NATIONAL TECHNICAL UNIVERSITY OF ATHENS
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EXPLORING THE POSSIBILITY OF USING FLAPPING FOILS AS A MARINE PROPULSOR


## DIPLOMA THESIS

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## Introduction

Throughout his whole history, man has sought in nature for inspiration, in order to find effective ways of transportation. From ancient mythology to science fiction, there are ornithopters and fish-like submarines. Many inventors have tried to imitate nature, but most have been discouraged by the complexity of the problem and the resulting difficulty in modeling it.

Over millions of years, fish have evolved swimming capabilities far superior in many ways to what has been achieved by nautical science and technology. Instinctively, they use their superbly streamlined bodies to exploit fluid-mechanical principles in ways naval architects today can only dream about, achieving extraordinary propulsion efficiencies, acceleration and maneuverability.

Dolphins, for example, dart through water with impressive grace and apparent ease,
playfully bursting through the waves as they follow ships cruising at 20 knots. Marine biologists have reported that yellowfin tuna caught on a fishing line can swim at speeds of at least 40 knots. The aggressive pike overcomes its prey with short bursts of acceleration that can exceed 20G.

While aeronautical technology has advanced rapidly over the past 100 years, nature's
flying machines, which have evolved over 150 million years, are still impressive. A simple comparison can astonish anyone. Humans move at top speeds of 3-4 body lengths per second, a race horse runs approximately 7 body lengths per second, and the fastest terrestrial animal, a cheetah, accomplishes 18 body lengths per second. A supersonic aircraft such as the SR 71 Blackbird travelling near Mach 3 ( $\sim 2000 \mathrm{mph}$ ) covers about 32 body lengths per second. Yet a common pigeon frequently attains speeds of 50 mph ; this converts to 75 body lengths per second. A European Starling (Sturnus vulgaris) is capable of flying at 120 and various species of Swifts over 140 body lengths per second. The roll rate of highly aerobatic aircraft (e.g., A-4 Skyhawk) is said to be approximately 720 degrees per second, while a Barn Swallow (Hirundo rustics) has a roll rate in excess of 5000 degrees per second. The maximum positive G-forces permitted in most general aviation aircraft is $4-5 G s$ and best military aircraft withstand 8 -10Gs. However, many birds have been calculated to routinely experience (i.e., hundreds of times each day) positive Gforces in excess of 10Gs and up to 14Gs.

## SCOPE - TARGET OF THIS THESIS

Biomimetics is a fast advancing region of science and engineering. As it is obvious to the most engineers, three million years of natural evolution have produced much more remarkable results than the last century of manmade engineering. Consequently, it is becoming clear that reverse engineering the nature can help us produce more efficient machines.

Following this trend, in order to make more efficient propulsion systems it is necessary to observe and imitate the most advanced sea creatures (specifically big fishes like tuna or sharks and sea mammals)

From the mechanics point of view, there have been many patents over the last decade and many kinds of mechanism that can imitate the movement of the fish.

All the above bring up the necessity of acquiring more knowledge on the hydrodynamics behind these remarkable "swimming machines" in order to optimize and control such systems, with final purpose the placement of ships under "flapping foil propulsion".

The scope of this thesis is to produce a CFD program capable of simulating most of flapping foil systems, verify the capability to produce reliable results, run systematic simulations and visualize them, in order to look for emergence of patterns that could lead to improvements. Additionally, there is also an attempt to start reverse engineering the fish and dolphin swimming.

For this purpose a CFD program was made, using a boundary element time stepping method with free wake, which can take any given foil (or foils) and place it under any harmonic flapping motion given by the user (within the restrictions described later on). The program can calculate pressures, consequently forces and moments, nondimentionalize them and produce the wake geometry in visualizable form.

For the production of the foil geometries another program was made which can produce the grid of any foil with naca section and the spanwise characteristics given by the user in parameters.

For the visualization of the results (and wake in specific) the commercial program Tecplot was used.

## HISTORICAL REFFERENCE ${ }^{1}$ :

According to Alexander in 'The history of fish mechanics', Aristotle considers in some of his works the anatomy and locomotion of swimming creatures. After this, however, it took a long time before fish mechanics really made progress.

From about 1700-1800 research on fish was not making much progress. Its revival was due to the further development of apparatuses for measuring pressures and to the invention of photography and film by which the movements of the fish could be recorded. Some names are A. Moreau regarding measurements at the bladder and E.J. Marey for the recording of fish movements.

After 1910 until 1950 research was done, not only by zoologists, but also by engineers, which yielded a fruitful collaboration. For instance, wooden models of fish were towed through water and their resistance measured. Also, this resistance was investigated by letting weighted dead fish sink, head first, in a tank and by using optical arrangements for measuring their velocity. In this period Breder in 'The locomotion of fishes' published a review of fish swimming in which he gave names to a number of swimming forms of which we mention the following three:

- the anguiliform which is named after the swimming of the eel;
- the carangiform, where the front part of the body has little flexibility and the flexural movements are confined to the rear half or the rear one-third of the body length;
- the balistiform, where the propulsion is caused by the synchronized movements of dorsal and anal fins, while the body and the caudal fin are held rigid, by which the latter is of no direct use for the propulsion of the fish.

For other forms of swimming reference can be made to Blake in his book 'Fish locomotion'.

In the years around 1935 J . Gray studied the waves travelling posteriorly along the body of some fish such as the eel or the whiting. For this he used a machine for artificially imparting a wave motion to a flexible model or a dead fish.

One of the early mathematically oriented investigators of the swimming of aquatic animals was G.I. Taylor, who published the article 'Analysis of the swimming of long and narrow animals' in which he developed a so-called resistive theory. A bending wave travels with constant speed along the body of the animal. The forces per unit of length of each element

[^0]of the swimming body are assumed to be the same as the resistance experienced per unit of length by a long cylinder with the same surface structure as that of the body and moving through the fluid with the same but now steady velocity and having the same inclination to the direction of the relative flow. This theory is suitable for the swimming of snakes, leeches and certain marine worms. Later theories are often of a different type, for instance the reactive theory. In contrast with the mentioned resistive theory, this type of theory considers the flow of the fluid outside the thin boundary layer on the fish's body. By this the inertial effects of the fluid are dominant and an inviscid fluid model can be used.

After Lighthill's seminal paper 'Note on the swimming of slender fish' and the fundamental paper of Wu 'Swimming of a waving plate', many other mathematically oriented articles using reactive theories followed. This led to hydrodynamics getting a strong foothold in research on the swimming of fish.

The history of modern research on flapping foil propulsion starts in 1936 with Gray's paradox. The famous paradox was formulated by J. Gray in his article 'Studies in animal locomotion VI. The propulsion powers of the dolphin'. Gray estimated among other things the power needed for a dolphin of length 1.82 m to swim at a speed of $10.1 \mathrm{~m} / \mathrm{s}$. He did this by calculating the dolphin's resistance by means of a drag coefficient based on a turbulent boundary layer. He found that the required power was possibly about seven times the estimated muscular power available for propulsion. This yields the paradox which is considered by a large number of investigators. The paradox is, however, rather difficult to tackle because of the lack of a common opinion among investigators on the influence of the swimming motion on the resistance of the body. Some opinions are that the resistance of the swimming body can be increased by a factor of three with respect to the resistance of the body when it glides motionless through the water; (see for instance M.J. Lighthill's 'Large-amplitude elongated-body theory of fish locomotion')

The term 'flapping' is usually applied in the context of the wing motion of birds and insects and consists of an oscillatory rolling motion of the wing about the shoulder joint with simultaneous change in the geometric pitch angle of the wing via a rotation of the wing about its spanwise axis. Pectoral fins of fish also exhibit essentially similar kinematics although large-scale passive as well as active deformation of these fins can significantly increase the complexity of the fin kinematics. In many studies, this so-called 'pitching and rolling' motion has been simplified to a 'pitching and heaving' motion wherein the rolling motion of the wing is replaced by a heaving motion. In addition to serving as a model for flapping wing/fin kinematics, pitch-and-heave also represents the essential kinematics of caudal-fin motion in carangiform propulsion (Lighthill 1975).

Past studies have successfully employed pitching-and-heaving foils as models of flapping wings and gained useful insight into the fluid dynamics of flapping flight as well as carangiform propulsion.

Katz \& Weihs (1978) analyzed the unsteady large amplitude linearized motion of a chordwise flexible slender $(A R<1)$ wing in inviscid incompressible fluid. The local deflections of the chord are calculated from the hydrodynamic forces acting on it, which are dependent on the foil shape. The problem was solved in a non-inertial system attached to the foil. In the statement of the unsteady Bernoulli's equation second order disturbances were neglected relative to quantities containing larger values. Another assumption in the analysis of the deflections was that the foil is linearly elastic, so that no "memory" effects have to be taken into account. The foil span is not allowed to bend under the action of the forces.

The slenderness assumptions lead to the neglecting of the term in the Laplace 's equation so as to return the flow in the so-called cross-flow plane.

Katz \& Weihs in this publication concluded that the thrust and efficiency increase when the heaving amplitude H/C grows. Another conclusion was that if the path curvature is increased (e.g. if the frequency grows while H/C remains constant) the efficiency will decrease. The thrust coefficient is highest when the phase difference is close to $90^{\circ}$.

Katz \& Weihs in a later publication discussed the wake roll-up and the Kutta condition for airfoils oscillating at high frequency. They showed that the Kutta condition can be applied for force and moment prediction in unsteady small amplitude non-separated flows even when the reduced frequency is well above 1. Wake roll-up calculations, based on the Kutta condition showed good agreement with available flow visualization data. It was concluded therefore that when trailing edge displacement is small ( $\mathrm{A} / \mathrm{C}<0.1$ ) the range of linearized theory calculations using the Kutta condition can be extended far beyond reduced frequencies of 1 (one). They also showed that in high frequency motions the contribution of the potential time derivative $\partial \Phi / \partial t$ to the lift becomes more important, i.e. force due to the acceleration of the surrounding fluid is considerably increased relative to the far wake influence.

Katz \& Weihs (1978) analyzed the problem of a thin foil with flexible chord of constant length $C$ which varies its shape passively owing to the hydrodynamic forces acting on it. The propulsor was taken to move in water at high Reynolds number so that the analysis could be based on incompressible potential theory.

The trajectory S was such that the flow disturbance caused by the foil stayed small and no point of the foil traverses the wake. The displacement of the foil was small $(h(x, t) \ll 1)$ so that the downwash velocity $\mathrm{w}(\mathrm{x}, \mathrm{t}) / \mathrm{V}(\mathrm{t}) \ll 1$ where $\mathrm{V}(\mathrm{t})$ is the velocity of the point where the body's frame of reference was attached. They assumed in the analysis that the foil was clamped at its leading edge and that its elastic behavior can be estimated by the cantilever model.

The wake model consisted of discrete vortices and after each time step its distortion as a result of the velocity field induced by the foil and its wake was estimated. In cases where the foil did not come close to its wake the influence was usually found to be negligible.

They concluded that for a rigid propulsor the thrust grows as $\mathrm{H} / \mathrm{C}$ increases. The wake deformation may be neglected in modestly oscillating motions (reduced frequencies smaller than 0.3 ). It was also obvious that a phase difference of $\pi / 2$ between the heaving and pitching motions gave both high thrust and high efficiency.

Zervos (1983) presented the model of a two dimensional propulsor having infinitely thin and flexible walls. At each instant the exact form of the walls and the vortex wake were taken into account. Large oscillating amplitudes were applied in order to achieve practical thrust levels. The results showed that the pressures were distributed in such a manner that at each instant, a propulsive force is created.

In order to have propulsion it was shown that Vo/C must be less than 1 where C is the velocity of wave propagation and Vo is the advance velocity of the foil. The influence of $\mathrm{I} / \lambda$ on the efficiency ( $\eta$ ) and propulsive coefficient (CT) is very important. In the aforementioned ratio $I$ is the length of the propulsor and $\lambda$ the wavelength of propagation so that $\mathrm{I} / \lambda$ is the number of waves covering the body. Its augmentation ameliorates the efficiency under the condition that does not get bigger than 2.0 but diminishes, at the same time the corresponding values of CT .

A/H produces a slight decrease of the efficiency, but it is accompanied with an important increase of the CT. In the above ratio $A$ is the total amplitude of the motion and $H$ is the distance covered in one period T0.

Poling \& Telionis (1986) offered some experimental evidence on the physical characteristics of unsteady flow in the neighborhood of a sharp trailing edge. They provided measurements of two periodic problems. The first was the classical pitching airfoil and the second was the flow over a fixed airfoil immersed in a periodic wake that represents essentially a periodic change on the angle of attack. The experimental data obtained indicated that for periodic flows with reduced frequencies larger than $\mathrm{k}=2$ and not very small amplitudes, the classical Kutta condition is never satisfied. As classical Kutta condition it is meant that the trailing stagnation streamline is tangent to the bisector of the wedge at the trailing edge. In the viscous region there was ample evidence of finite normal pressure gradients and therefore nonzero trailing edge loadings. It is also stated that for unsteady flow the loading near the trailing edge varies very sharply with the distance from the trailing edge. Even a few percent of the chord length may have a significant effect on global characteristics like instantaneous or averaged lift and drag.

Tuncer, Wang \& James Wu (1990) developed an integro-differential formulation of the Navier-Stokes equations. The formulation of the viscous flow analysis confined computations only to the viscous flow zone and lead to an efficient zonal solution procedure. In the simplified vortical flow analysis, computational demands were greatly reduced by the partial analytic evaluations. Vorticity transport equation was solved only in
the viscous flow zone. In addition, attached boundary layer and detached recirculating flow regions in the viscous flow zone were treated individually. On the other hand the integral equations for velocity permitted the velocity vector in the viscous flow zone to be evaluated explicitly. The results of the study showed that during the upstroke the computed lift coefficient increases linearly until the leading-edge vortex forms. The formation of the leading edge vortex then causes a steep increase in the lift. At maximum angle of attack the lift coefficient reached a local maximum as a result of the burst of the bubble at the trailing edge and the shedding of clockwise vorticity. Due to the suction generated by the trailing edge vortex, it subsequently rises to a second local maximum.

During the downstroke, following the shedding of the trailing-edge vortex the lift initially decreases rapidly. As the flow reattaches at the trailing edge and as the secondary vortex structures develop, the lift curve flattens. The minimum lift is observed just before the flow attaches fully on the upper surface. The development of the leading edge suction then drives the lift towards the steady state values. However, the low pressure aft of the midchord on the upper surface delays the recovery process.

For different reduced frequencies the evnts mentioned above occur at different angles of attack during the oscillatory motion, and as a result, the aerodynamic loading differs significantly. It was observed that as the reduced frequency increases, the flow reversal originates at a smaller angle of attack.

The conclusion was that the dynamics of the leading edge vortex has a dominant effect on the dynamic stall behavior. As the reduced frequency of the oscillatory motion increases, the formation of the leading edge vortex delays until higher angles of attack are reached.

Wang, Wu and Qian (1991) generalized the above two dimensional zonal procedure to treat three-dimensional general viscous flow problems. The three non-zero vorticity components in a three dimensional problem satisfy the vorticity divergence-free condition through a numerical filter mechanism. Flow around fast pitched flat plate wings were computed by the generalized zonal procedure. This is summarized as follows: In an external flow problem when the Reynolds number is not small, large potential region where the vorticity and hence all viscous effects are absent, coexists with flow zones where viscous effects are important. As a result the potential region can be solved uncoupled from the viscous one. The information about the flow in a removed potential flow region is not lost but is stored in the boundary velocity values that have already been counted for this region. With prescribed velocity boundary condition the zonal approach follows the development of the vorticity field. The solution advances from an initial time level at which the velocity and vorticity fields are known to a subsequent new time level by using a computational loop.

Numerical errors may accumulate and grow so that, over a period of time the divergence of the vorticity field becomes significantly different from zero, violating the physics of incompressible fluid. Numerical studies showed that the divergence or the numerical error of the vorticity field is greater if finer grids are used near the wing edges, especially near
the wing tip. A liftering mechanism based on the concept of vortex loop in space was devised to control the growth of the divergence of the vorticity field. At any instant of time, if a vorticity field can be approximated by vortex filaments in space where these filaments are the local summations of strengths of existing vortex loops in space, the vorticity field is regarded divergence-free in the numerical sense.

The conclusion of this study was that the generalized zonal approach was successfully used to study unsteady flows around stationary and rapidly pitched flat plate wings. The roles of the leading edge recirculating flow and tip vortices are identified to the contributions of the normal force experienced by the pitching wing.

Neil Bose (1993) presented a two dimensional time domain constant potential panel method used for the analysis of chordwise flexible oscillating hydrofoils as oscillating propulsors. The oscillating motions as well as the chordwise deflections were of large amplitude. The foil surface was discretized into panels following a cosine spacing over the chord and assuming a constant value of the doublet potential and source strength over each panel.

All memory effects were included in the foil wake which contains the shed vorticity from the foil but at a given time step this is fixed. The calculation proceeded in a series of time steps and the wake was made up of segments or panels. The wake panels were left in the fluid flow where they were formed. No attempt was made to allow the wake to move with the local induced flow. The first wake panel was assumed to leave the trailing edge along the bisector of the trailing edge angle. A linear variation of potential was applied on the wake panel immediately behind the trailing edge because this makes the calculation relatively insensitive to the time step size.

A method based on a linearized pressure coefficient was used for the Kutta condition formulation. A dummy doublet potential value $\Phi N+1$ was introduced at the upper surface at the trailing edge. The linearized pressure coefficient was included as an $\mathrm{N}+1$ th equation for the potential values. A first order differentiation was used for the term $\partial \Phi / \partial \mathrm{t}$ in the linearized version of Bernoulli for calculating the pressure coefficient.

The analysis lead to the conclusion that the propulsive efficiency increased as thrust coefficient reduced. Efficiency varied strongly with changes in heave amplitude ratio and pitch amplitude. It was also shown that flexibility increases propulsive efficiency but reduces thrust. In addition for a given thrust a flexible foil has a higher propulsive efficiency than a rigid foil.
M.S. Triantafyllou \& K. Streitlien (1995) presented closed form expressions for the force and moment on a Joukowski profile in arbitrary motion, surrounded by point vortices that are free to convect with the local flow. The foil shape was represented as the conformal mapping of a circle making use of the theory of complex functions.

The wake of the profile was discretized into point vortices and the circle theorem insured that the body boundary condition is satisfied everywhere on the foil. It was shown that the force and moment acting on the Joukowski profile consist of added mass terms as if the flow were free of vortices plus the summed effect of all vortices in the flow.

One of the example calculations presented concerned the large amplitude symmetric foil oscillation. The illustration showed that periodic time dependence was established in very short time, indicating that the added mass forces are dominant in this case.

Another interesting example illustrated was the case of a vortex released at a point upstream of a stationary cambered foil, convecting with the free stream. The force record obtained showed that the maximum lift on the foil occurs as the vortex passes over the trailing edge.

Sarpkaya (1975) presented a potential flow model of 2D vortex shedding behind an inclined plate. The calculated normal force coefficients were about $20 \%$ larger than those obtained experimentally.

Basu \& Hancock (1978) presented a numerical method developed for the calculation of the pressure distribution, and loads on a 2D airfoil undergoing an arbitrary unsteady motion in an inviscid incompressible flow.

Results of the algorithm were presented for a sudden change in incidence, a high frequency oscillation and entry into a sharp-edged gust.
M. Vezza \& R.A McD. Galbraith (1985), presented a model for the calculation of the incompressible, inviscid flow around an arbitary airfoil undergoing unsteady motion. The same authors extended their model to include fixed upper surface separation. The analysis was based on the assumption that the flow is irrotational over the entire region except at the body and its wake elements. The separation point was a necessary input into the algorithm. The pressure distribution predicted was compared with experimental results in the case of step change in incidence from 0 to 20.05 deg and agreement was evident.
V. Riziotis \& S. Voutsinas (1996), reported a 2D vortex type stall model. The separated flow over an airfoil was considered in constant large incidence and in pitching motion. The wake was represented by a set of freely moving vortex particles.

A result from various attempts to attack the dynamic stall was that the influence of the details of the shape of the hysterisis loop on the force coefficients was not significant.

Another conclusion was that the separation point delay loop is a dominant feature for the valid estimation of the forces acting on a pitching airfoil.

Most of these past studies have assumed that the aspect ratio of the foils is large and have therefore restricted their attention to two-dimensional flapping foil configurations. In experimental studies, this has been accomplished through the use of high-aspect-ratio foils (Koochesfahani 1989; Triantafyllou, Triantafyllou \& Grosenbaugh 1992) whereas numerical studies accomplish this by explicitly performing two-dimensional simulations that ignore any spanwise variability in the foil geometry and the flow field (Jones, Dohring \& Platzer 1998; Isogai, Shinmoto \& Watanabe 1999; Tuncer \& Platzer 2000; Wang 2000; Mittal, Uttukar \& Udaykumar 2002a; Mittal et al. 2003; Lewin \& Haj-Hariri 2003; Pedro, Suleman \& Djilali 2003; Guglielmini \& Blondeaux 2004).

The assumption of two-dimensionality has some validity for bird and insect flight where wings of many species tend to be of a relatively large aspect ratio. For instance, even a small bird such as a tree sparrow has a wing with an aspect ratio (denoted by symbol AR and defined as (span)2/(area)) of about 5 (Azuma 1992). Aspect ratio for soaring birds such as albatrosses can reach values as high as 18. Examples of relatively high-aspectratio wings also abound in the insect world; for instance, the aspect ratio of a bumblebee wing is about 6.3 (Usherwood \& Ellington 2002) and that of craneflies is about 11 (Ellington 1984). In contrast, the aspect ratio of fish pectoral fins tend to be generally smaller. For instance, the aspect ratios of four species of
labrid fishes range from about 1.5 to 3.5 (Walker \& Westneat 2002), whereas bluegill sunfish and ratfish have pectoral fins with aspect ratios of about 2.4 (Drucker \& Lauder 1999) and 2.2 (Combes \& Daniel 2001), respectively. Evolutionary pressure towards these smaller-aspect-ratios in pectoral fins is probably due to a number of different factors. Smaller fish that live in highly energetic habitats such as coral reefs and near-shore regions make extensive use of pectoral fins for propulsion as well as maneuvering and station-keeping. As discussed extensively by Walker \& Westneat (2002), the pectoral fin kinematics adopted by these fish can range all the way from a back-and-forth paddle-like motion (for braking, turning and fast starts) to flapping motion (for cruising). It has generally been understood that propulsive forces in paddling are drag-based for which low aspect-ratio fins are most appropriate. In contrast, flapping motion is considered to be associated with lift-based propulsion and this is expected to work best with higher-aspectratio wings/fins (Combes \& Daniel 2001; Walker \& Westneat 2002). Thus, just from a hydrodynamic point of view, fish pectoral fins would tend to be of a lower aspect ratio than insect and bird wings. In addition, large-aspect-ratio fins would require a stiffer and therefore heavier fin support structure since they would be subject to larger bending moments.

In this context, it might also be argued that the abundance of high-aspect-ratio wings in flying animals and the contrasting paucity of high-aspect-ratio pectoral fins in fish is primarily connected with water being three-orders of magnitude denser than air. A flapping fish fin would therefore experience significantly higher added-mass associated bending moments and a lower aspect ratio would tend to reduce this bending moment for fish fins.

However, many insects such as wasps, flies and bumblebees routinely flap their wings at frequencies exceeding 150 Hz (Azuma 1992) whereas flapping frequencies of fish pectoral fins seldom exceed 5Hz (Drucker \& Lauder 1999; Walker \& Westneat 2002). Since the added-mass force is proportional to the square of the flapping frequency, it is easy to see that added-mass associated moments experienced by the wings of many flying insects could be comparable with those experienced by the pectoral fins of fish.

Finally, many studies have shown that the wake structure is a critical determinant of the hydrodynamic performance of flapping foils. However, relatively little is known about the wake topology of finite and low-ratio foils. A systematic and comprehensive examination of the hydrodynamics of small-aspect-ratio (AR $\leq 4$ ) flapping foils would allow us to gain some insight into all these issues and this forms one of the motivations for the current study.

A number of studies have examined the fluid dynamics and force production of finite aspect-ratio flapping foils/wings. Usherwood \& Ellington (2002) have experimentally studied the fluid dynamics of a hawk moth wing model of aspect ratio 6.34 and numerical simulations (2d NS) of this same wing have been carried out by Liu et al. (1998). Dickinson and co-workers (Dickinson, Lehmann \& Sane 1999; Sane \& Dickinson 2001) have performed systematic experimental studies with a dynamically scaled fruit fly flapping wing with aspect ratio of about 3.8 and Ramamurti \& Sandberg (2002).

Detailed experiments of pectoral fin hydrodynamics in controlled experiments with swimming fish have also been carried out (Walker \& Westneat 1997; Drucker \& Lauder 2002). The comprehensive particle image velocimetry (PIV) measurements carried out for a swimming bluegill sunfish (Drucker \& Lauder 2002; Lauder et al.
2005) are of particular interest for the current study. In these experiments, the fish swims almost steadily in an incoming stream using only its pectoral fins. That the fish is swimming at very nearly a constant speed is confirmed by the fact that the body of the fish maintains its position to within a few millimetres over many fin strokes (Lauder et al. 2005). Thus, in this situation, the thrust produced by the fin is almost exactly balanced by the drag on the body of the fish. In this mode, fin hydrodynamics is primarily determined by the fin flapping frequency, fin amplitude and the flow speed which can be expressed in terms of a fin Strouhal number, normalized amplitude and fin Reynolds number. In general, for fish with different sizes and swimming speeds, these three non-dimensional parameters can vary over a wide range. Because of this, most studies that attempt to gain general insights into the performance of fins, flapping foils or flapping wings find it convenient to examine the problem in terms of these non dimensional parameters (Freymuth 1988; Triantafyllou et al. 1992; Anderson et al. 1998; Walker \& Westneat 2000; Wang 2000; Combes \& Daniel 2001; von Ellenrieder, Parker \& Soria 2003; Lewin \& Haj-Hariri 2003; Prempraneerach, Hover \& Triantafyllou 2003; Hover, Haugsdal \& Triantafyllou 2004; Blondeaux et al. 2005a, b; Techet et al. 2005) since this allows for the study of the flapping appendage without regard to the associated body. We have adopted a similar approach in the current study.

In the particular case of labriform ${ }^{2}$ propulsion, since the very near wake of the pectoral fin is not affected by the wake of the fish body, the fin near-wake can be examined in order to assess the thrust production of the fin. The study of Drucker \& Lauder (2002) showed that the pectoral fins of the sunfish produce a train of vortex rings which are associated with momentum addition in the fin wake and consequently to a production of force on the fin. Through modification in the fin gait, the fish can alter the axis and direction of travel of these vortex rings and through this, control the direction and magnitude of the forces and moments on the fin. Ramamurti et al. (2002) simulated (via NS) the flow associated with the pectoral fin of a bird-wrasse which was the subject of the study by Walker \& Westneat (1997) and examined in detail the flow structure and force production of this fin. Von Ellenrieder et al. (2003) examined the flow associated with a rectangular flapping foil of aspect ratio 3.0 at a Reynolds number of 163. The Strouhal number in this study varies from 0.2 to 0.35 and pitch angle amplitude from 0 to 20 degrees. The dye visualization study of von Ellenrieder et al. (2003) was conducted over a range of flapping amplitudes and frequencies and the effect of these parameters on the vortex topology was elucidated. They found that the wake of these flapping foils was dominated by sets of loops and rings and they describe the evolution of these vortex structures. This configuration was studied numerically by Blondeaux and co-workers (Guglielmini \& Blondeaux 2004; Blondeaux et al. 2005a, b). Blondeaux et al. $(2005 a, b)$ have examined the wake evolution at Strouhal numbers of 0.175 and 0.35 and the simulations show that a vortex ring is shed every halfcycle from the flapping foil. Also, Blondeaux et al. (2005a, b) indicate that as the Strouhal number is increased, there is an increased interaction between adjacent rings. The vortex structures in the numerical study were found to be different from those observed in the experiments of von Ellenrieder et al. (2003). In particular, Blondeaux et al. (2005a, b) point out that in contrast to the experiments, the simulations do not show the presence of distinct vortex loops in the wake associated with the trailing-edge vortex. Neither the experiments nor any of these simulations have examined the force generation by this flapping foil, therefore it is not clear if the foils are indeed generating thrust.

Buckholtz \& Smits (2006) is also of relevance to the current study. In this study, flow visualizations are used to examine the wake of a low-aspect-ratio pitching panel. The Strouhal number of the panel was 0.23 and the chord-based Reynolds number was 640. The experiments showed that the wake of the panel was dominated by vortex loops of alternating sign and a vortex skeleton model was proposed for the wake formation. Buckholtz \& Smits (2006) also observe that despite the lack of a leading edge, the wake behaviour is similar to that observed by Guglielmini \& Blondeaux (2004) for a pitchingheaving foil. This indicates that the underlying vortex dynamics of these configurations is quite robust.

Despite all of these previous works on finite aspect-ratio flapping foils/wings, the number of studies that have systematically examined the effect of aspect ratio on the fluid dynamics and force production of low-aspect-ratio foils is limited. Ahmadi \& Widnall (1986) used a linearized low-frequency unsteady lifting line theory to examine the energetics of wings

[^1]undergoing a combined pitch-and-heave motion. Aspect ratios in their study varied from 8 to 16. Cheng, Zhuang \& Tong (1991) used an unsteady vortex ring panel method to study the energetics and force production of undulating plates with aspect ratios ranging from 0.5 to 8.0. A key finding in their study was that the undulatory motion can reduce threedimensional effects and lead to good swimming performance. Usherwood \& Ellington (2002) have experimentally examined the effect of aspect ratio on the force generation by a rotating wing based on a hawk moth wing. The emphasis of this study was on insect and bird flight and the aspect ratios in their study varied from 4.53 to 15.84. Mittal et al. (2003) used Navier-Stokes simulations to examine the wake vortex topology for foils with aspect ratios of 1.27 and 2.55 undergoing a sinusoidal heaving motion at a Reynolds number of 100. They found that the wake of these foils was dominated by two sets of interconnected vortex rings that convect at an oblique angle to the wake centreline.

A study that requires special mention here is that of Combes \& Daniel (2001) who have examined the effect of aspect ratio and fin planform on the hydrodynamic performance of fins modelled after ratfish pectoral fins. They employ unsteady potential theory to predict the thrust and efficiency of wings of aspect ratios ranging from about 0.1 to higher than 10. The simulations do not include any tip effects since the model employed does not account for spanwise flow variations, but nevertheless, their calculation show that while thrust increases monotonically with aspect ratio, efficiency has non-monotonic variation up to aspect ratios of about 2 and a monotonic increase beyond that. Based on this analysis, Combes \& Daniel (2001) attempt to explain the wide variety of fin kinematics and morphologies that are observed in nature.
H. Dong, R. Mittal and F. M. Najjar (2006) performed a comprehensive analysis of the wake topology, force production and energetics of ellipsoidal flapping foils over a range of aspect ratios (AR), Strouhal frequencies (St), Reynolds numbers (Re) and pitch-bias angles. The full three-dimensional Navier-Stokes equations were solved numerically and therefore, all unsteady-viscous and spanwise effects were included. The simulations showed that the wake of thrust-producing finite-aspect-ratio flapping foils is dominated by two sets of interconnected vortex loops that induce a thrust producing jet. Analysis of the hydrodynamic performance of these flapping foils showed that the thrust coefficient increases monotonically with aspect-ratio and Strouhal number for all foils. Furthermore, all foils exhibit a clear maximum in propulsive efficiency with Strouhal number, although it is found that the peak efficiency decreases and the Strouhal number at which this maximum is achieved increases with decreasing aspect-ratio.

The factor of shape, which has also to be taken into account was experimentally explored by Luska Luznik and Neil Bose in "an experimental investigation of the propulsive thrust of oscillating foils of different planforms" (July 1997)

Pengfei Liu and Neil Bose also studied the hydrodynamic efficiency of a foil with aft-swept wing tips in "Hydrodynamic characteristics of a lunate shape oscillating propulsor" (November 1997)

Last but not least, Stefan Kern and Petros Koumoutsakos in "Simulations of optimized anguilliform swimming" (October 2006) investigated the hydrodynamics of anguilliform swimming motions using three-dimensional simulations of the fluid flow past a selfpropelled body. The motion of the body was obtained through an evolutionary algorithm used to optimize the swimming efficiency and the burst swimming speed. The fast and the efficient swimming mode both shed a double row of vortex rings responsible for the strong lateral jets observed in the wake. The results provided quantification of the vortex formation and shedding processes and enable the identification of the portions of the body that are responsible for the majority of thrust in anguilliform swimming. In burst swimming the tail is responsible for the majority of the thrust, while in efficient swimming the anterior part of the body also contributes to the thrust.

## Panel Methods: A Brief History

Aerodynamics Panel Methods were first investigated in the late 1950s. Since their initial development, they have been instrumental in the design, optimization and analysis of aircraft and aerodynamic bodies [15, 16, 107, 18, 22, 23]. A brief outline of some salient history of panel methods is presented:

BIEs in the Pre-1960s
Prior to the use of digital computers, basic analytical solutions to the potential flow Boundary Integral Equations were employed [24, 25, 26]. The principle of linear superposition of fundamental solutions such as point sources, and point doublets was used regularly to solve potential problems[10]. The field of panel methods was born in 1958, when Smith and Pierce from Douglas Aircraft Company used a discrete form of the boundary integral equations to solve for the potential flow around bodies of revolution [29].

1960s-1980s
With the success of the initial panel methods, the Smith group received support to continue development of panel methods for both two and three dimensional flow [78]. They pioneered the panel method solution to the lifting body problem in 2-Dimensions [13] and in 3-Dimensions [12]. The development continued to include higher order discretizations of the BEM approach in 2-Dimensions[11]. The Douglas group panel methods were almost exclusively of Neumann type, using either source or vorticity distributions over the surface [10]. In the 1970s, the Green's Theorem perturbation potential based Dirichiet problem was introduced by Morino [31]. There were also several variations of different complexity of the surface singularity boundary element method/membrane lattice approach [30, 99, 101].

The early panel methods were limited by computer memory and processing power. Some alleviation of computational complexity was achieved by using multipole expansions in place of analytical expressions for panel integral expressions for farfield evaluations; however, the methods still required the solution of a dense linear system.

1980s-1990s
During the 1980s, several low order three dimensional panel methods were developed [34, 35]. In addition to the low order methods (low order here referring to the constant basis function approximation of the solution), several high order implementations were also
developed. These high order methods were developed for the benefits of increased solution accuracy as well as for satisfying the solution continuity requirements imposed by supersonic flow applications. A combined Boeing and NASA effort resulted in PANAIR/A502, a quadratic basis, flat-sub-element high order panel method [94, 36]. Additionally, HISSS [37] a panel method based on PANAIR was developed. In the late 1980s PMARC was developed at NASA-Ames Research Center and was later released as a controlled access computer program. Although the 1980s brought with them great advances in computational power, limitations on computational time and memory still
prevented large-scale panel method solutions. Solutions with several thousand panels were routinely performed on large computers; however, due to the coarseness of surface discretizations, limitations on the practical use of panel methods existed. In addition to developments in three dimensional solvers, two dimensional panel methods were being developed and used heavily for inverse airfoil design [38, 27, 39, 28]. Furthermore, the use of boundary layer coupling was investigated for incorporating viscous effects[38, 39].

In the 1980s several algorithmic developments were also made which have had a significant impact on the development of panel methods. These developments included iterative solution methods, most notably the development of Krylov subspace iterative solvers [ 42, 43]. In addition to iterative solvers, several sparsification and acceleration routines were also developed in order to facilitate the rapid computation of matrix vector products of the dense BEM linear systems. The first category of fast methods involved multipole expansion approximations of the farfield influences [70, 86]. The second category of methods relied on rapidly approximating farfield interactions using a Cartesian background mesh[50, 68].

## 1990s - present

By the 1990s panel methods had largely given way to higher fidelity Navier-Stokes and Euler solvers [51]. Although Eulerian reference frame domain solvers were being heavily investigated, several Lagrangian based approaches were developed. The Vortex Particle Method was refined and further investigated for the simulation of largely vortical flow [54, 59, 56, 57, 60, 67]. The section which follows describes some of the history and development of vortex particle methods. Despite the promise of Navier-Stokes solvers, accurate viscous drag prediction remained an elusive task. In the 1990s several researchers started to consider the problem of 3-Dimensional Integral Boundary Layer Methods [40, 41] with some success.

The Fast Multipole Method was used and further developed in practical boundary element method solvers for many diverse disciplines [44, 49, 46]. In the early 1990s, the precorrected-FFT algorithm [84] was developed. The precorrected-FFT approach provided a kernel independent framework for the acceleration of BEM and N -body problems[45, 83]. In the 1990 s and 2000s several panel method codes continued the advancement of higher order approximations to the boundary integral equations[93, 95, 96, 97, 98, 101, 102, 103, 89], however, due to the complexity involved with higher order methods and the lack of robust and efficient integration techniques for higher order approaches, their adoption in the BEM community is limited in comparison with the popular constant collocation type approaches.

The present thesis takes the methods published by G. Politis in "Simulation of unsteady motion of a propeller in a fluid including free wake modeling" with slight improvements in the numerical methods.

Below, there is a table with the chronological list of the most known panel methods and their main features, taken form Katz and Plotkin "low speed Aerodynamics"

| Method | Geometry <br> of panel | Singularity <br> distribution | Boundary <br> conditions | Remarks |
| :--- | :--- | :--- | :--- | :--- |
| 1962, Douglas- <br> Neumann <br> 12.1 | Flat | Constant source | Neumann |  |
| 1966, Woodward I ${ }^{12.2}$ | Flat | Linear sources <br> Constant vortex | Neumann | $M>1$ |
| 1973, USSAERO ${ }^{12.3}$ | Flat | Linear sources <br> Linear vortex <br> Constant source | Neumann | Neumann |

## PARAMETERS OF MOVEMENT

In a foil with maximum chord length and maximum span ,moving at steady speed, and at an angle of attack, the parameters of relevance are (a) the geometric shape (rectangular, delta-shaped, etc.); (b) the aspect ratio (AR), defined to be equal to the ratio of an average span $s$ over an average chord $c$; (c) the angle of attack; and (d) the Reynolds number , $R e=U c / v$ where $v$ is the kinematic viscosity of the fluid.

The forces are classified as lift, the component perpendicular
to the velocity $U$, and drag, the component parallel to $U$.


Fig. a Pitching and heaving


Fig. b Pitching and rolling

In unsteadily moving foils, we must first parametrize the motion of the foil. For a foil of chord c , moving forward at average velocity U , and oscillating harmonically with a linear (heave) motion $\mathrm{h}(\mathrm{t})$, transversely to the velocity, and an angular (pitch) motion $\theta(\mathrm{t})$ (fig. a)

$$
\begin{gather*}
h t=h_{0} \sin 2 \pi f t  \tag{0.1}\\
\theta t=\theta_{m}+\theta_{0} \sin 2 \pi f t+\psi \tag{0.2}
\end{gather*}
$$

where $\psi$ is the phase angle between heave and pitch, $h_{o}$ the heave amplitude, $\theta_{0}$ the pitch amplitude, $\theta_{m}$ the average pitch angle, and $\omega=2 \pi f$ the frequency of oscillation

In the case of bird type flapping foil, the heave is substituted by a rolling motion and (1.1) is changed by (fig. b)

$$
\begin{equation*}
\varphi t=\varphi_{0} \sin 2 \pi f t \tag{0.3}
\end{equation*}
$$

Where $\varphi(t)$ the angular position and $\varphi_{o}$ the maximum rolling angle.
the term $h(t)$, that is used in the following, is substituted by the amplitude of motion at the $70 \%$ of span which is approximated by

$$
\begin{equation*}
h_{0.7} t=0.7 s \cdot \sin \varphi t \tag{0.4}
\end{equation*}
$$

Then we can define the following nondimensional parameters, in addition to those for a steadily moving foil:

1) heave to chord ratio $h^{*}=h / c$
2) maximum unsteady angle of attack $a_{\text {max }}$
3) reduced frequency $k=f c / U$
4) Strouhal number, defined as $=A f / U$, where $A$ is the width of the wake of the foil *
5) mean angle of attack, which is equal to $\theta_{m}$.
$6)$ the phase angle between the two motions $\psi$

* The Strouhal number is often approximated by taking $A=2 h_{o}$,
i.e.,

$$
\begin{equation*}
S t=\frac{A f}{U}=2 h_{0} f / U \tag{0.5}
\end{equation*}
$$

The maximum angle of attack is defined as the maximum value of the angle $\alpha(t)-\theta(t)$, where $\alpha(t)$ is defined as

$$
\begin{equation*}
a t=\operatorname{ATAN}\left(\frac{d h t}{d t} / U\right) \tag{0.6}
\end{equation*}
$$

Another nondimentional parameter, is $\theta_{0}$ which can also describe clearly any case and/or experiment. The maximum angle of attack was used mostly as a mean to make sure that there is no leading edge separation.

## FORCES, POWER AND EFFICIENCY

The instantaneous force $F$ can be decomposed into a transverse component, and an axial component $X$, with respect to the steady velocity $U$ and the instantaneous moment can be decomposed into (among others) a moment around the axis of pitching $M_{p}$ and a moment around the axis of rolling $M_{r}$ (for the case of bird type flight). The transverse force may contain a steady component, which can serve, as in steadily moving foils, to provide a constant lift force. The instantaneous axial force may be positive (thrust) or negative (drag); it typically contains a steady (average) component, $X_{m}$, which can be used to propel a body. When propulsion is the goal, a propulsive efficiency is defined as

$$
\begin{equation*}
\eta=X_{m} U / P_{M} \tag{0.7}
\end{equation*}
$$

Where $P_{m}$ is the time-averaged power needed to flap the foil. It must be noted that the force $X_{m}$ is the net average axial force acting on the foil.

Having the capability to measure forces and moments in all axes, $P_{m}$ can be calculated as

$$
\begin{equation*}
P_{m} t=Z \frac{d h t}{d t}+M_{p} \frac{d \theta t}{d t} \tag{0.8}
\end{equation*}
$$

for the pitching and heaving motion

$$
P_{m} t=M_{r} \frac{d \varphi t}{d t}+M_{p} \frac{d \theta t}{d t}
$$

for the pitching and rolling motion.

Then the mean propulsive efficiency till given time t is calculated as

$$
\begin{equation*}
\bar{\eta} t=\frac{\int_{0}^{t} X_{m} \tau U d \tau}{\int_{0}^{t} P_{m} \tau d \tau} \tag{0.10}
\end{equation*}
$$

## MATHEMATICAL FORMULATION

Representation theorems:

$$
\begin{equation*}
\Phi P=-\frac{1}{4 \pi} \int_{S} \frac{\sigma}{r} d S+\frac{1}{4 \pi} \int_{S} \mu \frac{\vec{n} \vec{r}}{r^{3}} d S \tag{1.1}
\end{equation*}
$$

Where: $\sigma=\vec{n} \cdot \nabla\left(\varphi^{-}-\varphi^{+}\right)$and $\mu=\varphi^{-}-\varphi^{+}$

The perturbation velocity can be calculated as:

$$
\vec{V}(P)=\frac{1}{4 \pi} \int_{S} \sigma \frac{\vec{r}}{r^{3}} d S+\frac{1}{4 \pi} \int_{S} \vec{n} \times \nabla \mu \times \frac{\vec{r}}{r^{3}} d S-\frac{1}{4 \pi} \oint \mu \frac{d \vec{\ell} \times \vec{r}}{r^{3}}(1.2)
$$



## MORINO FORMULATION

Kinematics:
Any motion can be analyzed into a Linear velocity $\vec{V}$ and an angular velocity $\vec{\Omega}$. Then, the velocity of any element of the body can be written as:

$$
\begin{equation*}
\vec{q}=\vec{V}+\vec{\Omega} \times \vec{r} \tag{1.3}
\end{equation*}
$$



Figure 1
For the point $P^{-}$inside body, as $P^{-} \rightarrow P^{1-}$ from the inside (point $P^{\prime-} \in S_{B}$ ), the expression for perturbation potential becomes (note the addition of $\frac{1}{2} \mu$, which is a result of the procedure to tackle the singularity of the kernel):

$$
\begin{equation*}
\Phi P^{\prime-}=\frac{1}{4 \pi} \int_{E} \frac{\sigma}{r} d S+\frac{1}{4 \pi} \int_{E-E_{0}} \mu \frac{\vec{n}^{+} \cdot \vec{r}}{r^{3}} d S+\frac{1}{2} \mu \tag{1.4}
\end{equation*}
$$

But according to Morino we have the internal condition:

$$
\begin{equation*}
\Phi P^{\prime^{-}}=0 \tag{1.5}
\end{equation*}
$$

Then (2.4) becomes:

$$
\begin{equation*}
\frac{1}{4 \pi} \int_{E} \frac{\sigma}{r} d S+\frac{1}{4 \pi} \int_{E-E_{0}} \mu \frac{\vec{n} \cdot \vec{r}}{r^{3}} d S+\frac{1}{2} \mu=0 \tag{1.6}
\end{equation*}
$$

we remind that: $\mu=\Phi^{-}-\Phi^{+} \sigma=\vec{n} \nabla \Phi^{+}-\Phi^{-}$
The no entrance condition on body is expressed as: $\vec{n}^{+} \nabla \Phi^{+}=\vec{q} \cdot \vec{n}^{+}$
Or using $\sigma=\overrightarrow{n_{i}} \nabla \quad \Phi-\Phi_{l}$ :

$$
\begin{equation*}
\sigma=\vec{n}^{+} \nabla \Phi^{+}-\vec{n}^{+} \nabla \Phi^{-}=\vec{n}^{+} \cdot \vec{q} \tag{1.8}
\end{equation*}
$$

Substituting the above to (2.4) we get:

$$
\begin{equation*}
\frac{1}{2} \mu+\frac{1}{4 \pi} \int_{S_{B}} \mu \frac{\vec{n} \cdot \vec{r}}{r^{3}} d S=-\left[-\frac{1}{4 \pi} \int_{S_{B}} \frac{\sigma}{r} d S\right]-\left[-\frac{1}{4 \pi} \int_{S_{W}} \mu \frac{\vec{n} \cdot \vec{r}}{r^{3}} d S\right] \tag{1.9}
\end{equation*}
$$

Where $S_{B}$ for body surface and $S_{W}$ for wake surface
Thus we have reached an Integral equation for the determination of $\mu=\Phi^{-}-\Phi^{+}$The equation above is a Fredholm integral equation of the second kind with a weakly singular integral. The existence and uniqueness of the solution for smooth domain is proved by Kress and Mikhlin ${ }^{3}$.

The induced velocity is now calculated as (where the sign notifies the inner or outer side):

$$
\begin{equation*}
\vec{V} P^{ \pm}=-\frac{1}{4 \pi} \int_{E} \sigma \frac{\vec{r}}{r^{3}} d S+\frac{1}{2} \vec{n} \cdot \sigma-\frac{1}{4 \pi} \int_{E} \vec{\gamma} \times \frac{\vec{r}}{r^{3}} d S \mp \frac{1}{2} \nabla_{S} \mu-\frac{1}{4 \pi} \int_{\partial E} \mu \frac{d \vec{\ell} \times \vec{r}}{r^{3}} \tag{2.9b}
\end{equation*}
$$

[^2]Calculation of pressures, Forces and Moments:
In Katz \& Plotkin "Low Speed Aerodynamics" p.376, rel. 13.23 we can find in a more expanded expression that:

$$
\begin{equation*}
\frac{p-p_{\infty}}{\rho}=-\frac{1}{2} \nabla \Phi^{2}-\vec{V}_{r e f} \cdot \nabla \Phi-\frac{\partial \Phi}{\partial t} \tag{1.10}
\end{equation*}
$$

Where $\frac{d \Phi}{d t}$ is evaluated at the local body system,
And: $\quad \vec{V}_{r e f}=-\left[\vec{V}_{0}+\vec{\Omega} \times \vec{r}\right]=-\vec{q}$
If we define:

$$
\begin{equation*}
\vec{Q}=\vec{V}_{r e f}+\nabla \Phi \tag{1.12}
\end{equation*}
$$

Where $\nabla \Phi$ the perturbation velocity, rel.(2.118) leads to:

$$
\begin{equation*}
\frac{P-P_{\infty}}{\rho}=\frac{V_{r e f}^{2}}{2}-\frac{Q^{2}}{2}-\frac{\partial \Phi}{\partial t} \tag{1.13}
\end{equation*}
$$

Or by using the definition of pressure coefficient $\left(C_{P}=\frac{P-P_{r e f}}{\frac{1}{2} \rho V_{r e f}^{2}}\right)$ :

$$
\begin{equation*}
C_{P}=\frac{P-P_{r e f}}{\frac{1}{2} \rho V_{r e f}^{2}}=1-\frac{Q^{2}}{2 V_{r e f}^{2}}-\frac{2}{V_{r e f}^{2}} \cdot \frac{\partial \Phi}{\partial t} \tag{1.14}
\end{equation*}
$$

Forces and moments can be calculated using:

$$
\begin{equation*}
\vec{F}=\int_{S_{B}} p \cdot \vec{n} d S \tag{1.15}
\end{equation*}
$$

And

$$
\begin{equation*}
\vec{M}=\int_{S_{B}} \overrightarrow{r_{i}} \times p \cdot \vec{n} d S \tag{1.16}
\end{equation*}
$$

Before moving on to the discretization and solution of the integral equation (2.9) it is important to clarify two things. The behavior and movement of the free wake and the Kutta condition on the trailing edge.

The domain space for $\mu$ in (2.9) consists of two parts. The body $S_{B}$ and the wake $S_{W}$, which carries the generated vorticity at the blade trailing edge, satisfying Helmholtz's equations. The wake, moves with the induced velocity on it. Equation (2.9b) gives the induced velocity on the two sides of the vortex sheet. The mean value of the two is:

$$
\rangle \vec{V}_{W}\left\langle=-\frac{1}{4 \pi} \int_{S} \sigma \frac{\vec{r}}{r^{3}} d S-\frac{1}{4 \pi} \int_{S} \vec{\gamma} \times \frac{\vec{r}}{r^{3}} d S-\frac{1}{4 \pi} \int_{S_{W}-\ell} \mu \times \frac{d \vec{\ell} \times \vec{r}}{r^{3}}(1.17)\right.
$$

An elegant proof of the above equation can be found in G.K Politis' "Simulation of unsteady motion of a propeller in a fluid including free wake modeling" and is given in Appendix 1 of it.

The Kutta condition is expressed as the relation that defines the potential in the first row of elements of the wake, which are adjacent to the trailing edge, at each time. Specifically the generated potential on the Kutta strip takes the mean value of the lower and the upper side of the foil that were calculated in the previous timestep. For the adjacent points P and $P^{\prime}$ (of the body and the wake respectively) and a given time $t$ this can be written as:

$$
\begin{equation*}
\varphi_{U} t-\left.\varphi_{L} t\right|_{P}=\mu t+\left.d t\right|_{P} \tag{1.18}
\end{equation*}
$$

## Discretization of the problem.

For the purposes of the present program, It has been chosen to use bilinear elements, comprised of four nodes and one control point in the center where the potential is carried. This type of elements is used for both the body and the wake.

With all the previous in mind and $\sigma=\vec{n} \cdot \vec{q}(2.9)$ can be written as:

$$
\begin{align*}
& \frac{1}{2} \mu+\frac{1}{4 \pi} \int_{\partial S_{B} t+\text { Kutta_Strip }(t)} \mu \frac{\vec{n} \cdot \vec{r}}{r^{3}} d S= \\
& =\frac{1}{4 \pi} \int_{\partial S_{B} t} \frac{\vec{n} \cdot \vec{q}}{r} d S-\frac{1}{4 \pi} \int_{\partial S_{W} t-\text { Kutta_Strip } \mu \frac{\vec{n} \cdot \vec{r}}{r^{3}} d S} \tag{1.19}
\end{align*}
$$

Assuming that there are $Z$ bodies With $N$ elements each and the wake of each is comprised of $M$ elements, $K$ of which are of the kutta strip, (1.19) can be written in the following discretized form:

$$
\begin{align*}
& \frac{1}{2} \mu_{i^{\prime}, n^{\prime}}+\frac{1}{4 \pi} \sum_{\substack{i=1, Z \\
n=1, N}} \mu_{i, n} \int_{E B_{i, n}} \frac{\vec{n} \cdot \vec{r}}{r^{3}} d S+\frac{1}{4 \pi} \sum_{\substack{i=1, Z \\
m=1, K}} \mu_{i, m} \int_{E W_{i, m}} \frac{\vec{n} \cdot \vec{r}}{r^{3}} d S= \\
& =\frac{1}{4 \pi} \sum_{\substack{i=1, Z \\
n=1, N}} \int_{E B_{i, n}} \frac{\vec{n} \cdot \vec{q}}{r} d S-\frac{1}{4 \pi} \sum_{\substack{i=1, Z \\
m=K+1, M}} \int_{E W_{i, m}} \mu \frac{\vec{n} \cdot \vec{r}}{r^{3}} d S \tag{1.20}
\end{align*}
$$

where $i^{\prime}$, $n^{\prime}$ denote control point position, $i^{\prime}=1, Z, n^{\prime}=1, N$. Constant dipole intensity $\mu_{i, n}$ for both blade and trailing vortex sheet region has been used. Regarding source distribution (first term in the right hand side), its intensity is analytically known at each time step, relation (2.103), and thus the surface integral can be
calculated numerically. Considering the second term on the right hand side, both $\mu$ and the geometry of its support W (trailing vortex sheets) are assumed to be known from previous time steps. As shown in equation (2.20) a kutta-strip is used as an additional unknown at each time step. Dipole intensities are then calculated by satisfying equation (2.20) at the Z*N centroids of body elements simultaneously with equation (2.18) relating dipole intensity of kutta-strip elements with dipole intensities of the directly adjacent body elements ( $Z^{*} K$ more equations). Thus body and kutta-strip dipole strengths can be calculated at each time step.

Then we use a constant dipole intensity, formula (2.17) for the calculation of the mean perturbation velocities on free vortex sheet, which degenerates to:
$\vec{V}_{W}=-\frac{1}{4 \pi} \sum_{\substack{i=1, Z \\ n=1, N}}\left(\int_{E B_{i, n}} \sigma \frac{\vec{r}}{r^{3}} d S+\int_{E B_{n, i}} \vec{\gamma} \times \frac{\vec{r}}{r^{3}} d S\right)-\frac{1}{4 \pi} \sum_{\substack{i=1, Z \\ m=1, M}} \int_{E W_{i, m}} \mu_{i . m} \times \frac{d \vec{\ell} \times \vec{r}}{r^{3}}(1$
where source and dipole intensities and integration surface $W$ are known from previous time steps. More specifically the first term is a surface integral with regular kernel for points $P \in W$ while the second and third terms in the right hand side are line integrals of known dipole strengths over the boundary line of each element.

Note that these integrals are equivalent to the well known Biot - Savart formula for the calculation of induced velocities from singular vortex lines. Thus the integrals on the right hand side of (2.21) can be calculated and mean induced velocities on the trailing vortex sheet can be found. In our implementation we use relation (2.21) to calculate velocities directly at grid points of the grid representing the free wake of each blade.

With the induced perturbation velocities known at each grid point, the new position of the nodes can be calculated by moving each node by $\vec{V}_{W} \quad p d t$ where p is the node.

After that, with all dipole strengths known, perturbation velocities can be calculated on the body by taking the surface gradient of the dipole strength and pressures forces and moments are calculated using (2.13), (2.15) and (2.16)

The final step of the algorithm is to move the body(ies) to the next position for the calculation of the next timestep according to the movement equations defined by the user.

## Treatment of integrals:

Almost in every BEM the researcher is faced with the calculation of some type of singular integrals. The appearance of the singularity in the kernel is always connected with the relative position of the control point and the integration point. In our formulation we have two types of singular integrals:

- Weakly singular integrals (improper integrals), appearing in the calculation of element self induction factors (or self influence coefficients) due to dipoles (left hand side of equation (2.116)) and in the calculation of the self part of the source distribution term (first term in the right hand side of equation (2.116)).
- Non-integrable (at $\mathrm{P} \equiv \mathrm{Q}$ ) in the usual sense line integrals of the Biot-Savart type, appearing in the right hand side of relation (24) (second and third terms). With the selected bilinear boundary elements the integration range of those integrals is a straight line (side of the bilinear element). Thus integrations as well as treatment of singularity can
be performed analytically as will be explained in the sequel.
The first term in the right hand side of equation (2.117) is regular since this integral is always evaluated on the trailing vortex wake where $P \neq Q$. (otherwise this term presents a strong Cauchy type surface singularity as $P \rightarrow Q$ ).

The numerical calculation of surface improper integrals is a simple procedure and can be achieved using common numerical integration rules and a cutoff function symmetric around the singularity (cutoff radius). In our implementation all improper integrals are calculated using adaptive Simpson quadrature. According to this the integrals are first transformed to iterated and then the integrations (in each direction) are performed using adaptive Simpson quadrature. Furthermore for the evaluation of the improper integrals for dipoles and sources the cutoff radius used is:
cutoff_radius $=0.01 \cdot \min ($ side_u,side_v)
where, side_u and side_v are the lengths of the two sides of the element for which the self integration is performed.

Regarding numerical handling of Biot-Savart type integrations (2nd and 3rd terms in relation (2.117)), consider a straight line vortex of unit strength starting at point $\vec{a}$ and ending at point $\vec{b}$.

Let also the position of the control point be denoted by $\vec{p}$. Then the induced velocity $\vec{v}$ at $\vec{p}$ from the line vortex defined by $\vec{a}, \vec{b}$ can be calculated analytically and is given by the following relation :

$$
\begin{equation*}
\vec{v}=s 2 \cdot \vec{l} \times \overrightarrow{a 1} / 4 \pi \tag{1.22}
\end{equation*}
$$

Where s2 $\vec{l}, \overrightarrow{a 1}$ can be calculated as follows:

$$
\begin{aligned}
& \overrightarrow{a 1}=\vec{a}-\vec{p}, \quad \overrightarrow{b 1}=\vec{b}-\vec{p} \\
& \vec{l}=\overrightarrow{b 1}-\overrightarrow{a 1} / d, \quad d=|\overrightarrow{b 1}-\overrightarrow{a 1}| \\
& s 0=|\overrightarrow{a 1}|^{2}, \quad s 1=\vec{l} \cdot \overrightarrow{a 1} \\
& f=\sqrt{d^{2}+2 \cdot d \cdot s 1+s 0} \quad, \quad g=s 0-s 1^{2} \\
& s 2=s 1 \cdot f-\sqrt{s 0} \cdot d+s 1 \quad / \sqrt{s 0} \cdot f \cdot g
\end{aligned}
$$

## Calculation of potential surface gradient:

Let $f_{n-1}, f_{n}, f_{n+1}$ denote the values of the potential at three consecutive nodal points P ${ }_{1}, \mathrm{P}_{2} \mathrm{P}_{3}$, (centroids of elements) on blade surface along either u (chordwise) or v (spanwise) directions. Let also $I_{1}=\left|P_{1} P_{2}\right|, I_{2}=\left|P_{2} P_{3}\right|$ be the curvilinear physical distances of $P_{1}, P_{2}, P_{3}$ along either $u$ or $v$ direction. These distances can be easily calculated using the element shape functions and the surface metric tensor.

Derivative at $P_{2}$ is calculated using the following formula (weighted central difference):

$$
\begin{aligned}
& \frac{\partial f}{\partial l_{\mathrm{u} \text { or } \mathrm{v}}}=\frac{f 1 \cdot l_{2}+f 2 \cdot l_{1}}{l_{2}+l_{1}} \\
& f 1=\frac{f_{n}-f_{n-1}}{l_{1}}, f 2=\frac{f_{n+1}-f_{n}}{l_{2}}
\end{aligned}
$$

and $l_{u \text { orv }}$ denotes the physical length along direction $u$ or $v$. At boundary points of $\partial L_{i}$ a usual forward or backward difference scheme has been used. By applying the above
relation along $u$ and $v$ directions, the corresponding physical potential derivatives can be found. Tangential perturbation velocity can then be calculated using:
$\vec{v}=\overrightarrow{e_{u}} \frac{\partial f}{\partial l_{u}}+\overrightarrow{e_{v}} \frac{\partial f}{\partial l_{v}}$
where $\overrightarrow{e_{u}}$ and $\overrightarrow{e_{v}}$ are unit vectors along the element local $u$ and $v$ directions and can be calculated using
$\overrightarrow{e_{u}}=\frac{\partial \vec{r} / d u}{|\partial \vec{r} / d u|}, \overrightarrow{e_{v}}=\frac{\partial \vec{r} / d v}{|\partial \vec{r} / d v|}$ where $\vec{r}$ the position vector at point $u, v$ of the element.

## Note on the Kutta condition.

It is said by many other authors, that the best kutta condition is a pressure kutta, which equalizes the pressures on both sides of the foil, which equivalently equalizes the velocities on each side on the trailing edge.

The above kutta condition has some drawbacks. First, it is "expensive" in terms of computational cost. Furthermore, in near stall cases (large angles of attack), the velocities (and pressures) cannot be equalized. Even in experiment, it is observed that there is a natural average smoothing of the velocities and pressures in order to satisfy the continuity of the fluid (which is not necessarily satisfied in nature either. e.g. cavitation), but the velocities and pressures on the sides of the trailing edge can be different.

The selected kutta condition, although it can lead to unequal pressures at the trailing edge, it can work faster and the integrated forces results show good agreement with calculations. Obviously, the subject shall be examined in more detail in the future.

## THE PROGRAMS

After having introduced the mathematical formulation and the arithmetical implementation of it, It is much easier to describe the solving program produced and its structure.

The program has been written in Fortran 95 with the intent to put all the capabilities and advantages of the language in use and make it evolvable. Consequently the program is written in a highly structured manner and quite all different groups of operations are separate subprograms. Thus, with only slight changes, the program can be put to solve for different types of motion, any part of the procedure can be omitted or the sequence can be changed and most important, any part of the program can be replaced by other compatible subprogram that might be using another method or have an improvement.

More specifically the program uses a separate packet of subprograms called "types" and a set of 30 subroutines. It also uses the module functionality of Fortran.

## TYPES:

This packet is a "toolkit" in which there are the definitions of all types of parameters and variables and the operations between them, which are defined as "elemental" subroutines or functions. The main difference of usage of elemental subprograms is that these operations can be performed on whole arrays or parts of them by using a single command in the main program. For example, the translation and/or the rotation of the moving body can be done with one command. The module also contains interpolation, integration, linear equation system solving and eucledian norm subprograms. There will be no further explanation of these subprograms, as they are basic numerical operations with no special interest.

## MAIN PROGRAM AND SUBROUTINES

As a result of the structured programming, the main program is less than 300 lines, about 50 of which are comments and more than 80 are the definition of variables. The following is a list of the subroutines in order of call along with a short description and a rough description of the algorithm.

Note: In the program the term particle occurs, but it does not imply the known from other methods particles, but is used as a general sense that contains bilinear elements and control points.

1. read_body_geom

The program starts with opening the file "i_bem" which contains the geometry of the foil used. Then the locations of the nodes are read, the elements and the mapping between them are created and the association between them is tested.
2. read_body_movement_param

The program then opens the file i_other_data where it reads the number of bodies, the linear velocity of the system, the timestep, number of steps and $h_{0}, \theta_{0}, \varphi_{0}$, as well as all frequencies and phase angles. Note that any combination of them is possible, even different movement for each body. Also there is no restriction for the number of bodies (except for the computing capabilities of the user). Also the heave amplitudes and the system velocity are given as vectors
3. initialize_body_properties

Then, from the same file, the program reads the initial position of the bodies along with the definition for the center of rotation and the point of reference for the calculation of moments.
4. tecpl_1_1

The program plots the initialized geometry in a format compatible with tecplot
5. read_trailing_vortex_parameters introduce trailing vortex sheet geometry and movement parameters
6. dipole_potential_1

Then, dipole potential induction factors are calculated

## TIME STEPPING STARTS HERE

7. Body trailing edge nodes are passed to the first strip of vortex wake nodes
8. The bodies are rotated according to the pitch equations
9. The bodies are translated according to the heave equations (if any)
10. The bodies are rotated according to the roll equations (if any)
11. dipole_potential_2

Then the right hand side free wake dipole term is found.
12. source_potential

Body source potential rhs forcing term is found

## 13. GELG

The linear equation system is solved for dipole strengths
14. check_solution

The correct of the solution of the linear system is checked
15. tecpl_4

Solution dipoles (coordinates and strength) are exported in a format compatible with Tecplot
16. tecpl_5

Radial circulation distribution is plotted
17. particle_vorticity

Body and free wake particle vorticity is calculated
18. perturbation_velocity_2

Perturbation velocity on control points is calculated
19. force_moment

Forces, and moments are calculated. Using these, The required power and the thrust power are calculated along with efficiency and nondimentionalized factors are written in the "factors" file
20. tecpl_6 Body pressures and velocities are plotted
21. tecpl_9 Surface pressures are plotted in 2-D form
22. vortex_wake_deformation

The new position of the free wake elements is calculated
---------END OF TIME STEP-
The time stepping loop repeats for the given number of steps
END OF PROGRAM

## GRID GENERATION:

For the generation of the foil grid, another program, "grid_wing_2a" was made.
The program calculates the geometry of 3D-Wing, based on NACA 4 and 5 digit airfoil sections and produce a surface grid of points. More specifically, the user types the number of NACA section, the chord length in the center of the wing, the span, the number of points in the chordwise and the spanwise direction the desired tapering of the wing, the skew angle, a skew curvature factor, and the desired position of the system of reference of the foil.
The program uses two subroutines.
The first returns the position of a point of a section, after having taken the NACA number, the chord length the chordwise position and whether the point is on the lower or the upper side. Thus, the main program, by calling this subroutine in a loop for the span, gets the whole section and just has to add the spanwise position of the section and position it according to the skew parameters given. For the tapering, the chord length is adjusted before being given to the subroutine. Inside these two loops needed for the generation of the geometry, there is an equation defining an iso-cosine distribution on the chord direction and an equal distance distribution in the span direction.
The second, takes the geometry and exports the "i_bem" in a compatible format with the program and the "tec_gridwing" file with the geometry in a compatible with tecplot format for the check of geometry before it is imported to the main program for the simulation.

Following, there are some examples of generated geometries to show the capabilities of the program. (Note that the program can work with any combination of parameters. Thus there might be some undesirable effects if extreme values are given)


Rectangular foil
Skew curvature and tapper


The first and the last are the geometries that will be used for the purposes of this thesis. The exact parameters for each simulation will be given with the simulation facts.

## SIMULATIONS MADE:

For the purposes of the present thesis, it was decided to make systematic runs of the program produced, with three parameters in mind. To produce results comparable with existing experimental data for verification, to have a clear image of the effect of each parameter of the experiment and to export ideas on possible improvements of the system under investigation.

For these reasons, and with the existing literature in mind (plus our computational and time limitations), it was decided that the heave to chord ratio ( $h^{*}=h / c$ ) and the phase angle $\psi$ would be kept with constant values 0.75 and 90 degrees respectively, as they have been taken in most of the known experiments. The NACA section will be 0012 for all the simulations. Additionally, the motion under investigation is the heaving and pitching motion and the system configurations are the one foil and the two symmetrically moving foils. The following table provides an overview of the cases tested.

| $\operatorname{Str}=0.15$ | $\operatorname{Str}=0.22$ | $\operatorname{Str}=0.26$ | $\operatorname{Str}=0.30$ | $\operatorname{Str}=0.40$ | $\operatorname{Str}=0.45$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\theta_{0}=0.1$ | $\theta_{0}=0.1$ | $\theta_{0}=0.1$ | $\theta_{0}=0.1$ | $\theta_{0}=0.3$ | $\theta_{0}=0.2$ |
| $\theta_{0}=0.2$ | $\theta_{0}=0.2$ | $\theta_{0}=0.2$ | $\theta_{0}=0.2$ | $\theta_{0}=0.4$ | $\theta_{0}=0.3$ |
| $\theta_{0}=0.3$ | $\theta_{0}=0.3$ | $\theta_{0}=0.3$ | $\theta_{0}=0.3$ | $\theta_{0}=0.5$ | $\theta_{0}=0.4$ |
|  | $\theta_{0}=0.4$ | $\theta_{0}=0.4$ | $\theta_{0}=0.4$ | $\theta_{0}=0.6$ |  |
|  |  |  | $\theta_{0}=0.5$ | $\theta_{0}=0.7$ |  |
|  |  | $\theta_{0}=0.6$ |  |  |  |
|  |  | $\theta_{0}=0.7$ |  |  |  |

The same pattern of simulations has been ran for the aspect ratios of $3,6,9$ for one and two foil configuration and there have been some runs (for comparison and verification reasons mainly) with the aspect ratios of 2 and 4 . The results will be compared and discussed further in the following chapters.

A fully detailed list with all the simulations made and the most important results in raw form can be found in appendix 1 .

## CONVERGENCE TEST:

All the main runs were made with a grid of 40 points in the chord direction (in order to "catch" the leading edge suction) and 20-40 in the span direction with respect to the AR The timestep was 0.05 sec . For the convergence test, there have been several runs of sparser grid in order to check if the trend is found even with sparser grid, and fewer (only 5 for time reasons) with a dense grid. Additionally, there have been several runs with a smaller timestep $(0.02 \mathrm{sec})$. There have also been some repeated runs with exactly the same input, in order to investigate the sensitivity to random numerical errors.

For the last case, all the differences in the results were below $0.1 \%$. Consequently, it is of no particular interest to plot any results or make any comparison.

## Grid sensitivity:

The table below shows a comparison of mean thrust coefficient and efficiency of three different grids used in a two foil configuration with AR=3 and Strouhal number 0.15 for three different pitch amplitudes, in order to see the behavior in non optimal cases too.

| Grid | $\theta_{0}(\mathrm{rad})$ | Ct | n |
| :---: | :---: | :---: | :---: |
| $60 \times 40$ | 0.1 | 0.158809 | 0.701 |
|  | 0.2 | 0.063651 | 0.998 |
|  | 0.3 | -0.03451 | 0.305 |
| $40 \times 30$ | 0.1 | 0.157909 | 0.698 |
|  | 0.2 | 0.063644 | 0.995 |
|  | 0.3 | -0.03442 | 0.301 |
| $30 \times 12$ | 0.1 | 0.059706 | 0.581 |
|  | 0.2 | 0.042741 | 0.903 |
|  | 0.3 | -0.04341 | 0.329 |

And the graph of it :


Note that the $60 \times 40$ and $40 \times 30$ cannot be distinguished in the graph as the difference is very small. Also the lower grid follows the trend very well, which means that low grid runs can be taken into account for optimization purposes.

The visualization of the wakes is the same even for the ones with the biggest difference.


## Effect of smaller timestep:

As it was said before, there have been several runs with a smaller timestep ( 0.02 sec ). The results produced were almost identical to the ones with the bigger timestep, in terms of mean values of thrust coefficient and efficiency. One interesting thing though, is that for the cases of small speeds and less simple geometries, the visualization of the wake is becoming "curly", as the distance between panels is too small and the round - off error causes problems in the wake deformation.


This is an aspect that has to be taken into consideration before setting any simulation, especially if there is a need for a good visualization.

## VERIFICATION OF RESULTS:

In order to verify the results of the program, experimental results had to be found and compared, as well as results from other methods in the existing literature. For this purpose, there were results extracted from the following papers:
"Forces on oscillating foils for propulsion and maneuvering" (Read, Hover, Triantafyllou 2002) for the one foil cases
"Exploring the possibility of placing traditional marine vessels under oscillating foil propulsion" (Czarnowski, Triantafyllou 1995) for the two foil cases
"Three dimensional flow structure and vorticity control in fish - like swimming" (Zhu, Wolfgang, Yue, Triantafyllou 2002) For the fish like foil cases.
"Wake topology and hydrodynamic performance of low aspect ratio flapping foils" (Dong, Mittal, Najjar 2006) for visualization comparison
"Computational and experimental investigation of flapping foil propulsion" (Jones, Lai, Tuncer, Platzer ISABMEC 2000) for comparison with experimental visualization.

The comparison with the systematic runs will be done in the discussion of the results.

Verifying the visualizations:
Below there are the iso-vorticity surfaces produced by the Navier-Stokes simulations of Dong, Mittal, Najjar And then the visualization of the results produced.



Note that there is a slight difference in the geometry of the foil. Yet, all other parameters are the same. What is interesting, is that the topology (size and angles) of the vortex rings are the same. Also, the last picture, shows very clearly the well known Karman Vortex street, and can be compared with the experimental visualization below.


## PHENOMENOLOGY OF THE FLAPPING FOIL PROPULSION:

All oscillatory motions of bodies in a fluid cause the body to shed a vorticity pattern called the Von Karman street after Theodore von Kármán the Hungarian - German - American engineer and physicist.

Such a pattern is the one below.


This is a typical thrust indicative Karman street. The wake rollup indicates vortex rings that induce a velocity which works as a jet providing thrust. The angle of the rings on the XY plane is determined by the Strouhal Number and the pitch amplitude (angle of attack).

The larger the Str , the larger the angle of the rings from the X axis, thus the more thrust is produced. On the other hand the efficiency drops. When $\theta_{0}$ goes beyond a certain point, the angles of attack become negative for the most of the duration of the phenomenon and then drag is produced causing the opposite angles to the vortex rings, as below:


The interest of the present thesis is on the thrust producing pattern. Consequently there will be no further investigation of the latter phenomenon.

## EFFECT OF $\theta_{0}$ ON THE PRODUCTION OF THRUST AND EFFICIENCY

The diagrams below show the efficiency (red) and mean thrust coefficient (blue) as a function of the pitch amplitude for all the Strouhal numbers tested for the AR=6


Note the point where the thrust coefficient becomes zero and then drag is being produced.
Also (and more important) note that for every str, there is a different $\theta_{0}$ where the efficiency is maximum.

The same pattern is recognized in all aspect ratios, with sole difference the magnitude as the AR changes (larger with larger ARs). Thus, it is judged not necessary to repeat these diagrams for all cases.

In addition to these, since the optimization for efficiency and thrust is the main purpose, the cases of maximum efficiency will be presented from now on, keeping in mind that a smaller $\theta_{0}$ gives slightly smaller efficiency, but larger thrust coefficient.

In order to compare the behavior of different aspect ratios and observe the effect of changing Strouhal number, the best and simplest possible diagram is a common diagram of Thrust coefficient and mean efficiency as functions of Strouhal number.


Additionally, there are the experimental results, comparison with which shows that the trends are followed very well and that the estimation of efficiency is lower than the experimental, which is on the safe side.

Further examination of the diagram, shows that the actual effect of the aspect ratio, is that it provides more thrust in larger ARs with practically the same efficiency. This effect gets stronger for larger Strouhal numbers.

On the other hand, examining the behavior of the system with respect to the Strouhal number, It is seen that the smaller the Str the bigger the efficiency, tending to $99 \%$ for $\operatorname{Str}=0.15$ and moving asymptotically to $100 \%$ for even lower values but with very low thrust coefficient, which with the addition of viscosity makes the thrust inexistent, giving an experimental value of zero efficiency.

To better understand the phenomena behind the previous diagram the following table of visualizations is given:
STR

What is easily seen, is that the aspect ratio makes small difference in the vortex pattern for small Strouhal numbers and as the Str is advancing, the change is that the larger aspect
ratio sheds vorticity of higher magnitude, causing a stronger rollup, thus producing more thrust.

## TWO FOILS CONFIGURATION:

The one foil configuration has one major disadvantage if used for ship propulsion. There is a strong oscillating force in the transverse direction. That problem was solved by using two foils moving symmetrically. In addition to that, the interaction between the two foils gives more thrust.

In order to compare the two configurations, the following diagram is produced for $\mathrm{AR}=9$


Again the trend is followed very well, only with the difference that the efficiency is overestimated. Yet, there should be slight reservations on the accuracy of the calculations of the experimental efficiency, as the experiments were done with older equipment and except for the thrust, everything else was measured indirectly.

The interesting thing is that the thrust coefficient is always larger for the two foil configuration and the mean (computed) efficiency is larger too. Keeping in mind that the factors are nondimentional parameters, it can be considered that the two cooperating foils behave as if they were of a larger AR and independent. This effect again is stronger for larger STRs. Visualizations for several Strouhal numbers can shed more light on the phenomenon:

| STR | Top view | Perspective view |
| :---: | :---: | :---: |
| 0.15 |  |  |
| 0.22 |  |  |
| 0.30 |  |  |



What is seen, is that the vorticity of the one foil acts as a ground effect to the other foil.

Then, it is important to present the effect of the pitch amplitude as it was done for the one foil, in order to justify the choice of presenting the results of one specific $\theta_{0}$ per Strouhal number.


If the time diagram of the simulation is looked into closely, there is another interesting thing to be taken into consideration:


In the diagram above, it is seen that the peaks are not aligned, but there is a second harmonic in the calculated values. This happens because the motion of each individual foil is not symmetric, but it has the induced ground effect on the one side. This is also what causes the overall increase of the thrust coefficient.

## FISH - LIKE FOIL:

Before going into the results of the simulations, it is important that the reader is provided with some background information. The larger of the fish and the sea mammals have an easily noticeable difference in the orientation of the foil and the smaller aspect ratio of the foils of mammals. In addition to these it is seen that fish have the ability to change the sweepback angle of their foil. An attempt to explain these effects is made below.

As the simulations so far have shown, most of the necessary power needed for the production of thrust, is for the heaving motion. On the other hand it has already been seen that larger aspect ratios provide more thrust for the same efficiency. In addition to these, the fact that the mammals have to be close to the surface in order to breathe, has to be kept in mind. With all these into consideration, a logical assumption is that mammals have their tails in horizontal orientation not only because it allows them to stay closer to the surface without losing thrust, but also because it allows them to take energy from the sea waves, as the waves can provide the heave. Thus, gaining this extra "free" energy, the mammals do not need the large aspect ratio foils.

On the other hand, the fishes live deeper below surface, so they need other ways to improve their abilities. The smaller fishes swim in close formations, gaining an increase of efficiency by means of the interaction presented in the two foil case. On the other hand, the larger fish uses another advantage that lies in the geometry of the tail and their ability to change it.

This effect is examined below. Taking the tuna as an example and having in mind the existence of some experimental results from Robo - tuna, it was decided to use the following geometry for the simulations.


It should also be noted that the geometry is not chosen by using any criterion other than the resemblance with the foil of fish. There certainly is room for improvements. The NACA section is again the 0012.

Comparison with the one foil:


Note that both cases are of the same aspect ratio. What can be clearly seen, is that the fish foil gives almost the same thrust with the rectangular foil, but with greater efficiency for some Strouhal numbers, while it tends to the behavior of the rectangular foil when outside that "area of advantage". The experimental data confirm this observation, but present this area to be smaller.

The next step is to compare the fish foil and the rectangular in the area of advantage and outside it in order to find the reason for it.


As it is clearly seen in the visualization above, the vortical pattern is completely changed. The Vortex rings have been replaced by tip vortices and the rest of vortex sheet has much smaller potential. If the fact that fish foils are very elastic at the tips is taken into account, it can be assumed that this vorticity is reduced further.

Another interesting thing is the behavior outside the area of advantage:


What can be seen in the above layout, is the same foil for the same STR but with different pitch amplitude. As it can be seen, in the off design condition, the foil sheds vorticity resembling to the rectangular foil, something that confirms the initial idea that it behaves like a rectangular foil outside the "area of advantage". Moreover, what is not so obvious, is that it resembles to the vorticity shed by larger AR rectangular foils and it can be confirmed by the much larger thrust coefficient for smaller $\theta_{0}$.

If the fact that fishes can change the sweepback angle of the foil is taken into account, it can be assumed that the "area of advantage" discussed above can change location after a change in the foil. This gives motive for further investigation and more systematic simulations for different sweepback angles in order to optimize the effect.

## CONCLUSION:

Returning to the initial question and the scope of the present thesis, the question has to be answered: "Can a ship be placed under flapping foil propulsion?"

The answer is that it is feasible, but there is a lot of investigation and design till then. The astonishing efficiencies are succeeded with very small thrust and where the thrust is adequate for ship propulsion, the efficiency is almost the same with a conventional propeller. On the other hand, the flapping foil system, as it is now, can take advantage of a much larger area behind the ship than a propeller and if the example of the dolphins is followed, it can even take energy from the waves to produce thrust directly. Moreover, the optimization of the geometry has just started, the effect of elasticity has not been investigated enough and the harmonic motions seem not to be the followed by the fishes, meaning that they may not be optimal.

As far as the program produced for the purposes of this thesis is concerned, it is found to be accurate enough to carry on more simulations for optimization purposes and can handle less simple geometries, as long as the grid is dense enough and the timestep is sized correctly.

## RECOMMENDATIONS FOR FUTURE WORK:

As said before, what can be done in the very immediate future, is to conduct more and systematic simulations with the fishlike foil and investigate the effect of varying sweepback angles. Moreover, other motion configurations (bird or insect like motion) or other number of foils could be tested. The evaluation of a pressure kutta is also something to be looked into.

Additionally, the elasticity could be simulated, or at least, the addition of small oscillating flaps on the tips could be tested.

The program could be improved further, by means of parallelization. There already are many libraries of parallel solvers for linear systems and other operations, like the calculation of gradients (induced velocities) can be broken into pieces for parallel processing. Even further, new technologies like GPU computing could be put into effect, at least for system solving, and vector operations by using existing libraries and replacing older subroutines.

Finally, an interesting assumption that should be examined closely (and experimentally) is the ability to produce thrust from the sea waves, by using the heave produced by them and a horizontal pitching foil.


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## REFFERENCES

[1] Effect of angle of attack profiles in flapping foil propulsion F.S. Hover*, $\varnothing$. Haugsdal, M.S. Triantafyllou
[2] Hydrodynamic Performance of Deformable Fish Fins and Flapping Foils M. Bozkurttas*, H. Dong., R. Mittal
[3] Design of a Flapping Foil Underwater Vehicle Stephen Licht, Franz Hover and Michael S. Triantafyllou
[4] Drag reduction in fish-like locomotion By D. S. BARRETT, M. S. TRIANTAFYLLOU
[5] experimental visualization of the near-boundary hydrodynamics about fish like swimming bodies Alexandra Hughes Techet (2001)
[6]Experiments on the reverse Benard-von Karman vortex street produced by a flapping foil R. Godoy-Diana_, J.L. Aider_ and J.E. Wesfrei
[7] exploring the possibility of placing traditional marine vessels under oscillating foil propulsion (MIT 1997)
[8] Flapping Membranes for Thrust Production * J.M. Jiménez, J.H.J Buchholz, A.E. Staples, J.J. Allen, and A.J. Smits
[9] The effect of chordwise flexibility on the thrust and efficiency of a flapping foil $P$. Prempraneerach, F.S. Hover and M.S. Triantafyllou
[10] Hess, J.L., Panel methods in computational fluid dynamics, Annu. Rev, of Fluid Mech., 22:255-74, 1990.
[11] Hess, J.L. Higher-order numerical solution of the integral equation for the two dimensional Neumann problem. Comput. Methods App!. Mesh. Eng., 2:1-15, 1973.
[12] Hess, J.L. The problem of three-dimensional lifting flow and its solution by means of surface singularity distribution. Comput. Methods App!. Mech. Eng. 4:283-3 19, 1974.
[13] Hess, J.L., Smith, A.M.O., Calculation of nonlfting potential flow about arbitrary three-dimensional bodies. J. Ship Res., 8:22-44, 1964.
[14] force and hydrodynamic measurements of a three dimentional flapping foil (MIT 2004)
[15] Cenko, A,, PAN AIR applictions to complex configurations, J. of Aircraft, 00218669, vol 20, no. 10, 887-892, 1983.
[16] Park, M.A., Green, L.L., Montgomery, R.C., Raney, D.L., Detennination of stability and control derivatives using computational fluid dynamics and automatic differentiation, AIAA-99-3136, 1999.
[17] Forces on oscillating uniform and tapered cylinders in crossflow
HOVER, A. H. TECHET AND M. S. TRIANTAFYLLOU
[18] Tinoco, E.N., Ball, D.N., Rice, F.A., PANAIR analysis of a transport high-lift configuration, J. Aircr., 24:181-86, 1987.
[19] Forces on oscillating foils for propulsion and maneuvering D.A. Read1, F.S. Hover*, M.S. Triantafyllou
[20] Computational and experimenta investigation of flapping-foil propulsion (isabmec2000)
[21] Katz, J., Plotkin, A., Low-speed aerodynamics: from wing theory to panel methods, McGraw-Hill, New York, 1991.
[22] Rosen, B.S., Laiosa, J.P., Davis, W.H., Stavetski, D., Splash Free-Surface Flow Code Methodology for Hydrodynamic Design and Analysis of IACC Yachts, 11 Chesapeake Sailing Yacht Symposium, Annapolis, 1993.
[23] Sciavounos, P.D., Ship flow simulation in calm water and waves, SWAN 1 Version 3.1, User Manual, Boston Marine Consulting Inc., 1999.
[24] Lamb, H, Hydrodynamics., London, Cambridge University Press, 6th ed., 1932.
[25] Kellogg, O.D., Foundations of Potential Theory, Dover Publications, 1954.
[26] Brazier, J.G., and Smith, A.M.O., Development of Nose and Tail Shapes in Incompressible, Irrotational Flow, Douglas Aircraft Co., Long Beach, CA, Rept. E.S.

0875, June 1947.
[27] Eppler, R., Airfoil Design and Data, Springer Verlag, Berlin, 1990.
[28] Selig, M.S., Maughmer, M.D., A multi-point inverse airfoil design method based on conformal mapping, AIAA- 1991-69,29th Aerospace Sciences Meeting and Exhibit,1991.
[29] Smith, A.M.O., Pierce, J., Exact solution of the Neumann problem. Calculation of plane and axially symmetric flows about or within arbitrary boundaries, Proc. U.S. Nati. Congr. Appl. Mech., 3rd, Providence, R.I., pp.807-15, 1958.
[30] Wake topology and hydrodynamic performance of low-aspect-ratio flapping foils By H. DONG1, R. MITTAL1 AND F. M. NAJJAR2 (2006)
[31] Morino, L., Luo, C.C., Subsonic potential aerodynamics for complex configurations. A general theory. AIAA J., 12:191-97.
[32] The Kármán gait: novel body kinematics of rainbow trout swimming in a vortex street James C. Liao1,*, David N. Beal2, George V. Lauder3 and Michael S. Triantafyllou4 (2002)
[33] kinematics and vortical wake patterns of rapidly maneuvering fish and flapping foils (MIT 2006)
[34] Coopersmith, R.M., Youngren, H.H., Bouchard, E.E., Quadrilateral Element Panel Method (QUADPAN), Theoretical Report (V. 3.0), Lockheed-California, ER, 1983.
[35] Maskew, B., USA ERO, A Timestepping Analysis Method for the Flow About Multiple Bodies in General Motions, User Manual, Technical Report, Analytical Methods, Inc., Redmond, WA, 1990.
[36] High order panel method paper
[37] Fomasier, L., HISS-A high order subsonic/supersonic singularity method for calculating linearized potential flow, 17th AIAA Fluid dynamics, plasma dynamics andlasers conference, 1984.
[38] Drela, M., XFOIL An Analysis and Design System for Low Reynolds Number Airfoils, Conference on Low Reynolds Number Aerodynamics, University of Notre Dame, June 1989.
[39] Liebeck, R.H., Subsonic Ai,foil Design, In, Applied Computational aerodynamics, Vol 125 Progress in Astronautics and Aeronautics, AIAA, pp. 133-165, 1990.
[40] Mughal, B., Drela, M., A calculation method for the three-dimensional boundary layer equations in integral form, AIAA Paper 93-0786, Reno, NV, 1993.
[41] Nishida, B., and Drela, M., Fully Simultaneous Coupling for three dimensional viscous/inviscid flows, AIAA-95-1 806-CR AIAA 13th Applied Aerodynamics Conference, June, 1995.
[42] Bulirsch, R., Stoer, J., The con gugaze-gradient method of Hesienes and \&iiefel,Section 8,7 in Introduction to Numerical Analysis., New York, Springer Verlag, 1991.
[43] Freund, R., Nachtigal, N. QMR: A quasi-minimal residual methodfor non-hermitian linear systems, Numer. Math., 60:315-339, 1991.
[44] Nabors, K, White, J.K., FasiCap: A Multipole Accelerated 3-D Extraction Program, iEEE Trans. On Comp. Aided Design, Vol. 10, No. 11, 1991.
[45] Aluru, N.R., Nadkami, V., White, J.K., A parallel precorrected-FFT based capacitance extraction program for signal integrity analysis, Proceedings of the 33rd Design Automation Conference, Las Vegas, Jun, 1996.
[46] Board, J.A., Hakura, Z.S., Elliot, W.S., Gray, D.C., Blanke, W.J., Leathrum, J.F.,Jr.Scalable implementations of multipole-accelerated algorithms for molecular
dynamics, Technical Report 94-002, Duke University, 1994.
[47] Willis, D.J., Lee, J., Coehlo, C.P.Bardhan, J, Hu, X., Ried, H., White, J.K., pFFT2+:
A Generic precorrected-FFT algorithm in C++: Users Manual, Work in progress.
[48] Frigo, M., Johnson, S.,FFTW Users Manual, Version 3.0.1, Massachusetts Institute Of Technology, 2003.
[49] Vassberg, J, C. A fast surface-panel method capable of solving million-element pro blems, 35th AIAA Aerospace Sciences Meeting and Exhibit,AIAA-97-0168, 1997.
[50] Passive propulsion in vortex wakes By D. N. BEAL1, F. S. HOVER1, M. S. TRIANTAFYLLOU1, J. C. LIAO2 AND G. V. LAUDER2 (2004)
[51] propulsion through wake synchronization using a flapping foil (cornel university 1995)
[52] Separation and Turbulence Control in Biomimetic Flows ALEXANDRA H. TECHET, FRANZ S. HOVER and MICHAEL S. TRIANTAFYLLOU (2003)
[53] Stereoscopic PIV measurements behind a 3D flapping foil producing thrust Parker, K.1, von Ellenrieder, K. D2 and Soria, J1 (2004)
[54] Cottet, G-H, Koumoutsakos, P.D.,Vortex Methods:Theory and Applications, Cambridge University Press, London, 2000.
[55] Transitions in the wake of a flapping foil (france feb 2007)
[56] velocity characteristics in the wake of an oscilating cylinder (MIT 2001)
[57] Vortical patterns behind a tapered cylinder oscillating transversely to a uniform flow By A. H. TECHET, F. S. HOVER, AND M. S. TRIANTAFYLLOU (1997)
[58] Vorticity Control in Fish-like Propulsion and Maneuvering M. S. TRIANTAFYLLOU, A. H. TECHET, Q. ZHU, D. N. BEAL, F. S. HOVER, AND D. K. P. YUE (2002)
[59] Leonard, A. Computing three-dimensional incompressible flows with vortex elements. Ann. Rev. Fluid Mech. 17:523-559, 1985.
[60] 0. M. Knio, A. F. Ghoniem, The three-dimensional structure of periodic vorticity
layers under non-symmetric conditions, J. Fluid Mech. 243, 353-392, 1992.
[61] Spalart, P.R., Two recent extensions of thevortex particle method, presented at the 22nd Aerospace Sciences Meeting and Exhibit, AIAA Paper 1984-343, Reno, January 1984.
[62] Voutsinas S.G., Belessis M.A., and Rados K.G., Investigation of the yawed operanon of wind turbines by means of a vortex particle method. AGARD-CP-552 FDP Symposium on Aerodynamics and Aeroacoustics of Rotorcraft, Berlin, Germany, Paper 11, 1995.
[63] Rehbach, C., Calcul numerique d'ecoulement tridimensionels instationaires avec nappes tourillonnaires, La Recherche Aerospatiale, pp.289-298, 1977.
[64] Huberson, S. Calcule d'ecoulements tridimensionneis instazionnaires incomprressibles par une methode particulaire. Journal de Mecaniques Theorique et Appliquee, 3(1): 805-819, 1984.
[65] Eldredge, J. D., Efficient tools for the simulation of flapping wing flows, presented at 43rd AIAA Aerospace Sciences Meeting, AIAA Paper 2005-0085, Reno, January2005.
[66] Y.M. Marzouk, A.F. Ghoniem, Mechanism of streamwise vorticity formation in a transverse jet, 40th Aerospace Sciences Meeting and Exhibit, AIAA-2002-1063,January 2002.
[67] A. Gharakhani, A. F. Ghoniem, Three-dimensional vortex simulation of time dependent incompressible internal viscous flows, J. Comput. Phys. 134:75-95, 1997.
[68] Anderson, C., A method of local corrections for computing the velocity field due to a distribution of vortex blobs. J. Comput. Phys., vol 62, 111-123, 1986.
[69] Ploumhans, P., Winckelmans, G.S., Vortex methods for high-resolution simulations of viscous flow past bi uff bodies of general geometry. J Comput Phys, 165:354-406, 2000.
[70] Appel, A.A., An efficient program for many body simulations', SIAM Journal of scientific and statistical computing, vol. 16, n. 1, pp85-IO3, 1985.
[71] Spring stiffness influence on an oscillating propulsor, M.M. Murraya, L.E. Howleb,
[72] Forces on oscillating foils for propulsion and maneuvering, D.A. Read1, F.S. Hover*, M.S. Triantafyllou
[73] Propulsive performance of three naturally occurring oscillating propeller planforms, PENGFEI LIU and N. BOSE
[74] Hydrodynamic characteristics of a lunate shape oscillating propulsor, Pengfei Liu, Neil Bose
[75] A simple model ofpropulsive oscillating foils, Laura Guglielmini, Paolo Blondeaux, Giovanna Vittori
[76] A three-dimensional wake impingement model and applications on tandem oscillating foils, Moqin Hea, Brain Veitcha, Neil Bose, Bruce Colbourne, Pengfei Liu
[77] McConnell AJ, Applications of Tensor Analysis, Dover Publications, 1957.
[78] Press WH, Teukolsky SA, Vetterling WT, Flannery BP, Numerical Recipes in Fortran, Cambridge University Press, 1986
[79] Sarpkaya T, Computational methods with vortices-The 1988 freeman scholar lecture, Journal of Fluids Engineering, vol.111/5, 1989.
[80] Mikhlin SG, Multidimensional singular integrals and integral equations, Pergamon press, 1965.
[81] G. Papaioannou, 'Numerical simulation of unsteady separated flow around airfoils with applications to the propulsion of aquatic animals', Diploma Thesis, NTUA, 1997
[82] W.J. McCroskey \& S.L. Pucci, 'Viscous-Inviscid interaction on oscillating airfoils in subsonic flow', AIAA Journal, February 1982.
[83] D.R. Poling \& D.P. Telionis, 'The response of airfoils to periodic disturbances - The unsteady Kutta Condition', AIAA Journal, 1986.
[84] J. Katz \& D. Weihs, 'Wake Roll up and the Kutta Condition for airfoils oscillating at high frequencies', AIAA Journal, 1981.
[85] A. Zervos \& G. Coulmy, 'Unsteady periodic motion of a flexible thin propulsor using the boundary element method', GAMM Conference, 1983.
[86] R. Gopalkrishnan, M.S. Triantafyllou, G.S. Triantafyllou \& D. Barrett, 'Active vorticity control in a shear flow using a flapping foil', J.Fluid. Mech., 1994.
[87] N. Bose, 'An explicit Kutta Condition for use with an unsteady two-dimensional constant potential panel method, AIAA Journal, 1994.
[88] J.C. Wu, C.M. Wang \& I.H. Tuncer, 'Theoretical and numerical studies of oscillating airfoils', AIAA Journal, 1990.
[89] X. Wang, J.N.Newman, and J.K.White, Robust Algorithms for Boun\&uy Element Integrals on Curved Surfaces, International Conference on Modeling and Simulation of Micro systems, 2000.
[90] J. N. Newman, Distribution of Sources and Normal Dipoles Over a Quadrilateral Panel, J. Eng. Math., 20, 1985.
[91] A. Techet, M. Triantafyllou, F. Hover, Review of Experimental Work in Biomimetic Foils, IEEE 2004
[92] O.D.Kellogg Foundations of Potential Theoiy,J. Springer, New York, 1929.
[93] C.Y. Hsm, J.E.Kerwin, and J.N.Newman, A high order panel method based on BSplines. Proceedings of Sixth International Conference on Numerical Ship Hydrodynamics, eds. V.C. Patel and F. Stem, National Academy Press: Washington, pp 133-151, 1993.
[94] A.E. Magnus, and M.A.Epton, PAN AIR- A Computer Program for predicting Subsonic or Supersonic Linear Potential Flows about Arbitrary Configurations Using a High Order Panel Method, Volume 1, Theory Document (Version 1.0), NASA CR-3251, 1980.
[95] C.S. Lee and J.E. Kerwin, A B-Spline Higher-Order Panel Method Applied to Two Dimensional L(fling Problems, Journal Of Ship Research, Volume 47, Number 4, pp 290298, 2003.
[96] P. Ramachandran, S.C. Rajan, M. Ramakrishna, A Fast Multipole Method for Higher Order Vortex Panels in Two Dimensions, SIAM Journal on Scientific Computing, Vol 26, Issue 5, pp. 1620-1642, 2005.
[97] H. Maniar, A B-Spline Based Higher Order Method in 3D, Presented at the $10^{\text {th }}$ Workshop on Water Waves and Floating Bodies, Oxford, England, 1995.
[98] H. Maniar, A Three Dimensional High Order Panel Method Based on B-splines, PhD. Thesis, Massachusetts Institute Of Technology, 1995.
[99] F.E. Ehiers, F.T.Johnson, and RE. Rubbert, A Higher Order Panel Method for Linearized Supersonic Flow. AIAA 76-381, July 1976.
[100] J. Hess, and A.M.O Smith, Calculation of Non-lifting potential flow about arbitrary three dimensional bodies, J. Ship Research, 8, pp 22-44, 1994.
[101] U. Lemma, V.Marchese,and L. Morino, High-order BEM for potential transonic flows, omputational Mechanics, Springer Verlag, Vol. 21, No. 3, pp. 243-252, 1998.
[102] J.H.M. Frijns, Improving the Accuracy of the Boundary Element Method by the use of Second-Order Interpolation Functions., IEEE Transactions on Biomedical Engineering, Vol. 47, No. 10, October 2000.
[103] A. Buchau, W. Rieger, and W.M.Rucker BEM Computations Using the Fast Multipole Method in Combination with Higher Order Elements and the Galerkin Method., IEEE Transactions on Magnetics, Vol. 37 No. 5, September 2001.
[104] J. A. Sparenberg, Survey of the mathematical theory of fish locomotion, JEM 2002

# APPENDIX 1 : SUMMARY OF ALL THE SIMULATIONS MADE 

## Extended data from some of the simulations are given in Appendix 2 $10 \times 14$

ONE FOIL
chord $=1.000$ span $=4.000, A=0.15$ its=200 blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=4.000, A=0.3$, its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=4.000, A=0.4$, its $=200$ blade no 1 strouhal $n o=0.30$ thrust coefficient $=$
chord $=1.000$ span $=4.000, A=0.5$ its $=200$ blade no 1 strouhal $n o=0.30$ thrust coefficient $=$
chord $=1.000$ span $=4.000, A=0.6$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient=
chord $=1.000$ span $=4.000, A=0.7$ its $=200$ blade no 1 strouhal $n o=0.30$ thrust coefficient $=$
chord $=1.000$ span $=4.000, A=0.2$ its $=200$ blade no 1 strouhal no $=0.45$ thrust coefficient $=$
chord= 1.000 span $=4.000$, $\mathrm{A}=0.3$ its= 200 blade no 1 strouhal no= 0.45 thrust coefficient=
chord= 1.000 span $=4.000$, $A=0.4$ its= 200 blade no 1 strouhal no $=0.45$ thrust coefficient=
chord $=1.000$ span $=4.000, A=0.5$ its= 200 blade no 1 strouhal $n o=0.45$ thrust coefficient $=$
chord= 1.000 span $=4.000$, $A=0.6$ its= 200 blade no 1 strouhal no $=0.30$ thrust coefficient=
chord= 1.000 span $=4.000$, $A=0.7$ its= 200 blade no 1 strouhal no= 0.45 thrust coefficient=
$40 \times 20$
chord= 1.000 span=2.000, $A=0.2$ its=200 blade no 1 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=2.000$, $A=0.3$ its= 200 blade no 1 strouhal $n o=0.22$ thrust coefficient $=$
chord $=1.000$ span $=2.000, A=0.4$ its $=200$ blade no 1 strouhal no $=0.22$ thrust coefficient=
chord $=1.000$ span $=2.000$, $A=0.2$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient=
chord $=1.000$ span $=2.000, A=0.3$ its $=200$ blade no 1 strouhal $n o=0.30$ thrust coefficient $=$
chord $=1.000$ span $=2.000, A=0.4$ its $=200$ blade no 1 strouhal $n o=0.30$ thrust coefficient $=$
chord= 1.000 span $=2.000$, $\mathrm{A}=0.5$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient=
chord= 1.000 span $=2.000$, $\mathrm{A}=0.6$ its= 200 blade no 1 strouhal no $=0.30$ thrust coefficient=
chord= 1.000 span $=2.000, A=0.7$ its= 200 blade no 1 strouhal no $=0.30$ thrust coefficient=
chord= 1.000 span $=2.000$, $A=0.2$ its= 200 blade no 1 strouhal no $=0.45$ thrust coefficient=
chord $=1.000$ span $=2.000, A=0.3$ its= 200 blade no 1 strouhal $n o=0.45$ thrust coefficient=
chord $=1.000$ span $=2.000, A=0.4$ its $=200$ blade no 1 strouhal no $=0.45$ thrust coefficient=
$40 \times 30$
chord= 1.000 span $=3.000, \mathrm{~A}=0.1$ its=200 blade no 1 strouhal $\mathrm{no}=0.15$ thrust coefficient $=$ chord $=1.000$ span $=3.000, A=0.2$ its $=200$ blade no 1 strouhal no $=0.15$ thrust coefficient $=$
chord $=1.000$ span $=3.000, A=0.3$ its $=200$ blade no 1 strouhal no $=0.15$ thrust coefficient= chord $=1.000$ span $=3.000$, $A=0.3$ its $=200$ blade no 1 strouhal no $=0.22$ thrust coefficient=
0.347008 efficiency $=$ mean 0.316 0.2888 efficiency $=$ mean 0.405
0.171 efficiency $=$ mean 0.390
-0.01 efficiency $=$ mean -0.122
-0.26455 efficiency $=$ mean 1.435
-0.57905 efficiency $=$ mean 1.422
0.815067 efficiency $=$ mean 0.276
0.840728 efficiency $=$ mean 0.319
0.792312 efficiency $=$ mean 0.351 0.661377 efficiency $=$ mean 0.354 -0.57905 efficiency $=$ mean 1.378 0.089867 efficiency= mean 0.107
0.185261 efficiency $=$ mean 0.601 0.086919 efficiency $=$ mean 0.700 -0.02306 efficiency $=$ mean 0.371 0.40017 efficiency $=$ mean 0.315 0.288875 efficiency $=$ mean 0.405 0.166759 efficiency $=$ mean 0.486 0.027867 efficiency $=$ mean 0.176 -0.26455 efficiency $=$ mean 1.424 -0.26455 efficiency $=$ mean 1.491 1.007558 efficiency $=$ mean 0.436 0.862225 efficiency $=$ mean 0.445 0.700492 efficiency $=$ mean 0.445 0.142404 efficiency $=$ mean 0.695 0.059359 efficiency $=$ mean 0.998 -0.02899 efficiency $=$ mean 0.286 0.103449 efficiency $=$ mean 0.659
chord= 1.000 span $=3.000, A=0.1$ its= 200 blade no 1 strouhal no $=0.26$ thrust coefficient $=$ chord $=1.000$ span $=3.000$, $A=0.2$ its= 200 blade no 1 strouhal no $=0.26$ thrust coefficient= chord $=1.000$ span $=3.000$, $A=0.3$ its $=200$ blade no 1 strouhal no $=0.26$ thrust coefficient= chord $=1.000$ span $=3.000$, $A=0.4$ its $=200$ blade no 1 strouhal no $=0.26$ thrust coefficient= chord $=1.000$ span $=3.000, A=0.1$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient $=$ chord $=1.000$ span $=3.000, A=0.2$ its $=200$ blade no 1 strouhal $n o=0.30$ thrust coefficient= chord $=1.000$ span $=3.000, A=0.3$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=3.000, \mathrm{~A}=0.4$ its=200 blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=3.000$, $\mathrm{A}=0.5$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=3.000$, $A=0.6$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=3.000$, $A=0.7$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=3.000$, $A=0.1$ its $=200$ blade no 1 strouhal no $=0.40$ thrust coefficient= chord= 1.000 span $=3.000$, $A=0.2$ its $=200$ blade no 1 strouhal no $=0.40$ thrust coefficient= chord $=1.000$ span $=3.000, A=0.3$ its $=200$ blade no 1 strouhal no $=0.40$ thrust coefficient= chord $=1.000$ span $=3.000, A=0.4$ its $=200$ blade no 1 strouhal no $=0.40$ thrust coefficient= chord $=1.000$ span $=3.000, A=0.5$ its $=200$ blade no 1 strouhal no $=0.40$ thrust coefficient= chord $=1.000$ span $=3.000$, $A=0.6$ its $=200$ blade no 1 strouhal no $=0.40$ thrust coefficient= chord $=1.000$ span $=3.000$, $A=0.7$ its $=200$ blade no 1 strouhal no $=0.40$ thrust coefficient= $40 \times 30$
two foil
chord= 1.000 span $=3.000, \mathrm{~A}=0.1$ its= 200 blade no 2 strouhal no $=0.15$ thrust coefficient=
chord $=1.000$ span $=3.000$, $A=0.2$ its $=200$ blade no 2 strouhal no $=0.15$ thrust coefficient= chord $=1.000$ span $=3.000, \mathrm{~A}=0.3$ its= 200 blade no 2 strouhal no $=0.15$ thrust coefficient= chord $=1.000$ span $=3.000, A=0.1$ its $=200$ blade no 2 strouhal no $=0.26$ thrust coefficient= chord= 1.000 span $=3.000$, $A=0.2$ its= 200 blade no 2 strouhal no $=0.26$ thrust coefficient= chord= 1.000 span $=3.000, A=0.3$ its=200 blade no 2 strouhal no $=0.26$ thrust coefficient= chord $=1.000$ span $=3.000$, $\mathrm{A}=0.4$ its $=200$ blade no 2 strouhal no $=0.26$ thrust coefficient= chord $=1.000$ span $=3.000, A=0.1$ its=200 blade no 2 strouhal $n o=0.30$ thrust coefficient $=$ chord $=1.000$ span $=3.000$, $A=0.2$ its $=200$ blade no 2 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=3.000$, $A=0.3$ its=200 blade no 2 strouhal no $=0.30$ thrust coefficient $=$ chord= 1.000 span $=3.000, A=0.4$ its= 200 blade no 2 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=3.000, \mathrm{~A}=0.5$ its=200 blade no 2 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=3.000, \mathrm{~A}=0.6$ its $=200$ blade no 2 strouhal $\mathrm{no}=0.30$ thrust coefficient $=$ chord $=1.000$ span $=3.000$, $\mathrm{A}=0.7$ its $=200$ blade no 2 strouhal no $=0.30$ thrust coefficient=
0.477818 efficiency $=$ mean 0.522 0.349452 efficiency $=$ mean 0.544 0.211954 efficiency $=$ mean 0.563 0.061968 efficiency $=$ mean 0.546 0.627773 efficiency $=$ mean 0.503 0.489249 efficiency $=$ mean 0.516 0.343417 efficiency $=$ mean 0.520 0.187276 efficiency $=$ mean 0.492 0.017911 efficiency $=$ mean 0.183 -0.167620 efficiency $=$ mean 1.220 -0.374285 efficiency $=$ mean 0.971 1.15195 efficiency $=$ mean 0.450 0.987889 efficiency $=$ mean 0.460 0.815378 efficiency $=$ mean 0.465 0.629993 efficiency $=$ mean 0.457 0.427705 efficiency $=$ mean 0.419 0.203891 efficiency $=$ mean 0.303 -0.04613 efficiency $=$ mean -0.085
0.157909 efficiency $=$ mean 0.698 0.063644 efficiency $=$ mean 0.995 -0.034420 efficiency $=$ mean 0.301 0.231373 efficiency $=$ mean 0.573 0.384969 efficiency $=$ mean 0.554 0.231372 efficiency $=$ mean 0.573 0.064756 efficiency $=$ mean 0.582 0.683506 efficiency $=$ mean 0.512 0.527583 efficiency $=$ mean 0.526 0.361131 efficiency $=$ mean 0.536 0.180596 efficiency $=$ mean 0.525 -0.017270 efficiency $=$ mean -0.326 -0.232690 efficiency $=$ mean 0.807 -0.464690 efficiency $=$ mean 0.800 12X30

TWO FOIL
chord= 1.000 span $=3.000$, $\mathrm{A}=0.1$ its=200 blade no 2 strouhal no $=0.15$ thrust coefficient= chord= 1.000 span $=3.000, A=0.2$ its= 200 blade no 2 strouhal no $=0.15$ thrust coefficient= chord= 1.000 span $=3.000$, $A=0.3$ its= 200 blade no 2 strouhal no $=0.15$ thrust coefficient=
0.059706 efficiency $=$ mean 0.581
0.042741 efficiency $=$ mean 0.903 -0.043412 efficiency $=$ mean 0.329
$10 \times 14$
one foil
chord $=1.000$ span $=4.000, A=0.2$ its $=200$ blade no 1 strouhal no $=0.45$ thrust coefficient= chord $=1.000$ span $=4.000, A=0.3$ its $=200$ blade no 1 strouhal $n o=0.30$ thrust coefficient $=$ chord $=1.000$ span $=4.000$, $A=0.4$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=4.000$, $\mathrm{A}=0.5$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=4.000, A=0.6$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= chord= 1.000 span $=4.000, A=0.7$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= chord= 1.000 span $=4.000, A=0.3$ its= 200 blade no 1 strouhal no $=0.45$ thrust coefficient= chord $=1.000$ span $=4.000, A=0.4$ its $=200$ blade no 1 strouhal no $=0.45$ thrust coefficient= chord $=1.000$ span $=4.000, A=0.5$ its $=200$ blade no 1 strouhal no $=0.45$ thrust coefficient= chord $=1.000$ span $=4.000, A=0.6$ its $=200$ blade no 1 strouhal no $=0.45$ thrust coefficient= $40 \times 20$
one foil
chord $=1.000$ span $=2.000$, $A=0.2$ its $=500$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=2.000$, $A=0.3$ its $=500$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=2.000$, $A=0.4$ its $=500$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=2.000, \mathrm{~A}=0.2$ its=200 blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=2.000, A=0.2$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= $40 \times 20$
two foil
chord= 1.000 span $=2.000, A=0.1$ its= 200 blade no 2 strouhal no $=-0.22$ thrust coefficient $=$ chord $=1.000$ span $=2.000, A=0.2$ its= 200 blade no 2 strouhal no $=-0.22$ thrust coefficient= chord $=1.000$ span $=2.000$, $\mathrm{A}=0.3$ its $=200$ blade no 2 strouhal no $=-0.22$ thrust coefficient $=$ chord $=1.000$ span $=2.000$, $\mathrm{A}=0.4$ its $=200$ blade no 2 strouhal no $=-0.22$ thrust coefficient $=$ chord $=1.000$ span $=2.000, A=0.2$ its $=200$ blade no 2 strouhal $n o=-0.30$ thrust coefficient $=$ chord $=1.000$ span $=2.000, A=0.3$ its $=200$ blade no 2 strouhal no $=-0.30$ thrust coefficient $=$ chord $=1.000$ span $=2.000, A=0.4$ its $=200$ blade no 2 strouhal $n o=-0.30$ thrust coefficient $=$ chord $=1.000$ span $=2.000, \mathrm{~A}=0.5$ its $=200$ blade no 2 strouhal $n o=-0.30$ thrust coefficient $=$ chord $=1.000$ span $=2.000$, $A=0.6$ its $=200$ blade no 2 strouhal no $=-0.30$ thrust coefficient $=$ chord $=1.000$ span $=2.000, A=0.7$ its $=200$ blade no 2 strouhal $n o=-0.30$ thrust coefficient $=$
0.815067 efficiency $=$ mean 0.280
0.288835 efficiency $=$ mean 0.406
0.347008 efficiency $=$ mean 0.320
-0.013610 efficiency $=$ mean -0.040
-0.264550 efficiency $=$ mean 1.701
-0.579050 efficiency $=$ mean 1.493 0.840728 efficiency $=$ mean 0.322 0.792312 efficiency $=$ mean 0.350 0.661377 efficiency $=$ mean 0.363 0.124066 efficiency $=$ mean 0.145
0.185261 efficiency $=$ mean 0.601 0.086919 efficiency $=$ mean 0.700 -0.023060 efficiency $=$ mean 0.371 0.400170 efficiency $=$ mean 0.320 0.288835 efficiency $=$ mean 0.406
0.298890 efficiency= mean 0.553 0.204582 efficiency $=$ mean 0.598 0.098567 efficiency $=$ mean 0.680 -0.082511 efficiency $=$ mean 0.379 0.417213 efficiency $=$ mean 0.500 0.292192 efficiency $=$ mean 0.509 0.151652 efficiency $=$ mean 0.478 -0.006800 efficiency $=$ mean -0.003
-0.183660 efficiency $=$ mean 1.154 -0.376540 efficiency $=$ mean 0.999
chord= 1.000 span $=2.000, A=0.2$ its= 200 blade no 2 strouhal no $=-0.45$ thrust coefficient $=$ chord= 1.000 span $=2.000, A=0.3$ its= 200 blade no 2 strouhal no $=-0.45$ thrust coefficient= chord= 1.000 span=2.000, A $=0.4$ its= 200 blade no 2 strouhal no $=-0.45$ thrust coefficient= $40 \times 28$
one foil
chord $=1.000$ span $=6.000, A=0.1$ its= 200 blade no 1 strouhal $n o=0.15$ thrust coefficient $=$ chord $=1.000$ span $=6.000, A=0.2$ its $=200$ blade no 1 strouhal no $=0.15$ thrust coefficient= chord $=1.000$ span $=6.000$, $\mathrm{A}=0.3$ its= 200 blade no 1 strouhal $\mathrm{no}=0.15$ thrust coefficient= chord $=1.000$ span $=6.000$, $\mathrm{A}=0.1$ its= 200 blade no 1 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=6.000, A=0.1$ its $=200$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.3$ its $=200$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.4$ its $=200$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord= 1.000 span $=6.000$, $A=0.1$ its= 200 blade no 1 strouhal no $=0.26$ thrust coefficient= chord= 1.000 span $=6.000, A=0.2$ its= 200 blade no 1 strouhal no $=0.26$ thrust coefficient= chord= 1.000 span $=6.000$, $A=0.3$ its= 200 blade no 1 strouhal no $=0.26$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.4$ its $=200$ blade no 1 strouhal no $=0.26$ thrust coefficient= chord= 1.000 span $=6.000$, $A=0.1$ its= 200 blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.2$ its= 200 blade no 1 strouhal $n o=0.30$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.3$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=6.000$, $\mathrm{A}=0.4$ its= 200 blade no 1 strouhal $\mathrm{no}=0.30$ thrust coefficient= chord $=1.000$ span $=6.000, \mathrm{~A}=0.5$ its=200 blade no 1 strouhal no $=0.30$ thrust coefficient $=$ chord $=1.000$ span $=6.000$, $A=0.6$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=6.000, \mathrm{~A}=0.7$ its=200 blade no 1 strouhal no $=0.30$ thrust coefficient $=$ chord $=1.000$ span $=6.000, A=0.1$ its $=200$ blade no 1 strouhal no $=0.40$ thrust coefficient= chord= 1.000 span $=6.000$, $A=0.2$ its= 200 blade no 1 strouhal no $=0.40$ thrust coefficient= chord= 1.000 span $=6.000, A=0.3$ its= 200 blade no 1 strouhal no $=0.40$ thrust coefficient $=$ chord $=1.000$ span $=6.000, A=0.4$ its $=200$ blade no 1 strouhal no $=0.40$ thrust coefficient= chord= 1.000 span $=6.000$, $\mathrm{A}=0.5$ its= 200 blade no 1 strouhal $\mathrm{no}=0.40$ thrust coefficient= chord $=1.000$ span $=6.000, A=0.6$ its $=200$ blade no 1 strouhal no $=0.40$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.7$ its $=200$ blade no 1 strouhal no $=0.40$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.2$ its $=500$ blade no 1 strouhal no $=0.45$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.3$ its $=500$ blade no 1 strouhal no $=0.45$ thrust coefficient= chord $=1.000$ span $=6.000, A=0.4$ its $=500$ blade no 1 strouhal $n o=0.45$ thrust coefficient $=$ 40×28
two foil
chord= 1.000 span $=6.000, A=0.1$ its $=200$ blade no 2 strouhal no $=0.15$ thrust coefficient=
1.106047 efficiency= mean 0.435 0.957126 efficiency= mean 0.445 0.788513 efficiency $=$ mean 0.446
0.175659 efficiency $=$ mean 0.677 0.063503 efficiency $=$ mean 1.045 -0.045030 efficiency $=$ mean 0.327 0.264172 efficiency $=$ mean 0.583 0.264172 efficiency $=$ mean 0.583 0.105345 efficiency $=$ mean 0.655 -0.005129 efficiency $=$ mean 0.456 0.582847 efficiency $=$ mean 0.523 0.410114 efficiency $=$ mean 0.539 0.231891 efficiency $=$ mean 0.554 0.076510 efficiency $=$ mean 0.461 0.676661 efficiency $=$ mean 0.369 0.614708 efficiency $=$ mean 0.505 0.418424 efficiency $=$ mean 0.505 0.210237 efficiency $=$ mean 0.468 -0.012940 efficiency $=$ mean -0.102 -0.256670 efficiency $=$ mean 0.972 -0.528210 efficiency $=$ mean 0.863 1.469389 efficiency $=$ mean 0.439 1.241258 efficiency $=$ mean 0.448 1.006080 efficiency $=$ mean 0.452 0.752813 efficiency $=$ mean 0.444 0.484220 efficiency $=$ mean 0.399 0.186077 efficiency $=$ mean 0.255 -0.149350 efficiency $=$ mean -0.442 1.488071 efficiency $=$ mean 0.429 1.228745 efficiency $=$ mean 0.436 0.952154 efficiency $=$ mean 0.438
chord= 1.000 span $=6.000$, $A=0.2$ its= 200 blade no 2 strouhal no $=0.15$ thrust coefficient= chord $=1.000$ span $=6.000$, $\mathrm{A}=0.3$ its= 200 blade no 2 strouhal $\mathrm{no}=0.15$ thrust coefficient $=$ chord $=1.000$ span $=6.000, \mathrm{~A}=0.1$ its=200 blade no 2 strouhal $\mathrm{no}=0.22$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.1$ its $=200$ blade no 2 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.3$ its $=200$ blade no 2 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.4$ its $=200$ blade no 2 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.1$ its $=200$ blade no 2 strouhal no $=0.26$ thrust coefficient= chord $=1.000$ span $=6.000, A=0.2$ its=200 blade no 2 strouhal $n o=0.26$ thrust coefficient $=$ chord $=1.000$ span $=6.000$, $\mathrm{A}=0.3$ its=200 blade no 2 strouhal $\mathrm{no}=0.26$ thrust coefficient $=$ chord $=1.000$ span $=6.000$, $\mathrm{A}=0.4$ its=200 blade no 2 strouhal no $=0.26$ thrust coefficient $=$ chord $=1.000$ span $=6.000, A=0.1$ its $=200$ blade no 2 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=6.000, \mathrm{~A}=0.2$ its $=200$ blade no 2 strouhal $\mathrm{no}=0.30$ thrust coefficient $=$ chord= 1.000 span $=6.000$, $\mathrm{A}=0.3$ its= 200 blade no 2 strouhal no $=0.30$ thrust coefficient= chord= 1.000 span $=6.000$, $\mathrm{A}=0.4$ its $=200$ blade no 2 strouhal no $=0.30$ thrust coefficient= chord= 1.000 span $=6.000, \mathrm{~A}=0.5$ its=200 blade no 2 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=6.000, A=0.6$ its $=200$ blade no 2 strouhal $n o=0.30$ thrust coefficient $=$ chord $=1.000$ span $=6.000$, $A=0.7$ its $=200$ blade no 2 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.1$ its $=200$ blade no 2 strouhal no $=0.40$ thrust coefficient= chord $=1.000$ span $=6.000, A=0.2$ its $=200$ blade no 2 strouhal no $=0.40$ thrust coefficient= chord $=1.000$ span $=6.000$, $\mathrm{A}=0.3$ its $=200$ blade no 2 strouhal no $=0.40$ thrust coefficient= chord $=1.000$ span $=6.000, \mathrm{~A}=0.4$ its=200 blade no 2 strouhal $\mathrm{no}=0.40$ thrust coefficient $=$ chord $=1.000$ span $=6.000$, $A=0.5$ its $=200$ blade no 2 strouhal no $=0.40$ thrust coefficient= chord $=1.000$ span $=6.000, \mathrm{~A}=0.6$ its $=200$ blade no 2 strouhal $\mathrm{no}=0.40$ thrust coefficient $=$ chord $=1.000$ span $=6.000$, $\mathrm{A}=0.7$ its $=200$ blade no 2 strouhal no $=0.40$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.2$ its $=500$ blade no 2 strouhal no $=0.45$ thrust coefficient= chord= 1.000 span $=6.000, A=0.3$ its $=500$ blade no 2 strouhal no $=0.45$ thrust coefficient= chord= 1.000 span $=6.000$, $A=0.4$ its $=500$ blade no 2 strouhal no $=0.45$ thrust coefficient= $40 \times 40$
two foil
chord $=1.000$ span $=9.000, \mathrm{~A}=0.1$ its $=200$ blade no 2 strouhal no $=-0.15$ thrust coefficient $=$ chord $=1.000$ span $=9.000, A=0.2$ its= 200 blade no 2 strouhal $n o=-0.15$ thrust coefficient $=$ chord $=1.000$ span $=9.000, A=0.3$ its= 200 blade no 2 strouhal no $=-0.15$ thrust coefficient $=$ chord $=1.000$ span $=9.000, A=0.1$ its $=200$ blade no 2 strouhal $n o=-0.22$ thrust coefficient $=$ chord $=1.000$ span $=9.000, A=0.2$ its $=200$ blade no 2 strouhal no $=-0.22$ thrust coefficient= chord $=1.000$ span $=9.000, A=0.3$ its $=200$ blade no 2 strouhal no $=-0.22$ thrust coefficient $=$ chord $=1.000$ span $=9.000$, $\mathrm{A}=0.4$ its= 200 blade no 2 strouhal no $=-0.22$ thrust coefficient=
0.029623 efficiency $=$ mean 1.065 -0.045030 efficiency $=$ mean 0.347 0.262622 efficiency $=$ mean 0.593 0.224672 efficiency $=$ mean 0.583 0.105345 efficiency $=$ mean 0.695 -0.005129 efficiency $=$ mean 0.476 0.582447 efficiency $=$ mean 0.543 0.410264 efficiency $=$ mean 0.559 0.262431 efficiency $=$ mean 0.564 0.072610 efficiency $=$ mean 0.491 0.626561 efficiency $=$ mean 0.379 0.614708 efficiency $=$ mean 0.535 0.418362 efficiency $=$ mean 0.525 0.210237 efficiency $=$ mean 0.488 -0.062940 efficiency $=$ mean -0.122 -0.252666 efficiency $=$ mean 0.992 -0.562210 efficiency $=$ mean 0.873 1.463593 efficiency $=$ mean 0.449 1.243658 efficiency $=$ mean 0.458 1.056080 efficiency $=$ mean 0.482 0.765443 efficiency $=$ mean 0.454 0.445720 efficiency $=$ mean 0.419 0.186077 efficiency $=$ mean 0.275 -0.146850 efficiency $=$ mean -0.452 1.484236 efficiency $=$ mean 0.439 1.227445 efficiency $=$ mean 0.466 0.964154 efficiency $=$ mean 0.488
0.233805 efficiency $=$ mean 0.686 0.077613 efficiency $=$ mean 0.966 -0.068530 efficiency $=$ mean 0.365 0.549084 efficiency $=$ mean 0.575 0.342408 efficiency $=$ mean 0.589 0.132435 efficiency $=$ mean 0.644 -0.082150 efficiency $=$ mean 0.473
chord= 1.000 span $=9.000, A=0.1$ its= 200 blade no 2 strouhal no $=-0.26$ thrust coefficient $=$ chord= 1.000 span $=9.000, A=0.2$ its= 200 blade no 2 strouhal no $=-0.26$ thrust coefficient= chord $=1.000$ span $=9.000, A=0.3$ its $=200$ blade no 2 strouhal no $=-0.26$ thrust coefficient $=$ chord $=1.000$ span $=9.000, A=0.4$ its $=200$ blade no 2 strouhal no $=-0.26$ thrust coefficient $=$ chord $=1.000$ span $=9.000, A=0.1$ its $=200$ blade no 2 strouhal $n o=-0.30$ thrust coefficient $=$ chord= 1.000 span $=9.000$, $A=0.2$ its= 200 blade no 2 strouhal no $=-0.30$ thrust coefficient= chord $=1.000$ span $=9.000, A=0.3$ its $=200$ blade no 2 strouhal $n o=-0.30$ thrust coefficient $=$ chord $=1.000$ span $=9.000, \mathrm{~A}=0.4$ its $=200$ blade no 2 strouhal no $=-0.30$ thrust coefficient $=$ chord $=1.000$ span $=9.000, A=0.5$ its $=200$ blade no 2 strouhal no $=-0.30$ thrust coefficient $=$ chord $=1.000$ span $=9.000, A=0.6$ its $=200$ blade no 2 strouhal no $=-0.30$ thrust coefficient $=$ chord $=1.000$ span $=9.000$, $A=0.7$ its= 200 blade no 2 strouhal no $=-0.30$ thrust coefficient= chord $=1.000$ span $=9.000, A=0.3$ its $=200$ blade no 2 strouhal no $=-0.40$ thrust coefficient $=$ chord $=1.000$ span $=9.000$, $\mathrm{A}=0.4$ its= 200 blade no 2 strouhal no $=-0.40$ thrust coefficient= chord $=1.000$ span $=9.000, A=0.6$ its $=200$ blade no 2 strouhal no $=-0.40$ thrust coefficient $=$ chord $=1.000$ span $=9.000, A=0.3$ its $=200$ blade no 2 strouhal no $=-0.45$ thrust coefficient $=$ chord $=1.000$ span $=9.000, A=0.4$ its $=200$ blade no 2 strouhal $n o=-0.45$ thrust coefficient $=$ chord $=1.000$ span $=9.000$, $\mathrm{A}=0.5$ its $=200$ blade no 2 strouhal no $=-0.45$ thrust coefficient= $40 \times 40$
one foil
chord= 1.000 span $=9.000, A=0.1$ its= 200 blade no 1 strouhal no $=0.15$ thrust coefficient= chord $=1.000$ span $=9.000, A=0.2$ its $=200$ blade no 1 strouhal no $=0.15$ thrust coefficient= chord $=1.000$ span $=9.000$, $\mathrm{A}=0.3$ its= 200 blade no 1 strouhal no $=0.15$ thrust coefficient= chord $=1.000$ span $=9.000$, $A=0.2$ its $=500$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=9.000$, $A=0.2$ its $=500$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord= 1.000 span $=9.000$, $\mathrm{A}=0.3$ its $=500$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord= 1.000 span $=9.000, A=0.4$ its $=500$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=9.000, \mathrm{~A}=0.1$ its=200 blade no 1 strouhal no $=0.26$ thrust coefficient $=$ chord $=1.000$ span $=9.000, A=0.2$ its= 200 blade no 1 strouhal no= 0.26 thrust coefficient= chord $=1.000$ span $=9.000, A=0.3$ its $=200$ blade no 1 strouhal $n o=0.26$ thrust coefficient $=$ chord $=1.000$ span $=9.000$, $A=0.4$ its $=200$ blade no 1 strouhal no $=0.26$ thrust coefficient= chord $=1.000$ span $=9.000, A=0.1$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=9.000, A=0.2$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=9.000$, $\mathrm{A}=0.3$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient= chord= 1.000 span $=9.000$, $\mathrm{A}=0.4$ its= 200 blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=9.000, \mathrm{~A}=0.5$ its=200 blade no 1 strouhal no $=0.30$ thrust coefficient $=$ chord $=1.000$ span $=9.000$, $\mathrm{A}=0.6$ its $=200$ blade no 1 strouhal no $=0.30$ thrust coefficient=
0.754899 efficiency $=$ mean 0.540 0.529107 efficiency $=$ mean 0.548 0.295776 efficiency $=$ mean 0.556 0.050647 efficiency $=$ mean 0.549 0.745644 efficiency $=$ mean 0.382 0.667211 efficiency $=$ mean 0.520 0.440692 efficiency $=$ mean 0.524 0.198434 efficiency $=$ mean 0.504 -0.060950 efficiency $=$ mean 1.624 -0.335850 efficiency $=$ mean 0.728 -0.628420 efficiency $=$ mean 0.733 1.148366 efficiency $=$ mean 0.464 0.821998 efficiency $=$ mean 0.461 0.092398 efficiency $=$ mean 0.205 1.641597 efficiency $=$ mean 0.441 1.292089 efficiency $=$ mean 0.439 0.927072 efficiency $=$ mean 0.416
0.197352 efficiency $=$ mean 0.668 0.069629 efficiency $=$ mean 0.975 -0.056810 efficiency $=$ mean 0.362 0.282446 efficiency $=$ mean 0.581 0.282446 efficiency $=$ mean 0.581 0.108837 efficiency $=$ mean 0.653 -0.072800 efficiency $=$ mean 0.473 0.656306 efficiency $=$ mean 0.517 0.463961 efficiency $=$ mean 0.531 0.264128 efficiency $=$ mean 0.540 0.051842 efficiency $=$ mean 0.487 0.676661 efficiency $=$ mean 0.369 0.614708 efficiency $=$ mean 0.505 0.418424 efficiency $=$ mean 0.505
0.210237 efficiency $=$ mean 0.468
-0.012940 efficiency $=$ mean -0.102
-0.256670 efficiency $=$ mean 0.972
chord= 1.000 span $=9.000$, $\mathrm{A}=0.7$ its= 200 blade no 1 strouhal no $=0.30$ thrust coefficient= chord $=1.000$ span $=9.000$, $\mathrm{A}=0.3$ its= 200 blade no 1 strouhal no $=0.40$ thrust coefficient= chord $=1.000$ span $=9.000, \mathrm{~A}=0.4$ its=200 blade no 1 strouhal no= 0.40 thrust coefficient $=$ chord $=1.000$ span $=9.000, A=0.6$ its $=200$ blade no 1 strouhal $n o=0.40$ thrust coefficient $=$ chord $=1.000$ span $=9.000, A=0.2$ its $=500$ blade no 1 strouhal $n o=0.45$ thrust coefficient $=$ chord $=1.000$ span $=9.000$, $A=0.3$ its $=500$ blade no 1 strouhal no $=0.45$ thrust coefficient= chord $=1.000$ span $=9.000, A=0.4$ its $=500$ blade no 1 strouhal $n o=0.45$ thrust coefficient $=$ $40 \times 20$
one foil
chord= 1.000 span $=2.000$, $A=0.2$ its $=500$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=2.000$, $A=0.3$ its $=500$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord= 1.000 span $=2.000$, $A=0.4$ its $=500$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord= 1.000 span $=2.000$, $A=0.2$ its $=500$ blade no 1 strouhal no $=0.45$ thrust coefficient= chord $=1.000$ span $=2.000$, $A=0.3$ its $=500$ blade no 1 strouhal no $=0.45$ thrust coefficient= chord= 1.000 span $=2.000$, $A=0.4$ its $=500$ blade no 1 strouhal no $=0.45$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.1$ its $=500$ blade no 1 strouhal no $=0.15$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.2$ its $=500$ blade no 1 strouhal no $=0.15$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.3$ its $=500$ blade no 1 strouhal no $=0.15$ thrust coefficient= chord $=1.000$ span $=6.000, A=0.1$ its $=500$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.3$ its $=500$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.4$ its= 500 blade no 1 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.1$ its $=500$ blade no 1 strouhal no $=0.26$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.2$ its $=500$ blade no 1 strouhal no $=0.26$ thrust coefficient= chord $=1.000$ span $=6.000, A=0.3$ its $=500$ blade no 1 strouhal no $=0.26$ thrust coefficient= 40X28
one foil fish
chord= 1.000 span $=6.000$, $A=0.1$ its= 200 blade no 1 strouhal no $=0.15$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.2$ its $=200$ blade no 1 strouhal no $=0.15$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.3$ its= 200 blade no 1 strouhal no $=0.15$ thrust coefficient= chord $=1.000$ span $=6.000, A=0.1$ its $=200$ blade no 1 strouhal no $=0.22$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.2$ its $=200$ blade no 1 strouhal $n o=0.22$ thrust coefficient= chord $=1.000$ span $=6.000$, $A=0.3$ its= 200 blade no 1 strouhal no $=0.22$ thrust coefficient $=$ chord $=1.000$ span $=6.000$, $A=0.1$ its $=200$ blade no 1 strouhal no $=0.40$ thrust coefficient= chord $=1.000$ span $=6.000$, $\mathrm{A}=0.2$ its= 200 blade no 1 strouhal $\mathrm{no}=0.40$ thrust coefficient $=$ chord= 1.000 span $=6.000$, $A=0.3$ its= 200 blade no 1 strouhal no $=0.40$ thrust coefficient=
-0.528210 efficiency $=$ mean 0.863 1.082879 efficiency $=$ mean 0.450 0.800534 efficiency $=$ mean 0.442 0.172739 efficiency $=$ mean 0.237 1.599689 efficiency $=$ mean 0.426 1.309736 efficiency $=$ mean 0.436 1.007398 efficiency $=$ mean 0.435
0.184622 efficiency $=$ mean 0.604 0.087381 efficiency $=$ mean 0.702 -0.027231 efficiency $=$ mean 0.372 1.007528 efficiency $=$ mean 0.437 0.864235 efficiency $=$ mean 0.448 0.700681 efficiency $=$ mean 0.448 0.176752 efficiency $=$ mean 0.681 0.063214 efficiency $=$ mean 1.047 -0.045432 efficiency $=$ mean 0.329 0.264867 efficiency $=$ mean 0.584 0.107545 efficiency $=$ mean 0.651 -0.005134 efficiency $=$ mean 0.457 0.583557 efficiency $=$ mean 0.523 0.412354 efficiency $=$ mean 0.535 0.231891 efficiency $=$ mean 0.553
0.175659 efficiency $=$ mean 0.677 0.063503 efficiency $=$ mean 1.045 -0.045030 efficiency $=$ mean 0.327 0.264172 efficiency $=$ mean 0.583 0.105345 efficiency $=$ mean 0.655 -0.005129 efficiency $=$ mean 0.456
1.469389 efficiency $=$ mean 0.439
1.241258 efficiency $=$ mean 0.448
1.006080 efficiency $=$ mean 0.452

## APPENDIX 2 :DETAILED RESULTS OF SIMULATIONS













| $\begin{gathered} \text { NACA } \\ \text { AR } \end{gathered}$ | $\begin{gathered} 0012 \\ 9 \end{gathered}$ | STR | 0.22 | $\theta_{0}$ | 0.2 |  |  | $\begin{gathered} \mathrm{Ct} \\ \mathrm{n} \end{gathered}$ | $\begin{gathered} 0.3224 \\ 0.589 \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ei |  |  | 1 |  |  |  |  |  |  |
| 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 |
|  |  |  |  |  |  |  |  |  |  |
| C | C | ¢ |  |  | $\hat{S}_{x}$ |  | ${\underset{c}{d}}^{\infty}$ | $\underbrace{\infty}_{c}$ |  |
| ${ }_{c}{ }^{\Gamma}$ | $\psi_{0}$ | \% | ) 「 | $\mathcal{K}_{2}{ }^{r}$ | $W^{2}$ | $\psi_{0}$ | $\frac{a}{b}$ | Q | ${ }_{S}$ |





| NACA <br> AR | $\begin{gathered} 0012 \\ 9 \end{gathered}$ | STR | 0.4 | $\theta_{0}$ | 0.3 |  |  | $\begin{gathered} \mathrm{Ct} \\ \mathrm{n} \end{gathered}$ | $\begin{gathered} 1.1483 \\ 0.464 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| E |  |  |  |  |  |  |  |  | Ct n A A in P P out <br> < |
| 100 | 110 | 120 | 130 | 140 | 150 | 160 | 170 | 180 | 190 |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |








[^0]:    ${ }^{1}$ For several paragraphs of the historical reference, credits are to be given to G Papaioannou [81], A. Techet [91], J. Sparenberg [104]

[^1]:    ${ }^{2}$ lift-based mode of swimming; that is, by flapping their pectoral fins

[^2]:    ${ }^{3}$ (Kress R. "Linear integral equations. Applied mathematical series", Mikhlin SG. "Multidimentional singular integrals and integral
    equations")

