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Διατμηματικό μεταπτυχιακό πρόγραμμα σπουδών

"Ναυτική και θαλάσσια τεχνολογία και επιστήμη"

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Αφιερώνεται στους
γονείς μου.

ΠΕΡΙΕΧΟΜΕΝΑ

<u>ΠΡΟΛΟΓΟΣ</u>

Η κυριότερη κατηγορία φορτίων που επάγονται σε μια θαλάσσια κατασκευή, είτε αυτή εδράζεται στον πυθμένα είτε όχι, από τη δράση των στοιχείων του περιβάλλοντος πάνω της, είναι εκείνη που προκαλείται από τους θαλάσσιους κυματισμούς. Ο υπολογισμός της οριζόντιας και κατακόρυφης δύναμης έκπτωσης που ασκείται πάνω στην κατασκευή αποτελεί αντικείμενο μελέτης πολλά χρόνια.

Για την επίλυση του προβλήματος υπολογισμού της δύναμης έκπτωσης γενικά χρησιμοποιούνται δύο μέθοδοι. Η "*μέθοδος μεταβολής της ορμής"* Σ.Α. Μαυράκος (1986), υπολογίζει τις δυνάμεις έκπτωσης χρησιμοποιώντας τη θεωρία διαταραχών. Ο τρόπος υπολογισμού της δύναμης έκπτωσης έχει μεγάλα πλεονεκτήματα, την αναλυτική επίλυση και τον άμεσο υπολογισμό αυτής.

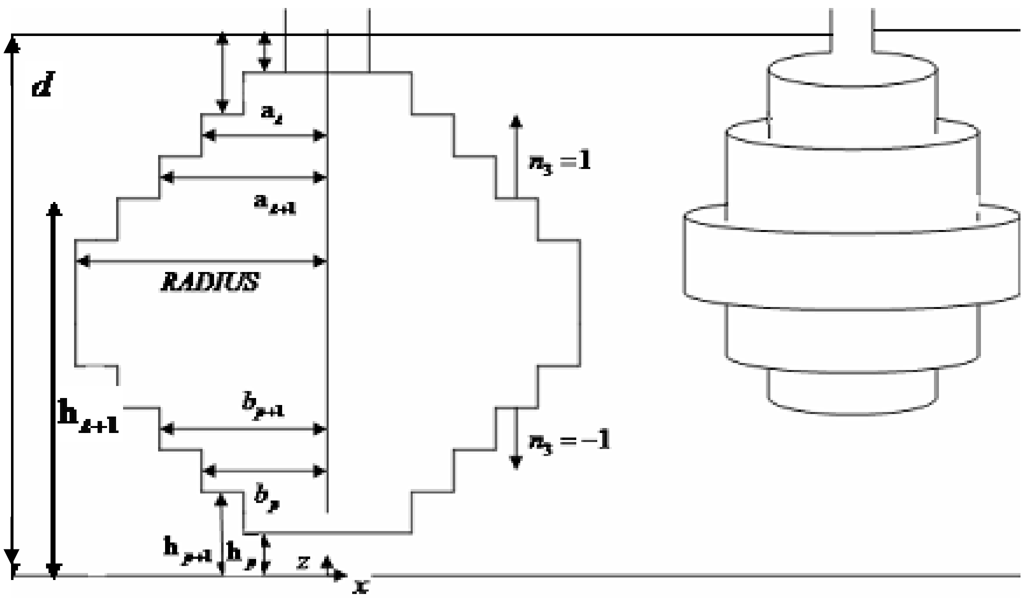
Ο δεύτερος τρόπος υπολογισμού είναι με τη μέθοδο της "απευθείας ολοκλήρωσης πάνω στο σώμα". Σύμφωνα με αυτή την μέθοδο το σώμα δεν εξετάζεται στο άπειρο, αλλά για τον υπολογισμό των δυνάμεων έκπτωσης ολοκληρώνουμε πάνω στο σώμα. Σκοπός της παρούσας εργασίας είναι ο υπολογισμός της οριζόντιας και κατακόρυφης δύναμης έκπτωσης με τη μέθοδο της απ' ευθείας ολοκλήρωσης καθώς και η σύνταξη μιας υπορουτίνας σε γλώσσα προγραμματισμού *Fortran* για τον άμεσο υπολογισμό των δυνάμεων έκπτωσης με την μέθοδο αυτή.

Η διπλωματική εργασία δεν θα ήταν δυνατόν να περατωθεί χωρίς την καταλυτική βοήθεια του Καθηγητή κ. Σ. Α. Μαυράκου, ο οποίος παρακολούθησε την προσπάθεια, υπέδειξε τις απαραίτητες διορθώσεις και συνέβαλε, με την εμπειρία του και την γνώση του, την ενεργητική συμπαράσταση και ενθάρρυνσή του, στην ολοκλήρωση της υπορουτίνας του προγράμματος. Θέλω να ευχαριστήσω τον κ. Ι. Θάνο για τη συνεργασία του στη σύγκριση των αποτελεσμάτων αυτής της εργασίας με δικά του προγράμματα, πράγμα που απαιτούσε χρόνο και υπομονή. Τέλος, ένα θερμό ευχαριστώ στον υποψήφιο διδάκτορα κ. Θ. Μαζαράκο για τις χρήσιμες συμβουλές και παρατηρήσεις που έκανε πάνω στη παρούσα εργασία.

$1^{ο}$ ΠΕΡΙΓΡΑΦΗ ΥΔΡΟΔΥΝΑΜΙΚΟΥ ΠΡΟΒΛΗΜΑΤΟΣ $1^{ης}$ ΤΑΞΗΣ

1.1 Εισαγωγή

Θεωρούμε στερεό αξονοσυμμετρικό σώμα που επιπλέει χωρίς περιορισμό παρουσία απλών αρμονικών κυματισμών. Η ροή του ρευστού γύρω από το σώμα θεωρείται ασυμπίεστη και αστρόβιλη ενώ τα φαινόμενα τριβής αγνοούνται (δηλαδή έχουμε ιδανικό ρευστό).



Ορίζουμε τρία πεδία τα οποία περιβάλλουν το σώμα:

$1^{ο}$ Πεδίο (I): Εξωτερικά του σώματος. Type I

$2^{ο}$ Πεδίο (II): Άνω του σώματος (Σώμα και ελεύθερη επιφάνεια). Type II

$3^{ο}$ Πεδίο (III): Κάτω του σώματος (Σώμα και πυθμένα). Type III

Τα όρια κάθε πεδίου είναι:

Πεδίο (I): $r \geq a \quad \kappa\alpha\iota \quad 0 \leq z \leq d$

Πεδίο (II): $a_\ell \leq r \leq a_{\ell+1} \quad \kappa\alpha\iota \quad d_\ell \leq z \leq d$

Πεδίο (III): $b_p \leq r \leq b_{p+1} \quad \kappa\alpha\iota \quad 0 \leq z \leq h_p$

Ο όγκος ελέγχου γύρω από το σώμα καθορίζεται από την ελεύθερη επιφάνεια η οποία είναι άπειρη προς όλες τις κατευθύνσεις, τη βρεχόμενη επιφάνεια του σώματος και τον πυθμένα που έχει άπειρη έκταση και πεπερασμένη απόσταση βάθους d από την ελεύθερη επιφάνεια. Χρησιμοποιούμε σύστημα κυλινδρικών συντεταγμένων *(r,θ,z)* με αρχή στον πυθμένα της θάλασσας και με τον θετικό ημιάξονα Oz κατακόρυφα προς τα πάνω.

Το σώμα το οποίο μελετάμε είναι συμμετρικό εκ περιστροφής περί τον κατακόρυφο άξονα. Επομένως μας απασχολούν μόνο τρεις κινήσεις του σώματος, η διαμήκης ταλάντωση x_1 (surge), η κατακόρυφη ταλάντωση x_3 (heave), και η περιστροφή περί τον εγκάρσιο άξονα x_5 (pitch).

Από την υπόθεσή μας ότι το ρευστό είναι ιδανικό, μπορούμε να περιγράψουμε το πεδίο ταχυτήτων του ρευστού γύρω από το σώμα, κάνοντας χρήση του δυναμικού ταχύτητας του οποίου το ανάδελτα δίνει την ταχύτητα του ρευστού σε κάθε σημείο του πεδίου.

Η πίεση ορίζεται από τον τύπο $P = -\rho \dfrac{\partial \Phi}{\partial t}$ και επομένως οι δυνάμεις F_x, F_z που ασκούνται στο σώμα στις x, y, z κατευθύνσεις δίνονται

$F_x = -\int_0^{2\pi}\int_{-b_i}^{0} P(a_i,\theta,z;t)\,a_i \cos\vartheta \quad dz\,d\vartheta$

$F_z = \int_0^{2\pi}\int_0^{a_i} P(r,\theta,-b_i,t)\,r \quad dr\,d\vartheta$

Ισχύει ότι το δυναμικό της ταχύτητας δίνεται από τη σχέση

$$\Phi = \Phi_0 + \Phi_7 + \Phi_R$$

Με Φ_0 παρουσιάζουμε το δυναμικό ταχύτητας του επερχόμενου αρμονικού κυματισμού και Φ_7 το δυναμικό περίθλασης για το συγκρατημένο ακίνητο σώμα.

Και $\quad \Phi_R = x_0\,\Phi_1 + z_0\,\Phi_3 + \alpha\varphi_0\,\Phi_5 = \xi_1\,\Phi_1 + \xi_3\,\Phi_3 + \xi_5\,\Phi_5$

Surge Heave Pitch

Όπου Φ_R είναι το δυναμικό ακτινοβολίας που προκύπτει από την εξαναγκασμένη ταλάντωση του σώματος στην *j*- κατεύθυνση.

Η ανύψωση της ελεύθερης επιφάνειας δίνεται

$$\zeta_0 = \mathrm{Re}\left[\frac{H}{2}\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\cos(m\theta)e^{-i\omega t}\right]$$

Το δυναμικό της ταχύτητας πρόσπτωσης απλού αρμονικού κυματισμού δίνεται από τη σχέση

$$\Phi_0(r,\theta,z,\mathrm{t}) = \mathrm{Re}[\ \varphi_0(\mathrm{r},\theta,\mathrm{z})e^{-i\omega t}]$$

$$\varphi_0(\mathrm{r},\theta,\mathrm{z})e^{-i\omega t} = -i\omega\frac{H}{2}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\left[\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\cos(m\theta)e^{-i\omega t}\right]$$

Όπου $\in_m$ σύμβολο Neumann [$\in_m = 1$ (m=0), $\in_m = 2$ ($\mathrm{m}\geq 1$)]

και $J_m(\kappa r)$ συνάρτηση Bessel, m-τάξης, πρώτου είδους.

Προσθέτοντας το δυναμικό ανύψωσης του απλού αρμονικού κυματισμού και το δυναμικό περίθλασης προκύπτει η παρακάτω σχέση

$$\varphi(r,\theta,z)\ e^{-i\omega t} = (\varphi_0\ (r,\theta,z) + \varphi_7\ (r,\theta,z))\ e^{-i\omega t} =$$

$$= -i\omega\frac{H}{2}d\left[\sum_{m=0}^{\infty}\in_m i^m \frac{1}{d}\Psi_{Dm}(r,z)\cos(m\theta)\right]e^{-i\omega t}\ .$$

Επομένως τα δυναμικά ακτινοβολίας δίνονται

$$\varphi_1(r,\theta,z)\ e^{-i\omega t} = -i\omega\frac{H}{2}d\left[\sum_{m=0}^{\infty}\in_m i^m \frac{1}{d}\Psi_{1m}(r,z)\cos(m\theta)\right]e^{-i\omega t}$$

$$\varphi_3(r,\theta,z)\ e^{-i\omega t} = -i\omega\frac{H}{2}d\left[\sum_{m=0}^{\infty}\in_m i^m \frac{1}{d}\Psi_{3m}(r,z)\cos(m\theta)\right]e^{-i\omega t}$$

$$\varphi_5(r,\theta,z)\ e^{-i\omega t} = -i\omega\frac{H}{2}d\left[\sum_{m=0}^{\infty}\in_m i^m \frac{1}{d}\Psi_{5m}(r,z)\cos(m\theta)\right]e^{-i\omega t}$$

Όταν θεωρούμε την ροή του ρευστού γύρω από το σώμα που προκαλείται από τις εξαναγκασμένες κινήσεις του στο αδιατάραχτο ρευστό βλέπουμε ότι αυτή θα είναι συμμετρική για την heave (surge, pitch) κίνηση. Οπότε αντίστοιχα για j=1,3,5 τα δυναμικά ακτινοβολίας γίνονται

$$\varphi_1(r,\theta,z)\ e^{-i\omega t} = \Psi_{11}(r,\theta)\cos\theta\ \ e^{-i\omega t}$$

$$\varphi_3(r,\theta,z)\ e^{-i\omega t} = \Psi_{30}(r,\theta)\ \ e^{-i\omega t}$$

$$\varphi_5(r,\theta,z)\ e^{-i\omega t} = \Psi_{51}(r,\theta)\cos\theta\ \ e^{-i\omega t}$$

Η συχνότητα ω και ο αριθμός κύματος κ συνδέονται με την εξίσωση της διασποράς.

$$\omega^2 = \kappa\ g\ tanh(\kappa d)$$

1.2 Οριακές συνθήκες

Τα επιμέρους μιγαδικά δυναμικά $\varphi_0(r,\theta,z)$, $\varphi_1(r,\theta,z)$, $\varphi_3(r,\theta,z)$, $\varphi_5(r,\theta,z)$, $\varphi_7(r,\theta,z)$ πρέπει να ικανοποιούν τις παρακάτω συνθήκες σε ολόκληρο τον όγκο ελέγχου του ρευστού

Εξίσωση Laplace $\nabla^2\Phi = 0$

Οριακή Συνθήκη Ελεύθερης Επιφάνειας $\omega^2\Phi - g\dfrac{\partial\Phi}{\partial z} = 0 \quad \mu\varepsilon \quad z = d,\ \ r = a$

Οριακή Συνθήκη Πυθμένα $\dfrac{\partial\Phi}{\partial z} = 0 \quad \gamma\iota\alpha \quad z = 0$

Οριακή Συνθήκη στο Άπειρο $\lim\limits_{r\to\infty}\sqrt{r}\,(\dfrac{\partial\phi_j}{\partial r} - i\kappa\phi_j) = 0$

όπου j=1,3,5,7.

Η συνθήκη του απείρου πρέπει να ικανοποιείται από τα δυναμικά περίθλασης και ακτινοβολίας εξασφαλίζοντας ότι δεν δημιουργείται διαταραχή από την κίνηση του σώματος σε άπειρη απόσταση από αυτό.

Η εξίσωση Laplace είναι η συνθήκη συνέχειας του ρευστού, ενώ η συνθήκη στον πυθμένα, εκφράζει την κάθετη μηδενική ταχύτητα του ρευστού που βρίσκεται σε επαφή με τον πυθμένα.

Η οριακή συνθήκη στην ελεύθερη επιφάνεια είναι η συνδυασμένη κινηματική και δυναμική συνθήκη που πρέπει να πληρεί το δυναμικό ταχύτητας του ρευστού στην επιφάνεια της θάλασσας.

Από τις παραπάνω συνθήκες προκύπτουν οι τελικές εκφράσεις των συναρτήσεων $\Psi_{Dm}(r,z)$ για τα πεδία (I), (II), (III).

1.3 Δυναμικά ταχύτητας περίθλασης και ακτινοβολίας

Το δυναμικό ταχύτητας πρώτης τάξης είναι υπέρθεση των δυναμικών περίθλασης και ακτινοβολίας, τα οποία αντίστοιχα περιγράφονται από τις συναρτήσεις δύο μεταβλητών $\Psi_{Dm}(r,z)$, $\Psi_{11}(r,z)$, $\Psi_{30}(r,z)$ $\Psi_{51}(r,z)$.

Οι συναρτήσεις $\Psi_{jm}(r,z)$ προσδιορίζονται για κάθε περιοχή του πεδίου ροής με την επίλυση του προβλήματος οριακών συνθηκών. Στη συνέχεια παρουσιάζονται οι τελικές εκφράσεις των συναρτήσεων $\Psi_{jm}(r,z)$ για τα τρία διαφορετικά πεδία (Ι), (ΙΙ), (ΙΙΙ). Kokkinowrachos, Mavrakos, Asorakos. (1986)

Πεδίο (Ι) $r \geq a \quad \kappa\alpha\iota \quad 0 \leq z \leq d$

$$\frac{1}{\delta_j}\Psi^1_{j,m}(r,z) = g^1_{j,m}(r,z) + \sum_{\substack{j=0 \\ a_j=a}}^{\infty} F_{j,ma}\frac{K_m(ar)}{K_m(a\mathrm{a})} z_a(z)$$

Όπου $g^1_{11}(r,z) = g^1_{30}(r,z) = g^1_{51}(r,z) = 0$.

$\delta_D = \delta_1 = \delta_3 = d \quad \kappa\alpha\iota \quad \delta_5 = d^2$.

Με α ρίζες της εξίσωσης $\dfrac{\omega^2}{g} + a\tan(ad) = 0$.

(Η οποία έχει μια φανταστική ρίζα (κ) και άπειρες πραγματικές (α_n).

$K_m(ar)$ τροποποιημένη συνάρτηση Bessel m-τάξης δεύτερου είδους.

$$g^1_{Dm}(r,z) = \left[J_m(\kappa r) - \frac{J_m(\kappa \mathrm{a})}{H_m(\kappa \mathrm{a})} H_m(\kappa r) \right] \frac{z_\kappa(z)}{dz_\kappa'(d)} .$$

$H_m(\kappa r)$ συνάρτηση Hankel m-τάξης πρώτου είδους.

$Z_\kappa(z) = N_\kappa^{-1/2}\cosh(\kappa z)$ και $N_\kappa = \dfrac{1}{2}\left[1 + \dfrac{\sinh(2\kappa d)}{2\kappa d}\right]$

$Z_\alpha(z) = N_\alpha^{-1/2}\cos(\alpha z)$ και $N_\alpha = \dfrac{1}{2}\left[1 + \dfrac{\sin(2ad)}{2ad}\right]$

Πεδίο (ΙΙ): $\mathrm{a}_{\ell} \le r \le \mathrm{a}_{\ell+1} \quad \kappa\alpha\iota \quad d_{\ell} \le z \le d$

$$\frac{1}{\delta_j}\Psi^{\ell}_{j,m}(r,z) = g^{\ell}_{j,m}(r,z) + \sum_{\substack{\ell=0 \\ a_\ell}}^{\infty} [\Re_{ma_\ell}(r)F_{j,ma_\ell} + \Re^*_{ma_\ell}(r)F^*_{j,ma_\ell}] z_{a_\ell}(z)$$

Όπου $g^{\ell}_{Dm}(r,z) = g_{11}(r,z) = 0$

$$g^{\ell}_{51}(r,z) = -\frac{r}{d^2}\left[(z-d) + \frac{g}{\omega^2}\right]$$

$$g^{\ell}_{30}(r,z) = \frac{z}{d} - 1 + \frac{g}{\omega^2 d}$$

$$\delta_D = \delta_1 = \delta_3 = d \quad \kappa\alpha\iota \quad \delta_5 = d^2$$

$$\Re_{ma_\ell}(r) = \frac{I_m(a_\ell r)K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell r)}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}$$

$$\Re^*{}_{ma_\ell}(r) = \frac{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell r) - I_m(a_\ell r)K_m(a_\ell \mathrm{a}_{\ell+1})}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}$$

$I_m(\alpha r)$ τροποποιημένη συνάρτηση Bessel m-τάξης πρώτου είδους.

$K_m(\alpha r)$ τροποποιημένη συνάρτηση Bessel m-τάξης δεύτερου είδους.

$$Z_{a_\ell}(\mathrm{z}) = N_{a_\ell}^{-1/2} \cos[a_\ell(z-d)] \text{ και } N_{a_\ell} = \frac{1}{2}\left[1 + \frac{\sin[2a_\ell(d-d_\ell)]}{2a_\ell(d-d_\ell)}\right]$$

Με a_ℓ ρίζες της $\dfrac{\omega^2}{g} + a_\ell \tan[a_\ell(d-d_\ell)] = 0$.

Πεδίο (ΙΙΙ): $b_p \le r \le b_{p+1} \quad \kappa\alpha\iota \quad 0 \le z \le h_p$

$$\frac{1}{\delta_j}\Psi^{p}_{j,m}(r,z) = g^{p}_{j,m}(r,z) + \sum_{n_p=0}^{\infty} \in_{n_p} [\Re_{mn_p}(r)F_{j,mn_p} + \Re^*_{mn_p}(r)F^*_{j,mn_p}]\cos(\frac{n_p \pi z}{h_p})$$

Όπου $g^{p}_{Dm}(r,z) = g_{11}(r,z) = 0$

$$g^{p}_{51}(r,z) = -\frac{r}{2h_p d^2}\left[z^2 - (\frac{1}{4})r^2\right]$$

$$g_{30}^{p}(r,z)=\frac{z^2-(\frac{1}{2})r^2}{2h_p d}$$

$$\delta_D=\delta_1=\delta_3=d \quad \kappa\alpha\iota \quad \delta_5=d^2$$

$$\Re_{mn_p}(r)=\frac{K_m(\frac{n_p\pi b_p}{h_p})I_m(\frac{n_p\pi r}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi r}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}$$

$$\Re^*_{mn_p}(r)=\frac{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi r}{h_p})-K_m(\frac{n_p\pi b_{p+1}}{h_p})I_m(\frac{n_p\pi r}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})} \text{ για } n_p\neq 0$$

$I_m(\alpha r)$ τροποποιημένη συνάρτηση Bessel m-τάξης πρώτου είδους.

$K_m(\alpha r)$ τροποποιημένη συνάρτηση Bessel m-τάξης δεύτερου είδους.

$$\Re_{m0_p}(r)=\frac{(\frac{r}{b_p})^m-(\frac{b_p}{r})^m}{(\frac{b_{p+1}}{b_p})^m-(\frac{b_p}{b_{p+1}})^m} \text{ και } \Re^*_{m0_p}(r)=\frac{(\frac{b_{p+1}}{r})^m-(\frac{r}{b_{p+1}})^m}{(\frac{b_{p+1}}{b_p})^m-(\frac{b_p}{b_{p+1}})^m} \text{ για } n_p=0.$$

Συνεπώς οι μορφές των επιλεγμένων λύσεων των συναρτήσεων $\Psi_{jm}(r,z)$ ικανοποιούν τις οριακές συνθήκες στα οριζόντια σύνορα των στοιχείων. Επιπλέον ικανοποιούνται οι κινηματικές συνθήκες στις κατακόρυφες πλευρές του σώματος όπως και οι συνθήκες συνέχειας για το δυναμικό, στις κατακόρυφες πλευρές των γειτονικών δακτυλιοειδών στοιχείων. Η αριθμητική διαδικασία που πρέπει να ακολουθηθεί για να προσδιοριστούν στις συναρτήσεις $\Psi_{jm}(r,z)$ οι συντελεστές Fourier δεν θα αναφερθεί στην παρούσα εργασία. Ο υπολογισμός των συντελεστών Fourier έχει γίνει από το πρόγραμμα *cylinder3.f* του κ. Σ.Α.Μαυράκου.

Στη συνέχεια δίνονται οι ολοκληρωμένες μορφές του δυναμικού πρώτης τάξης για δακτυλιοειδή στοιχεία στα πεδία (I), (II), (III).

1.4 Δυναμικό ταχύτητας 1ης τάξης για το πεδίο (I)

Η σχέση που μας δίνει το δυναμικό της ταχύτητας πρώτης τάξης για δακτυλιοειδή στοιχεία στο πεδίο (I): $r \geq \mathrm{a} \quad \kappa\alpha\iota \quad 0 \leq z \leq d \quad \varepsilon\iota\nu\alpha\iota$

$$\Phi(r,\theta,z;t) = \frac{-igH}{2\omega}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}e^{i(\kappa r\cos\theta)}e^{-i\omega t} - i\omega\frac{H}{2}d\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^{H} + \delta_{1,m}(\frac{x_0}{H/2}F_{1\kappa}^{S} +$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^{P}) + \in_m i^m(F_{m\kappa} - \frac{J_m(\kappa \mathrm{a})}{dz_\kappa^{\prime}(d)})]\frac{H_m(\kappa r)}{H_m(\kappa \mathrm{a})}\cos(m\theta)z_\kappa(z)e^{-i\omega t} - i\omega\frac{H}{2}d\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}$$

$$F_{0_a}^{H} + \delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^{S} + \frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_m i^m F_{m_a}]\frac{K_m(ar)}{K_m(a\mathrm{a})}\cos(m\theta)z_a(z)e^{-i\omega t} =$$

$$= \frac{-igH}{2\omega}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}e^{i(\kappa r\cos\theta)}e^{-i\omega t} - i\omega\frac{H}{2}dN_\kappa^{-1/2}\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^{H} + \delta_{1,m}(\frac{x_0}{H/2}F_{1\kappa}^{S} +$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^{P}) + \in_m i^m(F_{m\kappa} - \frac{J_m(\kappa \mathrm{a})}{dz_\kappa^{\prime}(d)})]\frac{H_m(\kappa r)}{H_m(\kappa \mathrm{a})}\cos(m\theta)\cosh(\kappa z)e^{-i\omega t} - i\omega\frac{H}{2}dN_a^{-1/2}$$

$$\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^{H} + \delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^{S} + \frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_m i^m F_{m_a}]\frac{K_m(ar)}{K_m(a\mathrm{a})}\cos(m\theta)\cos(az)$$

$$e^{-i\omega t}$$

Άρα

$$\varphi(r,\theta,z) = \frac{-igH}{2\omega}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}e^{i(\kappa r\cos\theta)} - i\omega\frac{H}{2}dN_\kappa^{-1/2}\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^{H} + \delta_{1,m}(\frac{x_0}{H/2}F_{1\kappa}^{S} +$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^{P}) + \in_m i^m(F_{m\kappa} - \frac{J_m(\kappa \mathrm{a})}{dz_\kappa^{\prime}(d)})]\frac{H_m(\kappa r)}{H_m(\kappa \mathrm{a})}\cos(m\theta)\cosh(\kappa z) - i\omega\frac{H}{2}dN_a^{-1/2}$$

$$\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^{H} + \delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^{S} + \frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_m i^m F_{m_a}]\frac{K_m(ar)}{K_m(a\partial)}\cos(m\theta)\cos(az)$$

Για ευκολία στις πράξεις αντικαθιστούμε όπου: $-i = e^{-i\frac{\pi}{2}}$

Χρησιμοποιώντας την ταυτότητα του Jacobi- Anger (Μαυράκος 1997) ισχύει ότι

$$e^{i\kappa r\cos\theta} = \sum_{m=0}^{\infty} \in_m i^m J_m(\kappa r)\cos(m\theta)\,.$$

Επομένως

φ(r,θ,z)=

$$= \frac{-igH}{2\omega}\frac{\cosh(\kappa z)}{\cosh(\kappa d)} \sum_{m=0}^{\infty} \in_m i^m J_m(\kappa r)\cos(m\theta) - i\ \omega\frac{H}{2} dN_{\kappa}^{-1/2} \sum_{m=0}^{\infty} [\delta_{0,m} \frac{z_0}{H/2} F_{0_\kappa}^{H} + \delta_{1,m} (\frac{x_0}{H/2}$$

$$F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2} F_{1\kappa}^{P}) + \in_m i^m (F_{m\kappa} - \frac{J_m(\kappa \mathrm{a})}{dz_\kappa{}'(d)})] \frac{H_m(\kappa r)}{H_m(\kappa \mathrm{a})}\cos(m\theta)\cosh(\kappa z) - i\ \omega \frac{H}{2} dN_a^{-1/2}$$

$$\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\ a_j=a}}^{\infty}[\delta_{0,m} \frac{z_0}{H/2}\ F_{0_a}^{H} + \delta_{1,m} (\frac{x_0}{H/2} F_{1_a}^{S} + \frac{d\phi_0}{H/2} F_{1_a}^{P}) + \in_m i^m F_{m_a}] \frac{K_m(ar)}{K_m(a\mathrm{a})}\cos(m\theta)\cos(az)\,.$$

Οι συναρτήσεις $\delta_{0,m}$ και $\delta_{1,m}$ δίνονται

$$\delta_{0,m} = \begin{Bmatrix} 1 & m=0 \\ & \\ 0 & m \neq 0 \end{Bmatrix} \qquad\qquad \delta_{1,m} = \begin{Bmatrix} 1 & m=1 \\ & \\ 0 & m \neq 1 \end{Bmatrix}.$$

Όπου $\frac{H}{2}$ είναι το ύψος κύματος και x_0, z_0, φ_0 τα μιγαδικά πλάτη των γενικευμένων κινήσεων του σώματος σε surge, heave, pitch. Οι δείκτες *S, H, P* αναφέρονται στις κινήσεις surge, heave, pitch αντίστοιχα.

1.5 Δυναμικό ταχύτητας 1ης τάξης για το πεδίο (ΙΙ)

Η σχέση που μας δίνει το δυναμικό της ταχύτητας πρώτης τάξης για δακτυλιοειδή στοιχεία στο πεδίο (ΙΙ): $\mathrm{a}_\ell \le r \le \mathrm{a}_{\ell+1} \quad \kappa\alpha\iota \quad d_\ell \le z \le d$ είναι

$$\Phi(r,\theta,z;t) = -i\omega\frac{H}{2}d\left[\frac{z_0}{\frac{H}{2}}\left(\frac{z-d}{d}+\frac{g}{\omega^2 d}\right)-d\,\frac{\phi_0}{\frac{H}{2}}r\left(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2}\right)\cos\theta\right]e^{-i\omega t} - i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)z_{\kappa_\ell}(z)+\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)z_{\kappa_\ell}(z)]\cos(m\theta)e^{-i\omega t} - i\omega\frac{H}{2}d\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$

$$z_{a_\ell}(z)+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)z_{a_\ell}(z)]]\cos(m\theta)e^{-i\omega t}$$

Στην παραπάνω σχέση ο δείκτης $\ell = 1,2,\ldots,\mathrm{L}$ αναφέρεται σε κάθε ένα από τα «από πάνω» δακτυλιοειδή στοιχεία.

Όπου $\Lambda_{m_{\kappa_\ell}} = \delta_{0,m}\frac{z_0}{H/2}F^H_{0_{\kappa_\ell}} + \delta_{1,m}\left(\frac{x_0}{H/2}F^S_{1_{\kappa_\ell}} + d\frac{\phi_0}{H/2}F^p_{1_{\kappa_\ell}}\right) + \in_m i^m F_{m_{\kappa_\ell}}$

$$\Lambda^*_{m_{\kappa_\ell}} = \delta_{0,m}\frac{z_0}{H/2}F^{*H}_{0_{\kappa_\ell}} + \delta_{1,m}\left(\frac{x_0}{H/2}F^{*S}_{1_{\kappa_\ell}} + d\frac{\phi_0}{H/2}F^{*p}_{1_{\kappa_\ell}}\right) + \in_m i^m F^*_{m_{\kappa_\ell}}$$

$$\Lambda_{m_{a_\ell}} = \delta_{0,m}\frac{z_0}{H/2}F^H_{0_{a_\ell}} + \delta_{1,m}\left(\frac{x_0}{H/2}F^S_{1_{a_\ell}} + d\frac{\phi_0}{H/2}F^p_{1_{a_\ell}}\right) + \in_m i^m F_{m_{a_\ell}}$$

$$\Lambda^*_{m_{a_\ell}} = \delta_{0,m}\frac{z_0}{H/2}F^{*H}_{0_{a_\ell}} + \delta_{1,m}\left(\frac{x_0}{H/2}F^{*S}_{1_{a_\ell}} + d\frac{\phi_0}{H/2}F^{*p}_{1_{a_\ell}}\right) + \in_m i^m F^*_{m_{a_\ell}}$$

$$z_{\kappa_\ell}(z) = N^{-1/2}_{\kappa_\ell}\cosh(\kappa_\ell(z-h_\ell))$$

$$N_{\kappa_\ell} = \frac{1}{2}\left[1+\frac{\sinh[2\kappa_\ell(d-d_\ell)]}{2\kappa_\ell(d-d_\ell)}\right]$$

$$z_{a_\ell}(z) = N^{-1/2}_{a_\ell}\cos(a_\ell(z-h_\ell))$$

$$N_{a_\ell} = \frac{1}{2}\left[1+\frac{\sin[2a_\ell(d-d_\ell)]}{2a_\ell(d-d_\ell)}\right]$$

$$\Re_{ma_\ell}(r) = \frac{I_m(a_\ell r)K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell r)}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}$$

$$\Re^{*}_{ma_\ell}(r) = \frac{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell r) - I_m(a_\ell r)K_m(a_\ell \mathrm{a}_{\ell+1})}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}$$

$$\Re_{m\kappa_\ell}(r) = \frac{J_m(\kappa_\ell r)Y_m(\mathrm{a}_\ell \kappa_\ell) - J_m(\mathrm{a}_\ell \kappa_\ell)Y_m(\kappa_\ell r)}{J_m(\kappa_\ell \mathrm{a}_{\ell+1})Y_m(\kappa_\ell \mathrm{a}_\ell) - J_m(\kappa_\ell \mathrm{a}_\ell)Y_m(\kappa_\ell \mathrm{a}_{\ell+1})}$$

$$\Re^{*}_{m\kappa_\ell}(r) = \frac{J_m(\kappa_\ell \mathrm{a}_{\ell+1})Y_m(\kappa_\ell r) - J_m(\kappa_\ell r)Y_m(\kappa_\ell \mathrm{a}_{\ell+1})}{J_m(\kappa_\ell \mathrm{a}_{\ell+1})Y_m(\kappa_\ell \mathrm{a}_\ell) - J_m(\kappa_\ell \mathrm{a}_\ell)Y_m(\kappa_\ell \mathrm{a}_{\ell+1})}$$

a_ℓ πραγματικές ρίζες της $\frac{\omega^2}{g} + a_\ell \tan(a_\ell(d - d_\ell)) = 0$

κ_ℓ φανταστική λύση της εξίσωσης διασποράς.

Επομένως

$$\varphi(r,\theta,z) = -i\omega\frac{H}{2}d\left[\frac{z_0}{\frac{H}{2}}\left(\frac{z-d}{d} + \frac{g}{\omega^2 d}\right) - d\,\frac{\phi_0}{\frac{H}{2}}\,r\left(\frac{z-d}{d^2} + \frac{g}{\omega^2 d^2}\right)\cos\theta\right] - i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)z_{\kappa_\ell}(z) + \Lambda^{*}_{m_{\kappa_\ell}}\Re^{*}_{m_{\kappa_\ell}}(r)z_{\kappa_\ell}(z)]\ \cos(m\theta) - i\omega\frac{H}{2}d\ \sum_{m=0}^{\infty}[\sum_{\substack{\ell=1 \\ a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$

$$z_{a_\ell}(z) + \Lambda^{*}_{m_{a_\ell}}\Re^{*}_{m_{a_\ell}}(r)z_{a_\ell}(z)]]\cos(m\theta).$$

Στην περίπτωση αυτή, το εκ περιστροφής αξονοσυμμετρικό σώμα δεν είναι πλήρως βυθισμένο. Υπάρχει δηλαδή, ένα δακτυλιοειδές στοιχείο, που διαπερνά την ελεύθερη επιφάνεια.

1.6 Δυναμικό ταχύτητας 1ης τάξης για το πεδίο (ΙΙΙ)

Η σχέση που μας δίνει το δυναμικό της ταχύτητας πρώτης τάξης για δακτυλιοειδή στοιχεία στο πεδίο (ΙΙΙ): $b_p \le r \le b_{p+1} \quad \kappa\alpha\iota \quad 0 \le z \le h_p$ είναι

$$\Phi(r,\theta,z;t) = -i\omega\frac{H}{2}d\left(\frac{z_0}{H/2}\frac{z^2-0{,}5r^2}{2h_p d} - d\,\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_p d^2}\cos\theta\right)e^{-i\omega t} - i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}\left[\sum_{n_p=0}^{\infty}\in_{n_p}\left(\Lambda_{mn_p}\Re_{mn_p}(r)+\Lambda^*_{mn_p}\Re^*_{mn_p}(r)\right)\cos\left(\frac{n_p\pi z}{h_p}\right)\right]\cos(m\theta)e^{-i\omega t}.$$

Ο δείκτης p αναφέρεται σε κάθε ένα δακτυλιοειδές στοιχείο. Το μέγεθος του p φτάνει από 1 έως και το μέγιστο αριθμό των «από κάτω» στοιχείων.

Όπου $\Lambda_{m_{n_p}} = \delta_{0,m}\frac{z_0}{H/2}F^H_{0_{n_p}} + \delta_{1,m}\left(\frac{x_0}{H/2}F^S_{1_{n_p}} + d\frac{\phi_0}{H/2}F^p_{1_{n_p}}\right) + \in_m i^m F_{m_{n_p}}$

$$\Lambda^*_{m_{n_p}} = \delta_{0,m}\frac{z_0}{H/2}F^{*H}_{0_{n_p}} + \delta_{1,m}\left(\frac{x_0}{H/2}F^{*S}_{1_{n_p}} + d\frac{\phi_0}{H/2}F^{*p}_{1_{n_p}}\right) + \in_m i^m F^*_{m_{n_p}}$$

$$\Re_{m_{n_p}}(r) = \frac{K_m\left(\frac{n_p\pi b_p}{h_p}\right)I_m\left(\frac{n_p\pi r}{h_p}\right) - I_m\left(\frac{n_p\pi b_p}{h_p}\right)K_m\left(\frac{n_p\pi r}{h_p}\right)}{I_m\left(\frac{n_p\pi b_{p+1}}{h_p}\right)K_m\left(\frac{n_p\pi b_p}{h_p}\right) - I_m\left(\frac{n_p\pi b_p}{h_p}\right)K_m\left(\frac{n_p\pi b_{p+1}}{h_p}\right)}$$

$$\Re^*_{m_{n_p}}(r) = \frac{I_m\left(\frac{n_p\pi b_{p+1}}{h_p}\right)K_m\left(\frac{n_p\pi r}{h_p}\right) - K_m\left(\frac{n_p\pi b_{p+1}}{h_p}\right)I_m\left(\frac{n_p\pi r}{h_p}\right)}{I_m\left(\frac{n_p\pi b_{p+1}}{h_p}\right)K_m\left(\frac{n_p\pi b_p}{h_p}\right) - I_m\left(\frac{n_p\pi b_p}{h_p}\right)K_m\left(\frac{n_p\pi b_{p+1}}{h_p}\right)}$$

b_p ακτίνα δακτυλιοειδούς στοιχείου (p)

$$\Re_{m_{0_p}}(r) = \frac{\left(\frac{r}{b_p}\right)^m - \left(\frac{b_p}{r}\right)^m}{\left(\frac{b_{p+1}}{b_p}\right)^m - \left(\frac{b_p}{b_{p+1}}\right)^m} \qquad \Re^*_{m_{0_p}}(r) = \frac{-\left(\frac{r}{b_{p+1}}\right)^m + \left(\frac{b_{p+1}}{r}\right)^m}{\left(\frac{b_{p+1}}{b_p}\right)^m - \left(\frac{b_p}{b_{p+1}}\right)^m}$$

$$\Re_{0_{0_p}}(r) = \frac{\ln\left(\frac{r}{b_p}\right)}{\ln\left(\frac{b_{p+1}}{b_p}\right)} \qquad \Re^*_{0_{0_p}}(r) = \frac{\ln\left(\frac{b_{p+1}}{r}\right)}{\ln\left(\frac{b_{p+1}}{b_p}\right)}$$

Επομένως

$$\varphi(r,\theta,z) = -i\omega\frac{H}{2}d\left(\frac{z_0}{H/2}\frac{z^2-0{,}5r^2}{2h_p d} - d\,\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_p d^2}\cos\theta\right) - i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}\Big[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{mn_p}\Re_{mn_p}(r) + \Lambda^*_{mn_p}\Re^*_{mn_p}(r))\cos(\frac{n_p\pi z}{h_p})\Big]\cos(m\theta)$$

Το δυναμικό ταχύτητας για το μεσαίο «από κάτω» κυλινδρικό στοιχείο, όταν το σώμα δεν στηρίζεται στον πυθμένα, δίνεται από τον τύπο:

$$\phi_M(\mathrm{r},\theta,\mathrm{z}) = -i\omega\frac{H}{2}d\left(\frac{z_0}{H/2}\frac{z^2-0{,}5r^2}{2h_p d} - d\,\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_p d^2}\cos\theta\right) - i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}\Big[(\Lambda_{m0_M}A_{m0_M}(r) + 2\sum_{n_M=1}^{\infty}\ \Lambda_{mn_M}A_{mn_M}(r))\cos(\frac{n_M\pi z}{h_p})\Big]\cos(m\theta)$$

Όπου $\Lambda_{mn_M} = \delta_{0,m}\frac{z_0}{H/2}F^H_{0_{0_M}} + \delta_{1,m}(\frac{x_0}{H/2}F^S_{1_{0_M}} + d\,\frac{\phi_0}{H/2}F^p_{1_{0_M}}) + \in_m i^m F_{m_{0_M}}$

$$A_{mn_M}(r) = \frac{I_m(\frac{n_M\pi r}{h_M})}{I_m(\frac{n_M\pi b_M}{h_M})}$$

με b_M : ακτίνα του μεσαίου στοιχείου.

h_M : ύψος του μεσαίου στοιχείου.

$A_{m0_M}(r) = (\frac{r}{b_M})^m$ Και ειδική περίπτωση είναι $A_{00_M}(r) = 1$

Όπου $\frac{\partial A_{m0_M}(r)}{\partial r} = \frac{m}{b_M}(\frac{r}{b_M})^{m-1}$ Ειδική περίπτωση είναι $\frac{\partial A_{00_M}(r)}{\partial r} = 0$

$$\frac{\partial A_{mn_M}(r)}{\partial r} = \frac{n_M\pi}{h_M}\frac{I_{m+1}(\frac{n_M\pi r}{h_M})}{I_m(\frac{n_M\pi b_M}{h_M})} + \frac{m}{r}\ \frac{I_m(\frac{n_M\pi r}{h_M})}{I_m(\frac{n_M\pi b_M}{h_M})}$$

2^{o} ΥΠΟΛΟΓΙΣΜΟΣ ΟΡΙΖΟΝΤΙΑΣ ΚΑΙ ΚΑΘΕΤΗΣ ΔΥΝΑΜΗΣ ΕΚΠΤΩΣΗΣ

2.1 Εισαγωγή

Η οριζόντια και κάθετη δύναμη έκπτωσης υπολογίζεται από τον τύπο του Pinkster (1979) και εκφράζεται με την μορφή

$$F = -\int_{WL} \frac{1}{2}\rho g \overline{\zeta_r^{(1)^2}}^T \bar{n} dl + M \overline{R^{(1)} \overline{X_g}''^{(1)}}^T - \int\int_{S_0} -\frac{1}{2}\rho \overline{\left|\nabla\Phi^{(1)}\right|^2}^T \bar{n} dS - \int\int_{S_0} -\rho \overline{(\overline{X}^{(1)} \nabla\Phi_t^{(1)})}^T \bar{n} dS$$

Όπου ρ είναι η πυκνότητα του νερού, g η επιτάχυνση της βαρύτητας, $\zeta_r^{(1)}$ η ανύψωση της ελεύθερης επιφάνειας, M η μάζα της κατασκευής, $R^{(1)}$ πίνακας που περιέχει τις γωνίες περιστροφής, $X_g^{(1)}$ οι μεταφορικές κινήσεις του κέντρου βάρους της κατασκευής, $\left|\nabla\Phi^{(1)}\right|$ το ανάδελτα του δυναμικού της ταχύτητας πρώτης τάξης για δακτυλιοειδή στοιχεία το οποίο δίνει την ταχύτητα του ρευστού σε κάθε σημείο του πεδίου και $\overline{X^{(1)}}$ το άνυσμα μετακίνησης από την παλιά θέση ισορροπίας .

Επομένως για τον υπολογισμό των δυνάμεων έκπτωσης στα πεδία I,II,III αρκεί να υπολογιστούν τα ολοκληρώματα $\int_{WL} \zeta_r^{(1)^2} \bar{n} dl$, $\int\int_{S_0} \left|\nabla\Phi^{(1)}\right|^2 \bar{n} dS$ και $[\int\int_{S_0} (\overline{X}^{(1)} \nabla\Phi_t^{(1)}) \bar{n} dS]$.

Στα επόμενα κεφάλαια υπολογίζονται τα παραπάνω ολοκληρώματα και για τα τρία πεδία.

2.2 Ανύψωση της ελεύθερης επιφάνειας

Για την ανύψωση της ελεύθερης επιφάνειας ισχύει ότι $\zeta(x,t) = \varepsilon\, \zeta^{(1)}(x,t) + \varepsilon\, \zeta^{(2)}(x,t)$

Όπου $\zeta^{(1)} = -\frac{1}{g}\Phi_t^1 \big|_{z=d}$

$$\zeta^{(2)} = -\frac{1}{g}(\Phi_t^{(2)} + \frac{1}{2}\nabla\Phi^{(1)}\nabla\Phi^{(1)} - \frac{1}{g}\Phi_t^{(1)}\Phi_{tz}^{(2)}) \big|_{z=d}\,.$$

Και από τη σχέση $\zeta_r^{(1)} = \zeta^{(1)} - X_3^{(1)WL}$

Όπου $X_3^{(1)WL} = X_{g_3}^{(1)} - X_5^{(1)} r\cos\theta$ η heave κίνηση η οποία είναι γνωστή από τη λύση του πρωτοτάξιου προβλήματος.

Επομένως

$$(\zeta_r^{(1)})^2 = (\zeta^{(1)} - X_3^{(1)WL})^2 = (-\frac{1}{g}\Phi_t^{(1)} - X_{g_3}^{(1)} + X_5^{(1)} r\cos\theta)^2$$

Όμως $\Phi(r,\theta,z,\mathrm{t}) = \mathrm{Re}[\ \varphi(r,\theta,z)\mathrm{e}^{-i\omega t}\ ]$.

Άρα $\Phi_t(r,\theta,z,\mathrm{t}) = \mathrm{Re}[\frac{\partial\{\phi(r,\theta,z)e^{-i\omega t}\}}{\partial t}] = \mathrm{Re}[(-i\omega)\ \varphi(r,\theta,z)\ \mathrm{e}^{-i\omega t}]$.

Επομένως

$$(\zeta_r^{(1)})^2 = (-\frac{1}{g}\mathrm{Re}\{(-i\omega)\phi(r,\theta,z)e^{-i\omega t}\} - X_{g_3}^{(1)} + X_5^{(1)} r\cos\theta)^2 =$$

$$= [\ \mathrm{Re}\left\{\frac{i\omega_j}{g}\phi(r,\theta,z)e^{-i\omega_j t}\right\} - \mathrm{Re}\left\{\ \left|X_{g_3}^{(1)}\right|e^{i\phi_3} - \left|X_5^{(1)}\right|e^{i\phi_5} r\cos\theta\right\}e^{-i\omega_\kappa t}\ ]^2.$$

Αν θέσουμε $\frac{i\omega_j}{g}\phi(r,\theta,z) = a$ και $\left|X_{g_3}^{(1)}\right|e^{i\phi_3} - \left|X_5^{(1)}\right|e^{i\phi_5} r\cos\theta = b$.

Τότε

$$(\zeta_r^{(1)})^2 = (\mathrm{Re}\{ae^{-i\omega_j t}\} - \mathrm{Re}\{be^{-i\omega_\kappa t}\})^2 = \mathrm{Re}\{ae^{-i\omega_j t}\}\ \mathrm{Re}\{ae^{-i\omega_j t}\} + \mathrm{Re}\{be^{-i\omega_\kappa t}\}\ \mathrm{Re}\{be^{-i\omega_\kappa t}\}\ -$$

$$-2\,\mathrm{Re}\{ae^{-i\omega_j t}\}\ \mathrm{Re}\{be^{-i\omega_\kappa t}\} = \frac{1}{2}\mathrm{Re}\{aa^*\} + \frac{1}{2}\mathrm{Re}\{bb^*\} + \frac{1}{2}\mathrm{Re}\{aae^{-i(2\omega_j)t}\} + \frac{1}{2}\mathrm{Re}\{bbe^{-i(2\omega_\kappa)t}\}\ -$$

$$-\,\mathrm{Re}\{ab^* e^{-i(\omega_j - \omega_\kappa)t}\} - \mathrm{Re}\{abe^{-i(\omega_j + \omega_\kappa)t}\} = \frac{1}{2}\mathrm{Re}\{a^2\} + \frac{1}{2}\mathrm{Re}\{a^2 e^{-i(2\omega_j)t}\} + \frac{1}{2}\mathrm{Re}\{b^2\} +$$

$\frac{1}{2}\text{Re}\{b^2 e^{-i(2\omega_\kappa)t}\}$ - $\text{Re}\{ab^* e^{-i(\omega_j-\omega_\kappa)t}\}$ - $\text{Re}\{abe^{-i(\omega_j+\omega_\kappa)t}\}$.

Οι δευτεροτάξιοι όροι παραλείπονται, για $\omega_\kappa = \omega_j = \omega$ και z = d, προκύπτει η σχέση

$$(\zeta_r^{(1)})^2 = \frac{1}{2}\text{Re}\left\{(\frac{i\omega}{g})\phi(r,\theta,d)(\frac{-i\omega}{g})\overline{\phi(r,\theta,d)}\right\} + \frac{1}{2}\text{Re}\left\{ \left[\left|X_{g_3}^{(1)}\right|e^{i\phi_3} - \left|X_5^{(1)}\right|e^{i\phi_5} r\cos\theta\right]^2\right\} -$$

$$- \text{Re}\left[\left\{(\frac{i\omega}{g})\phi(r,\theta,d)\right\}\left[\left|X_{g_3}^{(1)}\right|e^{i\phi_3} - \left|X_5^{(1)}\right|e^{i\phi_5} r\cos\theta\right]\right].$$

Όπου $\phi_{3,5}$: η διαφορά φάσης.

ω : η ιδιοσυχνότητα της κατασκευής.

$X_{g_3}^{(1)}$: μεταφορική κίνηση του κέντρου βάρους της κατασκευής στον άξονα OX_3

$X_5^{(1)}$: περιστροφή γύρω από τον άξονα GX_2

2.3 Υπολογισμός του όρου ($\overline{x}^{(1)}\nabla\Phi_t^{(1)}$)

Για τον υπολογισμό του όρου $\overline{x}^{(1)}\nabla\Phi_t^{(1)}$ ισχύει ότι

$$\overline{X}^{(1)} = \overline{X}_g^{(1)} + R^{(1)}\overline{X} = \overline{X}_g^{(1)} + (a^{(1)} \times \overline{r}) = \overline{X}_g^{(1)} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ X_4^{(1)} & X_5^{(1)} & X_6^{(1)} \\ X_1 & X_2 & X_3 \end{vmatrix} =$$

$$= i(X_{g_1} + X_5^{(1)}X_3 - X_6^{(1)}X_2) + j(X_{g_2} + X_6^{(1)}X_1 - X_4^{(1)}X_3) + k(X_{g_3} + X_4^{(1)}X_2 - X_5^{(1)}X_1)$$

Και $\overline{X}\nabla\Phi_t^{(1)} = (-i\omega)\overline{X}\nabla\Phi^{(1)}$

Όμως σε κυλινδρικές συντεταγμένες προκύπτει ότι

$$\nabla\Phi_t^{(1)} = \frac{\partial\phi(r,\theta,z)}{\partial r}\overline{e_r} + \frac{1}{r}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\overline{e_\theta} + \frac{\partial\phi(r,\theta,z)}{\partial z}\overline{e_k}$$

Όπου $\bar{i} = \cos\theta\overrightarrow{e_r} - \sin\theta\overrightarrow{e_\theta}$ και $\overline{j} = \sin\theta\overrightarrow{e_r} + \cos\theta\overrightarrow{e_\theta}$

Επομένως

$$\overline{X}_{(r,\theta,z)}^{(1)} = (\cos\theta\overrightarrow{e_r} - \sin\theta\overrightarrow{e_\theta})(X_{g_1} + X_5^{(1)}z) + \overline{k}(X_{g_3} - X_5^{(1)}r\cos\theta) =$$
$$= \overline{e_r}(X_{g_1}^{(1)} + X_5^{(1)}z)\cos\theta - \overline{e_\theta}(X_{g_1}^{(1)} + X_5^{(1)}z)\sin\theta + \overline{k}(X_{g_3} - X_5^{(1)}r\cos\theta)$$

Δηλαδή η παραπάνω σχέση γίνεται:

$$\overline{x}^{(1)}\nabla\Phi_t^{(1)} = [(X_{g_1}^{(1)} + X_5^{(1)}z)\frac{\partial\phi(r,\theta,z)}{\partial r}\cos\theta - (X_{g_1}^{(1)} + X_5^{(1)}z)\frac{1}{r}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\sin\theta$$
$$+ (X_{g_3}^{(1)} - X_5^{(1)}r\cos\theta)\frac{\partial\phi(r,\theta,z)}{\partial z}](-i\omega)]$$

Όπου $X_{g_1}^{(1)}$: μεταφορική κίνηση του κέντρου βάρους της κατασκευής στον άξονα OX_1

$X_{g_3}^{(1)}$: μεταφορική κίνηση του κέντρου βάρους της κατασκευής στον άξονα OX_3

$X_5^{(1)}$: περιστροφή γύρω από τον άξονα GX_2

r : η ακτίνα του $i-$στου στοιχείου

2.4 Υπολογισμός του όρου $M\,R^{(1)}\overline{X_g}^{(1)''}$

Για τον υπολογισμό των οριζόντιων και κάθετων δυνάμεων έκπτωσης, έχουμε δείξει στη σελίδα 16 ότι πρέπει να υπολογιστεί η τιμή της παράστασης: $M\,R^{(1)}\overline{X_g}^{(1)''}$.

Όμως ισχύει ότι $\xi_1 = \mathrm{Re}\{\xi_1 e^{-i\omega t}\}$

Παραγωγίζοντας προκύπτει ότι $\xi_1' = \mathrm{Re}\{(-i\omega)\xi_1 e^{-i\omega t}\}$ και $\xi_1'' = \mathrm{Re}\{(-\omega)^2\xi_1 e^{-i\omega t}\}$

Επίσης ισχύει ότι $M = \rho g \nabla$

Άρα $$MR^{(1)}\overline{X_g}^{(1)''} = MR^{(1)}\begin{bmatrix}\xi_1''\\ \xi_2''\\ \xi_3''\end{bmatrix} = \rho g\nabla\begin{bmatrix}0 & -X_6^{(1)} & X_5^{(1)}\\ X_6^{(1)} & 0 & -X_4^{(1)}\\ -X_5^{(1)} & X_4^{(1)} & 0\end{bmatrix}\begin{bmatrix}\xi_1''\\ 0\\ \xi_3''\end{bmatrix} = \rho g\nabla\begin{bmatrix}-\omega^2 X_5^{(1)}\xi_3\\ 0\\ \omega^2 X_5^{(1)}\xi_1\end{bmatrix}$$

Και η μέση τιμή ως προς τον χρόνο

$$\overline{MR^{(1)}\overline{X_g}^{(1)''}}^T = \rho g\nabla\begin{bmatrix}-\omega^2\overline{X_5^{(1)}\xi_3}^T\\ 0\\ \omega^2\overline{X_5^{(1)}\xi_1}^T\end{bmatrix} = \rho g\nabla\begin{bmatrix}-\omega^2(\overline{\xi_{3_{\mathrm{Re}}}X_{5_{\mathrm{Re}}}^{(1)}\cos^2(\omega t) + \xi_{3_{\mathrm{Im}}}X_{5_{\mathrm{Im}}}^{(1)}\sin^2(\omega t)}^T)\\ 0\\ \omega^2(\overline{\xi_{1_{\mathrm{Re}}}X_{5_{\mathrm{Re}}}^{(1)}\cos^2(\omega t) + \xi_{1_{\mathrm{Im}}}X_{5_{\mathrm{Im}}}^{(1)}\sin^2(\omega t)}^T)\end{bmatrix} =$$

$$= \rho g\nabla\begin{bmatrix}-\frac{1}{2}\omega^2(\xi_{3_{\mathrm{Re}}}X_{5_{\mathrm{Re}}}^{(1)} + \xi_{3_{\mathrm{Im}}}X_{5_{\mathrm{Im}}}^{(1)})\\ 0\\ \frac{1}{2}\omega^2(\xi_{1_{\mathrm{Re}}}X_{5_{\mathrm{Re}}}^{(1)} + \xi_{1_{\mathrm{Im}}}X_{5_{\mathrm{Im}}}^{(1)})\end{bmatrix} = \frac{1}{2}\rho g\nabla\,\mathrm{Re}\begin{bmatrix}-\omega^2\xi_3 X_5^{(1)^*}\\ 0\\ \omega^2\xi_1 X_5^{(1)^*}\end{bmatrix}$$

Όπου ω: η ιδιοσυχνότητα της κατασκευής.

ξ_1 : μεταφορικές κινήσεις του κέντρου βάρους της κατασκευής στον άξονα OX_1

ξ_3 : μεταφορικές κινήσεις του κέντρου βάρους της κατασκευής στον άξονα OX_3

$X_5^{(1)}$: περιστροφή γύρω από τον άξονα GX_2

ρ : πυκνότητα νερού

∇ : όγκος αξονοσυμμετρικού σώματος

2.5 Υπολογισμός του όρου $\left|\nabla\Phi^{(1)}\right|^2$

Ισχύει για το $\overline{\left|\nabla\Phi^{(1)}\right|^2}^T = \overline{\left[\frac{\partial\Phi^{(1)}}{\partial r}, \frac{1}{r}\frac{\partial\Phi^{(1)}}{\partial\theta}, \frac{\partial\Phi^{(1)}}{\partial z}\right]^2}^T$ ή $\overline{\left(\frac{\partial\Phi}{\partial r}\right)^2}^T + \frac{1}{r^2}\overline{\left(\frac{\partial\Phi}{\partial\theta}\right)^2}^T + \overline{\left(\frac{\partial\Phi}{\partial z}\right)^2}^T$.

Όπου $\overline{\left(\frac{\partial\Phi}{\partial r}\right)^2}^T = \frac{1}{2}\frac{\partial\Phi}{\partial r}\frac{\partial\overline{\Phi}}{\partial r}$

$$\overline{\left(\frac{\partial\Phi}{\partial\theta}\right)^2}^T = \frac{1}{2}\frac{1}{r^2}\frac{\partial\Phi}{\partial\theta}\frac{\partial\overline{\Phi}}{\partial\theta}$$

$$\overline{\left(\frac{\partial\Phi}{\partial z}\right)^2}^T = \frac{1}{2}\frac{\partial\Phi}{\partial z}\frac{\partial\overline{\Phi}}{\partial z}$$

Όμως –σελίδα 4– $\Phi(r,\theta,z,t) = \mathrm{Re}[\,\varphi(r,\theta,z)e^{-i\omega t}\,]$ άρα $\frac{\partial\Phi}{\partial r} = \frac{\partial\phi}{\partial r}e^{-i\omega t}$ και $\frac{\partial\overline{\Phi}}{\partial r} = \frac{\partial\overline{\phi}}{\partial r}e^{-i\omega t}$.

Όμοια $\frac{\partial\Phi}{\partial\theta} = \frac{\partial\phi}{\partial\theta}e^{-i\omega t}$ και $\frac{\partial\overline{\Phi}}{\partial\theta} = \frac{\partial\overline{\phi}}{\partial\theta}e^{-i\omega t}$

$\frac{\partial\Phi}{\partial z} = \frac{\partial\phi}{\partial z}e^{-i\omega t}$ και $\frac{\partial\overline{\Phi}}{\partial z} = \frac{\partial\overline{\phi}}{\partial z}e^{-i\omega t}$

3^{o} ΟΡΙΖΟΝΤΙΑ ΔΥΝΑΜΗ ΕΚΠΤΩΣΗΣ ΓΙΑ ΤΟ ΠΕΔΙΟ (Ι)

3.1 Υπολογισμός του όρου $\int_{d_2}^{d_1}(\int_0^{2\pi} \left|\nabla\Phi^{(1)}\right|^2)\overline{n}dS$ για το πεδίο (Ι)

Έχουμε αποδείξει στο Κεφάλαιο 1 –σελίδα 11– ότι η σχέση που μας δίνει το δυναμικό της ταχύτητας πρώτης τάξης για δακτυλιοειδή στοιχεία στην περιοχή (Ι): r≥ a $\kappa\alpha\iota \quad 0 \le z \le d$ είναι:

φ(*r,θ,z*)=

$$=\frac{-igH}{2\omega}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\cos(m\theta) - i\ \omega\frac{H}{2}dN_\kappa^{-1/2}\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,m}(\frac{x_0}{H/2}$$

$$F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa a)}{dz_\kappa'(d)})]\frac{H_m(\kappa r)}{H_m(\kappa a)}\cos(m\theta)\cosh(\kappa z) - i\ \omega\frac{H}{2}dN_a^{-1/2}$$

$$\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_m i^m F_{m_a}]\frac{K_m(ar)}{K_m(a a)}\cos(m\theta)\cos(az)\ .$$

Επομένως

$$\frac{\partial\phi(r,\theta,z)}{\partial r}=$$

$$=\frac{-igH}{2\omega}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\sum_{m=0}^{\infty}\in_m i^m\kappa J_m'(\kappa r)\cos(m\theta) - i\ \omega\frac{H}{2}dN_\kappa^{-1/2}\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,m}(\frac{x_0}{H/2}$$

$$F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa a)}{dz_\kappa'(d)})]\ \kappa\frac{H_m'(\kappa r)}{H_m(\kappa a)}\cos(m\theta)\cosh(\kappa z) - i\ \omega\frac{H}{2}dN_a^{-1/2}$$

$$\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_m i^m F_{m_a}]\ a\frac{K_m'(ar)}{K_m(a a)}\cos(m\theta)\cos(az)$$

Όμοια

$$\overline{\frac{\partial\phi(r,\theta,z)}{\partial r}}=$$

$$=\frac{igH}{2\omega}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\sum_{m=0}^{\infty}\in_m i^m\kappa\overline{J_m'(\kappa r)}\cos(m\theta) + i\ \omega\frac{H}{2}dN_\kappa^{-1/2}\sum_{m=0}^{\infty}\overline{[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,m}(\frac{x_0}{H/2}}$$

$$\overline{F^{S}{}_{1_\kappa}} \; \overline{+\frac{d\phi_0}{H/2}F^{P}_{1\kappa}) + \in_m i^m (F_{m\kappa} - \frac{J_m(\kappa \mathrm{a})}{dz'_\kappa(d)})]} \; \kappa \frac{\overline{H'_m(\kappa r)}}{H_m(\kappa \mathrm{a})} \cos(m\theta)\cosh(\kappa z) + i\,\omega \frac{H}{2} dN_a^{-1/2}$$

$$\sum_{m=0}^{\infty}[\sum_{a=0}^{\infty}[\delta_{0,m} \overline{\frac{z_0}{H/2}} \; \overline{F^{H}_{0_a}} + \delta_{1,m}(\overline{\frac{x_0}{H/2}F^{S}_{1_a}} \; \overline{+\frac{d\phi_0}{H/2}F^{P}_{1_a}) + \in_m i^m F_{m_a}}] \; a \frac{\overline{K'_m(ar)}}{K_m(a\mathrm{a})} \cos(m\theta)\cos(az)$$

Και

$$\overline{\left(\frac{\partial\phi(r,\theta,z)}{\partial r}\right)^2}^{T} = \frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial r}\frac{\partial\overline{\phi(r,\theta,z)}}{\partial r} =$$

$$= \frac{g^2H^2}{8\omega^2}\kappa^2(\frac{\cosh(\kappa z)}{\cosh(\kappa d)})^2 \sum_{m=0}^{\infty}\in_m i^m J'_m(\kappa r)\cos(m\theta) \sum_{n=0}^{\infty}\in_n \overline{J'_n(\kappa r)}\cos(n\theta) +$$

$$+\frac{gH^2}{8} dN_\kappa^{-1/2}\frac{\cosh(\kappa z)}{\cosh(\kappa d)} \sum_{m=0}^{\infty}\in_m i^m J'_m(\kappa r)\cos(m\theta) \sum_{n=0}^{\infty}\overline{[\delta_{0,n}\frac{z_0}{H/2}F^{H}_{0_\kappa} + \delta_{1,n}(\frac{x_0}{H/2}}$$

$$\overline{F^{S}{}_{1_\kappa}} \; \overline{+\frac{d\phi_0}{H/2}F^{P}_{1\kappa}) + \in_n i^n (F_{n\kappa} - \frac{J_n(\kappa \mathrm{a})}{dz'_\kappa(d)})]} \; \kappa \frac{\overline{H'_n(\kappa r)}}{H_n(\kappa \mathrm{a})} \cos(n\theta)\cosh(\kappa z) +$$

$$+\frac{gH^2}{8} d \frac{\cosh(\kappa z)}{\cosh(\kappa d)} \sum_{m=0}^{\infty}\in_m i^m J'_m(\kappa r)\cos(m\theta) \sum_{n=0}^{\infty}[\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,n} \overline{\frac{z_0}{H/2}} \; \overline{F^{H}_{0_a}} + \delta_{1,n}(\overline{\frac{x_0}{H/2}F^{S}_{1_a}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^{P}_{1_a}) + \in_n i^n F_{n_a}}] \; a \frac{\overline{K'_n(ar)}}{K_n(a\mathrm{a})} \cos(n\theta)\cos(az) +$$

$$+\frac{gH^2}{8} dN_\kappa^{-1/2}\frac{\cosh(\kappa z)}{\cosh(\kappa d)} \sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F^{H}_{0_\kappa} + \delta_{1,m}(\frac{x_0}{H/2} F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F^{P}_{1\kappa}) + \in_m i^m (F_{m\kappa} - \frac{J_m(\kappa \mathrm{a})}{dz'_\kappa(d)})] \; \kappa \frac{H'_m(\kappa r)}{H_m(\kappa \mathrm{a})} \cos(m\theta)\cosh(\kappa z) \sum_{n=0}^{\infty}\in_n i^n \kappa \overline{J'_n(\kappa r)}\cos(n\theta) +$$

$$+\frac{\omega^2H^2}{8} d^2 N_\kappa^{-1} \sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F^{H}_{0_\kappa} + \delta_{1,m}(\frac{x_0}{H/2} F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2}F^{P}_{1\kappa}) + \in_m i^m (F_{m\kappa} - \frac{J_m(\kappa \mathrm{a})}{dz'_\kappa(d)})]$$

$$\kappa \frac{H'_m(\kappa r)}{H_m(\kappa \mathrm{a})} \cos(m\theta)\cosh(\kappa z) \sum_{n=0}^{\infty}\overline{[\delta_{0,n}\frac{z_0}{H/2}F^{H}_{0_\kappa} + \delta_{1,n}(\frac{x_0}{H/2}} \; \overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^{P}_{1\kappa}) + \in_n i^n (F_{n\kappa} - \frac{J_n(\kappa \mathrm{a})}{dz'_\kappa(d)})]} \; \kappa \frac{\overline{H'_n(\kappa r)}}{H_n(\kappa \mathrm{a})} \cos(n\theta)\cosh(\kappa z) +$$

$$+\frac{\omega^2 H^2}{8} d^2 N_\kappa^{-1/2} \sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,m}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa \mathrm{a})}{dz'_\kappa(d)})]$$

$$\kappa\frac{H'_m(\kappa r)}{H_m(\kappa \mathrm{a})}\cos(m\theta)\cosh(\kappa z)\sum_{n=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,n}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,n}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_n i^n F_{n_a}]}\ a\frac{\overline{K'_n(ar)}}{K_n(a\mathrm{a})}\cos(n\theta)\cos(az)+$$

$$+\frac{gH^2}{8}d\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_m i^m F_{m_a}]$$

$$a\frac{K'_m(ar)}{K_m(a\mathrm{a})}\cos(m\theta)\cos(az)\sum_{n=0}^{\infty}\in_n i^n\kappa\overline{J'_n(\kappa r)}\cos(n\theta)+$$

$$+\frac{\omega^2 H^2}{8}d^2 N_\kappa^{-1/2}\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_m i^m F_{m_a}]$$

$$a\frac{K'_m(ar)}{K_m(a\mathrm{a})}\cos(m\theta)\cos(az)\sum_{n=0}^{\infty}[\overline{\delta_{0,n}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,n}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_n i^n(F_{n\kappa}-\frac{J_n(\kappa \mathrm{a})}{dz'_\kappa(d)})]}\ \kappa\frac{\overline{H'_n(\kappa r)}}{H_n(\kappa \mathrm{a})}\cos(n\theta)\cosh(\kappa z)+$$

$$+\frac{\omega^2 H^2}{8}d^2\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_m i^m F_{m_a}]$$

$$a\frac{K'_m(ar)}{K_m(a\mathrm{a})}\cos(m\theta)\cos(az)\sum_{n=0}^{\infty}[\sum_{\substack{i=1\\a_i=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,n}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,n}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_n i^n F_{n_a}]}\ a\frac{\overline{K'_n(ar)}}{K_n(a\mathrm{a})}\cos(n\theta)\cos(az).$$

Μένει να υπολογιστεί το $\int_{d_2}^{d_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial r}\right)^2}^T\cos\theta d\theta\right]RADIUS dz$. *(Παράρτημα Α*)

Όπου *RADIUS* η ακτίνα του εξωτερικού $i-$στου στοιχείου.

Πρώτα υπολογίζουμε το $\int_0^{2\pi}\left(\frac{\partial\phi}{\partial r}\right)^2\cos\theta d\theta$ και προκύπτει η

$$\int_0^{2\pi}\left(\frac{\partial\phi}{\partial r}\right)^2\cos\theta d\theta =$$

$$=\frac{g^2H^2}{8\omega^2}\left(\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\right)^2 [\in_0 i^0\kappa J_0'(\kappa r)\,\overline{\in_1 i^1\kappa J_1'(\kappa r)}\,\pi + \in_1 i^1\kappa J_1'(\kappa r)\,\overline{\in_0 i^0\kappa J_0'(\kappa r)}\,\pi +$$

$$+\in_1 i^1\kappa J_1'(\kappa r)\,\overline{\in_2 i^2\kappa J_2'(\kappa r)}\,\frac{\pi}{2} + \sum_{p=2,3,}^{\infty}\in_p i^p\kappa J_p'(\kappa r)[\kappa[\in_{p-1}\overline{i^{p-1}J_{p-1}'(\kappa r)} +$$

$$\in_{p+1} i^{p+1}\overline{J_{p+1}'(\kappa r)}]]\frac{\pi}{2}]] +$$

$$+\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}[\in_0 i^0\kappa J_0'(\kappa r)\,\overline{[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H + \delta_{1,1}(\frac{x_0}{H/2}}$$

$$\overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_1 i^1(F_{1\kappa} - \frac{J_1(\kappa a)}{dz_\kappa'(d)})]}\ \kappa\frac{\overline{H_1'(\kappa r)}}{H_1(\kappa a)}\cosh(\kappa z)\,\pi +$$

$$\in_1 i^1\kappa J_1'(\kappa r)\,\overline{[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H + \delta_{1,0}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_0 i^0(F_{1\kappa} - \frac{J_0(\kappa a)}{dz_\kappa'(d)})]}$$

$$\kappa\frac{\overline{H_0'(\kappa r)}}{H_0(\kappa a)}\cosh(\kappa z)\,\pi + \in_1 i^1\kappa J_1'(\kappa r)\,\overline{[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H + \delta_{1,2}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}} +$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_2 i^2(F_{1\kappa} - \frac{J_2(\kappa a)}{dz_\kappa'(d)})]}\ \kappa\frac{\overline{H_2'(\kappa r)}}{H_2(\kappa a)}\cosh(\kappa z)\frac{\pi}{2} +$$

$$+\sum_{p=2,3,}^{\infty}\in_p i^p\kappa J_p'(\kappa r)[\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H + \delta_{1,p-1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_{p-1} i^{p-1}(F_{p-1_\kappa} - \frac{J_{p-1}(\kappa a)}{dz_\kappa'(d)})]}\ \kappa\frac{\overline{H_{p-1}'(\kappa r)}}{H_{p-1}(\kappa a)}\cosh(\kappa z) +$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H + \delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_{p+1} i^{p+1}(F_{p+1_\kappa} - \frac{J_{p+1}(\kappa a)}{dz_\kappa'(d)})]}$$

$$\kappa\frac{\overline{H_{p+1}'(\kappa r)}}{H_{p+1}(\kappa a)}\cosh(\kappa z)]\frac{\pi}{2}] +$$

$$+\frac{gH^2}{8}d\frac{\cosh(\kappa z)}{\cosh(\kappa d)}[[\in_0 i^0\kappa J_0'(\kappa r)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,1}\frac{z_0}{H/2}}\,\overline{F_{0_a}^H+\delta_{1,1}(}\,\overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]}\,a\frac{\overline{K_1'(ar)}}{K_1(a\mathrm{a})}\cos(az)\,\pi+$$

$$+\in_1 i^1\kappa J_1'(\kappa r)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,0}\frac{z_0}{H/2}}\,\overline{F_{0_a}^H+\delta_{1,0}(}\,\overline{\frac{x_0}{H/2}F_{1_a}^S}\,\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_0 i^0F_{0_a}]}$$

$$a\frac{\overline{K_0'(ar)}}{K_0(a\mathrm{a})}\cos(az)\,\pi+\in_1 i^1\kappa J_1'(\kappa r)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,2}\frac{z_0}{H/2}}\,\overline{F_{0_a}^H+\delta_{1,2}(}\,\overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\varepsilon_2 i^2F_{2_a}]}\,a\frac{\overline{K_2'(ar)}}{K_2(a\mathrm{a})}\cos(az)\,\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}\in_p i^p\kappa J_p'(\kappa r)[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,p-1}\frac{z_0}{H/2}}\,\overline{F_{0_a}^H+\delta_{1,p-1}(}\,\overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p-1} i^{p-1}F_{p-1_a}]}\,a\frac{\overline{K_{p-1}'(ar)}}{K_{p-1}(a\mathrm{a})}\cos(az)+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,p+1}\frac{z_0}{H/2}}\,\overline{F_{0_a}^H+\delta_{1,p+1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\,\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p+1} i^{p+1}F_{p+1_a}]}\,a\frac{\overline{K_{p+1}'(ar)}}{K_{p+1}(a\mathrm{a})}\cos(az)\,]\,\frac{\pi}{2}]+$$

$$+\frac{gH^2}{8}N_\kappa^{-1/2}d\frac{\cosh(\kappa z)}{\cosh(\kappa d)}[[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa\mathrm{a})}{dz_\kappa'(d)})]\,\kappa\frac{H_0'(\kappa r)}{H_0(\kappa\mathrm{a})}\cosh(\kappa z)\in_1 i^1\kappa\overline{J_1'(\kappa r)}\pi+$$

$$[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa\mathrm{a})}{dz_\kappa'(d)})]\,\kappa\frac{H_1'(\kappa r)}{H_1(\kappa\mathrm{a})}\cosh(\kappa z)$$

$$\in_0 i^0\kappa\overline{J_0'(\kappa r)}\pi+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa\mathrm{a})}{dz_\kappa'(d)})]$$

$$\kappa\frac{H_1'(\kappa r)}{H_1(\kappa\mathrm{a})}\cosh(\kappa z)\in_2 i^2\kappa\overline{J_2'(\kappa r)}\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz_\kappa^{'}(d)})]\ \kappa\frac{H_p'(\kappa r)}{H_p(\kappa a)}\cosh(\kappa z)\,[[\in_{p-1} i^{p-1}\kappa\overline{J_{p-1}'(\kappa r)}+$$

$$\in_{p+1} i^{p+1}\kappa\overline{J_{p+1}'(\kappa r)}\ \frac{\pi}{2}]+$$

$$+\frac{\omega^2H^2}{8}d^2N_\kappa^{-1}[[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz_\kappa'(d)})]$$

$$\kappa\frac{H_0'(\kappa r)}{H_0(\kappa a)}\cosh(\kappa z)\ \overline{[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz_\kappa'(d)})]}$$

$$\kappa\frac{\overline{H_1'(\kappa r)}}{H_1(\kappa a)}\cosh(\kappa z)\pi+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz_\kappa'(d)})]$$

$$\kappa\frac{\overline{H_1'(\kappa r)}}{H_1(\kappa a)}\cosh(\kappa z)\ \overline{[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz_\kappa'(d)})]}$$

$$\kappa\frac{\overline{H_0'(\kappa r)}}{H_0(\kappa a)}\cosh(\kappa z)\pi+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz_\kappa'(d)})]$$

$$\kappa\frac{H_1'(\kappa r)}{H_1(\kappa a)}\cosh(\kappa z)\ \overline{[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa a)}{dz_\kappa'(d)})]}$$

$$\kappa\frac{\overline{H_2'(\kappa r)}}{H_2(\kappa a)}\cosh(\kappa z)\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz_\kappa'(d)})]$$

$$\kappa\frac{H_p'(\kappa r)}{H_p(\kappa a)}\cosh(\kappa z)\,[[\overline{\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa a)}{dz_\kappa'(d)})]}\ \kappa\frac{\overline{H_{p-1}'(\kappa r)}}{H_{p-1}(\kappa a)}\cosh(\kappa z)]+$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz_\kappa'(d)})]}$$

$$\kappa\frac{\overline{H_{p+1}'(\kappa r)}}{H_{p+1}(\kappa a)}\cosh(\kappa z)]]\frac{\pi}{2}]+$$

$$+\frac{\omega^2 H^2}{8} d^2 N_\kappa^{-1/2} [[\delta_{0,0} \frac{z_0}{H/2} F_{0_\kappa}^H + \delta_{1,0}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa} + \frac{d\phi_0}{H/2} F_{1\kappa}^P) + \in_0 i^0 (F_{0\kappa} - \frac{J_0(\kappa a)}{dz'_\kappa(d)})]$$

$$\kappa \frac{\overline{H'_0(\kappa r)}}{H_0(\kappa a)} \cosh(\kappa z)\ [\sum_{\substack{j=1 \\ a_j=a}}^{\infty} N_a^{-1/2} [\overline{\delta_{0,1} \frac{z_0}{H/2}}\ \overline{F_{0_a}^H + \delta_{1,1}(}\ \overline{\frac{x_0}{H/2} F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2} F_{1_a}^P) + \in_1 i^1 F_{1_a}}\ ]$$

$$a \frac{\overline{K'_1(ar)}}{K_1(aa)} \cos(az)\pi + [\delta_{0,1} \frac{z_0}{H/2} F_{0_\kappa}^H + \delta_{1,1}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa} + \frac{d\phi_0}{H/2} F_{1\kappa}^P) + \in_1 i^1 (F_{1\kappa} - \frac{J_1(\kappa a)}{dz'_\kappa(d)})]$$

$$\kappa \frac{H'_1(\kappa r)}{H_1(\kappa a)} \cosh(\kappa z)\ [\sum_{\substack{j=1 \\ a_j=a}}^{\infty} N_a^{-1/2} [\overline{\delta_{0,0} \frac{z_0}{H/2}}\ \overline{F_{0_a}^H + \delta_{1,0}(}\ \overline{\frac{x_0}{H/2} F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2} F_{1_a}^P) + \in_0 i^0 F_{0_a}}\ ]$$

$$a \frac{\overline{K'_0(ar)}}{K_0(aa)} \cos(az)\pi + [\delta_{0,1} \frac{z_0}{H/2} F_{0_\kappa}^H + \delta_{1,1}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa} + \frac{d\phi_0}{H/2} F_{1\kappa}^P) + \in_1 i^1 (F_{1\kappa} - \frac{J_1(\kappa a)}{dz'_\kappa(d)})]$$

$$\kappa \frac{H'_1(\kappa r)}{H_1(\kappa a)} \cosh(\kappa z)\ [\sum_{\substack{j=1 \\ a_j=a}}^{\infty} N_a^{-1/2} [\overline{\delta_{0,2} \frac{z_0}{H/2}}\ \overline{F_{0_a}^H + \delta_{1,2}(}\ \overline{\frac{x_0}{H/2} F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2} F_{1_a}^P) + \in_2 i^2 F_{2_a}}\ ]$$

$$a \frac{\overline{K'_2(ar)}}{K_2(aa)} \cos(az) \frac{\pi}{2} + \sum_{p=2,3,}^{\infty} [\delta_{0,p} \frac{z_0}{H/2} F_{0_\kappa}^H + \delta_{1,p}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2} F_{1\kappa}^P) + \in_p i^p (F_{p\kappa} - \frac{J_p(\kappa a)}{dz'_\kappa(d)})]\ \kappa \frac{H'_p(\kappa r)}{H_p(\kappa a)} \cosh(\kappa z)\ [[\sum_{\substack{j=1 \\ a_j=a}}^{\infty} N_a^{-1/2} [\overline{\delta_{0,p-1} \frac{z_0}{H/2}}\ \overline{F_{0_a}^H + \delta_{1,p-1}(}$$

$$\overline{\frac{x_0}{H/2} F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2} F_{1_a}^P) + \in_{p-1} i^{p-1} F_{p-1_a}}\ ]\ a \frac{\overline{K'_{p-1}(ar)}}{K_{p-1}(aa)} \cos(az) + [\sum_{\substack{j=1 \\ a_j=a}}^{\infty} N_a^{-1/2} [\overline{\delta_{0,p+1} \frac{z_0}{H/2}}$$

$$\overline{F_{0_a}^H + \delta_{1,p+1}(}\ \overline{\frac{x_0}{H/2} F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2} F_{1_a}^P) + \in_{p+1} i^{p+1} F_{p+1_a}}\ ]\ a \frac{\overline{K'_{p+1}(ar)}}{K_{p+1}(aa)} \cos(az)]\ \frac{\pi}{2}\ ] +$$

$$+\frac{gH^2}{8} d \frac{\cosh(\kappa z)}{\cosh(\kappa d)} [[\sum_{\substack{j=1 \\ a_j=a}}^{\infty} N_a^{-1/2} [\delta_{0,0} \frac{z_0}{H/2}\ F_{0_a}^H + \delta_{1,0}(\ \frac{x_0}{H/2} F_{1_a}^S + \frac{d\phi_0}{H/2} F_{1_a}^P) + \in_0 i^0 F_{0_a}\ ]$$

$$a \frac{K'_0(ar)}{K_0(aa)} \cos(az) \in_1 i^1 \kappa \overline{J'_1(\kappa r)} \pi + [\sum_{\substack{j=1 \\ a_j=a}}^{\infty} N_a^{-1/2} [\delta_{0,1} \frac{z_0}{H/2}\ F_{0_a}^H + \delta_{1,1}(\ \frac{x_0}{H/2} F_{1_a}^S$$

$$+\frac{d\phi_0}{H/2} F_{1_a}^P) + \in_1 i^1 F_{1_a}\ ]\ a \frac{K'_1(ar)}{K_1(aa)} \cos(az) \in_0 i^0 \kappa \overline{J'_0(\kappa r)} \pi + \sum_{\substack{j=1 \\ a_j=a}}^{\infty} N_a^{-1/2} [\delta_{0,1} \frac{z_0}{H/2}\ F_{0_a}^H + \delta_{1,1}($$

$$\frac{x_0}{H/2} F_{1_a}^S + \frac{d\phi_0}{H/2} F_{1_a}^P) + \in_1 i^1 F_{1_a}\ ]\ a \frac{K'_1(ar)}{K_1(aa)} \cos(az) \in_2 i^2 \kappa \overline{J'_2(\kappa r)} \frac{\pi}{2} +$$

$$\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_p i^p F_{p_a}]\; a\frac{K_p'(ar)}{K_p(a\mathrm{a})}\cos(az)[$$

$$\in_{p-1} i^{p-1}\kappa\overline{J'_{p-1}(\kappa r)}+\in_{p+1} i^{p+1}\kappa\overline{J'_{p+1}(\kappa r)}]\frac{\pi}{2}]+$$

$$+\frac{\omega^2H^2}{8}d^2N_\kappa^{-1/2}[[\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,0}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_0 i^0F_{0_a}]$$

$$a\frac{K_0'(ar)}{K_0(a\mathrm{a})}\cos(az)\,\overline{[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,1}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa\mathrm{a})}{dz'_\kappa(d)})]}$$

$$\kappa\frac{\overline{H_1'(\kappa r)}}{H_1(\kappa\mathrm{a})}\cosh(\kappa z)\pi+$$

$$+\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_1 i^1F_{1_a}]\; a\frac{K_1'(ar)}{K_1(a\mathrm{a})}\cos(az)$$

$$\overline{[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,0}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\;\kappa\frac{\overline{H_0'(\kappa r)}}{H_0(\kappa\mathrm{a})}\cosh(\kappa z)\pi+$$

$$+\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_1 i^1F_{1_a}]\; a\frac{K_1'(ar)}{K_1(a\mathrm{a})}\cos(az)$$

$$\overline{[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,2}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\;\kappa\frac{\overline{H_2'(\kappa r)}}{H_2(\kappa\mathrm{a})}\cosh(\kappa z)\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_p i^pF_{p_a}]\; a\frac{K_p'(ar)}{K_p(a\mathrm{a})}\cos(az)$$

$$\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,p-1}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa\mathrm{a})}{dz'_\kappa(d)})]}$$

$$\kappa\frac{\overline{H'_{p-1}(\kappa r)}}{H_{p-1}(\kappa\mathrm{a})}\cosh(\kappa z)+\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,p+1}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\;\kappa\frac{\overline{H'_{p+1}(\kappa r)}}{H_{p+1}(\kappa\mathrm{a})}\cosh(\kappa z)]\frac{\pi}{2}]+$$

$$+\frac{\omega^2 H^2}{8} d^2 [[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty} N_{a_\ell}^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}\ F_{0_{a_\ell}}^{H} + \delta_{1,0}(\ \frac{x_0}{H/2}F_{1_{a_\ell}}^{S} + \frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P}) + \in_0\ i^0 F_{0_{a_\ell}}\ ]$$

$$a_\ell\,\frac{K_0'(a_\ell r)}{K_0(a_\ell \mathrm{a})}\cos(a_\ell z)\ [\sum_{\substack{n=1\\ a_n}}^{\infty} N_{a_n}^{-1/2}[\overline{\delta_{0,1}\frac{z_0}{H/2}}\ \overline{F_{0_{a_n}}^{H} + \delta_{1,1}(}\ \overline{\frac{x_0}{H/2}F_{1_{a_n}}^{S}}\ \overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P}) + \in_1\ i^1 F_{1_{a_n}}}\ ]$$

$$a_n\,\frac{\overline{K_1'(a_n r)}}{\overline{K_1(a_n \mathrm{a})}}\cos(a_n z)\,\pi +$$

$$\sum_{\substack{\ell=1\\ a_\ell}}^{\infty} N_{a_\ell}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\ F_{0_{a_\ell}}^{H} + \delta_{1,1}(\ \frac{x_0}{H/2}F_{1_{a_\ell}}^{S} + \frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P}) + \in_1\ i^1 F_{1_{a_\ell}}\ ]\ a_\ell\,\frac{K_1'(a_\ell r)}{K_1(a_\ell \mathrm{a})}\cos(a_\ell z)$$

$$\sum_{\substack{n=1\\ a_n}}^{\infty} N_{a_n}^{-1/2}[\overline{\delta_{0,0}\frac{z_0}{H/2}}\ \overline{F_{0_{a_n}}^{H} + \delta_{1,0}(}\ \overline{\frac{x_0}{H/2}F_{1_{a_n}}^{S}}\ \overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P}) + \in_0\ i^0 F_{0_{a_n}}}\ ]\ a_n\,\frac{\overline{K_0'(a_n r)}}{\overline{K_0(a_n \mathrm{a})}}\cos(a_n z)\,\pi +$$

$$\sum_{\substack{\ell=1\\ a_\ell}}^{\infty} N_{a_\ell}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\ F_{0_{a_\ell}}^{H} + \delta_{1,1}(\ \frac{x_0}{H/2}F_{1_{a_\ell}}^{S} + \frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P}) + \in_1\ i^1 F_{1_{a_\ell}}\ ]\ a_\ell\,\frac{K_1'(a_\ell r)}{K_1(a_\ell \mathrm{a})}\cos(a_\ell z)$$

$$\sum_{\substack{n=1\\ a_n}}^{\infty} N_{a_n}^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F_{0_{a_n}}^{H} + \delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F_{1_{a_n}}^{S}}\ \overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P}) + \in_2\ i^2 F_{2_{a_n}}}\ ]\ a_n\,\frac{\overline{K_2'(a_n r)}}{\overline{K_2(a_n \mathrm{a})}}\cos(a_n z)\ \frac{\pi}{2} +$$

$$+\sum_{p=2,3,}^{\infty} [\sum_{\substack{\ell=1\\ a_\ell}}^{\infty} N_{a_\ell}^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}\ F_{0_{a_\ell}}^{H} + \delta_{1,p}(\ \frac{x_0}{H/2}F_{1_{a_\ell}}^{S} + \frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P}) + \in_p\ i^p F_{p_{a_\ell}}\ ]$$

$$a_\ell\,\frac{K_p'(a_\ell r)}{K_p(a_\ell \mathrm{a})}\cos(a_\ell z)\ [\sum_{\substack{n=1\\ a_n}}^{\infty} N_{a_n}^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F_{0_{a_n}}^{H} + \delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F_{1_{a_n}}^{S}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P}) + \in_{p-1}\ i^{p-1} F_{p-1_{a_n}}}\ ]\ a_n\,\frac{\overline{K_{p-1}'(a_n r)}}{\overline{K_{p-1}(a_n \mathrm{a})}}\cos(a_n z) + \sum_{\substack{n=1\\ a_n}}^{\infty} N_{a}^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F_{0_{a_n}}^{H} + \delta_{1,p+1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_{a_n}}^{S}}\ \overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P}) + \in_{p+1}\ i^{p+1} F_{p+1_{a_n}}}\ ]\ a_n\,\frac{\overline{K_{p+1}'(a_n r)}}{\overline{K_{p+1}(a_n \mathrm{a})}}\cos(a_n z)\,]\,\frac{\pi}{2}].$$

Στη συνέχεια υπολογίζω το ολοκλήρωμα $\int_{d_2}^{d_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial \Phi}{\partial r}\right)^2}^{T}\cos\theta d\theta\right] RADIUS dz$.

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$$\int_{d_2}^{d_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial r}\right)^2}^T \cos\theta d\theta\right] RADIUSdz =$$

$$=\frac{g^2H^2}{8\omega^2}\left(\frac{1}{\cosh(\kappa d)}\right)^2 [\in_0 i^0\kappa J_0'(\kappa r)\ \overline{\in_1 i^1\kappa J_1'(\kappa r)}\pi + \in_1 i^1\kappa J_1'(\kappa r)\ \overline{\in_0 i^0\kappa J_0'(\kappa r)}\pi +$$

$$+\in_1 i^1\kappa J_1'(\kappa r)\ \overline{\in_2 i^2\kappa J_2'(\kappa r)}\ \frac{\pi}{2} + \sum_{p=2,3,}^{\infty} \in_p i^p\kappa J_p'(\kappa r)\,[\,\kappa[\in_{p-1}\ \overline{i^{p-1}J_{p-1}'(\kappa r)} +$$

$$\in_{p+1}\ \overline{i^{p+1}J_{p+1}'(\kappa r)}]]\frac{\pi}{2}]]\ \underline{RADIUS}\left[\frac{2(d_1-d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{1}{\cosh(\kappa d)}[\in_0 i^0\kappa J_0'(\kappa r)\ \overline{[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz_\kappa'(d)})]}\ \kappa\frac{\overline{H_1'(\kappa r)}}{H_1(\kappa a)}\pi + \varepsilon_1 i^1\kappa J_1'(\kappa r)\ \overline{[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \varepsilon_0 i^0(F_{1\kappa}-\frac{J_0(\kappa a)}{dz_\kappa'(d)})]}\ \kappa\frac{\overline{H_0'(\kappa r)}}{H_0(\kappa a)}\pi + \varepsilon_1 i^1\kappa J_1'(\kappa r)\ \overline{[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}$$

$$\overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_2 i^2(F_{1\kappa}-\frac{J_2(\kappa a)}{dz_\kappa'(d)})]}\ \kappa\frac{\overline{H_2'(\kappa r)}}{H_2(\kappa a)}\frac{\pi}{2} + \sum_{p=2,3,}^{\infty}\in_p i^p\kappa J_p'(\kappa r)$$

$$[\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa a)}{dz_\kappa'(d)})]}$$

$$\kappa\frac{\overline{H_{p-1}'(\kappa r)}}{H_{p-1}(\kappa a)} + \overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz_\kappa'(d)})]}$$

$$\kappa\frac{\overline{H_{p+1}'(\kappa r)}}{H_{p+1}(\kappa a)}]\frac{\pi}{2}]\ \underline{RADIUS}\left[\frac{2(d_1-d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{gH^2}{8}d\frac{1}{\cosh(\kappa d)}[[\in_0 i^0\kappa J_0'(\kappa r)\sum_{\substack{j=1\\ a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,1}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P) + \in_1 i^1F_{1_a}]}\ a\frac{\overline{K_1'(ar)}}{K_1(aa)}\pi\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big]+\in_1 i^1\kappa J_1'(\kappa r)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,0}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,0}}$$

$$\overline{(\frac{x_0}{H/2}F_{1_a}^S}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_0 i^0F_{0_a}}\,]\,a\frac{\overline{K_0'(ar)}}{K_0(a\mathrm{a})}\pi$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big]+$$

$$+\in_1 i^1\kappa J_1'(\kappa r)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,2}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^S}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_2 i^2F_{2_a}}\,]\,a\frac{\overline{K_2'(ar)}}{K_2(a\mathrm{a})}\frac{\pi}{2}$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big]+$$

$$+\sum_{p=2,3}^{\infty}\in_p i^p\kappa J_p'(\kappa r)[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,p-1}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p-1} i^{p-1}F_{p-1_a}}\,]\,a\frac{\overline{K_{p-1}'(ar)}}{K_{p-1}(a\mathrm{a})}$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big]+\sum_{\substack{k=1\\a_k=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,p+1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p+1} i^{p+1}F_{p+1_a}}\,]\,a\frac{\overline{K_{p+1}'(ar)}}{K_{p+1}(a\mathrm{a})}$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big]\big]\frac{\pi}{2}]\ \underline{RADIUS}+$$

$$+\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{1}{\cosh(\kappa d)}[[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa\mathrm{a})}{dz_\kappa'(d)})]\,\kappa\frac{H_0'(\kappa r)}{H_0(\kappa\mathrm{a})}\in_1 i^1\kappa\overline{J_1'(\kappa r)}\pi+[[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}$$

$$F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\;\kappa\frac{H'_1(\kappa r)}{H_1(\kappa a)}\in_0\kappa\overline{J'_0(\kappa r)}\pi+[\delta_{0,1}\frac{z_0}{H/2}F^{H}_{0_\kappa}+\delta_{1,1}(\frac{x_0}{H/2}$$

$$F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\;\kappa\frac{H'_1(\kappa r)}{H_1(\kappa a)}\in_2 i^2\kappa\overline{J'_2(\kappa r)}\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F^{H}_{0_\kappa}+\delta_{1,p}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\;\kappa\frac{H'_p(\kappa r)}{H_p(\kappa a)}[[$$

$$\in_{p-1} i^{p-1}\kappa\overline{J'_{p-1}(\kappa r)}+\in_{p+1} i^{p+1}\kappa\overline{J'_{p+1}(\kappa r)}\,]\frac{\pi}{2}]\underline{RADIUS}$$

$$\left[\frac{2(d_1-d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{\omega^2H^2}{8}d^2N_\kappa^{-1}[[\delta_{0,0}\frac{z_0}{H/2}F^{H}_{0_\kappa}+\delta_{1,0}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]$$

$$\kappa\frac{H'_0(\kappa r)}{H_0(\kappa a)}\overline{[\delta_{0,1}\frac{z_0}{H/2}F^{H}_{0_\kappa}+\delta_{1,1}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]}$$

$$\kappa\overline{\overline{\frac{H'_1(\kappa r)}{H_1(\kappa a)}}}\pi+[\delta_{0,1}\frac{z_0}{H/2}F^{H}_{0_\kappa}+\delta_{1,1}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\;\kappa\frac{H'_1(\kappa r)}{H_1(\kappa a)}$$

$$\overline{[\delta_{0,0}\frac{z_0}{H/2}F^{H}_{0_\kappa}+\delta_{1,0}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]}$$

$$\kappa\overline{\overline{\frac{H'_0(\kappa r)}{H_0(\kappa a)}}}\pi+[\delta_{0,1}\frac{z_0}{H/2}F^{H}_{0_\kappa}+\delta_{1,1}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\;\kappa\frac{H'_1(\kappa r)}{H_1(\kappa a)}$$

$$\overline{[\delta_{0,2}\frac{z_0}{H/2}F^{H}_{0_\kappa}+\delta_{1,2}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa a)}{dz'_\kappa(d)})]}$$

$$\kappa\overline{\overline{\frac{H'_2(\kappa r)}{H_2(\kappa a)}}}\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F^{H}_{0_\kappa}+\delta_{1,p}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\;\kappa\frac{H'_p(\kappa r)}{H_p(\kappa a)}[$$

$$\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F^{H}_{0_\kappa}+\delta_{1,p-1}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa a)}{dz'_\kappa(d)})]}\;\kappa\overline{\overline{\frac{H'_{p-1}(\kappa r)}{H_{p-1}(\kappa a)}}}+$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1}i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})]}$$

$$\kappa\frac{\overline{H'_{p+1}(\kappa r)}}{H_{p+1}(\kappa a)}]]\frac{\pi}{2}]\ \underline{RADIUS}\ \left[\frac{2(d_1-d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{\omega^2H^2}{8}d^2N_\kappa^{-1/2}[[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]$$

$$\kappa\frac{H'_0(\kappa r)}{H_0(\kappa\partial)}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,1}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}}]\ a\frac{\overline{K'_1(ar)}}{K_1(a a)}\pi$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\ \kappa\frac{H'_1(\kappa r)}{H_1(\kappa a)}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,0}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,0}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_0 i^0F_{0_a}}]\ a\frac{\overline{K'_0(ar)}}{K_0(a a)}\pi\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\ \kappa\frac{H'_1(\kappa r)}{H_1(\kappa a)}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_2 i^2F_{2_a}}]\ a\frac{\overline{K'_2(ar)}}{K_2(a a)}\frac{\pi}{2}\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]+\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\ \kappa\frac{H'_p(\kappa r)}{H_p(\kappa a)}[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,p-1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p-1} i^{p-1}F_{p-1_a}}]\ a\frac{\overline{K'_{p-1}(ar)}}{K_{p-1}(a a)}$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]+[\sum_{\substack{k=1\\a_k=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^{H}+\delta_{1,p+1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^{S}}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P})}+\in_{p+1} i^{p+1}F_{p+1_a}]\ a\frac{\overline{K'_{p+1}(ar)}}{K_{p+1}(a\text{a})}$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]\frac{\pi}{2}]\ \underline{RADIUS}+$$

$$+\frac{gH^2}{8}d\frac{1}{\cosh(\kappa d)}[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,0}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_0 i^0F_{0_a}]$$

$$a\frac{K'_0(ar)}{K_0(a\text{a})}\Bigg[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]\in_1 i^1\kappa\overline{J'_1(\kappa r)}\pi+[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}$$

$$F_{0_a}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_1 i^1F_{1_a}]\ a\frac{K'_1(ar)}{K_1(a\text{a})}$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]\in_0 i^0\kappa\overline{J'_0(\kappa r)}\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}$$

$$F_{0_a}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_1 i^1F_{1_a}]\ a\frac{K'_1(ar)}{K_1(a\text{a})}$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]\in_2 i^2\kappa\overline{J'_2(\kappa r)}\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}$$

$$F_{0_a}^{H}+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_p i^pF_{p_a}]\ a\frac{K'_p(ar)}{K_p(a\text{a})}$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big][\in_{p-1} i^{p-1}\kappa\overline{J'_{p-1}(\kappa r)}+\in_{p+1} i^{p+1}\kappa\overline{J'_{p+1}(\kappa r)}\,]\frac{\pi}{2}$$

] <u>*RADIUS*</u> +

$$+\frac{\omega^2H^2}{8}d^2N_\kappa^{-1/2}[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,0}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_0 i^0F_{0_a}]\,a\frac{K_0'(ar)}{K_0(a\mathrm{a})}$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big]\overline{[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\;\kappa\frac{\overline{H_1'(\kappa r)}}{H_1(\kappa\mathrm{a})}\pi$$

$$+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]\,a\frac{K_1'(ar)}{K_1(a\mathrm{a})}$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big]\overline{[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\;\kappa\frac{\overline{H_0'(\kappa r)}}{H_0(\kappa\mathrm{a})}\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]\,a\frac{K_1'(ar)}{K_1(a\mathrm{a})}\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big]\overline{[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\;\kappa\frac{\overline{H_2'(\kappa r)}}{H_2(\kappa\mathrm{a})}\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,p}($$

$$\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^pF_{p_a}]\,a\frac{K_p'(ar)}{K_p(a\mathrm{a})}$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big]$$

$$[\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\;\kappa\frac{\overline{H_{p-1}'(\kappa r)}}{H_{p-1}(\kappa\mathrm{a})}+$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa \mathrm{a})}{dz'_\kappa(d)})]}\ \kappa\frac{\overline{H'_{p+1}(\kappa r)}}{H_{p+1}(\kappa \mathrm{a})}]$$

$$\frac{\pi}{2}]\ \underline{RADIUS}+$$

$$+\frac{\omega^2H^2}{8}d^2\,[[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,0}\frac{z_0}{H/2}\ F^H_{0_a}+\delta_{1,0}(\ \frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_0 i^0F_{0_a}]\ a\frac{K'_0(ar)}{K_0(a\mathrm{a})}$$

$$\overline{[\delta_{0,1}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,1}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_1 i^1F_{1_a}}]\ a\frac{\overline{K'_1(ar)}}{\overline{K_1(a\mathrm{a})}}$$

$$[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]+$$

$$[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n\\a_\ell\neq a_n}}^{\infty}\ N_{a_\ell}^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}\ F^H_{0_{a_\ell}}+\delta_{1,0}(\ \frac{x_0}{H/2}F^S_{1_{a_\ell}}+\frac{d\phi_0}{H/2}F^P_{1_{a_\ell}})+\in_0 i^0F_{0_{a_\ell}}]\ a_\ell\frac{K'_0(a_\ell r)}{K_0(a_\ell \mathrm{a})}$$

$$N_{a_n}^{-1/2}\overline{[\delta_{0,1}\frac{z_0}{H/2}}\ \overline{F^H_{0_{a_n}}+\delta_{1,1}(}\ \overline{\frac{x_0}{H/2}F^S_{1_{a_n}}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_{a_n}})+\in_1 i^1F_{1_{a_n}}}]\ a_n\frac{\overline{K'_1(a_nr)}}{\overline{K_1(a_n\mathrm{a})}}]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_nd_1)\sin(a_\ell d_1)-a_\ell\cos(a_nd_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_nd_2)]]\pi+$$

$$+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,1}\frac{z_0}{H/2}\ F^H_{0_a}+\delta_{1,1}(\ \frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_1 i^1F_{1_a}]\ a\frac{K'_1(ar)}{K_1(a\mathrm{a})}\ \overline{[\delta_{0,0}\frac{z_0}{H/2}}$$

$$\overline{F^H_{0_a}+\delta_{1,0}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_0 i^0F_{0_a}}]\ a\frac{\overline{K'_0(ar)}}{\overline{K_0(a\mathrm{a})}}\ [\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]$$

$$+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n\\a_\ell\neq a_n}}^{\infty}\ N_{a_\ell}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\ F^H_{0_{a_\ell}}+\delta_{1,1}(\ \frac{x_0}{H/2}F^S_{1_{a_p}}+\frac{d\phi_0}{H/2}F^P_{1_{a_\ell}})+\in_1 i^1F_{1_{a_\ell}}]\ a_\ell\frac{K'_1(a_\ell r)}{K_1(a_\ell \mathrm{a})}$$

$$N_{a_n}^{-1/2}\overline{[\delta_{0,0}\frac{z_0}{H/2}}\ \overline{F^H_{0_{a_n}}+\delta_{1,0}(}\ \overline{\frac{x_0}{H/2}F^S_{1_{a_n}}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_{a_n}})+\in_0 i^0F_{0_{a_n}}}]\ a_n\frac{\overline{K'_0(a_nr)}}{\overline{K_0(a_n\mathrm{a})}}]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_nd_1)\sin(a_\ell d_1)-a_\ell\cos(a_nd_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_nd_2)]]\pi+$$

$$+\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1}[\delta_{0,1}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]\ a\frac{K_1'(ar)}{K_1(a\text{a})}\ \overline{[\delta_{0,2}\frac{z_0}{H/2}}$$

$$\overline{F_{0_a}^H+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_2 i^2F_{2_a}]}\ a\overline{\frac{K_2'(ar)}{K_2(a\text{a})}}$$

$$[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]+$$

$$+[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\sum_{\substack{n=1\\ a_n\\ a_\ell\neq a_n}}^{\infty} N_{a_\ell}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\ F_{0_{a_\ell}}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_{a_\ell}}^S+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^P)+\in_1 i^1F_{1_{a_\ell}}]\ a_\ell\frac{K_1'(a_\ell r)}{K_1(a_\ell\text{a})}$$

$$N_{a_n}^{-1/2}\overline{[\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F_{0_{a_n}}^H+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F_{1_{a_n}}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^P)+\in_2 i^2F_{2_{a_n}}]}\ a_p\overline{\frac{K_2'(a_pr)}{K_2(a_p\text{a})}}]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_nd_1)\sin(a_\ell d_1)-a_\ell\cos(a_nd_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_nd_2)]]\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\ a_j=a}}^{\infty}N_a^{-1}[\delta_{0,p}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^pF_{p_a}]\ a\ \frac{K_p'(a\ r)}{K_p(a\ \text{a})}$$

$$\overline{[\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p-1} i^{p-1}F_{p-1_a}]}\ a\ \overline{\frac{K_{p-1}'(a\ r)}{K_{p-1}(a\ \text{a})}}$$

$$[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]+$$

$$+\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\sum_{\substack{n=1\\ a_n\\ a_\ell\neq a_n}}^{\infty} N_{a_\ell}^{-1/2}[\delta_{0,p}\ \frac{z_0}{H/2}\ F_{0_{a_\ell}}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1_{a_\ell}}^S+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^P)+\in_p i^pF_{p_{a_\ell}}]\ a_\ell\frac{K_p'(a_\ell r)}{K_p(a_\ell\text{a})}$$

$$N_{a_n}^{-1/2}\overline{[\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F_{0a_n}^H+\delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F_{1_{a_p}}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^P)+\in_{p-1} i^{p-1}F_{p-1_{a_n}}]}\ a_n\overline{\frac{K_{p-1}'(a_nr)}{K_{p-1}(a_n\text{a})}}]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_nd_1)\sin(a_\ell d_1)-a_\ell\cos(a_nd_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_nd_2)]\ ]\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_p i^p F_{p_a}]\, a\,\frac{K'_p(a\, r)}{K_p(a\, \mathrm{a})}$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}\,F_{0_a}^{H}+\delta_{1,p+1}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_{p+1} i^{p+1}F_{p+1_a}]}\, a\,\overline{\frac{K'_{p+1}(a\, r)}{K_{p+1}(a\, \mathrm{a})}}$$

$$[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]+$$

$$+\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\sum_{\substack{n=1\\ a_n\\ a_\ell\neq a_n}}^{\infty} N_{a_\ell}^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_{a_\ell}}^{H}+\delta_{1,p}(\frac{x_0}{H/2}F_{1_{a_\ell}}^{S}+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P})+\in_p i^p F_{p_{a_\ell}}]\, a_\ell\,\frac{K'_p(a_\ell\, r)}{K_p(a_\ell\, \mathrm{a})}$$

$$N_{a_n}^{-1/2}\overline{[\delta_{0,p+1}\frac{z_0}{H/2}\,F_{0a_n}^{H}+\delta_{1,p+1}(\frac{x_0}{H/2}F_{1_{a_n}}^{S}+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P})+\in_{p+1} i^{p+1}F_{p+1_{a_n}}]}\, a_n\,\overline{\frac{K'_{p+1}(a_n r)}{K_{p+1}(a_n \mathrm{a})}}$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_n d_1)\sin(a_\ell d_1)-a_\ell\cos(a_n d_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_n d_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_n d_2)]\,]\frac{\pi}{2}]\,\underline{RADIUS}$$

Όμοια θα υπολογίσουμε τον όρο $\frac{1}{r^2}\left(\frac{\partial\Phi}{\partial\theta}\right)^2=\frac{1}{r^2}\frac{1}{2}\frac{\partial\phi}{\partial\theta}\overline{\frac{\partial\phi}{\partial\theta}}$.

Ισχύει από το Κεφάλαιο 1 -σελίδα 11-, ότι

$\varphi(r,\theta,z)=$

$$=\frac{-igH}{2\omega}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\cos(m\theta)-i\,\omega\frac{H}{2}dN_\kappa^{-1/2}\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,m}(\frac{x_0}{H/2}$$

$$F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa \mathrm{a})}{dz'_\kappa(d)})]\,\frac{H_m(\kappa r)}{H_m(\kappa \mathrm{a})}\cos(m\vartheta)\cosh(\kappa z)-i\,\omega\frac{H}{2}dN_a^{-1/2}$$

$$\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\ a_j=a}}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_m i^m F_{m_a}]\,\frac{K_m(ar)}{K_m(a\mathrm{a})}\cos(m\vartheta)\cos(az)\,.$$

Επομένως

$$\frac{\partial\phi(r,\theta,z)}{\partial\theta}=$$

$$=\frac{igH}{2\omega}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\sin(m\theta)m + i\ \omega\frac{H}{2}dN_\kappa^{-1/2}\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,m}(\frac{x_0}{H/2}$$

$$F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa \mathrm{a})}{dz'_\kappa(d)})]\frac{H_m(\kappa r)}{H_m(\kappa \mathrm{a})}\sin(m\theta)m\cosh(\kappa z)+i\ \omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_m i^m F_{m_a}]$$

$$\mathrm{N}_a^{-1/2}\frac{K_m(ar)}{K_m(a\mathrm{a})}\sin(m\theta)m\cos(az).$$

Όμοια

$$\frac{\overline{\partial\phi(r,\theta,z)}}{\partial\theta}=$$

$$=\frac{-igH}{2\omega}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\sum_{m=0}^{\infty}\in_m i^m\overline{J_m(\kappa r)}\sin(m\theta)m - i\ \omega\frac{H}{2}dN_\kappa^{-1/2}\sum_{m=0}^{\infty}\overline{[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,m}(\frac{x_0}{H/2}}$$

$$\overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa \mathrm{a})}{dz'_\kappa(d)})]}\ \frac{\overline{H_m(\kappa r)}}{\overline{H_m(\kappa \mathrm{a})}}\sin(m\theta)m\cosh(\kappa z)-i\ \omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}\overline{[\delta_{0,m}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,m}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_m i^m F_{m_a}]}$$

$$\mathrm{N}_a^{-1/2}\frac{\overline{K_m(ar)}}{\overline{K_m(a\mathrm{a})}}\sin(m\theta)m\cos(az).$$

Και

$$\frac{1}{r^2}\left(\frac{\partial\Phi}{\partial\theta}\right)^2=\frac{1}{r^2}\frac{1}{2}\frac{\partial\phi}{\partial\theta}\frac{\overline{\partial\phi}}{\partial\theta}=$$

$$=\frac{1}{r^2}\frac{g^2H^2}{8\omega^2}\left(\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\right)^2\left[\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)m\sin(m\theta)\sum_{n=0}^{\infty}\in_n i^n \overline{J_n(\kappa r)}n\sin(n\theta)+\right.$$

$$+\frac{1}{r^2}\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\left[\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)m\sin(m\theta)\sum_{n=0}^{\infty}\overline{[\delta_{0,n}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,n}(\frac{x_0}{H/2}}\right.$$

$$\overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_n i^n(F_{n\kappa}-\frac{J_n(\kappa a)}{dz_\kappa'(d)})]}\ n\frac{\overline{H_n(\kappa r)}}{\overline{H_n(\kappa a)}}(\sin(n\theta))\cosh(\kappa z)+$$

$$+\frac{1}{r^2}\frac{gH^2}{8}d\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\left[\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)m\sin(m\theta)\sum_{m=0}^{\infty}\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,n}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,n}(}\right.$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_n i^n F_{n_a}]}\ \frac{\overline{K_n(ar)}}{\overline{K_n(a a)}}n\sin(n\theta)\cos(az)+$$

$$+\frac{1}{r^2}\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\left[\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,m}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa}\right.$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa a)}{dz_\kappa'(d)})]\frac{H_m(\kappa r)}{H_m(\kappa a)}m\cosh(\kappa z)\sin(m\theta)\sum_{n=0}^{\infty}\in_n i^n\overline{J_n(\kappa r)}n\sin(n\theta)+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2}{8}d^2N_\kappa^{-1}\left[\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,m}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa a)}{dz_\kappa'(d)})]\right.$$

$$\frac{H_m(\kappa r)}{H_m(\kappa a)}m\cosh(\kappa z)\sin(m\theta)\sum_{n=0}^{\infty}\overline{[\delta_{0,n}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,n}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_n i^n(F_{n\kappa}-\frac{J_n(\kappa a)}{dz_\kappa'(d)})]}\ \frac{\overline{H_n(\kappa r)}}{\overline{H_n(\kappa a)}}n\sin(n\theta)\cosh(\kappa z)+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2}{8}d^2N_\kappa^{-1/2}\left[\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,m}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa}\right.$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa a)}{dz_\kappa'(d)})]\frac{H_m(\kappa r)}{H_m(\kappa a)}m\cosh(\kappa z)\sin(m\theta)\sum_{n=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,n}\frac{z_0}{H/2}}$$

$$\overline{F_{0_a}^H+\delta_{1,n}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_n i^n F_{n_a}]}\ \frac{\overline{K_n(ar)}}{\overline{K_n(a a)}}n\sin(n\theta)\cos(az)+$$

$$+\frac{1}{r^2}\frac{gH^2}{8}d\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\left[\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,m}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,m}(\ \frac{x_0}{H/2}F_{1_a}^S\right.$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_m i^m F_{m_a}]\ \frac{K_m(ar)}{K_m(a a)}m\cos(az)\sin(m\theta)\sum_{n=0}^{\infty}\in_n\overline{i^n J_n(\kappa r)}n\sin(n\theta)+$$

$$+\frac{1}{r^2}\frac{\omega^2 H^2}{8} d^2 N_\kappa^{-1/2} [\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_m i^m F_{m_a}]$$

$$\frac{K_m(ar)}{K_m(a\text{a})} m\cos(az)\sin(m\theta)\sum_{n=0}^{\infty}\overline{[\delta_{0,n}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,n}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_n i^n(F_{n\kappa}-\frac{J_n(\kappa\text{a})}{dz'_\kappa(d)})]}\ \overline{\frac{H_n(\kappa r)}{H_n(\kappa\text{a})}} n\sin(n\theta)\cosh(\kappa z)+$$

$$+\frac{1}{r^2}\frac{\omega^2 H^2}{8} d^2 [\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_m i^m F_{m_a}]$$

$$\frac{K_m(ar)}{K_m(a\text{a})} m\cos(az)\cos(m\theta)\sum_{n=0}^{\infty}[\sum_{\substack{i=1\\ a_i=a}}^{\infty} N_a^{-1/2}[\delta_{0,n}\overline{\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,n}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_n i^n F_{n_a}]}\ \overline{\frac{K_n(ar)}{K_n(a\text{a})}} n\sin(n\theta)\cos(az)$$

Θα υπολογίσουμε το $\frac{1}{r^2}\int_{d_2}^{d_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial\theta}\right)^2}^T\cos\theta d\theta\right] RADIUS dz$.

Όπου $RADIUS$ η ακτίνα του $i-$στου εξωτερικού στοιχείου.

Πρώτα υπολογίζουμε το $\int_0^{2\pi}\frac{1}{r^2}\left(\frac{\partial\phi}{\partial\theta}\right)^2\cos\theta d\theta$ και προκύπτει η

$$\int_0^{2\pi}\frac{1}{r^2}\left(\frac{\partial\phi}{\partial\theta}\right)^2\cos\theta d\theta=$$

$$=\frac{1}{r^2}\frac{g^2H^2}{8\omega^2}(\frac{\cosh(\kappa z)}{\cosh(\kappa d)})^2[\in_1 i^1 J_1(\kappa r)1\ \overline{\in_2 i^2 J_2(\kappa r)}2\ \frac{\pi}{2}+\sum_{p=2,3,}^{\infty}\in_p i^p J_p(\kappa r)p\ [$$

$$\in_{p-1} i^{p-1}\overline{J_{p-1}(\kappa r)}(p-1)+\in_{p+1} i^{p+1}\overline{J_{p+1}(\kappa r)}(p+1)]\frac{\pi}{2}]]+$$

$$+\frac{1}{r^2}\frac{gH^2}{8} dN_\kappa^{-1/2}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}[\in_1 i^1 J_1(\kappa r)1\ \overline{[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}+$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_2 i^2(F_{1\kappa}-\frac{J_2(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\ \frac{\overline{H_2(\kappa r)}}{\overline{H_2(\kappa\mathrm{a})}}2\cosh(\kappa z)\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}\in_p i^p J_p(\kappa r)p\ [[\delta_{0,p-1}\overline{\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p-1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\ \frac{\overline{H_{p-1}(\kappa r)}}{\overline{H_{p-1}(\kappa\mathrm{a})}}(p-1)\cosh(\kappa z)+$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa\mathrm{a})}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_{p+1}(\kappa r)}}{\overline{H_{p+1}(\kappa\partial)}}(p+1)\cosh(\kappa z)]\frac{\pi}{2}]+$$

$$+\frac{1}{r^2}\frac{gH^2}{8}d\frac{\cosh(\kappa z)}{\cosh(\kappa d)}[\in_1 i^1 J_1(\kappa r)1\sum_{\substack{j=1\\ a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_2 i^2F_{2_a}]}\ \frac{\overline{K_2(ar)}}{\overline{K_2(a\mathrm{a})}}2\cos(az)\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}\in_p i^p J_p(\kappa r)p\ [\sum_{\substack{j=1\\ a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_{p-1} i^{p-1}F_{p-1_a}]}\ \frac{\overline{K_{p-1}(ar)}}{\overline{K_{p-1}(a\mathrm{a})}}(p-1)\cos(az)+\sum_{\substack{j=1\\ a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,p+1}(}$$

$$\overline{\frac{x_0}{H/2}F^S_{1_a}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_{p+1} i^{p+1}F_{p+1_a}]}\ \frac{\overline{K_{p+1}(ar)}}{\overline{K_{p+1}(a\mathrm{a})}}(p+1)\cos(az)]\frac{\pi}{2}]+$$

$$+\frac{1}{r^2}\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,1}(\frac{x_0}{H/2}F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa\mathrm{a})}{dz'_\kappa(d)})]\frac{H_1(\kappa r)}{H_1(\kappa\mathrm{a})}1\cosh(\kappa z)\in_2\overline{i^2J_2(\kappa r)}2\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p}(\frac{x_0}{H/2}F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa\mathrm{a})}{dz'_\kappa(d)})]$$

$$\frac{H_p(\kappa r)}{H_p(\kappa\mathrm{a})}p\cosh(\kappa z)[[\in_{p-1} i^{p-1}\overline{J_{p-1}(\kappa r)}(p-1)+\in_{p+1} i^{p+1}\overline{J_{p+1}(\kappa r)}(p+1)]\frac{\pi}{2}]+$$

$$+\frac{1}{r^2}\frac{\omega^2 H^2}{8} d^2 N_\kappa^{-1}\,[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\;F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa \mathrm{a})}{dz'_\kappa(d)})]$$

$$\frac{H_1(\kappa r)}{H_1(\kappa \mathrm{a})}1\cosh(\kappa z)\;[\overline{\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa \mathrm{a})}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_2(\kappa r)}}{\overline{H_2(\kappa \mathrm{a})}}2\cosh(\kappa z)\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}\;F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa \mathrm{a})}{dz'_\kappa(d)})]\;\frac{H_p(\kappa r)}{H_p(\kappa \mathrm{a})}p\cosh(\kappa z)\;[\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}$$

$$\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa \mathrm{a})}{dz'_\kappa(d)})]}\;\frac{\overline{H_{p-1}(\kappa r)}}{\overline{H_{p-1}(\kappa \mathrm{a})}}(p-1)\cosh(\kappa z)]+$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa \mathrm{a})}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_{p+1}(\kappa r)}}{\overline{H_{p+1}(\kappa \mathrm{a})}}(p+1)\cosh(\kappa z)]]\frac{\pi}{2}]+$$

$$+\frac{1}{r^2}\frac{\omega^2 H^2}{8} d^2 N_\kappa^{-1/2}\,[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\;F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa \mathrm{a})}{dz'_\kappa(d)})]$$

$$\frac{H_1(\kappa r)}{H_1(\kappa \mathrm{a})}1\cosh(\kappa z)\;[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,2}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^S}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_2 i^2 F_{2_a}]}$$

$$\frac{\overline{K_2(ar)}}{\overline{K_2(a\mathrm{a})}}2\cos(az)\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}\;F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa \mathrm{a})}{dz'_\kappa(d)})]\;\frac{H_p(\kappa r)}{H_p(\kappa \mathrm{a})}p\cosh(\kappa z)\;[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,p-1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p-1} i^{p-1}F_{p-1_a}]}\;\frac{\overline{K_{p-1}(ar)}}{\overline{K_{p-1}(a\mathrm{a})}}(p-1)\cos(az)+[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}$$

$$\overline{F_{0_a}^H+\delta_{1,p+1}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^S}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p+1} i^{p+1}F_{p+1_a}]}\;\frac{\overline{K_{p+1}(ar)}}{\overline{K_{p+1}(a\mathrm{a})}}(p+1)\cos(az)]\;\frac{\pi}{2}]+$$

$$+\frac{1}{r^2}\frac{gH^2}{8}d\frac{\cosh(\kappa z)}{\cosh(\kappa d)}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})}1\cos(az)\in_2 i^2\overline{J_2(\kappa r)}2\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^pF_{p_a}]\frac{K_p(ar)}{K_p(a\mathrm{a})}p\cos(az)$$

$$[\in_{p-1} i^{p-1}\overline{J_{p-1}(\kappa r)}(p-1)+\in_{p+1} i^{p+1}\overline{J_{p+1}(\kappa r)}(p+1)]\frac{\pi}{2}]+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2}{8}d^2N_\kappa^{-1/2}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})}1\cos(az)\,\overline{[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}\,\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa\mathrm{a})}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_2(\kappa r)}}{\overline{H_2(\kappa\mathrm{a})}}2\cosh(\kappa z)\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^S$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^pF_{p_a}]\frac{K_p(ar)}{K_p(a\mathrm{a})}p\cos(az)[\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\,\frac{\overline{H_{p-1}(\kappa r)}}{\overline{H_{p-1}(\kappa\mathrm{a})}}(p-1)\cosh(\kappa z)+$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}\,\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa\mathrm{a})}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_{p+1}(\kappa r)}}{\overline{H_{p+1}(\kappa\mathrm{a})}}(p+1)\cosh(\kappa z)]\frac{\pi}{2}]+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2}{8}d^2[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}N_{a_\ell}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_{a_\ell}}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_{a_\ell}}^S+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^P)+\in_1 i^1F_{1_{a_\ell}}]$$

$$\frac{K_1(a_\ell r)}{K_1(a_\ell\mathrm{a})}1\cos(a_\ell z)\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_n}^{-1/2}\overline{[\delta_{0,2}\frac{z_0}{H/2}}\,\overline{F_{0_{a_n}}^H+\delta_{1,2}(}\,\overline{\frac{x_0}{H/2}F_{1_{a_n}}^S}\,\overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^P)+\in_2 i^2F_{2_{a_n}}]}$$

$$\frac{\overline{K_2(a_nr)}}{\overline{K_2(a_n\mathrm{a})}}2\cos(a_nz)\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}N_{a_\ell}^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_{a_\ell}}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1_{a_\ell}}^S$$

$$+\frac{d\phi_0}{H/2}F^P_{1_{a_\ell}})+\in_p i^p F_{p_{a_\ell}}]\frac{K_p(a_\ell r)}{K_p(a_\ell \mathrm{a})}p\cos(a_\ell z)\,[\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_n}^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\,\overline{F^H_{0_{a_n}}+\delta_{1,p-1}}(\overline{\frac{x_0}{H/2}F^S_{1_{a_n}}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1_{a_n}})+\in_{p-1} i^{p-1}F_{p-1_{a_n}}]}\,\frac{\overline{K_{p-1}(a_n r)}}{\overline{K_{p-1}(a_n \mathrm{a})}}(p-1)\cos(a_n z)+\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_n}^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}$$

$$\overline{F^H_{0_{a_n}}+\delta_{1,p+1}}(\overline{\frac{x_0}{H/2}F^S_{1_{a_n}}}\,\overline{+\frac{d\phi_0}{H/2}F^P_{1_{a_n}})+\in_{p+1} i^{p+1}F_{p+1_{a_n}}]}\,\frac{\overline{K_{p+1}(a_n r)}}{K_{p+1}(a_n \mathrm{a})}(p+1)\cos(a_n z)\,]\frac{\pi}{2}].$$

Στη συνέχεια υπολογίζω το ολοκλήρωμα $\int_{d_2}^{d_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial\theta}\right)^2}^T\cos\theta d\theta\right]RADIUSdz$.

Όπου r η ακτίνα του $i-$στου εξωτερικού στοιχείου, δηλαδή

r = RADIUS

(*Παράρτημα ΣΤ*)

Επομένως προκύπτει ότι

$$\int_{d_2}^{d_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial\theta}\right)^2}^T\cos\theta d\theta\right]RADIUSdz=$$

$$=\frac{1}{r^2}\frac{g^2H^2}{8\omega^2}\left(\frac{1}{\cosh(\kappa d)}\right)^2[\in_1 i^1 J_1(\kappa r)1\,\overline{\in_2 i^2 J_2(\kappa r)}2\,\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}\in_p i^p J_p(\kappa r)p\,[$$

$$\in_{p-1} i^{p-1}(p-1)\overline{J_{p-1}(\kappa r)}+\in_{p+1} i^{p+1}(p+1)\overline{J_{p+1}(\kappa r)}]]\frac{\pi}{2}]]\underline{RADIUS}$$

$$\left[\frac{2(d_1-d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{1}{r^2}\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{1}{\cosh(\kappa d)}[\in_1 i^1 J_1(\kappa r)1\,[\overline{\delta_{0,2}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,2}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_2 i^2(F_{1\kappa}-\frac{J_2(\kappa \mathrm{a})}{dz'_\kappa(d)})]}\,\frac{\overline{H_2(\kappa r)}}{H_2(\kappa \mathrm{a})}2\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}\in_p i^p J_p(\kappa r)p$$

$$[[\overline{\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa a)}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_{p-1}(\kappa r)}}{\overline{H_{p-1}(\kappa a)}}(p-1)+[\overline{\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})]}\ \frac{\overline{H_{p+1}(\kappa r)}}{\overline{H_{p+1}(\kappa a)}}(p+1)]\frac{\pi}{2}]\underline{RADIUS}$$

$$\left[\frac{2(d_1-d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{1}{r^2}\frac{gH^2}{8}d\frac{1}{\cosh(\kappa d)}[\in_1 i^1 J_1(\kappa r)1\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_2 i^2F_{2_a}]}\ \frac{\overline{K_2(ar)}}{\overline{K_2(aa)}}2\frac{\pi}{2}\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]$$

$$+\sum_{p=2,3}^{\infty}\varepsilon_p i^p J_p(\kappa r)p[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p-1} i^{p-1}F_{p-1_a}]}$$

$$\frac{\overline{K_{p-1}(ar)}}{\overline{K_{p-1}(aa)}}(p-1)\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,p+1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p+1} i^{p+1}F_{p+1_a}]}\ \frac{\overline{K_{p+1}(ar)}}{\overline{K_{p+1}(aa)}}(p+1)$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]]\frac{\pi}{2}]\ \underline{RADIUS}+$$

$$+\frac{1}{r^2}\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{1}{\cosh(\kappa d)}[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\ \frac{H_1(\kappa r)}{H_1(\kappa a)}1\in_2\overline{i^2J_2(\kappa r)}2\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}\;F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\frac{H_p(\kappa r)}{H_p(\kappa a)}p[[$$

$$\in_{p-1} i^{p-1}(p-1)\overline{J_{p-1}(\kappa r)}+\in_{p+1} i^{p+1}(p+1)\overline{J_{p+1}(\kappa r)}]\frac{\pi}{2}]\underline{RADIUS}$$

$$\left[\frac{2(d_1-d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2}{8}d^2N_\kappa^{-1}[[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\;F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_1(\kappa r)}{H_1(\kappa a)}1\,\overline{[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa a)}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_2(\kappa r)}}{\overline{H_2(\kappa a)}}2\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}\;F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_p(\kappa r)}{H_p(\kappa a)}p\,[\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa a)}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_{p-1}(\kappa r)}}{\overline{H_{p-1}(\kappa a)}}(p-1)+\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})]}\;\frac{\overline{H_{p+1}(\kappa r)}}{\overline{H_{p+1}(\kappa a)}}(p+1)]]\frac{\pi}{2}]\underline{RADIUS}$$

$$\left[\frac{2(d_1-d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2}{8}d^2N_\kappa^{-1/2}[[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\;F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_1(\kappa r)}{H_1(\kappa a)}1\,[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,2}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^S}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_2 i^2F_{2_a}]}\;\frac{\overline{K_2(ar)}}{\overline{K_2(a a)}}2\frac{\pi}{2}$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]+\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}\;F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\frac{H_p(\kappa r)}{H_p(\kappa a)}p\,[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,p-1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S} \; \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p-1} i^{p-1}F_{p-1_a}]} \; \frac{\overline{K_{p-1}(ar)}}{K_{p-1}(a\mathrm{a})}(p-1)$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]+[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p+1}\overline{\frac{z_0}{H/2}} \; \overline{F_{0_a}^H+\delta_{1,p+1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S} \; \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p+1} i^{p+1}F_{p+1_a}]} \; \frac{\overline{K_{p+1}(ar)}}{K_{p+1}(a\mathrm{a})}(p+1)$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]\frac{\pi}{2}]\;\underline{RADIUS}+$$

$$+\frac{1}{r^2}\frac{gH^2}{8}d\frac{1}{\cosh(\kappa d)}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})}1\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]\in_2 i^2\overline{J_2(\kappa r)}2\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}$$

$$F_{0_a}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^pF_{p_a}]\frac{K_p(ar)}{K_p(a\mathrm{a})}p$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right][\in_{p-1} i^{p-1}\overline{J_{p-1}(\kappa r)}(p-1)+$$

$$\in_{p+1} i^{p+1}\overline{J_{p+1}(\kappa r)}(p+1)]\frac{\pi}{2}]\;\underline{RADIUS}+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2}{8}d^2N_\kappa^{-1/2}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})}1\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]\,\overline{[\delta_{0,2}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,2}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa a)}{dz'_\kappa(d)})]}\;\frac{\overline{H_2(\kappa r)}}{\overline{H_2(\kappa a)}}\,2\,\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F^H_{0_a}+\delta_{1,p}($$

$$\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_p i^p F_{p_a}]\,\frac{K_p(ar)}{K_p(a\mathrm{a})}\,p$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]$$

$$[\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p-1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa a)}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_{p-1}(\kappa r)}}{\overline{H_{p-1}(\kappa a)}}(p-1)+\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p+1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})]}\;\frac{\overline{H_{p+1}(\kappa r)}}{\overline{H_{p+1}(\kappa a)}}(p+1)\,]\,\frac{\pi}{2}\,]\;\underline{RADIUS}+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2}{8}d^2[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_a}+\delta_{1,1}(\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_1 i^1F_{1_a}]\,\frac{K_1(ar)}{K_1(a\mathrm{a})}1$$

$$\overline{[\delta_{0,2}\frac{z_0}{H/2}}\;\overline{F^H_{0_a}+\delta_{1,2}(}\;\overline{\frac{x_0}{H/2}F^S_{1_a}}\;\overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_2 i^2F_{2_a}]}\;\frac{\overline{K_2(ar)}}{\overline{K_2(a\mathrm{a})}}2$$

$$[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]+$$

$$+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_\ell}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F^H_{0a_\ell}+\delta_{1,1}(\frac{x_0}{H/2}F^S_{1_{a_\ell}}+\frac{d\phi_0}{H/2}F^P_{1_{a_\ell}})+\in_1 i^1F_{1_{a_\ell}}]\,\frac{K_1(a_\ell r)}{K_1(a_\ell \mathrm{a})}1$$

$$N_{a_n}^{-1/2}\overline{[\delta_{0,2}\frac{z_0}{H/2}}\;\overline{F^H_{0_{a_n}}+\delta_{1,2}(}\;\overline{\frac{x_0}{H/2}F^S_{1_{a_n}}}\;\overline{+\frac{d\phi_0}{H/2}F^P_{1_{a_n}})+\in_2 i^2F_{2_{a_n}}]}\;\frac{K_2(a_nr)}{K_2(a_n\mathrm{a})}2$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_nd_1)\sin(a_\ell d_1)-a_\ell\cos(a_nd_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_nd_2)]]\frac{\pi}{2}+$$

$$+\sum_{\substack{p=2,3,\\}}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty} N_a{}^{-1}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^p F_{p_a}]\frac{K_p(a\,r)}{K_p(a\,\mathrm{a})}p$$

$$\overline{[\delta_{0,p-1}\frac{z_0}{H/2}}\,\overline{F_{0_a}^H+\delta_{1,p-1}(}\,\overline{\frac{x_0}{H/2}}F_{1_a}^S\,\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p-1} i^{p-1}F_{p-1_a}]}\,\overline{\frac{K_{p-1}(a\,r)}{K_{p-1}(a\,\mathrm{a})}}(p-1)$$

$$[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]+$$

$$+\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n}}^{\infty} N_{a_\ell}^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_{a_\ell}}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1a_\ell}^S+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^P)+\in_p i^p F_{p_{a_\ell}}]\frac{K_p(a_\ell r)}{K_p(a_\ell \mathrm{a})}p$$

$$N_{a_n}^{-1/2}\overline{[\delta_{0,p-1}\frac{z_0}{H/2}}\,\overline{F_{0_{a_n}}^H+\delta_{1,p-1}(}\,\overline{\frac{x_0}{H/2}}F_{1a_n}^S\,\overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^P)+\in_{p-1} i^{p-1}F_{p-1_{a_n}}]}\,\overline{\frac{K_{p-1}(a_n r)}{K_{p-1}(a_n \mathrm{a})}}(p-1)]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_n d_1)\sin(a_\ell d_1)-a_\ell\cos(a_n d_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_n d_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_n d_2)]]\frac{\pi}{2}+$$

$$+\sum_{\substack{p=2,3,\\}}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty} N_a{}^{-1}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^p F_{p_a}]\frac{K_p(a\,r)}{K_p(a\,\mathrm{a})}p$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}}\,\overline{F_{0_a}^H+\delta_{1,p+1}(}\,\overline{\frac{x_0}{H/2}}F_{1_a}^S\,\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p+1} i^{p+1}F_{p+1_a}]}\,\overline{\frac{K_{p+1}(a\,r)}{K_{p+1}(a\,\mathrm{a})}}(p+1)$$

$$[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]+$$

$$+\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n}}^{\infty} N_{a_\ell}^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_{a_\ell}}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1a_\ell}^S+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^P)+\in_p i^p F_{p_{a_\ell}}]\frac{K_p(a_\ell r)}{K_p(a_\ell \mathrm{a})}p$$

$$N_{a_n}^{-1/2}\overline{[\delta_{0,p+1}\frac{z_0}{H/2}}\,\overline{F_{0a_n}^H+\delta_{1,p+1}(}\,\overline{\frac{x_0}{H/2}}F_{1_{a_n}}^S\,\overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^P)+\in_{p+1} i^{p+1}F_{p+1_{a_n}}]}\,\overline{\frac{K_{p+1}(a_n r)}{K_{p+1}(a_n \mathrm{a})}}(p+1)]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_n d_1)\sin(a_\ell d_1)-a_\ell\cos(a_n d_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_n d_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_n d_2)]]\frac{\pi}{2}]\ \underline{RADIUS}$$

Όμοια θα υπολογίσουμε τον όρο $\left(\frac{\partial \Phi}{\partial z}\right)^2 = \frac{1}{2}\frac{\partial \phi}{\partial z}\frac{\overline{\partial \phi}}{\partial z}$.

Ισχύει από το Κεφάλαιο 1 –σελίδα 11– ότι

φ(r,θ,z) =

$$= \frac{-igH}{2\omega}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\cos(m\theta) - i\ \omega\frac{H}{2}dN_{\kappa}^{-1/2}\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^{H} + \delta_{1,m}(\frac{x_0}{H/2}$$

$$F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2}F_{1\kappa}^{P}) + \in_m i^m (F_{m\kappa} - \frac{J_m(\kappa a)}{dz'_\kappa(d)})]\frac{H_m(\kappa r)}{H_m(\kappa a)}\cos(m\theta)\cosh(\kappa z) - i\ \omega\frac{H}{2}dN_a^{-1/2}$$

$$\sum_{m=0}^{\infty}[\sum_{\substack{j=1 \\ a_j=a}}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^{H} + \delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^{S} + \frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_m i^m F_{m_a}]\frac{K_m(ar)}{K_m(a\text{a})}\cos(m\theta)\cos(az).$$

Επομένως

$$\frac{\partial \phi(r,\theta,z)}{\partial z} =$$

$$= -\frac{igH}{2\omega}\frac{\sinh(\kappa z)}{\cosh(\kappa d)}\kappa\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\cos(m\theta) - i\ \omega\frac{H}{2}dN_{\kappa}^{-1/2}\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^{H} + \delta_{1,m}(\frac{x_0}{H/2}$$

$$F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2}F_{1\kappa}^{P}) + \in_m i^m (F_{m\kappa} - \frac{J_m(\kappa a)}{dz'_\kappa(d)})]\frac{H_m(\kappa r)}{H_m(\kappa a)}\cos(m\theta) m\sinh(\kappa z)\kappa + i\ \omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\sum_{\substack{j=1 \\ a_j=a}}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^{H} + \delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^{S} + \frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_m i^m F_{m_a}]$$

$$\text{N}_a^{-1/2}\frac{K_m(ar)}{K_m(a\text{a})}\cos(m\theta)a\sin(az).$$

Όμοια

$$\overline{\frac{\partial \phi(r,\theta,z)}{\partial z}} =$$

$$= \frac{igH}{2\omega}\frac{\sinh(\kappa z)}{\cosh(\kappa d)}\kappa\sum_{m=0}^{\infty}\in_m i^m \overline{J_m(\kappa r)}\cos(m\theta) + i\ \omega\frac{H}{2}dN_{\kappa}^{-1/2}\sum_{m=0}^{\infty}\overline{[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^{H} + \delta_{1,m}(\frac{x_0}{H/2}}$$

$$\overline{F^{S}{}_{1_\kappa}} + \overline{\frac{d\phi_0}{H/2}F_{1\kappa}^{P}) + \in_m i^m (F_{m\kappa} - \frac{J_m(\kappa a)}{dz'_\kappa(d)})]}\ \frac{\overline{H_m(\kappa r)}}{\overline{H_m(\kappa a)}}\cos(m\theta)\sinh(\kappa z)\kappa - i\ \omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}[\overline{\delta_{0,m}\frac{z_0}{H/2}}\ \overline{F_{0_a}^{H}+\delta_{1,m}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^{S}}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_m i^m F_{m_a}}]$$

$$N_a^{-1/2}\frac{\overline{K_m(ar)}}{K_m(a\text{a})}\cos(m\theta)\sin(az)a.$$

Και

$$\left(\frac{\partial\Phi}{\partial z}\right)^2=\frac{1}{2}\frac{\partial\phi}{\partial z}\frac{\overline{\partial\phi}}{\partial z}=$$

$$=\frac{g^2H^2}{8\omega^2}\left(\frac{\sinh(\kappa z)}{\cosh(\kappa d)}\right)^2\kappa^2\,[\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\cos(m\theta)\sum_{n=0}^{\infty}\in_n i^n\overline{J_n(\kappa r)}\cos(n\theta)+$$

$$+\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{\sinh(\kappa z)}{\cosh(\kappa d)}\kappa^2\,[\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\cos(m\theta)\sum_{n=0}^{\infty}\overline{[\delta_{0,n}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,n}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_n i^n(F_{n\kappa}-\frac{J_n(\kappa\text{a})}{dz'_\kappa(d)})]}\ \frac{\overline{H_n(\kappa r)}}{H_n(\kappa\text{a})}(\cos(n\theta))\sinh(\kappa z)]-$$

$$-\frac{gH^2}{8}d\frac{\sinh(\kappa z)}{\cosh(\kappa d)}\kappa\,[\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\cos(m\theta)\sum_{n=0}^{\infty}\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,n}\frac{z_0}{H/2}}\ \overline{F_{0_a}^{H}+\delta_{1,n}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^{S}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_n i^n F_{n_a}}]\ \frac{\overline{K_n(ar)}}{K_n(a\text{a})}a\cos(n\theta)\sin(az)+$$

$$+\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{\sinh(\kappa z)}{\cosh(\kappa d)}\kappa^2\,[\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,m}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa\text{a})}{dz'_\kappa(d)})]\ \frac{H_m(\kappa r)}{H_m(\kappa\text{a})}\sinh(\kappa z)\cos(m\theta)\sum_{n=0}^{\infty}\in_n i^n\overline{J_n(\kappa r)}\cos(n\theta)+$$

$$+\frac{\omega^2H^2}{8}d^2N_\kappa^{-1}\kappa^2\,[\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,m}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa\text{a})}{dz'_\kappa(d)})]$$

$$\frac{H_m(\kappa r)}{H_m(\kappa\text{a})}\sinh(\kappa z)\cos(m\theta)\sum_{n=0}^{\infty}\ \overline{[\delta_{0,n}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,n}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_n i^n(F_{n\kappa}-\frac{J_n(\kappa\text{a})}{dz'_\kappa(d)})]}\ \frac{\overline{H_n(\kappa r)}}{H_n(\kappa\text{a})}\cos(n\theta)\sinh(\kappa z)-$$

$$-\frac{\omega^2 H^2}{8} d^2 N_\kappa^{-1/2} \kappa \, [\sum_{m=0}^{\infty} [\delta_{0,m} \frac{z_0}{H/2} F_{0_\kappa}^H + \delta_{1,m} (\frac{x_0}{H/2} \, F^{S}{}_{1_\kappa}$$

$$\overline{+\frac{d\phi_0}{H/2} F_{1\kappa}^P) + \in_m i^m (F_{m\kappa} - \frac{J_m(\kappa a)}{dz'_\kappa(d)})]} \, \frac{H_m(\kappa r)}{H_m(\kappa a)} \sinh(\kappa z)\cos(m\theta) \sum_{n=0}^{\infty} [\sum_{\substack{j=1 \\ a_j=a}}^{\infty} N_a^{-1/2} [\overline{\delta_{0,n} \frac{z_0}{H/2}}$$

$$\overline{F_{0_a}^H + \delta_{1,n}(} \, \overline{\frac{x_0}{H/2} F_{1_a}^S} \, \overline{+\frac{d\phi_0}{H/2} F_{1_a}^P) + \in_n i^n F_{n_a}]} \, \overline{\frac{K_n(ar)}{K_n(a a)}} \cos(n\theta)\sin(az)a -$$

$$-\frac{gH^2}{8} d \frac{\sinh(\kappa z)}{\cosh(\kappa d)} \kappa \, [\sum_{m=0}^{\infty} [\sum_{\substack{j=1 \\ a_j=a}}^{\infty} N_a^{-1/2} [\delta_{0,m} \frac{z_0}{H/2} \, F_{0_a}^H + \delta_{1,m} (\frac{x_0}{H/2} F_{1_a}^S + \frac{d\phi_0}{H/2} F_{1_a}^P) + \in_m i^m F_{m_a}]$$

$$\frac{K_m(ar)}{K_m(a a)} \sin(az) a \cos(m\theta) \sum_{n=0}^{\infty} \in_n i^n \overline{J_n(\kappa r)} \cos(n\theta) -$$

$$-\frac{\omega^2 H^2}{8} d^2 N_\kappa^{-1/2} \kappa \, [\sum_{m=0}^{\infty} [\sum_{\substack{j=1 \\ a_j=a}}^{\infty} N_a^{-1/2} [\delta_{0,m} \frac{z_0}{H/2} \, F_{0_a}^H + \delta_{1,m} (\frac{x_0}{H/2} F_{1_a}^S + \frac{d\phi_0}{H/2} F_{1_a}^P) + \in_m i^m F_{m_a}]$$

$$\frac{K_m(ar)}{K_m(a a)} \sin(az) a \cos(m\theta) \sum_{n=0}^{\infty} \, \overline{[\delta_{0,n} \frac{z_0}{H/2} F_{0_\kappa}^H + \delta_{1,n} (\frac{x_0}{H/2}} \, \overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2} F_{1\kappa}^P) + \in_n i^n (F_{n\kappa} - \frac{J_n(\kappa a)}{dz'_\kappa(d)})]} \, \overline{\frac{H_n(\kappa r)}{H_n(\kappa a)}} \cos(n\theta)\sinh(\kappa z) +$$

$$+\frac{\omega^2 H^2}{8} d^2 \, [\sum_{m=0}^{\infty} [\sum_{\substack{\ell=1 \\ a_\ell}}^{\infty} N_{a_\ell}^{-1/2} [\delta_{0,m} \frac{z_0}{H/2} \, F_{0_{a_\ell}}^H + \delta_{1,m} (\frac{x_0}{H/2} F_{1_{a_\ell}}^S + \frac{d\phi_0}{H/2} F_{1_{a_\ell}}^P) + \in_m i^m F_{m_{a_\ell}}]$$

$$\frac{K_m(a_\ell r)}{K_m(a_\ell a)} a_\ell \sin(a_\ell z)\cos(m\theta) \, [\sum_{n=0}^{\infty} \, \sum_{\substack{n=1 \\ a_n}}^{\infty} N_{a_n}^{-1/2} [\overline{\delta_{0,n} \frac{z_0}{H/2}} \, \overline{F_{0_{a_n}}^H + \delta_{1,n}(} \, \overline{\frac{x_0}{H/2} F_{1_{a_n}}^S}$$

$$\overline{+\frac{d\phi_0}{H/2} F_{1_a}^P) + \in_n i^n F_{n_{a_n}}]} \, \overline{\frac{K_n(a_n r)}{K_n(a_n a)}} \cos(n\theta)\sin(a_n z) a_n]]$$

Μένει να υπολογιστεί το $\int_{d_2}^{d_1} \left[\int_0^{2\pi} \overline{\left(\frac{\partial \Phi}{\partial z}\right)^2}^T \cos\theta d\theta \right] RADIUS dz$. (*Παράρτημα B*)

Όπου *RADIUS* η ακτίνα του $i-$στου εξωτερικού στοιχείου.

Πρώτα υπολογίζουμε το $\int_0^{2\pi} \left(\frac{\partial \phi}{\partial z}\right)^2 \cos\theta d\theta$ και προκύπτει η

$$\int_0^{2\pi}\left(\frac{\partial\phi}{\partial z}\right)^2\cos\theta d\theta =$$

$$=\frac{g^2H^2}{8\omega^2}(\frac{\sinh(\kappa z)}{\cosh(\kappa d)})^2\kappa^2\,[\in_0 i^0 J_0(\kappa r)\in_1 i^1\overline{J_1(\kappa r)}\pi+\in_1 i^1 J_1(\kappa r)\in_0 i^0\overline{J_0(\kappa r)}\pi+\in_1 i^1 J_1(\kappa r)$$

$$\overline{\in_2 i^2 J_2(\kappa r)}\,\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}\in_p i^p J_p(\kappa r)\,[\in_{p-1} i^{p-1}\overline{J_{p-1}(\kappa r)}+\in_{p+1} i^{p+1}\overline{J_{p+1}(\kappa r)}]\frac{\pi}{2}]]+$$

$$+\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{\sinh(\kappa z)}{\cosh(\kappa d)}\kappa^2[\in_0 i^0 J_0(\kappa r)\,[\overline{\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}+$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz_\kappa'(d)})]}\,\frac{\overline{H_1(\kappa r)}}{H_1(\kappa a)}\sinh(\kappa z)\pi+\in_1 i^1 J_1(\kappa r)$$

$$[\overline{\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}\,\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{1\kappa}-\frac{J_0(\kappa a)}{dz_\kappa'(d)})]}\,\frac{\overline{H_0(\kappa r)}}{H_0(\kappa a)}\sinh(\kappa z)\pi+$$

$$\in_1 i^1 J_1(\kappa r)\,[\overline{\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}\,\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{1\kappa}-\frac{J_2(\kappa a)}{dz_\kappa'(d)})]}$$

$$\frac{\overline{H_2(\kappa r)}}{H_2(\kappa a)}\sinh(\kappa z)\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}\in_p i^p J_p(\kappa r)\,[[\overline{\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa a)}{dz_\kappa'(d)})]}\,\frac{\overline{H_{p-1}(\kappa r)}}{H_{p-1}(\kappa a)}\sinh(\kappa z)+$$

$$[\overline{\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}\,\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz_\kappa'(d)})]}$$

$$\frac{\overline{H_{p+1}(\kappa r)}}{H_{p+1}(\kappa a)}\sinh(\kappa z)]\frac{\pi}{2}]-$$

$$-\frac{gH^2}{8}d\frac{\sinh(\kappa z)}{\cosh(\kappa d)}\kappa\,[\in_0 i^0 J_0(\kappa r)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,1}\frac{z_0}{H/2}}\,\overline{F_{0_a}^H+\delta_{1,1}(}\,\overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1 F_{1_a}]}\,\frac{\overline{K_1(ar)}}{K_1(aa)}a\sin(az)\pi+\in_1 i^1 J_1(\kappa r)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,0}\frac{z_0}{H/2}}\,\overline{F_{0_a}^H+\delta_{1,0}(}$$

$$\overline{\frac{x_0}{H/2}F^S_{1_a}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_0 i^0 F_{0_a}]}\ \frac{\overline{K_0(ar)}}{\overline{K_0(a\mathrm{a})}}a\sin(az)\pi+\in_1 i^1 J_1(\kappa r)\sum_{\substack{j=1\\ a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,2}\overline{\frac{z_0}{H/2}}$$

$$\overline{F^H_{0_a}+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_2 i^2 F_{2_a}]}\ \frac{\overline{K_2(ar)}}{\overline{K_2(a\mathrm{a})}}a\sin(az)\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}\in_p i^p J_p(\kappa r)[\sum_{\substack{j=1\\ a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p-1}\overline{\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_{p-1} i^{p-1}F_{p-1_a}]}$$

$$\frac{\overline{K_{p-1}(ar)}}{\overline{K_{p-1}(a\mathrm{a})}}a\sin(az)+\sum_{\substack{i=1\\ a_i=a}}^{\infty}N_a^{-1/2}[\delta_{0,p+1}\overline{\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,p+1}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_{p+1} i^{p+1}F_{p+1_a}]}\ \frac{\overline{K_{p+1}(ar)}}{\overline{K_{p+1}(a\mathrm{a})}}\sin(az)a\,]\frac{\pi}{2}]+$$

$$+\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{\sinh(\kappa z)}{\cosh(\kappa d)}\kappa^2[[\delta_{0,0}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,0}(\frac{x_0}{H/2}F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa\mathrm{a})}{dz'_\kappa(d)})]\frac{H_0(\kappa r)}{H_0(\kappa\mathrm{a})}\sinh(\kappa z)\in_1 i^1\overline{J_1(\kappa r)}\pi+$$

$$[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,1}(\frac{x_0}{H/2}F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa\mathrm{a})}{dz'_\kappa(d)})]\frac{H_1(\kappa r)}{H_1(\kappa\mathrm{a})}\sinh(\kappa z)$$

$$\in_0\overline{J_0(\kappa r)}\pi+[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,1}(\frac{x_0}{H/2}F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa\mathrm{a})}{dz'_\kappa(d)})]$$

$$\frac{H_1(\kappa r)}{H_1(\kappa\mathrm{a})}\sinh(\kappa z)\in_2 i^2\overline{J_2(\kappa r)}\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p}(\frac{x_0}{H/2}F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa\mathrm{a})}{dz'_\kappa(d)})]\frac{H_p(\kappa r)}{H_p(\kappa\mathrm{a})}\sinh(\kappa z)[[\in_{p-1} i^{p-1}\overline{J_{p-1}(\kappa r)}+$$

$$\in_{p+1} i^{p+1}\overline{J_{p+1}(\kappa r)}]\frac{\pi}{2}]+$$

$$+\frac{\omega^2H^2}{8}d^2N_\kappa^{-1}\kappa^2[[\delta_{0,0}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,0}(\frac{x_0}{H/2}F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_0 i^0(\Im_{0\kappa}-\frac{J_0(\kappa\mathrm{a})}{dz'_\kappa(d)})]$$

$$\frac{H_0(\kappa r)}{H_0(\kappa\mathrm{a})}\sinh(\kappa z)[\overline{\delta_{0,1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa\mathrm{a})}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_1(\kappa r)}}{\overline{H_1(\kappa a)}}\sinh(\kappa z)\pi + [\delta_{0,1}\frac{z_0}{H/2}F^H_{0_\kappa} + \delta_{1,1}(\frac{x_0}{H/2}\,F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2}F^P_{1\kappa}) + \in_1 i^1(F_{1\kappa} - \frac{J_1(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_1(\kappa r)}{H_1(\kappa a)}\sinh(\kappa z)\,[\overline{\delta_{0,0}\frac{z_0}{H/2}F^H_{0_\kappa} + \delta_{1,0}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}} + \overline{\frac{d\phi_0}{H/2}F^P_{1\kappa}) + \in_0 i^0(F_{0\kappa} - \frac{J_0(\kappa a)}{dz'_\kappa(d)})}]$$

$$\frac{\overline{H_0(\kappa r)}}{\overline{H_0(\kappa a)}}\sinh(\kappa z)\pi + [\delta_{0,1}\frac{z_0}{H/2}F^H_{0_\kappa} + \delta_{1,1}(\frac{x_0}{H/2}\,F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2}F^P_{1\kappa}) + \in_1 i^1(F_{1\kappa} - \frac{J_1(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_1(\kappa r)}{H_1(\kappa a)}\sinh(\kappa z)\,[\overline{\delta_{0,2}\frac{z_0}{H/2}F^H_{0_\kappa} + \delta_{1,2}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}} + \overline{\frac{d\phi_0}{H/2}F^P_{1\kappa}) + \in_2 i^2(F_{2\kappa} - \frac{J_2(\kappa a)}{dz'_\kappa(d)})}]$$

$$\frac{\overline{H_2(\kappa r)}}{\overline{H_2(\kappa a)}}\sinh(\kappa z)\frac{\pi}{2} + \sum_{p=2,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F^H_{0_\kappa} + \delta_{1,p}(\frac{x_0}{H/2}\,F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F^P_{1\kappa}) + \in_p i^p(F_{p\kappa} - \frac{J_p(\kappa a)}{dz'_\kappa(d)})]\,\frac{H_p(\kappa r)}{H_p(\kappa a)}\sinh(\kappa z)\,[[\overline{\delta_{0,p-1}\frac{z_0}{H/2}F^H_{0_\kappa} + \delta_{1,p-1}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa}) + \in_{p-1} i^{p-1}(F_{p-1_\kappa} - \frac{J_{p-1}(\kappa a)}{dz'_\kappa(d)})]}\;\frac{\overline{H_{p-1}(\kappa r)}}{\overline{H_{p-1}(\kappa a)}}\sinh(\kappa z)] +$$

$$[\overline{\delta_{0,p+1}\frac{z_0}{H/2}F^H_{0_\kappa} + \delta_{1,p+1}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa}) + \in_{p+1} i^{p+1}(F_{p+1_\kappa} - \frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})}]$$

$$\frac{\overline{H_{p+1}(\kappa r)}}{\overline{H_{p+1}(\kappa a)}}\sinh(\kappa z)]]\frac{\pi}{2}] -$$

$$-\frac{\omega^2 H^2}{8}d^2 N_\kappa^{-1/2}\kappa\,[[\delta_{0,0}\frac{z_0}{H/2}F^H_{0_\kappa} + \delta_{1,0}(\frac{x_0}{H/2}\,F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2}F^P_{1\kappa}) + \in_0 i^0(F_{0\kappa} - \frac{J_0(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_0(\kappa r)}{H_0(\kappa a)}\sinh(\kappa z)\,[\sum_{\substack{j=1\\ a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,1}\frac{z_0}{H/2}}\;\overline{F^H_{0_a} + \delta_{1,1}(}\;\overline{\frac{x_0}{H/2}F^S_{1_a}}\;\overline{+\frac{d\phi_0}{H/2}F^P_{1_a}) + \in_1 i^1 F_{1_a}}]$$

$$\frac{\overline{K_1(ar)}}{\overline{K_1(aa)}}a\sin(az)\pi + [\delta_{0,1}\frac{z_0}{H/2}F^H_{0_\kappa} + \delta_{1,1}(\frac{x_0}{H/2}\,F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2}F^P_{1\kappa}) + \in_1 i^1(F_{1\kappa} - \frac{J_1(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_1(\kappa r)}{H_1(\kappa a)}\sinh(\kappa z)\,[\sum_{\substack{j=1\\ a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,0}\frac{z_0}{H/2}}\;\overline{F^H_{0_a} + \delta_{1,0}(}\;\overline{\frac{x_0}{H/2}F^S_{1_a}}\;\overline{+\frac{d\phi_0}{H/2}F^P_{1_a}) + \in_0 i^0 F_{0_a}}]$$

$$\frac{\overline{K_0(ar)}}{\overline{K_0(aa)}}a\sin(az)\pi + [\delta_{0,1}\frac{z_0}{H/2}F^H_{0_\kappa} + \delta_{1,1}(\frac{x_0}{H/2}\,F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2}F^P_{1\kappa}) + \in_1 i^1(F_{1\kappa} - \frac{J_1(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_1(\kappa r)}{H_1(\kappa a)}\sinh(\kappa z)\,[\sum_{\substack{j=1\\ a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\;\overline{F^H_{0_a} + \delta_{1,2}(}\;\overline{\frac{x_0}{H/2}F^S_{1_a}}\;\overline{+\frac{d\phi_0}{H/2}F^P_{1_a}) + \in_2 i^2 F_{2_a}}]$$

$$\frac{\overline{K_2(ar)}}{\overline{K_2(a\mathrm{a})}} a\sin(az)\frac{\pi}{2} + \sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^{H} + \delta_{1,p}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^{P}) + \in_p i^p (F_{p\kappa} - \frac{J_p(\kappa\mathrm{a})}{dz'_\kappa(d)})]\frac{H_p(\kappa r)}{H_p(\kappa\mathrm{a})}\sinh(\kappa z)[[\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}\overline{[\delta_{0,p-1}\frac{z_0}{H/2}}\,\overline{F_{0_a}^{H} + \delta_{1,p-1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^{S}}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_{p-1} i^{p-1}F_{p-1_a}]}\frac{\overline{K_{p-1}(ar)}}{\overline{K_{p-1}(a\mathrm{a})}} a\sin(az) + [\sum_{\substack{i=1\\ a_i=a}}^{\infty} N_a^{-1/2}\overline{[\delta_{0,p+1}\frac{z_0}{H/2}}$$

$$\overline{F_{0_a}^{H} + \delta_{1,p+1}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^{S}}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_{p+1} i^{p+1}F_{p+1_a}]}\frac{\overline{K_{p+1}(ar)}}{\overline{K_{p+1}(a\mathrm{a})}} a\sin(az)]\frac{\pi}{2}] -$$

$$-\frac{gH^2}{8}d\frac{\sinh(\kappa z)}{\cosh(\kappa d)}\kappa[[\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F_{0_a}^{H} + \delta_{1,0}(\frac{x_0}{H/2}F_{1_a}^{S} + \frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_0 i^0 F_{0_a}]$$

$$\frac{K_0(ar)}{K_0(a\mathrm{a})} a\sin(az) \in_1 i^1\overline{J_1(\kappa r)}\pi + [\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^{H} + \delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^{S}$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_1 i^1 F_{1_a}]\frac{K_1(ar)}{K_1(a\mathrm{a})} a\sin(az) \in_1 i^1\overline{J_1(\kappa r)}\pi + [\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^{H} + \delta_{1,1}($$

$$\frac{x_0}{H/2}F_{1_a}^{S} + \frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_1 i^1 F_{1_a}]\frac{K_1(ar)}{K_1(a\mathrm{a})} a\sin(az) \in_2 i^2\overline{J_2(\kappa r)}\frac{\pi}{2} +$$

$$\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^{H} + \delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^{S} + \frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_p i^p F_{p_a}]\frac{K_p(ar)}{K_p(a\mathrm{a})} a\sin(az)$$

$$[\in_{p-1} i^{p-1}\overline{J_{p-1}(\kappa r)} + \in_{p+1} i^{p+1}\overline{J_{p+1}(\kappa r)}]\frac{\pi}{2}] -$$

$$-\frac{\omega^2 H^2}{8}d^2 N_\kappa^{-1/2}\kappa[\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F_{0_a}^{H} + \delta_{1,0}(\frac{x_0}{H/2}F_{1_a}^{S} + \frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_0 i^0 F_{0_a}]$$

$$\frac{K_0(ar)}{K_0(a\mathrm{a})} a\sin(az)\,\overline{[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^{H} + \delta_{1,1}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P}) + \varepsilon_1 i^1 (F_{1\kappa} - \frac{J_1(\kappa\mathrm{a})}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_1(\kappa r)}}{\overline{H_1(\kappa\mathrm{a})}}\sinh(\kappa z)\pi + [\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^{H} + \delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^{S} + \frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_1 i^1 F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})} a\sin(az)\,\overline{[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^{H} + \delta_{1,0}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P}) + \in_0 i^0 (F_{0\kappa} - \frac{J_0(\kappa\mathrm{a})}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_0(\kappa r)}}{\overline{H_0(\kappa a)}}\sinh(\kappa z)\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})}a\sin(az)\,[\overline{\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa a)}{dz'_\kappa(d)})}]$$

$$\frac{\overline{H_2(\kappa r)}}{\overline{H_2(\kappa a)}}\sinh(\kappa z)\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^pF_{p_a}]$$

$$\frac{K_p(ar)}{K_p(a\mathrm{a})}a\sin(az)\,[[\overline{\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa a)}{dz'_\kappa(d)})}]\;\frac{\overline{H_{p-1}(\kappa r)}}{\overline{H_{p-1}(\kappa a)}}\sinh(\kappa z)+$$

$$[\overline{\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})}]$$

$$\frac{\overline{H_{p+1}(\kappa r)}}{\overline{H_{p+1}(\kappa a)}}\sinh(\kappa z)\,]\frac{\pi}{2}]+$$

$$+\frac{\omega^2H^2}{8}d^2\,[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}N_{a_\ell}^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F_{0_{a_\ell}}^H+\delta_{1,0}(\frac{x_0}{H/2}F_{1_{a_\ell}}^S+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^P)+\in_0 i^0F_{0_{a_\ell}}]$$

$$\frac{K_0(a_\ell r)}{K_0(a_\ell \mathrm{a})}a_\ell\sin(a_\ell z)\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_n}^{-1/2}[\overline{\delta_{0,1}\frac{z_0}{H/2}}\;\overline{F_{0_{a_n}}^H+\delta_{1,1}(}\;\overline{\frac{x_0}{H/2}F_{1_{a_n}}^S}\;\overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^P)+\in_1 i^1F_{1_{a_n}}}]$$

$$\frac{\overline{K_1(a_n r)}}{\overline{K_1(a_n \mathrm{a})}}a_n\sin(a_n z)\pi+\sum_{\substack{\ell=1\\a_\ell}}^{\infty}N_{a_\ell}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_{a_\ell}}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_{a_\ell}}^S+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^P)+\in_1 i^1F_{1_{a_\ell}}]$$

$$\frac{K_1(a_\ell r)}{K_1(a_\ell \mathrm{a})}a_\ell\sin(a_\ell z)\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_n}^{-1/2}[\overline{\delta_{0,0}\frac{z_0}{H/2}}\;\overline{F_{0_{a_n}}^H+\delta_{1,0}(}\;\overline{\frac{x_0}{H/2}F_{1_{a_n}}^S}\;\overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^P)+\in_0 i^0F_{0_{a_n}}}]$$

$$\frac{\overline{K_0(a_n r)}}{\overline{K_0(a_n \mathrm{a})}}a_n\sin(a_n z)\pi+\sum_{\substack{\ell=1\\a_\ell}}^{\infty}N_{a_\ell}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_{a_\ell}}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_{a_\ell}}^S+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^P)+\in_1 i^1F_{1_{a_\ell}}]$$

$$\frac{K_1(a_\ell r)}{K_1(a_\ell \mathrm{a})}a_\ell\sin(a_\ell z)\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_n}^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\;\overline{F_{0_{a_n}}^H+\delta_{1,2}(}\;\overline{\frac{x_0}{H/2}F_{1_{a_n}}^S}\;\overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^P)+\in_2 i^2F_{2_{a_n}}}]$$

$$\frac{\overline{K_2(a_n r)}}{\overline{K_2(a_n \mathrm{a})}}a_n\sin(a_n z)\,\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}N_{a_\ell}^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_{a_\ell}}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1_{a_\ell}}^S$$

$$+\frac{d\phi_0}{H/2}F^P_{1_{a_\ell}})+\in_p i^p F_{p_{a_\ell}}]\frac{K_p(a_\ell r)}{K_p(a_\ell \mathrm{a})}a_\ell \sin(a_\ell z)\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_n}^{-1/2}[\delta_{0,p-1}\overline{\frac{z_0}{H/2}}\overline{F^H_{0_{a_n}}}+\delta_{1,p-1}(\overline{\frac{x_0}{H/2}F^S_{1_{a_n}}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1_{a_n}})+\in_{p-1} i^{p-1}F_{p-1_{a_n}}]}\,\overline{\frac{K_{p-1}(a_n r)}{K_{p-1}(a_n \mathrm{a})}}a_n \sin(a_n z)+\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_n}^{-1/2}[\delta_{0,p+1}\overline{\frac{z_0}{H/2}}\overline{F^H_{0_{a_n}}}+\delta_{1,p+1}($$

$$\overline{\frac{x_0}{H/2}F^S_{1_{a_n}}}\overline{+\frac{d\phi_0}{H/2}F^P_{1_{a_n}})+\in_{p+1} i^{p+1}F_{p+1_{a_n}}]}\,\overline{\frac{K_{p+1}(a_n r)}{K_{p+1}(a_n \mathrm{a})}}a_n \sin(a_n z)]\frac{\pi}{2}].$$

Στη συνέχεια υπολογίζω το ολοκλήρωμα $\int_{d_2}^{d_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial z}\right)^2}^T\cos\theta d\theta\right]RADIUSdz$ και προκύπτει

$$\int_{d_2}^{d_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial z}\right)^2}^T\cos\theta d\theta\right]RADIUSdz=$$

$$=\frac{g^2H^2}{8\omega^2}\left(\frac{1}{\cosh(\kappa d)}\right)^2\kappa^2[\in_0 J_0(\kappa r)\,\overline{\in_1 iJ_1(\kappa r)}\pi+\in_1 i^1J_1(\kappa r)\,\overline{\in_0 J_0(\kappa r)}\pi+\in_1 iJ_1(\kappa r)$$

$$\overline{\in_2 i^2J_2(\kappa r)}\,\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}\in_p i^pJ_p(\kappa r)\,[\in_{p-1} i^{p-1}\overline{J_{p-1}(\kappa r)}+\in_{p+1} i^{p+1}\overline{J_{p+1}(\kappa r)}]]\frac{\pi}{2}]]\underline{RADIUS}$$

$$\left[\frac{2(-d_1+d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{1}{\cosh(\kappa d)}\kappa^2[\in_0 J_0(\kappa r)\,[\overline{\delta_{0,1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,1}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\,\overline{\frac{H_1(\kappa r)}{H_1(\kappa\mathrm{a})}}\pi+\in_1 iJ_1(\kappa r)\,[\overline{\delta_{0,0}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,0}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\,\overline{\frac{H_0(\kappa r)}{H_0(\kappa\mathrm{a})}}\pi+\in_1 iJ_1(\kappa r)\,[\overline{\delta_{0,2}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,2}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\,\overline{\frac{H_2(\kappa r)}{H_2(\kappa\mathrm{a})}}\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}\in_p i^pJ_p(\kappa r)$$

$$[[\overline{\delta_{0,p-1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p-1}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}\,\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\,\overline{\frac{H_{p-1}(\kappa r)}{H_{p-1}(\kappa\mathrm{a})}}+$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})]}\ \overline{\frac{H_{p+1}(\kappa r)}{H_{p+1}(\kappa a)}}]\frac{\pi}{2}]$$

$$\underline{RADIUS}\left[\frac{2(-d_1+d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]-$$

$$-\frac{gH^2}{8}d\frac{1}{\cosh(\kappa d)}\kappa[\in_0 J_0(\kappa r)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,1}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]}\ \overline{\frac{K_1(ar)}{K_1(a\text{a})}}a\pi\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(d1\kappa)\sin(ad1)-\kappa\cosh(d2\kappa)\sin(ad2)\right.$$

$$\left.-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\right]+\in_1 iJ_1(\kappa r)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,0}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,0}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_0 i^0F_{0_a}]}\ \overline{\frac{K_0(ar)}{K_0(a\text{a})}}a\pi$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\right]+$$

$$\in_1 iJ_1(\kappa r)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_2 i^2F_{2_a}]}\ \overline{\frac{K_2(ar)}{K_2(a\text{a})}}a$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\right]\frac{\pi}{2}+$$

$$+\sum_{p=2,3}^{\infty}\in_p i^pJ_p(\kappa r)[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p-1} i^{p-1}F_{p-1_a}]}$$

$$\overline{\frac{K_{p-1}(ar)}{K_{p-1}(a\text{a})}}a\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\right]+\sum_{\substack{i=1\\a_i=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,p+1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p+1} i^{p+1}F_{p+1_a}]}\ \overline{\frac{K_{p+1}(ar)}{K_{p+1}(a\text{a})}}a$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))]]\frac{\pi}{2}]\ \underline{RADIUS}+$$

$$+\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{1}{\cosh(\kappa d)}\kappa^2[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]\frac{H_0(\kappa r)}{H_0(\kappa a)}\in_1\overline{J_1(\kappa r)}\pi+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\frac{H_1(\kappa r)}{H_1(\kappa a)}\in_0\overline{J_0(\kappa r)}\pi+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\frac{H_1(\kappa r)}{H_1(\kappa a)}1\in_2\overline{J_2(\kappa r)}\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}$$

$$F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\frac{H_p(\kappa r)}{H_p(\kappa a)}[[\in_{p-1}\overline{J_{p-1}(\kappa r)}+\in_{p+1}\overline{J_{p+1}(\kappa r)}]\frac{\pi}{2}]$$

$$\underline{RADIUS}\left[\frac{2(-d_1+d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{\omega^2H^2}{8}d^2N_\kappa^{-1}\kappa^2[[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_0(\kappa r)}{H_0(\kappa a)}\overline{[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]}$$

$$\overline{\frac{\overline{H_1(\kappa r)}}{H_1(\kappa a)}}\pi+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\frac{H_1(\kappa r)}{H_1(\kappa a)}$$

$$\overline{[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]}\ \overline{\frac{\overline{H_0(\kappa r)}}{H_0(\kappa a)}}\pi+$$

$$[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\frac{H_1(\kappa r)}{H_1(\kappa a)}$$

$$\overline{[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa a)}{dz'_\kappa(d)})]}\ \overline{\frac{\overline{H_2(\kappa r)}}{H_2(\kappa a)}}\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\frac{H_p(\kappa r)}{H_p(\kappa a)}[$$

$$\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa a)}{dz'_\kappa(d)})]}\;\frac{\overline{H_{p-1}(\kappa r)}}{\overline{H_{p-1}(\kappa a)}}+$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_{p+1}(\kappa r)}}{\overline{H_{p+1}(\kappa a)}}]]\frac{\pi}{2}]\,\underline{RADIUS}\,\left[\frac{2(-d_1+d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]-$$

$$-\frac{\omega^2H^2}{8}d^2N_\kappa^{-1/2}\kappa\,[[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}\,F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_0(\kappa r)}{H_0(\kappa a)}\,[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,1}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,1}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^S}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}}\,]\,\frac{\overline{K_1(ar)}}{\overline{K_1(aa)}}a\pi$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\Big]+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\,F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\,\frac{H_1(\kappa r)}{H_1(\kappa a)}\,[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,0}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,0}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_0 i^0F_{0_a}}\,]\,\frac{\overline{K_0(ar)}}{\overline{K_0(aa)}}a\pi\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\Big]+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\,F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\,\frac{H_1(\kappa r)}{H_1(\kappa a)}\,[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,2}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,2}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_2 i^2F_{2_a}}\,]\,\frac{\overline{K_2(ar)}}{\overline{K_2(aa)}}a\frac{\pi}{2}\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\Big]+\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}\,F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\,\frac{H_p(\kappa r)}{H_p(\kappa a)}[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,p-1}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,p-1}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p-1} i^{p-1}F_{p-1_a}}\,]\,\frac{\overline{K_{p-1}(ar)}}{\overline{K_{p-1}(aa)}}a$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\Big]+[\sum_{\substack{i=1\\a_i=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^{H}}+\delta_{1,p+1}($$

$$\overline{\frac{x_0}{H/2}}F_{1_a}^{S}+\overline{\frac{d\phi_0}{H/2}F_{1_a}^{P}})+\in_{p+1}i^{p+1}F_{p+1_a}]\ \overline{\frac{K_{p+1}(ar)}{K_{p+1}(a\text{a})}}a$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\Big]\frac{\pi}{2}]\ \underline{RADIUS}\ -$$

$$-\frac{gH^2}{8}d\frac{1}{\cosh(\kappa d)}\kappa[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,0}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_0 i^0F_{0_a}]$$

$$\frac{K_0(ar)}{K_0(a\text{a})}a\Bigg[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\Big]\in_1 i\overline{J_1(\kappa r)}\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,1}($$

$$\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_1 i^1F_{1_a}]\frac{K_1(ar)}{K_1(a\text{a})}a$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\Big]\in_0\overline{J_0(\kappa r)}\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,1}($$

$$\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_1 i^1F_{1_a}]\frac{K_1(ar)}{K_1(a\text{a})}a$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\Big]\in_2 i^2\overline{J_2(\kappa r)}\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}$$

$$F_{0_a}^{H}+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_p i^pF_{p_a}]\frac{K_p(ar)}{K_p(a\text{a})}a$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\big][\in_{p-1} i^{p-1}\overline{J_{p-1}(\kappa r)}+\in_{p+1} i^{p+1}\overline{J_{p+1}(\kappa r)}]\ \frac{\pi}{2}]$$

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$$-\frac{\omega^2 H^2}{8}d^2 N_\kappa^{-1/2}\kappa\,[\sum_{\substack{j=1\\a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,0}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_0 i^0 F_{0_a}]$$

$$\frac{K_0(ar)}{K_0(a\text{a})}a\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\big]\overline{[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa\text{a})}{dz'_\kappa(d)})]}\ \frac{\overline{H_1(\kappa r)}}{\overline{H_1(\kappa\text{a})}}\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,1}(\ \frac{x_0}{H/2}F_{1_a}^S$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1 F_{1_a}]\ \frac{K_1(ar)}{K_1(a\text{a})}a\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\big]\overline{[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa\text{a})}{dz'_\kappa(d)})]}\ \frac{\overline{H_0(\kappa r)}}{\overline{H_0(\kappa\text{a})}}\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,1}(\ \frac{x_0}{H/2}F_{1_a}^S$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1 F_{1_a}]\ \frac{K_1(ar)}{K_1(a\text{a})}a\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\big]\overline{[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa\text{a})}{dz'_\kappa(d)})]}\ \frac{\overline{H_2(\kappa r)}}{\overline{H_2(\kappa\text{a})}}\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,p}($$

$$\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^p F_{p_a}]\ \frac{K_p(ar)}{K_p(a\text{a})}a$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\big]$$

$$[\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa\text{a})}{dz'_\kappa(d)})]}\ \frac{\overline{H_{p-1}(\kappa r)}}{\overline{H_{p-1}(\kappa\text{a})}}+$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})]}\ \overline{\frac{H_{p+1}(\kappa r)}{H_{p+1}(\kappa a)}}\ ]$$

$$\frac{\pi}{2}]\ \underline{RADIUS}+$$

$$+\frac{\omega^2H^2}{8}d^2\,[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,0}\frac{z_0}{H/2}\ F^H_{0_a}+\delta_{1,0}(\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_0 i^0F_{0_a}]\ \frac{K_0(ar)}{K_0(a\mathrm{a})}a$$

$$\overline{[\delta_{0,1}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,1}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_1 i^1F_{1_a}]}\ \overline{\frac{K_1(ar)}{K_1(a\mathrm{a})}}a$$

$$[\frac{2a(d_1-d_2)-\sin(2ad_1)+\sin(2ad_2)}{4a}]+$$

$$+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n}}^{\infty}\ N_{a_\ell}^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}\ F^H_{0_{a_\ell}}+\delta_{1,0}(\frac{x_0}{H/2}F^S_{1_{a_\ell}}+\frac{d\phi_0}{H/2}F^P_{1_{a_\ell}})+\in_0 i^0F_{0_{a_\ell}}]\ \frac{K_0(a_\ell r)}{K_0(a_\ell \mathrm{a})}a_\ell$$

$$N_{a_n}^{-1/2}\overline{[\delta_{0,1}\frac{z_0}{H/2}}\ \overline{F^H_{0_{a_n}}+\delta_{1,1}(}\ \overline{\frac{x_0}{H/2}F^S_{1_{a_n}}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_{a_n}})+\in_1 i^1F_{1_{a_n}}]}\ \overline{\frac{K_1(a_nr)}{K_1(a_n\mathrm{a})}}a_n$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_n\cos(a_nd_1)\sin(a_\ell d_1)-a_n\cos(a_nd_2)\sin(a_\ell d_2)-a_\ell\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_\ell\cos(a_\ell d_2)\sin(a_nd_2))]]\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,1}\frac{z_0}{H/2}\ F^H_{0_a}+\delta_{1,1}(\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_1 i^1F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})}a\ \overline{[\delta_{0,0}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,0}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_0 i^0F_{0_a}]}\ \overline{\frac{K_0(ar)}{K_0(a\mathrm{a})}}a$$

$$[\frac{2a(d_1-d_2)-\sin(2ad_1)+\sin(2ad_2)}{4a}]+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n}}^{\infty}\ N_{a_\ell}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\ F^H_{0_{a_\ell}}+\delta_{1,1}(\frac{x_0}{H/2}F^S_{1_{a_\ell}}$$

$$+\frac{d\phi_0}{H/2}F^P_{1_{a_\ell}})+\in_1 i^1F_{1_{a_\ell}}]\ \frac{K_1(a_\ell r)}{K_1(a_\ell \mathrm{a})}a_\ell\ N_{a_n}^{-1/2}\overline{[\delta_{0,0}\frac{z_0}{H/2}}\ \overline{F^H_{0_{a_n}}+\delta_{1,0}(}\ \overline{\frac{x_0}{H/2}F^S_{1_{a_n}}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1_{a_n}})+\in_0 i^0F_{0_{a_n}}]}\ \overline{\frac{K_0(a_nr)}{K_0(a_n\mathrm{a})}}a_n]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_n\cos(a_nd_1)\sin(a_\ell d_1)-a_n\cos(a_nd_2)\sin(a_\ell d_2)-a_\ell\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_\ell\cos(a_\ell d_2)\sin(a_nd_2))]]\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,1}\frac{z_0}{H/2}\ F^H_{0_a}+\delta_{1,1}(\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_1 i^1F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})}a\,[\overline{\delta_{0,2}\frac{z_0}{H/2}}\,\overline{F^H_{0_a}+\delta_{1,2}(}\,\overline{\frac{x_0}{H/2}F^S_{1_a}}\,\overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_2 i^2F_{2_a}}\,]\,\overline{\frac{K_2(ar)}{K_2(a\mathrm{a})}}a$$

$$[\frac{2a(d_1-d_2)-\sin(2ad_1)+\sin(2ad_2)}{4a}]+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n}}^{\infty}N^{-1/2}_{a_\ell}[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_{a_\ell}}+\delta_{1,1}(\frac{x_0}{H/2}F^S_{1_{a_\ell}}$$

$$+\frac{d\phi_0}{H/2}F^P_{1_{a_\ell}})+\in_1 i^1F_{1_{a_\ell}}]\,\frac{K_1(a_\ell r)}{K_1(a_\ell\mathrm{a})}a_\ell\;N^{-1/2}_{a_n}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\,\overline{F^H_{0_{a_n}}+\delta_{1,2}(}\,\overline{\frac{x_0}{H/2}F^S_{1_{a_n}}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1_{a_n}})+\in_2 i^2F_{2_{a_n}}}\,]\,\overline{\frac{K_2(a_nr)}{K_2(a_n\mathrm{a})}}a_n]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_n\cos(a_nd_1)\sin(a_\ell d_1)-a_n\cos(a_nd_2)\sin(a_\ell d_2)-a_\ell\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_\ell\cos(a_\ell d_2)\sin(a_nd_2)]]\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,p}\frac{z_0}{H/2}F^H_{0_a}+\delta_{1,p}(\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_p i^pF_{p_a}]\,\frac{K_p(a\,r)}{K_p(a\,\mathrm{a})}a$$

$$[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\,\overline{F^H_{0_a}+\delta_{1,p-1}(}\,\overline{\frac{x_0}{H/2}F^S_{1_a}}\,\overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_{p-1} i^{p-1}F_{p-1_a}}\,]\,\overline{\frac{K_{p-1}(a\,r)}{K_{p-1}(a\,\mathrm{a})}}a$$

$$[\frac{2a(d_1-d_2)-\sin(2ad_1)+\sin(2ad_2)}{4a}]+$$

$$+\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_\ell}{}^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F^H_{0_{a_\ell}}+\delta_{1,p}(\frac{x_0}{H/2}F^S_{1a_\ell}+\frac{d\phi_0}{H/2}F^P_{1_{a_\ell}})+\in_p i^pF_{p_{a_\ell}}]\,\frac{K_p(a_\ell r)}{K_p(a_\ell\mathrm{a})}a_\ell$$

$$N^{-1/2}_{a_n}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\,\overline{F^H_{0a_n}+\delta_{1,p-1}(}\,\overline{\frac{x_0}{H/2}F^S_{1a_n}}\,\overline{+\frac{d\phi_0}{H/2}F^P_{1_{a_n}})+\in_{p-1} i^{p-1}F_{p-1_{a_n}}}\,]\,\overline{\frac{K_{p-1}(a_nr)}{K_{p-1}(a_n\mathrm{a})}}a_n$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_n\cos(a_nd_1)\sin(a_\ell d_1)-a_n\cos(a_nd_2)\sin(a_\ell d_2)-a_\ell\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_\ell\cos(a_\ell d_2)\sin(a_nd_2)]\,]]\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,p}\frac{z_0}{H/2}F^H_{0_a}+\delta_{1,p}(\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_p i^pF_{p_a}]\,\frac{K_p(a\,r)}{K_p(a\,\mathrm{a})}a$$

$$[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H\ +\delta_{1,p+1}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p+1}\ i^{p+1}F_{p+1_a}}\ ]\ \frac{\overline{K_{p+1}(a\ r)}}{\overline{K_{p+1}(a\ \text{a})}}a$$

$$[\frac{2a(d_1-d_2)-\sin(2ad_1)+\sin(2ad_2)}{4a}]+$$

$$+\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n}}^{\infty}\ N_{a_\ell}{}^{-1/2}[\delta_{0,p}\ \ \frac{z_0}{H/2}\ F_{0_{a_\ell}}^H+\delta_{1,p}(\ \frac{x_0}{H/2}F_{1a_\ell}^S\ +\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^P)+\in_p\ i^pF_{pa_\ell}\ ]\ \frac{K_p(a_\ell r)}{K_p(a_\ell \text{a})}a_\ell$$

$$N_{a_n}^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F_{0a_n}^H+\delta_{1,p+1}(}\ \overline{\frac{x_0}{H/2}F_{1a_n}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^P)+\in_{p+1}\ i^{p+1}F_{p+1a_n}}\ ]\ \frac{\overline{K_{p+1}(a_n r)}}{\overline{K_{p+1}(a_n \text{a})}}a_n]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_n\cos(a_nd_1)\sin(a_\ell d_1)-a_n\cos(a_nd_2)\sin(a_\ell d_2)-a_\ell\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_\ell\cos(a_\ell d_2)\sin(a_nd_2)]\frac{\pi}{2}]$$ <u>*RADIUS*</u>

Συνοψίζοντας για το Πεδίο (Ι) το $\int_{d_2}^{d_1}\int_{0}^{2\pi} \left|\nabla\Phi^{(1)}\right|^2 \overline{n} \; dS =$

$$\int_{d_2}^{d_1}(\int_0^{2\pi}[\overline{(\frac{\partial\Phi}{\partial r})^2}^T + \frac{1}{r^2}\overline{(\frac{\partial\Phi}{\partial\theta})^2}^T + \overline{(\frac{\partial\Phi}{\partial z})^2}^T]\cos\theta \; d\theta)RADIUS \; dz =$$

$$= \frac{g^2H^2}{8\omega^2}(\frac{1}{\cosh(\kappa d)})^2[\in_0 \kappa J_0'(\kappa r)\,\overline{\in_1 i\kappa J_1'(\kappa r)}\,\pi + \in_1 i\kappa J_1'(\kappa r)\,\overline{\in_0 \kappa J_0'(\kappa r)}\,\pi +$$

$$+ \in_1 i\kappa J_1'(\kappa r)\,\overline{\in_2 i^2\kappa J_2'(\kappa r)}\,\frac{\pi}{2} + \sum_{p=2,3,}^{\infty}\in_p i^p\kappa J_p'(\kappa r)[\,\kappa[\,\in_{p-1} i^{p-1}\overline{J_{p-1}'(\kappa r)} +$$

$$\in_{p+1} i^{p+1}\overline{J_{p+1}'(\kappa r)}]]\frac{\pi}{2}]]\;\underline{RADIUS}\left[\frac{2(d_1-d_2)\kappa + \sinh(2\kappa d_1) - \sinh(2\kappa d_2)}{4\kappa}\right] +$$

$$+ \frac{gH^2}{8}dN_\kappa^{-1/2}\frac{1}{\cosh(\kappa d)}[\in_0 \kappa J_0'(\kappa r)\,\overline{[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H + \delta_{1,1}(\frac{x_0}{H/2}}$$

$$\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_1 i^1(F_{1\kappa} - \frac{J_1(\kappa a)}{dz_\kappa'(d)})]}\;\kappa\frac{\overline{H_1'(\kappa r)}}{H_1(\kappa a)}\pi + \varepsilon_1 i\kappa J_1'(\kappa r)\,\overline{[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H + \delta_{1,0}(\frac{x_0}{H/2}}$$

$$\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \varepsilon_0 i^0(F_{1\kappa} - \frac{J_0(\kappa a)}{dz_\kappa'(d)})]}\;\kappa\frac{\overline{H_0'(\kappa r)}}{H_0(\kappa a)}\pi + \varepsilon_1 i\kappa J_1'(\kappa r)\,\overline{[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H + \delta_{1,2}(\frac{x_0}{H/2}}$$

$$\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_2 i^2(F_{1\kappa} - \frac{J_2(\kappa a)}{dz_\kappa'(d)})]}\;\kappa\frac{\overline{H_2'(\kappa r)}}{H_2(\kappa a)}\frac{\pi}{2} + \sum_{p=2,3,}^{\infty}\in_p i^p\kappa J_p'(\kappa r)$$

$$[\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H + \delta_{1,p-1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_{p-1} i^{p-1}(F_{p-1_\kappa} - \frac{J_{p-1}(\kappa a)}{dz_\kappa'(d)})]}$$

$$\kappa\frac{\overline{H_{p-1}'(\kappa r)}}{H_{p-1}(\kappa a)} + \overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H + \delta_{1,p+1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_{p+1} i^{p+1}(F_{p+1_\kappa} - \frac{J_{p+1}(\kappa a)}{dz_\kappa'(d)})]}$$

$$\kappa\frac{\overline{H_{p+1}'(\kappa r)}}{H_{p+1}(\kappa a)}]\frac{\pi}{2}]\;\underline{RADIUS}\left[\frac{2(d_1-d_2)\kappa + \sinh(2\kappa d_1) - \sinh(2\kappa d_2)}{4\kappa}\right] +$$

$$+ \frac{gH^2}{8}d\frac{1}{\cosh(\kappa d)}[[\in_0 \kappa J_0'(\kappa r)\sum_{\substack{j=1\\ a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,1}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H + \delta_{1,1}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P) + \in_1 i^1 F_{1_a}}\,]\;a\frac{\overline{K_1'(ar)}}{K_1(aa)}\pi\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1) - a\cosh(\kappa d_2)\sin(ad_2))\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]+\in_1 i\kappa J_1'(\kappa r)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,0}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,0}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)}+\in_0 i^0F_{0_a}]\ a\frac{\overline{K_0'(ar)}}{\overline{K_0(a\text{a})}}\pi$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]+$$

$$+\in_1 i\kappa J_1'(\kappa r)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)}+\in_2 i^2F_{2_a}]\ a\frac{\overline{K_2'(ar)}}{\overline{K_2(a\text{a})}}\frac{\pi}{2}$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]+$$

$$+\sum_{p=2,3}^{\infty}\in_p i^p\kappa J_p'(\kappa r)[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p-1} i^{p-1}F_{p-1_a}]}\ a\frac{\overline{K_{p-1}'(ar)}}{\overline{K_{p-1}(a\text{a})}}$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]+\sum_{\substack{k=1\\a_k=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,p+1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p+1} i^{p+1}F_{p+1_a}]}\ a\frac{\overline{K_{p+1}'(ar)}}{\overline{K_{p+1}(a\text{a})}}$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]]\frac{\pi}{2}]\ \underline{RADIUS}+$$

$$+\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{1}{\cosh(\kappa d)}[[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa\text{a})}{dz_\kappa'(d)})]\ \kappa\frac{H_0'(\kappa r)}{H_0(\kappa\text{a})}\in_1 i\kappa\overline{J_1'(\kappa r)}\pi+[[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}$$

$$F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2} F^P_{1\kappa}) + \in_1 i^1 (F_{1\kappa} - \frac{J_1(\kappa a)}{dz'_\kappa(d)})] \; \kappa \frac{H'_1(\kappa r)}{H_1(\kappa a)} \in_0 \kappa \overline{J'_0(\kappa r)} \pi + [\delta_{0,1} \frac{z_0}{H/2} F^H_{0_\kappa} + \delta_{1,1}(\frac{x_0}{H/2}$$

$$F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2} F^P_{1\kappa}) + \in_1 i^1 (F_{1\kappa} - \frac{J_1(\kappa a)}{dz'_\kappa(d)})] \; \kappa \frac{H'_1(\kappa r)}{H_1(\kappa a)} \in_2 i^2 \kappa \overline{J'_2(\kappa r)} \frac{\pi}{2} +$$

$$\sum_{p=2,3,}^{\infty} [\delta_{0,p} \frac{z_0}{H/2} F^H_{0_\kappa} + \delta_{1,p}(\frac{x_0}{H/2} F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2} F^P_{1\kappa}) + \in_p i^p (F_{p\kappa} - \frac{J_p(\kappa a)}{dz'_\kappa(d)})] \; \kappa \frac{H'_p(\kappa r)}{H_p(\kappa a)} [[$$

$$\in_{p-1} i^{p-1} \kappa \overline{J'_{p-1}(\kappa r)} + \in_{p+1} i^{p+1} \kappa \overline{J'_{p+1}(\kappa r)} \,] \frac{\pi}{2}] \underline{RADIUS}$$

$$\left[\frac{2(d_1 - d_2)\kappa + \sinh(2\kappa d_1) - \sinh(2\kappa d_2)}{4\kappa} \right] +$$

$$+ \frac{\omega^2 H^2}{8} d^2 N_\kappa^{-1} [[\delta_{0,0} \frac{z_0}{H/2} F^H_{0_\kappa} + \delta_{1,0}(\frac{x_0}{H/2} F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2} F^P_{1\kappa}) + \in_0 i^0 (F_{0\kappa} - \frac{J_0(\kappa a)}{dz'_\kappa(d)})]$$

$$\kappa \frac{H'_0(\kappa r)}{H_0(\kappa a)} \overline{[\delta_{0,1} \frac{z_0}{H/2} F^H_{0_\kappa} + \delta_{1,1}(\frac{x_0}{H/2}} \; \overline{F^{S}{}_{1_\kappa}} \; \overline{+ \frac{d\phi_0}{H/2} F^P_{1\kappa}) + \in_1 i^1 (F_{1\kappa} - \frac{J_1(\kappa a)}{dz'_\kappa(d)})]}$$

$$\kappa \overline{\frac{H'_1(\kappa r)}{H_1(\kappa a)}} \pi + [\delta_{0,1} \frac{z_0}{H/2} F^H_{0_\kappa} + \delta_{1,1}(\frac{x_0}{H/2} F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2} F^P_{1\kappa}) + \in_1 i^1 (F_{1\kappa} - \frac{J_1(\kappa a)}{dz'_\kappa(d)})] \; \kappa \frac{H'_1(\kappa r)}{H_1(\kappa a)}$$

$$\overline{[\delta_{0,0} \frac{z_0}{H/2} F^H_{0_\kappa} + \delta_{1,0}(\frac{x_0}{H/2}} \; \overline{F^{S}{}_{1_\kappa}} \; \overline{+ \frac{d\phi_0}{H/2} F^P_{1\kappa}) + \in_0 i^0 (F_{0\kappa} - \frac{J_0(\kappa a)}{dz'_\kappa(d)})]}$$

$$\kappa \overline{\frac{H'_0(\kappa r)}{H_0(\kappa a)}} \pi + [\delta_{0,1} \frac{z_0}{H/2} F^H_{0_\kappa} + \delta_{1,1}(\frac{x_0}{H/2} F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2} F^P_{1\kappa}) + \in_1 i^1 (F_{1\kappa} - \frac{J_1(\kappa a)}{dz'_\kappa(d)})] \; \kappa \frac{H'_1(\kappa r)}{H_1(\kappa a)}$$

$$\overline{[\delta_{0,2} \frac{z_0}{H/2} F^H_{0_\kappa} + \delta_{1,2}(\frac{x_0}{H/2}} \; \overline{F^{S}{}_{1_\kappa}} \; \overline{+ \frac{d\phi_0}{H/2} F^P_{1\kappa}) + \in_2 i^2 (F_{2\kappa} - \frac{J_2(\kappa a)}{dz'_\kappa(d)})]}$$

$$\kappa \overline{\frac{H'_2(\kappa r)}{H_2(\kappa a)}} \frac{\pi}{2} +$$

$$\sum_{p=2,3,}^{\infty} [\delta_{0,p} \frac{z_0}{H/2} F^H_{0_\kappa} + \delta_{1,p}(\frac{x_0}{H/2} F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2} F^P_{1\kappa}) + \in_p i^p (F_{p\kappa} - \frac{J_p(\kappa a)}{dz'_\kappa(d)})] \; \kappa \frac{H'_p(\kappa r)}{H_p(\kappa a)} [$$

$$\overline{[\delta_{0,p-1} \frac{z_0}{H/2} F^H_{0_\kappa} + \delta_{1,p-1}(\frac{x_0}{H/2}} \; \overline{F^{S}{}_{1_\kappa}} \; \overline{+ \frac{d\phi_0}{H/2} F^P_{1\kappa}) + \in_{p-1} i^{p-1} (F_{p-1_\kappa} - \frac{J_{p-1}(\kappa a)}{dz'_\kappa(d)})]} \; \kappa \overline{\frac{H'_{p-1}(\kappa r)}{H_{p-1}(\kappa a)}} +$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})]}$$

$$\kappa\frac{\overline{H'_{p+1}(\kappa r)}}{H_{p+1}(\kappa a)}]]\frac{\pi}{2}]\ \underline{RADIUS}\ \left[\frac{2(d_1-d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{\omega^2H^2}{8}d^2N_\kappa^{-1/2}[[\delta_{0,0}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,0}(\frac{x_0}{H/2}F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]$$

$$\kappa\frac{H'_0(\kappa r)}{H_0(\kappa\partial)}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,1}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,1}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_1 i^1F_{1_a}]}\ a\frac{\overline{K'_1(ar)}}{K_1(a a)}\pi$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]+[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,1}(\frac{x_0}{H/2}F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\ \kappa\frac{H'_1(\kappa r)}{H_1(\kappa a)}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,0}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,0}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_0 i^0F_{0_a}]}\ a\frac{\overline{K'_0(ar)}}{K_0(aa)}\pi\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]+[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,1}(\frac{x_0}{H/2}F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\ \kappa\frac{H'_1(\kappa r)}{H_1(\kappa a)}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_2 i^2F_{2_a}]}\ a\frac{\overline{K'_2(ar)}}{K_2(aa)}\frac{\pi}{2}\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]+\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p}(\frac{x_0}{H/2}F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\ \kappa\frac{H'_p(\kappa r)}{H_p(\kappa a)}[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,p-1}(}$$

$$\overline{\frac{x_0}{H/2}F^S_{1_a}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_{p-1} i^{p-1}F_{p-1_a}]}\ a\frac{\overline{K'_{p-1}(ar)}}{K_{p-1}(aa)}$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]+[\sum_{\substack{k=1\\a_k=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^{H}+\delta_{1,p+1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^{S}}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P})}+\in_{p+1}i^{p+1}F_{p+1_a}]\ a\frac{\overline{K'_{p+1}(ar)}}{\overline{K_{p+1}(a\mathrm{a})}}$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]\frac{\pi}{2}]\ \underline{RADIUS}+$$

$$+\frac{gH^2}{8}d\frac{1}{\cosh(\kappa d)}[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,0}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_0 i^0F_{0_a}]$$

$$a\frac{K'_0(ar)}{K_0(a\mathrm{a})}\Bigg[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]\in_1 i\kappa\overline{J'_1(\kappa r)}\pi+[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}$$

$$F_{0_a}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_1 i^1F_{1_a}]\ a\frac{K'_1(ar)}{K_1(a\mathrm{a})}$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]\in_0\kappa\overline{J'_0(\kappa r)}\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}$$

$$F_{0_a}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_1 i^1F_{1_a}]\ a\frac{K'_1(ar)}{K_1(a\mathrm{a})}$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\Big]\in_2 i^2\kappa\overline{J'_2(\kappa r)}\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}$$

$$F_{0_a}^{H}+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_p i^pF_{p_a}]\ a\frac{K'_p(ar)}{K_p(a\mathrm{a})}$$

$$\Bigg[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big][\in_{p-1} i^{p-1}\kappa\overline{J'_{p-1}(\kappa r)}+\in_{p+1} i^{p+1}\kappa\overline{J'_{p+1}(\kappa r)}\,]\frac{\pi}{2}$$

] <u>RADIUS</u> +

$$+\frac{\omega^2H^2}{8}d^2N_\kappa^{-1/2}[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,0}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_0 i^0F_{0_a}]\,a\frac{K_0'(ar)}{K_0(a\text{a})}$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big][\overline{\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa\text{a})}{dz'_\kappa(d)})}]\,\kappa\frac{\overline{H_1'(\kappa r)}}{H_1(\kappa\text{a})}\pi$$

$$+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]\,a\frac{K_1'(ar)}{K_1(a\text{a})}$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big][\overline{\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa\text{a})}{dz'_\kappa(d)})}]\,\kappa\frac{\overline{H_0'(\kappa r)}}{H_0(\kappa\text{a})}\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]\,a\frac{K_1'(ar)}{K_1(a\text{a})}\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big][\overline{\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa\text{a})}{dz'_\kappa(d)})}]\,\kappa\frac{\overline{H_2'(\kappa r)}}{H_2(\kappa\text{a})}\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,p}($$

$$\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^pF_{p_a}]\,a\frac{K_p'(ar)}{K_p(a\text{a})}$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\big]$$

$$[[\overline{\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa\text{a})}{dz'_\kappa(d)})}]\,\kappa\frac{\overline{H_{p-1}'(\kappa r)}}{H_{p-1}(\kappa\text{a})}+$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F^{H}_{0_\kappa}+\delta_{1,p+1}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_{p+1}i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})]}\;\kappa\overline{\frac{H'_{p+1}(\kappa r)}{H_{p+1}(\kappa a)}}\;]$$

$$\frac{\pi}{2}]\;\underline{RADIUS}+$$

$$+\frac{\omega^2H^2}{8}d^2\;[[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,0}\frac{z_0}{H/2}\;F^{H}_{0_a}+\delta_{1,0}(\frac{x_0}{H/2}F^{S}_{1_a}+\frac{d\phi_0}{H/2}F^{P}_{1_a})+\in_0 i^0F_{0_a}]\;a\frac{K'_0(ar)}{K_0(aa)}$$

$$[\overline{\delta_{0,1}\frac{z_0}{H/2}}\;\overline{F^{H}_{0_a}+\delta_{1,1}(}\;\overline{\frac{x_0}{H/2}F^{S}_{1_a}}\;\overline{+\frac{d\phi_0}{H/2}F^{P}_{1_a})+\in_1 i^1F_{1_a}}\;]\;a\overline{\frac{K'_1(ar)}{K_1(aa)}}$$

$$[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]+$$

$$[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n\\a_\ell\neq a_n}}^{\infty}N_{a_\ell}^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}\;F^{H}_{0_{a_\ell}}+\delta_{1,0}(\frac{x_0}{H/2}F^{S}_{1_{a_\ell}}+\frac{d\phi_0}{H/2}F^{P}_{1_{a_\ell}})+\in_0 i^0F_{0_{a_\ell}}]\;a_\ell\frac{K'_0(a_\ell r)}{K_0(a_\ell a)}$$

$$N_{a_n}^{-1/2}[\overline{\delta_{0,1}\frac{z_0}{H/2}}\;\overline{F^{H}_{0_{a_n}}+\delta_{1,1}(}\;\overline{\frac{x_0}{H/2}F^{S}_{1_{a_n}}}\;\overline{+\frac{d\phi_0}{H/2}F^{P}_{1_{a_n}})+\in_1 i^1F_{1_{a_n}}}\;]\;a_n\overline{\frac{K'_1(a_nr)}{K_1(a_na)}}]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_nd_1)\sin(a_\ell d_1)-a_\ell\cos(a_nd_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_nd_2)]]\pi+$$

$$+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,1}\frac{z_0}{H/2}\;F^{H}_{0_a}+\delta_{1,1}(\frac{x_0}{H/2}F^{S}_{1_a}+\frac{d\phi_0}{H/2}F^{P}_{1_a})+\in_1 i^1F_{1_a}]\;a\frac{K'_1(ar)}{K_1(aa)}\;[\overline{\delta_{0,0}\frac{z_0}{H/2}}$$

$$\overline{F^{H}_{0_a}+\delta_{1,0}(}\;\overline{\frac{x_0}{H/2}F^{S}_{1_a}}\;\overline{+\frac{d\phi_0}{H/2}F^{P}_{1_a})+\in_0 i^0F_{0_a}}\;]\;a\overline{\frac{K'_0(ar)}{K_0(aa)}}\;[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]$$

$$+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n\\a_\ell\neq a_n}}^{\infty}N_{a_\ell}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\;F^{H}_{0_{a_\ell}}+\delta_{1,1}(\frac{x_0}{H/2}F^{S}_{1_{a_p}}+\frac{d\phi_0}{H/2}F^{P}_{1_{a_\ell}})+\in_1 i^1F_{1_{a_\ell}}]\;a_\ell\frac{K'_1(a_\ell r)}{K_1(a_\ell a)}$$

$$N_{a_n}^{-1/2}[\overline{\delta_{0,0}\frac{z_0}{H/2}}\;\overline{F^{H}_{0_{a_n}}+\delta_{1,0}(}\;\overline{\frac{x_0}{H/2}F^{S}_{1_{a_n}}}\;\overline{+\frac{d\phi_0}{H/2}F^{P}_{1_{a_n}})+\in_0 i^0F_{0_{a_n}}}\;]\;a_n\overline{\frac{K'_0(a_nr)}{K_0(a_na)}}]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_nd_1)\sin(a_\ell d_1)-a_\ell\cos(a_nd_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_nd_2)]]\pi+$$

$$+\sum_{\substack{j=1\\a_j=a}}^{\infty} N_a^{-1}[\delta_{0,1}\frac{z_0}{H/2}\, F_{0_a}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_1 i^1 F_{1_a}]\, a\frac{K_1'(ar)}{K_1(a\text{a})}\,\overline{[\delta_{0,2}\frac{z_0}{H/2}}$$

$$\overline{F_{0_a}^{H}+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^{S}}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_2 i^2 F_{2_a}]}\ a\overline{\frac{K_2'(ar)}{K_2(a\text{a})}}$$

$$[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]+$$

$$+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n\\a_\ell\neq a_n}}^{\infty} N_{a_\ell}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\, F_{0_{a_\ell}}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_{a_\ell}}^{S}+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P})+\in_1 i^1 F_{1_{a_\ell}}]\, a_\ell\frac{K_1'(a_\ell r)}{K_1(a_\ell\text{a})}$$

$$N_{a_n}^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F_{0_{a_n}}^{H}+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F_{1_{a_n}}^{S}}\ \overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P})+\in_2 i^2 F_{2_{a_n}}]}\ a_p\overline{\frac{K_2'(a_p r)}{K_2(a_p\text{a})}}]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_n d_1)\sin(a_\ell d_1)-a_\ell\cos(a_n d_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_n d_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_n d_2))]]\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty} N_a^{-1}[\delta_{0,p}\frac{z_0}{H/2}\, F_{0_a}^{H}+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_p i^p F_{p_a}]\, a\,\frac{K_p'(a\, r)}{K_p(a\ \text{a})}$$

$$\overline{[\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^{H}+\delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^{S}}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_{p-1} i^{p-1}F_{p-1_a}}]\ a\ \overline{\frac{K_{p-1}'(a\, r)}{K_{p-1}(a\ \text{a})}}$$

$$[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]+$$

$$+\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n\\a_\ell\neq a_n}}^{\infty} N_{a_\ell}^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}\, F_{0_{a_\ell}}^{H}+\delta_{1,p}(\frac{x_0}{H/2}F_{1_{a_\ell}}^{S}+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P})+\in_p i^p F_{p_{a_\ell}}]\, a_\ell\frac{K_p'(a_\ell r)}{K_p(a_\ell\text{a})}$$

$$N_{a_n}^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F_{0a_n}^{H}+\delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F_{1_{a_p}}^{S}}\ \overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P})+\in_{p-1} i^{p-1}F_{p-1_{a_n}}]}\ a_n\overline{\frac{K_{p-1}'(a_n r)}{K_{p-1}(a_n\text{a})}}]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_n d_1)\sin(a_\ell d_1)-a_\ell\cos(a_n d_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_n d_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_n d_2))]\,]\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty} N_a^{-1}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_p i^p F_{p_a}]\, a\,\frac{K'_p(a\,r)}{K_p(a\,\mathrm{a})}$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}}\,\overline{F_{0_a}^{H}+\delta_{1,p+1}(}\,\overline{\frac{x_0}{H/2}F_{1_a}^{S}}\,\overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_{p+1} i^{p+1}F_{p+1_a}}]\, a\,\frac{\overline{K'_{p+1}(a\,r)}}{\overline{K_{p+1}(a\,\mathrm{a})}}$$

$$[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]+$$

$$+\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n\\a_\ell\neq a_n}}^{\infty} N_{a_\ell}^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_{a_\ell}}^{H}+\delta_{1,p}(\frac{x_0}{H/2}F_{1_{a_\ell}}^{S}+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P})+\in_p i^p F_{p_{a_\ell}}]\, a_\ell\,\frac{K'_p(a_\ell\, r)}{K_p(a_\ell\,\mathrm{a})}$$

$$N_{a_n}^{-1/2}\overline{[\delta_{0,p+1}\frac{z_0}{H/2}}\,\overline{F_{0a_n}^{H}+\delta_{1,p+1}(}\,\overline{\frac{x_0}{H/2}F_{1_{a_n}}^{S}}\,\overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P})+\in_{p+1} i^{p+1}F_{p+1_{a_n}}}]\, a_n\,\frac{\overline{K'_{p+1}(a_n r)}}{\overline{K_{p+1}(a_n\mathrm{a})}}$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_nd_1)\sin(a_\ell d_1)-a_\ell\cos(a_nd_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_nd_2))]\,]\frac{\pi}{2}]\,\underline{RADIUS}$$

$$+$$

$$+\frac{1}{r^2}\frac{g^2H^2}{8\omega^2}(\frac{1}{\cosh(\kappa d)})^2[\in_1 iJ_1(\kappa r)1\,\overline{\in_2 i^2J_2(\kappa r)}2\,\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}\in_p i^pJ_p(\kappa r)p\,[$$

$$\in_{p-1} i^{p-1}(p-1)\overline{J_{p-1}(\kappa r)}+\in_{p+1} i^{p+1}(p+1)\overline{J_{p+1}(\kappa r)}]]\frac{\pi}{2}]\,]\underline{RADIUS}$$

$$\left[\frac{2(d_1-d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{1}{r^2}\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{1}{\cosh(\kappa d)}[\in_1 iJ_1(\kappa r)1\,\overline{[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,2}(\frac{x_0}{H/2}}\,\overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_2 i^2(F_{1\kappa}-\frac{J_2(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\,\frac{\overline{H_2(\kappa r)}}{\overline{H_2(\kappa\mathrm{a})}}2\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}\in_p i^pJ_p(\kappa r)p$$

$$[\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,p-1}(\frac{x_0}{H/2}}\,\overline{F^{S}{}_{1_\kappa}}\,\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa\mathrm{a})}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_{p-1}(\kappa r)}}{\overline{H_{p-1}(\kappa\mathrm{a})}}(p-1)+\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,p+1}(\frac{x_0}{H/2}}\,\overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_{p+1} i^{p+1}(F_{p+1_\kappa} - \frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})]}\ \overline{\frac{H_{p+1}(\kappa r)}{H_{p+1}(\kappa a)}}(p+1)]\frac{\pi}{2}]\underline{RADIUS}$$

$$\left[\frac{2(d_1-d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{1}{r^2}\frac{gH^2}{8}d\frac{1}{\cosh(\kappa d)}[\in_1 iJ_1(\kappa r)1\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_2 i^2F_{2_a}]}\ \overline{\frac{K_2(ar)}{K_2(a a)}}2\frac{\pi}{2}\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]$$

$$+\sum_{p=2,3}^{\infty}\varepsilon_p i^p J_p(\kappa r)p[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_{p-1} i^{p-1}F_{p-1_a}]}$$

$$\overline{\frac{K_{p-1}(ar)}{K_{p-1}(a a)}}(p-1)\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,p+1}(}$$

$$\overline{\frac{x_0}{H/2}F^S_{1_a}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_{p+1} i^{p+1}F_{p+1_a}]}\ \overline{\frac{K_{p+1}(ar)}{K_{p+1}(a a)}}(p+1)$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]]\frac{\pi}{2}]\ \underline{RADIUS}+$$

$$+\frac{1}{r^2}\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{1}{\cosh(\kappa d)}[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,1}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\frac{H_1(\kappa r)}{H_1(\kappa a)}1\in_2 i^2\overline{J_2(\kappa r)}2\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\frac{H_p(\kappa r)}{H_p(\kappa a)}p$$

$$[\in_{p-1} i^{p-1}(p-1)\overline{J_{p-1}(\kappa r)} + \in_{p+1} i^{p+1}(p+1)\overline{J_{p+1}(\kappa r)}\,]\frac{\pi}{2}]\underline{RADIUS}$$

$$\left[\frac{2(d_1-d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2}{8}d^2N_\kappa^{-1}[[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\;F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_1(\kappa r)}{H_1(\kappa a)}1\,[\overline{\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa a)}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_2(\kappa r)}}{\overline{H_2(\kappa a)}}2\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}\;F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_p(\kappa r)}{H_p(\kappa a)}p\,[[\overline{\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa a)}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_{p-1}(\kappa r)}}{\overline{H_{p-1}(\kappa a)}}(p-1)+[\overline{\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})]}\;\frac{\overline{H_{p+1}(\kappa r)}}{\overline{H_{p+1}(\kappa a)}}(p+1)]]\frac{\pi}{2}]\underline{RADIUS}$$

$$\left[\frac{2(d_1-d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2}{8}d^2N_\kappa^{-1/2}[[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\;F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_1(\kappa r)}{H_1(\kappa a)}1\,[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,2}(}\;\overline{\frac{x_0}{H/2}}F_{1_a}^S\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_2 i^2F_{2_a}]}\;\frac{\overline{K_2(ar)}}{\overline{K_2(a\mathrm{a})}}2\frac{\pi}{2}$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]+\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}\;F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\frac{H_p(\kappa r)}{H_p(\kappa a)}p\,[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,p-1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p-1} i^{p-1}F_{p-1_a}]}\;\frac{\overline{K_{p-1}(ar)}}{\overline{K_{p-1}(a\mathrm{a})}}(p-1)$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]+[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,p+1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p+1}i^{p+1}F_{p+1_a}}]\ \frac{\overline{K_{p+1}(ar)}}{\overline{K_{p+1}(a\mathrm{a})}}(p+1)$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]\frac{\pi}{2}]\ \underline{RADIUS}+$$

$$+\frac{1}{r^2}\frac{gH^2}{8}d\frac{1}{\cosh(\kappa d)}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})}1\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]\in_2 i^2\overline{J_2(\kappa r)}2\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}$$

$$F_{0_a}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^pF_{p_a}]\ \frac{K_p(ar)}{K_p(a\mathrm{a})}p$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right][\in_{p-1}i^{p-1}\overline{J_{p-1}(\kappa r)}(p-1)+$$

$$\in_{p+1}i^{p+1}\overline{J_{p+1}(\kappa r)}(p+1)]\frac{\pi}{2}]\ \underline{RADIUS}+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2}{8}d^2N_\kappa^{-1/2}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})}1\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right][\overline{\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa \mathrm{a})}{dz'_\kappa(d)})]}\ \frac{\overline{H_2(\kappa r)}}{\overline{\overline{H_2(\kappa \mathrm{a})}}}2\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,p}($$

$$\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^p F_{p_a}]\ \frac{K_p(ar)}{K_p(a\mathrm{a})}p$$

$$\left[\frac{1}{a^2+\kappa^2}(a\cosh(\kappa d_1)\sin(ad_1)-a\cosh(\kappa d_2)\sin(ad_2)+\right.$$

$$\left.+\kappa\cos(ad_1)\sinh(\kappa d_1)-\kappa\cos(ad_2)\sinh(\kappa d_2))\right]$$

$$[[\overline{\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa \mathrm{a})}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_{p-1}(\kappa r)}}{\overline{\overline{H_{p-1}(\kappa \mathrm{a})}}}(p-1)+[\overline{\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa \mathrm{a})}{dz'_\kappa(d)})]}\ \frac{\overline{H_{p+1}(\kappa r)}}{\overline{\overline{H_{p+1}(\kappa \mathrm{a})}}}(p+1)]\frac{\pi}{2}]\ \underline{RADIUS}+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2}{8}d^2[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,1}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]\ \frac{K_1(ar)}{K_1(a\mathrm{a})}1$$

$$[\overline{\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_2 i^2F_{2_a}]}\ \frac{\overline{K_2(ar)}}{\overline{\overline{K_2(a\mathrm{a})}}}2$$

$$[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]+$$

$$+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n}}^{\infty}\ N_{a_\ell}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\ F_{0a_\ell}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_{a_\ell}}^S+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^P)+\in_1 i^1F_{1_{a_\ell}}]\ \frac{K_1(a_\ell r)}{K_1(a_\ell \mathrm{a})}1$$

$$N_{a_n}^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F_{0_{a_n}}^H+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F_{1_{a_n}}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^P)+\in_2 i^2F_{2_{a_n}}]}\ \frac{K_2(a_nr)}{K_2(a_n\mathrm{a})}2$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_nd_1)\sin(a_\ell d_1)-a_\ell\cos(a_nd_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_nd_2)]]\frac{\pi}{2}+$$

$$+\sum_{\substack{p=2,3,\\ }}^{\infty}[\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^p F_{p_a}]\frac{K_p(a\,r)}{K_p(a\,\mathrm{a})}p$$

$$\overline{[\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p-1} i^{p-1}F_{p-1_a}]}\ \overline{\frac{K_{p-1}(a\,r)}{K_{p-1}(a\,\mathrm{a})}}(p-1)$$

$$[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]+$$

$$+\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\sum_{\substack{n=1\\ a_n}}^{\infty} N_{a_\ell}^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_{a_\ell}}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1a_\ell}^S+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^P)+\in_p i^p F_{p_{a_\ell}}]\frac{K_p(a_\ell r)}{K_p(a_\ell \mathrm{a})}p$$

$$N_{a_n}^{-1/2}\overline{[\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F_{0_{a_n}}^H+\delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F_{1a_n}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^P)+\in_{p-1} i^{p-1}F_{p-1_{a_n}}]}\ \overline{\frac{K_{p-1}(a_n r)}{K_{p-1}(a_n \mathrm{a})}}(p-1)]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_n d_1)\sin(a_\ell d_1)-a_\ell\cos(a_n d_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_n d_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_n d_2)]]\frac{\pi}{2}+$$

$$+\sum_{\substack{p=2,3,\\ }}^{\infty}[\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^p F_{p_a}]\frac{K_p(a\,r)}{K_p(a\,\mathrm{a})}p$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,p+1}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_{p+1} i^{p+1}F_{p+1_a}]}\ \overline{\frac{K_{p+1}(a\,r)}{K_{p+1}(a\,\mathrm{a})}}(p+1)$$

$$[\frac{2a(d_1-d_2)+\sin(2ad_1)-\sin(2ad_2)}{4a}]+$$

$$+\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\sum_{\substack{n=1\\ a_n}}^{\infty} N_{a_\ell}^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_{a_\ell}}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1a_\ell}^S+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^P)+\in_p i^p F_{p_{a_\ell}}]\frac{K_p(a_\ell r)}{K_p(a_\ell \mathrm{a})}p$$

$$N_{a_n}^{-1/2}\overline{[\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F_{0a_n}^H+\delta_{1,p+1}(}\ \overline{\frac{x_0}{H/2}F_{1_{a_n}}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^P)+\in_{p+1} i^{p+1}F_{p+1_{a_n}}]}\ \overline{\frac{K_{p+1}(a_n r)}{K_{p+1}(a_n \mathrm{a})}}(p+1)]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_\ell\cos(a_n d_1)\sin(a_\ell d_1)-a_\ell\cos(a_n d_2)\sin(a_\ell d_2)-a_n\cos(a_\ell d_1)\sin(a_n d_1)+$$

$$a_n\cos(a_\ell d_2)\sin(a_n d_2)]]\frac{\pi}{2}]\ \underline{RADIUS}$$

+

$$\frac{g^2H^2}{8\omega^2}\left(\frac{1}{\cosh(\kappa d)}\right)^2\kappa^2\,[\in_0 J_0(\kappa r)\,\overline{\in_1 iJ_1(\kappa r)}\pi+\in_1 iJ_1(\kappa r)\,\overline{\in_0 J_0(\kappa r)}\pi+\in_1 iJ_1(\kappa r)$$

$$\overline{\in_2 i^2J_2(\kappa r)}\,\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}\in_p i^pJ_p(\kappa r)\,[\in_{p-1} i^{p-1}\overline{J_{p-1}(\kappa r)}+\in_{p+1} i^{p+1}\overline{J_{p+1}(\kappa r)}]]\frac{\pi}{2}]]\underline{RADIUS}$$

$$\left[\frac{2(-d_1+d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]+$$

$$+\frac{gH^2}{8}dN_\kappa^{-1/2}\frac{1}{\cosh(\kappa d)}\kappa^2\,[\in_0 J_0(\kappa r)\,[\overline{\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]}\;\frac{\overline{H_1(\kappa r)}}{H_1(\kappa a)}\pi+\in_1 iJ_1(\kappa r)\,[\overline{\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]}\;\frac{\overline{H_0(\kappa r)}}{H_0(\kappa a)}\pi+\in_1 iJ_1(\kappa r)\,[\overline{\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa a)}{dz'_\kappa(d)})]}\;\frac{\overline{H_2(\kappa r)}}{H_2(\kappa a)}\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}\in_p i^pJ_p(\kappa r)$$

$$[[\overline{\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa a)}{dz'_\kappa(d)})]}\;\frac{\overline{H_{p-1}(\kappa r)}}{H_{p-1}(\kappa a)}+$$

$$[\overline{\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})]}\;\frac{\overline{H_{p+1}(\kappa r)}}{H_{p+1}(\kappa a)}]\frac{\pi}{2}]$$

$$\underline{RADIUS}\left[\frac{2(-d_1+d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]-$$

$$-\frac{gH^2}{8}d\frac{1}{\cosh(\kappa d)}\kappa\,[\in_0 J_0(\kappa r)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,1}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,1}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]}\;\frac{\overline{K_1(ar)}}{K_1(aa)}a\pi\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(d1\kappa)\sin(ad1)-\kappa\cosh(d2\kappa)\sin(ad2)\right.$$

$$\left.-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\right]+\in_1 iJ_1(\kappa r)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,0}\frac{z_0}{H/2}}\;\overline{F_{0_a}^H+\delta_{1,0}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_0 i^0F_{0_a}]}\;\frac{\overline{K_0(ar)}}{K_0(aa)}a\pi$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\right]+$$

$$\in_1 iJ_1(\kappa r) \sum_{\substack{j=1 \\ a_j=a}}^{\infty} N_a^{-1/2} [\overline{\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F_{0_a}^{H} + \delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^{S}}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_2 i^2 F_{2_a}}]\ \frac{\overline{K_2(ar)}}{\overline{K_2(a\text{a})}} a$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1) - \kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left. - a\cos(ad_1)\sinh(\kappa d_1) + a\cos(ad_2)\sinh(\kappa d_2))\right]\frac{\pi}{2} +$$

$$+\sum_{p=2,3}^{\infty} \in_p i^p J_p(\kappa r)\ [\sum_{\substack{j=1 \\ a_j=a}}^{\infty} N_a^{-1/2} [\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^{H} + \delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^{S}}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_{p-1} i^{p-1} F_{p-1_a}}]$$

$$\frac{\overline{K_{p-1}(ar)}}{\overline{K_{p-1}(a\text{a})}} a \left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1) - \kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left. - a\cos(ad_1)\sinh(\kappa d_1) + a\cos(ad_2)\sinh(\kappa d_2))\right] + \sum_{\substack{i=1 \\ a_i=a}}^{\infty} N_a^{-1/2} [\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^{H} + \delta_{1,p+1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^{S}}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P}) + \in_{p+1} i^{p+1} F_{p+1_a}}]\ \frac{\overline{K_{p+1}(ar)}}{\overline{K_{p+1}(a\text{a})}} a$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1) - \kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left. - a\cos(ad_1)\sinh(\kappa d_1) + a\cos(ad_2)\sinh(\kappa d_2))\right]]\frac{\pi}{2}]\ \underline{RADIUS} +$$

$$+\frac{gH^2}{8} dN_\kappa^{-1/2} \frac{1}{\cosh(\kappa d)} \kappa^2 [\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^{H} + \delta_{1,0}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^{P}) + \in_0 i^0 (F_{0\kappa} - \frac{J_0(\kappa\text{a})}{dz'_\kappa(d)})]\ \frac{H_0(\kappa r)}{H_0(\kappa\text{a})} \in_1 i\overline{J_1(\kappa r)}\pi + [\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^{H} + \delta_{1,1}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^{P}) + \in_1 i^1 (F_{1\kappa} - \frac{J_1(\kappa\text{a})}{dz'_\kappa(d)})]\ \frac{H_1(\kappa r)}{H_1(\kappa\text{a})} \in_0 \overline{J_0(\kappa r)}\pi + [\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^{H} + \delta_{1,1}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^{P}) + \in_1 i^1 (F_{1\kappa} - \frac{J_1(\kappa\text{a})}{dz'_\kappa(d)})]\ \frac{H_1(\kappa r)}{H_1(\kappa\text{a})} 1 \in_2 i^2 \overline{J_2(\kappa r)}\frac{\pi}{2} + \sum_{p=2,3,}^{\infty} [\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^{H} + \delta_{1,p}(\frac{x_0}{H/2}$$

$$F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2}F_{1\kappa}^{P}) + \in_p i^p (F_{p\kappa} - \frac{J_p(\kappa\text{a})}{dz'_\kappa(d)})]\ \frac{H_p(\kappa r)}{H_p(\kappa\text{a})} [[\in_{p-1} i^{p-1}\overline{J_{p-1}(\kappa r)} + \in_{p+1} i^{p+1}\overline{J_{p+1}(\kappa r)}]$$

$$\frac{\pi}{2}]\ \underline{RADIUS} \left[\frac{2(-d_1+d_2)\kappa + \sinh(2\kappa d_1) - \sinh(2\kappa d_2)}{4\kappa}\right] +$$

$$+\frac{\omega^2H^2}{8}d^2N_\kappa^{-1}\kappa^2[[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_0(\kappa r)}{H_0(\kappa a)}[\overline{\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]}$$

$$\overline{\frac{H_1(\kappa r)}{H_1(\kappa a)}}\pi+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\ \frac{H_1(\kappa r)}{H_1(\kappa a)}$$

$$[\overline{\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]}\ \overline{\frac{H_0(\kappa r)}{H_0(\kappa a)}}\pi+$$

$$[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\ \frac{H_1(\kappa r)}{H_1(\kappa a)}$$

$$[\overline{\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa a)}{dz'_\kappa(d)})]}\ \overline{\frac{H_2(\kappa r)}{H_2(\kappa a)}}\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\ \frac{H_p(\kappa r)}{H_p(\kappa a)}[$$

$$[\overline{\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa a)}{dz'_\kappa(d)})]}\ \overline{\frac{H_{p-1}(\kappa r)}{H_{p-1}(\kappa a)}}+$$

$$[\overline{\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})]}$$

$$\overline{\frac{H_{p+1}(\kappa r)}{H_{p+1}(\kappa a)}}]]\frac{\pi}{2}]\underline{RADIUS}\left[\frac{2(-d_1+d_2)\kappa+\sinh(2\kappa d_1)-\sinh(2\kappa d_2)}{4\kappa}\right]-$$

$$-\frac{\omega^2H^2}{8}d^2N_\kappa^{-1/2}\kappa[[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_0(\kappa r)}{H_0(\kappa a)}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,1}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}}]\ \overline{\frac{K_1(ar)}{K_1(aa)}}a\pi$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\right]+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\ \frac{H_1(\kappa r)}{H_1(\kappa a)}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,0}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,0}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_0 i^0F_{0_a}}]\ \overline{\frac{K_0(ar)}{K_0(aa)}}a\pi\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\big]+[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,1}(\frac{x_0}{H/2}F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\ \frac{H_1(\kappa r)}{H_1(\kappa a)}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,2}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_2 i^2F_{2_a}]}\ \overline{\frac{K_2(ar)}{K_2(aa)}}a\frac{\pi}{2}\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\big]+\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p}(\frac{x_0}{H/2}F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\ \frac{H_p(\kappa r)}{H_p(\kappa a)}[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_{p-1} i^{p-1}F_{p-1_a}]}\ \overline{\overline{\frac{K_{p-1}(ar)}{K_{p-1}(aa)}}}a$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\big]+[\sum_{\substack{i=1\\a_i=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,p+1}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,p+1}(}$$

$$\overline{\frac{x_0}{H/2}F^S_{1_a}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_{p+1} i^{p+1}F_{p+1_a}]}\ \overline{\overline{\frac{K_{p+1}(ar)}{K_{p+1}(aa)}}}a$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\big]\frac{\pi}{2}]\ \underline{RADIUS}\ -$$

$$-\frac{gH^2}{8}d\frac{1}{\cosh(\kappa d)}\kappa[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F^H_{0_a}+\delta_{1,0}(\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_0 i^0F_{0_a}]$$

$$\frac{K_0(ar)}{K_0(aa)}a\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\big]\in_1 i\overline{J_1(\kappa r)}\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_a}+\delta_{1,1}($$

$$\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_1 i^1F_{1_a}]\ \frac{K_1(ar)}{K_1(aa)}a$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\right]\in_0 \overline{J_0(\kappa r)}\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\,F_{0_a}^H+\delta_{1,1}($$

$$\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]\,\frac{K_1(ar)}{K_1(a\text{a})}a$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\right]\in_2 i^2\overline{J_2(\kappa r)}\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}$$

$$F_{0_a}^H+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_p i^pF_{p_a}]\,\frac{K_p(ar)}{K_p(a\text{a})}a$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\right][\in_{p-1} i^{p-1}\overline{J_{p-1}(\kappa r)}+\in_{p+1}\overline{i^{p+1}J_{p+1}(\kappa r)}\,]\frac{\pi}{2}]$$

<u>*RADIUS*</u>

$$-\frac{\omega^2H^2}{8}d^2N_\kappa^{-1/2}\kappa\,[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}\,F_{0_a}^H+\delta_{1,0}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_0 i^0F_{0_a}]$$

$$\frac{K_0(ar)}{K_0(a\text{a})}a\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\right]\overline{[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa\text{a})}{dz_\kappa'(d)})]}\;\frac{\overline{H_1(\kappa r)}}{\overline{H_1(\kappa\text{a})}}\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\,F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]\,\frac{K_1(ar)}{K_1(a\text{a})}a\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\right]\overline{[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa\text{a})}{dz_\kappa'(d)})]}\;\frac{\overline{H_0(\kappa r)}}{\overline{H_0(\kappa\text{a})}}\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}\,F_{0_a}^H+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^S$$

$$+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_1 i^1 F_{1_a}]\ \frac{K_1(ar)}{K_1(a\mathrm{a})}a\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\right]\overline{[\delta_{0,2}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,2}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\ \overline{\frac{H_2(\kappa r)}{H_2(\kappa\mathrm{a})}}\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F^H_{0_a}+\delta_{1,p}($$

$$\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_p i^p F_{p_a}]\ \frac{K_p(ar)}{K_p(a\mathrm{a})}a$$

$$\left[\frac{1}{a^2+\kappa^2}(\kappa\cosh(\kappa d_1)\sin(ad_1)-\kappa\cosh(\kappa d_2)\sin(ad_2)\right.$$

$$\left.-a\cos(ad_1)\sinh(\kappa d_1)+a\cos(ad_2)\sinh(\kappa d_2))\right]$$

$$[\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p-1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\ \overline{\frac{H_{p-1}(\kappa r)}{H_{p-1}(\kappa\mathrm{a})}}+$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^S{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\ \overline{\frac{H_{p+1}(\kappa r)}{H_{p+1}(\kappa\mathrm{a})}}]$$

$$\frac{\pi}{2}]\ \underline{RADIUS}+$$

$$+\frac{\omega^2H^2}{8}d^2[[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,0}\frac{z_0}{H/2}F^H_{0_a}+\delta_{1,0}(\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_0 i^0F_{0_a}]\ \frac{K_0(ar)}{K_0(a\mathrm{a})}a$$

$$\overline{[\delta_{0,1}\frac{z_0}{H/2}}\ \overline{F^H_{0_a}+\delta_{1,1}(}\ \overline{\frac{x_0}{H/2}F^S_{1_a}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_1 i^1F_{1_a}]}\ \overline{\frac{K_1(ar)}{K_1(a\mathrm{a})}}a$$

$$[\frac{2a(d_1-d_2)-\sin(2ad_1)+\sin(2ad_2)}{4a}]+$$

$$+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_\ell}^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F^H_{0_{a_\ell}}+\delta_{1,0}(\frac{x_0}{H/2}F^S_{1_{a_\ell}}+\frac{d\phi_0}{H/2}F^P_{1_{a_\ell}})+\in_0 i^0F_{0_{a_\ell}}]\ \frac{K_0(a_\ell r)}{K_0(a_\ell\mathrm{a})}a_\ell$$

$$N_{a_n}^{-1/2}\overline{[\delta_{0,1}\frac{z_0}{H/2}}\ \overline{F^H_{0_{a_n}}+\delta_{1,1}(}\ \overline{\frac{x_0}{H/2}F^S_{1_{a_n}}}\ \overline{+\frac{d\phi_0}{H/2}F^P_{1_{a_n}})+\in_1 i^1F_{1_{a_n}}]}\ \overline{\frac{K_1(a_nr)}{K_1(a_n\mathrm{a})}}a_n$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_n\cos(a_nd_1)\sin(a_\ell d_1)-a_n\cos(a_nd_2)\sin(a_\ell d_2)-a_\ell\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_\ell\cos(a_\ell d_2)\sin(a_nd_2)]]\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_a}+\delta_{1,1}(\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_1 i^1F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})}a\,[\overline{\delta_{0,0}\frac{z_0}{H/2}}\;\overline{F_{0_a}^{H}+\delta_{1,0}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^{S}}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_0 i^0F_{0_a}}\,]\;\overline{\overline{\frac{K_0(ar)}{K_0(a\mathrm{a})}}}a$$

$$[\frac{2a(d_1-d_2)-\sin(2ad_1)+\sin(2ad_2)}{4a}]+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_\ell}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_{a_\ell}}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_{a_\ell}}^{S}$$

$$+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P})+\in_1 i^1F_{1_{a_\ell}}]\frac{K_1(a_\ell r)}{K_1(a_\ell \mathrm{a})}a_\ell\;N_{a_n}^{-1/2}[\overline{\delta_{0,0}\frac{z_0}{H/2}}\;\overline{F_{0_{a_n}}^{H}+\delta_{1,0}(}\;\overline{\frac{x_0}{H/2}F_{1_{a_n}}^{S}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P})+\in_0 i^0F_{0_{a_n}}}\,]\;\overline{\overline{\frac{K_0(a_nr)}{K_0(a_n\mathrm{a})}}}a_n]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_n\cos(a_nd_1)\sin(a_\ell d_1)-a_n\cos(a_nd_2)\sin(a_\ell d_2)-a_\ell\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_\ell\cos(a_\ell d_2)\sin(a_nd_2)]]\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_1 i^1F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})}a\,[\overline{\delta_{0,2}\frac{z_0}{H/2}}\;\overline{F_{0_a}^{H}+\delta_{1,2}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^{S}}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_2 i^2F_{2_a}}\,]\;\overline{\overline{\frac{K_2(ar)}{K_2(a\mathrm{a})}}}a$$

$$[\frac{2a(d_1-d_2)-\sin(2ad_1)+\sin(2ad_2)}{4a}]+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_\ell}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_{a_\ell}}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_{a_\ell}}^{S}$$

$$+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P})+\in_1 i^1F_{1_{a_\ell}}]\frac{K_1(a_\ell r)}{K_1(a_\ell \mathrm{a})}a_\ell\;N_{a_n}^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\;\overline{F_{0_{a_n}}^{H}+\delta_{1,2}(}\;\overline{\frac{x_0}{H/2}F_{1_{a_n}}^{S}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P})+\in_2 i^2F_{2_{a_n}}}\,]\;\overline{\overline{\frac{K_2(a_nr)}{K_2(a_n\mathrm{a})}}}a_n]$$

$$[\frac{1}{a_\ell^2-a_n^2}(a_n\cos(a_nd_1)\sin(a_\ell d_1)-a_n\cos(a_nd_2)\sin(a_\ell d_2)-a_\ell\cos(a_\ell d_1)\sin(a_nd_1)+$$

$$a_\ell\cos(a_\ell d_2)\sin(a_nd_2)]]\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_p i^pF_{p_a}]\frac{K_p(a\,r)}{K_p(a\,\mathrm{a})}a$$

$$[\overline{\delta_{0,p-1}\frac{z_0}{H/2}}\;\overline{F_{0_a}^{H}+\delta_{1,p-1}(}\;\overline{\frac{x_0}{H/2}F_{1_a}^{S}}\;\overline{+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_{p-1} i^{p-1}F_{p-1_a}}\,]\;\overline{\overline{\frac{K_{p-1}(a\,r)}{K_{p-1}(a\,\mathrm{a})}}}a$$

$$[\frac{2a(d_1-d_2)-\sin(2ad_1)+\sin(2ad_2)}{4a}]+$$

$$
+\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\sum_{\substack{n=1\\ a_n}}^{\infty} N_{a_\ell}{}^{-1/2}[\delta_{0,p}\ \frac{z_0}{H/2}\ F^H_{0_{a_\ell}} + \delta_{1,p}(\ \frac{x_0}{H/2} F^S_{1a_\ell} + \frac{d\phi_0}{H/2} F^P_{1_{a_\ell}}) + \in_p\ i^p F_{p_{a_\ell}}\ ]\ \frac{K_p(a_\ell r)}{K_p(a_\ell \mathrm{a})} a_\ell
$$

$$
N_{a_n}^{-1/2}[\delta_{0,p-1}\overline{\frac{z_0}{H/2}}\ \overline{F^H_{0a_n} + \delta_{1,p-1}(}\ \overline{\frac{x_0}{H/2}} F^S_{1a_n}\ \overline{+\frac{d\phi_0}{H/2} F^P_{1_{a_n}}) + \in_{p-1}\ i^{p-1} F_{p-1_{a_n}}\ ]}\ \overline{\frac{K_{p-1}(a_n r)}{K_{p-1}(a_n \mathrm{a})}} a_n
$$

$$
[\frac{1}{a_\ell^2 - a_n^2}(a_n \cos(a_n d_1)\sin(a_\ell d_1) - a_n \cos(a_n d_2)\sin(a_\ell d_2) - a_\ell \cos(a_\ell d_1)\sin(a_n d_1) +
$$

$$
a_\ell \cos(a_\ell d_2)\sin(a_n d_2)]\ ]]\frac{\pi}{2} +
$$

$$
+\sum_{p=2,3,}^{\infty}[\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a{}^{-1}[\delta_{0,p}\frac{z_0}{H/2}\ F^H_{0_a} + \delta_{1,p}(\ \frac{x_0}{H/2} F^S_{1_a} + \frac{d\phi_0}{H/2} F^P_{1_a}) + \in_p\ i^p F_{p_a}\ ]\ \frac{K_p(a\ r)}{K_p(a\ \mathrm{a})} a
$$

$$
[\delta_{0,p+1}\overline{\frac{z_0}{H/2}}\ \overline{F^H_{0_a} + \delta_{1,p+1}(}\ \overline{\frac{x_0}{H/2}} F^S_{1_a}\ \overline{+\frac{d\phi_0}{H/2} F^P_{1_a}) + \in_{p+1}\ i^{p+1} F_{p+1_a}\ ]}\ \overline{\frac{K_{p+1}(a\ r)}{K_{p+1}(a\ \mathrm{a})}} a
$$

$$
[\frac{2a(d_1 - d_2) - \sin(2ad_1) + \sin(2ad_2)}{4a}]+
$$

$$
+\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\sum_{\substack{n=1\\ a_n}}^{\infty} N_{a_\ell}{}^{-1/2}[\delta_{0,p}\ \frac{z_0}{H/2}\ F^H_{0_{a_\ell}} + \delta_{1,p}(\ \frac{x_0}{H/2} F^S_{1a_\ell} + \frac{d\phi_0}{H/2} F^P_{1_{a_\ell}}) + \in_p\ i^p F_{pa_\ell}\ ]\ \frac{K_p(a_\ell r)}{K_p(a_\ell \mathrm{a})} a_\ell
$$

$$
N_{a_n}^{-1/2}[\delta_{0,p+1}\overline{\frac{z_0}{H/2}}\ \overline{F^H_{0a_n} + \delta_{1,p+1}(}\ \overline{\frac{x_0}{H/2}} F^S_{1a_n}\ \overline{+\frac{d\phi_0}{H/2} F^P_{1_{a_n}}) + \in_{p+1}\ i^{p+1} F_{p+1a_n}\ ]}\ \overline{\frac{K_{p+1}(a_n r)}{K_{p+1}(a_n \mathrm{a})}} a_n]
$$

$$
[\frac{1}{a_\ell^2 - a_n^2}(a_n \cos(a_n d_1)\sin(a_\ell d_1) - a_n \cos(a_n d_2)\sin(a_\ell d_2) - a_\ell \cos(a_\ell d_1)\sin(a_n d_1) +
$$

$$
a_\ell \cos(a_\ell d_2)\sin(a_n d_2)]\frac{\pi}{2}]\ \underline{\mathit{RADIUS}}
$$

3.2 Υπολογισμός του όρου $\int_{d_2}^{d_1}(\int_0^{2\pi} \overline{x}^{(1)}\nabla\Phi_t^{(1)})\overline{n}dS$ για το πεδίο (Ι)

Έχουμε δείξει από το Κεφάλαιο 2 –σελίδα 19– ότι

$$\overline{x}^{(1)}\nabla\Phi_t^{(1)} = [(X_{g_1}^{(1)} + X_5^{(1)}z)\frac{\partial\phi(r,\theta,z)}{\partial r}\cos\theta - (X_{g_1}^{(1)} + X_5^{(1)}z)\frac{1}{r}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\sin\theta$$
$$+ (X_{g_3}^{(1)} - X_5^{(1)}r\cos\theta)\frac{\partial\phi(r,\theta,z)}{\partial z}](-i\omega)]$$

Η οποία μεταβάλλεται ανάλογα σε ποιο πεδίο βρισκόμαστε ((Ι), (ΙΙ), (ΙΙΙ)) λόγω της μεταβολής της τιμής του *φ(r,θ,z)*

Από το Κεφάλαιο 2 –σελίδα 16– προκύπτει ότι για τον υπολογισμό της οριζόντιας δύναμης έκπτωσης πρέπει να υπολογίσουμε τον όρο

$$\int_{d_2}^{d_1}(\int_0^{2\pi}\overline{x}^{(1)}\nabla\Phi_t^{(1)})ndS = \int_{d_2}^{d_1}(\int_0^{2\pi}\overline{x}^{(1)}\nabla\Phi_t^{(1)}\cos\theta d\theta)RADIUSdz =$$

$$= \int_{d_2}^{d_1}[X_{g_1}^{(1)}\frac{\pi}{2}[-\frac{igH}{2\omega}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\in_2 i^2\kappa J_2^{'}(\kappa r) - i\,\omega\frac{H}{2}dN_\kappa^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H +$$

$$+\delta_{1,2}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_2 i^2(F_{2\kappa} - \frac{J_2(\kappa \mathrm{a})}{dz_\kappa'(d)})]\,\kappa\frac{H_2'(\kappa r)}{H_2(\kappa \mathrm{a})}\cosh(\kappa z) - i\,\omega\frac{H}{2}d$$

$$\sum_{\substack{j=1\\ a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}F_{0_a}^H + \delta_{1,2}(\frac{x_0}{H/2}F_{1_a}^S + \frac{d\phi_0}{H/2}F_{1_a}^P) + \in_2 i^2F_{2_a}]$$

$$a\frac{K_2'(ar)}{K_2(a\mathrm{a})}\cos(az)]]RADIUSdz +$$

$$+\int_{d_2}^{d_1}[X_5^{(1)}RADIUS\frac{\pi}{2}[z[-\frac{igH}{2\omega}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\in_2 i^2\kappa J_2^{'}(\kappa r) - i\,\omega\frac{H}{2}dN_\kappa^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H +$$

$$+\delta_{1,2}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa} + \frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_2 i^2(F_{2\kappa} - \frac{J_2(\kappa \mathrm{a})}{dz_\kappa'(d)})]\,\kappa\frac{H_2'(\kappa r)}{H_2(\kappa \mathrm{a})}\cosh(\kappa z) - i\,\omega\frac{H}{2}d$$

$$\sum_{\substack{j=1\\a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,2}(\ \frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_2\ i^2F_{2_a}]$$

$$a\frac{K_2'(ar)}{\overline{K_2(a\text{a})}}\cos(az)]]RADIUSdz+$$

$$-\frac{1}{r}\int_{d_2}^{d_1}[X_{g_1}^{(1)}\ [\frac{igH}{2\omega}\frac{\cosh(\kappa z)}{\cosh(\kappa d)}\in_2\ i^2J_2(\kappa r)2\frac{\pi}{2}+i\,\omega\frac{H}{2}dN_\kappa^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+$$

$$+\delta_{1,2}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2\ i^2(F_{2\kappa}-\frac{J_2(\kappa\text{a})}{dz_\kappa'(d)})]\ \frac{H_2(\kappa r)}{\overline{H_2(\kappa\text{a})}}\cosh(\kappa z)2\frac{\pi}{2}+i\ \omega\frac{H}{2}d$$

$$\sum_{\substack{j=1\\a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,2}(\ \frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_2\ i^2F_{2_a}]$$

$$\frac{K_2(ar)}{\overline{K_2(a\text{a})}}\cos(az)2\frac{\pi}{2}]]RADIUSdz+$$

$$-\frac{1}{r}\int_{d_2}^{d_1}[X_5^{(1)}\ [\frac{igH}{2\omega}\frac{\cosh(\kappa z)z}{\cosh(\kappa d)}\in_2\ i^2J_2(\kappa r)2\frac{\pi}{2}+i\,\omega\frac{H}{2}dN_\kappa^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+$$

$$+\delta_{1,2}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2\ i^2(F_{2\kappa}-\frac{J_2(\kappa\text{a})}{dz_\kappa'(d)})]\ \frac{H_2(\kappa r)}{\overline{H_2(\kappa\text{a})}}z\cosh(\kappa z)2\frac{\pi}{2}+i\ \omega\frac{H}{2}d$$

$$\sum_{\substack{j=1\\a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,2}(\ \frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_2\ i^2F_{2_a}]$$

$$\frac{K_2(ar)}{\overline{K_2(a\text{a})}}z\cos(az)2\frac{\pi}{2}]]RADIUSdz+$$

$$+\int_{d_2}^{d_1}[X_{g_3}^{(1)}RADIUS\pi[-\frac{igH}{2\omega}\frac{\sinh(\kappa z)}{\cosh(\kappa d)}\kappa\in_1\ i^1J_1\ (\kappa r)-i\,\omega\frac{H}{2}dN_\kappa^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+$$

$$+\delta_{1,0}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1\ i^1(F_{1\kappa}-\frac{J_0(\kappa\text{a})}{dz_\kappa'(d)})]\ \kappa\frac{H_1(\kappa r)}{\overline{H_1(\kappa\text{a})}}\sinh(\kappa z)+i\,\omega\frac{H}{2}d$$

$$\sum_{\substack{j=1\\a_j=a}}^{\infty} N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}\ F_{0_a}^H+\delta_{1,0}(\ \frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1\ i^1F_{1_a}]$$

$$a\frac{\overline{K_1(ar)}}{\overline{K_1(a\text{a})}}\sin(az)]]RADIUSdz-$$

$$-\int_{d_2}^{d_1}[X_5^{(1)}RADIUS^2\ \frac{\pi}{2}[-\frac{igH}{2\omega}\frac{\sinh(\kappa z)}{\cosh(\kappa d)}\kappa\in_2\ i^2J_2\ (\kappa r)-i\,\omega\frac{H}{2}dN_\kappa^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+$$

$$+\delta_{1,0}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa a)}{dz'_\kappa(d)})]\ \kappa\frac{H_2(\kappa r)}{H_2(\kappa a)}\sinh(\kappa z)+i\,\omega\frac{H}{2}d$$

$$\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}\ F^{H}_{0_a}+\delta_{1,0}(\ \frac{x_0}{H/2}F^{S}_{1_a}+\frac{d\phi_0}{H/2}F^{P}_{1_a})+\in_2 i^2F_{2_a}]$$

$$a\frac{K_2(ar)}{K_2(a\mathrm{a})}\sin(az)]]RADIUSdz=$$

$$=X^{(1)}_{g_1}\frac{\pi}{2}RADIUS\,[-\frac{igH}{2\omega}\frac{1}{\cosh(\kappa d)}\in_2 i^2J'_2(\kappa r)\kappa(\frac{\sinh(\kappa d_1)-\sinh(\kappa d_2)}{\kappa})$$

$$-i\,\omega\frac{H}{2}dN_\kappa^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}F^{H}_{0_\kappa}+\delta_{1,2}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa a)}{dz'_\kappa(d)})]$$

$$\kappa\frac{H'_2(\kappa r)}{H_2(\kappa a)}(\frac{\sinh(\kappa d_1)-\sinh(\kappa d_2)}{\kappa})-i\ \omega\frac{H}{2}d\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}\ F^{H}_{0_a}+\delta_{1,2}(\ \frac{x_0}{H/2}F^{S}_{1_a}$$

$$+\frac{d\phi_0}{H/2}F^{P}_{1_a})+\in_2 i^2F_{2_a}]\ a\frac{K'_2(ar)}{K_2(a\mathrm{a})}(\frac{\sin(\alpha d_1)-\sin(\alpha d_2)}{\alpha})]]+$$

$$+X^{(1)}_5RADIUS\frac{\pi}{2}[\ -\frac{igH}{2\omega}\frac{1}{\cosh(\kappa d)}\in_2 i^2\kappa J'_2(\kappa r)(\frac{-\cosh(\kappa d_1)+\cosh(\kappa d_2)}{\kappa^2}+$$

$$\frac{\kappa d_1\sinh(\kappa d_1)-\kappa d_2\sinh(\kappa d_2)}{\kappa^2})-i\,\omega\frac{H}{2}dN_\kappa^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}F^{H}_{0_\kappa}+\delta_{1,2}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F^{P}_{1\kappa})+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa a)}{dz'_\kappa(d)})]\ \kappa\frac{H'_2(\kappa r)}{H_2(\kappa a)}(\frac{-\cosh(\kappa d_1)+\cosh(\kappa d_2)}{\kappa^2}+$$

$$\frac{\kappa d_1\sinh(\kappa d_1)-\kappa d_2\sinh(\kappa d_2)}{\kappa^2})-i\ \omega\frac{H}{2}d\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}\ F^{H}_{0_a}+\delta_{1,2}(\ \frac{x_0}{H/2}F^{S}_{1_a}$$

$$+\frac{d\phi_0}{H/2}F^{P}_{1_a})+\in_2 i^2F_{2_a}]\ a\frac{K'_2(ar)}{K_2(a\mathrm{a})}(\frac{\cos(ad_1)-\cos(ad_2)}{a^2}+\frac{ad_1\sin(\kappa d_1)-ad_2\sinh(ad_2)}{a^2})+$$

$$-\frac{1}{RADIUS}X^{(1)}_{g_1}\pi RADIUS\,[\frac{igH}{2\omega}\frac{1}{\cosh(\kappa d)}\in_2 i^2J_2(\kappa r)(\frac{\sinh(\kappa d_1)-\sinh(\kappa d_2)}{\kappa})$$

$$+i\,\omega\frac{H}{2}dN_{\kappa}^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,2}(\frac{x_0}{H/2}\;F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa\mathrm{a})}{dz_\kappa'(d)})]$$

$$\frac{H_2(\kappa r)}{\overline{\overline{H_2(\kappa\mathrm{a})}}}(\frac{\sinh(\kappa d_1)-\sinh(\kappa d_2)}{\kappa})+i\;\omega\frac{H}{2}d\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}\;F_{0_a}^{H}+\delta_{1,2}(\;\frac{x_0}{H/2}F_{1_a}^{S}$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_2 i^2F_{2_a}]\;a\frac{K_2(ar)}{\overline{\overline{K_2(a\mathrm{a})}}}(\frac{-\sin(\alpha d_1)+\sin(\alpha d_2)}{\alpha})]]+$$

$$-\frac{1}{RADIUS}X_5^{(1)}RADIUS\pi[\;\frac{igH}{2\omega}\frac{1}{\cosh(\kappa d)}\in_2 i^2J_2(\kappa r)(\frac{-\cosh(\kappa d_1)+\cosh(\kappa d_2)}{\kappa^2}+$$

$$\frac{\kappa d_1\sinh(\kappa d_1)-\kappa d_2\sinh(\kappa d_2)}{\kappa^2})+i\,\omega\frac{H}{2}dN_{\kappa}^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,2}(\frac{x_0}{H/2}\;F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa\mathrm{a})}{dz_\kappa'(d)})]\;\frac{H_2(\kappa r)}{\overline{\overline{H_2(\kappa\mathrm{a})}}}(\frac{-\cosh(\kappa d_1)+\cosh(\kappa d_2)}{\kappa^2}+$$

$$\frac{\kappa d_1\sinh(\kappa d_1)-\kappa d_2\sinh(\kappa d_2)}{\kappa^2})+i\;\omega\frac{H}{2}d\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}\;F_{0_a}^{H}+\delta_{1,2}(\;\frac{x_0}{H/2}F_{1_a}^{S}$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_2 i^2F_{2_a}]\;\frac{K_2(ar)}{\overline{\overline{K_2(a\mathrm{a})}}}(\frac{\cos(ad_1)-\cos(ad_2)}{a^2}+\frac{ad_1\sin(\kappa d_1)-ad_2\sinh(ad_2)}{a^2})+$$

$$+X_{g_3}^{(1)}RADIUS\pi[\;-\frac{igH}{2\omega}\frac{1}{\cosh(\kappa d)}\kappa\in_1 i^1J_1(\kappa r)(\frac{\cosh(\kappa d_1)-\cosh(\kappa d_2)}{\kappa})$$

$$-i\,\omega\frac{H}{2}dN_{\kappa}^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,0}(\frac{x_0}{H/2}\;F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_1 i^1(F_{1\kappa}-\frac{J_0(\kappa\mathrm{a})}{dz_\kappa'(d)})]$$

$$\kappa\frac{H_1(\kappa r)}{\overline{\overline{H_1(\kappa\mathrm{a})}}}(\frac{\cosh(\kappa d_1)-\cosh(\kappa d_2)}{\kappa})+i\,\omega\frac{H}{2}d\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}\;F_{0_a}^{H}+\delta_{1,0}(\;\frac{x_0}{H/2}F_{1_a}^{S}$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_1 i^1F_{1_a}]\;a\frac{\overline{K_1(ar)}}{\overline{\overline{K_1(a\mathrm{a})}}}(-\frac{\cos(\alpha d_1)+\cos(\alpha d_2)}{\alpha})]]-$$

$$-X_5^{(1)}RADIUS^2[\;-\frac{igH}{2\omega}\frac{1}{\cosh(\kappa d)}\kappa(\in_2 i^2J_2(\kappa r)\frac{\pi}{2}+\in_0 i^0J_0(\kappa r)\pi)$$

$$(\frac{\cosh(\kappa d_1)-\cosh(\kappa d_2)}{\kappa})-i\,\omega\frac{H}{2}dN_{\kappa}^{-1/2}([\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,0}(\frac{x_0}{H/2}\;F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa\mathrm{a})}{dz_\kappa'(d)})]\;\kappa\frac{H_2(\kappa r)}{\overline{\overline{H_2(\kappa\mathrm{a})}}}\frac{\pi}{2}+[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,0}(\frac{x_0}{H/2}\;F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa\mathrm{a})}{dz_\kappa'(d)})]\;\kappa\frac{H_0(\kappa r)}{\overline{\overline{H_0(\kappa\mathrm{a})}}}\pi)\;(\frac{\cosh(\kappa d_1)-\cosh(\kappa d_2)}{\kappa})+i\,\omega\frac{H}{2}d$$

$$\sum_{\substack{j=1\\ a_j=a}}^{\infty} N_a^{-1/2}([\delta_{0,0}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,0}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_2 i^2F_{2_a}]a\frac{K_2(ar)}{K_2(a\mathrm{a})}\frac{\pi}{2}$$

$$(\frac{-\cos(ad_1)+\cos(ad_2)}{a})+\delta_{0,0}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,0}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_0 i^0F_{0_a}]$$

$$a\frac{K_0(ar)}{K_0(a\mathrm{a})}\pi)\ (\frac{-\cos(ad_1)+\cos(ad_2)}{a})$$

Όμως για τον υπολογισμός της δύναμης έκπτωσης πρέπει να υπολογίσουμε τον όρο

$$\overline{\overline{x}^{(1)}\nabla\Phi_t^{(1)}}^{T}=\omega(\overline{X}_{\mathrm{Re}}^{(1)}\cos(\omega t)+\overline{X}_{\mathrm{Im}}^{(1)}\sin(\omega t))(\nabla\phi_{\mathrm{Im}}^{(1)}\cos(\omega t)-\nabla\phi_{\mathrm{Re}}^{(1)}\sin(\omega t))=$$

$$=\overline{\omega(\overline{X}_{\mathrm{Re}}^{(1)}\nabla\phi_{\mathrm{Im}}^{(1)}\cos^2(\omega t)-\overline{X}_{\mathrm{Im}}^{(1)}\nabla\phi_{\mathrm{Re}}^{(1)}\sin^2(\omega t)+(-\overline{X}_{\mathrm{Re}}^{(1)}\nabla\phi_{\mathrm{Re}}^{(1)}+\overline{X}_{\mathrm{Im}}^{(1)}\nabla\phi_{\mathrm{Im}}^{(1)})\cos(\omega t)\sin(\omega t))}^{t}=$$

$$=\frac{1}{2}\omega(\overline{X}_{\mathrm{Re}}^{(1)}\nabla\phi_{\mathrm{Im}}^{(1)}-\overline{X}_{\mathrm{Im}}^{(1)}\nabla\phi_{\mathrm{Re}}^{(1)})=\frac{1}{2}\omega\,\mathrm{Im}(X^{(1)^*}\nabla\phi^{(1)})$$

Όπου $r=\mathrm{a}=RADIUS$: η ακτίνα του $i-$στου εξωτερικού στοιχείου

$\overline{X}^{(1)}$: το άνυσμα μετακίνησης από την παλιά θέση ισορροπίας

$\overline{X_g^{(1)}}$: οι μεταφορικές κινήσεις του κέντρου βάρους της κατασκευής

3.3 Υπολογισμός του όρου $\int_{WL}(\zeta_r^{(1)})^2 n_1 dl$ για το πεδίο (I)

Έχουμε αποδείξει στο Κεφάλαιο 2 –σελίδα 17– για την ανύψωση της ελεύθερης επιφάνειας ότι

$$(\zeta_r^{(1)})^2 = \frac{1}{2}\mathrm{Re}\left\{(\frac{i\omega}{g})\phi(r,\theta,d)(\frac{-i\omega}{g})\overline{\phi(r,\theta,d)}\right\} + \frac{1}{2}\mathrm{Re}\left\{ \left|X_{g_3}^{(1)} - X_5^{(1)} r\cos\theta\right|^2\right\} -$$

$$- \mathrm{Re}\left[\left\{(\frac{i\omega}{g})\phi(r,\theta,d)\right\}\{X_{g_3}^{(1)} - X_5^{(1)} r\cos\theta\}\right]$$

Επομένως

$$\int_{WL}(\zeta_r^{(1)})^2 n_1 dl = \int_{WL}[\frac{1}{2}\mathrm{Re}\left\{(\frac{i\omega}{g})\phi(r,\theta,d)(\frac{-i\omega}{g})\overline{\phi(r,\theta,d)}\right\} + \frac{1}{2}\mathrm{Re}\left\{ \left|X_{g_3}^{(1)} - X_5^{(1)} r\cos\theta\right|^2\right\} -$$

$$- \mathrm{Re}\left[\left\{(\frac{i\omega}{g})\phi(r,\theta,d)\right\}\{X_{g_3}^{(1)} - X_5^{(1)} r\cos\theta\}\right] =$$

$$= \int_{WL}[\frac{1}{2}\mathrm{Re}\left\{\frac{\omega^2}{g^2}\phi(r,\theta,d)\overline{\phi(r,\theta,d)}\right\} + \frac{1}{2}\mathrm{Re}\left\{ \left|X_{g_3}^{(1)} - X_5^{(1)} r\cos\theta\right|^2\right\} - \mathrm{Re}[\left\{\frac{i\omega}{g}\phi(r,\theta,d)\right\}$$

$$\{X_{g_3}^{(1)} - X_5^{(1)} r\cos\theta\}\,]\,n_1 dl$$

Με $n_1 = cos\theta$ και $dl = RADIUS\ d\theta$.

Όπου $RADIUS$: η ακτίνα του $i-$στου εξωτερικού στοιχείου.

Επομένως

$$\int_{WL}(\zeta_r^{(1)})^2 n_1 dl =$$

$$= \int_{WL}[\frac{1}{2}\mathrm{Re}\left\{\frac{\omega^2}{g^2}\phi(r,\theta,d)\overline{\phi(r,\theta,d)}\right\}\cos\theta \ \ RADIUS \ \ d\theta +$$

$$+ \int_{WL}[\frac{1}{2}\mathrm{Re}\left\{ \left|X_{g_3}^{(1)} - X_5^{(1)} r\cos\theta\right|^2\right\}\cos\theta \ \ RADIUS \ \ d\theta -$$

$$- \int_{WL}[\mathrm{Re}[\left\{\frac{i\omega}{g}\phi(r,\theta,d)\right\}\{X_{g_3}^{(1)} - X_5^{(1)} r\cos\theta\}\,]\cos\theta \ \ RADIUS \ \ d\theta$$

Δηλαδή

$$\int_{WL}(\zeta_r^{(1)})^2 n_1 dl =$$

$$\int_{WL}(\frac{1}{2}\mathrm{Re}(\frac{\omega^2}{g^2}(\frac{g^2H^2}{4\omega^2}\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\cos(m\theta)\sum_{n=0}^{\infty}\in_n i^n\overline{J_n(\kappa r)}\cos(n\theta)+$$

$$+\frac{gH^2}{4}dN_\kappa^{-1/2}\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\cos(m\theta)\sum_{n=0}^{\infty}\overline{[\delta_{0,n}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,n}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_n i^n(F_{n\kappa}-\frac{J_n(\kappa a)}{dz'_\kappa(d)})]}\,\frac{\overline{H_n(\kappa r)}}{\overline{H_n(\kappa a)}}\cos(n\theta)\cosh(\kappa d)+$$

$$+\frac{gH^2}{4}d\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\cos(m\theta)\sum_{n=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,n}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,n}(\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_n i^n F_{n_a}]}\,\frac{\overline{K_n(ar)}}{\overline{K_n(a\partial)}}\cos(n\theta)\cos(ad)+$$

$$+\frac{gH^2}{4}dN_\kappa^{-1/2}\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,m}(\frac{x_0}{H/2}F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_m(\kappa r)}{H_m(\kappa a)}\cos(m\theta)\cosh(\kappa d)\sum_{n=0}^{\infty}\varepsilon_n i^n\overline{J_n(\kappa r)}\cos(n\theta)+$$

$$+\frac{\omega^2H^2d^2}{4}N_\kappa^{-1}\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,m}(\frac{x_0}{H/2}F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_m(\kappa r)}{H_m(\kappa a)}\cos(m\theta)\cosh(\kappa d)\sum_{n=0}^{\infty}\overline{[\delta_{0,n}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,n}(\frac{x_0}{H/2}}\,\overline{F^S{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_n i^n(F_{n\kappa}-\frac{J_n(\kappa a)}{dz'_\kappa(d)})]}\,\frac{\overline{H_n(\kappa r)}}{\overline{H_n(\kappa a)}}\cos(n\theta)\cosh(\kappa d)+$$

$$+\frac{\omega^2H^2d^2}{4}N_\kappa^{-1/2}\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,m}(\frac{x_0}{H/2}F^S{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_m(\kappa r)}{H_m(\kappa a)}\cos(m\theta)\cosh(\kappa d)\sum_{n=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,n}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,n}(\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_n i^n F_{n_a}]}\,\frac{\overline{K_n(ar)}}{\overline{K_n(a\partial)}}\cos(n\theta)\cos(ad)+$$

$$+\frac{gH^2}{4}d\sum_{n=0}^{\infty}\in_n i^n\overline{J_n(\kappa r)}\cos(n\theta)\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^S$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_m i^m F_{m_a}]\frac{K_m(ar)}{K_m(a\text{a})}\cos(m\theta)\cos(az)+$$

$$+\frac{\omega^2H^2d^2}{4}N_\kappa^{-1/2}\sum_{n=0}^{\infty}\overline{[\delta_{0,n}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,n}(\frac{x_0}{H/2}}\,\overline{F^{S}{}_{1_\kappa}}\,\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_n i^n(F_{n\kappa}-\frac{J_n(\kappa\text{a})}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_n(\kappa r)}}{\overline{H_n(\kappa\text{a})}}\cos(n\theta)\cosh(\kappa d)\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^S$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_m i^m F_{m_a}]\frac{K_m(ar)}{K_m(a\text{a})}\cos(m\theta)\cos(az)+$$

$$+\frac{\omega^2H^2d^2}{4}\sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,m}\frac{z_0}{H/2}F_{0_a}^H+\delta_{1,m}(\frac{x_0}{H/2}F_{1_a}^S+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_m i^m F_{m_a}]$$

$$\frac{K_m(ar)}{K_m(a\text{a})}\cos(m\theta)\cos(az)\sum_{n=0}^{\infty}[\sum_{\substack{i=1\\a_i=a}}^{\infty}\overline{N_a^{-1/2}[\delta_{0,n}\frac{z_0}{H/2}}\,\overline{F_{0_a}^H+\delta_{1,n}(}\,\overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_n i^n F_{n_a}]}\,\frac{\overline{K_n(ar)}}{\overline{K_n(a\text{a})}}\cos(n\theta)\cos(ad))\cos\theta\quad RADIUS\quad d\theta+$$

$$+\int_{WL}[\frac{1}{2}\text{Re}(X_{g_3}^{(1)^2}+X_5^{(1)^2}r^2(\cos\theta)^2-2X_{g_3}X_5 r\cos\theta)\cos\theta\quad RADIUS\quad d\theta+$$

$$\int_{WL}\text{Re}(\frac{i\omega}{g}[(\frac{-igH}{2\omega}1\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\cos(m\theta)\,X_{g_3}+\frac{igH}{2\omega}1\sum_{m=0}^{\infty}\in_m i^m J_m(\kappa r)\cos(m\theta)\,\text{X}_5$$

$$r\cos\theta)+(-i\,\omega\frac{H}{2}N_\kappa^{-1/2}\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,m}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa\text{a})}{dz'_\kappa(d)})]\frac{H_m(\kappa r)}{H_m(\kappa\text{a})}\cos(m\theta)\cosh(\kappa d)\,X_{g_3}+$$

$$i\omega\frac{H}{2}N_\kappa^{-1/2}\sum_{m=0}^{\infty}[\delta_{0,m}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,m}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_m i^m(F_{m\kappa}-\frac{J_m(\kappa\text{a})}{dz'_\kappa(d)})]$$

$$\frac{H_m(\kappa r)}{H_m(\kappa\partial)}\cos(m\theta)\cosh(\kappa d)\;\text{X}_5\,r\cos\theta)+$$

$$+(-i\,\omega\frac{H}{2}d\ \sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,m}\frac{z_0}{H/2}\ F_{0_a}^{H}+\delta_{1,m}(\ \frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_m\ i^m F_{m_a}]$$

$$\frac{K_m(ar)}{K_m(a\text{a})}\cos(m\theta)\cos(ad)\ X_{g_3}+i\,\omega\frac{H}{2}d\ \sum_{m=0}^{\infty}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,m}\frac{z_0}{H/2}\ F_{0_a}^{H}+\delta_{1,m}(\ \frac{x_0}{H/2}F_{1_a}^{S}$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_m\ i^m F_{m_a}]\ \frac{K_m(ar)}{K_m(a\text{a})}\cos(m\theta)\cos(ad)\ \mathrm{X}_5\, r\cos\theta)]\cos\theta\ \ RADIUS\ \ d\theta\,.$$

Από *Παράρτημα Α*, θα έχουμε

$$\int_0^{2\pi}(\zeta_r^{(1)})^2\cos\theta d\theta\ \ RADIUS=$$

$$=\frac{1}{2}\mathrm{Re}(\frac{\omega^2}{g^2}(\frac{g^2H^2}{4\omega^2}[\ \in_0\ J_0(\kappa r)\in_1\ i\overline{J_1(\kappa r)\pi}+\in_1\ iJ_1(\kappa r)\in_0\ \overline{J_0(\kappa r)\pi}+\in_1\ iJ_1(\kappa r)\in_2\ i^2\overline{J_2(\kappa r)}\frac{\pi}{2}$$

$$+\sum_{P=2,3,}^{\infty}\in_p\ i^pJ_p(\kappa r)\ [\in_{p+1}\ i^{p+1}\overline{J_{p+1}(\kappa r)}+\in_{p-1}\ i^{p-1}\overline{J_{p-1}(\kappa r)}\ ]\ \frac{\pi}{2}]\,\underline{RADIUS}+$$

$$+\frac{gH^2}{4}dN_\kappa^{-1/2}[\ \in_0\ J_0(\kappa r)\ \overline{[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_1\ i^1(F_{1\kappa}-\frac{J_1(\kappa\text{a})}{dz_\kappa'(d)})]}\ \frac{\overline{H_1(\kappa r)}}{\overline{H_1(\kappa\text{a})}}\cosh(\kappa d)\pi\ +\in_1\ iJ_1(\kappa r)$$

$$\overline{\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,0}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_0\ i^0(F_{0\kappa}-\frac{J_0(\kappa\text{a})}{dz_\kappa'(d)})]}\ \frac{\overline{H_0(\kappa r)}}{\overline{H_0(\kappa\text{a})}}\cosh(\kappa d)\pi$$

$$+\in_1\ iJ_1(\kappa r)\ \overline{\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,2}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_2\ i^2(F_{2\kappa}-\frac{J_2(\kappa\text{a})}{dz_\kappa'(d)})]}$$

$$\frac{\overline{H_2(\kappa r)}}{\overline{H_2(\kappa\text{a})}}\cosh(\kappa d)\frac{\pi}{2}+\ \sum_{P=2,3,}^{\infty}\in_p\ i^pJ_p(\kappa r)$$

$$\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_{p+1}\ i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa\text{a})}{dz_\kappa'(d)})]}$$

$$\frac{\overline{H_{p+1}(\kappa r)}}{\overline{H_{p+1}(\kappa\text{a})}}\cosh(\kappa d)+[\overline{\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,p-1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_{p-1}\ i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa\text{a})}{dz_\kappa'(d)})]}\ \frac{\overline{H_{p-1}(\kappa r)}}{\overline{H_{p-1}(\kappa\text{a})}}\cosh(\kappa d)\ ]\frac{\pi}{2}\ ]\ \underline{RADIUS\ }+\frac{gH^2}{4}d$$

$$[\in_0 J_0(\kappa r) \sum_{\substack{j=1\\a_j=a}}^{\infty} N_a^{-1/2}[\overline{\delta_{0,1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H + \delta_{1,1}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P) + \in_1 i^1 F_{1_a}]}$$

$$\frac{\overline{K_1(ar)}}{\overline{K_1(a\text{a})}}\cos(ad)\pi + \in_1 iJ_1(\kappa r) \sum_{\substack{j=1\\a_j=a}}^{\infty} N_a^{-1/2}[\overline{\delta_{0,0}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H + \delta_{1,0}(}\ \overline{\frac{x_0}{H/2}F_{1_a}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_a}^P) + \in_0 i^0 F_{0_a}]}\ \frac{\overline{K_0(ar)}}{\overline{K_0(a\text{a})}}\cos(ad)\pi + \in_1 iJ_1(\kappa r) \sum_{\substack{j=1\\a_j=a}}^{\infty} N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H + \delta_{1,2}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P) + \in_2 i^2 F_{2_a}]}\ \frac{\overline{K_2(ar)}}{\overline{K_2(a\text{a})}}\cos(ad)\frac{\pi}{2} + \sum_{P=2,3,}^{\infty} \in_p i^p J_p(\kappa r)\,[$$

$$\sum_{\substack{\ell=1\\a_\ell}}^{\infty} N_{a_\ell}^{-1/2}[\overline{\delta_{0,P+1}\frac{z_0}{H/2}}\ \overline{F_{0_{a_\ell}}^H + \delta_{1,P+1}(}\ \overline{\frac{x_0}{H/2}F_{1_{a_\ell}}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^P) + \in_{P+1} i^{P+1} F_{P+1_{a_\ell}}]}$$

$$\frac{\overline{K_{P+1}(a_\ell r)}}{\overline{K_{P+1}(a_\ell \text{a})}}\cos(a_\ell d) + \sum_{\substack{n=1\\a_n}}^{\infty} N_{a_n}^{-1/2}[\overline{\delta_{0,P-1}\frac{z_0}{H/2}}\ \overline{F_{0_{a_n}}^H + \delta_{1,P-1}(}\ \overline{\frac{x_0}{H/2}F_{1_{a_n}}^S}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_{a_n}}^P) + \in_{P-1} i^{P-1} F_{P-1_{a_n}}]}\ \frac{\overline{K_{P-1}(a_n r)}}{\overline{K_{P-1}(a_n \text{a})}}\cos(a_n d)]\frac{\pi}{2}]$$ <u>*RADIUS*</u>

$$+\frac{gH^2}{4} dN_\kappa^{-1/2}[\,[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H + \delta_{1,0}(\frac{x_0}{H/2}$$

$$F^S{}_{1_\kappa} + \frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_0 i^0 (F_{0\kappa} - \frac{J_0(\kappa \text{a})}{dz_\kappa'(d)})]\ \frac{H_0(\kappa r)}{H_0(\kappa \text{a})}\cosh(\kappa d) \in_1 \overline{iJ_1(\kappa r)\pi} +$$

$$[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H + \delta_{1,1}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa} + \frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_1 i^1 (F_{1\kappa} - \frac{J_1(\kappa \text{a})}{dz_\kappa'(d)})]\ \frac{H_1(\kappa r)}{H_1(\kappa \text{a})}\cosh(\kappa d)$$

$$\in_0 \overline{J_0(\kappa r)\pi} + [\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H + \delta_{1,1}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa} + \frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_1 i^1 (F_{1\kappa} - \frac{J_1(\kappa \text{a})}{dz_\kappa'(d)})]$$

$$\frac{H_1(\kappa r)}{H_1(\kappa \text{a})}\cosh(\kappa d) \in_2 i^2 \overline{J_2(\kappa r)}\frac{\pi}{2} + \sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H + \delta_{1,p}(\frac{x_0}{H/2}\ F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P) + \in_p i^p (F_{p\kappa} - \frac{J_p(\kappa \text{a})}{dz_\kappa'(d)})]\ \frac{H_p(\kappa r)}{H_p(\kappa \text{a})}\cosh(\kappa d)\,[\in_{p+1} i^{p+1}\overline{J_{p+1}(\kappa r)} +$$

$$\in_{p-1} i^{p-1}\overline{J_{p-1}(\kappa r)}]\frac{\pi}{2}]$$ <u>*RADIUS*</u> +

$$+\frac{\omega^2 H^2 d^2}{4} N_\kappa^{-1}[[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_0(\kappa r)}{H_0(\kappa a)}\cosh(\kappa d)\ \overline{[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]}$$

$$\overline{\frac{\overline{H_1(\kappa r)}}{H_1(\kappa a)}}\cosh(\kappa d)\pi+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_1(\kappa r)}{H_1(\kappa a)}\cosh(\kappa d)\ \overline{[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]}$$

$$\overline{\frac{\overline{H_0(\kappa r)}}{H_0(\kappa a)}}\cosh(\kappa d)\pi+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]$$

$$\frac{H_1(\kappa r)}{H_1(\kappa a)}\cosh(\kappa d)\ \overline{[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,2}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa a)}{dz'_\kappa(d)})]}$$

$$\overline{\frac{\overline{H_2(\kappa r)}}{H_2(\kappa a)}}\cosh(\kappa d)\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\ \frac{H_p(\kappa r)}{H_p(\kappa a)}\cosh(\kappa d)\ \overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p+1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa a)}{dz'_\kappa(d)})]}\ \frac{\overline{H_{p+1}(\kappa r)}}{H_{p+1}(\kappa a)}\cosh(\kappa d)+$$

$$\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,p-1}(\frac{x_0}{H/2}}\ \overline{F^{S}{}_{1_\kappa}}\ \overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa a)}{dz'_\kappa(d)})]}$$

$$\overline{\frac{\overline{H_{p-1}(\kappa r)}}{H_{p-1}(\kappa a)}}\cosh(\kappa d)]\ \frac{\pi}{2}]\ \underline{RADIUS}+\frac{\omega^2 H^2 d^2}{4}N_\kappa^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,0}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa a)}{dz'_\kappa(d)})]\ \frac{H_0(\kappa r)}{H_0(\kappa a)}\cosh(\kappa d)\ \sum_{\substack{j=1\\ a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,1}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_1 i^1F_{1_a}]}\ \frac{\overline{K_1(ar)}}{\overline{K_1(aa)}}\cos(ad)\pi+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F_{1\kappa}^P)+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\ \frac{H_1(\kappa r)}{H_1(\kappa a)}\cosh(\kappa d)\ \sum_{\substack{j=1\\ a_j=a}}^{\infty}N_a^{-1/2}\overline{[\delta_{0,0}\frac{z_0}{H/2}}\ \overline{F_{0_a}^H+\delta_{1,0}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_a}^S}\ \overline{+\frac{d\phi_0}{H/2}F_{1_a}^P)+\in_0 i^0F_{0_a}]}\ \frac{\overline{K_0(ar)}}{\overline{K_0(aa)}}\cos(ad)\pi+[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^H+\delta_{1,1}(\frac{x_0}{H/2}\ F^{S}{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]\frac{H_1(\kappa r)}{H_1(\kappa a)}\cosh(\kappa d)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\overline{\delta_{0,2}\frac{z_0}{H/2}}\;\overline{F^H_{0_a}+\delta_{1,2}(}$$

$$\overline{\frac{x_0}{H/2}F^S_{1_a}}\;\overline{+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_2 i^2F_{2_a}]}\;\frac{\overline{K_2(ar)}}{\overline{K_2(a\mathrm{a})}}\cos(ad)\frac{\pi}{2}+\sum_{p=2,3}^{\infty}[\delta_{0,p}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,p}(\frac{x_0}{H/2}F^S{}_{1_\kappa}$$

$$+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_p i^p(F_{p\kappa}-\frac{J_p(\kappa a)}{dz'_\kappa(d)})]\frac{H_p(\kappa r)}{H_p(\kappa a)}\cosh(\kappa d)[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}N_{a_\ell}^{-1/2}[\overline{\delta_{0,P+1}\frac{z_0}{H/2}}\;\overline{F^H_{0_{a_\ell}}+\delta_{1,P+1}(}$$

$$\overline{\frac{x_0}{H/2}F^S_{1_{a_\ell}}}\;\overline{+\frac{d\phi_0}{H/2}F^P_{1_{a_\ell}})+\in_{P+1} i^{P+1}F_{P+1_{a_\ell}}]}\;\frac{\overline{K_{P+1}(a_\ell r)}}{K_{P+1}(a_\ell \mathrm{a})}\cos(a_\ell d)+\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_n}^{-1/2}[\overline{\delta_{0,P-1}\frac{z_0}{H/2}}$$

$$\overline{F^H_{0_{a_n}}+\delta_{1,P-1}(}\;\overline{\frac{x_0}{H/2}F^S_{1_{a_n}}}\;\overline{+\frac{d\phi_0}{H/2}F^P_{1_{a_n}})+\in_{P-1} i^{P-1}F_{P-1_{a_n}}]}\;\frac{\overline{K_{P-1}(a_n r)}}{K_{P-1}(a_n \mathrm{a})}\cos(a_n d)]\frac{\pi}{2}]\ \underline{RADIUS}+$$

$$+\frac{gH^2}{4}d[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F^H_{0_a}+\delta_{1,0}(\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_0 i^0F_{0_a}]\frac{K_0(ar)}{K_0(a\mathrm{a})}\cos(ad)$$

$$\in_1 i\overline{J_1(\kappa r)\pi}+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_a}+\delta_{1,1}(\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_1 i^1F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})}\cos(ad)\in_0\overline{J_0(\kappa r)\pi}+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_a}+\delta_{1,1}(\frac{x_0}{H/2}F^S_{1_a}$$

$$+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_1 i^1F_{1_a}]\frac{K_1(ar)}{K_1(a\mathrm{a})}\cos(ad)\in_2 i^2\overline{J_2(\kappa r)}\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}$$

$$F^H_{0_a}+\delta_{1,p}(\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_p i^pF_{p_a}]\frac{K_p(ar)}{K_p(a\mathrm{a})}\cos(ad)[\in_{p+1} i^{p+1}\overline{J_{p+1}(\kappa r)}+$$

$$\in_{p-1} i^{p-1}\overline{J_{p-1}(\kappa r)}]\frac{\pi}{2}]\underline{RADIUS}+$$

$$\frac{\omega^2H^2d^2}{4}N_\kappa^{-1/2}[\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F^H_{0_a}+\delta_{1,0}(\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_0 i^0F_{0_a}]$$

$$\frac{K_0(ar)}{K_0(a\mathrm{a})}\cos(ad)\ \overline{[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_\kappa}+\delta_{1,1}(\frac{x_0}{H/2}}\;\overline{F^S{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F^P_{1\kappa})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa a)}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_1(\kappa r)}}{\overline{H_1(\kappa a)}}\cosh(\kappa d)\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F^H_{0_a}+\delta_{1,1}(\frac{x_0}{H/2}F^S_{1_a}+\frac{d\phi_0}{H/2}F^P_{1_a})+\in_1 i^1F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})}\cos(ad)\,\overline{[\delta_{0,0}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,0}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_0 i^0(F_{0\kappa}-\frac{J_0(\kappa\mathrm{a})}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_0(\kappa r)}}{\overline{H_0(\kappa\mathrm{a})}}\cosh(\kappa d)\pi+\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_1 i^1F_{1_a}]$$

$$\frac{K_1(ar)}{K_1(a\mathrm{a})}\cos(ad)\,\overline{[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,2}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa\mathrm{a})}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_2(\kappa r)}}{\overline{H_2(\kappa\mathrm{a})}}\cosh(\kappa d)\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}\;\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,p}(\frac{x_0}{H/2}F_{1_a}^{S}$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_p i^pF_{p_a}]\;\frac{K_p(ar)}{K_p(a\mathrm{a})}\cos(ad)\,\overline{[\delta_{0,p+1}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,p+1}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_{p+1} i^{p+1}(F_{p+1_\kappa}-\frac{J_{p+1}(\kappa\mathrm{a})}{dz'_\kappa(d)})]}\;\frac{\overline{H_{p+1}(\kappa r)}}{\overline{H_{p+1}(\kappa\mathrm{a})}}\cosh(\kappa d)+$$

$$\overline{[\delta_{0,p-1}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,p-1}(\frac{x_0}{H/2}}\;\overline{F^{S}{}_{1_\kappa}}\;\overline{+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_{p-1} i^{p-1}(F_{p-1_\kappa}-\frac{J_{p-1}(\kappa\mathrm{a})}{dz'_\kappa(d)})]}$$

$$\frac{\overline{H_{p-1}(\kappa r)}}{\overline{H_{p-1}(\kappa\mathrm{a})}}\cosh(\kappa d)]\;\frac{\pi}{2}]\;\underline{\mathit{RADIUS}}+$$

$$+\frac{\omega^2H^2d^2}{8}[\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_n}^{-1/2}[\delta_{0,0}\frac{z_0}{H/2}F_{0_{a_n}}^{H}+\delta_{1,0}(\frac{x_0}{H/2}F_{1_{a_n}}^{S}+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P})+\in_0 i^0F_{0_{a_n}}]$$

$$\frac{K_0(a_nr)}{K_0(a_n\mathrm{a})}\cos(a_nd)\sum_{\substack{\ell=1\\a_\ell}}^{\infty}N_{a_\ell}^{-1/2}\overline{[\delta_{0,1}\frac{z_0}{H/2}}\;\overline{F_{0_{a_\ell}}^{H}+\delta_{1,1}(}\;\overline{\frac{x_0}{H/2}F_{1_{a_\ell}}^{S}}\;\overline{+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P})+\in_1 i^1F_{1_{a_\ell}}]}$$

$$\frac{\overline{K_1(a_\ell r)}}{\overline{K_1(a_\ell\mathrm{a})}}\cos(a_\ell d)\pi+\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_n}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_{a_n}}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_{a_n}}^{S}+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P})+\in_1 i^1F_{1_{a_n}}]$$

$$\frac{K_1(a_nr)}{K_1(a_n\mathrm{a})}\cos(a_nd)\sum_{\substack{\ell=1\\a_\ell}}^{\infty}N_{a_\ell}^{-1/2}\overline{[\delta_{0,0}\frac{z_0}{H/2}}\;\overline{F_{0_{a_\ell}}^{H}+\delta_{1,0}(}\;\overline{\frac{x_0}{H/2}F_{1_{a_\ell}}^{S}}\;\overline{+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P})+\in_0 i^0F_{0_{a_\ell}}]}$$

$$\frac{\overline{K_0(a_\ell r)}}{\overline{K_0(a_\ell\mathrm{a})}}\cos(a_\ell d)\pi+\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_n}^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_{a_n}}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_{a_n}}^{S}+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P})+\in_1 i^1F_{1_{a_n}}]$$

$$\frac{K_1(a_nr)}{K_1(a_n\mathrm{a})}\cos(a_nd)\sum_{\substack{\ell=1\\a_\ell}}^{\infty}N_{a_\ell}^{-1/2}\overline{[\delta_{0,2}\frac{z_0}{H/2}}\;\overline{F_{0_{a_\ell}}^{H}+\delta_{1,2}(}\;\overline{\frac{x_0}{H/2}F_{1_{a_\ell}}^{S}}\;\overline{+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P})+\in_2 i^2F_{2_{a_\ell}}]}$$

$$\overline{\frac{K_2(a_\ell r)}{K_2(a_\ell \mathrm{a})}}\cos(a_\ell d)\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}\sum_{\substack{n=1\\a_n}}^{\infty}N_{a_n}^{-1/2}[\delta_{0,p}\frac{z_0}{H/2}F_{0_{a_n}}^{H}+\delta_{1,p}(\frac{x_0}{H/2}F_{1_{a_n}}^{S}$$

$$+\frac{d\phi_0}{H/2}F_{1_{a_n}}^{P})+\in_p i^p F_{p_{a_n}}]\frac{K_p(a_n r)}{K_p(a_n \mathrm{a})}\cos(a_n d)[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}N_{a_\ell}^{-1/2}\overline{[\delta_{0,P+1}\frac{z_0}{H/2}}\,\overline{F_{0_{a_\ell}}^{H}+\delta_{1,P+1}(}\,\overline{\frac{x_0}{H/2}F_{1_{a_\ell}}^{S}}$$

$$\overline{+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P})+\in_{P+1}i^{P+1}F_{P+1_{a_\ell}}]}\,\overline{\frac{K_{P+1}(a_\ell r)}{K_{P+1}(a_\ell \mathrm{a})}}\cos(a_\ell d)+\sum_{\substack{\ell=1\\a_\ell}}^{\infty}N_{a_\ell}^{-1/2}\overline{[\delta_{0,P-1}\frac{z_0}{H/2}}\,\overline{F_{0_{a_\ell}}^{H}+\delta_{1,P-1}(}$$

$$\overline{\frac{x_0}{H/2}F_{1_{a_\ell}}^{S}}\,\overline{+\frac{d\phi_0}{H/2}F_{1_{a_\ell}}^{P})+\in_{P-1}i^{P-1}F_{P-1_{a_\ell}}]}\,\overline{\frac{K_{P-1}(a_\ell r)}{K_{P-1}(a_\ell \mathrm{a})}}\cos(a_\ell d)]\frac{\pi}{2}]\ \underline{RADIUS})+$$

$$+\frac{1}{2}\mathrm{Re}(-2X_{g_3}X_5 r\pi)\,\underline{RADIUS}+$$

$$+\mathrm{Re}(\frac{i\omega}{g}[(\frac{-igH}{2\omega}\in_1 iJ_1(\kappa r)\pi+\frac{igH}{2\omega}\in_2 i^2 J_2(\kappa r)X_5 r\frac{\pi}{2})+(-i\omega\frac{H}{2}dN_\kappa^{-1/2})$$

$$[\delta_{0,1}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_1 i^1(F_{1\kappa}-\frac{J_1(\kappa \mathrm{a})}{dz_\kappa'(d)})]\frac{H_1(\kappa r)}{H_1(\kappa \mathrm{a})}\cosh(\kappa d)\ X_{g_3}\pi+$$

$$+i\omega\frac{H}{2}dN_\kappa^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}F_{0_\kappa}^{H}+\delta_{1,2}(\frac{x_0}{H/2}F^{S}{}_{1_\kappa}+\frac{d\phi_0}{H/2}F_{1\kappa}^{P})+\in_2 i^2(F_{2\kappa}-\frac{J_2(\kappa \mathrm{a})}{dz_\kappa'(d)})]$$

$$\frac{H_2(\kappa r)}{H_2(\kappa \mathrm{a})}\cosh(\kappa d)\ X_5 r\frac{\pi}{2})+(-i\omega\frac{H}{2}d)\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,1}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,1}(\frac{x_0}{H/2}F_{1_a}^{S}$$

$$+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\varepsilon_1 i^1 F_{1_a}]\frac{K_1(ar)}{K_1(a\mathrm{a})}\cos(ad)\ X_{g_3}\pi+i\omega\frac{H}{2}d\sum_{\substack{j=1\\a_j=a}}^{\infty}N_a^{-1/2}[\delta_{0,2}\frac{z_0}{H/2}F_{0_a}^{H}+\delta_{1,2}($$

$$\frac{x_0}{H/2}F_{1_a}^{S}+\frac{d\phi_0}{H/2}F_{1_a}^{P})+\in_2 i^2 F_{2_a}]\frac{K_2(ar)}{K_2(a\mathrm{a})}\cos(ad)\ X_5 r\frac{\pi}{2})])\,\underline{RADIUS}$$

Όπου X_{g_3} : μεταφορική κίνηση του κέντρου βάρους της κατασκευής στον άξονα OX_3

X_5 : περιστροφή γύρω από τον άξονα GX_2

r : η ακτίνα του εξωτερικού, $i-$στού, στοιχείου.

3.4 Υπολογισμός της οριζόντιας δύναμης έκπτωσης για το πεδίο (Ι)

Η οριζόντια δύναμη έκπτωσης για το Πεδίο (Ι) υπολογίζεται στο Κεφάλαιο 2 –σελίδα 16– από τη σχέση

$$F_X = -\int_{WL} \frac{1}{2}\rho g \zeta_r^{(1)2}\bar{n}dl + MR^{(1)}\overline{X_g^{(1)}}'' - \iint_{S_0} -\frac{1}{2}\rho\left|\nabla\Phi^{(1)}\right|^2\bar{n}dS - \iint_{S_0} -\rho(\overline{X}^{(1)}\nabla\Phi_t^{(1)})\bar{n}dS =$$

$$= -\frac{1}{2}\rho g\,[\underline{\int_{WL}\zeta_r^{(1)2}\bar{n}dl}\,] + MR^{(1)}\overline{X_g^{(1)}}'' + \frac{1}{2}\rho\,[\underline{\iint_{S_0}\left|\nabla\Phi^{(1)}\right|^2\bar{n}dS}\,] + \rho\,[\underline{\iint_{S_0}(\overline{X}^{(1)}\nabla\Phi_t^{(1)})\bar{n}dS}\,].$$

Όμως οι παραστάσεις $\underline{\int_{WL}\zeta_r^{(1)2}\bar{n}dl}$ $\quad\underline{\iint_{S_0}\left|\nabla\Phi^{(1)}\right|^2\bar{n}dS}$ $\quad\underline{[\iint_{S_0}(\overline{X}^{(1)}\nabla\Phi_t^{(1)})\bar{n}dS]}$

είναι γνωστές από τα προηγούμενα (σελίδα 96, σελίδα 69, σελίδα 91 αντίστοιχα)

Όπου *ρ:* η πυκνότητα νερού.

g: η επιτάχυνση της βαρύτητας.

M: η μάζα του αξονοσυμμετρικού σώματος.

$R^{(1)}$: ο πίνακας που περιέχει τις γωνίες περιστροφής.

$X_g^{(1)}$: οι μεταφορικές κινήσεις του κέντρου βάρους της κατασκευής.

ω: η ιδιοσυχνότητα της κατασκευής.

φ: η διαφορά φάσης.

Επομένως μπορούμε να υπολογίσουμε την οριζόντια δύναμη έκπτωσης F_X για το Πεδίο (Ι).

4^{o} ΟΡΙΖΟΝΤΙΑ ΔΥΝΑΜΗ ΕΚΠΤΩΣΗΣ ΓΙΑ ΤΟ ΠΕΔΙΟ (ΙΙ)

4.1 Υπολογισμός του όρου $\int_{d_2}^{d_1}(\int_0^{2\pi} \left|\nabla\Phi^{(1)}\right|^2)\overline{n}dS$ για το πεδίο (ΙΙ)

Έχουμε αποδείξει στο Κεφάλαιο 1 –σελίδα 12– ότι η σχέση που μας δίνει το δυναμικό της ταχύτητας πρώτης τάξης για δακτυλιοειδή στοιχεία στο πεδίο (ΙΙ): $a_\ell \le r \le a_{\ell+1}$ $\kappa\alpha\iota$ $d_\ell \le z \le d$ είναι

$$\Phi(r,\theta,z;t) = -i\omega\frac{H}{2}d\left[\frac{z_0}{\frac{H}{2}}(\frac{z-d}{d}+\frac{g}{\omega^2 d}) - d\frac{\phi_0}{\frac{H}{2}}r(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\cos\theta\right]e^{-i\omega t} - i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)z_{\kappa_\ell}(z) + \Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)z_{\kappa_\ell}(z)]\cos(m\theta)e^{-i\omega t} - i\omega\frac{H}{2}d\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$

$$z_{a_\ell}(z) + \Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)z_{a_\ell}(z)]]\cos(m\theta)e^{-i\omega t}$$

Και αντίστοιχα

$$\varphi(r,\theta,z) = -i\omega\frac{H}{2}d\left[\frac{z_0}{\frac{H}{2}}(\frac{z-d}{d}+\frac{g}{\omega^2 d}) - d\frac{\phi_0}{\frac{H}{2}}r(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\cos\theta\right] - i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)z_{\kappa_\ell}(z) + \Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)z_{\kappa_\ell}(z)]\cos(m\theta) - i\omega\frac{H}{2}d\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$

$$z_{a_\ell}(z) + \Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)z_{a_\ell}(z)]]\cos(m\theta)$$

Και

$$\frac{\partial\phi(r,\theta,z)}{\partial r} = -i\omega\frac{H}{2}d\left[-d\frac{\phi_0}{\frac{H}{2}}(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\cos\theta\right] - i\omega\frac{H}{2}d\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\frac{\partial\Re_{m_{\kappa_\ell}}(r)}{\partial r} +$$

$$\Lambda^*_{m_{\kappa_\ell}}\frac{\partial\Re^*_{m_{\kappa_\ell}}(r)}{\partial r}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\cos(m\theta) - i\omega\frac{H}{2}d\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\frac{\partial\Re_{m_{a_\ell}}(r)}{\partial r}$$

$$+\Lambda^*_{m_{a_\ell}}\frac{\partial\Re^*_{m_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\cos(m\theta)$$

Όπου

ℓ : άνω στοιχείο του αξονοσυμμετρικού σώματος

$$\frac{\partial \Re_{m_{\kappa_\ell}}(r)}{\partial r} = \kappa_\ell \frac{J_m(\kappa_\ell \mathrm{a}_\ell)Y_{m+1}(\kappa_\ell r) - J_{m+1}(\kappa_\ell r)Y_m(\kappa_\ell \mathrm{a}_\ell)}{J_m(\kappa_\ell \mathrm{a}_{\ell+1})Y_m(\kappa_\ell \mathrm{a}_\ell) - J_m(\kappa_\ell \mathrm{a}_\ell)Y_m(\kappa_\ell \mathrm{a}_{\ell+1})} + $$
$$+ \frac{m}{r}\frac{J_m(\kappa_\ell r)Y_m(\mathrm{a}_\ell \kappa_\ell) - J_m(\mathrm{a}_\ell \kappa_\ell)Y_m(\kappa_\ell r)}{J_m(\kappa_\ell \mathrm{a}_{\ell+1})Y_m(\kappa_\ell \mathrm{a}_\ell) - J_m(\kappa_\ell \mathrm{a}_\ell)Y_m(\kappa_\ell \mathrm{a}_{\ell+1})}$$

$$\frac{\partial \Re^*_{m_{\kappa_\ell}}(r)}{\partial r} = \kappa_\ell \frac{Y_m(\kappa_\ell \mathrm{a}_{\ell+1})J_{m+1}(\kappa_\ell r) - Y_{m+1}(\kappa_\ell r)J_m(\kappa_\ell \mathrm{a}_{\ell+1})}{J_m(\kappa_\ell \mathrm{a}_{\ell+1})Y_m(\kappa_\ell \mathrm{a}_\ell) - Y_m(\kappa_\ell \mathrm{a}_{\ell+1})J_m(\kappa_\ell \mathrm{a}_\ell)} + $$
$$+ \frac{m}{r}\frac{J_m(\kappa_\ell \mathrm{a}_{\ell+1})Y_m(\kappa_\ell r) - J_m(\kappa_\ell r)Y_m(\kappa_\ell \mathrm{a}_{\ell+1})}{J_m(\kappa_\ell \mathrm{a}_{\ell+1})Y_m(\kappa_\ell \mathrm{a}_\ell) - J_m(\kappa_\ell \mathrm{a}_\ell)Y_m(\kappa_\ell \mathrm{a}_{\ell+1})}$$

$$\frac{\partial \Re_{m_{a_\ell}}(r)}{\partial r} = a_\ell \frac{K_m(a_\ell \mathrm{a}_\ell)I_{m+1}(a_\ell r) + K_{m+1}(a_\ell r)I_m(a_\ell \mathrm{a}_\ell)}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(\alpha_\ell \mathrm{a}_{\ell+1})} + $$
$$+ \frac{m}{r}\frac{I_m(a_\ell r)K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell r)}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}$$

$$\frac{\partial \Re^*_{m_{a_\ell}}(r)}{\partial r} = -a_\ell \frac{I_m(a_\ell \mathrm{a}_{\ell+1})K_{m+1}(a_\ell r) + K_m(a_\ell \mathrm{a}_{\ell+1})I_m(a_\ell r)}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(\kappa_\ell \mathrm{a}_{\ell+1})} + $$
$$+ \frac{m}{r}\frac{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell r) - I_m(a_\ell r)K_m(a_\ell \mathrm{a}_{\ell+1})}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}$$

Και αντίστοιχα

$$\frac{\partial \overline{\phi(r,\theta,z)}}{\partial r} = i\omega \frac{H}{2} d \left[-d \frac{\overline{\phi_0}}{\frac{H}{2}} \left(\frac{z-d}{d^2} + \frac{g}{\omega^2 d^2}\right)\cos\theta \right] + i\omega \frac{H}{2} d \sum_{m=0}^{\infty} \overline{[\Lambda_{m_{\kappa_\ell}} \frac{\partial \Re_{m_{\kappa_\ell}}(r)}{\partial r} +}$$
$$\overline{\Lambda^*_{m_{\kappa_\ell}} \frac{\partial \Re^*_{m_{\kappa_\ell}}(r)}{\partial r}}] N_{\kappa_\ell}^{-1/2} \cosh(\kappa_\ell (z-h_\ell)) \cos(m\theta) + i\omega \frac{H}{2} d \sum_{m=0}^{\infty} [\sum_{\substack{\ell=1 \\ a_\ell}}^{\infty} \overline{[\Lambda_{m_{a_\ell}} \frac{\partial \Re_{m_{a_\ell}}(r)}{\partial r}}$$

$$\overline{+\Lambda^*_{m_{a_\ell}} \frac{\partial \Re^*_{m_{a_\ell}}(r)}{\partial r}}] N_{a_\ell}^{-1/2} \cos(a_\ell(z-h_\ell))] \cos(m\theta)$$

Άρα

$$\overline{\left(\frac{\partial \phi(r,\theta,z)}{\partial r}\right)^2}^T = \frac{1}{2}\frac{\partial \phi(r,\theta,z)}{\partial r}\frac{\partial \overline{\phi(r,\theta,z)}}{\partial r} =$$

$$= \frac{1}{2}\frac{\omega^2 H^2 d^2}{4}[\, d^2 \frac{\phi_0}{H/2}\frac{\overline{\phi_0}}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})^2(\cos\theta)^2] - \frac{1}{2}\frac{\omega^2 H^2 d^2}{4}$$

$$\left[d\frac{\phi_0}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\cos\theta\right]\sum_{m=0}^{\infty}[\overline{\Lambda_{m_{\kappa_\ell}}\frac{\partial \Re_{m_{\kappa_\ell}}(r)}{\partial r}} + \overline{\Lambda^*_{m_{\kappa_\ell}}\frac{\partial \Re^*_{m_{\kappa_\ell}}(r)}{\partial r}}] N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))$$

$$\cos(\mathrm{m}\theta)\,] - \frac{1}{2}\frac{\omega^2 H^2 d^2}{4}\left[d\frac{\phi_0}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\cos\theta\right]\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\overline{\Lambda_{m_{a_\ell}}\frac{\partial \Re_{m_{a_\ell}}(r)}{\partial r}}$$

$$\overline{+\Lambda^*_{m_{a_\ell}}\frac{\partial \Re^*_{m_{a_\ell}}(r)}{\partial r}}] N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\,]\cos(m\theta)] - \frac{1}{2}\frac{\omega^2 H^2 d^2}{4}\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\frac{\partial \Re_{m_{\kappa_\ell}}(r)}{\partial r} +$$

$$\Lambda^*_{m_{\kappa_\ell}}\frac{\partial \Re^*_{m_{\kappa_\ell}}(r)}{\partial r}] N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\cos(m\theta)\left[d\frac{\overline{\phi_0}}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\cos\theta\right] - \frac{1}{2}\frac{\omega^2 H^2 d^2}{4}$$

$$\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\frac{\partial \Re_{m_{a_\ell}}(r)}{\partial r} + \Lambda^*_{m_{a_\ell}}\frac{\partial \Re^*_{m_{a_\ell}}(r)}{\partial r}] N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\cos(m\theta)$$

$$\left[d\frac{\overline{\phi_0}}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\cos\theta\right] + \frac{1}{2}\frac{\omega^2 H^2 d^2}{4}[\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\frac{\partial \Re_{m_{\kappa_\ell}}(r)}{\partial r} +$$

$$\Lambda^*_{m_{\kappa_\ell}}\frac{\partial \Re^*_{m_{\kappa_\ell}}(r)}{\partial r}] N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\cos(m\theta) + \sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\frac{\partial \Re_{m_{a_\ell}}(r)}{\partial r}$$

$$+\Lambda^*_{m_{a_\ell}}\frac{\partial \Re^*_{m_{a_\ell}}(r)}{\partial r}] N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\cos(m\theta)][\sum_{n=0}^{\infty}[\overline{\Lambda_{n_{\kappa_\ell}}\frac{\partial \Re_{n_{\kappa_\ell}}(r)}{\partial r}} +$$

$$\overline{\Lambda^*_{n_{\kappa_\ell}}\frac{\partial \Re^*_{n_{\kappa_\ell}}(r)}{\partial r}}] N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\cos(n\theta) + \sum_{n=0}^{\infty}[\sum_{\substack{n=1\\ a_n}}^{\infty}[\overline{\Lambda_{n_{a_n}}\frac{\partial \Re_{n_{a_n}}(r)}{\partial r}}$$

$$\overline{+\Lambda^*_{n_{a_n}}\frac{\partial \Re^*_{n_{a_n}}(r)}{\partial r}}] N_{a_n}^{-1/2}\cos(a_n(z-h_\ell))\,]\cos(n\theta)]$$

Και $\int_{d_2}^{d_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial r}\right)^2}^T\cos\theta d\theta\right]\mathrm{a}_\ell dz$.

Πρώτα υπολογίζουμε το $\int_0^{2\pi}\left(\frac{\partial\phi}{\partial r}\right)^2\cos\theta d\theta$ οπότε

$$\int_0^{2\pi}\left(\frac{\partial\phi}{\partial r}\right)^2\cos\theta d\theta =$$

$$=-\frac{\omega^2H^2d^2}{8}\left[d\frac{\phi_0}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\right][[\Lambda_{2_{\kappa_\ell}}\overline{\frac{\partial\Re_{2_{\kappa_\ell}}(r)}{\partial r}}+\Lambda^*_{2_{\kappa_\ell}}\overline{\frac{\partial\Re^*_{2_{\kappa_\ell}}(r)}{\partial r}}]N_{\kappa_\ell}^{-1/2}$$

$$\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\overline{\frac{\partial\Re_{2_{a_\ell}}(r)}{\partial r}}+\Lambda^*_{2_{a_\ell}}\overline{\frac{\partial\Re^*_{2_{a_\ell}}(r)}{\partial r}}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]\frac{\pi}{2}]-$$

$$-\frac{\omega^2H^2d^2}{8}\left[d\frac{\overline{\phi_0}}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\right][\Lambda_{2_{\kappa_\ell}}\frac{\partial\Re_{2_{\kappa_\ell}}(r)}{\partial r}+\Lambda^*_{2_{\kappa_\ell}}\frac{\partial\Re^*_{2_{\kappa_\ell}}(r)}{\partial r}]N_{\kappa_\ell}^{-1/2}$$

$$\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\frac{\partial\Re_{2_{a_\ell}}(r)}{\partial r}+\Lambda^*_{2_{a_\ell}}\frac{\partial\Re^*_{2_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]\frac{\pi}{2}]+\frac{\omega^2H^2d^2}{8}$$

$$[[[\Lambda_{0_{\kappa_\ell}}\frac{\partial\Re_{0_{\kappa_\ell}}(r)}{\partial r}+\Lambda^*_{0_{\kappa_\ell}}\frac{\partial\Re^*_{0_{\kappa_\ell}}(r)}{\partial r}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{0_{a_\ell}}\frac{\partial\Re_{0_{a_\ell}}(r)}{\partial r}+$$

$$\Lambda^*_{0_{a_\ell}}\frac{\partial\Re^*_{0_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))][[\Lambda_{1_{\kappa_\ell}}\overline{\frac{\partial\Re_{1_{\kappa_\ell}}(r)}{\partial r}}+\Lambda^*_{1_{\kappa_\ell}}\overline{\frac{\partial\Re^*_{1_{\kappa_\ell}}(r)}{\partial r}}]N_{\kappa_\ell}^{-1/2}$$

$$\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\overline{\frac{\partial\Re_{1_{a_\ell}}(r)}{\partial r}}+\Lambda^*_{1_{a_\ell}}\overline{\frac{\partial\Re^*_{1_{a_\ell}}(r)}{\partial r}}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]\pi+$$

$$[\Lambda_{1_{\kappa_\ell}}\frac{\partial\Re_{1_{\kappa_\ell}}(r)}{\partial r}+\Lambda^*_{1_{\kappa_\ell}}\frac{\partial\Re^*_{1_{\kappa_\ell}}(r)}{\partial r}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\frac{\partial\Re_{1_{a_\ell}}(r)}{\partial r}+$$

$$\Lambda^*_{1_{a_\ell}}\frac{\partial\Re^*_{1_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))][[\Lambda_{0_{\kappa_\ell}}\overline{\frac{\partial\Re_{0_{\kappa_\ell}}(r)}{\partial r}}+\Lambda^*_{0_{\kappa_\ell}}\overline{\frac{\partial\Re^*_{0_{\kappa_\ell}}(r)}{\partial r}}]N_{\kappa_\ell}^{-1/2}$$

$$\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{0_{a_\ell}}\frac{\partial\Re_{0_{a_\ell}}(r)}{\partial r}}+\overline{\Lambda^*_{0_{a_\ell}}\frac{\partial\Re^*_{0_{a_\ell}}(r)}{\partial r}}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]\pi+$$

$$[\Lambda_{1_{\kappa_\ell}}\frac{\partial\Re_{1_{\kappa_\ell}}(r)}{\partial r}+\Lambda^*_{1_{\kappa_\ell}}\frac{\partial\Re^*_{1_{\kappa_\ell}}(r)}{\partial r}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\frac{\partial\Re_{1_{a_\ell}}(r)}{\partial r}+$$

$$\Lambda^*_{1_{a_\ell}}\frac{\partial\Re^*_{1_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]][[\overline{\Lambda_{2_{\kappa_\ell}}\frac{\partial\Re_{2_{\kappa_\ell}}(r)}{\partial r}}+\overline{\Lambda^*_{2_{\kappa_\ell}}\frac{\partial\Re^*_{2_{\kappa_\ell}}(r)}{\partial r}}]N_{\kappa_\ell}^{-1/2}$$

$$\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{2_{a_\ell}}\frac{\partial\Re_{2_{a_\ell}}(r)}{\partial r}}+\overline{\Lambda^*_{2_{a_\ell}}\frac{\partial\Re^*_{2_{a_\ell}}(r)}{\partial r}}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\,[\Lambda_{p_{\kappa_\ell}}\frac{\partial\Re_{p_{\kappa_\ell}}(r)}{\partial r}+\Lambda^*_{p_{\kappa_\ell}}\frac{\partial\Re^*_{p_{\kappa_\ell}}(r)}{\partial r}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\frac{\partial\Re_{p_{a_\ell}}(r)}{\partial r}+$$

$$\Lambda^*_{p_{a_\ell}}\frac{\partial\Re^*_{p_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]][[[\overline{\Lambda_{p+1_{\kappa_\ell}}\frac{\partial\Re_{p+1_{\kappa_\ell}}(r)}{\partial r}}+\overline{\Lambda^*_{p+1_{\kappa_\ell}}\frac{\partial\Re^*_{p+1_{\kappa_\ell}}(r)}{\partial r}}]N_{\kappa_\ell}^{-1/2}$$

$$\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{p+1_{a_\ell}}\frac{\partial\Re_{p+1_{a_\ell}}(r)}{\partial r}}+\overline{\Lambda^*_{p+1_{a_\ell}}\frac{\partial\Re^*_{p+1_{a_\ell}}(r)}{\partial r}}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]+$$

$$\overline{[\Lambda_{p-1_{\kappa_\ell}}\frac{\partial\Re_{p-1_{\kappa_\ell}}(r)}{\partial r}}+\overline{\Lambda^*_{p-1_{\kappa_\ell}}\frac{\partial\Re^*_{p-1_{\kappa_\ell}}(r)}{\partial r}}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\overline{[\Lambda_{p-1_{a_\ell}}\frac{\partial\Re_{p-1_{a_\ell}}(r)}{\partial r}}+$$

$$\overline{\Lambda^*_{p-1_{a_\ell}}\frac{\partial\Re^*_{p-1_{a_\ell}}(r)}{\partial r}}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]]\frac{\pi}{2}]$$

Στη συνέχεια υπολογίζουμε το $\int_{d_2}^{d_1}\left[\overline{\left(\frac{\partial\Phi}{\partial r}\right)^2}^{T}\right]ndS$. (*Παράρτημα Α*)

Για ευκολία στον υπολογισμό συμψηφίσαμε στο άθροισμα και τη μια φανταστική ρίζα και τις άπειρες πραγματικές ρίζες της εξίσωσης της διασποράς.

Επομένως

$$\int_{d_2}^{d_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial r}\right)^2}^T \cos\theta d\theta\right]\mathrm{a}_\ell dz =$$

$$=-\frac{\omega^2 H^2 d^2}{8}\left[\frac{\phi_0}{H/2}\frac{1}{d}\left[\left[\sum_{\substack{\ell=0\\ a_\ell}}^{\infty}\left[\Lambda_{2_{a_\ell}}\overline{\frac{\partial\Re_{2_{a_\ell}}(r)}{\partial r}}+\overline{\Lambda^*_{2_{a_\ell}}\frac{\partial\Re^*_{2_{a_\ell}}(r)}{\partial r}}\right]N_{a_\ell}^{-1/2}\left[\frac{1}{a^2}[\cos(a(-d_1+h))\right.\right.\right.$$

$$-\cos(a(-d_2+h))-ad_1\sin(a(-d_1+h))+$$

$$+ad_2\sin(a(-d_2+h))]]\frac{\pi}{2}+\left[d\frac{\phi_0}{H/2}(\frac{-1}{d}+\frac{g}{\omega^2 d^2})\right][[\sum_{\substack{\ell=0\\ a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\overline{\frac{\partial\Re_{2_{a_\ell}}(r)}{\partial r}}+$$

$$\overline{\Lambda^*_{2_{a_\ell}}\frac{\partial\Re^*_{2_{a_\ell}}(r)}{\partial r}}]N_{a_\ell}^{-1/2}\left[\frac{-\sin(a(-d_1+h))}{a}+\frac{\sin(a(-d_2+h))}{a}\right]\frac{\pi}{2}]\mathrm{a}_\ell-$$

$$-\frac{\omega^2 H^2 d^2}{8}\left[\frac{\overline{\phi_0}}{H/2}\frac{1}{d}[[\sum_{\substack{\ell=0\\ a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\frac{\partial\Re_{2_{a_\ell}}(r)}{\partial r}+\Lambda^*_{2_{a_\ell}}\frac{\partial\Re^*_{2_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}[\frac{1}{a^2}[\cos(a(-d_1+h))\right.$$

$$-\cos(a(-d_2+h))-ad_1\sin(a(-d_1+h))++ad_2\sin(a(-d_2+h))]]\frac{\pi}{2}+$$

$$\left[d\frac{\overline{\phi_0}}{H/2}(\frac{-1}{d}+\frac{g}{\omega^2 d^2})\right][[\sum_{\substack{\ell=0\\ a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\frac{\partial\Re_{2_{a_\ell}}(r)}{\partial r}+\Lambda^*_{2_{a_\ell}}\frac{\partial\Re^*_{2_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\left[\frac{-\sin(a(-d_1+h))+}{a}\right.$$

$$\left.\frac{+\sin(a(-d_2+h))}{a}\right]\frac{\pi}{2}]\mathrm{a}_\ell+$$

$$+\frac{\omega^2 H^2 d^2}{8}[[[\sum_{\substack{\ell=0\\ a_\ell\\ a_\ell\neq a_n}}^{\infty}\sum_{\substack{n=0\\ a_n}}^{\infty}[\Lambda_{1_{a_\ell}}\frac{\partial\Re_{1_{a_\ell}}(r)}{\partial r}+\Lambda^*_{1_{a_\ell}}\frac{\partial\Re^*_{1_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}[\Lambda_{0_{a_n}}\overline{\frac{\partial\Re_{0_{a_n}}(r)}{\partial r}}+$$

$$\overline{\Lambda^*_{0_{a_n}}\frac{\partial\Re^*_{0_{a_n}}(r)}{\partial r}}]\ N_{a_n}^{-1/2}[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]+[\sum_{\substack{\ell=0\\ a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\frac{\partial\Re_{1_{a_\ell}}(r)}{\partial r}+$$

$$\Lambda^*_{1_{a_\ell}} \frac{\partial \Re^*_{1_{a_\ell}}(r)}{\partial r}] N_{a_\ell}^{-1/2} \overline{[\Lambda_{0_{a_\ell}} \frac{\partial \Re_{0_{a_\ell}}(r)}{\partial r}} + \overline{\Lambda^*_{0_{a_\ell}} \frac{\partial \Re^*_{0_{a_\ell}}(r)}{\partial r}}]$$

$$[\frac{2a_\ell(d_1-d_2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}]\pi +$$

$$+[\sum_{\substack{\ell=0\\a_\ell}}^{\infty} \overline{[\Lambda_{1_{a_\ell}} \frac{\partial \Re_{1_{a_\ell}}(r)}{\partial r}} + \overline{\Lambda^*_{1_{a_\ell}} \frac{\partial \Re^*_{1_{a_\ell}}(r)}{\partial r}}] N_{a_\ell}^{-1/2} [\Lambda_{0_{a_\ell}} \frac{\partial \Re_{0_{a_\ell}}(r)}{\partial r} + \Lambda^*_{0_{a_\ell}} \frac{\partial \Re^*_{0_{a_\ell}}(r)}{\partial r}]$$

$$N_{a_\ell}^{-1/2}\left[\frac{2a_\ell(d_1-d_2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}\right] + [\sum_{\substack{\ell=0\\a_\ell\\a_\ell\neq a_n}}^{\infty} \sum_{\substack{n=0\\a_n}}^{\infty} \overline{[\Lambda_{1_{a_\ell}} \frac{\partial \Re_{1_{a_\ell}}(r)}{\partial r}} +$$

$$\overline{\Lambda^*_{1_{a_\ell}} \frac{\partial \Re^*_{1_{a_\ell}}(r)}{\partial r}}] N_{a_\ell}^{-1/2} [\Lambda_{0_{a_n}} \frac{\partial \Re_{0_{a_n}}(r)}{\partial r} + \Lambda^*_{0_{a_n}} \frac{\partial \Re^*_{0_{a_n}}(r)}{\partial r}] N_{a_n}^{-1/2}$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]\,]\pi + +[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}} \frac{\partial \Re_{1_{a_\ell}}(r)}{\partial r} +$$

$$\Lambda^*_{1_{a_\ell}} \frac{\partial \Re^*_{1_{a_\ell}}(r)}{\partial r}] N_{a_\ell}^{-1/2} \overline{[\Lambda_{2_{a_\ell}} \frac{\partial \Re_{2_{a_\ell}}(r)}{\partial r}} + \overline{\Lambda^*_{2_{a_\ell}} \frac{\partial \Re^*_{2_{a_\ell}}(r)}{\partial r}}]$$

$$N_{a_\ell}^{-1}\left[\frac{2a_\ell(d_1-d_2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}\right] + [\sum_{\substack{\ell=0\\a_\ell\\a_\ell\neq a_n}}^{\infty} \sum_{\substack{n=0\\a_n}}^{\infty}[\Lambda_{1_{a_\ell}} \frac{\partial \Re_{1_{a_\ell}}(r)}{\partial r} +$$

$$\Lambda^*_{1_{a_\ell}} \frac{\partial \Re^*_{1_{a_\ell}}(r)}{\partial r}] N_{a_\ell}^{-1/2} \overline{[\Lambda_{2_{a_n}} \frac{\partial \Re_{2_{a_n}}(r)}{\partial r}} + \overline{\Lambda^*_{2_{a_n}} \frac{\partial \Re^*_{2_{a_n}}(r)}{\partial r}}] N_{a_n}^{-1/2}$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2} +$$

$$\sum_{p=2,3}^{\infty}[\sum_{\substack{\ell=0\\ a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\frac{\partial\Re_{p_{a_\ell}}(r)}{\partial r}+\Lambda^*_{p_{a_\ell}}\frac{\partial\Re^*_{p_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\,\overline{[\Lambda_{p+1_{a_\ell}}\frac{\partial\Re_{p+1_{a_\ell}}(r)}{\partial r}}+\overline{\Lambda^*_{p+1_{a_\ell}}\frac{\partial\Re^*_{p+1_{a_\ell}}(r)}{\partial r}}]$$

$$N_{a_\ell}^{-1/2}\left[\frac{2a_\ell(d_1-d_2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}\right]+\sum_{\substack{n=0\\ a_n}}^{\infty}[\sum_{\substack{\ell=0\\ a_\ell\\ a_n\neq a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\frac{\partial\Re_{p_{a_\ell}}(r)}{\partial r}+$$

$$\Lambda^*_{p_{a_\ell}}\frac{\partial\Re^*_{p_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\,\overline{[\Lambda_{p+1_{a_n}}\frac{\partial\Re_{p+1_{a_n}}(r)}{\partial r}}+\overline{\Lambda^*_{p+1_{a_n}}\frac{\partial\Re^*_{p+1_{a_n}}(r)}{\partial r}}]N_{a_n}^{-1/2}$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}+[\sum_{p=2,3}^{\infty}\sum_{\substack{\ell=0\\ a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\frac{\partial\Re_{p_{a_\ell}}(r)}{\partial r}+$$

$$\Lambda^*_{p_{a_\ell}}\frac{\partial\Re^*_{p_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\,\overline{[\Lambda_{p-1_{a_\ell}}\frac{\partial\Re_{p-1_{a_\ell}}(r)}{\partial r}}+\overline{\Lambda^*_{p-1_{a_\ell}}\frac{\partial\Re^*_{p-1_{a_\ell}}(r)}{\partial r}}]$$

$$N_{a_\ell}^{-1/2}\left[\frac{2a_\ell(d_1-d_2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}\right]+[\sum_{\substack{n=0\\ a_n}}^{\infty}\sum_{\substack{\ell=0\\ a_\ell\\ a_n\neq a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\frac{\partial\Re_{p_{a_\ell}}(r)}{\partial r}+$$

$$\Lambda^*_{p_{a_\ell}}\frac{\partial\Re^*_{p_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\,\overline{[\Lambda_{p-1_{a_n}}\frac{\partial\Re_{p-1_{a_n}}(r)}{\partial r}}+\overline{\Lambda^*_{p-1_{a_n}}\frac{\partial\Re^*_{p-1_{a_n}}(r)}{\partial r}}]N_{a_n}^{-1/2}$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}]]\mathrm{a}_\ell.$$

Όπου a_ℓ η ακτίνα του $\ell-$στου «από πάνω» στοιχείου.

Στη συνέχεια υπολογίζουμε το

$$\frac{1}{r}\frac{\partial\phi(r,\theta,z)}{\partial\theta}=\frac{1}{r}\,(-i\omega\frac{H}{2}d)\left[d\,\frac{\phi_0}{\frac{H}{2}}r(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\sin\theta\right]+i\omega\frac{H}{2}d\,[\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)]mN_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\ \sin(m\theta)+\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$

$$+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)]mN_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\ \sin(m\theta)].$$

Και όμοια

$$\frac{1}{r}\frac{\partial\overline{\phi(r,\theta,z)}}{\partial\theta}=\frac{1}{r}\,(i\omega\frac{H}{2}d)\left[d\,\frac{\overline{\phi_0}}{\frac{H}{2}}r(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\sin\theta\right]-i\omega\frac{H}{2}d\,\sum_{m=0}^{\infty}[\overline{\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+}$$

$$\overline{\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)}]mN_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\ \sin(m\theta)+\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)}]mN_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\ \sin(m\theta)].$$

Επομένως

$$\frac{1}{r^2}\left(\frac{\partial\phi(r,\theta,z)}{\partial\theta}\right)^2=\frac{1}{r^2}\frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\frac{\overline{\partial\phi(r,\theta,z)}}{\partial\theta}=$$

$$=\frac{1}{2r^2}\frac{\omega^2H^2d^2}{4}[\ d^2\,\frac{\phi_0}{H/2}\frac{\overline{\phi_0}}{H/2}r^2(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})^2(\sin\theta)^2]-\frac{1}{2r^2}\frac{\omega^2H^2d^2}{4}$$

$$\left[d\,\frac{\phi_0}{H/2}r(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\sin\theta\right][\sum_{m=0}^{\infty}[\overline{\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)}]mN_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))$$

$$\sin(m\theta)+\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)}\ \overline{+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)}]mN_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\ \sin(m\theta)]]-$$

$$-\frac{1}{2r^2}\frac{\omega^2H^2d^2}{4}\left[d\frac{\overline{\phi_0}}{H/2}r(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\sin\theta\right][\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)]mN_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\sin(m\theta)+\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$

$$+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)]mN_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\sin(m\theta)]]$$

$$+\frac{1}{2r^2}\frac{\omega^2H^2d^2}{4}[\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)]mN_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\sin(m\theta)+$$

$$\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)]mN_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\sin(m\theta)]$$

$$[\sum_{n=0}^{\infty}[\overline{\Lambda_{n_{\kappa_\ell}}\Re_{n_{\kappa_\ell}}(r)+}\,\overline{\Lambda^*_{n_{\kappa_\ell}}\Re^*_{n_{\kappa_\ell}}(r)}]nN_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\sin(n\theta)$$

$$+\sum_{n=0}^{\infty}[\sum_{\substack{n=1\\a_n}}^{\infty}[\overline{\Lambda_{n_{a_n}}\Re_{n_{a_n}}(r)}\,\overline{+\Lambda^*_{n_{a_n}}\Re^*_{n_{a_n}}(r)}]nN_{a_n}^{-1/2}\cos(a_n(z-h_\ell))\ \sin(n\theta)]]].$$

Όμοια υπολογίζουμε

$$\int_0^{2\pi}\left(\frac{\partial\phi}{\partial\theta}\right)^2\cos\theta d\theta=$$

$$=-\frac{1}{r^2}\frac{\omega^2H^2d^2}{8}\left[d\frac{\phi_0}{H/2}r(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\right][[\overline{\Lambda_{2_{\kappa_\ell}}\Re_{2_{\kappa_\ell}}(r)+}\,\overline{\Lambda^*_{2_{\kappa_\ell}}\Re^*_{2_{\kappa_\ell}}(r)}]2N_{\kappa_\ell}^{-1/2}$$

$$\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)}\,\overline{+\Lambda^*_{2_{a_\ell}}\Re^*_{2_{a_\ell}}(r)}]2N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\,]]\frac{\pi}{2}-$$

$$-\frac{1}{r^2}\frac{\omega^2H^2d^2}{8}\left[d\frac{\overline{\phi_0}}{H/2}r(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\right][[\Lambda_{2_{\kappa_\ell}}\Re_{2_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{2_{\kappa_\ell}}\Re^*_{2_{\kappa_\ell}}(r)]2N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)$$

$$+\Lambda^*{}_{2_{a_\ell}}\Re^*{}_{2_{a_\ell}}(r)]2N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\,]]\frac{\pi}{2}+\frac{1}{r^2}\frac{\omega^2H^2d^2}{8}[[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+$$

$$\Lambda^*{}_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]1N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)+\Lambda^*{}_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)]1N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\,]$$

$$[[\overline{\Lambda_{2_{\kappa_\ell}}\Re_{2_{\kappa_\ell}}(r)}+\overline{\Lambda^*{}_{2_{\kappa_\ell}}\Re^*_{2_{\kappa_\ell}}(r)}]2N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\overline{\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*{}_{2_{a_\ell}}\Re^*{}_{2_{a_\ell}}(r)}]2N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\,]]\frac{\pi}{2}+$$

$$[\sum_{p=2,3,}^{\infty}[\Lambda_{p_{\kappa_\ell}}\Re_{p_{\kappa_\ell}}(r)+\Lambda^*{}_{p_{\kappa_\ell}}\Re^*_{p_{\kappa_\ell}}(r)]pN_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\overline{\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)}$$

$$+\Lambda^*{}_{p_{a_\ell}}\Re^*{}_{p_{a_\ell}}(r)]pN_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\,]]$$

$$[[\overline{\Lambda_{p+1_{\kappa_\ell}}\Re_{p+1_{\kappa_\ell}}(r)}+\overline{\Lambda^*{}_{p+1_{\kappa_\ell}}\Re^*_{p+1_{\kappa_\ell}}(r)}](p+1)N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\overline{\Lambda_{p+1_{a_\ell}}\Re_{p+1_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*{}_{p+1_{a_\ell}}\Re^*{}_{p+1_{a_\ell}}(r)}](p+1)N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\,]+[[\overline{\Lambda_{p-1_{\kappa_\ell}}\Re_{p-1_{\kappa_\ell}}(r)+}$$

$$\overline{\Lambda^*{}_{p-1_{\kappa_\ell}}\Re^*_{p-1_{\kappa_\ell}}(r)}](p-1)N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\overline{\Lambda_{p-1_{a_\ell}}\Re_{p-1_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*{}_{p-1_{a_\ell}}\Re^*{}_{p-1_{a_\ell}}(r)}](p-1)N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\,]]\frac{\pi}{2}]$$

Στη συνέχεια υπολογίζουμε το $\int_{d_2}^{d_1}\left[\overline{\left(\frac{\partial\Phi}{\partial\theta}\right)^2}\right]^T ndS$, συμψηφίζοντας στο άθροισμα και την μια φανταστική ρίζα και τις άπειρες ρίζες της διασποράς.

Επίσης r είναι η ακτίνα του $\ell-$στου «από πάνω» στοιχείου. Δηλαδή

$$r=\mathrm{a}_\ell$$

Από το *Παράρτημα Α* προκύπτει

$$\int_{d_2}^{d_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial\theta}\right)^2}^T\cos\theta d\theta\right]\mathrm{a}_\ell dz=$$

$$=-\frac{1}{r^2}\frac{\omega^2H^2d^2}{8}\left[\frac{\phi_0}{H/2}r\frac{1}{d}\right][\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\overline{\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)}\ \overline{+\Lambda^*{}_{2_{a_\ell}}\Re^*{}_{2_{a_\ell}}(r)}]2N_{a_\ell}^{-1/2}[\frac{1}{a_\ell{}^2}[\cos(a_\ell(-d_1+h)$$

$$-\cos(a_\ell(-d_2+h))-a_\ell d_1\sin(a_\ell(-d_1+h))+a_\ell d_2\sin(a_\ell(-d_2+h))]]\frac{\pi}{2}+$$

$$\left[d\frac{\phi_0}{H/2}r(\frac{-1}{d}+\frac{g}{\omega^2d^2})\right][\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\overline{\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)}\ \overline{+\Lambda^*{}_{2_{a_\ell}}\Re^*{}_{2_{a_\ell}}(r)}]2N_{a_\ell}^{-1/2}\left[\frac{-\sin(a_\ell(-d_1+h))+}{a_\ell}\right.$$

$$\left.\frac{+\sinh(a_\ell(-d_2+h))}{a_\ell}\right]\quad]\frac{\pi}{2}]\mathrm{a}_\ell-$$

$$-\frac{1}{r^2}\frac{\omega^2H^2d^2}{8}\left[\frac{\overline{\phi_0}}{H/2}r\frac{1}{d}\right][\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)+\Lambda^*{}_{2_{a_\ell}}\Re^*{}_{2_{a_\ell}}(r)]2N_{a_\ell}^{-1/2}[\frac{1}{a_\ell{}^2}[\cos(a_\ell(-d_1+h)$$

$$-\cos(a_\ell(-d_2+h))-a_\ell d_1\sin(a_\ell(-d_1+h))+a_\ell d_2\sin(a_\ell(-d_2+h))]]\frac{\pi}{2}+$$

$$\left[d\frac{\overline{\phi_0}}{H/2}r(\frac{-1}{d}+\frac{g}{\omega^2d^2})\right][\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)+\Lambda^*{}_{2_{a_\ell}}\Re^*{}_{2_{a_\ell}}(r)]2N_{a_\ell}^{-1/2}\left[\frac{-\sin(a_\ell(-d_1+h))+}{a_\ell}\right.$$

$$\left.\frac{+\sinh(a_\ell(-d_2+h))}{a_\ell}\right]]\frac{\pi}{2}]\quad\mathrm{a}_\ell+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2d^2}{8}[[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\overline{\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)}+\Lambda^*{}_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\ [\overline{\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)}\ \overline{+\Lambda^*{}_{2_{a_\ell}}\Re^*{}_{2_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}$$

$$\left[\frac{2a_\ell(d_1-d_2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}\right][\sum_{\substack{\ell=0\\a_\ell}}^{\infty}\sum_{\substack{n=0\\a_n}}^{\infty}\ [\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)$$

$$+\Lambda^*{}_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\ [\overline{\Lambda_{2_{a_n}}\Re_{2_{a_n}}(r)}\ \overline{+\Lambda^*{}_{2_{a_n}}\Re^*{}_{2_{a_n}}(r)}]N_{a_n}^{-1/2}$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}+$$

$$+[\sum_{p=2,3}^{\infty}\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)+\Lambda^*{}_{p_{a_\ell}}\Re^*{}_{p_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{p+1_{a_\ell}}\Re_{p+1_{a_\ell}}(r)}\,\overline{+\Lambda^*{}_{p+1_{a_\ell}}\Re^*{}_{p+1_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}$$

$$p(p+1)\left[\frac{2a_\ell(d_1-d_2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}\right]+[\sum_{\substack{n=0\\a_n}}^{\infty}\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)$$

$$+\Lambda^*{}_{p_{a_\ell}}\Re^*{}_{p_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{p+1_{a_n}}\Re_{p+1_{a_n}}(r)}\,\overline{+\Lambda^*{}_{p+1_{a_n}}\Re^*{}_{p+1_{a_n}}(r)}]N_{a_n}^{-1/2}p(p+1)$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}+$$

$$+[\sum_{p=2,3}^{\infty}[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)+\Lambda^*{}_{p_{a_\ell}}\Re^*{}_{p_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\,[\Lambda_{p-1_{a_\ell}}\Re_{p-1_{a_\ell}}(r)+\Lambda^*{}_{p-1_{a_\ell}}\Re^*{}_{p-1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}$$

$$p(p-1)\left[\frac{2a_\ell(d1-d2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}\right]+[\sum_{\substack{n=0\\a_n}}^{\infty}\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)$$

$$+\Lambda^*{}_{p_{a_\ell}}\Re^*{}_{p_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{p-1_{a_n}}\Re_{p-1_{a_n}}(r)}\,\overline{+\Lambda^*{}_{p-1_{a_n}}\Re^*{}_{p-1_{a_n}}(r)}]N_{a_n}^{-1/2}p(p-1)$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}\;]\;\;\mathrm{a}_\ell\,.$$

Όπου a_ℓ η ακτίνα του $\ell-$στου «από πάνω» στοιχείου.

Τέλος υπολογίζουμε το

$$\frac{\partial\phi(r,\theta,z)}{\partial z}=(-i\omega\frac{H}{2}d)\left[\frac{z_0}{H/2}\frac{1}{d}-d\frac{\phi_0}{H/2}r(\frac{1}{d^2})\cos\theta\right]-i\omega\frac{H}{2}d\,[\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+$$

$$\Lambda^*{}_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell\;\cos(m\theta)\text{ - }\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$

$$+\Lambda^*{}_{m_{a_\ell}}\Re^*{}_{m_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell\;\cos(m\theta)]].$$

Και

$$\frac{\partial\overline{\phi(r,\theta,z)}}{\partial z}=(i\omega\frac{H}{2}d)\left[\frac{z_0}{H/2}\frac{1}{d}-d\frac{\overline{\phi_0}}{H/2}r(\frac{1}{d^2})\cos\theta\right]+i\omega\frac{H}{2}d\,[\sum_{m=0}^{\infty}[\overline{\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+}$$

$$\overline{\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell\;\cos(m\theta)\;-\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell\;\cos(m\theta)]]$$

$$\left(\frac{\partial\phi(r,\theta,z)}{\partial z}\right)^2=\frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial z}\frac{\overline{\partial\phi(r,\theta,z)}}{\partial z}=$$

$$=\frac{1}{2}\frac{\omega^2H^2d^2}{4}\left[\frac{z_0}{H/2}\frac{1}{d}-\frac{\phi_0}{H/2}r(\frac{1}{d})\cos\theta\right]\left[\frac{z_0}{H/2}\frac{1}{d}-\frac{\overline{\phi_0}}{H/2}r(\frac{1}{d})\cos\theta\right]+\frac{1}{2}\frac{\omega^2H^2d^2}{4}$$

$$\left[\frac{z_0}{H/2}\frac{1}{d}-\frac{\phi_0}{H/2}r(\frac{1}{d})\cos\theta\right][\sum_{m=0}^{\infty}[\overline{\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+}\;\overline{\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell$$

$$\cos(m\theta)-\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)}\;\overline{+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell\;\cos(m\theta)]]+$$

$$+\frac{1}{2}\frac{\omega^2H^2d^2}{4}\left[\frac{z_0}{H/2}\frac{1}{d}-\frac{\overline{\phi_0}}{H/2}r(\frac{1}{d})\cos\theta\right][\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell\;\cos(m\theta)-\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$

$$+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell\;\cos(m\theta)]]+\frac{1}{2}\frac{\omega^2H^2d^2}{4}[\sum_{m=0}^{\infty}[\overline{\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+}$$

$$\overline{\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell\;\cos(m\theta)-\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell\;\cos(m\theta)]]\,[\sum_{n=0}^{\infty}[\Lambda_{n_{\kappa_\ell}}\Re_{n_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{n_{\kappa_\ell}}\Re^*_{n_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell\cos(n\theta)-\sum_{n=0}^{\infty}[\sum_{\substack{n=1\\a_n}}^{\infty}[\Lambda_{n_{a_n}}\Re_{n_{a_n}}(r)$$

$$+\Lambda^*_{n_{a_n}}\Re^*_{n_{a_n}}(r)]N_{a_n}^{-1/2}\sin(a_n(z-h_\ell))a_n\cos(n\theta)]]].$$

Και με τον ίδιο τρόπο υπολογίζουμε το ολοκλήρωμα

$$\int_0^{2\pi}\left(\frac{\partial\phi}{\partial z}\right)^2\cos\theta d\theta=$$

$$=\frac{\omega^2H^2d^2}{8}\left[-\frac{z_0}{H/2}\frac{1}{d}\frac{\overline{\phi_0}}{H/2}r(\frac{1}{d^2})\pi-\frac{z_0}{H/2}\frac{1}{d}\frac{\phi_0}{H/2}r(\frac{1}{d^2})\pi\right]+$$

$$+\frac{\omega^2H^2d^2}{8}[\left[\frac{z_0}{H/2}\frac{1}{d}\pi\right][[\overline{\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell$$

$$-[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)}\ \overline{+\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]]-\left[\frac{\phi_0}{H/2}r(\frac{1}{d})\frac{\pi}{2}\right][[\overline{\Lambda_{2_{\kappa_\ell}}\Re_{2_{\kappa_\ell}}(r)+}$$

$$\overline{\Lambda^*_{2_{\kappa_\ell}}\Re^*_{2_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell-[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*_{2_{a_\ell}}\Re^*_{2_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]]]+$$

$$+\frac{\omega^2H^2d^2}{8}[\left[\frac{z_0}{H/2}\frac{1}{d}\pi\right][[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+\Lambda^*_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell-$$

$$-[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)+\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]]-\left[\frac{\overline{\phi_0}}{H/2}r(\frac{1}{d})\frac{\pi}{2}\right][[\Lambda_{2_{\kappa_\ell}}\Re_{2_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{2_{\kappa_\ell}}\Re^*_{2_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell-[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)$$

$$+\Lambda^*_{2_{a_\ell}}\Re^*_{2_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]]]+$$

$$+\frac{\omega^2H^2d^2}{8}[[\Lambda_{0_{\kappa_\ell}}\Re_{0_{\kappa_\ell}}(r)+\Lambda^*_{0_{\kappa_\ell}}\Re^*_{0_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell-[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{0_{a_\ell}}\Re_{0_{a_\ell}}(r)$$

$$+\Lambda^*_{0_{a_\ell}}\Re^*_{0_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]]$$

$$[[\overline{\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*{}_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell -[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\overline{\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*{}_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]]\pi+[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+$$

$$\Lambda^*{}_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell -[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)$$

$$+\Lambda^*{}_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]][[\overline{\Lambda_{0_{\kappa_\ell}}\Re_{0_{\kappa_\ell}}(r)+}$$

$$\overline{\Lambda^*{}_{0_{\kappa_\ell}}\Re^*_{0_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell -[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\overline{\Lambda_{0_{a_\ell}}\Re_{0_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*{}_{0_{a_\ell}}\Re^*{}_{0_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]]\pi+[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+$$

$$\Lambda^*{}_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell -[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)$$

$$+\Lambda^*{}_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]][[\overline{\Lambda_{2_{\kappa_\ell}}\Re_{2_{\kappa_\ell}}(r)+}$$

$$\overline{\Lambda^*{}_{2_{\kappa_\ell}}\Re^*_{2_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell -[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\overline{\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*{}_{2_{a_\ell}}\Re^*{}_{2_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]]\frac{\pi}{2}+\sum_{p=2,3,}^{\infty}[[\Lambda_{p_{\kappa_\ell}}\Re_{p_{\kappa_\ell}}(r)+$$

$$\Lambda^*{}_{p_{\kappa_\ell}}\Re^*_{p_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell -[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)$$

$$+\Lambda^*{}_{p_{a_\ell}}\Re^*{}_{p_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]][[[\overline{\Lambda_{p+1_{\kappa_\ell}}\Re_{p+1_{\kappa_\ell}}(r)+}$$

$$\overline{\Lambda^*{}_{p+1_{\kappa_\ell}}\Re^*_{p+1_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell -[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\overline{\Lambda_{p+1_{a_\ell}}\Re_{p+1_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*{}_{p+1_{a_\ell}}\Re^*{}_{p+1_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]]+[[\overline{\Lambda_{p-1_{\kappa_\ell}}\Re_{p-1_{\kappa_\ell}}(r)+}$$

$$\overline{\Lambda^*{}_{p-1_{\kappa_\ell}}\Re^*_{p-1_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell -[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\overline{\Lambda_{p-1_{a_\ell}}\Re_{p-1_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*{}_{p-1_{a_\ell}}\Re^*{}_{p-1_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]]\frac{\pi}{2}].$$

Στη συνέχεια υπολογίζουμε το $\int_{d_2}^{d_1}\left[\overline{\left(\frac{\partial\Phi}{\partial z}\right)^2}^T\right]ndS$, συμψηφίζοντας στο άθροισμα και τη μια φανταστική ρίζα και τις άπειρες ρίζες της εξίσωσης διασποράς.

Από το *Παράρτημα Β* προκύπτει

$$\int_{d_2}^{d_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial z}\right)^2}^T\cos\theta d\theta\right]\mathrm{a}_\ell dz=$$

$$=\frac{\omega^2H^2d^2}{8}\left[-\frac{z_0}{H/2}\frac{1}{d}\frac{\overline{\phi_0}}{H/2}r(\frac{1}{d^2})\pi-\frac{z_0}{H/2}\frac{1}{d}\frac{\phi_0}{H/2}r(\frac{1}{d^2})\pi\right](\mathrm{d}_1\text{-}\mathrm{d}_2)\,\mathrm{a}_\ell+$$

$$+\frac{\omega^2H^2d^2}{8}[\left[\frac{z_0}{H/2}\frac{1}{d}\pi\right][[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\overline{\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)}\;\overline{+\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}a_\ell\left[\frac{-\cos(a_\ell(-d_1+h))+}{a_\ell}\right.$$

$$\left.\frac{+\cos(a_\ell(-d_2+h))}{a_\ell}\right]]-\left[\frac{\phi_0}{H/2}r(\frac{1}{d})\frac{\pi}{2}\right][[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\overline{\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)}\;\overline{+\Lambda^*_{2_{a_\ell}}\Re^*_{2_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}a_\ell$$

$$\left[\frac{-\cos(a_\ell(-d_1+h))}{a_\ell}+\frac{\cos(a_\ell(-d_2+h))}{a_\ell}\right]]]\,\mathrm{a}_\ell+$$

$$+\frac{\omega^2H^2d^2}{8}[\left[\frac{z_0}{H/2}\frac{1}{d}\pi\right][\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)+\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}a_\ell\left[\frac{-\cos(a_\ell(-d_1+h))}{a_\ell}+\right.$$

$$\left.\frac{\cos(a_\ell(-d_2+h))}{a_\ell}\right]]-\left[\frac{\overline{\phi_0}}{H/2}r(\frac{1}{d})\frac{\pi}{2}\right][[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)+\Lambda^*_{2_{a_\ell}}\Re^*_{2_{a_\ell}}(r)]N_{a_\ell}^{-1/2}a_\ell$$

$$\left[\frac{-\cos(a_\ell(-d_1+h))}{a_\ell}+\frac{\cos(a_\ell(-d_2+h))}{a_\ell}\right]]]]\,\mathrm{a}_\ell+$$

$$+\frac{\omega^2H^2d^2}{8}[[[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{0_{a_\ell}}\Re_{0_{a_\ell}}(r)a_\ell+\Lambda^*_{0_{a_\ell}}\Re^*_{0_{a_\ell}}(r)a_\ell]N_{a_\ell}^{-1/2}\;[\overline{\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)}\;\overline{+\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}a_\ell$$

$$\left[\frac{2a_\ell(d_1-d_2)-\sin[2a_\ell(d_1-h)]+\sin[2a_\ell(d_2-h)]}{4a_\ell}\right]+[\sum_{\substack{n=0\\a_n}}^{\infty}\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{0_{a_\ell}}\Re_{0_{a_\ell}}(r)a_\ell+$$

$$\Lambda^*_{0_{a_\ell}}\Re^*_{0_{a_\ell}}(r)a_\ell]N_{a_\ell}^{-1/2}\;[\overline{\Lambda_{1_{a_n}}\Re_{1_{a_n}}(r)}\;\overline{+\Lambda^*_{1_{a_n}}\Re^*_{1_{a_n}}(r)}]N_{a_n}^{-1/2}a_n$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell - a_n)(-d_1+h)] + \sin[(a_\ell - a_n)(-d_2+h)]}{(a_\ell - a_n)}\right] +$$

$$\frac{1}{2}\left[\frac{\sin[(a_\ell + a_n)(-d_1+h)] - \sin[(a_\ell + a_n)(-d_2+h)]}{(a_\ell + a_n)}\right]]]\pi +$$

$$+[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)a_\ell + \Lambda^*{}_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)a_\ell]N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{0_{a_\ell}}\Re_{0_{a_\ell}}(r)}\ \overline{+\Lambda^*{}_{0_{a_\ell}}\Re^*{}_{0_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}a_\ell$$

$$\left[\frac{2a_\ell(d_1-d_2) - \sin[2a_\ell(d_1-h)] + \sin[2a_\ell(d_2-h)]}{4a_\ell}\right] + [\sum_{\substack{n=0\\a_n}}^{\infty}\ \sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)a_\ell +$$

$$\Lambda^*{}_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)a_\ell]N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{0_{a_n}}\Re_{0_{a_n}}(r)}\ \overline{+\Lambda^*{}_{0_{a_n}}\Re^*{}_{0_{a_n}}(r)}]N_{a_n}^{-1/2}a_n$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell - a_n)(-d_1+h)] + \sin[(a_\ell - a_n)(-d_2+h)]}{(a_\ell - a_n)}\right] +$$

$$\frac{1}{2}\left[\frac{\sin[(a_\ell + a_n)(-d_1+h)] - \sin[(a_\ell + a_n)(-d_2+h)]}{(a_\ell + a_n)}\right]]]\pi +$$

$$+[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)a_\ell + \Lambda^*{}_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)a_\ell]N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)}\ \overline{+\Lambda^*{}_{2_{a_\ell}}\Re^*{}_{2_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}a_\ell$$

$$\left[\frac{2a_\ell(d_1-d_2) - \sin[2a_\ell(d_1-h)] + \sin[2a_\ell(d_2-h)]}{4a_\ell}\right] + [\sum_{\substack{n=0\\a_n}}^{\infty}\ \sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)a_\ell +$$

$$\Lambda^*{}_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)a_\ell]N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{2_{a_n}}\Re_{2_{a_n}}(r)}\ \overline{+\Lambda^*{}_{2_{a_n}}\Re^*{}_{2_{a_n}}(r)}]N_{a_n}^{-1/2}a_n$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell - a_n)(-d_1+h)] + \sin[(a_\ell - a_n)(-d_2+h)]}{(a_\ell - a_n)}\right] +$$

$$\frac{1}{2}\left[\frac{\sin[(a_\ell + a_n)(-d_1+h)] - \sin[(a_\ell + a_n)(-d_2+h)]}{(a_\ell + a_n)}\right]]]\frac{\pi}{2} +$$

$$+\sum_{p=2,3}^{\infty}\ [\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)a_\ell + \Lambda^*{}_{p_{a_\ell}}\Re^*_{p_{a_\ell}}(r)a_\ell]\ N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{p+1_{a_\ell}}\Re_{p+1_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*{}_{p+1_{a_\ell}}\Re^*{}_{p+1_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}a_\ell\left[\frac{2a_\ell(d_1-d_2) - \sin[2a_\ell(d_1-h)] + \sin[2a_\ell(d_2-h)]}{4a_\ell}\right] +$$

$$+\sum_{\substack{n=0\\a_n}}^{\infty}\ \sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)a_\ell + \Lambda^*{}_{p_{a_\ell}}\Re^*_{p_{a_\ell}}(r)a_\ell]\ N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{p+1_{a_n}}\Re_{p+1_{a_n}}(r)}$$

$$\overline{+\Lambda^{*}{}_{p+1_{a_n}}\Re^{*}{}_{p+1_{a_n}}(r)}]N_{a_n}^{-1/2}a_n\,[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{\sin[(a_\ell+a_n)(-d_1+h)]-\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}+$$

$$+\sum_{p=2,3}^{\infty}[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)a_\ell+\Lambda^{*}{}_{p_{a_\ell}}\Re^{*}_{p_{a_\ell}}(r)a_\ell]\;N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{p-1_{a_\ell}}\Re_{p-1_{a_\ell}}(r)}$$

$$\overline{+\Lambda^{*}{}_{p-1_{a_\ell}}\Re^{*}{}_{p-1_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}a_\ell\left[\frac{2a_\ell(d_1-d_2)-\sin[2a_\ell(d_1-h)]+\sin[2a_\ell(d_2-h)]}{4a_\ell}\right]+$$

$$+\sum_{\substack{n=0\\a_n}}^{\infty}\;\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)a_\ell+\Lambda^{*}{}_{p_{a_\ell}}\Re^{*}_{p_{a_\ell}}(r)a_\ell]\;N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{p-1_{a_n}}\Re_{p-1_{a_n}}(r)}$$

$$\overline{+\Lambda^{*}{}_{p-1_{a_n}}\Re^{*}{}_{p-1_{a_n}}(r)}]N_{a_n}^{-1/2}a_n\,[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{\sin[(a_\ell+a_n)(-d_1+h)]-\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}]]\mathrm{a}_\ell.$$

Συνοψίζοντας για την <u>Πεδίο (ΙΙ)</u> το $\int_{d_2}^{d_1}\int_0^{2\pi} \left|\nabla\Phi^{(1)}\right|^2 \overline{n} \quad dS =$

$$\int_{d_2}^{d_1}(\int_0^{2\pi}[\overline{(\frac{\partial\Phi}{\partial r})^2}^T + \frac{1}{r^2}\overline{(\frac{\partial\Phi}{\partial\theta})^2}^T + \overline{(\frac{\partial\Phi}{\partial z})^2}^T]\cos\theta \quad d\theta) \quad \mathrm{a}_\ell \quad dz =$$

$$= -\frac{\omega^2 H^2 d^2}{8}\left[\frac{\phi_0}{H/2}\frac{1}{d}[[\sum_{\substack{\ell=0 \\ a_\ell}}^{\infty}[\overline{\Lambda_{2_{a_\ell}}\frac{\partial\Re_{2_{a_\ell}}(r)}{\partial r}} + \overline{\Lambda^*_{2_{a_\ell}}\frac{\partial\Re^*_{2_{a_\ell}}(r)}{\partial r}}]N_{a_\ell}^{-1/2}[\frac{1}{a^2}[\cos(a(-d_1+h))\right.$$

$$-\cos(a(-d_2+h)) - ad_1\sin(a(-d_1+h)) +$$

$$+ad_2\sin(a(-d_2+h))]]\frac{\pi}{2} + \left[d\frac{\phi_0}{H/2}(\frac{-1}{d}+\frac{g}{\omega^2 d^2})\right][[\sum_{\substack{\ell=0 \\ a_\ell}}^{\infty}[\overline{\Lambda_{2_{a_\ell}}\frac{\partial\Re_{2_{a_\ell}}(r)}{\partial r}} +$$

$$\overline{\Lambda^*_{2_{a_\ell}}\frac{\partial\Re^*_{2_{a_\ell}}(r)}{\partial r}}]N_{a_\ell}^{-1/2}\left[\frac{-\sin(a(-d_1+h))}{a} + \frac{\sin(a(-d_2+h))}{a}\right]\frac{\pi}{2}]\mathrm{a}_\ell -$$

$$-\frac{\omega^2 H^2 d^2}{8}\left[\frac{\overline{\phi_0}}{H/2}\frac{1}{d}[[\sum_{\substack{\ell=0 \\ a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\frac{\partial\Re_{2_{a_\ell}}(r)}{\partial r} + \Lambda^*_{2_{a_\ell}}\frac{\partial\Re^*_{2_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}[\frac{1}{a^2}[\cos(a(-d_1+h))\right.$$

$$-\cos(a(-d_2+h)) - ad_1\sin(a(-d_1+h)) + + ad_2\sin(a(-d_2+h))]]\frac{\pi}{2} +$$

$$\left[d\frac{\overline{\phi_0}}{H/2}(\frac{-1}{d}+\frac{g}{\omega^2 d^2})\right][[\sum_{\substack{\ell=0 \\ a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\frac{\partial\Re_{2_{a_\ell}}(r)}{\partial r} + \Lambda^*_{2_{a_\ell}}\frac{\partial\Re^*_{2_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\left[\frac{-\sin(a(-d_1+h))+}{a}\right.$$

$$\left.\frac{+\sin(a(-d_2+h))}{a}\right]\frac{\pi}{2}]\mathrm{a}_\ell +$$

$$+\frac{\omega^2 H^2 d^2}{8}[[[\sum_{\substack{\ell=0 \\ a_\ell \\ a_\ell \neq a_n}}^{\infty}\sum_{\substack{n=0 \\ a_n}}^{\infty}[\Lambda_{1_{a_\ell}}\frac{\partial\Re_{1_{a_\ell}}(r)}{\partial r} + \Lambda^*_{1_{a_\ell}}\frac{\partial\Re^*_{1_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}[\overline{\Lambda_{0_{a_n}}\frac{\partial\Re_{0_{a_n}}(r)}{\partial r}} +$$

$$\overline{\Lambda^*_{0_{a_n}}\frac{\partial\Re^*_{0_{a_n}}(r)}{\partial r}}] \; N_{a_n}^{-1/2}[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]+[\sum_{\substack{\ell=0 \\ a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\frac{\partial\Re_{1_{a_\ell}}(r)}{\partial r} +$$

$$\Lambda^*_{1_{a_\ell}} \frac{\partial \Re^*_{1_{a_\ell}}(r)}{\partial r}] N_{a_\ell}^{-1/2} \overline{[\Lambda_{0_{a_\ell}} \frac{\partial \Re_{0_{a_\ell}}(r)}{\partial r}} + \overline{\Lambda^*_{0_{a_\ell}} \frac{\partial \Re^*_{0_{a_\ell}}(r)}{\partial r}}]$$

$$[\frac{2a_\ell(d_1-d_2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}]\pi +$$

$$+[\sum_{\substack{\ell=0\\a_\ell}}^{\infty} \overline{[\Lambda_{1_{a_\ell}} \frac{\partial \Re_{1_{a_\ell}}(r)}{\partial r}} + \overline{\Lambda^*_{1_{a_\ell}} \frac{\partial \Re^*_{1_{a_\ell}}(r)}{\partial r}}] N_{a_\ell}^{-1/2} [\Lambda_{0_{a_\ell}} \frac{\partial \Re_{0_{a_\ell}}(r)}{\partial r} + \Lambda^*_{0_{a_\ell}} \frac{\partial \Re^*_{0_{a_\ell}}(r)}{\partial r}]$$

$$N_{a_\ell}^{-1/2}\left[\frac{2a_\ell(d_1-d_2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}\right] + [\sum_{\substack{\ell=0\\a_\ell\\a_\ell\neq a_n}}^{\infty} \sum_{\substack{n=0\\a_n}}^{\infty} \overline{[\Lambda_{1_{a_\ell}} \frac{\partial \Re_{1_{a_\ell}}(r)}{\partial r}} +$$

$$\overline{\Lambda^*_{1_{a_\ell}} \frac{\partial \Re^*_{1_{a_\ell}}(r)}{\partial r}}] N_{a_\ell}^{-1/2} [\Lambda_{0_{a_n}} \frac{\partial \Re_{0_{a_n}}(r)}{\partial r} + \Lambda^*_{0_{a_n}} \frac{\partial \Re^*_{0_{a_n}}(r)}{\partial r}] N_{a_n}^{-1/2}$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right] +$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\pi + +[\sum_{\substack{\ell=0\\a_\ell}}^{\infty} [\Lambda_{1_{a_\ell}} \frac{\partial \Re_{1_{a_\ell}}(r)}{\partial r} +$$

$$\Lambda^*_{1_{a_\ell}} \frac{\partial \Re^*_{1_{a_\ell}}(r)}{\partial r}] N_{a_\ell}^{-1/2} \overline{[\Lambda_{2_{a_\ell}} \frac{\partial \Re_{2_{a_\ell}}(r)}{\partial r}} + \overline{\Lambda^*_{2_{a_\ell}} \frac{\partial \Re^*_{2_{a_\ell}}(r)}{\partial r}}]$$

$$N_{a_\ell}^{-1}\left[\frac{2a_\ell(d_1-d_2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}\right] + [\sum_{\substack{\ell=0\\a_\ell\\a_\ell\neq a_n}}^{\infty} \sum_{\substack{n=0\\a_n}}^{\infty} [\Lambda_{1_{a_\ell}} \frac{\partial \Re_{1_{a_\ell}}(r)}{\partial r} +$$

$$\Lambda^*_{1_{a_\ell}} \frac{\partial \Re^*_{1_{a_\ell}}(r)}{\partial r}] N_{a_\ell}^{-1/2} \overline{[\Lambda_{2_{a_n}} \frac{\partial \Re_{2_{a_n}}(r)}{\partial r}} + \overline{\Lambda^*_{2_{a_n}} \frac{\partial \Re^*_{2_{a_n}}(r)}{\partial r}}] N_{a_n}^{-1/2}$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right] +$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2} +$$

$$\sum_{\substack{p=2,3}}^{\infty}[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\frac{\partial\Re_{p_{a_\ell}}(r)}{\partial r}+\Lambda^*_{p_{a_\ell}}\frac{\partial\Re^*_{p_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\overline{[\Lambda_{p+1_{a_\ell}}\frac{\partial\Re_{p+1_{a_\ell}}(r)}{\partial r}}+\overline{\Lambda^*_{p+1_{a_\ell}}\frac{\partial\Re^*_{p+1_{a_\ell}}(r)}{\partial r}}]$$

$$N_{a_\ell}^{-1/2}\left[\frac{2a_\ell(d_1-d_2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}\right]+\sum_{\substack{n=0\\a_n}}^{\infty}[\sum_{\substack{\ell=0\\a_\ell\\a_n\neq a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\frac{\partial\Re_{p_{a_\ell}}(r)}{\partial r}+$$

$$\Lambda^*_{p_{a_\ell}}\frac{\partial\Re^*_{p_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\overline{[\Lambda_{p+1_{a_n}}\frac{\partial\Re_{p+1_{a_n}}(r)}{\partial r}}+\overline{\Lambda^*_{p+1_{a_n}}\frac{\partial\Re^*_{p+1_{a_n}}(r)}{\partial r}}]N_{a_n}^{-1/2}$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}+[\sum_{p=2,3}^{\infty}\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\frac{\partial\Re_{p_{a_\ell}}(r)}{\partial r}+$$

$$\Lambda^*_{p_{a_\ell}}\frac{\partial\Re^*_{p_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\overline{[\Lambda_{p-1_{a_\ell}}\frac{\partial\Re_{p-1_{a_\ell}}(r)}{\partial r}}+\overline{\Lambda^*_{p-1_{a_\ell}}\frac{\partial\Re^*_{p-1_{a_\ell}}(r)}{\partial r}}]$$

$$N_{a_\ell}^{-1/2}\left[\frac{2a_\ell(d_1-d_2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}\right]+[\sum_{\substack{n=0\\a_n}}^{\infty}\sum_{\substack{\ell=0\\a_\ell\\a_n\neq a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\frac{\partial\Re_{p_{a_\ell}}(r)}{\partial r}+$$

$$\Lambda^*_{p_{a_\ell}}\frac{\partial\Re^*_{p_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\overline{[\Lambda_{p-1_{a_n}}\frac{\partial\Re_{p-1_{a_n}}(r)}{\partial r}}+\overline{\Lambda^*_{p-1_{a_n}}\frac{\partial\Re^*_{p-1_{a_n}}(r)}{\partial r}}]N_{a_n}^{-1/2}$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}]]\mathrm{a}_\ell+$$

$$+$$

$$-\frac{1}{r^2}\frac{\omega^2H^2d^2}{8}\left[\frac{\phi_0}{H/2}r\frac{1}{d}\right][\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\overline{\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)}\ \overline{+\Lambda^*_{2_{a_\ell}}\Re^*_{2_{a_\ell}}(r)}]2N_{a_\ell}^{-1/2}[\frac{1}{{a_\ell}^2}[\cos(a_\ell(-d_1+h)$$

$$-\cos(a_\ell(-d_2+h))-a_\ell d_1\sin(a_\ell(-d_1+h))+a_\ell d_2\sin(a_\ell(-d_2+h))]]\frac{\pi}{2}+$$

$$\left[d\frac{\phi_0}{H/2}r(\frac{-1}{d}+\frac{g}{\omega^2d^2})\right][\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\overline{\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)}\ \overline{+\Lambda^*_{2_{a_\ell}}\Re^*_{2_{a_\ell}}(r)}]2N_{a_\ell}^{-1/2}\left[\frac{-\sin(a_\ell(-d_1+h))+}{a_\ell}\right.$$

$$\left.\frac{+\sinh(a_\ell(-d_2+h))}{a_\ell}\right] \quad]\frac{\pi}{2}]\mathrm{a}_\ell -$$

$$-\frac{1}{r^2}\frac{\omega^2H^2d^2}{8}\left[\frac{\overline{\phi_0}}{H/2}r\frac{1}{d}\right][\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)+\Lambda^*{}_{2_{a_\ell}}\Re^*{}_{2_{a_\ell}}(r)]2N_{a_\ell}^{-1/2}[\frac{1}{a_\ell{}^2}[\cos(a_\ell(-d_1+h)$$

$$-\cos(a_\ell(-d_2+h))-a_\ell d_1\sin(a_\ell(-d_1+h))+a_\ell d_2\sin(a_\ell(-d_2+h))]]\frac{\pi}{2}+$$

$$\left[d\frac{\overline{\phi_0}}{H/2}r(\frac{-1}{d}+\frac{g}{\omega^2d^2})\right][\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)+\Lambda^*{}_{2_{a_\ell}}\Re^*{}_{2_{a_\ell}}(r)]2N_{a_\ell}^{-1/2}\left[\frac{-\sin(a_\ell(-d_1+h))+}{a_\ell}\right.$$

$$\left.\frac{+\sinh(a_\ell(-d_2+h))}{a_\ell}\right]]\frac{\pi}{2}] \quad \mathrm{a}_\ell +$$

$$+\frac{1}{r^2}\frac{\omega^2H^2d^2}{8}[[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}\overline{[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)}+\Lambda^*{}_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\,\overline{[\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)}\,\overline{+\Lambda^*{}_{2_{a_\ell}}\Re^*{}_{2_{a_\ell}}(r)]}N_{a_\ell}^{-1/2}$$

$$\left[\frac{2a_\ell(d_1-d_2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}\right][\sum_{\substack{\ell=0\\a_\ell}}^{\infty}\sum_{\substack{n=0\\a_n}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)$$

$$+\Lambda^*{}_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\,\overline{[\Lambda_{2_{a_n}}\Re_{2_{a_n}}(r)}\,\overline{+\Lambda^*{}_{2_{a_n}}\Re^*{}_{2_{a_n}}(r)]}N_{a_n}^{-1/2}$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}+$$

$$+[\sum_{p=2,3}^{\infty}\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)+\Lambda^*{}_{p_{a_\ell}}\Re^*{}_{p_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\,\overline{[\Lambda_{p+1_{a_\ell}}\Re_{p+1_{a_\ell}}(r)}\,\overline{+\Lambda^*{}_{p+1_{a_\ell}}\Re^*{}_{p+1_{a_\ell}}(r)]}N_{a_\ell}^{-1/2}$$

$$p(p+1)\left[\frac{2a_\ell(d_1-d_2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}\right]+[\sum_{\substack{n=0\\a_n}}^{\infty}\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)$$

$$+\Lambda^*{}_{p_{a_\ell}}\Re^*{}_{p_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\,\overline{[\Lambda_{p+1_{a_n}}\Re_{p+1_{a_n}}(r)}\,\overline{+\Lambda^*{}_{p+1_{a_n}}\Re^*{}_{p+1_{a_n}}(r)]}N_{a_n}^{-1/2}p(p+1)$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}+$$

$$+[\sum_{\substack{p=2,3}}^{\infty}[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)+\Lambda^*{}_{p_{a_\ell}}\Re^*{}_{p_{a_\ell}}(r)]N_{a_\ell}^{-1/2}[\Lambda_{p-1_{a_\ell}}\Re_{p-1_{a_\ell}}(r)+\Lambda^*{}_{p-1_{a_\ell}}\Re^*{}_{p-1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}$$

$$p(p-1)\left[\frac{2a_\ell(d1-d2)+\sin(2a_\ell(d_1-h))-\sin(2a_\ell(d_2-h))}{4a_\ell}\right]+[\sum_{\substack{n=0\\a_n}}^{\infty}\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)$$

$$+\Lambda^*{}_{p_{a_\ell}}\Re^*{}_{p_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\overline{[\Lambda_{p-1_{a_n}}\Re_{p-1_{a_n}}(r)}\ \overline{+\Lambda^*{}_{p-1_{a_n}}\Re^*{}_{p-1_{a_n}}(r)]}N_{a_n}^{-1/2}p(p-1)$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{-\sin[(a_\ell+a_n)(-d_1+h)]+\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}\ ]\ \ a_\ell+$$

$$+$$

$$+\frac{\omega^2H^2d^2}{8}\left[-\frac{z_0}{H/2}\frac{1}{d}\frac{\overline{\phi_0}}{H/2}r(\frac{1}{d^2})\pi-\frac{z_0}{H/2}\frac{1}{d}\frac{\phi_0}{H/2}r(\frac{1}{d^2})\pi\right](\mathrm{d_1\text{-}d_2})\,a_\ell+$$

$$+\frac{\omega^2H^2d^2}{8}[\left[\frac{z_0}{H/2}\frac{1}{d}\pi\right][[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}\overline{[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)}\ \overline{+\Lambda^*{}_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)]}N_{a_\ell}^{-1/2}a_\ell\left[\frac{-\cos(a(-d_1+h))+}{a}\right.$$

$$\left.\frac{+\cos(a(-d_2+h))}{a}\right]]-\left[\frac{\phi_0}{H/2}r(\frac{1}{d})\frac{\pi}{2}\right][[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}\overline{[\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)}\ \overline{+\Lambda^*{}_{2_{a_\ell}}\Re^*{}_{2_{a_\ell}}(r)]}N_{a_\ell}^{-1/2}a_\ell$$

$$\left[\frac{-\cos(a(-d_1+h))}{a}+\frac{\cos(a(-d_2+h))}{a}\right]]]\,a_\ell+$$

$$+\frac{\omega^2H^2d^2}{8}[\left[\frac{z_0}{H/2}\frac{1}{d}\pi\right][\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)+\Lambda^*{}_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}a_\ell\left[\frac{-\cos(a(-d_1+h))}{a}+\right.$$

$$\left.\frac{\cos(a(-d_2+h))}{a}\right]]-\left[\frac{\overline{\phi_0}}{H/2}r(\frac{1}{d})\frac{\pi}{2}\right][[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)+\Lambda^*{}_{2_{a_\ell}}\Re^*{}_{2_{a_\ell}}(r)]N_{a_\ell}^{-1/2}a_\ell$$

$$\left[\frac{-\cos(a(-d_1+h))}{a}+\frac{\cos(a(-d_2+h))}{a}\right]]]]\,a_\ell+$$

$$+\frac{\omega^2H^2d^2}{8}[[[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{0_{a_\ell}}\Re_{0_{a_\ell}}(r)a_\ell+\Lambda^*{}_{0_{a_\ell}}\Re^*_{0_{a_\ell}}(r)a_\ell]N_{a_\ell}^{-1/2}\ \overline{[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)}\ \overline{+\Lambda^*{}_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)]}N_{a_\ell}^{-1/2}a_\ell$$

$$\left[\frac{2a_\ell(d_1-d_2)-\sin[2a_\ell(d_1-h)]+\sin[2a_\ell(d_2-h)]}{4a_\ell}\right]+[\sum_{\substack{n=0\\a_n}}^{\infty}\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{0_{a_\ell}}\Re_{0_{a_\ell}}(r)a_\ell+$$

$$\Lambda^*_{0_{a_\ell}}\Re^*_{0_{a_\ell}}(r)a_\ell]N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{1_{a_n}}\Re_{1_{a_n}}(r)}\,\overline{+\Lambda^*_{1_{a_n}}\Re^*_{1_{a_n}}(r)}]N_{a_n}^{-1/2}a_n$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{\sin[(a_\ell+a_n)(-d_1+h)]-\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\pi+$$

$$+[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)a_\ell+\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)a_\ell]N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{0_{a_\ell}}\Re_{0_{a_\ell}}(r)}\,\overline{+\Lambda^*_{0_{a_\ell}}\Re^*_{0_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}a_\ell$$

$$\left[\frac{2a_\ell(d_1-d_2)-\sin[2a_\ell(d_1-h)]+\sin[2a_\ell(d_2-h)]}{4a_\ell}\right]+[\sum_{\substack{n=0\\a_n}}^{\infty}\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)a_\ell+$$

$$\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)a_\ell]N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{0_{a_n}}\Re_{0_{a_n}}(r)}\,\overline{+\Lambda^*_{0_{a_n}}\Re^*_{0_{a_n}}(r)}]N_{a_n}^{-1/2}a_n$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{\sin[(a_\ell+a_n)(-d_1+h)]-\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\pi+$$

$$+[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)a_\ell+\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)a_\ell]N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)}\,\overline{+\Lambda^*_{2_{a_\ell}}\Re^*_{2_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}a_\ell$$

$$\left[\frac{2a_\ell(d_1-d_2)-\sin[2a_\ell(d_1-h)]+\sin[2a_\ell(d_2-h)]}{4a_\ell}\right]+[\sum_{\substack{n=0\\a_n}}^{\infty}\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)a_\ell+$$

$$\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)a_\ell]N_{a_\ell}^{-1/2}\,[\overline{\Lambda_{2_{a_n}}\Re_{2_{a_n}}(r)}\,\overline{+\Lambda^*_{2_{a_n}}\Re^*_{2_{a_n}}(r)}]N_{a_n}^{-1/2}a_n$$

$$[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{\sin[(a_\ell+a_n)(-d_1+h)]-\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}+$$

$$+\sum_{\substack{p=2,3}}^{\infty}[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)a_\ell+\Lambda^*_{p_{a_\ell}}\Re^*_{p_{a_\ell}}(r)a_\ell]\;N_{a_\ell}^{-1/2}\;[\overline{\Lambda_{p+1_{a_\ell}}\Re_{p+1_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*_{p+1_{a_\ell}}\Re^*_{p+1_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}a_\ell\left[\frac{2a_\ell(d_1-d_2)-\sin[2a_\ell(d_1-h)]+\sin[2a_\ell(d_2-h)]}{4a_\ell}\right]+$$

$$+\sum_{\substack{n=0\\a_n}}^{\infty}\;\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)a_\ell+\Lambda^*_{p_{a_\ell}}\Re^*_{p_{a_\ell}}(r)a_\ell]\;N_{a_\ell}^{-1/2}\;[\overline{\Lambda_{p+1_{a_n}}\Re_{p+1_{a_n}}(r)}$$

$$\overline{+\Lambda^*_{p+1_{a_n}}\Re^*_{p+1_{a_n}}(r)}]N_{a_n}^{-1/2}a_n\,[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{\sin[(a_\ell+a_n)(-d_1+h)]-\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}+$$

$$+\sum_{\substack{p=2,3}}^{\infty}[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)a_\ell+\Lambda^*_{p_{a_\ell}}\Re^*_{p_{a_\ell}}(r)a_\ell]\;N_{a_\ell}^{-1/2}\;[\overline{\Lambda_{p-1_{a_\ell}}\Re_{p-1_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*_{p-1_{a_\ell}}\Re^*_{p-1_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}a_\ell\left[\frac{2a_\ell(d_1-d_2)-\sin[2a_\ell(d_1-h)]+\sin[2a_\ell(d_2-h)]}{4a_\ell}\right]+$$

$$+\sum_{\substack{n=0\\a_n}}^{\infty}\;\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)a_\ell+\Lambda^*_{p_{a_\ell}}\Re^*_{p_{a_\ell}}(r)a_\ell]\;N_{a_\ell}^{-1/2}\;[\overline{\Lambda_{p-1_{a_n}}\Re_{p-1_{a_n}}(r)}$$

$$\overline{+\Lambda^*_{p-1_{a_n}}\Re^*_{p-1_{a_n}}(r)}]N_{a_n}^{-1/2}a_n\,[\frac{1}{2}\left[\frac{-\sin[(a_\ell-a_n)(-d_1+h)]+\sin[(a_\ell-a_n)(-d_2+h)]}{(a_\ell-a_n)}\right]+$$

$$\frac{1}{2}\left[\frac{\sin[(a_\ell+a_n)(-d_1+h)]-\sin[(a_\ell+a_n)(-d_2+h)]}{(a_\ell+a_n)}\right]]]\frac{\pi}{2}]]\mathrm{a}_\ell.$$

4.2 Υπολογισμός του όρου $\int_{d_2}^{d_1}(\int_0^{2\pi} \overline{x}^{(1)}\nabla\Phi_t^{(1)})\overline{n}dS$ για το πεδίο (ΙΙ)

Έχουμε δείξει στο Κεφάλαιο 2 –σελίδα 19– ότι

$$\overline{x}^{(1)}\nabla\Phi_t^{(1)} = [(X_{g_1}^{(1)} + X_5^{(1)}z)\frac{\partial\phi(r,\theta,z)}{\partial r}\cos\theta - (X_{g_1}^{(1)} + X_5^{(1)}z)\frac{1}{r}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\sin\theta$$
$$+ (X_{g_3}^{(1)} - X_5^{(1)}r\cos\theta)\frac{\partial\phi(r,\theta,z)}{\partial z}](-i\omega)]$$

Δηλαδή

$$\overline{x}^{(1)}\nabla\Phi_t^{(1)} = (X_{g_1}^{(1)} + X_5^{(1)}z)\ (-i\omega)\ [-i\omega\frac{H}{2}d\left[-d\frac{\phi_0}{H/2}(\frac{z-d}{d^2} + \frac{g}{\omega^2 d^2})\cos\theta\right] - i\omega\frac{H}{2}d$$
$$\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\frac{\partial\Re_{m_{\kappa_\ell}}(r)}{\partial r} + \Lambda^*_{m_{\kappa_\ell}}\frac{\partial\Re^*_{m_{\kappa_\ell}}(r)}{\partial r}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\cos(m\theta) - i\omega\frac{H}{2}d\ [$$
$$\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\frac{\partial\Re_{m_{a_\ell}}(r)}{\partial r} + \Lambda^*_{m_{a_\ell}}\frac{\partial\Re^*_{m_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\cos(m\theta)]\cos(\theta) -$$

$$-(X_{g_1}^{(1)} + X_5^{(1)}z)\ (-i\omega)\ \frac{1}{r}\ (-i\omega\frac{H}{2}d)\left[d\frac{\phi_0}{\frac{H}{2}}r(\frac{z-d}{d^2} + \frac{g}{\omega^2 d^2})\sin\theta\right] + i\omega\frac{H}{2}d$$
$$[\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r) + \Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)]mN_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\ \sin(m\theta) + \sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$
$$+ \Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)]mN_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\ \sin(m\theta)]\sin(\theta) +$$
$$+ (X_{g_3}^{(1)} - X_5^{(1)}r\cos\theta)(-i\omega)\ [(-i\omega\frac{H}{2}d)\left[\frac{z_0}{H/2}\frac{1}{d} - d\frac{\phi_0}{H/2}r(\frac{1}{d^2})\cos\theta\right] - i\omega\frac{H}{2}d$$
$$\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r) + \Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell\ \cos(m\theta) + \sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$
$$+ \Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell\ \cos(m\theta)]]$$

Θα υπολογίσουμε τον όρο

$$\int_{d_2}^{d_1}(\int_0^{2\pi}\overline{x}^{(1)}\nabla\Phi_t^{(1)})ndS=\int_{d_2}^{d_1}(\int_0^{2\pi}\overline{x}^{(1)}\nabla\Phi_t^{(1)}\cos\theta d\theta)\mathrm{a}_\ell dz=$$

$$=X_{g_1}^{(1)}(-i\omega)[-i\omega\frac{H}{2}d\left[-d\frac{\phi_0}{H/2}(\frac{-d}{d^2}(\frac{d_1^{\,2}-d_2^{\,2}}{2})+(\frac{-d}{d^2}+\frac{g}{\omega^2d^2})(d_1-d_2))\pi\right]$$

$$-i\omega\frac{H}{2}d\,[\Lambda_{2_{\kappa_\ell}}\frac{\partial\Re_{2_{\kappa_\ell}}(r)}{\partial r}+\Lambda^*_{2_{\kappa_\ell}}\frac{\partial\Re^*_{2_{\kappa_\ell}}(r)}{\partial r}]\,N_{\kappa_\ell}^{-1/2}\kappa_\ell(\frac{\sinh(\kappa_\ell d_1)-\sinh(\kappa_\ell d_2)}{\kappa_\ell})\frac{\pi}{2}$$

$$-i\omega\frac{H}{2}d\frac{\pi}{2}\,[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\frac{\partial\Re_{2_{a_\ell}}(r)}{\partial r}+\Lambda^*_{2_{a_\ell}}\frac{\partial\Re^*_{2_{a_\ell}}(r)}{\partial r}]\,N_{a_\ell}^{-1/2}\alpha_\ell(\frac{\sin(a_\ell d_1)-\sin(a_\ell d_2)}{a_\ell})]+$$

$$+X_5^{(1)}(-i\omega)[-i\omega\frac{H}{2}d\left[-d\frac{\phi_0}{H/2}(\frac{-d}{d^2}(\frac{d_1^{\,3}-d_2^{\,3}}{3})+(\frac{-d}{d^2}+\frac{g}{\omega^2d^2})(\frac{d_1^{\,2}-d_2^{\,2}}{2}))\pi\right]$$

$$-i\omega\frac{H}{2}d\frac{\pi}{2}[\Lambda_{2_{\kappa_\ell}}\frac{\partial\Re_{2_{\kappa_\ell}}(r)}{\partial r}+\Lambda^*_{2_{\kappa_\ell}}\frac{\partial\Re^*_{2_{\kappa_\ell}}(r)}{\partial r}]\,N_{\kappa_\ell}^{-1/2}\kappa_\ell(\frac{-\cosh(\kappa_\ell d_1)+\cosh(\kappa_\ell d_2)}{\kappa_\ell^{\,2}}+$$

$$\frac{\kappa d_1\sinh(\kappa_\ell d_1)-\kappa d_2\sinh(\kappa_\ell d_2)}{\kappa_\ell^{\,2}})-i\omega\frac{H}{2}d\,\frac{\pi}{2}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\frac{\partial\Re_{2_{a_\ell}}(r)}{\partial r}+\Lambda^*_{2_{a_\ell}}\frac{\partial\Re^*_{2_{a_\ell}}(r)}{\partial r}]$$

$$N_{a_\ell}^{-1/2}\alpha_\ell(\frac{\cos(a_\ell d_1)-\cos(a_\ell d_2)}{a_\ell^{\,2}}+\frac{a_\ell d_1\sin(a_\ell d_1)-a_\ell d_2\sinh(a_\ell d_2)}{a_\ell^{\,2}})]\;\;\mathrm{a}_\ell+$$

$$-X_{g_1}^{(1)}(-i\omega)\,\frac{1}{r}\,[i\omega\frac{H}{2}d\left[-d\frac{\phi_0}{H/2}(\frac{-d}{d^2}(\frac{d_1^{\,2}-d_2^{\,2}}{2})+(\frac{-d}{d^2}+\frac{g}{\omega^2d^2})(d_1-d_2))\pi\right]$$

$$+i\omega\frac{H}{2}d\,[\Lambda_{2_{\kappa_\ell}}\Re_{2_{\kappa_\ell}}(r)+\Lambda^*_{2_{\kappa_\ell}}\Re^*_{2_{\kappa_\ell}}(r)]2N_{\kappa_\ell}^{-1/2}\frac{\pi}{2}\,(\frac{\sinh(\kappa_\ell d_1)-\sinh(\kappa_\ell d_2)}{\kappa_\ell})+$$

$$[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)+\Lambda^*_{2_{a_\ell}}\Re^*_{2_{a_\ell}}(r)]2N_{a_\ell}^{-1/2}\frac{\pi}{2}\;\;(\frac{\sin(a_\ell d_1)-\sin(a_\ell d_2)}{a_\ell})]\;\;\mathrm{a}_\ell-$$

$$-X_5^{(1)}(-i\omega)\,\frac{1}{r}\,[i\omega\frac{H}{2}d\left[-d\frac{\phi_0}{H/2}(\frac{-d}{d^2}(\frac{d_1^{\,3}-d_2^{\,3}}{3})+(\frac{-d}{d^2}+\frac{g}{\omega^2d^2})(\frac{d_1^{\,2}-d_2^{\,2}}{2}))\pi\right]$$

$$+i\omega\frac{H}{2}d\,[\Lambda_{2_{\kappa_\ell}}\Re_{2_{\kappa_\ell}}(r)+\Lambda^*_{2_{\kappa_\ell}}\Re^*_{2_{\kappa_\ell}}(r)]2N_{\kappa_\ell}^{-1/2}\frac{\pi}{2}\,(\frac{-\cosh(\kappa_\ell d_1)+\cosh(\kappa_\ell d_2)}{\kappa_\ell^{\ 2}}+$$

$$\frac{\kappa d_1\sinh(\kappa_\ell d_1)-\kappa d_2\sinh(\kappa_\ell d_2)}{\kappa_\ell^{\ 2}})+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)+\Lambda^*_{2_{a_\ell}}\Re^*_{2_{a_\ell}}(r)]2N_{a_\ell}^{-1/2}\frac{\pi}{2}$$

$$(\frac{\cos(a_\ell d_1)-\cos(a_\ell d_2)}{a_\ell^{\ 2}}+\frac{a_\ell d_1\sin(a_\ell d_1)-a_\ell d_2\sinh(a_\ell d_2)}{a_\ell^{\ 2}})]\ \ \mathrm{a}_\ell+$$

$$+X_{g_3}^{(1)}(-i\omega)(-i\omega\frac{H}{2}d)\left[-d\frac{\phi_0}{H/2}r(\frac{1}{d^2})\pi\right](d_1-d_2)-i\omega\frac{H}{2}d\,[[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\kappa_\ell\frac{1}{\kappa_\ell}(\cosh(\kappa_\ell d_1)-\cosh(\kappa_\ell d_2))\pi+\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)+\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)]$$

$$N_{a_\ell}^{-1/2}(-\cos(a_\ell d_1)+\cos(a_\ell d_1))a_\ell\frac{1}{a_\ell}\pi]\ \ \mathrm{a}_\ell+$$

$$+(-X_5^{(1)}\mathrm{a}_\ell)(-i\omega)\left[\frac{z_0}{H/2}\frac{1}{d}\pi\right](d_1-d_2)-i\omega\frac{H}{2}d\,[[\Lambda_{2_{\kappa_\ell}}\Re_{2_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{2_{\kappa_\ell}}\Re^*_{2_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\kappa_\ell\frac{1}{\kappa_\ell}(\cosh(\kappa_\ell d_1)-\cosh(\kappa_\ell d_2))\frac{\pi}{2}+[\Lambda_{0_{\kappa_\ell}}\Re_{0_{\kappa_\ell}}(r)+\Lambda^*_{0_{\kappa_\ell}}\Re^*_{0_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\kappa_\ell\frac{1}{\kappa_\ell}$$

$$(\cosh(\kappa_\ell d_1)-\cosh(\kappa_\ell d_2))\pi]+\frac{i\omega H}{2}d[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{0_{a_\ell}}\Re_{0_{a_\ell}}(r)+\Lambda^*_{0_{a_\ell}}\Re^*_{0_{a_\ell}}(r)]$$

$$N_{a_\ell}^{-1/2}(-\cos(a_\ell d_1)+\cos(a_\ell d_1))a_\ell\frac{1}{a_\ell}\pi]+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)+\Lambda^*_{2_{a_\ell}}\Re^*_{2_{a_\ell}}(r)]$$

$$N_{a_\ell}^{-1/2}(-\cos(a_\ell d_1)+\cos(a_\ell d_1))a_\ell\frac{1}{a_\ell}\frac{\pi}{2}]]\ \ \mathrm{a}_\ell\,.$$

Όμως για τον υπολογισμός της δύναμης έκπτωσης πρέπει να υπολογίσουμε τον όρο

$$\overline{\overline{x}^{(1)}\nabla\Phi_t^{(1)}}^{T}=\omega(\overline{X}^{(1)}_{\mathrm{Re}}\cos(\omega t)+\overline{X}^{(1)}_{\mathrm{Im}}\sin(\omega t))(\nabla\phi^{(1)}_{\mathrm{Im}}\cos(\omega t)-\nabla\phi^{(1)}_{\mathrm{Re}}\sin(\omega t))=$$

$$=\overline{\omega(\overline{X}^{(1)}_{\mathrm{Re}}\nabla\phi^{(1)}_{\mathrm{Im}}\cos^2(\omega t)-\overline{X}^{(1)}_{\mathrm{Im}}\nabla\phi^{(1)}_{\mathrm{Re}}\sin^2(\omega t)+(-\overline{X}^{(1)}_{\mathrm{Re}}\nabla\phi^{(1)}_{\mathrm{Re}}+\overline{X}^{(1)}_{\mathrm{Im}}\nabla\phi^{(1)}_{\mathrm{Im}})\cos(\omega t)\sin(\omega t))}^{t}=$$

$$=\frac{1}{2}\omega(\overline{X}^{(1)}_{\mathrm{Re}}\nabla\phi^{(1)}_{\mathrm{Im}}-\overline{X}^{(1)}_{\mathrm{Im}}\nabla\phi^{(1)}_{\mathrm{Re}})=\frac{1}{2}\omega\,\mathrm{Im}(X^{(1)^*}\nabla\phi^{(1)})$$

Όπου $\overline{X_g^{(1)}}$: οι μεταφορικές κινήσεις του κέντρου βάρους της κατασκευής.

$X_5^{(1)}$: περιστροφή γύρω από τον άξονα GX_2

ϕ_0 : μιγαδικό πλάτος της ταλαντωτικής κίνησης του σώματος σε pitch.

$a_\ell = r$: η ακτίνα του $\ell -$στου «από πάνω» στοιχείου.

4.3 Υπολογισμός του όρου $\int_{WL}(\zeta_r^{(1)})^2 n_1 dl$ για το πεδίο (II)

Έχουμε αποδείξει στο Κεφάλαιο 2 –σελίδα 17– για την ανύψωση της ελεύθερης επιφάνειας ότι

$$(\zeta_r^{(1)})^2 = \frac{1}{2}\mathrm{Re}\left\{(\frac{i\omega}{g})\phi(r,\theta,d)(\frac{-i\omega}{g})\overline{\phi(r,\theta,d)}\right\} + \frac{1}{2}\mathrm{Re}\left\{\left|X_{g_3}^{(1)} - X_5^{(1)} r\cos\theta\right|^2\right\} -$$

$$- \mathrm{Re}\left[\left\{(\frac{i\omega}{g})\phi(r,\theta,d)\right\}\{X_{g_3}^{(1)} - X_5^{(1)} r\cos\theta\}\right].$$

Επομένως αφού $n_1 = cos\theta$ και $dl = a_\ell d\theta$

Προκύπτει ότι

$$\int_{WL}(\zeta_r^{(1)})^2 n_1 dl = \int_0^{2\pi}(\zeta_r^{(1)})^2 \cos\theta \quad a_\ell \quad d\theta =$$

$$= \frac{1}{2}\mathrm{Re}\left(\frac{\omega^2}{g^2}\right)\left[\frac{-z_0\overline{\phi_0}rg^2}{\omega^2}\pi - \frac{\omega^2 H^2 d^2}{4}\frac{z_0}{H/2}\frac{g}{\omega^2 d}\,[\overline{[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r) +} \overline{\Lambda^*_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)}]\right.$$

$$N_\kappa^{-1/2}\cosh(\kappa(d-h)) + (\overline{\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{1a_\ell}\Re_{1_{a_\ell}}(r) +} \overline{\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)}]\; N_a^{-1/2}\cos(a(d-h)))]\pi -$$

$$\left[\frac{-z_0\phi_0 rg^2}{\omega^2}\pi - \frac{\omega^2 H^2 d^2}{4}\frac{d\phi_0}{H/2}r\frac{g}{\omega^2 d^2}\pi\;\overline{[[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r) +}\;\overline{\Lambda^*_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)}]\;N_\kappa^{-1/2}\cosh(\kappa(d-h)) +\right.$$

$$+ (\overline{\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\Lambda_{1a_\ell}\Re_{1_{a_\ell}}(r) +}\overline{\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)}]\;N_a^{-1/2}\cos(a(d-h)))] + [\frac{\omega^2 H^2 d^2}{4}\frac{z_0}{H/2}\frac{g}{\omega^2 d}\pi]$$

$$[[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r) + \Lambda^*_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]\;N_\kappa^{-1/2}\cosh(\kappa(d-h)) + (\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{1a_\ell}\Re_{1_{a_\ell}}(r) + \Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)]$$

$$N_a^{-1/2}\cos(a(d-h))) - [\frac{\omega^2 H^2 d^2}{4}\frac{d\overline{\phi_0}}{H/2}r\frac{g}{\omega^2 d^2}\frac{\pi}{2}]\;[\Lambda_{2_{\kappa_\ell}}\Re_{2_{\kappa_\ell}}(r) + \Lambda^*_{2_{\kappa_\ell}}\Re^*_{2_{\kappa_\ell}}(r)]$$

$$\left. N_\kappa^{-1/2}\cosh(\kappa(d-h)) + (\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{2a_\ell}\Re_{2_{a_\ell}}(r) + \Lambda^*_{2_{a_\ell}}\Re^*_{2_{a_\ell}}(r)]\;N_a^{-1/2}\cos(a(d-h)))\right]\; a_\ell +$$

$$+\frac{\omega^2 H^2 d^2}{4}\Big[\ [\Lambda_{0_{\kappa_\ell}}\Re_{0_{\kappa_\ell}}(r)+\Lambda^*{}_{0_{\kappa_\ell}}\Re^*_{0_{\kappa_\ell}}(r)]\ \overline{[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*{}_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]}$$

$$N_\kappa^{-1}\cosh(\kappa(d-h))^2\pi+[\Lambda_{0_{\kappa_\ell}}\Re_{0_{\kappa_\ell}}(r)+\Lambda^*{}_{0_{\kappa_\ell}}\Re^*_{0_{\kappa_\ell}}(r)]\ N_\kappa^{-1/2}\cosh(\kappa(d-h))\ (\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\overline{[\Lambda_{1a_\ell}}\,\overline{\Re_{1_{a_\ell}}(r)}$$

$$+\overline{\Lambda^*{}_{1_{a_\ell}}}\,\overline{\Re^*_{1_{a_\ell}}(r)]}\ N_{a_\ell}^{-1/2}\cos(a_\ell(d-h))\pi+(\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{0a_\ell}\Re_{0_{a_\ell}}(r)+\Lambda^*{}_{0_{a_\ell}}\Re^*_{0_{a_\ell}}(r)]\ \overline{[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+}$$

$$\overline{\Lambda^*{}_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]}\ N_\kappa^{-1/2}\cosh(\kappa(d-h))\ N_{a_\ell}^{-1/2}\cos(a_\ell(d-h)))\pi+(\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{0a_\ell}\Re_{0_{a_\ell}}(r)+$$

$$\Lambda^*{}_{0_{a_\ell}}\Re^*_{0_{a_\ell}}(r)]\ N_{a_\ell}^{-1/2}\cos(a_\ell(d-h)))\ (\sum_{\substack{n=1\\ a_n}}^{\infty}\overline{[\Lambda_{1a_n}}\,\overline{\Re_{1_{a_n}}(r)}+\overline{\Lambda^*{}_{1_{a_n}}}\,\overline{\Re^*_{1_{an}}(r)]}$$

$$N_{a_n}^{-1/2}\cos(a_n(d-h))\pi+[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+\Lambda^*{}_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]\ \overline{[\Lambda_{0_{\kappa_\ell}}\Re_{0_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*{}_{0_{\kappa_\ell}}\Re^*_{0_{\kappa_\ell}}(r)]}$$

$$N_\kappa^{-1}\cosh(\kappa(d-h))^2\pi+[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+\Lambda^*{}_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]\ N_\kappa^{-1/2}\cosh(\kappa(d-h))\ (\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\overline{[\Lambda_{0a_\ell}}\,\overline{\Re_{0_{a_\ell}}(r)}$$

$$+\overline{\Lambda^*{}_{0_{a_\ell}}}\,\overline{\Re^*_{0_{a_\ell}}(r)]}\ N_{a_\ell}^{-1/2}\cos(a_\ell(d-h))\pi+$$

$$(\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{1a_\ell}\Re_{1_{a_\ell}}(r)+\Lambda^*{}_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)]\ \overline{[\Lambda_{0_{\kappa_\ell}}\Re_{0_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*{}_{0_{\kappa_\ell}}\Re^*_{0_{\kappa_\ell}}(r)]}\ N_\kappa^{-1/2}\cosh(\kappa(d-h))$$

$$N_{a_\ell}^{-1/2}\cos(a_\ell(d-h)))\pi+(\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{1a_\ell}\Re_{1_{a_\ell}}(r)+\Lambda^*{}_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)]\ N_{a_\ell}^{-1/2}\cos(a_\ell(d-h)))$$

$$(\sum_{\substack{n=1\\ a_n}}^{\infty}\overline{[\Lambda_{0a_n}}\,\overline{\Re_{0_{a_n}}(r)}+\overline{\Lambda^*{}_{0_{a_n}}}\,\overline{\Re^*_{0_{a_n}}(r)]}\ N_{a_n}^{-1/2}\cos(a_n(d-h))\pi+\frac{\pi}{2}[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+\Lambda^*{}_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]$$

$$\overline{[\Lambda_{2_{\kappa_\ell}}\Re_{2_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*{}_{2_{\kappa_\ell}}\Re^*_{2_{\kappa_\ell}}(r)]}\ N_\kappa^{-1}\cosh(\kappa(d-h))^2+\frac{\pi}{2}[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+\Lambda^*{}_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]$$

$$N_\kappa^{-1/2}\cosh(\kappa(d-h))\ (\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\overline{[\Lambda_{2a_\ell}}\,\overline{\Re_{2_{a_\ell}}(r)}+\overline{\Lambda^*{}_{2_{a_\ell}}}\,\overline{\Re^*_{2_{a_\ell}}(r)]}\ N_{a_\ell}^{-1/2}\cos(a_\ell(d-h))+$$

$$\frac{\pi}{2}\,(\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{1a_\ell}\Re_{1_{a_\ell}}(r)+\Lambda^{*}{}_{1_{a_\ell}}\Re^{*}_{1_{a_\ell}}(r)]\,\overline{[\Lambda_{2_{\kappa_\ell}}\Re_{2_{\kappa_\ell}}(r)+}\;\overline{\Lambda^{*}{}_{2_{\kappa_\ell}}\Re^{*}_{2_{\kappa_\ell}}(r)]}\;N_{\kappa}^{-1/2}\cosh(\kappa(d-h))$$

$$N_{a_\ell}^{-1/2}\cos(a_\ell(d-h)))+\frac{\pi}{2}\,(\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{1a_\ell}\Re_{1_{a_\ell}}(r)+\Lambda^{*}{}_{1_{a_\ell}}\Re^{*}_{1_{a_\ell}}(r)]\;N_{a_\ell}^{-1/2}\cos(a_\ell(d-h)))$$

$$(\sum_{\substack{n=1\\a_n}}^{\infty}[\overline{\Lambda_{2a_n}}\,\overline{\Re_{2_{a_n}}(r)}+\overline{\Lambda^{*}{}_{2_{a_n}}}\,\overline{\Re^{*}_{2_{a_n}}(r)}]\;N_{a_n}^{-1/2}\cos(a_n(d-h))+$$

$$\sum_{p=2,3,}^{\infty}\frac{\pi}{2}\,[\Lambda_{p_{\kappa_\ell}}\Re_{p_{\kappa_\ell}}(r)+\Lambda^{*}{}_{p_{\kappa_\ell}}\Re^{*}_{p_{\kappa_\ell}}(r)]\,\overline{[\Lambda_{p+1_{\kappa_\ell}}\Re_{p+1_{\kappa_\ell}}(r)+}\;\overline{\Lambda^{*}{}_{p+1_{\kappa_\ell}}\Re^{*}_{p+1_{\kappa_\ell}}(r)]}$$

$$N_{\kappa}^{-1}\cosh(\kappa(d-h))^{2}+\frac{\pi}{2}[\Lambda_{p_{\kappa_\ell}}\Re_{p_{\kappa_\ell}}(r)+\Lambda^{*}{}_{p_{\kappa_\ell}}\Re^{*}_{p_{\kappa_\ell}}(r)]\;N_{\kappa}^{-1/2}\cosh(\kappa(d-h))$$

$$(\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{p+1a_\ell}}\,\overline{\Re_{p+1_{a_\ell}}(r)}+\overline{\Lambda^{*}{}_{p+1_{a_\ell}}}\,\overline{\Re^{*}_{p+1_{a_\ell}}(r)}]\;N_{a_\ell}^{-1/2}\cos(a_\ell(d-h))+\frac{\pi}{2}\,(\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)+$$

$$\Lambda^{*}{}_{p_{a_\ell}}\Re^{*}_{p_{a_\ell}}(r)]\,\overline{[\Lambda_{p+1_{\kappa_\ell}}\Re_{p+1_{\kappa_\ell}}(r)+}\;\overline{\Lambda^{*}{}_{p+1_{\kappa_\ell}}\Re^{*}_{p+1_{\kappa_\ell}}(r)]}\;N_{\kappa}^{-1/2}\cosh(\kappa(d-h))$$

$$N_{a_\ell}^{-1/2}\cos(a_\ell(d-h)))+\frac{\pi}{2}\,(\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)+\Lambda^{*}{}_{p_{a_\ell}}\Re^{*}_{p_{a_\ell}}(r)]\;N_{a_\ell}^{-1/2}\cos(a_\ell(d-h)))$$

$$(\sum_{\substack{n=1\\a_n}}^{\infty}[\overline{\Lambda_{p+1a_n}}\,\overline{\Re_{p+1_{a_n}}(r)}+\overline{\Lambda^{*}{}_{p+1_{a_n}}\Re^{*}_{p+1_{a_n}}(r)}]\;N_{a_n}^{-1/2}\cos(a_n(d-h))+\frac{\pi}{2}[\Lambda_{p_{\kappa_\ell}}\Re_{p_{\kappa_\ell}}(r)+$$

$$\Lambda^{*}{}_{p_{\kappa_\ell}}\Re^{*}_{p_{\kappa_\ell}}(r)]\,\overline{[\Lambda_{p-1_{\kappa_\ell}}\Re_{p-1_{\kappa_\ell}}(r)+}\;\overline{\Lambda^{*}{}_{p-1_{\kappa_\ell}}\Re^{*}_{p-1_{\kappa_\ell}}(r)]}\;N_{\kappa}^{-1}\cosh(\kappa(d-h))^{2}+\frac{\pi}{2}[\Lambda_{p_{\kappa_\ell}}\Re_{p_{\kappa_\ell}}(r)+$$

$$\Lambda^{*}{}_{p_{\kappa_\ell}}\Re^{*}_{p_{\kappa_\ell}}(r)]\;N_{\kappa}^{-1/2}\cosh(\kappa(d-h))\;(\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{p-1a_\ell}}\,\overline{\Re_{p-1_{a_\ell}}(r)}+\overline{\Lambda^{*}{}_{p-1_{a_\ell}}}\,\overline{\Re^{*}_{p-1_{a_\ell}}(r)}]$$

$$N_{a_\ell}^{-1/2}\cos(a_\ell(d-h))+\frac{\pi}{2}\,(\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)+\Lambda^{*}{}_{p_{a_\ell}}\Re^{*}_{p_{a_\ell}}(r)]\,\overline{[\Lambda_{p-1_{\kappa_\ell}}\Re_{p-1_{\kappa_\ell}}(r)+}$$

$$\overline{\Lambda^{*}{}_{p-1_{\kappa_\ell}}\Re^{*}_{p-1_{\kappa_\ell}}(r)]}\;N_{\kappa}^{-1/2}\cosh(\kappa(d-h))\;N_{a_\ell}^{-1/2}\cos(a_\ell(d-h)))+\frac{\pi}{2}\,(\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{p_{a_\ell}}\Re_{p_{a_\ell}}(r)+$$

$$\Lambda^*{}_{p_{a_\ell}} \Re^*_{p_{a_\ell}}(r)]\ N_{a_\ell}^{-1/2}\cos(a_\ell(d-h)))\ (\sum_{\substack{n=1\\a_n}}^{\infty}[\overline{\Lambda_{p-1_{a_n}}}\,\overline{\Re_{p-1_{a_n}}(r)}+\overline{\Lambda^*{}_{p-1_{a_n}}}\,\overline{\Re^*_{p-1_{a_n}}(r)}]$$

$$N_{a_n}^{-1/2}\cos(a_n(d-h))\ \Big]\ a_\ell +$$

$$+\frac{1}{2}\mathrm{Re}(-2X_{g_3}X_5 r\pi)\ \ a_\ell -$$

$$-\mathrm{Re}\left(\frac{i\omega}{g}\right)X_{g_3}(-i\omega\frac{H}{2}d\left(\frac{-d\phi_0}{H/2}r\left(\frac{g}{\omega^2 d^2}\right)\pi\right)-i\omega\frac{H}{2}d\pi(\ [\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+\Lambda^*{}_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]$$

$$N_\kappa^{-1/2}\cosh(\kappa(d-h))+(\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{1a_\ell}\Re_{1_{a_\ell}}(r)+\Lambda^*{}_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)]\ N_a^{-1/2}\cos(a_\ell(d-h)))\quad a_\ell +$$

$$+\mathrm{Re}(\frac{i\omega}{g}X_5 r(-i\omega\frac{H}{2}d\left(\frac{z_0}{H/2}\frac{g}{\omega^2 d}\right)\pi-i\omega\frac{H}{2}d\frac{\pi}{2}(\ [\Lambda_{2_{\kappa_\ell}}\Re_{2_{\kappa_\ell}}(r)+\Lambda^*{}_{2_{\kappa_\ell}}\Re^*_{2_{\kappa_\ell}}(r)]$$

$$N_\kappa^{-1/2}\cosh(\kappa(d-h))+(\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{2a_\ell}\Re_{2_{a_\ell}}(r)+\Lambda^*{}_{2_{a_\ell}}\Re^*_{2_{a_\ell}}(r)]\ N_a^{-1/2}\cos(a(d-h)))\quad \mathrm{a}_\ell$$

Όπου X_{g_3} : μεταφορική κίνηση του κέντρου βάρους της κατασκευής στον άξονα OX_3.

X_5 : περιστροφή γύρω από τον άξονα GX_2.

r : η ακτίνα του $\ell-$στου «από πάνω» στοιχείου.

a_ℓ : η ακτίνα του $\ell-$στου «από πάνω» στοιχείου.

4.4 Υπολογισμός της οριζόντιας δύναμης έκπτωσης για το πεδίο (II)

Η οριζόντια δύναμη έκπτωσης F_X για το Πεδίο (II) υπολογίζεται Κεφάλαιο 2 –σελίδα 16 – από τη σχέση

$$F_X = -\int_{WL} \frac{1}{2}\rho g \zeta_r^{(1)2} \bar{n} dl + MR^{(1)} \overline{X_g^{(1)}}'' - \iint_{S_0} -\frac{1}{2}\rho \left|\nabla\Phi^{(1)}\right|^2 \bar{n} dS - \iint_{S_0} -\rho(\overline{X}^{(1)} \nabla\Phi_t^{(1)})\bar{n} dS =$$

$$= -\frac{1}{2}\rho g \, [\underline{\int_{WL} \zeta_r^{(1)2} \bar{n} dl}\,] + MR^{(1)} \overline{X_g^{(1)}}'' + \frac{1}{2}\rho \, [\underline{\iint_{S_0} \left|\nabla\Phi^{(1)}\right|^2 \bar{n} dS}\,] + \rho \, [\underline{\iint_{S_0} (\overline{X}^{(1)} \nabla\Phi_t^{(1)})\bar{n} dS}\,] .$$

Όμως οι παραστάσεις $\underline{\int_{WL} \zeta_r^{(1)2} \bar{n} dl}$ $\underline{\iint_{S_0} \left|\nabla\Phi^{(1)}\right|^2 \bar{n} dS}$ $\underline{[\iint_{S_0} (\overline{X}^{(1)} \nabla\Phi_t^{(1)})\bar{n} dS]}$

είναι γνωστές από τα προηγούμενα (σελίδα 136, σελίδα 125, σελίδα 132, αντίστοιχα)

Όπου ρ: η πυκνότητα νερού.

g: η επιτάχυνση της βαρύτητας.

M: μάζα αξονοσυμμετρικού σώματος.

$R^{(1)}$: ο πίνακας που περιέχει τις γωνίες περιστροφής.

$X_g^{(1)}$: οι ευθύγραμμες μεταφορικές κινήσεις του κέντρου βάρους της κατασκευής

Επομένως μπορούμε να υπολογίσουμε την οριζόντια δύναμη έκπτωσης F_X για το Πεδίο (II).

5^o ΟΡΙΖΟΝΤΙΑ ΔΥΝΑΜΗ ΕΚΠΤΩΣΗΣ ΓΙΑ ΤΟ ΠΕΔΙΟ (ΙΙΙ)

5.1 Υπολογισμός του όρου $\int_{h_2}^{h_1}(\int_0^{2\pi} \left|\nabla\Phi^{(1)}\right|^2)\overline{n}dS$ για το πεδίο (ΙΙΙ)

Έχουμε αποδείξει στο Κεφάλαιο 1 –σελίδα 14– ότι η σχέση που μας δίνει το δυναμικό της ταχύτητας πρώτης τάξης για δακτυλιοειδή στοιχεία στο πεδίο (ΙΙΙ): $b_p \le r \le b_{p+1} \quad \kappa\alpha\iota \quad 0 \le z \le h_p$ είναι

$$\Phi(r,\theta,z;\text{t}) = -i\omega\frac{H}{2}d(\frac{z_0}{H/2}\frac{z^2-0{,}5r^2}{2h_p d} - d\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_p d^2}\cos\theta)e^{-i\omega t} - i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\Re_{mn_p}(r)+\Lambda^*_{mn_p}\Re^*_{mn_p}(r))\cos(\frac{n_p\pi z}{h_p})]\cos(m\theta)e^{-i\omega t}.$$

Επομένως

$$\varphi(r,\theta,z) = -i\omega\frac{H}{2}d(\frac{z_0}{H/2}\frac{z^2-0{,}5r^2}{2h_p d} - d\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_p d^2}\cos\theta) - i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\Re_{mn_p}(r)+\Lambda^*_{mn_p}\Re^*_{mn_p}(r))\cos(\frac{n_p\pi z}{h_p})]\cos(m\theta).$$

Και

$$\frac{\partial\phi(r,\theta,z)}{\partial r} = -i\omega\frac{H}{2}d(\frac{z_0}{H/2}\frac{-r}{2h_p d} - d\frac{\phi_0}{H/2}\frac{(z^2-0{,}75r^2)}{2h_p d^2}\cos\theta) - i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\frac{\partial\Re_{mn_p}(r)}{\partial r}+\Lambda^*_{mn_p}\frac{\partial\Re^*_{mn_p}(r)}{\partial r})\cos(\frac{n_p\pi z}{h_p})]\cos(m\theta).$$

Όπου

$$\frac{\partial \Re_{mn_p}(r)}{\partial r}=\frac{n_p\pi}{h_p}\frac{K_m(\frac{n_p\pi b_p}{h_p})I_{m+1}(\frac{n_p\pi r}{h_p})+I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi r}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}+$$

$$+\frac{m}{r}\frac{K_m(\frac{n_p\pi b_p}{h_p})I_m(\frac{n_p\pi r}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi r}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}.$$

$$\frac{\partial \Re^*_{mn_p}(r)}{\partial r}=-\frac{n_p\pi}{h_p}\frac{I_m(\frac{n_p\pi b_p}{h_p})K_{m+1}(\frac{n_p\pi r}{h_p})+K_m(\frac{n_p\pi b_{p+1}}{h_p})I_{m+1}(\frac{n_p\pi r}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}+$$

$$+\frac{m}{r}\frac{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi r}{h_p})-K_m(\frac{n_p\pi b_{p+1}}{h_p})I_m(\frac{n_p\pi r}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}.$$

Ειδικές περιπτώσεις είναι

$$\frac{\partial \Re_{0_{0_p}}(r)}{\partial r}=\frac{1}{r}\frac{1}{\ln(\frac{b_{p+1}}{b_p})},\qquad \frac{\partial \Re^*_{0_{0_p}}(r)}{\partial r}=-\frac{1}{r}\frac{1}{\ln(\frac{b_{p+1}}{b_p})}$$

$$\frac{\partial \Re_{m_{0_p}}(r)}{\partial r}=m\frac{\frac{1}{b_p}\left(\frac{r}{b_p}\right)^{m-1}+\frac{b_p}{r^2}\left(\frac{b_p}{r}\right)^{m-1}}{\left(\frac{b_{p+1}}{b_p}\right)^m-\left(\frac{b_p}{b_{p+1}}\right)^m},\qquad \frac{\partial \Re^*_{m_{0_p}}(r)}{\partial r}=m\frac{\frac{1}{b_{p+1}}\left(\frac{r}{b_p}\right)^{m-1}+\frac{b_{p+1}}{r^2}\left(\frac{b_{p+1}}{r}\right)^{m-1}}{\left(\frac{b_{p+1}}{b_p}\right)^m-\left(\frac{b_p}{b_{p+1}}\right)^m}$$

Και

$$\frac{\partial \overline{\phi(r,\theta,z)}}{\partial r}=i\omega\frac{H}{2}d(-\frac{z_0}{H/2}\frac{r}{2h_pd}-d\frac{\overline{\phi_0}}{H/2}\frac{(z^2-0,75r^2)}{2h_pd^2}\cos\theta)+i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\overline{\frac{\partial \Re_{mn_p}(r)}{\partial r}}+\Lambda^*_{mn_p}\overline{\frac{\partial \Re^*_{mn_p}(r)}{\partial r}})\cos(\frac{n_p\pi z}{h_p})]\cos(m\theta).$$

Επομένως

$$\overline{\left(\frac{\partial\phi(r,\theta,z)}{\partial r}\right)^2}^T = \frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial r}\frac{\overline{\partial\phi(r,\theta,z)}}{\partial r} =$$

$$\frac{1}{2}\omega^2\frac{H^2}{4}d^2(-\frac{z_0}{H/2}\frac{r}{2h_p d} - d\frac{\overline{\phi_0}}{H/2}\frac{(z^2-0{,}75r^2)}{2h_p d^2}\cos\theta)$$

$$(\frac{z_0}{H/2}\frac{-r}{2h_p d} - d\frac{\phi_0}{H/2}\frac{(z^2-0{,}75r^2)}{2h_p d^2}\cos\theta) +$$

$$+\frac{1}{2}\omega^2\frac{H^2}{4}d^2(\frac{z_0}{H/2}\frac{-r}{2h_p d} - d\frac{\phi_0}{H/2}\frac{(z^2-0{,}75r^2)}{2h_p d^2}\cos\theta)\ \sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\overline{\in_{n_p}(\Lambda_{mn_p}\frac{\partial\Re_{mn_p}(r)}{\partial r}+}$$

$$\overline{\Lambda^*_{mn_p}\frac{\partial\Re^*_{mn_p}(r)}{\partial r}})\cos(\frac{n_p\pi z}{h_p})]\cos(m\theta)+$$

$$+\frac{1}{2}\omega^2\frac{H^2}{4}d^2(-\frac{z_0}{H/2}\frac{r}{2h_p d} - d\frac{\overline{\phi_0}}{H/2}\frac{(z^2-0{,}75r^2)}{2h_p d^2}\cos\theta)\ \sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\frac{\partial\Re_{mn_p}(r)}{\partial r}+$$

$$\Lambda^*_{mn_p}\frac{\partial\Re^*_{mn_p}(r)}{\partial r})\cos(\frac{n_p\pi z}{h_p})]\cos(m\theta)+$$

$$+\frac{1}{2}\omega^2\frac{H^2}{4}d^2\ \sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\overline{\Lambda_{mn_p}\frac{\partial\Re_{mn_p}(r)}{\partial r}}+\overline{\Lambda^*_{mn_p}\frac{\partial\Re^*_{mn_p}(r)}{\partial r}})\cos(\frac{n_p\pi z}{h_p})]\cos(m\theta)$$

$$\sum_{n=0}^{\infty}[\sum_{n_q=0}^{\infty}\in_{n_p}(\Lambda_{nn_q}\frac{\partial\Re_{nn_q}(r)}{\partial r}+\Lambda^*_{nn_q}\frac{\partial\Re^*_{nn_q}(r)}{\partial r})\cos(\frac{n_q\pi z}{h_p})]\cos(n\theta).$$

Για τον υπολογισμό της drift δύναμης πρέπει να βρεθεί η τιμή του ολοκληρώματος $\int_{h2}^{h1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial r}\right)^2}^T\cos\theta d\theta\right]b_\ell dz$. (*Παράρτημα A* και *B*)

Πρώτα υπολογίζουμε το $\int_0^{2\pi}\left(\frac{\partial\phi}{\partial r}\right)^2\cos\theta d\theta$ και προκύπτει η

$$\int_0^{2\pi}\left(\frac{\partial\phi}{\partial r}\right)^2\cos\theta d\theta =$$

$$=\pi\omega^2\frac{H^2}{8}d^2(\frac{z_0 r}{H/2}\frac{1}{2h_p d}\frac{d\overline{\phi_0}}{H/2}\frac{(z^2-0{,}75r^2)}{2h_p d^2}+\frac{z_0 r}{H/2}\frac{1}{2h_p d}\frac{d\phi_0}{H/2}\frac{(z^2-0{,}75r^2)}{2h_p d^2}+$$

$$+\pi\omega^2\frac{H^2}{8}d^2[(-\frac{z_0}{H/2}\frac{r}{2h_pd})[\sum_{n_p=0}^{\infty}\in_{n_p}\overline{(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}}+\overline{\Lambda^*_{1n_p}\frac{\partial\Re^*_{1n_p}(r)}{\partial r}})\cos(\frac{n_p\pi z}{h_p})]-$$

$$-d\frac{\phi_0}{H/2}\frac{(z^2-0{,}75r^2)}{2h_pd^2}[\sum_{n_p=0}^{\infty}\in_{n_p}\overline{(\Lambda_{2n_p}\frac{\partial\Re_{2n_p}(r)}{\partial r}}+\overline{\Lambda^*_{2n_p}\frac{\partial\Re^*_{2n_p}(r)}{\partial r}})\cos(\frac{n_p\pi z}{h_p})\frac{1}{2}]+$$

$$+\pi\omega^2\frac{H^2}{8}d^2[(-\frac{z_0}{H/2}\frac{r}{2h_pd})[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*_{1n_p}\frac{\partial\Re^*_{1n_p}(r)}{\partial r})\cos(\frac{n_p\pi z}{h_p})]-$$

$$-d\frac{\overline{\phi_0}}{H/2}\frac{(z^2-0{,}75r^2)}{2h_pd^2}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{2n_p}\frac{\partial\Re_{2n_p}(r)}{\partial r}+\Lambda^*_{2n_p}\frac{\partial\Re^*_{2n_p}(r)}{\partial r})\cos(\frac{n_p\pi z}{h_p})\frac{1}{2}]+$$

$$+\omega^2\frac{H^2}{8}d^2[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{0n_p}\frac{\partial\Re_{0n_p}(r)}{\partial r}+\Lambda^*_{0n_p}\frac{\partial\Re^*_{0n_p}(r)}{\partial r})\cos(\frac{n_p\pi z}{h_p})\sum_{n_q=0}^{\infty}\in_{n_q}\overline{(\Lambda_{1n_q}\frac{\partial\Re_{1n_q}(r)}{\partial r}}+$$

$$\overline{\Lambda^*_{1n_q}\frac{\partial\Re^*_{1n_q}(r)}{\partial r}})\cos(\frac{n_q\pi z}{h_p})\pi]+\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*_{1n_p}\frac{\partial\Re^*_{1n_p}(r)}{\partial r})\cos(\frac{n_p\pi z}{h_p})$$

$$\sum_{n_q=0}^{\infty}\in_{n_q}\overline{(\Lambda_{0n_q}\frac{\partial\Re_{0n_q}(r)}{\partial r}}+\overline{\Lambda^*_{0n_q}\frac{\partial\Re^*_{0n_q}(r)}{\partial r}})\cos(\frac{n_q\pi z}{h_p})\pi]+\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+$$

$$\Lambda^*_{1n_p}\frac{\partial\Re^*_{1n_p}(r)}{\partial r})\cos(\frac{n_p\pi z}{h_p})\sum_{n_q=0}^{\infty}\in_{n_q}\overline{(\Lambda_{2n_q}\frac{\partial\Re_{2n_q}(r)}{\partial r}}+\overline{\Lambda^*_{2n_q}\frac{\partial\Re^*_{2n_q}(r)}{\partial r}})\cos(\frac{n_q\pi z}{h_p})\frac{\pi}{2}]+$$

$$\sum_{p=2,3,}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{pn_p}\frac{\partial\Re_{pn_p}(r)}{\partial r}+\Lambda^*{}_{pn_p}\frac{\partial\Re^*{}_{pn_p}(r)}{\partial r})\cos(\frac{n_p\pi z}{h_p})$$

$$[\sum_{n_q=0}^{\infty}\in_{n_q}\overline{(\Lambda_{p+1n_q}\frac{\partial\Re_{p+1n_q}(r)}{\partial r}}\overline{+\Lambda^*{}_{p+1n_q}\frac{\partial\Re^*{}_{p+1n_q}(r)}{\partial r}})\cos(\frac{n_q\pi z}{h_p})+$$

$$\sum_{n_q=0}^{\infty}\in_{n_q}\overline{(\Lambda_{p-1n_q}\frac{\partial\Re_{p-1n_q}(r)}{\partial r}}\overline{+\Lambda^*{}_{p-1n_q}\frac{\partial\Re^*{}_{p-1n_q}(r)}{\partial r}})\cos(\frac{n_q\pi z}{h_p})\frac{\pi}{2}]$$

Και στη συνέχεια υπολογίζουμε το $\int_{h_2}^{h_1}\left[\overline{\left(\frac{\partial\Phi}{\partial r}\right)^2}^T\right]ndS$.

Επομένως

$$\int_{h_2}^{h_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial r}\right)^2}^T\cos\theta d\theta\right]b_pdz=$$

$$=-\frac{\pi\omega^2H^2d^2}{8}[\frac{z_0}{H/2}\frac{r}{2h_pd}\frac{d\overline{\phi_0}}{H/2}\frac{1}{2h_pd^2}[\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3}-0{,}75r^2(h_1-h_2)]+\frac{z_0}{H/2}\frac{r}{2h_pd}$$

$$\frac{d\phi_0}{H/2}\frac{1}{2h_pd^2}[\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3}-0{,}75r^2(h_1-h_2)]b_p+$$

$$-\frac{\pi\omega^2H^2d^2}{8}[-\frac{z_0r}{H/2}\frac{1}{2h_pd}[\sum_{n_p=1}^{\infty}\in_{n_p}\overline{(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*_{1n_p}\frac{\partial\Re^*_{1n_p}(r)}{\partial r})}\frac{h_p}{n_p\pi}$$

$$[\sin(\frac{n_p\pi}{h_p}h_1)-\sin(\frac{n_p\pi}{h_p}h_2)]+\in_0\overline{(\Lambda_{1_0}\frac{\partial\Re_{1_0}(r)}{\partial r}+\Lambda^*_{1_0}\frac{\partial\Re^*{}_{1_0}(r)}{\partial r})}(h_1-h_2)-\frac{d\phi_0}{H/2}\frac{1}{2h_pd}[$$

$$\sum_{n_p=1}^{\infty}\in_{n_p}\overline{(\Lambda_{2n_p}\frac{\partial\Re_{2n_p}(r)}{\partial r}}+\overline{\Lambda^*_{2n_p}\frac{\partial\Re^*_{2n_p}(r)}{\partial r})}(\frac{h_p}{n_p\pi})^2[-\frac{2n_p\pi}{h_p}(h_2)\cos(\frac{n_p\pi}{h_p}h_2)+\frac{2n_p\pi}{h_p}(h_1)$$

$$\cos(\frac{n_p\pi}{h_p}h_2)-(-2+(\frac{n_p\pi}{h_p})^2(h_2)^2\sin\left(\frac{n_p\pi}{h_p}h_2\right))+(-2+(\frac{n_p\pi}{h_p})^2(h_1)^2\sin\left(\frac{n_p\pi}{h_p}h_1\right))]\frac{1}{2}+$$

$$+\in_0\overline{(\Lambda_{2_0}\frac{\partial\Re_{2_0}(r)}{\partial r}+\Lambda^*{}_{2_0}\frac{\partial\Re^*{}_{2_0}(r)}{\partial r})}(\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3})\frac{1}{2}+\frac{d\phi_0}{H/2}\frac{0{,}75r^2}{2h_pd^2}\frac{1}{2}[$$

$$\sum_{n_p=1}^{\infty}\in_{n_p}\overline{(\Lambda_{2n_p}\frac{\partial\Re_{2n_p}(r)}{\partial r}}+\overline{\Lambda^*_{2n_p}\frac{\partial\Re^*_{2n_p}(r)}{\partial r})}(\frac{h_p}{n_p\pi})[\sin(\frac{n_p\pi}{h_p}h_1)-\sin(\frac{n_p\pi}{h_p}h_2)]+$$

$$+\in_0\overline{(\Lambda_{2_0}\frac{\partial\Re_{2_0}(r)}{\partial r}+\Lambda^*{}_{2_0}\frac{\partial\Re^*{}_{2_0}(r)}{\partial r})}(h_1-h_2)]]b_p+$$

$$-\frac{\pi\omega^2H^2d^2}{8}[-\frac{z_0r}{H/2}\frac{1}{2h_pd}[\sum_{n_p=1}^{\infty}\in_{n_p}(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*_{1n_p}\frac{\partial\Re^*_{1n_p}(r)}{\partial r})\frac{h_p}{n_p\pi}$$

$$[\sin(\frac{n_p\pi}{h_p}h_1)-\sin(\frac{n_p\pi}{h_p}h_2)]+\in_0(\Lambda_{1_0}\frac{\partial\Re_{1_0}(r)}{\partial r}+\Lambda^*_{1_0}\frac{\partial\Re^*{}_{1_0}(r)}{\partial r})(h_1-h_2)-\frac{d\overline{\phi_0}}{H/2}\frac{1}{2h_pd}[$$

$$\sum_{n_p=1}^{\infty}\in_{n_p}(\Lambda_{2n_p}\frac{\partial\Re_{2n_p}(r)}{\partial r}+\Lambda^*_{2n_p}\frac{\partial\Re^*_{2n_p}(r)}{\partial r})(\frac{h_p}{n_p\pi})^2[-\frac{2n_p\pi}{h_p}(h_2)\cos(\frac{n_p\pi}{h_p}h_2)+\frac{2n_p\pi}{h_p}(h_1)$$

$$\cos(\frac{n_p\pi}{h_p}h_2)-(-2+(\frac{n_p\pi}{h_p})^2(h_2)^2\sin\left(\frac{n_p\pi}{h_p}h_2\right))+(-2+(\frac{n_p\pi}{h_p})^2(h_1)^2\sin\left(\frac{n_p\pi}{h_p}h_1\right))]\frac{1}{2}+$$

$$+\in_0(\Lambda_{2_0}\frac{\partial\Re_{2_0}(r)}{\partial r}+\Lambda^*{}_{2_0}\frac{\partial\Re^*{}_{2_0}(r)}{\partial r})(\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3})\frac{1}{2}+\frac{d\overline{\phi_0}}{H/2}\frac{0{,}75r^2}{2h_pd^2}\frac{1}{2}[$$

$$\sum_{n_p=1}^{\infty}\in_{n_p}(\Lambda_{2n_p}\frac{\partial\Re_{2n_p}(r)}{\partial r}+\Lambda^*_{2n_p}\frac{\partial\Re^*_{2n_p}(r)}{\partial r})(\frac{h_p}{n_p\pi})[\sin(\frac{n_p\pi}{h_p}h_1)-\sin(\frac{n_p\pi}{h_p}h_2)]+$$

$$+\in_0(\Lambda_{2_0}\frac{\partial\Re_{2_0}(r)}{\partial r}+\Lambda^*{}_{2_0}\frac{\partial\Re^*{}_{2_0}(r)}{\partial r})(h_1-h_2)]]b_p+$$

$$-\frac{\omega^2H^2d^2}{8}[[\sum_{n_p=1}^{\infty}[\in_{n_p}(\Lambda_{0n_p}\frac{\partial\Re_{0n_p}(r)}{\partial r}+\Lambda^*{}_{0n_p}\frac{\partial\Re^*{}_{0n_p}(r)}{\partial r})$$

$$\overline{\in_{n_p}(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*{}_{1n_p}\frac{\partial\Re^*{}_{1n_p}(r)}{\partial r})}[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+[$$

$$\sum_{n_q=1}^{\infty}\sum_{\substack{n_p=1\\n_p\neq n_q}}^{\infty}[\in_{n_p}(\Lambda_{0n_p}\frac{\partial\Re_{0n_p}(r)}{\partial r}+\Lambda^*{}_{0n_p}\frac{\partial\Re^*{}_{0n_p}(r)}{\partial r})\overline{\in_{n_q}(\Lambda_{1n_q}\frac{\partial\Re_{1n_q}(r)}{\partial r}+\Lambda^*{}_{1n_q}\frac{\partial\Re^*{}_{1n_q}(r)}{\partial r})}]$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+$$

$$+\in_0{}^2(\Lambda_{0_0}\frac{\partial\Re_{0_0}(r)}{\partial r}+\Lambda^*{}_{0_0}\frac{\partial\Re^*{}_{0_0}(r)}{\partial r})\overline{(\Lambda_{1_0}\frac{\partial\Re_{1_0}(r)}{\partial r}+\Lambda^*{}_{1_0}\frac{\partial\Re^*{}_{1_0}(r)}{\partial r})}(h_1-h_2)]\pi+$$

$$+\sum_{n_p=1}^{\infty}[\in_{n_p}(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*{}_{1n_p}\frac{\partial\Re^*{}_{1n_p}(r)}{\partial r})\overline{\in_{n_p}(\Lambda_{0n_p}\frac{\partial\Re_{0n_p}(r)}{\partial r}+\Lambda^*{}_{0n_p}\frac{\partial\Re^*{}_{0n_p}(r)}{\partial r})}$$

$$[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}^{\infty}\sum_{\substack{n_q=1\\n_p\neq n_q}}^{\infty}[\in_{n_p}(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*{}_{1n_p}\frac{\partial\Re^*{}_{1n_p}(r)}{\partial r})\overline{\in_{n_q}(\Lambda_{0n_q}\frac{\partial\Re_{0n_{pq}}(r)}{\partial r}+\Lambda^*{}_{0n_q}\frac{\partial\Re^*{}_{0n_q}(r)}{\partial r})}]$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+$$

$$+\in_0{}^2\,(\Lambda_{1_0}\frac{\partial\Re_{1_0}(r)}{\partial r}+\Lambda^*_{1_0}\frac{\partial\Re^*{}_{1_0}(r)}{\partial r})\,\overline{(\Lambda_{0_0}\frac{\partial\Re_{0_0}(r)}{\partial r}+\Lambda^*{}_{0_0}\frac{\partial\Re^*{}_{0_0}(r)}{\partial r}})(h_1-h_2)]\pi+$$

$$+\sum_{n_p=1}^{\infty}[\in_{n_p}\,(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*_{1n_p}\frac{\partial\Re^*{}_{1n_p}(r)}{\partial r})\,\overline{\in_{n_p}\,(\Lambda_{2n_p}\frac{\partial\Re_{2n_p}(r)}{\partial r}+\Lambda^*{}_{2n_p}\frac{\partial\Re^*{}_{2n_p}(r)}{\partial r})}$$

$$[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}^{\infty}\sum_{\substack{n_q=1\\n_p\neq n_q}}^{\infty}[\in_{n_p}\,(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*_{1n_p}\frac{\partial\Re^*{}_{1n_p}(r)}{\partial r})\,\overline{\in_{n_q}\,(\Lambda_{2n_q}\frac{\partial\Re_{2n_q}(r)}{\partial r}+\Lambda^*{}_{2n_q}\frac{\partial\Re^*{}_{2n_q}(r)}{\partial r})}]$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+$$

$$+\in_0{}^2\,(\Lambda_{1_0}\frac{\partial\Re_{1_0}(r)}{\partial r}+\Lambda^*_{1_0}\frac{\partial\Re^*{}_{1_0}(r)}{\partial r})\,\overline{(\Lambda_{2_0}\frac{\partial\Re_{2_0}(r)}{\partial r}+\Lambda^*{}_{2_0}\frac{\partial\Re^*{}_{2_0}(r)}{\partial r}})(h_1-h_2)]\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}[\sum_{n_p=1}^{\infty}\in_{n_p}\,(\Lambda_{pn_p}\frac{\partial\Re_{pn_p}(r)}{\partial r}+\Lambda^*{}_{pn_p}\frac{\partial\Re^*{}_{pn_p}(r)}{\partial r})$$

$$\overline{\in_{n_p}\,(\Lambda_{p+1n_p}\frac{\partial\Re_{p+1n_p}(r)}{\partial r}}\overline{+\Lambda^*{}_{p+1n_p}\frac{\partial\Re^*{}_{p+1n_p}(r)}{\partial r})}$$

$$[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}^{\infty}\sum_{\substack{n_q=1\\n_p\neq n_q}}^{\infty}[\in_{n_p}\,(\Lambda_{pn_p}\frac{\partial\Re_{pn_p}(r)}{\partial r}+\Lambda^*{}_{pn_p}\frac{\partial\Re^*{}_{pn_p}(r)}{\partial r})$$

$$\overline{\in_{n_q}\,(\Lambda_{p+1n_q}\frac{\partial\Re_{p+1n_q}(r)}{\partial r}}\overline{+\Lambda^*{}_{p+1n_q}\frac{\partial\Re^*{}_{p+1n_q}(r)}{\partial r})}$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+$$

$$+\in_0{}^2\;(\Lambda_{p_0}\frac{\partial\Re_{p_0}(r)}{\partial r}+\Lambda^*{}_{p_0}\frac{\partial\Re^*{}_{p_0}(r)}{\partial r})\;\overline{(\Lambda_{p+1_0}\frac{\partial\Re_{p+1_0}(r)}{\partial r}+\Lambda^*{}_{p+1_0}\frac{\partial\Re^*{}_{p+1_0}(r)}{\partial r})}(h_1-h_2)]]\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}[\sum_{n_p=1}^{\infty}\in_{n_p}\;(\Lambda_{pn_p}\frac{\partial\Re_{pn_p}(r)}{\partial r}+\Lambda^*{}_{pn_p}\frac{\partial\Re^*{}_{pn_p}(r)}{\partial r})$$

$$\in_{n_p}\;\overline{(\Lambda_{p-1n_p}\frac{\partial\Re_{p-1n_p}(r)}{\partial r}+\Lambda^*{}_{p-1n_p}\frac{\partial\Re^*{}_{p-1n_p}(r)}{\partial r})}$$

$$[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}^{\infty}\sum_{\substack{n_q=1\\ n_p\neq n_q}}^{\infty}[\in_{n_p}\;(\Lambda_{pn_p}\frac{\partial\Re_{pn_p}(r)}{\partial r}+\Lambda^*{}_{pn_p}\frac{\partial\Re^*{}_{pn_p}(r)}{\partial r})$$

$$\in_{n_q}\;\overline{(\Lambda_{p-1n_q}\frac{\partial\Re_{p-1n_q}(r)}{\partial r}+\Lambda^*{}_{p-1n_q}\frac{\partial\Re^*{}_{p-1n_q}(r)}{\partial r})}$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+$$

$$+\in_0{}^2\;(\Lambda_{p_0}\frac{\partial\Re_{p_0}(r)}{\partial r}+\Lambda^*{}_{p_0}\frac{\partial\Re^*{}_{p_0}(r)}{\partial r})\;\overline{(\Lambda_{p-1_0}\frac{\partial\Re_{p-1_0}(r)}{\partial r}+\Lambda^*{}_{p-1_0}\frac{\partial\Re^*{}_{p-1_0}(r)}{\partial r})}(h_1-h_2)]]\frac{\pi}{2}]b_p$$

Όπου b_p ακτίνα του $p-$ στου «από κάτω» στοιχείου.

Όμοια και για τον υπολογισμό $\frac{1}{r^2}\left(\frac{\partial\phi(r,\theta,z)}{\partial\theta}\right)^2=\frac{1}{r^2}\frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\overline{\frac{\partial\phi(r,\theta,z)}{\partial\theta}}$

Έχουμε

$$\frac{1}{r}\frac{\partial\phi(r,\theta,z)}{\partial\theta}=-i\omega\frac{H}{2}d\frac{1}{r}(+d\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_pd^2}\sin\theta)+i\omega\frac{H}{2}d\frac{1}{r}$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\;(\Lambda_{mn_p}\Re_{mn_p}(r)m+\Lambda^*_{mn_p}\Re^*_{mn_p}(r)m)\cos(\frac{n_p\pi z}{h_p})]\sin(m\theta).$$

Και

$$\frac{1}{r}\overline{\frac{\partial\phi(r,\theta,z)}{\partial\theta}} = i\omega\frac{H}{2}d\frac{1}{r}(d\frac{\overline{\phi_0}}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_pd^2}\sin\theta) - i\omega\frac{H}{2}d\frac{1}{r}$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\overline{\in_{n_p}(\Lambda_{mn_p}\Re_{mn_p}(r)m} + \overline{\Lambda^*_{mn_p}\Re^*_{mn_p}(r)m)}\cos(\frac{n_p\pi z}{h_p})]\sin(m\theta).$$

Άρα ισχύει ότι

$$\frac{1}{r^2}\left(\frac{\partial\phi(r,\theta,z)}{\partial\theta}\right)^2 = \frac{1}{r^2}\frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\overline{\frac{\partial\phi(r,\theta,z)}{\partial\theta}} =$$

$$= \frac{1}{2}\omega^2\frac{H^2}{4}d^2\frac{1}{r^2}(d\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_pd^2}\sin\theta)\,(d\frac{\overline{\phi_0}}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_pd^2}\sin\theta) -$$

$$-\frac{1}{2}\omega^2\frac{H^2}{4}d^2\frac{1}{r^2}(d\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_pd^2}\sin\theta)\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\overline{\in_{n_p}(\Lambda_{mn_p}\Re_{mn_p}(r)m} +$$

$$\overline{\Lambda^*_{mn_p}\Re^*_{mn_p}(r)m)}\cos(\frac{n_p\pi z}{h_p})]\sin(m\theta) - \frac{1}{2}\omega^2\frac{H^2}{4}d^2\frac{1}{r^2}(d\frac{\overline{\phi_0}}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_pd^2}\sin\theta)$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\Re_{mn_p}(r)m + \Lambda^*_{mn_p}\Re^*_{mn_p}(r)m)\cos(\frac{n_p\pi z}{h_p})]\sin(m\theta) + \frac{1}{2}\omega^2\frac{H^2}{4}d^2\frac{1}{r^2}$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\Re_{mn_p}(r)m + \Lambda^*_{mn_p}\Re^*_{mn_p}(r)m)\cos(\frac{n_p\pi z}{h_p})]\sin(m\theta)$$

$$\sum_{n=0}^{\infty}[\sum_{n_q=0}^{\infty}\overline{\in_{n_q}(\Lambda_{nn_q}\Re_{nn_q}(r)n} + \overline{\Lambda^*_{nn_q}\Re^*_{nn_q}(r)n)}\cos(\frac{n_q\pi z}{h_p})]\sin(n\theta)$$

Και $\int_{h_2}^{h_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial\theta}\right)^2}^T\cos\theta d\theta\right]b_\ell dz$.

Πρώτα υπολογίζουμε το $\int_0^{2\pi}\left(\frac{\partial\phi}{\partial\theta}\right)^2\cos\theta d\theta$ και προκύπτει η

$$\int_0^{2\pi}\left(\frac{\partial\phi}{\partial\theta}\right)^2\cos\theta d\theta=$$

$$=-\omega^2\frac{H^2}{8}d^2\frac{1}{r^2}\pi(d\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_pd^2}\frac{1}{2})\,[\sum_{n_p=0}^{\infty}\overline{\in_{n_p}\ (\Lambda_{2n_p}\Re_{2n_p}(r)2+}$$

$$\overline{\Lambda^*_{2n_p}\Re^*_{2n_p}(r)2)}\cos(\frac{n_p\pi z}{h_p})]-\omega^2\frac{H^2}{8}d^2\frac{1}{r^2}\pi(d\frac{\overline{\phi_0}}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_pd^2}\frac{1}{2})$$

$$[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{2n_p}\Re_{2n_p}(r)2+\ \Lambda^*_{2n_p}\Re^*_{2n_p}(r)2)\cos(\frac{n_p\pi z}{h_p})]+\omega^2\frac{H^2}{8}d^2\frac{1}{r^2}[$$

$$[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{1n_p}\Re_{1n_p}(r)1+\ \Lambda^*_{1n_p}\Re^*_{1n_p}(r)1)\cos(\frac{n_p\pi z}{h_p})]\,[\sum_{n_q=0}^{\infty}\overline{\in_{n_q}\ (\Lambda_{2n_q}\Re_{2n_q}(r)2+}$$

$$\overline{\Lambda^*_{2n_q}\Re^*_{2n_q}(r)2)}\cos(\frac{n_q\pi z}{h_p})]\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\ \sum_{n_p=0}^{\infty}[\in_{n_p}\ (\Lambda_{pn_p}\Re_{pn_p}(r)p+\ \Lambda^*_{pn_p}\Re^*_{pn_p}(r)p)\cos(\frac{n_p\pi z}{h_p})]$$

$$[\sum_{n_q=0}^{\infty}\overline{\in_{n_q}\ (\Lambda_{p+1n_q}\Re_{p+1n_q}(r)(p+1)}+\overline{\Lambda^*_{p+1n_q}\Re^*_{p+1n_q}(r)(p+1))}\cos(\frac{n_q\pi z}{h_p})]+$$

$$[\sum_{n_q=0}^{\infty}\overline{\in_{n_q}\ (\Lambda_{p-1n_q}\Re_{p-1n_q}(r)(p-1)}+\overline{\Lambda^*_{p-1n_q}\Re^*_{p-1n_q}(r)(p-1))}\cos(\frac{n_q\pi z}{h_p})]]\frac{\pi}{2}]$$

Όπου r η ακτίνα του $p-$στου «από κάτω» στοιχείου.

Στη συνέχεια υπολογίζουμε το $\int_{h_2}^{h_1}\left[\overline{\left(\frac{\partial\Phi}{\partial\theta}\right)^2}^T\right]ndS$. (*Παράρτημα Α*)

Επομένως

$$\int_{h_2}^{h_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial\theta}\right)^2}^T\cos\theta d\theta\right]b_pdz=$$

$$=+\omega^2\frac{H^2}{8}d^2\frac{1}{r^2}\pi(d\frac{\phi_0}{H/2}\frac{r}{2h_pd^2})\,[\sum_{n_p=1}^{\infty}\overline{\in_{n_p}\ (\Lambda_{2n_p}\Re_{2n_p}(r)}+\overline{\Lambda^*_{2n_p}\Re^*_{2n_p}(r))}\ (\frac{h_p}{n_p\pi})^2(-2\frac{n_p\pi}{h_p}h_1$$

$$\cos[\frac{n_p\pi}{h_p}h_1]+2\frac{n_p\pi}{h_p}h_2\cos[\frac{n_p\pi}{h_p}h_2]-(-2+(\frac{n_p\pi}{h})^2(h_1)^2)\sin[\frac{n_p\pi}{h_p}h_1]+(-2+(\frac{n_p\pi}{h})^2(h_2)^2)$$

$$\sin[\frac{n_p\pi}{h_p}h_2])+[\overline{\in_0(\Lambda_{2_0}\Re_{2_0}(r)+}\overline{\Lambda^*_{2_0}\Re^*_{2_0}(r)}]\,(\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3})\,]-d\,\frac{\phi_0}{H/2}\frac{r0{,}25r^2}{2h_pd^2}$$

$$[\sum_{n_p=1}^{\infty}\overline{\in_{n_p}(\Lambda_{2n_p}\Re_{2n_p}(r)+}\overline{\Lambda^*_{2n_p}\Re^*_{2n_p}(r))}\,(\frac{h_p}{n_p\pi})\,(\sin(\frac{n_p\pi}{h_p}h_1)-\sin(\frac{n_p\pi}{h_p}h_2))+$$

$$[\overline{\in_0(\Lambda_{2_0}\Re_{2_0}(r)+}\overline{\Lambda^*_{2_0}\Re^*_{2_0}(r)}]\,(h_1-h_2)]]\;b_p+$$

$$+\omega^2\frac{H^2}{8}d^2\frac{1}{r^2}\pi(d\frac{\overline{\phi_0}}{H/2}\frac{r}{2h_pd^2})\,[\sum_{n_p=1}^{\infty}\in_{n_p}(\Lambda_{2n_p}\Re_{2n_p}(r)+\Lambda^*_{2n_p}\Re^*_{2n_p}(r))(\frac{h_p}{n_p\pi})^2(-2\frac{n_p\pi}{h_p}h_1$$

$$\cos[\frac{n_p\pi}{h_p}h_1]+2\frac{n_p\pi}{h_p}h_2\cos[\frac{n_p\pi}{h_p}h_2]-(-2+(\frac{n_p\pi}{h})^2(h_1)^2)\sin[\frac{n_p\pi}{h_p}h_1]+(-2+(\frac{n_p\pi}{h})^2(h_2)^2)$$

$$\sin[\frac{n_p\pi}{h_p}h_2])+[\in_0(\Lambda_{2_0}\Re_{2_0}(r)+\Lambda^*_{2_0}\Re^*_{2_0}(r)]\,(\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3})\,]-d\,\frac{\overline{\phi_0}}{H/2}\frac{r0{,}25r^2}{2h_pd^2}$$

$$[\sum_{n_p=1}^{\infty}\in_{n_p}(\Lambda_{2n_p}\Re_{2n_p}(r)+\Lambda^*_{2n_p}\Re^*_{2n_p}(r))(\frac{h_p}{n_p\pi})\,(\sin(\frac{n_p\pi}{h_p}h_1)-\sin(\frac{n_p\pi}{h_p}h_2))+[\in_0(\Lambda_{2_0}\Re_{2_0}(r)+$$

$$\Lambda^*_{2_0}\Re^*_{2_0}(r)]\,(h_1-h_2)]]b_p+$$

$$+\omega^2\frac{H^2}{8}d^2\frac{1}{r^2}\Bigg[\;[[\sum_{n_p=1}^{\infty}[\in_{n_p}(\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))\in_{n_p}\overline{(\Lambda_{2n_p}\Re_{2n_p}(r)+}$$

$$\overline{\Lambda^*_{2n_p}\Re^*_{2n_p}(r))}2\,[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}\sum_{\substack{n_q=1\\ n_q\neq n_p}}^{\infty}[\in_{n_p}(\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))\in_{n_q}\overline{(\Lambda_{2n_q}\Re_{2n_q}(r)+}\overline{\Lambda^*_{2n_q}\Re^*_{2n_q}(r))}2\,2$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+\in_0(\Lambda_{1_0}\Re_{1_0}(r)+\Lambda^*_{1_0}\Re^*_{1_0}(r))$$

$$\in_0\overline{(\Lambda_{2_0}\Re_{2_0}(r)+}\overline{\Lambda^*_{2_0}\Re^*_{2_0}(r))}2(h_1-h_2)]\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\sum_{n_p=1}^{\infty}[\in_{n_p}(\Lambda_{pn_p}\Re_{pn_p}(r)+\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\in_{n_p}(\overline{\Lambda_{p+1n_p}\Re_{p+1n_p}(r)+}$$

$$\overline{\Lambda^*_{p+1n_p}\Re^*_{p+1n_p}(r))}\pi(p+1)p][\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}\sum_{\substack{n_q=1\\n_q\neq n_p}}^{\infty}[\in_{n_p}(\Lambda_{pn_p}\Re_{pn_p}(r)+\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\in_{n_q}(\overline{\Lambda_{p+1n_q}\Re_{p+1n_q}(r)+}$$

$$\overline{\Lambda^*_{p+1n_q}\Re^*_{p+1n_q}(r))}(p+1)p]$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+$$

$$+\in_0(\Lambda_{p_0}\Re_{p_0}(r)+\Lambda^*_{p_0}\Re^*_{p_0}(r))\in_0(\overline{\Lambda_{p+1_0}\Re_{p+1_0}(r)+}\ \overline{\Lambda^*_{p+1_0}\Re^*_{p+1_0}(r))}(p+1)p(h_1-h_2)]\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\sum_{n_p=1}^{\infty}[\in_{n_p}(\Lambda_{pn_p}\Re_{pn_p}(r)+\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\in_{n_p}(\overline{\Lambda_{p-1n_p}\Re_{p-1n_p}(r)+}$$

$$\overline{\Lambda^*_{p-1n_p}\Re^*_{p-1n_p}(r))}\pi(p-1)p][\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}\sum_{\substack{n_q=1\\n_q\neq n_p}}^{\infty}[\in_{n_p}(\Lambda_{pn_p}\Re_{pn_p}(r)+\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\in_{n_q}(\overline{\Lambda_{p-1n_q}\Re_{p-1n_q}(r)+}$$

$$\overline{\Lambda^*_{p-1n_q}\Re^*_{p-1n_q}(r))}(p-1)p]$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+\in_0(\Lambda_{p_0}\Re_{p_0}(r)+$$

$$\Lambda^*_{p_0}\Re^*_{p_0}(r))\in_0(\overline{\Lambda_{p-1_0}\Re_{p-1_0}(r)+}\ \overline{\Lambda^*_{p-1_0}\Re^*_{p-1_0}(r))}(p-1)p]\frac{\pi}{2}(h_1-h_2)]b_p$$

Όπου r η ακτίνα του $p-$στου «από κάτω» στοιχείου.

Και b_p η ακτίνα του $p-$στου «από κάτω» στοιχείου.

Τέλος υπολογίζουμε το

$$\frac{\partial\phi(r,\theta,z)}{\partial z} = -i\omega\frac{H}{2}d(\frac{z_0}{H/2}\frac{2z}{2h_p d} - d\frac{\phi_0}{H/2}\frac{r2z}{2h_p d^2}\cos\theta) + i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\Re_{mn_p}(r) + \Lambda^*_{mn_p}\Re^*_{mn_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]\cos(m\theta).$$

Και

$$\frac{\partial\overline{\phi(r,\theta,z)}}{\partial z} = i\omega\frac{H}{2}d(\frac{z_0}{H/2}\frac{2z}{2h_p d} - d\frac{\overline{\phi_0}}{H/2}\frac{r2z}{2h_p d^2}\cos\theta) - i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\overline{(\Lambda_{mn_p}\Re_{mn_p}(r)} + \overline{\Lambda^*_{mn_p}\Re^*_{mn_p}(r))}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]\cos(m\theta).$$

Δηλαδή

$$\left(\frac{\partial\phi(r,\theta,z)}{\partial z}\right)^2 = \frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial z}\frac{\overline{\partial\phi(r,\theta,z)}}{\partial z} =$$

$$= \frac{1}{2}\omega^2\frac{H^2}{4}d^2(\frac{z_0}{H/2}\frac{2z}{2h_p d} - d\frac{\phi_0}{H/2}\frac{r2z}{2h_p d^2}\cos\theta)\,(\frac{z_0}{H/2}\frac{2z}{2h_p d} - d\frac{\overline{\phi_0}}{H/2}\frac{r2z}{2h_p d^2}\cos\theta) -$$

$$-\frac{1}{2}\omega^2\frac{H^2}{4}d^2(\frac{z_0}{H/2}\frac{2z}{2h_p d} - d\frac{\phi_0}{H/2}\frac{r2z}{2h_p d^2}\cos\theta)\,\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\overline{(\Lambda_{mn_p}\Re_{mn_p}(r)} +$$

$$\overline{\Lambda^*_{mn_p}\Re^*_{mn_p}(r))}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]\cos(m\theta)\; -\frac{1}{2}\omega^2\frac{H^2}{4}d^2\,(\frac{z_0}{H/2}\frac{2z}{2h_p d} - d\frac{\overline{\phi_0}}{H/2}\frac{r2z}{2h_p d^2}\cos\theta)$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\Re_{mn_p}(r) + \Lambda^*_{mn_p}\Re^*_{mn_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]\cos(m\theta) + \frac{1}{2}\omega^2\frac{H^2}{4}d^2$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\Re_{mn_p}(r)+\Lambda^*_{mn_p}\Re^*_{mn_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]\cos(m\theta)$$

$$\sum_{n=0}^{\infty}[\sum_{n_q=0}^{\infty}\in_{n_q}(\overline{\Lambda_{nn_q}\Re_{nn_q}(r)}+\overline{\Lambda^*_{nn_q}\Re^*_{nn_q}(r)})\sin(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}]\cos(n\theta).$$

Και $\int_{h_2}^{h_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial z}\right)^2}^T\cos\theta d\theta\right]b_\ell dz$.

Πρώτα υπολογίζουμε το $\int_0^{2\pi}\left(\frac{\partial\phi}{\partial z}\right)^2\cos\theta d\theta$. (*Παράρτημα Β*)

Επομένως

$$\int_0^{2\pi}\left(\frac{\partial\phi}{\partial z}\right)^2\cos\theta d\theta=$$

$$=\omega^2\frac{H^2}{8}d^2\pi(-\frac{z_0}{H/2}\frac{2z}{2h_pd}d\frac{\overline{\phi_0}}{H/2}\frac{r2z}{2h_pd^2}-\frac{z_0}{H/2}\frac{2z}{2h_pd}d\frac{\phi_0}{H/2}\frac{r2z}{2h_pd^2})-$$

$$-\omega^2\frac{H^2}{8}d^2\pi[(\frac{z_0}{H/2}\frac{2z}{2h_pd})\sum_{n_p=0}^{\infty}\in_{n_p}(\overline{\Lambda_{1n_p}\Re_{1n_p}(r)}+\overline{\Lambda^*_{1n_p}\Re^*_{1n_p}(r)})\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}-$$

$$-d\frac{\phi_0}{H/2}\frac{r2z}{2h_pd^2}\sum_{n_p=0}^{\infty}\in_{n_p}(\overline{\Lambda_{2n_p}\Re_{2n_p}(r)}+\overline{\Lambda^*_{2n_p}\Re^*_{2n_p}(r)})\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}\frac{1}{2}]-\omega^2\frac{H^2}{8}d^2\pi[$$

$$(\frac{z_0}{H/2}\frac{2z}{2h_pd})\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}-d\frac{\overline{\phi_0}}{H/2}\frac{r2z}{2h_pd^2}$$

$$\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{2n_p}\Re_{2n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}\frac{1}{2}+\omega^2\frac{H^2}{8}d^2[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{0n_p}\Re_{0n_p}(r)+$$

$$\Lambda^*_{0n_p}\Re^*_{0n_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}\sum_{n_q=0}^{\infty}\in_{n_q}(\overline{\Lambda_{1n_q}\Re_{1n_q}(r)}+\overline{\Lambda^*_{1n_q}\Re^*_{1n_q}(r)})\sin(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}]\pi+$$

$$\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}\sum_{n_q=0}^{\infty}\in_{n_q}(\overline{\Lambda_{0n_q}\Re_{0n_q}(r)}+$$

$$\overline{\Lambda^*_{0n_q}\Re^*_{0n_q}(r)})\sin(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}]\pi+[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\sum_{n_q=0}^{\infty}\in_{n_q}\ (\overline{\Lambda_{2n_q}\Re_{2n_q}(r)}+\overline{\Lambda^*_{2n_q}\Re^*_{2n_q}(r))}\sin(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}]\frac{\pi}{2}+\ \sum_{p=2,3,}^{\infty}[\ \sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{pn_p}\Re_{pn_p}(r)+$$

$$\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}\ [\sum_{n_q=0}^{\infty}\in_{n_q}\ (\overline{\Lambda_{p+1n_q}\Re_{p+1n_q}(r)}+\overline{\Lambda^*_{p+1n_q}\Re^*_{p+1n_q}(r))}$$

$$\sin(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}]\frac{\pi}{2}+\ \sum_{n_q=0}^{\infty}\in_{n_q}\ (\overline{\Lambda_{p-1n_q}\Re_{p-1n_q}(r)}+\overline{\Lambda^*_{p-1n_q}\Re^*_{p-1n_q}(r))}\ \sin(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}]\frac{\pi}{2}].$$

$$\int_{h_2}^{h_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial z}\right)^2}^{T}\cos\theta d\theta\right]b_p dz=$$

$$=-\omega^2\frac{H^2}{8}d^2\pi(-\frac{z_0}{H/2}\frac{4}{2h_pd}d\frac{\overline{\phi_0}}{H/2}\frac{r}{2h_pd^2})\ [\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3}]-\frac{z_0}{H/2}\frac{4}{2h_pd}d\frac{\phi_0}{H/2}\frac{r}{2h_pd^2}$$

$$[\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3}]b_p-$$

$$+\omega^2\frac{H^2}{8}d^2\pi(\frac{z_0}{H/2}\frac{2}{2h_pd}\ \sum_{n_p=0}^{\infty}\in_{n_p}\ (\overline{\Lambda_{1n_p}\Re_{1n_p}(r)}+\overline{\Lambda^*_{1n_p}\Re^*_{1n_p}(r))}(\frac{h_p}{n_p\pi})\ [-\frac{n_p\pi}{h_p}h_2\cos(\frac{n_p\pi}{h_p}h_2)$$

$$+\frac{n_p\pi}{h_p}h_1\cos(\frac{n_p\pi}{h_p}h_1)+\sin(\frac{n_p\pi}{h_p}h_2)-\sin(\frac{n_p\pi}{h_p}h_1)]-d\frac{\phi_0}{H/2}\frac{r2}{2h_pd^2}\ \sum_{n_p=0}^{\infty}\in_{n_p}\ (\overline{\Lambda_{2n_p}\Re_{2n_p}(r)}+$$

$$\overline{\Lambda^*_{2n_p}\Re^*_{2n_p}(r))}\frac{h_p}{n_p\pi}]\ [-\frac{n_p\pi}{h_p}h_2\cos(\frac{n_p\pi}{h_p}h_2)\ +\frac{n_p\pi}{h_p}h_1\cos(\frac{n_p\pi}{h_p}h_1)+$$

$$+\sin(\frac{n_p\pi}{h_p}h_2)-\sin(\frac{n_p\pi}{h_p}h_1))\frac{1}{2}]b_p+$$

$$+\omega^2\frac{H^2}{8}d^2\pi(\frac{z_0}{H/2}\frac{2}{2h_pd}\ \sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))(\frac{h_p}{n_p\pi})\ [-\frac{n_p\pi}{h_p}h_2\cos(\frac{n_p\pi}{h_p}h_2)$$

$$+\frac{n_p\pi}{h_p}h_1\cos(\frac{n_p\pi}{h_p}h_1)+\sin(\frac{n_p\pi}{h_p}h_2)-\sin(\frac{n_p\pi}{h_p}h_1)]-d\frac{\overline{\phi_0}}{H/2}\frac{r2}{2h_pd^2}\ \sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{2n_p}\Re_{2n_p}(r)+$$

$$\Lambda^*_{2n_p}\Re^*_{2n_p}(r))(\frac{h_p}{n_p\pi})\ [-\frac{n_p\pi}{h_p}h_2\cos(\frac{n_p\pi}{h_p}h_2)\ +\frac{n_p\pi}{h_p}h_1\cos(\frac{n_p\pi}{h_p}h_1)+$$

$$+\sin(\frac{n_p\pi}{h_p}h_2)-\sin(\frac{n_p\pi}{h_p}h_1))\frac{1}{2}]b_p+$$

$$+\omega^2 \frac{H^2}{8} d^2 [\, [\sum_{n_p=1}^{\infty} \in_{n_p} (\Lambda_{0n_p} \Re_{0n_p}(r) + \Lambda^*_{0n_p} \Re^*_{0n_p}(r)) \in_{n_p} (\overline{\Lambda_{1n_p} \Re_{1n_p}(r)} +$$

$$\overline{\Lambda^*_{1n_p} \Re^*_{1n_p}(r)})(\frac{n_p\pi}{h_p})^2]\, [\frac{2\frac{n_p\pi}{h_p}(h_1-h_2) - \sin(2\frac{n_p\pi}{h_p}h_1) + \sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}] +$$

$$\sum_{n_p=1}^{\infty}\sum_{n_q=1}^{\infty} \in_{n_p} (\Lambda_{0n_p} \Re_{0n_p}(r) + \Lambda^*_{0n_p} \Re^*_{0n_p}(r)) \in_{n_q} (\overline{\Lambda_{1n_q} \Re_{1n_q}(r)} + \overline{\Lambda^*_{1n_q} \Re^*_{1n_q}(r)})(\frac{n_p\pi}{h_p})(\frac{n_q\pi}{h_p})$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2 - (\frac{n_q\pi}{h_p})^2}[\frac{n_q\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1) - (\frac{n_q\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2) -$$

$$-\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1) + \frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]]\pi +$$

$$[\sum_{n_p=1}^{\infty} \in_{n_p} (\Lambda_{1n_p} \Re_{1n_p}(r) + \Lambda^*_{1n_p} \Re^*_{1n_p}(r)) \in_{n_p} (\overline{\Lambda_{0n_p} \Re_{0n_p}(r)} + \overline{\Lambda^*_{0n_p} \Re^*_{0n_p}(r)})(\frac{n_p\pi}{h_p})^2$$

$$[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2) - \sin(2\frac{n_p\pi}{h_p}h_1) + \sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}] + \sum_{n_p=1}^{\infty}\sum_{n_q=1}^{\infty} \in_{n_p} (\Lambda_{1n_p} \Re_{1n_p}(r) + \Lambda^*_{1n_p} \Re^*_{1n_p}(r))$$

$$\in_{n_q} (\overline{\Lambda_{0n_q} \Re_{0n_q}(r)} + \overline{\Lambda^*_{0n_q} \Re^*_{0n_q}(r)})(\frac{n_p\pi}{h_p})(\frac{n_q\pi}{h_p})$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2 - (\frac{n_q\pi}{h_p})^2}[\frac{n_q\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1) - (\frac{n_q\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2) -$$

$$-\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1) + \frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]]\pi +$$

$$[\sum_{n_p=1}^{\infty} \in_{n_p} (\Lambda_{1n_p} \Re_{1n_p}(r) + \Lambda^*_{1n_p} \Re^*_{1n_p}(r)) \in_{n_p} (\overline{\Lambda_{2n_p} \Re_{2n_p}(r)} + \overline{\Lambda^*_{2n_p} \Re^*_{2n_p}(r)})(\frac{n_p\pi}{h_p})^2$$

$$[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2) - \sin(2\frac{n_p\pi}{h_p}h_1) + \sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}] + \sum_{n_p=1}^{\infty}\sum_{n_q=1}^{\infty} \in_{n_p} (\Lambda_{1n_p} \Re_{1n_p}(r) + \Lambda^*_{1n_p} \Re^*_{1n_p}(r))$$

$$\in_{n_q}\ (\overline{\Lambda_{2n_q}\Re_{2n_q}(r)}+\overline{\Lambda^*_{2n_q}\Re^*_{2n_q}(r)})(\frac{n_p\pi}{h_p})(\frac{n_p\pi}{h_p})$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_q\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_q\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]]\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}[[\sum_{n_p=1}^{\infty}\in_{n_p}\ (\Lambda_{pn_p}\Re_{pn_p}(r)+\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\in_{n_p}\ (\overline{\Lambda_{p+1n_p}\Re_{p+1n_p}(r)}+$$

$$\overline{\overline{\Lambda_{p+1n_p}}\,\overline{\Re_{p+1n_p}(r)}})(\frac{n_p\pi}{h_p})^2\ [\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)-\sin(2\frac{n_p\pi}{h_p}h_1)+\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}^{\infty}\sum_{n_q=1}^{\infty}\ \in_{n_p}\ (\Lambda_{pn_p}\Re_{pn_p}(r)+\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\ \varepsilon_{n_q}(\overline{\Lambda_{p+1n_q}\Re_{p+1n_q}(r)}+$$

$$\overline{\Lambda^*_{p+1n_q}\Re^*_{p+1n_q}(r)})(\frac{n_p\pi}{h_p})(\frac{n_p\pi}{h_p})$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_q\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_q\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]]\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[[\sum_{n_p=1}^{\infty}\in_{n_p}\ (\Lambda_{pn_p}\Re_{pn_p}(r)+\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\ \varepsilon_{n_p}(\overline{\Lambda_{p-1n_p}\Re_{p-1n_p}(r)}+\overline{\overline{\Lambda_{p-1n_p}}\,\overline{\Re_{p-1n_p}(r)}})(\frac{n_p\pi}{h_p})^2$$

$$[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)-\sin(2\frac{n_p\pi}{h_p}h_1)+\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+\sum_{n_p=1}^{\infty}\sum_{n_q=1}^{\infty}\ \in_{n_p}\ (\Lambda_{pn_p}\Re_{pn_p}(r)+$$

$$\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\ \varepsilon_{n_q}(\overline{\Lambda_{p-1n_q}\Re_{p-1n_q}(r)}+\overline{\Lambda^*_{p-1n_q}\Re^*_{p-1n_q}(r)})(\frac{n_p\pi}{h_p})(\frac{n_q\pi}{h_p})$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_q\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_q\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]]\frac{\pi}{2}]b_p\,.$$

Όπου b_p η ακτίνα του $p-$ στου «από κάτω» στοιχείου.

Συνοψίζοντας για το <u>Πεδίο (ΙΙΙ)</u> το $\int_{h_2}^{h_1}\int_{0}^{2\pi} \left|\nabla\Phi^{(1)}\right|^2 \overline{n}\ \ dS =$

$$\int_{h_2}^{h_1}(\int_{0}^{2\pi}[\overline{(\frac{\partial\Phi}{\partial r})^2}^T + \frac{1}{r^2}\overline{(\frac{\partial\Phi}{\partial\theta})^2}^T + \overline{(\frac{\partial\Phi}{\partial z})^2}^T]\cos\theta\ \ d\theta)b_p\ \ dz =$$

$$= -\frac{\pi\omega^2 H^2 d^2}{8}[\frac{z_0}{H/2}\frac{r}{2h_p d}\frac{d\overline{\phi_0}}{H/2}\frac{1}{2h_p d^2}[\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3}-0{,}75r^2(h_1-h_2)] + \frac{z_0}{H/2}\frac{r}{2h_p d}$$

$$\frac{d\phi_0}{H/2}\frac{1}{2h_p d^2}[\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3}-0{,}75r^2(h_1-h_2)]b_p +$$

$$-\frac{\pi\omega^2 H^2 d^2}{8}[-\frac{z_0 r}{H/2}\frac{1}{2h_p d}[\sum_{n_p=1}^{\infty}\in_{n_p}\overline{(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}} + \overline{\Lambda^*_{1n_p}\frac{\partial\Re^*_{1n_p}(r)}{\partial r}})\ \frac{h_p}{n_p\pi}$$

$$[\sin(\frac{n_p\pi}{h_p}h_1)-\sin(\frac{n_p\pi}{h_p}h_2)] + \in_0\overline{(\Lambda_{1_0}\frac{\partial\Re_{1_0}(r)}{\partial r}+\Lambda^*{}_{1_0}\frac{\partial\Re^*{}_{1_0}(r)}{\partial r}})(h_1-h_2) - \frac{d\phi_0}{H/2}\frac{1}{2h_p d}[$$

$$\sum_{n_p=1}^{\infty}\in_{n_p}\overline{(\Lambda_{2n_p}\frac{\partial\Re_{2n_p}(r)}{\partial r}} + \overline{\Lambda^*_{2n_p}\frac{\partial\Re^*_{2n_p}(r)}{\partial r}})\ (\frac{h_p}{n_p\pi})^2[-\frac{2n_p\pi}{h_p}(h_2)\cos(\frac{n_p\pi}{h_p}h_2)+\frac{2n_p\pi}{h_p}(h_1)$$

$$\cos(\frac{n_p\pi}{h_p}h_2)-(-2+(\frac{n_p\pi}{h_p})^2(h_2)^2\sin\left(\frac{n_p\pi}{h_p}h_2\right))+(-2+(\frac{n_p\pi}{h_p})^2(h_1)^2\sin\left(\frac{n_p\pi}{h_p}h_1\right))]\frac{1}{2}+$$

$$+\in_0\overline{(\Lambda_{2_0}\frac{\partial\Re_{2_0}(r)}{\partial r}+\Lambda^*{}_{2_0}\frac{\partial\Re^*{}_{2_0}(r)}{\partial r}})(\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3})\frac{1}{2} + \frac{d\phi_0}{H/2}\frac{0{,}75r^2}{2h_p d^2}\frac{1}{2}[$$

$$\sum_{n_p=1}^{\infty}\in_{n_p}\overline{(\Lambda_{2n_p}\frac{\partial\Re_{2n_p}(r)}{\partial r}} + \overline{\Lambda^*_{2n_p}\frac{\partial\Re^*_{2n_p}(r)}{\partial r}})\ (\frac{h_p}{n_p\pi})\ [\sin(\frac{n_p\pi}{h_p}h_1)-\sin(\frac{n_p\pi}{h_p}h_2)]+$$

$$+\in_0\overline{(\Lambda_{2_0}\frac{\partial\Re_{2_0}(r)}{\partial r}+\Lambda^*{}_{2_0}\frac{\partial\Re^*{}_{2_0}(r)}{\partial r}})(h_1-h_2)]]b_p +$$

$$-\frac{\pi\omega^2 H^2 d^2}{8}[-\frac{z_0 r}{H/2}\frac{1}{2h_p d}[\sum_{n_p=1}^{\infty}\in_{n_p}(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r} + \Lambda^*_{1n_p}\frac{\partial\Re^*_{1n_p}(r)}{\partial r})\ \frac{h_p}{n_p\pi}$$

$$[\sin(\frac{n_p\pi}{h_p}h_1)-\sin(\frac{n_p\pi}{h_p}h_2)] + \in_0(\Lambda_{1_0}\frac{\partial\Re_{1_0}(r)}{\partial r}+\Lambda^*{}_{1_0}\frac{\partial\Re^*{}_{1_0}(r)}{\partial r})(h_1-h_2) - \frac{d\overline{\phi_0}}{H/2}\frac{1}{2h_p d}[$$

$$\sum_{n_p=1}^{\infty}\in_{n_p}(\Lambda_{2n_p}\frac{\partial\Re_{2n_p}(r)}{\partial r} + \Lambda^*_{2n_p}\frac{\partial\Re^*_{2n_p}(r)}{\partial r})\ (\frac{h_p}{n_p\pi})^2[-\frac{2n_p\pi}{h_p}(h_2)\cos(\frac{n_p\pi}{h_p}h_2)+\frac{2n_p\pi}{h_p}(h_1)$$

$$\cos(\frac{n_p\pi}{h_p}h_2)-(-2+(\frac{n_p\pi}{h_p})^2(h_2)^2\sin\left(\frac{n_p\pi}{h_p}h_2\right))+(-2+(\frac{n_p\pi}{h_p})^2(h_1)^2\sin\left(\frac{n_p\pi}{h_p}h_1\right))]\frac{1}{2}+$$

$$+\in_0(\Lambda_{2_0}\frac{\partial\Re_{2_0}(r)}{\partial r}+\Lambda^*_{2_0}\frac{\partial\Re^*_{2_0}(r)}{\partial r})(\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3})\frac{1}{2}+\frac{d\overline{\phi_0}}{H/2}\frac{0{,}75r^2}{2h_pd^2}\frac{1}{2}[$$

$$\sum_{n_p=1}^{\infty}\in_{n_p}(\Lambda_{2n_p}\frac{\partial\Re_{2n_p}(r)}{\partial r}+\Lambda^*_{2n_p}\frac{\partial\Re^*_{2n_p}(r)}{\partial r})(\frac{h_p}{n_p\pi})[\sin(\frac{n_p\pi}{h_p}h_1)-\sin(\frac{n_p\pi}{h_p}h_2)]+$$

$$+\in_0(\Lambda_{2_0}\frac{\partial\Re_{2_0}(r)}{\partial r}+\Lambda^*_{2_0}\frac{\partial\Re^*_{2_0}(r)}{\partial r})(h_1-h_2)]]b_p+$$

$$-\frac{\omega^2H^2d^2}{8}[[\sum_{n_p=1}^{\infty}[\in_{n_p}(\Lambda_{0n_p}\frac{\partial\Re_{0n_p}(r)}{\partial r}+\Lambda^*_{0n_p}\frac{\partial\Re^*_{0n_p}(r)}{\partial r})$$

$$\overline{\in_{n_p}(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*_{1n_p}\frac{\partial\Re^*_{1n_p}(r)}{\partial r})}[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+[$$

$$\sum_{n_q=1}^{\infty}\sum_{\substack{n_p=1\\n_p\neq n_q}}^{\infty}[\in_{n_p}(\Lambda_{0n_p}\frac{\partial\Re_{0n_p}(r)}{\partial r}+\Lambda^*_{0n_p}\frac{\partial\Re^*_{0n_p}(r)}{\partial r})\overline{\in_{n_q}(\Lambda_{1n_q}\frac{\partial\Re_{1n_q}(r)}{\partial r}+\Lambda^*_{1n_q}\frac{\partial\Re^*_{1n_q}(r)}{\partial r})}]$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+$$

$$+\in_0{}^2(\Lambda_{0_0}\frac{\partial\Re_{0_0}(r)}{\partial r}+\Lambda^*_{0_0}\frac{\partial\Re^*_{0_0}(r)}{\partial r})\overline{(\Lambda_{1_0}\frac{\partial\Re_{1_0}(r)}{\partial r}+\Lambda^*_{1_0}\frac{\partial\Re^*_{1_0}(r)}{\partial r})}(h_1-h_2)]\pi+$$

$$+\sum_{n_p=1}^{\infty}[\in_{n_p}(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*_{1n_p}\frac{\partial\Re^*_{1n_p}(r)}{\partial r})\overline{\in_{n_p}(\Lambda_{0n_p}\frac{\partial\Re_{0n_p}(r)}{\partial r}+\Lambda^*_{0n_p}\frac{\partial\Re^*_{0n_p}(r)}{\partial r})}$$

$$[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}^{\infty}\sum_{\substack{n_q=1\\n_p\neq n_q}}^{\infty}[\in_{n_p}(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*_{1n_p}\frac{\partial\Re^*_{1n_p}(r)}{\partial r})\overline{\in_{n_q}(\Lambda_{0n_q}\frac{\partial\Re_{0n_{pq}}(r)}{\partial r}+\Lambda^*_{0n_q}\frac{\partial\Re^*_{0n_q}(r)}{\partial r})}]$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+$$

$$+\in_0{}^2\ (\Lambda_{1_0}\frac{\partial\Re_{1_0}(r)}{\partial r}+\Lambda^*_{1_0}\frac{\partial\Re^*{}_{1_0}(r)}{\partial r})\ \overline{(\Lambda_{0_0}\frac{\partial\Re_{0_0}(r)}{\partial r}+\Lambda^*_{0_0}\frac{\partial\Re^*{}_{0_0}(r)}{\partial r})}(h_1-h_2)]\pi+$$

$$+\sum_{n_p=1}^{\infty}[\in_{n_p}\ (\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*_{1n_p}\frac{\partial\Re^*{}_{1n_p}(r)}{\partial r})\ \overline{\in_{n_p}\ (\Lambda_{2n_p}\frac{\partial\Re_{2n_p}(r)}{\partial r}+\Lambda^*_{2n_p}\frac{\partial\Re^*{}_{2n_p}(r)}{\partial r})}$$

$$[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}^{\infty}\sum_{\substack{n_q=1\\ n_p\neq n_q}}^{\infty}[\in_{n_p}\ (\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*_{1n_p}\frac{\partial\Re^*{}_{1n_p}(r)}{\partial r})\ \overline{\in_{n_q}\ (\Lambda_{2n_q}\frac{\partial\Re_{2n_q}(r)}{\partial r}+\Lambda^*_{2n_q}\frac{\partial\Re^*{}_{2n_q}(r)}{\partial r})}]$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+$$

$$+\in_0{}^2\ (\Lambda_{1_0}\frac{\partial\Re_{1_0}(r)}{\partial r}+\Lambda^*_{1_0}\frac{\partial\Re^*{}_{1_0}(r)}{\partial r})\ \overline{(\Lambda_{2_0}\frac{\partial\Re_{2_0}(r)}{\partial r}+\Lambda^*_{2_0}\frac{\partial\Re^*{}_{2_0}(r)}{\partial r})}(h_1-h_2)]\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}[\sum_{n_p=1}^{\infty}\in_{n_p}\ (\Lambda_{pn_p}\frac{\partial\Re_{pn_p}(r)}{\partial r}+\Lambda^*_{pn_p}\frac{\partial\Re^*{}_{pn_p}(r)}{\partial r})$$

$$\overline{\in_{n_p}\ (\Lambda_{p+1n_p}\frac{\partial\Re_{p+1n_p}(r)}{\partial r}+\Lambda^*_{p+1n_p}\frac{\partial\Re^*{}_{p+1n_p}(r)}{\partial r})}$$

$$[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}^{\infty}\sum_{\substack{n_q=1\\ n_p\neq n_q}}^{\infty}[\in_{n_p}\ (\Lambda_{pn_p}\frac{\partial\Re_{pn_p}(r)}{\partial r}+\Lambda^*_{pn_p}\frac{\partial\Re^*{}_{pn_p}(r)}{\partial r})$$

$$\in_{n_q} \overline{(\Lambda_{p+1n_q} \frac{\partial \Re_{p+1n_q}(r)}{\partial r}} \overline{+\Lambda^*_{p+1n_q} \frac{\partial \Re^*_{p+1n_q}(r)}{\partial r})}$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+$$

$$+\in_0{}^2\ (\Lambda_{p_0}\frac{\partial \Re_{p_0}(r)}{\partial r}+\Lambda^*_{p_0}\frac{\partial \Re^*{}_{p_0}(r)}{\partial r})\ \overline{(\Lambda_{p+1_0}\frac{\partial \Re_{p+1_0}(r)}{\partial r}+\Lambda^*{}_{p+1_0}\frac{\partial \Re^*{}_{p+1_0}(r)}{\partial r})}(h_1-h_2)]]\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}[\sum_{n_p=1}^{\infty}\in_{n_p}(\Lambda_{pn_p}\frac{\partial \Re_{pn_p}(r)}{\partial r}+\Lambda^*{}_{pn_p}\frac{\partial \Re^*{}_{pn_p}(r)}{\partial r})$$

$$\in_{n_p} \overline{(\Lambda_{p-1n_p} \frac{\partial \Re_{p-1n_p}(r)}{\partial r}} \overline{+\Lambda^*_{p-1n_p} \frac{\partial \Re^*_{p-1n_p}(r)}{\partial r})}$$

$$[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}^{\infty}\sum_{\substack{n_q=1\\ n_p\neq n_q}}^{\infty}[\in_{n_p}(\Lambda_{pn_p}\frac{\partial \Re_{pn_p}(r)}{\partial r}+\Lambda^*{}_{pn_p}\frac{\partial \Re^*{}_{pn_p}(r)}{\partial r})$$

$$\in_{n_q} \overline{(\Lambda_{p-1n_q} \frac{\partial \Re_{p-1n_q}(r)}{\partial r}} \overline{+\Lambda^*_{p-1n_q} \frac{\partial \Re^*_{p-1n_q}(r)}{\partial r})}$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+$$

$$+\in_0{}^2\ (\Lambda_{p_0}\frac{\partial \Re_{p_0}(r)}{\partial r}+\Lambda^*_{p_0}\frac{\partial \Re^*{}_{p_0}(r)}{\partial r})\ \overline{(\Lambda_{p-1_0}\frac{\partial \Re_{p-1_0}(r)}{\partial r}+\Lambda^*{}_{p-1_0}\frac{\partial \Re^*{}_{p-1_0}(r)}{\partial r})}(h_1-h_2)]]\frac{\pi}{2}]b_p$$

$$+$$

$$+\omega^2\frac{H^2}{8}d^2\frac{1}{r^2}\pi(d\frac{\phi_0}{H/2}\frac{r}{2h_pd^2})\ [\sum_{n_p=1}^{\infty}\overline{\in_{n_p}(\Lambda_{2n_p}\Re_{2n_p}(r)+}\ \overline{\Lambda^*_{2n_p}\Re^*_{2n_p}(r))}\ (\frac{h_p}{n_p\pi})^2(-2\frac{n_p\pi}{h_p}h_1$$

$$\cos[\frac{n_p\pi}{h_p}h_1]+2\frac{n_p\pi}{h_p}h_2\cos[\frac{n_p\pi}{h_p}h_2]-(-2+(\frac{n_p\pi}{h})^2(h_1)^2)\sin[\frac{n_p\pi}{h_p}h_1]+(-2+(\frac{n_p\pi}{h})^2(h_2)^2)$$

$$\sin[\frac{n_p\pi}{h_p}h_2])+[\overline{\in_0(\Lambda_{2_0}\Re_{2_0}(r)}+\overline{\Lambda^*_{2_0}\Re^*_{2_0}(r)}]\,(\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3})\,]-d\,\frac{\phi_0}{H/2}\frac{r0{,}25r^2}{2h_pd^2}$$

$$[\sum_{n_p=1}^{\infty}\overline{\in_{n_p}(\Lambda_{2n_p}\Re_{2n_p}(r)}+\overline{\Lambda^*_{2n_p}\Re^*_{2n_p}(r))}\,(\frac{h_p}{n_p\pi})\,(\sin(\frac{n_p\pi}{h_p}h_1)-\sin(\frac{n_p\pi}{h_p}h_2))+$$

$$[\overline{\in_0(\Lambda_{2_0}\Re_{2_0}(r)}+\overline{\Lambda^*_{2_0}\Re^*_{2_0}(r)}]\,(h_1-h_2)]]\;\;b_p+$$

$$+\omega^2\frac{H^2}{8}d^2\frac{1}{r^2}\pi(d\frac{\overline{\phi_0}}{H/2}\frac{r}{2h_pd^2})\,[\sum_{n_p=1}^{\infty}\in_{n_p}(\Lambda_{2n_p}\Re_{2n_p}(r)+\Lambda^*_{2n_p}\Re^*_{2n_p}(r))(\frac{h_p}{n_p\pi})^2(-2\frac{n_p\pi}{h_p}h_1$$

$$\cos[\frac{n_p\pi}{h_p}h_1]+2\frac{n_p\pi}{h_p}h_2\cos[\frac{n_p\pi}{h_p}h_2]-(-2+(\frac{n_p\pi}{h})^2(h_1)^2)\sin[\frac{n_p\pi}{h_p}h_1]+(-2+(\frac{n_p\pi}{h})^2(h_2)^2)$$

$$\sin[\frac{n_p\pi}{h_p}h_2])+[\in_0(\Lambda_{2_0}\Re_{2_0}(r)+\Lambda^*_{2_0}\Re^*_{2_0}(r)]\,(\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3})\,]-d\,\frac{\overline{\phi_0}}{H/2}\frac{r0{,}25r^2}{2h_pd^2}$$

$$[\sum_{n_p=1}^{\infty}\in_{n_p}(\Lambda_{2n_p}\Re_{2n_p}(r)+\Lambda^*_{2n_p}\Re^*_{2n_p}(r))(\frac{h_p}{n_p\pi})\,(\sin(\frac{n_p\pi}{h_p}h_1)-\sin(\frac{n_p\pi}{h_p}h_2))+[\in_0(\Lambda_{2_0}\Re_{2_0}(r)+$$

$$\Lambda^*_{2_0}\Re^*_{2_0}(r)]\,(h_1-h_2)]]b_p+$$

$$+\omega^2\frac{H^2}{8}d^2\frac{1}{r^2}\Bigg[\;[[\sum_{n_p=1}^{\infty}[\in_{n_p}(\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))\in_{n_p}\overline{(\Lambda_{2n_p}\Re_{2n_p}(r)+}$$

$$\overline{\Lambda^*_{2n_p}\Re^*_{2n_p}(r))}2\,[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}\sum_{\substack{n_q=1\\n_q\neq n_p}}^{\infty}[\in_{n_p}(\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))\in_{n_q}\overline{(\Lambda_{2n_q}\Re_{2n_q}(r)+}\overline{\Lambda^*_{2n_q}\Re^*_{2n_q}(r))2}\,2$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+\in_0(\Lambda_{1_0}\Re_{1_0}(r)+\Lambda^*_{1_0}\Re^*_{1_0}(r))$$

$$\in_0\overline{(\Lambda_{2_0}\Re_{2_0}(r)+}\overline{\Lambda^*_{2_0}\Re^*_{2_0}(r))2}(h_1-h_2)]\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\sum_{n_p=1}^{\infty}[\in_{n_p}(\Lambda_{pn_p}\Re_{pn_p}(r)+\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\in_{n_p}(\overline{\Lambda_{p+1n_p}\Re_{p+1n_p}(r)+}$$

$$\overline{\Lambda^*_{p+1n_p}\Re^*_{p+1n_p}(r))}\pi(p+1)p][\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}\sum_{\substack{n_q=1\\n_q\neq n_p}}^{\infty}[\in_{n_p}(\Lambda_{pn_p}\Re_{pn_p}(r)+\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\in_{n_q}(\overline{\Lambda_{p+1n_q}\Re_{p+1n_q}(r)+}$$

$$\overline{\Lambda^*_{p+1n_q}\Re^*_{p+1n_q}(r))}(p+1)p]$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+$$

$$+\in_0(\Lambda_{p_0}\Re_{p_0}(r)+\Lambda^*_{p_0}\Re^*_{p_0}(r))\in_0(\overline{\Lambda_{p+1_0}\Re_{p+1_0}(r)+}\ \overline{\Lambda^*_{p+1_0}\Re^*_{p+1_0}(r))}(p+1)p(h_1-h_2)]\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[\sum_{n_p=1}^{\infty}[\in_{n_p}(\Lambda_{pn_p}\Re_{pn_p}(r)+\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\in_{n_p}(\overline{\Lambda_{p-1n_p}\Re_{p-1n_p}(r)+}$$

$$\overline{\Lambda^*_{p-1n_p}\Re^*_{p-1n_p}(r))}\pi(p-1)p][\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)+\sin(2\frac{n_p\pi}{h_p}h_1)-\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}\sum_{\substack{n_q=1\\n_q\neq n_p}}^{\infty}[\in_{n_p}(\Lambda_{pn_p}\Re_{pn_p}(r)+\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\in_{n_q}(\overline{\Lambda_{p-1n_q}\Re_{p-1n_q}(r)+}$$

$$\overline{\Lambda^*_{p-1n_q}\Re^*_{p-1n_q}(r))}(p-1)p]$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_p\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_p\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_q\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]+\in_0(\Lambda_{p_0}\Re_{p_0}(r)+$$

$$\Lambda^*_{p_0}\Re^*_{p_0}(r))\in_0(\overline{\Lambda_{p-1_0}\Re_{p-1_0}(r)+}\ \overline{\Lambda^*_{p-1_0}\Re^*_{p-1_0}(r))}(p-1)p]\frac{\pi}{2}(h_1-h_2)]b_p+$$

+

$$-\omega^2 \frac{H^2}{8} d^2 \pi (-\frac{z_0}{H/2} \frac{4}{2h_p d} d \frac{\overline{\phi_0}}{H/2} \frac{r}{2h_p d^2})\, [\frac{(h_1)^3}{3} - \frac{(h_2)^3}{3}] - \frac{z_0}{H/2} \frac{4}{2h_p d} d \frac{\phi_0}{H/2} \frac{r}{2h_p d^2}$$

$$[\frac{(h_1)^3}{3} - \frac{(h_2)^3}{3}] b_p -$$

$$+\omega^2 \frac{H^2}{8} d^2 \pi (\frac{z_0}{H/2} \frac{2}{2h_p d} \sum_{n_p=0}^{\infty} \in_{n_p} (\overline{\Lambda_{1n_p} \Re_{1n_p}(r)} + \overline{\Lambda^*_{1n_p} \Re^*_{1n_p}(r)})(\frac{h_p}{n_p \pi})\, [-\frac{n_p \pi}{h_p} h_2 \cos(\frac{n_p \pi}{h_p} h_2)$$

$$+\frac{n_p \pi}{h_p} h_1 \cos(\frac{n_p \pi}{h_p} h_1) + \sin(\frac{n_p \pi}{h_p} h_2) - \sin(\frac{n_p \pi}{h_p} h_1)] - d \frac{\phi_0}{H/2} \frac{r2}{2h_p d^2} \sum_{n_p=0}^{\infty} \in_{n_p} (\overline{\Lambda_{2n_p} \Re_{2n_p}(r)} +$$

$$\overline{\Lambda^*_{2n_p} \Re^*_{2n_p}(r)}) \frac{h_p}{n_p \pi}]\, [-\frac{n_p \pi}{h_p} h_2 \cos(\frac{n_p \pi}{h_p} h_2) + \frac{n_p \pi}{h_p} h_1 \cos(\frac{n_p \pi}{h_p} h_1) +$$

$$+\sin(\frac{n_p \pi}{h_p} h_2) - \sin(\frac{n_p \pi}{h_p} h_1)) \frac{1}{2}] b_p +$$

$$+\omega^2 \frac{H^2}{8} d^2 \pi (\frac{z_0}{H/2} \frac{2}{2h_p d} \sum_{n_p=0}^{\infty} \in_{n_p} (\Lambda_{1n_p} \Re_{1n_p}(r) + \Lambda^*_{1n_p} \Re^*_{1n_p}(r))(\frac{h_p}{n_p \pi})\, [-\frac{n_p \pi}{h_p} h_2 \cos(\frac{n_p \pi}{h_p} h_2)$$

$$+\frac{n_p \pi}{h_p} h_1 \cos(\frac{n_p \pi}{h_p} h_1) + \sin(\frac{n_p \pi}{h_p} h_2) - \sin(\frac{n_p \pi}{h_p} h_1)] - d \frac{\overline{\phi_0}}{H/2} \frac{r2}{2h_p d^2} \sum_{n_p=0}^{\infty} \in_{n_p} (\Lambda_{2n_p} \Re_{2n_p}(r) +$$

$$\Lambda^*_{2n_p} \Re^*_{2n_p}(r))(\frac{h_p}{n_p \pi})\, [-\frac{n_p \pi}{h_p} h_2 \cos(\frac{n_p \pi}{h_p} h_2) + \frac{n_p \pi}{h_p} h_1 \cos(\frac{n_p \pi}{h_p} h_1) +$$

$$+\sin(\frac{n_p \pi}{h_p} h_2) - \sin(\frac{n_p \pi}{h_p} h_1)) \frac{1}{2}] b_p +$$

$$+\omega^2 \frac{H^2}{8} d^2 [\, [\sum_{n_p=1}^{\infty} \in_{n_p} (\Lambda_{0n_p} \Re_{0n_p}(r) + \Lambda^*_{0n_p} \Re^*_{0n_p}(r)) \in_{n_p} (\overline{\Lambda_{1n_p} \Re_{1n_p}(r)} +$$

$$\overline{\Lambda^*_{1n_p} \Re^*_{1n_p}(r)})(\frac{n_p \pi}{h_p})^2]\, [\frac{2\frac{n_p \pi}{h_p}(h_1 - h_2) - \sin(2\frac{n_p \pi}{h_p} h_1) + \sin(2\frac{n_p \pi}{h_p} h_2)}{4\frac{n_p \pi}{h_p}}] +$$

$$\sum_{n_p=1}^{\infty}\sum_{n_q=1}^{\infty} \in_{n_p} (\Lambda_{0n_p}\Re_{0n_p}(r)+\Lambda^*_{0n_p}\Re^*_{0n_p}(r)) \in_{n_q} (\overline{\Lambda_{1n_q}\Re_{1n_q}(r)}+\overline{\Lambda^*_{1n_q}\Re^*_{1n_q}(r)})(\frac{n_p\pi}{h_p})(\frac{n_q\pi}{h_p})$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_q\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_q\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]]\pi+$$

$$[\sum_{n_p=1}^{\infty}\in_{n_p} (\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r)) \in_{n_p} (\overline{\Lambda_{0n_p}\Re_{0n_p}(r)}+\overline{\Lambda^*_{0n_p}\Re^*_{0n_p}(r)})(\frac{n_p\pi}{h_p})^2$$

$$[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)-\sin(2\frac{n_p\pi}{h_p}h_1)+\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+\sum_{n_p=1}^{\infty}\sum_{n_q=1}^{\infty} \in_{n_p} (\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))$$

$$\in_{n_q} (\overline{\Lambda_{0n_q}\Re_{0n_q}(r)}+\overline{\Lambda^*_{0n_q}\Re^*_{0n_q}(r)})(\frac{n_p\pi}{h_p})(\frac{n_q\pi}{h_p})$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_q\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_q\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]]\pi+$$

$$[\sum_{n_p=1}^{\infty}\in_{n_p} (\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r)) \in_{n_p} (\overline{\Lambda_{2n_p}\Re_{2n_p}(r)}+\overline{\Lambda^*_{2n_p}\Re^*_{2n_p}(r)})(\frac{n_p\pi}{h_p})^2$$

$$[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)-\sin(2\frac{n_p\pi}{h_p}h_1)+\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+\sum_{n_p=1}^{\infty}\sum_{n_q=1}^{\infty} \in_{n_p} (\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))$$

$$\in_{n_q} (\overline{\Lambda_{2n_q}\Re_{2n_q}(r)}+\overline{\Lambda^*_{2n_q}\Re^*_{2n_q}(r)})(\frac{n_p\pi}{h_p})(\frac{n_p\pi}{h_p})$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_q\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_q\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]]\frac{\pi}{2}+$$

$$+\sum_{p=2,3,}^{\infty}[[\sum_{n_p=1}^{\infty}\in_{n_p}(\Lambda_{pn_p}\Re_{pn_p}(r)+\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\in_{n_p}(\overline{\Lambda_{p+1n_p}\Re_{p+1n_p}(r)}+$$

$$\overline{\overline{\Lambda_{p+1n_p}}\,\overline{\Re_{p+1n_p}(r)}})(\frac{n_p\pi}{h_p})^2\,[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)-\sin(2\frac{n_p\pi}{h_p}h_1)+\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+$$

$$\sum_{n_p=1}^{\infty}\sum_{n_q=1}^{\infty}\in_{n_p}(\Lambda_{pn_p}\Re_{pn_p}(r)+\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\,\varepsilon_{n_q}(\overline{\Lambda_{p+1n_q}\Re_{p+1n_q}(r)}+$$

$$\overline{\Lambda^*_{p+1n_q}\Re^*_{p+1n_q}(r)})(\frac{n_p\pi}{h_p})(\frac{n_p\pi}{h_p})$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_q\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_q\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]]\frac{\pi}{2}+$$

$$\sum_{p=2,3,}^{\infty}[[\sum_{n_p=1}^{\infty}\in_{n_p}(\Lambda_{pn_p}\Re_{pn_p}(r)+\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\,\varepsilon_{n_p}(\overline{\Lambda_{p-1n_p}\Re_{p-1n_p}(r)}+\overline{\overline{\Lambda_{p-1n_p}}\,\overline{\Re_{p-1n_p}(r)}})(\frac{n_p\pi}{h_p})^2$$

$$[\frac{2\frac{n_p\pi}{h_p}(h_1-h_2)-\sin(2\frac{n_p\pi}{h_p}h_1)+\sin(2\frac{n_p\pi}{h_p}h_2)}{4\frac{n_p\pi}{h_p}}]+\sum_{n_p=1}^{\infty}\sum_{n_q=1}^{\infty}\in_{n_p}(\Lambda_{pn_p}\Re_{pn_p}(r)+$$

$$\Lambda^*_{pn_p}\Re^*_{pn_p}(r))\,\varepsilon_{n_q}(\overline{\Lambda_{p-1n_q}\Re_{p-1n_q}(r)}+\overline{\Lambda^*_{p-1n_q}\Re^*_{p-1n_q}(r)})(\frac{n_p\pi}{h_p})(\frac{n_q\pi}{h_p})$$

$$\frac{1}{(\frac{n_p\pi}{h_p})^2-(\frac{n_q\pi}{h_p})^2}[\frac{n_q\pi}{h_p}\cos(\frac{n_q\pi}{h_p}h_1)\sin(\frac{n_p\pi}{h_p}h_1)-(\frac{n_q\pi}{h_p}h_1)\cos(\frac{n_q\pi}{h_p}h_2)\sin(\frac{n_p\pi}{h_p}h_2)-$$

$$-\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_1)\sin(\frac{n_q\pi}{h_p}h_1)+\frac{n_p\pi}{h_p}\cos(\frac{n_p\pi}{h_p}h_2)\sin(\frac{n_q\pi}{h_p}h_2)]]\frac{\pi}{2}]b_p\,.$$

5.2 Υπολογισμός του όρου $\int_{h_2}^{h_1}(\int_0^{2\pi} \overline{x}^{(1)}\nabla\Phi_t^{(1)})\overline{n}dS$ για το πεδίο (III)

Έχουμε δείξει στο Κεφάλαιο 2 –σελίδα 19– ότι

$$\overline{x}^{(1)}\nabla\Phi_t^{(1)} = [(X_{g_1}^{(1)} + X_5^{(1)}z)\frac{\partial\phi(r,\theta,z)}{\partial r}\cos\theta - (X_{g_1}^{(1)} + X_5^{(1)}z)\frac{1}{r}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\sin\theta$$
$$+(X_{g_3}^{(1)} - X_5^{(1)}r\cos\theta)\frac{\partial\phi(r,\theta,z)}{\partial z}](-i\omega)]$$

Δηλαδή

$$\overline{x}^{(1)}\nabla\Phi_t^{(1)} = (X_{g_1}^{(1)} + X_5^{(1)}z)\ (-i\omega)\ [-i\omega\frac{H}{2}d(-\frac{z_0}{H/2}\frac{r}{2h_pd} - d\frac{\phi_0}{H/2}\frac{(z^2-0{,}75r^2)}{2h_pd^2}\cos\theta) -$$

$$-i\omega\frac{H}{2}d\ \sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{mn_p}\frac{\partial\Re_{mn_p}(r)}{\partial r} + \Lambda_{mn_p}^*\frac{\partial\Re_{mn_p}^*(r)}{\partial r})\cos(\frac{n_p\pi z}{h_p})]\cos(m\theta)]\cos(\theta) -$$

$$-(X_{g_1}^{(1)} + X_5^{(1)}z)\ (-i\omega)\ \frac{1}{r}[-i\omega\frac{H}{2}d(d\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_pd^2}\sin\theta) + i\omega\frac{H}{2}d$$
$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{mn_p}\Re_{mn_p}(r) + \Lambda_{mn_p}^*\Re_{mn_p}^*(r))\cos(\frac{n_p\pi z}{h_p})m]\sin(m\theta)]]\sin(\theta) +$$

$$+(X_{g_3}^{(1)} - X_5^{(1)}r\cos\theta)(-i\omega)\ [i\omega\frac{H}{2}d(\frac{z_0}{H/2}\frac{2z}{2h_pd} - d\frac{\phi_0}{H/2}\frac{r2z}{2h_pd^2}\cos\theta) - i\omega\frac{H}{2}d$$
$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{mn_p}\Re_{mn_p}(r) + \Lambda_{mn_p}^*\Re_{mn_p}^*(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]\cos(m\theta)]]$$

Θα υπολογίσουμε τον όρο

$$\int_{h_2}^{h_1}(\int_0^{2\pi}\overline{x}^{(1)}\nabla\Phi_t^{(1)})ndS = \int_{h_2}^{h_1}(\int_0^{2\pi}\overline{x}^{(1)}\nabla\Phi_t^{(1)}\cos\theta d\theta)b_pdz =$$

$$=(X_{g_1}^{(1)})\ (-i\omega)\ [i\omega\frac{H}{2}d(-d\frac{\phi_0}{H/2}[\frac{1}{2h_p d^2}(\frac{d_1^{\ 3}-d_2^3}{3})-\frac{(0{,}75b_p^{\ 2})}{2h_p d^2}(d_1-d_2)]\pi)-i\omega\frac{H}{2}d$$

$$[\sum_{n_p=1}^{\infty}\in_{n_p}\ (\Lambda_{2n_p}\frac{\partial\Re_{2n_p}(r)}{\partial r}+\Lambda_{2n_p}^{*}\frac{\partial\Re_{2n_p}^{*}(r)}{\partial r})(\sin(\frac{n_p\pi d_1}{h_p})-\sin(\frac{n_p\pi d_2}{h_p})\frac{h_p}{n_p\pi}\frac{\pi}{2}+$$

$$[\in_0\ (\Lambda_{20}\frac{\partial\Re_{20}(r)}{\partial r}+\Lambda_{20}^{*}\frac{\partial\Re_{20}^{*}(r)}{\partial r})]\frac{\pi}{2}(d_1-d_2)]\ \ b_p+$$

$$+(X_5^{(1)}b_p^2)\ (-i\omega)\ [i\omega\frac{H}{2}d(-d\frac{\phi_0}{H/2}[\frac{1}{2h_p d^2}(\frac{d_1^{\ 4}-d_2^4}{4})-\frac{(0{,}75b_p^{\ 2})}{2h_p d^2}(\frac{d_1^{\ 2}-d_2^{\ 2}}{2})]\pi)-i\omega\frac{H}{2}d$$

$$[[\sum_{n_p=1}^{\infty}\in_{n_p}\ (\Lambda_{2n_p}\frac{\partial\Re_{2n_p}(r)}{\partial r}+\Lambda_{2n_p}^{*}\frac{\partial\Re_{2n_p}^{*}(r)}{\partial r})((\sin(\frac{n_p\pi d_1}{h_p})-\sin(\frac{n_p\pi d_2}{h_p})+$$

$$\frac{n_p\pi d_1}{h_p}\cos(\frac{n_p\pi d_1}{h_p})-\frac{n_p\pi d_2}{h_p}\cos(\frac{n_p\pi d_2}{h_p}))\frac{h_p}{n_p\pi}]\frac{\pi}{2}+[\in_0\ (\Lambda_{20}\frac{\partial\Re_{20}(r)}{\partial r}+$$

$$\Lambda_{20}^{*}\frac{\partial\Re_{20}^{*}(r)}{\partial r})(\frac{d_1^{\ 2}}{2}-\frac{d_2^{\ 2}}{2})]]\ \ b_p+$$

$$-(X_{g_1}^{(1)})\ (-i\omega)\ \frac{1}{r}[-i\omega\frac{H}{2}d(d\frac{\phi_0}{H/2}(\frac{r}{2h_p d^2}(\frac{d_1^{\ 3}-d_2^{\ 3}}{3})-\frac{0{,}25r^2)}{2h_p d^2}(d_1-d_2)\pi)+i\omega\frac{H}{2}d$$

$$[\sum_{n_p=1}^{\infty}\in_{n_p}\ (\Lambda_{2n_p}\Re_{2n_p}(r)+\Lambda_{2n_p}^{*}\Re_{2n_p}^{*}(r))2\frac{\pi}{2}]\ (\sin(\frac{n_p\pi d_1}{h_p})-\sin(\frac{n_p\pi d_2}{h_p})\frac{h_p}{n_p\pi}+$$

$$[\in_0\ (\Lambda_{20}\Re_{20}(r)+\Lambda_{20}^{*}\Re_{20}^{*}(r))2\frac{\pi}{2}(d_1-d_2)]]\ \ b_p$$

$$-(X_5^{(1)})\ (-i\omega)\ \frac{1}{r}[-i\omega\frac{H}{2}d(d\frac{\phi_0}{H/2}(\frac{r}{2h_p d^2}(\frac{d_1^{\ 4}-d_2^{\ 4}}{4})-\frac{0{,}25r^2)}{2h_p d^2}(\frac{d_1^{\ 2}-d_2^{\ 2}}{2})\pi)+i\omega\frac{H}{2}d$$

$$[\sum_{n_p=1}^{\infty}\in_{n_p}\ (\Lambda_{2n_p}\Re_{2n_p}(r)+\Lambda_{2n_p}^{*}\Re_{2n_p}^{*}(r))2\frac{\pi}{2}]\ ((\sin(\frac{n_p\pi d_1}{h_p})-\sin(\frac{n_p\pi d_2}{h_p})+$$

$$\frac{n_p\pi d_1}{h_p}\cos(\frac{n_p\pi d_1}{h_p})-\frac{n_p\pi d_2}{h_p}\cos(\frac{n_p\pi d_2}{h_p}))\frac{h_p}{n_p\pi}]\frac{\pi}{2}+[\in_0\ (\Lambda_{20}\Re_{20}(r)+$$

$$\Lambda_{20}^{*}\Re_{20}^{*}(r))2\frac{\pi}{2}(\frac{d_1^{\ 2}-d_2^{\ 2}}{2})]]\ \ b_p+$$

$$+(X_{g_3}^{(1)})\ (-i\omega)\ [i\omega\frac{H}{2}d(-d\frac{\phi_0}{H/2}[\frac{1}{2h_p d^2}(\frac{d_1^{\ 3}-d_2^3}{3})-\frac{(0{,}75b_p^{\ 2})}{2h_p d^2}(d_1-d_2)]\frac{\pi}{2})+i\omega\frac{H}{2}d$$

$$[[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{2n_p}\Re_{2n_p}(r)+\Lambda_{2n_p}^{*}\Re_{2n_p}^{*}(r))(\cos(\frac{n_p\pi d_1}{h_p})-\cos(\frac{n_p\pi d_2}{h_p})]\frac{\pi}{2}+$$

$$[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{0n_p}\Re_{0n_p}(r)+\Lambda_{0n_p}^{*}\Re_{0n_p}^{*}(r))(\cos(\frac{n_p\pi d_1}{h_p})-\cos(\frac{n_p\pi d_2}{h_p})]\pi]]\ b_p$$

Όμως για τον υπολογισμός της δύναμης έκπτωσης πρέπει να υπολογίσουμε τον όρο

$$\overline{\overline{x}^{(1)}\nabla\Phi_t^{(1)}}^{T}=\omega(\overline{X}_{\mathrm{Re}}^{(1)}\cos(\omega t)+\overline{X}_{\mathrm{Im}}^{(1)}\sin(\omega t))(\nabla\phi_{\mathrm{Im}}^{(1)}\cos(\omega t)-\nabla\phi_{\mathrm{Re}}^{(1)}\sin(\omega t))=$$

$$=\overline{\omega(\overline{X}_{\mathrm{Re}}^{(1)}\nabla\phi_{\mathrm{Im}}^{(1)}\cos^2(\omega t)-\overline{X}_{\mathrm{Im}}^{(1)}\nabla\phi_{\mathrm{Re}}^{(1)}\sin^2(\omega t)+(-\overline{X}_{\mathrm{Re}}^{(1)}\nabla\phi_{\mathrm{Re}}^{(1)}+\overline{X}_{\mathrm{Im}}^{(1)}\nabla\phi_{\mathrm{Im}}^{(1)})\cos(\omega t)\sin(\omega t))}^{t}=$$

$$=\frac{1}{2}\omega(\overline{X}_{\mathrm{Re}}^{(1)}\nabla\phi_{\mathrm{Im}}^{(1)}-\overline{X}_{\mathrm{Im}}^{(1)}\nabla\phi_{\mathrm{Re}}^{(1)})=\frac{1}{2}\omega\,\mathrm{Im}(X^{(1)^*}\nabla\phi^{(1)})$$

Όπου $\overline{X_g^{(1)}}$: οι μεταφορικές κινήσεις του κέντρου βάρους της κατασκευής.

$X_5^{(1)}$: περιστροφή γύρω από τον άξονα GX_2

ϕ_0 : μιγαδικό πλάτος της ταλαντωτικής κίνησης του σώματος σε pitch.

b_p η ακτίνα του $p-$στου «από κάτω» όρου.

5.3 Υπολογισμός της οριζόντιας δύναμης έκπτωσης για το πεδίο (III)

Η οριζόντια δύναμη έκπτωσης F_X για το Πεδίο (III) Κεφάλαιο 2 –σελίδα 16– υπολογίζεται από τη σχέση

$$F_X = -\int_{WL} \frac{1}{2}\rho g \zeta_r^{(1)2} \bar{n} dl + MR^{(1)} \overline{X_g^{(1)}}'' - \iint_{S_0} -\frac{1}{2}\rho \left|\nabla\Phi^{(1)}\right|^2 \bar{n} dS - \iint_{S_0} -\rho(\overline{X}^{(1)} \nabla\Phi_t^{(1)})\bar{n} dS =$$

$$= MR^{(1)} \overline{X_g^{(1)}}'' + \frac{1}{2}\rho\, [\underline{\iint_{S_0} \left|\nabla\Phi^{(1)}\right|^2 \bar{n} dS}\,] + \rho\, [\underline{\iint_{S_0} (\overline{X}^{(1)} \nabla\Phi_t^{(1)})\bar{n} dS}\,].$$

Όμως οι παραστάσεις $\underline{\iint_{S_0} \left|\nabla\Phi^{(1)}\right|^2 \bar{n} dS}$ $\quad \underline{[\iint_{S_0} (\overline{X}^{(1)} \nabla\Phi_t^{(1)})\bar{n} dS]}$

είναι γνωστές από τα προηγούμενα (σελίδα 159, σελίδα 168).

Όπου *ρ:* η πυκνότητα νερού.

g: η επιτάχυνση της βαρύτητας.

M: η μάζα αξονοσυμμετρικού σώματος.

$R^{(1)}$: ο πίνακας που περιέχει τις γωνίες περιστροφής.

$X_g^{(1)}$: οι μεταφορικές κινήσεις του κέντρου βάρους της κατασκευής.

ω : η ιδιοσυχνότητα της κατασκευής.

φ : η διαφορά φάσης.

Επομένως μπορούμε να υπολογίσουμε την οριζόντια δύναμη έκπτωσης F_X για το Πεδίο (III).

6^{o} ΚΑΤΑΚΟΡΥΦΗ ΔΥΝΑΜΗ ΕΚΠΤΩΣΗΣ ΓΙΑ ΤΟ ΠΕΔΙΟ (Ι)

6.1 Υπολογισμός της κατακόρυφης δύναμης έκπτωσης για το πεδίο (Ι)

Η κατακόρυφη δύναμη έκπτωσης F_Z για το Πεδίο (Ι) είναι ίση με μηδέν.

7^o ΚΑΤΑΚΟΡΥΦΗ ΔΥΝΑΜΗ ΕΚΠΤΩΣΗΣ ΓΙΑ ΤΟ ΠΕΔΙΟ (ΙΙ)

7.1 Υπολογισμός του όρου $\int_{a_\ell}^{a_{\ell+1}} (\int_0^{2\pi} \left|\nabla\Phi^{(1)}\right|^2)\bar{n}dS$ για το πεδίο (ΙΙ)

Έχουμε αποδείξει στο Κεφάλαιο 4, ότι για δακτυλιοειδή στοιχεία στο πεδίο (ΙΙ): $a_\ell \le r \le a_{\ell+1}$ $\kappa\alpha\iota$ $d_\ell \le z \le d$ η σχέση που μας δίνει το $(\left|\nabla\Phi^{(1)}\right|^2)$ της ταχύτητας πρώτης τάξης είναι

$$(\left|\nabla\Phi^{(1)}\right|^2) =$$

$$= \overline{\left(\frac{\partial\phi(r,\theta,z)}{\partial r}\right)^2}^T + \overline{\frac{1}{r^2}\left(\frac{\partial\phi(r,\theta,z)}{\partial\theta}\right)^2}^T + \overline{\left(\frac{\partial\phi(r,\theta,z)}{\partial z}\right)^2}^T =$$

$$= \frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial r}\frac{\partial\overline{\phi(r,\theta,z)}}{\partial r} + \frac{1}{r^2}\frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\frac{\overline{\partial\phi(r,\theta,z)}}{\partial\theta} + \frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial z}\frac{\overline{\partial\phi(r,\theta,z)}}{\partial z} =$$

$$= \frac{1}{2}\frac{\omega^2H^2d^2}{4}[\, d^2\frac{\phi_0}{H/2}\frac{\overline{\phi_0}}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})^2(\cos\theta)^2\,] - \frac{1}{2}\frac{\omega^2H^2d^2}{4}$$

$$\left[d\frac{\phi_0}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\cos\theta\right]\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\overline{\frac{\partial\Re_{m_{\kappa_\ell}}(r)}{\partial r}} + \overline{\Lambda^*_{m_{\kappa_\ell}}\frac{\partial\Re^*_{m_{\kappa_\ell}}(r)}{\partial r}}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))$$

$$\cos(m\theta)\,] - \frac{1}{2}\frac{\omega^2H^2d^2}{4}\left[d\frac{\phi_0}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\cos\theta\right]\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}\overline{[\Lambda_{m_{a_\ell}}\frac{\partial\Re_{m_{a_\ell}}(r)}{\partial r}}$$

$$\overline{+\Lambda^*_{m_{a_\ell}}\frac{\partial\Re^*_{m_{a_\ell}}(r)}{\partial r}}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\,]\cos(m\theta)] - \frac{1}{2}\frac{\omega^2H^2d^2}{4}\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\frac{\partial\Re_{m_{\kappa_\ell}}(r)}{\partial r} +$$

$$\Lambda^*_{m_{\kappa_\ell}}\frac{\partial\Re^*_{m_{\kappa_\ell}}(r)}{\partial r}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\cos(m\theta)\left[d\frac{\overline{\phi_0}}{H/2}r(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\cos\theta\right]$$

$$-\frac{1}{2}\frac{\omega^2H^2d^2}{4}\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\frac{\partial\Re_{m_{a_\ell}}(r)}{\partial r} + \Lambda^*_{m_{a_\ell}}\frac{\partial\Re^*_{m_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\cos(m\theta)$$

$$\left[d\frac{\overline{\phi_0}}{H/2} r(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\cos\theta \right] + \frac{1}{2}\frac{\omega^2 H^2 d^2}{4} [\sum_{m=0}^{\infty} [\Lambda_{m_{\kappa_\ell}} \frac{\partial \Re_{m_{\kappa_\ell}}(r)}{\partial r} +$$

$$\Lambda^*_{m_{\kappa_\ell}} \frac{\partial \Re^*_{m_{\kappa_\ell}}(r)}{\partial r}] N_{\kappa_\ell}^{-1/2} \cosh(\kappa_\ell (z-h_\ell)) \cos(m\theta) + \sum_{m=0}^{\infty} [\sum_{\substack{\ell=1 \\ a_\ell}}^{\infty} [\Lambda_{m_{a_\ell}} \frac{\partial \Re_{m_{a_\ell}}(r)}{\partial r}$$

$$+ \Lambda^*_{m_{a_\ell}} \frac{\partial \Re^*_{m_{a_\ell}}(r)}{\partial r}] N_{a_\ell}^{-1/2} \cos(a_\ell (z-h_\ell)) \cos(m\theta)] [\sum_{n=0}^{\infty} \overline{[\Lambda_{n_{\kappa_\ell}} \frac{\partial \Re_{n_{\kappa_\ell}}(r)}{\partial r}} +$$

$$\overline{\Lambda^*_{n_{\kappa_\ell}} \frac{\partial \Re^*_{n_{\kappa_\ell}}(r)}{\partial r}}] N_{\kappa_\ell}^{-1/2} \cosh(\kappa_\ell (z-h_\ell)) \cos(n\theta) + \sum_{n=0}^{\infty} [\sum_{\substack{i=1 \\ a_i}}^{\infty} \overline{[\Lambda_{n_{a_i}} \frac{\partial \Re_{n_{a_i}}(r)}{\partial r}}$$

$$+ \overline{\Lambda^*_{n_{a_i}} \frac{\partial \Re^*_{n_{a_i}}(r)}{\partial r}}] N_{a_i}^{-1/2} \cos(a_i (z-h_\ell))] \cos(n\theta)] +$$

$$+ \frac{1}{2r^2}\frac{\omega^2 H^2 d^2}{4} [d^2 \frac{\phi_0}{H/2}\frac{\overline{\phi_0}}{H/2} r^2 (\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})^2 (\sin\theta)^2] - \frac{1}{2r^2}\frac{\omega^2 H^2 d^2}{4}$$

$$\left[d\frac{\phi_0}{H/2} r(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\sin\theta \right] [\sum_{m=0}^{\infty} \overline{[\Lambda_{m_{\kappa_\ell}} \Re_{m_{\kappa_\ell}}(r) +} \overline{\Lambda^*_{m_{\kappa_\ell}} \Re^*_{m_{\kappa_\ell}}(r)}] m N_{\kappa_\ell}^{-1/2} \cosh(\kappa_\ell (z-h_\ell))$$

$$\sin(m\theta) + \sum_{m=0}^{\infty} [\sum_{\substack{\ell=1 \\ a_\ell}}^{\infty} \overline{[\Lambda_{m_{a_\ell}} \Re_{m_{a_\ell}}(r)} \overline{+ \Lambda^*_{m_{a_\ell}} \Re^*_{m_{a_\ell}}(r)}] m N_{a_\ell}^{-1/2} \cos(a_\ell (z-h_\ell)) \sin(m\theta)]] -$$

$$- \frac{1}{2r^2}\frac{\omega^2 H^2 d^2}{4} \left[d\frac{\overline{\phi_0}}{H/2} r(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\sin\theta \right] [\sum_{m=0}^{\infty} [\Lambda_{m_{\kappa_\ell}} \Re_{m_{\kappa_\ell}}(r) +$$

$$\Lambda^*_{m_{\kappa_\ell}} \Re^*_{m_{\kappa_\ell}}(r)] m N_{\kappa_\ell}^{-1/2} \cosh(\kappa_\ell (z-h_\ell)) \sin(m\theta) + \sum_{m=0}^{\infty} [\sum_{\substack{\ell=1 \\ a_\ell}}^{\infty} [\Lambda_{m_{a_\ell}} \Re_{m_{a_\ell}}(r)$$

$$+ \Lambda^*_{m_{a_\ell}} \Re^*_{m_{a_\ell}}(r)] m N_{a_\ell}^{-1/2} \cos(a_\ell (z-h_\ell)) \sin(m\theta)]]$$

$$+ \frac{1}{2r^2}\frac{\omega^2 H^2 d^2}{4} [\sum_{m=0}^{\infty} [\Lambda_{m_{\kappa_\ell}} \Re_{m_{\kappa_\ell}}(r) + \Lambda^*_{m_{\kappa_\ell}} \Re^*_{m_{\kappa_\ell}}(r)] m N_{\kappa_\ell}^{-1/2} \cosh(\kappa_\ell (z-h_\ell)) \sin(m\theta) +$$

$$\sum_{m=0}^{\infty} [\sum_{\substack{\ell=1 \\ a_\ell}}^{\infty} [\Lambda_{m_{a_\ell}} \Re_{m_{a_\ell}}(r) + \Lambda^*_{m_{a_\ell}} \Re^*_{m_{a_\ell}}(r)] m N_{a_\ell}^{-1/2} \cos(a_\ell (z-h_\ell)) \sin(m\theta)]$$

$$[\sum_{n=0}^{\infty}[\overline{\Lambda_{n_{\kappa_\ell}}\Re_{n_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*_{n_{\kappa_\ell}}\Re^*_{n_{\kappa_\ell}}(r)}]nN_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\sin(n\theta)$$

$$+\sum_{n=0}^{\infty}[\sum_{\substack{i=1\\a_i}}^{\infty}[\overline{\Lambda_{n_{a_i}}\Re_{n_{a_i}}(r)}\ \overline{+\Lambda^*_{n_{a_i}}\Re^*_{n_{a_i}}(r)]}nN_{a_i}^{-1/2}\cos(a_i(z-h_\ell))\ \sin(n\theta)]]]+$$

$$+\frac{1}{2}\frac{\omega^2H^2d^2}{4}\left[\frac{z_0}{H/2}\frac{1}{d}-\frac{\phi_0}{H/2}r(\frac{1}{d})\cos\theta\right]\left[\frac{z_0}{H/2}\frac{1}{d}-\frac{\overline{\phi_0}}{H/2}r(\frac{1}{d})\cos\theta\right]+\frac{1}{2}\frac{\omega^2H^2d^2}{4}$$

$$\left[\frac{z_0}{H/2}\frac{1}{d}-\frac{\phi_0}{H/2}r(\frac{1}{d})\cos\theta\right][\sum_{m=0}^{\infty}[\overline{\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell$$

$$\cos(m\theta)-\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)}\ \overline{+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell\ \cos(m\theta)]]+$$

$$+\frac{1}{2}\frac{\omega^2H^2d^2}{4}\left[\frac{z_0}{H/2}\frac{1}{d}-\frac{\overline{\phi_0}}{H/2}r(\frac{1}{d})\cos\theta\right][\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell\ \cos(m\theta)-\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$

$$+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell\ \cos(m\theta)]]+\frac{1}{2}\frac{\omega^2H^2d^2}{4}[\sum_{m=0}^{\infty}[\overline{\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+}$$

$$\overline{\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell\ \cos(m\theta)-\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell\ \cos(m\theta)]][\sum_{n=0}^{\infty}[\Lambda_{n_{\kappa_\ell}}\Re_{n_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{n_{\kappa_\ell}}\Re^*_{n_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell\ \cos(n\theta)-\sum_{n=0}^{\infty}[\sum_{\substack{i=1\\a_i}}^{\infty}[\Lambda_{n_{a_i}}\Re_{n_{a_i}}(r)$$

$$+\Lambda^*_{n_{a_i}}\Re^*_{n_{a_i}}(r)]N_{a_i}^{-1/2}\sin(a_i(z-h_\ell))a_i\ \cos(n\theta)]]].$$

Για τον υπολογισμό της κατακόρυφης δύναμης έκπτωσης F_Z στο πεδίο II, θα υπολογίσουμε τον όρο

$$\int_{a_\ell}^{a_{\ell+1}}(\int_0^{2\pi}\left|\nabla\Phi^{(1)}\right|^2)\overline{n_3}dS$$

Όπου a_i η ακτίνα του $i-$στου «από πάνω» στοιχείου

$\overline{n_3}=1$

$dS = rd\theta dr$

r η ακτίνα του $i-$στου «από πάνω» στοιχείου

Επομένως

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}}(\int_0^{2\pi}\left|\nabla\Phi^{(1)}\right|^2)\overline{n_3}dS=\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}}(\int_0^{2\pi}\left|\nabla\Phi^{(1)}\right|^2)1\ \ r\ \ d\theta dr.$$

Υπολογίζουμε τη σχέση

$$\int_0^{2\pi}(\left|\nabla\Phi^{(1)}\right|^2)1\ \ r\ \ d\theta=$$

$$=\frac{\omega^2H^2d^2}{8}\pi[\ d^2\frac{\phi_0}{H/2}\frac{\overline{\phi_0}}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})^2]r-$$

$$-\frac{\omega^2H^2d^2}{8}\pi\left[d\frac{\phi_0}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\right]\overline{[\Lambda_{1_{\kappa_\ell}}\frac{\partial\Re_{1_{\kappa_\ell}}(r)}{\partial r}+}$$

$$\overline{\Lambda^*_{1_{\kappa_\ell}}\frac{\partial\Re^*_{1_{\kappa_\ell}}(r)}{\partial r}}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))]r-$$

$$-\frac{\omega^2H^2d^2}{8}\pi\left[d\frac{\phi_0}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\cos\theta\right][\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{1_{a_\ell}}\frac{\partial\Re_{1_{a_\ell}}(r)}{\partial r}}+$$

$$\overline{\Lambda^*_{1_{a_\ell}}\frac{\partial\Re^*_{1_{a_\ell}}(r)}{\partial r}}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]r-\frac{\omega^2H^2d^2}{8}\pi\ [\Lambda_{1_{\kappa_\ell}}\frac{\partial\Re_{1_{\kappa_\ell}}(r)}{\partial r}+$$

$$\Lambda^*_{1_{\kappa_\ell}}\frac{\partial\Re^*_{1_{\kappa_\ell}}(r)}{\partial r}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))]r\left[d\frac{\overline{\phi_0}}{H/2}r(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\cos\theta\right]-\frac{\omega^2H^2d^2}{8}\pi$$

$$[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\frac{\partial\Re_{1_{a_\ell}}(r)}{\partial r}+\Lambda^*_{1_{a_\ell}}\frac{\partial\Re^*_{1_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))r\left[d\frac{\overline{\phi_0}}{H/2}r(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\cos\theta\right]+$$

$$+\frac{\omega^2H^2d^2}{8}\pi\ [\sum_{m=1}^{\infty}\Bigg[\ [[\Lambda_{m_{\kappa_\ell}}\frac{\partial\Re_{m_{\kappa_\ell}}(r)}{\partial r}+\Lambda^*_{m_{\kappa_\ell}}\frac{\partial\Re^*_{m_{\kappa_\ell}}(r)}{\partial r}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))]+$$

$$[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\frac{\partial\Re_{m_{a_\ell}}(r)}{\partial r}+\Lambda^*_{m_{a_\ell}}\frac{\partial\Re^*_{m_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\cos(m\theta)][[\overline{\Lambda_{m_{\kappa_\ell}}\frac{\partial\Re_{m_{\kappa_\ell}}(r)}{\partial r}}+$$

$$\overline{\Lambda^*_{m_{\kappa_\ell}} \frac{\partial \Re^*_{m_{\kappa_\ell}}(r)}{\partial r}}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\,]+[\sum_{\substack{n=1\\a_n}}^{\infty}[\overline{\Lambda_{m_{a_n}} \frac{\partial \Re_{m_{a_n}}(r)}{\partial r}}+$$

$$\left.\overline{\Lambda^*_{m_{a_n}} \frac{\partial \Re^*_{m_{a_n}}(r)}{\partial r}}]N_{a_n}^{-1/2}\cos(a_n(z-h_\ell))]\right]+$$

$$2\left[\ [[\Lambda_{0_{\kappa_\ell}} \frac{\partial \Re_{0_{\kappa_\ell}}(r)}{\partial r}+\Lambda^*_{0_{\kappa_\ell}} \frac{\partial \Re^*_{0_{\kappa_\ell}}(r)}{\partial r}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\,]+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{0_{a_\ell}} \frac{\partial \Re_{0_{a_\ell}}(r)}{\partial r}+\right.$$

$$\Lambda^*_{0_{a_\ell}} \frac{\partial \Re^*_{0_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\,][[\overline{\Lambda_{0_{\kappa_\ell}} \frac{\partial \Re_{0_{\kappa_\ell}}(r)}{\partial r}}+$$

$$\overline{\Lambda^*_{0_{\kappa_\ell}} \frac{\partial \Re^*_{0_{\kappa_\ell}}(r)}{\partial r}}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\,]+[\sum_{\substack{n=1\\a_n}}^{\infty}[\overline{\Lambda_{0_{a_n}} \frac{\partial \Re_{0_{a_n}}(r)}{\partial r}}+$$

$$\left.\overline{\Lambda^*_{0_{a_n}} \frac{\partial \Re^*_{0_{a_n}}(r)}{\partial r}}]N_{a_n}^{-1/2}\cos(a_n(z-h_\ell))]\right]]r+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2d^2}{8}\pi[\ d^2\frac{\phi_0}{H/2}\frac{\overline{\phi_0}}{H/2}r^2(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})^2]r-\frac{1}{r^2}\frac{\omega^2H^2d^2}{8}\pi$$

$$\left[d\frac{\phi_0}{H/2}r(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\right][[\overline{\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)}]1N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+$$

$$+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)}\ \overline{+\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)}]1N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]]r-$$

$$-\frac{1}{r^2}\frac{\omega^2H^2d^2}{8}\pi\left[d\frac{\overline{\phi_0}}{H/2}r(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\right][[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]1N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)$$

$$+\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)]1N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]]r+$$

$$+\frac{1}{r^2}\frac{\omega^2H^2d^2}{8}\pi[\sum_{m=1}^{\infty}\Big[\ [[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)]mN_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+$$

$$\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)]mN_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\,]$$

$$[[\overline{\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)}]mN_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+\sum_{\substack{n=1\\a_n}}^{\infty}[\overline{\Lambda_{m_{a_n}}\Re_{m_{a_n}}(r)}$$

$$\left.\overline{+\Lambda^*_{m_{a_n}}\Re^*_{m_{a_n}}(r)}]mN_{a_n}^{-1/2}\cos(a_n(z-h_\ell))]\ \right]r+$$

$$+\frac{\omega^2H^2d^2}{8}\left[\frac{(z_0)^2}{(H/2)^2}\frac{1}{d^2}\right](2\pi-0)r+$$

$$+\frac{\omega^2H^2d^2}{8}\pi\left[\frac{d^2\phi_0}{H/2}\frac{\overline{\phi_0}}{H/2}r^2(\frac{1}{d^2})^2\right]r+$$

$$+\frac{\omega^2H^2d^2}{8}\pi\left[-\frac{\phi_0}{H/2}r(\frac{1}{d})\right][[\overline{\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell-$$

$$-\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)}\ \overline{+\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]r+$$

$$+\frac{\omega^2H^2d^2}{8}\pi\left[-\frac{\overline{\phi_0}}{H/2}r(\frac{1}{d})\right][[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+\Lambda^*_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell-$$

$$-\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)+\Lambda^*_{1_{a_\ell}}\Re^*_{1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]r+$$

$$+\frac{\omega^2H^2d^2}{8}\pi\,[\sum_{m=1}^{\infty}\Big[\ [[\overline{\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell-$$

$$-\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)}\ \overline{+\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]\,[[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell-\sum_{\substack{n=1\\a_n}}^{\infty}[\Lambda_{m_{a_n}}\Re_{m_{a_n}}(r)$$

$$+\Lambda^*_{m_{a_n}}\Re^*_{m_{a_n}}(r)]N_{a_n}^{-1/2}\sin(a_n(z-h_\ell))a_n]+2\ [[\overline{\Lambda_{0_{\kappa_\ell}}\Re_{0_{\kappa_\ell}}(r)+}$$

$$\overline{\Lambda^*_{0_{\kappa_\ell}}\Re^*_{0_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell-\sum_{\substack{n=1\\a_n}}^{\infty}[\overline{\Lambda_{0_{a_n}}\Re_{0_{a_n}}(r)}$$

$$\overline{+\Lambda^*_{0_{a_n}}\Re^*_{0_{a_n}}(r)}]N_{a_n}^{-1/2}\sin(a_n(z-h_\ell))a_n]\,[[\Lambda_{0_{\kappa_\ell}}\Re_{0_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{0_{\kappa_\ell}}\Re^*_{0_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell-\sum_{\substack{n=1\\a_n}}^{\infty}[\Lambda_{0_{a_n}}\Re_{0_{a_n}}(r)$$

$$+\Lambda^*_{0_{a_n}}\Re^*_{0_{a_n}}(r)]N_{a_n}^{-1/2}\sin(a_n(z-h_\ell))a_n]\Bigg]r$$

Στη συνέχεια υπολογίζουμε το $\int_{a_\ell}^{a_{\ell+1}}(\int_0^{2\pi}\left|\nabla\Phi^{(1)}\right|^2)1\ \ r\ \ d\theta dr$.

Για ευκολία στον υπολογισμό συμψηφίσαμε στο άθροισμα και την μια φανταστική ρίζα και τις άπειρες ρίζες της εξίσωσης διασποράς.

$$\int_{a_\ell}^{a_{\ell+1}}(\int_0^{2\pi}\left|\nabla\Phi^{(1)}\right|^2)1\ \ r\ \ d\theta dr=$$

$$=\frac{\omega^2H^2d^2}{8}\pi[\ d^2\frac{\phi_0}{H/2}\frac{\overline{\phi_0}}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})^2]\frac{1}{2}((a_{\ell+1})^2-(a_\ell)^2)-$$

$$-\frac{\omega^2H^2d^2}{8}\pi\left[d\frac{\phi_0}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\right][\int_{a_\ell}^{a_{\ell+1}}r\,[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}\overline{[\Lambda_{1_{a_\ell}}\frac{\partial\Re_{1_{a_\ell}}(r)}{\partial r}+}$$

$$\overline{\Lambda^*_{1_{a_\ell}}\frac{\partial\Re^*_{1_{a_\ell}}(r)}{\partial r}}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]dr- \tag{A}$$

$$-\frac{\omega^2H^2d^2}{8}\pi\left[d\frac{\overline{\phi_0}}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2d^2})\cos\theta\right][\int_{a_\ell}^{a_{\ell+1}}r[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\frac{\partial\Re_{1_{a_\ell}}(r)}{\partial r}+\Lambda^*_{1_{a_\ell}}\frac{\partial\Re^*_{1_{a_\ell}}(r)}{\partial r}]$$

$$N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]dr+ \tag{B}$$

$$+\frac{\omega^2H^2d^2}{8}\pi\,[\sum_{m=1}^{\infty}\Bigg[\int_{a_\ell}^{a_{\ell+1}}r[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\frac{\partial\Re_{m_{a_\ell}}(r)}{\partial r}+\Lambda^*_{m_{a_\ell}}\frac{\partial\Re^*_{m_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\,]$$

$$\sum_{\substack{n=0\\a_n}}^{\infty}[\overline{\Lambda_{m_{a_n}}\frac{\partial\Re_{m_{a_n}}(r)}{\partial r}}+\overline{\Lambda^*_{m_{a_n}}\frac{\partial\Re^*_{m_{a_n}}(r)}{\partial r}}]N_{a_n}^{-1/2}\cos(a_n(z-h_\ell))]]\Bigg]dr+ \tag{Γ}$$

$$2\left[\int_{a_\ell}^{a_{\ell+1}} r[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{0_{a_\ell}} \frac{\partial \Re_{0_{a_\ell}}(r)}{\partial r} + \Lambda^*_{0_{a_\ell}} \frac{\partial \Re^*_{0_{a_\ell}}(r)}{\partial r}] N_{a_\ell}^{-1/2} \cos(a_\ell(z-h_\ell)) \sum_{\substack{n=0\\a_n}}^{\infty}[\Lambda_{0_{a_n}} \overline{\frac{\partial \Re_{0_{a_n}}(r)}{\partial r}} + \right.$$

$$\left. \overline{\Lambda^*_{0_{a_n}} \frac{\partial \Re^*_{0_{a_\ell}}(r)}{\partial r}}] N_{a_n}^{-1/2} \cos(a_n(z-h_\ell))]] \right] dr +$$

$$+ \frac{\omega^2 H^2 d^2}{8} \pi [d^2 \frac{\phi_0}{H/2} \frac{\overline{\phi_0}}{H/2} (\frac{z-d}{d^2} + \frac{g}{\omega^2 d^2})^2] \frac{1}{2} ((a_{\ell+1})^2 - (a_\ell)^2) -$$

$$- \frac{\omega^2 H^2 d^2}{8} \pi \left[d \frac{\phi_0}{H/2} (\frac{z-d}{d^2} + \frac{g}{\omega^2 d^2}) \right] \int_{a_\ell}^{a_{\ell+1}} [\sum_{\substack{\ell=0\\a_\ell}}^{\infty} [\overline{\Lambda_{1_{a_\ell}} \Re_{1_{a_\ell}}(r)} \; \overline{+ \Lambda^*_{1_{a_\ell}} \Re^*_{1_{a_\ell}}(r)}]$$

$$N_{a_\ell}^{-1/2} \cos(a_\ell(z-h_\ell))]] dr - \qquad (\Delta)$$

$$- \frac{\omega^2 H^2 d^2}{8} \pi \left[d \frac{\overline{\phi_0}}{H/2} (\frac{z-d}{d^2} + \frac{g}{\omega^2 d^2}) \right] \int_{a_\ell}^{a_{\ell+1}} [[\sum_{a_\ell=0}^{\infty} [\Lambda_{1_{a_\ell}} \Re_{1_{a_\ell}}(r)$$

$$+ \Lambda^*_{1_{a_\ell}} \Re^*_{1_{a_\ell}}(r)] 1 N_{a_\ell}^{-1/2} \cos(a_\ell(z-h_\ell))]] dr + \qquad (E)$$

$$+ \frac{\omega^2 H^2 d^2}{8} \pi \sum_{m=1}^{\infty} \left[\int_{a_\ell}^{a_{\ell+1}} m^2 \frac{1}{r} [\sum_{\substack{\ell=0\\a_\ell}}^{\infty} [\Lambda_{m_{a_\ell}} \Re_{m_{a_\ell}}(r) + \Lambda^*_{m_{a_\ell}} \Re^*_{m_{a_\ell}}(r)] N_{a_\ell}^{-1/2} \cos(a_\ell(z-h_\ell))] \right.$$

$$\left. \sum_{\substack{n=0\\a_n}}^{\infty} [\overline{\Lambda_{m_{a_n}} \Re_{m_{a_n}}(r)} \; \overline{+ \Lambda^*_{m_{a_n}} \Re^*_{m_{a_n}}(r)}] m N_{a_n}^{-1/2} \cos(a_n(z-h_\ell))] dr \right] + \qquad (Z)$$

$$+ \frac{\omega^2 H^2 d^2}{8} \pi \left[\frac{(z_0)^2}{(H/2)^2} \frac{1}{d^2} \right] \frac{2}{2} ((a_{\ell+1})^2 + (a_\ell)^2) +$$

$$+ \frac{\omega^2 H^2 d^2}{8} \pi \left[\frac{d^2 \phi_0}{H/2} \frac{\overline{\phi_0}}{H/2} (\frac{1}{d^2})^2 \right] \frac{1}{4} ((a_{\ell+1})^4 - (a_\ell)^4) +$$

$$+ \frac{\omega^2 H^2 d^2}{8} \pi \left[- \frac{\phi_0}{H/2} (\frac{1}{d}) \right] \int_{a_\ell}^{a_{\ell+1}} r^2 [\sum_{\substack{\ell=0\\a_\ell}}^{\infty} [\overline{\Lambda_{1_{a_\ell}} \Re_{1_{a_\ell}}(r)}$$

$$\overline{+ \Lambda^*_{1_{a_\ell}} \Re^*_{1_{a_\ell}}(r)}] N_{a_\ell}^{-1/2} \sin(a_\ell(z-h_\ell)) a_\ell] dr + \qquad (H)$$

$$+ \frac{\omega^2 H^2 d^2}{8} \pi \left[- \frac{\overline{\phi_0}}{H/2} (\frac{1}{d}) \right] \int_{a_\ell}^{a_{\ell+1}} r^2 [\sum_{\substack{\ell=0\\a_\ell}}^{\infty} [\Lambda_{1_{a_\ell}} \Re_{1_{a_\ell}}(r)$$

$$+ \Lambda^*_{1_{a_\ell}} \Re^*_{1_{a_\ell}}(r)] N_{a_\ell}^{-1/2} \sin(a_\ell(z-h_\ell)) a_\ell] dr + \qquad (\Theta)$$

$$+\frac{\omega^2 H^2 d^2}{8}\pi \sum_{m=1}^{\infty}\Bigg[\int_{a_\ell}^{a_{\ell+1}} r[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\overline{\Lambda_{m_{a_\ell}} \Re_{m_{a_\ell}}(r)} \overline{+\Lambda^*{}_{m_{a_\ell}} \Re^*{}_{m_{a_\ell}}(r)}] N_{a_\ell}^{-1/2} \sin(a_\ell(z-h_\ell))a_\ell]$$

$$[\sum_{\substack{n=0\\a_n}}^{\infty}[\Lambda_{m_{a_n}} \Re_{m_{a_n}}(r) + \Lambda^*{}_{m_{a_n}} \Re^*{}_{m_{a_n}}(r)]\ N_{a_\ell}^{-1/2} \sin(a_\ell(z-h_\ell))a_\ell]dr + \qquad \text{(I)}$$

$$2\int_{a_\ell}^{a_{\ell+1}} r \sum_{a_\ell=0}^{\infty}[\overline{\Lambda_{0_{a_\ell}} \Re_{0_{a_\ell}}(r)} \overline{+\Lambda^*{}_{0_{a_\ell}} \Re^*{}_{0_{a_\ell}}(r)}] N_{a_\ell}^{-1/2} \sin(a_\ell(z-h_\ell))a_\ell]\,[\sum_{a_\ell=0}^{\infty}[\Lambda_{0_{a_\ell}} \Re_{0_{a_\ell}}(r)$$

$$+\Lambda^*{}_{0_{a_\ell}} \Re^*{}_{0_{a_\ell}}(r)]\ N_{a_\ell}^{-1/2} \sin(a_\ell(z-h_\ell))a_\ell]dr \Bigg]$$

Στη συνέχεια, από το *Παράρτημα Γ,* υπολογίζουμε τα ολοκληρώματα.

A)

$$[\int_{a_\ell}^{a_{\ell+1}} r \sum_{a_\ell=0}^{\infty}[\overline{\Lambda_{1_{a_\ell}} \frac{\partial \Re_{1_{a_\ell}}(r)}{\partial r}} + \overline{\Lambda^*_{1_{a_\ell}} \frac{\partial \Re^*_{1_{a_\ell}}(r)}{\partial r}}] N_{a_\ell}^{-1/2} \cos(a_\ell(z-h_\ell))]dr =$$

$$=\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\overline{\frac{\Lambda_{1_{a_\ell}}[a_\ell K_1(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}}\right]+\left[\overline{\frac{\Lambda^*{}_{1_{a_\ell}}[a_\ell K_1(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}}\right]$$

$$\left[\frac{1}{a_\ell^{\,2}}[\overline{2I_0(\mathrm{a}_\ell a_\ell)-2I_0(\mathrm{a}_{\ell+1}a_\ell)-\mathrm{a}_\ell a_\ell I_1(\mathrm{a}_\ell \kappa_\ell)+\mathrm{a}_{\ell+1}a_\ell I_1(\mathrm{a}_{\ell+1}a_\ell)}]\right]+$$

$$+\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\overline{\frac{\Lambda_{1_{a_\ell}}[a_\ell I_1(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}}\right]-\left[\overline{\frac{\Lambda^*{}_{1_{a_\ell}}[a_\ell I_1(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}}\right]$$

$$\int_{a_\ell}^{a_{\ell+1}} rK_2^*(a_\ell r)dr +$$

$$+\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\overline{\frac{\Lambda_{1_{a_\ell}}[K_1(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}}\right]-\left[\overline{\frac{\Lambda^*{}_{1_{a_\ell}}[K_1(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}}\right]$$

$$\left[\overline{\frac{I_0(\mathrm{a}_{\ell+1}a_\ell)-I_0(\mathrm{a}_\ell a_\ell)}{a_\ell}}\right]+$$

$$+\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{\Lambda_{1_{a_\ell}}[-1I_1(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]+\left[\frac{\Lambda^*_{1_{a_\ell}}[1I_1(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{-K_0(\mathrm{a}_{\ell+1}a_\ell)+K_0(\mathrm{a}_\ell a_\ell)}{a_\ell}\right]$$

B)

$$[\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} r[\sum_{a_\ell=0}^{\infty}[\Lambda_{1_{a_\ell}}\frac{\partial\Re_{1_{a_\ell}}(r)}{\partial r}+\Lambda^*_{1_{a_\ell}}\frac{\partial\Re^*_{1_{a_\ell}}(r)}{\partial r}]\,N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]dr=$$

$$=\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{\Lambda_{1_{a_\ell}}[a_\ell K_1(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]+\left[\frac{\Lambda^*_{1_{a_\ell}}[a_\ell K_1(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{1}{a_\ell^{\,2}}[2I_0(\mathrm{a}_\ell a_\ell)-2I_0(\mathrm{a}_{\ell+1}a_\ell)-\mathrm{a}_\ell a_\ell I_1(\mathrm{a}_\ell\kappa_\ell)+\mathrm{a}_{\ell+1}a_\ell I_1(\mathrm{a}_{\ell+1}a_\ell)]\right]+$$

$$+\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{\Lambda_{1_{a_\ell}}[a_\ell I_1(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]-\left[\frac{\Lambda^*_{1_{a_\ell}}[a_\ell I_1(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} rK_2(a_\ell r)dr+$$

$$+\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{\Lambda_{1_{a_\ell}}[K_1(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]-\left[\frac{\Lambda^*_{1_{a_\ell}}[K_1(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{I_0(\mathrm{a}_{\ell+1}a_\ell)-I_0(\mathrm{a}_\ell a_\ell)}{a_\ell}\right]+$$

$$+\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{\Lambda_{1_{a_\ell}}[-1I_1(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]+\left[\frac{\Lambda^*_{1_{a_\ell}}[1I_1(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{-K_0(\mathrm{a}_{\ell+1}a_\ell)+K_0(\mathrm{a}_\ell a_\ell)}{a_\ell}\right]$$

Γ)

$$\left[\int_{a_\ell}^{a_{\ell+1}} r[\sum_{\substack{\ell=0\\ a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}} \frac{\partial \Re_{m_{a_\ell}}(r)}{\partial r} + \Lambda^{*}_{m_{a_\ell}} \frac{\partial \Re^{*}_{m_{a_\ell}}(r)}{\partial r}] N_{a_\ell}^{-1/2} \cos(a_\ell(z-h_\ell))\,][\overline{\sum_{\substack{n=0\\ a_n}}^{\infty}[\Lambda_{m_{a_n}} \frac{\partial \Re_{m_{a_n}}(r)}{\partial r} +}\right.$$

$$\left.\overline{\Lambda^{*}_{m_{a_n}} \frac{\partial \Re^{*}_{m_{a_n}}(r)}{\partial r}}] N_{a_n}^{-1/2} \cos(a_n(z-h_\ell))]] \right] dr =$$

$$= \sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty} \sum_{\substack{\ell_2=0\\ a_{\ell_2}}}^{\infty} [$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1} K_m(a_{\ell_1} \mathrm{a}_\ell) \cos(a_{\ell_1}(z-h_\ell)) \mathrm{N}_{a_{\ell_1}}^{-1/2}] - \Lambda^{*}_{m_{a_{\ell_1}}}[a_{\ell_1} K_m(a_{\ell_1} \mathrm{a}_{\ell+1}) \cos(a_{\ell_1}(z-h_\ell)) \mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1} \mathrm{a}_{\ell+1}) K_m(a_{\ell_1} \mathrm{a}_\ell) - I_m(a_{\ell_1} \mathrm{a}_\ell) K_m(a_{\ell_1} \mathrm{a}_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_2}}}[a_{\ell_2} K_m(a_{\ell_2} \mathrm{a}_\ell) \cos(a_{\ell_2}(z-h_\ell)) \mathrm{N}_{a_{\ell_2}}^{-1/2}] + \Lambda^{*}_{m_{a_{\ell_2}}}[a_{\ell_2} K_m(a_{\ell_2} \mathrm{a}_{\ell+1}) \cos(a_{\ell_2}(z-h_\ell)) \mathrm{N}_{a_{\ell_2}}^{-1/2}]}{I_m(a_{\ell_2} \mathrm{a}_{\ell+1}) K_m(a_{\ell_2} \mathrm{a}_\ell) - I_m(a_{\ell_2} \mathrm{a}_\ell) K_m(a_{\ell_2} \mathrm{a}_{\ell+1})}}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} r I_{m+1}(a_{\ell_1} r) \overline{I_{m+1}(a_{\ell_2} r)} dr] +$$

$$+ \sum_{a_{\ell_1}=0}^{\infty} \sum_{\substack{a_{\ell_2}=0\\ a_{\ell_1} \neq a_{\ell_2}}}^{\infty} [$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1} K_m(a_{\ell_1} \mathrm{a}_\ell) \cos(a_{\ell_1}(z-h_\ell)) \mathrm{N}_{a_{\ell_1}}^{-1/2}] - \Lambda^{*}_{m_{a_{\ell_1}}}[a_{\ell_1} K_m(a_{\ell_1} \mathrm{a}_{\ell+1}) \cos(a_{\ell_1}(z-h_\ell)) \mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1} \mathrm{a}_{\ell+1}) K_m(a_{\ell_1} \mathrm{a}_\ell) - I_m(a_{\ell_1} \mathrm{a}_\ell) K_m(a_{\ell_1} \mathrm{a}_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_2}}}[a_{\ell_2} I_m(a_{\ell_2} \mathrm{a}_\ell) \cos(a_{\ell_2}(z-h_\ell)) \mathrm{N}_{a_{\ell_2}}^{-1/2}] - \Lambda^{*}_{m_{a_{\ell_2}}}[a_{\ell_2} I_m(a_{\ell_2} \mathrm{a}_{\ell+1}) \cos(a_{\ell_2}(z-h_\ell)) \mathrm{N}_{a_{\ell_2}}^{-1/2}}{I_m(a_{\ell_2} \mathrm{a}_{\ell+1}) K_m(a_{\ell_2} \mathrm{a}_\ell) - I_m(a_{\ell_2} \mathrm{a}_\ell) K_m(a_{\ell_2} \mathrm{a}_{\ell+1})}}\right]$$

$$\left[\frac{\mathrm{a}_{\ell+1}}{a_{\ell_1}{}^2 - a_{\ell_2}^2} \{a_{\ell_1} I_{m+2}(a_{\ell_1} \mathrm{a}_{\ell+1}) \overline{K_{m+1}(a_{\ell_2} \mathrm{a}_{\ell+1})} - a_{\ell_2} I_{m+1}(a_{\ell_1} \mathrm{a}_{\ell+1}) \overline{K_{m+2}(a_{\ell_2} \mathrm{a}_{\ell+1})}\} -\right.$$

$$\left.- \frac{\mathrm{a}_\ell}{a_{\ell_1}{}^2 - a_{\ell_2}^2} \{a_{\ell_1} I_{m+2}(a_{\ell_1} \mathrm{a}_\ell) \overline{K_{m+1}(a_{\ell_2} \mathrm{a}_\ell)} - a_{\ell_2} I_{m+1}(a_{\ell_1} \mathrm{a}_\ell) \overline{K_{m+2}(a_{\ell_2} \mathrm{a}_\ell)}\} \right] +$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}K_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[a_{\ell_1}K_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\overline{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} rI_{m+1}(a_{\ell_1}r)\overline{K_{m+1}(a_{\ell_1}r)}dr]+$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\ a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}K_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[a_{\ell_1}K_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\overline{\Lambda_{m_{a_{\ell_2}}}[mK_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_2}}}[mK_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]}}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} rI_{m+1}(a_{\ell_1}r)\overline{I_m(a_{\ell_2}r)}dr+$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\ a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}K_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_\ell}}[a_{\ell_1}K_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\overline{\Lambda_{m_{a_{\ell_2}}}[-mI_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_2}}}[mI_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}}}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\mathrm{a}_{\ell+1}}{a_{\ell_1}{}^2-a_{\ell_2}^2}\{a_{\ell_1}I_{m+2}(a_{\ell_1}\mathrm{a}_{\ell+1})\overline{K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}-a_{\ell_2}I_{m+1}(a_{\ell_1}\mathrm{a}_{\ell+1})\overline{K_{m+1}(a_{\ell_2}\mathrm{a}_{\ell+1})}\}-\right.$$

$$-\frac{\mathrm{a}_\ell}{{a_{\ell_1}}^2-a_{\ell_2}^2}\{a_{\ell_1}I_{m+2}(a_{\ell_1}\mathrm{a}_\ell)\overline{K_m(a_{\ell_2}\mathrm{a}_\ell)}-a_{\ell_2}I_{m+1}(a_{\ell_1}\mathrm{a}_\ell)\overline{K_{m+1}(a_{\ell_2}\mathrm{a}_\ell)}\}\Bigg]+$$

$$+\sum_{\substack{\ell_1=0\\a_{\ell_1}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}K_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_\ell}}[a_{\ell_1}K_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_1}}}[-mI_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_1}}}[mI_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} rI_{m+1}(a_{\ell_1}r)\overline{K_m(a_{\ell_1}r)}dr]+$$

$$+\sum_{\substack{\ell_1=0\\a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_2}}}[a_{\ell_2}K_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_2}}}[a_{\ell_2}K_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}}\right]$$

$$\Bigg[\frac{\mathrm{a}_{\ell+1}}{{a_{\ell_1}}^2-a_{\ell_2}^2}\{a_{\ell_1}\overline{I_{m+2}(a_{\ell_1}\mathrm{a}_{\ell+1})}K_{m+1}(a_{\ell_2}\mathrm{a}_{\ell+1})-a_{\ell_2}\overline{I_{m+1}(a_{\ell_1}\mathrm{a}_{\ell+1})}K_{m+2}(a_{\ell_2}\mathrm{a}_{\ell+1})\}-$$

$$-\frac{\mathrm{a}_\ell}{{a_{\ell_1}}^2-a_{\ell_2}^2}\{a_{\ell_1}\overline{I_{m+2}(a_{\ell_1}\mathrm{a}_\ell)}K_{m+1}(a_{\ell_2}\mathrm{a}_\ell)-a_{\ell_2}\overline{I_{m+1}(a_{\ell_1}\mathrm{a}_\ell)}K_{m+2}(a_{\ell_2}\mathrm{a}_\ell)\}\Bigg]+$$

$$+\sum_{\substack{\ell_1=0\\a_{\ell_1}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}K_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[a_{\ell_1}K_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} r\overline{I_{m+1}(a_{\ell_1}r)}K_{m+1}(a_{\ell_1}r)dr]+$$

$$+\sum_{\substack{\ell_1=0\\a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[a_\ell I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\overline{\Lambda_{m_{a_{\ell_2}}}[a_{\ell_2}I_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_2}}}[a_{\ell_2}I_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}}}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\mathrm{a}_{\ell+1}}{a_{\ell_1}{}^2-a_{\ell_2}^2}\{a_{\ell_1}\overline{K_{m+2}(a_{\ell_1}\mathrm{a}_{\ell+1})}K_{m+1}(a_{\ell_2}\mathrm{a}_{\ell+1})-a_{\ell_2}\overline{K_{m+1}(a_{\ell_1}\mathrm{a}_{\ell+1})}K_{m+2}(a_{\ell_2}\mathrm{a}_{\ell+1})\}-\right.$$

$$\left.-\frac{\mathrm{a}_\ell}{a_{\ell_1}{}^2-a_{\ell_2}^2}\{a_{\ell_1}\overline{K_{m+2}(a_{\ell_1}\mathrm{a}_\ell)}K_{m+1}(a_{\ell_2}\mathrm{a}_\ell)-a_{\ell_2}\overline{K_{m+1}(a_{\ell_1}\mathrm{a}_\ell)}K_{m+2}(a_{\ell_2}\mathrm{a}_\ell)\}\right]+$$

$$+\sum_{\substack{\ell_1=0\\a_{\ell_1}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[a_\ell I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\overline{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_1}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\frac{1}{2}(\mathrm{a}_n{}^2(-K_{m+1}(a\mathrm{a}_n)\overline{K_{m+1}(a\mathrm{a}_n)}+K_m(a\mathrm{a}_n)\overline{K_{m+2}(a\mathrm{a}_n)})+$$

$$a_{n+1}{}^2(K_{m+1}(a\mathrm{a}_{n+1})\overline{K_{m+1}(a\mathrm{a}_{n+1})}-K_m(a\mathrm{a}_{n+1})\overline{K_{m+2}(a\mathrm{a}_{n+1})}+$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\ a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_\ell}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\Lambda_{m_{a_{\ell_2}}}[mK_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_2}}}[mK_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\mathrm{a}_{\ell+1}}{a_{\ell_1}{}^2-a_{\ell_2}^2}\{a_{\ell_1}\overline{I_{m+1}(a_{\ell_1}\mathrm{a}_{\ell+1})}K_{m+1}(a_{\ell_2}\mathrm{a}_{\ell+1})-a_{\ell_2}\overline{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})}K_{m+2}(a_{\ell_2}\mathrm{a}_{\ell+1})\}-\right.$$

$$\left.-\frac{\mathrm{a}_\ell}{a_{\ell_1}{}^2-a_{\ell_2}^2}\{a_{\ell_1}\overline{I_{m+1}(a_{\ell_1}\mathrm{a}_\ell)}K_{m+1}(a_{\ell_2}\mathrm{a}_\ell)-a_{\ell_2}\overline{I_m(a_{\ell_1}\mathrm{a}_\ell)}K_{m+2}(a_{\ell_2}\mathrm{a}_\ell)\}\right]+$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_\ell}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} r\overline{I_m(a_{\ell_1}r)}K_{m+1}(a_{\ell_1}r)dr]+$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\ a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\Lambda_{m_{a_{\ell_2}}}[-mI_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_2}}}[mI_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{a_{\ell+1}}{a_{\ell_1}{}^2 - a_{\ell_2}^2}\{a_{\ell_1}\overline{K_{m+1}(a_{\ell_1}a_{\ell+1})}K_{m+1}(a_{\ell_2}a_{\ell+1}) - a_{\ell_2}\overline{K_m(a_{\ell_1}a_{\ell+1})}K_{m+2}(a_{\ell_2}a_{\ell+1})\} -\right.$$

$$\left. - \frac{a_{\ell}}{a_{\ell_1}{}^2 - a_{\ell_2}^2}\{a_{\ell_1}\overline{K_{m+1}(a_{\ell_1}a_{\ell})}K_{m+1}(a_{\ell_2}a_{\ell}) - a_{\ell_2}\overline{K_m(a_{\ell_1}a_{\ell})}K_{m+2}(a_{\ell_2}a_{\ell})\} \right] +$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}a_{\ell})\cos(a_{\ell_1}(z-h_{\ell}))N_{a_{\ell_1}}^{-1/2}] - \Lambda^*{}_{m_{a_{\ell_1}}}[a_{\ell_1}I_m(a_{\ell_1}a_{\ell+1})\cos(a_{\ell_1}(z-h_{\ell}))N_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}a_{\ell+1})K_m(a_{\ell_1}a_{\ell}) - I_m(a_{\ell_1}a_{\ell})K_m(a_{\ell_1}a_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_1}}}[-mI_m(a_{\ell_1}a_{\ell})\cos(a_{\ell_1}(z-h_{\ell}))N_{a_{\ell_1}}^{-1/2}] + \Lambda^*{}_{m_{a_{\ell_1}}}[mI_m(a_{\ell_1}a_{\ell+1})\cos(a_{\ell_1}(z-h_{\ell}))N_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}a_{\ell+1})K_m(a_{\ell_1}a_{\ell}) - I_m(a_{\ell_1}a_{\ell})K_m(a_{\ell_1}a_{\ell+1})}}\right]$$

$$\int_{a_{\ell}}^{a_{\ell+1}} r\overline{K_{m+1}(a_{\ell_1}r)}K_m(a_{\ell_1}r)dr] +$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\ a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}a_{\ell})\cos(a_{\ell_1}(z-h_{\ell}))N_{a_{\ell_1}}^{-1/2}] - \Lambda^*{}_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}a_{\ell+1})\cos(a_{\ell_1}(z-h_{\ell}))N_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}a_{\ell+1})K_m(a_{\ell_1}a_{\ell}) - I_m(a_{\ell_1}a_{\ell})K_m(a_{\ell_1}a_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_2}}}[a_{\ell_2}K_m(a_{\ell_2}a_{\ell})\cos(a_{\ell_2}(z-h_{\ell}))N_{a_{\ell_2}}^{-1/2}] - \Lambda^*{}_{m_{a_{\ell_2}}}[a_{\ell_2}K_m(a_{\ell_2}a_{\ell+1})\cos(a_{\ell_2}(z-h_{\ell}))N_{a_{\ell_2}}^{-1/2}]}{I_m(a_{\ell_2}a_{\ell+1})K_m(a_{\ell_2}a_{\ell}) - I_m(a_{\ell_2}a_{\ell})K_m(a_{\ell_2}a_{\ell+1})}}\right]$$

$$\int_{a_{\ell}}^{a_{\ell+1}} rI_m(a_{\ell_1}r)\overline{I_{m+1}(a_{\ell_2}r)}dr +$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\ a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}a_{\ell})\cos(a_{\ell_1}(z-h_{\ell}))N_{a_{\ell_1}}^{-1/2}] - \Lambda^*{}_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}a_{\ell+1})\cos(a_{\ell_1}(z-h_{\ell}))N_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}a_{\ell+1})K_m(a_{\ell_1}a_{\ell}) - I_m(a_{\ell_1}a_{\ell})K_m(a_{\ell_1}a_{\ell+1})}\right]$$

$$\left[\frac{\Lambda_{m_{a_{\ell_2}}}[a_{\ell_2} I_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}] - \Lambda^*{}_{m_{a_{\ell_2}}}[a_{\ell_2} I_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell) - I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\mathrm{a}_{\ell+1}}{a_{\ell_1}{}^2 - a_{\ell_2}^2}\{a_{\ell_1} I_{m+1}(a_{\ell_1}\mathrm{a}_{\ell+1})\overline{K_{m+1}(a_{\ell_2}\mathrm{a}_{\ell+1})} - a_{\ell_2} I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\overline{K_{m+2}(a_{\ell_2}\mathrm{a}_{\ell+1})}\} - \right.$$

$$\left. -\frac{\mathrm{a}_\ell}{a_{\ell_1}{}^2 - a_{\ell_2}^2}\{a_{\ell_1} I_{m+1}(a_{\ell_1}\mathrm{a}_\ell)\overline{K_{m+1}(a_{\ell_2}\mathrm{a}_\ell)} - a_{\ell_2} I_m(a_{\ell_1}\mathrm{a}_\ell)\overline{K_{m+2}(a_{\ell_2}\mathrm{a}_\ell)}\} \right] +$$

$$+\sum_{\substack{\ell_1=0 \\ a_{\ell_1}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}] - \Lambda^*{}_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell) - I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1} I_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}] - \Lambda^*{}_{m_{a_{\ell_1}}}[a_{\ell_1} I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell) - I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} rI_m(a_{\ell_1}r)\overline{K_{m+1}(a_{\ell_1}r)}dr +$$

$$+\sum_{\substack{\ell_1=0 \\ a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0 \\ a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}] - \Lambda^*{}_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell) - I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\Lambda_{m_{a_{\ell_2}}}[mK_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}] - \Lambda^*{}_{m_{a_{\ell_2}}}[mK_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell) - I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} rI_m(a_{\ell_1}r)\overline{I_m(a_{\ell_2}r)}dr +$$

$$+\sum_{\substack{\ell_1=0 \\ a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0 \\ a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\Lambda_{m_{a_{\ell_2}}}[-mI_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_2}}}[mI_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\mathrm{a}_{\ell+1}}{a_{\ell_1}{}^2-a_{\ell_2}^2}\{a_{\ell_1}I_{m+1}(a_{\ell_1}\mathrm{a}_{\ell+1})\overline{K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}-a_{\ell_2}I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\overline{K_{m+1}(a_{\ell_2}\mathrm{a}_{\ell+1})}\}-\right.$$

$$\left.-\frac{\mathrm{a}_\ell}{a_{\ell_1}{}^2-a_{\ell_2}^2}\{a_{\ell_1}I_{m+1}(a_{\ell_1}\mathrm{a}_\ell)\overline{K_m(a_{\ell_2}\mathrm{a}_\ell)}-a_{\ell_2}I_m(a_{\ell_1}\mathrm{a}_\ell)\overline{K_{m+1}(a_{\ell_2}\mathrm{a}_\ell)}\}\quad\right]+$$

$$+\sum_{\substack{\ell_1=0\\a_{\ell_1}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-mI_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_1}}}[mI_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} rI_m(a_{\ell_1}r)\overline{K_m(a_{\ell_1}r)}dr+$$

$$+\sum_{\substack{\ell_1=0\\a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-mI_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_1}}}[mI_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\Lambda_{m_{a_{\ell_2}}}[a_{\ell_2}K_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_2}}}[a_{\ell_2}K_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\mathrm{a}_{\ell+1}}{a_{\ell_1}{}^2-a_{\ell_2}^2}\{a_{\ell_1}\overline{I_{m+2}(a_{\ell_1}\mathrm{a}_{\ell+1})}K_m(a_{\ell_2}\mathrm{a}_{\ell+1})-a_{\ell_2}\overline{I_{m+1}(a_{\ell_1}\mathrm{a}_{\ell+1})}K_{m+1}(a_{\ell_2}\mathrm{a}_{\ell+1})\}-\right.$$

$$-\frac{\mathrm{a}_\ell}{a_{\ell_1}{}^2-a_{\ell_2}^2}\{a_{\ell_1}\overline{I_{m+2}(a_{\ell_1}\mathrm{a}_\ell)}K_m(a_{\ell_2}\mathrm{a}_\ell)-a_{\ell_2}\overline{I_{m+1}(a_{\ell_1}\mathrm{a}_\ell)}K_{m+1}(a_{\ell_2}\mathrm{a}_\ell)\}\Bigg]+$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-mI_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]+\Lambda^*_{m_{a_{\ell_1}}}[mI_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1}K_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*_{m_{a_{\ell_1}}}[a_{\ell_1}K_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} r\overline{I_{m+1}(a_{\ell_1}r)}K_m(a_{\ell_1}r)dr+$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\ a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-mI_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]+\Lambda^*_{m_{a_{\ell_1}}}[mI_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_\ell}}[a_{\ell_2}I_m(a_\ell\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]-\Lambda^*_{m_{a_{\ell_2}}}[a_{\ell_2}I_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}}\right]$$

$$\Bigg[\frac{\mathrm{a}_{\ell+1}}{a_{\ell_1}{}^2-a_{\ell_2}^2}\{a_{\ell_1}\overline{K_{m+2}(a_{\ell_1}\mathrm{a}_{\ell+1})}K_m(a_{\ell_2}\mathrm{a}_{\ell+1})-a_{\ell_2}\overline{K_{m+1}(a_{\ell_1}\mathrm{a}_{\ell+1})}K_{m+1}(a_{\ell_2}\mathrm{a}_{\ell+1})\}-$$

$$-\frac{\mathrm{a}_\ell}{a_{\ell_1}{}^2-a_{\ell_2}^2}\{a_{\ell_1}\overline{K_{m+2}(a_{\ell_1}\mathrm{a}_\ell)}K_m(a_{\ell_2}\mathrm{a}_\ell)-a_{\ell_2}\overline{K_{m+1}(a_{\ell_1}\mathrm{a}_\ell)}K_{m+1}(a_{\ell_2}\mathrm{a}_\ell)\}\Bigg]+$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-mI_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]+\Lambda^*_{m_{a_{\ell_1}}}[mI_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[a_{\ell_1} I_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}] - \Lambda^*{}_{m_{a_{\ell_1}}}[a_{\ell_1} I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell) - I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} r\overline{K_{m+1}(a_{\ell_1}r)}K_m(a_{\ell_1}r)dr +$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\ a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-mI_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}] + \Lambda^*{}_{m_{a_{\ell_1}}}[mI_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell) - I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\Lambda_{m_{a_{\ell_2}}}[mK_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}] - \Lambda^*{}_{m_{a_{\ell_2}}}[mK_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell) - I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\mathrm{a}_{\ell+1}}{a_{\ell_1}{}^2 - a_{\ell_2}^2}\{a_{\ell_1}\overline{I_{m+1}(a_{\ell_1}\mathrm{a}_{\ell+1})}K_m(a_{\ell_2}\mathrm{a}_{\ell+1}) - a_{\ell_2}\overline{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})}K_{m+1}(a_{\ell_2}\mathrm{a}_{\ell+1})\} - \right.$$

$$\left. -\frac{\mathrm{a}_\ell}{a_{\ell_1}{}^2 - a_{\ell_2}^2}\{a_{\ell_1}\overline{I_{m+1}(a_{\ell_1}\mathrm{a}_\ell)}K_m(a_{\ell_2}\mathrm{a}_\ell) - a_{\ell_2}\overline{I_m(a_{\ell_1}\mathrm{a}_\ell)}K_{m+1}(a_{\ell_2}\mathrm{a}_\ell)\} \right] +$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-mI_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}] + \Lambda^*{}_{m_{a_{\ell_1}}}[mI_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell) - I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}] - \Lambda^*{}_{m_{a_{\ell_1}}}[mK_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell) - I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} r\overline{I_{m1}(a_{\ell_1}r)}K_m(a_{\ell_1}r)dr +$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\ a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-mI_m(a_{\ell_1}\partial_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_1}}}[mI_m(a_{\ell_1}\partial_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\partial_{\ell+1})K_m(a_{\ell_1}\partial_\ell)-I_m(a_{\ell_1}\partial_\ell)K_m(a_{\ell_1}\partial_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_2}}}[-mI_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_2}}}[mI_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}}\right]$$

$$\left[\frac{\mathrm{a}_{\ell+1}}{{a_{\ell_1}}^2-a_{\ell_2}^2}\{a_{\ell_1}\overline{K_{m+1}(a_{\ell_1}\mathrm{a}_{\ell+1})}K_m(a_{\ell_2}\mathrm{a}_{\ell+1})-a_{\ell_2}\overline{K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}K_{m+1}(a_{\ell_2}\mathrm{a}_{\ell+1})\}-\right.$$

$$\left.-\frac{\mathrm{a}_\ell}{{a_{\ell_1}}^2-a_{\ell_2}^2}\{a_{\ell_1}\overline{K_{m+1}(a_{\ell_1}\mathrm{a}_\ell)}K_m(a_{\ell_2}\mathrm{a}_\ell)-a_{\ell_2}\overline{K_m(a_{\ell_1}\mathrm{a}_\ell)}K_{m+1}(a_{\ell_2}\mathrm{a}_\ell)\}\right]+$$

$$+\sum_{\substack{\ell_1=0\\a_{\ell_1}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-mI_m(a_{\ell_1}\partial_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_1}}}[mI_m(a_{\ell_1}\partial_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\partial_{\ell+1})K_m(a_{\ell_1}\partial_\ell)-I_m(a_{\ell_1}\partial_\ell)K_m(a_{\ell_1}\partial_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_1}}}[-mI_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_1}}}[mI_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}}\right]$$

$$\frac{1}{2}({\mathrm{a}_n}^2(-K_m(a\mathrm{a}_n)\overline{K_m(a\mathrm{a}_n)}+K_{m-1}(a\mathrm{a}_n)\overline{K_{m+1}(a\mathrm{a}_n)})+$$
$${a_{n+1}}^2(K_{m+1}(a\mathrm{a}_{n+1})\overline{K_{m+1}(a\mathrm{a}_{n+1})}-K_{m-1}(a\mathrm{a}_{n+1})\overline{K_{m+1}(a\mathrm{a}_{n+1})}$$

(Δ)

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}}[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\overline{\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)}\ \overline{+\Lambda^*{}_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)]}\ N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]]dr=$$

$$=\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\overline{\frac{\Lambda_{1_{a_\ell}}[K_1(a_\ell\mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]-\Lambda^*{}_{1_{a_\ell}}[K_1(a_\ell\mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell\mathrm{a}_{\ell+1})K_1(a_\ell\mathrm{a}_\ell)-I_1(a_\ell\mathrm{a}_\ell)K_1(a_\ell\mathrm{a}_{\ell+1})}}\right]$$

$$\left[\frac{I_0(\mathrm{a}_{\ell+1}a_\ell)-I_0(\mathrm{a}_\ell a_\ell)}{a_\ell}\right]+$$

$$+\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{\Lambda_{1_{a_\ell}}[-I_1(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]+\Lambda^*{}_{1_{a_\ell}}[I_1(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{-K_0(\mathrm{a}_{\ell+1}a_\ell)+K_0(\mathrm{a}_\ell a_\ell)}{a_\ell}\right]$$

(E)

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}}[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)+\Lambda^*{}_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)]\ N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]]dr=$$

$$=\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{\Lambda_{1_{a_\ell}}[K_1(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]-\Lambda^*{}_{1_{a_\ell}}[K_1(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{I_0(\mathrm{a}_{\ell+1}a_\ell)-I_0(\mathrm{a}_\ell a_\ell)}{a_\ell}\right]+$$

$$+\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{\Lambda_{1_{a_\ell}}[-I_1(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]+\Lambda^*{}_{1_{a_\ell}}[I_1(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{-K_0(\mathrm{a}_{\ell+1}a_\ell)+K_0(\mathrm{a}_\ell a_\ell)}{a_\ell}\right]$$

(Z)

$$\int_{a_\ell}^{a_{\ell+1}}\left[\ \frac{1}{r}[\sum_{\substack{\ell=0\\ a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)+\Lambda^*{}_{m_{a_\ell}}\Re^*{}_{m_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\,][\sum_{\substack{n=0\\ a_n}}^{\infty}[\overline{\Lambda_{m_{a_n}}\Re_{m_{a_n}}(r)}\right.$$

$$\left.\overline{+\Lambda^*{}_{m_{a_n}}\Re^*{}_{m_{a_n}}(r)]}mN_{a_n}^{-1/2}\cos(a_n(z-h_\ell))]dr\right]=$$

$$=\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\ a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[K_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[K_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_2}}}[K_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_2}}}[K_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}}\right]$$

$$\int_{a_\ell}^{a_{\ell+1}}\frac{1}{r}I_m(a_{\ell_1}r)\overline{I_m(a_{\ell_2}r)}dr+$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\ a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[K_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[K_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_2}}}[-I_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_2}}}[I_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}}\right]$$

$$\int_{a_\ell}^{a_{\ell+1}}\frac{1}{r}I_m(a_{\ell_1}r)\overline{K_m(a_{\ell_2}r)}dr+$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\ a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-I_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_1}}}[I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_2}}}[K_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_2}}}[K_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}}\frac{1}{r}K_m(a_{\ell_1}r)\overline{I_m(a_{\ell_2}r)}dr+$$

$$+\sum_{\substack{\ell_1=0\\a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-I_m(a_{\ell_1}\mathrm{a}_\ell)\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_1}}}[I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\cos(a_{\ell_1}(z-h_\ell))\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_2}}}[-I_m(a_{\ell_2}\mathrm{a}_\ell)\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_2}}}[I_m(a_{\ell_2}\mathrm{a}_{\ell+1})\cos(a_{\ell_2}(z-h_\ell))\mathrm{N}_{a_{\ell_2}}^{-1/2}}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}}\frac{1}{r}K_m(a_{\ell_1}r)\overline{K_m(a_{\ell_2}r)}dr$$

(H)

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}}r^2\,[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\overline{\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)}\;\overline{+\Lambda^*{}_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]dr=$$

$$=\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\overline{\frac{\Lambda_{1_{a_\ell}}[K_1(a_\ell\mathrm{a}_\ell)\sin(a_\ell(z-h_\ell))a_\ell\mathrm{N}_{a_\ell}^{-1/2}]-\Lambda^*{}_{1_{a_\ell}}[K_1(a_\ell\mathrm{a}_{\ell+1})\sin(a_\ell(z-h_\ell))a_\ell\mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell\mathrm{a}_{\ell+1})K_1(a_\ell\mathrm{a}_\ell)-I_1(a_\ell\mathrm{a}_\ell)K_1(a_\ell\mathrm{a}_{\ell+1})}}\right]$$

$$\left[\overline{\frac{-\mathrm{a}^2{}_\ell I_2(\mathrm{a}_\ell a_\ell)+\mathrm{a}^2{}_{\ell+1}I_2(\mathrm{a}_{\ell+1}a_\ell)}{a_\ell}}\right]+$$

$$+\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\overline{\frac{\Lambda_{1_{a_\ell}}[-I_1(a_\ell \mathrm{a}_\ell)\sin(a_\ell(z-h_\ell))a_\ell \mathrm{N}_{a_\ell}^{-1/2}]+\Lambda^*{}_{1_{a_\ell}}[I_1(a_\ell \mathrm{a}_{\ell+1})\sin(a_\ell(z-h_\ell))a_\ell \mathrm{N}_{a_\ell}^{-1/2}}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} r^2\overline{K_1\,(a_\ell r)}dr$$

(Θ)

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} r^2\,[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\mathfrak{R}_{1_{a_\ell}}(r)\,+\Lambda^*{}_{1_{a_\ell}}\mathfrak{R}^*{}_{1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]dr=$$

$$=\sum_{a_\ell=0}^{\infty}[\left[\frac{\Lambda_{1_{a_\ell}}[K_1(a_\ell \mathrm{a}_\ell)\sin(a_\ell(z-h_\ell))a_\ell \mathrm{N}_{a_\ell}^{-1/2}]-\Lambda^*{}_{1_{a_\ell}}[K_1(a_\ell \mathrm{a}_{\ell+1})\sin(a_\ell(z-h_\ell))a_\ell \mathrm{N}_{a_\ell}^{-1/2}]}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{-\mathrm{a}^2{}_\ell I_2(\mathrm{a}_\ell a_\ell)+\mathrm{a}^2{}_{\ell+1}I_2(\mathrm{a}_{\ell+1}a_\ell)}{a_\ell}\right]+$$

$$+\sum_{a_\ell=0}^{\infty}[\left[\frac{\Lambda_{1_{a_\ell}}[-I_1(a_\ell \mathrm{a}_\ell)\sin(a_\ell(z-h_\ell))a_\ell \mathrm{N}_{a_\ell}^{-1/2}]+\Lambda^*{}_{1_{a_\ell}}[I_1(a_\ell \mathrm{a}_{\ell+1})\sin(a_\ell(z-h_\ell))a_\ell \mathrm{N}_{a_\ell}^{-1/2}}{I_1(a_\ell \mathrm{a}_{\ell+1})K_1(a_\ell \mathrm{a}_\ell)-I_1(a_\ell \mathrm{a}_\ell)K_1(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} r^2 K_1\,(a_\ell r)dr$$

(I)

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}}\Big[\quad r[\sum_{\substack{n=0\\a_n}}^{\infty}[\overline{\Lambda_{m_{a_n}}\mathfrak{R}_{m_{a_n}}(r)}\;\overline{+\Lambda^*{}_{m_{a_n}}\mathfrak{R}^*{}_{m_{a_n}}(r)}]N_{a_n}^{-1/2}\sin(a_n(z-h_\ell))a_n]$$

$$[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\mathfrak{R}_{m_{a_\ell}}(r)+\Lambda^*{}_{m_{a_\ell}}\mathfrak{R}^*{}_{m_{a_\ell}}(r)]\;N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]dr=$$

$$= \sum_{\substack{\ell=0 \\ a_\ell}}^{\infty} \sum_{\substack{n=0 \\ a_n}}^{\infty} [$$

$$\left[\frac{\Lambda_{m_{a_\ell}} [K_m(a_\ell \mathrm{a}_\ell) \sin(a_\ell (z - h_\ell)) a_\ell \mathrm{N}_{a_\ell}^{-1/2}] - \Lambda^*{}_{m_{a_\ell}} [K_m(a_\ell \mathrm{a}_{\ell+1}) \sin(a_\ell (z - h_\ell)) a_\ell \mathrm{N}_{a_\ell}^{-1/2}]}{I_m(a_\ell \mathrm{a}_{\ell+1}) K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell) K_m(a_\ell \mathrm{a}_{\ell+1})} \right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_n}} [K_m(a_n \mathrm{a}_\ell) \sin(a_n (z - h_\ell)) a_n \mathrm{N}_{a_n}^{-1/2}] - \Lambda^*{}_{m_{a_n}} [K_m(a_n \mathrm{a}_{\ell+1}) \sin(a_n (z - h_\ell)) a_n \mathrm{N}_{a_n}^{-1/2}]}{I_m(a_n \mathrm{a}_{\ell+1}) K_m(a_n \mathrm{a}_\ell) - I_m(a_n \mathrm{a}_\ell) K_m(a_n \mathrm{a}_{\ell+1})}} \right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} r I_m(a_\ell r) \overline{I_m(a_n r)} dr] +$$

$$+ \sum_{\substack{\ell_1=0 \\ a_{\ell_1}}}^{\infty} \sum_{\substack{\ell_2=0 \\ a_{\ell_2}}}^{\infty} [$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}} [K_m(a_{\ell_1} \mathrm{a}_\ell) \sin(a_{\ell_1} (z - h_\ell)) a_{\ell_1} \mathrm{N}_{a_{\ell_1}}^{-1/2}] - \Lambda^*{}_{m_{a_{\ell_1}}} [K_m(a_{\ell_1} \mathrm{a}_{\ell+1}) \sin(a_{\ell_1} (z - h_\ell)) a_{\ell_1} \mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1} \mathrm{a}_{\ell+1}) K_m(a_{\ell_1} \mathrm{a}_\ell) - I_m(a_{\ell_1} \mathrm{a}_\ell) K_m(a_{\ell_1} \mathrm{a}_{\ell+1})} \right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_2}}} [-I_m(a_{\ell_2} \mathrm{a}_\ell) \sin(a_{\ell_2} (z - h_\ell)) a_{\ell_2} \mathrm{N}_{a_{\ell_2}}^{-1/2}] + \Lambda^*{}_{m_{a_{\ell_2}}} [I_m(a_{\ell_2} \mathrm{a}_{\ell+1}) \sin(a_{\ell_2} (z - h_\ell)) a_{\ell_2} \mathrm{N}_{a_{\ell_2}}^{-1/2}}{I_m(a_{\ell_2} \mathrm{a}_{\ell+1}) K_m(a_{\ell_2} \mathrm{a}_\ell) - I_m(a_{\ell_2} \mathrm{a}_\ell) K_m(a_{\ell_2} \mathrm{a}_{\ell+1})}} \right]$$

$$\left[\frac{\mathrm{a}_{\ell+1}}{a_{\ell_1}{}^2 - a_{\ell_2}^2} \{ a_{\ell_1} \overline{I_{m+1}(a_{\ell_1} \mathrm{a}_{\ell+1})} K_m(a_{\ell_2} \mathrm{a}_{\ell+1}) - a_{\ell_2} \overline{I_m(a_{\ell_1} \mathrm{a}_{\ell+1})} K_{m+1}(a_{\ell_2} \mathrm{a}_{\ell+1}) \} - \right.$$

$$\left. - \frac{\mathrm{a}_\ell}{a_{\ell_1}{}^2 - a_{\ell_2}^2} \{ a_{\ell_1} \overline{I_{m+1}(a_{\ell_1} \mathrm{a}_\ell)} K_m(a_{\ell_2} \mathrm{a}_\ell) - a_{\ell_2} \overline{I_m(a_{\ell_1} \mathrm{a}_\ell)} K_{m+1}(a_{\ell_2} \mathrm{a}_\ell) \} \right] +$$

$$+ \sum_{\substack{\ell_1=0 \\ a_{\ell_1}}}^{\infty} [$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}} [K_m(a_{\ell_1} \mathrm{a}_\ell) \sin(a_{\ell_1} (z - h_\ell)) a_{\ell_1} \mathrm{N}_{a_{\ell_1}}^{-1/2}] - \Lambda^*{}_{m_{a_{\ell_1}}} [K_m(a_{\ell_1} \mathrm{a}_{\ell+1}) \sin(a_{\ell_1} (z - h_\ell)) a_{\ell_1} \mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1} \mathrm{a}_{\ell+1}) K_m(a_{\ell_1} \mathrm{a}_\ell) - I_m(a_{\ell_1} \mathrm{a}_\ell) K_m(a_{\ell_1} \mathrm{a}_{\ell+1})} \right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_1}}} [-I_m(a_{\ell_1} \mathrm{a}_\ell) \sin(a_{\ell_1} (z - h_\ell)) a_{\ell_1} \mathrm{N}_{a_{\ell_1}}^{-1/2}] + \Lambda^*{}_{m_{a_{\ell_1}}} [I_m(a_{\ell_1} \mathrm{a}_{\ell+1}) \sin(a_{\ell_1} (z - h_\ell)) a_{\ell_1} \mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1} \mathrm{a}_{\ell+1}) K_m(a_{\ell_1} \mathrm{a}_\ell) - I_m(a_{\ell_1} \mathrm{a}_\ell) K_m(a_{\ell_1} \mathrm{a}_{\ell+1})}} \right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} r I_m(a_{\ell_1} r) \overline{K_m(a_{\ell_1} r)} dr] +$$

$$+\sum_{\substack{\ell_1=0\\a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-I_m(a_{\ell_1}\mathrm{a}_\ell)\sin(a_{\ell_1}(z-h_\ell))a_{\ell_1}\mathrm{N}_{a_{\ell_1}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_1}}}[I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\sin(a_{\ell_1}(z-h_\ell))a_{\ell_1}\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_2}}}[K_m(a_{\ell_2}\mathrm{a}_\ell)\sin(a_{\ell_2}(z-h_\ell))a_{\ell_2}\mathrm{N}_{a_{\ell_2}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_2}}}[K_m(a_{\ell_2}\mathrm{a}_{\ell+1})\sin(a_{\ell_2}(z-h_\ell))a_{\ell_2}\mathrm{N}_{a_{\ell_2}}^{-1/2}]}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}}\right]$$

$$\left[\frac{\mathrm{a}_{\ell+1}}{a_{\ell_1}{}^2-a_{\ell_2}^2}\{a_{\ell_1}\overline{I_{m+1}(a_{\ell_1}\mathrm{a}_{\ell+1})}K_m(a_{\ell_2}\mathrm{a}_{\ell+1})-a_{\ell_2}\overline{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})}K_{m+1}(a_{\ell_2}\mathrm{a}_{\ell+1})\}-\right.$$

$$\left.-\frac{\mathrm{a}_\ell}{a_{\ell_1}{}^2-a_{\ell_2}^2}\{a_{\ell_1}\overline{I_{m+1}(a_{\ell_1}\mathrm{a}_\ell)}K_m(a_{\ell_2}\mathrm{a}_\ell)-a_{\ell_2}\overline{I_m(a_{\ell_1}\mathrm{a}_\ell)}K_{m+1}(a_{\ell_2}\mathrm{a}_\ell)\}\right]+$$

$$+\sum_{\substack{\ell_1=0\\a_{\ell_1}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-I_m(a_{\ell_1}\mathrm{a}_\ell)\sin(a_{\ell_1}(z-h_\ell))a_{\ell_1}\mathrm{N}_{a_{\ell_1}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_1}}}[I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\sin(a_{\ell_1}(z-h_\ell))a_{\ell_1}\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_1}}}[K_m(a_{\ell_1}\mathrm{a}_\ell)\sin(a_{\ell_1}(z-h_\ell))a_{\ell_1}\mathrm{N}_{a_{\ell_1}}^{-1/2}]-\Lambda^*{}_{m_{a_{\ell_1}}}[K_m(a_{\ell_1}\mathrm{a}_{\ell+1})\sin(a_{\ell_1}(z-h_\ell))a_{\ell_1}\mathrm{N}_{a_{\ell_1}}^{-1/2}]}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}}\right]$$

$$\int_{\mathrm{a}_\ell}^{\mathrm{a}_{\ell+1}} rK_m(a_{\ell_1}r)\overline{I_m(a_{\ell_1}r)}dr]+$$

$$+\sum_{\substack{\ell_1=0\\a_{\ell_1}}}^{\infty}\sum_{\substack{\ell_2=0\\a_{\ell_2}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-I_m(a_{\ell_1}\mathrm{a}_\ell)\sin(a_{\ell_1}(z-h_\ell))a_{\ell_1}\mathrm{N}_{a_{\ell_1}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_1}}}[I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\sin(a_{\ell_1}(z-h_\ell))a_{\ell_1}\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_\ell)-I_m(a_{\ell_1}\mathrm{a}_\ell)K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_2}}}[-I_m(a_{\ell_2}\mathrm{a}_\ell)\sin(a_{\ell_2}(z-h_\ell))a_{\ell_2}\mathrm{N}_{a_{\ell_2}}^{-1/2}]+\Lambda^*{}_{m_{a_{\ell_2}}}[I_m(a_{\ell_2}\mathrm{a}_{\ell+1})\sin(a_{\ell_2}(z-h_\ell))a_{\ell_2}\mathrm{N}_{a_{\ell_2}}^{-1/2}}{I_m(a_{\ell_2}\mathrm{a}_{\ell+1})K_m(a_{\ell_2}\mathrm{a}_\ell)-I_m(a_{\ell_2}\mathrm{a}_\ell)K_m(a_{\ell_2}\mathrm{a}_{\ell+1})}}\right]$$

$$\left[\frac{\mathrm{a}_{\ell+1}}{a_{\ell_1}{}^2 - a_{\ell_2}^2}\{a_{\ell_1}\overline{K_{m+1}(a_{\ell_1}\mathrm{a}_{\ell+1})}K_m(a_{\ell_2}\mathrm{a}_{\ell+1}) - a_{\ell_2}\overline{K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}K_{m+1}(a_{\ell_2}\mathrm{a}_{\ell+1})\} -\right.$$

$$\left.-\frac{\mathrm{a}_{\ell}}{a_{\ell_1}{}^2 - a_{\ell_2}^2}\{a_{\ell_1}\overline{K_{m+1}(a_{\ell_1}\mathrm{a}_{\ell})}K_m(a_{\ell_2}\mathrm{a}_{\ell}) - a_{\ell_2}\overline{K_m(a_{\ell_1}\mathrm{a}_{\ell})}K_{m+1}(a_{\ell_2}\mathrm{a}_{\ell})\}\right]$$

$$+\sum_{\substack{\ell_1=0\\ a_{\ell_1}}}^{\infty}[$$

$$\left[\frac{\Lambda_{m_{a_{\ell_1}}}[-I_m(a_{\ell_1}\mathrm{a}_{\ell})\sin(a_{\ell_1}(z-h_{\ell}))a_{\ell_1}\mathrm{N}_{a_{\ell_1}}^{-1/2}] + \Lambda^*{}_{m_{a_{\ell_1}}}[I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\sin(a_{\ell_1}(z-h_{\ell}))a_{\ell_1}\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_{\ell}) - I_m(a_{\ell_1}\mathrm{a}_{\ell})K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}\right]$$

$$\left[\overline{\frac{\Lambda_{m_{a_{\ell_1}}}[-I_m(a_{\ell_1}\mathrm{a}_{\ell})\sin(a_{\ell_1}(z-h_{\ell}))a_{\ell_1}\mathrm{N}_{a_{\ell_1}}^{-1/2}] + \Lambda^*{}_{m_{a_{\ell_1}}}[I_m(a_{\ell_1}\mathrm{a}_{\ell+1})\sin(a_{\ell_1}(z-h_{\ell}))a_{\ell_1}\mathrm{N}_{a_{\ell_1}}^{-1/2}}{I_m(a_{\ell_1}\mathrm{a}_{\ell+1})K_m(a_{\ell_1}\mathrm{a}_{\ell}) - I_m(a_{\ell_1}\mathrm{a}_{\ell})K_m(a_{\ell_1}\mathrm{a}_{\ell+1})}}\right]$$

$$\int_{\mathrm{a}_{\ell}}^{\mathrm{a}_{\ell+1}} rK_m(a_{\ell_1}r)\overline{K_m(a_{\ell_1}r)}dr]$$

7.2 Υπολογισμός του όρου $\int_{a_2}^{a_1}(\int_0^{2\pi} \overline{x}^{(1)}\nabla\Phi_t^{(1)})\overline{n}dS$ για το πεδίο (ΙΙ)

Έχουμε δείξει στο Κεφάλαιο 2 –σελίδα 19– ότι

$$\overline{x}^{(1)}\nabla\Phi_t^{(1)} = [(X_{g_1}^{(1)} + X_5^{(1)}z)\frac{\partial\phi(r,\theta,z)}{\partial r}\cos\theta - (X_{g_1}^{(1)} + X_5^{(1)}z)\frac{1}{r}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\sin\theta$$
$$+ (X_{g_3}^{(1)} - X_5^{(1)}r\cos\theta)\frac{\partial\phi(r,\theta,z)}{\partial z}](-i\omega)]$$

Επομένως

$$\overline{x}^{(1)}\nabla\Phi_t^{(1)} = (X_{g_1}^{(1)} + X_5^{(1)}z)\ (-i\omega)\ [-i\omega\frac{H}{2}d\left[-d\frac{\phi_0}{H/2}(\frac{z-d}{d^2} + \frac{g}{\omega^2 d^2})\cos\theta\right] - i\omega\frac{H}{2}d$$
$$\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\frac{\partial\Re_{m_{\kappa_\ell}}(r)}{\partial r} + \Lambda^*_{m_{\kappa_\ell}}\frac{\partial\Re^*_{m_{\kappa_\ell}}(r)}{\partial r}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\cos(m\theta) - i\omega\frac{H}{2}d\ [$$
$$\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\frac{\partial\Re_{m_{a_\ell}}(r)}{\partial r} + \Lambda^*_{m_{a_\ell}}\frac{\partial\Re^*_{m_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\cos(m\theta)]\cos(\theta) -$$

$$-(X_{g_1}^{(1)} + X_5^{(1)}z)\ (-i\omega)\ \frac{1}{r}\ (-i\omega\frac{H}{2}d)\left[d\frac{\phi_0}{\frac{H}{2}}r(\frac{z-d}{d^2} + \frac{g}{\omega^2 d^2})\sin\theta\right] + i\omega\frac{H}{2}d$$
$$[\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r) + \Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)]mN_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\ \sin(m\theta) + \sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$
$$+ \Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)]mN_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\ \sin(m\theta)]\sin(\theta) +$$

$$+ (X_{g_3}^{(1)} - X_5^{(1)}r\cos\theta)(-i\omega)\ [-i\omega\frac{H}{2}d\left[\frac{z_0}{H/2}\frac{1}{d} - d\frac{\phi_0}{H/2}r(\frac{1}{d^2})\cos\theta\right] - i\omega\frac{H}{2}d\ [$$
$$\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r) + \Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell\ \cos(m\theta) + \sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$
$$+ \Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell\ \cos(m\theta)]].$$

Θα υπολογίσουμε τον όρο

$$\int_{d2}^{d1}(\int_{0}^{2\pi}\overline{x}^{(1)}\nabla\Phi_t^{(1)})ndS=\int_{a_2}^{a_1}(\int_{0}^{2\pi}\overline{x}^{(1)}\nabla\Phi_t^{(1)}d\theta)rdr$$

Πρώτα υπολογίζουμε το ολοκλήρωμα

$$\int_{0}^{2\pi}\overline{x}^{(1)}\nabla\Phi_t^{(1)}\cos(\theta)d\theta=$$

$$=[(X_{g_1}^{(1)}+X_5^{(1)}z)\ (-i\omega)\,[-i\omega\frac{H}{2}d\left[-d\,\frac{\phi_0}{H/2}(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\pi\right]-i\omega\frac{H}{2}d\ [\Lambda_{2_{\kappa_\ell}}\ \frac{\partial\Re_{2_{\kappa_\ell}}(r)}{\partial r}+$$

$$\Lambda^*_{2_{\kappa_\ell}}\ \frac{\partial\Re^*_{2_{\kappa_\ell}}(r)}{\partial r}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\frac{\pi}{2}-i\omega\frac{H}{2}d\ \sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\ \frac{\partial\Re_{2_{a_\ell}}(r)}{\partial r}+$$

$$\Lambda^*_{2_{a_\ell}}\ \frac{\partial\Re^*_{2_{a_\ell}}(r)}{\partial r}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))\frac{\pi}{2}]]+$$

$$-(X_{g_1}^{(1)}+X_5^{(1)}z)\ (-i\omega)\ \frac{1}{r}[\ (i\omega\frac{H}{2}d)\left[-d\,\frac{\phi_0}{H/2}r(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\pi\right]+i\omega\frac{H}{2}d$$

$$[[\Lambda_{2_{\kappa_\ell}}\Re_{2_{\kappa_\ell}}(r)+\ \Lambda^*_{2_{\kappa_\ell}}\Re^*_{2_{\kappa_\ell}}(r)]2N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))\frac{\pi}{2}+[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{2_{a_\ell}}\Re_{2_{a_\ell}}(r)$$

$$+\Lambda^*_{2_{a_\ell}}\Re^*_{2_{a_\ell}}(r)]2N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]\frac{\pi}{2}]+$$

$$+(X_{g_3}^{(1)}-X_5^{(1)}\mathrm{a}_\ell)(-i\omega)\ [(-i\omega\frac{H}{2}d)\left[\frac{z_0}{H/2}d2\pi\right]-i\omega\frac{H}{2}d\,[[\Lambda_{0_{\kappa_\ell}}\Re_{0_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{0_{\kappa_\ell}}\Re^*_{0_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell+\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{0_{a_\ell}}\Re_{0_{a_\ell}}(r)+\Lambda^*_{0_{a_\ell}}\Re^*_{0_{a_\ell}}(r)]$$

$$N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))]]\ \ \mathrm{a}_\ell$$

Από το *Παράρτημα Γ*, προκύπτει ότι

$$\int_{a_2}^{a_1}\left(\int_0^{2\pi}\overline{x}^{(1)}\nabla\Phi_t^{(1)}d\theta\right)rdr =$$

$$=[(X_{g_1}^{(1)}+X_5^{(1)}z)\ (-i\omega)\,[-i\omega\frac{H}{2}d\left[-d\frac{\phi_0}{H/2}\left(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2}\right)\pi\right](a_{\ell+1}-a_\ell)$$

$$-i\omega\frac{H}{2}d[$$

$$\left[\frac{\Lambda_{2_{\kappa_\ell}}[\kappa_\ell J_2(\kappa_\ell a_\ell)\cosh(\kappa_\ell(z-h_\ell))N_{\kappa_\ell}^{-1/2}]-\Lambda^*{}_{2_{\kappa_\ell}}[\kappa_\ell J_2(\kappa_\ell a_{\ell+1})\cosh(\kappa_\ell(z-h_\ell))N_{\kappa_\ell}^{-1/2}}{J_2(\kappa_\ell a_{\ell+1})Y_2(\kappa_\ell a_\ell)-J_2(\kappa_\ell a_\ell)Y_2(\kappa_\ell a_{\ell+1})}\right]$$

$$\int_{a_\ell}^{a_{\ell+1}} rY_3(\kappa_\ell r)dr +$$

$$+\left[\frac{\Lambda_{2_{\kappa_\ell}}[-\kappa_\ell Y_2(\kappa_\ell a_\ell)\cosh(\kappa_\ell(z-h_\ell))N_{\kappa_\ell}^{-1/2}]+\Lambda^*{}_{2_{\kappa_\ell}}[\kappa_\ell Y_2(\kappa_\ell a_{\ell+1})\cosh(\kappa_\ell(z-h_\ell))N_{\kappa_\ell}^{-1/2}]}{J_2(\kappa_\ell a_{\ell+1})Y_2(\kappa_\ell a_\ell)-J_2(\kappa_\ell a_\ell)Y_2(\kappa_\ell a_{\ell+1})}\right]$$

$$\int_{a_\ell}^{a_{\ell+1}} rJ_3(\kappa_\ell r)dr +$$

$$+\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\left[\frac{\Lambda_{2_{a_\ell}}[a_\ell K_2(a_\ell a_\ell)\cos(a_\ell(z-h_\ell))N_{a_\ell}^{-1/2}]+\Lambda^*{}_{2_{a_\ell}}[a_\ell K_2(a_\ell a_{\ell+1})\cos(a_\ell(z-h_\ell))N_{a_\ell}^{-1/2}]}{I_2(a_\ell a_{\ell+1})K_2(a_\ell a_\ell)-I_2(a_\ell a_\ell)K_2(a_\ell a_{\ell+1})}\right]$$

$$\int_{a_\ell}^{a_{\ell+1}} rI_3(a_\ell r)dr +$$

$$+\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\left[\frac{\Lambda_{2_{a_\ell}}[a_\ell I_2(a_\ell a_\ell)\cos(a_\ell(z-h_\ell))N_{a_\ell}^{-1/2}]-\Lambda^*{}_{2_{a_\ell}}[a_\ell I_2(a_\ell a_{\ell+1})\cos(a_\ell(z-h_\ell))N_{a_\ell}^{-1/2}}{I_2(a_\ell a_{\ell+1})K_2(a_\ell a_\ell)-I_2(a_\ell a_\ell)K_2(a_\ell a_{\ell+1})}\right]$$

$$\int_{a_\ell}^{a_{\ell+1}} rK_3(a_\ell r)dr +$$

$$-(X_{g_1}^{(1)}+X_5^{(1)}z)\ (-i\omega)\,[(i\omega\frac{H}{2}d)\left[-d\frac{\phi_0}{H/2}\left(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2}\right)\pi\right](a_{\ell+1}-a_\ell)$$

$$+i\omega\frac{H}{2}d$$

$$\left[\frac{\Lambda_{2_{\kappa_\ell}}[Y_2(\kappa_\ell a_\ell)\cosh(\kappa_\ell(z-h_\ell))N_{\kappa_\ell}^{-1/2}]-\Lambda^*{}_{2_{\kappa_\ell}}[Y_2(\kappa_\ell a_{\ell+1})\cosh(\kappa_\ell(z-h_\ell))N_{\kappa_\ell}^{-1/2}}{J_2(\kappa_\ell a_{\ell+1})Y_2(\kappa_\ell a_\ell)-J_2(\kappa_\ell a_\ell)Y_2(\kappa_\ell a_{\ell+1})}\right]$$

$$\int_{a_\ell}^{a_{\ell+1}} Y_2(\kappa_\ell r)dr +$$

$$+\left[\frac{\Lambda_{2_{\kappa_\ell}}[-J_2(\kappa_\ell a_\ell)\cosh(\kappa_\ell(z-h_\ell))N_{\kappa_\ell}^{-1/2}] + \Lambda^*_{2_{\kappa_\ell}}[J_2(\kappa_\ell a_{\ell+1})\cosh(\kappa_\ell(z-h_\ell))N_{\kappa_\ell}^{-1/2}]}{J_2(\kappa_\ell a_{\ell+1})Y_2(\kappa_\ell a_\ell) - J_2(\kappa_\ell a_\ell)Y_2(\kappa_\ell a_{\ell+1})}\right]$$

$$\int_{a_\ell}^{a_{\ell+1}} J_2(\kappa_\ell r)dr +$$

$$+\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\left[\left[\frac{\Lambda_{2_{a_\ell}}[K_2(a_\ell a_\ell)\cos(a_\ell(z-h_\ell))N_{a_\ell}^{-1/2}] - \Lambda^*_{2_{a_\ell}}[K_2(a_\ell a_{\ell+1})\cos(a_\ell(z-h_\ell))N_{a_\ell}^{-1/2}]}{I_2(a_\ell a_{\ell+1})K_2(a_\ell a_\ell) - I_2(a_\ell a_\ell)K_2(a_\ell a_{\ell+1})}\right]\right.$$

$$\int_{a_\ell}^{a_{\ell+1}} I_2(a_\ell r)dr +$$

$$+\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\left[\left[\frac{-\Lambda_{2_{a_\ell}}[a_\ell I_2(a_\ell a_\ell)\cos(a_\ell(z-h_\ell))N_{a_\ell}^{-1/2}] + \Lambda^*_{2_{a_\ell}}[a_\ell I_2(a_\ell a_{\ell+1})\cos(a_\ell(z-h_\ell))N_{a_\ell}^{-1/2}}{I_2(a_\ell a_{\ell+1})K_2(a_\ell a_\ell) - I_2(a_\ell a_\ell)K_2(a_\ell a_{\ell+1})}\right]\right.$$

$$\int_{a_\ell}^{a_{\ell+1}} K_2(a_\ell r)dr +$$

$$+(X_{g_3}^{(1)} - X_5^{(1)}a_\ell)(-i\omega)\,(-i\omega\frac{H}{2}d)\left[\frac{z_0}{H/2}d2\pi\right]+$$

$$+(X_{g_3}^{(1)} - X_5^{(1)}a_\ell)(-i\omega)\,[-i\omega\frac{H}{2}d[$$

$$\left[\frac{\Lambda_{0_{\kappa_\ell}}[Y_0(\kappa_\ell a_\ell)\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell N_{\kappa_\ell}^{-1/2}] - \Lambda^*_{0_{\kappa_\ell}}[Y_0(\kappa_\ell a_{\ell+1})\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell N_{\kappa_\ell}^{-1/2}}{J_0(\kappa_\ell a_{\ell+1})Y_0(\kappa_\ell a_\ell) - J_0(\kappa_\ell a_\ell)Y_0(\kappa_\ell a_{\ell+1})}\right]$$

$$\left[\frac{-a_\ell J_1(a_\ell\kappa_\ell) + a_{\ell+1}J_1(a_{\ell+1}\kappa_\ell)}{\kappa_\ell}\right]+$$

$$+\left[\frac{-\Lambda_{0_{\kappa_\ell}}[J_0(\kappa_\ell a_\ell)\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell N_{\kappa_\ell}^{-1/2}] + \Lambda^*_{0_{\kappa_\ell}}[J_0(\kappa_\ell a_{\ell+1})\sinh(\kappa_\ell(z-h_\ell))\kappa_\ell N_{\kappa_\ell}^{-1/2}}{J_0(\kappa_\ell a_{\ell+1})Y_0(\kappa_\ell a_\ell) - J_0(\kappa_\ell a_\ell)Y_0(\kappa_\ell a_{\ell+1})}\right]$$

$$\left[\frac{-a_\ell Y_1(a_\ell\kappa_\ell) + a_{\ell+1}Y_1(a_{\ell+1}\kappa_\ell)}{\kappa_\ell}\right]-$$

$$-\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\left[\frac{\Lambda_{0_{a_\ell}}[a_\ell K_0(a_\ell a_\ell)\sin(a_\ell(z-h_\ell))N_{a_\ell}^{-1/2}] - \Lambda^*_{0_{a_\ell}}[a_\ell K_0(a_\ell a_{\ell+1})\sin(a_\ell(z-h_\ell))N_{a_\ell}^{-1/2}]}{I_0(a_\ell a_{\ell+1})K_0(a_\ell a_\ell) - I_0(a_\ell a_\ell)K_0(a_\ell a_{\ell+1})}\right]$$

$$\left[\frac{-\mathrm{a}_\ell I_1(a_\ell \mathrm{a}_\ell)+\mathrm{a}_{\ell+1} I_1(\mathrm{a}_{\ell+1} a_\ell)}{a_\ell}\right]-$$

$$-\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}\left[\frac{\Lambda_{0_{a_\ell}}[-a_\ell I_0(a_\ell \mathrm{a}_\ell)\sin(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]+\Lambda^*_{0_{a_\ell}}[a_\ell I_0(a_\ell \mathrm{a}_{\ell+1})\sin(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_0(a_\ell \mathrm{a}_{\ell+1})K_0(a_\ell \mathrm{a}_\ell)-I_0(a_\ell \mathrm{a}_\ell)K_0(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$\left[\frac{\mathrm{a}_\ell K_1(\mathrm{a}_\ell a_\ell)-\mathrm{a}_{\ell+1} K_1(\mathrm{a}_{\ell+1} a_\ell)}{a_\ell}\right]$$

Όμως για τον υπολογισμός της δύναμης έκπτωσης πρέπει να υπολογίσουμε τον όρο

$$\overline{\overline{x}^{(1)}\nabla\Phi_t^{(1)}}^T=\omega(\overline{X}^{(1)}_{\mathrm{Re}}\cos(\omega t)+\overline{X}^{(1)}_{\mathrm{Im}}\sin(\omega t))(\nabla\phi^{(1)}_{\mathrm{Im}}\cos(\omega t)-\nabla\phi^{(1)}_{\mathrm{Re}}\sin(\omega t))=$$

$$=\overline{\omega(\overline{X}^{(1)}_{\mathrm{Re}}\nabla\phi^{(1)}_{\mathrm{Im}}\cos^2(\omega t)-\overline{X}^{(1)}_{\mathrm{Im}}\nabla\phi^{(1)}_{\mathrm{Re}}\sin^2(\omega t)+(-\overline{X}^{(1)}_{\mathrm{Re}}\nabla\phi^{(1)}_{\mathrm{Re}}+\overline{X}^{(1)}_{\mathrm{Im}}\nabla\phi^{(1)}_{\mathrm{Im}})\cos(\omega t)\sin(\omega t))}^t=$$

$$=\frac{1}{2}\omega(\overline{X}^{(1)}_{\mathrm{Re}}\nabla\phi^{(1)}_{\mathrm{Im}}-\overline{X}^{(1)}_{\mathrm{Im}}\nabla\phi^{(1)}_{\mathrm{Re}})=\frac{1}{2}\omega\,\mathrm{Im}(X^{(1)^*}\nabla\phi^{(1)})$$

7.3 Υπολογισμός του όρου $\int_{WL}(\zeta_r^{(1)})^2 n_1 dl$ για το πεδίο (ΙΙ)

Έχουμε αποδείξει για την ανύψωση της ελεύθερης επιφάνειας στο Κεφάλαιο 2 –σελίδα 17– ότι

$$(\zeta_r^{(1)})^2 = \frac{1}{2}\mathrm{Re}\left\{(\frac{i\omega}{g})\phi(r,\theta,d)(\frac{-i\omega}{g})\overline{\phi(r,\theta,d)}\right\} + \frac{1}{2}\mathrm{Re}\left\{ \left|X_{g_3}^{(1)} - X_5^{(1)} r\cos\theta\right|^2 \right\} -$$

$$- \mathrm{Re}\left[\left\{(\frac{i\omega}{g})\phi(r,\theta,d)\right\}\{X_{g_3}^{(1)} - X_5^{(1)} r\cos\theta\}\right].$$

Επομένως αφού $n_3 = 1$ και $dl = rd\theta$

Επομένως

$$(\zeta_r^{(1)})^2 =$$

$$= \frac{1}{2}\mathrm{Re}\Bigg\{ (\frac{\omega}{g})^2 \Bigg[-i\omega\frac{H}{2}d\left[\frac{z_0}{\frac{H}{2}}(\frac{z-d}{d}+\frac{g}{\omega^2 d}) - d\frac{\phi_0}{\frac{H}{2}}r(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\cos\theta\right] - i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)z_{\kappa_\ell}(z) + \Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)z_{\kappa_\ell}(z)]\cos(m\theta) - i\omega\frac{H}{2}d\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$

$$z_{a_\ell}(z) + \Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)z_{a_\ell}(z)]]\cos(m\theta)\Bigg]$$

$$\Bigg[i\omega\frac{H}{2}d\left[\frac{z_0}{\frac{H}{2}}(\frac{z-d}{d}+\frac{g}{\omega^2 d}) - d\frac{\overline{\phi_0}}{\frac{H}{2}}r(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\cos\theta\right] + i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\overline{\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)z_{\kappa_\ell}(z)} + \overline{\Lambda^*_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)z_{\kappa_\ell}(z)}]\cos(m\theta) + i\omega\frac{H}{2}d\sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\a_\ell}}^{\infty}[\overline{\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)}$$

$$\overline{z_{a_\ell}(z)} + \overline{\Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)z_{a_\ell}(z)}]]\cos(m\theta)\Bigg]\Bigg\} +$$

$$+\frac{1}{2}\mathrm{Re}\left\{ \left|X_{g_3}^{(1)} - X_5^{(1)} r\cos\theta\right|^2 \right\} -$$

$$- \text{Re}\ [\Big\{ \ (\frac{i\omega}{g})[-i\omega\frac{H}{2}d\left[\frac{z_0}{\frac{H}{2}}(\frac{z-d}{d}+\frac{g}{\omega^2 d})-d\,\frac{\phi_0}{\frac{H}{2}}r(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\cos\theta\right] - \text{i}\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)z_{\kappa_\ell}(z)+\ \Lambda^*{}_{m_{\kappa_\ell}}\Re^*_{m_{\kappa_\ell}}(r)z_{\kappa_\ell}(z)]\ \cos(m\theta)\text{ - i}\omega\frac{H}{2}d\ \sum_{m=0}^{\infty}[\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\ \Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)$$

$$z_{a_\ell}(z)+\Lambda^*{}_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)z_{a_\ell}(z)]\ ]\cos(m\theta)]\ \Big\}\ \{X^{(1)}_{g_3}-X^{(1)}_5 r\cos\theta\}\ ].$$

Προκύπτει δηλαδή ότι

$$\int_{WL}(\zeta_r^{(1)})^2 n_3 dl=\int_0^{2\pi}(\zeta_r^{(1)})^2\quad r\quad d\theta=$$

$$=\frac{1}{2}\text{Re}\Big\{\ (\frac{\omega}{g})^2\ [\omega^2\frac{H^2}{4}d^2\left[\frac{z_0}{H/2}(\frac{z-d}{d}+\frac{g}{\omega^2 d})\right]^2 r2\pi+\ \omega^2\frac{H^2}{4}d^2\left[\frac{z_0}{H/2}(\frac{z-d}{d}+\frac{g}{\omega^2 d})\right]$$

$$[[\overline{\Lambda_{0_{\kappa_\ell}}\Re_{0_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*{}_{0_{\kappa_\ell}}\Re^*_{0_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))+\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\overline{\Lambda_{0_{a_\ell}}\Re_{0_{a_\ell}}(r)}$$

$$\overline{+\Lambda^*{}_{0_{a_\ell}}\Re^*{}_{0_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]r2\pi+\left[d^2\frac{\phi_0\overline{\phi_0}}{H/2H/2}r^2(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})^2\right]\pi-$$

$$-i\omega\frac{H}{2}d\left[d\frac{\phi_0}{H/2}r(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\right][[\overline{\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+}\ \overline{\Lambda^*_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))$$

$$+\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\overline{\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)}\ \overline{+\Lambda^*_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]r\pi+$$

$$\omega^2\frac{H^2}{4}d^2\left[\frac{z_0}{H/2}(\frac{z-d}{d}+\frac{g}{\omega^2 d})\right][[\Lambda_{0_{\kappa_\ell}}\Re_{0_{\kappa_\ell}}(r)+\ \Lambda^*{}_{0_{\kappa_\ell}}\Re^*_{0_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))$$

$$+\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{0_{a_\ell}}\Re_{0_{a_\ell}}(r)\ +\Lambda^*{}_{0_{a_\ell}}\Re^*{}_{0_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]r2\pi+$$

$$i\omega\frac{H}{2}d\left[d\frac{\overline{\phi_0}}{H/2}r(\frac{z-d}{d^2}+\frac{g}{\omega^2 d^2})\right][[\Lambda_{1_{\kappa_\ell}}\Re_{1_{\kappa_\ell}}(r)+\ \Lambda^*_{1_{\kappa_\ell}}\Re^*_{1_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))$$

$$+\sum_{\substack{\ell=1\\ a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}}\Re_{1_{a_\ell}}(r)\ +\Lambda^*_{1_{a_\ell}}\Re^*{}_{1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]r\pi+\ \omega^2\frac{H^2}{4}d^2\ \sum_{m=0}[[\Lambda_{m_{\kappa_\ell}}\Re_{m_{\kappa_\ell}}(r)+$$

$$\Lambda^*_{m_{\kappa_\ell}} \Re^*_{m_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell)) + \sum_{\substack{\ell=1 \\ a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}} \Re_{m_{a_\ell}}(r)$$

$$+ \Lambda^*_{m_{a_\ell}} \Re^*_{m_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))] [[\overline{\Lambda_{m_{\kappa_\ell}} \Re_{m_{\kappa_\ell}}(r) +}\ \overline{\Lambda^*_{m_{\kappa_\ell}} \Re^*_{m_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell))$$

$$+ \sum_{\substack{\ell=1 \\ a_\ell}}^{\infty}[\overline{\Lambda_{m_{a_\ell}} \Re_{m_{a_\ell}}(r)}\ \overline{+ \Lambda^*_{m_{a_\ell}} \Re^*_{m_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]]r\pi + \omega^2 \frac{H^2}{4} d^2\, 2[[\Lambda_{0_{\kappa_\ell}} \Re_{0_{\kappa_\ell}}(r) +$$

$$\Lambda^*_{0_{\kappa_\ell}} \Re^*_{0_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell)) + \sum_{\substack{\ell=1 \\ a_\ell}}^{\infty}[\Lambda_{0_{a_\ell}} \Re_{0_{a_\ell}}(r) + \Lambda^*_{0_{a_\ell}} \Re^*_{0_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]$$

$$[[\overline{\Lambda_{0_{\kappa_\ell}} \Re_{0_{\kappa_\ell}}(r) +}\ \overline{\Lambda^*_{0_{\kappa_\ell}} \Re^*_{0_{\kappa_\ell}}(r)}]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell)) + \sum_{\substack{\ell=1 \\ a_\ell}}^{\infty}[\overline{\Lambda_{0_{a_\ell}} \Re_{0_{a_\ell}}(r)}$$

$$\left.\overline{+ \Lambda^*_{0_{a_\ell}} \Re^*_{0_{a_\ell}}(r)}]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]]r \right\} +$$

$$+\frac{1}{2}\mathrm{Re}\left\{2\pi (X_{g_3}^{(1)})^2 r + (X_5^{(1)})^2 r^3 \pi\right\} -$$

$$-\mathrm{Re}\left[\left\{ \left(\frac{i\omega}{g}\right) [-i\omega \frac{H}{2} d\left[\frac{z_0}{H/2}\left(\frac{z-d}{d} + \frac{g}{\omega^2 d}\right)\right] r2\pi - i\omega \frac{H}{2} d\ [[\Lambda_{0_{\kappa_\ell}} \Re_{0_{\kappa_\ell}}(r) + \right.\right.$$

$$\Lambda^*_{0_{\kappa_\ell}} \Re^*_{0_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell)) + \sum_{\substack{\ell=1 \\ a_\ell}}^{\infty}[\Lambda_{0_{a_\ell}} \Re_{0_{a_\ell}}(r)$$

$$\left.+ \Lambda^*_{0_{a_\ell}} \Re^*_{0_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]r2\pi] \right\}] +$$

$$+\mathrm{Re}\left[\left\{ \left(\frac{i\omega}{g}\right) X_5^{(1)} r^2\ [i\omega \frac{H}{2} d\left[d\frac{\phi_0}{H/2} r\left(\frac{z-d}{d} + \frac{g}{\omega^2 d^2}\right)\right]\pi - i\omega \frac{H}{2} d\ [[\Lambda_{1_{\kappa_\ell}} \Re_{1_{\kappa_\ell}}(r) + \right.\right.$$

$$\Lambda^*_{1_{\kappa_\ell}} \Re^*_{1_{\kappa_\ell}}(r)]N_{\kappa_\ell}^{-1/2}\cosh(\kappa_\ell(z-h_\ell)) + \sum_{\substack{\ell=1 \\ a_\ell}}^{\infty}[\Lambda_{1_{a_\ell}} \Re_{1_{a_\ell}}(r)$$

$$\left.+ \Lambda^*_{1_{a_\ell}} \Re^*_{1_{a_\ell}}(r)]N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]\pi] \right\}].$$

Όπου $R^{(1)}$: ο πίνακας που περιέχει τις γωνίες περιστροφής.

X_{g_3} :ευθύγραμμη μεταφορική κίνηση του κέντρου βάρους της κατασκευής στον άξονα OX_3 .

X_5 : περιστροφή γύρω από τον άξονα GX_2.

h_ℓ : η απόσταση του $\ell -$στου στοιχείου από τον πυθμένα.

7.4 Υπολογισμός της κατακόρυφης δύναμης έκπτωσης για το πεδίο (ΙΙ)

Η κατακόρυφη δύναμη έκπτωσης F_Z για το Πεδίο (ΙΙ), Κεφάλαιο 2 –σελίδα 16– υπολογίζεται από τη σχέση

$$F_Z = -\int_{WL} \frac{1}{2}\rho g \zeta_r^{(1)2}\bar{n}dl + MR^{(1)}\overline{X_g^{(1)}}'' - \iint_{S_0} -\frac{1}{2}\rho\left|\nabla\Phi^{(1)}\right|^2\bar{n}dS - \iint_{S_0} -\rho(\overline{X}^{(1)}\nabla\Phi_t^{(1)})\bar{n}dS =$$

$$= -\frac{1}{2}\rho g\,[\underline{\int_{WL} \zeta_r^{(1)2}\bar{n}dl}\,] + MR^{(1)}\overline{X_g^{(1)}}'' + \frac{1}{2}\rho\,[\underline{\iint_{S_0}\left|\nabla\Phi^{(1)}\right|^2\bar{n}dS}\,] + \rho\,[\underline{\iint_{S_0}(\overline{X}^{(1)}\nabla\Phi_t^{(1)})\bar{n}dS}\,].$$

Όμως οι παραστάσεις $\underline{\int_{WL} \zeta_r^{(1)2}\bar{n}dl}$ $\underline{\iint_{S_0}\left|\nabla\Phi^{(1)}\right|^2\bar{n}dS}$ $\underline{[\iint_{S_0}(\overline{X}^{(1)}\nabla\Phi_t^{(1)})\bar{n}dS]}$

είναι γνωστές από τα προηγούμενα. (Σελίδα 206, σελίδα 179 και σελίδα 201 αντίστοιχα)

Όπου *ρ:* η πυκνότητα νερού.

g: η επιτάχυνση της βαρύτητας.

M : η μάζα αξονοσυμμετρικού σώματος.

$R^{(1)}$: ο πίνακας που περιέχει τις γωνίες περιστροφής.

$X_g^{(1)}$: οι μεταφορικές κινήσεις του κέντρου βάρους της κατασκευής.

ω : η ιδιοσυχνότητα της κατασκευής.

Επομένως μπορούμε να υπολογίσουμε την κατακόρυφη δύναμη έκπτωσης F_Z για το Πεδίο (ΙΙ).

8^{o} ΚΑΤΑΚΟΡΥΦΗ ΔΥΝΑΜΗ ΕΚΠΤΩΣΗΣ ΓΙΑ ΤΟ ΠΕΔΙΟ (ΙΙΙ)

8.1Υπολογισμός του όρου $\int_{b_p}^{b_{p+1}} (\int_0^{2\pi} \left|\nabla\Phi^{(1)}\right|^2)\overline{n}dS$ για το πεδίο (ΙΙΙ)

Έχουμε αποδείξει στο Κεφάλαιο 5 ότι για δακτυλιοειδή στοιχεία στο πεδίο (ΙΙΙ): $b_p \le r \le b_{p+1} \quad \kappa\alpha\iota \quad 0 \le z \le h_p$ η σχέση που μας δίνει το $(\left|\nabla\Phi^{(1)}\right|^2)$ της ταχύτητας πρώτης τάξης είναι

$$(\left|\nabla\Phi^{(1)}\right|^2) =$$

$$= \overline{\left(\frac{\partial\phi(r,\theta,z)}{\partial r}\right)^2}^T + \overline{\frac{1}{r^2}\left(\frac{\partial\phi(r,\theta,z)}{\partial\theta}\right)^2}^T + \overline{\left(\frac{\partial\phi(r,\theta,z)}{\partial z}\right)^2}^T =$$

$$= \frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial r}\frac{\partial\overline{\phi(r,\theta,z)}}{\partial r} + \frac{1}{r^2}\frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\frac{\partial\overline{\phi(r,\theta,z)}}{\partial\theta} + \frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial z}\frac{\partial\overline{\phi(r,\theta,z)}}{\partial z} =$$

$$= \frac{1}{2}\omega^2\frac{H^2}{4}d^2(-\frac{z_0}{H/2}\frac{r}{2h_p d} - d\frac{\overline{\phi_0}}{H/2}\frac{(z^2-0{,}75r^2)}{2h_p d^2}\cos\theta)$$

$$(\frac{z_0}{H/2}\frac{-r}{2h_p d} - d\frac{\phi_0}{H/2}\frac{(z^2-0{,}75r^2)}{2h_p d^2}\cos\theta) +$$

$$+\frac{1}{2}\omega^2\frac{H^2}{4}d^2(\frac{z_0}{H/2}\frac{-r}{2h_p d} - d\frac{\phi_0}{H/2}\frac{(z^2-0{,}75r^2)}{2h_p d^2}\cos\theta)\ \overline{\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\frac{\partial\Re_{mn_p}(r)}{\partial r} +}$$

$$\overline{\Lambda^*_{mn_p}\frac{\partial\Re^*_{mn_p}(r)}{\partial r}})\cos(\frac{n_p\pi z}{h_p})]\cos(m\theta) +$$

$$+\frac{1}{2}\omega^2\frac{H^2}{4}d^2(-\frac{z_0}{H/2}\frac{r}{2h_p d} - d\frac{\overline{\phi_0}}{H/2}\frac{(z^2-0{,}75r^2)}{2h_p d^2}\cos\theta)\ \sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\frac{\partial\Re_{mn_p}(r)}{\partial r} +$$

$$\Lambda^*_{mn_p}\frac{\partial\Re^*_{mn_p}(r)}{\partial r})\cos(\frac{n_p\pi z}{h_p})]\cos(m\theta) + \frac{1}{2}\omega^2\frac{H^2}{4}d^2\ \overline{\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\frac{\partial\Re_{mn_p}(r)}{\partial r} +}$$

$$\overline{\Lambda^*_{mn_p}\frac{\partial\Re^*_{mn_p}(r)}{\partial r}})\cos(\frac{n_p\pi z}{h_p})]\cos(m\theta)\sum_{n=0}^{\infty}[\sum_{n_q=0}^{\infty}\in_{n_q}(\Lambda_{nn_q}\frac{\partial\Re_{nn_q}(r)}{\partial r} + \Lambda^*_{nn_q}\frac{\partial\Re^*_{nn_q}(r)}{\partial r})\cos(\frac{n_q\pi z}{h_p})]$$

$$\cos(n\theta) +$$

$$+\frac{1}{2}\omega^2\frac{H^2}{4}d^2\frac{1}{r^2}(d\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_p d^2}\sin\theta)\;(d\frac{\overline{\phi_0}}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_p d^2}\sin\theta)-$$

$$-\frac{1}{2}\omega^2\frac{H^2}{4}d^2\frac{1}{r^2}(d\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_p d^2}\sin\theta)\;\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\overline{\in_{n_p}\;(\Lambda_{mn_p}\Re_{mn_p}(r)m+}$$

$$\overline{\Lambda^*_{mn_p}\Re^*_{mn_p}(r)m)}\cos(\frac{n_p\pi z}{h_p})]\sin(m\theta)\;-\frac{1}{2}\omega^2\frac{H^2}{4}d^2\frac{1}{r^2}(d\frac{\overline{\phi_0}}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_p d^2}\sin\theta)$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\;(\Lambda_{mn_p}\Re_{mn_p}(r)m+\Lambda^*_{mn_p}\Re^*_{mn_p}(r)m)\cos(\frac{n_p\pi z}{h_p})]\sin(m\theta)\;+\frac{1}{2}\omega^2\frac{H^2}{4}d^2\frac{1}{r^2}$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\;(\Lambda_{mn_p}\Re_{mn_p}(r)m+\Lambda^*_{mn_p}\Re^*_{mn_p}(r)m)\cos(\frac{n_p\pi z}{h_p})]\sin(m\theta)$$

$$\sum_{n=0}^{\infty}[\sum_{n_q=0}^{\infty}\overline{\in_{n_q}\;(\Lambda_{nn_q}\Re_{nn_q}(r)n}+\overline{\Lambda^*_{nn_q}\Re^*_{nn_q}(r)n)}\cos(\frac{n_q\pi z}{h_p})]\sin(n\theta)+$$

$$+\frac{1}{2}\omega^2\frac{H^2}{4}d^2(\frac{z_0}{H/2}\frac{2z}{2h_p d}-d\frac{\phi_0}{H/2}\frac{r2z}{2h_p d^2}\cos\theta)\;(\frac{z_0}{H/2}\frac{2z}{2h_p d}-d\frac{\overline{\phi_0}}{H/2}\frac{r2z}{2h_p d^2}\cos\theta)-$$

$$-\frac{1}{2}\omega^2\frac{H^2}{4}d^2(\frac{z_0}{H/2}\frac{2z}{2h_p d}-d\frac{\phi_0}{H/2}\frac{r2z}{2h_p d^2}\cos\theta)\;\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\overline{\in_{n_p}\;(\Lambda_{mn_p}\Re_{mn_p}(r)+}$$

$$\overline{\Lambda^*_{mn_p}\Re^*_{mn_p}(r))}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]\cos(\mathrm{m\theta})\;-\frac{1}{2}\omega^2\frac{H^2}{4}d^2\;(\frac{z_0}{H/2}\frac{2z}{2h_p d}-d\frac{\overline{\phi_0}}{H/2}\frac{r2z}{2h_p d^2}\cos\theta)$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\;(\Lambda_{mn_p}\Re_{mn_p}(r)+\Lambda^*_{mn_p}\Re^*_{mn_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]\cos(m\theta)+\;\frac{1}{2}\omega^2\frac{H^2}{4}d^2$$

$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\;(\Lambda_{mn_p}\Re_{mn_p}(r)+\Lambda^*_{mn_p}\Re^*_{mn_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]\cos(m\theta)$$

$$\sum_{n=0}^{\infty}[\sum_{n_q=0}^{\infty}\overline{\in_{n_q}\;(\Lambda_{nn_q}\Re_{nn_q}(r)}+\overline{\Lambda^*_{nn_q}\Re^*_{nn_q}(r))}\sin(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}]\cos(n\theta).$$

Για τον υπολογισμό της κατακόρυφης δύναμης έκπτωσης F_Z θα υπολογίσουμε τον όρο

$$\int_{b_p}^{b_{p+1}}(\int_0^{2\pi}\left|\nabla\Phi^{(1)}\right|^2)\overline{n_3}dS$$

Όπου b_i η ακτίνα του $i-$στου στοιχείου

$\overline{n_3} = -1$

$dS = rd\theta dr.$

Επομένως

$$\int_{b_p}^{b_{p+1}} (\int_0^{2\pi} \left|\nabla\Phi^{(1)}\right|^2) \overline{n_3} dS = \int_{b_p}^{b_{p+1}} (\int_0^{2\pi} \left|\nabla\Phi^{(1)}\right|^2)(-1) \quad r \quad d\theta dr.$$

Από το *Παράρτημα Δ*, υπολογίζουμε τη σχέση

$$\int_0^{2\pi} (\left|\nabla\Phi^{(1)}\right|^2)(-1) \quad r \quad d\theta =$$

$$= \omega^2 \frac{H^2}{8} d^2 \pi (\frac{z_0{}^2}{(H/2)^2} \frac{r^2}{(2h_p d)^2} 2 + d^2 \frac{\phi_0 \overline{\phi_0}}{(H/2)(H/2)} \frac{(z^2 - 0{,}75r^2)^2}{(2h_p d^2)^2})(-1)r +$$

$$+ \omega^2 \frac{H^2}{8} d^2 \pi [(\frac{z_0}{H/2} \frac{-r}{2h_p d}) \sum_{n_p=0}^{\infty} \in_{n_p} \overline{(\Lambda_{0n_p} \frac{\partial \Re_{0n_p}(r)}{\partial r}} + \overline{\Lambda^*_{0n_p} \frac{\partial \Re^*_{0n_p}(r)}{\partial r}}) \cos(\frac{n_p \pi z}{h_p}) \frac{1}{\pi} -$$

$$- d \frac{\phi_0}{H/2} \frac{(z^2 - 0{,}75r^2)}{2h_p d^2} \sum_{n_p=0}^{\infty} \in_{n_p} \overline{(\Lambda_{1n_p} \frac{\partial \Re_{1n_p}(r)}{\partial r}} + \overline{\Lambda^*_{1n_p} \frac{\partial \Re^*_{1n_p}(r)}{\partial r}}) \cos(\frac{n_p \pi z}{h_p})](-1)r +$$

$$+ \omega^2 \frac{H^2}{8} d^2 \pi (-\frac{z_0}{H/2} \frac{r}{2h_p d}) \sum_{n_p=0}^{\infty} \in_{n_p} (\Lambda_{0n_p} \frac{\partial \Re_{0n_p}(r)}{\partial r} + \Lambda^*_{0n_p} \frac{\partial \Re^*_{0n_p}(r)}{\partial r}) \cos(\frac{n_p \pi z}{h_p}) \frac{1}{\pi} -$$

$$- d \frac{\overline{\phi_0}}{H/2} \frac{(z^2 - 0{,}75r^2)}{2h_p d^2} \sum_{n_p=0}^{\infty} \in_{n_p} (\Lambda_{1n_p} \frac{\partial \Re_{1n_p}(r)}{\partial r} + \Lambda^*_{1n_p} \frac{\partial \Re^*_{1n_p}(r)}{\partial r}) \cos(\frac{n_p \pi z}{h_p})](-1)r +$$

$$+ \omega^2 \frac{H^2}{8} d^2 \pi [\sum_{m=1}^{\infty} [\sum_{n_p=0}^{\infty} \in_{n_p} \overline{(\Lambda_{mn_p} \frac{\partial \Re_{mn_p}(r)}{\partial r}} + \overline{\Lambda^*_{mn_p} \frac{\partial \Re^*_{mn_p}(r)}{\partial r}}) \cos(\frac{n_p \pi z}{h_p})]$$

$$[\sum_{n_p=0}^{\infty} \in_{n_p} (\Lambda_{mn_p} \frac{\partial \Re_{mn_p}(r)}{\partial r} + \Lambda^*_{mn_p} \frac{\partial \Re^*_{mn_p}(r)}{\partial r}) \cos(\frac{n_p \pi z}{h_p})] + [\sum_{n_p=0}^{\infty} \in_{n_p} \overline{(\Lambda_{0n_p} \frac{\partial \Re_{0n_p}(r)}{\partial r} +}$$

$$\overline{\Lambda^*_{0n_p} \frac{\partial \Re^*_{0n_p}(r)}{\partial r}}) \cos(\frac{n_p \pi z}{h_p})] [\sum_{n_p=0}^{\infty} \in_{n_p} (\Lambda_{0n_p} \frac{\partial \Re_{0n_p}(r)}{\partial r} + \Lambda^*_{0n_p} \frac{\partial \Re^*_{0n_p}(r)}{\partial r}) \cos(\frac{n_p \pi z}{h_p})]](-1)r +$$

$$+ \omega^2 \frac{H^2}{8} d^2 \pi (d^2 \frac{\phi_0 \overline{\phi_0}}{H/2 H/2} \frac{(z^2 - 0{,}25r^2)^2}{(2h_p d^2)^2})(-1)r -$$

$$-\omega^2\frac{H^2}{8}d^2\pi(d\frac{\phi_0}{H/2}\frac{(z^2-0{,}25r^2)}{2h_p d^2})\ \sum_{n_p=0}^{\infty}\overline{\in_{n_p}\ (\Lambda_{1n_p}\Re_{1n_p}(r)1}+$$

$$\overline{\Lambda^*_{1n_p}\Re^*_{1n_p}(r)1)}\cos(\frac{n_p\pi z}{h_p})](-1)-$$

$$-\omega^2\frac{H^2}{8}d^2\pi(d\frac{\overline{\phi_0}}{H/2}\frac{(z^2-0{,}25r^2)}{2h_p d^2})\ \sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{1n_p}\Re_{1n_p}(r)1+$$

$$\Lambda^*_{1n_p}\Re^*_{1n_p}(r)1)\cos(\frac{n_p\pi z}{h_p})](-1)+$$

$$+\omega^2\frac{H^2}{8}d^2\frac{1}{r^2}\pi\sum_{m=1}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{mn_p}\Re_{mn_p}(r)+\ \Lambda^*_{mn_p}\Re^*_{mn_p}(r))\cos(\frac{n_p\pi z}{h_p})]$$

$$[\sum_{n_p=0}^{\infty}\overline{\in_{n_p}\ (\Lambda_{mn_p}\Re_{mn_p}(r)}+\overline{\Lambda^*_{mn_p}\Re^*_{mn_p}(r))}\cos(\frac{n_p\pi z}{h_p})]m^2](-1)r+$$

$$+\omega^2\frac{H^2}{8}d^2\pi(\frac{{z_0}^2}{(H/2)^2}\frac{(2z)^2}{(2h_p d)^2}2+d^2\frac{\phi_0}{H/2}\frac{\overline{\phi_0}}{H/2}r^2\frac{(2z)^2}{(2h_p d^2)^2})(-1)r-$$

$$-\omega^2\frac{H^2}{8}d^2\pi[(\frac{z_0}{H/2}\frac{2z}{2h_p d})\ \sum_{n_p=0}^{\infty}\in_{n_p}\ (\overline{\Lambda_{0n_p}\Re_{0n_p}(r)}+\overline{\Lambda^*_{0n_p}\Re^*_{0n_p}(r))}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}\frac{1}{\pi}-$$

$$-d\frac{\phi_0}{H/2}\frac{r2z}{2h_p d^2}\ \sum_{n_p=0}^{\infty}\in_{n_p}\ (\overline{\Lambda_{1n_p}\Re_{1n_p}(r)}+\overline{\Lambda^*_{1n_p}\Re^*_{1n_p}(r))}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}](-1)r-$$

$$-\omega^2\frac{H^2}{8}d^2\pi[(\frac{z_0}{H/2}\frac{2z}{2h_p d})\ \sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{0n_p}\Re_{0n_p}(r)+\Lambda^*_{0n_p}\Re^*_{0n_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}\frac{1}{\pi}-$$

$$-d\frac{\overline{\phi_0}}{H/2}\frac{r2z}{2h_p d^2}\ \sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}](-1)r+$$

$$+\omega^2\frac{H^2}{8}d^2\pi[\sum_{m=1}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{mn_p}\Re_{mn_p}(r)+\ \Lambda^*_{mn_p}\Re^*_{mn_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]$$

$$[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\overline{\Lambda_{mn_p}\Re_{mn_p}(r)}+\overline{\Lambda^*_{mn_p}\Re^*_{mn_p}(r))}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]+[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{0n_p}\Re_{0n_p}(r)+$$

$$\Lambda^*_{0n_p}\Re^*_{0n_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]\ [\sum_{n_p=0}^{\infty}\in_{n_p}\ (\overline{\Lambda_{0n_p}\Re_{0n_p}(r)}+\overline{\Lambda^*_{0n_p}\Re^*_{0n_p}(r))}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]](-1)r$$

Επομένως

$$\int_{b_p}^{b_{p+1}} (\int_0^{2\pi} \left|\nabla\Phi^{(1)}\right|^2)\overline{n_3}dS = \int_{b_p}^{b_{p+1}} (\int_0^{2\pi} \left|\nabla\Phi^{(1)}\right|^2)(-1) \quad r \quad d\theta dr =$$

$$= (-1)\omega^2 \frac{H^2}{8} d^2\pi \Big\{ \quad \frac{{z_0}^2}{(H/2)^2}\frac{2}{(2h_p d)^2}(\frac{(b_{p+1})^3}{3} - \frac{(b_p)^3}{3}) + d^2 \frac{\phi_0\overline{\phi_0}}{(H/2)(H/2)}\frac{1}{(2h_p d^2)^2}$$

$$(z^4(\frac{(b_{p+1})^2}{2} - \frac{(b_p)^2}{2}) - d^2 \frac{\phi_0\overline{\phi_0}}{(H/2)(H/2)}\frac{2}{(2h_p d^2)^2} 0{,}75 z^2 (\frac{(b_{p+1})^4}{4} - \frac{(b_p)^4}{4}) +$$

$$+ d^2 \frac{\phi_0\overline{\phi_0}}{(H/2)(H/2)}\frac{1}{(2h_p d^2)^2} 0{,}75(\frac{(b_{p+1})^6}{6} - \frac{(b_p)^6}{6}) \quad \Big\} +$$

$$+ (-1)\omega^2 \frac{H^2}{8} d^2\pi \Big\{ \quad (-\frac{z_0}{H/2}\frac{1}{2h_p d}\frac{1}{\pi}) \sum_{n_p=0}^{\infty} (\int_{b_p}^{b_{p+1}} \overline{\in_{n_p} \ (\Lambda_{0n_p} \frac{\partial \Re_{0n_p}(r)}{\partial r} +} \qquad \text{(A)}$$

$$\overline{\Lambda^*_{0n_p} \frac{\partial \Re^*_{0n_p}(r)}{\partial r}}) \cos(\frac{n_p \pi z}{h_p})] r^2 dr) - d \frac{\phi_0}{H/2}\frac{z^2}{2h_p d^2} \sum_{n_p=0}^{\infty} (\int_{b_p}^{b_{p+1}} \overline{\in_{n_p} \ (\Lambda_{1n_p} \frac{\partial \Re_{1n_p}(r)}{\partial r} +} \qquad (\Gamma)$$

$$\overline{\Lambda^*_{1n_p} \frac{\partial \Re^*_{1n_p}(r)}{\partial r}}) \cos(\frac{n_p \pi z}{h_p})] r dr + d \frac{\phi_0}{H/2}\frac{0{,}75}{2h_p d^2} \sum_{n_p=0}^{\infty} (\int_{b_p}^{b_{p+1}} \overline{\in_{n_p} \ (\Lambda_{1n_p} \frac{\partial \Re_{1n_p}(r)}{\partial r} +} \qquad \text{(E)}$$

$$\overline{\Lambda^*_{1n_p} \frac{\partial \Re^*_{1n_p}(r)}{\partial r}}) \cos(\frac{n_p \pi z}{h_p})] r^3 dr \quad \Big\} +$$

$$+ (-1)\omega^2 \frac{H^2}{8} d^2\pi \Big\{ \quad (-\frac{z_0}{H/2}\frac{1}{2h_p d}\frac{1}{\pi}) \sum_{n_p=0}^{\infty} (\int_{b_p}^{b_{p+1}} \in_{n_p} \ (\Lambda_{0n_p} \frac{\partial \Re_{0n_p}(r)}{\partial r} + \qquad \text{(B)}$$

$$\Lambda^*_{0n_p} \frac{\partial \Re^*_{0n_p}(r)}{\partial r}) \cos(\frac{n_p \pi z}{h_p})] r^2 dr - d \frac{\overline{\phi_0}}{H/2}\frac{z^2}{2h_p d^2} \sum_{n_p=0}^{\infty} (\int_{b_p}^{b_{p+1}} \in_{n_p} \ (\Lambda_{1n_p} \frac{\partial \Re_{1n_p}(r)}{\partial r} + \qquad (\Delta)$$

$$\Lambda^*_{1n_p} \frac{\partial \Re^*_{1n_p}(r)}{\partial r}) \cos(\frac{n_p \pi z}{h_p})] r dr + d \frac{\overline{\phi_0}}{H/2}\frac{0{,}75}{2h_p d^2} \sum_{n_p=0}^{\infty} (\int_{b_p}^{b_{p+1}} \in_{n_p} \ (\Lambda_{1n_p} \frac{\partial \Re_{1n_p}(r)}{\partial r} + \qquad \text{(Z)}$$

$$\Lambda^*_{1n_p} \frac{\partial \Re^*_{1n_p}(r)}{\partial r}) \cos(\frac{n_p \pi z}{h_p})] r^3 dr \quad \Big\} +$$

$$+ (-1)\omega^2 \frac{H^2}{8} d^2\pi \ \Big\{ \ \sum_{m=1}^{\infty} [\int_{b_p}^{b_{p+1}} \sum_{n_p=0}^{\infty} \overline{\in_{n_p} \ (\Lambda_{mn_p} \frac{\partial \Re_{mn_p}(r)}{\partial r}} + \overline{\Lambda^*_{mn_p} \frac{\partial \Re^*_{mn_p}(r)}{\partial r}}) \cos(\frac{n_p \pi z}{h_p})] \qquad \text{(H)}$$

$$[\sum_{n_q=0}^{\infty}\in_{n_q}(\Lambda_{mn_q}\frac{\partial\Re_{mn_q}(r)}{\partial r}+\Lambda^*_{mn_q}\frac{\partial\Re^*_{mn_q}(r)}{\partial r})\cos(\frac{n_q\pi z}{h_p})]rdr+[\int_{b_p}^{b_{p+1}}\overline{\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{0n_p}\frac{\partial\Re_{0n_p}(r)}{\partial r}+}$$

$$\overline{\Lambda^*_{0n_p}\frac{\partial\Re^*_{0n_p}(r)}{\partial r}})\cos(\frac{n_p\pi z}{h_p})][\sum_{n_q=0}^{\infty}\in_{n_p}(\Lambda_{0n_q}\frac{\partial\Re_{0n_q}(r)}{\partial r}+\Lambda^*_{0n_q}\frac{\partial\Re^*_{0n_q}(r)}{\partial r})\cos(\frac{n_q\pi z}{h_p})]]rdr+$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{d^2\frac{\phi_0\overline{\phi_0}}{H/2H/2}\frac{1}{(2h_pd^2)^2})\left\{z^4(\frac{(b_{p+1})^2}{2}-\frac{(b_p)^2}{2})-0{,}5z^2\right.\right.$$

$$(\frac{(b_{p+1})^4}{4}-\frac{(b_p)^4}{4})+0{,}25^2(\frac{(b_{p+1})^6}{6}-\frac{(b_p)^6}{6})\left.\left.\right\}\right\}+ \qquad (\Theta)$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{d\frac{\phi_0}{H/2}\frac{z^2}{2h_pd^2}\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}\overline{\in_{n_p}(\Lambda_{1n_p}\Re_{1n_p}(r)}+\overline{\Lambda^*_{1n_p}\Re^*_{1n_p}(r)})\cos(\frac{n_p\pi z}{h_p})]dr-\right.$$

$$\left.-d\frac{\phi_0}{H/2}\frac{0{,}25}{2h_pd^2}\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}\overline{\in_{n_p}(\Lambda_{1n_p}\Re_{1n_p}(r)}+\overline{\Lambda^*_{1n_p}\Re^*_{1n_p}(r)})\cos(\frac{n_p\pi z}{h_p})]r^2dr\right\}+ \qquad (K)$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{d\frac{\overline{\phi_0}}{H/2}\frac{z^2}{2h_pd^2}\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))\cos(\frac{n_p\pi z}{h_p})]dr-\right.$$

$$\left.-d\frac{\overline{\phi_0}}{H/2}\frac{0{,}25}{2h_pd^2}\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))\cos(\frac{n_p\pi z}{h_p})]r^2dr\right\}+ \qquad (I)(\Lambda)$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\frac{1}{r^2}\pi\left\{\sum_{m=1}^{\infty}[m^2\int_{b_p}^{b_{p+1}}(\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\Re_{mn_p}(r)+\Lambda^*_{mn_p}\Re^*_{mn_p}(r))\cos(\frac{n_p\pi z}{h_p})\right.$$

$$\sum_{n_q=0}^{\infty}\overline{\in_{n_q}(\Lambda_{mn_q}\Re_{mn_q}(r)}+\overline{\Lambda^*_{mn_q}\Re^*_{mn_q}(r)})\cos(\frac{n_q\pi z}{h_p}))rdr] \qquad (M)$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{\frac{{z_0}^2}{(H/2)^2}\frac{(2z)^2}{(2h_pd)^2}2(\frac{(b_{p+1})^2}{2}-\frac{(b_p)^2}{2})+d^2\frac{\phi_0}{H/2}\frac{\overline{\phi_0}}{H/2}\right.$$

$$\left.\frac{(2z)^2}{(2h_pd^2)^2}(\frac{(b_{p+1})^4}{4}-\frac{(b_p)^4}{4})\right\}+$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{\frac{z_0}{H/2}\frac{2z}{2h_pd}\frac{1}{\pi}\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}\in_{n_p}(\overline{\Lambda_{0n_p}\Re_{0n_p}(r)}+\right. \qquad (N)$$

$$\overline{\Lambda^*_{0n_p}\Re^*_{0n_p}(r)})\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}rdr-d\frac{\phi_0}{H/2}\frac{2z}{2h_pd^2}\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}\in_{n_p}(\overline{\Lambda_{1n_p}\Re_{1n_p}(r)}+$$

$$\left.\overline{\Lambda^*_{1n_p}\Re^*_{1n_p}(r)})\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}r^2dr\right\}+ \qquad (O)$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{\frac{z_0}{H/2}\frac{2z}{2h_p d}\frac{1}{\pi}\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{0n_p}\Re_{0n_p}(r)+\right. \quad (\Xi)$$

$$\Lambda^*_{0n_p}\Re^*_{0n_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}rdr-d\frac{\overline{\phi_0}}{H/2}\frac{2z}{2h_p d^2}\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{1n_p}\Re_{1n_p}(r)+$$

$$\left.\Lambda^*_{1n_p}\Re^*_{1n_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}r^2dr\right\}+ \quad (\Pi)$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{\sum_{m=1}^{\infty}[\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{mn_p}\Re_{mn_p}(r)+\Lambda^*_{mn_p}\Re^*_{mn_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}\right. \quad (P)$$

$$\sum_{n_q=0}^{\infty}\in_{n_q}(\overline{\Lambda_{mn_q}\Re_{mn_q}(r)}+\overline{\Lambda^*_{mn_q}\Re^*_{mn_q}(r)})\sin(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}rdr]+2[\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{0n_p}\Re_{0n_p}(r)+$$

$$\Lambda^*_{0n_p}\Re^*_{0n_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}\sum_{n_q=0}^{\infty}\in_{n_q}(\overline{\Lambda_{0n_q}\Re_{0n_q}(r)}+\overline{\Lambda^*_{0n_q}\Re^*_{0n_q}(r)})\sin(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}]rdr$$

Στη συνέχεια θα υπολογίσουμε τα ολοκληρώματα, χρησιμοποιώντας το *Παράρτημα Ζ.*

A)

$$\int_{b_p}^{b_{p+1}} \sum_{n_p=0}^{\infty} (\overline{\in_{n_p} (\Lambda_{0n_p} \frac{\partial \Re_{0n_p}(r)}{\partial r}} + \overline{\Lambda^*_{0n_p} \frac{\partial \Re^*_{0n_p}(r)}{\partial r}}) \cos(\frac{n_p \pi z}{h_p})] r^2 dr) =$$

$$= \sum_{n_p=1}^{\infty} \left[\overline{\frac{\in_{n_p} \Lambda_{on_p} \frac{n_p\pi}{h_p} \mathrm{K}_0(\frac{n_p\pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{on_p} \frac{n_p\pi}{h_p} \mathrm{K}_0(\frac{n_p\pi b_p}{h_p})}{I_0(\frac{n_p\pi b_{p+1}}{h_p}) K_0(\frac{n_p\pi b_p}{h_p}) - I_0(\frac{n_p\pi b_p}{h_p}) K_0(\frac{n_p\pi b_{p+1}}{h_p})}} \right] \cos(\frac{n_p\pi z}{h_p})$$

$$\left[\overline{\frac{-b_p{}^2 I_2(\frac{b_p n_p \pi}{h_p}) + b_{p+1}{}^2 I_2(\frac{b_{p+1} n_p \pi}{h_p})}{(\frac{n_p\pi}{h_p})}} \right] +$$

$$+ \sum_{n_p=1}^{\infty} \left[\overline{\frac{\in_{n_p} \Lambda_{on_p} \frac{n_p\pi}{h_p} I_0(\frac{n_p\pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{on_p} \frac{n_p\pi}{h_p} I_0(\frac{n_p\pi b_p}{h_p})}{I_0(\frac{n_p\pi b_{p+1}}{h_p}) K_0(\frac{n_p\pi b_p}{h_p}) - I_0(\frac{n_p\pi b_p}{h_p}) K_0(\frac{n_p\pi b_{p+1}}{h_p})}} \right] \cos(\frac{n_p\pi z}{h_p}) \int_{b_p}^{b_{p+1}} r^2 \overline{K_1(\frac{n_p\pi r}{h_p})} dr +$$

$$\overline{\left[\in_{0_p} (\Lambda_{00_p} \frac{1}{\ln(\frac{b_{p+1}}{b_p})} - \Lambda^*_{00_p} \frac{1}{\ln(\frac{b_{p+1}}{b_p})}) \right]} (\frac{b_{p+1}^2 - b_p^2}{2})$$

B)

$$\int_{b_p}^{b_{p+1}} \sum_{n_p=0}^{\infty} (\in_{n_p} (\Lambda_{0n_p} \frac{\partial \Re_{0n_p}(r)}{\partial r} + \Lambda^*_{0n_p} \frac{\partial \Re^*_{0n_p}(r)}{\partial r}) \cos(\frac{n_p \pi z}{h_p})] r^2 dr =$$

$$=\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{on_p}\frac{n_p\pi}{h_p}\mathrm{K}_0(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{on_p}\frac{n_p\pi}{h_p}\mathrm{K}_0(\frac{n_p\pi b_p}{h_p})}{I_0(\frac{n_p\pi b_{p+1}}{h_p})K_0(\frac{n_p\pi b_p}{h_p})-I_0(\frac{n_p\pi b_p}{h_p})K_0(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\frac{-b_p{}^2 I_2(\frac{b_p n_p\pi}{h_p})+b_{p+1}{}^2 I_2(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right]+$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{on_p}\frac{n_p\pi}{h_p}I_0(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{on_p}\frac{n_p\pi}{h_p}I_0(\frac{n_p\pi b_p}{h_p})}{I_0(\frac{n_p\pi b_{p+1}}{h_p})K_0(\frac{n_p\pi b_p}{h_p})-I_0(\frac{n_p\pi b_p}{h_p})K_0(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\int_{b_p}^{b_{p+1}} r^2 K_1(\frac{n_p\pi r}{h_p})dr+$$

$$\left[\in_{0_p}\ (\Lambda_{00_p}\frac{1}{\ln(\frac{b_{p+1}}{b_p})}-\Lambda^*_{00_p}\frac{1}{\ln(\frac{b_{p+1}}{b_p})})\right](\frac{b_{p+1}^2-b_p^2}{2})$$

Γ)

$$\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}(\in_{n_p}\ \overline{(\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}}+\overline{\Lambda^*_{1n_p}\frac{\partial\Re^*_{1n_p}(r)}{\partial r}})\cos(\frac{n_p\pi z}{h_p})]rdr=$$

$$=\sum_{n_p=1}^{\infty}\left[\frac{\overline{\in_{n_p}\ \Lambda_{1n_p}\frac{n_p\pi}{h_p}K_1(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{1n_p}\frac{n_p\pi}{h_p}K_1(\frac{n_p\pi b_{p+1}}{h_p})}}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\overline{\frac{1}{(\frac{n_p\pi}{h_p})^2}\left[2I_0(\frac{b_p n_p\pi}{h_p})-2I_0(\frac{b_{p+1}n_p\pi}{h_p})-b_p I_1(\frac{b_p n_p\pi}{h_p})+b_{p+1}I_1(\frac{b_{p+1}n_p\pi}{h_p})\right]}\right]+$$

$$+\sum_{n_p=1}^{\infty}\left[\overline{\frac{\in_{n_p}\ \Lambda_{1n_p}\frac{n_p\pi}{h_p}I_1(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{1n_p}\frac{n_p\pi}{h_p}I_1(\frac{n_p\pi b_p}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}}\right]\cos(\frac{n_p\pi z}{h_p})\int_{b_p}^{b_{p+1}}\overline{rK_2(\frac{n_p\pi r}{h_p})}dr+$$

$$+\sum_{n_p=1}^{\infty}\left[\overline{\frac{\in_{n_p}\ \Lambda_{1n_p}K_1(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{1n_p}K_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\overline{\frac{-I_0(\frac{b_pn_p\pi}{h_p})+I_0(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}}\right]+$$

$$+\sum_{n_p=1}^{\infty}\left[\overline{\frac{-\in_{n_p}\ \Lambda_{1n_p}I_1(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\ \Lambda^*_{1n_p}I_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\overline{\frac{K_0(\frac{b_pn_p\pi}{h_p})-K_0(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}}\right]+\left[\overline{\in_{0_p}\ (\Lambda_{10_p}\frac{\frac{1}{b_p}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}}+\Lambda^*_{10_p}\frac{\frac{1}{b_{p+1}}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}})}\right](\frac{b^2{}_{p+1}-b^2{}_p}{2})+$$

$$+\left[\overline{\in_{0_p}\ (\Lambda_{10_p}\frac{\frac{1}{b_p}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}}+\Lambda^*_{10_p}\frac{\frac{1}{b_{p+1}}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}})}\right](\ln(b_{p+1}-b_p))$$

Δ)

$$\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}(\in_{n_p}\ (\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*_{1n_p}\frac{\partial\Re^*_{1n_p}(r)}{\partial r})\cos(\frac{n_p\pi z}{h_p})]rdr=$$

$$= \sum_{n_p=1}^{\infty} \left[\frac{\in_{n_p} \Lambda_{1n_p} \frac{n_p\pi}{h_p} K_1(\frac{n_p\pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{1n_p} \frac{n_p\pi}{h_p} K_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p}) - I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})} \right] \cos(\frac{n_p\pi z}{h_p})$$

$$\left[\frac{1}{(\frac{n_p\pi}{h_p})^2} \left[2I_0(\frac{b_p n_p\pi}{h_p}) - 2I_0(\frac{b_{p+1} n_p\pi}{h_p}) - b_p I_1(\frac{b_p n_p\pi}{h_p}) + b_{p+1} I_1(\frac{b_{p+1} n_p\pi}{h_p}) \right] \right] +$$

$$+ \sum_{n_p=1}^{\infty} \left[\frac{\in_{n_p} \Lambda_{1n_p} \frac{n_p\pi}{h_p} I_1(\frac{n_p\pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{1n_p} \frac{n_p\pi}{h_p} I_1(\frac{n_p\pi b_p}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p}) - I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})} \right] \cos(\frac{n_p\pi z}{h_p}) \int_{b_p}^{b_{p+1}} rK_2(\frac{n_p\pi r}{h_p})dr +$$

$$+ \sum_{n_p=1}^{\infty} \left[\frac{\in_{n_p} \Lambda_{1n_p} K_1(\frac{n_p\pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{1n_p} K_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p}) - I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})} \right] \cos(\frac{n_p\pi z}{h_p})$$

$$\left[\frac{-I_0(\frac{b_p n_p\pi}{h_p}) + I_0(\frac{b_{p+1} n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})} \right] +$$

$$+ \sum_{n_p=1}^{\infty} \left[\frac{-\in_{n_p} \Lambda_{1n_p} I_1(\frac{n_p\pi b_p}{h_p}) + \in_{n_p} \Lambda^*_{1n_p} I_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p}) - I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})} \right] \cos(\frac{n_p\pi z}{h_p})$$

$$\left[\frac{K_0(\frac{b_p n_p\pi}{h_p}) - K_0(\frac{b_{p+1} n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})} \right] + \left[\in_{0_p} (\Lambda_{10_p} \frac{\frac{1}{b_p}}{\frac{b_{p+1}}{b_p} - \frac{b_p}{b_{p+1}}} + \Lambda^*_{10_p} \frac{\frac{1}{b_{p+1}}}{\frac{b_{p+1}}{b_p} - \frac{b_p}{b_{p+1}}}) \right] (\frac{b^2{}_{p+1} - b^2{}_p}{2}) +$$

$$+ \left[\in_{0_p} (\Lambda_{10_p} \frac{\frac{1}{b_p}}{\frac{b_{p+1}}{b_p} - \frac{b_p}{b_{p+1}}} + \Lambda^*_{10_p} \frac{\frac{1}{b_{p+1}}}{\frac{b_{p+1}}{b_p} - \frac{b_p}{b_{p+1}}}) \right] (\ln(b_{p+1} - b_p))$$

E)

$$\int_{b_p}^{b_{p+1}} \sum_{n_p=0}^{\infty} (\overline{\in_{n_p} (\Lambda_{1n_p} \frac{\partial \Re_{1n_p}(r)}{\partial r}} + \overline{\Lambda^*_{1n_p} \frac{\partial \Re^*_{1n_p}(r)}{\partial r}}) \cos(\frac{n_p \pi z}{h_p})] r^3 dr \quad \Big\} =$$

$$= \sum_{n_p=1}^{\infty} \left[\overline{\frac{\in_{n_p} \Lambda_{1n_p} \frac{n_p\pi}{h_p} K_1(\frac{n_p\pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{1n_p} \frac{n_p\pi}{h_p} K_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p}) - I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}} \right] \cos(\frac{n_p\pi z}{h_p})$$

$$\left[\overline{\frac{-b_p{}^3 I_3(\frac{b_p n_p \pi}{h_p}) + b^3_{p+1} I_3(\frac{b_{p+1} n_p \pi}{h_p})}{(\frac{n_p\pi}{h_p})}} \right] +$$

$$+ \sum_{n_p=1}^{\infty} \left[\overline{\frac{\in_{n_p} \Lambda_{1n_p} \frac{n_p\pi}{h_p} I_1(\frac{n_p\pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{1n_p} \frac{n_p\pi}{h_p} I_1(\frac{n_p\pi b_p}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p}) - I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}} \right] \cos(\frac{n_p\pi z}{h_p}) \int_{b_p}^{b_{p+1}} r^3 \overline{K_2(\frac{n_p\pi r}{h_p})} dr +$$

$$+ \sum_{n_p=1}^{\infty} \left[\overline{\frac{\in_{n_p} \Lambda_{1n_p} K_1(\frac{n_p\pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{1n_p} K_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p}) - I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}} \right] \cos(\frac{n_p\pi z}{h_p})$$

$$\left[\overline{\frac{-b_p{}^2 I_2(\frac{b_p n_p \pi}{h_p}) + b^2_{p+1} I_2(\frac{b_{p+1} n_p \pi}{h_p})}{(\frac{n_p\pi}{h_p})}} \right] +$$

$$+ \sum_{n_p=1}^{\infty} \left[\overline{\frac{-\in_{n_p} \Lambda_{1n_p} I_1(\frac{n_p\pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{1n_p} I_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p}) - I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}} \right] \cos(\frac{n_p\pi z}{h_p}) \int_{b_p}^{b_{p+1}} r^2 \overline{K_1(\frac{n_p\pi r}{h_p})} dr +$$

$$+\left[\overline{\in_{0_p}\ (\Lambda_{10_p}\frac{\frac{1}{b_p}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}}+\Lambda^*_{10_p}\frac{\frac{1}{b_{p+1}}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}})}\right](\frac{b^4{}_{p+1}-b^4{}_p}{4})+$$

$$+\left[\overline{\in_{0_p}\ (\Lambda_{10_p}\frac{\frac{1}{b_p}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}}+\Lambda^*_{10_p}\frac{\frac{1}{b_{p+1}}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}})}\right](\frac{b^2_{p+1}-b^2_p}{2}))$$

Z)

$$\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}(\in_{n_p}\ (\Lambda_{1n_p}\frac{\partial\Re_{1n_p}(r)}{\partial r}+\Lambda^*_{1n_p}\frac{\partial\Re^*_{1n_p}(r)}{\partial r})\cos(\frac{n_p\pi z}{h_p})]r^3dr\ \Big\}=$$

$$=\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{1n_p}\frac{n_p\pi}{h_p}K_1(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*{}_{1n_p}\frac{n_p\pi}{h_p}K_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\frac{-b_p{}^3I_3(\frac{b_pn_p\pi}{h_p})+b^3_{p+1}I_3(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right]+$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{1n_p}\frac{n_p\pi}{h_p}I_1(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*{}_{1n_p}\frac{n_p\pi}{h_p}I_1(\frac{n_p\pi b_p}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\int_{b_p}^{b_{p+1}}r^3K_2(\frac{n_p\pi r}{h_p})dr+$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{1n_p}K_1(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*{}_{1n_p}K_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\frac{-b_p{}^2 I_2(\frac{b_p n_p \pi}{h_p}) + b_{p+1}^2 I_2(\frac{b_{p+1} n_p \pi}{h_p})}{(\frac{n_p \pi}{h_p})}\right] +$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_p}\ \Lambda_{1n_p} I_1(\frac{n_p \pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*_{1n_p} I_1(\frac{n_p \pi b_{p+1}}{h_p})}{I_1(\frac{n_p \pi b_{p+1}}{h_p})K_1(\frac{n_p \pi b_p}{h_p}) - I_1(\frac{n_p \pi b_p}{h_p})K_1(\frac{n_p \pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p \pi z}{h_p}) \int_{b_p}^{b_{p+1}} r^2 K_1(\frac{n_p \pi r}{h_p}) dr +$$

$$+\left[\in_{0_p}\ (\Lambda_{10_p} \frac{\frac{1}{b_p}}{\frac{b_{p+1}}{b_p} - \frac{b_p}{b_{p+1}}} + \Lambda^*_{10_p} \frac{\frac{1}{b_{p+1}}}{\frac{b_{p+1}}{b_p} - \frac{b_p}{b_{p+1}}})\right](\frac{b^4{}_{p+1} - b^4{}_p}{4}) +$$

$$+\left[\in_{0_p}\ (\Lambda_{10_p} \frac{\frac{1}{b_p}}{\frac{b_{p+1}}{b_p} - \frac{b_p}{b_{p+1}}} + \Lambda^*_{10_p} \frac{\frac{1}{b_{p+1}}}{\frac{b_{p+1}}{b_p} - \frac{b_p}{b_{p+1}}})\right](\frac{b_{p+1}^2 - b_p^2}{2}))$$

H)

$$\int_{b_p}^{b_{p+1}} [\sum_{n_p=0}^{\infty} \overline{\in_{n_p}\ (\Lambda_{mn_p} \frac{\partial \Re_{mn_p}(r)}{\partial r}} + \overline{\Lambda^*_{mn_p} \frac{\partial \Re^*_{mn_p}(r)}{\partial r}}) \cos(\frac{n_p \pi z}{h_p}) \sum_{n_q=0}^{\infty} \in_{n_p}\ (\Lambda_{mn_q} \frac{\partial \Re_{mn_q}(r)}{\partial r} +$$

$$\Lambda^*_{mn_q} \frac{\partial \Re^*_{mn_q}(r)}{\partial r}) \cos(\frac{n_q \pi z}{h_p})] r dr =$$

$$= \sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p} K_m(\frac{n_p \pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*_{mn_p} K_m(\frac{n_p \pi b_{p+1}}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p})K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p})K_m(\frac{n_p \pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p \pi z}{h_p}) \frac{n_p \pi}{h_p}$$

$$\overline{\left[\frac{\in_{n_p}\ \Lambda_{mn_p} K_m(\frac{n_p \pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*_{mn_p} K_m(\frac{n_p \pi b_{p+1}}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p})K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p})K_m(\frac{n_p \pi b_{p+1}}{h_p})}\right]} \cos(\frac{n_p \pi z}{h_p}) \frac{n_p \pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} rI_{m+1}(\frac{n_p\pi r}{h_p})\overline{I_{m+1}(\frac{n_p\pi r}{h_p})}dr +$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*{}_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\overline{\frac{\in_{n_p}\ \Lambda_{mn_p}I_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*{}_{mn_p}I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}\left[\frac{b_{p+1}}{2(\frac{n_p\pi}{h_p})^2}\left\{(\frac{n_p\pi}{h_p})\right.\right.$$

$$I_{m+2}(\frac{n_p\pi}{h_p}b_{p+1})\overline{K_{m+1}(\frac{n_p\pi}{h_p}b_{p+1})}-\frac{n_p\pi}{h_p}I_{m+1}(\frac{n_p\pi}{h_p}b_{p+1})\overline{K_{m+2}(\frac{n_p\pi}{h_p}b_{p+1})}\}-\frac{b_p}{2(\frac{n_p\pi}{h_p})^2}\left\{(\frac{n_p\pi}{h_p})\right.$$

$$\left.I_{m+2}(\frac{n_p\pi}{h_p}b_p)\overline{K_{m+1}(\frac{n_p\pi}{h_p}b_p)}-\frac{n_p\pi}{h_p}I_{m+1}(\frac{n_p\pi}{h_p}b_p)\overline{K_{m+2}(\frac{n_p\pi}{h_p}b_p)}\}\right]+$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*{}_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\overline{\frac{\in_{n_p}\ \Lambda_{mn_p}mK_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*{}_{mn_p}mK_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} I_{m+1}(\frac{n_p\pi r}{h_p})\overline{I_m(\frac{n_p\pi r}{h_p})}dr +$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*{}_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\overline{\frac{-m\in_{n_p}\ \Lambda_{mn_p}I_m(\frac{n_p\pi b_p}{h_p})+m\in_{n_p}\ \Lambda^*{}_{mn_p}I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} I_{m+1}(\frac{n_p\pi r}{h_p})\overline{K_m(\frac{n_p\pi r}{h_p})}dr +$$

$$+\sum_{n_p=1}^{\infty}\left[\overline{\frac{\in_{n_p}\Lambda_{mn_p}I_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\Lambda^*_{mn_p}I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\frac{\in_{n_p}\Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\Lambda^*_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}\left[\frac{b_{p+1}}{2(\frac{n_p\pi}{h_p})^2}\left\{(\frac{n_p\pi}{h_p})\right.\right.$$

$$\overline{I_{m+2}(\frac{n_p\pi}{h_p}b_{p+1})}K_{m+1}(\frac{n_p\pi}{h_p}b_{p+1})-\frac{n_p\pi}{h_p}\overline{I_{m+1}(\frac{n_p\pi}{h_p}b_{p+1})}K_{m+2}(\frac{n_p\pi}{h_p}b_{p+1})\}-\frac{b_p}{2(\frac{n_p\pi}{h_p})^2}\left\{(\frac{n_p\pi}{h_p})\right.$$

$$\left.\overline{I_{m+2}(\frac{n_p\pi}{h_p}b_p)}K_{m+1}(\frac{n_p\pi}{h_p}b_p)-\frac{n_p\pi}{h_p}\overline{I_{m+1}(\frac{n_p\pi}{h_p}b_p)}K_{m+2}(\frac{n_p\pi}{h_p}b_p)\}\right]+$$

$$+\sum_{n_p=1}^{\infty}\left[\overline{\frac{\in_{n_p}\Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\Lambda^*_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\frac{\in_{n_p}\Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\Lambda^*_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} K_{m+1}(\frac{n_p\pi r}{h_p})\overline{K_{m+1}(\frac{n_p\pi r}{h_p})}dr +$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\Lambda_{mn_p}I_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\Lambda^*_{mn_p}I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\frac{\in_{n_p}\ \Lambda_{mn_p} m K_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*_{mn_p} m K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} K_{m+1}(\frac{n_p\pi r}{h_p})\ \overline{I_m(\frac{n_p\pi r}{h_p})} dr +$$

$$\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p} I_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*_{mn_p} I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\frac{-m \in_{n_p}\ \Lambda_{mn_p} I_m(\frac{n_p\pi b_p}{h_p}) + m \in_{n_p}\ \Lambda^*_{mn_p} I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} K_{m+1}(\frac{n_p\pi r}{h_p})\ \overline{K_m(\frac{n_p\pi r}{h_p})} dr$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p} m K_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*_{mn_p} m K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\frac{\in_{n_p}\ \Lambda_{mn_p} K_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*_{mn_p} K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} I_m(\frac{n_p\pi r}{h_p})\ \overline{I_{m+1}(\frac{n_p\pi r}{h_p})} dr +$$

$$\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p} m K_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*_{mn_p} m K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\frac{\in_{n_p}\ \Lambda_{mn_p} I_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*_{mn_p} I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} I_m(\frac{n_p\pi r}{h_p})\ \overline{K_{m+1}(\frac{n_p\pi r}{h_p})}dr +$$

$$\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p} mK_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*_{mn_p} mK_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\overline{\left[\frac{\in_{n_p}\ \Lambda_{mn_p} mK_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*_{mn_p} mK_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]}\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} I_m(\frac{n_p\pi r}{h_p})\ \overline{I_m(\frac{n_p\pi r}{h_p})}dr +$$

$$\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p} mK_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*_{mn_p} mK_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\overline{\left[\frac{-m\in_{n_p}\ \Lambda_{mn_p} I_m(\frac{n_p\pi b_p}{h_p}) + m\in_{n_p}\ \Lambda^*_{mn_p} I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]}\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} I_m(\frac{n_p\pi r}{h_p})\ \overline{K_m(\frac{n_p\pi r}{h_p})}dr +$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{-m\in_{n_p}\ \Lambda_{mn_p} I_m(\frac{n_p\pi b_p}{h_p}) + m\in_{n_p}\ \Lambda^*_{mn_p} I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\overline{\left[\frac{\in_{n_p}\ \Lambda_{mn_p} K_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*{}_{mn_p} K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]}\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} K_m(\frac{n_p\pi r}{h_p})\ \overline{I_{m+1}(\frac{n_p\pi r}{h_p})} dr +$$

$$\sum_{n_p=1}^{\infty}\left[\frac{-m \in_{n_p}\ \Lambda_{mn_p} I_m(\frac{n_p\pi b_p}{h_p}) + m \in_{n_p}\ \Lambda^*{}_{mn_p} I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\overline{\left[\frac{\in_{n_p}\ \Lambda_{mn_p} I_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*{}_{mn_p} I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]}\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} K_m(\frac{n_p\pi r}{h_p})\ \overline{K_{m+1}(\frac{n_p\pi r}{h_p})} dr +$$

$$\sum_{n_p=1}^{\infty}\left[\frac{-m \in_{n_p}\ \Lambda_{mn_p} I_m(\frac{n_p\pi b_p}{h_p}) + m \in_{n_p}\ \Lambda^*{}_{mn_p} I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\overline{\left[\frac{\in_{n_p}\ \Lambda_{mn_p} m K_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*{}_{mn_p} m K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]}\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} K_m(\frac{n_p\pi r}{h_p})\ \overline{I_m(\frac{n_p\pi r}{h_p})} dr +$$

$$\sum_{n_p=1}^{\infty}\left[\frac{-m \in_{n_p}\ \Lambda_{mn_p} I_m(\frac{n_p\pi b_p}{h_p}) + m \in_{n_p}\ \Lambda^*{}_{mn_p} I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\frac{-m\in_{n_p}\Lambda_{mn_p}I_m(\frac{n_p\pi b_p}{h_p})+m\in_{n_p}\Lambda^*_{mn_p}I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} K_m(\frac{n_p\pi r}{h_p})\ \overline{K_m(\frac{n_p\pi r}{h_p})}dr+$$

$$\int_{b_p}^{b_{p+1}}[\sum_{\substack{n_p=1\\ n_p\neq n_q}}^{\infty}\sum_{n_q=1}^{\infty}\left\{\left[\frac{\in_{n_p}\Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\Lambda^*_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}\right.$$

$$I_{m+1}(\frac{n_p\pi r}{h_p})+\left[\frac{\in_{n_p}\Lambda_{mn_p}I_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\Lambda^*_{mn_p}I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$K_{m+1}(\frac{n_p\pi r}{h_p})+\left[\frac{\in_{n_p}\Lambda_{mn_p}mK_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\Lambda^*_{mn_p}mK_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$I_m(\frac{n_p\pi r}{h_p})+\left[\frac{-m\in_{n_p}\Lambda_{mn_p}I_m(\frac{n_p\pi b_p}{h_p})+m\in_{n_p}\Lambda^*_{mn_p}I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left.K_m(\frac{n_p\pi r}{h_p})\right\}\left\{\overline{\left[\frac{\in_{n_q}\Lambda_{mn_q}K_m(\frac{n_q\pi b_p}{h_p})-\in_{n_q}\Lambda^*_{mn_q}K_m(\frac{n_q\pi b_{p+1}}{h_p})}{I_m(\frac{n_q\pi b_{p+1}}{h_p})K_m(\frac{n_q\pi b_p}{h_p})-I_m(\frac{n_q\pi b_p}{h_p})K_m(\frac{n_q\pi b_{p+1}}{h_p})}\right]}\cos(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}\right.$$

$$I_{m+1}(\frac{n_q\pi r}{h_p})+\left[\frac{\in_{n_q}\ \Lambda_{mn_q}I_m(\frac{n_q\pi b_p}{h_p})-\in_{n_q}\ \Lambda^*{}_{mn_p}I_m(\frac{n_q\pi b_{p+1}}{h_p})}{I_m(\frac{n_q\pi b_{p+1}}{h_p})K_m(\frac{n_q\pi b_p}{h_p})-I_m(\frac{n_q\pi b_p}{h_p})K_m(\frac{n_q\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}$$

$$K_{m+1}(\frac{n_q\pi r}{h_p})+\left[\frac{\in_{n_q}\ \Lambda_{mn_q}mK_m(\frac{n_q\pi b_p}{h_p})-\in_{n_q}\ \Lambda^*{}_{mn_q}mK_m(\frac{n_q\pi b_{p+1}}{h_p})}{I_m(\frac{n_q\pi b_{p+1}}{h_p})K_m(\frac{n_q\pi b_p}{h_p})-I_m(\frac{n_q\pi b_p}{h_p})K_m(\frac{n_q\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}$$

$$I_m(\frac{n_q\pi r}{h_p})+\left[\frac{-m\in_{n_q}\ \Lambda_{mn_q}I_m(\frac{n_q\pi b_p}{h_p})+m\in_{n_q}\ \Lambda^*{}_{mn_q}I_m(\frac{n_q\pi b_{p+1}}{h_p})}{I_m(\frac{n_q\pi b_{p+1}}{h_p})K_m(\frac{n_q\pi b_p}{h_p})-I_m(\frac{n_q\pi b_p}{h_p})K_m(\frac{n_q\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}$$

$$K_m(\frac{n_q\pi r}{h_p})\ \Big\}$$

Θ)

$$\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}\overline{\in_{n_p}(\Lambda_{1n_p}\Re_{1n_p}(r)}+\overline{\Lambda^*_{1n_p}\Re^*_{1n_p}(r)})\cos(\frac{n_p\pi z}{h_p})]dr=$$

$$=\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{1n_p}K_1(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{1n_p}K_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\frac{-I_0(\frac{b_pn_p\pi}{h_p})+I_0(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right]+$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_p}\ \Lambda_{1n_p} I_1(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\ \Lambda^*_{1n_p} I_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\frac{K_0(\frac{b_p n_p\pi}{h_p})-K_0(\frac{b_{p+1} n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right]+\left[\in_{0_p}\ (\frac{\Lambda_{1_{0_p}}\frac{1}{b_p}-\Lambda^*_{1_{0_p}}\frac{1}{b_{p+1}}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}})\right](\frac{b^3{}_{p+1}-b^3{}_p}{3})-$$

$$-\left[\in_{0_p}\ (\frac{\Lambda_{1_{0_p}} b_p-\Lambda^*_{1_{0_p}} b_{p+1}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}})\right](b_{p+1}-b_p)$$

I)

$$\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))\cos(\frac{n_p\pi z}{h_p})]dr=$$

$$=\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{1n_p} K_1(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{1n_p} K_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\frac{-I_0(\frac{b_p n_p\pi}{h_p})+I_0(\frac{b_{p+1} n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right]+$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_p}\ \Lambda_{1n_p}I_1(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\ \Lambda^*_{1n_p}I_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\frac{K_0(\frac{b_p n_p\pi}{h_p})-K_0(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right]+\left[\in_{0_p}\ (\frac{\Lambda_{1_{0_p}}\frac{1}{b_p}-\Lambda^*_{1_{0_p}}\frac{1}{b_{p+1}}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}})\right](\frac{b^3{}_{p+1}-b^3{}_p}{3})-$$

$$-\left[\in_{0_p}\ (\frac{\Lambda_{1_{0_p}}b_p-\Lambda^*_{1_{0_p}}b_{p+1}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}})\right](b_{p+1}-b_p)$$

K)

$$\left.\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}\overline{\in_{n_p}\ (\Lambda_{1n_p}\Re_{1n_p}(r)}+\overline{\Lambda^*_{1n_p}\Re^*_{1n_p}(r))}\cos(\frac{n_p\pi z}{h_p})]r^2dr\ \right\}=$$

$$=\sum_{n_p=1}^{\infty}\left[\overline{\frac{\in_{n_p}\ \Lambda_{1n_p}K_1(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{1n_p}K_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\overline{\frac{-b_p^2I_2(\frac{b_p n_p\pi}{h_p})+b_{p+1}^2I_2(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}}\right]+$$

$$+\sum_{n_p=1}^{\infty}\left[\overline{\frac{-\in_{n_p}\ \Lambda_{1n_p}I_1(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\ \Lambda^*_{1n_p}I_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}}\right]\cos(\frac{n_p\pi z}{h_p})\ \int_{b_p}^{b_{p+1}}r^2\overline{K_1(\frac{n_p\pi r}{h_p})}dr+$$

$$+\left[\in_{0_p}\left(\frac{\Lambda_{1_{0_p}}\frac{1}{b_p}-\Lambda^*_{1_{0_p}}\frac{1}{b_{p+1}}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}}\right)\right]\left(\frac{b^4{}_{p+1}-b^4{}_p}{4}\right)-\left[\in_{0_p}\left(\frac{\Lambda_{1_{0_p}}b_p-\Lambda^*_{1_{0_p}}b_{p+1}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}}\right)\right]\left(\frac{b^2{}_{p+1}-b^2{}_p}{2}\right)$$

Λ)

$$\left.\int_{b_p}^{b_{p+1}}\sum_{n_p=0}^{\infty}\in_{n_p}(\Lambda_{1n_p}\Re_{1n_p}(r)+\Lambda^*_{1n_p}\Re^*_{1n_p}(r))\cos(\frac{n_p\pi z}{h_p})]r^2dr\right\}=$$

$$=\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\Lambda_{1n_p}K_1(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\Lambda^*_{1n_p}K_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\frac{-b_p^2I_2(\frac{b_pn_p\pi}{h_p})+b_{p+1}^2I_2(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right]+$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_p}\Lambda_{1n_p}I_1(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\Lambda^*_{1n_p}I_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\int_{b_p}^{b_{p+1}}r^2K_1(\frac{n_p\pi r}{h_p})dr+$$

$$+\left[\in_{0_p}\left(\frac{\Lambda_{1_{0_p}}\frac{1}{b_p}-\Lambda^*_{1_{0_p}}\frac{1}{b_{p+1}}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}}\right)\right]\left(\frac{b^4{}_{p+1}-b^4{}_p}{4}\right)-\left[\in_{0_p}\left(\frac{\Lambda_{1_{0_p}}b_p-\Lambda^*_{1_{0_p}}b_{p+1}}{\frac{b_{p+1}}{b_p}-\frac{b_p}{b_{p+1}}}\right)\right]\left(\frac{b^2{}_{p+1}-b^2{}_p}{2}\right)$$

M)

$$\int_{b_p}^{b_{p+1}}\Big(\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{mn_p}\Re_{mn_p}(r)+\Lambda^*_{mn_p}\Re^*_{mn_p}(r))\cos(\frac{n_p\pi z}{h_p})\ \sum_{n_p=0}^{\infty}\overline{\in_{n_p}\ (\Lambda_{mn_p}\Re_{mn_p}(r)+}$$

$$\overline{\Lambda^*_{mn_p}\Re^*_{mn_p}(r))}\cos(\frac{n_p\pi z}{h_p})\Big)rdr]=$$

$$=\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*{}_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\overline{\left[\frac{\in_{n_p}\ \Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*{}_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]}\cos(\frac{n_p\pi z}{h_p})$$

$$\int_{b_p}^{b_{p+1}}I_m(\frac{n_p\pi r}{h_p})\ \overline{I_m(\frac{n_p\pi r}{h_p})}rdr+$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*{}_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\overline{\left[\frac{-\in_{n_p}\ \Lambda_{mn_p}I_m(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\ \Lambda^*{}_{mn_p}I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]}\cos(\frac{n_p\pi z}{h_p})\left[\frac{b_{p+1}}{2(\frac{n_p\pi}{h_p})^2}\Big\{(\frac{n_p\pi}{h_p})\right.$$

$$I_{m+1}(\frac{n_p\pi}{h_p}b_{p+1})\overline{K_m(\frac{n_p\pi}{h_p}b_{p+1})}-\frac{n_p\pi}{h_p}I_m(\frac{n_p\pi}{h_p}b_{p+1})\overline{K_m(\frac{n_p\pi}{h_p}b_{p+1})}\Big\}-\frac{b_p}{2(\frac{n_p\pi}{h_p})^2}\Big\{(\frac{n_p\pi}{h_p})$$

$$\left.I_{m+1}(\frac{n_p\pi}{h_p}b_p)\overline{K_m(\frac{n_p\pi}{h_p}b_p)}-\frac{n_p\pi}{h_p}I_m(\frac{n_p\pi}{h_p}b_p)\overline{K_m(\frac{n_p\pi}{h_p}b_p)}\Big\}\right]+$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_p}\ \Lambda_{mn_p} I_m(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\ \Lambda^*_{mn_p} I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\overline{\frac{\in_{n_p}\ \Lambda_{mn_p} K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{mn_p} K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}}\right]\cos(\frac{n_p\pi z}{h_p})\left[\frac{b_{p+1}}{2(\frac{n_p\pi}{h_p})^2}\left\{(\frac{n_p\pi}{h_p})\right.\right.$$

$$\overline{I_{m+1}(\frac{n_p\pi}{h_p}b_{p+1})}K_m(\frac{n_p\pi}{h_p}b_{p+1})-\frac{n_p\pi}{h_p}\overline{I_m(\frac{n_p\pi}{h_p}b_{p+1})}K_m(\frac{n_p\pi}{h_p}b_{p+1})\Big\}-\frac{b_p}{2(\frac{n_p\pi}{h_p})^2}\left\{(\frac{n_p\pi}{h_p})\right.$$

$$\left.\left.\overline{I_{m+1}(\frac{n_p\pi}{h_p}b_p)}K_m(\frac{n_p\pi}{h_p}b_p)-\frac{n_p\pi}{h_p}\overline{I_m(\frac{n_p\pi}{h_p}b_p)}K_m(\frac{n_p\pi}{h_p}b_p)\right\}\right]+$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_p}\ \Lambda_{mn_p} I_m(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\ \Lambda^*_{mn_p} I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\overline{\frac{-\in_{n_p}\ \Lambda_{mn_p} I_m(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\ \Lambda^*_{mn_p} I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\int_{b_p}^{b_{p+1}} I_m(\frac{n_p\pi r}{h_p})\ \overline{K_m(\frac{n_p\pi r}{h_p})}rdr+$$

$$+\sum_{n_p=1}^{\infty}\sum_{\substack{n_q=1\\ n_p\neq n_q}}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p} K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{mn_p} K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\left[\overline{\frac{\in_{n_q}\ \Lambda_{mn_q} K_m(\frac{n_q\pi b_q}{h_q})-\in_{n_q}\ \Lambda^*_{mn_q} K_m(\frac{n_q\pi b_{q+1}}{h_q})}{I_m(\frac{n_q\pi b_{q+1}}{h_q})K_m(\frac{n_q\pi b_q}{h_q})-I_m(\frac{n_q\pi b_q}{h_q})K_m(\frac{n_q\pi b_{q+1}}{h_q})}}\right]\sin(\frac{n_q\pi z}{h_q})\frac{n_q\pi z}{h_q}$$

$$\int_{b_p}^{b_{p+1}} I_m(\frac{n_p\pi r}{h_p})\ \overline{I_m(\frac{n_q\pi r}{h_q})} rdr +$$

$$\sum_{n_p=1}^{\infty}\ \sum_{\substack{n_q=1\\ n_p\neq n_q}}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^{*}{}_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\overline{\left[\frac{-\in_{n_q}\ \Lambda_{mn_q}I_m(\frac{n_q\pi b_p}{h_p})+\in_{n_q}\ \Lambda^{*}{}_{mn_q}I_m(\frac{n_q\pi b_{p+1}}{h_q})}{I_m(\frac{n_q\pi b_{p+1}}{h_p})K_m(\frac{n_q\pi b_p}{h_p})-I_m(\frac{n_q\pi b_p}{h_p})K_m(\frac{n_q\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_q\pi z}{h_p})}$$

$$\int_{b_p}^{b_{p+1}} I_m(\frac{n_p\pi r}{h_p})\ \overline{K_m(\frac{n_q\pi r}{h_q})} rdr +$$

$$+\sum_{n_p=1}^{\infty}\ \sum_{\substack{n_q=1\\ n_q\neq n_p}}^{\infty}\left[\frac{-\in_{n_q}\ \Lambda_{mn_q}I_m(\frac{n_q\pi b_p}{h_q})+\in_{n_q}\ \Lambda^{*}{}_{mn_q}I_m(\frac{n_q\pi b_{p+1}}{h_q})}{I_m(\frac{n_q\pi b_{p+1}}{h_p})K_m(\frac{n_q\pi b_p}{h_p})-I_m(\frac{n_q\pi b_p}{h_p})K_m(\frac{n_q\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_q\pi z}{h_p})$$

$$\overline{\left[\frac{\in_{n_p}\ \Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^{*}{}_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})}$$

$$\int_{b_p}^{b_{p+1}} K_m(\frac{n_q\pi r}{h_q})\ \overline{I_m(\frac{n_p\pi r}{h_p})} rdr +$$

$$+\sum_{n_q=1}^{\infty}\ \sum_{\substack{n_p=1\\ n_q\neq n_p}}^{\infty}\left[\frac{-\in_{n_p}\ \Lambda_{mn_p}I_m(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\ \Lambda^{*}{}_{mn_p}I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})$$

$$\overline{\left[\frac{-\in_{n_q}\ \Lambda_{mn_q}I_m(\frac{n_q\pi b_q}{h_q})+\in_{n_q}\ \Lambda^{*}{}_{mn_q}I_m(\frac{n_q\pi b_{q+1}}{h_q})}{I_m(\frac{n_q\pi b_{q+1}}{h_q})K_m(\frac{n_q\pi b_q}{h_q})-I_m(\frac{n_q\pi b_q}{h_q})K_m(\frac{n_q\pi b_{q+1}}{h_q})}\right]\cos(\frac{n_q\pi z}{h_q})}$$

$$\int_{b_p}^{b_{p+1}} K_m(\frac{n_p \pi r}{h_q}) \overline{K_m(\frac{n_q \pi r}{h_q})} r dr.$$

N)

$$\int_{b_p}^{b_{p+1}} \sum_{n_p=0}^{\infty} \in_{n_p} (\overline{\Lambda_{0n_p} \Re_{0n_p}(r)} + \overline{\Lambda^*_{0n_p} \Re^*_{0n_p}(r)}) \sin(\frac{n_p \pi z}{h_p}) \frac{n_p \pi}{h_p} r dr =$$

$$= -\sum_{n_p=1}^{\infty} \left[\overline{\frac{\in_{n_p} \Lambda_{on_p} \frac{n_p \pi}{h_p} \mathrm{K}_0(\frac{n_p \pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{on_p} \frac{n_p \pi}{h_p} \mathrm{K}_0(\frac{n_p \pi b_p}{h_p})}{I_0(\frac{n_p \pi b_{p+1}}{h_p}) K_0(\frac{n_p \pi b_p}{h_p}) - I_0(\frac{n_p \pi b_p}{h_p}) K_0(\frac{n_p \pi b_{p+1}}{h_p})}} \right] \sin(\frac{n_p \pi z}{h_p})$$

$$\left[\overline{\frac{-b_p I_1(\frac{b_p n_p \pi}{h_p}) + b_{p+1} I_1(\frac{b_{p+1} n_p \pi}{h_p})}{(\frac{n_p \pi}{h_p})}} \right] -$$

$$-\sum_{n_p=1}^{\infty} \left[\overline{\frac{-\in_{n_p} \Lambda_{on_p} \frac{n_p \pi}{h_p} I_0(\frac{n_p \pi b_p}{h_p}) + \in_{n_p} \Lambda^*_{on_p} \frac{n_p \pi}{h_p} I_0(\frac{n_p \pi b_p}{h_p})}{I_0(\frac{n_p \pi b_{p+1}}{h_p}) K_0(\frac{n_p \pi b_p}{h_p}) - I_0(\frac{n_p \pi b_p}{h_p}) K_0(\frac{n_p \pi b_{p+1}}{h_p})}} \right] \sin(\frac{n_p \pi z}{h_p})$$

$$\left[\overline{\frac{b_p K_1(\frac{b_p n_p \pi}{h_p}) - b_{p+1} K_1(\frac{b_{p+1} n_p \pi}{h_p})}{(\frac{n_p \pi}{h_p})}} \right]$$

Ξ)

$$\int_{b_p}^{b_{p+1}} \sum_{n_p=0}^{\infty} \in_{n_p} (\Lambda_{0n_p}\Re_{0n_p}(r) + \Lambda^*_{0n_p}\Re^*_{0n_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p} r dr =$$

$$= -\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\Lambda_{on_p}\frac{n_p\pi}{h_p}\mathrm{K}_0(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\Lambda^*_{on_p}\frac{n_p\pi}{h_p}\mathrm{K}_0(\frac{n_p\pi b_p}{h_p})}{I_0(\frac{n_p\pi b_{p+1}}{h_p})K_0(\frac{n_p\pi b_p}{h_p}) - I_0(\frac{n_p\pi b_p}{h_p})K_0(\frac{n_p\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_p\pi z}{h_p})$$

$$\left[\frac{-b_p I_1(\frac{b_p n_p\pi}{h_p}) + b_{p+1}I_1(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right] -$$

$$-\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_p}\Lambda_{on_p}\frac{n_p\pi}{h_p}I_0(\frac{n_p\pi b_p}{h_p}) + \in_{n_p}\Lambda^*_{on_p}\frac{n_p\pi}{h_p}I_0(\frac{n_p\pi b_p}{h_p})}{I_0(\frac{n_p\pi b_{p+1}}{h_p})K_0(\frac{n_p\pi b_p}{h_p}) - I_0(\frac{n_p\pi b_p}{h_p})K_0(\frac{n_p\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_p\pi z}{h_p})$$

$$\left[\frac{b_p K_1(\frac{b_p n_p\pi}{h_p}) - b_{p+1}K_1(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right]$$

O)

$$\int_{b_p}^{b_{p+1}} \sum_{n_p=0}^{\infty} \in_{n_p} (\overline{\Lambda_{1n_p}\Re_{1n_p}(r)} + \overline{\Lambda^*_{1n_p}\Re^*_{1n_p}(r)})\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p} r^2 dr \left.\right\} =$$

$$= -\sum_{n_p=1}^{\infty}\left[\frac{\overline{\in_{n_p}\Lambda_{1n_p}K_1(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\Lambda^*_{1n_p}K_1(\frac{n_p\pi b_{p+1}}{h_p})}}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p}) - I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\frac{-b_p^2 I_2(\frac{b_p n_p \pi}{h_p}) + b_{p+1}^2 I_2(\frac{b_{p+1} n_p \pi}{h_p})}{(\frac{n_p \pi}{h_p})}\right] -$$

$$-\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_p}\ \Lambda_{1n_p} I_1(\frac{n_p \pi b_p}{h_p}) + \in_{n_p}\ \Lambda^*_{1n_p} I_1(\frac{n_p \pi b_{p+1}}{h_p})}{I_1(\frac{n_p \pi b_{p+1}}{h_p}) K_1(\frac{n_p \pi b_p}{h_p}) - I_1(\frac{n_p \pi b_p}{h_p}) K_1(\frac{n_p \pi b_{p+1}}{h_p})}\right] \sin(\frac{n_p \pi z}{h_p}) \frac{n_p \pi}{h_p} \int_{b_p}^{b_{p+1}} r^2 \overline{K_1(\frac{n_p \pi r}{h_p})} dr$$

Π)

$$\left.\int_{b_p}^{b_{p+1}} \sum_{n_p=0}^{\infty} \in_{n_p} (\Lambda_{1n_p} \Re_{1n_p}(r) + \Lambda^*_{1n_p} \Re^*_{1n_p}(r)) \sin(\frac{n_p \pi z}{h_p}) \frac{n_p \pi}{h_p} r^2 dr \ \right\} =$$

$$= -\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{1n_p} K_1(\frac{n_p \pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*_{1n_p} K_1(\frac{n_p \pi b_{p+1}}{h_p})}{I_1(\frac{n_p \pi b_{p+1}}{h_p}) K_1(\frac{n_p \pi b_p}{h_p}) - I_1(\frac{n_p \pi b_p}{h_p}) K_1(\frac{n_p \pi b_{p+1}}{h_p})}\right] \sin(\frac{n_p \pi z}{h_p}) \frac{n_p \pi}{h_p}$$

$$\left[\frac{-b_p^2 I_2(\frac{b_p n_p \pi}{h_p}) + b_{p+1}^2 I_2(\frac{b_{p+1} n_p \pi}{h_p})}{(\frac{n_p \pi}{h_p})}\right] -$$

$$-\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_p}\ \Lambda_{1n_p} I_1(\frac{n_p \pi b_p}{h_p}) + \in_{n_p}\ \Lambda^*_{1n_p} I_1(\frac{n_p \pi b_{p+1}}{h_p})}{I_1(\frac{n_p \pi b_{p+1}}{h_p}) K_1(\frac{n_p \pi b_p}{h_p}) - I_1(\frac{n_p \pi b_p}{h_p}) K_1(\frac{n_p \pi b_{p+1}}{h_p})}\right] \sin(\frac{n_p \pi z}{h_p}) \frac{n_p \pi}{h_p} \int_{b_p}^{b_{p+1}} r^2 K_1(\frac{n_p \pi r}{h_p}) dr$$

P)

$$\sum_{m=1}^{\infty}[\int_{b_p}^{b_{p+1}} \sum_{n_p=0}^{\infty} \in_{n_p} (\Lambda_{mn_p}\Re_{mn_p}(r) + \Lambda^*_{mn_p}\Re^*_{mn_p}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p} \sum_{n_p=0}^{\infty} \in_{n_p} (\overline{\Lambda_{mn_p}\Re_{mn_p}(r)} +$$

$$\overline{\Lambda^*_{mn_p}\Re^*_{mn_p}(r)})\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p} rdr] =$$

$$= \sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p} \Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\overline{\left[\frac{\in_{n_p} \Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} I_m(\frac{n_p\pi r}{h_p})\ \overline{I_m(\frac{n_p\pi r}{h_p})} rdr +$$

$$+ \sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p} \Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\overline{\left[\frac{-\in_{n_p} \Lambda_{mn_p}I_m(\frac{n_p\pi b_p}{h_p}) + \in_{n_p} \Lambda^*_{mn_p}I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}\left[\frac{b_{p+1}}{2(\frac{n_p\pi}{h_p})^2}\left\{(\frac{n_p\pi}{h_p})\right.\right.$$

$$I_{m+1}(\frac{n_p\pi}{h_p}b_{p+1})\overline{K_m(\frac{n_p\pi}{h_p}b_{p+1})} - \frac{n_p\pi}{h_p}I_m(\frac{n_p\pi}{h_p}b_{p+1})\overline{K_m(\frac{n_p\pi}{h_p}b_{p+1})}\} - \frac{b_p}{2(\frac{n_p\pi}{h_p})^2}\left\{(\frac{n_p\pi}{h_p})\right.$$

$$\left.I_{m+1}(\frac{n_p\pi}{h_p}b_p)\overline{K_m(\frac{n_p\pi}{h_p}b_p)} - \frac{n_p\pi}{h_p}I_m(\frac{n_p\pi}{h_p}b_p)\overline{K_m(\frac{n_p\pi}{h_p}b_p)}\}\right] +$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_p}\ \Lambda_{mn_p} I_m(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\ \Lambda^*_{mn_p} I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\frac{\in_{n_p}\ \Lambda_{mn_p} K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{mn_p} K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}\left[\frac{b_{p+1}}{2(\frac{n_p\pi}{h_p})^2}\left\{(\frac{n_p\pi}{h_p})\right.\right.$$

$$\overline{I_{m+1}(\frac{n_p\pi}{h_p}b_{p+1})}K_m(\frac{n_p\pi}{h_p}b_{p+1})-\frac{n_p\pi}{h_p}\overline{I_m(\frac{n_p\pi}{h_p}b_{p+1})}K_m(\frac{n_p\pi}{h_p}b_{p+1})\Big\}-\frac{b_p}{2(\frac{n_p\pi}{h_p})^2}\left\{(\frac{n_p\pi}{h_p})\right.$$

$$\left.\left.\overline{I_{m+1}(\frac{n_p\pi}{h_p}b_p)}K_m(\frac{n_p\pi}{h_p}b_p)-\frac{n_p\pi}{h_p}\overline{I_m(\frac{n_p\pi}{h_p}b_p)}K_m(\frac{n_p\pi}{h_p}b_p)\right\}\right]+$$

$$+\sum_{n_p\neq n_q}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p} K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{mn_p} K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\frac{\in_{n_q}\ \Lambda_{mn_q} K_m(\frac{n_q\pi b_q}{h_q})-\in_{n_q}\ \Lambda^*_{mn_q} K_m(\frac{n_q\pi b_{q+1}}{h_q})}{I_m(\frac{n_q\pi b_{q+1}}{h_q})K_m(\frac{n_q\pi b_q}{h_q})-I_m(\frac{n_q\pi b_q}{h_q})K_m(\frac{n_q\pi b_{q+1}}{h_q})}\right]\sin(\frac{n_q\pi z}{h_q})\frac{n_q\pi}{h_q}$$

$$\int_{b_p}^{b_{p+1}} I_m(\frac{n_p\pi r}{h_p})\ \overline{I_m(\frac{n_q\pi r}{h_p})}rdr+$$

$$+\sum_{n_p\neq n_q}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{mn_p} K_m(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{mn_p} K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\frac{-\in_{n_q}\ \Lambda_{mn_q} I_m(\frac{n_q\pi b_p}{h_p})+\in_{n_q}\ \Lambda^*_{mn_q} I_m(\frac{n_q\pi b_{p+1}}{h_p})}{I_m(\frac{n_q\pi b_{p+1}}{h_p})K_m(\frac{n_q\pi b_p}{h_p})-I_m(\frac{n_q\pi b_p}{h_p})K_m(\frac{n_q\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} I_m(\frac{n_p\pi r}{h_p})\ \overline{K_m(\frac{n_q\pi r}{h_p})} rdr +$$

$$\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_q}\ \Lambda_{mn_q} I_m(\frac{n_q\pi b_p}{h_p}) + \in_{n_q}\ \Lambda^*{}_{mn_q} I_m(\frac{n_q\pi b_{p+1}}{h_p})}{I_m(\frac{n_q\pi b_{p+1}}{h_p})K_m(\frac{n_q\pi b_p}{h_p}) - I_m(\frac{n_q\pi b_p}{h_p})K_m(\frac{n_q\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}$$

$$\overline{\left[\frac{\in_{n_p}\ \Lambda_{mn_p} K_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\ \Lambda^*{}_{mn_p} K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} K_m(\frac{n_q\pi r}{h_p})\ \overline{I_m(\frac{n_p\pi r}{h_p})} rdr +$$

$$\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_q}\ \Lambda_{mn_q} I_m(\frac{n_q\pi b_p}{h_p}) + \in_{n_q}\ \Lambda^*{}_{mn_q} I_m(\frac{n_q\pi b_{p+1}}{h_p})}{I_m(\frac{n_q\pi b_{p+1}}{h_p})K_m(\frac{n_q\pi b_p}{h_p}) - I_m(\frac{n_q\pi b_p}{h_p})K_m(\frac{n_q\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}$$

$$\overline{\left[\frac{-\in_{n_p}\ \Lambda_{mn_p} I_m(\frac{n_p\pi b_p}{h_p}) + \in_{n_p}\ \Lambda^*{}_{mn_p} I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} K_m(\frac{n_p\pi r}{h_p})\ \overline{K_m(\frac{n_p\pi r}{h_p})} rdr.$$

8.2 Υπολογισμός του όρου $\int_{b_p}^{b_{p+1}}(\int_0^{2\pi}\overline{x}^{(1)}\nabla\Phi_t^{(1)})\overline{n}dS$ για το πεδίο (ΙΙΙ)

Έχουμε δείξει στο Κεφάλαιο 2 –σελίδα 19– ότι

$$\overline{x}^{(1)}\nabla\Phi_t^{(1)}=[(X_{g_1}^{(1)}+X_5^{(1)}z)\frac{\partial\phi(r,\theta,z)}{\partial r}\cos\theta-(X_{g_1}^{(1)}+X_5^{(1)}z)\frac{1}{r}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\sin\theta$$
$$+(X_{g_3}^{(1)}-X_5^{(1)}r\cos\theta)\frac{\partial\phi(r,\theta,z)}{\partial z}](-i\omega)]$$

Δηλαδή

$$\overline{x}^{(1)}\nabla\Phi_t^{(1)}=(X_{g_1}^{(1)}+X_5^{(1)}z)\ (-i\omega)\ [-i\omega\frac{H}{2}d(-\frac{z_0}{H/2}\frac{r}{2h_pd}-d\frac{\phi_0}{H/2}\frac{(z^2-0{,}75r^2)}{2h_pd^2}\cos\theta)-$$
$$-i\omega\frac{H}{2}d\ \sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{mn_p}\frac{\partial\Re_{mn_p}(r)}{\partial r}+\Lambda_{mn_p}^{*}\frac{\partial\Re_{mn_p}^{*}(r)}{\partial r})\cos(\frac{n_p\pi z}{h_p})]\cos(m\theta)]\cos(\theta)-$$
$$-(X_{g_1}^{(1)}+X_5^{(1)}z)\ (-i\omega)\ \frac{1}{r}[-i\omega\frac{H}{2}d(d\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_pd^2}\sin\theta)+i\omega\frac{H}{2}d$$
$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{mn_p}\Re_{mn_p}(r)+\Lambda_{mn_p}^{*}\Re_{mn_p}^{*}(r))\cos(\frac{n_p\pi z}{h_p})m]\sin(m\theta)]]\sin(\theta)+$$
$$+(X_{g_3}^{(1)}-X_5^{(1)}r\cos\theta)(-i\omega)\ [i\omega\frac{H}{2}d(\frac{z_0}{H/2}\frac{2z}{2h_pd}-d\frac{\phi_0}{H/2}\frac{r2z}{2h_pd^2}\cos\theta)-i\omega\frac{H}{2}d$$
$$\sum_{m=0}^{\infty}[\sum_{n_p=0}^{\infty}\in_{n_p}\ (\Lambda_{mn_p}\Re_{mn_p}(r)+\Lambda_{mn_p}^{*}\Re_{mn_p}^{*}(r))\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]\cos(m\theta)]]$$

Θα υπολογίσουμε τον όρο

$$\int_{b_p}^{b_{p+1}}(\int_0^{2\pi}\overline{x}^{(1)}\nabla\Phi_t^{(1)})ndS=\int_{b_p}^{b_{p+1}}(\int_0^{2\pi}\overline{x}^{(1)}\nabla\Phi_t^{(1)}d\theta)rdr=$$

$$=(X_{g_1}^{(1)}+X_5^{(1)}z)\ (-i\omega)\Big[\ -i\omega\frac{H}{2}d\Big\{\ -\frac{z_0}{H/2}\frac{2\pi}{2h_pd}(\frac{(b_{p+1})^3}{3}-\frac{(b_p)^3}{3})+$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{2n_p}K_2(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{2n_p}K_2(\frac{n_p\pi b_{p+1}}{h_p})}{I_2(\frac{n_p\pi b_{p+1}}{h_p})K_2(\frac{n_p\pi b_p}{h_p})-I_2(\frac{n_p\pi b_p}{h_p})K_2(\frac{n_p\pi b_{p+1}}{h_p})}\right]\frac{\pi}{2}\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\int_{b_p}^{b_{p+1}} rI_3(\frac{n_p\pi r}{h_p})dr+$$

$$+\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{2n_p}I_2(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{2n_p}I_2(\frac{n_p\pi b_{p+1}}{h_p})}{I_2(\frac{n_p\pi b_{p+1}}{h_p})K_2(\frac{n_p\pi b_p}{h_p})-I_2(\frac{n_p\pi b_p}{h_p})K_2(\frac{n_p\pi b_{p+1}}{h_p})}\right]\frac{\pi}{2}\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left.\int_{b_p}^{b_{p+1}} rK_3(\frac{n_p\pi r}{h_p})dr\right\}+$$

$$+X^{(1)}_{g_3}(-i\omega)\ [-i\omega\frac{H}{2}d\left\{\ \frac{z_0}{H/2}\frac{2\pi}{2h_pd}2z(\frac{(b_{p+1})^2}{2}-\frac{(b_p)^2}{2})-\right.$$

$$-\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{2n_p}K_2(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{2n_p}K_2(\frac{n_p\pi b_{p+1}}{h_p})}{I_2(\frac{n_p\pi b_{p+1}}{h_p})K_2(\frac{n_p\pi b_p}{h_p})-I_2(\frac{n_p\pi b_p}{h_p})K_2(\frac{n_p\pi b_{p+1}}{h_p})}\right]\frac{\pi}{2}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\frac{-b_pI_3(\frac{b_pn_p\pi}{h_p})+b_{p+1}I_3(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right]-$$

$$-\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_p}\ \Lambda_{2n_p}I_2(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\ \Lambda^*_{2n_p}I_2(\frac{n_p\pi b_{p+1}}{h_p})}{I_2(\frac{n_p\pi b_{p+1}}{h_p})K_2(\frac{n_p\pi b_p}{h_p})-I_2(\frac{n_p\pi b_p}{h_p})K_2(\frac{n_p\pi b_{p+1}}{h_p})}\right]\frac{\pi}{2}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left.\left[\frac{b_pK_3(\frac{b_pn_p\pi}{h_p})-b_{p+1}K_3(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right]\ \right\}\ ]+$$

$$-(X^{(1)}_{g_1}+X^{(1)}_5z)\ (-i\omega)\ [-i\omega\frac{H}{2}d(d\frac{\phi_0}{H/2}\frac{(z^2-0,25r^2)}{2h_pd^2}\pi)(b_{p+1}-b_p)+$$

$$+i\omega\frac{H}{2}d\left\{\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\Lambda_{2n_p}K_2(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\Lambda^*_{2n_p}K_2(\frac{n_p\pi b_{p+1}}{h_p})}{I_2(\frac{n_p\pi b_{p+1}}{h_p})K_2(\frac{n_p\pi b_p}{h_p})-I_2(\frac{n_p\pi b_p}{h_p})K_2(\frac{n_p\pi b_{p+1}}{h_p})}\right]\frac{\pi}{2}\cos(\frac{n_p\pi z}{h_p})\right.$$

$$\int_{b_p}^{b_{p+1}} I_2(\frac{n_p\pi r}{h_p})dr+$$

$$\left.+\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_p}\Lambda_{2n_p}I_2(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\Lambda^*_{2n_p}I_2(\frac{n_p\pi b_{p+1}}{h_p})}{I_2(\frac{n_p\pi b_{p+1}}{h_p})K_2(\frac{n_p\pi b_p}{h_p})-I_2(\frac{n_p\pi b_p}{h_p})K_2(\frac{n_p\pi b_{p+1}}{h_p})}\right]\frac{\pi}{2}\cos(\frac{n_p\pi z}{h_p})\int_{b_p}^{b_{p+1}} K_2(\frac{n_p\pi r}{h_p})dr\right\}+$$

$$+X_{g_3}^{(1)}(-i\omega)\,[-i\omega\frac{H}{2}d\left\{\frac{z_0}{H/2}\frac{2\pi}{2h_pd}2z(\frac{(b_{p+1})^2}{2}-\frac{(b_p)^2}{2})-\right.$$

$$-\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\Lambda_{2n_p}K_2(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\Lambda^*_{2n_p}K_2(\frac{n_p\pi b_{p+1}}{h_p})}{I_2(\frac{n_p\pi b_{p+1}}{h_p})K_2(\frac{n_p\pi b_p}{h_p})-I_2(\frac{n_p\pi b_p}{h_p})K_2(\frac{n_p\pi b_{p+1}}{h_p})}\right]\frac{\pi}{2}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\frac{-b_pI_3(\frac{b_pn_p\pi}{h_p})+b_{p+1}I_3(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right]-$$

$$-\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_p}\Lambda_{2n_p}I_2(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\Lambda^*_{2n_p}I_2(\frac{n_p\pi b_{p+1}}{h_p})}{I_2(\frac{n_p\pi b_{p+1}}{h_p})K_2(\frac{n_p\pi b_p}{h_p})-I_2(\frac{n_p\pi b_p}{h_p})K_2(\frac{n_p\pi b_{p+1}}{h_p})}\right]\frac{\pi}{2}\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left.\left[\frac{b_pK_3(\frac{b_pn_p\pi}{h_p})-b_{p+1}K_3(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right]\right\}\Bigg]+$$

$$+(-X_5^{(1)} r\cos\theta)(-i\omega)\ [-i\omega\frac{H}{2}d\left\{(-d\frac{\phi_0}{H/2}\frac{r2z}{2h_p d^2}\pi)-\right.$$

$$-\sum_{n_p=1}^{\infty}\left[\frac{\in_{n_p}\ \Lambda_{1n_p}K_1(\frac{n_p\pi b_p}{h_p})-\in_{n_p}\ \Lambda^*_{1n_p}K_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left[\frac{-b_p I_1(\frac{b_p n_p\pi}{h_p})+b_{p+1}I_1(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right]-$$

$$-\sum_{n_p=1}^{\infty}\left[\frac{-\in_{n_p}\ \Lambda_{1n_p}I_1(\frac{n_p\pi b_p}{h_p})+\in_{n_p}\ \Lambda^*_{1n_p}I_1(\frac{n_p\pi b_{p+1}}{h_p})}{I_1(\frac{n_p\pi b_{p+1}}{h_p})K_1(\frac{n_p\pi b_p}{h_p})-I_1(\frac{n_p\pi b_p}{h_p})K_1(\frac{n_p\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$\left.\left[\frac{b_p K_1(\frac{b_p n_p\pi}{h_p})-b_{p+1}K_1(\frac{b_{p+1}n_p\pi}{h_p})}{(\frac{n_p\pi}{h_p})}\right]\right\}\ ]+$$

Όπου b_p : η ακτίνα του $p-$στου «από κάτω» στοιχείου

h_p : η απόσταση του $p-$στου «από κάτω» στοιχείου από τον πυθμένα

Όμως για τον υπολογισμός της δύναμης έκπτωσης πρέπει να υπολογίσουμε τον όρο

$$\overline{\overline{x}^{(1)}\nabla\Phi_t^{(1)}}^T=\omega(\overline{X}_{\mathrm{Re}}^{(1)}\cos(\omega t)+\overline{X}_{\mathrm{Im}}^{(1)}\sin(\omega t))(\nabla\phi_{\mathrm{Im}}^{(1)}\cos(\omega t)-\nabla\phi_{\mathrm{Re}}^{(1)}\sin(\omega t))=$$

$$=\overline{\omega(\overline{X}_{\mathrm{Re}}^{(1)}\nabla\phi_{\mathrm{Im}}^{(1)}\cos^2(\omega t)-\overline{X}_{\mathrm{Im}}^{(1)}\nabla\phi_{\mathrm{Re}}^{(1)}\sin^2(\omega t)+(-\overline{X}_{\mathrm{Re}}^{(1)}\nabla\phi_{\mathrm{Re}}^{(1)}+\overline{X}_{\mathrm{Im}}^{(1)}\nabla\phi_{\mathrm{Im}}^{(1)})\cos(\omega t)\sin(\omega t))}^t=$$

$$=\frac{1}{2}\omega(\overline{X}_{\mathrm{Re}}^{(1)}\nabla\phi_{\mathrm{Im}}^{(1)}-\overline{X}_{\mathrm{Im}}^{(1)}\nabla\phi_{\mathrm{Re}}^{(1)})=\frac{1}{2}\omega\,\mathrm{Im}(X^{(1)^*}\nabla\phi^{(1)})$$

8.3 Υπολογισμός της κατακόρυφης δύναμης έκπτωσης για το πεδίο (ΙΙΙ)

Η κατακόρυφη δύναμη έκπτωσης F_Z για το Πεδίο (ΙΙΙ) Κεφάλαιο 2 –σελίδα 17– υπολογίζεται από τη σχέση

$$F_Z = MR^{(1)}\overline{X^{(1)}{}_g}'' - \iint_{S_0} -\frac{1}{2}\rho\left|\nabla\Phi^{(1)}\right|^2 \overline{n} dS - \iint_{S_0} -\rho(\overline{X}^{(1)}\nabla\Phi_t^{(1)})\overline{n} dS =$$

$$= MR^{(1)}\overline{X^{(1)}{}_g}'' + \frac{1}{2}\rho\,[\underline{\iint_{S_0}\left|\nabla\Phi^{(1)}\right|^2 \overline{n} dS}] + \rho\,[\underline{\iint_{S_0}(\overline{X}^{(1)}\nabla\Phi_t^{(1)})\overline{n} dS}].$$

Όμως οι παραστάσεις $\underline{\iint_{S_0}\left|\nabla\Phi^{(1)}\right|^2 \overline{n} dS}$ $\quad \underline{[\iint_{S_0}(\overline{X}^{(1)}\nabla\Phi_t^{(1)})\overline{n} dS]}$

είναι γνωστές από τα προηγούμενα (σελίδα 213 και σελίδα 243, αντίστοιχα).

Όπου ρ: πυκνότητα νερού

g: η επιτάχυνση της βαρύτητας

M : η μάζα αξονοσυμμετρικού σώματος

$R^{(1)}$: ο πίνακας που περιέχει τις γωνίες περιστροφής

$X_g^{(1)}$: οι ευθύγραμμες μεταφορικές κινήσεις του κέντρου βάρους της κατασκευής

Επομένως μπορούμε να υπολογίσουμε την κατακόρυφη δύναμη έκπτωσης, F_Z για την Περιοχή (ΙΙΙ).

8.4 Υπολογισμός του όρου $\int_{h_2}^{h_1}(\int_0^{2\pi} \left|\nabla\Phi^{(1)}\right|^2)\bar{n}dS$ για το μεσαίο στοιχείο στο πεδίο (III).

Έχουμε αποδείξει στο Κεφάλαιο 1 –σελίδα 14– ότι η σχέση που μας δίνει το δυναμικό της ταχύτητας πρώτης τάξης για το μεσαίο δακτυλιοειδές στοιχείο στο πεδίο (III) όταν το σώμα δεν στηρίζεται στον πυθμένα, δίνεται από τον τύπο:

$$\phi_M\,(\mathrm{r},\theta,\mathrm{z}) = -i\omega\frac{H}{2}d(\frac{z_0}{H/2}\frac{z^2-0{,}5r^2}{2h_M d} - d\,\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_M d^2}\cos\theta) - i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[(\Lambda_{m0_M}A_{m0_M}(r) + 2\sum_{n_M=1}^{\infty}\Lambda_{mn_M}A_{mn_M}(r))\cos(\frac{n_M\pi z}{h_M})]\cos(m\theta).$$

Και

$$\frac{\partial\phi(r,\theta,z)}{\partial r} = -i\omega\frac{H}{2}d(\frac{z_0}{H/2}\frac{-r}{2h_M d} - d\,\frac{\phi_0}{H/2}\frac{(z^2-0{,}75r^2)}{2h_M d^2}\cos\theta) - i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[(\Lambda_{m0_M}\frac{\partial A_{m0_M}(r)}{\partial r} + 2\sum_{n_M=1}^{\infty}\Lambda_{mn_M}\frac{\partial A_{mn_M}(r)}{\partial r})\cos(\frac{n_M\pi z}{h_M})]\cos(m\theta).$$

Όπου $\dfrac{\partial A_{m0_M}(r)}{\partial r} = \dfrac{m}{b_M}(\dfrac{r}{b_M})^{m-1}$

$$\frac{\partial A_{mn_M}(r)}{\partial r} = \frac{n_M\pi}{h_M}\frac{I_{m+1}(\frac{n_M\pi r}{h_M})}{I_m(\frac{n_M\pi b_M}{h_M})} + \frac{m}{r}\frac{I_m(\frac{n_M\pi r}{h_M})}{I_m(\frac{n_M\pi b_M}{h_M})}$$

Ειδικές περιπτώσεις είναι

$$\frac{\partial A_{0_{0_M}}(r)}{\partial r} = 0$$

Και

$$\frac{\partial \overline{\phi(r,\theta,z)}}{\partial r} = i\omega \frac{H}{2} d(-\frac{z_0}{H/2}\frac{r}{2h_M d} - d\frac{\overline{\phi_0}}{H/2}\frac{(z^2-0,75r^2)}{2h_M d^2}\cos\theta) + i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[(\Lambda_{m0_M}\overline{\frac{\partial A_{m0_M}(r)}{\partial r}} + 2\sum_{n_M=1}^{\infty}\Lambda_{mn_M}\overline{\frac{\partial A_{mn_M}(r)}{\partial r}})\cos(\frac{n_M \pi z}{h_M})]\cos(m\theta).$$

Επομένως

$$\overline{\left(\frac{\partial \phi(r,\theta,z)}{\partial r}\right)^2}^T = \frac{1}{2}\frac{\partial \phi(r,\theta,z)}{\partial r}\frac{\partial \overline{\phi(r,\theta,z)}}{\partial r} =$$

$$= \frac{1}{2}\omega^2\frac{H^2}{4}d^2(-\frac{z_0}{H/2}\frac{r}{2h_M d} - d\frac{\overline{\phi_0}}{H/2}\frac{(z^2-0,75r^2)}{2h_M d^2}\cos\theta)$$

$$(\frac{z_0}{H/2}\frac{-r}{2h_M d} - d\frac{\phi_0}{H/2}\frac{(z^2-0,75r^2)}{2h_M d^2}\cos\theta) +$$

$$+\frac{1}{2}\omega^2\frac{H^2}{4}d^2(\frac{z_0}{H/2}\frac{-r}{2h_M d} - d\frac{\phi_0}{H/2}\frac{(z^2-0,75r^2)}{2h_M d^2}\cos\theta)\sum_{m=0}^{\infty}\overline{[(\Lambda_{m0_M}\frac{\partial A_{m0_M}(r)}{\partial r}} +$$

$$2\sum_{n_M=1}^{\infty}\Lambda_{mn_M}\overline{\frac{\partial A_{mn_M}(r)}{\partial r}})\cos(\frac{n_M \pi z}{h_M})]\cos(\mathrm{m}\theta) +$$

$$+\frac{1}{2}\omega^2\frac{H^2}{4}d^2(-\frac{z_0}{H/2}\frac{r}{2h_M d} - d\frac{\overline{\phi_0}}{H/2}\frac{(z^2-0,75r^2)}{2h_M d^2}\cos\theta)\sum_{m=0}^{\infty}[(\Lambda_{m0_M}\frac{\partial A_{m0_M}(r)}{\partial r} +$$

$$2\sum_{n_M=1}^{\infty}\Lambda_{mn_M}\frac{\partial A_{mn_M}(r)}{\partial r})\cos(\frac{n_M \pi z}{h_M})]\cos(m\theta) +$$

$$+\frac{1}{2}\omega^2\frac{H^2}{4}d^2\sum_{m=0}^{\infty}[(\Lambda_{m0_M}\frac{\partial A_{m0_M}(r)}{\partial r} + 2\sum_{n_M=1}^{\infty}\Lambda_{mn_M}\frac{\partial A_{mn_M}(r)}{\partial r})\cos(\frac{n_M \pi z}{h_M})]\cos(m\theta)$$

$$\sum_{n=0}^{\infty}[(\Lambda_{n0_N}\overline{\frac{\partial A_{n0_N}(r)}{\partial r}} + 2\sum_{n_N=1}^{\infty}\Lambda_{nn_N}\overline{\frac{\partial A_{nn_N}(r)}{\partial r}})\cos(\frac{n_N \pi z}{h_M})]\cos(n\theta).$$

Και $\int_{h_2}^{h_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial \Phi}{\partial r}\right)^2}^T d\theta\right](-1)r dr$.

Πρώτα υπολογίζουμε το $\int_0^{2\pi}\left(\frac{\partial \phi}{\partial r}\right)^2 d\theta$ και προκύπτει η

$$\int_0^{2\pi}\left(\frac{\partial\phi}{\partial r}\right)^2 d\theta =$$

$$= \omega^2\frac{H^2}{8}d^2\pi(-\frac{{z_0}^2}{(H/2)^2}\frac{r^2}{(2h_M d)^2}2 + d^2\frac{\phi_0\overline{\phi_0}}{(H/2)(H/2)}\frac{(z^2-0{,}75r^2)^2}{(2h_M d^2)}) +$$

$$+\omega^2\frac{H^2}{8}d^2\pi[(\frac{z_0}{H/2}\frac{-r}{2h_M d})\sum_{n_M=1}^{\infty}\overline{(\Lambda_{00_M}\frac{\partial A_{00_M}(r)}{\partial r}} + \overline{2\Lambda_{0n_M}\frac{\partial A_{0n_M}(r)}{\partial r}})\cos(\frac{n_M\pi z}{h_M})\frac{1}{\pi} -$$

$$-d\frac{\phi_0}{H/2}\frac{(z^2-0{,}75r^2)}{2h_M d^2}\sum_{n_M=1}^{\infty}\overline{(\Lambda_{10_M}\frac{\partial A_{10_M}(r)}{\partial r}} + \overline{2\Lambda_{1n_M}\frac{\partial A_{1n_M}(r)}{\partial r}})\cos(\frac{n_M\pi z}{h_M})] +$$

$$+\omega^2\frac{H^2}{8}d^2\pi(-\frac{z_0}{H/2}\frac{r}{2h_M d})\sum_{n_M=1}^{\infty}(\Lambda_{00_M}\frac{\partial A_{00_M}(r)}{\partial r} + 2\Lambda_{0n_M}\frac{\partial A_{0n_M}(r)}{\partial r})\cos(\frac{n_M\pi z}{h_M})\frac{1}{\pi} -$$

$$-d\frac{\overline{\phi_0}}{H/2}\frac{(z^2-0{,}75r^2)}{2h_M d^2}\sum_{n_M=1}^{\infty}(\Lambda_{10_M}\frac{\partial A_{10_M}(r)}{\partial r} + 2\Lambda_{1n_M}\frac{\partial A_{1n_M}(r)}{\partial r})\cos(\frac{n_M\pi z}{h_M})] +$$

$$+\omega^2\frac{H^2}{8}d^2\pi[\sum_{m=0}^{\infty}[\sum_{n_M=1}^{\infty}\overline{(\Lambda_{m0_M}\frac{\partial A_{m0_M}(r)}{\partial r}} + \overline{2\Lambda_{mn_M}\frac{\partial A_{mn_M}(r)}{\partial r}})\cos(\frac{n_M\pi z}{h_M})]$$

$$[\sum_{n_N=1}^{\infty}(\Lambda_{m0_N}\frac{\partial\Re_{m0_N}(r)}{\partial r} + 2\Lambda_{mn_N}\frac{\partial\Re_{mn_N}(r)}{\partial r})\cos(\frac{n_N\pi z}{h_M})] + 2[\sum_{n_M=1}^{\infty}\overline{(\Lambda_{00_M}\frac{\partial A_{00_M}(r)}{\partial r}} +$$

$$\overline{2\Lambda_{0n_M}\frac{\partial A_{0n_M}(r)}{\partial r}})\cos(\frac{n_M\pi z}{h_M})][\sum_{n_N=1}^{\infty}(\Lambda_{0n_p}\frac{\partial A_{00_N}(r)}{\partial r} + 2\Lambda_{0n_N}\frac{\partial\Re_{0n_N}(r)}{\partial r})\cos(\frac{n_N\pi z}{h_M})]]$$

Και στη συνέχεια υπολογίζουμε το $\int_{h_2}^{h_1}\left[\overline{\left(\frac{\partial\Phi}{\partial r}\right)^2}^T\right]ndS$

Επομένως

$$\int_{h_2}^{h_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial r}\right)^2}^T d\theta\right](-1)rdr =$$

$$= (-1)\omega^2\frac{H^2}{8}d^2\pi\left\{\frac{{z_0}^2}{(H/2)^2}\frac{2}{(2h_M d)^2}(\frac{(h_1)^3}{3} - \frac{(h_2)^3}{3}) + d^2\frac{\phi_0\overline{\phi_0}}{(H/2)(H/2)}\frac{1}{(2h_m d^2)^2}\right.$$

$$(z^4(\frac{(h_1)^2}{2} - \frac{(h_2)^2}{2}) - d^2\frac{\phi_0\overline{\phi_0}}{(H/2)(H/2)}\frac{2}{(2h_m d^2)^2}0{,}75z^2(\frac{(h_1)^4}{4} - \frac{(h_2)^4}{4}) +$$

$$+d^2\frac{\phi_0\overline{\phi_0}}{(H/2)(H/2)}\frac{1}{(2h_M d^2)^2}0,75(\frac{(h_1)^6}{6}-\frac{(h_2)^6}{6})\Bigg\}+$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\Bigg\{(-\frac{z_0}{H/2}\frac{1}{2h_M d}\frac{1}{\pi})\sum_{n_M=1}^{\infty}(\overline{\int_{h_2}^{h_1}(\Lambda_{00_M}\frac{\partial A_{00_M}(r)}{\partial r}+} \quad \text{(A)}$$

$$\overline{2\Lambda_{0n_M}\frac{\partial A_{0n_M}(r)}{\partial r}})\cos(\frac{n_M\pi z}{h_M})]r^2dr)-d\frac{\phi_0}{H/2}\frac{z^2}{2h_M d^2}\sum_{n_M=1}^{\infty}(\int_{h_2}^{h_1}\overline{(\Lambda_{10_M}\frac{\partial A_{10_M}(r)}{\partial r}+} \quad (\Gamma)$$

$$\overline{2\Lambda_{1n_M}\frac{\partial A_{1n_M}(r)}{\partial r}})\cos(\frac{n_M\pi z}{h_M})]rdr+d\frac{\phi_0}{H/2}\frac{0,75}{2h_M d^2}\sum_{n_M=1}^{\infty}(\int_{h_2}^{h_1}\overline{(\Lambda_{10_M}\frac{\partial A_{10_M}(r)}{\partial r}+} \quad \text{(E)}$$

$$\overline{2\Lambda_{1n_M}\frac{\partial A_{1n_M}(r)}{\partial r}})\cos(\frac{n_M\pi z}{h_M})]r^3dr\Bigg\}+$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\Bigg\{(-\frac{z_0}{H/2}\frac{1}{2h_M d}\frac{1}{\pi})\sum_{n_M=1}^{\infty}(\int_{h_2}^{h_1}(\Lambda_{00_M}\frac{\partial A_{00_M}(r)}{\partial r}+ \quad \text{(B)}$$

$$2\Lambda_{0n_M}\frac{\partial A_{0n_M}(r)}{\partial r})\cos(\frac{n_M\pi z}{h_M})]r^2dr-d\frac{\overline{\phi_0}}{H/2}\frac{z^2}{2h_M d^2}\sum_{n_p=0}^{\infty}(\int_{h_2}^{h_1}(\Lambda_{10_M}\frac{\partial A_{10_M}(r)}{\partial r}+ \quad (\Delta)$$

$$2\Lambda_{1n_M}\frac{\partial A_{1n_M}(r)}{\partial r})\cos(\frac{n_M\pi z}{h_M})]rdr+d\frac{\overline{\phi_0}}{H/2}\frac{0,75}{2h_M d^2}\sum_{n_p=0}^{\infty}(\int_{h_2}^{h_1}(\Lambda_{10_M}\frac{\partial A_{10_M}(r)}{\partial r}+ \quad \text{(Z)}$$

$$2\Lambda_{1n_M}\frac{\partial A_{1n_M}(r)}{\partial r})\cos(\frac{n_M\pi z}{h_M})]r^3dr\Bigg\}+$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\Bigg\{\sum_{m=1}^{\infty}[\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}\overline{(\Lambda_{m0_M}\frac{\partial A_{m0_M}(r)}{\partial r}}+\overline{2\Lambda_{mn_M}\frac{\partial\Re_{mn_M}(r)}{\partial r}})\cos(\frac{n_M\pi z}{h_M})] \quad \text{(H)}$$

$$[\sum_{n_N=1}^{\infty}(\Lambda_{m0_N}\frac{\partial A_{m0_N}(r)}{\partial r}+2\Lambda_{mn_N}\frac{\partial A_{mn_N}(r)}{\partial r})\cos(\frac{n_N\pi z}{h_M})]rdr+[\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}\overline{(\Lambda_{00_M}\frac{\partial\Re_{00_M}(r)}{\partial r}+}$$

$$\overline{2\Lambda_{0n_M}\frac{\partial A_{0n_M}(r)}{\partial r}})\cos(\frac{n_M\pi z}{h_M})][\sum_{n_N=1}^{\infty}(\Lambda_{00_N}\frac{\partial A_{00_N}(r)}{\partial r}+2\Lambda_{0n_N}\frac{\partial A_{0n_N}(r)}{\partial r})\cos(\frac{n_N\pi z}{h_M})]]rdr$$

Όπου r η ακτίνα του $M-$στου μεσαίου στοιχείου.

Όμοια και για τον υπολογισμό $\frac{1}{r^2}\left(\frac{\partial\phi(r,\theta,z)}{\partial\theta}\right)^2=\frac{1}{r^2}\frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\overline{\frac{\partial\phi(r,\theta,z)}{\partial\theta}}$

Έχουμε

$$\frac{1}{r}\frac{\partial\phi(r,\theta,z)}{\partial\theta} = -i\omega\frac{H}{2}d\frac{1}{r}(+d\frac{\phi_0}{H/2}\frac{r(z^2-0,25r^2)}{2h_M d^2}\sin\theta) + i\omega\frac{H}{2}d\frac{1}{r}$$

$$\sum_{m=0}^{\infty}[(\Lambda_{m0_M}A_{m0_M}(r) + 2\sum_{n_M=1}^{\infty}\Lambda_{mn_M}A_{mn_M}(r))\cos(\frac{n_M\pi z}{h_M})]\sin(m\theta).$$

Και

$$\frac{1}{r}\overline{\frac{\partial\phi(r,\theta,z)}{\partial\theta}} = i\omega\frac{H}{2}d\frac{1}{r}(d\frac{\overline{\phi_0}}{H/2}\frac{r(z^2-0,25r^2)}{2h_M d^2}\sin\theta) - i\omega\frac{H}{2}d\frac{1}{r}$$

$$\sum_{m=0}^{\infty}[\overline{(\Lambda_{m0_M}A_{m0_M}(r)} + 2\sum_{n_M=1}^{\infty}\overline{\Lambda_{mn_M}A_{mn_M}(r)})\cos(\frac{n_M\pi z}{h_M})]\sin(m\theta).$$

Άρα ισχύει ότι

$$\frac{1}{r^2}\left(\frac{\partial\phi(r,\theta,z)}{\partial\theta}\right)^2 = \frac{1}{r^2}\frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\overline{\frac{\partial\phi(r,\theta,z)}{\partial\theta}} =$$

$$= \frac{1}{2}\omega^2\frac{H^2}{4}d^2\frac{1}{r^2}(d\frac{\phi_0}{H/2}\frac{r(z^2-0,25r^2)}{2h_M d^2}\sin\theta)\,(d\frac{\overline{\phi_0}}{H/2}\frac{r(z^2-0,25r^2)}{2h_M d^2}\sin\theta) -$$

$$-\frac{1}{2}\omega^2\frac{H^2}{4}d^2\frac{1}{r^2}(d\frac{\phi_0}{H/2}\frac{r(z^2-0,25r^2)}{2h_M d^2}\sin\theta)\,\sum_{m=0}^{\infty}[\overline{(\Lambda_{m0_M}A_{m0_M}(r) +}$$

$$2\sum_{n_M=1}^{\infty}\overline{\Lambda_{mn_M}A_{mn_M}(r)})\cos(\frac{n_M\pi z}{h_M})]\sin(m\theta) - \frac{1}{2}\omega^2\frac{H^2}{4}d^2\frac{1}{r^2}(d\frac{\overline{\phi_0}}{H/2}\frac{r(z^2-0,25r^2)}{2h_M d^2}\sin\theta)$$

$$\sum_{m=0}^{\infty}[(\Lambda_{m0_M}A_{m0_M}(r) + 2\sum_{n_M=1}^{\infty}\Lambda_{mn_M}A_{mn_M}(r))\cos(\frac{n_M\pi z}{h_M})]\sin(m\theta) + \frac{1}{2}\omega^2\frac{H^2}{4}d^2\frac{1}{r^2}$$

$$\sum_{m=0}^{\infty}[\overline{(\Lambda_{m0_M}A_{m0_M}(r)} + 2\sum_{n_M=1}^{\infty}\overline{\Lambda_{mn_M}A_{mn_M}(r)})\cos(\frac{n_M\pi z}{h_M})]\sin(m\theta)$$

$$\sum_{n=0}^{\infty}[(\Lambda_{m0_N}A_{m0_N}(r) + 2\sum_{n_N=1}^{\infty}\Lambda_{mn_N}A_{mn_N}(r))\cos(\frac{n_N\pi z}{h_M})]\sin(n\theta).$$

Και $\int_{h_2}^{h_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial\theta}\right)^2}^{T}d\theta\right](-1)rdr$.

Πρώτα υπολογίζουμε το $\int_0^{2\pi}\left(\frac{\partial\phi}{\partial\theta}\right)^2 rd\theta$ και προκύπτει η

$$\int_0^{2\pi}\left(\frac{\partial\phi}{\partial\theta}\right)^2 rd\theta =$$

$$=\omega^2\frac{H^2}{8}d^2\pi(d^2\frac{\phi_0\overline{\phi_0}}{H/2H/2}\frac{(z^2-0{,}25r^2)^2}{(2h_M d^2)^2})(-1)r-$$

$$-\omega^2\frac{H^2}{8}d^2\pi(d\frac{\phi_0}{H/2}\frac{(z^2-0{,}25r^2)}{2h_M d^2})\sum_{n_M=1}^{\infty}\overline{(\Lambda_{10_M}A_{10_M}(r)}+2\overline{\Lambda_{1n_M}A_{1n_M}(r)1)}\cos(\frac{n_M\pi z}{h_M})](-1)-$$

$$-\omega^2\frac{H^2}{8}d^2\pi(d\frac{\overline{\phi_0}}{H/2}\frac{(z^2-0{,}25r^2)}{2h_M d^2})\sum_{n_M=1}^{\infty}(\Lambda_{10_M}A_{10_M}(r)+2\Lambda_{1n_M}A_{1n_M}(r)1)\cos(\frac{n_M\pi z}{h_M})](-1)+$$

$$+\omega^2\frac{H^2}{8}d^2\frac{1}{r^2}\pi\sum_{m=1}^{\infty}[\sum_{n_M=1}^{\infty}(\Lambda_{m0_M}A_{m0_M}(r)+2\Lambda_{mn_M}A_{mn_M}(r))\cos(\frac{n_M\pi z}{h_M})]\,[\sum_{n_N=1}^{\infty}\overline{(\Lambda_{m0_N}A_{m0_N}(r)}+$$

$$\overline{2\Lambda_{mn_N}\Re_{mn_N}(r))}\cos(\frac{n_N\pi z}{h_M})]m^2](-1)r$$

Και στη συνέχεια υπολογίζουμε το $\int_{h_2}^{h_1}\left[\overline{\left(\frac{\partial\Phi}{\partial\theta}\right)^2}^T\right]ndS$

Επομένως

$$\int_{h_2}^{h_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial\theta}\right)^2}^T d\theta\right](-1)rdr=$$

$$=(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{\ d^2\frac{\phi_0\overline{\phi_0}}{H/2H/2}\frac{1}{(2h_M d^2)^2})\left\{\ z^4(\frac{(h_1)^2}{2}-\frac{(h_2)^2}{2})-0{,}5z^2\right.\right.$$

$$\left.\left.(\frac{(h_1)^4}{4}-\frac{(h_2)^4}{4})+0{,}25^2(\frac{(h_1)^6}{6}-\frac{(h_2)^6}{6})\ \right\}\right\}+ \qquad (\Theta)$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{\ d\frac{\phi_0}{H/2}\frac{z^2}{2h_M d^2}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}\overline{(\Lambda_{10_M}A_{10_M}(r)}+2\overline{\Lambda_{1n_M}A_{1n_M}(r))}\cos(\frac{n_M\pi z}{h_M})]dr-\right.$$

$$\left.-d\frac{\phi_0}{H/2}\frac{0{,}25}{2h_M d^2}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}\overline{(\Lambda_{10_M}A_{10_M}(r)}+2\overline{\Lambda_{1n_M}A_{1n_M}(r))}\cos(\frac{n_M\pi z}{h_M})]r^2dr\ \right\}+ \qquad (K)$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{\ d\frac{\overline{\phi_0}}{H/2}\frac{z^2}{2h_M d^2}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}(\Lambda_{10_M}A_{10_M}(r)+2\Lambda_{1n_M}A_{1n_M}(r))\cos(\frac{n_M\pi z}{h_M})]dr-\right.$$

$$-d\frac{\overline{\phi_0}}{H/2}\frac{0,25}{2h_M d^2}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}(\Lambda_{10_M}A_{10_M}(r)+2\Lambda_{1n_M}A_{1n_M}(r))\cos(\frac{n_M\pi z}{h_M})]r^2dr\ \Bigg\}+ \qquad (\text{I})(\Lambda)$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\frac{1}{r^2}\pi\Bigg\{\ \sum_{m=1}^{\infty}[m^2\int_{h_2}^{h_1}(\sum_{n_M=1}^{\infty}(\Lambda_{m0_M}A_{m0_M}(r)+2\Lambda_{mn_M}A_{mn_M}(r))\cos(\frac{n_M\pi z}{h_M})$$

$$\sum_{n_N=1}^{\infty}\overline{(\Lambda_{m0_N}A_{m0_N}(r)}+2\overline{\Lambda_{mn_N}A_{mn_N}(r))}\cos(\frac{n_N\pi z}{h_M}))rdr] \qquad (\text{M})$$

Όπου r η ακτίνα του $M-$στου μεσαίου στοιχείου.

Τέλος υπολογίζουμε το

$$\frac{\partial\phi(r,\theta,z)}{\partial z}=-i\omega\frac{H}{2}d(\frac{z_0}{H/2}\frac{2z}{2h_M d}-d\frac{\phi_0}{H/2}\frac{r2z}{2h_M d^2}\cos\theta)+i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[(\Lambda_{m0_M}A_{m0_M}(r)+2\sum_{n_M=1}^{\infty}\Lambda_{mn_M}A_{mn_M}(r))\ \sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}]\cos(m\theta).$$

Και

$$\frac{\partial\overline{\phi(r,\theta,z)}}{\partial z}=i\omega\frac{H}{2}d(\frac{z_0}{H/2}\frac{2z}{2h_M d}-d\frac{\overline{\phi_0}}{H/2}\frac{r2z}{2h_M d^2}\cos\theta)-i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[\overline{(\Lambda_{m0_M}A_{m0_M}(r)}+2\sum_{n_M=1}^{\infty}\overline{\Lambda_{mn_M}A_{mn_M}(r))}\ \sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}]\cos(m\theta).$$

Δηλαδή

$$\left(\frac{\partial\phi(r,\theta,z)}{\partial z}\right)^2=\frac{1}{2}\frac{\partial\phi(r,\theta,z)}{\partial z}\frac{\overline{\partial\phi(r,\theta,z)}}{\partial z}=$$

$$=\frac{1}{2}\omega^2\frac{H^2}{4}d^2(\frac{z_0}{H/2}\frac{2z}{2h_M d}-d\frac{\phi_0}{H/2}\frac{r2z}{2h_M d^2}\cos\theta)\ (\frac{z_0}{H/2}\frac{2z}{2h_M d}-d\frac{\overline{\phi_0}}{H/2}\frac{r2z}{2h_M d^2}\cos\theta)-$$

$$-\frac{1}{2}\omega^2\frac{H^2}{4}d^2(\frac{z_0}{H/2}\frac{2z}{2h_M d}-d\frac{\phi_0}{H/2}\frac{r2z}{2h_M d^2}\cos\theta)\sum_{m=0}^{\infty}[\overline{(\Lambda_{m0_M}A_{m0_M}(r)+}2\sum_{n_M=1}^{\infty}\overline{\Lambda_{mn_M}A_{mn_M}(r)})$$

$$\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}]\cos(\mathrm{m}\theta)-\frac{1}{2}\omega^2\frac{H^2}{4}d^2(\frac{z_0}{H/2}\frac{2z}{2h_p d}-d\frac{\overline{\phi_0}}{H/2}\frac{r2z}{2h_p d^2}\cos\theta)$$

$$\sum_{m=0}^{\infty}[(\Lambda_{m0_M}A_{m0_M}(r)+2\sum_{n_M=1}^{\infty}\Lambda_{mn_M}A_{mn_M}(r))\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}]\cos(m\theta)+$$

$$+\frac{1}{2}\omega^2\frac{H^2}{4}d^2\sum_{m=0}^{\infty}[\overline{(\Lambda_{m0_M}A_{m0_M}(r)+}2\sum_{n_M=1}^{\infty}\overline{\Lambda_{mn_M}A_{mn_M}(r)})\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}]\cos(m\theta)$$

$$\sum_{n=0}^{\infty}[(\Lambda_{n0_N}A_{n0_N}(r)+2\sum_{n_N=1}^{\infty}\Lambda_{nn_N}A_{nn_N}(r))\sin(\frac{n_N\pi z}{h_M})\frac{n_N\pi}{h_M}]\cos(n\theta).$$

Και $\int_{h_2}^{h_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial z}\right)^2}^T d\theta\right]r(-1)dr$.

Πρώτα υπολογίζουμε το $\int_0^{2\pi}\left(\frac{\partial\phi}{\partial z}\right)^2 rd\theta$ και προκύπτει η

$$\int_0^{2\pi}\left(\frac{\partial\phi}{\partial z}\right)^2 rd\theta=$$

$$=\omega^2\frac{H^2}{8}d^2\pi(\frac{z_0{}^2}{(H/2)^2}\frac{(2z)^2}{(2h_M d)^2}2+d^2\frac{\phi_0}{H/2}\frac{\overline{\phi_0}}{H/2}r^2\frac{(2z)^2}{(2h_M d^2)^2})(-1)r-$$

$$-\omega^2\frac{H^2}{8}d^2\pi[(\frac{z_0}{H/2}\frac{2z}{2h_M d})\sum_{n_M=1}^{\infty}(\overline{\Lambda_{00_M}A_{00_M}(r)}+2\overline{\Lambda_{0n_M}A_{0n_M}(r)})\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}\frac{1}{\pi}-$$

$$-d\frac{\phi_0}{H/2}\frac{r2z}{2h_M d^2}\sum_{n_M=1}^{\infty}(\overline{\Lambda_{10_M}A_{10_M}(r)}+2\overline{\Lambda_{1n_M}A_{1n_M}(r)})\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}](-1)r-$$

$$-\omega^2\frac{H^2}{8}d^2\pi[(\frac{z_0}{H/2}\frac{2z}{2h_M d})\sum_{n_M=1}^{\infty}(\Lambda_{0n_M}A_{0n_M}(r)+\overline{\Lambda_{0n_M}}\,\overline{A_{0n_M}(r)})\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}\frac{1}{\pi}-$$

$$-d\frac{\overline{\phi_0}}{H/2}\frac{r2z}{2h_M d^2}\sum_{n_M=1}^{\infty}(\Lambda_{10_M}A_{10_M}(r)+2\Lambda_{1n_M}A_{1n_M}(r))\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}](-1)r+$$

$$+\omega^2\frac{H^2}{8}d^2\pi[\sum_{m=1}^{\infty}[\sum_{n_M=1}^{\infty}(\Lambda_{m0_M}A_{m0_M}(r)+2\Lambda_{mn_M}A_{mn_M}(r))\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}]$$

$$[\sum_{n_N=1}(\overline{\Lambda_{m0_N}A_{m0_N}(r)}+\overline{2\Lambda_{mn_N}\Re_{mn_N}(r)})\sin(\frac{n_N\pi z}{h_M})\frac{n_N\pi}{h_M}]+2[\sum_{n_M=1}^{\infty}(\Lambda_{00_M}A_{00_M}(r)+$$

$$2\Lambda_{0n_M}A_{0n_M}(r))\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}]\ [\sum_{n_N=1}^{\infty}(\overline{\Lambda_{00_N}A_{00_N}(r)}+\overline{2\Lambda_{0n_N}A_{0n_N}(r)})\sin(\frac{n_N\pi z}{h_M})\frac{n_N\pi}{h_M}]](-1)r$$

Και

$$\int_{h_2}^{h_1}\left[\int_0^{2\pi}\overline{\left(\frac{\partial\Phi}{\partial z}\right)^2}^T rd\theta\right](-1)dr=$$

$$=(-1)\omega^2\frac{H^2}{8}d^2\pi\Bigg\{\ \frac{z_0{}^2}{(H/2)^2}\frac{(2z)^2}{(2h_Md)^2}2(\frac{(h_1)^2}{2}-\frac{(h_2)^2}{2})+d^2\frac{\phi_0}{H/2}\frac{\overline{\phi_0}}{H/2}$$

$$\frac{(2z)^2}{(2h_Md^2)^2}(\frac{(h_1)^4}{4}-\frac{(h_2)^4}{4})\ \Bigg\}+$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\Bigg\{\ \frac{z_0}{H/2}\frac{2z}{2h_Md}\frac{1}{\pi}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}(\overline{\Lambda_{00_M}A_{00_M}(r)}+ \qquad \text{(N)}$$

$$+\overline{2\Lambda_{0n_M}A_{0n_M}(r)})\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}rdr-d\frac{\phi_0}{H/2}\frac{2z}{2h_Md^2}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}(\overline{\Lambda_{10_M}A_{10_M}(r)}+$$

$$+\overline{2\Lambda_{1n_M}A_{1n_M}(r)})\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}r^2dr\ \Bigg\}+ \qquad \text{(O)}$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\Bigg\{\ \frac{z_0}{H/2}\frac{2z}{2h_pd}\frac{1}{\pi}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}(\Lambda_{00_M}A_{00_M}(r)+ \qquad (\Xi)$$

$$+2\Lambda_{0n_M}A_{0n_M}(r))\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}rdr-d\frac{\overline{\phi_0}}{H/2}\frac{2z}{2h_Md^2}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}(\Lambda_{10_M}A_{10_M}(r)+$$

$$+2\Lambda_{1n_M}A_{1n_M}(r))\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}r^2dr\ \Bigg\}+ \qquad (\Pi)$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\Bigg\{\ \sum_{m=1}^{\infty}[\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}(\Lambda_{m0_M}A_{m0_M}(r)+2\Lambda_{mn_M}A_{mn_M}(r))\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M} \qquad \text{(P)}$$

$$\sum_{n_N=1}^{\infty}(\overline{\Lambda_{m0_N} A_{m0_N}(r)} + 2\overline{\Lambda_{mn_N}\Re_{mn_N}(r)})\sin(\frac{n_N \pi z}{h_M})\frac{n_N \pi}{h_M} rdr] + [\int_{h_2}^{h_1} \sum_{n_M=1}^{\infty}(\Lambda_{00_M} A_{00_M}(r) +$$

$$+ \Lambda_{0n_M} A_{0n_M}(r))\sin(\frac{n_M \pi z}{h_M})\frac{n_M \pi}{h_M} \sum_{n_N=1}^{\infty}(\overline{\Lambda_{00_N} A_{00_N}(r)} + 2\overline{\Lambda_{0n_N}\Re_{0n_N}(r)})\sin(\frac{n_N \pi z}{h_M})\frac{n_N \pi}{h_M}]rdr$$

Συνοψίζοντας για το μεσαίο στοιχείο στο <u>Πεδίο (ΙΙΙ)</u> το $\int_{h_2}^{h_1}\int_0^{2\pi} \left|\nabla\Phi^{(1)}\right|^2 \overline{n} \; dS =$

$$\int_{h_2}^{h_1}(\int_0^{2\pi}[\overline{(\frac{\partial\Phi}{\partial r})^2}^T + \frac{1}{r^2}\overline{(\frac{\partial\Phi}{\partial\theta})^2}^T + \overline{(\frac{\partial\Phi}{\partial z})^2}^T]r \; d\theta)(-1) \; dr =$$

$$= (-1)\omega^2\frac{H^2}{8}d^2\pi\Big\{ \frac{{z_0}^2}{(H/2)^2}\frac{2}{(2h_M d)^2}(\frac{(h_1)^3}{3}-\frac{(h_2)^3}{3}) + d^2\frac{\phi_0\overline{\phi_0}}{(H/2)(H/2)}\frac{1}{(2h_m d^2)^2}$$

$$(z^4(\frac{(h_1)^2}{2}-\frac{(h_2)^2}{2}) - d^2\frac{\phi_0\overline{\phi_0}}{(H/2)(H/2)}\frac{2}{(2h_m d^2)^2}0{,}75z^2(\frac{(h_1)^4}{4}-\frac{(h_2)^4}{4}) +$$

$$+ d^2\frac{\phi_0\overline{\phi_0}}{(H/2)(H/2)}\frac{1}{(2h_M d^2)^2}0{,}75(\frac{(h_1)^6}{6}-\frac{(h_2)^6}{6}) \Big\} +$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\Big\{ (-\frac{z_0}{H/2}\frac{1}{2h_M d}\frac{1}{\pi}) \sum_{n_M=1}^{\infty}(\overline{\int_{h_2}^{h_1} (\Lambda_{00_M}\frac{\partial A_{00_M}(r)}{\partial r} +} \quad \text{(A)}$$

$$\overline{2\Lambda_{0n_M}\frac{\partial A_{0n_M}(r)}{\partial r}})\cos(\frac{n_M\pi z}{h_M})]r^2 dr) - d\frac{\phi_0}{H/2}\frac{z^2}{2h_M d^2}\sum_{n_M=1}^{\infty}(\int_{h_2}^{h_1}\overline{(\Lambda_{10_M}\frac{\partial A_{10_M}(r)}{\partial r}} + \quad (\Gamma)$$

$$\overline{2\Lambda_{1n_M}\frac{\partial A_{1n_M}(r)}{\partial r}})\cos(\frac{n_M\pi z}{h_M})]rdr + d\frac{\phi_0}{H/2}\frac{0{,}75}{2h_M d^2}\sum_{n_M=1}^{\infty}(\int_{h_2}^{h_1}\overline{(\Lambda_{10_M}\frac{\partial A_{10_M}(r)}{\partial r}} + \quad \text{(E)}$$

$$\overline{2\Lambda_{1n_M}\frac{\partial A_{1n_M}(r)}{\partial r}})\cos(\frac{n_M\pi z}{h_M})]r^3 dr \Big\} +$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\Big\{ (-\frac{z_0}{H/2}\frac{1}{2h_M d}\frac{1}{\pi}) \sum_{n_M=1}^{\infty}(\int_{h_2}^{h_1} (\Lambda_{00_M}\frac{\partial A_{00_M}(r)}{\partial r} + \quad \text{(B)}$$

$$2\Lambda_{0n_M}\frac{\partial A_{0n_M}(r)}{\partial r})\cos(\frac{n_M\pi z}{h_M})]r^2 dr - d\frac{\overline{\phi_0}}{H/2}\frac{z^2}{2h_M d^2}\sum_{n_p=0}^{\infty}(\int_{h_2}^{h_1}(\Lambda_{10_M}\frac{\partial A_{10_M}(r)}{\partial r} + \quad (\Delta)$$

$$2\Lambda_{1n_M}\frac{\partial A_{1n_M}(r)}{\partial r})\cos(\frac{n_M\pi z}{h_M})]rdr + d\frac{\overline{\phi_0}}{H/2}\frac{0{,}75}{2h_M d^2}\sum_{n_p=0}^{\infty}(\int_{h_2}^{h_1}(\Lambda_{10_M}\frac{\partial A_{10_M}(r)}{\partial r} + \quad \text{(Z)}$$

$$2\Lambda_{1n_M}\frac{\partial A_{1n_M}(r)}{\partial r})\cos(\frac{n_M\pi z}{h_M})]r^3 dr \Big\} +$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\Big\{ \sum_{m=1}^{\infty}[\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}\overline{(\Lambda_{m0_M}\frac{\partial A_{m0_M}(r)}{\partial r}} + \overline{2\Lambda_{mn_M}\frac{\partial\Re_{mn_M}(r)}{\partial r}})\cos(\frac{n_M\pi z}{h_M})] \quad \text{(H)}$$

$$[\sum_{n_N=1}^{\infty}(\Lambda_{m0_N}\frac{\partial A_{m0_N}(r)}{\partial r}+2\Lambda_{mn_N}\frac{\partial A_{mn_N}(r)}{\partial r})\cos(\frac{n_N\pi z}{h_M})]rdr+[\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}\overline{(\Lambda_{00_M}\frac{\partial\Re_{00_M}(r)}{\partial r}+}$$

$$\overline{2\Lambda_{0n_M}\frac{\partial A_{0n_M}(r)}{\partial r}})\cos(\frac{n_M\pi z}{h_M})][\sum_{n_N=1}^{\infty}(\Lambda_{00_N}\frac{\partial A_{00_N}(r)}{\partial r}+2\Lambda_{0n_N}\frac{\partial A_{0n_N}(r)}{\partial r})\cos(\frac{n_N\pi z}{h_M})]]rdr$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{\ d^2\frac{\phi_0\overline{\phi_0}}{H/2H/2}\frac{1}{(2h_Md^2)^2})\left\{\ z^4(\frac{(h_1)^2}{2}-\frac{(h_2)^2}{2})-0{,}5z^2\right.\right.$$

$$\left.\left.(\frac{(h_1)^4}{4}-\frac{(h_2)^4}{4})+0{,}25^2(\frac{(h_1)^6}{6}-\frac{(h_2)^6}{6})\ \right\}\right\}+ \qquad (\Theta)$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{\ d\frac{\phi_0}{H/2}\frac{z^2}{2h_Md^2}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}\overline{(\Lambda_{10_M}A_{10_M}(r)}+2\overline{\Lambda_{1n_M}A_{1n_M}(r))}\cos(\frac{n_M\pi z}{h_M})]dr-\right.$$

$$\left.-d\frac{\phi_0}{H/2}\frac{0{,}25}{2h_Md^2}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}\overline{(\Lambda_{10_M}A_{10_M}(r)}+2\overline{\Lambda_{1n_M}A_{1n_M}(r))}\cos(\frac{n_M\pi z}{h_M})]r^2dr\ \right\}+ \qquad (K)$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{\ d\frac{\overline{\phi_0}}{H/2}\frac{z^2}{2h_Md^2}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}(\Lambda_{10_M}A_{10_M}(r)+2\Lambda_{1n_M}A_{1n_M}(r))\cos(\frac{n_M\pi z}{h_M})]dr-\right.$$

$$\left.-d\frac{\overline{\phi_0}}{H/2}\frac{0{,}25}{2h_Md^2}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}(\Lambda_{10_M}A_{10_M}(r)+2\Lambda_{1n_M}A_{1n_M}(r))\cos(\frac{n_M\pi z}{h_M})]r^2dr\ \right\}+ \qquad (I)(\Lambda)$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\frac{1}{r^2}\pi\left\{\ \sum_{m=1}^{\infty}[m^2\int_{h_2}^{h_1}(\sum_{n_M=1}^{\infty}(\Lambda_{m0_M}A_{m0_M}(r)+2\Lambda_{mn_M}A_{mn_M}(r))\cos(\frac{n_M\pi z}{h_M})\right.$$

$$\sum_{n_N=1}^{\infty}\overline{(\Lambda_{m0_N}A_{m0_N}(r)}+2\overline{\Lambda_{mn_N}A_{mn_N}(r))}\cos(\frac{n_N\pi z}{h_M}))rdr]+ \qquad (M)$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{\ \frac{{z_0}^2}{(H/2)^2}\frac{(2z)^2}{(2h_Md)^2}2(\frac{(h_1)^2}{2}-\frac{(h_2)^2}{2})+d^2\frac{\phi_0}{H/2}\frac{\overline{\phi_0}}{H/2}\right.$$

$$\left.\frac{(2z)^2}{(2h_Md^2)^2}(\frac{(h_1)^4}{4}-\frac{(h_2)^4}{4})\ \right\}+$$

$$+(-1)\omega^2\frac{H^2}{8}d^2\pi\left\{\ \frac{z_0}{H/2}\frac{2z}{2h_Md}\frac{1}{\pi}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}\overline{(\Lambda_{00_M}A_{00_M}(r)}+\right. \qquad (N)$$

$$+2\overline{\Lambda_{0n_M}A_{0n_M}(r))}\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}rdr-d\frac{\phi_0}{H/2}\frac{2z}{2h_Md^2}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}\overline{(\Lambda_{10_M}A_{10_M}(r)}+$$

$$\left.+2\overline{\Lambda_{1n_M}A_{1n_M}(r))}\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}r^2dr\ \right\}+ \qquad (O)$$

$$+(-1)\omega^2 \frac{H^2}{8} d^2\pi \left\{ \frac{z_0}{H/2} \frac{2z}{2h_p d} \frac{1}{\pi} \int_{h_2}^{h_1} \sum_{n_M=1}^{\infty} (\Lambda_{00_M} A_{00_M}(r) + \right. \quad (\Xi)$$

$$+2\Lambda_{0n_M} A_{0n_M}(r))\sin(\frac{n_M \pi z}{h_M})\frac{n_M \pi}{h_M} rdr - d\frac{\overline{\phi_0}}{H/2}\frac{2z}{2h_M d^2}\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}(\Lambda_{10_M} A_{10_M}(r) +$$

$$\left. +2\Lambda_{1n_M} A_{1n_M}(r))\sin(\frac{n_M \pi z}{h_M})\frac{n_M \pi}{h_M} r^2 dr \right\} + \quad (\Pi)$$

$$+(-1)\omega^2 \frac{H^2}{8} d^2\pi \left\{ \sum_{m=1}^{\infty} [\int_{h_2}^{h_1} \sum_{n_M=1}^{\infty} (\Lambda_{m0_M} A_{m0_M}(r) + 2\Lambda_{mn_M} A_{mn_M}(r))\sin(\frac{n_M \pi z}{h_M})\frac{n_M \pi}{h_M} \right. \quad (P)$$

$$\sum_{n_N=1}^{\infty}(\overline{\Lambda_{m0_N} A_{m0_N}(r)} + 2\overline{\Lambda_{mn_N}\Re_{mn_N}(r)})\sin(\frac{n_N \pi z}{h_M})\frac{n_N \pi}{h_M} rdr] + [\int_{h_2}^{h_1}\sum_{n_M=1}^{\infty}(\Lambda_{00_M} A_{00_M}(r) +$$

$$+\Lambda_{0n_M} A_{0n_M}(r))\sin(\frac{n_M \pi z}{h_M})\frac{n_M \pi}{h_M} \sum_{n_N=1}^{\infty}(\overline{\Lambda_{00_N} A_{00_N}(r)} + 2\overline{\Lambda_{0n_N}\Re_{0n_N}(r)})\sin(\frac{n_N \pi z}{h_M})\frac{n_N \pi}{h_M}] rdr$$

Ο υπολογισμός των ολοκληρωμάτων (Α)-(Ρ) έχει γίνει στη σελίδα 210.

8.5 Υπολογισμός του όρου $\int_{h_2}^{h_1}(\int_0^{2\pi} \overline{x}^{(1)}\nabla\Phi_t^{(1)})\overline{n}dS$ για το μεσαίο στοιχείο στο πεδίο (ΙΙΙ)

Έχουμε δείξει στο Κεφάλαιο 2 –σελίδα 19– ότι

$$\overline{x}^{(1)}\nabla\Phi^{(1)} = [(X_{g_1}^{(1)} + X_5^{(1)}z)\frac{\partial\phi(r,\theta,z)}{\partial r}\cos\theta - (X_{g_1}^{(1)} + X_5^{(1)}z)\frac{1}{r}\frac{\partial\phi(r,\theta,z)}{\partial\theta}\sin\theta$$
$$+ (X_{g_3}^{(1)} - X_5^{(1)}r\cos\theta)\frac{\partial\phi(r,\theta,z)}{\partial z}](-i\omega)]$$

Δηλαδή

$$\overline{x}^{(1)}\nabla\Phi_t^{(1)} = [(X_{g_1}^{(1)} + X_5^{(1)}z)\ (-i\omega)[\ -i\omega\frac{H}{2}d(-\frac{z_0}{H/2}\frac{r}{2h_M d} - d\frac{\phi_0}{H/2}\frac{(z^2-0,75r^2)}{2h_M d^2}\cos\theta) -$$

$$-i\omega\frac{H}{2}d\ \sum_{m=0}^{\infty}[(\Lambda_{m0_M}\frac{\partial A_{m0_M}(r)}{\partial r} + 2\sum_{n_M=1}^{\infty}\Lambda_{mn_M}\frac{\partial A_{mn_M}(r)}{\partial r})\cos(\frac{n_M\pi z}{h_M})]\cos(m\theta)\,]\cos(\theta) -$$

$$-(X_{g_1}^{(1)} + X_5^{(1)}z)\ (-i\omega)[\ -i\omega\frac{H}{2}d\frac{1}{r}(d\frac{\phi_0}{H/2}\frac{r(z^2-0,25r^2)}{2h_M d^2}\sin\theta) + i\omega\frac{H}{2}d\frac{1}{r}$$

$$\sum_{m=0}^{\infty}[(\Lambda_{m0_M}A_{m0_M}(r) + 2\sum_{n_M=1}^{\infty}\ \Lambda_{mn_M}A_{mn_M}(r))\cos(\frac{n_M\pi z}{h_M})]\sin(m\theta)\,]\sin(\theta) +$$

$$+(X_{g_3}^{(1)} - X_5^{(1)}r\cos\theta)(-i\omega)\ [-i\omega\frac{H}{2}d(\frac{z_0}{H/2}\frac{2z}{2h_M d} - d\frac{\phi_0}{H/2}\frac{r2z}{2h_M d^2}\cos\theta) + i\omega\frac{H}{2}d$$

$$\sum_{m=0}^{\infty}[(\Lambda_{m0_M}A_{m0_M}(r) + 2\sum_{n_M=1}^{\infty}\Lambda_{mn_M}A_{mn_M}(r))\ \sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}]\cos(m\theta)]].$$

Θα υπολογίσουμε τον όρο

$$\int_0^{2\pi}(\overline{x}^{(1)}\nabla\Phi_t^{(1)})rd\theta =$$

$$= (X_{g_1}^{(1)} + X_5^{(1)}z)\ (-i\omega)\ [-i\omega\frac{H}{2}d(-d\frac{\phi_0}{H/2}\frac{(z^2-0,75r^2)}{2h_M d^2}\pi) - (\Lambda_{20_M}\frac{\partial A_{20_M}(r)}{\partial r} +$$

$$2\sum_{n_M=1}^{\infty}\Lambda_{2n_M}\frac{\partial A_{2n_M}(r)}{\partial r})\cos(\frac{n_M\pi z}{h_M})1)\frac{\pi}{2}r +$$

$$-(X_{g_1}^{(1)}+X_5^{(1)}z)\ (-i\omega)[\ -i\omega\frac{H}{2}d\frac{1}{r}(d\frac{\phi_0}{H/2}\frac{r(z^2-0{,}25r^2)}{2h_M d^2}\pi)+i\omega\frac{H}{2}d\frac{1}{r}\ [(\Lambda_{20_M}A_{20_M}(r)+$$

$$2\sum_{n_M=1}^{\infty}\ \Lambda_{2n_M}A_{2n_M}(r))\cos(\frac{n_M\pi z}{h_M})]\ \frac{\pi}{2}r+$$

$$+X_{g_3}^{(1)}(-i\omega)\ [-i\omega\frac{H}{2}d(\frac{z_0}{H/2}\frac{2z}{2h_M d}2\pi-(\Lambda_{00_M}A_{00_M}(r)+2\sum_{n_M=1}^{\infty}\Lambda_{0n_M}A_{0n_M}(r))$$

$$\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M})r+$$

$$+(-X_5^{(1)}b_p)(-i\omega)\ [-i\omega\frac{H}{2}d(-d\frac{\phi_0}{H/2}\frac{r2z}{2h_M d^2}\pi)-(\Lambda_{10_M}A_{10_M}(r)+2\sum_{n_M=1}^{\infty}\Lambda_{1n_M}A_{1n_M}(r))$$

$$\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M})r]$$

Επομένως

$$\int_{h_2}^{h_1}(\int_0^{2\pi}\bar{x}^{(1)}\nabla\Phi_t^{(1)})ndS=\int_{h_2}^{h_1}(\int_0^{2\pi}\bar{x}^{(1)}\nabla\Phi_t^{(1)}rd\theta)(-1)dr=$$

$$+2\sum_{n_M=1}^{\infty}\left[\frac{\Lambda_{0n_M}\dfrac{n_M\pi}{h_M}}{I_0(\dfrac{n_M\pi b_M}{h_M})}\right]\cos(\frac{n_M\pi z}{h_M})\int_{h_2}^{h_1}rI_1(\frac{n_M\pi r}{h_M})dr+$$

$$=(X_{g_1}^{(1)}+X_5^{(1)}z)\ (-i\omega)\ [-i\omega\frac{H}{2}d[(-d\frac{\phi_0}{H/2}(\frac{z^2}{2h_M d^2}(h_1-h_2)-\frac{0{,}75}{2h_M d^2}(\frac{h_1^{\ 3}-h_2^{\ 3}}{3}))\pi)-$$

$$-2\sum_{n_M=1}^{\infty}\left[\frac{\Lambda_{2n_M}\dfrac{n_M\pi}{h_M}}{I_2(\dfrac{n_M\pi b_M}{h_M})}\right]\cos(\frac{n_M\pi z}{h_M})]\int_{h_2}^{h_1}rI_3(\frac{n_M\pi r}{h_M})dr+$$

$$-(X_{g_1}^{(1)}+X_5^{(1)}z)\ (-i\omega)[\ [-i\omega\frac{H}{2}d[(d\frac{\phi_0}{H/2}(\frac{z^2}{2h_M d^2}(h_1-h_2)-\frac{0{,}25}{2h_M d^2}(\frac{h_1^{\ 3}-h_2^{\ 3}}{3}))\pi)+$$

$$+2\sum_{n_M=1}^{\infty}\left[\frac{\Lambda_{2n_M}\dfrac{n_M\pi}{h_M}}{I_2(\dfrac{n_M\pi b_M}{h_M})}\right]\cos(\frac{n_M\pi z}{h_M})]\int_{h_2}^{h_1}I_2(\frac{n_M\pi r}{h_M})dr+$$

$$+X_{g_3}^{(1)}(-i\omega)\ [-i\omega\frac{H}{2}d\left\{\ \frac{z_0}{H/2}\frac{2\pi}{2h_M d}2z(\frac{(h_1)^2}{2}-\frac{(h_2)^2}{2})-\right.$$

$$\left.-\sum_{n_M=1}^{\infty}\left[\Lambda_{00_M}+\frac{2\Lambda_{0n_M}}{I_0(\frac{n_M\pi b_M}{h_M})}\right]\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}\int_{h_2}^{h_1}rI_0(\frac{n_M\pi r}{h_M})dr\ \right\}+$$

$$+(-X_5^{(1)}b_p)(-i\omega)\ [-i\omega\frac{H}{2}d(-d\frac{\phi_0}{H/2}\frac{2z}{2h_M d^2}\pi)(\frac{(h_1)^2}{2}-\frac{(h_2)^2}{2})-$$

$$\left.-\sum_{n_M=1}^{\infty}\left[\Lambda_{10_M}+\frac{2\Lambda_{1n_M}}{I_1(\frac{n_M\pi b_M}{h_M})}\right]\sin(\frac{n_M\pi z}{h_M})\frac{n_M\pi}{h_M}\int_{h_2}^{h_1}rI_1(\frac{n_M\pi r}{h_M})dr\ \right\}\ ](-1)$$

Όμως για τον υπολογισμός της δύναμης έκπτωσης πρέπει να υπολογίσουμε τον όρο

$$\overline{\overline{x}^{(1)}\nabla\Phi_t^{(1)}}^T=\omega(\overline{X}_{\mathrm{Re}}^{(1)}\cos(\omega t)+\overline{X}_{\mathrm{Im}}^{(1)}\sin(\omega t))(\nabla\phi_{\mathrm{Im}}^{(1)}\cos(\omega t)-\nabla\phi_{\mathrm{Re}}^{(1)}\sin(\omega t))=$$

$$=\overline{\omega(\overline{X}_{\mathrm{Re}}^{(1)}\nabla\phi_{\mathrm{Im}}^{(1)}\cos^2(\omega t)-\overline{X}_{\mathrm{Im}}^{(1)}\nabla\phi_{\mathrm{Re}}^{(1)}\sin^2(\omega t)+(-\overline{X}_{\mathrm{Re}}^{(1)}\nabla\phi_{\mathrm{Re}}^{(1)}+\overline{X}_{\mathrm{Im}}^{(1)}\nabla\phi_{\mathrm{Im}}^{(1)})\cos(\omega t)\sin(\omega t))}^t=$$

$$=\frac{1}{2}\omega(\overline{X}_{\mathrm{Re}}^{(1)}\nabla\phi_{\mathrm{Im}}^{(1)}-\overline{X}_{\mathrm{Im}}^{(1)}\nabla\phi_{\mathrm{Re}}^{(1)})=\frac{1}{2}\omega\,\mathrm{Im}(X^{(1)^*}\nabla\phi^{(1)})$$

Όπου $R^{(1)}$: ο πίνακας που περιέχει τις γωνίες περιστροφής

h_m : η απόσταση του μεσαίου κάτω στοιχείου από τον πυθμένα

8.6 Υπολογισμός της κατακόρυφης δύναμης έκπτωσης για το μεσαίο στοιχείο στο πεδίο (III)

Η κατακόρυφη δύναμη έκπτωσης, F_Z για το μεσαίο στοιχείο στο Πεδίο (III) Κεφάλαιο 2 –σελίδα 17– υπολογίζεται από τη σχέση

$$F_Z = MR^{(1)} \overline{X^{(1)}{}_g}'' - \iint_{S_0} -\frac{1}{2}\rho\left|\nabla\Phi^{(1)}\right|^2 \bar{n} dS - \iint_{S_0} -\rho(\overline{X}^{(1)}\nabla\Phi_t^{(1)})\bar{n} dS =$$

$$= M\,\mathrm{Re}[R^{(1)}e^{i\phi}\ (-\omega^2 X_g^{(1)})e^{-i\phi}\ ] + \frac{1}{2}\rho\,[\underline{\iint_{S_0}\left|\nabla\Phi^{(1)}\right|^2 \bar{n} dS}\,] + \rho\,[\underline{\iint_{S_0}(\overline{X}^{(1)}\nabla\Phi_t^{(1)})\bar{n} dS}\,].$$

Όμως οι παραστάσεις $\underline{\iint_{S_0}\left|\nabla\Phi^{(1)}\right|^2 \bar{n} dS}$ $\quad$ $\underline{[\iint_{S_0}(\overline{X}^{(1)}\nabla\Phi_t^{(1)})\bar{n} dS]}$

είναι γνωστές από τα προηγούμενα.(Σελίδα 258, σελίδα 261 αντίστοιχα)

Όπου ρ :πυκνότητα νερού.

g :επιτάχυνση της βαρύτητας.

M :μάζα αξονοσυμμετρικού σώματος.

$R^{(1)}$: ο πίνακας που περιέχει τις γωνίες περιστροφής.

$X_g^{(1)}$ *:* οι μεταφορικές κινήσεις του κέντρου βάρους της κατασκευής.

ω : η ιδιοσυχνότητα της κατασκευής.

φ : διαφορά φάσης.

Επομένως μπορούμε να υπολογίσουμε την κατακόρυφη δύναμη έκπτωσης F_Z για το μεσαίο στοιχείο στο Πεδίο (III).

ΠΑΡΑΡΤΗΜΑ Α

Α) Ισχύει ότι

$$[\sum_{n_1=0}^{\infty} M_{p_{n_1}}(r)\cos(n_1\theta)\ \sum_{n_2=0}^{\infty} M_{p+1_{n_2}}(r)\cos(n_2\theta)]\cos\theta =$$

$$= \sum_{\substack{n=0 \\ n_1=n_2=n}}^{\infty} [M_{p_n}(r) M_{p+1_n}(r)\cos(n\theta)^2 \cos\theta] +$$

$$\sum_{\substack{n_1=0 \\ n_1\neq n_2}}^{\infty} \sum_{n_2=0}^{\infty} [M_{p_{n_1}}(r)\cos(n_1\theta) M_{p+1_{n_2}}(r)\cos(n_2\theta)]\cos\theta$$

Ολοκληρώνοντας ως προς θ, στο 0 – 2π, την παραπάνω σχέση, έχουμε:

$$\int_0^{2\pi} \sum_{\substack{n=0 \\ n_1=n_2=n}}^{\infty} [M_{p_n}(r) M_{p+1_n}(r)\cos(n\theta)^2]\cos\theta d\theta +$$

$$\int_0^{2\pi} \sum_{\substack{n_1=0 \\ n_1\neq n_2}}^{\infty} \sum_{n_2=0}^{\infty} [M_{p_{n_1}}(r)\cos(n_1\theta) M_{p+1_{n_2}}(r)\cos(n_2\theta)]\cos\theta d\theta$$

Όμως ισχύει ότι

$$\int_0^{2\pi} \cos(n\theta)\cos(n\theta)\cos\theta dz = \frac{n\sin(4n\pi)}{-1+4n^2} = 0$$

$$\int_0^{2\pi} \cos(m\theta)\cos(n\theta)\cos\theta dz = \frac{1}{4}\left(\frac{2(m-n)\sin(2\pi(n-m))}{-1+(m+n)^2} + \frac{2(n+m)\sin(2\pi(m+n))}{(-1+m+n)(1+m+n)}\right)$$

Για το δεύτερο ολοκλήρωμα οι αριθμητές είναι πάντα μηδέν.

Διακρίνουμε τις παρακάτω περιπτώσεις για το που μηδενίζονται οι παρονομαστές.

$m-n=-1$ ή $m-n=1$ δηλ. $m=-1+n$, $m=1+n$

$-1+m+n=0$ ή $1+m+n=0$ δηλ. $m=1-n$, $m=-1-n$

Δηλαδή

	$m=-1+n$	$m=1+n$	$m=1-n$	$m=-1-n$
n=0	m<0	m=1	m=1	m<0

n=1	m=0	m=2	m=0	m<0
n=2	m=1	m=3	m<0	m<0
n=3	m=2	m=4	m<0	m<0
n=4	m=3	m=5	m<0	m<0

Παρατηρούμε πως για n>1 το m παίρνει τιμές $m=-1+n\,, m=1+n$.

Όταν n=0 και m=1, το άθροισμα γίνεται

$$\int_0^{2\pi}\sum_{\substack{n_1=0\\n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n_1}}(r)\cos(n_1\theta)M_{p+1_{n_2}}(r)\cos(n_2\theta)]\cos\theta d\theta=M_{p_0}(r)M_{p+1_1}(r)\pi$$

Όταν n=1 και m=0, το άθροισμα γίνεται

$$\int_0^{2\pi}\sum_{\substack{n_1=0\\n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n_1}}(r)\cos(n_1\theta)M_{p+1_{n_2}}(r)\cos(n_2\theta)]\cos\theta d\theta=M_{p_1}(r)M_{p+1_0}(r)\pi$$

Όταν n=1 και m=2, το άθροισμα γίνεται

$$\int_0^{2\pi}\sum_{\substack{n_1=0\\n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n_1}}(r)\cos(n_1\theta)M_{p+1_{n_2}}(r)\cos(n_2\theta)]\cos\theta d\theta=M_{p_1}(r)M_{p+1_2}(r)\pi$$

Όταν n>1, το άθροισμα γίνεται

$$\int_0^{2\pi}\sum_{\substack{n_1=0\\n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n_1}}(r)\cos(n_1\theta)M_{p+1_{n_2}}(r)\cos(n_2\theta)]\cos\theta d\theta=$$

$$\int_0^{2\pi}\sum_{\substack{n_1=0\\n_1\neq n_2}}^{\infty}[M_{p_{n_1}}(r)\cos(n_1\theta)M_{p+1_{n_1+1}}(r)\cos((n_1+1)\theta)]\cos\theta d\theta=$$

$$=\sum_{\substack{n_1=0\\n_1\neq n_2}}^{\infty}[M_{p_{n_1}}(r)M_{p+1_{n_1+1}}(r)\frac{1}{8}(4\pi+(\frac{1}{n_1}+\frac{1}{1+n_1})\sin(4n_1\pi))]=$$

$$=\sum_{\substack{n_1=0\\n_1\neq n_2}}^{\infty}[M_{p_{n_1}}(r)M_{p+1_{n_1+1}}(r)\frac{\pi}{2}$$

Β) Ισχύει ότι

$$[\sum_{n_1=0}^{\infty} M_{p_{n_1}}(r)\cos(n_1\theta)\ \sum_{n_2=0}^{\infty} M_{p+1_{n_2}}(r)\cos(n_2\theta)] =$$

$$= \sum_{\substack{n=0\\ n_1=n_2=n}}^{\infty}[M_{p_n}(r)M_{p+1_n}(r)\cos(n\theta)^2] + \sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n1}}(r)\cos(n_1\theta)M_{p+1_{n_2}}(r)\cos(n_2\theta)]$$

Ολοκληρώνοντας ως προς θ, στο 0 – 2π, την παραπάνω σχέση, έχουμε:

$$\int_0^{2\pi} \sum_{\substack{n=0\\ n_1=n_2=n}}^{\infty}[M_{p_n}(r)M_{p+1_n}(r)\cos(n\theta)^2]d\theta +$$

$$\int_0^{2\pi} \sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n_1}}(r)\cos(n_1\theta)M_{p+1_{n_2}}(r)\cos(n_2\theta)]d\theta =$$

$$= \sum_{\substack{n=1\\ n_1=n_2=n}}^{\infty}[M_{p_n}(r)M_{p+1_n}(r)\pi + (M_{p_0}(r)M_{p+1_0}(r)) +$$

$$\sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n1}}(r)M_{p+1_{n_2}}(r)\int_0^{2\pi}[\frac{1}{2}\cos(n_1-n_2)\theta + \frac{1}{2}\cos(n_1+n_2)\theta]d\theta =$$

$$= \sum_{\substack{n=1\\ n_1=n_2=n}}^{\infty}[M_{p_n}(r)M_{p+1_n}(r)\pi + (M_{p_0}(r)M_{p+1_0}(r))$$

Αν θέλουμε να ολοκληρώσουμε ως προς z στο $d_2 - d_1$, τότε

$$\int_{d_2}^{d_1} \sum_{\substack{n=0\\ n_1=n_2=n}}^{\infty}[M_{p_n}(r)M_{p+1_n}(r)\cos(nz)^2]dz +$$

$$\int_{d_2}^{d_1} \sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n1}}(r)\cos(n_1 z)M_{p+1_{n_2}}(r)\cos(n_2 z)]dz =$$

$$= \sum_{\substack{n=1\\ n_1=n_2=n}}^{\infty}[M_{p_n}(r)M_{p+1_n}(r)[\frac{2n(d_1-d_2)+\sin(2nd_1)-\sin(2nd_2)}{4n}] + (M_{p_0}(r)M_{p+1_0}(r)) +$$

$$\sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n1}}(r)M_{p+1_{n_2}}(r)\int_{d_2}^{d_1}[\frac{1}{2}\cos(n_1-n_2)z+\frac{1}{2}\cos(n_1+n_2)z]dz=$$

$$=\sum_{\substack{n=1\\ n_1=n_2=n}}^{\infty}[M_{p_n}(r)M_{p+1_n}(r)[\frac{2n(d_1-d_2)+\sin(2nd_1)-\sin(2nd_2)}{4n}]+(M_{p_0}(r)M_{p+1_0}(r))+$$

$$\sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n1}}(r)M_{p+1_{n_2}}(r)[\frac{1}{2}\frac{\sin((n_1+n_2)d_1)-\sin((n_1+n_2)d_2)}{(n_1+n_2)}+$$

$$+\frac{1}{2}\frac{\sin((n_1-n_2)d_1)-\sin((n_1-n_2)d_2)}{(n_1-n_2)}]]$$

Γ) Ισχύει ότι

$$[\sum_{n_1=0}^{\infty} M_{p_{n_1}}(r)\sin(n_1\theta)\ \sum_{n_2=0}^{\infty} M_{p+1_{n_2}}(r)\sin(n_2\theta)] =$$

$$= \sum_{\substack{n=0 \\ n_1=n_2=n}}^{\infty} [M_{p_n}(r) M_{p+1_n}(r)\sin(n\theta)^2] + \sum_{\substack{n_1=0 \\ n_1\neq n_2}}^{\infty} \sum_{n_2=0}^{\infty} [M_{p_{n1}}(r)\sin(n_1\theta) M_{p+1_{n_2}}(r)\sin(n_2\theta)]$$

Ολοκληρώνοντας ως προς θ, στο 0 – 2π, την παραπάνω σχέση, έχουμε:

$$\int_0^{2\pi} \sum_{\substack{n=0 \\ n_1=n_2=n}}^{\infty} [M_{p_n}(r) M_{p+1_n}(r)\sin(n\theta)^2] d\theta +$$

$$\int_0^{2\pi} \sum_{\substack{n_1=0 \\ n_1\neq n_2}}^{\infty} \sum_{n_2=0}^{\infty} [M_{p_{n_1}}(r)\sin(n_1\theta) M_{p+1_{n_2}}(r)\sin(n_2\theta)] d\theta =$$

$$= \sum_{\substack{n=1 \\ n_1=n_2=n}}^{\infty} [M_{p_n}(r) M_{p+1_n}(r)\pi +$$

$$\sum_{\substack{n_1=0 \\ n_1\neq n_2}}^{\infty} \sum_{n_2=0}^{\infty} [M_{p_{n1}}(r) M_{p+1_{n_2}}(r) \int_0^{2\pi} [\frac{1}{2}\cos(n_1-n_2)\theta - \frac{1}{2}\cos(n_1+n_2)\theta] d\theta =$$

$$= \sum_{\substack{n=1 \\ n_1=n_2=n}}^{\infty} [M_{p_n}(r) M_{p+1_n}(r)\pi$$

Αν θέλουμε να ολοκληρώσουμε ως προς z στο $d_2 - d_1$, τότε

$$\int_{d_2}^{d_1} \sum_{\substack{n=0 \\ n_1=n_2=n}}^{\infty} [M_{p_n}(r) M_{p+1_n}(r)\sin(nz)^2] dz +$$

$$\int_{d_2}^{d_1} \sum_{\substack{n_1=0 \\ n_1\neq n_2}}^{\infty} \sum_{n_2=0}^{\infty} [M_{p_{n_1}}(r)\sin(n_1 z) M_{p+1_{n_2}}(r)\sin(n_2 z)] dz =$$

$$= \sum_{\substack{n=1 \\ n_1=n_2=n}}^{\infty} [M_{p_n}(r) M_{p+1_n}(r) [\frac{2n(d_1-d_2) - \sin(2nd_1) + \sin(2nd_2)}{4n}] +$$

$$\sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n1}}(r)M_{p+1_{n_2}}(r)\int_{d_2}^{d_1}[\frac{1}{2}\cos(n_1-n_2)z-\frac{1}{2}\cos(n_1+n_2)z]dz=$$

$$=\sum_{\substack{n=1\\ n_1=n_2=n}}^{\infty}[M_{p_n}(r)M_{p+1_n}(r)[\frac{2n(d_1-d_2)-\sin(2nd_1)+\sin(2nd_2)}{4n}]+$$

$$\sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n1}}(r)M_{p+1_{n_2}}(r)[\frac{1}{2}\frac{\sin((n_1+n_2)d_1)-\sin((n_1+n_2)d_2)}{(n_1+n_2)}-$$

$$-\frac{1}{2}\frac{\sin((n_1-n_2)d_1)-\sin((n_1-n_2)d_2)}{(n_1-n_2)}]]$$

ΠΑΡΑΡΤΗΜΑ Β

Ισχύει ότι

$$[\sum_{n_1=0}^{\infty} M_{p_{n_1}}(r)\sin(n_1\theta)\ \sum_{n_2=0}^{\infty} M_{p+1_{n_2}}(r)\sin(n_2\theta)]\cos\theta =$$

$$= \sum_{\substack{n=0\\ n_1=n_2=n}}^{\infty} [M_{p_n}(r)M_{p+1_n}(r)\sin(n\theta)^2\cos\theta] +$$

$$\sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty} \sum_{n_2=0}^{\infty} [M_{p_{n1}}(r)\sin(n_1\theta)M_{p+1_{n_2}}(r)\sin(n_2\theta)]\cos\theta$$

Ολοκληρώνοντας ως προς θ, στο 0 – 2π, την παραπάνω σχέση, έχουμε:

$$\int_0^{2\pi} \sum_{\substack{n=0\\ n_1=n_2=n}}^{\infty} [M_{p_n}(r)M_{p+1_n}(r)\sin(n\theta)^2]\cos\theta d\theta +$$

$$\int_0^{2\pi} \sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty} \sum_{n_2=0}^{\infty} [M_{p_{n_1}}(r)\sin(n_1\theta)M_{p+1_{n_2}}(r)\sin(n_2\theta)]\cos\theta d\theta$$

Όμως ισχύει ότι

$$\int_0^{2\pi} \sin(n\theta)\sin(n\theta)\cos\theta dz = \frac{n\sin(4n\pi)}{-1+4n^2} = 0$$

$$\int_0^{2\pi} \sin(m\theta)\sin(n\theta)\cos\theta dz = \frac{1}{4}(\frac{2(m-n)\sin(2\pi(n-m))}{-1+(m+n)^2} - \frac{2(n+m)\sin(2\pi(m+n))}{(-1+m+n)(1+m+n)})$$

Για το δεύτερο ολοκλήρωμα οι αριθμητές είναι πάντα μηδέν.

Διακρίνουμε τις παρακάτω περιπτώσεις για το που μηδενίζονται οι παρονομαστές.

$m-n=-1$ ή $m-n=1$ δηλ. $m=-1+n$, $m=1+n$

$-1+m+n=0$ ή $1+m+n=0$ δηλ. $m=1-n$, $m=-1-n$

Δηλαδή

	$m=-1+n$	$m=1+n$	$m=1-n$	$m=-1-n$
n=0	m<0	m=1	m=1	m<0

n=1	m=0	m=2	m=0	m<0
n=2	m=1	m=3	m<0	m<0
n=3	m=2	m=4	m<0	m<0
n=4	m=3	m=5	m<0	m<0

Παρατηρούμε πως για n>1 το m παίρνει τιμές $m=-1+n\,, m=1+n$.

Όταν n=0 και m=1, το άθροισμα γίνεται

$$\int_0^{2\pi}\sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n_1}}(r)\sin(n_1\theta)M_{p+1_{n_2}}(r)\sin(n_2\theta)]\cos\theta d\theta=M_{p_0}(r)M_{p+1_1}(r)\pi$$

Όταν n=1 και m=0, το άθροισμα γίνεται

$$\int_0^{2\pi}\sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n_1}}(r)\sin(n_1\theta)M_{p+1_{n_2}}(r)\sin(n_2\theta)]\cos\theta d\theta=M_{p_1}(r)M_{p+1_0}(r)\pi$$

Όταν n=1 και m=2, το άθροισμα γίνεται

$$\int_0^{2\pi}\sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n_1}}(r)\sin(n_1\theta)M_{p+1_{n_2}}(r)\sin(n_2\theta)]\cos\theta d\theta=M_{p_1}(r)M_{p+1_2}(r)\pi$$

Όταν n>1, το άθροισμα γίνεται

$$\int_0^{2\pi}\sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}\sum_{n_2=0}^{\infty}[M_{p_{n_1}}(r)\sin(n_1\theta)M_{p+1_{n_2}}(r)\sin(n_2\theta)]\cos\theta d\theta=$$

$$\int_0^{2\pi}\sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}[M_{p_{n_1}}(r)\sin(n_1\theta)M_{p+1_{n_1+1}}(r)\sin((n_1+1)\theta)]\cos\theta d\theta=$$

$$=\sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}[M_{p_{n_1}}(r)M_{p+1_{n_1+1}}(r)\frac{1}{8}(4\pi-(\frac{1}{n_1}+\frac{1}{1+n_1})\sin(4n_1\pi))]=$$

$$=\sum_{\substack{n_1=0\\ n_1\neq n_2}}^{\infty}[M_{p_{n_1}}(r)M_{p+1_{n_1+1}}(r)\frac{\pi}{2}$$

ΠΑΡΑΡΤΗΜΑ Γ

ΠΕΔΙΟ (ΙΙ)

$$\underline{M_m(r,z)} = \underline{\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r)} \underline{+\Lambda^*{}_{m_{a_\ell}}\Re^*{}_{m_{a_\ell}}(r)]} \underline{N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell)) =}$$

$$=[\sum_{\substack{\ell=0\\a_\ell}}^{\infty}\left[\frac{\Lambda_{m_{a_\ell}}I_m(a_\ell r)K_m(a_\ell \mathrm{a}_\ell)-\Lambda_{m_{a_\ell}}K_m(a_\ell r)I_m(a_\ell \mathrm{a}_\ell)}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell)-I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]+$$

$$+\sum_{\substack{\ell=0\\a_\ell}}^{\infty}\left[\frac{\Lambda^*{}_{m_{a_\ell}}I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell r)-\Lambda^*{}_{m_{a_\ell}}K_m(a_\ell \mathrm{a}_{\ell+1})I_m(a_\ell r)}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell)-I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]]\, N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell)) =$$

$$=\sum_{\substack{\ell=0\\a_\ell}}^{\infty}\left[\frac{\Lambda_{m_{a_\ell}}K_m(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}-\Lambda^*{}_{m_{a_\ell}}K_m(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell)-I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$I_m(a_\ell r)+$$

$$+\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{-\Lambda_{m_{a_\ell}}I_m(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}+\Lambda^*{}_{m_{a_\ell}}I_m(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell)-I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$K_m(a_\ell r).$$

$$\underline{\frac{\partial M_m(r,z)}{\partial r}} = \underline{\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}} \frac{\partial \Re_{m_{a_\ell}}(r)}{\partial r}} \underline{\pm \Lambda^*_{m_{a_\ell}} \frac{\partial \Re^*_{m_{a_\ell}}(r)}{\partial r}]} \underline{N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))] =}$$

$$= [\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}[\, a_\ell \frac{K_m(a_\ell \mathrm{a}_\ell)I_{m+1}(a_\ell r) + K_{m+1}(a_\ell r)I_m(a_\ell \mathrm{a}_\ell)}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(\kappa_\ell \mathrm{a}_{\ell+1})} +$$

$$+ \frac{m}{r}\frac{I_m(a_\ell r)K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell r)}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}]] +$$

$$+ \sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda^*_{m_{a_\ell}}[\, -a_\ell \frac{I_m(a_\ell \mathrm{a}_{\ell+1})K_{m+1}(a_\ell r) + K_m(a_\ell \mathrm{a}_{\ell+1})I_m(a_\ell r)}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_{\ell+1})K_m(\kappa_\ell \mathrm{a}_\ell)} +$$

$$+ \frac{m}{r}\frac{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell r) - I_m(a_\ell r)K_m(a_\ell \mathrm{a}_{\ell+1})}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}]]]\; N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))] =$$

$$= \sum_{\substack{\ell=0\\a_\ell}}^{\infty}\left[\frac{\Lambda_{m_{a_\ell}}[a_\ell K_m(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}] + \Lambda^*_{m_{a_\ell}}[a_\ell K_m(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$I_{m+1}(a_\ell r) +$$

$$+ \sum_{\substack{\ell=0\\a_\ell}}^{\infty}\left[\frac{\Lambda_{m_{a_\ell}}[a_\ell I_m(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}] - \Lambda^*_{m_{a_\ell}}[a_\ell I_m(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$K_{m+1}(a_\ell r) +$$

$$+ \sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{\Lambda_{m_{a_\ell}}[mK_m(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}] - \Lambda^*_{m_{a_\ell}}[mK_m(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$I_m(a_\ell r)\frac{1}{r} +$$

$$+ \sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{\Lambda_{m_{a_\ell}}[-mI_m(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}] + \Lambda^*_{m_{a_\ell}}[mI_m(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$K_m(a_\ell r)\frac{1}{r}$$

$$\frac{\partial M_m(r,z)}{\partial z} = \sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r) + \Lambda^*{}_{m_{a_\ell}}\Re^*{}_{m_{a_\ell}}(r)]\, N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell =$$

$$= \sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{\Lambda_{m_{a_\ell}}[K_m(a_\ell \mathrm{a}_\ell)\sin(a_\ell(z-h_\ell))a_\ell \mathrm{N}_{a_\ell}^{-1/2}] - \Lambda^*{}_{m_{a_\ell}}[K_m(a_\ell \mathrm{a}_{\ell+1})\sin(a_\ell(z-h_\ell))a_\ell \mathrm{N}_{a_\ell}^{-1/2}]}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$I_m(a_\ell r) -$$

$$-\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{\Lambda_{m_{a_\ell}}[-I_m(a_\ell \mathrm{a}_\ell)\sin(a_\ell(z-h_\ell))a_\ell \mathrm{N}_{a_\ell}^{-1/2}] + \Lambda^*{}_{m_{a_\ell}}[I_m(a_\ell \mathrm{a}_{\ell+1})\sin(a_\ell(z-h_\ell))a_\ell \mathrm{N}_{a_\ell}^{-1/2}}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$K_m(a_\ell r)$$

$$M_m(r,z)\,\overline{M_m(r,z)} = [\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r) + \Lambda^*_{m_{a_\ell}}\Re^*_{m_{a_\ell}}(r)]\,N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]$$

$$[\sum_{\substack{n=0\\a_n}}^{\infty}[\overline{\Lambda_{m_{a_n}}\Re_{m_{a_n}}(r)}\;\overline{+\Lambda^*_{m_{a_n}}\Re^*_{m_{a_n}}(r)]}\;\overline{N_{a_n}^{-1/2}\cos(a_n(z-h_\ell))]}.$$

Το οποίο σύμφωνα με τη σελίδα 273 γίνεται

$$M_m(r,z)\,\overline{M_m(r,z)} = \left[\Gamma_{ma_\ell} I_m(a_\ell r) + \Delta_{ma_\ell} K_m(a_\ell r)\right]\left[\overline{\Gamma_{ma_\ell}}\,\overline{I_m(a_n r)} + \overline{\Delta_{ma_\ell}}\,\overline{K_m(a_n r)}\right] =$$

$$= \Gamma_{ma_\ell}\overline{\Gamma_{ma_n}} I_m(a_\ell r)\overline{I_m(a_n r)} + \Gamma_{ma_\ell}\overline{\Delta_{ma_n}} I_m(a_\ell r)\overline{K_m(a_n r)} + \Delta_{ma_\ell}\overline{\Gamma_{ma_n}} K_m(a_\ell r)\overline{I_m(a_n r)} +$$

$$\Delta_{ma_\ell}\overline{\Delta_{ma_n}} K_m(a_\ell r)\overline{K_m(a_n r)}$$

Όπου

$$\Gamma_{ma_\ell} = \sum_{\substack{\ell=0\\a_\ell}}^{\infty}\left[\frac{\Lambda_{m_{a_\ell}} K_m(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2} - \Lambda^*_{m_{a_\ell}} K_m(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$\Delta_{ma_\ell} = \sum_{\substack{\ell=0\\a_\ell}}^{\infty}\left[\frac{-\Lambda_{m_{a_\ell}} I_m(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2} + \Lambda^*_{m_{a_\ell}} I_m(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]$$

Και

$$\overline{\Gamma_{ma_n}} = \sum_{\substack{n=0\\a_n}}^{\infty}\overline{\left[\frac{\Lambda_{m_{a_n}} K_m(a_n \mathrm{a}_\ell)\cos(a_n(z-h_\ell))\mathrm{N}_{a_n}^{-1/2} - \Lambda^*_{m_{a_n}} K_m(a_n \mathrm{a}_{\ell+1})\cos(a_n(z-h_\ell))\mathrm{N}_{a_n}^{-1/2}}{I_m(a_n \mathrm{a}_{\ell+1})K_m(a_n \mathrm{a}_\ell) - I_m(a_n \mathrm{a}_\ell)K_m(a_n \mathrm{a}_{\ell+1})}\right]}$$

$$\overline{\Delta_{ma_n}} = \sum_{\substack{n=0\\a_n}}^{\infty}\left[\frac{\overline{-\Lambda_{m_{a_n}} I_m(a_n \mathrm{a}_\ell)\cos(a_n(z-h_\ell))\mathrm{N}_{a_n}^{-1/2} + \Lambda^*_{m_{a_n}} I_m(a_n \mathrm{a}_{\ell+1})\cos(a_n(z-h_\ell))\mathrm{N}_{a_n}^{-1/2}}}{I_m(a_n \mathrm{a}_{\ell+1})K_m(a_n \mathrm{a}_\ell) - I_m(a_n \mathrm{a}_\ell)K_m(a_n \mathrm{a}_{\ell+1})}\right]$$

$$\frac{\partial M_m(r,z)}{\partial z}\overline{\frac{\partial M_m(r,z)}{\partial z}} = [\sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}}\Re_{m_{a_\ell}}(r) + \Lambda^*{}_{m_{a_\ell}}\Re^*{}_{m_{a_\ell}}(r)]\ N_{a_\ell}^{-1/2}\sin(a_\ell(z-h_\ell))a_\ell]$$

$$\sum_{\substack{n=0\\a_n}}^{\infty}[\overline{\Lambda_{m_{a_n}}\Re_{m_{a_n}}(r)}\ \overline{+\Lambda^*{}_{m_{a_n}}\Re^*{}_{m_{a_n}}(r)]}\ \overline{N_{a_n}^{-1/2}\sin(a_n(z-h_\ell))a_n]}$$

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$$\frac{\partial M_m(r,z)}{\partial z}\overline{\frac{\partial M_m(r,z)}{\partial z}} = \left[\Gamma_{ma_\ell z}I_m(a_\ell r) + \Delta_{ma_\ell z}K_m(a_\ell r)\right]\left[\overline{\Gamma_{ma_n z}}\,\overline{I_m(a_n r)} + \overline{\Delta_{ma_n z}}\,\overline{K_m(a_n r)}\right] =$$

$$= \Gamma_{ma_\ell z}\overline{\Gamma_{ma_n z}}I_m(a_\ell r)\overline{I_m(a_n r)} + \Gamma_{ma_\ell z}\overline{\Delta_{ma_n z}}I_m(a_\ell r)\overline{K_m(a_n r)} + \Delta_{ma_\ell z}\overline{\Gamma_{ma_n z}}K_m(a_\ell r)\overline{I_m(a_n r)} +$$

$$\Delta_{ma_\ell z}\overline{\Delta_{ma_\ell z}}K_m(a_\ell r)\overline{K_m(a_n r)}$$

Όπου

$$\Gamma_{ma_\ell z} = \sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{\Lambda_{m_{a_\ell}}[K_m(a_\ell \mathrm{a}_\ell)\sin(a_\ell(z-h_\ell))a_\ell \mathrm{N}_{a_\ell}^{-1/2}] - \Lambda^*{}_{m_{a_\ell}}[K_m(a_\ell \mathrm{a}_{\ell+1})\sin(a_\ell(z-h_\ell))a_\ell \mathrm{N}_{a_\ell}^{-1/2}]}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$\Delta_{ma_\ell z} = \sum_{\substack{\ell=0\\a_\ell}}^{\infty}[\left[\frac{\Lambda_{m_{a_\ell}}[-I_m(a_\ell \mathrm{a}_\ell)\sin(a_\ell(z-h_\ell))a_\ell \mathrm{N}_{a_\ell}^{-1/2}] + \Lambda^*{}_{m_{a_\ell}}[I_m(a_\ell \mathrm{a}_{\ell+1})\sin(a_\ell(z-h_\ell))a_\ell \mathrm{N}_{a_\ell}^{-1/2}}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]$$

Και

$$\overline{\Gamma_{ma_n z}} = \sum_{\substack{n=0\\a_n}}^{\infty}[\overline{\left[\frac{\Lambda_{m_{a_n}}[K_m(a_n \mathrm{a}_\ell)\sin(a_n(z-h_\ell))a_n \mathrm{N}_{a_n}^{-1/2}] - \Lambda^*{}_{m_{a_n}}[K_m(a_n \mathrm{a}_{\ell+1})\sin(a_n(z-h_\ell))a_n \mathrm{N}_{a_n}^{-1/2}]}{I_m(a_n \mathrm{a}_{\ell+1})K_m(a_n \mathrm{a}_\ell) - I_m(a_n \mathrm{a}_\ell)K_m(a_n \mathrm{a}_{\ell+1})}\right]}$$

$$\overline{\Delta_{ma_n z}} = \sum_{\substack{n=0\\a_n}}^{\infty}[\overline{\left[\frac{\Lambda_{m_{a_n}}[-I_m(a_n \mathrm{a}_\ell)\sin(a_n(z-h_\ell))a_n \mathrm{N}_{a_n}^{-1/2}] + \Lambda^*{}_{m_{a_n}}[I_m(a_n \mathrm{a}_{\ell+1})\sin(a_n(z-h_\ell))a_n \mathrm{N}_{a_n}^{-1/2}}{I_m(a_n \mathrm{a}_{\ell+1})K_m(a_n \mathrm{a}_\ell) - I_m(a_n \mathrm{a}_\ell)K_m(a_n \mathrm{a}_{\ell+1})}\right]}$$

$$\frac{\partial M_m(r,z)}{\partial r}\overline{\frac{\partial M_m(r,z)}{\partial r}} = [\sum_{\substack{\ell=0\\ a_\ell}}^{\infty}[\Lambda_{m_{a_\ell}} \frac{\partial \Re_{m_{a_\ell}}(r)}{\partial r} + \Lambda^*_{m_{a_\ell}} \frac{\partial \Re^*_{m_{a_\ell}}(r)}{\partial r}]\ N_{a_\ell}^{-1/2}\cos(a_\ell(z-h_\ell))]$$

$$\sum_{\substack{n=0\\ a_n}}^{\infty}[\overline{\Lambda_{m_{a_n}} \frac{\partial \Re_{m_{a_n}}(r)}{\partial r}} + \overline{\Lambda^*_{m_{a_n}} \frac{\partial \Re^*_{m_{a_n}}(r)}{\partial r}}]\ \overline{N_{a_n}^{-1/2}\cos(a_n(z-h_\ell))}]$$

Το οποίο σύμφωνα με τη σελίδα 274 γίνεται

$$\frac{\partial M_m(r,z)}{\partial r}\overline{\frac{\partial M_m(r,z)}{\partial r}} =$$

$$= E_{ma_\ell}\overline{E_{ma_n}} I_{m+1}(a_\ell r)\overline{I_{m+1}(a_n r)} + E_{ma_\ell}\overline{F_{ma_n}} I_{m+1}(a_\ell r)\overline{K_{m+1}(a_n r)} + E_{ma_\ell}\overline{G_{ma_n}} I_{m+1}(a_\ell r)\overline{I_m(a_n r)} +$$

$$E_{ma_\ell}\overline{H_{ma_n}} I_{m+1}(a_\ell r)\overline{K_m(a_n r)} + F_{ma_\ell}\overline{E_{ma_n}} K_{m+1}(a_\ell r)\overline{I_{m+1}(a_n r)} + F_{ma_\ell}\overline{F_{ma_n}} K_{m+1}(a_\ell r)\overline{K_{m+1}(a_n r)} +$$

$$F_{ma_\ell}\overline{G_{ma_n}} K_{m+1}(a_\ell r)\overline{I_m(a_n r)} + F_{ma_\ell}\overline{H_{ma_n}} K_{m+1}(a_\ell r)\overline{K_m(a_n r)} + G_{ma_\ell}\overline{E_{ma_n}} I_m(a_\ell r)\overline{I_{m+1}(a_n r)} +$$

$$G_{ma_\ell}\overline{F_{ma_n}} I_m(a_\ell r)\overline{K_{m+1}(a_n r)} + G_{ma_\ell}\overline{G_{ma_n}} I_m(a_\ell r)\overline{I_m(a_n r)} + G_{ma_\ell}\overline{H_{ma_n}} I_m(a_\ell r)\overline{K_m(a_n r)} +$$

$$+ H_{ma_\ell}\overline{E_{ma_n}} K_m(a_\ell r)\overline{I_{m+1}(a_n r)} + H_{ma_\ell}\overline{F_{ma_n}} K_m(a_\ell r)\overline{K_{m+1}(a_n r)} + H_{ma_\ell}\overline{G_{ma_n}} K_m(a_\ell r)\overline{I_m(a_n r)} +$$

$$H_{ma_\ell}\overline{H_{ma_n}} K_m(a_\ell r)\overline{K_m(a_n r)}$$

Όπου

$$E_{ma_\ell} =$$

$$= \sum_{\substack{\ell=0\\ a_\ell}}^{\infty}\left[\frac{\Lambda_{m_{a_\ell}}[a_\ell K_m(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}] + \Lambda^*_{m_{a_\ell}}[a_\ell K_m(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$F_{ma_\ell} =$$

$$= \sum_{\substack{\ell=0\\ a_\ell}}^{\infty}\left[\frac{\Lambda_{m_{a_\ell}}[a_\ell I_m(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}] - \Lambda^*_{m_{a_\ell}}[a_\ell I_m(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})}\right]$$

$$G_{ma_\ell} =$$

$$= \sum_{\substack{\ell=0 \\ a_\ell}}^{\infty} [\left[\frac{\Lambda_{m_{a_\ell}} [mK_m(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}] - \Lambda^*{}_{m_{a_\ell}} [mK_m(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}]}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})} \right]$$

$$H_{ma_\ell} =$$

$$= \sum_{\substack{\ell=0 \\ a_\ell}}^{\infty} [\left[\frac{\Lambda_{m_{a_\ell}} [-mI_m(a_\ell \mathrm{a}_\ell)\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}] + \Lambda^*{}_{m_{a_\ell}} [mI_m(a_\ell \mathrm{a}_{\ell+1})\cos(a_\ell(z-h_\ell))\mathrm{N}_{a_\ell}^{-1/2}}{I_m(a_\ell \mathrm{a}_{\ell+1})K_m(a_\ell \mathrm{a}_\ell) - I_m(a_\ell \mathrm{a}_\ell)K_m(a_\ell \mathrm{a}_{\ell+1})} \right]$$

ΠΑΡΑΡΤΗΜΑ Δ

ΠΕΔΙΟ (III)

$$\underline{M_m(r,z)} = \underline{\sum_{n_p=0}^{\infty} \in_{n_p} [\Lambda_{m_{n_p}} \Re_{m_{n_p}}(r)} \underline{+\Lambda^*_{m_{n_p}} \Re^*_{m_{n_p}}(r)]} \underline{\cos(\frac{n_p \pi z}{h_p}) =}$$

$$= [\sum_{n_p=0} \left[\frac{\in_{n_p} \Lambda_{mn_p} K_m(\frac{n_p \pi b_p}{h_p}) I_m(\frac{n_p \pi r}{h_p}) - \in_{n_p} \Lambda_{mn_p} I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi r}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right] +$$

$$\left[\frac{\Lambda^*_{mn_p} I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi r}{h_p}) - \Lambda^*_{mn_p} K_m(\frac{n_p \pi b_{p+1}}{h_p}) I_m(\frac{n_p \pi r}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right]]\cos(\frac{n_p \pi r}{h_p}) =$$

$$= \sum_{n_p=0} \left[\frac{\in_{n_p} \Lambda_{mn_p} K_m(\frac{n_p \pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{mn_p} K_m(\frac{n_p \pi b_{p+1}}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right] \cos(\frac{n_p \pi z}{h_p}) I_m(\frac{n_p \pi r}{h_p}) +$$

$$\sum_{n_p=0} \left[\frac{\in_{n_p} \Lambda^*_{mn_p} I_m(\frac{n_p \pi b_{p+1}}{h_p}) - \in_{n_p} \Lambda_{mn_p} I_m(\frac{n_p \pi b_p}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right] \cos(\frac{n_p \pi z}{h_p}) K_m(\frac{n_p \pi r}{h_p})$$

$$\frac{\partial M_m(r,z)}{\partial r} = \sum_{n_p=0}^{\infty} \in_{n_p} [\Lambda_{m_{n_p}} \frac{\partial \Re_{m_{n_p}}(r)}{\partial r} + \Lambda^*_{m_{n_p}} \frac{\partial \Re^*_{m_{n_p}}(r)}{\partial r}] \cos(\frac{n_p \pi z}{h_p}) =$$

$$= \sum_{n_p=0} \in_{n_p} [\Lambda_{mn_p} \frac{n_p \pi}{h_p} \left[\frac{K_m(\frac{n_p \pi b_p}{h_p}) I_{m+1}(\frac{n_p \pi r}{h_p}) + I_m(\frac{n_p \pi b_p}{h_p}) K_{m+1}(\frac{n_p \pi r}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right] +$$

$$\Lambda_{mn_p} \frac{m}{r} \left[\frac{K_m(\frac{n_p \pi b_p}{h_p}) I_m(\frac{n_p \pi r}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi r}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right] +$$

$$\Lambda^*_{mn_p} \frac{-n_p \pi}{h_p} \left[\frac{I_m(\frac{n_p \pi b_p}{h_p}) K_{m+1}(\frac{n_p \pi r}{h_p}) + K_m(\frac{n_p \pi b_{p+1}}{h_p}) I_{m+1}(\frac{n_p \pi r}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right] +$$

$$\Lambda^*_{mn_p} \frac{m}{r} \left[\frac{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi r}{h_p}) - K_m(\frac{n_p \pi b_{p+1}}{h_p}) I_m(\frac{n_p \pi r}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right]] \cos(\frac{n_p \pi z}{h_p}) =$$

$$= \sum_{n_p=0} \left[\frac{\in_{n_p} \Lambda_{mn_p} K_m(\frac{n_p \pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{mn_p} K_m(\frac{n_p \pi b_{p+1}}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right] \cos(\frac{n_p \pi z}{h_p}) \frac{n_p \pi}{h_p} I_m(\frac{n_p \pi r}{h_p}) +$$

$$\sum_{n_p=0} \left[\frac{\in_{n_p} \Lambda_{mn_p} I_m(\frac{n_p \pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{mn_p} I_m(\frac{n_p \pi b_{p+1}}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right] \cos(\frac{n_p \pi z}{h_p}) \frac{n_p \pi}{h_p} K_{m+1}(\frac{n_p \pi r}{h_p}) +$$

$$\sum_{n_p=0} \left[\frac{\in_{n_p} \Lambda_{mn_p} m K_m(\frac{n_p \pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{mn_p} m K_m(\frac{n_p \pi b_{p+1}}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right] \cos(\frac{n_p \pi z}{h_p}) \frac{n_p \pi}{h_p} I_m(\frac{n_p \pi r}{h_p}) \frac{1}{r} +$$

$$\sum_{n_p=0}\left[\frac{-m\in_{n_p}\Lambda_{mn_p}I_m(\frac{n_p\pi b_p}{h_p})+m\in_{n_p}\Lambda^*_{mn_p}I_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p})-I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\cos(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}K_m(\frac{n_p\pi r}{h_p})\frac{1}{r}$$

$$\frac{\partial M_m(r,z)}{\partial z} = \sum_{n_p=0}^{\infty} \in_{n_p} [\Lambda_{m_{n_p}} \Re_{m_{n_p}}(r) + \Lambda^*_{m_{n_p}} \Re^*_{m_{n_p}}(r)] \sin(\frac{n_p \pi z}{h_p}) \frac{n_p \pi}{h_p} =$$

$$= -\sum_{n_p=1} \left[\frac{\in_{n_p} \Lambda_{mn_p} K_m(\frac{n_p \pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{mn_p} K_m(\frac{n_p \pi b_{p+1}}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right] \sin(\frac{n_p \pi z}{h_p}) \frac{n_p \pi}{h_p} I_m(\frac{n_p \pi r}{h_p}) -$$

$$-\sum_{n_p=1} \left[\frac{\in_{n_p} \Lambda^*_{mn_p} I_m(\frac{n_p \pi b_{p+1}}{h_p}) - \in_{n_p} \Lambda_{mn_p} I_m(\frac{n_p \pi b_p}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right] \sin(\frac{n_p \pi z}{h_p}) \frac{n_p \pi}{h_p} K_m(\frac{n_p \pi r}{h_p})$$

$$\underline{M_m(r,z)\ \overline{M_m(r,z)}} = [\underline{\sum_{n_p=0}^{\infty} \in_{n_p} [\Lambda_{m_{n_p}} \Re_{m_{n_p}}(r)} \underline{+ \Lambda^*_{m_{n_p}} \Re^*_{m_{n_p}}(r)]} \underline{\cos(\frac{n_p \pi z}{h_p})]}$$

$$\underline{[\sum_{n_q=0}^{\infty} \overline{\in_{n_q} [\Lambda_{m_{n_q}} \Re_{m_{n_q}}(r)}} \underline{\overline{+ \Lambda^*_{m_{n_q}} \Re^*_{m_{n_q}}(r)]}} \underline{\overline{\cos(\frac{n_q \pi z}{h_p})}]}$$

Το οποίο σύμφωνα με τη σελίδα 280 γίνεται

$$M_m(r,z)\ \overline{M_m(r,z)} = [\sum_{n_p=0}^{\infty} A_{mn_p} I_m(\frac{n_p \pi r}{h_p}) + \sum_{n_p=0}^{\infty} B_{mn_p} K_m(\frac{n_p \pi r}{h_p})] [\sum_{n_q=0}^{\infty} \overline{A_{mn_q}} I_m(\overline{\frac{n_q \pi r}{h_p}}) +$$

$$\sum_{n_q=0}^{\infty} \overline{B_{mn_q}} K_m(\overline{\frac{n_q \pi r}{h_p}})] =$$

$$= \sum_{n_p=0}^{\infty} A_{mn_p} I_m(\frac{n_p \pi r}{h_p}) \sum_{n_q=0}^{\infty} \overline{A_{mn_q}} I_m(\overline{\frac{n_q \pi r}{h_p}}) + \sum_{n_p=0}^{\infty} A_{mn_p} I_m(\frac{n_p \pi r}{h_p}) \sum_{n_q=0}^{\infty} \overline{B_{mn_q}} K_m(\overline{\frac{n_q \pi r}{h_p}}) +$$

$$\sum_{n_p=0}^{\infty} B_{mn_p} K_m(\frac{n_p \pi r}{h_p}) \sum_{n_q=0}^{\infty} \overline{A_{mn_q}} I_m(\overline{\frac{n_q \pi r}{h_p}}) + \sum_{n_p=0}^{\infty} B_{mn_p} K_m(\frac{n_p \pi r}{h_p}) \sum_{n_q=0}^{\infty} \overline{B_{mn_p}} K_m(\overline{\frac{n_q \pi r}{h_p}}) =$$

$$= \sum_{n_p=0}^{\infty} A_{mn_p} \overline{A_{mn_p}} I_m(\frac{n_p \pi r}{h_p}) \overline{I_m(\frac{n_p \pi r}{h_p})} + \sum_{\substack{n_p=0 \\ n_p \neq n_q}}^{\infty} \sum_{n_q=0}^{\infty} A_{mn_p} \overline{A_{mn_q}} I_m(\frac{n_p \pi r}{h_p}) \overline{I_m(\frac{n_q \pi r}{h_p})} +$$

$$\sum_{n_p=0}^{\infty} A_{mn_p} \overline{B_{mn_p}} I_m(\frac{n_p \pi r}{h_p}) \overline{K_m(\frac{n_p \pi r}{h_p})} + \sum_{\substack{n_p=0 \\ n_p \neq n_q}}^{\infty} \sum_{n_q=0}^{\infty} A_{mn_p} \overline{B_{mn_q}} I_m(\frac{n_p \pi r}{h_p}) \overline{K_m(\frac{n_q \pi r}{h_p})} +$$

$$\sum_{n_p=0}^{\infty} B_{mn_p} \overline{A_{mn_p}} K_m(\frac{n_p \pi r}{h_p}) \overline{I_m(\frac{n_p \pi r}{h_p})} + \sum_{\substack{n_p=0 \\ n_p \neq n_q}}^{\infty} \sum_{n_q=0}^{\infty} B_{mn_p} \overline{A_{mn_q}} K_m(\frac{n_p \pi r}{h_p}) \overline{I_m(\frac{n_q \pi r}{h_p})} +$$

$$\sum_{n_p=0}^{\infty} B_{mn_p} \overline{B_{mn_p}} K_m(\frac{n_p \pi r}{h_p}) \overline{K_m(\frac{n_p \pi r}{h_p})} + \sum_{\substack{n_p=0 \\ n_p \neq n_q}}^{\infty} \sum_{n_q=0}^{\infty} B_{mn_p} \overline{B_{mn_q}} K_m(\frac{n_p \pi r}{h_p}) \overline{K_m(\frac{n_q \pi r}{h_p})}$$

Όπου

$$A_{nm_p} = \left[\frac{\in_{n_p} \Lambda_{mn_p} K_m(\frac{n_p \pi b_p}{h_p}) - \in_{n_p} \Lambda^*_{mn_p} K_m(\frac{n_p \pi b_{p+1}}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right] \cos(\frac{n_p \pi z}{h_p})$$

$$B_{nm_p} = \left[\frac{- \in_{n_p} \Lambda_{mn_p} I_m(\frac{n_p \pi b_p}{h_p}) + \in_{n_p} \Lambda^*_{mn_p} I_m(\frac{n_p \pi b_{p+1}}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right] \cos(\frac{n_p \pi z}{h_p})$$

Και

$$\overline{A_{nm_q}} = \left[\frac{\in_{n_q} \Lambda_{mn_q} K_m(\frac{n_q \pi b_p}{h_p}) - \in_{n_q} \Lambda^*_{mn_q} K_m(\frac{n_q \pi b_{p+1}}{h_p})}{I_m(\frac{n_q \pi b_{p+1}}{h_p}) K_m(\frac{n_q \pi b_p}{h_p}) - I_m(\frac{n_q \pi b_p}{h_p}) K_m(\frac{n_q \pi b_{p+1}}{h_p})} \right] \cos(\frac{n_q \pi z}{h_p})$$

$$\overline{B_{nm_q}} = \left[\frac{- \in_{n_q} \Lambda_{mn_q} I_m(\frac{n_q \pi b_p}{h_p}) + \in_{n_q} \Lambda^*_{mn_q} I_m(\frac{n_q \pi b_{p+1}}{h_p})}{I_m(\frac{n_q \pi b_{p+1}}{h_p}) K_m(\frac{n_q \pi b_p}{h_p}) - I_m(\frac{n_q \pi b_p}{h_p}) K_m(\frac{n_q \pi b_{p+1}}{h_p})} \right] \cos(\frac{n_q \pi z}{h_p})$$

$$\underline{\frac{\partial M_m(r,z)}{\partial z}\overline{\frac{\partial M_m(r,z)}{\partial z}}} = [\underline{\sum_{n_p=0}^{\infty}\in_{n_p}[\Lambda_{m_{n_p}}\Re_{m_{n_p}}(r)}\ \underline{+\Lambda^*_{m_{n_p}}\Re^*_{m_{n_p}}(r)]}\ \underline{\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}]}$$

$$\underline{[\sum_{n_q=0}^{\infty}\overline{\in_{n_q}}\,\overline{[\Lambda_{m_{n_q}}\Re_{m_{n_q}}(r)}}\ \underline{\overline{+\Lambda^*_{m_{n_q}}\Re^*_{m_{n_q}}(r)]}}\ \underline{\overline{\sin(\frac{n_q\pi z}{h_p})\frac{n_q\pi}{h_p}}]}$$

Το οποίο σύμφωνα με τη σελίδα 283 γίνεται

$$\frac{\partial M_m(r,z)}{\partial z}\overline{\frac{\partial M_m(r,z)}{\partial z}} = [\sum_{n_p=0}^{\infty}A_{mn_p}I_m(\frac{n_p\pi r}{h_p}) + \sum_{n_p=0}^{\infty}B_{mn_p}K_m(\frac{n_p\pi r}{h_p})]\ [\sum_{n_q=0}^{\infty}\overline{A_{mn_q}}I_m\overline{(\frac{n_q\pi r}{h_p})} +$$

$$\sum_{n_q=0}^{\infty}\overline{B_{mn_p}}K_m\overline{(\frac{n_p\pi r}{h_p})}] =$$

$$= \sum_{n_p=0}^{\infty}A_{mn_p}I_m(\frac{n_p\pi r}{h_p})\ \sum_{n_q=0}^{\infty}\overline{A_{mn_q}}I_m\overline{(\frac{n_q\pi r}{h_p})} + \sum_{n_p=0}^{\infty}A_{mn_p}I_m(\frac{n_p\pi r}{h_p})\ \sum_{n_q=0}^{\infty}\overline{B_{mn_q}}K_m\overline{(\frac{n_q\pi r}{h_p})} +$$

$$\sum_{n_p=0}^{\infty}B_{mn_p}K_m(\frac{n_p\pi r}{h_p})\ \sum_{n_q=0}^{\infty}\overline{A_{mn_p}}I_m\overline{(\frac{n_q\pi r}{h_p})} + \sum_{n_p=0}^{\infty}B_{mn_p}K_m(\frac{n_p\pi r}{h_p})\ \sum_{n_q=0}^{\infty}\overline{B_{mn_p}}K_m\overline{(\frac{n_q\pi r}{h_p})} =$$

$$= \sum_{n_p=0}^{\infty}A_{mn_p}\overline{A_{mn_p}}I_m(\frac{n_p\pi r}{h_p})\overline{I_m(\frac{n_p\pi r}{h_p})} + \sum_{\substack{n_p=0\\ n_p\neq n_q}}^{\infty}\sum_{n_q=0}^{\infty}A_{mn_p}\overline{A_{mn_q}}I_m(\frac{n_p\pi r}{h_p})\overline{I_m(\frac{n_q\pi r}{h_p})} +$$

$$\sum_{n_p=0}^{\infty}A_{mn_p}\overline{B_{mn_p}}I_m(\frac{n_p\pi r}{h_p})\overline{K_m(\frac{n_p\pi r}{h_p})} + \sum_{\substack{n_p=0\\ n_p\neq n_q}}^{\infty}\sum_{n_q=0}^{\infty}A_{mn_p}\overline{B_{mn_q}}I_m(\frac{n_p\pi r}{h_p})\overline{K_m(\frac{n_q\pi r}{h_p})} +$$

$$\sum_{n_p=0}^{\infty}B_{mn_p}\overline{A_{mn_p}}K_m(\frac{n_p\pi r}{h_p})\overline{I_m(\frac{n_p\pi r}{h_p})} + \sum_{\substack{n_p=0\\ n_p\neq n_q}}^{\infty}\sum_{n_q=0}^{\infty}B_{mn_p}\overline{A_{mn_q}}K_m(\frac{n_p\pi r}{h_p})\overline{I_m(\frac{n_q\pi r}{h_p})} +$$

$$\sum_{n_p=0}^{\infty}B_{mn_p}\overline{B_{mn_p}}K_m(\frac{n_p\pi r}{h_p})\overline{K_m(\frac{n_p\pi r}{h_p})} + \sum_{\substack{n_p=0\\ n_p\neq n_q}}^{\infty}\sum_{n_q=0}^{\infty}B_{mn_p}\overline{B_{mn_q}}K_m(\frac{n_p\pi r}{h_p})\overline{K_m(\frac{n_q\pi r}{h_p})}$$

Όπου

$$A_{nm_p} = \left[\frac{\in_{n_p}\Lambda_{mn_p}K_m(\frac{n_p\pi b_p}{h_p}) - \in_{n_p}\Lambda^*_{mn_p}K_m(\frac{n_p\pi b_{p+1}}{h_p})}{I_m(\frac{n_p\pi b_{p+1}}{h_p})K_m(\frac{n_p\pi b_p}{h_p}) - I_m(\frac{n_p\pi b_p}{h_p})K_m(\frac{n_p\pi b_{p+1}}{h_p})}\right]\sin(\frac{n_p\pi z}{h_p})\frac{n_p\pi}{h_p}$$

$$B_{nm_p} = \left[\frac{- \in_{n_p} \Lambda_{mn_p} I_m(\frac{n_p \pi b_p}{h_p}) + \in_{n_p} \Lambda^*_{mn_p} I_m(\frac{n_p \pi b_{p+1}}{h_p})}{I_m(\frac{n_p \pi b_{p+1}}{h_p}) K_m(\frac{n_p \pi b_p}{h_p}) - I_m(\frac{n_p \pi b_p}{h_p}) K_m(\frac{n_p \pi b_{p+1}}{h_p})} \right] \sin(\frac{n_p \pi z}{h_p}) \frac{n_p \pi}{h_p}$$

Και

$$\overline{A_{nm_q}} = \left[\overline{\frac{\in_{n_q} \Lambda_{mn_q} K_m(\frac{n_q \pi b_p}{h_p}) - \in_{n_q} \Lambda^*_{mn_q} K_m(\frac{n_q \pi b_{p+1}}{h_p})}{I_m(\frac{n_q \pi b_{p+1}}{h_p}) K_m(\frac{n_q \pi b_p}{h_p}) - I_m(\frac{n_q \pi b_p}{h_p}) K_m(\frac{n_q \pi b_{p+1}}{h_p})}} \right] \sin(\frac{n_q \pi z}{h_p}) \frac{n_q \pi}{h_p}$$

$$\overline{B_{nm_q}} = \left[\overline{\frac{- \in_{n_q} \Lambda_{mn_q} I_m(\frac{n_q \pi b_p}{h_p}) + \in_{n_q} \Lambda^*_{mn_q} I_m(\frac{n_q \pi b_{p+1}}{h_p})}{I_m(\frac{n_q \pi b_{p+1}}{h_p}) K_m(\frac{n_q \pi b_p}{h_p}) - I_m(\frac{n_q \pi b_p}{h_p}) K_m(\frac{n_q \pi b_{p+1}}{h_p})}} \right] \sin(\frac{n_q \pi z}{h_p}) \frac{n_q \pi}{h_p}$$

$$\frac{\partial M_m(r,z)}{\partial r}\,\overline{\frac{\partial M_m(r,z)}{\partial r}} = [\sum_{n_p=0}^{\infty} \in_{n_p} [\Lambda_{m_{n_p}} \frac{\partial \Re_{m_{n_p}}(r)}{\partial r} + \Lambda^{*}_{m_{n_p}} \frac{\partial \Re^{*}_{m_{n_p}}(r)}{\partial r}] \cos(\frac{n_p \pi z}{h_p})]$$

$$[\sum_{n_q=0}^{\infty} \overline{\in_{n_q}} [\overline{\Lambda_{m_{n_q}} \frac{\partial \Re_{m_{n_q}}(r)}{\partial r}} + \overline{\Lambda^{*}_{m_{n_q}} \frac{\partial \Re^{*}_{m_{n_q}}(r)}{\partial r}}] \overline{\cos(\frac{n_q \pi z}{h_p})}]$$

Το οποίο σύμφωνα με τη σελίδα 281 γίνεται

$$\frac{\partial M_m(r,z)}{\partial r}\,\overline{\frac{\partial M_m(r,z)}{\partial r}} = [\sum_{n_p=0}^{\infty} A_{mn_p} I_{m+1}(\frac{n_p \pi r}{h_p}) + B_{mn_p} K_{m+1}(\frac{n_p \pi r}{h_p}) + \Gamma_{mn_p} I_m(\frac{n_p \pi r}{h_p})\frac{1}{r} +$$

$$\Delta_{mn_p} K_m(\frac{n_p \pi r}{h_p})\frac{1}{r}]\,[\sum_{n_q=0}^{\infty} \overline{A_{mn_p}} I_{m+1}\overline{(\frac{n_q \pi r}{h_p})} + \overline{B_{mn_q}}\,\overline{K_{m+1}(\frac{n_q \pi r}{h_p})} + \overline{\Gamma_{mn_q}}\,\overline{I_m(\frac{n_q \pi r}{h_p})}\frac{1}{r} +$$

$$\overline{\Delta_{mn_q}}\,\overline{K_m(\frac{n_q \pi r}{h_p})}\frac{1}{r}] =$$

$$= \sum_{n_p=0}^{\infty} [\, A_{mn_p} \overline{A_{mn_p}} I_{m+1}(\frac{n_p \pi r}{h_p}) \overline{I_{m+1}(\frac{n_p \pi r}{h_p})} + A_{mn_p} \overline{B_{mn_p}} I_{m+1}(\frac{n_p \pi r}{h_p}) \overline{K_{m+1}(\frac{n_p \pi r}{h_p})} +$$

$$A_{mn_p} \overline{\Gamma_{mn_p}} I_{m+1}(\frac{n_p \pi r}{h_p}) \overline{I_m(\frac{n_p \pi r}{h_p})}\frac{1}{r} + A_{mn_p} \overline{\Delta_{mn_p}} I_{m+1}(\frac{n_p \pi r}{h_p}) \overline{K_m(\frac{n_p \pi r}{h_p})}\frac{1}{r} +$$

$$B_{mn_p} \overline{A_{mn_p}} K_{m+1}(\frac{n_p \pi r}{h_p}) \overline{I_{m+1}(\frac{n_p \pi r}{h_p})} + B_{mn_p} \overline{B_{mn_p}} K_{m+1}(\frac{n_p \pi r}{h_p}) \overline{K_{m+1}(\frac{n_p \pi r}{h_p})} +$$

$$B_{mn_p} \overline{\Gamma_{mn_p}} K_{m+1}(\frac{n_p \pi r}{h_p}) \overline{I_m(\frac{n_p \pi r}{h_p})}\frac{1}{r} + B_{mn_p} \overline{\Delta_{mn_p}} K_{m+1}(\frac{n_p \pi r}{h_p}) \overline{K_m(\frac{n_p \pi r}{h_p})}\frac{1}{r} +$$

$$\Gamma_{mn_p} \overline{A_{mn_p}} I_m(\frac{n_p \pi r}{h_p}) \overline{I_{m+1}(\frac{n_p \pi r}{h_p})}\frac{1}{r} + \Gamma_{mn_p} \overline{B_{mn_p}} I_m(\frac{n_p \pi r}{h_p}) \overline{K_{m+1}(\frac{n_p \pi r}{h_p})}\frac{1}{r} +$$

$$\Gamma_{mn_p} \overline{\Gamma_{mn_p}} I_m(\frac{n_p \pi r}{h_p}) \overline{I_m(\frac{n_p \pi r}{h_p})}\frac{1}{r} + \Gamma_{mn_p} \overline{\Delta_{mn_p}} I_m(\frac{n_p \pi r}{h_p}) \overline{K_m(\frac{n_p \pi r}{h_p})}\frac{1}{r} +$$

$$\Delta_{mn_p} \overline{A_{mn_p}} K_m(\frac{n_p \pi r}{h_p}) \overline{I_{m+1}(\frac{n_p \pi r}{h_p})}\frac{1}{r} + \Delta_{mn_p} \overline{B_{mn_p}} K_m(\frac{n_p \pi r}{h_p}) \overline{K_{m+1}(\frac{n_p \pi r}{h_p})}\frac{1}{r} +$$

$$\Delta_{mn_p}\overline{\Gamma_{mn_p}}K_m(\frac{n_p\pi r}{h_p})\overline{I_m(\frac{n_p\pi r}{h_p})}\frac{1}{r}+\Delta_{mn_p}\overline{\Delta_{mn_p}}K_m(\frac{n_p\pi r}{h_p})\overline{K_m(\frac{n_p\pi r}{h_p})}\frac{1}{r}]+$$

$$\sum_{n_p=0}^{\infty}\sum_{\substack{n_q=0\\ n_p\neq n_q}}^{\infty}[\,[\,A_{mn_p}I_{m+1}(\frac{n_p\pi r}{h_p})+B_{mn_p}K_{m+1}(\frac{n_p\pi r}{h_p})+\Gamma_{mn_p}I_m(\frac{n_p\pi r}{h_p})+\Delta_{mn_p}K_m(\frac{n_p\pi r}{h_p})]$$

$$[\overline{A_{mn_q}}\overline{I_{m+1}(\frac{n_q\pi r}{h_p})}+\overline{B_{mn_q}}\overline{K_{m+1}(\frac{n_q\pi r}{h_p})}+\overline{\Gamma_{mn_q}}\overline{I_m(\frac{n_q\pi r}{h_p})}+\overline{\Delta_{mn_q}}\overline{K_m(\frac{n_q\pi r}{h_p})}]]$$

ΠΑΡΑΡΤΗΜΑ Ε

Υπενθυμίζουμε ότι

$$\mathrm{Re}\{ae^{-i\omega_a t}\}\ \mathrm{Re}\{be^{-i\omega_b t}\} =$$

$$= \mathrm{Re}\{(a_r + ia_{im})(\cos(\omega_a t) - i\sin(\omega_a t)\}\ \mathrm{Re}\{(b_r + ib_{im})(\cos(\omega_b t) - i\sin(\omega_b t)\} =$$

$$= (a_r \cos(\omega_a t) + a_{im}\sin(\omega_a t))\ (b_r \cos(\omega_b t) + b_{im}\sin(\omega_b t)) =$$

$$= a_r b_r \cos(\omega_a t)\cos(\omega_b t) + b_r a_{im}\sin(\omega_a t)\cos(\omega_b t) + a_r b_{im}\cos(\omega_a t)\sin(\omega_b t) +$$

$$+ a_{im} b_{im}\sin(\omega_a t)\sin(\omega_b t) =$$

$$= \frac{1}{2}a_r b_r[\cos(\omega_a - \omega_b)t + \cos(\omega_a + \omega_b)t] + \frac{1}{2}a_{im}b_{im}[\cos(\omega_a - \omega_b)t - \cos(\omega_a + \omega_b)t] +$$

$$+ \frac{1}{2}a_r b_{im}[\sin(-\omega_a + \omega_b)t + \sin(\omega_a + \omega_b)t] + \frac{1}{2}b_r a_{im}[\sin(\omega_a - \omega_b)t + \sin(\omega_a + \omega_b)t] =$$

$$= \frac{1}{2}(a_r b_r + a_{im}b_{im})\cos(\omega_a - \omega_b)t + \frac{1}{2}(a_r b_r - a_{im}b_{im})\cos(\omega_a + \omega_b)t +$$

$$+ \frac{1}{2}(b_r a_{im} - a_r b_{im})\sin(\omega_a - \omega_b)t + \frac{1}{2}(b_r a_{im} + a_r b_{im})\sin(\omega_a + \omega_b)t =$$

$$= \frac{1}{2}\mathrm{Re}(ab^* e^{-i(\omega_a - \omega_b)t}) + \frac{1}{2}\mathrm{Re}(abe^{-i(\omega_a + \omega_b)t})$$

ΠΑΡΑΡΤΗΜΑ ΣΤ

Παρακάτω παραθέτουμε μερικά χρήσιμα ολοκληρώματα

$$\int_0^{2\pi} \cos\theta d\theta = 0 \qquad \int_0^{2\pi} \sin\theta d\theta = 0$$

$$\int_0^{2\pi} \cos\theta^2 d\theta = \pi \qquad \int_0^{2\pi} \cos\theta^3 d\theta = 0$$

$$\int_0^{2\pi} \cos\theta \cos(m\theta) d\theta = \begin{cases} 0, m \neq 1 \\ \pi, m = 1 \end{cases} \qquad \int_0^{2\pi} \cos\theta^2 \cos(m\theta) d\theta = \begin{cases} 0, m \neq 2 \\ \frac{\pi}{2}, m = 2 \end{cases}$$

$$\int_0^{2\pi} \cos(m\theta)^2 d\theta = \pi + \frac{\sin(4m\pi)}{4m} \qquad \int_0^{2\pi} \sin(m\theta)^2 d\theta = \pi - \frac{\sin(4m\pi)}{4m}$$

$$\int_0^{2\pi} \cos(m\theta) d\theta = -\frac{\sin(m\pi)}{m} \qquad \int_0^{2\pi} \sin(m\theta) d\theta = \frac{\cos(m\pi)}{m}$$

$$\int_0^{2\pi} \cos\theta \sin(\theta)^2 d\theta = 0 \qquad \int_0^{2\pi} \cos\theta \sin\theta \sin(m\theta) d\theta = \begin{cases} 0, m \neq 2 \\ \frac{\pi}{2}, m = 2 \end{cases}$$

$$\int_0^{2\pi} \cos\theta \cos(m\theta)^2 d\theta = 0$$

$$\int_{d_2}^{d_1} \cosh(kz)^2 dz = \frac{2(d_1 - d_2)k + \sinh(2kd_1) - \sinh(2kd_2)}{4k}$$

$$\int_{d_2}^{d_1} \cosh(kz)\cos(az) dz = \frac{1}{a^2 + k^2}(a\cosh(d_1 k)\sin(d_1 a) - a\cosh(d_2 k)\sin(d_2 a) + $$
$$+ k\cos(ad_1)\sinh(kd_1) - k\cos(ad_2)\sinh(ad_2))$$

$$\int_{d_2}^{d_1} \cos(az)\cos(bz) dz = \frac{1}{a^2 - b^2}(a\cos(bd_1)\sin(ad_1) - a\cos(bd_2)\sin(ad_2) - $$
$$- b\cos(ad_1)\sin(bd_1) + b\cos(ad_2)\sin(bd_2))$$

$$\int_{d_2}^{d_1} \cos(az)^2 dz = \frac{2(d_1 - d_2)a + \sin(2ad_1) - \sin(2ad_2)}{4a}$$

$$\int_{d_2}^{d_1} \sinh(kz)^2 \, dz = \frac{2(-d_1 + d_2)k + \sinh(2kd_1) - \sinh(2kd_2)}{4k}$$

$$\int_{d_2}^{d_1} \sinh(kz)\sin(az)dz = \frac{1}{a^2 + k^2}(k\cosh(d_1k)\sin(d_1a) - k\cosh(d_2k)\sin(d_2a) -$$

$$- a\cos(ad_1)\sinh(kd_1) + a\cos(ad_2)\sinh(ad_2))$$

$$\int_{d_2}^{d_1} \sin(az)\sin(bz)dz = \frac{1}{a^2 - b^2}(b\cos(bd_1)\sin(ad_1) - b\cos(bd_2)\sin(ad_2) -$$

$$- a\cos(ad_1)\sin(bd_1) + a\cos(ad_2)\sin(bd_2))$$

$$\int_{d_2}^{d_1} \sin(az)^2 \, dz = \frac{2(d_1 - d_2)a - \sin(2ad_1) + \sin(2ad_2)}{4a}$$

ΠΑΡΑΡΤΗΜΑ Ζ

Παρακάτω παραθέτουμε μερικά χρήσιμα ολοκληρώματα των Bessel συναρτήσεων.

$$\int_{a_n}^{a_{n+1}} J_1(\kappa r)dr = \frac{1}{\kappa}(J_0(\kappa a_n) - J_0(\kappa a_{n+1})) \qquad \int_{a_n}^{a_{n+1}} Y_1(\kappa r)dr = \frac{1}{\kappa}(Y_0(\kappa a_n) - Y_0(\kappa a_{n+1}))$$

$$\int_{a_n}^{a_{n+1}} I_1(ar)dr = \frac{1}{a}(-I_0(aa_n) + I_0(aa_{n+1})) \qquad \int_{a_n}^{a_{n+1}} K_1(ar)dr = \frac{1}{a}(K_0(aa_n) - K_0(aa_{n+1}))$$

$$\int_{a_n}^{a_{n+1}} rJ_2(\kappa r)dr = \frac{1}{\kappa^2}(2J_0(\kappa a_n) - 2J_0(\kappa a_{n+1}) + \kappa a_n J_1(\kappa a_n) - \kappa a_{n+1} J_1(\kappa a_{n+1}))$$

$$\int_{a_n}^{a_{n+1}} rI_2(\kappa r)dr = \frac{1}{a^2}(2I_0(aa_n) - 2I_0(aa_{n+1}) - aa_n I_1(aa_n) + aa_{n+1} I_1(aa_{n+1}))$$

$$\int_{a_n}^{a_{n+1}} r^2 J_1(\kappa r)dr = \frac{1}{\kappa}(-a_n{}^2 J_2(\kappa a_n) + a_{n+1}{}^2 J_2(\kappa a_{n+1}))$$

$$\int_{a_n}^{a_{n+1}} r^2 I_1(\kappa r)dr = \frac{1}{a}(-a_n{}^2 I_2(aa_n) + a_{n+1}{}^2 I_2(aa_{n+1}))$$

$$\int_{a_n}^{a_{n+1}} r^3 I_2(\kappa r)dr = \frac{1}{a}(-a_n{}^3 I_3(aa_n) + a_{n+1}{}^3 I_3(aa_{n+1}))$$

$$\int_{a_n}^{a_{n+1}} rJ_0(\kappa r)dr = \frac{1}{\kappa}(-a_n J_1(\kappa a_n) + a_{n+1} J_1(\kappa a_{n+1}))$$

$$\int_{a_n}^{a_{n+1}} rI_0(\kappa r)dr = \frac{1}{a}(-a_n I_1(aa_n) + aa_{n+1} I_1(aa_{n+1}))$$

$$\int_{a_n}^{a_{n+1}} rY_0(\kappa r)dr = \frac{1}{\kappa}(-a_n Y_1(\kappa a_n) + a_{n+1} Y_1(\kappa a_{n+1}))$$

$$\int_{a_n}^{a_{n+1}} rK_0(\kappa r)dr = \frac{1}{a}(a_n K_1(aa_n) - aa_{n+1} K_1(aa_{n+1}))$$

$$\int_{\mathrm{a}_n}^{\mathrm{a}_{n+1}} rK_m(ar)\overline{K_m(ar)}dr = \frac{1}{2}(\mathrm{a}_n{}^2(-K_m(a\mathrm{a}_n)\overline{K_m(a\mathrm{a}_n)} + K_{m-1}(a\mathrm{a}_n)\overline{K_{m+1}(a\mathrm{a}_n)}) +$$

$$a_{n+1}{}^2(K_{m+1}(a\mathrm{a}_{n+1})\overline{K_{m+1}(a\mathrm{a}_{n+1})} - K_{m-1}(a\mathrm{a}_{n+1})\overline{K_{m+1}(a\mathrm{a}_{n+1})}$$

$$\int_{\mathrm{a}_n}^{\mathrm{a}_{n+1}} rK_{m+1}(ar)\overline{K_{m+1}(ar)}dr = \frac{1}{2}(\mathrm{a}_n{}^2(-K_{m+1}(a\mathrm{a}_n)\overline{K_{m+1}(a\mathrm{a}_n)} + K_m(a\mathrm{a}_n)\overline{K_{m+2}(a\mathrm{a}_n)}) +$$

$$a_{n+1}{}^2(K_{m+1}(a\mathrm{a}_{n+1})\overline{K_{m+1}(a\mathrm{a}_{n+1})} - K_m(a\mathrm{a}_{n+1})\overline{K_{m+2}(a\mathrm{a}_{n+1})}$$

<u>ΔΙΑΓΡΑΜΜΑΤΑ</u>

Στη συνέχεια παρουσιάζονται τα διαγράμματα της οριζόντιας δύναμης έκπτωσης σε ακίνητο, απλό ή σύνθετο, κυλινδρικό σώμα που εδράζεται ή όχι, στον πυθμένα καθώς και σε κινούμενο απλό κυλινδρικό σώμα που ακουμπά στον πυθμένα ή επιπλέει.
Η μέθοδος που ακολουθήθηκε για την εξαγωγή των αποτελεσμάτων είναι η «*μέθοδος της απ' ευθείας ολοκλήρωσης*» και ο προγραμματισμός έγινε σε γλώσσα προγραμματισμού Fortran. Οι τιμές της οριζόντιας δύναμης έκπτωσης με την παραπάνω μέθοδο, συγκρίνονται με τα αποτελέσματα με τη «*μέθοδο μεταβολής της ορμής*» από το πρόγραμμα *cylinder3.f* του κ. Σ.Α. Μαυράκου, με τα αποτελέσματα με τη «*μέθοδο της απ' ευθείας ολοκλήρωσης*» από το πρόγραμμα *Sec.f* του κ. Ι. Θάνου, καθώς και με αποτελέσματα του υποψήφιου διδάκτορα κ. Θ. Μαζαράκου.

Οι περιπτώσεις που λαμβάνονται υπόψη είναι :

Α) Ενός κατακόρυφου κυλίνδρου που εδράζεται στον πυθμένα (Σχήμα 1), σε βαθύ, ρηχό και ενδιάμεσου βάθους νερό.

Β) Ενός απλού κυλινδρικού σώματος που επιπλέει (Σχήμα 2), χωρίς να κινείται, σε βαθύ και ενδιάμεσου βάθους νερό.

Γ) Ενός σύνθετου κυλινδρικού σώματος με κάτω σκαλοπάτι που επιπλέει (Σχήμα 3), χωρίς να κινείται, σε βαθύ και ενδιάμεσου βάθους νερό.

Δ) Ενός σύνθετου κυλινδρικού σώματος με άνω σκαλοπάτι που επιπλέει (Σχήμα 4), χωρίς να κινείται, σε βαθύ, ρηχό και ενδιάμεσου βάθους νερό.

Ε) Ενός κινούμενου κατακόρυφου κυλίνδρου που ακουμπά στον πυθμένα (Σχήμα 1), σε βαθύ και ενδιάμεσου βάθους νερό.

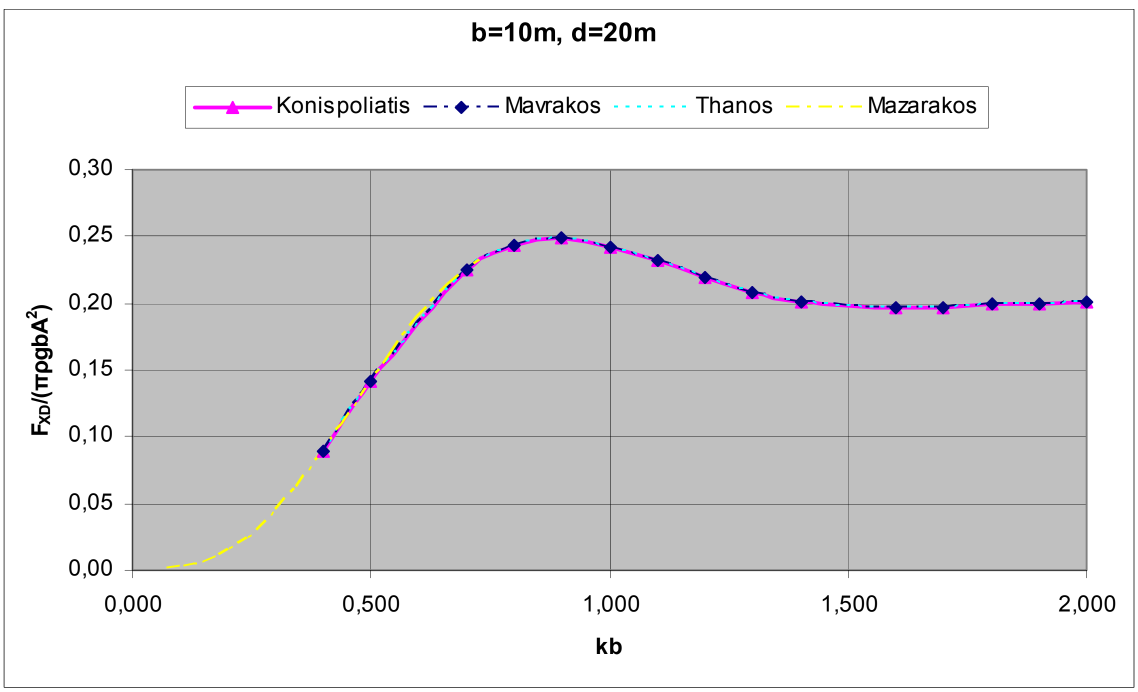
ΣΤ) Ενός απλού κινούμενου κυλινδρικού σώματος που επιπλέει (Σχήμα 2), σε βαθύ και ενδιάμεσου βάθους νερό.

Στα διαγράμματα, στον άξονα των x, εμφανίζεται η ποσότητα *κb* (κ : κυματαριθμός, b : μέγιστη ακτίνα κυλινδρικού στοιχείου) και στον άξονα των y, η οριζόντια δύναμη έκπτωσης, αδιαστατοποιημένη ως προς τις ποσότητες της πυκνότητας του νερού ρ, του π, της επιτάχυνσης της βαρύτητας g, της μέγιστης ακτίνας κυλινδρικού στοιχείου (*RADIUS*) και του ύψους κύματος στο τετράγωνο ($\left[\frac{H}{2}\right]^2$), όπου στα διαγράμματα συμβολίζεται με A^2.

ΔΥΝΑΜΗ ΕΚΠΤΩΣΗΣ ΣΕ **ΠΑΚΤΩΜΕΝΟ ΚΥΛΙΝΔΡΟ** ΣΕ ΜΕΣΟΥ ΒΑΘΟΥΣ ΝΕΡΟ

κb	Konispoliatis	Mavrakos	Thanos
0,400	0,089	0,089	0,089
0,500	0,141	0,141	0,141
0,700	0,225	0,225	0,225
0,800	0,244	0,244	0,244
0,900	0,248	0,248	0,248
1,000	0,243	0,243	0,243
1,100	0,232	0,232	0,232
1,200	0,219	0,219	0,219
1,300	0,208	0,208	0,208
1,400	0,201	0,201	0,201
1,600	0,196	0,196	0,196
1,700	0,197	0,197	0,197
1,800	0,199	0,199	0,199
1,900	0,200	0,200	0,200
2,000	0,201	0,201	0,201

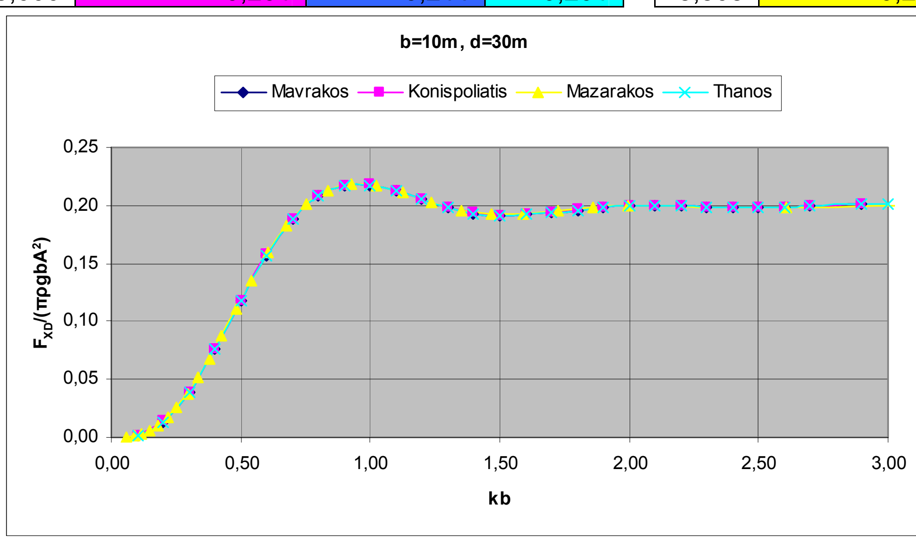
κb	Mazarakos
0,072	0,001
0,108	0,002
0,145	0,006
0,182	0,011
0,221	0,019
0,261	0,030
0,302	0,045
0,345	0,063
0,390	0,084
0,438	0,108
0,488	0,135
0,542	0,163
0,599	0,189
0,661	0,213
0,728	0,232



ΔΥΝΑΜΗ ΕΚΠΤΩΣΗΣ ΣΕ **ΠΑΚΤΩΜΕΝΟ ΚΥΛΙΝΔΡΟ** ΣΕ ΡΗΧΟ ΝΕΡΟ

κb	Konispoliatis	Mavrakos	Thanos
0,100	0,002	0,002	0,002
0,200	0,014	0,014	0,014
0,300	0,039	0,039	0,039
0,400	0,076	0,076	0,076
0,500	0,118	0,118	0,118
0,600	0,158	0,157	0,157
0,700	0,189	0,188	0,188
0,800	0,209	0,208	0,208
0,900	0,218	0,217	0,217
1,000	0,218	0,217	0,217
1,100	0,213	0,212	0,212
1,200	0,205	0,205	0,205
1,300	0,198	0,198	0,198
1,400	0,193	0,193	0,193
1,500	0,192	0,191	0,191
1,600	0,192	0,192	0,192
1,700	0,194	0,194	0,194
1,800	0,197	0,196	0,196
1,900	0,199	0,198	0,198
2,000	0,200	0,199	0,199
2,100	0,200	0,199	0,199
2,200	0,199	0,199	0,199
2,300	0,198	0,198	0,198
2,400	0,198	0,198	0,198
2,500	0,198	0,198	0,198
2,600	0,199	0,198	0,198
2,700	0,200	0,199	0,199
2,900	0,201	0,201	0,201
3,000	0,201	0,211	0,201

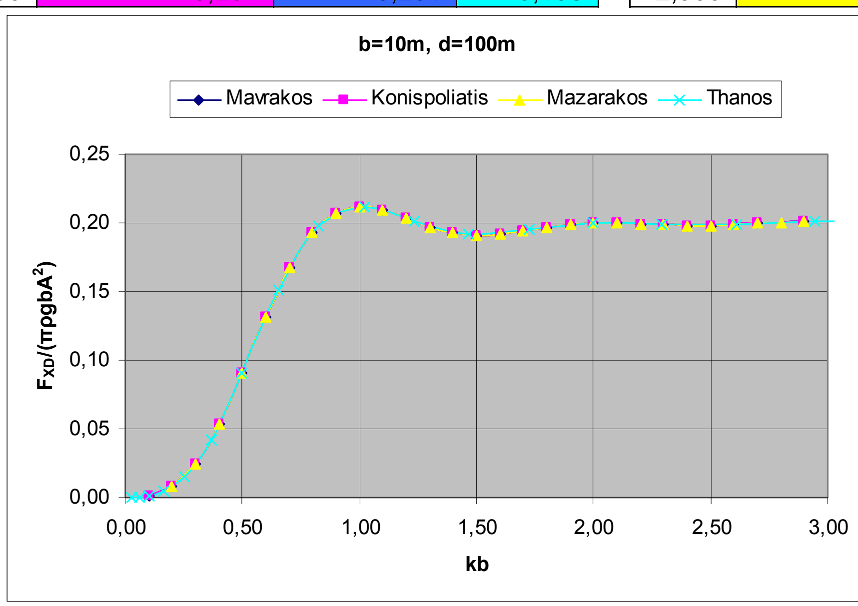
κb	Mazarakos
0,059	0,000
0,088	0,001
0,119	0,003
0,151	0,006
0,183	0,011
0,218	0,017
0,254	0,026
0,293	0,037
0,334	0,051
0,379	0,068
0,428	0,088
0,481	0,110
0,540	0,135
0,605	0,159
0,675	0,182
0,753	0,201
0,837	0,213
0,927	0,218
1,024	0,217
1,126	0,211
1,235	0,203
1,349	0,196
1,468	0,192
1,593	0,192
1,723	0,195
1,858	0,198
1,998	0,200
2,610	0,199
3,303	0,201



ΔΥΝΑΜΗ ΕΚΠΤΩΣΗΣ ΣΕ **ΠΑΚΤΩΜΕΝΟ ΚΥΛΙΝΔΡΟ** ΣΕ ΒΑΘΥ ΝΕΡΟ

κb	Konispoliatis	Mavrakos	Thanos
0,100	0,001	0,001	0,000
0,200	0,008	0,008	0,000
0,300	0,024	0,024	0,001
0,400	0,053	0,053	0,005
0,500	0,091	0,091	0,016
0,600	0,132	0,132	0,042
0,700	0,168	0,168	0,091
0,800	0,193	0,193	0,152
0,900	0,208	0,207	0,198
1,000	0,212	0,212	0,212
1,100	0,209	0,209	0,201
1,200	0,203	0,203	0,191
1,300	0,197	0,197	0,195
1,400	0,193	0,193	0,200
1,500	0,191	0,191	0,198
1,600	0,192	0,192	0,199
1,700	0,194	0,194	0,201
1,800	0,197	0,197	0,201
1,900	0,199	0,199	0,202
2,000	0,200	0,200	0,203
2,100	0,200	0,200	0,203
2,200	0,199	0,199	0,204
2,300	0,198	0,198	0,205
2,400	0,198	0,198	0,206
2,500	0,198	0,198	0,209
2,600	0,199	0,199	0,212
2,700	0,200	0,200	0,211
2,900	0,201	0,201	0,198

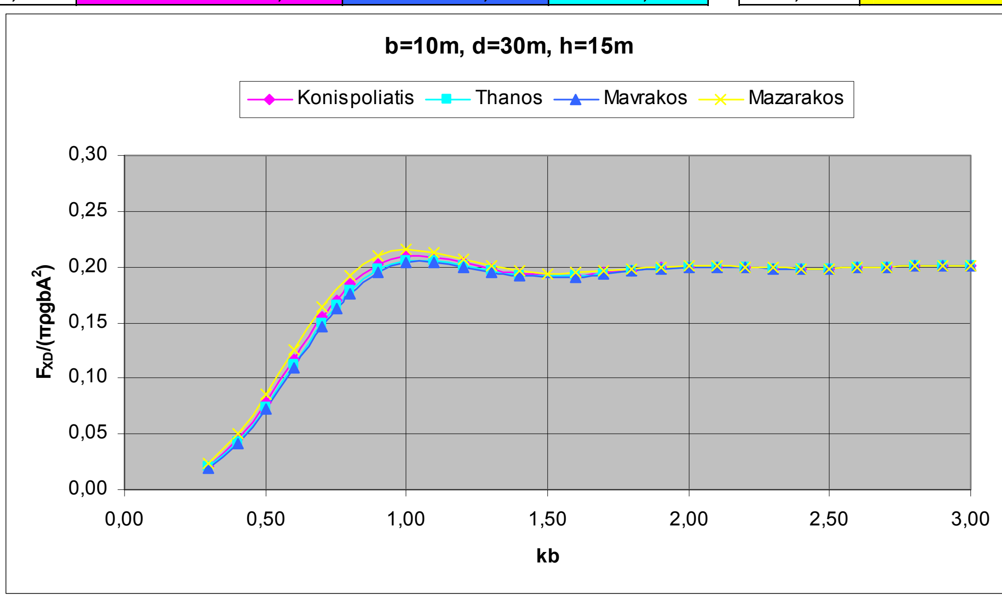
κb	Mazarakos
0,200	0,008
0,300	0,024
0,400	0,053
0,500	0,091
0,600	0,132
0,700	0,168
0,800	0,193
0,900	0,208
1,000	0,212
1,100	0,209
1,200	0,203
1,300	0,197
1,400	0,193
1,500	0,191
1,600	0,192
1,700	0,194
1,800	0,197
1,900	0,199
2,000	0,200
2,100	0,200
2,200	0,199
2,300	0,198
2,400	0,198
2,500	0,198
2,600	0,199
2,700	0,200
2,800	0,201
2,900	0,201



ΚΟΜΜΕΝΟΣ ΚΥΛΙΝΔΡΟΣ ΠΟΥ ΕΠΙΠΛΕΕΙ ΣΕ ΜΕΣΟΥ ΒΑΘΟΥΣ ΝΕΡΟ

κb	Konispoliatis	Mavrakos	Thanos
0,300	0,021	0,019	0,020
0,400	0,045	0,042	0,043
0,500	0,078	0,073	0,074
0,600	0,117	0,110	0,112
0,700	0,154	0,146	0,149
0,750	0,171	0,162	0,165
0,800	0,184	0,176	0,179
0,900	0,202	0,195	0,197
1,000	0,209	0,204	0,206
1,100	0,209	0,204	0,206
1,200	0,204	0,200	0,201
1,300	0,198	0,195	0,196
1,400	0,194	0,192	0,192
1,600	0,193	0,191	0,192
1,700	0,195	0,194	0,194
1,800	0,197	0,196	0,197
1,900	0,199	0,198	0,199
2,000	0,200	0,199	0,200
2,100	0,200	0,199	0,200
2,200	0,199	0,199	0,199
2,300	0,199	0,198	0,198
2,400	0,198	0,198	0,198
2,500	0,198	0,198	0,198
2,600	0,199	0,199	0,199
2,700	0,200	0,200	0,200
2,800	0,201	0,200	0,201
2,900	0,201	0,201	0,201
3,000	0,201	0,201	0,201

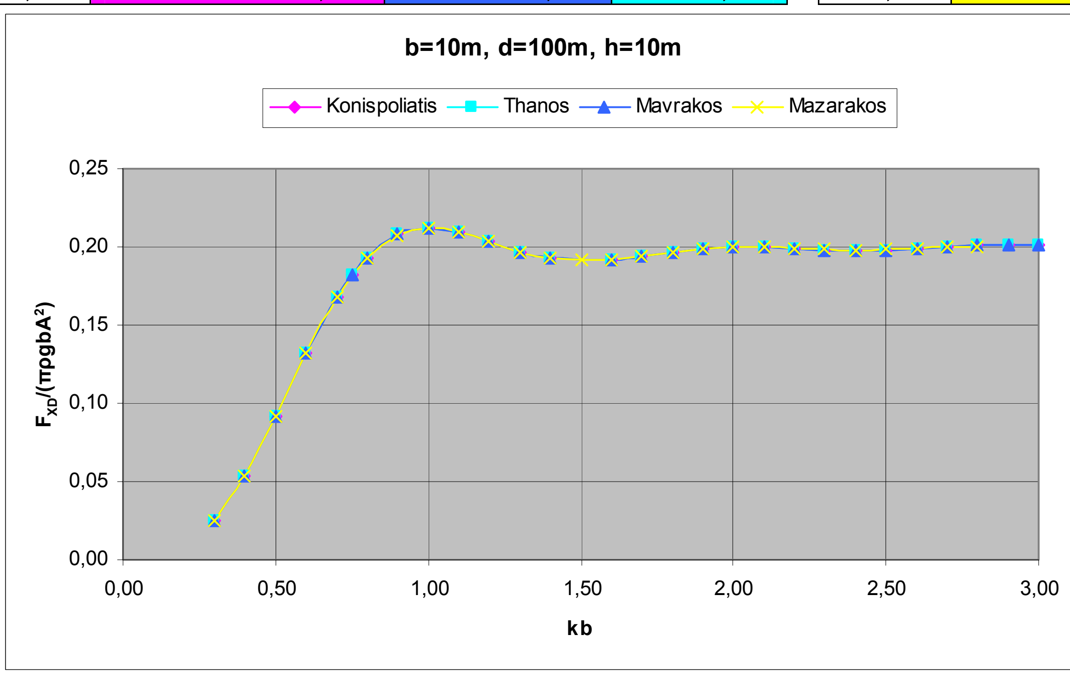
κb	Mazarakos
0,300	0,023
0,400	0,050
0,500	0,085
0,600	0,126
0,700	0,164
0,800	0,193
0,900	0,210
1,000	0,215
1,100	0,213
1,200	0,208
1,300	0,201
1,400	0,196
1,500	0,194
1,600	0,194
1,700	0,196
1,800	0,198
1,900	0,200
2,000	0,201
2,100	0,200
2,200	0,200
2,300	0,199
2,400	0,198
2,500	0,198
2,600	0,199
2,700	0,200
2,800	0,201
2,900	0,201
3,000	0,201



ΚΟΜΜΕΝΟΣ ΚΥΛΙΝΔΡΟΣ ΠΟΥ ΕΠΙΠΛΕΕΙ ΣΕ ΒΑΘΥ ΝΕΡΟ

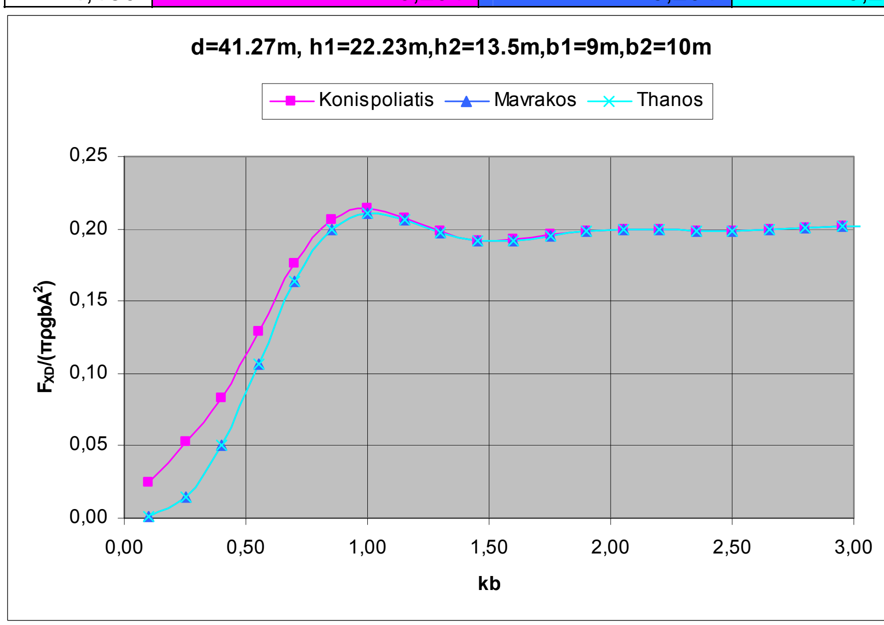
κb	Konispoliatis	Mavrakos	Thanos
0,300	0,025	0,025	0,025
0,400	0,053	0,053	0,053
0,500	0,091	0,091	0,091
0,600	0,132	0,132	0,132
0,700	0,168	0,168	0,168
0,750	0,182	0,182	0,182
0,800	0,193	0,193	0,193
0,900	0,208	0,208	0,208
1,000	0,212	0,212	0,212
1,100	0,209	0,209	0,209
1,200	0,203	0,203	0,203
1,300	0,197	0,197	0,197
1,400	0,193	0,193	0,193
1,600	0,192	0,192	0,192
1,700	0,194	0,194	0,194
1,800	0,197	0,197	0,197
1,900	0,199	0,199	0,199
2,000	0,200	0,200	0,200
2,100	0,200	0,200	0,200
2,200	0,199	0,199	0,199
2,300	0,198	0,198	0,198
2,400	0,198	0,198	0,198
2,500	0,198	0,198	0,198
2,600	0,199	0,199	0,199
2,700	0,200	0,200	0,200
2,800	0,201	0,201	0,201

κb	Mazarakos
0,300	0,025
0,400	0,053
0,500	0,091
0,600	0,132
0,700	0,168
0,800	0,193
0,900	0,208
1,000	0,212
1,100	0,209
1,200	0,203
1,300	0,197
1,400	0,193
1,500	0,191
1,600	0,192
1,700	0,194
1,800	0,197
1,900	0,199
2,000	0,200
2,100	0,200
2,200	0,199
2,300	0,198
2,400	0,198
2,500	0,198
2,600	0,199
2,700	0,200
2,800	0,201



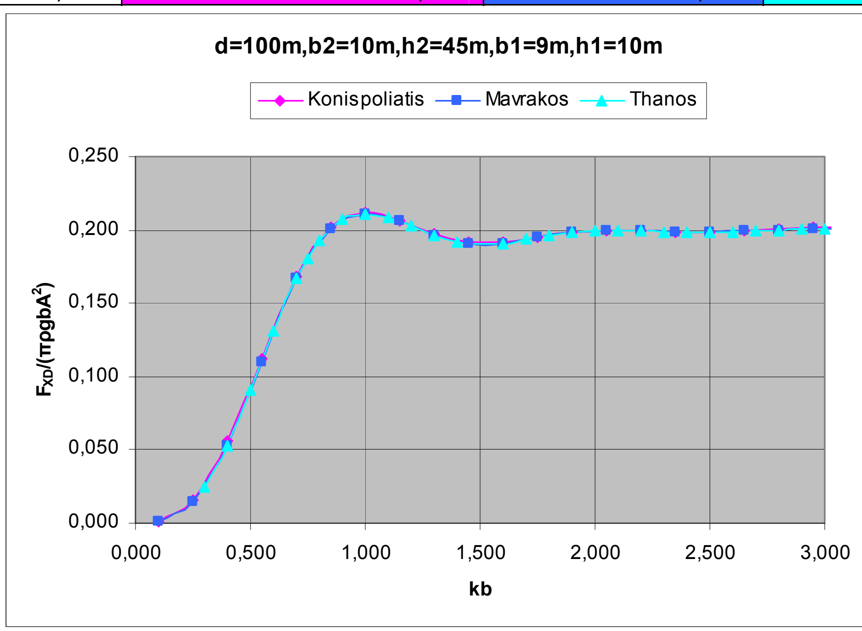
ΚΥΛΙΝΔΡΟΣ ΜΕ ΚΑΤΩ ΣΚΑΛΟΠΑΤΙ ΣΕ ΜΕΣΟΥ ΒΑΘΟΥΣ ΝΕΡΟ

κb	Konispoliatis	Mavrakos	Thanos
0,100	0,025	0,001	0,001
0,250	0,052	0,014	0,014
0,400	0,083	0,050	0,050
0,550	0,128	0,107	0,107
0,700	0,176	0,164	0,164
0,850	0,206	0,200	0,200
1,000	0,214	0,211	0,211
1,150	0,208	0,206	0,206
1,300	0,198	0,197	0,197
1,450	0,192	0,191	0,191
1,600	0,192	0,192	0,192
1,750	0,196	0,195	0,195
1,900	0,199	0,199	0,199
2,050	0,200	0,200	0,200
2,200	0,199	0,199	0,199
2,350	0,198	0,198	0,198
2,500	0,198	0,198	0,198
2,650	0,199	0,199	0,199
2,800	0,201	0,201	0,201
2,950	0,201	0,201	0,201
3,100	0,202	0,202	0,202
3,250	0,201	0,201	0,201
3,400	0,202	0,202	0,202
3,550	0,202	0,202	0,202
3,700	0,203	0,203	0,203
3,850	0,203	0,203	0,203
4,000	0,203	0,203	0,203
4,150	0,204	0,204	0,204



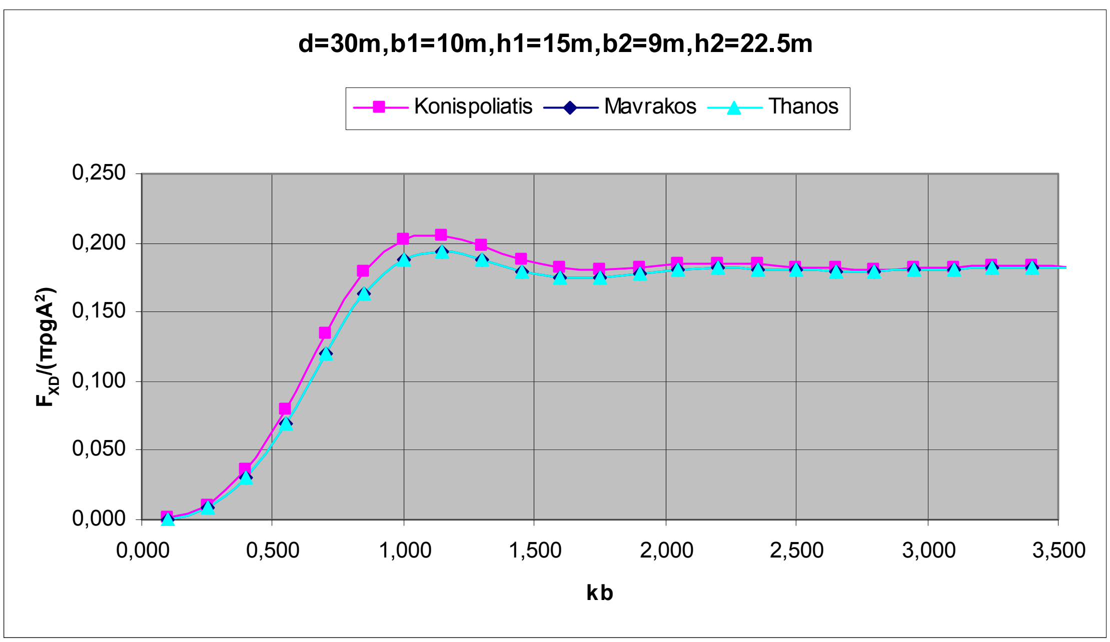
ΚΥΛΙΝΔΡΟΣ ΜΕ ΚΑΤΩ ΣΚΑΛΟΠΑΤΙ ΣΕ ΒΑΘΥ ΝΕΡΟ

κb	Konispoliatis	Mavrakos	Thanos
0,100	0,001	0,001	0,025
0,250	0,015	0,015	0,053
0,400	0,056	0,053	0,091
0,550	0,112	0,110	0,131
0,700	0,168	0,167	0,167
0,850	0,202	0,201	0,181
1,000	0,212	0,211	0,193
1,150	0,206	0,206	0,207
1,300	0,197	0,196	0,211
1,450	0,192	0,191	0,208
1,600	0,192	0,191	0,203
1,750	0,196	0,195	0,196
1,900	0,199	0,198	0,192
2,050	0,200	0,199	0,191
2,200	0,199	0,199	0,194
2,350	0,198	0,198	0,196
2,500	0,198	0,198	0,198
2,650	0,199	0,199	0,199
2,800	0,201	0,200	0,199
2,950	0,201	0,201	0,199
3,100	0,202	0,201	0,198
3,250	0,201	0,201	0,198
3,400	0,202	0,201	0,198
3,550	0,202	0,201	0,198
3,700	0,203	0,202	0,199
3,850	0,203	0,203	0,200
4,000	0,203	0,203	0,201
4,150	0,204	0,203	0,201



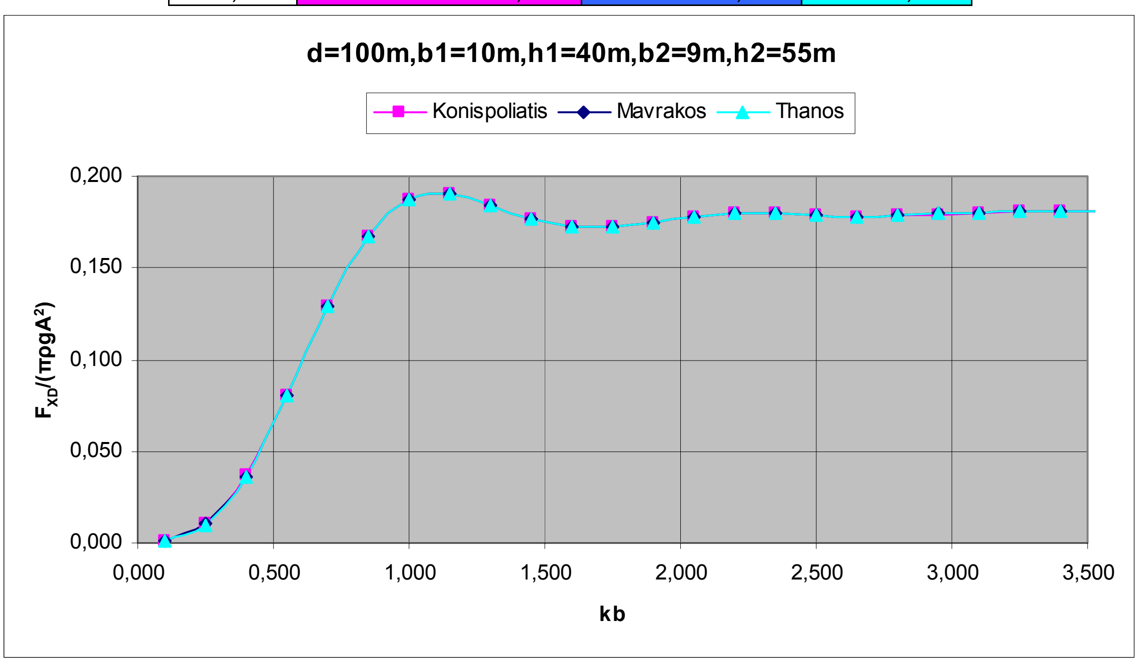
ΚΥΛΙΝΔΡΟΣ ΜΕ ΑΝΩ ΣΚΑΛΟΠΑΤΙ ΣΕ ΜΕΣΟΥ ΒΑΘΟΥΣ ΝΕΡΟ

κb	Konispoliatis	Mavrakos	Thanos
0,100	0,001	0,001	0,001
0,250	0,010	0,008	0,008
0,400	0,035	0,031	0,031
0,550	0,080	0,070	0,070
0,700	0,134	0,120	0,120
0,850	0,180	0,164	0,164
1,000	0,203	0,188	0,188
1,150	0,205	0,193	0,193
1,300	0,197	0,187	0,187
1,450	0,188	0,180	0,180
1,600	0,182	0,175	0,175
1,750	0,181	0,175	0,175
1,900	0,183	0,178	0,178
2,050	0,185	0,180	0,180
2,200	0,185	0,182	0,182
2,350	0,184	0,181	0,181
2,500	0,183	0,180	0,180
2,650	0,182	0,179	0,179
2,800	0,181	0,179	0,179
2,950	0,182	0,180	0,180
3,100	0,182	0,181	0,181
3,250	0,183	0,182	0,182
3,400	0,183	0,182	0,182
3,550	0,183	0,182	0,182
3,700	0,182	0,182	0,182
3,850	0,182	0,182	0,182



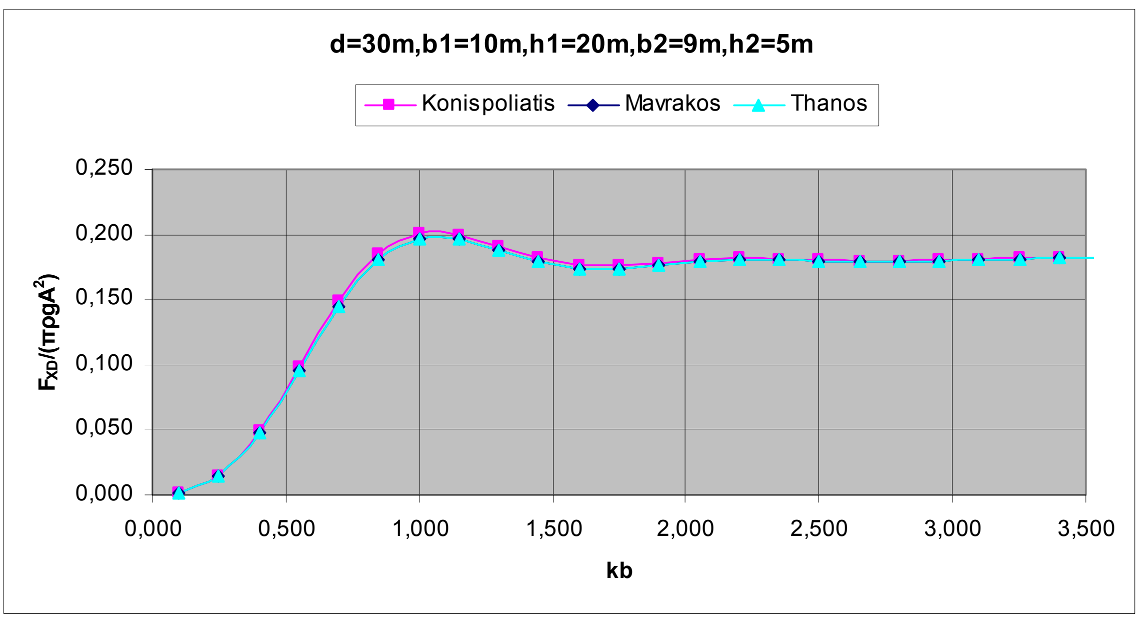
ΚΥΛΙΝΔΡΟΣ ΜΕ ΑΝΩ ΣΚΑΛΟΠΑΤΙ ΣΕ ΒΑΘΥ ΝΕΡΟ

κb	Konispoliatis	Mavrakos	Thanos
0,100	0,001	0,001	0,001
0,250	0,010	0,010	0,009
0,400	0,037	0,037	0,036
0,550	0,080	0,080	0,080
0,700	0,129	0,129	0,129
0,850	0,167	0,167	0,167
1,000	0,187	0,187	0,187
1,150	0,190	0,190	0,190
1,300	0,184	0,184	0,184
1,450	0,177	0,177	0,177
1,600	0,173	0,173	0,173
1,750	0,172	0,172	0,172
1,900	0,175	0,175	0,175
2,050	0,178	0,178	0,178
2,200	0,180	0,180	0,180
2,350	0,180	0,180	0,180
2,500	0,179	0,179	0,179
2,650	0,178	0,178	0,178
2,800	0,179	0,179	0,179
2,950	0,179	0,179	0,179
3,100	0,180	0,180	0,180
3,250	0,181	0,181	0,181
3,400	0,181	0,181	0,181
3,550	0,181	0,181	0,181
3,700	0,181	0,181	0,181
3,850	0,182	0,182	0,182



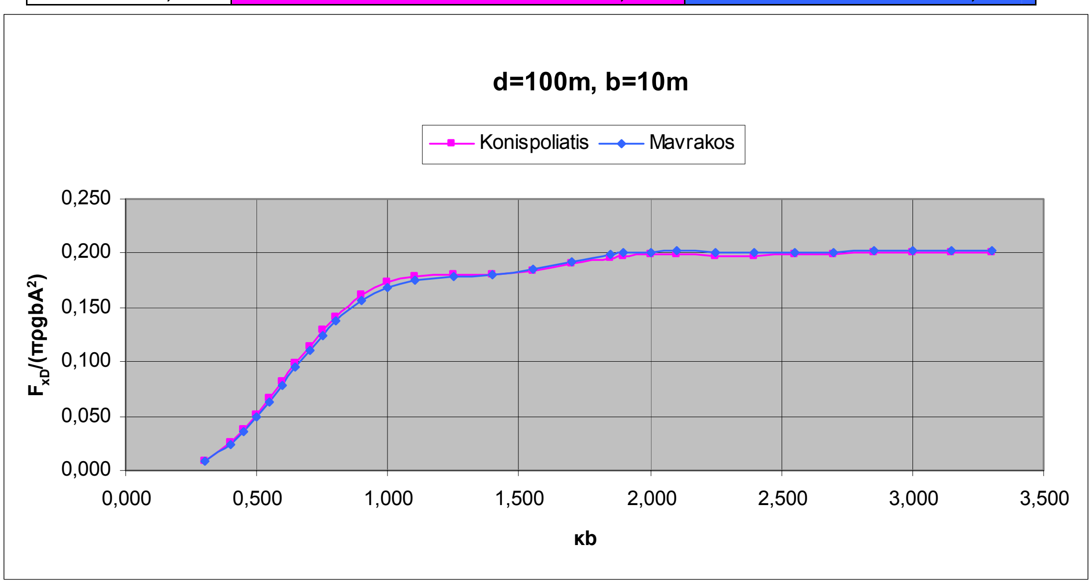
ΚΥΛΙΝΔΡΟΣ ΜΕ ΑΝΩ ΣΚΑΛΟΠΑΤΙ ΣΕ ΡΗΧΟ ΝΕΡΟ

κb	Konispoliatis	Mavrakos	Thanos
0,100	0,001	0,001	0,001
0,250	0,015	0,014	0,014
0,400	0,049	0,048	0,048
0,550	0,099	0,096	0,096
0,700	0,149	0,145	0,145
0,850	0,185	0,180	0,180
1,000	0,201	0,196	0,196
1,150	0,200	0,196	0,196
1,300	0,191	0,188	0,188
1,450	0,182	0,179	0,179
1,600	0,177	0,174	0,174
1,750	0,176	0,174	0,174
1,900	0,178	0,176	0,176
2,050	0,180	0,179	0,179
2,200	0,182	0,180	0,180
2,350	0,181	0,180	0,180
2,500	0,180	0,179	0,179
2,650	0,179	0,179	0,179
2,800	0,179	0,179	0,179
2,950	0,180	0,180	0,180
3,100	0,181	0,181	0,181
3,250	0,181	0,181	0,181
3,400	0,182	0,181	0,181
3,550	0,182	0,181	0,181
3,700	0,182	0,181	0,181
3,850	0,182	0,182	0,182



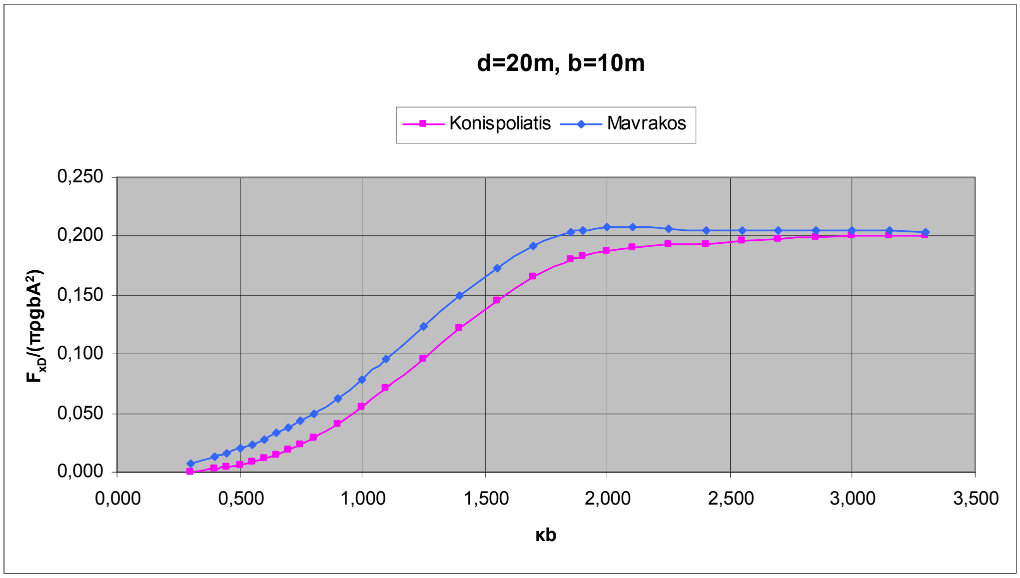
ΚΙΝΟΥΜΕΝΟΣ ΚΥΛΙΝΔΡΟΣ ΠΟΥ ΑΚΟΥΜΠΑ ΣΤΟΝ ΠΥΘΜΕΝΑ ΣΕ **ΒΑΘΥ** ΝΕΡΟ

κa	Konispoliatis	Mavrakos
0,300	0,009	0,009
0,400	0,026	0,024
0,450	0,037	0,036
0,500	0,051	0,049
0,550	0,066	0,063
0,600	0,082	0,079
0,650	0,098	0,095
0,700	0,114	0,110
0,750	0,129	0,124
0,800	0,141	0,137
0,900	0,161	0,157
1,000	0,173	0,169
1,100	0,178	0,175
1,250	0,180	0,179
1,400	0,180	0,181
1,550	0,184	0,186
1,700	0,190	0,193
1,850	0,195	0,199
1,900	0,197	0,200
2,000	0,198	0,201
2,100	0,199	0,202
2,250	0,198	0,201
2,400	0,198	0,200
2,550	0,198	0,200
2,700	0,199	0,201
2,850	0,201	0,202
3,000	0,201	0,202
3,150	0,201	0,202
3,300	0,201	0,202



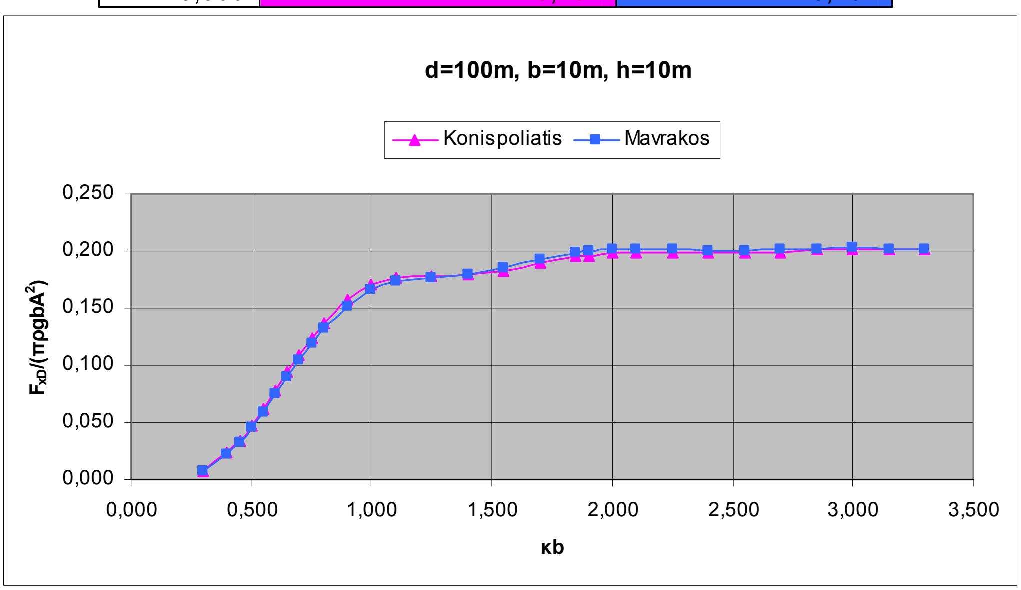
ΚΙΝΟΥΜΕΝΟΣ ΚΥΛΙΝΔΡΟΣ ΠΟΥ ΑΚΟΥΜΠΑ ΣΤΟΝ ΠΥΘΜΕΝΑ ΣΕ **ΡΗΧΟ** ΝΕΡΟ

κa	Konispoliatis	Mavrakos
0,300	0,000	0,007
0,400	0,002	0,013
0,450	0,004	0,016
0,500	0,006	0,020
0,550	0,008	0,024
0,600	0,012	0,028
0,650	0,015	0,033
0,700	0,019	0,038
0,750	0,024	0,044
0,800	0,029	0,050
0,900	0,041	0,063
1,000	0,055	0,079
1,100	0,071	0,096
1,250	0,096	0,123
1,400	0,122	0,150
1,550	0,146	0,173
1,700	0,166	0,192
1,850	0,180	0,203
1,900	0,183	0,205
2,000	0,188	0,208
2,100	0,191	0,208
2,250	0,193	0,207
2,400	0,194	0,205
2,550	0,196	0,205
2,700	0,197	0,205
2,850	0,199	0,205
3,000	0,200	0,205
3,150	0,201	0,205
3,300	0,201	0,204



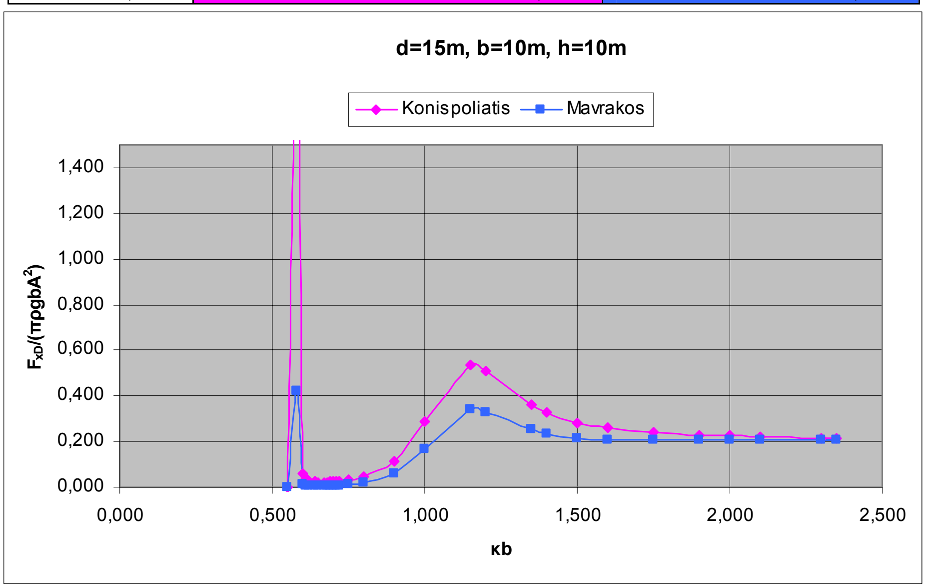
ΚΙΝΟΥΜΕΝΟΣ ΚΟΜΜΕΝΟΣ ΚΥΛΙΝΔΡΟΣ ΣΕ **ΒΑΘΥ** ΝΕΡΟ

κα	Konispoliatis	Mavrakos
0,300	0,007	0,008
0,400	0,023	0,022
0,450	0,034	0,033
0,500	0,048	0,045
0,550	0,062	0,059
0,600	0,078	0,074
0,650	0,094	0,090
0,700	0,109	0,105
0,750	0,124	0,119
0,800	0,137	0,132
0,900	0,157	0,152
1,000	0,170	0,166
1,100	0,176	0,173
1,250	0,178	0,177
1,400	0,179	0,180
1,550	0,183	0,186
1,700	0,189	0,193
1,850	0,195	0,199
1,900	0,196	0,200
2,000	0,198	0,202
2,100	0,198	0,202
2,250	0,198	0,201
2,400	0,198	0,200
2,550	0,198	0,200
2,700	0,199	0,201
2,850	0,201	0,202
3,000	0,201	0,203
3,150	0,201	0,202
3,300	0,201	0,202



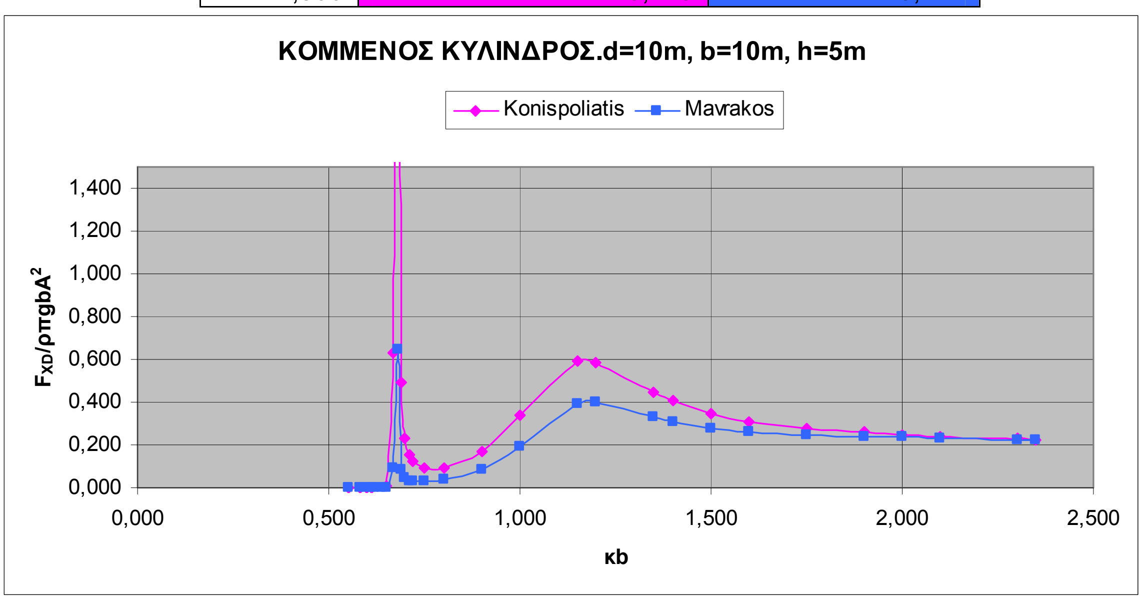
ΚΙΝΟΥΜΕΝΟΣ ΚΟΜΜΕΝΟΣ ΚΥΛΙΝΔΡΟΣ ΣΕ **ΜΕΣΟΥ** ΒΑΘΟΥΣ ΝΕΡΟ

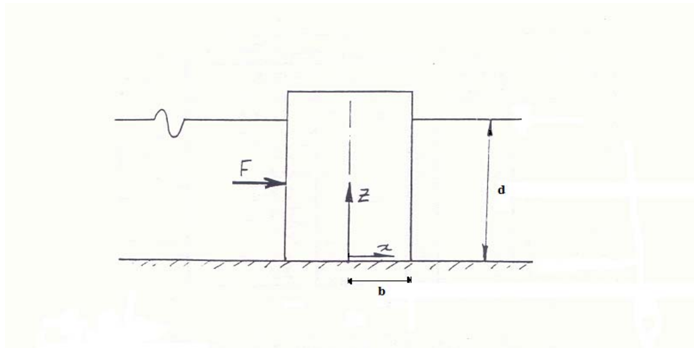
κb	Konispoliatis	Mavrakos
0,550	0,002	0,000
0,580	2,060	0,419
0,600	0,063	0,014
0,610	0,039	0,009
0,620	0,030	0,007
0,640	0,024	0,006
0,650	0,023	0,006
0,670	0,023	0,007
0,680	0,023	0,007
0,690	0,024	0,007
0,700	0,025	0,008
0,710	0,026	0,009
0,720	0,028	0,009
0,750	0,033	0,013
0,800	0,048	0,021
0,900	0,114	0,059
1,000	0,285	0,165
1,150	0,537	0,342
1,200	0,506	0,331
1,350	0,359	0,254
1,400	0,326	0,237
1,500	0,283	0,216
1,600	0,258	0,208
1,750	0,239	0,206
1,900	0,230	0,209
2,000	0,226	0,210
2,100	0,222	0,210
2,350	0,213	0,207
2,300	0,215	0,208



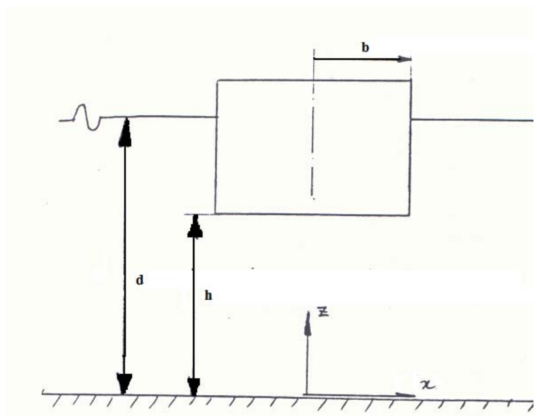
ΚΙΝΟΥΜΕΝΟΣ ΚΟΜΜΕΝΟΣ ΚΥΛΙΝΔΡΟΣ ΣΕ **ΡΗΧΟ** ΝΕΡΟ

κb	Konispoliatis	Mavrakos
0,550	0,000	0,001
0,580	-0,001	0,001
0,600	-0,003	0,001
0,610	-0,003	0,001
0,620	-0,004	0,001
0,640	-0,004	0,001
0,650	0,005	0,002
0,670	0,629	0,092
0,680	4,086	0,643
0,690	0,493	0,086
0,700	0,231	0,045
0,710	0,156	0,034
0,720	0,124	0,029
0,750	0,093	0,027
0,800	0,095	0,036
0,900	0,168	0,083
1,000	0,337	0,193
1,150	0,592	0,389
1,200	0,584	0,397
1,350	0,446	0,330
1,400	0,405	0,308
1,500	0,345	0,276
1,600	0,308	0,258
1,750	0,276	0,245
1,900	0,258	0,239
2,000	0,249	0,236
2,100	0,242	0,232
2,350	0,227	0,223
2,300	0,229	0,224

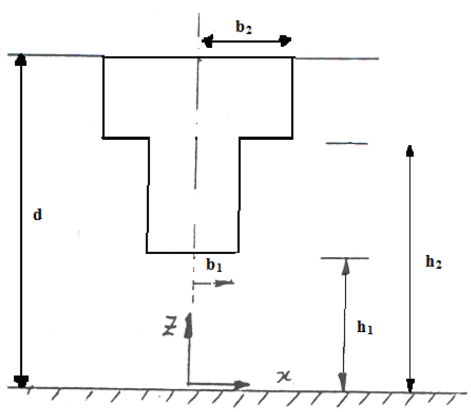




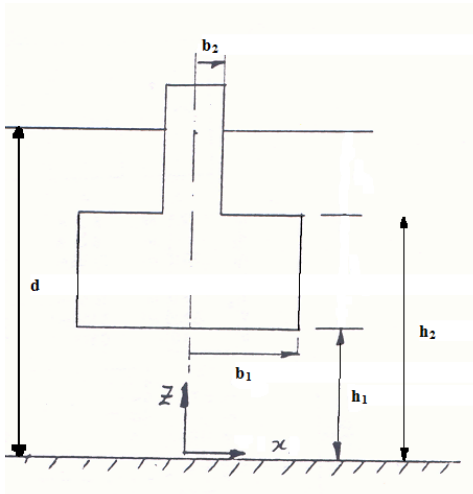
Σχήμα 1. Κατακόρυφος κύλινδρος που εδράζεται στον πυθμένα



Σχήμα 2. Κατακόρυφος κύλινδρος που επιπλέει.



Σχήμα 3. Σύνθετος κύλινδρος με κάτω σκαλοπάτι.



Σχήμα 4. Σύνθετος κύλινδρος με άνω σκαλοπάτι.

<u>**ΒΙΒΛΙΟΓΡΑΦΙΑ**</u>

1. Kokkinowrachos, K., Mavrakos, S., Asorakos, S (1986). *Behaviour of vertical bodies of revolution in waves*, Ocean Engng, **Vol**. 13, No 6, pp 505-538.
2. Mavrakos, S (1988). *The vertical drift force and pitch moment on axisymmetric bodies in regular waves*, Applied Ocean Research, **Vol**. 10, No. 4.
3. Kriebel, L (1990). *Nonlinear wave interaction with a vertical circular cylinder. Part I: Diffraction theory*, Ocean Engng, **Vol**. 17, No 4, pp 345-377.
4. Pinkster, A (5-19 March 1979). *Drift forces, mooring problems*, Second WEGEMT Graduate school advanced aspects of offshore engineering.
5. Μαυράκος Σ (1997). *Σημειώσεις μαθήματος μελέτη και σχεδίαση πλωτών κατασκευών*, Ε.Μ.Π.
6. Mavrakos, S (1985). *Wave loads on stationary floating bottomless cylindrical body with finite wall thickness*, Applied ocean research, **Vol**. 7, No. 4.
7. Thanos I (1989). *Wellenkrafte zweiter ordnung auf rotationssymmetrische korper mit vertikaler aches*.
8. Μπαλάσκας Χ (1987). *Δυνάμεις δευτέρας τάξης*, Διπλωματική εργασία, Ε.Μ.Π.