t_d} - \sin \frac{\pi t_y}{t_d} \right) \right) \right]$ $\dot{u}(t) = A_g \left[-\frac{\eta}{\pi} (t - t_y) - \frac{t_d}{\pi} \left(\cos \frac{\pi t}{t_d} - \cos \frac{\pi t_y}{t_d} \right) \right]$ $\ddot{u}(t) = A_g \left[\sin \frac{\pi t}{t_d} - \eta \right]$	$u(t) = A_g \left[-\frac{\eta}{2} (t - t_y)^2 + \frac{t_d}{\pi} \left((t - t_d) + (t - t_y) \cos \frac{\pi t_y}{t_d} + \frac{t_d}{\pi^2} \sin \frac{\pi t_y}{t_d} \right) \right]$ $\dot{u}(t) = A_g \left[-\eta (t - t_y) + \frac{t_d}{\pi} \left(1 + \cos \frac{\pi t_y}{t_d} \right) \right]$ $\ddot{u}(t) = -A_g \eta$
Normalized Peak Displacement Response [†]	$\frac{u_{\max}}{\Delta u_g} = 1 - \frac{\eta \pi (\tau_m - \tau_y)^2 - (\cos \pi \tau_y - 1) \cdot (\tau_m - \tau_y)}{[1 + H(\tau_m - 1)] \cdot \tau_m - \tau_y - H(\tau_m - 1) - \frac{1}{\pi} [\{1 - H(\tau_m - 1)\} \cdot \sin \pi \tau_m - \sin \pi \tau_y]}$	

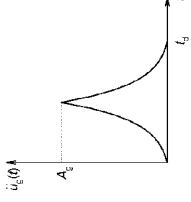
[†] $\Delta u_g - u_g(\tau_m) - u_g(\tau_y)$ defines incremental ground displacement between times of yielding and peak displacement

B1.3 Triangular Pulse

Time of Yielding	$\tau_y = \frac{\eta}{2}$	
Peak Velocity Response	$\frac{V_m}{ppV} = (1-\eta)^2$	$\tau_{vm} = 1 - \frac{\eta}{2}$
Time of Maximum Displacement	$\tau_m^2 + (\eta - 2)\tau_m + \tau_y^2 - \eta\tau_y + \frac{1}{2} = 0$	$\tau_m \geq 1$
		$\tau_m = \tau_y - \frac{1}{\eta}(\tau_y^2 - \frac{1}{2})$
		$t \leq t_d$
	$u(t) = A_g[-\frac{\eta}{2}(t-t_y)^2 - \frac{t^3}{3t_d} + t^2 + \frac{t_d^2}{12} + \frac{2t^3}{3t_d} - \frac{t \cdot t_d}{2} - \frac{t \cdot t_y^2}{2t_d}]$	$t \geq t_d$
Block Response	$\dot{u}(t) = A_g[-\eta(t-t_y) - \frac{t_d}{2} + 2t - (t^2 + t_y^2)/t_d]$	$\dot{u}(t) = A_g[-\eta(t-t_y) - \frac{t_d}{2} - t_y^2/t_d]$
	$\ddot{u}(t) = A_g[2(1-t/t_d) - \eta]$	$\ddot{u}(t) = -A_g\eta$
Normalized Peak Displacement Response †	$\frac{u_{max}}{\Delta u_g} = 1 - \frac{(\tau_m - \tau_y)(\tau_y^2 + \frac{\eta}{2}(\tau_m - \tau_y))}{1/12 - \tau_m/2 + \tau_m^2 - \tau_m^3/3 - \tau_y^3/3 - (1/3 + \tau_m^2 - \tau_m^3/3 - \tau_m) \cdot H(\tau_m - 1)}$	

† $\Delta u_g = u_g(\tau_m) - u_g(\tau_y)$ defines incremental ground displacement between times of yielding and peak displacement

B1.4 Exponential Pulse

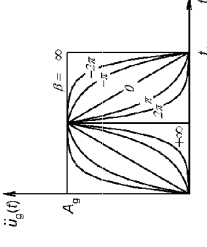
Time of Yielding	$\tau_y = \frac{1}{2\pi} \ln[1 - \eta(1 - e^\pi)]$	
Peak Velocity Response	$\frac{V_m}{PPV} = \frac{1}{e^\pi - \pi - 1} [-e^{2\pi\tau_y} + \pi(\eta - 1)(1 - 2\tau_y) + e^\pi(1 - \pi\eta + 2\pi\eta\tau_y)]$ $\tau_{vm} = 1 - \tau_y$	
Time of Maximum Displacement	$\tau_m = \tau_y + \frac{1}{\eta(1 - e^\pi)} \left[1 + \frac{1}{2\pi} (1 + e^{2\pi\tau_y}) - \frac{e^\pi}{\pi} \tau_y \right]$	$\tau_m \geq 1$
	$(\tau_m - \tau_y)(1 - \eta(1 - e^\pi)) + \frac{1}{2\pi} (e^{2\pi(1 - \tau_m)} + e^{2\pi\tau_y} - 2e^\pi) = 0$	
	$t_d / 2 \leq t \leq t_d$	$t \geq t_d$
Response Functions	$u(t) = \frac{A_g}{4\pi^2(1 - e^\pi)} [2\pi^2(t - t_y)^2(1 - \eta(1 - e^\pi)) + 2\pi e^\pi(1 - 2\tau_m) + 2\pi e^{2\pi t_y / t_d}(t - t_y)t_d - (e^{2\pi(1 - t/t_d)} - e^{2\pi t_y / t_d})t_d^2]$ $\dot{u}(t) = \frac{A_g}{2\pi(1 - e^\pi)} [2\pi(t - t_y)(1 - \eta(1 - e^\pi)) - t_d(2e^\pi - e^{2\pi(1 - t/t_d)} - e^{2\pi t_y / t_d})]$ $\ddot{u}(t) = \frac{A_g}{1 - e^\pi} [1 - e^{2\pi(1 - t/t_d)} - \eta(1 - e^\pi)]$	$u(t) = \frac{A_g}{4\pi^2(1 - e^\pi)} [-t_d^2 + 2\pi t_d(t - t_d) + e^{2\pi t_y / t_d}(t_d + 2\pi(t - t_y))t_d - 2\pi^2((t_d - t_y)(t_d + t_y - 2t) + \eta(t - t_y)^2) + 2\pi e^\pi(t_d(t_d - 2t) + \eta\pi(t - t_y)^2)]$ $\dot{u}(t) = \frac{A_g}{2\pi(1 - e^\pi)} [1 - 2e^\pi + e^{2\pi t_y / t_d} + 2\pi - 2\pi(t_y + \eta(t - t_y) - \eta e^\pi(t - t_y))]$ $\ddot{u}(t) = -A_g\eta$

B1.4 Exponential Pulse (cont'd)

Normalized Peak Displacement †	$\frac{u_{\max}}{\Delta u_g} = 1 - \frac{2\pi(\tau_m - \tau_y)(1 - e^{2\pi\tau_y} + 2\pi\tau_y + \eta\pi(\tau_m - \tau_y)(1 - e^\pi))}{-e^{-2\pi(\tau_m - 1)} + e^{2\pi\tau_y} + 2\pi e^\pi(2\tau_m - 1) + 2\pi(\tau_m - \tau_y)(1 + \pi(\tau_m + \tau_y)) - 1 - e^{-2\pi(\tau_m - 1)} + 2\pi(\tau_m - 1)(\pi(\tau_m - 1) - 1) \cdot H(\tau_m - 1)}$

† $\Delta u_g = u_g(\tau_m) - u_g(\tau_y)$ defines incremental ground displacement between times of yielding and peak block displacement

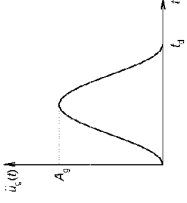
B1.5 Generalized Exponential Pulse

Time of Yielding	$\tau_y = \frac{1}{2\beta} \ln[1 - \eta(1 - e^\beta)]$	
Peak Velocity Response	$\frac{V_m}{PPV} = \frac{1}{e^\beta - \beta - 1} [-e^{2\beta\tau_y} + \beta(\eta - 1)(1 - 2\tau_y) + e^\beta(1 - \beta\eta + 2\beta\eta\tau_y)]$ $\tau_{ym} = 1 - \tau_y$	
Time of Maximum Displacement	$1/2 \leq \tau_m \leq 1$ $(\tau_m - \tau_y)(1 - \eta(1 - e^\beta)) + \frac{1}{2\beta}(e^{2\beta(1-\tau_m)} + e^{2\beta\tau_y} - 2e^\beta) = 0$ $t_d/2 \leq t \leq t_d$	$\tau_m \geq 1$ $\tau_m = \tau_y + \frac{1}{\eta(1 - e^\beta)} \left[1 + \frac{1}{2\beta} (1 + e^{2\beta\tau_y}) - \frac{e^\beta}{\beta} - \tau_y \right]$
Response Functions	$u(t) = \frac{A_g}{4\beta^2(1 - e^\beta)} [2\beta^2(t - t_y)^2(1 - \eta(1 - e^\beta)) + 2\beta e^\beta(1 - 2\tau_m) + 2\beta e^{2\beta\tau_y/t_d}(t - t_y)(t_d - (e^{2\beta(1-t/t_d)} - e^{2\beta\tau_y/t_d})t_d)^2]$ $\dot{u}(t) = \frac{A_g}{2\beta(1 - e^\beta)} [2\beta(t - t_y)(1 - \eta(1 - e^\beta)) - t_d(2e^\beta - e^{2\beta(1-t/t_d)} - e^{2\beta\tau_y/t_d})]$ $\ddot{u}(t) = \frac{A_g}{1 - e^\beta} [1 - e^{2\beta(1-t/t_d)} - \eta(1 - e^\beta)]$	$u(t) = \frac{A_g}{4\beta^2(1 - e^\beta)} [-t_d^2 + 2\beta t_d(t - t_d) + e^{2\beta\tau_y/t_d}(t_d + 2\beta(t - t_y))t_d - 2\beta^2((t_d - t_y)(t_d + t_y - 2t) + \eta(t - t_y)^2) + 2\beta e^\beta(t_d(t_d - 2t) + \eta\beta(t - t_y)^2)]$ $\dot{u}(t) = \frac{A_g}{2\beta(1 - e^\beta)} [1 - 2e^\beta + e^{2\beta\tau_y/t_d} + 2\beta - 2\beta(t_y + \eta(1 - t_y) - \eta e^\beta(t - t_y))]$ $\ddot{u}(t) = -A_g\eta$

B1.5 Generalized Exponential Pulse (cont'd)

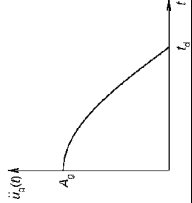
Normalized Peak Displacement †	$\frac{u_{\max}}{\Delta u_g} = 1 - \frac{2\beta(\tau_m - \tau_y)[1 - e^{2\beta\tau_y} + 2\beta\tau_y + \eta\beta(\tau_m - \tau_y)(1 - e^\beta)]}{-e^{-2\beta(\tau_m - 1)} + e^{2\beta\tau_y} - 2\beta e^\beta(2\tau_m - 1) + 2\beta(\tau_m - 1)[1 + \beta(\tau_m + \tau_y)] - [1 - e^{-2\beta(\tau_m - 1)} + 2\beta(\tau_m - 1)(\beta(\tau_m - 1) - 1)] \cdot H(\tau_m - 1)}$
† $\Delta u_g = u_g(\tau_m) - u_g(\tau_y)$ defines incremental ground displacement between times of yielding and peak block displacement	

B1.6 Versed-Sine Pulse

Time of Yielding	$\tau_y = \frac{1}{2\pi} \cos^{-1}[1 - 2\eta]$	
Peak Velocity Response	$\frac{V_m}{PPV} = \frac{1}{\pi} \sin 2\pi\tau_y + (1 - 2\eta)(1 - 2\tau_y)$	$\tau_{ym} = 1 - \tau_y$
		
		$\tau_m \geq 1$
Time of Maximum Displacement	$(\tau_m - \tau_y)\left(\frac{1}{2} - \eta\right) = \frac{1}{4\pi}(\sin 2\pi\tau_m - \sin 2\pi\tau_y)$	$\tau_m = \tau_y\left(1 - \frac{1}{2\eta}\right) + \frac{1}{4\pi\eta} \sin 2\pi\tau_y + \frac{1}{2\eta}$
Normalized Peak Displacement †	$\frac{u_{\max}}{\Delta u_g} = 1 - \frac{2\pi(\tau_m - \tau_y)[2\pi(\tau_y + \eta(\tau_m - \tau_y)) - \sin 2\pi\tau_y]}{-\cos 2\pi\tau_y - 2\pi^2\tau_y^2 + (2\pi^2\tau_m^2 + \cos 2\pi\tau_m)[1 - H(\tau_m - 1)] + (1 - 2\pi^2(1 - 2\tau_m))H(\tau_m - 1)}$	

† $\Delta u_g = u_g(\tau_m) - u_g(\tau_y)$ $u_g(\tau_y)$ defines incremental ground displacement between times of yielding and peak block displacement

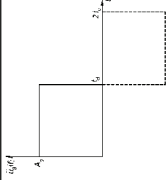
B1.7 Cosinusoidal Pulse

Time of Yielding	$\tau_y = 0$	
Peak Velocity Response	$\frac{V_m}{V_g} = \sqrt{1 - \eta^2} - \eta \cos^{-1} \eta$ $\tau_{vm} = \frac{2}{\pi} \cos^{-1} \eta$	
Time of Maximum Displacement	$\tau_m \leq 1$ $\sin \pi \frac{\tau_m}{2} = \frac{\eta \pi}{2}$	$\tau_m \geq 1$ $\tau_m = \frac{2}{\eta \pi}$
Normalized Peak Displacement †	$\frac{u_{\max}}{\Delta u_g} = 1 - \frac{1}{2 - \eta(\pi - 2)}$	

† $\Delta u_g = u_g(\tau_m) - u_g(\tau_y)$ defines incremental ground displacement between times of yielding and peak block displacement

B.2 Full-Cycle

B2.1 Rectangular Pulse

<i>Time of Yielding</i>	$\tau_y = 0$	
<i>Peak Velocity Response</i>	$\frac{V_m}{PPV} = 1 - \eta$ $\tau_{vm} = 1$	
<i>Time of Maximum Displacement</i>	$0 \leq \tau_m \leq 1$ <i>Doesn't apply</i>	$1 \leq \tau_m \leq 2$ $\tau_m = \frac{2}{1 + \eta}$
<i>Block Response</i>	$u(t) = A_g \frac{(1 - \eta)}{2} t^2$ $\dot{u}(t) = A_g (1 - \eta) t$ $\ddot{u}(t) = A_g (1 - \eta)$	$u(t) = A_g (2 \cdot t \cdot t_d - t_d^2 - \frac{(1 + \eta) \cdot t^2}{2})$ $\dot{u}(t) = A_g [2 \cdot t_d - (1 + \eta) \cdot t]$ $\ddot{u}(t) = -A_g (1 + \eta)$
<i>Normalized Peak Displacement Response</i> †	$\frac{u_{\max}}{\Delta u_g} = 1 - \eta \frac{2}{1 - \eta(\eta - 2)}$	

† $\Delta u_g = u_g(\tau_m) - u_g(\tau_y)$ defines incremental ground displacement between times of yielding and peak displacement

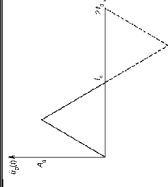
B2.2 Sinusoidal Pulse

Time of Yielding	$\tau_y = \frac{1}{\pi} \sin^{-1} \eta$	
Peak Velocity Response	$\frac{V_m}{V_g} = \cos \pi \tau_y + \eta \pi (\tau_y - 1/2)$ $\tau_{ym} = 1 - \tau_y$	
Time of Maximum Displacement	$0.5 < \tau_m \leq 2$ $\cos \pi \tau_y - \cos \pi \tau_m = \eta \pi (\tau_m - \tau_y)$ $0 < t \leq 2t_d$	
Block Response	$u(t) = A_g \left[-\frac{\eta}{2} (t - t_y)^2 + \frac{t_d}{\pi} ((t - t_y) \cos \frac{\pi t_y}{t_d} - \frac{t_d}{\pi} (\sin \frac{\pi t}{t_d} - \sin \frac{\pi t_y}{t_d})) \right]$ $\ddot{u}(t) = A_g \left[-\frac{\eta}{\pi} (t - t_y) - \frac{t_d}{\pi} \left(\cos \frac{\pi t}{t_d} - \cos \frac{\pi t_y}{t_d} \right) \right]$ $\ddot{u}(t) = A_g \left[\sin \frac{\pi t}{t_d} - \eta \right]$	$u(t) = A_g \left[-\frac{\eta}{2} (t - t_y)^2 + \frac{t_d}{\pi} (-(t - t_d) + (t - t_y) \cos \frac{\pi t_y}{t_d} + \frac{t_d}{\pi} \sin \frac{\pi t_y}{t_d}) \right]$ $\ddot{u}(t) = A_g \left[-\eta (t - t_y) + \frac{t_d}{\pi} (-1 + \cos \frac{\pi t_y}{t_d}) \right]$ $\ddot{u}(t) = -A_g \eta$
Normalized Peak Displacement Response †	$\frac{u_{\max}}{\Delta u_g} = 1 - \frac{\frac{\eta \pi}{2} (\tau_m - \tau_y)^2 - (\cos \pi \tau_y - 1) \cdot (\tau_m - \tau_y)}{\tau_m - \tau_y - \frac{1}{\pi} (\sin \pi \tau_m - \sin \pi \tau_y)}$	

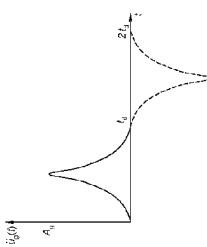
† $\Delta u_g = u_g(\tau_m) - u_g(\tau_y)$ defines incremental ground displacement between times of yielding and peak displacement

B2.3 Triangular Pulse

Time of Yielding	$\tau_y = \frac{\eta}{2}$	
Peak Velocity Response	$\frac{V_m}{PPV} = (1 - \eta)^2$	$\tau_{vm} = 1 - \frac{\eta}{2}$
	$1/2 \leq \tau_m \leq 3/2$	$3/2 \leq \tau_m \leq 2$
Time of Maximum Displacement	$\tau_m^2 + (\eta - 2)\tau_m + \tau_y^2 - \eta\tau_y + \frac{1}{2} = 0$	$\tau_m^2 - (\eta + 4)\tau_m - \tau_y^2 + \eta\tau_y - 4 = 0$
	$t_d/2 \leq t \leq 3t_d/2$	$3t_d/2 \leq t \leq 2t_d$
Block Response	$u(t) = A_g \left[-\frac{\eta}{2} (t - t_y)^2 - \frac{t^3}{3t_d} + \frac{t_d^2}{12} + \frac{2t_y^3}{3t_d} - \frac{t \cdot t_d}{2} - \frac{t \cdot t_y^2}{2t_d} \right]$ $\dot{u}(t) = A_g \left[-\eta(t - t_y) - \frac{t_d}{2} + 2t - (t^2 + t_y^2)/t_d \right]$	$u(t) = A_g \left[-\frac{\eta}{2} (t - t_y)^2 + \frac{t^3}{3t_d} - \frac{13t_d^2}{6} + \frac{2t_y^3}{3t_d} + 4t \cdot t_d - 2t^2 - \frac{t \cdot t_y^2}{t_d} \right]$ $\dot{u}(t) = A_g \left[-\eta(t - t_y) + (t - 2t_d)^2/t_d - t_d^2/t_d \right]$
Normalized Peak Displacement Response †	$\frac{u_{\max}}{\Delta u_g} = 1 - \frac{(\tau_m - \tau_y)[\tau_y^2 + \frac{\eta}{2}(\tau_m - \tau_y)]}{1/12 - \tau_m/2 + \tau_m^2 - \tau_y^3/3 - \tau_y^3/3 - (2\tau_m - 3)^3/12 \cdot H(\tau_m - 2/3)}$	$\ddot{u}(t) = A_g[2(1 - t/t_d) - \eta]$
† $\Delta u_g = u_g(\tau_m) - u_g(\tau_y)$ defines incremental ground displacement between times of yielding and peak displacement		



B2.4 Exponential Pulse

Time of Yielding	$\tau_y = \frac{1}{2\pi} \ln[1 - \eta(1 - e^\pi)]$	
Peak Velocity Response	$\frac{V_m}{ppV} = \frac{1}{e^\pi - \pi - 1} [-e^{2\pi\tau_y} + \pi(\eta - 1)(1 - 2\tau_y) + e^\pi(1 - \pi\eta + 2\pi\eta\tau_y)]$ $\tau_{vm} = 1 - \tau_y$	
Time of Maximum Displacement	$(\tau_m - \tau_y)(1 - \eta(1 - e^\pi)) + \frac{1}{2\pi}(e^{2\pi(1-\tau_m)} + e^{2\pi\tau_y} - 2e^\pi) = 0$, $1/2 \leq \tau_m \leq 1$ $-\tau_m(1 + \eta(1 - e^\pi)) - \tau_y(1 - \eta(1 - e^\pi)) + \frac{1}{2\pi}(e^{-2\pi(1-\tau_m)} + e^{2\pi\tau_y} + 4\pi - 2e^\pi) = 0$, $1 \leq \tau_m \leq 3/2$ $-\tau_m(1 + \eta(1 - e^\pi)) - \tau_y(1 - \eta(1 - e^\pi)) + \frac{1}{2\pi}(-e^{2\pi(2-\tau_m)} + e^{2\pi\tau_y} + 4\pi) = 0$, $3/2 \leq \tau_m \leq 2$	
Block Response	$u(t) = \frac{A_g}{4\pi^2(1 - e^\pi)} [2\pi^2(t - t_y)^2(1 - \eta(1 - e^\pi)) + 2\pi e^{2\pi t_y/t_d}(t - t_y)t_d - (e^{2\pi(1-t/t_d)} - e^{2\pi t_y/t_d})t_d^2]$ $u(t) = \frac{A_g}{4\pi^2(1 - e^\pi)} [-2\pi^2\eta(t - t_y)^2(1 - e^\pi) + 2\pi e^{2\pi t_y/t_d}(t - t_y)t_d - 2\pi(2e^\pi t \cdot t_d + \pi(t^2 - 4t \cdot t_d + 2t \cdot t_y - t_y^2)) - (2 + 4\pi^2 - 2\pi e^\pi - e^{-2\pi(1-t/t_d)} - e^{2\pi t_y/t_d})t_d^2]$	$t_d/2 \leq t \leq t_d$ $t_d \leq t \leq 3t_d/2$

$$\begin{aligned} \ddot{u}(t) &= \frac{A_g}{2\pi(1-e^\pi)} [2\pi(t-t_y)(1-\eta(1-e^\pi)) - \\ &\quad - t_d(2e^\pi - e^{2\pi(1-t/t_d)} - e^{2\pi_y/t_d})] \\ \ddot{u}(t) &= \frac{A_g}{1-e^\pi} [1 - e^{2\pi(1-t/t_d)} - \eta(1-e^\pi)] \end{aligned}$$

$$3t_d/2 \leq t \leq 2t_d$$

$$\begin{aligned} u(t) &= \frac{A_g}{4\pi^2(1-e^\pi)} [-2\pi^2\eta(t-t_y)^2(1-e^\pi) + \\ &\quad 2\pi e^{2\pi t_y/t_d}(t-t_y)t_d - 2\pi^2(t^2 - 4t \cdot t_d + 2t \cdot t_y - t_y^2)) \\ &\quad - (2 + 4\pi^2 + 4\pi e^\pi - e^{2\pi(2-t/t_d)} - e^{2\pi t_y/t_d})t_d^2] \\ \dot{u}(t) &= \frac{A_g}{2\pi(1-e^\pi)} [2\pi(\eta(t-t_y)(e^\pi - 1) - t - t_y + 2t_d) \\ &\quad - t_d(e^{2\pi(2-t/t_d)} - e^{2\pi t_y/t_d})] \end{aligned}$$

**Block
Response**

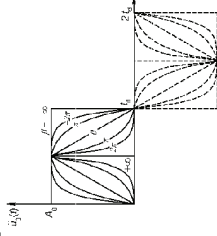
$$\ddot{u}(t) = \frac{A_g}{1 - e^\pi} [1 - e^{2\pi(2 - t/t_d)} + \eta(1 - e^\pi)]$$

$$\frac{u_{\text{max}}}{\Delta u_g} = 1 - \frac{2\pi(\tau_m - \tau_y)|1 - e^{2\pi\tau_y} + 2\pi\tau_y + \eta\pi(\tau_m - \tau_y)(1 - e^\pi)|}{\{e^{-2\pi(\tau_n-1)} + e^{2\pi\tau_y} - 2\pi e^\pi(2\tau_m-1) + 2\pi(\tau_m - \tau_y)|1 + \pi(\tau_m + \tau_y)| + e^{-2\pi(\tau_n-1)} + e^{2\pi(\tau_m-1)} - 4\pi^2(1-\tau_m)^2 - 2|H(\tau_m-1) + [e^{-2\pi(\tau_n-2)} - e^{2\pi(\tau_n-1)} + 2\pi e^\pi(2\tau_m-3)] \cdot H(\tau_m-3/2)\}}$$

 $\dagger \Delta u_g = u_g(\tau_m) - u_g(\tau_y)$ defines incremental ground displacement between times of yielding and peak block displacement

B2.5 Generalized Exponential Pulse

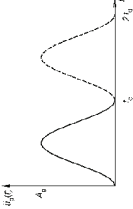
Time of Yielding	$\tau_y = \frac{1}{2\beta} \ln[1 - \eta(1 - e^\beta)]$	
Peak Velocity Response	$\frac{V_m}{PPV} = \frac{1}{e^\beta - \beta - 1} [-e^{2\beta\tau_y} + \beta(\eta - 1)(1 - 2\tau_y) + e^\beta(1 - \beta\eta + 2\beta\eta\tau_y)]$	$\tau_{ym} = 1 - \tau_y$
Time of Maximum Displacement	$(\tau_m - \tau_y)(1 - \eta(1 - e^\beta)) + \frac{1}{2\beta}(e^{2\beta(1 - \tau_m)} + e^{2\beta\tau_y} - 2e^\beta) = 0, \quad 1/2 \leq \tau_m \leq 1$ $-\tau_m(1 + \eta(1 - e^\beta)) - \tau_y(1 - \eta(1 - e^\beta)) + \frac{1}{2\beta}(e^{-2\beta(1 - \tau_m)} + e^{2\beta\tau_y} + 4\beta - 2e^\beta) = 0, \quad 1 \leq \tau_m \leq 3/2$ $-\tau_m(1 + \eta(1 - e^\beta)) - \tau_y(1 - \eta(1 - e^\beta)) + \frac{1}{2\beta}(-e^{2\beta(2 - \tau_m)} + e^{2\beta\tau_y} + 4\beta) = 0, \quad 3/2 \leq \tau_m \leq 2$	
Block Response	$u(t) = \frac{A_g}{4\beta^2(1 - e^\beta)} [2\beta^2(t - t_y)^2(1 - \eta(1 - e^\beta)) + 2\beta e^{2\beta t_y/t_d}(t - t_y)t_d - (e^{2\beta(1 - t/t_d)} - e^{2\beta t_y/t_d})t_d^2]$ $u(t) = \frac{A_g}{4\beta^2(1 - e^\beta)} [-2\beta^2\eta(t - t_y)^2(1 - e^\beta) + 2\beta e^{2\beta t_y/t_d}(t - t_y)t_d - 2\beta(2e^\beta t \cdot t_d + \beta(t^2 - 4t \cdot t_d + 2t \cdot t_y - t_y^2)) - (2 + 4\beta^2 - 2\beta e^\beta - e^{-2\beta(1 - t/t_d)} - e^{2\beta t_y/t_d})t_d^2]$	$t_d/2 \leq t \leq t_d$ $t_d \leq t \leq 3t_d/2$



B2.5 Generalized Exponential Pulse (cont'd)

	$\dot{u}(t) = \frac{A_g}{2\beta(1-e^\beta)} [2\beta(t-t_y)(1-\eta(1-e^\beta)) - t_d(2e^\beta - e^{2\beta(1-t/t_d)} - e^{2\beta t_y/t_d})]$ $\ddot{u}(t) = \frac{A_g}{2\beta(1-e^\beta)} [2\beta(\eta(t-t_y)(e^\beta - 1) - t - t_y + 2t_d) - t_d(2e^\beta - e^{2\beta(1-t/t_d)} - e^{2\beta t_y/t_d})]$
	$\ddot{u}(t) = \frac{A_g}{1-e^\beta} [1 - e^{2\beta(1-t/t_d)} - \eta(1-e^\beta)]$ $\ddot{u}(t) = \frac{A_g}{1-e^\beta} [-1 + e^{-2\beta(1-t/t_d)} - \eta(1-e^\beta)]$
	$3t_d/2 \leq t \leq 2t_d$
	$u(t) = \frac{A_g}{4\beta^2(1-e^\beta)} [-2\beta^2\eta(t-t_y)^2(1-e^\beta) + 2\beta e^{2\beta t_y/t_d}(t-t_y)t_d - 2\beta^2(t^2 - 4t \cdot t_d + 2t \cdot t_y - t_y^2) - (2 + 4\beta^2 + 4\beta e^\beta - e^{2\beta(2-t/t_d)} - e^{2\beta t_y/t_d})t_d^2]$
Block Response	$\dot{u}(t) = \frac{A_g}{2\beta(1-e^\beta)} [2\beta(\eta(t-t_y)(e^\beta - 1) - t - t_y + 2t_d) - t_d(e^{2\beta(2-t/t_d)} - e^{2\beta t_y/t_d})]$ $\ddot{u}(t) = \frac{A_g}{1-e^\beta} [1 - e^{2\beta(2-t/t_d)} + \eta(1-e^\beta)]$
Normalized Peak Displacement	$\frac{u_{\max}}{\Delta u_g} = 1 - \frac{2\beta(\tau_m - \tau_y)[1 - e^{2\beta\tau_y} + 2\beta\tau_y + \eta\beta(\tau_m - \tau_y)(1-e^\beta)]}{\{-e^{-2\beta(\tau_m-1)} + e^{2\beta\tau_y} - 2\beta e^\beta(2\tau_m-1) + 2\beta(\tau_m - \tau_y)[1 + \beta(\tau_m + \tau_y)] + e^{-2\beta(\tau_m-1)} + e^{2\beta(\tau_m-1)} - 4\beta^2(1-\tau_m)^2 - 2 \cdot H(\tau_m-1) + [e^{-2\beta(\tau_m-2)} - e^{2\beta(\tau_m-1)} + 2\beta e^\beta(2\tau_m-3)] \cdot H(\tau_m-3/2)\}}$
†	† $\Delta u_g = u_g(\tau_m) - u_g(\tau_y)$ defines incremental ground displacement between times of yielding and peak block displacement

B2.6 Versed-Sine Pulse

Time of Yielding	$\tau_y = \frac{1}{2\pi} \cos^{-1}[1 - 2\eta]$	
Peak Velocity Response	$\frac{V_m}{V_g} = \frac{1}{\pi} \sin 2\pi\tau_y + (1 - 2\eta)(1 - 2\tau_y)$ $\tau_{vm} = 1 - \tau_y$	
Time of Maximum Displacement*	$0.5 < \tau_m \leq 1$	$1 < \tau_m \leq 2$
	$\sin 2\pi\tau_y - \sin 2\pi\tau_m = 2\pi(\tau_m - \tau_y)(2\eta - 1)$	$\sin 2\pi\tau_y - \sin 2\pi\tau_m = 2\pi[(\tau_m - \tau_y)(2\eta + 1) + 2(\tau_y - 1)]$
Block Response	$0 < t \leq t_d$	
	$u(t) = \frac{A_g t_d^2}{4} \left[\left(\frac{t}{t_d} - \frac{t_y}{t_d} \right)^2 (1 - 2\eta) + \frac{1}{2\pi^2} \left(\cos 2\pi \frac{t}{t_d} - \cos 2\pi \frac{t_y}{t_d} \right) + \frac{1}{\pi} \left(\frac{t}{t_d} - \frac{t_y}{t_d} \right) \sin 2\pi \frac{t_y}{t_d} \right]$ $\dot{u}(t) = \frac{A_g t_d}{2} \left[\left(\frac{t}{t_d} - \frac{t_y}{t_d} \right) (1 - 2\eta) - \frac{1}{2\pi} \left(\sin 2\pi \frac{t}{t_d} - \sin 2\pi \frac{t_y}{t_d} \right) \right]$ $\ddot{u}(t) = \frac{A_g}{2} [1 - 2\eta - \cos 2\pi \frac{t}{t_d}]$	
	$t_d < t \leq 2t_d$	
	$u(t) = \frac{A_g t_d^2}{4} \left[\frac{1}{\pi^2} - \left(\frac{t}{t_d} - \frac{t_y}{t_d} \right)^2 (1 + 2\eta) + 4 \frac{t}{t_d} - 2 - 4 \frac{t_y}{t_d^2} + 2 \frac{t_y^2}{t_d^2} \right]$ $-\frac{1}{2\pi^2} \left(\cos 2\pi \frac{t}{t_d} + \cos 2\pi \frac{t_y}{t_d} \right) + \frac{1}{\pi} \left(\frac{t}{t_d} - \frac{t_y}{t_d} \right) \sin 2\pi \frac{t_y}{t_d}$ $\dot{u}(t) = \frac{A_g t_d}{2} \left[2 \left(1 - \frac{t_y}{t_d} \right) - \left(\frac{t}{t_d} - \frac{t_y}{t_d} \right) (1 + 2\eta) + \frac{1}{2\pi} \left(\sin 2\pi \frac{t}{t_d} + \sin 2\pi \frac{t_y}{t_d} \right) \right]$ $\ddot{u}(t) = -\frac{A_g}{2} [1 + 2\eta - \cos 2\pi \frac{t}{t_d}]$	

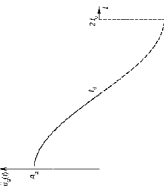
B2.6 Versed-SIne Pulse (cont'd)

	$u(t) = \frac{A_g t_d^2}{4} \left[\frac{1}{2\pi^2} - 2\eta \left(\frac{t}{t_d} - \frac{t_y}{t_d} \right)^2 + 2 - 2 \frac{t_y}{t_d} + \frac{t_y^2}{t_d^2} \right]$ $- \frac{1}{2\pi^2} \cos 2\pi \frac{t_y}{t_d} + \frac{1}{\pi} \left(\frac{t}{t_d} - \frac{t_y}{t_d} \right) \sin 2\pi \frac{t_y}{t_d}$ $\dot{u}(t) = -\frac{A_g t_d}{2} \frac{t_y}{t_d} \left[\frac{t_y}{t_d} + 2\eta \left(\frac{t}{t_d} - \frac{t_y}{t_d} \right) - \frac{1}{2\pi} \sin 2\pi \frac{t_y}{t_d} \right]$ $\ddot{u}(t) = -A_g \eta$
$t \geq 2t_d$	
Normalized Peak Displacement Response ^{**†}	$\frac{u_{m1}}{\Delta u_g} = 1 - \frac{(\tau_m - \tau_y)[2\pi(\tau_y + \eta(\tau_m - \tau_y)) - \sin 2\pi\tau_y]}{\pi(\tau_m^2 - \tau_y^2) + \frac{1}{2\pi}[\cos 2\pi\tau_m - \cos 2\pi\tau_y] + (\frac{1}{\pi}(1 - \cos 2\pi\tau_m) - 2\pi(1 - \tau_m)^2)H(\tau_m - 1)}$

^{*} for the versed-sine pulse τ_m in this table denotes τ_{m1} and u_m , u_{m1} , that is response at first stop.

[†] $\Delta u_g = u_g(\tau_m) - u_g(\tau_y)$ defines incremental ground displacement between times of yielding and peak displacement

B2.7 Cosinusoidal Pulse

<i>Time of Yielding</i>	$\tau_y = 0$	
<i>Peak Velocity Response</i>	$\frac{V_m}{V_g} = \sqrt{1 - \eta^2} - \eta \cos^{-1} \eta$ $\tau_{vm} = \frac{2}{\pi} \cos^{-1} \eta$	
<i>Time of Maximum Displacement</i>	$0 < \tau_m \leq 2$ $\sin \pi \frac{\tau_m}{2} = \frac{\eta \pi}{2} \tau_m$	
<i>Block Response</i>	$0 < t \leq 2t_d$ $u(t) = A_g \left(\frac{2t_d}{\pi} \right)^2 \left[1 - \frac{\eta}{2} \left(\frac{\pi t}{2t_d} \right)^2 - \cos \frac{\pi t}{2t_d} \right]$ $\dot{u}(t) = A_g \left[\frac{2t_d}{\pi} \sin \frac{\pi t}{2t_d} - \eta t \right]$ $\ddot{u}(t) = A_g \left[\cos \frac{\pi t}{2t_d} - \eta \right]$	$u(t) = 2A_g \left(\frac{2t_d}{\pi} \right)^2 \left[1 - \eta \left(\frac{\pi t}{4t_d} \right)^2 \right]$ $\dot{u}(t) = -A_g \eta t$ $\ddot{u}(t) = -A_g \eta$
<i>Normalized Peak Displacement Response</i> †	$\frac{u_{\max}}{\Delta u_g} = 1 - \eta \left(\frac{\pi \tau_m}{4} \right)^2 \sin \left(\frac{\pi \tau_m}{4} \right)^{-2}$	

† $\Delta u_g = u_g(\tau_m) - u_g(\tau_y)$ defines incremental ground displacement between times of yielding and peak displacement

Appendix C

Tabulated and Graphical Results for Yielding Oscillator

C.1 Rectangular Pulse

Table C.1.1 Classification of oscillators subjected to a rectangular pulse based on times of yielding and peak deformation.

Classification	I peak deformation during pulse ($\tau_m \leq 1$)	II peak deformation after pulse ($\tau_m > 1$)
A yielding during pulse ($\tau_y \leq 1$)	$\eta \leq 2, f \geq 1/2$ $f \geq \frac{1}{2\pi} \left[\cos^{-1}(1-\eta) - \frac{1}{1-\eta} \sin[\cos^{-1}(1-\eta)] \right]$	$\eta \leq 2, f \geq 1/2$ or $\eta \leq 1 - \cos(2\pi f), f < 1/2$ $f < \frac{1}{2\pi} \left[\cos^{-1}(1-\eta) - \frac{1}{1-\eta} \sin[\cos^{-1}(1-\eta)] \right]$
C yielding after pulse ($\tau_y > 1$)	not applicable	$1 - \cos(2\pi f) < \eta \leq 2 \sin(\pi f),$ $f < 1/2$
D no yielding	$\eta > 2, f \geq 1/2$	$\eta > 2 \sin(\pi f), f < 1/2$

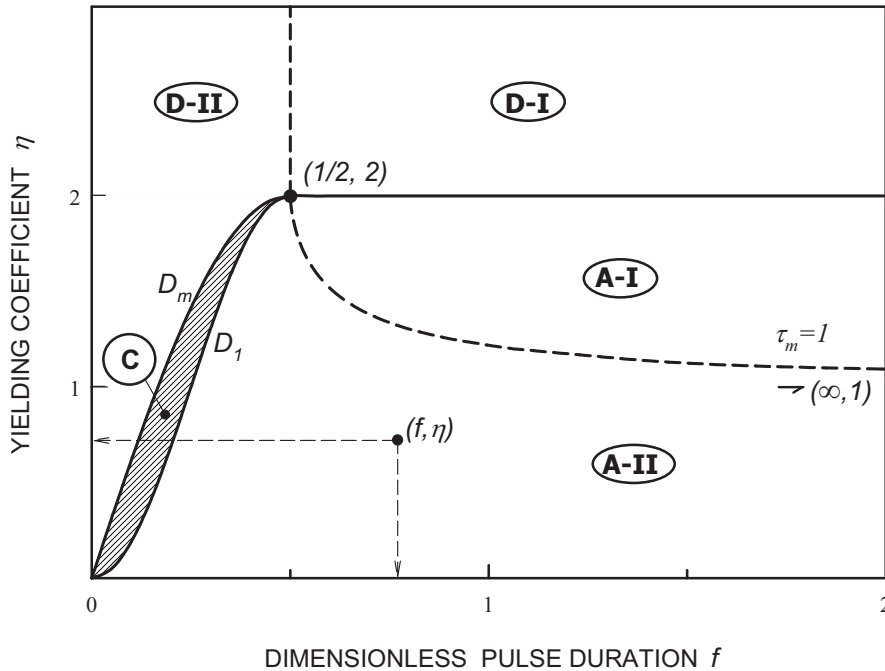
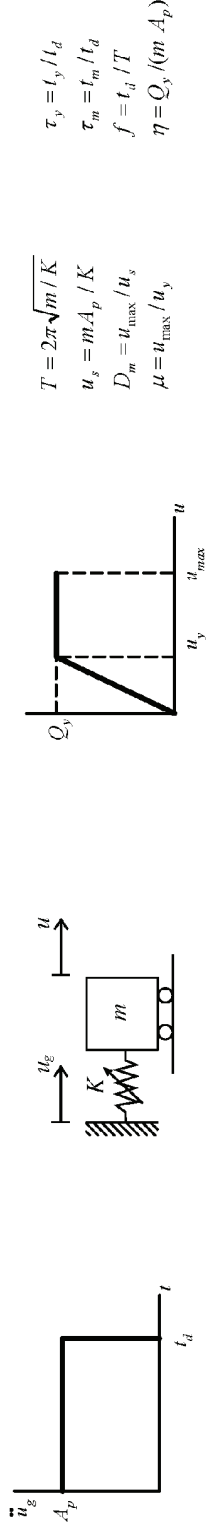


Figure C.1.1 Graphical representation of time of yielding and peak response for an undamped elastoplastic oscillator subjected to a rectangular pulse. [$D_1 = 1 - \cos(2\pi f)$ denotes the elastic response amplification factor at the end of the pulse.]

Table C.1.2 Summary of closed-form solutions for an undamped elastoplastic oscillator subjected to a rectangular pulse.



	I peak deformation during pulse ($\tau_m \leq 1$)	II peak deformation after pulse ($\tau_m > 1$)
A yielding during pulse ($\tau_y \leq 1$)	$\tau_y = \cos^{-1}[(1-\eta)/(2\pi f)]$ $\tau_m = \tau_y + \frac{1}{(\eta-1)} \left[\frac{1}{2\pi f} \sin(2\pi f \tau_y) \right]$ $\mu = 1 + \frac{4\pi^2 f^2}{1 - \cos(2\pi f \tau_y)} \left[\frac{E_r - \tau_y^2 / 2 - \eta(\tau_m - \tau_y)^2 / 2 + (\sin(2\pi f \tau_y))}{2\pi f (\tau_m - \tau_y)} \right], \quad E_r = \tau_m^2 / 2$	$\tau_y = \text{same as in A-I}$ $\tau_m = \tau_y + \frac{1}{\eta} \left[\frac{1}{2\pi f} \sin(2\pi f \tau_y) + 1 - \tau_y \right]$ $\mu = \text{same as in A-I, but } E_r = \tau_m - 1/2$
C yielding after pulse ($\tau_y > 1$)	<i>not applicable</i>	$\tau_y = \frac{1}{2\pi f} \sin^{-1} \left[\frac{\eta}{2 \sin(\pi f)} \right] + \frac{1}{2}$ $\tau_m = \tau_y + \frac{1}{\pi f \eta} \sin(\pi f) \cos[\pi f (2\tau_y - 1)]$ $\mu = 1 + \frac{\sin(\pi f) \sin[\pi f (2\tau_y - 1)]}{4\pi^2 f^2} \left[\frac{1 - \eta(\tau_m - \tau_y)}{2\pi f} + \frac{1}{2\pi f} \sin(\pi f) \cos[\pi f (2\tau_y - 1)] \right] (\tau_m - \tau_y)$
D no yielding	$\tau_m = \frac{1}{2f}$ $D_m = 1 - \cos(2\pi f \tau_m) = 2$	$\tau_m = \frac{1}{2} + \frac{1}{4f}$ $D_m = 2 \sin(\pi f) \sin[\pi f (2\tau_m - 1)] = 2 \sin(\pi f)$

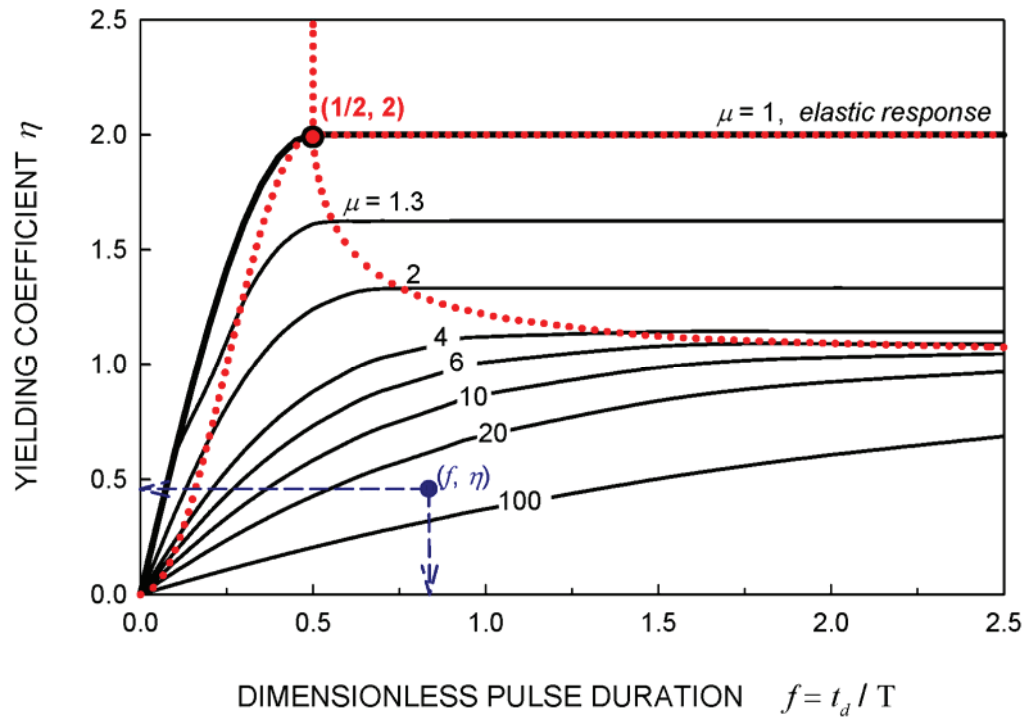


Figure C.1.2 Contour representation of ductility demand for an undamped elastic-perfectly plastic oscillator subjected to a rectangular pulse.

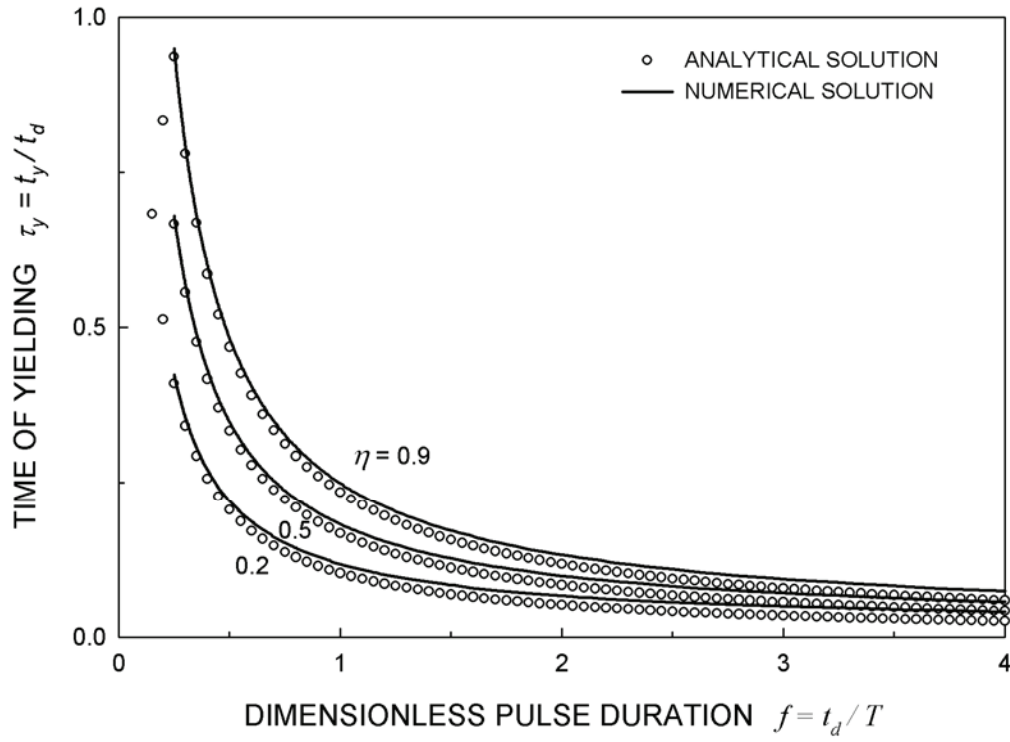


Figure C.1.3 Dimensionless time of yielding for an undamped elastic-perfectly plastic oscillator subjected to a rectangular pulse.

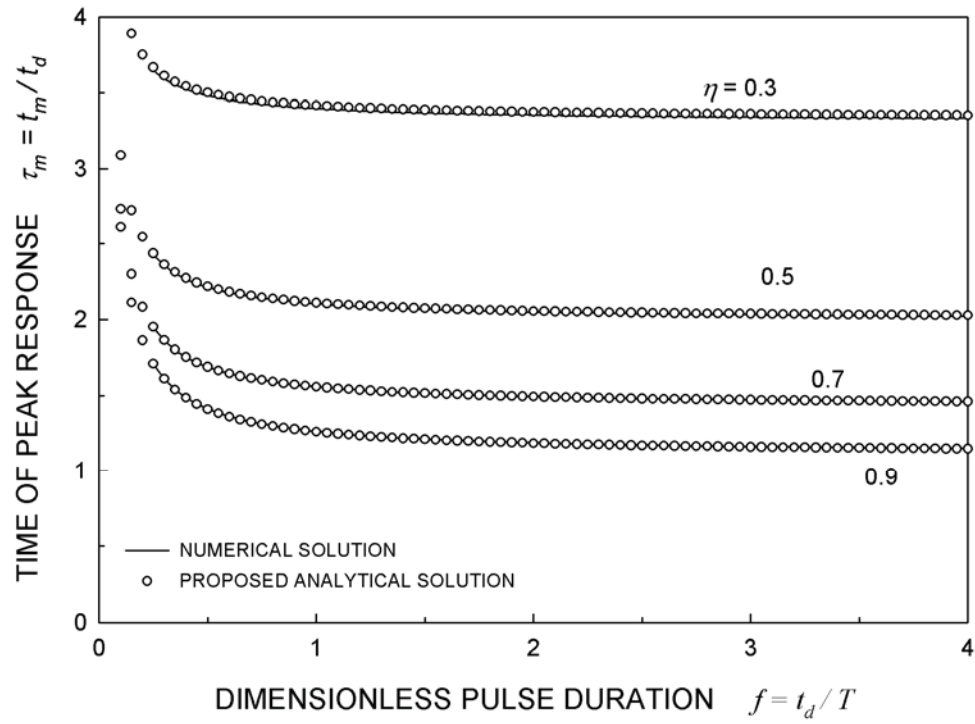


Figure C.1.4 Time of peak response for an undamped elastic - perfectly plastic oscillator subjected to a rectangular pulse.

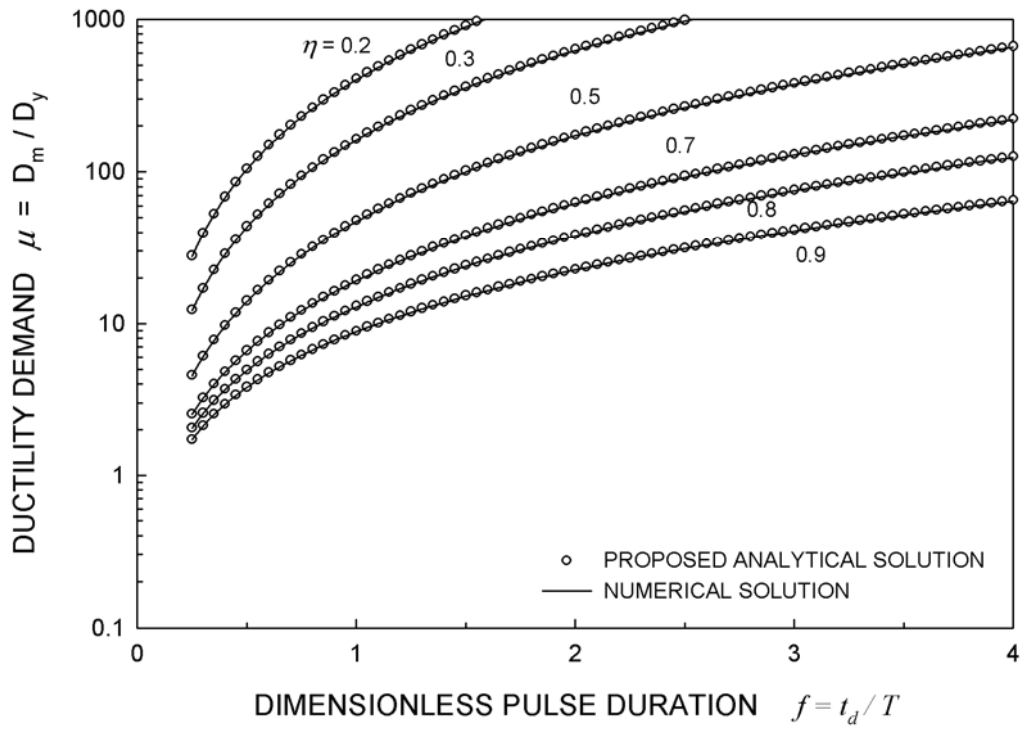


Figure C.1.5 Ductility demand for an undamped elastic-perfectly plastic oscillator subjected to a rectangular pulse. Comparison of the exact numerical solution with the proposed approximate solution in Table C.1.2.

C.2 Linear Pulse (*Increasing Ramp*)

Table C.2.1 Classification of oscillators subjected to a linear ramp pulse, based on times of yielding and peak deformation.

Classification	II peak deformation after pulse ($\tau_m > 1$)
A yielding during pulse ($\tau_y \leq 1$)	$\eta \leq 1 - \frac{1}{2\pi f} \sin 2\pi f, \quad f \leq 1$ $\eta \leq \frac{1}{2\pi f} \sqrt{2 + 4\pi^2 f^2 - 2 \cos 2\pi f - 4\pi f \sin 2\pi f}, \quad f \geq 1$
C yielding after pulse ($\tau_y > 1$)	$0 \leq f \leq 1$ $1 - \frac{1}{2\pi f} \sin 2\pi f \leq \eta \leq \frac{1}{2\pi f} \sqrt{2 + 4\pi^2 f^2 - 2 \cos 2\pi f - 4\pi f \sin 2\pi f}$
D no yielding	$\eta \geq \frac{1}{2\pi f} \sqrt{2 + 4\pi^2 f^2 - 2 \cos 2\pi f - 4\pi f \sin 2\pi f}$

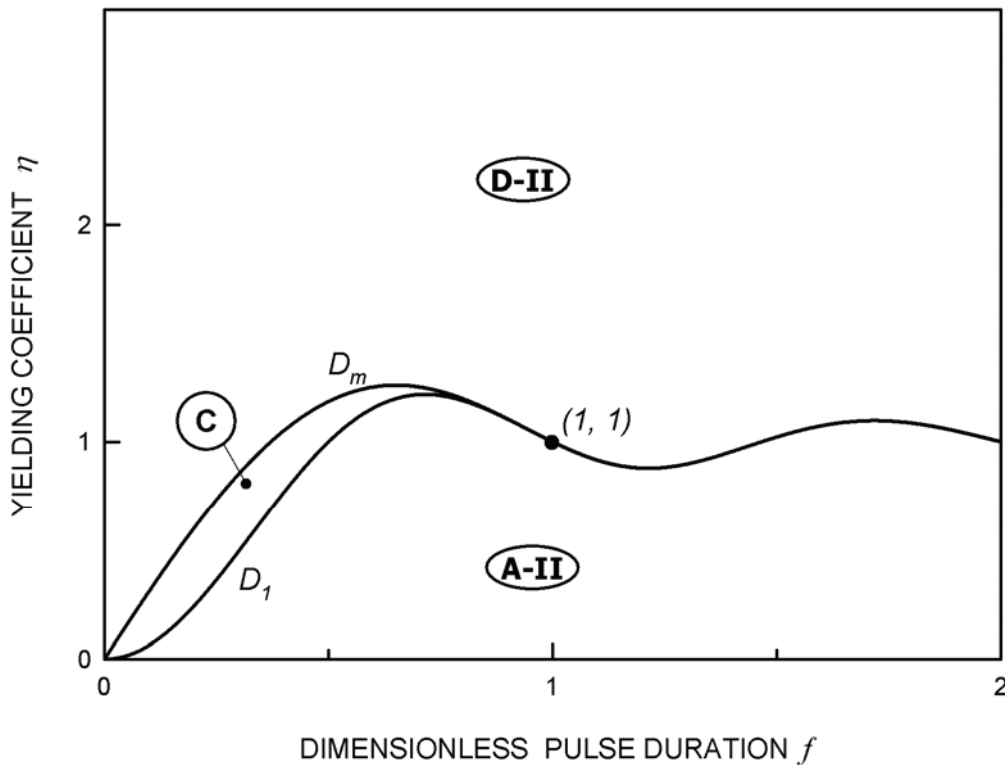


Figure C.2.1 Graphical representation of time of yielding and peak response for an undamped elastoplastic oscillator subjected to a linear ramp pulse. [D_1 denotes the elastic response amplification factor at the end of the pulse.]

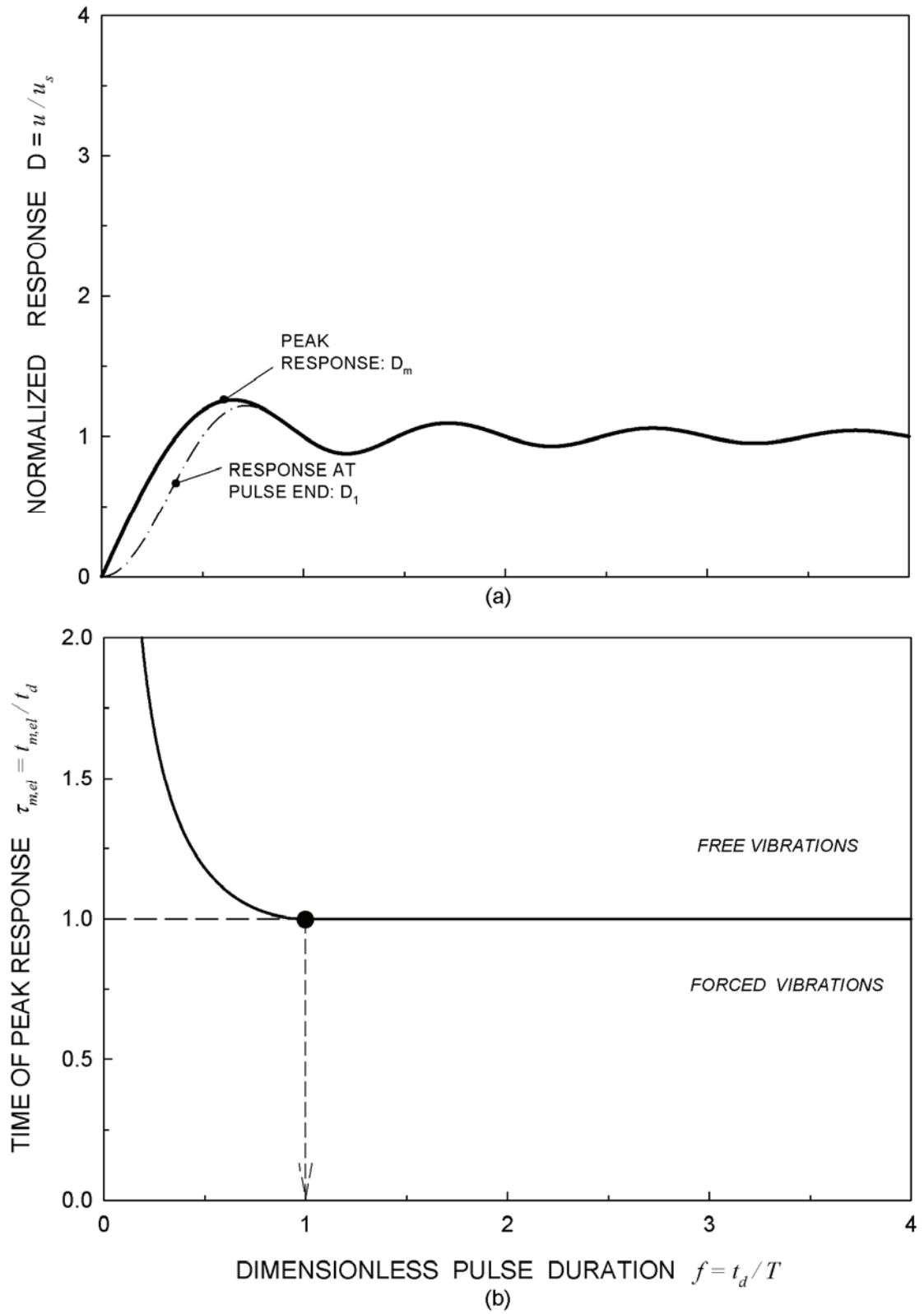
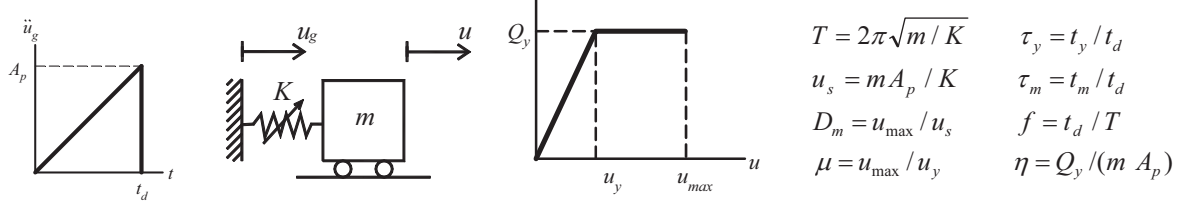


Figure C.2.2 Normalized peak displacement response (a) and corresponding time (b) of an undamped elastic oscillator subjected to a ramp acceleration pulse.

Table C.2.2 Parameter definition and summary of closed-form solutions for an undamped elastoplastic oscillator subjected to a linear ramp pulse (R1).

	II peak deformation after pulse ($\tau_m > 1$)
A yielding during pulse ($\tau_y \leq 1$)	$\tau_y = \frac{\eta - \tau_y^* \cos 2\pi f \tau_y^* + \frac{1}{2\pi f} \sin 2\pi f \tau_y^*}{1 - \cos 2\pi f \tau_y^*}, \quad \tau_y^* = \sqrt[3]{\frac{6\eta}{4\pi^2 f^2}}$ $\tau_m = \tau_y + \frac{1}{\eta} \left[\frac{1}{(2\pi f)^2} (1 - \cos 2\pi f \tau_y) - \frac{1}{2} \tau_y^2 + \frac{1}{2} \right]$ $\mu = 1 + \frac{(2\pi f)^2}{\eta} \left(\frac{1}{2} \tau_m - \frac{1}{3} - \frac{1}{6} \tau_y^3 \right) - 2\pi^2 f^2 (\tau_m - \tau_y)^2 - \frac{1}{\eta} [2\pi^2 f^2 \tau_y^2 + \cos 2\pi f \tau_y - 1] (\tau_m - \tau_y)$
C yielding after pulse ($\tau_y > 1$)	$\tau_y = \frac{1}{2\pi f} \left[\tan^{-1} \frac{\sin 2\pi f - 2\pi f \cos 2\pi f}{\cos 2\pi f + 2\pi f \sin 2\pi f - 1} + \sin^{-1} \frac{2\pi f \eta}{\sqrt{2 + 4\pi^2 f^2 - 2\cos 2\pi f - 4\pi f \sin 2\pi f}} \right]$ $\tau_m = \tau_y + \frac{1}{\eta} \left[\left(\frac{1}{4\pi^2 f^2} (-1 + \cos 2\pi f) + \frac{1}{2\pi f} \sin 2\pi f \right) \cos 2\pi f \tau_y + \left(\frac{1}{4\pi^2 f^2} \sin 2\pi f - \frac{1}{2\pi f} \cos 2\pi f \right) \sin 2\pi f \tau_y \right]$ $\mu = 1 - 2\pi^2 f^2 (\tau_m - \tau_y)^2 + \frac{1}{\eta} [(\cos 2\pi f - 1 + 2\pi f \sin 2\pi f) \cos 2\pi f \tau_y + (\sin 2\pi f - 2\pi f \cos 2\pi f) \sin 2\pi f \tau_y] (\tau_m - \tau_y)$
D no yielding	$\tau_m = \frac{1}{2\pi f} \left[\pi k + \frac{\pi}{2} + \tan^{-1} \frac{2\pi f \cos 2\pi f - \sin 2\pi f}{1 - \cos 2\pi f - 2\pi f \sin 2\pi f} \right], \quad k = \begin{cases} 0, & f < 0.371 \\ 1, & 0.371 < f < 1 \end{cases}$ $D_m = \frac{1}{2\pi f} \sqrt{2 + 4\pi^2 f^2 - 2\cos 2\pi f - 4\pi f \sin 2\pi f}$

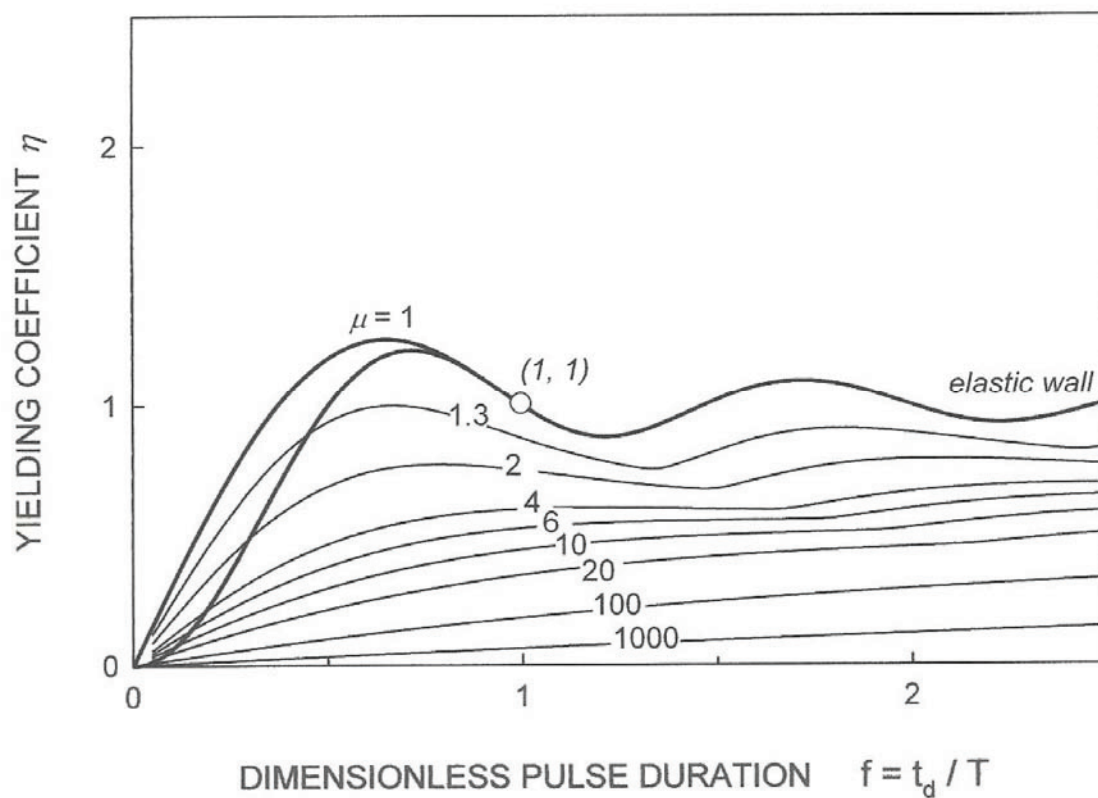


Figure C.2.3 Contour representation of ductility demand for an undamped elastic-perfectly plastic oscillator subjected to a linear ramp pulse.

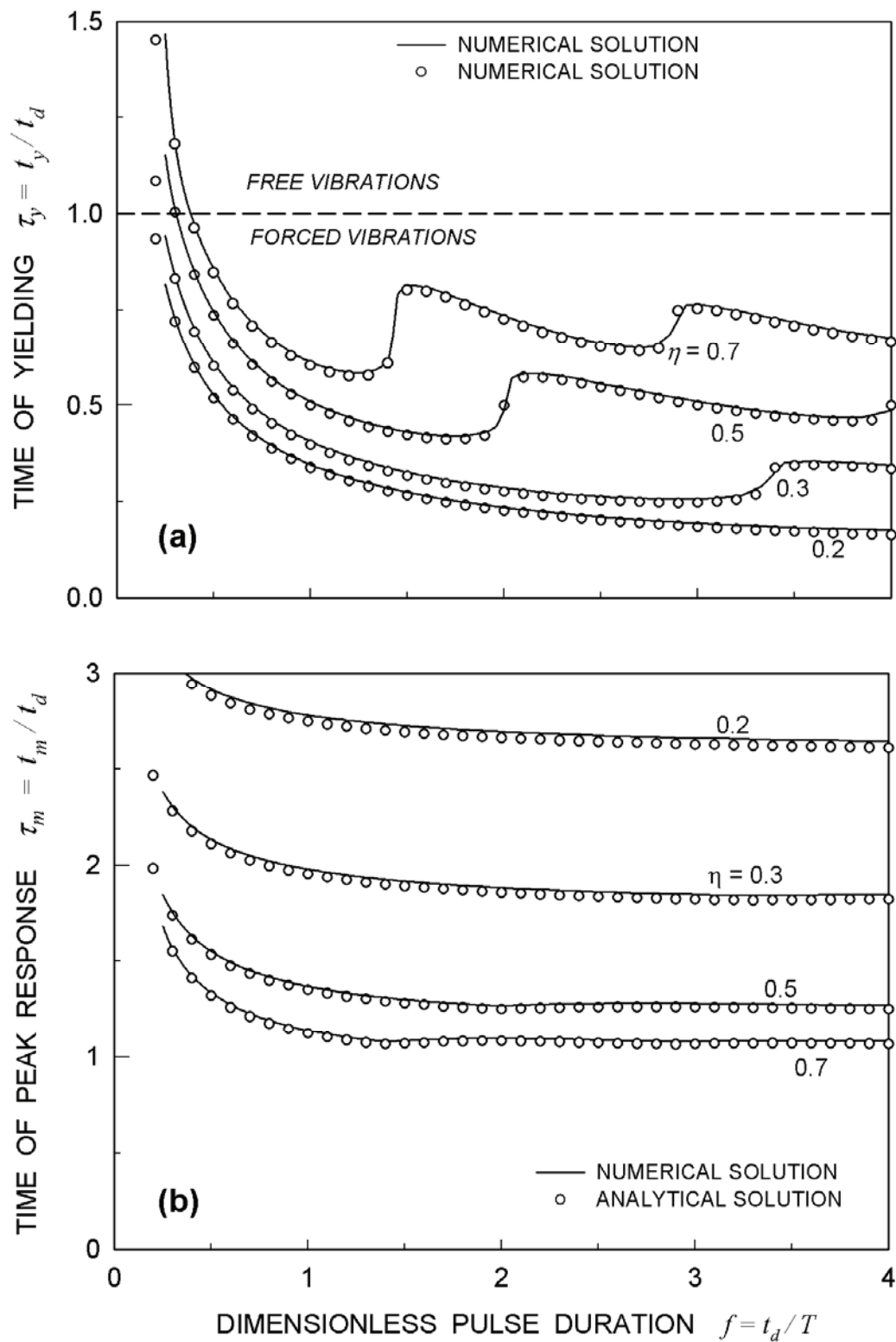


Figure C.2.4 Dimensionless time of yielding (a) and time of peak response; (b) for an undamped elastic-perfectly plastic oscillator subjected to a ramp pulse (R1).

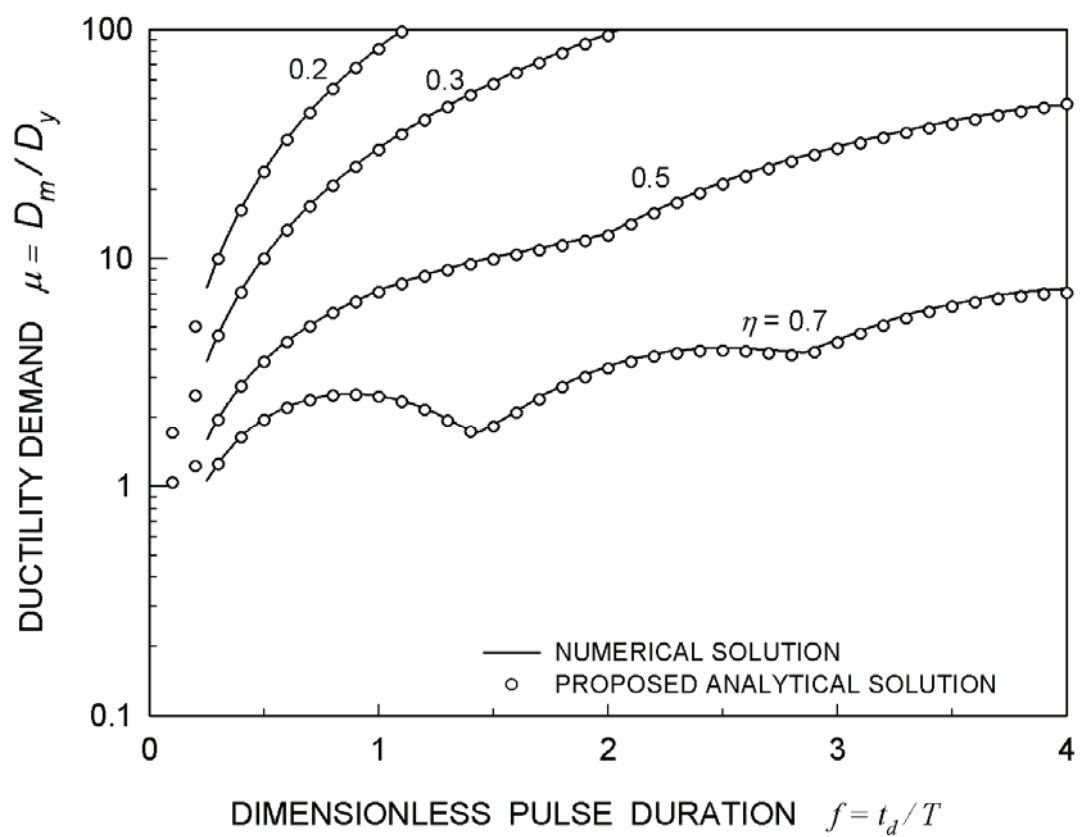


Figure C.2.5 Ductility demand for an undamped elastic-perfectly plastic oscillator subjected to a ramp pulse (R1).

C.3 Suddenly Applied Triangular Pulse (*Decreasing Ramp*)

Table C.3.1 Classification of oscillators, subjected to a rectangular pulse, based on times of yielding and peak deformation.

<i>Classification</i>	I peak deformation during pulse ($\tau_m \leq 1$)	II peak deformation after pulse ($\tau_m > 1$)
B yielding during pulse ($\tau_y \leq 1$)	$f \geq 0.371$ $\eta(\tau_m = 1) \leq \eta \leq 1 - \chi_I + \frac{1}{2\pi f}(\sqrt{1 - \chi_I^2} - \cos^{-1} \chi_I)$	$\eta \leq \eta(\tau_m = 1), f \geq 0.371$ $\eta \leq \frac{1}{2\pi f} \sin 2\pi f - \cos 2\pi f, f \leq 0.371$
C yielding after pulse ($\tau_y > 1$)	<i>not applicable</i>	$f \leq 0.371$ $\frac{1}{2\pi f} \sin 2\pi f - \cos 2\pi f \leq \eta \leq \frac{1}{\pi f} \sin \pi f \cos(\pi f - \chi_{II}) - \cos \chi_{II}$
D no yielding	$\eta \geq 1 - \chi_I + \frac{1}{2\pi f}(\sqrt{1 - \chi_I^2} - \cos^{-1} \chi_I),$ $f \geq 0.371$ where $\chi_I = \frac{1 - (2\pi f)^2}{1 + (2\pi f)^2}$	$\eta \geq \frac{1}{\pi f} \sin \pi f \cos(\pi f - \chi_{II}) - \cos \chi_{II}, f \leq 0.371$ where $\chi_{II} = \sqrt{2\pi f} \sqrt{1 + \frac{\sin 2\pi f - 2\pi f}{\sqrt{2 + (2\pi f)^2} - 2 \cos 2\pi f - 4\pi f \sin 2\pi f}}$

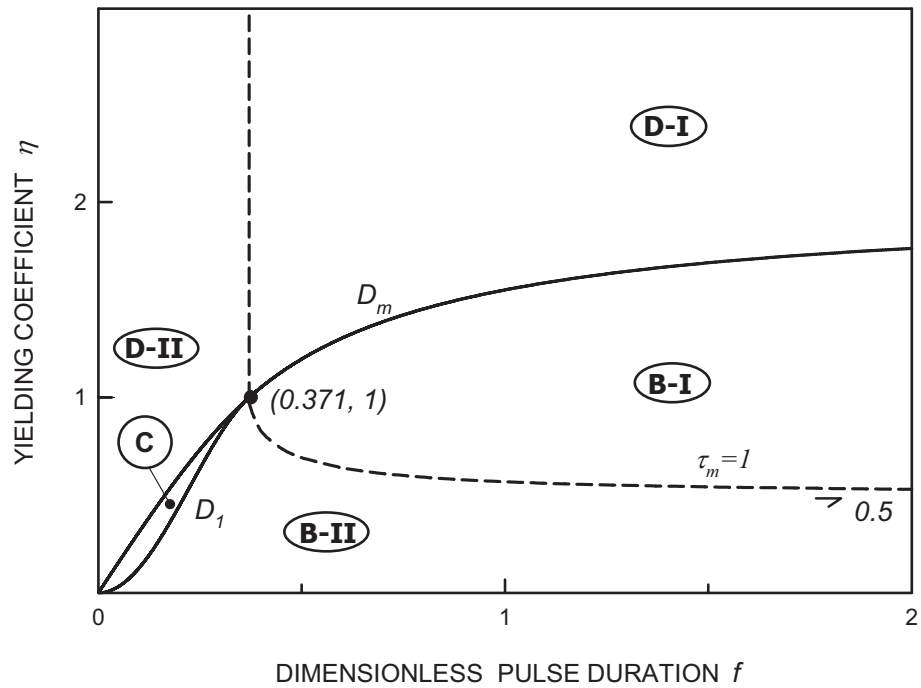


Figure C.3.1 Graphical representation of time of yielding and peak response for an undamped elastoplastic oscillator subjected to an inverse ramp acceleration pulse. [D_1 denotes the elastic response amplification factor at the end of the pulse.]

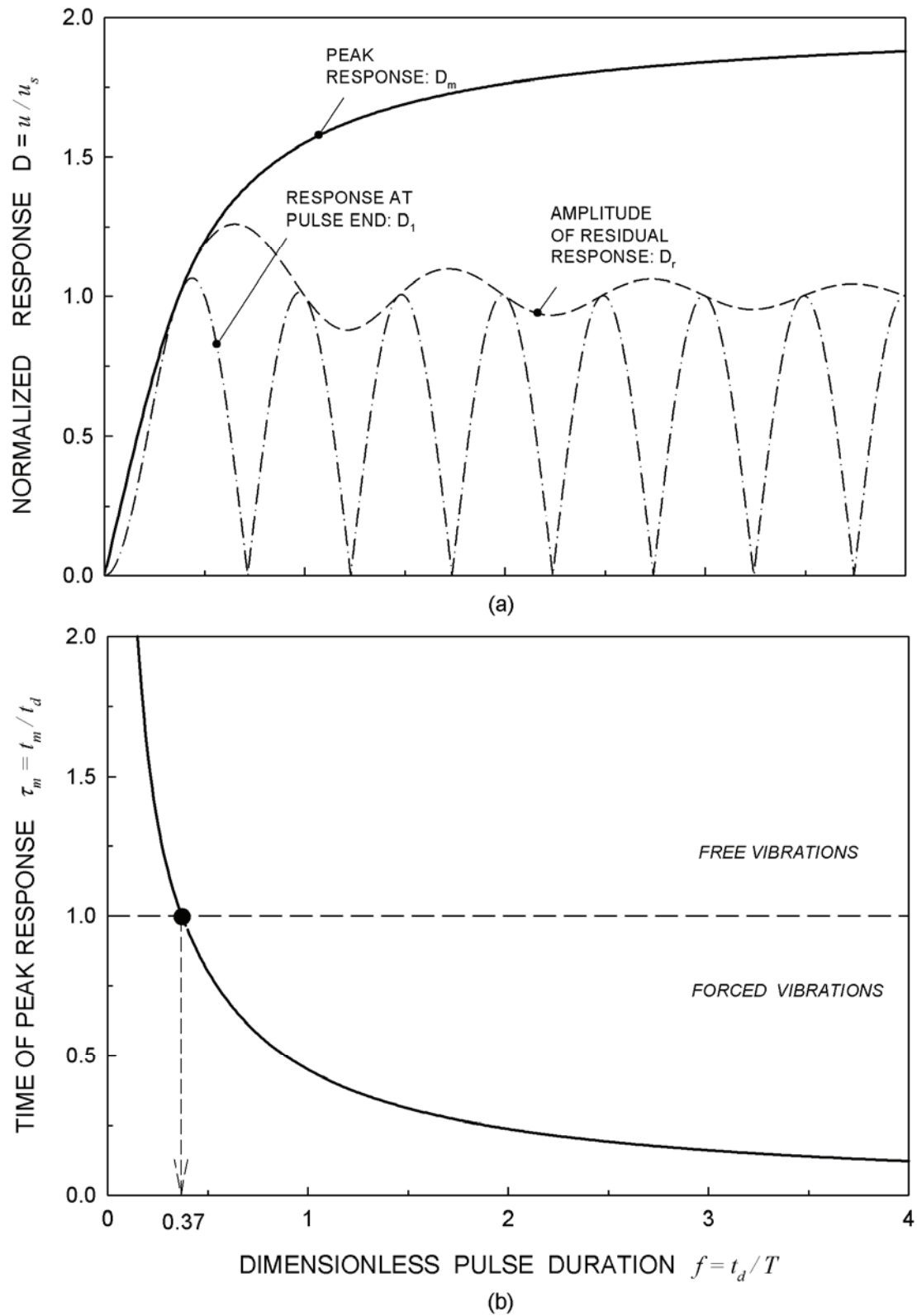
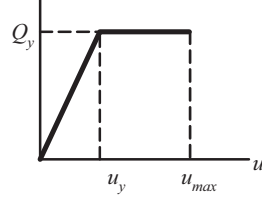


Figure C.3.2 Normalized peak displacement response (a) and corresponding time (b) of an undamped elastic oscillator subjected to an inverse ramp acceleration pulse.

Table C.3.2 Summary of closed-form solutions for an undamped elastoplastic oscillator subjected to a suddenly applied triangular pulse.

	I peak deformation during pulse ($\tau_m \leq 1$)
B yielding during pulse ($\tau_y \leq 1$)	$\tau_y = \frac{\eta - 1 - \left(\frac{1}{2\pi f} - 2\pi f \tau_y^* \right) \sin(2\pi f \tau_y^*) + (\tau_y^* + 1) \cos(2\pi f \tau_y^*)}{\cos(2\pi f \tau_y^*) + 2\pi f \sin(2\pi f \tau_y^*) - 1}, \quad \tau_y^* \approx \sqrt{\frac{\eta}{2(\pi f)^2}}$ $\tau_m = 1 - \eta + \frac{\sqrt{-1 + 2\pi^2 f^2 (-1 + \eta + \tau_y)^2 + \cos(2\pi f \tau_y) + 2\pi f \sin(2\pi f \tau_y)}}{\pi f \sqrt{2}}$ $\mu = 1 + \frac{2\pi^2 f^2}{\eta} E_{dr} - 2\pi^2 f^2 (\tau_m - \tau_y)^2 + \frac{1}{\eta} \left[\frac{2\pi^2 f^2 \tau_y^2 - 4\pi^2 f^2 \tau_y + \cos(2\pi f \tau_y) + 2\pi f \sin(2\pi f \tau_y) - 1}{\cos(2\pi f \tau_y) + 2\pi f \sin(2\pi f \tau_y) - 1} \right] (\tau_m - \tau_y),$ $E_{dr} = \left[\left(1 - \frac{\tau_m}{3} \right) \tau_m^2 - \left(1 - \frac{\tau_y}{3} \right) \tau_y^2 \right]$
C yielding after pulse ($\tau_y > 1$)	<i>not applicable</i>
D no yielding	$\tau_m = \frac{1}{\pi f} \cos^{-1} \left[\frac{1}{\sqrt{1 + (2\pi f)^2}} \right]$ $D_m = \frac{1}{2\pi f} \sin(2\pi f \tau_m) - \cos(2\pi f \tau_m) - \tau_m + 1$



$$T = 2\pi\sqrt{m/K}$$

$$u_s = m A_p / K$$

$$D_m = u_{\max} / u_s$$

$$\mu = u_{\max} / u_y$$

$$\tau_y = t_y / t_d$$

$$\tau_m = t_m / t_d$$

$$f = t_d / T$$

$$\eta = Q_y / (m A_p)$$

II

peak deformation after pulse ($\tau_m > 1$)

$\tau_y = \text{same as in } \mathbf{B-I}$

$$\tau_m = \tau_y + \frac{1}{\eta} \left[\frac{1}{(2\pi f)^2} [\cos(2\pi f \tau_y) - 1] + \frac{1}{2\pi f} \sin(2\pi f \tau_y) + \frac{1}{2} - \tau_y + \frac{1}{2} \tau_y^2 \right]$$

$$\mu = \text{same as in } \mathbf{B-I}, \text{ but } E_{dr} = \frac{1}{3} (\tau_y^3 - 3\tau_y^2 + 3\tau_m - 1)$$

$$\tau_y = \frac{1}{2\pi f} \left[\tan^{-1} \left[\frac{2\pi f - \sin(2\pi f)}{1 - \cos(2\pi f)} \right] + \sin^{-1} \left[\frac{2\pi f \eta}{\sqrt{2 + (2\pi f)^2 - 2\cos(2\pi f) - 4\pi f \sin(2\pi f)}} \right] \right]$$

$$\tau_m = \tau_y + \frac{1}{\eta(2\pi f)^2} [(1 - \cos(2\pi f)) \cos(2\pi f \tau_y) + (2\pi f - \sin(2\pi f)) \sin(2\pi f \tau_y)]$$

$$\mu = 1 - 2(\pi f)^2 (\tau_m - \tau_y)^2 + \frac{1}{\eta} [\cos(2\pi f \tau_y) + 2\pi f \sin(2\pi f \tau_y) - \cos[2\pi f(1 - \tau_y)]] (\tau_m - \tau_y)$$

$$\tau_m = \frac{1}{\pi f} \cos^{-1} \left[\sqrt{\frac{1}{2} - \frac{2\pi f - \sin(2\pi f)}{2\sqrt{2 + (2\pi f)^2 - 2\cos(2\pi f) - 4\pi f \sin(2\pi f)}}} \right]$$

$$D_m = \frac{1}{2\pi f} [\sin(2\pi f \tau_m) - \sin[2\pi f(\tau_m - 1)]] - \cos(2\pi f \tau_m)$$

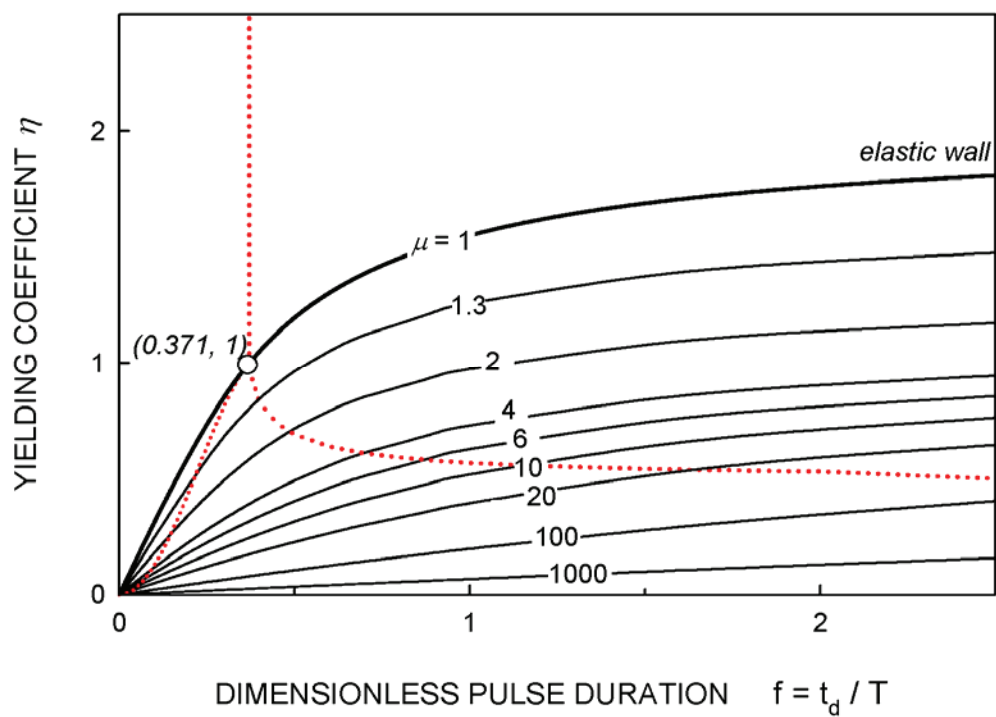


Figure C.3.3 Contour representation of ductility demand for an undamped elastic-perfectly plastic oscillator subjected to a suddenly applied triangular pulse.

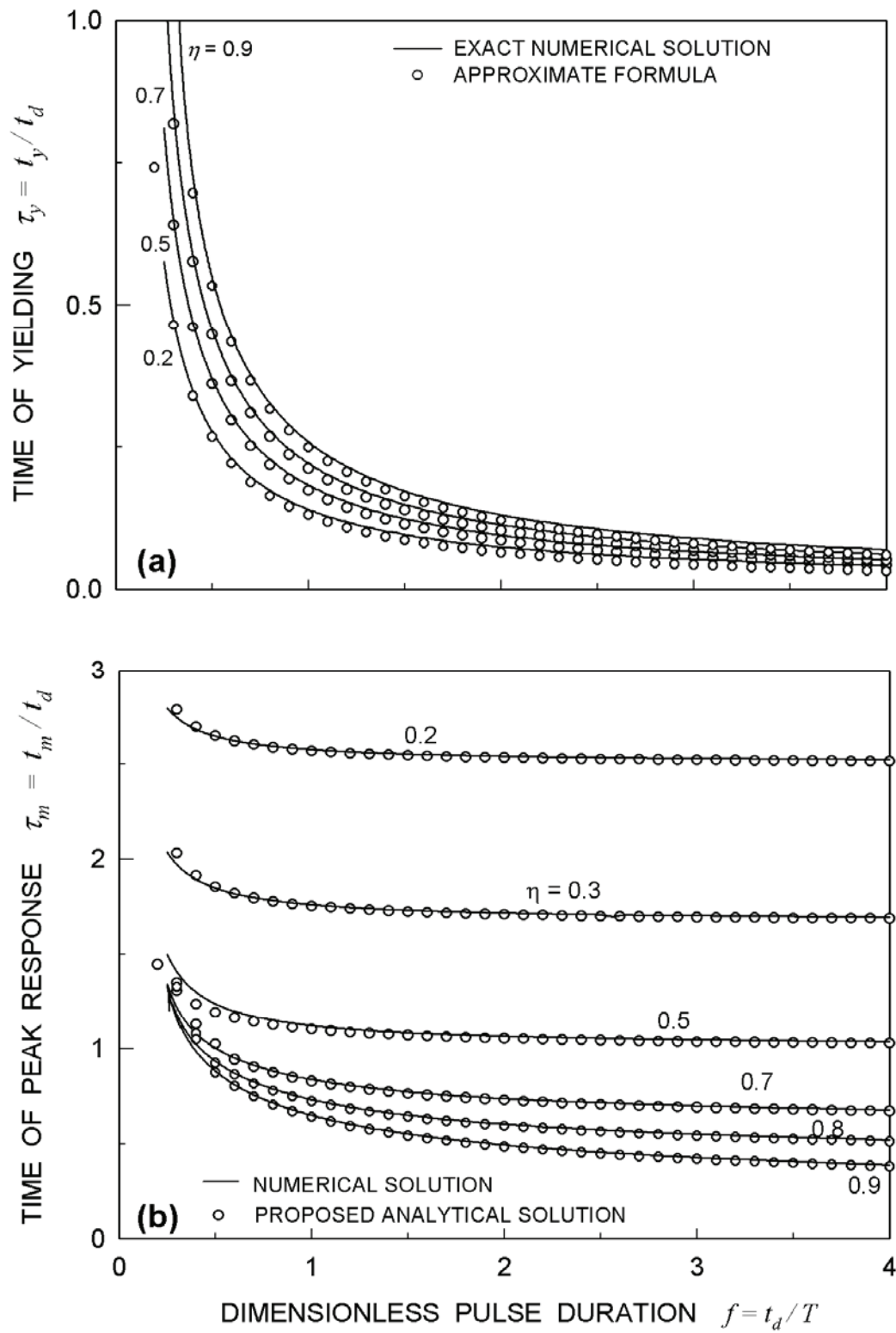


Figure C.3.4 Dimensionless time of yielding (a) and time of peak response; (b) for an undamped elastic-perfectly plastic oscillator subjected to an inverse ramp pulse.

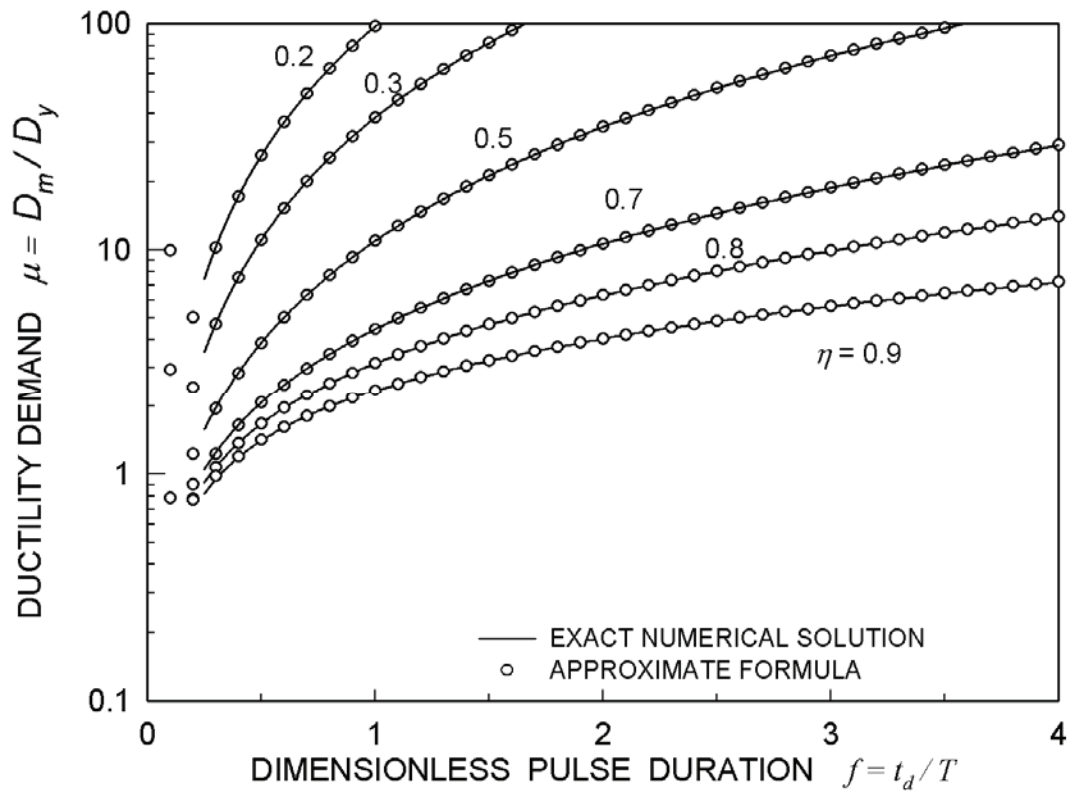


Figure C.3.5 Ductility demand for an undamped elastic-perfectly plastic oscillator subjected to an inverse ramp pulse.

Appendix D

Additional Material on Rocking Response

D.1 Closed – Form Solution for the Limit of Mode 1 for $t_i < 2t_d$

Equation (5.48) can be solved analytically to provide a closed-form solution for the Limit of Mode 1 for systems exhibiting impact during the course of the pulse.

$$\eta = c + \frac{e^{-2f}}{3} \left[-2 + e^f \left\{ -2(1 + \varepsilon^2) + e^f (3 - 2(e^f - 2)\varepsilon^2) \right\} \right] - \frac{e^{-4f}}{9c} \left[-1 - 4e^f (e^f - 1) \left\{ -2 + \varepsilon (e^f - 1) \left[3 - 2(e^f - 2)\varepsilon + e^f (e^f - 1)^2 \varepsilon^3 \right] \right\} \right] \quad (D.1a)$$

Where ε is the coefficient of restitution and

$$c = \frac{1}{3} e^{2f} c_1 + \frac{\sqrt{3}}{3} e^{-4f} c_2^{1/6} \quad (D.1b)$$

$$c_1 = \left[\begin{aligned} &1 + 48e^{8f} \varepsilon^6 - 8e^{9f} \varepsilon^6 - 12e^{7f} \varepsilon^4 (-1 + 10\varepsilon^2) + 4e^{6f} \varepsilon^3 (-9 - 18\varepsilon + 40\varepsilon^3) \\ &- 3e^f (5 + \varepsilon(12 + 5\varepsilon)) - 24e^{5f} \varepsilon^2 (-2 + \varepsilon(-6 - 7\varepsilon + 5\varepsilon^3)) \\ &- 6e^{2f} (-2 + \varepsilon(-6 + \varepsilon(-3 + 6\varepsilon + 4\varepsilon^2))) + 12e^{4f} \varepsilon (-3 + \varepsilon(-9 + 2\varepsilon(-9 - 8\varepsilon + 2\varepsilon^3))) \\ &+ e^{3f} (1 + \varepsilon(36 + \varepsilon(57 + 4\varepsilon(36 + 27\varepsilon - 2\varepsilon^3)))) \end{aligned} \right] \quad (D.1c)$$

$$c_2 = -e^{13f} (e^f - 1)^2 \left[1 + \varepsilon \left\{ 1 + 2e^f \left[-1 + (e^f - 1)\varepsilon \right] \right\} \right]^2 \times \left[\begin{aligned} &1 + (-10 + \varepsilon)\varepsilon - 16e^{5f} \varepsilon^4 + 4e^{6f} \varepsilon^4 + 4e^{4f} \varepsilon^2 (-2 + \varepsilon + 6\varepsilon^2) + \\ &2 + e^f \left\{ +e^f (18 + 2\varepsilon[12 + (7 - 2\varepsilon)\varepsilon]) - 4e^{3f} \varepsilon (-3 + \varepsilon[-8 + \varepsilon(3 + 4\varepsilon)]) + \right. \\ &\left. + e^{2f} (-19 + \varepsilon[-26 + \varepsilon(-39 + 4\varepsilon(3 + \varepsilon))) \right\} \end{aligned} \right] \quad (D.1d)$$

are dimensionless expressions.

In the limit case of $\alpha \rightarrow 0$ the coefficient of restitution tends to 1 and Eq. (D.1) simplifies to

$$\eta = c + \frac{e^{-2f}}{3} \left[-2 + e^f \left\{ -4 + e^f (3 - 2(e^f - 2)) \right\} \right] - \frac{e^{-4f}}{9c} \left[-1 - 4e^f (e^f - 1) \left\{ -2 + (e^f - 1) \left[3 - 2(e^f - 2) + e^f (e^f - 1)^2 \right] \right\} \right] \quad (\text{D.2a})$$

where now

$$c = \frac{1}{3} e^{2f} (1 - 66e^f + 6e^{2f} + 338e^{3f} - 504e^{4f} + 240e^{5f} + 52e^{6f} - 108e^{7f} + 48e^{8f} - 8e^{9f}) + \frac{\sqrt{3}}{3} e^{\frac{11f}{6}} (e^f - 1) (-8 + 32e^f - 208e^{2f} + 272e^{3f} - 64e^{4f} - 80e^{5f} + 64e^{6f} - 16e^{7f})^{1/6} \quad (\text{D.2b})$$

D.2 Function Study of Rocking Response

(a) Systems Overturning with *Mode 0*:

- Area O_1 : $\theta(t_d) > \alpha + \alpha_g$

t	0	t_d	$2t_d$	$+\infty$
$\text{sgn } \ddot{\theta}(t)$	+	+	+	
$\text{sgn } \dot{\theta}(t)$	+	+	+	
$\text{sgn } \theta(t)$	+	+	+	

- Area O_2 : $\theta(t_d) < \alpha + \alpha_g$ and $t_d < t_e < 2t_d$

t	0	t_d	$2t_d$	$+\infty$
$\text{sgn } \ddot{\theta}(t)$	+	−	+	+
$\text{sgn } \dot{\theta}(t)$	+	+	+	+
$\text{sgn } \theta(t)$	+	+	+	+

- Area O_3 : $\theta(t_d) < \alpha + \alpha_g$ and $\theta(2t_d) > \alpha$

t	0	t_d	$2t_d$	$+\infty$
$\text{sgn } \ddot{\theta}(t)$	+	−	+	
$\text{sgn } \dot{\theta}(t)$	+	+	+	
$\text{sgn } \theta(t)$	+	+	+	

- Area O_4 : $\theta(t_d) < \alpha + \alpha_g$, $\theta(2t_d) > \alpha$ and $t_d < t_{s1} < 2t_d$, $t_{s2} > 2t_d$

t	0	t_d	$2t_d$	$+\infty$
$\text{sgn } \ddot{\theta}(t)$	+	−	+	+
$\text{sgn } \dot{\theta}(t)$	+	+	−	+
$\text{sgn } \theta(t)$	+	+	+	+

(b) Systems Surviving *Mode 0*:

- Area S_1 : $\theta(t_d) < \alpha + \alpha_g$, $\theta(2t_d) > \alpha$, $t_e > 2t_d$ and $t_d < t_s < 2t_d$

t	0	t_d	$2t_d$	$+\infty$
$\text{sgn } \ddot{\theta}(t)$	+	−	+	−
$\text{sgn } \dot{\theta}(t)$	+	+	−	−
$\text{sgn } \theta(t)$	+	+	+	+

impact

- Area S_2 : $\theta(t_d) < \alpha + \alpha_g$, $\theta(2t_d) < \alpha$ and $t_d < t_s < 2t_d$

t	0	t_d	$2t_d$	$+\infty$
$\text{sgn } \ddot{\theta}(t)$	+	−	−	
$\text{sgn } \dot{\theta}(t)$	+	+	−	−
$\text{sgn } \theta(t)$	+	+	+	+

impact

- Area S_3 : $\theta(t_d) < \alpha + \alpha_g$, $t_d < t_s < 2t_d$ and $\theta(t_s) < \alpha + \alpha_g$

t	0	t_d	$2t_d$	$+\infty$
$\text{sgn } \ddot{\theta}(t)$	+	−	−	
$\text{sgn } \dot{\theta}(t)$	+	+	−	
$\text{sgn } \theta(t)$	+	+	+	

impact

(a) Systems Overturning with Mode 1:

- Area E_1 : during forward rocking: $|\theta(2t_d)| \geq |\alpha|$, $2t_d < t_{e1} < t_i$; during backward rocking: $t_i > 2t_d$, $t_{e2} > t_i$

t	t_d	$2t_d$	t_i	$+\infty$
$\text{sgn } \ddot{\theta}(t)$		+	−	+
$\text{sgn } \dot{\theta}(t)$		−	−	−
$\text{sgn } \theta(t)$		+	−	−

- Area E_2 : during forward rocking: $|\theta(2t_d)| < |\alpha|$; during backward rocking: $t_i > 2t_d$, $t_e > t_i$

t	t_d	$2t_d$	t_i	$+\infty$
$\text{sgn } \ddot{\theta}(t)$		−	+	−
$\text{sgn } \dot{\theta}(t)$		−	−	−
$\text{sgn } \theta(t)$		+	−	−

- Area E_3 : during backward rocking: $t_d < t_i < 2t_d$, $t_e > 2t_d$

t	t_d	t_i	$2t_d$	$+\infty$
$\text{sgn } \ddot{\theta}(t)$		−	−	+
$\text{sgn } \dot{\theta}(t)$		−	−	−
$\text{sgn } \theta(t)$		+	−	−

(b) Systems Surviving Mode 1:

- Area S_1 : during forward rocking: $\theta(t_d) < \alpha + \alpha_g$, $\theta(2t_d) > \alpha$, $2t_d < t_e < t_i$; during backward rocking: $t_i > 2t_d$, and $t_s > t_i$

t	t_d	$2t_d$	t_i	$+\infty$
$\text{sgn } \ddot{\theta}(t)$		+	−	+
$\text{sgn } \dot{\theta}(t)$		−	−	+
$\text{sgn } \theta(t)$		+	−	−

impact

- Area S_2 : during forward rocking: $\theta(t_d) < \alpha + \alpha_g$, $\theta(2t_d) < \alpha$; during backward rocking: $t_i > 2t_d$, $t_s > t_i$

t	t_d	$2t_d$	t_i		$+\infty$
$\text{sgn } \ddot{\theta}(t)$		—	+	+	<i>impact</i>
$\text{sgn } \dot{\theta}(t)$		—	—	+	
$\text{sgn } \theta(t)$		+	—	—	

- Area S_3 : during backward rocking: $t_d < t_i < 2t_d$, $t_s > 2t_d$

t	t_d	t_i	$2t_d$		$+\infty$
$\text{sgn } \ddot{\theta}(t)$		—	—	+	+
$\text{sgn } \dot{\theta}(t)$		—	—	—	+
$\text{sgn } \theta(t)$		+	—	—	—

impact

D.3 Impulsive Forces during Impact

Solving the governing differential equation of rocking motion (Eqs 5.1a,b) requires handling a number of nonlinearities stemming from: (1) the transition from one rocking pole to the other following an impact (P to P' in Fig. 5.1); (2) the energy dissipation during impact which results to a discontinuity in angular velocity $\dot{\theta}$; and (3) the geometric nonlinearities expressed by the trigonometric terms in the left and right –hand side of the equation.

The transition from one rocking pole to the other is described analytically via the signum function $[\text{sgn}(x)]$ which operates on the gravitational term. On the other hand, the discontinuity in angular velocity can be incorporated into the differential equation via a Dirac- δ function centered at $\theta = 0$. Possible difficulties may also arise during numerical manipulations in presence of discontinuous excitation and response functions such as signum, Dirac- δ , and Heaviside. It will be shown that these can be well approximated by continuous counterparts by means of pertinent penalty factors.

To this end, an elegant approach has been proposed by Prieto *et al* (2004), a summary of which is, *mutatis mutandis*, provided below.

The non-linear equations 5.1a,b governing rocking motion can be written in compact form as

$$I_o \ddot{\theta} + mgR \text{sgn}(\theta) \sin(\alpha - \theta \text{sgn}(\theta)) = +m\ddot{u}_g R \cos(\alpha - \theta \text{sgn}(\theta)) + M_\delta \quad (\text{D.3})$$

where M_δ is a directed impulsive moment to account for energy dissipation during impact.

Besides dimensional homogeneity, there are three basic requirements to be fulfilled by M_δ :

- (a) The moment is concentrated at space (or time)
- (b) The moment depends on angular velocity in such a way that its effect produces a sudden change in the value of velocity at the critical point equal to ε – times its value.
- (c) The moment reduces velocity for both directions of motion.

Condition (c) is met by the use of a $\text{sgn}(\dot{\theta})$ multiplier to ensure that the impulsive moment always opposes angular velocity. To fulfill requirements (a) and (b) the following functional form is considered

$$M_\delta = I_o A \dot{\theta}(t)^2 \delta(\theta) \text{sgn}(\dot{\theta}) \quad (\text{D.4})$$

which is dimensionally homogeneous* with $\delta(\theta)$ being a Dirac- δ function and A a dimensionless constant. Assuming, without loss of generality, positive angular velocity at impact, Eqs (D.3) and (D.4) yield

$$I_o \ddot{\theta} + (\text{other terms}) = I_o A \dot{\theta}(t)^2 \delta(\theta) \quad (\text{D.5})$$

Introducing the chain rule of differentiation

$$\ddot{\theta} = \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta} \frac{d\theta}{dt} = \frac{d\dot{\theta}}{d\theta} \dot{\theta} \quad (\text{D.6})$$

Eq. (D.5) can, after simplification and rearrangement, be cast in the separable form

$$\frac{d\dot{\theta}}{\dot{\theta}} = A\delta(\theta)d\theta \quad (\text{D.7})$$

in which all terms except for the highest derivative (angular acceleration) and the impulsive forces have been neglected.

Upon integration in the interval shortly before and after impact yields

$$\int_{\theta^-}^{\theta^+} \frac{d\dot{\theta}}{\dot{\theta}} = \int_{\theta^-}^{\theta^+} A\delta(\theta)d\theta = A \quad (\text{D.8})$$

from which it is possible to deduce

$$A = \ln\left(\frac{\dot{\theta}_{\theta^+}}{\dot{\theta}_{\theta^-}}\right) = \ln(\varepsilon) \quad (\text{D.9})$$

ε being the familiar coefficient of restitution. Accordingly Eq. (D.4) becomes

$$M_\delta = I_o \ln(\varepsilon) \dot{\theta}(t)^2 \delta(\theta) \quad (\text{D.10})$$

For numerical purposes, the delta function can be well approximated by the common exponential form, tantamount to Gaussian distribution of standard deviation n

$$\delta(\theta) = \frac{1}{n\sqrt{\pi}} e^{-\frac{\theta^2}{n^2}} \quad (\text{D.11})$$

where n is used here as a positive penalty parameter with the property $n \ll 1$.

Along the same vein, the signum function in Eq. (D.4) can be approximated by

$$\text{sgn}(\theta) = \tanh\left(\frac{\theta}{n}\right) \quad (\text{D.12})$$

with n having the same meaning as before.

In light of the above analysis, the non-linear equation of motion takes the form

* This can be readily proven by the dimensional equation $M_\delta \propto I_o \theta^\alpha \dot{\theta}^\beta \ddot{\theta}^\gamma$ under the reasonable hypothesis that moment is independent of acceleration, hence $\alpha=0$, $\beta=2$, $\gamma=0$.

$$I_o \ddot{\theta} + mgR \operatorname{sgn}(\theta) \sin[\alpha - \theta \operatorname{sgn}(\theta)] = m \ddot{u}_g R \cos[\alpha - \theta \operatorname{sgn}(\theta)] + I_o \ln(\varepsilon) \dot{\theta}^2 \delta(\theta) \operatorname{sgn}(\dot{\theta}) \quad (\text{D.13a})$$

or, upon division by I_o

$$\ddot{\theta} + p^2 \operatorname{sgn}(\theta) \sin[\alpha - \theta \operatorname{sgn}(\theta)] = + \frac{\ddot{u}_g}{g} p^2 \cos[\alpha - \theta \operatorname{sgn}(\theta)] + \ln(\varepsilon) \dot{\theta}^2 \delta(\theta) \operatorname{sgn}(\dot{\theta}) \quad (\text{D.13b})$$

where $p^2 = 3g/4R$ is the familiar intrinsic frequency of the rectangular block at hand.

The above equation can be further simplified via the changes in dependent and independent variables $x = \theta/\alpha$ and $\tau = t/t_d$, in conjunction with the familiar normalizations η

$= \alpha/\alpha_g$ (dimensionless uplift strength), $f = p t_d$ (dimensionless pulse duration),

$\ddot{\psi}_g(\tau) = \ddot{u}_g/(a_g \times g)$ (dimensionless ground acceleration), to give

$$\ddot{x} + \frac{f^2}{\alpha} \operatorname{sgn}(x) \sin[\alpha(1 - x \operatorname{sgn}(x))] = \frac{f^2}{\eta} \cos[\alpha(1 - x \operatorname{sgn}(x))] \ddot{\psi}_g(\tau) + \ln(\varepsilon) \dot{x}^2 \delta(x) \operatorname{sgn}(\dot{x}) \quad (\text{D.14})$$

in which the scaling property $\delta(\alpha x) = \delta(x)/|\alpha|$ (Bracewell 1986) has been employed.

In the special case of a slender block, $\alpha^2 \ll \alpha$, Eq. (D.14) simplifies to

$$\ddot{x} - f^2 x + f^2 \operatorname{sgn}(x) = f^2 \frac{1}{\eta} \ddot{\psi}_g(\tau) + \ln(\varepsilon) \dot{x}^2 \delta(x) \operatorname{sgn}(\dot{x}) \quad (\text{D.15})$$

which, together with Eq. (D.14), are suitable for numerical treatment. It should be noted that the presence of the impulsive term in Eq. (D.15) obliterates linear dependence between θ on α identified on Chapter 4, as the restitution coefficient ε is a nonlinear function of α (Eq. 4.8). If, however, ε is assumed to be independent of α , as done in this study, the similarity holds.

Another useful result is obtained by integrating Eq. (D.5) from θ^- to θ^+ to evaluate the integral of a Dirac – delta function across a velocity discontinuity

$$\int_{\theta^-}^{\theta^+} \dot{\theta}^2 \delta(\theta) d\theta = \frac{\dot{\theta}_{t_i^+}^2 - \dot{\theta}_{t_i^-}^2}{\ln \dot{\theta}_{t_i^+}^2 - \ln \dot{\theta}_{t_i^-}^2} \quad (\text{D.16a})$$

$$\int_{\theta^-}^{\theta^+} \dot{\theta}^2 \delta(\theta) d\theta = \frac{r-1}{\ln r} \dot{\theta}_{t_i^-}^2 \quad (\text{D.16b})$$

which, evidently, yields the anticipated values $\dot{\theta}_{t_i^-}$ at $r \rightarrow 1$ and 0 at $r \rightarrow 0$. Note that the above solution is different from the intuitive result $(r+1)\dot{\theta}_{t_i^-}/2$ i.e., the average value between $\dot{\theta}_{t_i^-}$ and $\dot{\theta}_{t_i^+}$, which does not satisfy the second of the above conditions.

Another significant simplification introduced by the above formulation lies in the ability of the impulsive moment to “damp out” the restoring moment before uplift. This suggests that the simulation is self standing and does not require an uplift criterion such as the one expressed by $\eta < 1$ in Eq. (4.3).

Appendix E

Shift Theorem Derivations

E.1 Proof of Equations (6.16) and (6.17)

The fundamental relation,

$$\dot{u}_g(t) = \int_0^t \ddot{u}_g dt + \dot{u}_g(0) \quad (\text{E.1})$$

can be decomposed as follows

$$\dot{u}_g(t) = \int_0^{t_y} \ddot{u}_g dt + \int_{t_y}^t \ddot{u}_g dt + \dot{u}_g(0) = \int_{t_y}^t \ddot{u}_g dt + \dot{u}_g(t_y) \quad (\text{E.2})$$

By means of the definitions in Eqs (6.12-6.15), the above result can be cast in the simpler form

$$\Delta \dot{u}_g(t) = \int_{t_y}^t \ddot{u}_g dt = \int_0^{\Delta t} \ddot{u}_g dt \quad (\text{E.3})$$

Adding and subtracting a_y in the integrand of the above equation, one gets

$$\Delta \dot{u}_g = \int_0^{\Delta t} (\ddot{u}_g - a_y) dt + a_y \int_0^{\Delta t} dt \quad (\text{E.4})$$

from which Eq. (6.17) can be readily obtained. Equation (6.16) can be derived in an analogous manner.

E.2 Proof of Equation (7.2b)

The equation of motion after yielding ($t > t_y$) of an undamped elastic-perfectly plastic system with yield acceleration $a_y = F_y/m$, is described by the piece-wise linear differential equation:

$$\ddot{u} + a_y \operatorname{sgn}(\dot{u}) = \ddot{u}_g(t) \quad (\text{E.5})$$

subject to the initial conditions $u(t_y+) = u_y$ and $\dot{u}(t_y+) = \dot{u}_y$.

Taking the Laplace transform of Eq. (E.5), the solution is obtained by the familiar expression

$$s^2 \mathcal{L}(u) = \mathcal{L}(\ddot{u}_g(t) - a_y \operatorname{sgn}(\dot{u})) - s u_y + \dot{u}_y \quad (\text{E.6})$$

which incorporates the initial conditions at $t = t_y$, $\mathcal{L}(u)$ and s being the dependent and independent variables in the Laplace domain.

Setting $t^* = t - t_y$, the right-hand side of Eq. (E.6) can be cast in the alternative form

$$s^2 \mathcal{L}(u) = \mathcal{L}(\ddot{u}_g(t^*) - a_y \operatorname{sgn}(\dot{u}) + \dot{\delta}(t^*) u_y + \delta(t^*) \dot{u}_y) \quad (\text{E.7})$$

in which the initial conditions are incorporated in the excitation function. This property is evident as

$$\int_0^\infty \dot{\delta}(t^*) u_y e^{-st^*} dt^* = -s u_y \quad \text{and} \quad \int_0^\infty \delta(t^*) \dot{u}_y e^{-st^*} dt^* = \dot{u}_y \quad (\text{E.8a,b})$$

Inverting formula (E.7) back to the time domain yields Eq. (7.2b).

E.3 Tables

Table E.1. Near-Fault Ground Motion Database*

No	Event	Date	Station	M_w	Closest Distance (km)	A_g (g)	V_g (cm/s)	D_g (cm)
1	Parkfield, CA, USA	27/6/1966	C02	6.20	0.10	0.476	75.1	22.5
2	San Fernando, CA, USA	9/2/1971	PCD	6.55	3.00	1.291	120.0	31.1
3	Gazli, USSR	17/5/1976	KAR	6.80	3.00	0.653	60.0	32.8
4	Bucharest, Romania	4/3/1977	BRI	7.27	190.00	0.204	74.9	20.9
5	Tabas, Iran	16/9/1978	TAB	7.11	1.20	0.846	122.0	87.9
6	Coyote Lake, CA, USA	8/6/1979	GA6	5.63	1.20	0.444	47.5	9.3
7	Imperial Valley, CA, USA	15/10/1979	E04	6.50	6.00	0.356	78.3	58.6
8			E05		2.70	0.375	91.8	61.5
9			E06		0.30	0.442	112.0	66.8
10			E07		1.80	0.462	109.0	45.8
11			EMO		1.20	0.378	115.0	40.1
12	Morgan Hill, CA, USA	24/4/1984	HAL	6.15	2.00	0.314	39.8	6.6
13	Palm Springs, CA, USA	8/7/1986	NPS	6.09	4.00	0.670	73.6	11.9
14			DSP		6.40	0.343	29.2	7.1
15	Whittier Narrows, CA, USA	10/10/1987	DOW	5.93	16.40	0.211	30.7	3.9
16			NWK		15.70	0.229	20.0	2.6
17	Superstition Hills, CA, USA	24/11/1987	PTS	6.40	0.70	0.433	109.0	51.6
18			ELC		13.60	0.298	52.0	22.1
19	Loma Prieta, USA	17/10/1989	LGP	6.90	3.00	0.646	102.0	36.6
20			STG		8.30	0.365	56.4	29.8
21	Sierra Madre, CA, USA	28/6/1991	COG	5.56	9.40	0.145	28.7	32.0
22	Erzican, Turkey	13/3/1992	ERZ	6.63	2.00	0.484	95.2	30.4
23	Landers, CA, USA	28/6/1992	LUC	7.20	1.10	0.734	114.0	174.0
24	Northridge, CA, USA	17/1/1994	JFA	6.70	5.20	0.393	105.0	45.1
25			RRS		6.00	0.887	173.0	31.8
26			SCG		5.10	0.588	134.0	44.2
27			SCH		5.00	0.889	122.0	37.8
28			NWS		5.30	0.412	117.0	42.8
29	Aigion, Greece	15/6/1995	AEG long	6.33	6.00	0.499	40.9	8.2
30			AEG tran		6.00	0.546	52.0	9.0
31	Izmit, Turkey	17/8/1999	ARC	7.40	14.00	0.134	44.3	65.6
32			SKR		3.10	0.407	80.3	209.7
33			YPT		2.60	0.230	87.8	185.6
34			GBZ SN		11.00	0.268	41.4	33.6
35			GBZ SP		11.00	0.145	28.7	32.0
36	Chi-Chi, Taiwan	20/9/1999	TCU052	7.60	0.80	0.506	270.0	853.9
37			TCU068		0.20	0.469	380.0	1086.9
38			TCU075		0.60	0.330	115.0	173.0
39			TCU076		2.30	0.425	88.0	128.8
40			TCU129		1.50	1.000	68.0	127.0

* after Mavroeidis & Papageorgiou 2003

Table E.2 Comparison of peak plastic input motion parameters considering only major pulses (Method A) and by rigorous integration of the whole earthquake record (Method B); $\alpha_y=0.10g$

No.	Ground Motion	(g) (cm/s) (cm)	No. of effective pulses	Method A				Method B		Deviation
				Pulse	Pulse	Pulse	Pulse	Peak value (A)	Peak value (B)	
1	Parkfield, CA, USA	A_{gp}	4	-0.501	-0.202	-0.416	-0.212	0.501	0.501	0.00
		V_{gp}		50.693	-2.369	59.282	-17.182	59.282	59.63	0.58
		D_{gp}		11.161	-0.08	19.365	-3.939	30.446	29.52	-3.14
2	San Fernando, CA, USA	A_{gp}	1	0.61				0.61	1.326	54.00
		V_{gp}		95				95	94.66	-0.36
		D_{gp}		52.7				52.7	43.19	-22.02
3	Gazzli, USSR	A_{gp}	2	-0.631	-0.389			0.631	0.631	0.00
		V_{gp}		40.746	31.267			40.746	36.67	-11.12
		D_{gp}		5.807	8.426			14.053	16.56	15.14
4	Bucharest, Romania	A_{gp}	1	0.262				0.262	0.26	-0.77
		V_{gp}		-43.046				43.046	43.06	0.03
		D_{gp}		-18.611				18.611	18.62	0.05
5	Tabas, Iran	A_{gp}	2	0.952	-0.516			0.952	0.952	0.00
		V_{gp}		-80.403	12.744			80.403	85.09	5.51
		D_{gp}		-71.125	1.537			71.125	64.09	-10.98
6	Coyote Lake, USA	A_{gp}	1	-0.41				0.41	0.399	-2.76
		V_{gp}		32.26				32.26	32.39	0.40
		D_{gp}		7.723				7.723	7.26	-6.38
7	Imperial Valley E04, USA	A_{gp}	1	-0.26				0.26	0.26	0.00
		V_{gp}		-48.991				48.991	49.19	0.40
		D_{gp}		-36.175				36.157	35.77	-1.08
8	Imperial Valley E05, USA	A_{gp}	3	0.435	0.235		0.24	0.435	0.41	-6.10
		V_{gp}		24.5	52.6		47.7	52.6	56.66	7.17
		D_{gp}		3.95	-44.6		24	40.6	49.34	17.71
9	Imperial Valley E06, USA	A_{gp}	2	-0.339	-0.178			0.339	0.339	0.00
		V_{gp}		-64.363	12.743			64.363	64.2	-0.25
		D_{gp}		-70.211	1.884			70.211	59.83	-17.35
10	Imperial Valley E07, USA	A_{gp}	1	-0.526				0.526	0.526	0.00
		V_{gp}		75.91				75.91	75.96	0.07
		D_{gp}		57.802				57.802	57.85	0.08

Table E.2 Comparison of peak plastic input motion parameters considering only major pulses (Method A) and by rigorous integration of the whole earthquake record (Method B); $\alpha_y=0.10g$ (*continued*)

11	Imperial Valley EMO, USA	A_{gp}	-0.396	0.396	0.396	0.00
		V_{gp}	41.756	41.756	41.77	0.03
		D_{gp}	30.25	30.25	30.26	0.03
12	Morgan Hill, USA	A_{gp}	0.412	0.412	0.412	0.00
		V_{gp}	-32.586	32.586	32.62	0.10
		D_{gp}	-7.376	7.376	8.84	16.56
13	N.Palm Springs NPS, USA	A_{gp}	0.4941	0.4942	0.4941	-0.02
		V_{gp}	66.579	66.579	66.93	0.52
		D_{gp}	23.279	23.279	22.96	-1.39
14	N.Palm Springs DSP, USA	A_{gp}	0.2673	0.2673	0.283	5.55
		V_{gp}	-9.725	9.725	9.81	0.87
		D_{gp}	-0.896	0.896	0.767	-16.82
15	Whittier Narrows DOW, USA	A_{gp}	-0.3188	0.3188	0.315	-1.21
		V_{gp}	18.29	18.29	18.29	0.00
		D_{gp}	4.795	4.795	4.602	-4.19
16	Whittier Narrows NWK, USA	A_{gp}	-0.131	0.15	0.15	0.00
		V_{gp}	6.23	-15.795	15.86	0.41
		D_{gp}	0.903	-2.693	1.79	4.79
17	Superstition Hill PTS, USA	A_{gp}	0.555	-0.335	0.555	0.00
		V_{gp}	-49.564	105.22	104.76	-0.44
		D_{gp}	-24.681	71.906	47.225	8.28
18	Superstition Hill ELS, USA	A_{gp}	-0.1437	0.4579	0.4579	0.00
		V_{gp}	5.508	28.087	28.101	0.05
		D_{gp}	0.784	-6.405	5.448	-3.18
19	Loma Prieta LGP, USA	A_{gp}	-0.5829	-0.6097	0.6097	29.60
		V_{gp}	27.507	-94.267	94.267	9.88
		D_{gp}	7.315	-52.468	45.153	-3.82
20	Loma Prieta STG, USA	A_{gp}	0.385	-0.413	0.412	-0.24
		V_{gp}	-30.848	-26.512	30.894	0.15
		D_{gp}	-8.723	-4.61	13.333	-15.54

Table E.2 Comparison of peak plastic input motion parameters considering only major pulses (Method A) and by rigorous integration of the whole earthquake record (Method B); $\alpha_y=0.10g$ (*continued*)

21	Sierra Madre, USA	A_{gp}	-0.402	0.402	0.351	-14.53
		V_{gp}	8.009	8.009	8.629	7.19
		D_{gp}	0.677	0.677	0.673	-0.59
22	Erzikan, Turkey	A_{gp}	0.4153	0.4153	0.4153	0.00
		V_{gp}	89.892	89.892	90.029	0.15
		D_{gp}	48.025	48.025	47.511	-1.08
23	Landers, USA	A_{gp}	-0.698	0.6522	0.661	-5.60
		V_{gp}	22.926	-60.827	62.997	3.44
		D_{gp}	2.723	-56.443	49.795	-7.88
24	Northridge JFA, USA	A_{gp}	0.3747	0.3747	0.5	25.06
		V_{gp}	-67.258	67.258	67.411	0.23
		D_{gp}	-25.489	25.489	28.481	10.51
25	Northridge RRS, USA	A_{gp}	0.867	0.738	0.863	-0.46
		V_{gp}	127	-102	125.22	-1.42
		D_{gp}	36.1	-34.8	35.59	-1.43
26	Northridge NWS, USA	A_{gp}	0.3549	0.3549	0.3549	0.00
		V_{gp}	109.861	109.861	109.93	0.06
		D_{gp}	78.155	78.155	75.135	-4.02
27	Northridge SCG, USA	A_{gp}	0.6603	0.6603	0.646	-2.21
		V_{gp}	-95.686	95.686	95.715	0.03
		D_{gp}	-41.386	41.386	41.413	0.07
28	Northridge SCH, USA	A_{gp}	0.9283	0.9283	0.9283	0.00
		V_{gp}	-87.054	87.054	87.07	0.02
		D_{gp}	-26.463	26.463	29.926	11.57
29	Aegion Long, Greece	A_{gp}	0.4199	0.4199	0.426	1.43
		V_{gp}	-52.075	52.075	52.142	0.13
		D_{gp}	-9.862	9.862	9.205	-7.14
30	Aegion Tran, Greece	A_{gp}	0.56	0.543	0.56	0.00
		V_{gp}	-37.4	36	37.64	0.64
		D_{gp}	-8.5	4.76	8.24	-3.16

Table E.2 Comparison of peak plastic input motion parameters considering only major pulses (Method A) and by rigorous integration of the whole earthquake record (Method B); $\alpha_y=0.10g$ (*continued*)

31	Izmit ARC, Turkey	A_{gp}	0.0548	0.0548	0.0526	-4.18
		V_{gp}	0.876	0.876	0.896	2.23
		D_{gp}	0.033	0.033	0.0223	-47.98
32	Izmit SKR, Turkey	A_{gp}	-0.327	0.271	0.428	23.60
		V_{gp}	12.096	-17.396	18.07	3.73
		D_{gp}	1.089	-3.429	2.279	-2.68
33	Izmit GBZ SN, Turkey	A_{gp}	-0.204	-0.1213	0.204	0.00
		V_{gp}	13.746	6.352	13.746	0.38
		D_{gp}	1.786	0.889	2.675	-3.28
34	Izmit GBZ SP, Turkey	A_{gp}	0.128		0.128	-24.27
		V_{gp}	-2.038		2.038	1.31
		D_{gp}	-0.144		0.144	4.64
35	Izmit YPT, Turkey	A_{gp}	-0.196	-0.174	0.196	0.00
		V_{gp}	-11.929	-12.961	12.961	10.74
		D_{gp}	-3.674	-6.735	10.409	11.21
36	Chi-Chi TCU052, Taiwan	A_{gp}	-0.3888		0.3888	0.00
		V_{gp}	78.268		78.268	0.02
		D_{gp}	38.577		38.577	0.04
37	Chi-Chi TCU068, Taiwan	A_{gp}	-0.159	-0.362	0.362	0.00
		V_{gp}	66.5	108.9	108.9	0.06
		D_{gp}	29.5	257.2	264.54	-8.49
38	Chi-Chi TCU075, Taiwan	A_{gp}	-0.2334		0.2334	21.68
		V_{gp}	-51.37		51.37	2.36
		D_{gp}	-28.254		28.254	5.35
39	Chi-Chi TCU076, Taiwan	A_{gp}	-0.2167	-0.2964	0.2964	12.82
		V_{gp}	11.201	14.074	14.074	0.33
		D_{gp}	2.229	1.818	4.047	-5.69
40	Chi-Chi TCU129, Taiwan	A_{gp}	0.9817	-0.634	0.9817	0.00
		V_{gp}	-56.757	-44.053	56.757	0.16
		D_{gp}	-9.386	-12.185	21.571	-13.41

Table E.3 Comparison of peak plastic input motion parameters considering only major pulses (Method A) and by rigorous integration of the whole earthquake record (Method B); $\alpha_y=0.20g$

No.	Ground Motion	(g) (cm/s) (cm)	No. of effective pulses	Method A				Method B		Deviation (B)-(A)/(B)%
				Pulse	Pulse	Pulse	Pulse	Peak value (A)	Peak value (B)	
				1	2	3	4			
1	Parkfield, CA, USA	A_{gp}		-0.502	0.35	-0.419	0.184	0.502	0.467	-7.49
		V_{gp}	4	29.886	-11.875	27.488	-12.366	29.886	29.91	0.08
		D_{gp}		4.976	-0.956	7.912	-2.88	11.932	11.26	-5.97
2	San Fernando, CA, USA	A_{gp}		0.51				0.51	1.316	61.25
		V_{gp}	1	55				55	54.212	-1.45
		D_{gp}		20.4				20.4	11.76	-73.47
3	Gazzli, USSR	A_{gp}		-0.713	-0.293	-0.502		0.713	0.731	2.46
		V_{gp}	3	25.158	-5.172	13.403		25.158	25.3	0.56
		D_{gp}		3.093	-0.157	1.109		4.045	3.67	-10.22
4	Bucharest, Romania		-							
5	Tabas, Iran	A_{gp}		0.957	0.999	-0.532		0.999	1.052	5.04
		V_{gp}	3	-53.671	-45.155	-44.498		53.671	52.36	-2.50
		D_{gp}		-9.408	-5.508	-10.391		25.307	23.2	-9.08
6	Coyote Lake, USA	A_{gp}		0.285				0.285	0.285	0.00
		V_{gp}	1	-20.101				20.101	20.16	0.29
		D_{gp}		-2.272				2.272	2.05	-10.83
7	Imperial Valley E04, USA	A_{gp}		-0.16				0.16	0.16	0.00
		V_{gp}	1	-10.45				10.45	10.54	0.85
		D_{gp}		-1.858				1.858	1.94	4.23
8	Imperial Valley E05, USA	A_{gp}		0.264	0.141	0.14		0.264	0.392	32.65
		V_{gp}	3	10.5	-8.78	9.06		10.5	11.1	5.41
		D_{gp}		1.1	-2.52	2.77		1.42	1.75	18.86
9	Imperial Valley E06, USA	A_{gp}		-0.239				0.239	0.347	31.12
		V_{gp}	1	-21.643				21.643	21.7	0.26
		D_{gp}		-4.213				4.213	4.14	-1.76
10	Imperial Valley E07, USA	A_{gp}		0.236				0.263	0.264	0.38
		V_{gp}	1	47.387				47.387	47.42	0.07
		D_{gp}		13.442				13.442	13.55	0.80

Table E.3 Comparison of peak plastic input motion parameters considering only major pulses (Method A) and by rigorous integration of the whole earthquake record (Method B); $\alpha_y=0.20g$ (*continued*)

11	Imperial Valley EMO, USA	A_{gp}	-0.115	0.115	0.115	0.00
		V_{gp}	11.625	11.625	11.67	0.39
		D_{gp}	2.048	2.048	2.06	0.58
12	Morgan Hill, USA	A_{gp}	-0.156	0.156	0.216	27.78
		V_{gp}	6.494	6.494	6.53	0.55
		D_{gp}	0.695	0.695	0.653	-6.43
13	N. Palm Springs NPS, USA	A_{gp}	-0.5811	0.5811	0.581	-0.02
		V_{gp}	22.173	22.173	22.26	0.39
		D_{gp}	6.293	6.293	6.33	0.58
14	N. Palm Springs DSP, USA	A_{gp}	-0.2482	0.2482	0.1913	-29.74
		V_{gp}	2.286	2.286	2.3	0.61
		D_{gp}	0.086	0.086	0.103	16.50
15	Whittier Narrows DOW, USA	A_{gp}	-0.0421	0.0421	0.0353	-19.26
		V_{gp}	0.504	0.504	0.71	29.01
		D_{gp}	0.034	0.034	0.037	8.11
16	Whittier Narrows NWK, USA	A_{gp}	0.1368	0.1638	0.1638	0.00
		V_{gp}	-3.09	3.09	3.107	0.55
		D_{gp}	-0.244	0.244	0.2467	1.09
17	Supersition Hill PTS, USA	A_{gp}	-0.3295	0.3295	0.3295	0.00
		V_{gp}	71.419	71.419	71.57	0.21
		D_{gp}	36.658	36.658	38.29	4.26
18	Supersition Hill ELS, USA	A_{gp}	0.2231	0.2231	0.3633	38.59
		V_{gp}	-9.682	9.682	9.699	0.18
		D_{gp}	1.07	1.07	1.075	0.47
19	Loma Prieta LGP, USA	A_{gp}	0.678	0.678	0.766	11.49
		V_{gp}	-70.425	70.425	70.833	0.58
		D_{gp}	-21.016	21.016	18.258	-15.11
20	Loma Prieta STG, USA	A_{gp}	0.312	-0.313	0.312	-0.32
		V_{gp}	-11.329	-16.342	16.379	0.23
		D_{gp}	-1.169	-1.506	2.496	-7.17

Table E.3 Comparison of peak plastic input motion parameters considering only major pulses (Method A) and by rigorous integration of the whole earthquake record (Method B); $\alpha_y=0.20g$ (*continued*)

21	Sicra Madrc, USA	A_{gp}	-0.2026	0.2026	0.322	37.08
		V_{gp}	1.477	1.477	2.385	38.07
		D_{gp}	0.041	0.041	0.0593	30.86
22	Erzikan, Turkey	A_{gp}	-0.4171	0.4171	0.4	-4.28
		V_{gp}	50.89	50.89	51.01	0.24
		D_{gp}	19.949	19.949	20.026	0.38
23	Landers, USA	A_{gp}	-0.5268	0.5268	0.7774	32.24
		V_{gp}	-29.214	29.214	29.289	0.26
		D_{gp}	-6.329	6.329	6.874	7.93
24	Northridge JFA, USA	A_{gp}	0.3706	0.3706	0.475	21.98
		V_{gp}	33.765	33.765	33.877	0.33
		D_{gp}	6.012	6.012	6.012	0.00
25	Northridge RRS, USA	A_{gp}	0.967	0.967	0.963	-0.42
		V_{gp}	83.2	120	120.59	0.49
		D_{gp}	18.4	23.1	26.46	12.70
26	Northridge NWS, USA	A_{gp}	-0.3058	0.3058	0.2956	-3.45
		V_{gp}	58.007	58.007	58.05	0.07
		D_{gp}	27.515	27.515	27.535	0.07
27	Northridge SCG, USA	A_{gp}	0.3806	0.3806	0.526	27.64
		V_{gp}	-49.241	49.241	49.282	0.08
		D_{gp}	-14.516	14.516	14.526	0.07
28	Northridge SCH, USA	A_{gp}	1.0167	1.0167	1.005	-1.16
		V_{gp}	-42.387	42.387	50.457	15.99
		D_{gp}	-10.825	10.825	10.852	0.25
29	Aegion Long, Greece	A_{gp}	0.4953	0.4953	0.4953	0.00
		V_{gp}	-33.022	33.022	33.098	0.23
		D_{gp}	-4.176	4.176	4.738	11.86
30	Aegion Tran, Greece	A_{gp}	0.49	0.64	0.64	0.00
		V_{gp}	-15.4	36.8	37.26	1.23
		D_{gp}	-3.06	3.06	3.09	0.97

Table E.3 Comparison of peak plastic input motion parameters considering only major pulses (Method A) and by rigorous integration of the whole earthquake record (Method B); $\alpha_y=0.20g$ (*continued*)

[illegible]

Table E.4 Comparison of peak plastic input motion parameters considering only major pulses (Method A) and by rigorous integration of the whole earthquake record (Method B); $\alpha_y=0.30g$

No.	Ground Motion	(g) (cm/s) (cm)	No. of effective pulses	Method A				Method B		Deviation
				Pulse	Pulse	Pulse	Pulse	Peak value (A)	Peak value (B)	
				1	2	3	4			
1	Parkfield, CA, USA	A_{gp}		-0.387	0.324	0.125		0.387	0.349	-10.89
		V_{gp}	4	14.195	-4.395	10.448		14.195	14.21	0.11
		D_{gp}		1.635	-0.234	1.474		2.875	2.89	0.52
2	San Fernando, CA, USA	A_{gp}		0.41	0.926	0.519		0.926	0.926	0.00
		V_{gp}	3	26	-38.8	9.63		38.8	43.09	9.96
		D_{gp}		4.85	-5.53	0.77		4.85	7.64	36.52
3	Gazzli, USSR	A_{gp}		-0.675				0.675	0.61	-10.66
		V_{gp}	1	12.836				12.836	12.9	0.50
		D_{gp}		1.363				1.363	1.367	0.29
4	Bucharest, Romania		-							
5	Tabas, Iran	A_{gp}		0.826	1.152	0.125		1.152	1.137	-1.32
		V_{gp}	3	36.013	29.087	10.448		36.013	45.08	20.11
		D_{gp}		5.631	3.11	4.49		12.961	14.46	10.37
6	Coyote Lake, USA	A_{gp}		0.385				0.385	0.377	-2.12
		V_{gp}	1	-7.578				7.758	7.64	-1.54
		D_{gp}		-0.501				0.501	0.44	-13.86
7	Imperial Valley E04, USA	A_{gp}		0.085				0.085	0.083	-2.41
		V_{gp}	1	-1.697				1.697	1.728	1.79
		D_{gp}		-0.078				0.078	0.082	4.88
8	Imperial Valley E05, USA	A_{gp}		0.146	0.043	0.108		0.146	0.201	27.36
		V_{gp}	3	2.6	-0.547	1.23		2.6	2.66	2.26
		D_{gp}		0.12	-0.225	0.259		0.154	0.105	-46.67
9	Imperial Valley E06, USA	A_{gp}		0.18				0.18	0.174	-3.45
		V_{gp}	1	-7.782				7.782	7.84	0.74
		D_{gp}		-0.63				0.63	0.64	1.56
10	Imperial Valley E07, USA	A_{gp}		-0.31				0.31	0.31	0.00
		V_{gp}	1	23.591				23.591	23.63	0.17
		D_{gp}		4.386				4.386	4.4	0.32

Table E.4 Comparison of peak plastic input motion parameters considering only major pulses (Method A) and by rigorous integration of the whole earthquake record (Method B); $\alpha_y=0.30g$ (*continued*)

[illegible]

[illegible]

Table E.4 Comparison of peak plastic input motion parameters considering only major pulses (Method A) and by rigorous integration of the whole earthquake record (Method B); $\alpha_y=0.30g$ (*continued*)

[illegible]

Table E.5 Comparison of peak plastic input motion parameters considering only major pulses (Method A) and by rigorous integration of the whole earthquake record (Method B); $\alpha_y=0.40g$

No.	Ground Motion	(g) (cm/s) (cm)	No. of effective pulses	Method A				Method B		Deviation
				Pulse	Pulse	Pulse	Pulse	Peak value (A)	Peak value (B)	
				1	2	3	4			
1	Parkfield, CA, USA	A_{gp}	4	-0.228	-0.068			0.228	0.181	-25.97
		V_{gp}		3.679	0.824			3.679	3.78	2.67
		D_{gp}		0.231	0.037			0.268	0.29	7.59
2	San Fernando, CA, USA	A_{gp}	2	0.31	0.826			0.826	0.826	0.00
		V_{gp}		13.8	-30			30	5.93	-405.90
		D_{gp}		1.31	-3.33			2.02	0.284	-611.27
3	Gazzli, USSR	A_{gp}	1	-0.293				0.293	0.474	38.19
		V_{gp}		5.901				5.901	5.93	0.49
		D_{gp}		0.277				0.277	0.284	2.46
4	Bucharest, Romania		-							
5	Tabas, Iran	A_{gp}	3	0.577	1.199	0.492		1.199	1.199	0.00
		V_{gp}		-19.914	-17.678	-13.41		19.914	30.57	34.86
		D_{gp}		-2.34	-1.888	-1.602		5.83	7.62	23.49
6	Coyote Lake, USA	A_{gp}	1	0.122				0.122	0.117	-4.27
		V_{gp}		-0.814				0.814	0.84	3.10
		D_{gp}		-0.024				0.024	0.03	20.00
7	Imperial Valley E04, USA		-							
8	Imperial Valley E05, USA		-							
9	Imperial Valley E06, USA	A_{gp}	1	0.123				0.122	0.085	-43.53
		V_{gp}		-0.689				0.689	0.72	4.31
		D_{gp}		-0.016				0.016	0.018	11.11
10	Imperial Valley E07, USA	A_{gp}	1	-0.191				0.191	0.172	-11.05
		V_{gp}		5.131				5.131	5.17	0.75
		D_{gp}		0.563				0.563	0.57	1.23

– E18 –

Table E.5 Comparison of peak plastic input motion parameters considering only major pulses (Method A) and by rigorous integration of the whole earthquake record (Method B); $\alpha_y=0.40g$ (*continued*)

[illegible]

Table E.6 Peak plastic input motion parameters for different levels of yielding acceleration after rigorous integration (Method B).

ground motion	(g) (cm/s) (cm)	original waveform	peak plastic motions (Eq.6.19)			
			$a_y = 0.1g$	$a_y = 0.2g$	$a_y = 0.3g$	$a_y = 0.4g$
full – cycle sinusoid	A_g	0.5	0.6	0.614	0.484	0.276
	V_g	156	110	70.6	38	13.3
	D_g	78	44.6	23.3	9.8	2.3
Pacoima Dam	A_g	1.23	1.13	1.03	0.926	0.826
	V_g	113	95.6	54	40.4	28.5
	D_g	35.4	45	12.5	7.3	5
Imperial Valley	A_g	0.379	0.435	0.264	0.146	-
	V_g	90.5	56.7	11.1	2.6	-
	D_g	63.1	49.3	1.7	0.11	-
Erzican	A_g	0.515	0.415	0.42	0.303	0.12
	V_g	84	90	51	20.3	8.1
	D_g	27.7	48	20	6.1	0.82
Rinaldi	A_g	0.838	0.867	0.967	0.989	0.961
	V_g	166	127	121	116	93
	D_g	28.2	36.3	25	30.8	24.3
Aegion	A_g	0.538	0.56	0.64	0.71	0.372
	V_g	51	37.6	37.2	21.5	8.34
	D_g	9.6	8.5	3.08	1.98	0.62
Chi-Chi	A_g	0.462	0.362	0.281	0.162	0.25
	V_g	263	109	28.3	4.53	0.97
	D_g	430	265	15.8	0.36	0.022

