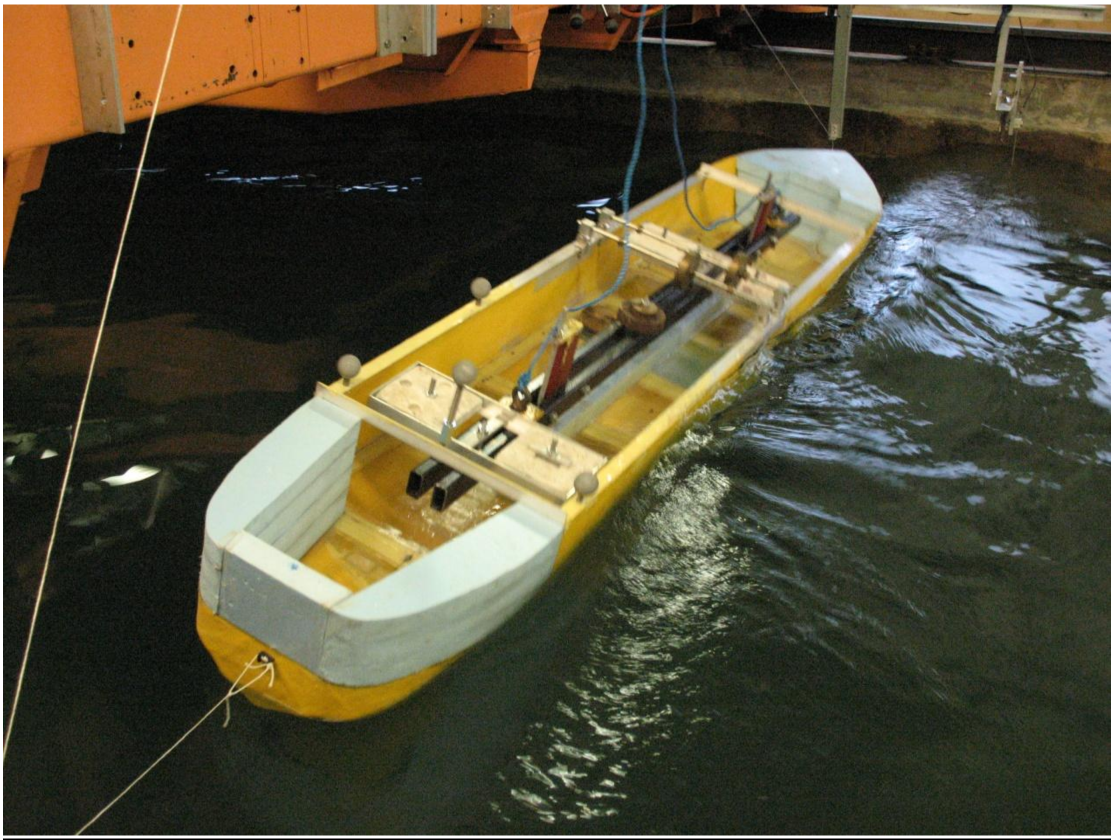
Department of Naval Architecture and Marine Engineering

Universities of Strathclyde and Glasgow

National Technical University of Athens

Diploma thesis

“Physical verification of predictions of time to capsize after Ro-Pax ships collisions and quantification of uncertainty”



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Abstract

Definition of the problem and aim of thesis

Tank experiments are the most accurate method to describe a physical phenomenon and a reliable way to solve many naval architecture problems. In this project we can notice the repeatability of the results is in specific wave conditions defined each time we do the experiments by a different realization of the waves as well as establishing uncertainty of predictions.

By performing physical tests on survivability of a Ro-Pax ship the main aim of this project is to derive data for validation of the technique for prediction of survival time (time available for evacuation) after a collision damage.

Such tests have never been performed to date on any ship, as there is no commercial interest for such tests on one hand and on the other, the knowledge of the approach to quantifying time to capsize has only been recently derived in SAFEDOR project with absolutely no prior attempts referenced to date.

1.1 Historical background

The question as to how to quantify ship's stability has been addressed since ca 250 B.C. by Archimedes, although the first attempts to crystallize these principles were only made in the 17th/18th century, when a concept of ship stability measure, the metacentric height, or **GM** as it is known today, was introduced by Paul Hoste in 1698. This concept was later elaborated further in a more widely acknowledged exposition by Pierre Bouguer, who produced in 1746 the actual term "metacentre". Leonhard Euler focused in 1749 on the righting moment at a particular angle of heel as a better measure of stability, but it was George Atwood who eventually demonstrated in 1798 that such measure can be derived for any angle, inventing thereby the **GZ** curve. Milestones on stability quantification thereafter were set by Canon Moseley's concept of using the area under a GZ curve as a better measure of ship stability in 1850, and Jaakko Rahola's propositions to use a function of GZ curve to express the ability of a ship to stay in functional equilibrium after flooding in 1939.

As advances in identifying "stability" parameters progressed, the legislation process for implementation of any such "technicalities" has surprisingly been slow, even though the need for some "legal" safety instrument was realised for many centuries.

First attempts to introduce governmental intervention were in place since ancient ages, e.g. ban on sailing in winter (15th September to 26th May) in Rome during the Roman Empire (27 BC – AD 476 / 1453), in force in some places even as late as the 18th century, or the first recorded regulations on load line in middle ages (cross marked on each ship) in Venice in 1255, or the first system of survey inspections imposed by The Recesses of the Diet of the Hanseatic League of 1412. However, only during the Industrial Revolution of 19th century with the introduction of steam-powered engines onboard ships, steel hulls and rapid escalation of sea trade to the dimension of an "industry", the true face of risk faced by shipping started to show: during the winter of 1820 alone, more than two thousand ships were wrecked in the North Sea, causing the death of twenty thousand people, in one year!, with some 700-800 ships being lost annually in the UK on average.

Such loss toll has prompted the main maritime nations of the time, France and UK, to exercise their policy making powers to introduce accident-preventative regulations, to great opposition from the industry. Of note are Colbert's Naval Ordinance, instituted by a Royal Declaration of 17th August 1779 in France, which introduced again the office of huissier-visiteur, a **surveyor**, and the Merchant Shipping Act of 1850 (reinforced by the government in 1854, amended by the Act of 21 December 1906) in the United Kingdom, which obliged the Board of Trade to monitor, regulate and control all aspects of safety and working conditions of seamen. The latter implemented also the load line requirements, which were applied to all vessels, including foreign ships visiting UK ports.

It was the sinking of the **Titanic,** however, on 14 April 1912 after colliding with an iceberg and causing the death of some 1,500 people that provided the catalyst for the adoption on 20 January 1914 of the first International Convention for the Safety of Life at Sea (SOLAS), which gained international recognition. The SOLAS Convention was subsequently revised and adopted four times since then, namely in **1929**, in **1948**, in **1960** and **1974**, with the latter still in force today and allowing a flexible process of revisions through amendment procedures included in Article VIII.

It is worth noting that although the provisions of SOLAS 1914 prescribed requirements on **margin line** and **factor of subdivision**, in addressing the state of a damaged ship, the Convention did not even mention the concept of stability. It was the third Convention of 1948, which referred to **stability** explicitly in Chapter II B Regulation 7, and subsequently SOLAS 1960, which actually prescribed a specific requirement on one parameter of stability after flooding, $GM \geq 0.05m$ and then finally SOLAS 1974, which adopted Rahola's proposals of using properties of the GZ curve to measure stability. In principle, Rahola's approach forms the basis for amendments of technical requirements on stability ever since, [10], applied in various frameworks for adherence to the following SOLAS'74 goal:

Table 1 SOLAS 1974 Chapter II-1

"The subdivision of passenger ships into watertight compartments must be such that after ***assumed*** damage to the ship's hull the vessel will remain afloat and stable."

Rahola's use of GZ curve properties to quantify stability are the core of even the most modern amendment to SOLAS 1974 criteria of stability for a ship in a damaged state. This subtlety can easily escape attention, since the overall framework of stability assessment of a damaged ship, based on the Kurt Wendel's concepts of probabilistic index of subdivision **A**, is rather a complex mathematical construct, with the basic details not discernible. The framework is also a major step-change in the philosophy of stability standardisation. However, it seems that such an implicit reliance on Raholas measures is a major obstacle for practical disclosure of the meaning of stability standards, as no common-sense interpretations are possible, regardless of the acclaimed rationality of the overall framework. Rahola himself has stressed: *"When beginning to study the stability arm curve material … in detail, one immediately observes that the quality of the curves varies very much. One can therefore not apply any systematic method of comparison but must be content with the endeavour to determine for certain stability factors such values as have been judged to be sufficient or not in investigations of accidents that have occurred*". But, what is sufficient?

Today's standards do not offer explicit answer. The profession seems to be content with an implicit comparative criterion, whereby a required index **R** is put forward as an acceptance instrument (ultimately as "a" stability measure), without clear explanation as to what is implied if the criterion is met or in which sense is the goal of Table 1 catered ofr. In essence, the question "what does **A=R** mean", has not been explicitly disclosed. These notes summarise an approach devised in the SAFEDOR project, and which is hoped to allow for a simple yet comprehensive quantification of the phenomenon of ship stability, or more specifically ship's vulnerability to flooding, in the context of or explicitly expressed as a contribution to risk to human life. As such, it is suggested that the meaning of "stability" is far more comprehendible than has been possible by use of any other interpretation offered yet.

Table 3 LMIU 1987 - 2004, 21 fatal flooding accidents plus accidents of Al Salam[8] and Sea Diamond.

year	ship name	number of fatalities	total number of persons onboard	rate of fatalities
1987	Herald of Free Enterprise	193	459	42.05%
2000	Ciudad de Ceuta	6	290	2.07%
2000	Ytong I	1	10	10.00%
1999	Asia South Korea	56	606	9.24%
2004	Sam-Son	111	113	98.23%
1993	Jan Heweliusz	55	63	87.30%
1994	Estonia	852	989	86.15%
1996	Bukoba	869	1000	86.90%
1996	Gurita	338	378	89.42%
1992	Royal Pacific	3	514	0.58%
2000	Express Samina	94	500	18.80%
1991	Moby Prince	140	141	99.29%
1994	Al-Qamar Al-Saudi Al-Mis	21	527	3.98%
1998	Princess of the Orient	94	430	21.86%
1991	Salem Express	464	1310	35.42%
1999	Sleipner	20	85	23.53%
2001	Pamyat Merkuriya	26	51	50.98%
2002	Le Joola	180	1034	17.41%
2003	San Nicolas	28	228	12.28%
2002	Mercuri 2	49	51	96.08%
2003	Wimala Dharma	60	148	40.54%
2006	Al-Salam Boccaccio 98			
2007	Sea Diamond			

Figure 10 **Modern** cruise vessel Sea Diamond, April 6^{th} 2007.

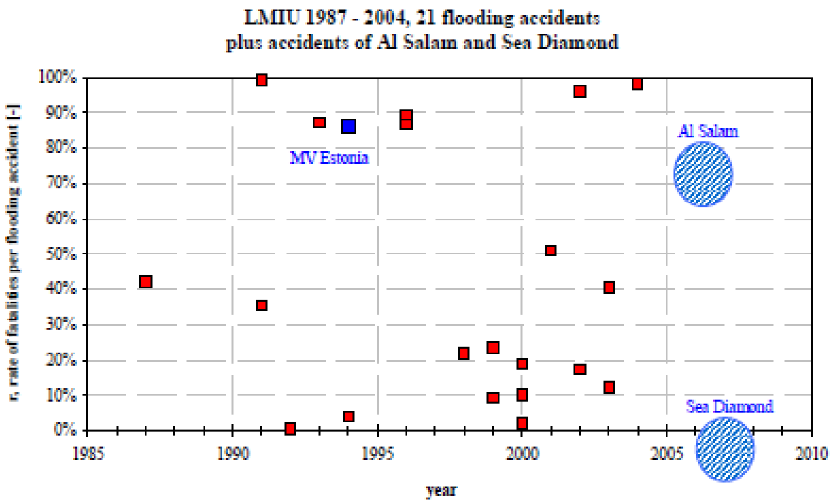
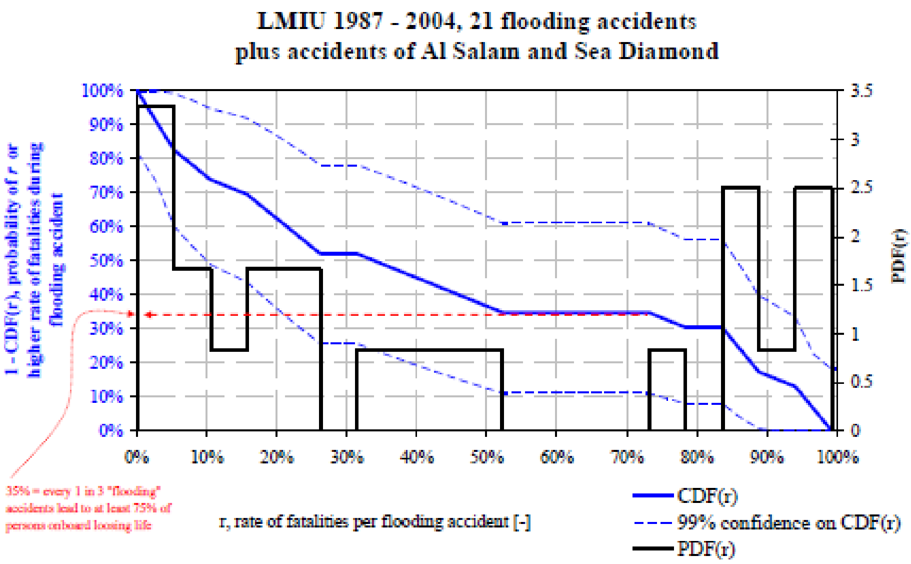


Figure 11 Historical record of the rate of fatalities in a fatal flooding accident, LMIU, 1985-2004 (red), and indication of accidents of Al Salam and Sea Diamond.



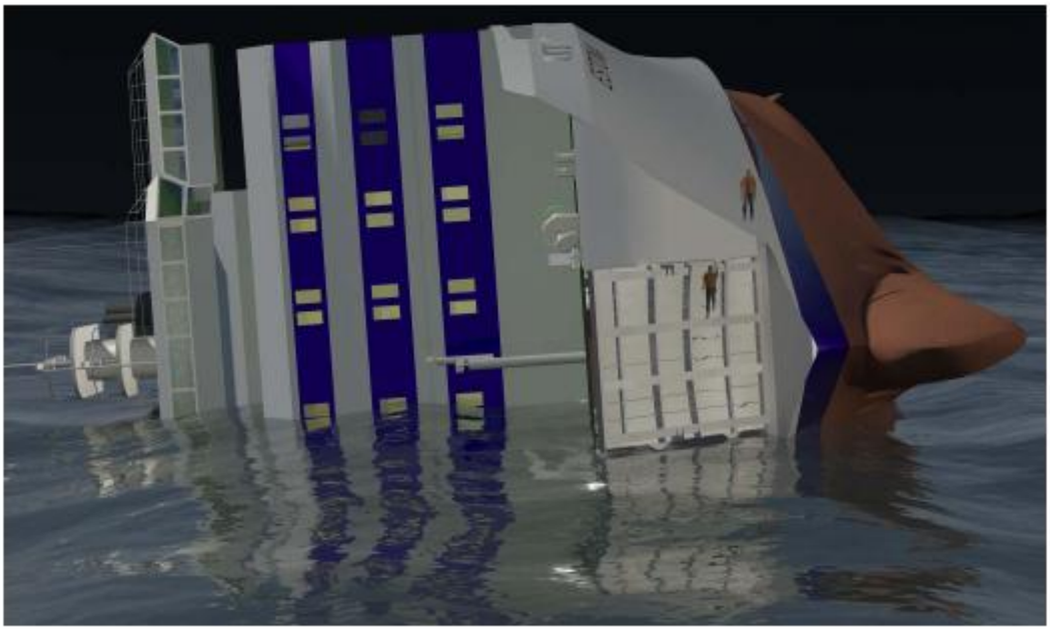


Figure 15 A dramatic moment of passengers abandoning the ship during the process of foundering. The ship lost stability due to flooding!

On the basis of accident records in the recent past, it can be expected that on average every three years there is a ship flooding accident where most, 75% or more, of persons onboard would lose their lives, given that one accident takes place every year on average. Not every accident is a rapid loss of stability, see Figure 10. Some 3,000 lives were lost since MV Estonia.

It becomes rather enlightening why accidents of the dimension of MV Estonia, where 853 people died on the night of 27/28th September 1994, see Figure 15, is possible to take place almost on regular basis. Figure 15 A dramatic moment of passengers abandoning the ship during the process of foundering. The ship lost stability due to flooding! Therefore, quantification of stability by means of marginal probability distribution for number of fatalities that can result from ship's inherent vulnerability to flooding (deficient stability), equation (18), affords a far more direct interpretation of SOLAS regulations on provision of stability. Such a stability measure can be directly compared with other loss scenarios in terms of contribution to risk to life (2), which instrument can be considered as the most objective basis to establish relative effectiveness of different safety regulations, which in turn can allow for more appropriate allocation of design/legislation and enforcement resources to effectively provide safety on ships. Also of note is the intermediate model (12) which itself is a major step towards achieving earlier objectives of IMO, MSC 75/4, 8th February 2001 setting that:

"*… analytical relationship between* ***time to sink {capsize}*** *and residual damage stability be developed … the methodology should make use of probabilistic principles as necessary to be compatible and used in conjunction with the future probabilistic harmonised stability calculation methods*".

This new knowledge puts immediately in question the order of 25 − 35% for *pN (N max)*for passenger ships, which cannot possibly be considered satisfactory. Perhaps the first step in revising this status quo of damaged ship stability should be the SOLAS 1974 goal, which could be rephrased as shown in Table 4. Table 4 (Proposed) SOLAS 1974 Chapter II-1, compare to Table 1.

“The subdivision of passenger ships into watertight compartments must be such that after ***any feasible*** damage to the ship's hull the vessel will remain afloat and stable”

A reasonable implementation of such a goal would be requirement for the rate with which catastrophic ship flooding accidents occur to be lowered to **no more than 1%** from the historical average level of some 35% per every fatal flooding accident, so to ensure that such accidents do not repeat in one hundred years on average 10, 11. To fulfil this requirement, in turn, steps should be taken to raise ship stability standards substantially with the target to minimise *pN (N max)*<1%.

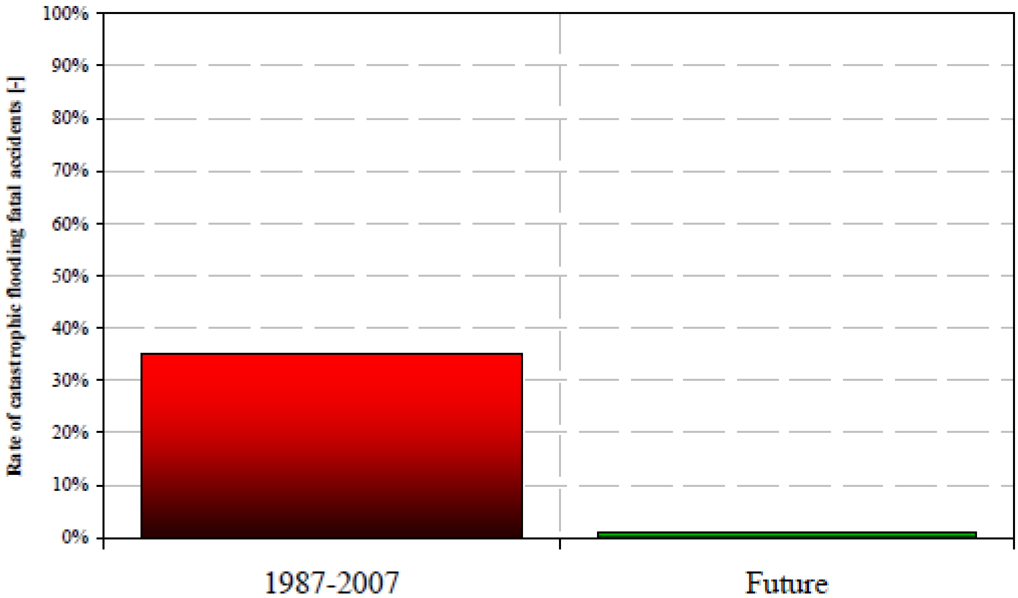


Figure 16 Attaining a goal of not more than 1% rate of occurrence of catastrophic flooding accidents would ensure that such accidents do not occur within hundred years on average[11].

1.2 Stockholm agreement & Ro-Ro Ships

The Ro-Ro concept provides the capability to carry a wide variety of cargoes in the same ship, thus being able to offer a competitive frequency with minimum port infrastructure or special shore-based equipment. Short sea routes are dominated by Ro-Ro ships with lorries, trailers, train wagons, containers, trade cars and passengers being transferred from the “outer” regions (UK, Ireland, Scandinavia and Finland) to the “main” land (continental Europe). In addition to that, in the Southern Europe corridors, the Ro-Ro freight service is progressively increasing in volume. The case for a long distance Ro-Ro service to provide a European maritime highway has also been made several times before. This is particularly relevant and important in respect of fast sea transportation where again Ro-Ro ferries play a prominent role. The main concern with the Ro-Ro ship design, whether justified or not, relates to safety and with safety becoming of paramount importance, it is vital that a rational approach to safety is demonstrated, validated and adopted. This is the right way to ensuring both the survival and a meaningful evolution of Ro-Ro ships in the future.

Along these lines, the maritime industry is acutely aware of recent shipping casualties involving Ro-Ro ferries, which have resulted in severe loss of life. Standards for Ro-Ro ship configuration, construction and operation have come under close scrutiny and new legislation has been put into place aimed at improving the safety of these vessels, notably SOLAS ’90, as the new global standard for all existing ferries with dates of compliance ranging from 1st October 1998 to 1st October 2010 depending on a combination of the vessel’s A/Amax[1] value, the number of persons carried and age. However, since the great majority of Ro-Ro passenger ferries were designed and built prior to the coming-into-force of SOLAS ’90, it is hardly surprising that few of them comply with the new requirements. Furthermore, concerted action to address the water-on-deck problem in the wake of the *Estonia* tragedy led IMO to set up a Panel of Experts (PoE) to consider the issues carefully and make suitable recommendations.

[1] The A/A_{max} calculation procedure, [2] is a simplified version of the probabilistic damage stability calculation of ships, [3] and was adopted by IMO as a means of trying to compare the survivability of one vessel against another in order to achieve a hierarchy for phasing-in purposes. It is not a survivability standard.

However, the complexity of the problem and the need to take swift action to reassure the public that appropriate steps are taken to avoid a repeat of the *Estonia* disaster influenced and shaped to a large extent both the initial and final proposals. In this pace of developments and following considerable deliberations and debate, a new requirement for damage stability has been agreed among North West European Nations to account for the risk of accumulation of water on the Ro-Ro deck. This new requirement, known as the ***Stockholm Agreement***, ameliorates the original proposals by demanding that a vessel satisfies SOLAS ’90 requirements (allowing only for minor relaxation) with, in addition, water on deck by considering a constant height rather than a constant amount of water as was originally intended. The dates of compliance with the provisions of the agreement range from April 1st, 1997 to October 1st, 2002. However, in view of the uncertainties in the current state of knowledge concerning the ability of a vessel to survive damage in a given sea state, an alternative route has also been allowed which provides a non-prescriptive way of ensuring compliance, through the “*Equivalence*” route, by performing model experiments in accordance with the Model Test Method of SOLAS ’95 Resolution 14.

Deriving from systematic research over the past twelve years, numerical simulation models have been developed, capable of predicting with good engineering accuracy the capsizal resistance of a damaged ship, of any type and compartmentation, in a realistic environment whilst accounting for progressive flooding. A comprehensive calibration/validation programme has allowed for sufficient confidence to be built up, rendering the developed models a valuable design “tool”. This, in turn, offered the ferry industry the attractive possibility of utilising such “tools” to assessing the damage survivability of ferry safety by using numerical simulation programs to effectively plan or, in time, replace the model tests, the so called “*Numerical Equivalence*” route. Numerical simulation readily allows for a systematic identification of the most cost-effective and survivability effective solutions to improving ferry safety and hence offers a means for overcoming the deficiency of the physical model tests route in searching for optimum solutions and an indispensable “tool” for the planning and undertaking of such tests. The close involvement of ferry owners/operators in North Western Europe with research projects in the wake of the *Herald of Free Enterprise* and the *Estonia* accidents was instrumental in nurturing industry to firmly accept the “*Numerical Equivalence*” route as a viable alternative for assessing Ro-Ro vessel survivability. This afforded SU-SSRC a unique opportunity to develop in close collaboration with NTUA-SDL a rational approach to ferry safety with the capability of attending to the needs of the shipping industry cost-effectively and led to the establishment of what is termed a “Total Stability Assessment” (TSA) procedure. The procedure comprises assessment of a vessel’s survivability utilising all the currently available instruments, namely: A.265 (VIII) + amendments (probabilistic procedure), SOLAS ’90, Stockholm Agreement (prescriptive criteria) and safety “Equivalence” tests by means of physical model experiments and numerical simulations (performance-based criteria). A schematic illustration is provided in Figure 1. The tightening of legislation described above is coupled with serious considerations at IMO for regular application of risk assessment methods, for example, the *Formal Safety Assessment*. In this context, considerable attention has been focusing on the application of probabilistic procedures of damage stability assessment for the evaluation of Ro-Ro vessels and it appears more than likely that developments in the foreseeable future will most certainly adopt a framework of a probabilistic description. The regulatory regime described in the foregoing has understandably left the shipping industry in a state of confusion and uncertainty concerning the available options, approaches and optimum choice to ensure compliance and to ascertain the level of safety attained with regard to any such choice.

Stated specifically, a ship owner today is faced with the following choices concerning safety standards:

(i) Deterministic (SOLAS ’90) Vs probabilistic (A.265 (VIII))
(ii) Prescriptive (SOLAS ’90 + 50) Vs performance based (physical model experiments or numerical simulations)

Standards in each group are assumed to ensure an “equivalent” level of safety, correspondingly, whilst a serious attempt to demonstrate such equivalence is totally lacking. Adding to the confusion is the fact that the dates of compliance with deterministic/prescriptive standards are decided on the basis of a simplified probabilistic approach (calculation of A/A_{max}). In response to the challenge presented by this state of affairs, the maritime industry, slowly but steadily, appears to be favouring the model experiments route, implicitly demonstrating a preference towards performance-based safety standards over deterministic static stability standards when addressing the damage survivability of new concept designs. Not only is the introduction of performance standards a major development in assessing safety but it is also seen as beneficial from the industry as these readily allow consideration of alternative designs as well as a rapid implementation of technological innovation.

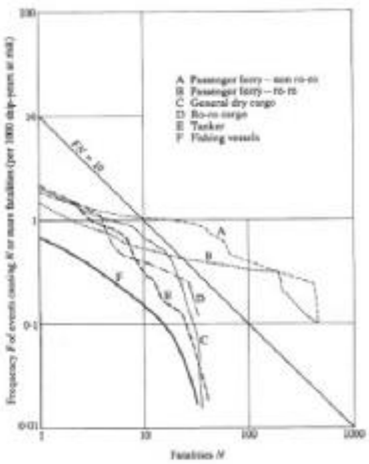
1.3 The probabilistic aspect

It is proposed for the sake of simplicity that the risk is understood as a *"chance of a loss"*, whereby the "chance" is quantified by means of various statistics of the loss, and "the loss" is measured by an integer number of fatalities, *N* (no type of injury is considered). Two commonly used statistics of the loss will be considered2, namely "FN curve" and "PLL", as explained below.

$$F_N(N) = \sum_{i=N}^{N_{max}} fr_N(i) \qquad (1)$$

cumulative distribution of frequency for occurrence of N or more number of fatalities per ship per year, known as an "F-N curve"

Where max *N* is the total number of persons considered (e.g. number of crew, number of passengers or both, onboard the ship), and *fr* (*N*) *N* is the frequency of occurrence of exactly *N* number of fatalities per ship per year, given by equation (3).



The second statistic often considered is the expected number of fatalities, *E*(*N*), given by

$$Risk_{PLL} \equiv E(N) \equiv PLL \equiv \sum_{i=1}^{N_{max}} F_N(i) \equiv \sum_{i=1}^{N_{max}} i \cdot fr_N(i) \qquad (2)$$

The frequency of occurrence of exactly *N* number of fatalities per ship per year, *fr* (*N*)*N* , can be obtained from a form of Bayes' theorem on total "frequency", namely:

$$fr_N(N) = \sum_{j=1}^{n_{hz}} fr_{HZ}(hz_j) \cdot p_{N|HZ}(N|hz_j) \qquad (3)$$

Where hz_n is the total number of loss scenarios considered as exhaustively contributing to risk to life, and *hz j* represents an event of the occurrence of a chain of events *HZ* , (a loss scenario), identifiable by any of the following principal hazards:

Table 2 Principal hazards

j	Principal hazards, hz_j	Average historical frequency of its occurrence, $fr_{HZ}(hz_j)$
1	Collision and flooding	2.58e-3 , ref [17]
2	Grounding and flooding	
3	Fire	
4	Intact Stability Loss	
5	… etc	

Modelling basis

As a first step in introducing the model, consider an event of exactly N=100 number of fatalities occurring as a result of flooding of an extent *D* = *d* of a ship during a collision scenario *HZ* = *hz*1 . For convenience let the following notation be assumed in relation to occurrence of scenario *hz*1 :

$$p_{X|HZ}(x|hz_1) \equiv p_X(x)$$

The joint probability mass of a compound event $D \cap N$can be then written, based on Bayes' theorem, as:

$$p_{D\&N}(d \cap N) = p_D(d) \cdot p_{N|D}(N|d)$$

Where *pr* (*N* *d*)*N*\ *D* is conditional probability mass for occurrence of exactly N number of fatalities, given damage extent *D* = *d* occurred.

Assigning probabilities

The question arises as to which events should be considered to meaningfully quantify the marginal distribution of probability for number of fatalities occurring due to flooding in a ship-to-ship collision? An answer can be derived from today's practice, namely from the developed and adopted concept of international standard of ship stability.

The standard stipulates that stability should be assessed with respect to occurrence of (a) ship loading condition, *W* , (b) ship flooding extent, *D*, (c) the environment, *E* and (d) the ship geometry. As a step in expressing stability in terms of risk, this assumption leads to the sought format of the conditional probability mass for number of fatalities, *p*(*N*) *N* , as follows:

$$p_N(N) = \sum_{\Omega \setminus N} p_{W\&D\&E\&N}(w \cap d \cap e \cap N) \qquad (4)$$

$$p_{W\&D\&E\&N}(w \cap d \cap e \cap N) = p_W(w) \cdot p_D(d) \cdot p_{E|D}(e|d) \cdot p_{N|W\&D\&E}(N|w \cap d \cap e) \qquad (5)$$

Where all the relevant models of (5) are presented below.
Ship loading condition
The probability distribution for loading conditions a ship operates at can be assigned as:

$$p_W(w = light) = w_l = 0.2$$
$$p_W(w = partial) = w_p = 0.4 \qquad (6)$$
$$p_W(w = subdivision) = w_s = 0.4$$

Note that it has become customary to introduce a one-letter notation for the probability, and hence care should be taken in interpretations. If more accurate information is available on the actually expected distribution of occurrence of given loading conditions, then the above assignment should be replaced.

<u>Flooding extent and ship geometry</u>

Assigning of the distribution of probability for flooding extent, $p(d)_D$, has been based on the historical data on possible damage characteristics, location, length, penetration and height, as shown in the figure below.

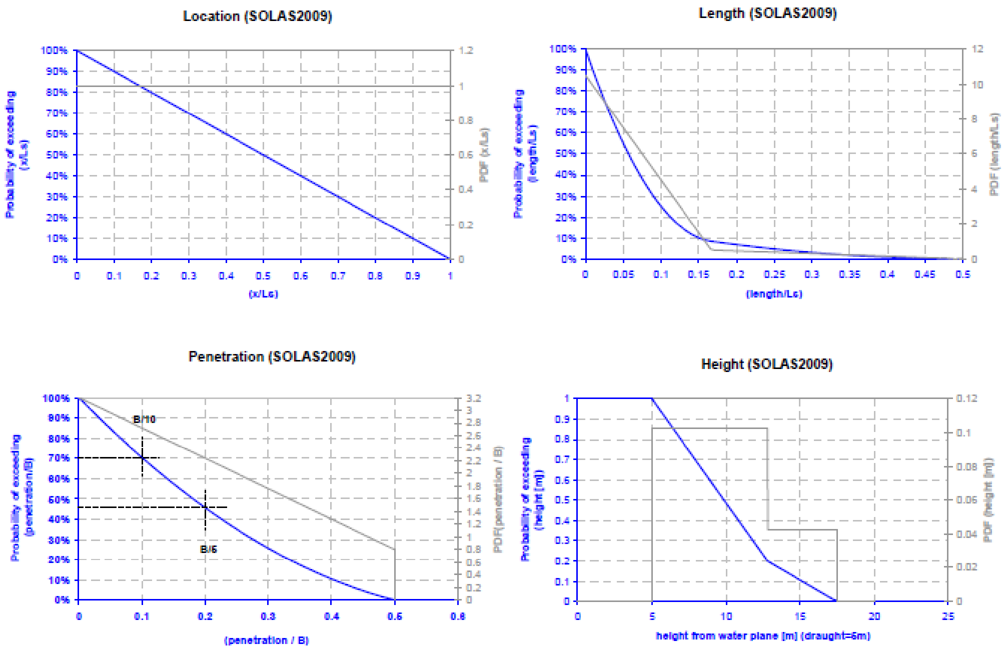
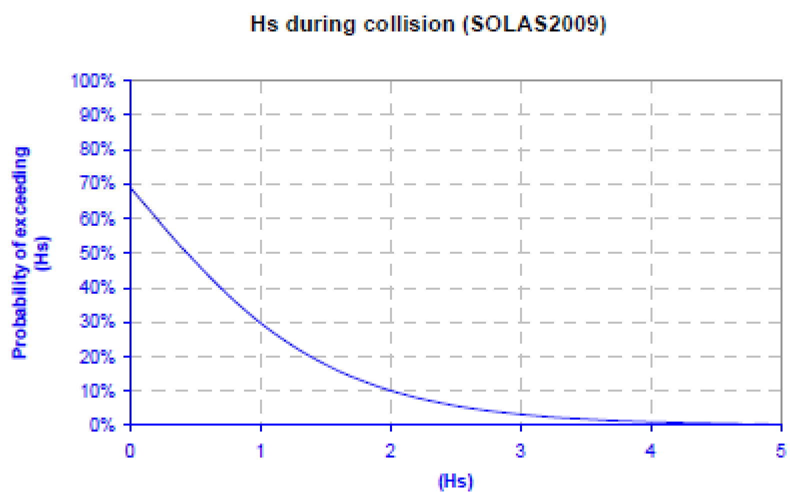


Figure 2 Probability distributions for damage characteristics

<u>The environment</u>

Assignment of the probability distribution for the environmental conditions expressed in terms of significant wave height Hs, $e = \{Hs\}$, and which a ship is expected to encounter in an event of a collision, $p_{E|D}(Hs|d) \equiv p_{E|HZ\&D}(Hs|hz_1 \cap d)$, has also been based on historical data, as shown in figure below.



<u>Number of fatalities and ship geometry</u>

The model for assigning conditional probability mass for number of fatalities $p_{N|W\&D\&E}(N|w \cap d \cap e)$, given loading condition *W* = *w* , flooding extent *D* = *d* and environment *E* = *e* occurred, can be developed from:

$$p_{N|W\&D\&E}(N|w \cap d \cap e) = f_{T|W\&D\&E}(t_{fail}(N)|w \cap d \cap e) \cdot |\partial t_{fail}(N)| \qquad (8)$$

Where:

$$\partial t_{fail}(N) = t_{fail}(N) - t_{fail}(N-1) \qquad (9)$$

$$t_{fail}(N) = N_{fail}^{-1}(t) \qquad (10)$$

$$N_{fail}(t) = N_{\max} - N_{evac}(t) \qquad (11)$$

And where the term $N(t)_{evac}$ is the number of passengers evacuated within time *t* , referred to as an “evacuation completion curve”, see figure below. Such a curve can effectively be estimated on the basis of numerical simulations. Note that ship geometry is implicitly accounted for in $N(t)_{evac}$.

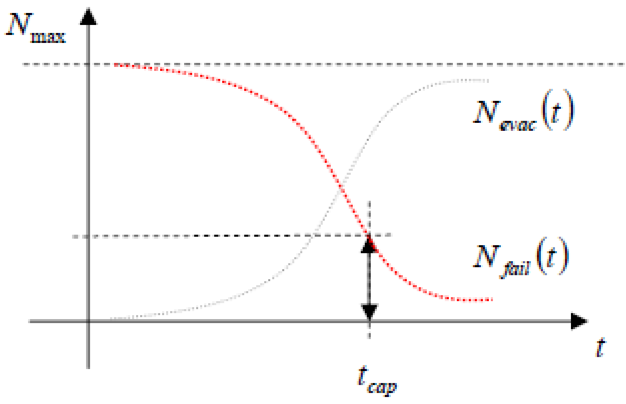


Figure 4 Evacuation completion curve

The function $f_{T|W\&D\&E}(t|w\cap d\cap e)$, is distribution of probability density for random variable expressing the time during which a ship of given geometry will capsize as a result of flooding of extent *D* = *d* in an environment *E* = *e* and at loading *W* = *w* . Let a set of matrix numerators (indexes) describe a number of variables as follows:

i denotes a successive loading condition of the set given by equation (6), *j* denotes a successive flooding extent of the set of *flood n* possible cases, and *k* denotes a successive environmental condition of the set of *Hs n* given by the equation (7), for each of *k Hs* , where

$0 < Hs_k \leq 4m$, $Hs_k = k \cdot 4 \cdot n_{Hs}^{-1}$.

Let also denote the mathematical $f_{T|W\&D\&E}(t|w\cap d\cap e)=f_{i,j,k}(t)$, i.e. in terms of numerical notation. It has been shown in [20] that the distribution of probability density for time to capsize, *f* (*t*)*i*, *j*,*k* , in a given flooding case, environment and loading can be described by the following model (12)

$$f_{i,j,k}(t) = -\ln(\varepsilon_{i,j,k}) \cdot (\varepsilon_{i,j,k})^{\frac{t}{t_0}} \cdot t_0^{-1} \qquad (12)$$

$$t_0 = 30 \text{ min} \qquad (13)$$

$$\varepsilon_{i,j,k} = 1 - \Phi\left(\frac{Hs_k - Hs_{crit}(s_{ij})}{\sigma_r(Hs_{crit}(s_{ij}))}\right)$$

$$\sigma_r(Hs_{crit}) = 0.039 \cdot Hs_{crit} + 0.049$$

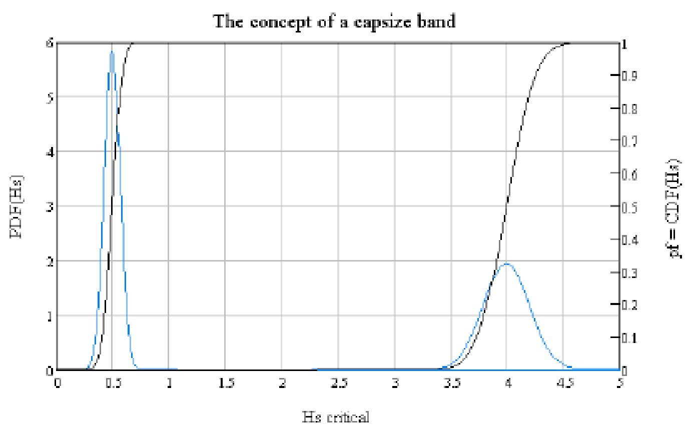


Figure 5 The approximated spread in the rate of capsizing, $p_f = \Phi(Hs)$, within a capsize band.

The relationship between the significant wave heights at the instant of collision, here equal to the critical sea state6, and the “ s ” factors is given by equation (16)

$$Hs_{crit}(s)=\begin{cases}0 & if \quad s\leq 0.3093\\ \dfrac{0.16-\ln(-\ln(s))}{1.2} & if \quad s>0.3093\end{cases}$$

Where $s=(\frac{GZmax*Range}{0,12*16})^{1/4}$

Finally, the model (4) for assigning of marginal probability mass distribution $p_{N|HZ}(N/hz_1)$ for a number of fatalities N that can occur as a result of scenario of flooding due to a collision and subsequent loss of stability, $hz1$, can be presented in a numerically disclosed form (18), as follows:

$$p_{N|HZ}(N|hz_1)=\sum_{i}^{3}\sum_{j}^{n_{flood}}\sum_{k}^{n_{Hs}}w_i\cdot p_j\cdot e_k\cdot c_{i,j,k}(N) \qquad (18)$$

where

$$c_{i,j,k}(N)=\left(-\ln(\varepsilon_{i,j,k})\cdot(\varepsilon_{i,j,k})^{\frac{t_{fail}(N)}{t_0}}\right)\cdot\frac{\left|\partial t_{fail}(N)\right|}{t_0} \qquad (17)$$

Quantification of uncertainty

Quantification of the sampling error is based here on the continuity-corrected version of the Clopper-Pearson confidence interval[1] on the binomial proportion, **Error! Reference source not found.** This confidence interval is referred to as a sampling error, and details of its implementation are outlined below.

The probability $C = (1-\alpha)$ for occurrence of proportion $\hat{p} = \frac{x}{n}$, (occurrence of x instances of interest in n number of trials), can be assigned based on an inverse of the regularised incomplete Beta function given by (1).

$I_\beta(a,b,x) = \dfrac{\int_0^x t^{a-1} \cdot (1-t)^{b-1} \cdot dt}{\int_0^1 t^{a-1} \cdot (1-t)^{b-1} \cdot dt}$	(1)

Namely, given the desired probability C, an interval that contains the "true" binomial proportion $\hat{p}$ can be found as $[\hat{p}_{lo}; \hat{p}_{up}]$, where $\hat{p}_{lo} = \frac{x_{lo}}{n}$ and $\hat{p}_{up} = \frac{x_{up}}{n}$, and where the number of occurrences, x_{lo} and x_{up}, derive from the inverse solutions to equations (2) and (3), respectively:

$I_\beta\left(n - x_{lo} + \frac{1}{2},\quad x_{lo} + \frac{1}{2};\quad 1 - \hat{p}\right) = \frac{\alpha}{2}$	(2)
$I_\beta\left(n - x_{up} + \frac{1}{2},\quad x_{up} + \frac{1}{2};\quad 1 - \hat{p}\right) = 1 - \frac{\alpha}{2}$	(3)

In case of cumulative probability function for random variable X, the above proportions refer to maximum number of occurrences of random variable X up to the specific value x.

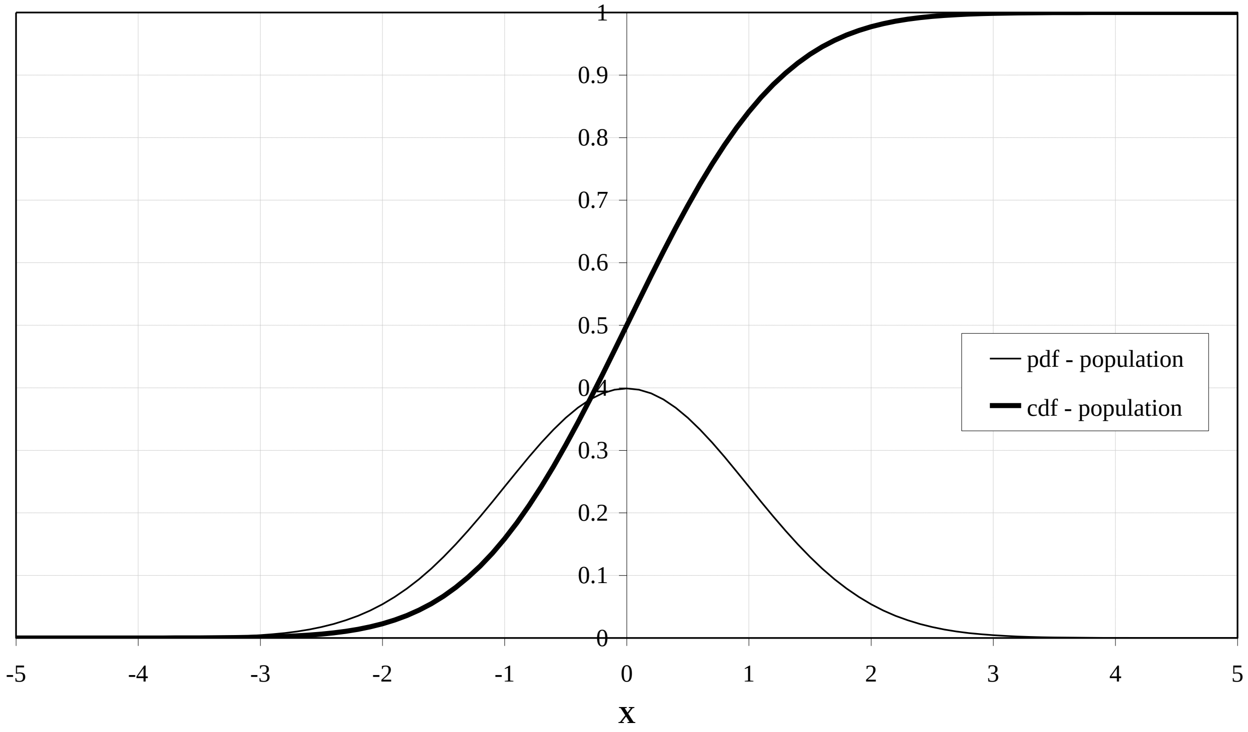


Figure 1 An example of probability distribution for the population of random variable X, where X~N(0,1).

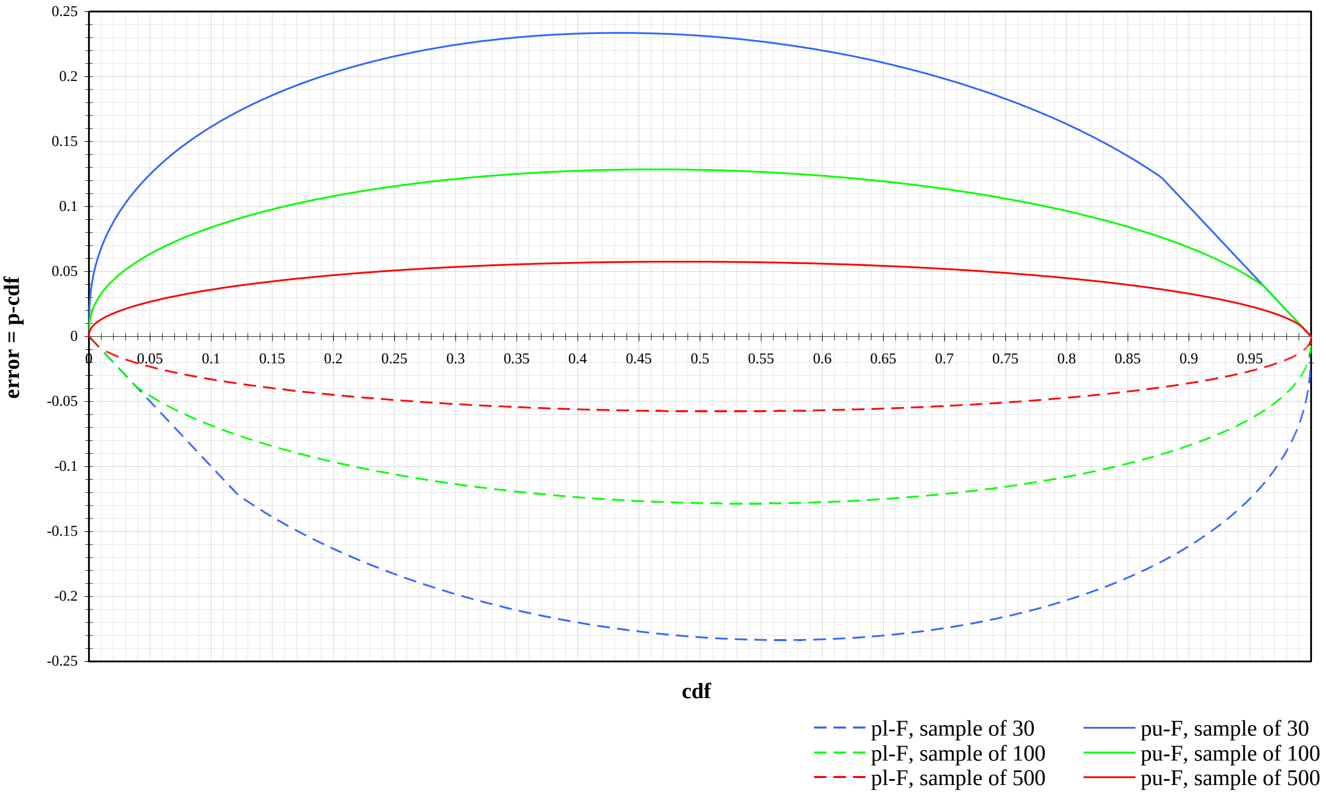


Figure 2 Continuity-corrected Clopper-Pearson 99% confidence interval on binomial proportion for n=30, n=100 and n=500. The interval is referred to as a sampling error.

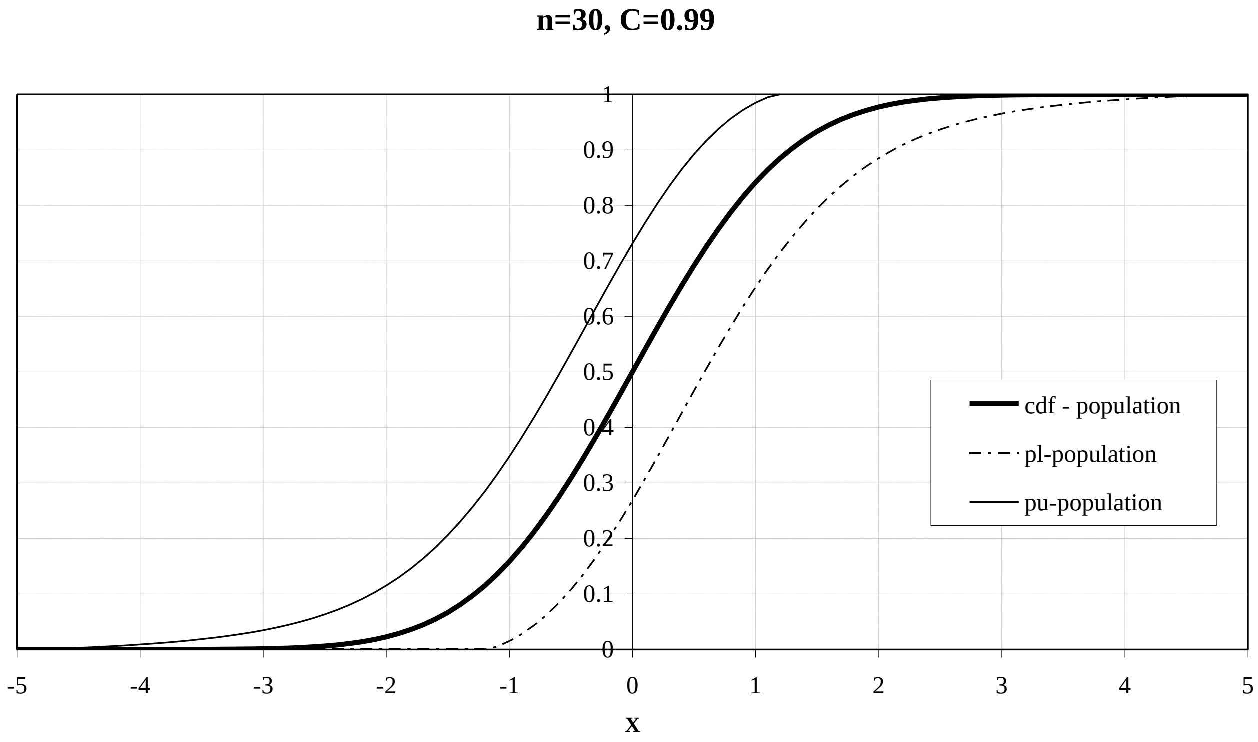


Figure 3 A 99% sampling error on the cdf for the population of random variable X. Sample size n=30.

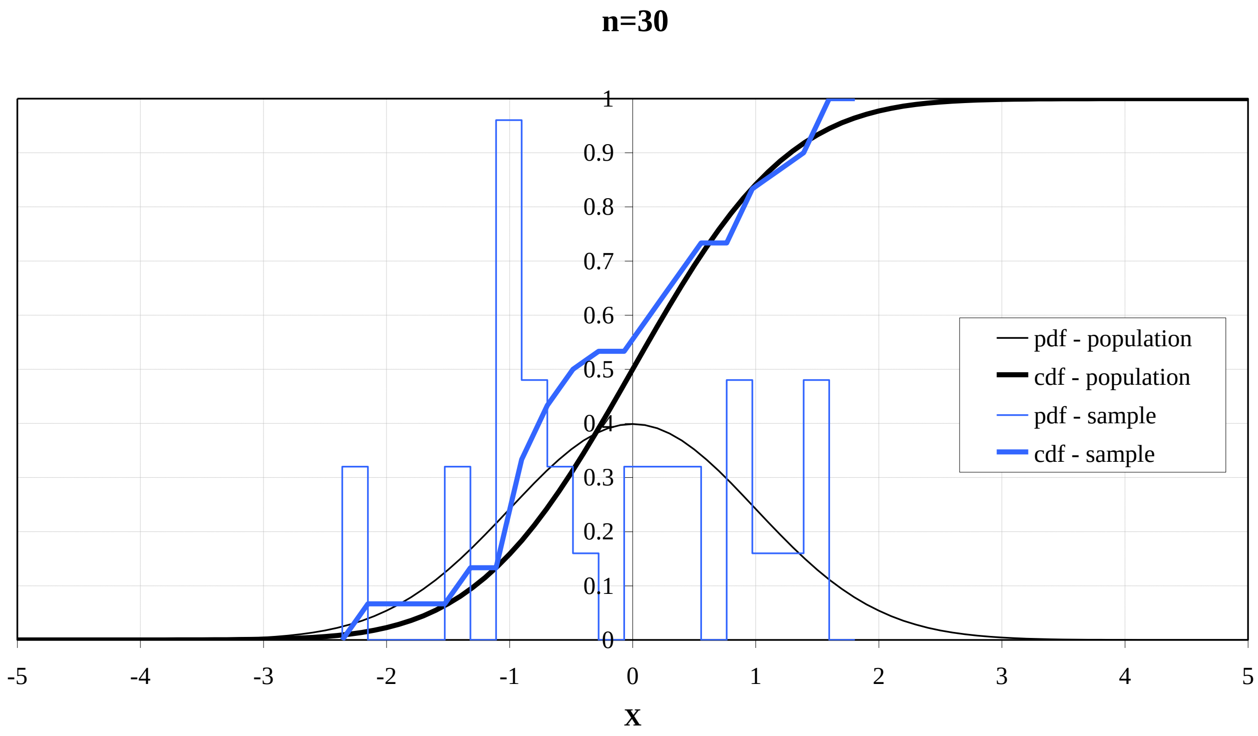


Figure 4 An example of probability distribution for a randomly generated sample of variable X. Sample size n=30.

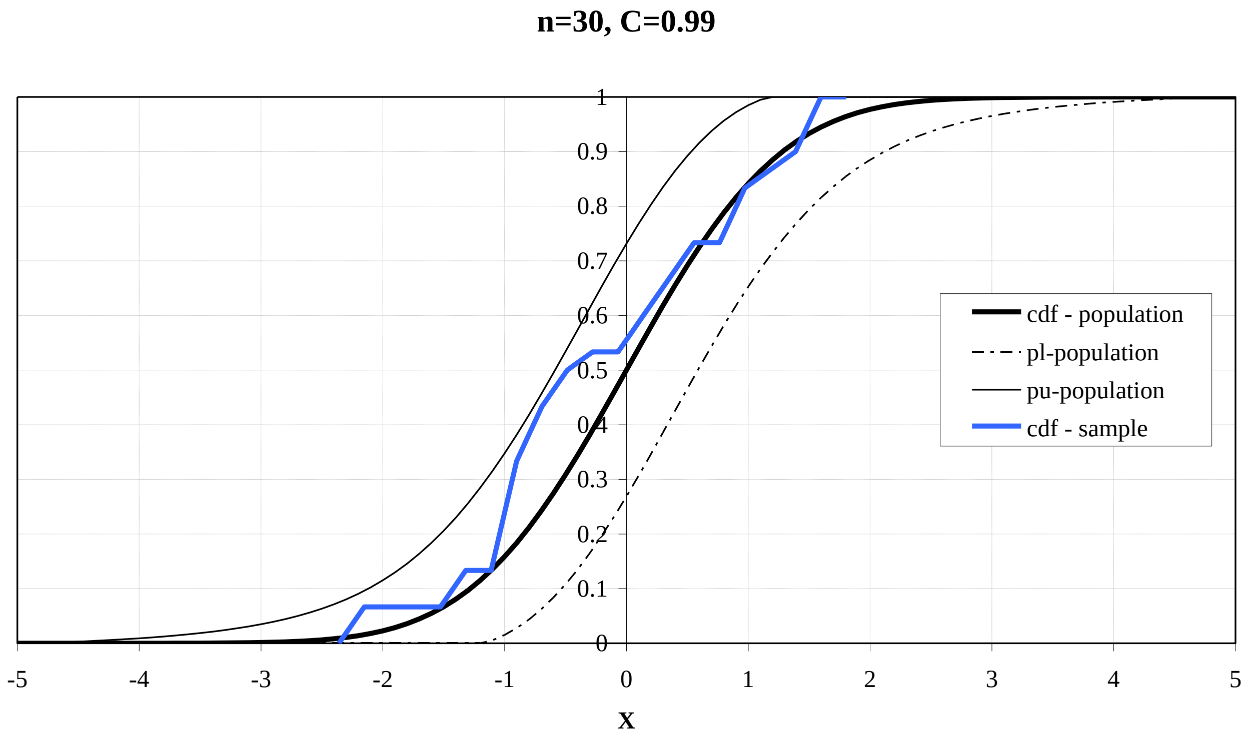
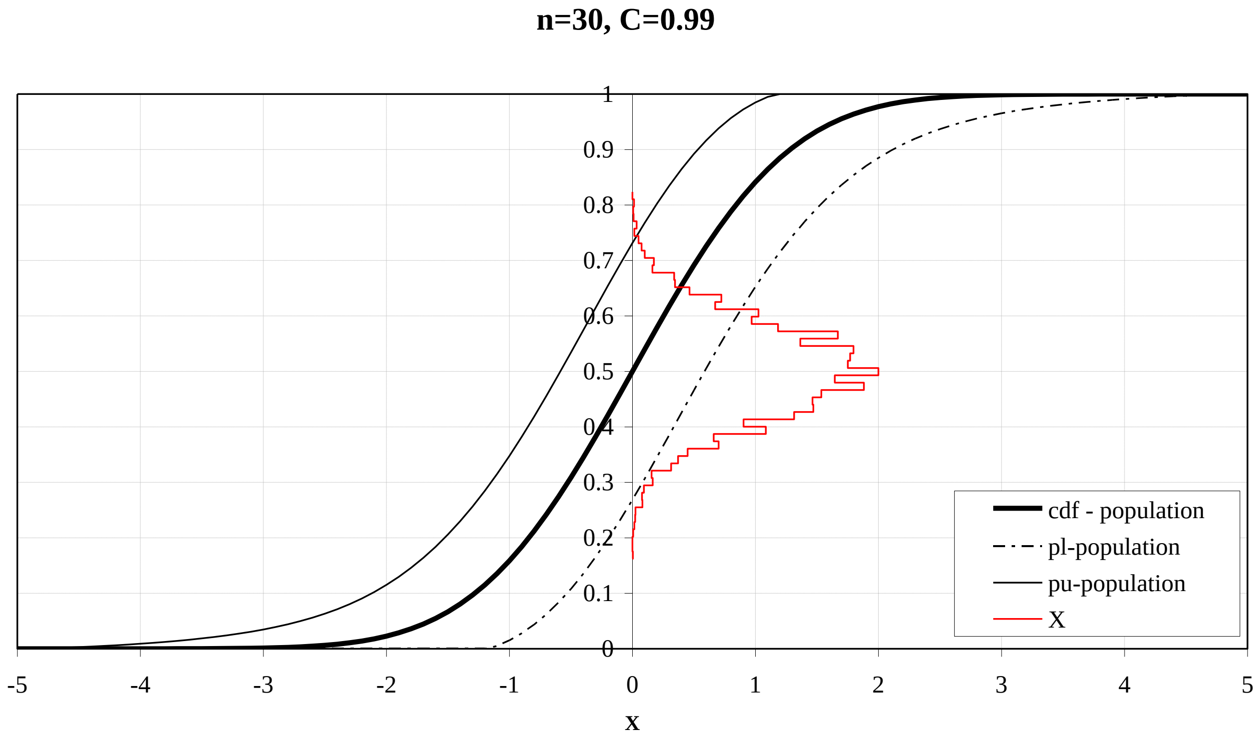


Figure 5 The cdf assigned on the basis of the sample of 30 will be contained within the 0.5% quantiles around the cdf for the population, if it is known.



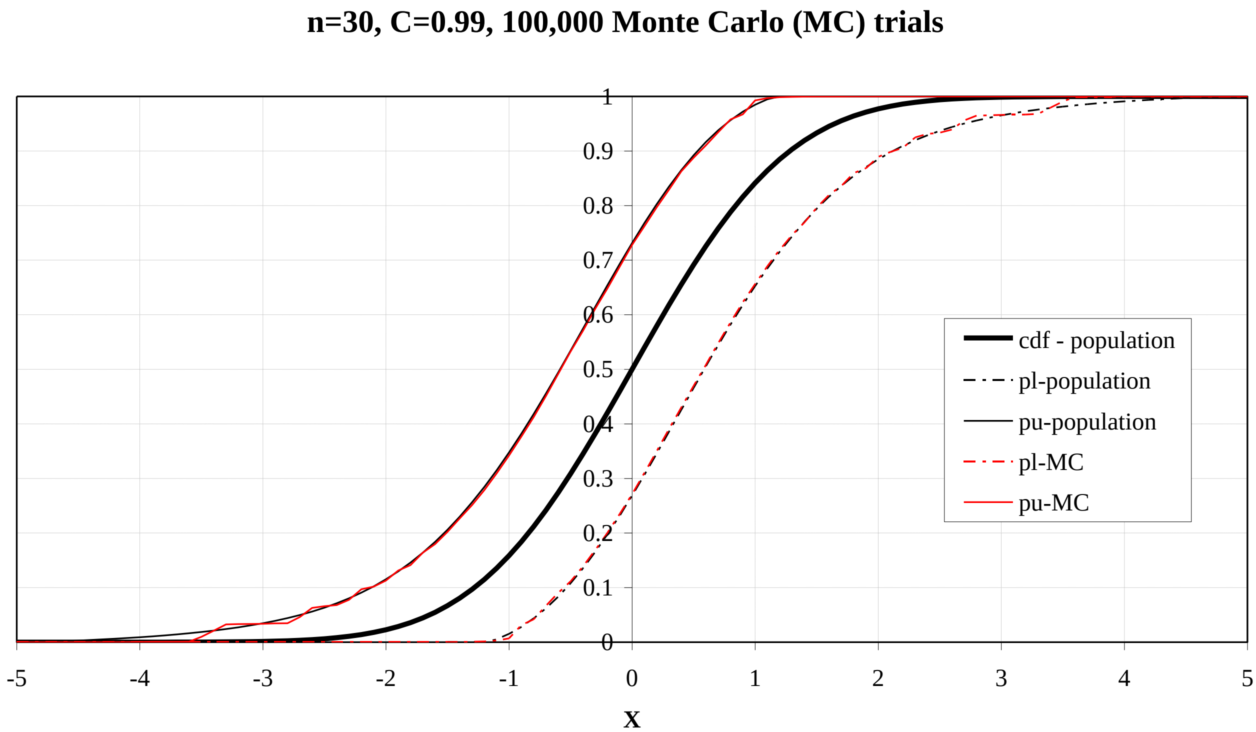


Figure 6 A Monte-Carlo experiment confirms that only in about 1000 occasions out of 100,000 samples of 30 elements drawn randomly from population N(0,1), would the cdf for any of the samples be beyond the 0.5% quantiles off any value of the cdf for the population. Sample size n=30.

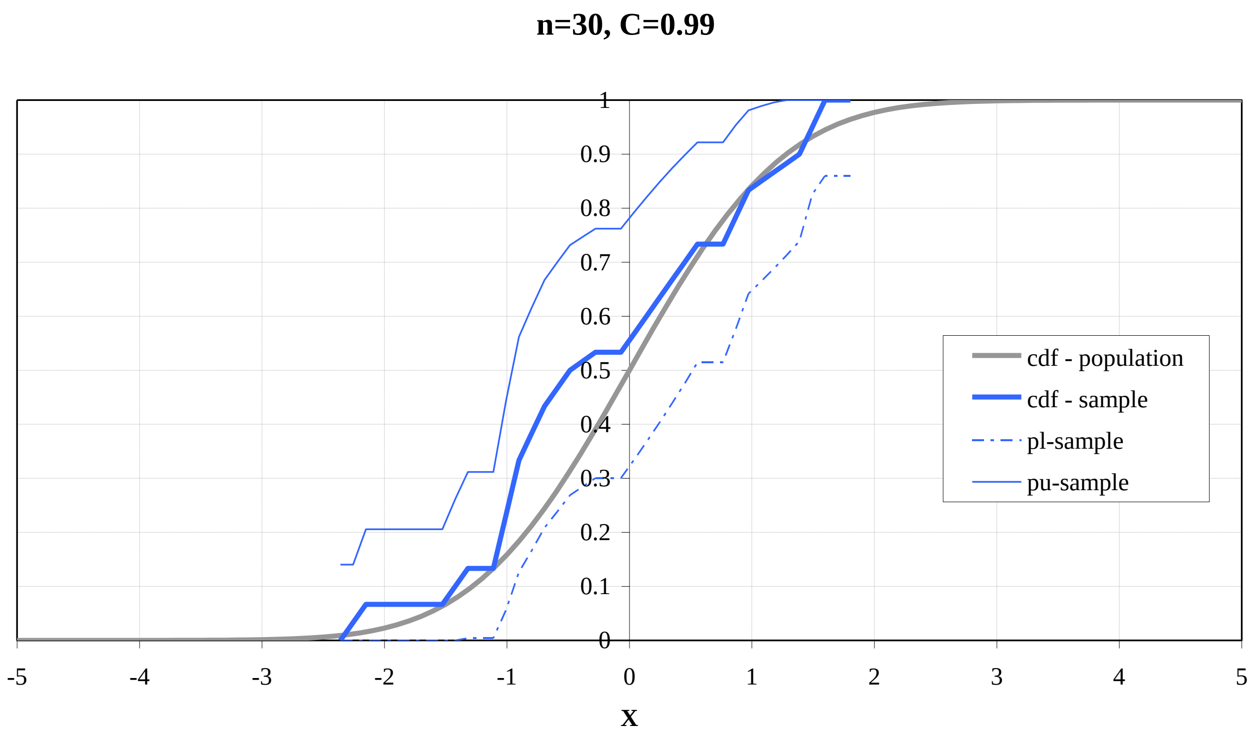


Figure 7 Since the cdf of a population is never known, the sampling error allows to derive the interval around the sample cdf within which the population cdf can be expected with C probability. Sample size n=30.

Approach Method

2.1 the actual target

As it is already mentioned, the main aim of this project is to derive data for validation of technique for prediction of survival time after a collision damage, as well as establishing uncertainty of predictions. To this end, the following specific objectives will be targeted:

1. Gain an in-depth understanding of the Stockholm agreement rules and follow them in the whole experimental process.
2. Use an existing model of a Ro-Pax ship according the Stockholm agreement regulations.
3. Find the Hs bounds for the 100% capsizing and the 100% of survivability of the boat.
4. Perform 20 runs for each 0.2 m of Hs in between the bounds and make a statistical analysis of the results
5. Create the histograms with the probability distribution function for the time to capsize for each Hs.
6. Calculate the CDF from the histograms for the time to capsize for each Hs
7. Compare the CDF curve we create with the analytical CDF curve.
8. Create the uncertainty bounds for the experimental CDF curves.
9. Construct the CDF(Hs) (which shows the probability for capsize in each Hs) according our experimental results.
10. Compare this curve with the analytical CDF(Hs) curve.
11. Derive conclusions.

2.2 The experiment set up

Before the experiments begins, it must be ensured that the measuring organs are measuring the model motions with the appropriate precision, the wave maker creates the correct waveform and the calculation of the models centre of gravity with the inclining test has been completed.

Wave maker

The wave maker installed in the Ship Hydrodynamics Centre's Towing Tank is a hydraulically operated wet-backed, single flap type paddle that is positioned at the end of the tank. The computer software that controls the wave maker paddle provides the ability to produce a wide variety of wave forms, including: regular waves, most commonly known irregular wave spectra and user defined wave spectra or sequences. A large range of waveforms can be generated in the Towing Tank include:

Regular Waves
The wave makers in each facility can readily generate regular waves where the selected discrete wave frequency and height remain constant.

Wave Spectra

The wave makers in each facility can generate wave sequences as defined by the following idealised standard irregular wave spectra:

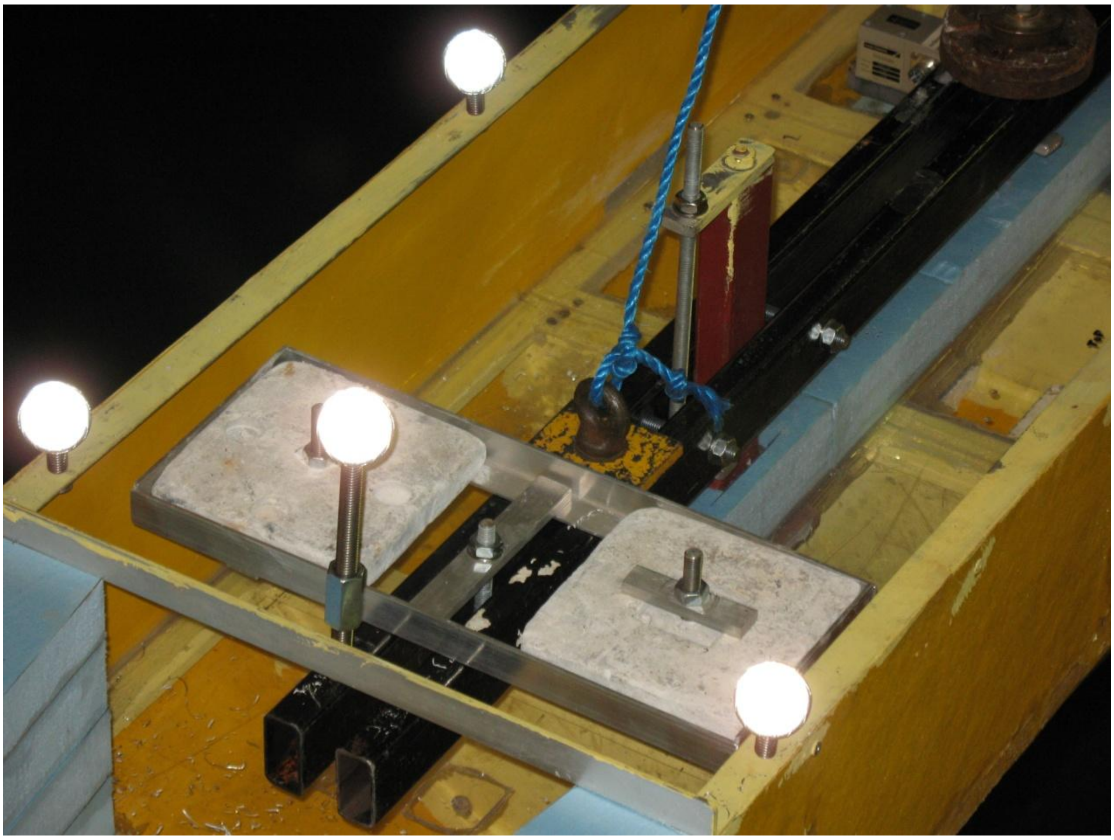
- Jonswap
- Pierson-Moskowitz
- ITTC (International Towing Tank Conference)
- ISSC (International Ship Structures Conference)
- Bretschneider/Mitsuyasu
- Neumann
- Darbyshire Coastal
- Darbyshire Ocean
- Top Hat/Pink
- BTTP

The produced waves form is: $\zeta = \sum_{n=1}^{N} A_i * \sin(\omega_i t - k_i x + \varepsilon_i)$. By changing the random number ε_i, we can have a different realization of a specific wave. These different realizations, are being used for the statistical analysis for the number of capsizes in each sea condition (Hs, ω_p). During the experiment it must be ensured that the wanted waveform is also produced as it wanted. For this reason, the produced waves are tested in the tank in calm water conditions, without the model.

The wave maker was tested while the water was calm in the tank, and the produced waves had the expecting waveform.

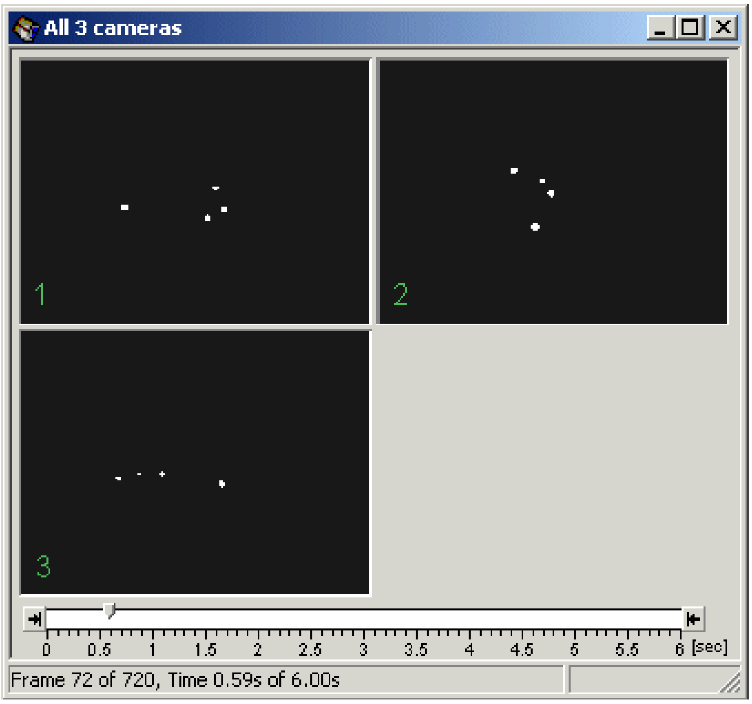
Capture system

The motion capture system which is used during the experiment is a set of three cameras facing the model from different view angles defining the measuring volume. These cameras identify the motion of four markers placed on the model. The change of the position of the three markers on the boat, relatively to the Oxyz axis system which is manually defined, gives the required information about the roll, surge, sway, pitch, yaw, heave of the boat.

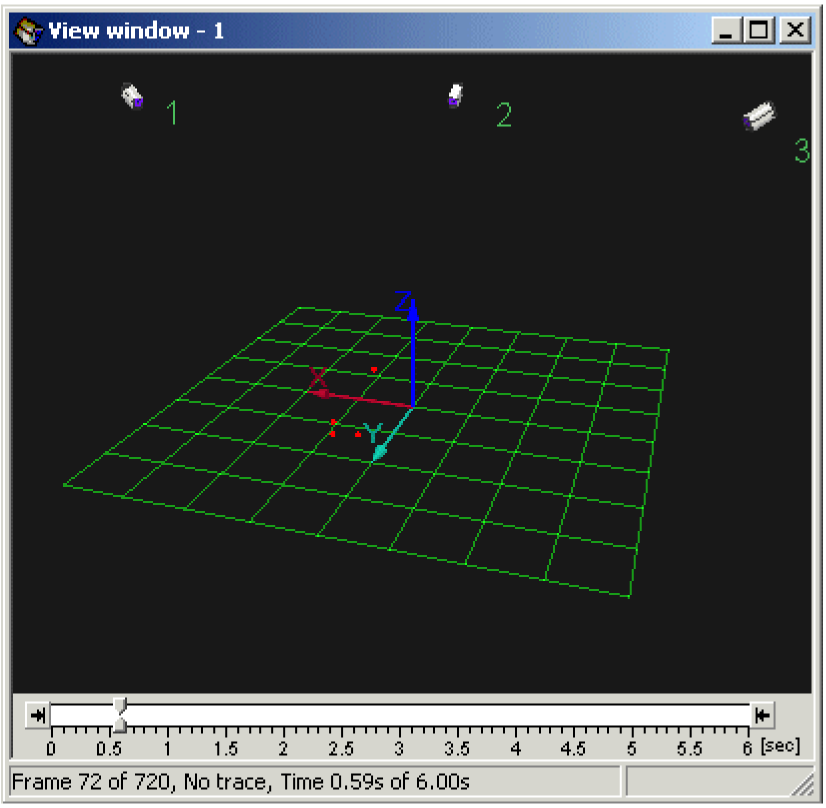


All these information plus the carriage speed and the travel and fixed waveform the wave makers produce are transferred through 9 canals to a computer software named Spike.

The first step for the camera calibration is to define the Oxyz axis system. Firstly we hide the markers that are on the model and we place on it four markers located on the same layer (shape Γ), then the cameras are being adjusted one at a time, so every camera can see all of the markers.



The next step is to ensure that cameras can see a specific object's length with the same value, whatever the position of the object. For this reason, a stick with two markers on each side is being used. The distance between the markers is fixed, 50 cm. The markers are getting moved around the measuring volume for a specific time period 10 sec. During this period the camera system is taking samples for the distance between the markers. When the calibration is done a window with the calibration results is shown. The results will tell if the calibration passed and some calibration quality results. If the calibration fails, the cameras have to be readjusted.



More information about the camera calibration, are included in the manual of the motion capture camera QTM (qualisys track manager). This specific brand was used during the whole experimental process for this project.

Inclining test

The purpose of this procedure is to determine the centre of gravity of the model ship. To determine the centre of gravity we use the inclinometer. This device is connected with the boat and measures the first zero point. The first zero point is the roll angle which the boat has after the ballasting. Then the starboard roll ($Tan\theta_{starboard}$) angle is measured after moving a specific weight, the exact value of which is known, from one side of the boat to the other, in a specific distance. As a next step the weight is placed back to the initial position and the second zero point ($Tan\theta_{port}$) is measured. After that, the port angle is measured as soon as the weights are moved to the port side, at the same distance as before.

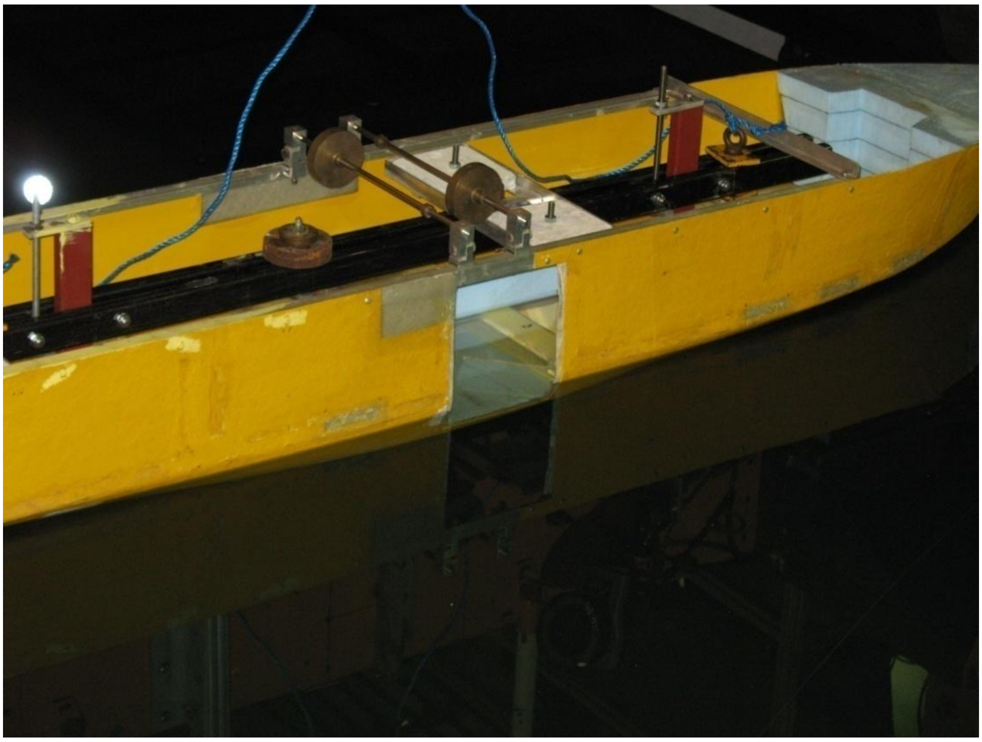
$$GM = w*d/(W_F*Tan\vartheta)$$

w = Inclining Weights
d = Distance Weights Were Moved
W_F = Displacement of model (with Inclining Weights)
$Tan\vartheta = (Tan\vartheta_{starboard} + Tan\vartheta_{port})/2$

The KM is already known, so KG can easily be calculated from the following equation: KG=KM-GM

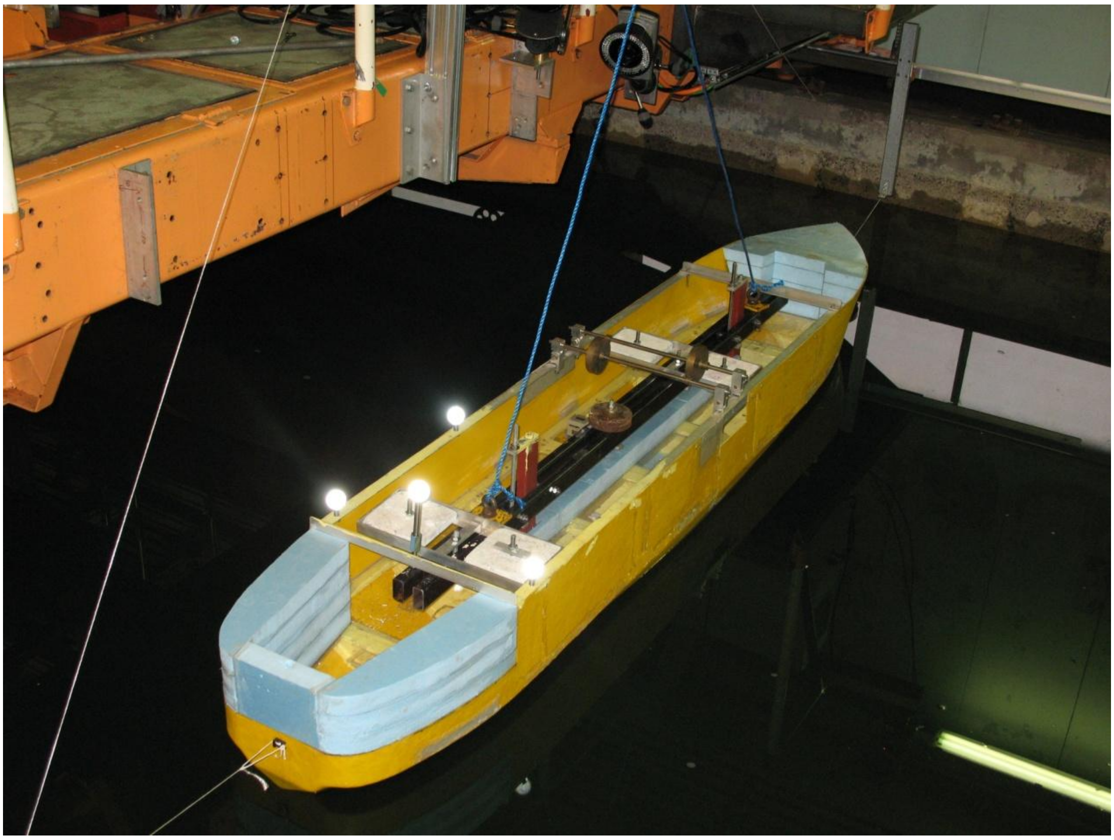
2.3 The model

The model used for the experiments, is an already existing model in the laboratory of ship stability of Glasgow and Strathclyde Universities. The name of the model is Pentalina and it has already been used for several other damage stability researches.



Original Pentalina is a passenger/ Ro-Ro cargo ship which was built in May of 1970 and belongs to the shipping company Pentland ferries. This ship, is currently covering the run between the Isle of Lewis and the Scottish mainland.

The model has a removable part on the hull.



During the experiments this part is being removed so that way we can simulate the flooding after the collision in the car deck. The damaged side of the ship was selected to be on the starboard side. The general particulars of the model and the ship are given in the following table:

GENERAL PARTICULARS

Model Scale = 1: 27

Longitudinal dimensions are measured from frame 0
Vertical dimensions are measured from baseline

	SHIP (m)	MODEL (mm)	NOTES	CHECKED
MAIN DIMENSIONS				
LOA	74.00	2740.7	of ship	
LMOD	73.65	2727.8	of model	
LBP	70.10	2596.3		
BMLD	13.40	496.3	to hull	
BMLD	NA	NA	to sponsons	
DMLD	9.80	363.0	to top of model	

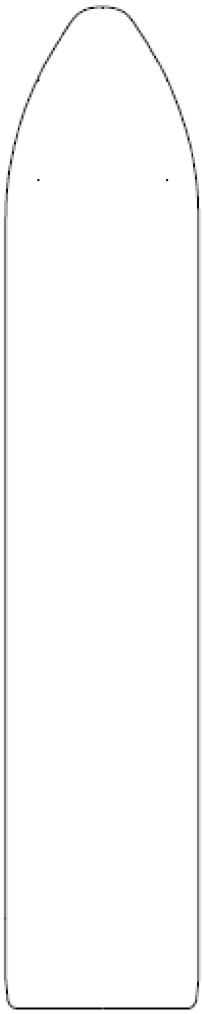
WORST SOLAS DAMAGE			
Damage Case:	Pentalina B - D5.DBS		
Damage Location:	Mid	Aft / Mid / Fwd	
Damage Side:	Starboard	Port / Starboard	
Trim Condition:	Even Keel		

LONGITUDINAL POSITIONS OF WORST SOLAS DAMAGE CASE				
Aft Bulkhead	32.92	1219.3	Fr 54	
Mid Bulkhead	NA	NA	-	
Fwd Bulkhead	40.23	1490.0	Fr 66	

CARDECK TRANSVERSE BARRIERS				
Dist from AP	NA	NA	Aft barrier	*0.0*
Dist from AP	NA	NA	Fwd barrier	*0.0*
Dist from AP	NA	NA		*0.0*
Height	NA	NA	above car deck	*0.0*

INTACT DRAUGHTS - WORST SOLAS DAMAGE

FP		
Full Size	3.60	m
Model	133.3	mm
Check	*0.0*	mm



Port

Midship		
Full Size	3.60	m
Model	133.3	mm
Check	*0.0*	mm

Quarter		
Full Size	3.60	m
Model	133.3	mm
Check	*0.0*	mm

AP		
Full Size	3.60	m
Model	133.3	mm
Check	*0.0*	mm

Starboard

Midship		
Full Size	3.60	m
Model	133.3	mm
Check	*0.0*	mm

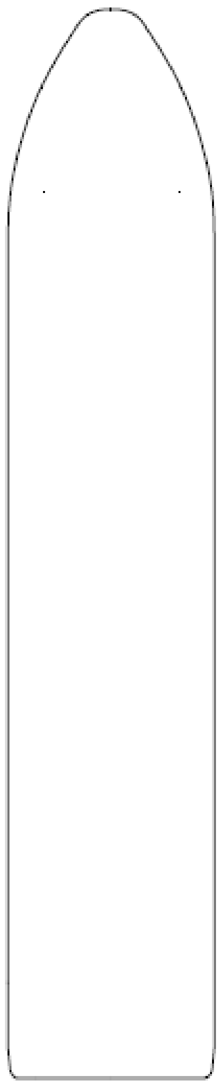
Quarter		
Full Size	3.60	m
Model	133.3	mm
Check	*0.0*	mm

AP		
Full Size	3.60	m
Model	133.3	mm
Check	*0.0*	mm

DAMAGE DRAUGHTS - WORST SOLAS DAMAGE

FP		
Full Size	3.91	m
Model	144.6	mm
Check	*0.0*	mm

Static Heel	
1.000	degree(s)



Port

Midship		
Full Size	3.70	m
Model	136.9	mm
Check	*0.0*	mm

Quarter		
Full Size	3.66	m
Model	135.4	mm
Check	*0.0*	mm

AP		
Full Size	3.66	m
Model	135.7	mm
Check	*0.0*	mm

Starboard

Midship		
Full Size	3.93	m
Model	145.4	mm
Check	*0.0*	mm

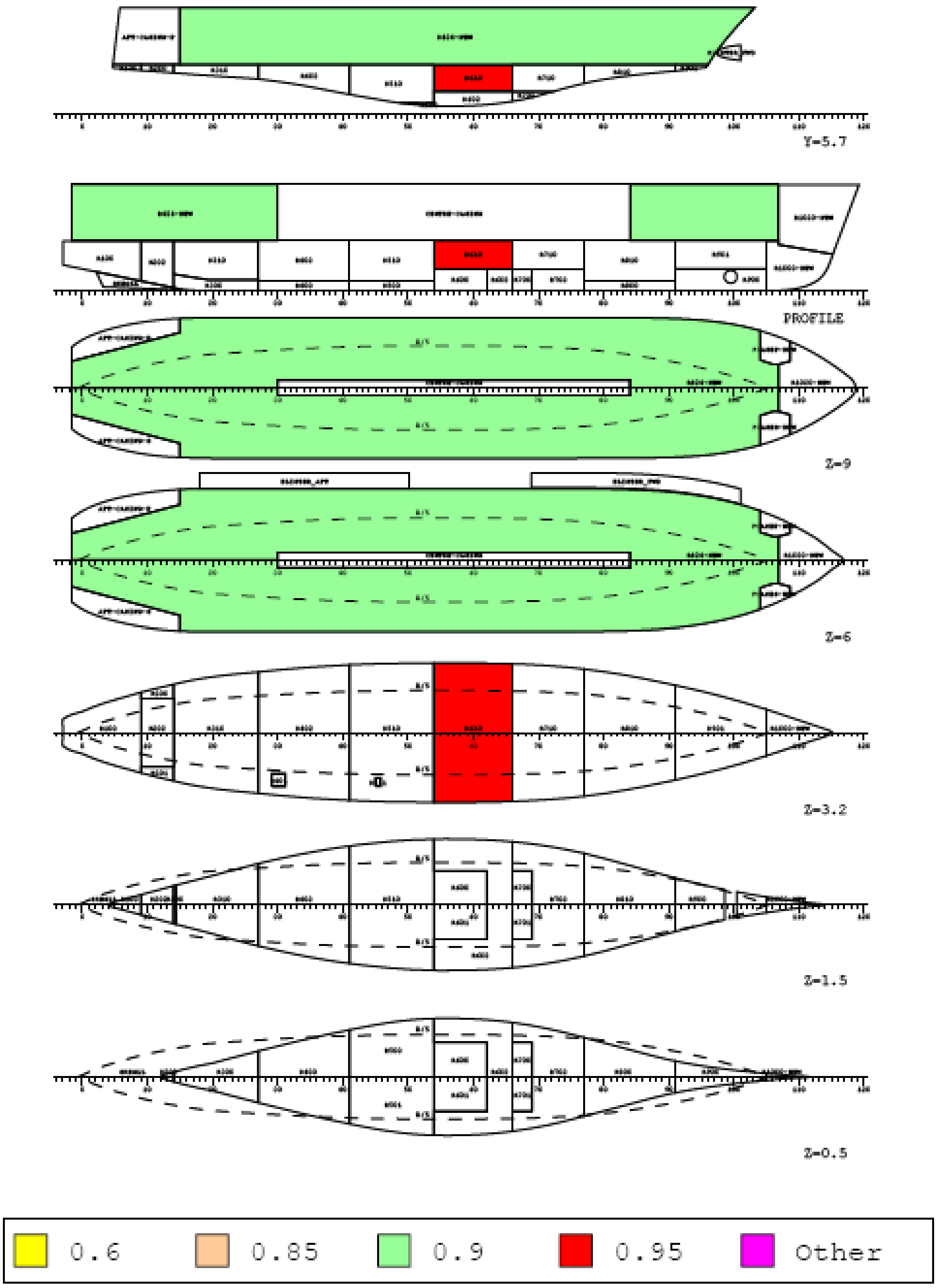
Quarter		
Full Size	3.87	m
Model	143.4	mm
Check	*0.0*	mm

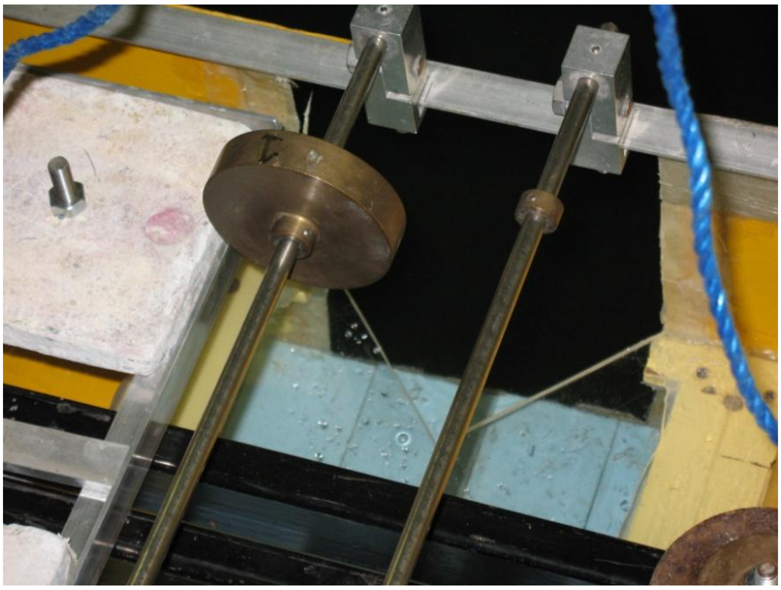
AP		
Full Size	3.77	m
Model	139.7	mm
Check	*0.0*	mm

WORST SOLAS DAMAGE

	SHIP (m)	MODEL (mm)	NOTES	CHECKED
DAMAGE OPENING				
DMLD	4.57	169.3	Cardeck at V	
Length	5.16	191.1	3%Ls + 3.0	
Dist from AP	36.58	1354.8		
B/5	3.87	143.3	from CL	
	2.83	104.8	from BEXT	

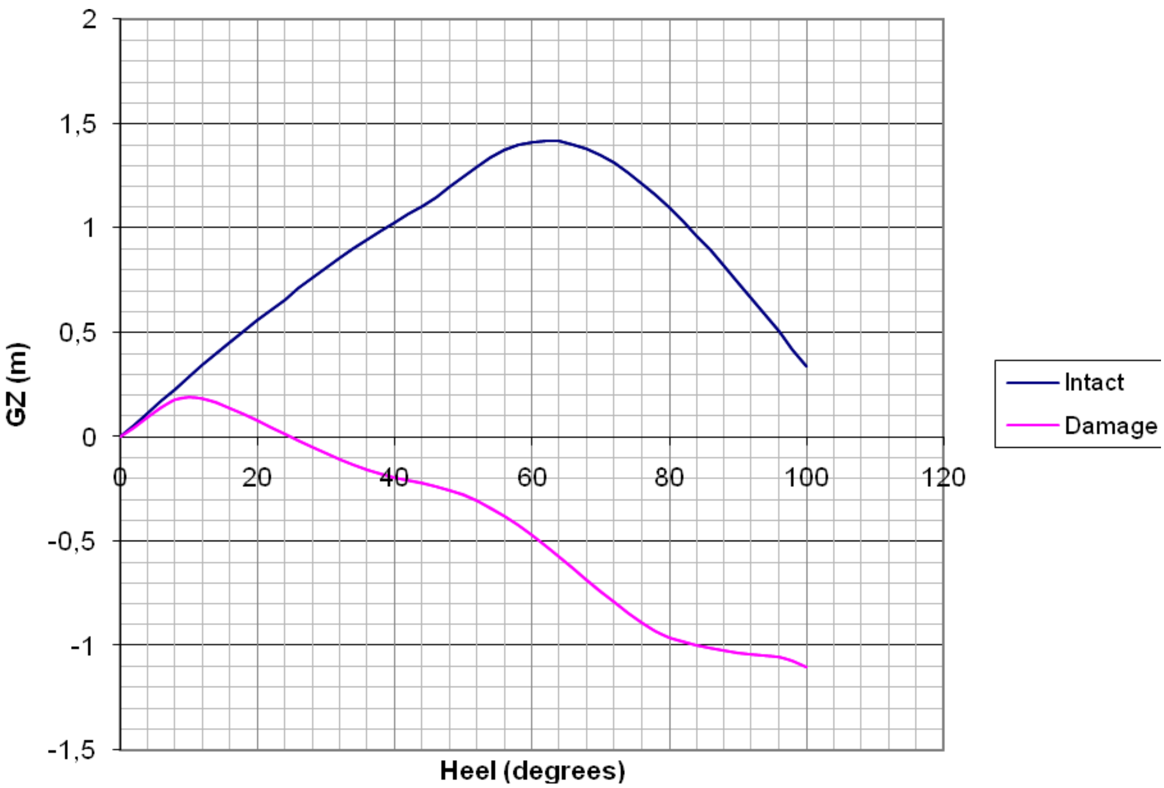
Damaged Compartments							
Room	Permeability	Volume	XCG	YCG	ZCG	Moulded Volume	
R610	0,95	157,1	36,57	0	2,94	165,3	
R424-NEW	0,9	0	-	-	-	0	
Floating Position							
Draught Fwd (TF)	3,904	m		Heel angle	0	Deg	
Draught (T)	3,811	m		Trim	0,187	m	(Forward)
Draught Aft (TA)	3,717	m		Trim angle	0,153	Deg	(Forward)

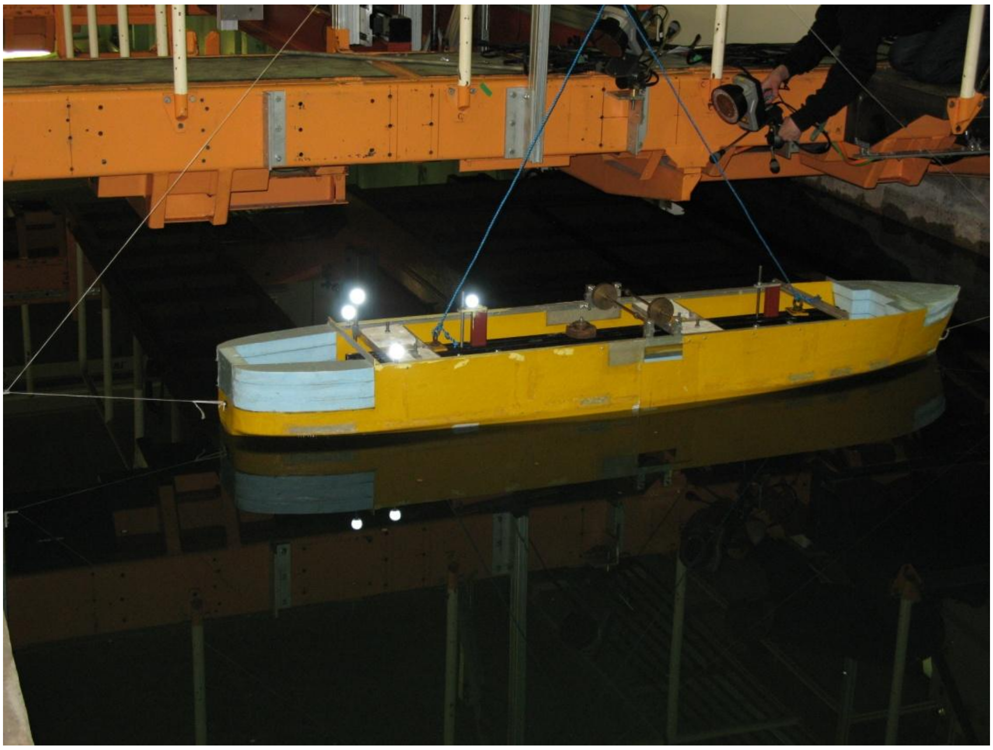

Y=5.7
PROFILE
Z=9
Z=6
Z=3.2
Z=1.5
Z=0.5
0.6
0.85
0.9
0.95
Other



INITIAL CONDIDIONS		(Ship)
Draught	3,6	m
Trim	0	m
Heel	0	deg
Displacement	1915,9	te
KM	6,681	m
KG	**5,146**	**m**
GM	1,535	m

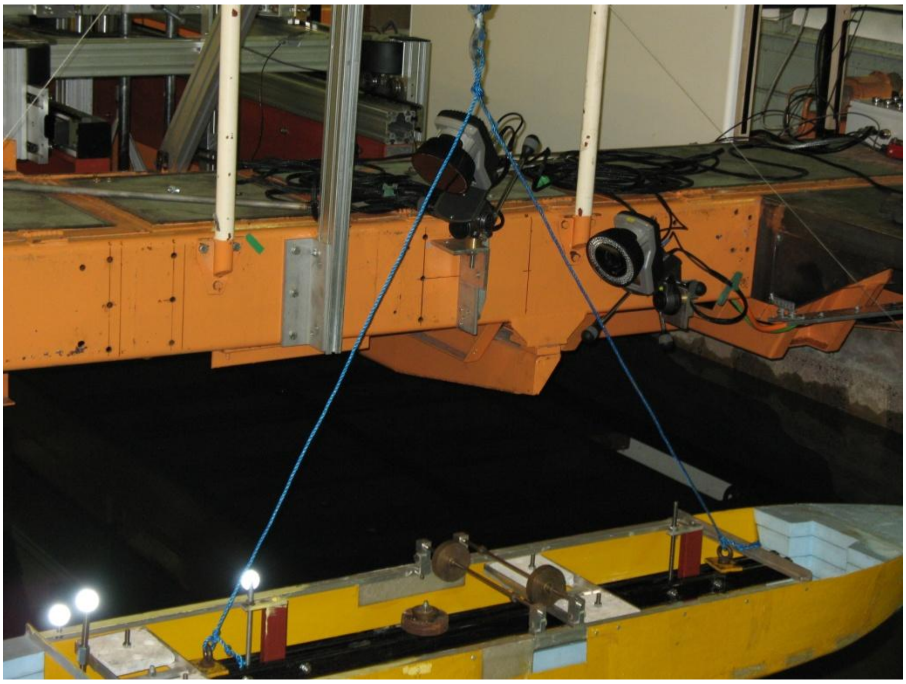
For this particular loading condition the GZ curve is as follows for the intact and the damage condition:





2.4 The experimental process

After the wave maker test, the calibration of the motion capture system and the inclining test, the experiment takes place. The model is placed under the carriage, vertical to the direction of the carriage movement. The damage is situated windward (the side in which the wave from the wave maker meets the boat). The boat is free to do all the six of the ship motions, but because of the limited breadth of the tank and the only one direction waves, the yaw motion of the boat must be limited. A string is tied on the fore and aft part of the model so every time yaw increases rapidly, a technician on the carriage can bring it in the initial condition by pulling the string. The carriage is moving with the required speed, so that every time the model is in the same position under the carriage. The model is also tied with a rope, so every time it is going to capsize, the technician on the carriage pulls the rope to avoid the capsize, as it would be extremely difficult to recover the model from the bottom of the tank. Whenever it is necessary to pull the rope, we consider the case as a capsize.

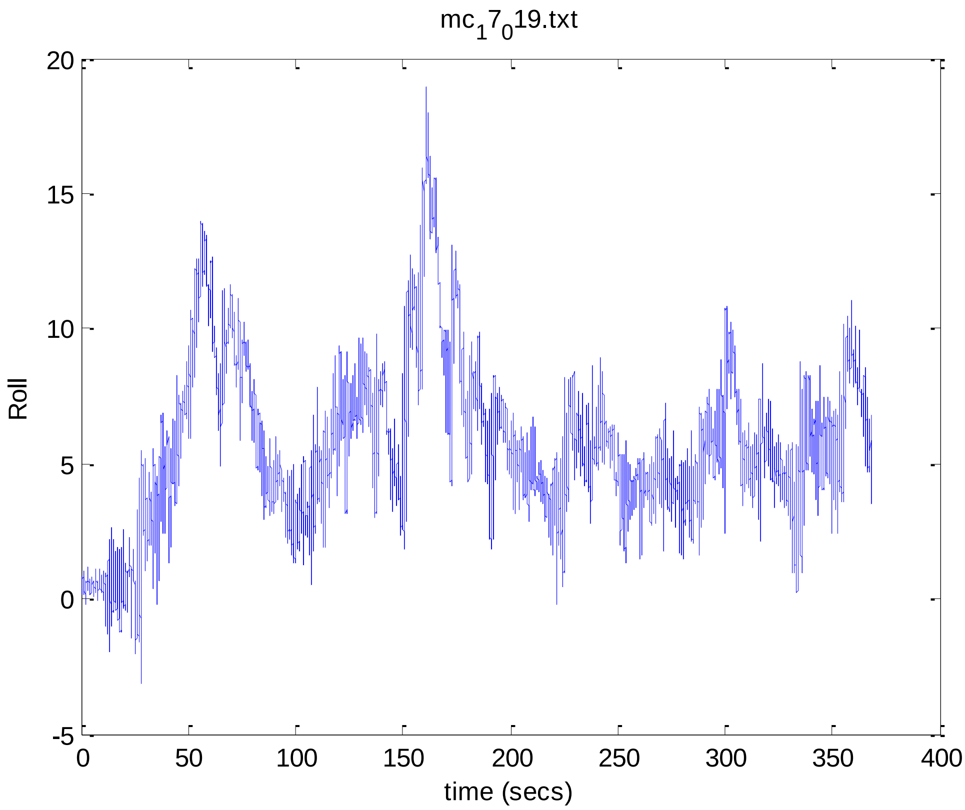


In each run, the wave maker and the software for displaying and saving the nine principal variables of the experiment (roll, pitch, yaw, heave, surge, sway, carriage speed, fixed waveform, travel waveform) are activated first. The sample rate of the motions is 237 Hz. The carriage is following the boat with the appropriate speed. The duration of each run is approximately six minutes, because according to the Stockholm agreement regulations, the required time for all passengers to abandon the ship orderly before sinking is 30 minutes. The equivalent time for the model is $(30)^{1/2}$= 5,46 minutes. Whenever the model is ready to capsize during these 5,45 min, the man on the carriage pulls the rope to avoid it and we consider the case as a capsize, if not, we consider the case as a survival. A case is also considered as a capsize if the roll motion exceeds 20 degrees for a long period according to the Stockholm agreement's regulations for physical experiments.

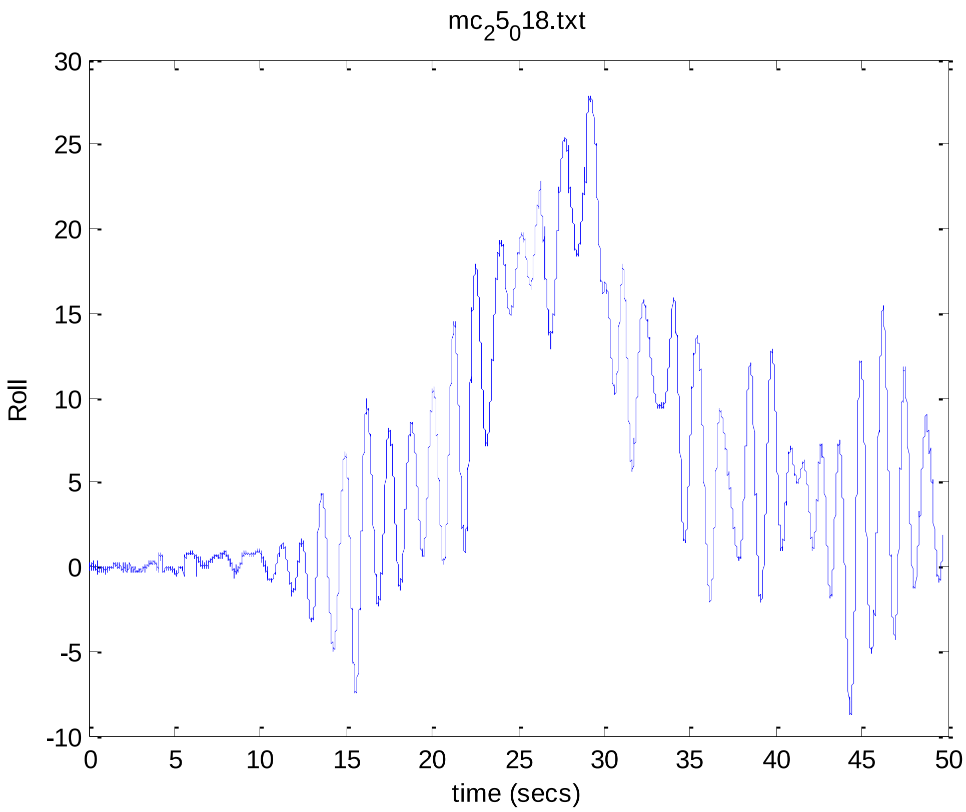
The first runs purpose were to identify the two bounds of Hs, in which the model capsizes 100% and 100% of survivals. These bounds found to be Hs=1,5m (full scale) and Hs=2,5m. The Tp for each case is considered to be for short waves $T_p = 4*\sqrt{H_s}$ (sec), and the spectrum which is used is JONSWAP spectrum with γ=3,3.

In between the bounds, 20 runs are completed for every 0,2 m of Hs with several capsizes and survivals. All the data from the software Spike are collected and they are exported to txt files. The statistical results for the 20 runs for each Hs are presented at the following table:

Hs	Capsizes (%)
1,5	*0*
1,7	*5*
1,9	*25*
2,1	*45*
2,3	*90*
2,5	*100*



Roll(degrees)-time(seconds) for a case of survival for Hs=1,7 m



Roll(degrees)-time(seconds) for a case of capsize for Hs=2,5 m

3.Results

3.1 Hs bounds for the 100% capsizing and the 100% of survivability of the boat.

Initially experiments were performed in high values of Hs. The first experiments were at Hs=3m. While the percentage of capsize was 100%, the Hs was decreasing each time for 0,2 m. The upper bound was found to be Hs= 2,5 m according the experiments. Under 2,5 metres Hs, there is a percentage from 0-90 % of capsizes. The wave height in which we have no capsizes is 1,5 m.

Finally, the bounds were found to be Hs=**1,5** and **2,5** m.

3.2 Perform 20 runs for each 0.2 m of Hs in between the bounds and make a statistical analysis of the results

There were 20 runs for the following Hs: 1,5m 1,7m 1,9m 2,1m 2,3m and there were 18 runs for Hs=2,5m. The exported txt files including information for 9 data that are mentioned earlier (roll, pitch, heave, yaw, surge, sway, carriage speed, travel and fixed wave) are used to calculate the actual time for capsize for each case. The data include useless noise information in the beginning for the first 10 seconds approximately. For this reason a primary question must be answered. What time should be considered as the start of the experiment?

The value itself is not of grave importance as long as it is consistent for all the experiments. The first step for this procedure is to calculate the average of the first 4 seconds for the travel and fixed waveform and take out this value from all the data. This way we can exclude some of the noise and calibration errors which are included in the wave data. The maximum height is then found for the first 4 seconds. The first waveform peak whose value is 1,5 times the maximum wave height from the first 4 seconds was finally chosen as a starting point.

The instant when the roll angle of the model exceeds 20 degrees and increases rapidly, is considered as the capsize point. The time of the capsize point minus the time of the start of the experiment is the actual time for capsize. When no capsize occurs, time is considered to be over 30 min and is noted as 1800 sec. The results for time to capsize for each run are shown in the following table:

Hs 1.50	Hs 1.70	Hs 1.90	Hs 2.10	Hs 2.30	Hs 2.50
Sec	sec	sec	sec	sec	sec
1800	1800	720,07	543,56	542,83	435,75
1800	1800	1800	1800	1236,7	373,36
1800	1800	1800	1800	421,22	393
1800	1800	1800	423,3	366,36	180,79
1800	958,61	1800	485,28	189,21	295,02
1800	1800	1800	1800	307,19	240,32
1800	1800	1800	174,67	343,52	334,94
1800	1800	723,12	685,21	659,69	221,13
1800	1800	558,22	685,26	220,28	115,41
1800	1800	1800	1800	1361,2	291,8
1800	1800	1800	1800	298,57	233,83
1800	1800	1800	1800	1800	960,94
1800	1800	602,8	585,87	585,74	481,64
1800	1800	1800	1800	1800	365,35
1800	1800	1800	1800	496,73	264,59
1800	1800	1800	1800	424,22	81,867
1800	1800	1800	1800	308,37	325,19
1800	1800	580,74	288,38	155,58	572,1
1800	1800	1800	1800	610,01	
1800	1800	1800	805,05	647,77	

3.4 Histograms with the probability distribution function for the time to capsize for each Hs

According to the table above, the histograms with the probability distribution function for capsizing are created for each Hs. In the same graph is included the analytical probability distribution function according to the equation

$$\frac{-\ln(1-pf(Hs))*(1-pf(Hs))t/to}{to}$$

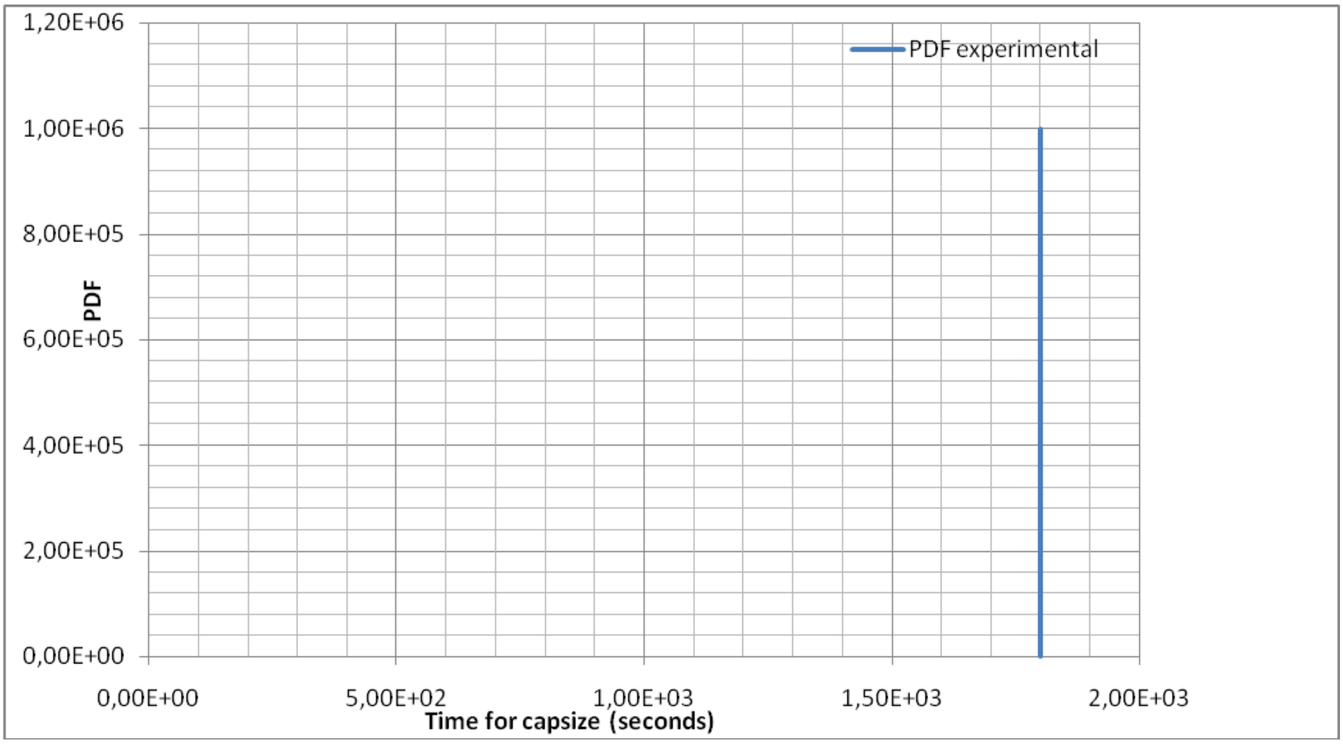
.

Where **pf**: the probability for capsize according to the experiments

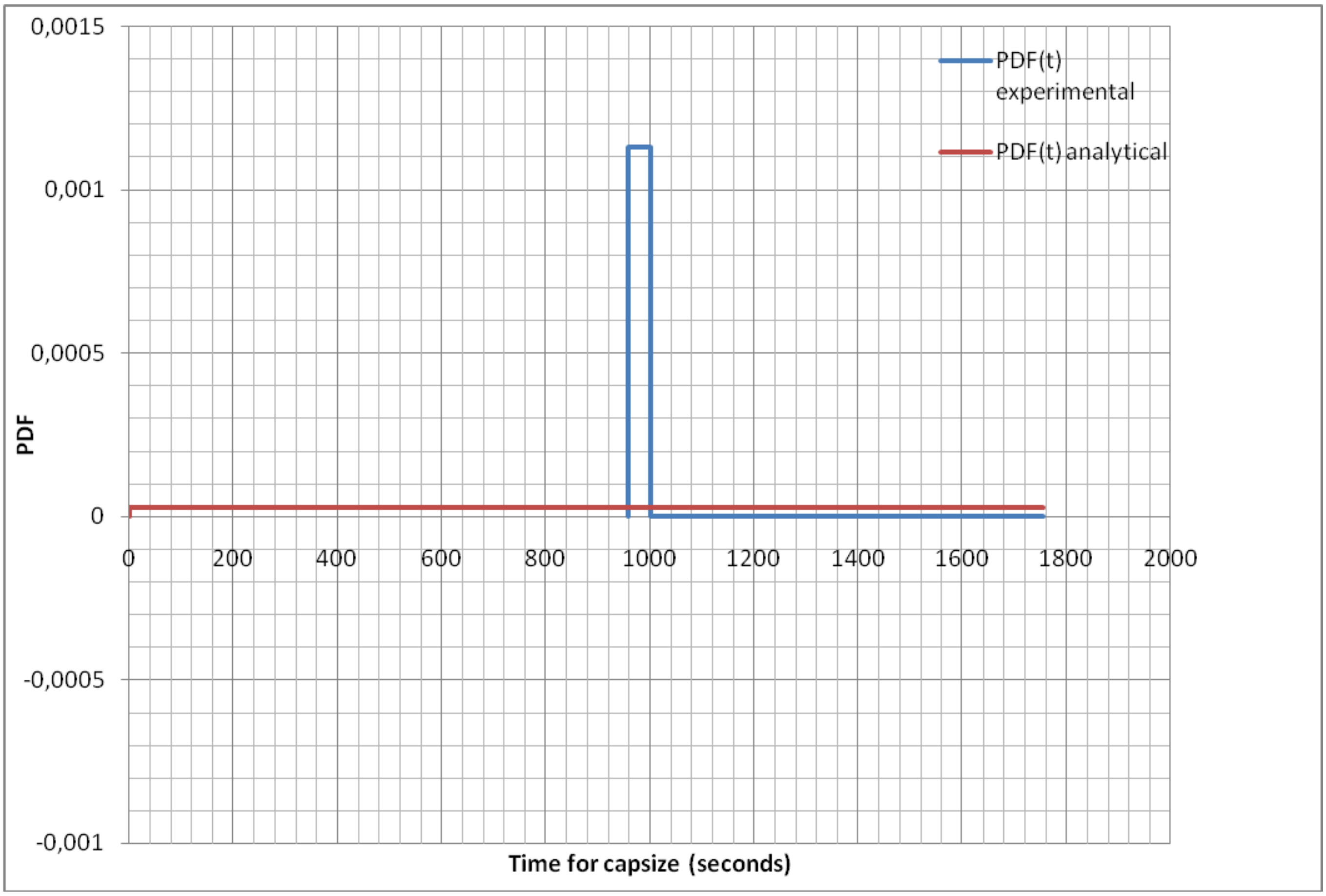
Hs(m)	pf
1,5	0
1,7	0,05
1,9	0,25
2,1	0,45
2,3	0,9
2,5	1

t_o: the elapsed time for capsize (1800 for the cases that we don't have 100% capsizes and maximum time for capsize for the case of 100% of capsizes). For the Hs=2,5 m condition we use instead of pf=1, pf=0,99 sto we don't have zero in the ln equation.

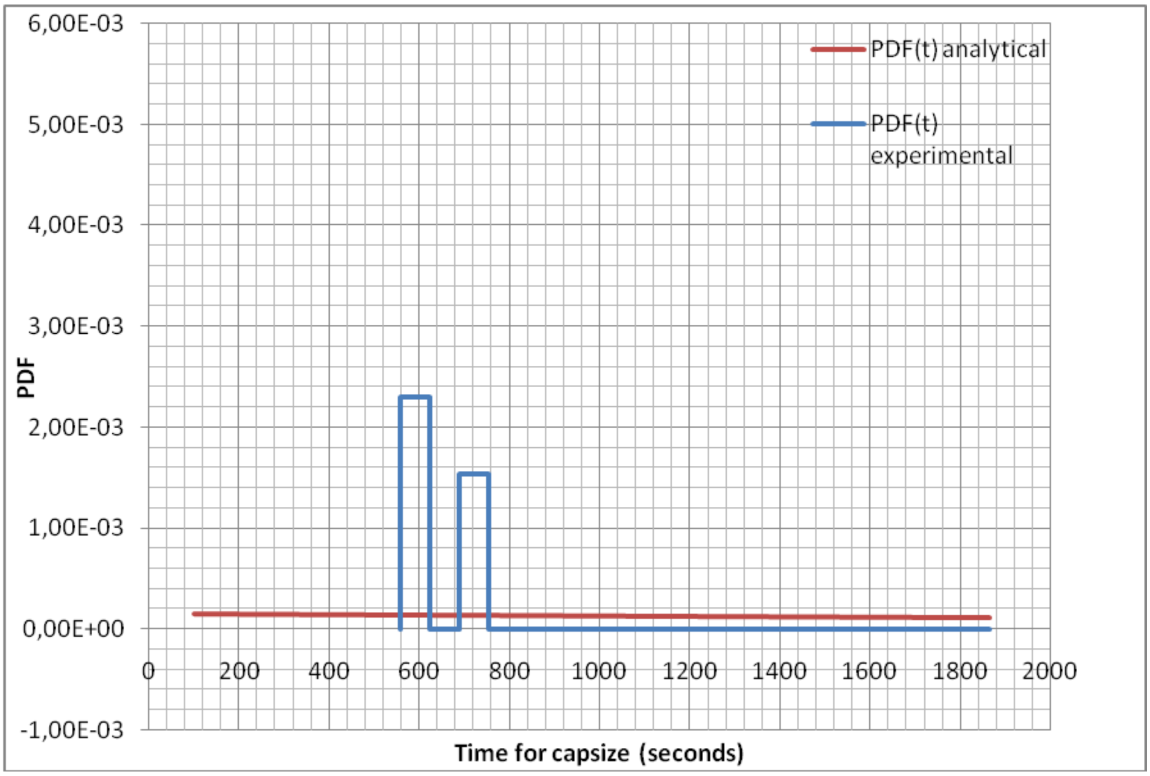
Hs=1,5m


1,20E+06
1,00E+06
8,00E+05
6,00E+05
4,00E+05
2,00E+05
0,00E+00
PDF
PDF experimental
0,00E+00
5,00E+02
1,00E+03
1,50E+03
2,00E+03
Time for capsize (seconds)

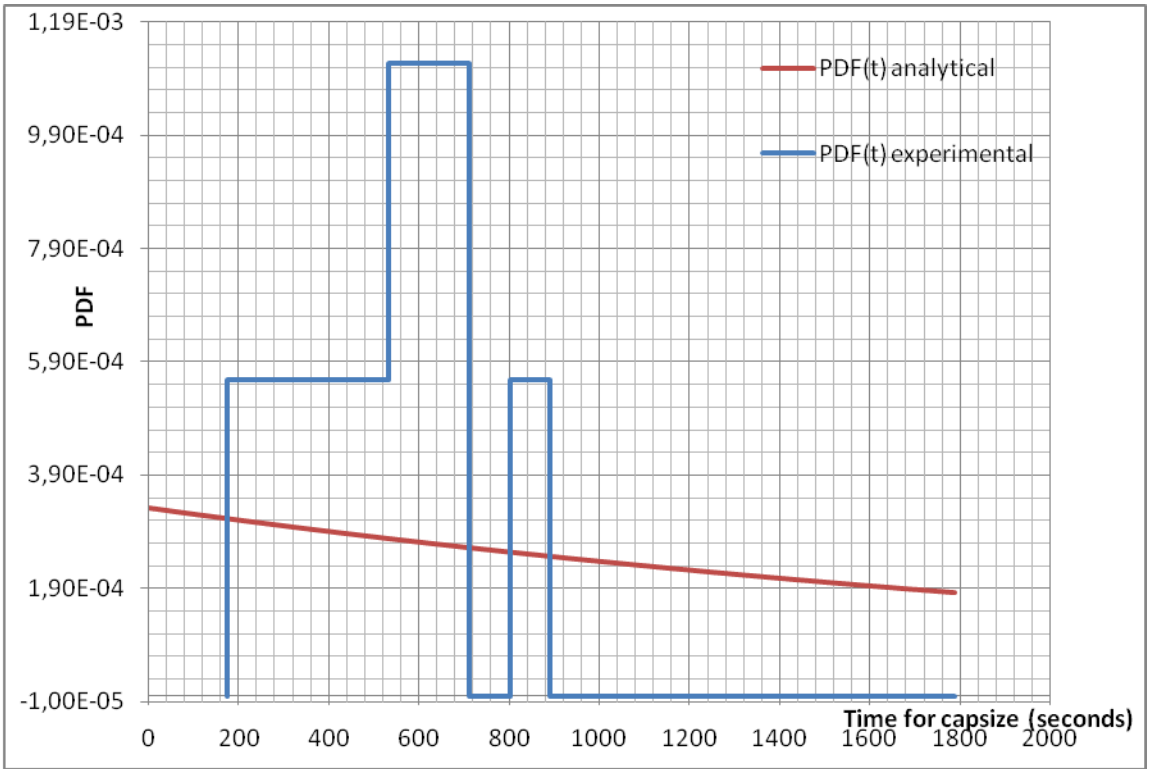
Hs=1,7m

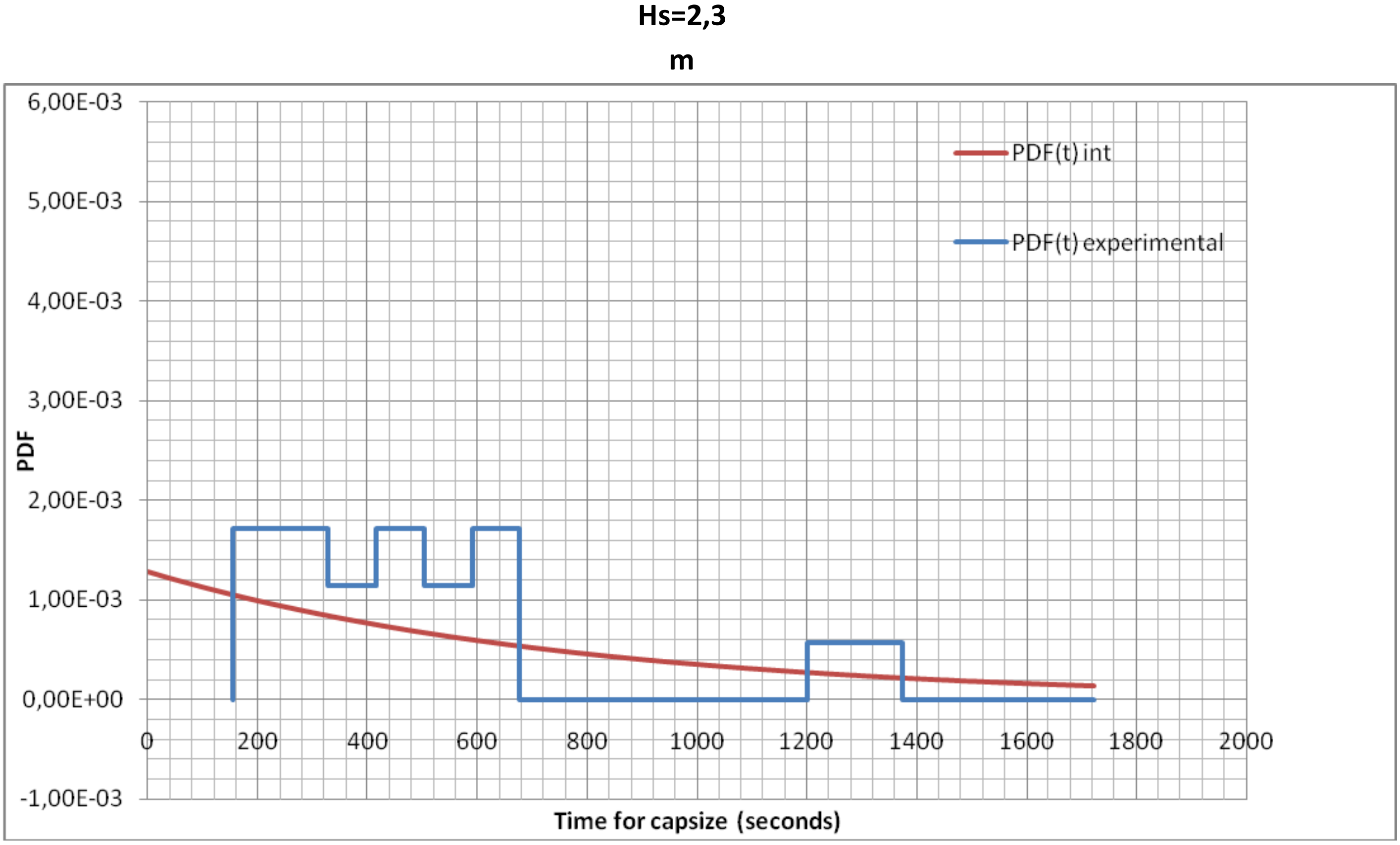

0,0015
0,001
0,0005
0
-0,0005
-0,001
PDF
PDF(t) experimental
PDF(t) analytical
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Time for capsize (seconds)

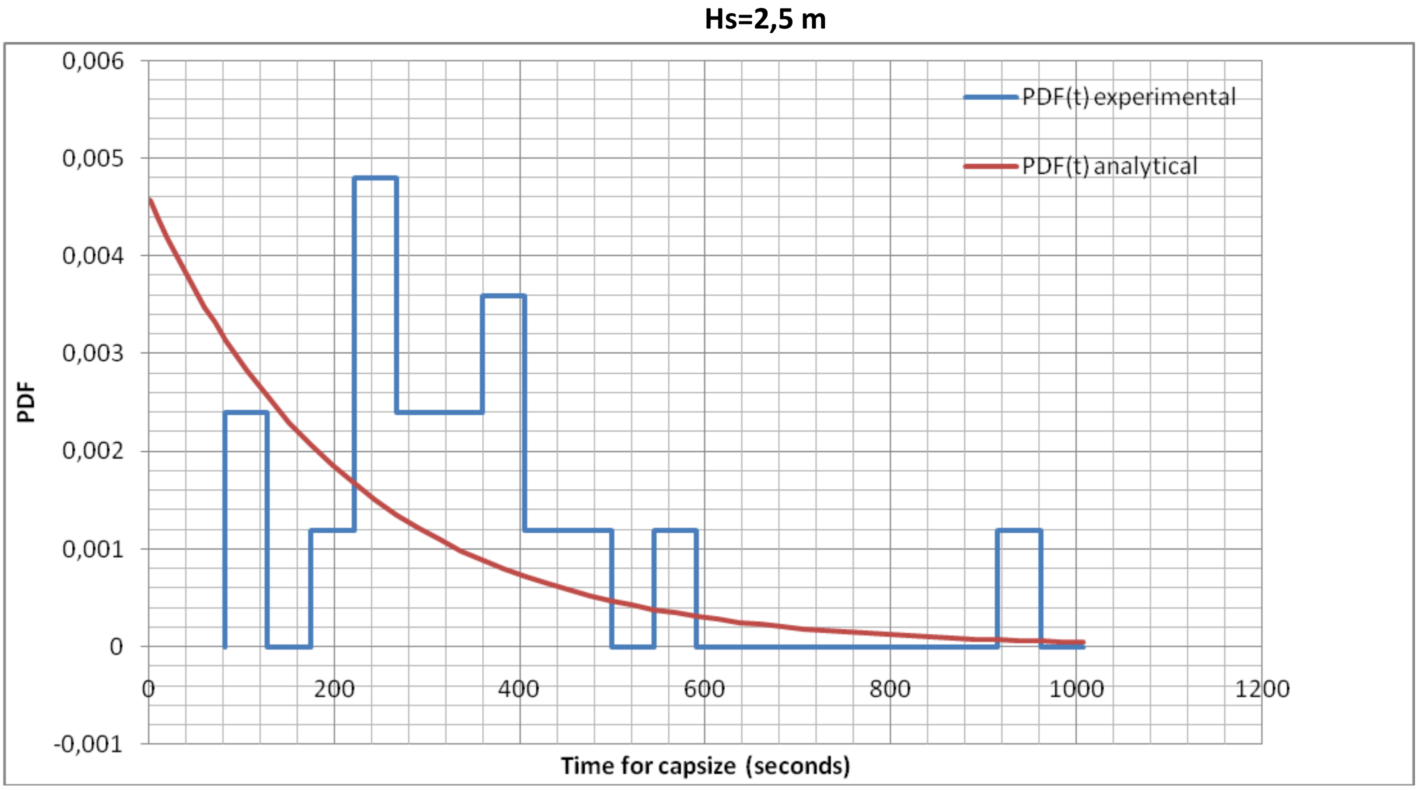
Hs=1,9m


6,00E-03
5,00E-03
4,00E-03
3,00E-03
2,00E-03
1,00E-03
0,00E+00
-1,00E-03
PDF
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Time for capsize (seconds)
PDF(t) analytical
PDF(t) experimental

Hs=2,1 m


1,19E-03
9,90E-04
7,90E-04
5,90E-04
3,90E-04
1,90E-04
-1,00E-05
PDF
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Time for capsize (seconds)
PDF(t) analytical
PDF(t) experimental


Hs=2,3
m
6,00E-03
5,00E-03
4,00E-03
3,00E-03
2,00E-03
1,00E-03
0,00E+00
-1,00E-03
PDF
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Time for capsize (seconds)
PDF(t) int
PDF(t) experimental


Hs=2,5 m
0,006
0,005
0,004
0,003
0,002
0,001
0
-0,001
PDF
0
200
400
600
800
1000
1200
Time for capsize (seconds)
PDF(t) experimental
PDF(t) analytical

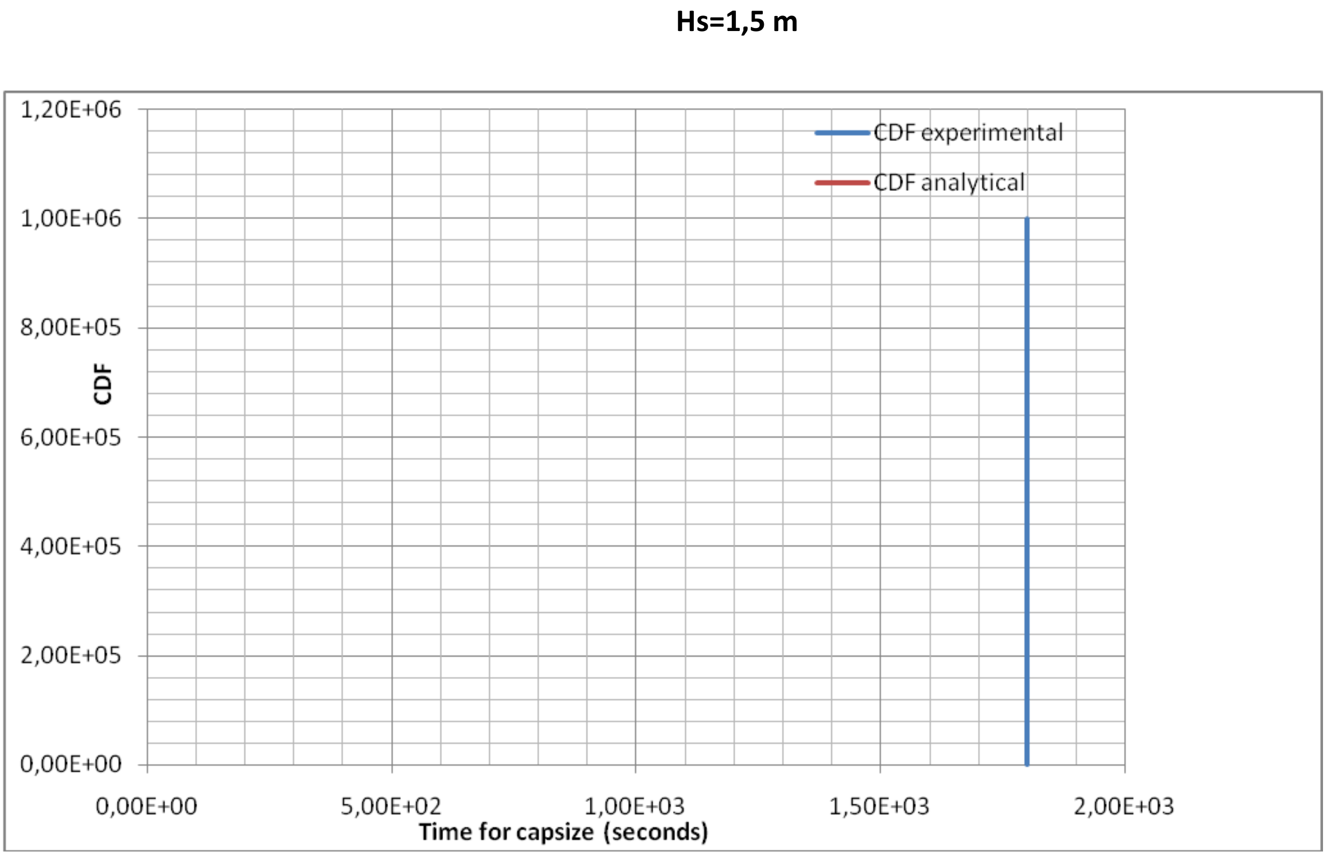
3.4 CDF(t) for the time to capsize for each Hs and uncertainty bounds

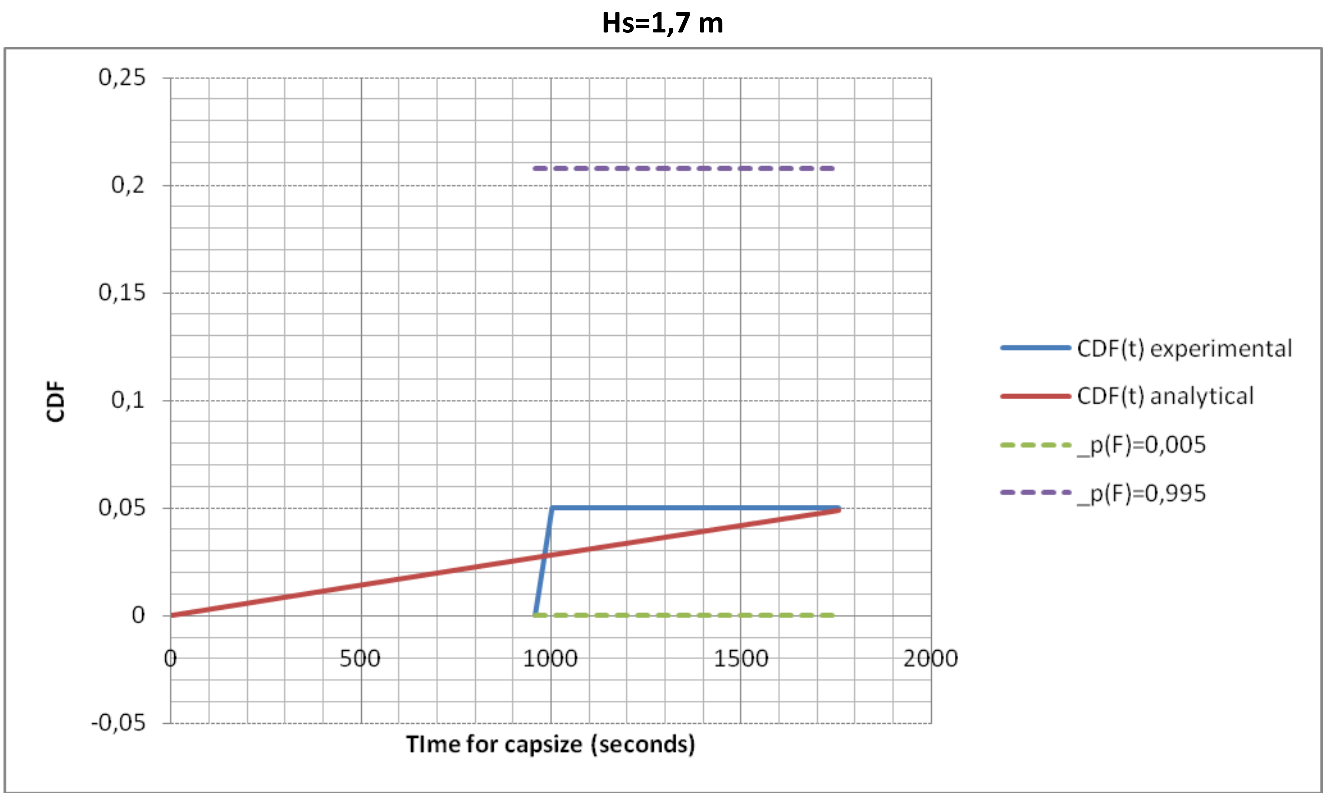
The following CDF(t) curves are presented, are the result of the integration of the PDF(t) for each sea condition.

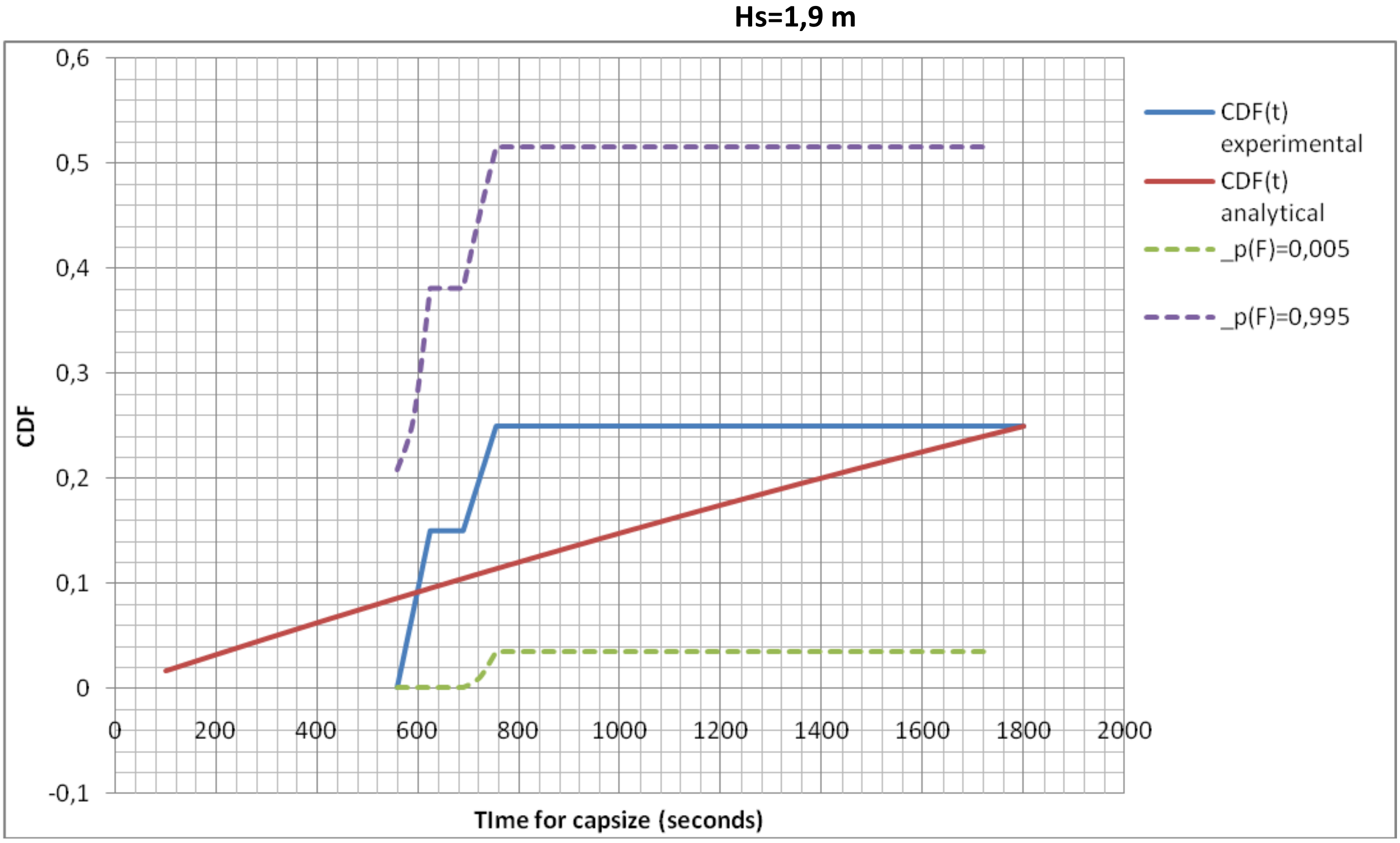
CDF(t|Hs)= $\int_{o}^{t} PDF(\tau|Hs)d\tau$

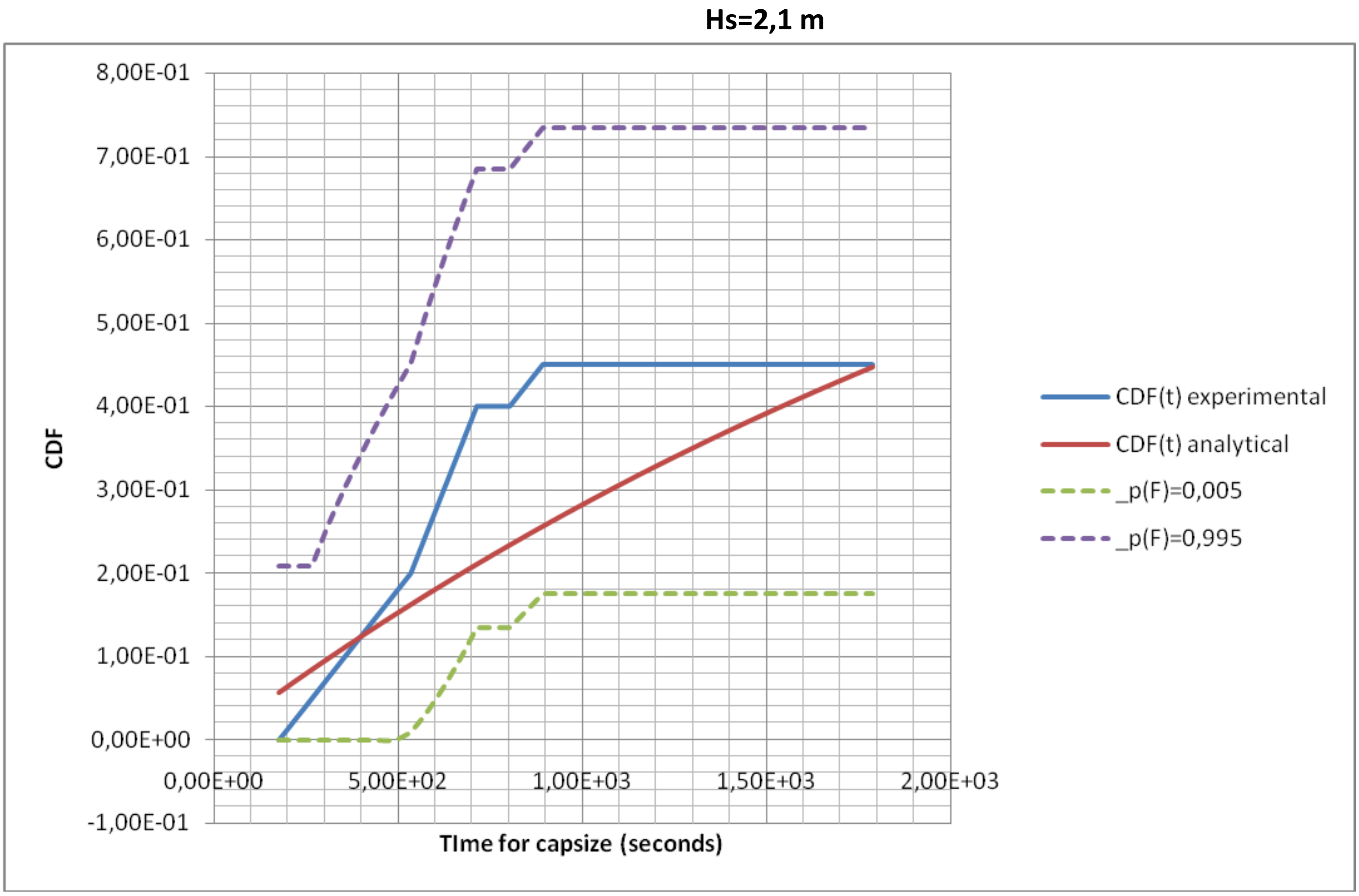
The experimental CDF(t) curve is calculated as the numerical integration of PDF(t) experimental. The analytical CDF(t) curve is calculated as the analytical integration of PDF(t) analytical. The equation of the analytical CDF(t) is the following: CDF(t)= $\int_{o}^{t} PDF(\tau|Hs)d\tau$ = 1-{(1-pf(Hs))$^{t/to}$*1/to}

The graphs include also the uncertainty bounds for 99% confidence. The calculation of these bounds is explained analytically in page 19.

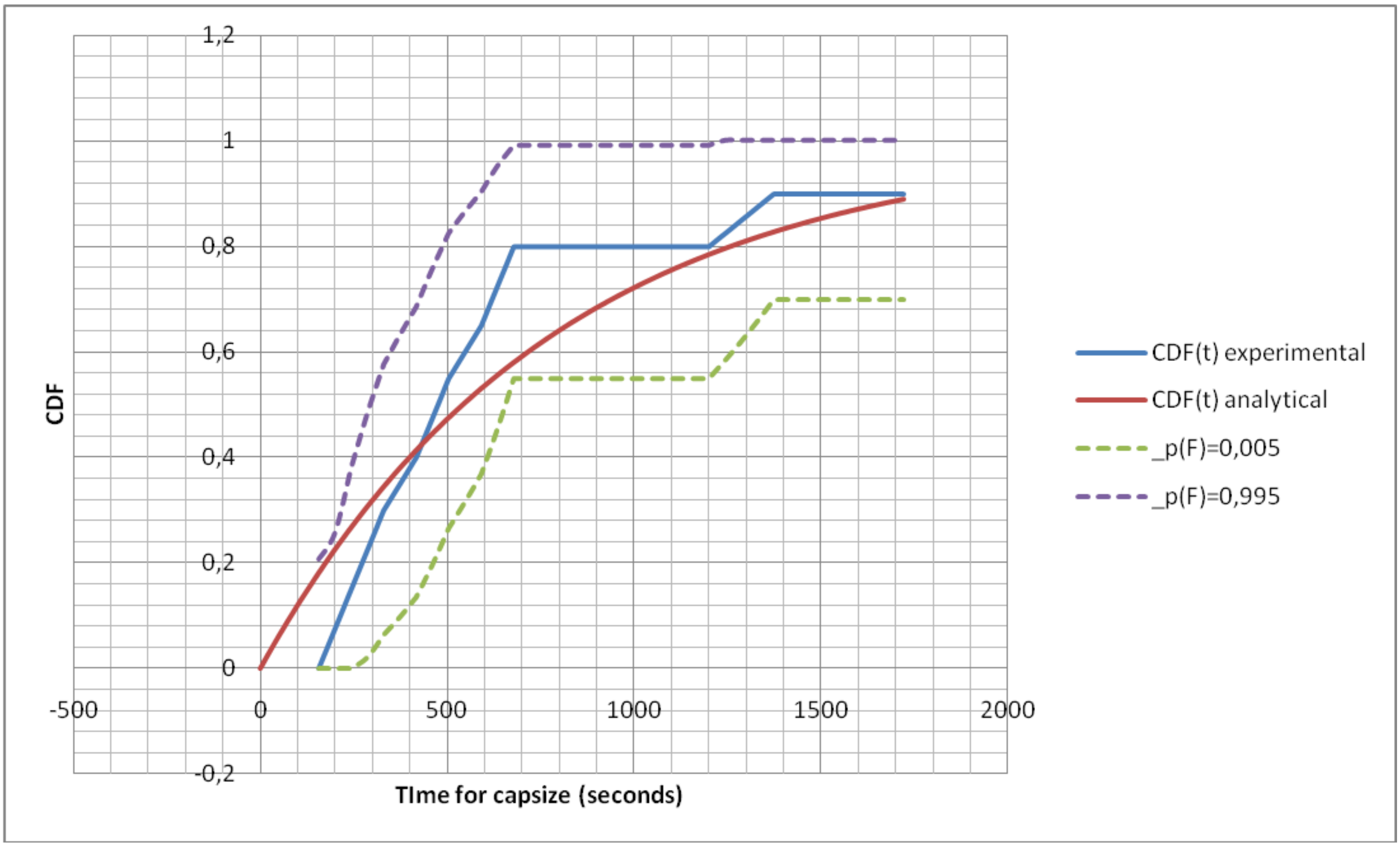

Hs=1,5 m
CDF experimental
CDF analytical
CDF
1,20E+06
1,00E+06
8,00E+05
6,00E+05
4,00E+05
2,00E+05
0,00E+00
0,00E+00
5,00E+02
1,00E+03
1,50E+03
2,00E+03
Time for capsize (seconds)


Hs=1,7 m
CDF
0,25
0,2
0,15
0,1
0,05
0
-0,05
0
500
1000
1500
2000
TIme for capsize (seconds)
CDF(t) experimental
CDF(t) analytical
_p(F)=0,005
_p(F)=0,995

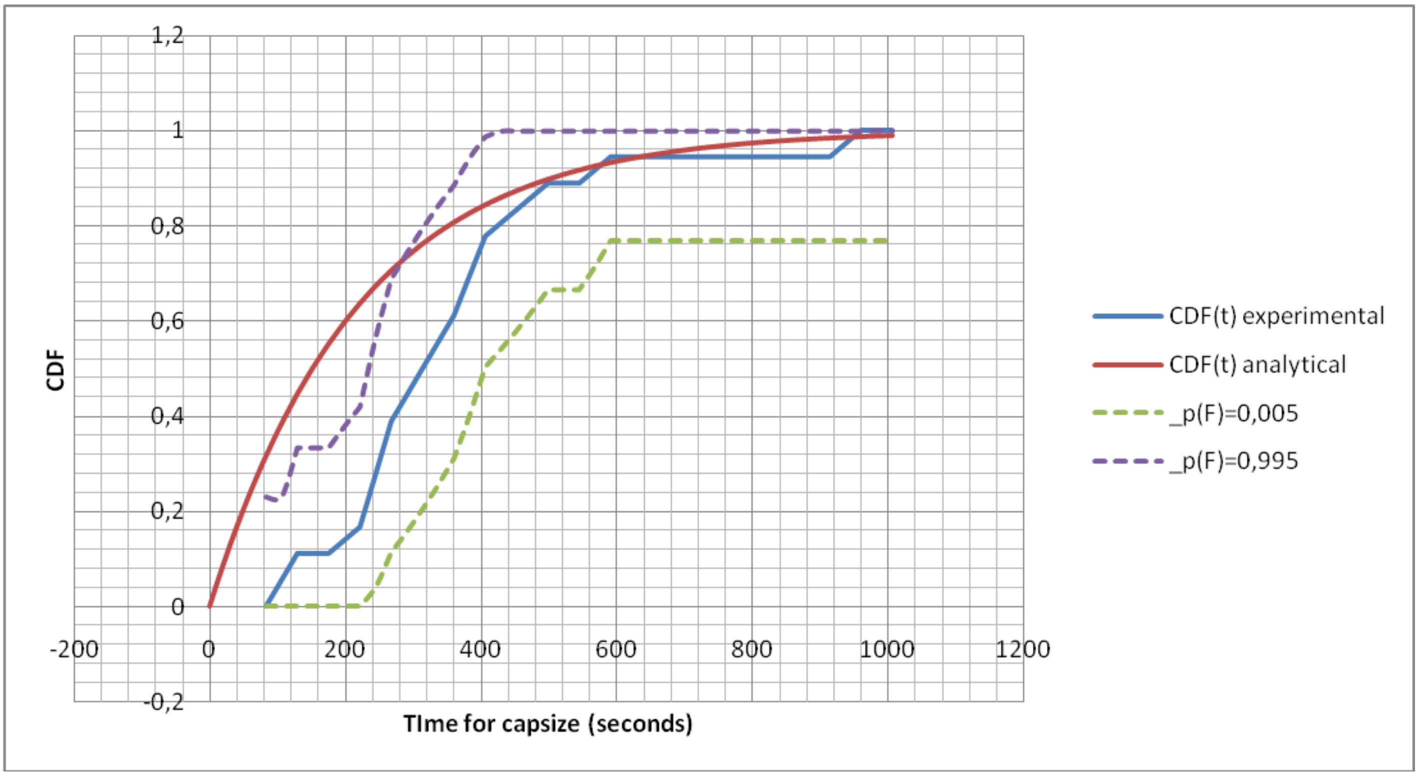

Hs=1,9 m
0,6
0,5
0,4
0,3
0,2
0,1
0
-0,1
CDF
0
200
400
600
800
1000
1200
1400
1600
1800
2000
TIme for capsize (seconds)
CDF(t) experimental
CDF(t) analytical
_p(F)=0,005
_p(F)=0,995


Hs=2,1 m
8,00E-01
7,00E-01
6,00E-01
5,00E-01
4,00E-01
3,00E-01
2,00E-01
1,00E-01
0,00E+00
-1,00E-01
CDF
0,00E+00
5,00E+02
1,00E+03
1,50E+03
2,00E+03
TIme for capsize (seconds)
CDF(t) experimental
CDF(t) analytical
_p(F)=0,005
_p(F)=0,995

Hs=2,3m


1,2
1
0,8
0,6
0,4
0,2
0
-0,2
CDF
-500
0
500
1000
1500
2000
TIme for capsize (seconds)
CDF(t) experimental
CDF(t) analytical
_p(F)=0,005
_p(F)=0,995

Hs=2,5 m


1,2
1
0,8
0,6
0,4
0,2
0
-0,2
CDF
-200
0
200
400
600
800
1000
1200
TIme for capsize (seconds)
CDF(t) experimental
CDF(t) analytical
_p(F)=0,005
_p(F)=0,995

3.5 Compare of the experimental CDF curve with the analytical CDF curve.

The CDF curves above, show the probability for a ship to sink in a specific time period. The one curve is showing the experimental results and the second is from the analytical equation $1-\{(1-pf(Hs))^{t/to}*1/to\}$.

As it is noticeable, these curves are close together but because of the small repetition of the experiments on each Hs condition, they are not the same. This has to deal with the low number of repetitions of the experiments. It's expected with a larger number of experiments to have a better approach, closer to the analytical curve but this is not jet proved.

It is also noticeable on graphs, that the uncertainty has a different value for each Hs condition. The following table shows the difference between the uncertainty bounds in percentage which define how accurate is the result for the maximum time to capsize.

Hs	Uncertainty
1,5	***0,21***
1,7	***0,21***
1,9	***0,27***
2,1	***0,29***
2,3	***0,21***
2,5	***0,21***

We notice that the uncertainty near the critical Hs is more intense than in the bounds. This reflects the way we measure the uncertainty, as the difference between the expected CDF values.

3.6 CDF(Hs) curve (which shows the probability for capsize in each Hs) according to our experimental results.

The graph 3.6.1 shows the survivability curve of the model according to the Glasgow University's tank experimental results.

Hs	Capsizes (%)
1,5	0
1,7	5
1,9	25
2,1	45
2,3	90
2,5	100

The graph also include the expecting survivability curve $f_{i,j,k}(t)$ according to SOLAS 2009. The survivability curve according to SOLAS is calculated as a cumulative normal distribution of Hs, with mean value $Hs_{critical}$

$$f_{i,j,k}(t) = -\ln(\varepsilon_{i,j,k}) \cdot (\varepsilon_{i,j,k})^{\frac{t}{t_0}} \cdot t_0^{-1}$$

$$t_0 = 30 \text{ min}$$

$$\varepsilon_{i,j,k} = 1 - \Phi\left(\frac{Hs_k - Hs_{crit}(s_{ij})}{\sigma_r(Hs_{crit}(s_{ij}))}\right)$$

$$\sigma_r(Hs_{crit}) = 0.039 \cdot Hs_{crit} + 0.049$$

The term $\varepsilon_{i,j,k}$ (with σ_r) represents the phenomenon of the capsize band, that is the spread of sea states where the vessel might capsize. The s_{ij} is the probability of survival, calculated according to the following equation and implicitly encoding information about ships geometry.

$$Hs_{crit}(s) = \begin{cases} 0 & if \quad s \le 0.3093 \\ \dfrac{0.16 - \ln(-\ln(s))}{1.2} & if \quad s > 0.3093 \end{cases}$$

Regulation 7-2.3 of IMO MSC.216(82)

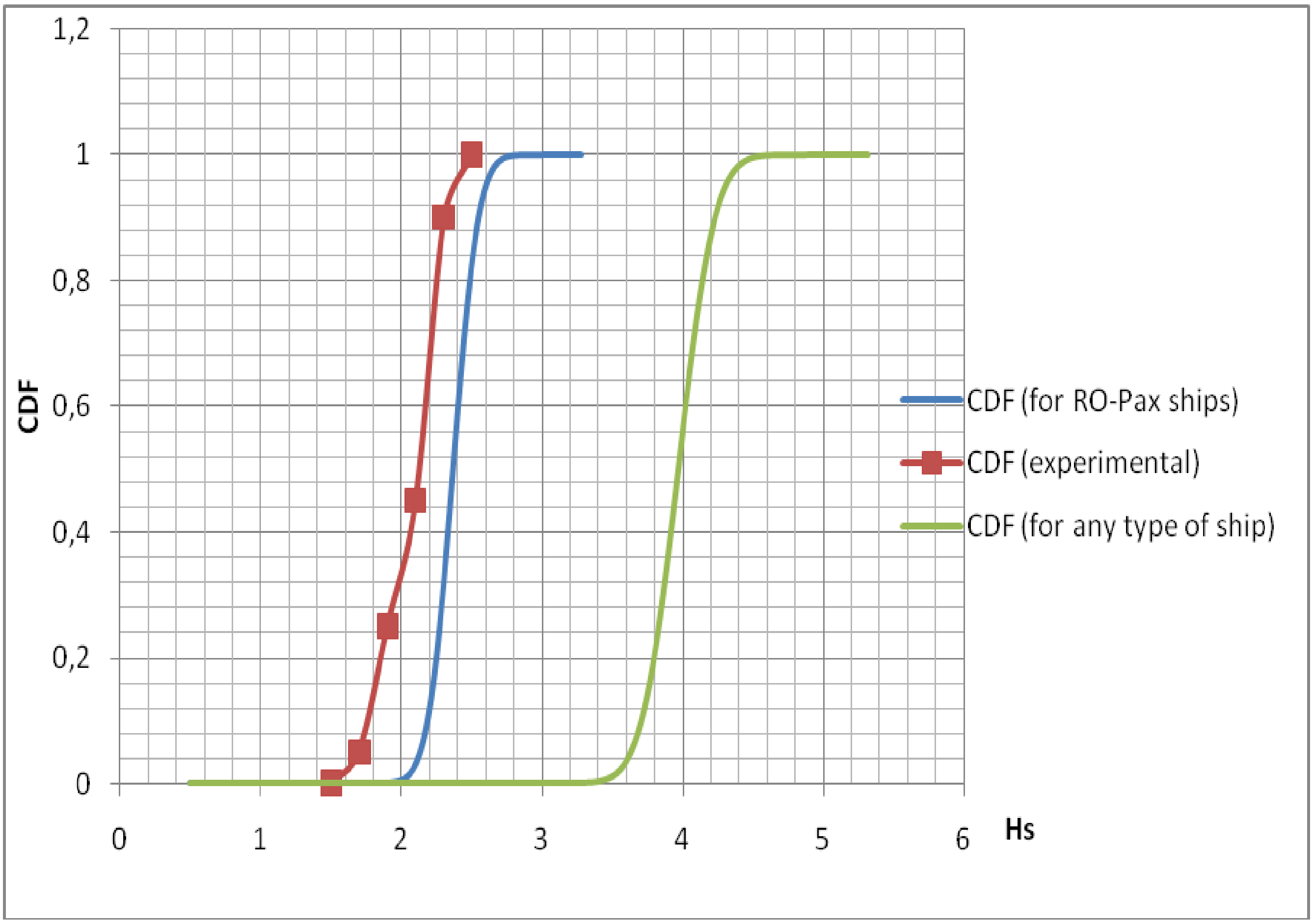
Where $s = \left(\frac{GZmax * Range}{0{,}12 * 16}\right)^{1/4}$

Where if GZmax>0,12 then is set GZmax=0,12

And when Range>16 degrees then is set Range=16

GZmax of the model was found to be 0,19 and Range found to be 25 degrees. So they are set GZmax=0,12 and Range=16 degrees, so "s" is equal to 1. But ln(0)=-∞, so "s" is considered as 0,99.

s(survivability)	**0,99**
Hs(critical)	**3,966791**
σ	**0,203705**

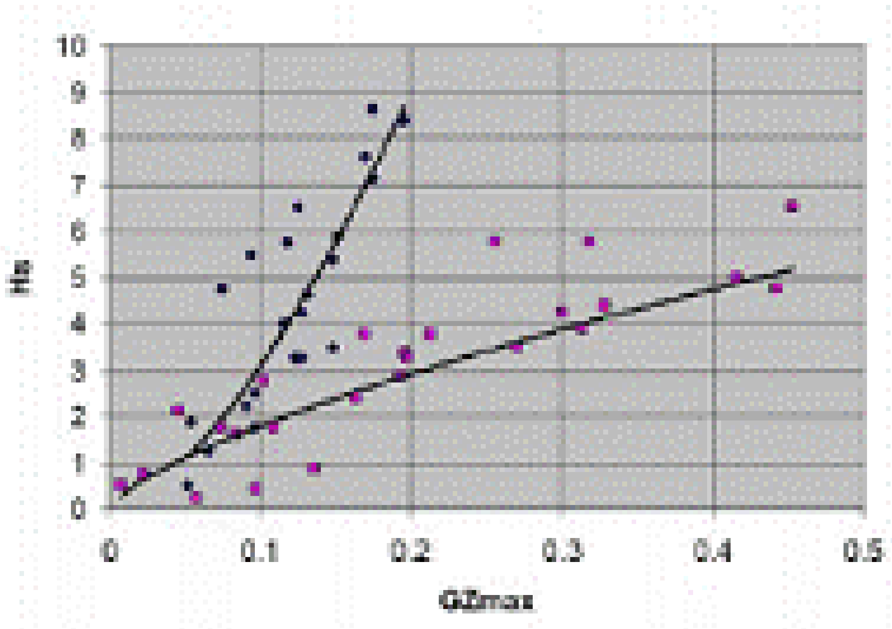


3.6.1 CDF(Hs) graph according to the experiments and SOLAS 2009

As can be seen on the graph below, a RO-RO ship only survives a sea state of 4 meters after flooding, if GZmax in this flooding case approaches an approximate value of 0.25m or above. So the $Hs_{critical}$ equation is converted to

$$Hs_{crit}(s) = \begin{cases} 0 & if \quad s \leq 0.3093 \\ \dfrac{0.16 - \ln(-\ln(s))}{\mathbf{2.5}} & if \quad s > 0.3093 \end{cases}$$

Where $S=\{GZmax*Range/(0,25*16)\}^{1/4}$



3.6.2 Critical Hs for a specific GZmax for any type of ship (black), for RO-Pax ships (purple)

3.7 Comparison of the experimental CDF(Hs) curve with the analytical CDF(Hs) curve.

As it is obvious in the graph above, there is a noticeable difference between the experimental and the analutical CDF(Hs) (for any type of ship) curve on the $Hs_{critical}$.

The difference between the $Hs_{critical}$ that is derived from SOLAS equations (for any type of ship) and the one that is calculated from the experiments is **45%** more with SOLAS approach which shows that SOLAS expects a survival when it's actually a capsize case. Considering the equation between the Hs and s factor, for the experimental results Hs critical 2,15 gives s factor **0,901** opposing to the 0,99 that is calculated according to SOLAS.

On the other hand, if we consider RO-Pax ships as an additional category, (because as we can see in 3.6.2 graph RO-Pax ships are less stable than other types of ships) with GZmax=0,25m for survival in each case, the theoretical CDF(Hs) curve is very similar to the experimental and the error is just 9,13%. However, the critical sea state is 2.36 meters much less than 4 meters, as it should be.

The experiments include a single ship loading condition (specific KG). If more loading conditions were tested, the graph CDF(Hs) would be more accurate as there was going to be a band of possible values of CDF for each Hs and the uncertainty would be calculated so that it would be safer to compare with the SOLAS analytical CDF(Hs) curve.

Many disagreements and critisism of SOLAS probabilistic approach can be found. Also, many investigations for alternative solutions have been tested. A recent project about this specific research is the HARDER project. The report covers 'investigations and proposed formulations for the factor s'.

Although, several other reasons can lead to this difference of results. These could be errors during the inclining test and wrong KG value calculation.

Conclusion

Based on the results and arguments presented in this project the following conclusions can be drawn:

- The experimental PDF curve tends to be similar with the analytical curve. The differences are caused from the relatively small number of experiments
- The same conclusion is derived for the CDF curves.
- The CDF values uncertainty is more intense near the critical Hs values. As we measure the CDF uncertainty in a specific Hs condition it is expected as it can easily been noticed that is going to be greater than in other Hs conditions because the band is bigger near the 50% capsize point than near the bounds there where the uncertainty tend to be near zero
- The Hs condition bounds for capsize are different from the expected bounds according SOLAS. Also the critical Hs for capsize is different from the expected critical Hs according SOLAS.
- Not all type of ships have the same response in every sea state. RO-Pax and cruise ships have greater probability to capsize than other type of ships, so they should be considered as a separate category.

Further investigation

For a further investigation for a PHD project, the following alternatives could be subdued:

- More experiments for each Hs condition and more Hs conditions could be tested for more detailed results.
- Tank tests for more than one ship loading condition by changing the KG of the model which was used, so it can be constructed a uncertainty band for the CDF-Hs curve.
- Tank tests for more than one damage case, by changing the longitudinal position of the flooded department, but always according to the Stockholm agreement's tank test regulations, so the effects of the flooding position could be noticed.
- Change the model so the effects of the ship geometry details can be noticed.

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Appendix

CDF and PDF offsets for each Hs condition

Hs1,50	PDF	CDF	_p(F)=0,005	_p(F)=0,995
1,80E+03	0,00E+00	0,00E+00	2,33E-11	2,08E-01
1,80E+03	1,00E+06	0,00E+00	2,33E-11	2,08E-01
1,80E+03	1,00E+06	5,00E-01	2,19E-01	7,81E-01
1,80E+03	1,00E+06	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00

1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00
1,80E+03	0,00E+00	1,00E+00	7,92E-01	1,00E+00

Hs1,70	**PDF**	**CDF**	**_p(F)=0,005**	**_p(F)=0,995**
9,59E+02	0,00E+00	0,00E+00	2,33E-11	2,08E-01
9,59E+02	1,13E-03	0,00E+00	2,33E-11	2,08E-01
9,81E+02	1,13E-03	2,50E-02	2,33E-11	2,08E-01
1,00E+03	1,13E-03	5,00E-02	2,33E-11	2,08E-01
1,00E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,03E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,05E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,05E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,07E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,09E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,09E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,11E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,14E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,14E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,16E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,18E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,18E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,20E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,22E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,22E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,25E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,27E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,27E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,29E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,31E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,31E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,34E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,36E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,36E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,38E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,40E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,40E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,42E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,45E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,45E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,47E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,49E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,49E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,51E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,53E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,53E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01

1,56E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,58E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,58E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,60E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,62E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,62E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,65E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,67E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,67E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,69E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,71E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,71E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,73E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01
1,76E+03	0,00E+00	5,00E-02	2,33E-11	2,08E-01

Hs1,90	**PDF**	**CDF**	**_p(F)=0,005**	**_p(F)=0,995**
5,58E+02	0,00E+00	0,00E+00	2,33E-11	2,08E-01
5,58E+02	2,30E-03	0,00E+00	2,33E-11	2,08E-01
5,91E+02	2,30E-03	7,50E-02	2,33E-11	2,58E-01
6,24E+02	2,30E-03	1,50E-01	2,33E-11	3,81E-01
6,24E+02	0,00E+00	1,50E-01	2,33E-11	3,81E-01
6,56E+02	0,00E+00	1,50E-01	2,33E-11	3,81E-01
6,89E+02	0,00E+00	1,50E-01	2,33E-11	3,81E-01
6,89E+02	1,53E-03	1,50E-01	2,33E-11	3,81E-01
7,22E+02	1,53E-03	2,00E-01	9,44E-03	4,51E-01
7,54E+02	1,53E-03	2,50E-01	3,42E-02	5,16E-01
7,54E+02	0,00E+00	2,50E-01	3,42E-02	5,16E-01
7,87E+02	0,00E+00	2,50E-01	3,42E-02	5,16E-01
8,20E+02	0,00E+00	2,50E-01	3,42E-02	5,16E-01
8,20E+02	0,00E+00	2,50E-01	3,42E-02	5,16E-01
8,52E+02	0,00E+00	2,50E-01	3,42E-02	5,16E-01
8,85E+02	0,00E+00	2,50E-01	3,42E-02	5,16E-01
8,85E+02	0,00E+00	2,50E-01	3,42E-02	5,16E-01
9,18E+02	0,00E+00	2,50E-01	3,42E-02	5,16E-01
9,50E+02	0,00E+00	2,50E-01	3,42E-02	5,16E-01
9,50E+02	0,00E+00	2,50E-01	3,42E-02	5,16E-01
9,83E+02	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,02E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,02E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,05E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,08E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,08E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,11E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,15E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,15E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01

1,18E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,21E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,21E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,24E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,28E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,28E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,31E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,34E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,34E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,38E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,41E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,41E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,44E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,47E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,47E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,51E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,54E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,54E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,57E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,60E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,60E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,64E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,67E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,67E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,70E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,73E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,73E+03	0,00E+00	2,50E-01	3,42E-02	5,16E-01
1,77E+03	0,00E+00	2,50E-01	3,41E-01	8,84E-01
1,80E+03	0,00E+00	2,50E-01	7,92E-01	1,00E+00
1,80E+03	0,00E+00	2,50E-01	7,92E-01	1,00E+00
1,83E+03	0,00E+00	2,50E-01	7,92E-01	1,00E+00
1,87E+03	0,00E+00	2,50E-01	7,92E-01	1,00E+00

Hs2,10	PDF	CDF	_p(F)=0,005	_p(F)=0,995
1,75E+02	0,00E+00	0,00E+00	2,33E-11	2,08E-01
1,75E+02	5,58E-04	0,00E+00	2,33E-11	2,08E-01
2,19E+02	5,58E-04	2,50E-02	2,33E-11	2,08E-01
2,64E+02	5,58E-04	5,00E-02	2,33E-11	2,08E-01
2,64E+02	5,58E-04	5,00E-02	2,33E-11	2,08E-01
3,09E+02	5,58E-04	7,50E-02	2,33E-11	2,58E-01
3,54E+02	5,58E-04	1,00E-01	2,33E-11	3,02E-01
3,54E+02	5,58E-04	1,00E-01	2,33E-11	3,02E-01
3,99E+02	5,58E-04	1,25E-01	2,33E-11	3,43E-01
4,43E+02	5,58E-04	1,50E-01	2,33E-11	3,81E-01
4,43E+02	5,58E-04	1,50E-01	2,33E-11	3,81E-01
4,88E+02	5,58E-04	1,75E-01	2,33E-11	4,17E-01
5,33E+02	5,58E-04	2,00E-01	9,44E-03	4,51E-01
5,33E+02	1,12E-03	2,00E-01	9,44E-03	4,51E-01
5,78E+02	1,12E-03	2,50E-01	3,42E-02	5,16E-01
6,22E+02	1,12E-03	3,00E-01	6,39E-02	5,76E-01
6,22E+02	1,12E-03	3,00E-01	6,39E-02	5,76E-01
6,67E+02	1,12E-03	3,50E-01	9,76E-02	6,32E-01
7,12E+02	1,12E-03	4,00E-01	1,35E-01	6,85E-01
7,12E+02	0,00E+00	4,00E-01	1,35E-01	6,85E-01
7,57E+02	0,00E+00	4,00E-01	1,35E-01	6,85E-01
8,02E+02	0,00E+00	4,00E-01	1,35E-01	6,85E-01
8,02E+02	5,58E-04	4,00E-01	1,35E-01	6,85E-01
8,46E+02	5,58E-04	4,25E-01	1,55E-01	7,10E-01
8,91E+02	5,58E-04	4,50E-01	1,75E-01	7,34E-01
8,91E+02	0,00E+00	4,50E-01	1,75E-01	7,34E-01
9,36E+02	0,00E+00	4,50E-01	1,75E-01	7,34E-01
9,81E+02	0,00E+00	4,50E-01	1,75E-01	7,34E-01
9,81E+02	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,03E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,07E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,07E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,12E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,16E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,16E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,20E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,25E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,25E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,29E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,34E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,34E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,38E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,43E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,43E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,47E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,52E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,52E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,56E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,61E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01

1,61E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,65E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,70E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,70E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,74E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01
1,79E+03	0,00E+00	4,50E-01	1,75E-01	7,34E-01

Hs2,30	**PDF**	**CDF**	**_p(F)=0,005**	**_p(F)=0,995**
1,56E+02	0,00E+00	0,00E+00	2,33E-11	2,08E-01
1,56E+02	1,72E-03	0,00E+00	2,33E-11	2,08E-01
1,99E+02	1,72E-03	7,50E-02	2,33E-11	2,58E-01
2,43E+02	1,72E-03	1,50E-01	2,33E-11	3,81E-01
2,43E+02	1,72E-03	1,50E-01	2,33E-11	3,81E-01
2,86E+02	1,72E-03	2,25E-01	2,11E-02	4,84E-01
3,30E+02	1,72E-03	3,00E-01	6,39E-02	5,76E-01
3,30E+02	1,15E-03	3,00E-01	6,39E-02	5,76E-01
3,73E+02	1,15E-03	3,50E-01	9,76E-02	6,32E-01
4,17E+02	1,15E-03	4,00E-01	1,35E-01	6,85E-01
4,17E+02	1,72E-03	4,00E-01	1,35E-01	6,85E-01
4,60E+02	1,72E-03	4,75E-01	1,97E-01	7,58E-01
5,04E+02	1,72E-03	5,50E-01	2,66E-01	8,25E-01
5,04E+02	1,15E-03	5,50E-01	2,66E-01	8,25E-01
5,47E+02	1,15E-03	6,00E-01	3,15E-01	8,65E-01
5,91E+02	1,15E-03	6,50E-01	3,68E-01	9,02E-01
5,91E+02	1,72E-03	6,50E-01	3,68E-01	9,02E-01
6,34E+02	1,72E-03	7,25E-01	4,53E-01	9,52E-01
6,78E+02	1,72E-03	8,00E-01	5,49E-01	9,91E-01
6,78E+02	0,00E+00	8,00E-01	5,49E-01	9,91E-01
7,22E+02	0,00E+00	8,00E-01	5,49E-01	9,91E-01
7,65E+02	0,00E+00	8,00E-01	5,49E-01	9,91E-01
7,65E+02	0,00E+00	8,00E-01	5,49E-01	9,91E-01
8,09E+02	0,00E+00	8,00E-01	5,49E-01	9,91E-01
8,52E+02	0,00E+00	8,00E-01	5,49E-01	9,91E-01
8,52E+02	0,00E+00	8,00E-01	5,49E-01	9,91E-01
8,96E+02	0,00E+00	8,00E-01	5,49E-01	9,91E-01
9,39E+02	0,00E+00	8,00E-01	5,49E-01	9,91E-01
9,39E+02	0,00E+00	8,00E-01	5,49E-01	9,91E-01
9,83E+02	0,00E+00	8,00E-01	5,49E-01	9,91E-01
1,03E+03	0,00E+00	8,00E-01	5,49E-01	9,91E-01
1,03E+03	0,00E+00	8,00E-01	5,49E-01	9,91E-01
1,07E+03	0,00E+00	8,00E-01	5,49E-01	9,91E-01

1,11E+03	0,00E+00	8,00E-01	5,49E-01	9,91E-01
1,11E+03	0,00E+00	8,00E-01	5,49E-01	9,91E-01
1,16E+03	0,00E+00	8,00E-01	5,49E-01	9,91E-01
1,20E+03	0,00E+00	8,00E-01	5,49E-01	9,91E-01
1,20E+03	5,74E-04	8,00E-01	5,49E-01	9,91E-01
1,24E+03	5,74E-04	8,25E-01	5,83E-01	1,00E+00
1,29E+03	5,74E-04	8,50E-01	6,19E-01	1,00E+00
1,29E+03	5,74E-04	8,50E-01	6,19E-01	1,00E+00
1,33E+03	5,74E-04	8,75E-01	6,57E-01	1,00E+00
1,37E+03	5,74E-04	9,00E-01	6,98E-01	1,00E+00
1,37E+03	0,00E+00	9,00E-01	6,98E-01	1,00E+00
1,42E+03	0,00E+00	9,00E-01	6,98E-01	1,00E+00
1,46E+03	0,00E+00	9,00E-01	6,98E-01	1,00E+00
1,46E+03	0,00E+00	9,00E-01	6,98E-01	1,00E+00
1,51E+03	0,00E+00	9,00E-01	6,98E-01	1,00E+00
1,55E+03	0,00E+00	9,00E-01	6,98E-01	1,00E+00
1,55E+03	0,00E+00	9,00E-01	6,98E-01	1,00E+00
1,59E+03	0,00E+00	9,00E-01	6,98E-01	1,00E+00
1,64E+03	0,00E+00	9,00E-01	6,98E-01	1,00E+00
1,64E+03	0,00E+00	9,00E-01	6,98E-01	1,00E+00
1,68E+03	0,00E+00	9,00E-01	6,98E-01	1,00E+00
1,72E+03	0,00E+00	9,00E-01	6,98E-01	1,00E+00

Hs2,50	**PDF**	**CDF**	**_p(F)=0,005**	**_p(F)=0,995**
8,19E+01	0,00E+00	0,00E+00	2,33E-11	2,30E-01
8,19E+01	2,40E-03	0,00E+00	2,33E-11	2,30E-01
1,05E+02	2,40E-03	5,56E-02	2,33E-11	2,30E-01
1,28E+02	2,40E-03	1,11E-01	2,33E-11	3,33E-01
1,28E+02	0,00E+00	1,11E-01	2,33E-11	3,33E-01
1,51E+02	0,00E+00	1,11E-01	2,33E-11	3,33E-01
1,74E+02	0,00E+00	1,11E-01	2,33E-11	3,33E-01
1,74E+02	1,20E-03	1,11E-01	2,33E-11	3,33E-01
1,98E+02	1,20E-03	1,39E-01	2,33E-11	3,78E-01
2,21E+02	1,20E-03	1,67E-01	2,33E-11	4,19E-01
2,21E+02	4,80E-03	1,67E-01	2,33E-11	4,19E-01
2,44E+02	4,80E-03	2,78E-01	4,02E-02	5,65E-01
2,67E+02	4,80E-03	3,89E-01	1,14E-01	6,89E-01
2,67E+02	2,40E-03	3,89E-01	1,14E-01	6,89E-01
2,90E+02	2,40E-03	4,44E-01	1,57E-01	7,44E-01
3,13E+02	2,40E-03	5,00E-01	2,05E-01	7,95E-01
3,13E+02	2,40E-03	5,00E-01	2,05E-01	7,95E-01
3,36E+02	2,40E-03	5,56E-01	2,56E-01	8,43E-01
3,59E+02	2,40E-03	6,11E-01	3,11E-01	8,86E-01
3,59E+02	3,60E-03	6,11E-01	3,11E-01	8,86E-01
3,83E+02	3,60E-03	6,94E-01	4,02E-01	9,43E-01

4,06E+02	3,60E-03	7,78E-01	5,04E-01	9,88E-01
4,06E+02	1,20E-03	7,78E-01	5,04E-01	9,88E-01
4,29E+02	1,20E-03	8,06E-01	5,42E-01	1,00E+00
4,52E+02	1,20E-03	8,33E-01	5,81E-01	1,00E+00
4,52E+02	1,20E-03	8,33E-01	5,81E-01	1,00E+00
4,75E+02	1,20E-03	8,61E-01	6,22E-01	1,00E+00
4,98E+02	1,20E-03	8,89E-01	6,67E-01	1,00E+00
4,98E+02	0,00E+00	8,89E-01	6,67E-01	1,00E+00
5,21E+02	0,00E+00	8,89E-01	6,67E-01	1,00E+00
5,45E+02	0,00E+00	8,89E-01	6,67E-01	1,00E+00
5,45E+02	1,20E-03	8,89E-01	6,67E-01	1,00E+00
5,68E+02	1,20E-03	9,17E-01	7,16E-01	1,00E+00
5,91E+02	1,20E-03	9,44E-01	7,70E-01	1,00E+00
5,91E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
6,14E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
6,37E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
6,37E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
6,60E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
6,83E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
6,83E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
7,06E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
7,30E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
7,30E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
7,53E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
7,76E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
7,76E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
7,99E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
8,22E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
8,22E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
8,45E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
8,68E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
8,68E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
8,92E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
9,15E+02	0,00E+00	9,44E-01	7,70E-01	1,00E+00
9,15E+02	1,20E-03	9,44E-01	7,70E-01	1,00E+00
9,38E+02	1,20E-03	9,72E-01	7,70E-01	1,00E+00
9,61E+02	1,20E-03	1,00E+00	7,70E-01	1,00E+00
9,61E+02	0,00E+00	1,00E+00	7,70E-01	1,00E+00
9,84E+02	0,00E+00	1,00E+00	7,70E-01	1,00E+00
1,01E+03	0,00E+00	1,00E+00	7,70E-01	1,00E+00