National Technical University of Athens

School of Mechanical Engineering

Section of Mechanical Design and Automatic Control

Event-Based Model Predictive Controllers

DOCTORAL DISSERTATION

BY

ALINA M. EQTAMI

Thesis Supervisor: Kostas J. Kyriakopoulos, Professor

Athens, July 2013

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Η έγκριση της διδακτορικής διατριβής από την Ανώτατη Σχολή Μηχανολόγων Μηχανικών του Εθνικού Μετσόβιου Πολυτεχνείου δεν υποδηλώνει αποδοχή των γνωμών του σθγγραφέα (Ν.5343/1932, Άρθρο 202).

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Αλίνα Μ. Εκτάμι

Αθήνα, Ιούλιος 2013

Στην γιαγιά μου Χρυσούλα Κεφάλα

που μου λείπει απεριόριστα.

Ευχαριστίες

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Τετάρτη/19/4/'88

Περιμένοντάς ένα σπουδαίο μήνυμα

Αγρυπνούσα κάθε βράδυ. Περίμενα το Ρομπότ μου. Μου το είχαν τάξει για τα Χριστούγεννα, όμως το ανάβαλαν για το Πάσχα. Διάβαζα... Επαιζα.... Αλλά τίποτα. Το ήθελα και το θέλω πάρα πολύ. Ψυθίριζα «Αχ να μου το έφερναν». Κάθε φορά που χτυπούσε κουδούνι κάποιος, εγώ αμέσως άνοιγα την πόρτα. Τα βράδια έκλεινα τα μάτια - Με σβυστό το φώς για να προσπαθήσω να μπώ κι εγώ μέσα στο όνειρό μου - κι έβλεπα το ρομπότ να μου φέρνει το πρωϊνό στο κρεβάτι. Μια φορά στο κρεβάτι που πήγε να με φιλήσει μου είπε πως αύριο θα το έφερναν κι εγώ πέταξα τις κουβέρτες κι φίλησα την μαμά και τον μπαμπά. Την άλλη μέρα το ονειρό μου έγινε αληθινό. Το ρομπότ ήταν Απίθανο!

Πολύ καλά

Αλίνα Εκτάμι

Event-Based Model Predictive Controllers

by

Alina Eqtami

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requirements for the degree of
Doctor of Philosophy

Abstract

This thesis, mainly lie within the area of Model Predictive Control (MPC) which is a well-known control methodology and the emerging field of event-based control. In particular, this work has been focused on event-based designs of various MPC schemes. Although MPC schemes have conspicuous advantages they are considered to be computationally demanding so it seems useful to update the optimal control recalculation as rarely as possible. To achieve that, we use event-based designs which appear to improve the requirements for computation resources (eg. more efficient resource allocation), and, at the same time, preserve the stability and the convergence properties of the system. Moreover, extensions on this approach can in fact be helpful in networks to decrease control traffic.

In the early stages of our research we were first acquainted with the MPC framework for controlling real robotic systems. We applied the (Nonlinear)MPC framework and tackled the problem of driving a manipulator that initially does not interact with the environment to a desired position and then apply a desired force on a planar surface. The transition from the no-contact case to the contact case was smooth and no impact effects occurred, see Chapter 9. Although MPC can be considered a natural candidate for more "humane-like" control methodologies, it was apparent that it was computationally demanding. This realization led us to the question *"How often do we need to compute the control law?"*. At that point the idea of using the control trajectory that MPC provides in open-loop fashion when it is needed, was already known, [Bem98]. However, it did not provide guarantees on how large delays can be handled without resulting to instability.

The event-based designs for general nonlinear systems was the framework that provided us with the theoretic tools to tackle the fundamental question on how large the inter-sampling times of Model Predictive Controllers can be. A stepping stone was to provide the general event-based control design that was presented for continuous systems, [Tab07], to its discrete-counterpart. That is given in Chapter 2, where an event-triggered strategy is proposed for general discrete-time systems and also more specific results are derived for linear discrete-time systems. Moreover, some results are given in the context of self-triggered control. The first approach for event-triggered MPC is also presented for linear systems.

Using the above framework, in Chapter 3 we combine the event-triggered framework

with MPC and derive some results on how often to compute the control law. The condition that is monitored in order to find whether or not the control law should be computed is not an ad-hoc criterion. On the contrary, the notion of Input-to-State stability (ISS) is used in order to derive a triggering condition which is based on a measurement error. This approach results to less conservative results in terms of computation, with respect to the traditional time-triggering scheme. Also, stability and convergence properties of the closed-loop system are preserved. In particular, we can derive that the system is ISS with respect to measurements errors and that the solution of the closed-loop system converges to a bounded set. Note also, that the systems in hand are constrained nonlinear systems subject to additive disturbances. In Chapter 3 the centralized, the decentralized case as well as the linear time invariant case are treated while in Chapter 4, the Event-Triggered MPC (ET-MPC) is utilized for deriving triggering conditions for a team of agents that are cooperating in a common environment.

In the aforementioned Chapters we are considering dynamics in the discrete-time domain, while in Chapter 6 the continuous counterpart was presented. Note that, although the basic idea is the same (the goal is to compute less frequently the control law) the derivation is significantly different.

In Chapter 5, an enhanced ET-MPC scheme is presented. The idea is to measure the error in order not only to check if the triggering condition still holds, but also to utilize this error in order to "correct" the control input during the inter-sampling periods. This is achieved using tools from second variation methodologies (perturbation analysis).

In the subsequent Chapters, the extension of the event-based MPC framework to a Self-Triggering MPC (ST-MPC) setup is presented. Using this framework the need for continuously measuring the error and checking the triggering condition is relaxed. Specific results on ST-MPC are given for an underwater nonholonomic vehicle, see Chapter 7. The validity of the results have been proven with an experiment in the Control Systems Laboratory, NTUA, [HaEDK13]. Finally, in Chapter 8, a team of agents are considered, that are being controlled locally through ST-MPC. This set-up provides interesting results in terms of reduced communication between the agents.

Thesis Supervisor: Kostas J. Kyriakopoulos
Title: Professor

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Chapter 1

Overview

In this Chapter we are going to present the relevant, more recent, literature review with respect to Event-Based Model Predictive Controllers. Moreover, the motivation and the contributions of this Dissertation are thoroughly discussed.

Firstly, an overview of the Model Predictive Control framework is given. The attributes as well as the drawbacks of this particular methodology are highlighted. Then, the relatively new idea of event-based control is introduced while some recent publications are discussed. Given these ideas along with a teleoperation scenario leads us to the main question that we try to answer in this work, i.e., the motivation is presented. Finally, the Event-Based Model Predictive Control scheme is outlined and the most fundamental contributions of the dissertation are discussed as well as some relevant publications.

1.1 Model Predictive Control

The Model Predictive Control framework has been developed considerably over the last years. The reason for this success can be attributed to the fact that Model Predictive Control is perhaps, the most general way of posing a control problem in the time domain. Moreover, as a finite horizon is used, constraints and in general non-linear processes, can be handled. Literature related to the Model Predictive Control framework is abundant, thus only some recent books are going to be mentioned, i.e., [Wan09], [GP11], [FFB07], [dRAG$^+$10], [CLdlPn11], [Mac00], [Zhe10], [BC10], [MAR09], [KH05], [AZ00], [KC] and [CB00].

We are going to outline the basic idea of a Nonlinear Model Predictive Controller. Assume the following general nonlinear system:

$$x_{k+1} = f(x_k, u_k) \tag{1.1}$$

where $x_k \in \mathbb{R}^n$ denotes the system's state and $u_k \in \mathbb{R}^m$ is the control vector for each time instant $k \in \mathbb{Z}_{\geq 0}$. Function $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is assumed to be continuous and without loss of generality we also assume $f(0,0) = 0$. The NMPC involves solving on-line a finite-horizon, open-loop optimal control problem, based on the state measurement x_k provided by the plant. The optimal control problem involves minimizing a cost function $J_N(\cdot)$ with respect to a control sequence $u_F(k) \triangleq [u(k|k), u(k+1|k), \dots, u(k+N-1|k)]$. The OCP for the system (1.1), is given by

$$\min_{u_F(k)} J_N(x_k, u_F(k)) \tag{1.2}$$

and is subject to system constraints. The positive integer $N \in \mathbb{Z}_{>0}$ denotes the prediction horizon. In the classic NMPC strategy the control law is updated at each time-step k and

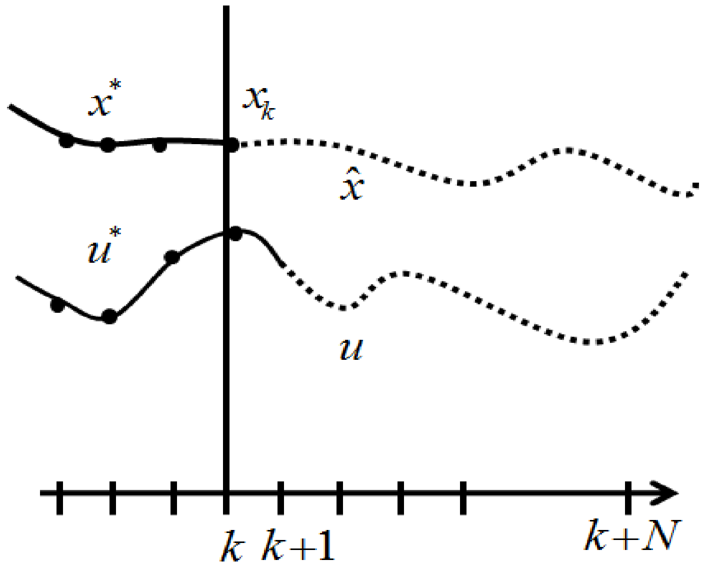

Figure 1-1: Model Predictive Control: The main idea.

the control input that is applied to the system is given by $u_k = u^*(k|k)$, where $u^*(k|k)$ is the first column of the optimal solution provided by the optimal control problem (1.2). The algorithm that describes the NMPC is given in the table “Algorithm 1”. Furthermore, the whole set-up is borne out in Fig. 1-1. The algorithm for the Predictive Control can be summarized as follows

Algorithm 1 Classic MPC

1: Time equals to k.
2: **while** $x_k \notin X_f$ **do**
3: Measure the current state of the plant x_k.
4: Compute the open-loop optimal control sequence $u^* : [k, k+N]$.
5: Apply the control input: $u(k) = u^*(k|k)$.
6: $k = k+1$.
7: **end while**
8: Continue from 3.

As it is apparent the NMPC framework can deal with nonlinearities as well as constraints. Also, it should be pointed out that NMPC have some kind of inherent robustness due to the (not explicit) feedback policy. These are the attributes that make NMPC to be widely used. However, there are some drawbacks. These are comprised mainly to the solution of the optimal control problem. It is widely known that the solution of a constraint optimal control problem can be computationally demanding. In some cases, the computational cost may be as high, that make the use of NMPC practically prohibitive. This is a part of the discussion that will take place in the sequel as the computational cost of NMPC was one of the motivations for this work.

1.2 Event-Based Control

The formulation of event-based control schemes is a flourishing field in the recent years. The key attribute of these approaches is that the decision for the execution of the control task is not made ad-hoc, but it is based on a certain condition of the state of the system. More particularly, the decision for sampling in event-based schemes takes into account state or output feedback in order to sample as infrequently as possible. This results to a more flexible aperiodic sampling, while preserving necessary properties of the system such as stability and convergence. Notice, that the particular formulation for sampling can lead to the alleviation of energy consumption, to an improvement on the requirements for computational resources and may lead to a significant reduction of the network traffic in network control systems. Thus, it can be proven to be less conservative with respect to the constant sampling where the worst case scenario is considered.

Notice that the event-triggered control for discrete-time systems is going to be treated extensively in Chapter 2 of this Thesis, [EDK10]. The formulation of the event-based control in particular for discrete-time systems is a contribution of this thesis, nevertheless, quite a few papers have appeared mostly in the continuous-time frame. More specifically, here we are going to present some of the most recent paper that deal with the event and self triggered control framework.

The time-triggered paradigm is the most dominant scheduling rule for the control community. This assumes a constant sampling period. There is no particular policy or rule in order to reach to this sampling period. The main idea is to sample as frequently as possible in order to take into account the worst case scenario. The block diagram of the traditional time-triggered case is depicted in Fig. 1-2.

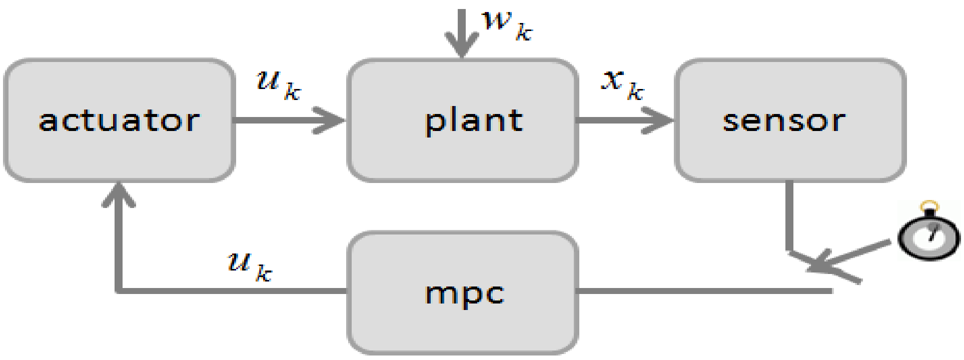

Figure 1-2: Time-Triggered Case.

Some papers deal with the comparison of the time-triggered versus the event-triggered policies and reach to conclusions on the pros and cons of both approaches. A comparison of time-driven and event-driven control for stochastic systems favoring the latter can be found in [AB02], [Åst08].

As it is already mentioned the event-based policies for triggering has gained much attention in the recent years. The block diagram that depicts the event-based approach is given in Fig. 1-3. An introductory paper is [HJT12], where many issues on event-triggered control is outlined. In [Tab07], the control actuation is triggered whenever a certain error

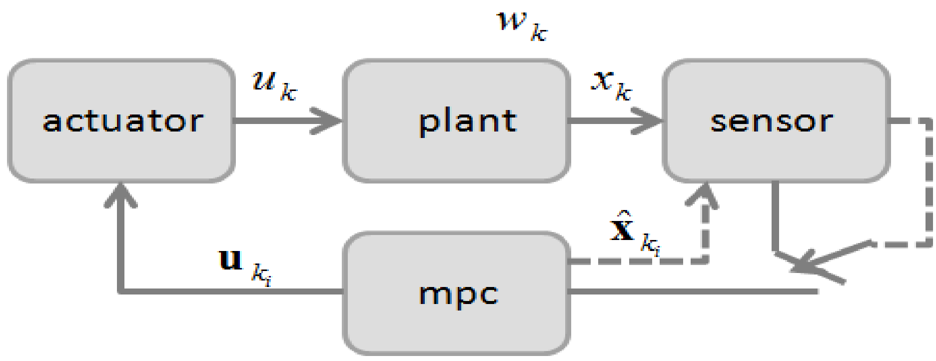

Figure 1-3: Event-Triggered Case.

norm becomes large enough to overtake the norm of the state of the plant. The nominal system is assumed to be Input-to-State stable [Son08], with respect to measurement errors and then, tools from perturbation analysis of nonlinear systems are used in order to analyze the performance of the event-driven system. The results revealed that the proposed scheme can maintain desired levels of performance. This framework is a stepping stone for this Thesis, as the discrete-time counterpart is presented and utilized throughout this work. In a similar context as [Tab07], the authors in [JT08] have resorted to an event-driven policy for sensor/ actuator networks, which resulted in less energy consumption.

An alternative approach to event-driven control, for perturbed linear systems can be found in [HSB07]. In the case of event-triggered feedback nonlinear systems another approach can be seen in [WL08a]. In [CBJ09], an event-driven optimization-based control of hybrid systems with integral continuous-time dynamics was presented. An adaptive stabilization of Model-Based Networked Control Systems was given in [GA11a]. Some case studies for event-driven control are given in [SHB07]. Related works can be found also in [VML08a], [GA11b], [HD13],[JAT09], [JT09], [WL09c], [MH11], [JT11], [SDJ11], [WL09a], [YZA10].

The event-based approach has been extended to decentralized and multi-agent frameworks. Event-driven strategies for multi-agent systems are presented in [DJ09a] and a cooperative scenario in [DJ09b]. Event-triggered and self-triggered stabilization of distributed networked control systems was given in [PTNA11] as well as in [DF09]. Moreover, the event-based framework is utilized to stochastic control also. Stochastic event-triggered strategies regarding sensing and actuation for networked control systems have been stated in [RJJ08].

It was mentioned that most of the works on event-based control are regarding continuous-time systems. However, some papers have appeared that assume the discrete-time framework, [LL10b] and [MM10].

Event-driven techniques require the constant measurement of the current state of the plant, in order to decide when the control execution must be triggered. In the case of self-triggered control [WL09b], [AT10], only the last state measurement needs to be known for determining the next time instant where the state must be measured again so that the control law is computed and the actuators are updated. A first attempt has been made for linear systems in [VFM05] and recently for systems with finite-gain $\mathscr{L}_2$ stability, [WL08b]. Some particular classes of nonlinear systems, namely state-dependent homogeneous systems and polynomial systems, under self-triggered policy have been presented in [AT10]. Related approaches can be found in [LL09].

In [LCHZ07], the authors present full-information self-triggered $\mathscr{H}_\infty$ controllers. In the context of linear systems the paper [JAT09] deal with the tradeoffs between the the computational resources required for the self-triggered implementation and the resulting performance. Moreover, in [JT08] the same approach is used for sensor/actuators networks. The Input-to-State stability of the self-triggered framework is revisited in [JT09].

1.3 Contributions

The NMPC strategy is a widely used control strategy for constrained systems. Even though formulating a control problem under NMPC is intuitively attractive, the computation of the control law is considerably demanding. Motivated by this fact, an event-based framework for this kind of controllers has been investigated, in order to reduce the number of times the control input should be computed. This event-based approach exploits the fact that predictive controllers provide a control sequence for a prediction horizon. The main idea is that the control sequence provided by the controller is applied to the system in an open-loop fashion between actuator updates. Figure 1-4 depicts the main idea for the Event-based

NMPC framework.

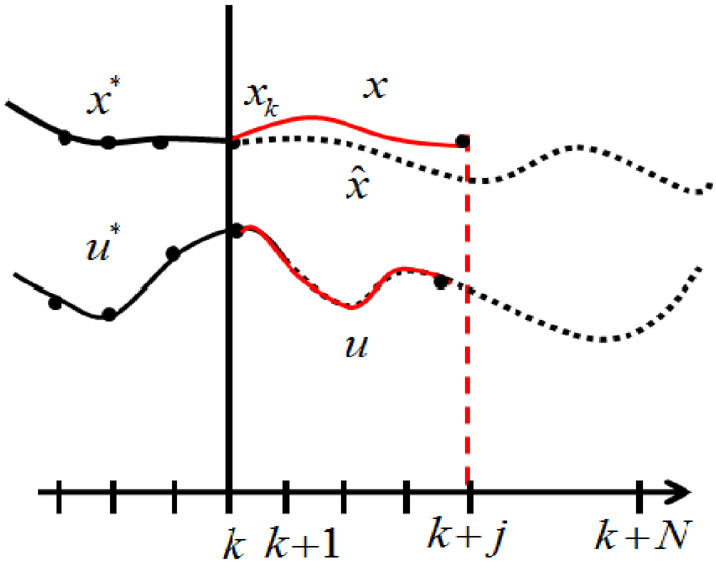

Figure 1-4: Event-Based Model Predictive Control: The main idea.

It should be pointed out that the triggering conditions depend on an error $e(\cdot)$. This error, depends on the actual state of the system $x(\cdot)$ and the predicted state of the system $\hat{x}(\cdot)$, that can be calculated by forward integration of the model (1.1). The difference between the predicted and the actual state is due to disturbances as well as uncertainties induced by the model of the system. The algorithm for the Event-Based Model Predictive Control can be summarized as follows

This approach resulted to a number of publications, indicatively, [EDK10], [EDK11a], [EDK11b], [EDK12b], [EDK12a], [EHaDK13] and [HaEDK13].

1.3.1 Event-Based Model Predictive Control

The field of event-based MPC is quite new, however, some relevant works have already been presented. In [VKFF09], the NMPC framework for continuous-time Network Control systems is event-based, but the system is assumed to be nominal and the triggering condition was not discussed. Also, an event-driven MPC scheme for a particular class of integral continuous-time hybrid automata was presented in [BCJ06] and in [BB08]. More recently, an algorithm for event-based optimal feedback control was given in [GM09] and

Algorithm 2 Event-Based MPC

1: **while** $x_k \notin X_f$ **do**
2: Measure x_k.
3: Calculate the MPC control law: $u_F^*(k)$
Buffer: (Zero-based Indexing)
$u_F^* = [u^*(k|k), u^*(k+1|k), \dots, u^*(k+N-1|k)]$.
$\hat{x}_F = [x_k, \hat{x}(k+1|k), \dots, \hat{x}(k+N-1|k)]$.
4: $k_0 \leftarrow k$.
5: Error: $e(k|k_0) = ||x_k - \hat{x}_F(k-k_0)|| = 0$.
6: **while** Error does not violate triggering conditions **do**
7: Apply $u^*(k-k_0)$.
At next step $k \leftarrow k+1$.
Measure x_k.
Calculate Error: $e(k|k_0)$.
8: **end while**
9: Continue from 3.
10: **end while**

in [SLH10] an event-based estimator between the sensor and the MPC controller was introduced in order to decouple the triggering events and the control algorithm. Other related papers for linear systems can be found in [IHF09], [GPW09]. Event-triggered output feedback control of finite horizon discrete-time multi-dimensional linear processes [LL10a].

As the event-based set-up for MPC controllers has just started to gain attention only a few results have been presented for the self-triggered MPC. In the context of self-triggered MPC, an analysis was presented in [HQSJ12] for Network Scheduling. The authors focus on discrete-time LTI systems and they propose a cost function of the MPC that depends on the control performance and the cost for sampling. In [BGH12], a self-triggered MPC framework was presented for constrained discrete-time linear systems. The MPC controller is designed to maintain some specific optimality levels while the control input that is sent to the actuators is the current control value and not the trajectory of the optimal inputs as is the case in the current paper. An approach for network control systems which is extended to continuous time systems, but not in the area of MPC, proposes a self-triggered selection based on quadratic programming, [MOndlPn+11]. There, the authors present an analysis that leads to an optimization problem for maximizing the intersampling period.

Chapter 2

Event-Triggered Control for Discrete-Time Systems

In this Chapter, event-triggered strategies for control of discrete-time systems are proposed and analyzed. The plant is assumed to be Input-to-State Stable with respect to measurement errors and the control law is updated once a triggering condition involving the norm of a measurement error is violated. The results are also extended to a self-triggered formulation, where the next control updates are decided at the previous ones, thus relaxing the need for continuous monitoring of the measurement error. The overall framework is then used in a novel Model Predictive Control approach. Finally, the results are illustrated through simulated examples.

2.1 Introduction

Traditional approaches to sampling for feedback control involve a time-periodic decision ruling. However, this might be a conservative choice. In some cases, equidistant sampling can be prohibitive to attain certain goals. For example, the issues of limited resource and insufficient communication bandwidth for decentralized control of large scale systems, or even the case of inadequate computation power for fast systems, are problems that often have to be dealt with. A recent approach is to sample only when is needed. Even though we need to relax the periodicity for computation of the control law, we still need to preserve necessary properties of the system, such as stability and convergence. It is therefore of great interest to build mechanisms for sampling that do not rely on periodicity or time-triggering techniques. As a result, in recent years the issue of event-driven feedback and sampling, has been developed. The key attribute of these approaches, is that the decision for the execution of the control task is not made ad-hoc, but it is based on a certain condition of the state of the system.

Almost all of the event-based approaches that was mentioned in Chapter 1, Section 1.2, have been performed in the continuous-time frame. The contribution of this Chapter is to show how the event-triggered, as well as the self-triggered techniques can be implemented over the discrete-time frame. The main assumption used for the event-triggered policies, is the Input-to-State property of the plant. For general nonlinear discrete-time systems the ISS characterization was first introduced in [JSW99], and later refined in [JW01]. For sampled-data systems notions of ISS stability can be found in [NTS99], while in [HJND05] minimal ISS gains and transient bounds are given for discrete nonlinear systems.

2.2 Event-Triggered Control

In this section we are going to introduce the event-triggered formulation for a general non-linear system in its discrete-time framework and we will state the general event-triggering rule for sampling.

Consider a control system in the discrete time-domain of the general form

$$x(k+1) = f(x(k), u(k)) \tag{2.1}$$

where $x \in \mathbb{R}^n$ is the state, $x(k+1) \in \mathbb{R}^n$ is the successor state, and $u \in \mathbb{R}^m$ are the control values for each time instant $k \in \mathbb{Z}_+$. The vector field $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is assumed to be continuous. Also assume without loss of generality, that the origin is an equilibrium point for (2.1), i.e., $f(0,0) = 0$. Let the system (2.1) be continuously stabilizable by a continuous feedback law of the form $u(k) = w(x(k))$, with $w : \mathbb{R}^n \to \mathbb{R}^m$. Then, system (2.1) is ISS-stabilizable with respect to measurement errors $e(k)$, i.e., there is feedback control law,

$p : \mathbb{R}^n \to \mathbb{R}^m$ of the form $u(k) = p(x(k)+e(k))$ that renders the closed-loop system

$$x(k+1) = f(x(k), p(x(k)+e(k))) \tag{2.2}$$

Input-to-State (ISS) stable with respect to measurement errors $e(k)$, [JW01]. As in classic Lyapunov theory a system that is ISS-stable, admits an ISS-Lyapunov function, [Kha02]. A continuous function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is an ISS-Lyapunov function for the system (2.2) if there exist $\mathscr{K}_\infty$ functions α_1, α_2, such that

$$\alpha_1(||x||) \leq V(x) \leq \alpha_2(||x||) \qquad \forall x \in \mathbb{R}^n \tag{2.3}$$

and for some α that is class $\mathscr{K}_\infty$ function, and γ that is class $\mathscr{K}$, $V(x)$ also satisfies

$$V(f(x(k), p(x(k)+e(k)))) - V(x(k)) \leq -\alpha(||x(k)||) + \gamma(||e(k)||) \tag{2.4}$$

Assume, now, that in the event-triggered setup the control is updated at the discrete time instants

$$k_0, k_1, k_2, \ldots$$

The control law is then defined as follows

$$u(k) = p(x(k_i)), \qquad k \in [k_i, k_{i+1}) \tag{2.5}$$

i.e., it remains constant in the inter-execution interval. We assume that at the sampling instant k_i with $k_i > 0$, the state variable vector $x(k_i)$ is available through measurement and provides the current plant information. Defining the state measurement error in this interval, as follows

$$e(k) = x(k_i) - x(k), \qquad k \in [k_i, k_{i+1}) \tag{2.6}$$

we get that the stabilizable feedback control law is given by

$$p(x(k_i)) = p(x(k)+e(k))$$

with $k \in [k_i, k_{i+1})$ and the closed-loop equation of system (2.1) becomes

$$x(k+1) = f(x(k), p(x(k) + e(k))) \tag{2.7}$$

System (2.7) remains ISS-stable, if $e(k)$ satisfies

$$\gamma(||e||) \leq \sigma a(||x||) \tag{2.8}$$

with $0 < \sigma < 1$. Invoking this rule into equation (2.4), it becomes

$$V(f(x(k), p(x(k) + e(k)))) - V(x(k)) \leq (\sigma - 1)a(||x||) \tag{2.9}$$

with V still guaranteed to be decreasing. Hence, the control law should be updated when the condition

$$\gamma(||e||) \leq \sigma a(||x||) \tag{2.10}$$

is violated.

Theorem 1 *Consider the system (2.1) and assume that the previously presented assumptions hold. Then the control law (2.5) with the event-triggered ruling (2.10), asymptotically stabilizes the system to the origin.*

2.3 Linear Discrete-Time Systems

Using the general notion for event-trigger policy, presented above, we specialize to discrete-time, time-invariant linear systems. In this case, the triggering policy is found to be also linear with respect to the state of the plant. The system under consideration is

$$x(k+1) = Ax(k) + Bu(k) \tag{2.11}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and the input $u(k) \in \mathbb{R}^m$ is defined in $k \in \mathbb{Z}_+$. The pair (A,B) is considered to be stabilizable, which means that there exists a matrix K so that the eigenvalues of $A+BK$ are inside the unit disc, which yields the system (2.11) to be also ISS-stabilizable. ISS properties of linear discrete-time systems were provided in [JW01].

Input-to-State stabilizability of system (2.11), implies that there exists a stabilizing feedback control law $u(k) = K(x(k)+e(k))$ where $e(\cdot)$ is the measurement error seen as a new input and K is an appropriate matrix, defined above. The compensated closed-loop system can be described by the equation

$$x(k+1) = (A+BK)x(k) + BKe(k) \tag{2.12}$$

System (2.12) admits a quadratic ISS-Lyapunov function of the form

$$V(x) = x^T P x \tag{2.13}$$

Function V is considered to be positive-definite, radially unbounded and satisfies property (2.3) with $\alpha_1(r) = \lambda_{min}(P)r^2$ and $\alpha_2(r) = \lambda_{max}(P)r^2$, with $\lambda_{min}(P)$, $\lambda_{max}(P)$ being the smallest and the largest eigenvalue of matrix P, respectively. Given a symmetric and positive-definite matrix Q, let P be the unique positive definite solution to

$$(A+BK)^T P(A+BK) - P = -Q \tag{2.14}$$

The difference of the ISS-Lyapunov function is

$$V(x(k+1)) - V(x(k)) = -x^T(k)Qx(k) + 2x^T(A+BK)^T PBKe(k) + e^T(k)K^T B^T PBKe(k) \tag{2.15}$$

Then, the property (2.4), holds with

$$\alpha(||x||) = \frac{1}{2}\lambda_{min}(Q)||x||^2$$
$$\gamma(||e||) = (\frac{2||(A+BK)^T PBK||^2}{\lambda_{min}(Q)} + ||K^T B^T PBK||)||e||^2$$

In an event-triggered formulation of system (2.12), with ISS-Lyapunov equation of the form (2.13) the control updates should be enforced when

$$||e|| \leq (4\frac{||(A+BK)^T PBK||^2}{\sigma\lambda_{min}^2(Q)} + 2\frac{||K^T B^T PBK||}{\sigma\lambda_{min}(Q)})^{-1}||x|| \tag{2.16}$$

with $0 < \sigma < 1$, is violated. Thus (2.16) is the linear equivalent of (2.10). We are going to utilize the following notation

$$\mu = (4\frac{||(A+BK)^T PBK||^2}{\sigma\lambda_{min}^2(Q)} + 2\frac{||K^T B^T PBK||}{\sigma\lambda_{min}(Q)})^{-1}$$

in the next subsections.

2.3.1 Time Elapsed Between Consecutive Executions

In the sequel, a result on the minimum time between two consecutive executions is presented for the linear case. We note here that non-trivial lower bounds on the inter-execution times, i.e. bounds strictly larger than one, are not suitable for the systems considered here due to their discrete time nature. A proposition providing sufficient conditions for non-trivial inter-execution times is given in the following paragraphs.

We consider now the state as well as the error to evolve with time. In view of equation (2.6), the system described in equation (2.12) now becomes

$$x(k+1) = Ax(k) + BKx(k_i) \tag{2.17}$$

where k_i is the latest actuation update instant. We set the vector $c_1 = BKx(k_i) = const.$, and thus the solution of (2.17) is

$$x(k) = A^k x(k_i) + \sum_{j=0}^{k-1} A^{k-1-j} c_1 \tag{2.18}$$

It is straightforward to see, that the error at the next discrete time instant is given by $e(k+1) = x(k_i) - x(k+1)$. Thus, equation (2.12) with some manipulation becomes

$$e(k+1) = (A+BK)e(k) + (I-A-2BK)x(k_i) \tag{2.19}$$

The solution of this linear nonhomogeneous equation, is given by

$$e(k) = \sum_{j=0}^{k-1} (A+BK)^{k-1-j} c_2 \tag{2.20}$$

with $c_2 = (I - A - 2BK)x(k_i) = const.$ and $e(k_i) = 0$.

Define now the minimum $k = k^*$ that violates condition (2.16), i.e.,

$$k^* = \arg\min_{k\in\mathbb{N}} \{ || \sum_{j=0}^{k-1} (A+BK)^{k-1-j} c_2 || \geq \mu || A^k x(k_i) + \sum_{j=0}^{k-1} A^{k-1-j} c_1 || \} \tag{2.21}$$

Proposition 1 *Consider the system (2.12) and assume that (2.21) has a solution $k^* > 1$ for all k_i. Then the event-triggered rule (2.16) is non-trivial, in the sense that it takes at least two steps for the next controller update.*

2.3.2 Self-Triggered Control

Another view for finding sampling periods is the self-triggered formulation. Motivated by the corresponding self-triggered notion which was originally proposed by for the continuous-time systems in [JT08], here we are going to provide their discrete analogues. Using this kind of implementation, inter-execution times are provided as in the event-triggered implementation, but in this case no continuous monitoring of the plant's state is required. We shall write the system (2.12) in a state-space representation by eliminating variable $x(k_i)$, while treating $e(k)$ as a new state variable

$$\begin{bmatrix} x_{k+1} \\ e_{k+1} \end{bmatrix} = \begin{bmatrix} A+BK & BK \\ I-A-BK & I-BK \end{bmatrix} \begin{bmatrix} x_k \\ e_k \end{bmatrix}$$

We define $y = [x(k)^\top \, e(k)^\top]^\top$ and

$$C = \begin{bmatrix} A+BK & BK \\ I-A-BK & I-BK \end{bmatrix}$$

This system in a stack vector form becomes a linear homogeneous system. In particular, it can be rewritten as

$$y(k+1) = Cy(k) \qquad \text{with} \qquad y_0 = y_{k_0} \tag{2.22}$$

The initial conditions of this system, at each sampling period, is $y_{k_0} = [x(k_i)^T, 0]^T$, and the solution of (2.22) is of the form

$$y(k) = C^{k-k_i} y_{k_0}$$

As in the general event-triggered formulation, the difference of the ISS-Lyapunov function must be negative, and an inequality of the form (2.10) must exist. As we saw in (2.16), at the linear case this inequality becomes also linear. In view of (2.16), while making some easy manipulations, this inequality can be rewritten as

$$||e(k)||^2 + ||x(k)||^2 \leq \mu^2 ||x(k)||^2 + ||x(k)||^2 \Rightarrow ||y(k)||^2 \leq (1+\mu^2)||\tilde{I}y(k)||^2 \tag{2.23}$$

with $\tilde{I} = \begin{bmatrix} I & 0 \end{bmatrix}$. Similarly to the derivation of Proposition 1, define the minimum $k = k^{**}$ that violates condition (2.23), i.e.,

$$k^{**} = \arg\min_{k \in \mathbb{N}} \{ ||C^{k-k_i} y_{k_0}||^2 \geq (1+\mu^2)||\tilde{I}C^{k-k_i} y_{k_0}||^2 \} \tag{2.24}$$

We now can state the following result for the inter-execution times in this formulation:

Proposition 2 *Consider the system (2.12) and assume that (2.24) has a solution $k^{**} > 1$ for all k_i. Then the self-triggered rule (2.23) is non-trivial, in the sense that it takes at least two steps for the next controller update.*

It is worth noting from equation (2.24), that only the current state of the plant is required to compute the next execution time of the control, thus at each time instant it is known when the next sampling time is going to take place.

2.4 Event-Triggered Control: Another Approach

In this section we propose another event-triggered strategy. The approach is valid for a smaller class of nonlinear systems that satisfy stronger stability conditions. Recall

$$x(k+1) = f(x(k), p(x(k)+e(k)))$$

and assume that the following assumption holds:

Assumption 1 *There exist positive constants $L, L_1, a, \gamma \geq 0$, a C^1 function $W : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$, and $\mathscr{K}_\infty$ functions α_1, α_2 such that*

$$\|f(x(k), p(x(k)+e(k)))\| \leq L\|x(k)\| + L\|e(k)\| \tag{2.25}$$

$$\alpha_1(||x||) \leq \; W(x) \leq \alpha_2(||x||) \qquad \forall x \in \mathbb{R}^n \tag{2.26}$$

$$W(f(x(k), p(x(k)+e(k)))) - W(x(k)) \leq -aW(x(k)) + \gamma\|e(k)\| \tag{2.27}$$

$$\alpha_1^{-1}(||x||) \leq L_1||x|| \tag{2.28}$$

Let k_i be the last update time. For each $k \in [k_i, k_{i+1})$, we can then compute $e(k+1) = x(k_i) - x(k+1)$, so that $\|e(k+1)\| \leq \|x(k+1)\|$, and thus $\|e(k+1)\| \leq L\|x(k)\| + L\|e(k)\|$. Further note that $x(k) = x(k_i) - e(k)$, so that $\|e(k+1)\| \leq 2L\|e(k)\| + L\|x(k_i)\|$. Recalling that $e(k_i) = 0$, the comparison principle for discrete-time systems (see for example, Proposition 1 in [BG95]) yields

$$\|e(k)\| \leq \frac{(2L)^k - 1}{2(2L-1)}\|x(k_i)\| \tag{2.29}$$

for all $k \in [k_i, k_{i+1})$. Equation (2.27) then yields

$$W(f(x(k), p(x(k)+e(k)))) - W(x(k)) \leq -aW(x(k)) + \gamma\frac{(2L)^k - 1}{2(2L-1)}\|x(k_i)\| \tag{2.30}$$

From (2.26), (2.28) we also have

$$\|x(k_i)\| \leq \alpha_1^{-1}(||W(x(k_i))||) \leq L_1 W(x(k_i))$$

Denoting $\psi(k) = \gamma \frac{(2L)^k - 1}{2(2L-1)} L_1$ we get $W(x(k+1)) \leq (1-a)W(x(k)) + \psi(k)W(x(k_i))$. Using again the comparison principle of [BG95] and assuming $a < 1$, we get

$$W(x(k)) \leq \frac{1-(1-a)^k}{a(1-a)} \psi(k) W(x(k_i)) \tag{2.31}$$

Similarly to [WL08a], assume that events are triggered according to

$$W(x(k)) = -\xi W(x(k_i))(k - k_i) + W(x(k_i)) \tag{2.32}$$

the right hand side equation is strictly decreasing for $\xi > 0$ and thus convergence is guaranteed. Define now the minimum $k = k^{***}$ as follows

$$k^{***} = \arg\min_{k \in \mathbb{N}} \{-\xi(k - k_i) + 1 \geq \frac{1-(1-a)^k}{a(1-a)} \psi(k)\} \tag{2.33}$$

Then, using (2.31), (2.32), a sufficient condition for a nontrivial interexecution time is given by $k^{***} > 1$. Note that in this case the result holds only for the restricted class of nonlinear systems satisfying Assumption 1.

2.5 Event-triggered Model Predictive Control for LTI Systems

In this section, we provide initial results on the main motivation behind the study of event-driven strategies for discrete-time systems, namely, the application on computing the inter-sample times in a Model Predictive Control framework.

Consider that the feedback control that we use to stabilize the plant is computed with a Model Predictive Control (abbr. MPC) formulation. It is widely known that this approach is computationally demanding, in the sense that at each sampling period a finite-time optimal control problem must be solved. In this paper we propose an alternative approach based on the event-triggered framework described previously that may be used to reduce the computational load of the MPC framework.

MPC is an implicit feedback policy, thus, the event-trigger condition defined in (2.16), cannot be directly used. In order to reach in an MPC event-trigger policy, we will use the results as well as the notation of [JM02], where ISS properties of linear MPC were investigated. Specifically, in [JM02] the authors deal with linear systems and they prove that the closed-loop system with a receding horizon feedback is globally ISS, when the system is open-loop stable and when input constraints are present. In the case of unstable system, though, the same results apply, but have local nature.

We consider the same linear system as in (2.11). We assume that the prediction horizon is N. The solution of the optimization MPC problem is the optimal sequence

$$u^o(x) = \{u^o(0;x), u^o(1;x), \dots, u^o(N-1;x)\}$$

Consider a set X_r, over which there exist a feasible and stabilizing control, and thus, application of this feasible controller results in feasible state trajectories. Consider, also, a controllability set X_n, i.e., the set of all initial conditions that can be steered into the set X_r in N steps or less, where the MPC feedback controller is defined. The optimization problem has the following cost function

$$V_N^*(x) = \min \sum_{i=0}^{N-1} (x(k)^T Q x(k) + u^T(k) R u(k))) + F(x(N)) \tag{2.34}$$

where $Q > 0$ and $R > 0$ are appropriate performance functions. With particular choices of the terminal state function $F(\cdot)$ and the set X_r, it can be proved that the open-loop stable system (2.11), under the receding horizon feedback $\kappa_N(x) = u^o(0;x)$ can be rendered exponentially stable. The closed-loop system is a piecewise affine system which is stable with a piecewise and differentiable quadratic Lyapunov function $V_N^*(x) = \bar{x}^T P_{i(x)} \bar{x}$, where $\bar{x} = [x, 1]^\top$, and $i(\cdot)$ is a switching function that maps the state space to a finite set of indices labeling the polytopic partitions of the state space.

In [JM02], the authors proved that the receding horizon scheme globally ISS stabilizes stable linear systems with input constraints, with respect to additive disturbance. In an event-triggered formulation, the error, defined as the difference in (2.6), can be considered as the additive disturbance. Thus the closed-loop system becomes

$$x(k+1) = (A - B\kappa_N(x))x(k) + B\kappa_N(x)e(k) = \bar{A}_{i(x)}x(k) + B\kappa_N(x)e(k) \tag{2.35}$$

where $\bar{A}_{i(x)}$ is the closed-loop matrix corresponding to the $i-$th partition of the state space, and i is the switching function that maps the state space to a finite set of indices corresponding to different polytopic regions where the active constraints do not change.

Constant sampling implies a zero error and global exponential stability for the closed-loop system

$$V_N^*(A_{i(x)}x) - V_N^*(x) \leq -C_q||x||^2 \tag{2.36}$$

where $C_q > 0$ is the rate of the exponential decay. The differentiability of the Lyapunov function implies

$$||\frac{V_N^*(x)}{x}|| = ||P_{i(x)}x|| \leq \tilde{L}||x|| \tag{2.37}$$

where $\tilde{L} = \max_i \lambda_{\max}(P_i)$. The maximum is taken over all possible partitions, and $\lambda_{\max}$ is the largest singular value of P_i. Following a similar procedure as in [JM02] we obtain the following result, which shows that the Lyapunov function defined in (2.34) is also an ISS Lyapunov function

$$V_N^*(x(k+1)) - V_N^*(x(k)) \leq (-C_q + \varepsilon\tilde{L}C_A)||x(k)||^2 + (1+\frac{1}{\varepsilon})\tilde{L}C_AC_B||e(k)||^2 \tag{2.38}$$

where $C_A = max_i||\bar{A}_{i(x)}||$ and we let C_B to be defined as $C_B = ||B\kappa_N(x)||^2$. Thus we reach to a conclusion for the event-triggered formulation for a system under a model predictive control strategy

Theorem 2 *The controller updates with event-triggered formulation for a linear system as* (2.11) *under receding horizon control, can be implemented when*

$$||e(k)||^2 \leq \Theta||x(k)||^2 \tag{2.39}$$

is violated, with $\Theta = \frac{(1+\frac{1}{\varepsilon})\tilde{L}C_AC_B}{\sigma(C_q - \varepsilon\tilde{L}C_A)}$ *and* $0 < \sigma < 1$.

2.6 Examples

In this section we provide some simulation results in order to assess the efficiency of the proposed event-triggered, as well as self-triggered stabilizing controllers in the linear case.

The process we consider is a linear, unstable, discrete-time system described by

$$x(k+1) = Ax(k) + Bu(k) \tag{2.40}$$

where matrix $A = \begin{bmatrix} 0.1 & 1.2 \\ 0.007 & 1.05 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 300 & 200 \\ 0.5 & 0.001 \end{bmatrix}$. The control sequence is considered to be optimal and can be determined from an LQR problem, which minimizes a cost function of the form

$$J(u) = \sum_{k=0}^{\infty} [x^{\top}(k)Qx(k) + u^{\top}(k)Ru(k)] \tag{2.41}$$

with performance matrices $Q = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix}$ and $R = \begin{bmatrix} 0.01 & 0.01 \\ 0.01 & 0.01 \end{bmatrix}$. The linear state feedback control law is written in analytical form as $u^*(k) = Kx(k)$, where the matrix K is given by $K = -[R + B^{\top}PB]^{-1}B^{\top}PA$. The matrix P is the unique, symmetric, and positive-definite solution of the discrete-time algebraic Riccati equation $P = A^{\top}P(A+BK) + Q$. We are going to use P in the quadratic ISS-Lyapunov equation, with $V(x) = x^{\top}Px$ being the Lyapunov function candidate. We also define another matrix $\tilde{Q}$ which satisfies the following equation

$$P - (A+BK)^{\top}P(A+BK) = \tilde{Q} \tag{2.42}$$

For the particular problem (2.40) and the event-trigger policy given in (2.16) we choose $\sigma = 0.98$. Then the constant μ has the value $\mu = 0.2934$. Assume, also, that the initial state conditions are $x_0 = [-0.2, 0.5]^{\top}$ and that we want to stabilize system (2.40) at the equilibrium.

Figure 2-1 depicts the norm of the error $||e(k)||$. This stays below the specified state-depended threshold, as given in (2.16) and is represented by the blue solid line in the figure. It can be witnessed that using this event-trigger policy, which is conservative, we

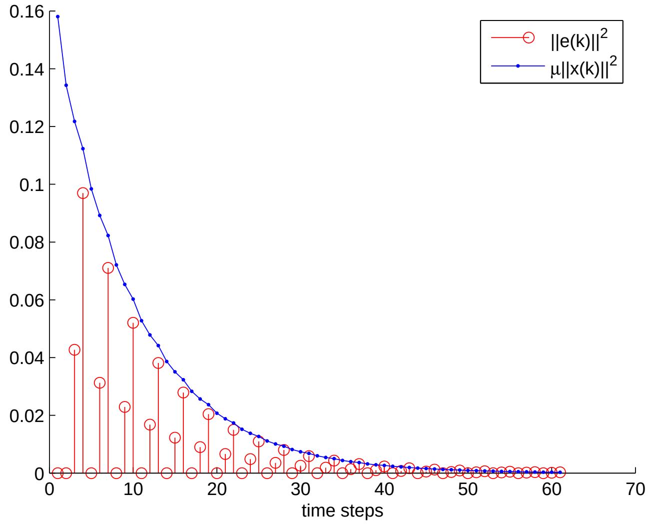

Figure 2-1: Evolution of the error norm in the event-triggered case. Red stems represent the evolution of the error norm $||e(k)||$ which stays below the state-dependent threshold $\mu||x(k)||$ which is represented by the blue line in the Figure.

can sample in periodic fashion, every three steps.

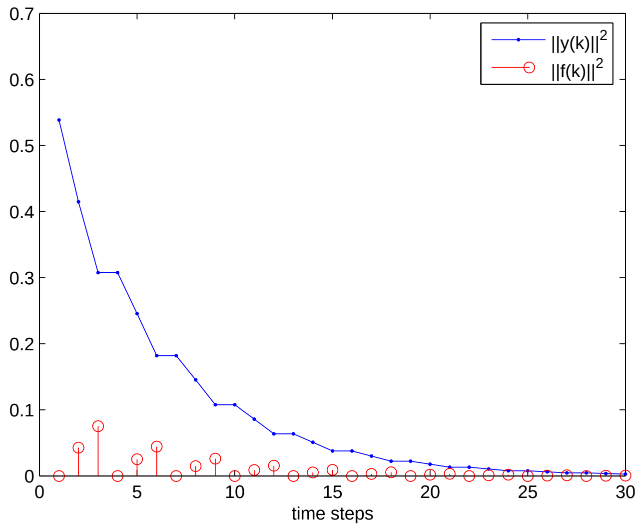

Figure 2-2: Self-trigger framework. The red stems represent $||y(k)||^2$, while the blue solid line represents $f(k) = (1+\mu^2)||\tilde{I}y(k)||^2$ both from (2.23).

The next Figure depicts the sampling of system (2.40) under the self-triggered framework. In order to better visualize when sampling takes place, under the self-trigger policy,

Figure 2-2, depicts the difference

$$D \triangleq ||y(k)||^2 - (1+\mu^2)||\tilde{I}y(k)||^2 \tag{2.43}$$

where we used (2.23). When D, represented by the blue stems, is below zero, there is no need for sampling, or in other words, there is sampling when the blue stems are above the zero line. From the simulations is apparent that the system converges under the event-triggered and self-triggered control frameworks.

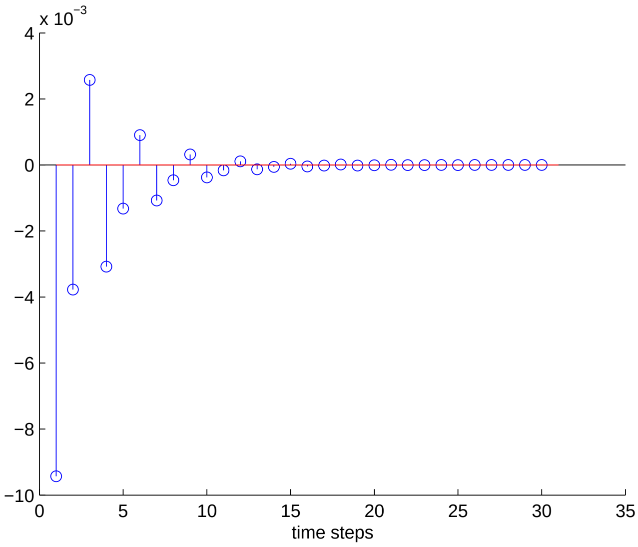

Figure 2-3: Blue stems represent the difference given in (2.43).

2.7 Conclusions

In this Chapter, event-triggered strategies for control of discrete-time systems were proposed and analyzed. Similarly to the continuous-time case, the plant is assumed input-to-state stable with respect to measurement errors and the control law is updated once a triggering condition involving the norm of a measurement error is violated. We considered both nonlinear and linear plant and sufficient condition for non-trivial inter-execution times were derived. The results were also extended to a self-triggered formulation, where the next control updates are decided at the previous ones, thus relaxing the need for continuous monitoring of the measurement error. The overall framework was then used in a novel Model Predictive Control approach.

A straightforward direction of research involves further integration of the event-triggered approach with the Model Predictive Control framework. This will be presented in the subsequent Chapters for both the discrete-time and the continuous-time case.

Chapter 3

Event-Based Methodologies of Model Predictive Controllers for Discrete-time Systems

In this Chapter, novel event-triggered strategies for the design of Model Predictive Controller (MPC) are presented. The MPC framework consists in finding the solution to a constraint optimal-control problem at every time-step. The case of triggering the optimization of the MPC only when is needed, is investigated. The centralized case is treated first and the results are then extended to a decentralized formulation. The event-based framework for the for linear systems is next. Sufficient conditions for triggering the MPC laws are given and the results are illustrated through simulated examples.

3.1 Introduction

The problem addressed here is the event-driven control of general nonlinear discrete-time systems under NMPC. A number of schemes are treated in this Chapter, namely, the centralized case for general uncertain systems, a fully decentralized case with no exchange of information between the agents and finally the linear case. Sufficient conditions for triggering the predictive control law are given for each of these cases. Since the systems in consideration are uncertain, in order to prove stability, a procedure similar to the analysis for Input-to-State (ISS) stability for MPC is going to be used. Some relevant citations of ISS MPC schemes are [MRS06], [MAC02], [PRMP09], [FMP$^+$08], [RMS07], [Laz06].

3.2 Event-based Nonlinear MPC

Given the general nonlinear system (2.1) and under some specifically stated assumptions, one can design a model predictive controller that stabilizes the system to the origin. NMPC involves solving on-line a finite-horizon, open-loop optimal control problem (abbr. OCP), based on the state measurement x_k provided by the plant. The OCP consists in minimizing a cost function $J_N(\cdot)$ with respect to a control sequence $u_F(k) \triangleq [u(k|k), u(k+1|k), \ldots, u(k+N-1|k)]$. The OCP for the system (2.1), is given by

$$\min_{u_F(k)} J_N(x_k, u_F(k)) \tag{3.1}$$

and is subject to system constraints. The positive integer $N \in \mathbb{Z}_{>0}$ denotes the prediction horizon.

In the classic NMPC strategy the control law is updated at each time-step k and the control input that is applied to the system is given by $u_k = u^*(k|k)$, where $u^*(k|k)$ is the first column of the optimal solution provided by the OCP (3.1). Solving an OCP on-line is generally a non-trivial task and is considered computationally demanding. The need to relax the periodicity of the control updates leads to an event-based scheme.

In the event-based setup a portion of the optimal sequence and not necessarily only the first term, might be applied to the system. The described formulation is depicted in Fig. 3-3. In this case, the optimal control sequence is re-calculated at the discrete time instants

$$\{t_0, t_1, t_2, \ldots, t_i, \ldots\} \subseteq \{k_0, k_1, k_2, \ldots, k_j, \ldots\} \qquad t_i, k_j, i, j \in \mathbb{Z}_{\geq 0}$$

Assume that for every triggering instant t_i a new OCP is triggered too, and that t_i coincides with k_j. During the time span $[t_i, t_{i+1})$, where t_{i+1} is the next triggering instant, the control law provided at $t_i \equiv k_j$ is implemented in open-loop fashion. For illustrative purposes assume that $t_{i+1} - t_i = \delta_i$. The time period δ_i will then satisfy $1 \leq \delta_i \leq N-1$.

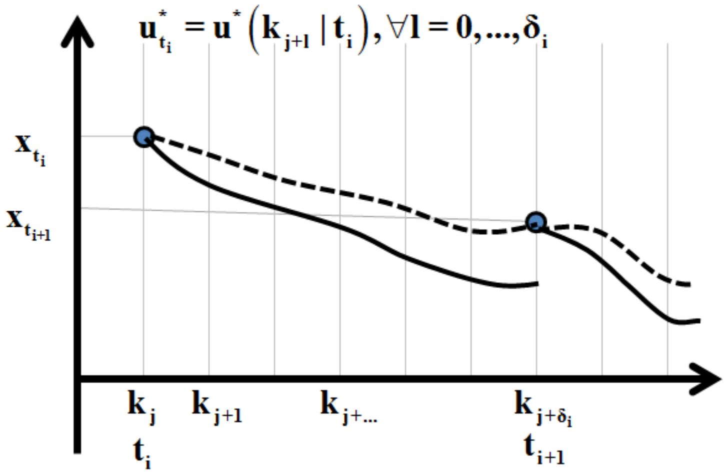

Figure 3-1: The event-triggered NMPC problem formulation. The black doted line represents the actual state sequence and the black solid line represents the system's predicted sequence. For time span $[t_i, t_{i+1})$ the system is controlled in open-loop fashion. The control sequence applied is $u^*(k_{j+l}|t_i)$ for $l = 0, \ldots, \delta_i$. Violation of the triggering condition, defines the next triggering instant $t_{i+1} \equiv k_{i+\delta_i}$.

Similarly to the general event-triggered setup that was described in the previous Chapter, an error is defined and a triggering condition is stated. The specifics for particular systems are given in the sequel Sections.

3.3 Event-based NMPC for Centralized Discrete-time Systems

In real systems and applications the model that describes the system may be inaccurate as a result of disturbances or uncertainties. In the subsequent Sections we are going to treat the problem of reaching to a triggering condition for systems with additive disturbance under a NMPC law. This event-based setup leads to relaxing the periodicity of the control updates while the system reaches to a desired, bounded, set.

3.3.1 Problem Statement for the Centralized Case of ET-NMPC

The idea is to find a triggering condition for a nonlinear system with additive disturbances using a similar approach as of the general framework that was previously presented in

Chapter 2. Thus, an ISS stability analysis for a nonlinear system under a NMPC law is going to be introduced next. In [MAC02] it is rigorously proven that the closed-loop system of a nonlinear system with a NMP Controller is ISS stable with respect to the uncertainties. In the following a modification of the ISS analysis of [MAC02] is properly addressed in order to attain the triggering condition.

Consider the nonlinear system of the form

$$x_{k+1} = f(x_k, u_k) + w_k \tag{3.2}$$

where $x_k \in \mathbb{R}^n$ denotes the system's state, $u_k \in \mathbb{R}^m$ the control variables and $w_k \in \mathscr{W} \subseteq \mathbb{R}^n$ is the additive disturbance. Assume that $\mathscr{W}$ is a compact set, containing the origin, and that the admissible set of uncertainties is bounded for some $\gamma > 0$ i.e., $||w_k|| \leq \gamma$. The state and control variables are constrained as

$$x_k \in \mathscr{X}, \qquad u_k \in \mathscr{U} \qquad k \in \mathbb{Z}_{\geq 0} \tag{3.3}$$

For control design purposes the nominal model of the system (2.1) is used, which is assumed to be locally Lipschitz in x in the domain $\mathscr{X} \times \mathscr{U}$, with Lipschitz constant L_f. To facilitate the analysis, a double subscript notation will be used hereafter

$$\hat{x}(k+j+1|k) = f(\hat{x}(k+j|k), u_{k+j})$$

where the nominal model (2.1) is used. The term $\hat{x}(k+j+1|k)$ is the predicted state at time-step $k+j+1$, based on the measured state of the real system at time-step k i.e., $x_k = \hat{x}(k|k)$ and the term u_{k+j} is the applied control sequence for time-step k until time-step $k+j$.

The uncertainty term of the real system can cause discrepancies between the predicted state given from the nominal model (2.1) subject to a specific sequence of inputs and the actual state, given from (3.2) for the same sequence of inputs. In order to account for this mismatch the error e is introduced in the analysis. The error e is defined as the norm of the difference between the predicted and the real evolution of the state. In the sequel, the double subscript notation will be reserved for the error, too. The error $e(k+j|k)$ will particularly be defined as

$$e(k+j|k) = ||x_{k+j} - \hat{x}(k+j|k)|| \tag{3.4}$$

It has been shown in [MAC02] that

([MAC02], Lemma 1) Lemma 1 *Given the nominal system* (2.1) *and the perturbed system* (3.2) *and for a given sequence of inputs, the difference between the nominal prediction of the state* $\hat{x}(k+j|k)$ *and the real state of the system* x_{k+j} *is bounded by* $||\hat{x}(k+j|k) - x_{k+j}|| \leq \frac{L_f^j - 1}{L_f - 1}\gamma$.

It is straightforward to prove that the error $e(k+j|k)$ is also bounded by the same bound, and that $e(k|k) \equiv 0$.

In the following, the NMP Controller is going to be introduced along with some specific assumptions that are fundamental in order to prove stability of the overall scheme. The OCP (3.1), of the NMPC is given by

$$\min_{u_F(k)} J_N(x_k, u_F(k)) = \min_{u_F(k)} \sum_{i=0}^{i=N-1} L(\tilde{x}(k+i|k), u(k+i|k)) + V(\tilde{x}(k+N|k)) \tag{3.5a}$$

subject to

$$\tilde{x}(k+j|k) \in \mathscr{X}_j \qquad u(k+j|k) \in \mathscr{U} \qquad \tilde{x}(k+N|k) \in \mathscr{X}_f \qquad \forall j = 1, \ldots, N-1 \tag{3.5b}$$

The notation $\tilde{\cdot}$ is used to denote the controller internal variables. Also we have, $\tilde{x}(k|k) = x_k$.

The set $\mathscr{X}_f$ denotes the terminal constraint set with $\mathscr{X}_f \subset \mathscr{X}$ to be closed and $0 \in \mathscr{X}_f$. The system under control is perturbed, thus the terminal constraint set is computed as a subset of an admissible positively invariant set $\mathscr{X}_f^n$ for the nominal system, [MAC02]. Notice also, that the state constraint set $\mathscr{X}$ of the nominal system, is being replaced by a restricted constraint set $\mathscr{X}_j$. This state constraints' tightening for the nominal system guarantees that the evolution of the real system will be admissible for all time, [MAC02]. Given Lemma 1 where a bound on the state prediction error is evaluated, we set $\mathscr{X}_j = \mathscr{X} \sim \mathbb{B}_j$ where $\mathbb{B}_j = \{x \in \mathbb{R}^n : ||x|| \leq \frac{L_f^j - 1}{L_f - 1}\gamma\}$. This ensures the satisfaction of the original state constraints under the worst case uncertainty. The set operator "$\sim$" denotes the Pontryagin difference, i.e., given two sets $A, B \in \mathbb{R}^n$ the Pontryagin difference set C, is defined as $C = A \sim B \triangleq \{x \in \mathbb{R}^n : x + \xi \in A, \forall \xi \in B\}$.

The following assumptions for the NMPC formulation are stated:

Assumption 2 *The stage cost $L(x,u)$ is such that $L(0,0) = 0$ and $\underline{L}(||x||) \leq L(x,u)$, $\forall x \in \mathscr{X}$ and $\forall u \in \mathscr{U}$ where $\underline{L}$ is a $\mathscr{K}_\infty$-function. Furthermore, $L(\cdot)$ is Lipschitz continuous with respect to x and u in $\mathscr{X} \times \mathscr{U}$, with Lipschitz constants $L_c \in \mathbb{R}_{\geq 0}$ and $L_{c_u} \in \mathbb{R}_{\geq 0}$, respectively.*

Assumption 3 *Assume that there is a local stabilizing controller $h(x_k)$ for the set $\mathscr{X}_f^n$. The associated Lyapunov function $V(\cdot)$ has the following properties*

$$V(f(x_k, h(x_k)) - V(x_k) \leq -L(x_k, h(x_k)) \qquad \forall x_k \in \mathscr{X}_f^n \tag{3.6}$$

and is Lipschitz in $x \in \mathscr{X}_f^n$, with Lipschitz constant $L_V \in \mathbb{R}_{>0}$.

For the auxiliary control law $h(x_k)$ we make the following assumptions

Assumption 4 *There is $h(x) \in \mathscr{U}$, $||h(x)|| \leq L_h||x||$, $L_h > 0$, $\forall x \in \mathscr{X}_f^n$. Also, we have $||f(x,h(x))|| \leq L_{f_h}||x||$, $L_{f_h} > 0$, $\forall x \in \mathscr{X}_f^n$.*

We also assume that:

Assumption 5 *The set $\mathscr{X}_f^n$ is given by $\mathscr{X}_f^n = \{x \in \mathbb{R}^n : V(x) \leq \alpha\}$ and the set $\mathscr{X}_f$ is such that $\mathscr{X}_f = \{x \in \mathbb{R}^n : V(x) \leq \alpha_v\}$ and that for all $x \in \mathscr{X}_f^n$ we have $f(x,h(x)) \in \mathscr{X}_f$, with $\alpha \geq \alpha_v$.*

We are now ready to state the problem statement for the centralized case of the event-based NMPC:

Problem Statement 1 *Consider the system* (3.2) *that is subject to constraints* (3.3). *The objective is (i) to design a feedback control law provided by* (3.5a)-(3.5b) *such that ISS-stability with respect to disturbances is achieved while state and control constraints are satisfied and (ii) to find the event-based condition for triggering the control updates while satisfying convergence and stability criteria.*

3.3.2 Feasibility

Let $\mathscr{X}^{MPC}$ be the set containing all the state vectors for which a feasible control sequence exists, i.e. a control sequence of the form $[u(k|k),\dots,u(k+N-1|k)]$ that satisfies all the constraints of the optimal control problem. Assume that at $t_i = k-1$ an event is triggered, thus an OCP is solved and new control sequence is provided. More specifically, solving the OCP of the NMPC (3.5a)-(3.5b) at a time step $k-1$ results in an optimal control trajectory $u_F^*(k-1) \triangleq [u^*(k-1|k-1),\dots,u^*(k+N-2|k-1)]$. Now, consider the control sequences $\bar{u}_F(k+m)$, for the subsequent time steps $k+m$ with $m=0,\dots,N-1$, based on the optimal solution at $k-1$, $u_F^*(k-1)$, i.e., for $m=0,\dots,N-1$

$$\bar{u}_F(k+m) = \bar{u}(k+j|k+m) = \begin{cases} u^*(k+j|k-1) & \text{for} \quad j=m,\dots,N-2 \\ h(\hat{x}(k+j|k+m)) & \text{for} \quad j=N-1,\dots,N+m-1 \end{cases} \tag{3.7}$$

Notice that the time-steps $k+m$ are the discrete-time instants after the time-step of the triggering instant t_i, thus they can be written as $[k-1,k,k+1,\dots,k+N-2] \equiv [t_i,t_i+1,t_i+2,\dots,t_i+N-1]$. In order to derive feasibility it is essential to show that $\hat{x}(k+N|k+m) \in \mathscr{X}_f$ for all $m=0,\dots,N-1$. Nevertheless, we begin showing that $\hat{x}(k+N-1|k+m) \in \mathscr{X}_f^n$. With the help of (3.4) and Lemma 1, it can be obtained that

$$\begin{aligned}
&||\hat{x}(k+N-1|k) - \hat{x}(k+N-1|k-1)|| \leq L_f^{N-1} e(k|k-1) \\
&||\hat{x}(k+N-1|k+1) - \hat{x}(k+N-1|k-1)|| \leq L_f^{N-2} e(k+1|k-1) \\
&\vdots \\
&||\hat{x}(k+N-1|k+m) - \hat{x}(k+N-1|k-1)|| \leq L_f^{(N-1)-m} e(k+m|k-1)
\end{aligned} \tag{3.8}$$

From the Lipschitz property of $V(\cdot)$ (Assumption 3) and (3.8), we get:

$$\begin{aligned}
&V(\hat{x}(k+N-1|k+m)) - V(\hat{x}(k+N-1|k-1)) \leq \\
&L_V ||\hat{x}(k+N-1|k+m) - \hat{x}(k+N-1|k-1)|| \leq L_V L_f^{(N-1)-m} e(k+m|k-1)
\end{aligned} \tag{3.9}$$

Noticing that $\hat{x}(k+N-1|k-1) \in \mathscr{X}_f$ and from Assumption 5, we get:

$$V(\hat{x}(k+N-1|k+m)) \leq \alpha_v + L_V L_f^{(N-1)-m} e(k+m|k-1) \tag{3.10}$$

It should hold that $V(\hat{x}(k+N-1|k+m)) \leq \alpha$, i.e., $\hat{x}(k+N-1|k+m) \in \mathscr{X}_f^n$, thus

$$\alpha_v + L_V L_f^{(N-1)-m} e(k+m|k-1) \leq \alpha \Rightarrow e(k+m|k-1) \leq \frac{\alpha - \alpha_v}{L_V L_f^{(N-1)-m}} \tag{3.11}$$

If this is the case, then by applying the local controller $h(\cdot)$ we get $\hat{x}(k+N|k+m) \in \mathscr{X}_f$ for all $m = 0, \ldots, N-1$. It should be pointed out that (3.11) is one of the triggering conditions that is going to be proposed in the next sections. From the feasibility of the initial trajectory $u_F^*(k-1)$ and Assumption 4, it follows that for $m = 0, \ldots, N-1$ we have $\bar{u}(k+j|k+m) \in \mathscr{U}$.

Nevertheless, another approach, similar to the [MAC02] would have been as follows:

Remark 1 *Taking into consideration Lemma 1, we get that*

$$||\hat{x}(k+N-1|k+m) - \hat{x}(k+N-1|k-1)|| \leq L_f^{(N-1)-m} \frac{L_f^{m+1}-1}{L_f - 1} \gamma$$

Making similar derivations as above, it can be concluded that

$$\gamma \leq \frac{(\alpha - \alpha_v)(L_f - 1)}{L_V L_f^{(N-1)-m} (L_f^{m+1} - 1)} \tag{3.12}$$

which states that the set $\mathscr{X}^{MPC}$ can be proven to be robustly positively invariant if the uncertainties are bounded by (3.12) *for all $m = 0, \ldots, N-1$. However, in this section we consider that the error can be measured, thus we imposed the aforementioned triggering condition* (3.11)*. Notice that* (3.11)-(3.12) *should still hold for $m = 0$, for the problem to be meaningful, in the sense that it should be feasible at least in the time-triggered case.*

3.3.3 Input-to-State Stability under the Event-based NMPC set-up

In this section, the stability of the proposed event-based NMPC is presented and then the triggering condition is given using a similar approach as of Chapter 2. The stability proof of the NMP Controller is based on the initial feasibility property and the decrease of a suitable Lyapunov function. The former was proven in the previous Section while the latter will be addressed in this Section.

In [LHT12] it was shown that every discrete-time system that admits a continuous Lyapunov function is inherently ISS on a robustly positively invariant compact set with respect to both inner and outer perturbations:

Lemma 2 *([LHT12], Theorem IV.4) Let* $\mathbb{X},\mathbb{E},\mathbb{D} \subseteq \mathbb{R}^n$ *and let* $\alpha_1,\alpha_2,\alpha_3 \in \mathscr{K}_\infty$, $\sigma_1,\sigma_2 \in \mathscr{K}$, $\mathbb{X} \subseteq \mathbb{R}^n$ *with* $0 \in int(\mathbb{X})$. *Let* $J : \mathbb{X} \to \mathbb{R}_+$ *be a function with* $J(0) = 0$ *and consider the following inequalities:*

$$\alpha_1(||x||) \leq J(x) \leq \alpha_2(||x||) \tag{3.13a}$$

$$J(\Psi(x,e,d)) - J(x) \leq -\alpha_3(||x||) + \sigma_1(||e||) + \sigma_2(||d||) \tag{3.13b}$$

(i) Assume that $\mathbb{X}$ *is a robustly positively invariant set for the perturbed system* $x(k+1) = \Psi(x(k),e(k),d(k))$ *for all* $e(k) \in \mathbb{E}$ *and* $d(k) \in \mathbb{D}$ *and (ii) assume that the inequalities* (3.13a)-(3.13b) *hold for all* $x \in \mathbb{X}, e \in \mathbb{E}$ *and all* $d \in \mathbb{D}$. *Then, the system* $x(k+1) = \Psi(x(k),e(k),d(k))$ *is ISS in* $\mathbb{X}$ *for inputs in* $\mathbb{E}$ *and* $\mathbb{D}$.

In this Chapter the optimal cost is employed as a Lyapunov function candidate for the Input-to-State Stability analysis of the event-based MPC. However, notice that here we only consider outer perturbations. At time step $k-1$, the optimal cost is denoted as $J_N^*(k-1) = J_N(x_{k-1}, u_F^*(k-1))$. Analogously, the optimal cost at a time step $k+m$ with $m \in \{0,\dots,N-1\}$ is denoted as $J_N^*(k+m)$. Then the difference of these costs is given by:

$$\Delta J_m^* = J_N^*(k+m) - J_N^*(k-1) \tag{3.14}$$

The next lemma can now be stated

Lemma 3 *Consider the system* (3.2) *subject to* (3.3) *and assume that Assumptions 2-5 hold. Then, the difference between the optimal cost at time-step $k+m$ and the optimal cost at time-step $k-1$, for all $m \in \{0,1,\dots,N-1\}$, is bounded by:*

$$\Delta J_m^* \leq L_{Z_m} e(k+m|k-1) - \sum_{i=0}^{m} \underline{L}(||x_{k-1-i+m}||) \tag{3.15}$$

where L_{Z_m} is given by $L_{Z_m} = L_V L_f^{(N-1)-m} + L_C \frac{L_f^{(N-1)-m}-1}{L_f-1}$.

Proof We are going to employ the feasible control law (3.7). From (3.1) and (3.7), we denote $\bar{J_N}(k+m) \triangleq J_N(x_{k+m}, \bar{u_F}(k+m))$ to be the "feasible" cost. The difference between the cost of a feasible sequence at time-step $k+m$ and the optimal cost at time $k-1$ is indicated by:

$$\Delta J_m = \bar{J_N}(k+m) - J_N^*(k-1) \tag{3.16}$$

First, the difference (3.16) is calculated for $m=0$. Then the calculation will be repeated for $m=1$, and finally the general rule for random m will be stated. Finally, the bound of the difference between optimal costs (3.15) will be provided. Notice that analogously to the "feasible" cost, we denote as $\bar{x}(k+j+1|k+m)$ the state of the nominal system (2.1) at time-step $k+j+1$ having applied the control sequence (3.7) from time-step $k+m$ until $k+j$.

For $m=0$ the difference (3.16) is given by:

$$\begin{aligned}
\Delta J_0 = \bar{J_N}(k) - J_N^*(k-1) = \sum_{i=0}^{N-1} \{ & L(\bar{x}(k+i|k), \bar{u}(k+i|k)) \\
& - L(\hat{x}(k+i-1|k-1), u^*(k+i-1|k-1))\} + V(\bar{x}(k+N|k)) - V(\hat{x}(k+N-1|k-1)) \\
= \sum_{i=0}^{N-2} \{ & L(\bar{x}(k+i|k), \bar{u}(k+i|k)) - L(\hat{x}(k+i|k-1), u^*(k+i|k-1))\} - L(x_{k-1}, u_{k-1}) \\
& + L(\bar{x}(k+N-1|k), h(\bar{x}(k+N-1|k))) + V(\bar{x}(k+N|k)) - V(\hat{x}(k+N-1|k-1))
\end{aligned} \tag{3.17}$$

From definition of (3.7) we have $\bar{u}(k+i|k) = u^*(k+i|k-1)$ for all $i \in \{0,\dots,N-2\}$.

Imposing this control law for $m = 0$ to the nominal system, we get:

$$||\hat{x}(k+j|k) - \hat{x}(k+j|k-1)|| \leq L_f^j e(k|k-1) \tag{3.18}$$

where the error defined by (3.4), is given by $e(k|k-1) = ||x_k - \hat{x}(k|k-1)||$. The following result can be derived by induction; given (3.18) and the Lipschitz property from Assumption 2, the difference between the running costs is bounded by:

$$L(\bar{x}(k+j|k), \bar{u}(k+j|k)) - L(\hat{x}(k+j|k-1), u^*(k+j|k-1)) \leq L_c L_f^j e(k|k-1) \tag{3.19}$$

From the feasibility, it was derived that $\hat{x}(k+N-1|k) \in \mathscr{X}_f^n$. Thus, Assumption 3 can now be applied as follows:

$$V(\bar{x}(k+N|k)) - V(\bar{x}(k+N-1|k)) + L(\bar{x}(k+N-1|k), h(\bar{x}(k+N-1|k))) \leq 0 \tag{3.20}$$

Consider also (3.9), for $m = 0$, and Assumption 2. Substituting the above expressions to (3.17), the following can be derived:

$$\Delta J_0 \leq L_{Z_0} e(k|k-1) - \underline{L}(||x_{k-1}||) \tag{3.21}$$

with $L_{Z_0} = L_V L_f^{N-1} + L_c \frac{L_f^{N-1} - 1}{L_f - 1}$.

For $m = 1$ the difference (3.16) becomes:

$$\begin{aligned}
\Delta J_1 = \bar{J}_N(k+1) - J_N^*(k-1) = \sum_{i=0}^{N-1} \{L(\bar{x}(k+i+1|k+1), \bar{u}(k+i+1|k+1)) \\
- L(\hat{x}(k+i-1|k-1), u^*(k+i-1|k-1))\} + V(\bar{x}(k+N+1|k+1)) \\
- V(\hat{x}(k+N-1|k-1)) = \sum_{i=0}^{N-3} \{L(\bar{x}(k+i+1|k+1), \bar{u}(k+i+1|k+1)) \\
- L(\hat{x}(k+i+1|k-1), u^*(k+i+1|k-1))\} - L(x_{k-1}, u_{k-1}) - L(x_k, u_k) \\
+ L(\bar{x}(k+N-1|k+1), h(\bar{x}(k+N-1|k+1))) + V(\bar{x}(k+N+1|k+1)) \\
+ L(\bar{x}(k+N|k+1), h(\bar{x}(k+N|k+1))) - V(\hat{x}(k+N-1|k-1))
\end{aligned} \tag{3.22}$$

Noticing that $\bar{x}(k+N|k+1) \in \mathscr{X}_f \subseteq \mathscr{X}_f^n$ and $\bar{x}(k+N-1|k+1) \in \mathscr{X}_f^n$, we apply the inequality from Assumption 2. Thus, we get

$$V(\bar{x}(k+N+1|k+1)) - V(\bar{x}(k+N|k+1)) + L(\bar{x}(k+N|k+1), h(\bar{x}(k+N|k+1))) \leq 0$$
$$V(\bar{x}(k+N|k+1)) - V(\bar{x}(k+N-1|k+1)) + L(\bar{x}(k+N-1|k+1), h(\bar{x}(k+N|k+1))) \leq 0$$

Moreover,

$$V(\bar{x}(k+N-1|k+1)) - V(\hat{x}(k+N-1|k-1)) \leq L_V L_f^{N-2} e(k+1|k-1) \tag{3.23}$$

Substituting these expressions to (3.22), it can be concluded that ΔJ_1 is bounded by

$$\Delta J_1 \leq L_{Z_1} e(k+1|k-1) - \underline{L}(||x_{k-1}||) - \underline{L}(||x_k||) \tag{3.24}$$

with $L_{Z_1} = L_V L_f^{N-2} + L_c \frac{L_f^{N-2}-1}{L_f-1}$.

From the above it can be concluded using the same calculations that (3.16) can be generalized for any $m \in \{0, \ldots, N-1\}$. Moreover, the optimality of the solution results to

$$J_N^*(k+m) - J_N^*(k-1) \leq \bar{J}_N(k+m) - J_N^*(k-1)$$

and the proof is completed.

Note that, for the optimal cost we have

$$J_N^*(k) \leq \bar{J}_N(k) \leq (L_c + L_{c_u} L_h) \frac{L_{f_h}^N - 1}{L_{f_h} - 1} ||x_k|| + L_V L_{f_h}^N ||x_k|| \tag{3.25}$$

Also we have

$$J_N^*(k) \geq \underline{L}(||x_k||) \tag{3.26}$$

Hence, there exist $\mathscr{K}_\infty$-functions $\alpha_1(||x_k||)$ and $\alpha_2(||x_k||)$ such that (3.13a) is satisfied. We are going to need a decreasing Lyapunov function to assert the ISS of the overall scheme. This will me ascertained by the triggering conditions that we will provide next.

3.3.4 Triggering Conditions for the Event-based NMPC Framework

We are now ready to reach to the triggering condition. The ISS property of the system is utilized based on the general rule given in (2.10). For time step k, the triggering condition is

$$L_{Z_0} \cdot e(k|k-1) \leq \sigma \underline{L}(||x_{k-1}||) \tag{3.27}$$

having considered the inequality (3.21). The next OCP is thus triggered whenever condition (3.27) is violated, otherwise the control law from (3.7) is used for $m = 0$. However, in order to ensure that the system remains stable using the control law (3.7) for $m \geq 0$, there are few more things to consider. In order to maintain stability we must ensure that ΔJ_m^* is strictly decreasing for all $m \geq 0$. The system can use the control law (3.7), as long as

$$\Delta J_{m+1}^* \leq \Delta J_m^* \tag{3.28}$$

In this case the convergence of the closed-loop system is guaranteed, as it is depicted in Fig.3-2.

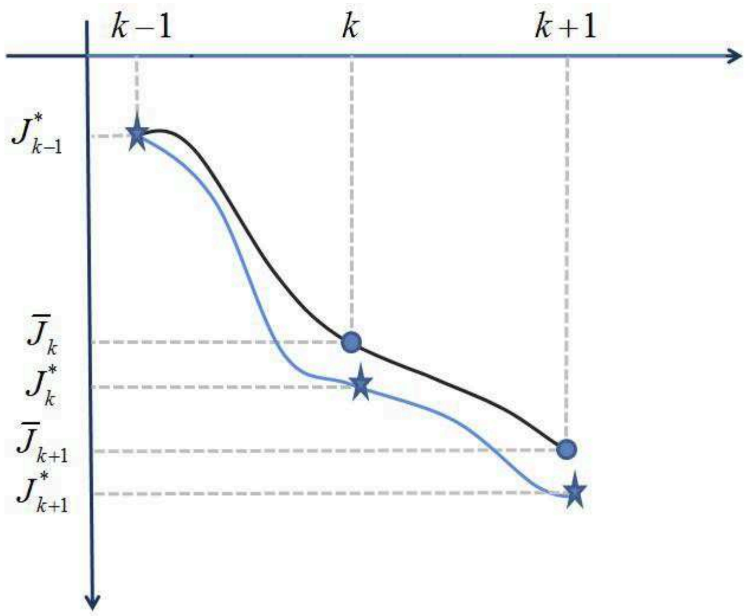

Figure 3-2: The triggering condition. The black solid line represents the cost of the feasible sequence $\bar{J}_N(\cdot)$ for time steps $k-1$, k and $k+1$, and the blue solid line represents the optimal cost $J_N^*(\cdot)$ at the same time steps.

Consequently, the triggering rule can be stated as

$$L_{Z_m} \cdot e(k+m|k-1) \leq \sigma \sum_{i=0}^{m} \underline{L}(||x_{k-1-i+m}||) \tag{3.29a}$$

and

$$L_{Z_{m+1}} \cdot e(k+m+1|k-1) - \sigma \underline{L}(||x_{k-1+m}||) \leq L_{Z_m} \cdot e(k+m|k-1) \tag{3.29b}$$

The next OCP is triggered whenever condition (3.29a) or (3.29b) are violated. The previous analysis guarantees that the closed loop system will have the same convergence properties as in [MAC02]. We are ready to state the main Theorem

Theorem 3 *Consider the system* (3.2)*, subject to* (3.3) *under an NMPC strategy and assume that the previously presented Assumptions 2-5 hold. Then the NMPC control law given by* (3.5a)-(3.5b) *along with the triggering rule* (3.29a)-(3.29b) *and* (3.11)*, drives the closed loop system towards a compact set where it is ultimately bounded.*

3.4 Event-based NMPC for Decentralized Discrete-time Systems

In the following, the proposed framework for finding event-triggering condition for sampling is extended to a general system which is composed by the interconnection of M local subsystems, each one controlled by a NMPC law. The framework is considered to be fully decentralized, i.e., there is no information exchange between the subsystems. However, the effect of interconnections are considered as perturbation terms in the system models. The ISS stability with respect to the uncertainty imposed by the neighboring subsystems, is properly modified in order to reach to a triggering condition, similarly to the previous Sections.

3.4.1 Problem Statement for the Decentralized Case of ET-NMPC

Each of the subsystem is modeled as a perturbed dynamic discrete-time nonlinear system

$$x_s(k+1) = f_s(x_s(k), u_s(k)) + g_s(x(k)) + \psi_s(k) \qquad \forall k \in \mathbb{Z}_{\geq 0} \tag{3.30}$$

where $s = 1, \ldots, M$ is the number of the subsystems. The state of the s-th subsystem is denoted as $x_s(k) \in \mathbb{R}^{n_s}$, $u_s(k) \in \mathbb{R}^{m_s}$ is the control variable and $\psi_s(k) \in \mathbb{R}^{n_s}$ is the additive disturbance. The overall state is given as $x(k) \triangleq [x_1(k), x_2(k), \ldots, x_M(k)] \in \mathbb{R}^n$ with $n = \sum_{s=1}^{M} n_s$. The term $g_s(\cdot)$ denotes the influence of the M subsystems on the s-subsystem and that $g_s(0) = 0$. Assume that there exist M positive constants $L_{g_s,j}$ such that $||g_s(x)|| \leq \sum_{j=1}^{M} L_{g_s,j}||x_j||$. The states, the control inputs and the disturbances are required to fulfil the following constraints

$$x_s \in \mathscr{X}_s \subseteq \mathbb{R}^{n_s} \qquad u_s(k) \in \mathscr{U}_s \subseteq \mathbb{R}^{m_s} \qquad w_s \triangleq \{g_s(x) + \psi_s\} \in \mathscr{W}_s \subseteq \mathbb{R}^{n_s} \tag{3.31}$$

where $\mathscr{X}_s$, $\mathscr{U}_s$ are compact sets, all of them containing the origin as an interior point. We assume that $f_s(\cdot)$ are Lipschitz continuous with Lipschitz constants L_{f_s}, for all $x_s \in \mathscr{X}_s$, $u_s \in \mathscr{U}_s$ and that $f_s(0,0) = 0$. For each subsystem, the sum of interaction term and the disturbance term are restricted to belong to a compact set $\mathscr{W}_s$, with $||w_s|| \leq \gamma_s$, while the overall state $x \in \mathscr{X} \triangleq \mathscr{X}_1 \times \cdots \times \mathscr{X}_M$.

The whole system formed by the M local subsystems can be written as

$$x(k+1) = f(x(k), u(k)) + g(x(k)) + \psi(k) \qquad k \in \mathbb{Z}_{\geq 0} \tag{3.32}$$

where $f(x,u) \triangleq [f_1(x_1,u_1), \ldots, f_M(x_M,u_M)]$, $g(x) \triangleq [g_1(x), \ldots, g_M(x)]$ and $\psi \triangleq [\psi_1, \ldots, \psi_M]$. Moreover, the nominal model of the system is also considered

$$x_s(k+1) = f_s(x_s(k), u_s(k)) \tag{3.33}$$

Since there are mismatches between the real subsystem (3.30) and the nominal subsystem (3.33), we are introducing the error $e_s(k+j|k)$ of the s-th subsystem

$$e_s(k+j|k) = ||x_{s,k+j} - \hat{x}_s(k+j|k)|| \tag{3.34}$$

where $x_{s,k+j}$ is the state of the subsystem s, measured at time step $k+j$, and $\hat{x}_s(k+j|k)$ is the predicted state of the same subsystem computed by (3.33) at the same time step.

Each subsystem s, is controlled locally by an MPC law. As follows, each MPC is computed as the solution of an OCP problem. The OCP for the nominal system (3.33), is obtained by locally minimizing at time instant k with respect to a control sequence $u_{sF}(k) \triangleq [u_s(k|k), \ldots, u_s(k+N_s-1|k)]$, the following performance index

$$\min_{u_{sF}(k)} J_s(x_s, u_{sF}(k)) = \min_{u_{sF}(k)} \sum_{i=0}^{N_s-1} L_s(\tilde{x}_s(k+i|k), u_s(k+i|k)) + V_s(\tilde{x}_s(k+N_s|k)) \tag{3.35a}$$

subject to

$$\tilde{x}_s(k+j|k) \in \mathscr{X}_{j_s} \qquad u_s(k+j|k) \in \mathscr{U}_s \qquad \tilde{x}_s(k+N_s|k) \in \mathscr{X}_{f_s} \tag{3.35b}$$

for all $j = 1, \ldots, N_s - 1$. With N_s to be the prediction horizon and $\mathscr{X}_{f_s}$ to be the terminal constraint set, for the s-th subsystem. As in the centralized case, the state constraint set $\mathscr{X}_s$ is being replaced with a restricted set $\mathscr{X}_{j_s}$.

The necessary assumptions for the decentralized MPC schemes are stated next.

Assumption 6 *The stage cost $L_s(x_s, u_s)$ is Lipschitz continuous in $\mathscr{X}_s \times \mathscr{U}_s$, with a Lipschitz constant L_{c_s}. Let $L_s(0,0) = 0$, and assume that $L_s(x_s, u_s) \geqslant \underline{L}_s(||x_s||)$ where $\underline{L}_s$ is a class $\mathscr{K}_\infty$-function.*

Assumption 7 *Let the terminal region $\mathscr{X}_{f_s}$ from* (3.35b) *be a subset of an admissible positively invariant set $\mathscr{X}_s$ of the nominal system. Assume that there is a local controller h_s for the terminal state $\mathscr{X}_{f_s}$. The associated terminal penalty $V_s(\cdot)$ has the following property $\alpha_{V_s}(||x_s||) \leqslant V_s(x_s) \leqslant \beta_{V_s}(||x_s||)$ for all $x_s \in \mathscr{X}_{f_s}$, where α_{V_s} and β_{V_s} are class $\mathscr{K}_\infty$-functions. We also assume that $V_s(f_s(x_s, h_s))) - V_s(x_s) \leqslant -L_s(x_s, h_s(x_s))$, $\forall x_s \in \mathscr{X}_{f_s}$ and that V_s is Lipschitz in $\mathscr{X}_{f_s}$, with Lipschitz constant L_{V_s}.*

Problem Statement 2 *Consider the system* (3.32) *formed by a number of subsystems* (3.30)*, that are subject to constraints* (3.31)*. The objective is to design a feedback control law for each of the subsystem s, computed by* (3.35a)-(3.35b)*, such that ISS-stability with respect to uncertainties is achieved. Then find the event-based conditions for triggering the control updates for each of the subsystems, while satisfying convergence and stability criteria.*

3.4.2 Triggering Condition for the Decentralized NMPC Case

Finding the triggering condition for each of the subsystems (3.30) under a decentralized NMPC control law of the form (3.35a)-(3.35b) can be treated as an extension of the centralized case. The proof is rather straightforward and is omitted. Therefore, the triggering rule for each of the subsystems s is given by

$$L_{Z_{s,j}} \cdot e_s(k+j|k-1) \leq \sigma \sum_{i=0}^{j} \underline{L}_s(||x_{s,k-1-i+j}||) \tag{3.36a}$$

and

$$L_{Z_{s,j+1}} \cdot e_s(k+j+1|k-1) - \sigma \underline{L}_s(||x_{s,k-1+j}||) \leq L_{Z_{s,j}} \cdot e_s(k+j|k-1) \tag{3.36b}$$

with

$$L_{Z_{s,j}} = L_{V_s} L_{f_s}^{(N_s-1)-j} + L_{C_s} \frac{L_{f_s}^{(N_s-1)-j}}{L_{f_s} - 1}$$

The next OCP is triggered whenever condition (3.36a) or (3.36b) is violated. The next Theorem can now be stated

Theorem 4 *Consider the subsystem* (3.30)*, subject to* (3.31) *under a decentralized NMPC strategy and assume that the previously presented Assumption 4 and Assumption 5, holds. Then the NMPC control law given by* (3.35a)-(3.35b) *along with the triggering rule* (3.36a)-(3.36b)*, drives the closed loop system to a compact set where it is ultimately bounded.*

3.5 Event-triggered MPC for Linear Discrete-time Systems

In this Section we are going to provide the framework of event-based MPC for linear systems. The approach is an extension of the previously presented event-based MPC schemes for centralized systems, specifically stated for Piecewise Affine (PWA) systems. In Chapter 2 we utilized the classic time-triggered MPC where the control law is given by the first term of the control sequence provided by the OCP of the MPC. However, the triggering is based on a condition of an error and the control law is not renewed at each sampling instant as in the classic MPC. These two different approaches, i.e., the one presented here and the one that was presented in Chapter 2, differ from each other in the sense that in the former the control input remains constant between triggering events and is equal to the first term of the optimal control sequence and in the latter the control sequence from the OCP is applied to the plant between triggering events.

3.5.1 An Event-Based MPC Scheme for Discontinuous PWA Systems

The event-based setup for centralized MPC that was previously presented, is specified in this Section for PWA systems. Thus, the main assumptions and the OCP remains the same and is not presented here. This Section will be consistent with [FAB07], [Laz06], where an ISS analysis for PWA systems under MPC laws was presented.

In this section we consider the class of discrete-time piecewise affine systems. Consider the nominal model of the form

$$x_{k+1} = f(x_k, u_k) = A_j x_k + B_j u_k \tag{3.37}$$

Assume that the states are constrained in the compact set $\mathscr{X} \subseteq \mathbb{R}^n$ and the input vectors are constrained in the compact set $\mathscr{U} \subseteq \mathbb{R}^m$, both of them containing the origin in their interior. The perturbed model is of the form

$$x_{k+1} = A_j x_k + B_j u_k + w_k \tag{3.38}$$

where $x_k \in \Omega_j$. There is $w_k \in \mathscr{W} \subset \mathbb{R}^n$, $k \in \mathbb{Z}_{\geq 0}$, $A_j \in \mathbb{R}^{n\times n}$, $B_j \in \mathbb{R}^{n\times m}$, $j \in \mathscr{S}$ with

$\mathscr{S} = \{1,2,\ldots,s\}$ to be a finite set of indices. The collection $\{\Omega_j | j \in \mathscr{S}\}$ defines a partition of $\mathscr{X}$ and there is $int(\Omega_j) \cap int(\Omega_i) = \varnothing$ for $j \neq i$. Each Ω_j is assumed to be polyhedron. We assume that the origin is an equilibrium state for (5.1). Let $||w_k|| \leq \gamma$, with $\gamma > 0$. The OCP problem of the MPC is the same as the general centralized case (3.5a)-(3.5b), where the stage cost is given by $L(x,u) = ||Qx|| + ||Ru||$ and the terminal cost is given by $V(x) = ||Px||$, with $Q \in \mathbb{R}^{q\times n}$, $R \in \mathbb{R}^{r\times n}$, $P \in \mathbb{R}^{p\times n}$ to be known matrices that have full-column rank. Due to full-column rank of Q there exists $q > 0$ such that $||Qx|| \geq q||x||$ for all x.

In this setting we assume the auxiliary controller $h(\cdot)$ to be of the form: $h(x) = K_j x$ where $x \in \Omega_j$ and $K_j \in \mathbb{R}^{m\times n}$ with $j \in \mathscr{S}$. Let $\eta = \max_{j\in\mathscr{S}} ||A_j||$, $\xi = ||P||$ and define $\mathbb{B}_i = \{x \in \mathbb{R}^n : ||x|| \leq \gamma \sum_{p=0}^{i-1} \eta^p\}$. The constraints tightening technique is used here as well, hence we define the constraint set from (3.5b) to be $\mathscr{X}_i = \cup_{j\in\mathscr{S}} \{\Omega_j \sim \mathbb{B}_i\} \subseteq \mathscr{X}$, for $i = 1,\ldots,N-1$.

It is relatively easy to make the connection of the linear system with the general non-linear system given in the previous Section. We define the error to be

$$e(k|k-1) = ||x_k - \hat{x}(k|k-1)|| \tag{3.39}$$

which yields

$$||\hat{x}(k+i|k) - x(k+i|k-1)|| \leq \eta^i e(k|k-1) \tag{3.40}$$

For (3.19) and (3.20) we have

$$L(\overline{x}(k+i|k), \overline{u}(k+i|k)) - L(\hat{x}(k+i|k-1), u^*(k+i|k-1)) \leq ||Q||\eta^i e(k|k-1) \tag{3.41}$$

and

$$V(\overline{x}(k+N-1|k)) - V(\hat{x}(k+N-1|k-1)) \leq \xi\eta^{N-1} \cdot e(k|k-1) \tag{3.42}$$

Conducting the same analysis as in the centralized case of event-based MPC we get the following triggering rule for systems (5.2)

$$L_j^{linear} \cdot e(k+j|k-1) \leq \sigma \sum_{i=0}^{j} q||x_{k-1-i+j}|| \tag{3.43a}$$

and

$$L_{j+1}^{linear} \cdot e(k+j+1|k-1) - \sigma q||x_{k-1+j}|| \leq L_j^{linear} \cdot e(k+j|k-1) \tag{3.43b}$$

with $L_j^{linear} = \xi\eta^{(N-1)-j} + ||Q||\frac{\eta^{(N-1)-j}-1}{\eta-1}$.

3.6 Simulation Examples

In this Section, two different examples are given in order to depict the efficacy of the proposed Event-based scheme of Model Predictive Controllers.

3.6.1 Example 1: Robotic Manipulator

In this section, a simulated example of the proposed design on a robotic manipulator is presented. The objective is to provide an efficient NMPC controller, triggered whenever (3.29a) or (3.29b) is violated, in order to stabilize the robotic manipulator, in a desired equilibrium configuration. Consider a general manipulator of r degrees of freedom (d.o.f.), which does not interact with the environment. The joint-space dynamic model of these types of manipulators is described as

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau \tag{3.44}$$

where B is the inertia matrix, C is the Coriolis term, g is the gravity term, F is a positive definite diagonal matrix of viscous friction coefficients at the joints, $q = [q_1, \ldots, q_r]$, $\dot{q} = [\dot{q}_1, \ldots, \dot{q}_r]$ and $\ddot{q} = [\ddot{q}_1, \ldots, \ddot{q}_r]$ are the vectors of the arm joint position, velocity and acceleration, respectively. Finally, $\tau \in \mathbb{R}^r$ are the joint torque inputs. We consider a two-link, planar robotic manipulator, $r = 2$ with no friction effects for simplicity. For illustrative purposes the numerical values of the parameters are taken as in [LK97]. The NMPC controller that is used, is the one described in [Yoo02], with prediction horizon $N = 10$.

In the control affine, state-space model of the manipulator, the state accounts for $x = [q_1, q_2, \dot{q}_1, \dot{q}_2]$. The initial state is $x_{\text{initial}} = [\pi/2, 0, 0, 0]$ and the desired state is $x_{\text{desired}} = [0, 0, 0, 0]$. In Fig.3-3, the norm of the distance between the state of the system and the desired state is depicted. The simulation shows that the system (3.44), under a NMPC strategy, using the triggering condition (3.29a)-(3.29b), converges to the final state in the nominal case. In the perturbed case the system converges to a bounded set around the origin.

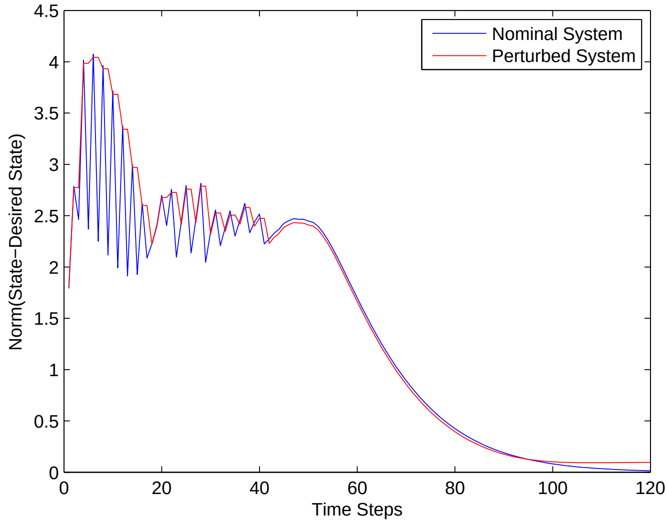

Figure 3-3: The norm of the distance between the state of the system (3.44) and the desired state, i.e. dist $= ||x - x_{\text{desired}}||$. The blue line represents the distance of the nominal system, while the red line represents the distance of the perturbed system, under an additive disturbance.

The next Fig. 3-4, depicts the triggering moments, during the NMPC strategy. It can be witnessed that using the event-triggered policy, the inter-calculation times are strictly larger than one when the system is far away from the equilibrium, until about the 80th time step. After the 80th time step, the system has practically converged to the desired equilibrium.

3.6.2 Example 2: Underwater Vehicle

In this section, a simulated example of the proposed framework for a nonholonomic robot is presented. The objective is to control the robot using an event-based NMPC law, in

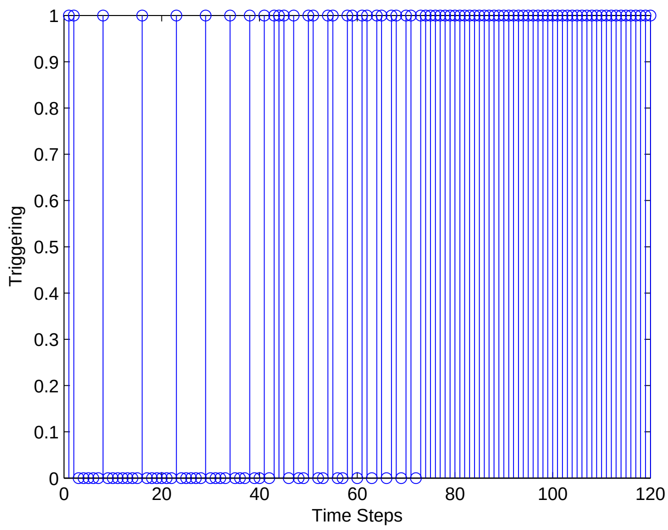

Figure 3-4: Triggering instants. When the triggering axis has the value 1, the NMPC algorithm is triggered. For value 0, the NMPC law is implemented on the system in an open-loop fashion.

order to reach a desired terminal constraint set. Let the motion of the robot be governed by unicycle kinematics with respect to a global cartesian coordinate frame G. The discrete-time perturbed kinematic model is given by:

$$\begin{bmatrix} \chi_{k+1} \\ y_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} \chi_k + dt \cos\theta_k \upsilon_k \\ y_k + dt \sin\theta_k \upsilon_k \\ \theta_k + dt\,\omega_k \end{bmatrix} + \begin{bmatrix} w_\chi \\ w_y \\ 0 \end{bmatrix}$$

where $x = [\chi, y, \theta]^\top$ is the state vector comprised by the position of the robot (χ, y) and the orientation θ with respect to G. The vector $u = [\upsilon, \omega]^\top$ denotes the control inputs. The robot is equipped with an onboard camera with limited angle-of-view and laser pointers that provide the state vector x of the robot with respect to G. The requirements imposed by the sensors are captured by the following constraints

$$\begin{aligned} -y + \chi \tan(\theta - \frac{\alpha}{2}) - y_T &\geq 0 \\ y - \chi \tan(\theta + \frac{\alpha}{2}) - y_T &\geq 0 \\ R_{\max}^2 - \chi^2 - y^2 &\geq 0 \end{aligned}$$

where α is the angle-of-view of the camera and $R_{\max}$ is the maximum distance of the vehicle with respect to the target. These requirements, along with a saturation bound in the velocity, impose the constraints of the problem, i.e. $||\bar{u}|| \triangleq \sqrt{\upsilon^2+\omega^2} \leq 2$. Furthermore we assume that additive disturbances are bounded by $||w|| \leq 0.7$. The discretization time is $dt = 0.1$ and the cost function is of quadratic form, i.e., $L(x,u) = x^\top Qx + u^\top Ru$ with $Q = diag[3,4,0.05]$ and $R = diag[0.9,1]$. The prediction horizon is $N = 20$ and the constant ν is taken equal to 0.9. The initial position of the robot is $x_{\text{initial}} = [-7,2,-\pi/6]^\top$ and the desired position is $x_d = [0,0,0]^\top$. The simulation shows that the states of the perturbed system under the event-based NMPC framework are converging to the terminal constraint set, see Fig.3-5 and Fig.3-6(a). Figure 3-6(b) is capturing the triggering instants. It can

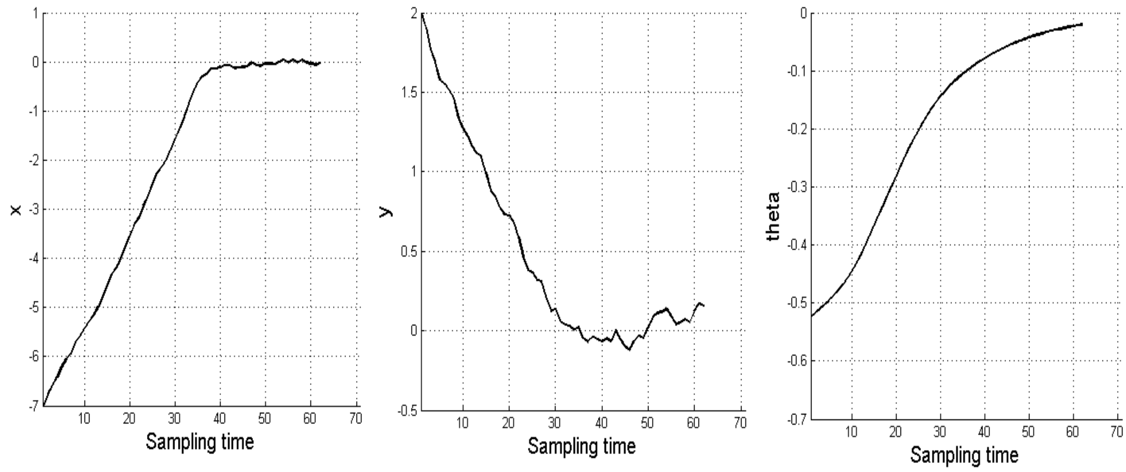

Figure 3-5: The evolution of the state trajectories $x = [\chi, y, \theta]^\top$ with respect to the sampling time.

be witnessed that using the event-triggered policy an overall reduction of the computation times is achieved. Furthermore it is apparent that the inter-calculation times are more scarce when the system is away from the desired point and that they become more frequent when the system approaches the terminal set.

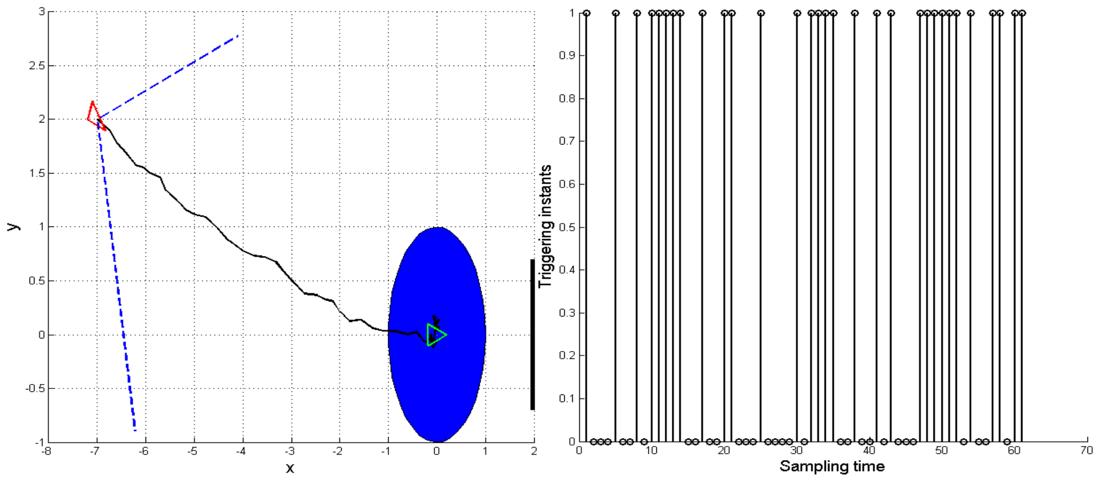

Figure 3-6: (a) The state trajectory of the nonholonomic robot. (b) The triggering instants. When the vertical axis has the value 1, the NMPC is triggered. For value 0 the control law is implemented in an open-loop fashion.

3.7 Conclusions

We provided an event-triggered formulation of model predictive control based systems. The main idea is to trigger the solution of the optimal control problem only when it is needed, and not at every time-step as in the case of classic discrete time MPC. The event-based strategy is possible to alleviate the computational burden of a MPC framework. Sufficient conditions for triggering the MPC laws were given in a number of frameworks. The results were illustrated through simulated examples.

The next step is to find triggering conditions in a set-up where the information between the agents is not considered as a disturbance but it is explicitly taken into consideration. This will be treated in the next Chapter.

Chapter 4

Event-Based Model Predictive Control for the Cooperation of Distributed Agents

This Chapter proposes an event-based framework for the control of a team of cooperating distributed agents. The agents are dynamically decoupled and they are controlled locally by Nonlinear Model Predictive Controllers (NMPC). The event-driven framework allows for triggering the solution of the optimal control problem of the NMPC only when it is needed. The scheduling of the control updates for each of the agents depends on an error of the state information received from the neighboring agents. Sufficient conditions for triggering are provided and the results are illustrated through a simulated example.

4.1 Introduction

The control of many interacting subsystems has gained much interest in the recent years. Formulating the problem of control of such large-scale systems under a NMPC framework is an efficient approach because of the inherent virtues of these kind of controllers. NMPC controllers can handle nonlinearities and offer the possibility of incorporating control and state constraints. Related results on NMPC for large-scale systems can be found in [DM06], [FMP+08], [KBB06], [RHLM09], [RMS07], [RH04] and in the review paper [Sca09] and the number of papers quoted therein.

When implementing decentralized control laws, the communication schemes between interacting subsystems as well as the controllers' design, are aspects that should be taken into consideration. The actuation updates can be either periodic or can be determined by certain events. The first one might be a conservative choice since in this case, stability is guaranteed based on a worst-case scenario. On the contrary, in the event-based approaches the decision for the execution of the control task depends on the state of the system. This methodology may lead to an overall reduction on the number of the control updates which might be desirable when the system has limited resources. The computation of the control law of the NMPC controller is rather demanding particularly when large-scale systems are of consideration. Motivated by this fact, an event-based framework for this kind of controllers is investigated in order to reduce the number of times the control input should be computed. Under the proposed scheme the control law of the NMPC is not updated at each sampling instant but rather, the already computed control sequence is implemented to the plant until an event occurs. The problem addressed here is the control of a team of cooperating agents operating in the same environment. Each agent is a nonlinear discrete-time system and no dynamic coupling between the agents is assumed. The agents are controlled by local NMPC controllers which depend not only on local information, but also on the information of the neighboring agents.

The contribution of this Chapter relies in finding sufficient conditions for triggering in the case of a team of cooperative nonlinear subsystems. Each one of the subsystems has its own triggering condition which depends on the local state information and an error of the state information of their neighbors. The stability, and particularly the Input-to-State (ISS) stability, of a system of cooperating agents under NMPC has been presented in [FMP+08]. The authors consider the classic time-driven NMPC where the optimization problem is solved at each sampling instant. In this work we appropriately modify the formulation presented in [FMP+08] in order to reach a triggering condition. Moreover, unlike [FMP+08] where the predicted dynamics of the neighbors are considered to be of decreasing "importance" during the prediction horizon, in this work the error of the prediction is included in the triggering condition.

Notice that in Chapter 3, a triggering condition for a decentralized NMPC was given. That approach consisted of describing the effect of the interconnection among the subsystems as disturbances acting on local models and a robust NMPC approach was utilized in order to reach to the triggering condition. The aforementioned scheme is applicable to systems where the interaction between the agents is limited, opposed to the proposed framework of this Chapter, where the interactions between the agents are taken explicitly into account.

4.2 Problem Statement

In the following, triggering conditions for distributed agents which operate in a common environment under local NMPC control laws, are going to be presented. This general framework was used in [FMP$^+$08], where it is proven that each one of the subsystems is ISS stable with respect to the delayed state information received by a group of neighboring agents. The aforementioned result is being appropriately modified in this Chapter in order to reach to the triggering conditions for each one of the subsystems.

Consider a general system which is composed by M local subsystems. The dynamics of the subsystems are described by a nonlinear discrete-time equation

$$x_{k+1}^i = f^i(x_k^i, u_k^i) \tag{4.1}$$

with $k \in \mathbb{Z}_{\geq 0}$ and $i = 1, \ldots, M$. The state of subsystem i is denoted by $x_k^i \in \mathbb{R}^{n^i}$, while $u_k^i \in \mathbb{R}^{m^i}$ denotes the control variable. Assume that $f^i(0,0) = 0$ and suppose that the agents evolve on the same discrete-time space. The state and the control vectors are required to fulfill the following constraints

$$x_k^i \in \mathscr{X}^i \qquad u_k^i \in \mathscr{U}^i \tag{4.2}$$

where $\mathscr{X}^i$ is a compact set of $\mathbb{R}^{n^i}$ and $\mathscr{U}^i$ is a compact set of $\mathbb{R}^{m^i}$, all of them containing the origin as an interior point.

Given the system (4.1), the predicted state is denoted by

$$\hat{x}^i(k+l+1|k) = f^i(\hat{x}^i(k+l|k), u^i_{k+l}) \tag{4.3}$$

This notation will be equipped hereafter and it accounts for the predicted state at time $k+l+1$ with $l \in \mathbb{Z}_{\geq 0}$, based on the measurement of the state at time $k+l$ while using a control input u^i_{k+l}.

4.2.1 NMPC for Cooperative Control

As already mentioned, a distributed control structure is assumed in our scenario, where each one of the subsystems is controlled by a local NMPC controller. Even though the agents are dynamically decoupled, the fact that they operate in the same environment imposes a "cooperative" factor. This will be evident in the design of the local NMPC controllers where cooperative cost functions as well as information exchange between agents are assumed.

A partially connected framework is considered in this paper, i.e., the information is transmitted from any local controller, only to a given subset of the others. More precisely, each agent $\mathscr{A}^i, \forall i = 1, \ldots, M$ exchanges state information with a set of neighboring agents $\mathscr{G}^i \triangleq \{\mathscr{A}^j, j \in G^i\}$, where G^i denotes the set of indexes identifying the agents belonging to the set $\mathscr{G}^i$. The state information received by an agent $\mathscr{A}^i$ at time step k, can be written in stack vector form as

$$w^i_k \triangleq col(x^j_k, j \in G^i) \tag{4.4}$$

with

$$w^i_k \in \mathscr{W}^i \triangleq \Pi_{j \in G^i} \mathscr{X}^j \tag{4.5}$$

Note that any agent knows the state of the agents in its neighborhood without delay. In a subsequent section, the presence of transmission delays is going to be discussed as well.

In the centralized NMPC the control law is computed by solving a finite-horizon, open-loop optimal control problem (OCP), based on the state measurement provided by the plant. In the distributed case though, each agent $\mathscr{A}^i$ solves an OCP based not only on its state measurements x^i_k, but also on the information vector of the neighbors w^i_k. The optimal

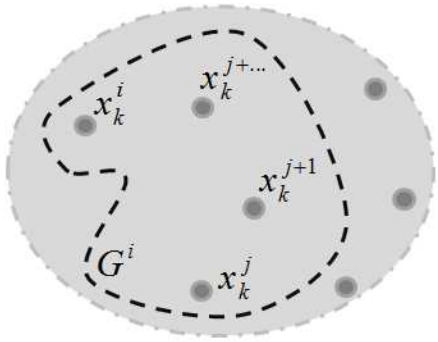

Figure 4-1: Team of agents.

problem, consists in minimizing, with respect to a control sequence $u_F^i(k) \triangleq [u^i(k|k), u^i(k+1|k), \ldots, u^i(k+N^i-1|k)]$, a cost function $J_N^i(x_k^i, u_F^i(k), w_k^i)$. Thus, the OCP for the system (4.1), is given by

$$\min_{u_F^i(k)} J_N^i(x_k^i, u_F^i(k), w_k^i) = \tag{4.6a}$$

$$\min_{u_F^i(k)} \sum_{l=0}^{l=N^i-1} \{L^i(\tilde{x}^i(k+l|k), u^i(k+l|k)) + Q^i(\tilde{x}^i(k+l|k), w^i(k+l|k))\} + V^i(\tilde{x}^i(k+N^i|k))$$

subject to

$$\tilde{x}^i(k+l|k) \in \mathscr{X}^i \qquad \forall l = 1, \ldots, N^i-1 \tag{4.6b}$$

$$u^i(k+l|k) \in \mathscr{U}^i \qquad \forall l = 0, \ldots, N^i-1 \tag{4.6c}$$

$$w^i(k+l|k) \in \mathscr{W}^i \qquad \forall l = 0, \ldots, N^i-1 \tag{4.6d}$$

$$\tilde{x}^i(k+N^i|k) \in \mathscr{X}_f^i \tag{4.6e}$$

where $\mathscr{X}_f^i$ denotes the terminal constraint set and $\tilde{\cdot}$ denotes the controller internal variables with $\tilde{x}^i(k|k) = x_k^i$. The positive integer $N^i \in \mathbb{Z}_{\geq 0}$, denotes the prediction horizon.

The vector $w^i(k+l|k)$ for $l = 0, \ldots, N^i-1$, denotes the prediction of the neighbors' states. Since, only $w^i(k|k) \triangleq w_k^i$ is known to the agent $\mathscr{A}^i$, the following is assumed

$$w^i(k+l|k) = w^i(k|k) \qquad \forall l = 0, \ldots, N^i-1 \tag{4.7}$$

Namely, each agent assumes that its neighbors maintain the same state during the prediction horizon. This assumption is utilized in order to solve the OCP, (4.6a)-(4.6e). Obviously, this is not the case due to the individual agent dynamics. The use of the event-triggered framework will enable us to overcome this limitation.

Some standard stability conditions for the design parameters of the NMPC must be introduced, in order to assert that NMPC strategy results in a stabilizing controller.

Assumption 8 *The stage cost* $L^i(x^i,u^i)$ *is Lipschitz continuous in* $\mathscr{X}^i \times \mathscr{U}^i$ *and it holds that* $L^i(0,0)=0$. *Moreover, there is a* $\mathscr{K}_\infty$*-function* r^i, *such that* $L^i(x^i_k,u^i_k) \geq r^i(||x^i_k||)$.

Assumption 9 *The running cost* $Q^i(x^i,w^i)$ *is such that* $Q^i(x^i,w^i) \geq 0$. *Moreover,* Q^i *is Lipschitz continuous in* $\mathscr{X}^i \times \mathscr{W}^i$, *with Lipschitz constants* L^i_{qx} *and* L^i_{qw}, *respectively.*

Assumption 10 *Let the terminal set* $\mathscr{X}^i_f$ *be such that* $\mathscr{X}^i_f \subset \mathscr{X}^i$, $\mathscr{X}^i_f$ *to be closed, and* $0 \in \mathscr{X}^i_f$. *Assume that there is a locally stabilizing controller* $h^i(x_k)$ *for the terminal set. The associated Lyapunov function* $V^i(\cdot)$ *has the following property*

$$V^i(f^i(x^i_k,h^i(x^i_k))) - V^i(x^i_k) \leq -L^i(x^i_k,h^i(x^i_k)) - Q^i(x^i_k,w^i_k)\ \forall x^i \in \mathscr{X}^i_f \text{ and } \forall w^i \in \mathscr{W}^i$$

4.3 Event-Based NMPC

In this section a triggering condition for each one of the agents $\mathscr{A}^i$, will be provided. Before tackling this problem though, some concepts about the event-based approach for a distributed NMPC scheme will be given.

Consider a generic time-instant k. The solution of the OCP (4.6a)-(4.6e) provides an optimal control sequence $u^{i*}(k+l|k)$ for $l=0,\ldots,N^i-1$. In the classic NMPC strategy only the first term of the optimal solution $u^i(k|k) \triangleq u^i_k$ is applied to the system and the control law is updated for the next time-step $k+1$. In the event-based framework some portion of the optimal solution $[u^{i*}(k|k),\ldots,u^{i*}(k+\hat{l}|k)]$ with $\hat{l} \in [0,N^i-1]$, is applied to the plant in an open-loop fashion, provided some stability conditions are fulfilled. The event-based strategy is used in order to enlarge, as much as possible, the inter-calculation period of the NMPC. This results to the overall reduction of the control updates which is desirable in numerous occasions, for example energy consumption reasons.

4.3.1 ISS Stability With Respect to Measurement Errors of the Neighbors

In order to find a triggering condition for the distributed system under the NMPC control laws, the ISS properties of the systems will be used. A similar problem of cooperative control of a team of agents under local NMPC controllers, was addressed in [FMP+08] and the stability analysis was carried out using ISS properties. Moreover, the stability of the team of agents was also proven. A modification of the analysis proposed by [FMP+08] will be used in the following approach, in order to reach to the triggering condition of each of the agents.

Consider an event, triggered at time-step $k-1$, which provide an optimal sequence $u_F^{i*}(k-1)$. Consider also, control trajectories $\bar{u}_F^i(k+m)$, for time steps $m = 0,\dots,N^i-1$, based on the optimal solution in $k-1$,

$$\bar{u}^i(k+t|k+m) = \begin{cases} u^{i*}(k+t|k-1) & \text{for } t = m,\dots,N^i-2 \\ h^i(\hat{x}^i(k+N^i-1|k+m)) & \text{for } t = m+N^i-1 \end{cases} \tag{4.8}$$

These control sequences are admissible and in general suboptimal. From the feasibility of $u_F^{i*}(k-1)$ it follows that for all $m = 0,\dots,N^i-1$ we have $\bar{u}^i(k+t|k+m) \in \mathscr{U}^i$ and $\hat{x}^i(k+N^i|k+m) \in \mathscr{X}_f^i$.

The optimal cost at the triggering instant $k-1$ is denoted by $J_N^{i*}(k-1)$ and the cost of the feasible sequence at a time step $t \in [0, N^i-1]$ is indicated by $\bar{J}_N^i(k+t)$. The difference between these costs is

$$\Delta J_t^i = \bar{J}_N^i(k+t) - J_N^{i*}(k-1) \tag{4.9}$$

The next lemma can now be stated

Lemma 4 *Consider the system* (4.1) *subject to* (4.2) *and assume that Assumptions 8-10,* *hold. The difference* (4.9) *is bounded by*

$$\Delta J_t^i \leq (N^i - t - 2) L_{qw}^i e_w^i (k+t|k-1) - \sum_{\rho=0}^{t} \{ r^i(||x_{k-\rho+t}||) \} \tag{4.10}$$

with the error e_w^i *defined as*

$$e_w^i(k+\tilde{l}|k-1) = ||w^i(k+l|k+\tilde{l}) - w^i(k+l|k-1)|| = ||w_{k+\tilde{l}}^i - w_{k-1}^i|| \tag{4.11}$$

For all $l, \tilde{l} = 0, \ldots, N^i - 1$ *and with* $l \geq \tilde{l}$. *The state information* w_k^i *is from* (4.4) *and is subject to* (4.5).

Proof Firstly it is shown that (4.10) holds for $t = 0$. The calculation is then repeated for $t = 1$, and eventually the general rule for random t will be stated.

For $t = 0$ the difference (4.9) is

$$\begin{aligned}
&\Delta J_0^i = \bar{J}_N^i(k) - J_N^{i*}(k-1) = \sum_{l=0}^{N^i-1} \{ L^i(\bar{x}^i(k+l|k), \bar{u}^i(k+l|k)) + Q^i(\bar{x}^i(k+l|k), w^i(k+l|k)) \\
&- L^i(\hat{x}^i(k+l-1|k-1), u^{i*}(k+l-1|k-1)) - Q^i(\hat{x}^i(k+l-1|k-1), w^i(k+l-1|k-1)) \} \\
&+ V^i(\bar{x}^i(k+N^i|k)) - V^i(\hat{x}^i(k+N^i-1|k-1)) = \sum_{l=0}^{N^i-2} \{ L^i(\bar{x}^i(k+l|k), \bar{u}^i(k+l|k)) \\
&- L^i(\hat{x}^i(k+l|k-1), u^{i*}(k+l|k-1)) + Q^i(\bar{x}^i(k+l|k), w^i(k+l|k)) \\
&- Q^i(\hat{x}^i(k+l|k-1), w^i(k+l|k-1)) \} + L^i(\bar{x}^i(k+N^i-1|k), h^i(\bar{x}^i(k+N^i-1|k)) \\
&- L^i(x_{k-1}^i, u_{k-1}^i) - Q^i(x_{k-1}^i, w_{k-1}^i) + Q^i(\bar{x}^i(k+N^i-1|k), w^i(k+N^i-1|k)) \\
&+ V^i(\bar{x}^i(k+N^i|k)) - V^i(\hat{x}^i(k+N^i-1|k-1))
\end{aligned} \tag{4.12}$$

Where $\bar{x}^i(k+l+1|k+m)$ is the state of the subsystem i at time step $k+l+1$ with $l \in \mathbb{Z}_{\geq 0}$ and $m \in [0, N^i - 1]$ while using a feasible control input from (4.8). It is important to note that since stability of the nominal system is considered, the predicted state $\hat{x}(\cdot)$ and the "feasible" state $\bar{x}(\cdot)$, computed at the same time-step are coinciding.

Using the inequality of Assumption 10, the following result can be obtained

$$V^i(\vec{x^i}(k+N^i|k)) - V^i(\vec{x^i}(k+N^i-1|k)) + L^i(\vec{x^i}(k+N^i-1|k), h^i(\vec{x^i}(k+N^i-1|k)))$$
$$+ Q^i(\vec{x^i}(k+N^i-1|k), w^i(k+N^i-1|k))) \le 0 \tag{4.13}$$

Since nominal stability is considered in this case, we have

$$V^i(\vec{x^i}(k+N^i-1|k)) \equiv V^i(\hat{x}^i(k+N^i-1|k-1)) \tag{4.14}$$

From (4.8) we have $\vec{u^i}(k+l|k) = u^{i*}(k+l|k-1)$ for $l = 0, \ldots, N^i-2$, so imposing this control law for $m = 0$ to (4.1), we get

$$L^i(\vec{x^i}(k+l|k), \vec{u^i}(k+l|k)) = L^i(\hat{x}^i(k+l|k-1), u^{i*}(k+l|k-1)) \; \forall l = 0, \ldots, N^i-2 \tag{4.15}$$

Notice that using Assumption 9 as well as (4.11), we obtain

$$Q^i(\vec{x^i}(k+l|k), w^i(k+l|k)) - Q^i(\hat{x}^i(k+l|k-1), w^i(k+l|k-1))$$
$$\le ||Q^i(\cdot, w^i(k+l|k)) - Q^i(\cdot, w^i(k+l|k-1))||$$
$$\le L^i_{qw}||w^i(k+l|k) - w^i(k+l|k-1)|| \le L^i_{qw} \cdot e^i_w(k|k-1) \tag{4.16}$$

Substituting (4.11), (4.13), (4.14), (4.15), (4.16) to (4.12) and utilizing Assumption 8, the following is derived

$$\Delta J^i_0 \le -L^i(x^i_{k-1}, u^i_{k-1}) - Q^i(x^i_{k-1}, w^i_{k-1}) + \sum_{l=0}^{N^i-2} \{L^i_{qw} \cdot e^i_w(k+l|k-1)\}$$
$$\le (N^i-2)L^i_{qw} \cdot e^i_w(k|k-1) - r^i(||x^i_{k-1}||) \tag{4.17}$$

For $t=1$ the difference (4.9) becomes

$$\Delta J_1^i = \bar{J}_N^i(k+1) - J_N^{i*}(k-1) = \sum_{l=0}^{N^i-1}\{L^i(\bar{x}^i(k+l+1|k+1), \bar{u}^i(k+l+1|k+1))$$
$$- L^i(\hat{x}^i(k+l-1|k-1), u^{i*}(k+l-1|k-1)) + Q^i(\bar{x}^i(k+l+1|k+1), w^i(k+l+1|k+1))$$
$$- Q^i(\hat{x}^i(k+l-1|k-1), w^i(k+l-1|k-1))\} + V^i(\bar{x}^i(k+N^i+1|k+1))$$
$$- V^i(\hat{x}^i(k+N^i-1|k-1)) = \sum_{l=0}^{N^i-3}\{L^i(\bar{x}^i(k+l+1|k+1), \bar{u}^i(k+l+1|k+1))$$
$$- L^i(\hat{x}^i(k+l+1|k-1), u^{i*}(k+l+1|k-1)) + Q^i(\bar{x}^i(k+l+1|k+1), w^i(k+l+1|k+1))$$
$$- Q^i(\hat{x}^i(k+l+1|k-1), w^i(k+l+1|k-1))\}$$
$$+ L^i(\bar{x}^i(k+N^i-1|k+1), h^i(\bar{x}^i(k+N^i-1|k+1))$$
$$+ Q^i(\bar{x}^i(k+N^i-1|k+1), w^i(k+N^i-1|k+1) - L^i(x_{k-1}^i, u_{k-1}^i) - Q^i(x_{k-1}^i, w_{k-1}^i)$$
$$+ L^i(\bar{x}^i(k+N^i|k+1), h^i(\bar{x}^i(k+N^i|k+1)) + Q^i(\bar{x}^i(k+N^i|k+1), w^i(k+N^i|k+1)$$
$$- L^i(x_k^i, u_k^i) - Q^i(x_k^i, u_k^i) + V^i(\bar{x}^i(K+N^i-1|k+1))$$
$$+ V^i(\bar{x}^i(k+N^i+1|k+1)) - V^i(\bar{x}^i(k+N^i|k+1)) - V^i(\hat{x}^i(k+N^i-1|k-1))$$
$$+ V^i(\bar{x}^i(k+N^i|k+1)) - V^i(\bar{x}^i(K+N^i-1|k+1)) \tag{4.18}$$

Using similar arguments as in the case of $t=0$, it can be concluded that the difference ΔJ_1^i is bounded by

$$\Delta J_1^i \leq -L^i(x_{k-1}^i, u_{k-1}^i) - Q^i(x_{k-1}^i, w_{k-1}^i) - L^i(x_k^i, u_k^i) - Q^i(x_k^i, u_k^i) \tag{4.19}$$
$$+ \sum_{l=0}^{N^i-3}\{L_{qw}^i e_w^i(k+1+l|k-1)\} \leq (N^i-3)L_{qw}^i \cdot e_w^i(k+1|k-1) - r^i(||x_{k-1}^i||) - r^i(||x_k^i||)$$

From the above it can be concluded using the same procedure, that for random $t \in [0, N^i-1]$ the difference $\Delta J_t^i = \bar{J}_N^i(k+t) - J_N^{i*}(k-1)$, is given from (4.10), and hence the proof is completed.

System (4.1), subject to (4.2), which satisfies the Assumptions 8-10, is ISS stable with respect to measurement errors of the neighboring agents, under an NMPC strategy. This can be concluded by the optimality of the solution that results to

$$J_N^{i*}(k) - J_N^{i*}(k-1) \leq \Delta J_0^i \leq (N^i - 2)L_{qw}^i \cdot e_w^i(k|k-1) - r^i(||x_{k-1}^i||) \tag{4.20}$$

Notice that J_N^{i*} is an ISS Lyapunov function of the system (4.1) under an NMPC framework. This result has been proven in [FMP$^+$08] relaying to similar assumptions as in this Chapter, thus the proof is omitted.

Remark 2 *The error* (4.11) *can be seen as the error between the predicted and the actual trajectory of the neighboring agents. From equation* (4.11) *considering* $\tilde{l} = 0$*, then* $w^i(k+l|k)$ *and* $w^i(k+l|k-1)$ *are the predicted states of the neighbors at time* $k+l$*. If we set* w_{k+l}^i *to be the actual state of the systems at time* $k+l$*, it can be proven that*

$$e_w^i(k|k-1) = ||w^i(k+l|k) - w_{k+l}^i - (w^i(k+l|k-1) - w_{k+l}^i)||$$

which is the difference on the errors between the predicted and the real trajectories of the neighboring agents.

4.3.2 Triggering Condition for the NMPC

In the following, the triggering condition will be provided. Consider that at time t an event is triggered. In order to maintain the ISS property (4.20) of the system, the Lyapunov function $J_N^{i*}(\cdot)$ must be decreasing. Suppose that the error is restricted to satisfy

$$(N^i - 2)L_{qw}^i e_w^i(k|k-1) \leq \sigma r^i(||x_{k-1}^i||) \tag{4.21}$$

with $0 < \sigma < 1$. Plugging in (4.21) to (4.20) we get

$$J_N^{i*}(k) - J_N^{i*}(k-1) \leq (\sigma - 1)r^i(||x_{k-1}^i||) \tag{4.22}$$

This suggests that provided $\sigma < 1$, the ISS property of the system is still guaranteed.

This triggering rule states that when (4.21) is violated, the OCP is solved again using the current measurement of the state, as the initial state. If (4.21) is not violated, the control law from (4.8) is used for $m = 0$.

The triggering rule (4.21), is only valid in the first step. In order to ensure that the system remains stable while using control law (4.8) for $m \geq 0$ some additional restrictions for the difference (4.9) must be stated. According to [MAC02] and the proof of Theorem 1, optimality of the solution is not necessary to guarantee convergence of the closed-loop system. Thus, in order to maintain stability we must ensure that ΔJ_t^i is strictly decreasing for all $m \geq 0$. Hence, the system can use the control law (4.8), as long as

$$\Delta J_{t+1}^i \leq \Delta J_t^i \tag{4.23}$$

In this case the convergence of the closed-loop system is guaranteed.

Consequently, the triggering rule can be stated as

$$(N^i - t - 2)L_{qw}^i e_w^i(k+t|k-1) \leq \sigma \sum_{\rho=0}^{t} \{r^i(||x_{k-\rho+t}^i||)\} \tag{4.24a}$$

and

$$(N^i - t - 2)L_{qw}^i e_w^i(k+t|k-1) - \sigma r^i(||x_{k+t}^i||) \leq (N^i - t - 1)L_{qw}^i e_w^i(k+t-1|k-1) \tag{4.24b}$$

The next OCP is triggered whenever condition (4.24a) or (4.24b) is violated. Note that it must hold that $N^i \geq 2$. This is a necessity since for prediction horizons $N^i < 2$, the controller would only provide one step ahead, thus this would result triggering at every time step.

The previous analysis guarantees that the closed loop system will have the same convergence properties as in [FMP+08]. However, the OCP in the case of this paper is not calculated at each time instant, but only when the triggering condition is violated. Thus the convergence to a compact set and ultimate boundedness properties of [FMP+08] are preserved in the event-triggered formulation:

Theorem 5 *Consider a locally controlled agent $\mathscr{A}^i$ for all $i = 1, \ldots, M$ with dynamics described by* (4.1), *subject to* (4.2), *and assume also that the previously presented Assump-*

tions 8-10, hold. Then the NMPC control law given by (4.6a)-(4.6e) *with the neighboring' state information* (4.4) *to be subject to* (4.5), *along with the triggering rule* (4.24a)-(4.24b) *drives the closed loop system towards a compact set where it is ultimately bounded.*

4.3.3 Delays

When large scale systems are considered, it is expected that there will be a delay during the exchange of the information between the cooperating agents. Using some assumptions, a triggering condition of each agent of the distributed dynamic system similar to (4.24a)-(4.24b), will be defined in the following, in the presence of communication delays.

Assume that agent $\mathscr{A}^i$ receives from each neighboring agent $\mathscr{A}^j \in \mathscr{G}^i$ the value of its state with a delay of Δ_{ij}. The delayed state information of the neighbors, received by agent $\mathscr{A}^i$, is

$$w^i_{k-\Delta_{ij}} \triangleq col(x^j_{k-\Delta_{ij}}, j \in G^i)$$

Assume that Δ_{ij} is such that

$$||w^i_k - w^i_{k-\Delta_{ij}}|| \leq \gamma_{ij} \tag{4.25}$$

If delays are present, then (4.10) is modified as follows

$$\Delta J^i_t \leq (N^i - t - 2)L^i_{qw} e^i_{wd}(k+t|k-1) - \sum_{\rho=0}^{t} \{r^i(||x_{k-\rho+t}||)\} \tag{4.26}$$

with $e^i_{wd}(k+t|k-1) = ||w^i_{k+t-\Delta_{ij}} - w^i_{k-1-\Delta_{ij}}||$. Notice that using the reverse triangle inequality, it yields that

$$e^i_w(k+t|k-1) - e^i_{wd}(k+t|k-1) = ||w^i_{k+t} - w^i_{k-1}|| - ||w^i_{k+t-\Delta_{ij}} - w^i_{k-1-\Delta_{ij}}|| \tag{4.27}$$
$$\leq ||w^i_{k+t} - w^i_{k-1} - w^i_{k-1-\Delta_{ij}} + w^i_{k+t-\Delta_{ij}}|| \leq ||w^i_{k+t} - w^i_{k+t-\Delta_{ij}}|| + ||w^i_{k-1} - w^i_{k-1-\Delta_{ij}}|| \leq 2\gamma_{ij}$$

Thus,

$$e^i_w(k+t|k-1) \leq e^i_{wd}(k+t|k-1) + 2\gamma_{ij} \tag{4.28}$$

Substituting (4.28) to (4.26), it can be obtained that

$$\Delta J_t^i \leq (N^i - t - 2)L_{qw}^i(e_{wd}^i(k+t|k-1) + 2\gamma_{ij}) - \sum_{\rho=0}^{t} \{r^i(||x_{k-\rho+t}||)\} \tag{4.29}$$

Finding the triggering condition in the presence of communication delays using similar approach as in the previous section, is straightforward and is omitted.

4.4 Example: Three Cooperating Agents

In this Section, a simulated example of the proposed event-based framework is presented. Three agents moving in $\mathbb{R}^2$ is considered. Each one of the agents is controlled by a local MPC controller while exchanging state information with the neighboring agents without delays.

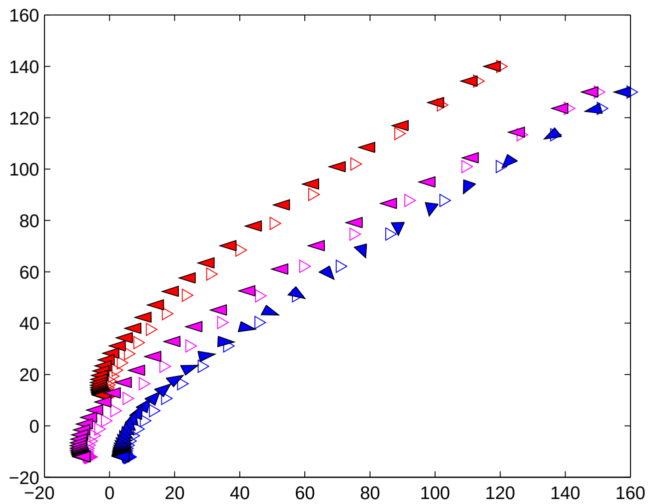

Figure 4-2: Trajectories of the team of agents. The filled triangles represent the agents under the event-based NMPC, while the empty triangles represent the agents under time-based NMPC.

The objective of the agents is to reach a desired configuration, while they keep some relative distance between them. For illustrative purposes, the numerical values of the parameters of the system that was taken into account, are taken as in [FPP04], where the classic time-driven MPC was considered. The simulation results are reported in Fig. 4-2 where the trajectories of the three agents are depicted. The filled triangles represent the agents under the event-based framework and the empty triangles represent the agents under classic time-driven MPC. Note, that the orientation of the agents is only depicted in the event-driven case. It can be witnessed that the event-driven as well as the time-driven approach results in comparable performance, i.e., clearly in both cases there is coordinated behavior of the team of the UAV's and collision was always avoided.

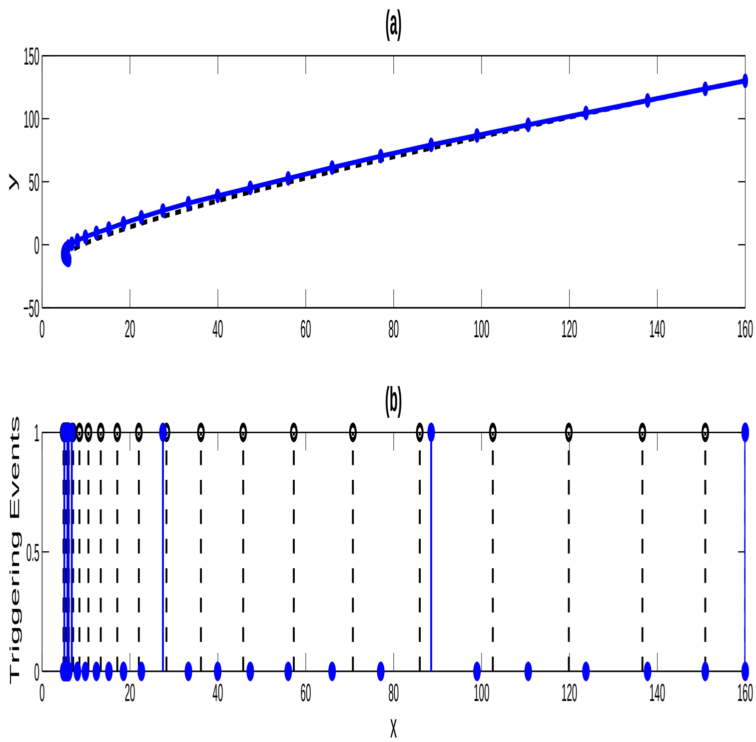

Figure 4-3: (a) Trajectory of an agent. The blue solid line represents the trajectory of the agent under the event-based NMPC, while the black dashed line, represents the trajectory of the same agent, under the classic NMPC. (b)Triggering instants. When the triggering axis has the value 1, the OCP of the NMPC is triggered. For value 0, the NMPC law is implemented on the system in an open-loop fashion. The blue solid stems represent the triggering instants of the event-based NMPC, while the black dashed stems represent the sampling instants of the classic time-based NMPC.

The next simulation represents the trajectory of a single agent, in the same cooperative scenario. In Fig.4-3(a), the trajectory of the agent is shown in both the event-driven and the time-driven case. Figure 4-3(b) depicts the triggering instants. The black dashed line represents the triggering instants in the time-driven case and the blue solid line depicts the event-triggered policy. In this example, it is evident that the inter-calculation times are strictly larger than one. Namely, with the event-triggered strategy the control updates are significantly lower. Moreover, in both cases, the final configurations of the agents were reached in the same number of sampling instants.

4.5 Conclusions

In Chapter, an event-based framework for the control of a team of cooperating distributed agents under NMPC controllers was proposed and analyzed. The event-based formulation consists of triggering the solution of the OCP of the NMPC, only when an event occurs. During the inter-event period the control sequence provided from the previous triggering event is used in an open-loop fashion. This even-based scheme is favorable in a number of occasions, because it is possible to reduce the number of times the control law should be computed. This results to the alleviation of the energy consumption.

Chapter 5

Aperiodic Model Predictive Control via Perturbation Analysis

In this Chapter, an enhanced event-based scheme for model predictive control (MPC) of constrained discrete-time systems with additive disturbances is investigated. The recalculation of the MPC control law is triggered whenever an event depending on the error of the measured state with respect to the nominal state of the system occurs. Between the controller updates, the last computed control trajectory is applied to the system, in conjunction with a correction term. This term consists of a perturbation solution of the nominal system which itself depends on the aforementioned error. The overall framework yields less conservative results with respect to the previous Chapters. The results are illustrated through a simulated example.

5.1 Introduction

The problem addressed here is the event-driven control of a general nonlinear discrete-time system with additive disturbances, under an NMPC framework. Since the system in consideration is uncertain, in order to prove stability, a similar procedure as in the case of Input-to-State (ISS) stability analysis for MPC, is going to be used. As in Chapter 3, the error between the real state of the system and the predicted state given by the nominal model, is monitored. However, the control law that is applied to the plant, during the inter-event times, is the previously computed control law in conjunction with a correction term. This term utilizes a perturbation solution of the optimal trajectory and is explicitly dependent on the aforementioned error. With this approach the controller has some kind of additional "intelligence" and reacts to the measured error. Thus, the contribution of this approach relies in finding sufficient conditions for triggering in the case of uncertain discrete-time systems under an NMPC control law with a correction term. Notice, that perturbation analysis of predictive controllers has been presented in [GSK07], [GGR05] and [WHM09].

5.2 Problem Formulation

Consider the nonlinear discrete-time dynamic system

$$x_{k+1} = f(x_k, u_k) \tag{5.1}$$

where $x_k \in \mathbb{R}^n$ denotes the system's state and $u_k \in \mathbb{R}^m$ is the control vector. The state and control variables are subject to the following constraints

$$x_k \in X, \qquad u_k \in U, \qquad k \in \mathbb{Z}_{\geq 0} \tag{5.2}$$

where X is a closed subset of $\mathbb{R}^n$ and U is a compact subset of $\mathbb{R}^m$, both of them containing the origin as an interior point. Assume that $f(0,0) = 0$ and that $f(x,u)$ is locally Lipschitz with respect to x and u in the domain $X \times U$, with Lipschitz constants L_{f_x} and L_{f_u}, respectively. The predicted state of the system at a time step $k+j+1$ with $j \in \mathbb{Z}_{\geq 0}$ can be found by the nominal model of the system i.e. (5.1), and is denoted as

$$\hat{x}(k+j+1|k) = f(\hat{x}(k+j|k), u_{k+j})$$

where u_{k+j} is a control sequence for time $[k, k+j]$, and $x_k = \hat{x}(k|k)$ is the measured state of the system at time step k.

In a realistic formulation though, modeling errors, uncertainties and disturbances may exist. Thus, a perturbed version of (5.1) is going to be considered as well. The perturbed system is described as

$$x_{k+1} = f(x_k, u_k) + w_k \tag{5.3}$$

with $w_k \in W \subseteq \mathbb{R}^n$ to be the additive disturbance and W to be a compact set containing the origin. The admissible set of uncertainties are bounded, thus

$$w_k \in W, \qquad ||w_k|| \leq \gamma \tag{5.4}$$

It is apparent that the uncertainty of system (5.3) can cause discrepancies between the predicted state sequence given from (5.1) and the actual state sequence of the system. This divergence can be quantified in terms of an error. Therefor, the error $e(k+j|k)$ is introduced in the analysis and is denoted as

$$e(k+j|k) = ||x_{k+j} - \hat{x}(k+j|k)|| \tag{5.5}$$

5.3 Event-Based NMPC via Perturbation Analysis

In the classic NMPC strategy, the control law is updated at each time-step k. The control input that is applied to the system is the first term of the optimal control sequence provided by the NMPC. However, in the event-triggered setup the rest of the optimal sequence might be used as well, provided that the real evolution of the system stays close to the predicted by means of the nominal model. In Chapter 3 the last computed control sequence was applied to the system in an open-loop fashion, during the inter-event times. The error between the real state sequence and the predicted sequence was monitored in order to trigger an event. On the other hand, in this approach, the last computed control law along with a correction term is applied to the system during the triggering events. The correction term can be found as the approximation solution of the MPC, it is easily computable and corrects the nominal solution. Hence, this term is applied in order to account for the error on-line.

In the following, a perturbation analysis is conducted in order to reach to the the analytic expression of the correction term. Moreover, the convergence and stability properties of the overall scheme of the NMPC with the neighboring extremals approach are proven.

5.3.1 Neighboring Extremals

The solution of an optimal control problem, when perturbations in the initial state are present, can be approximated using the optimal perturbation analysis approach. Namely, if there is a perturbation $dx(k)$ in the initial condition, the resulting optimal solution can be approximated by $\hat{x}(k) + dx(k)$ and $u^*(k) + du(k)$. The real state of the system x_k at time step k, can be found by

$$x_k = \hat{x}(k|k-1) + dx(k) \Rightarrow dx(k) = x_k - \hat{x}(k|k-1) \tag{5.6}$$

and for time step $k+j$ we have respectively

$$dx(k+j) = x_{k+j} - \hat{x}(k+j|k-1)$$

The perturbation analysis for constrained, discrete-time MPC is treated in the Appendix. The neighboring extremal path method, developed in [BH75], is adopted. It holds that

$$du(k+j) = K^*(k+j)dx(k+j) \tag{5.7}$$

In order to find $K^*(k+j)$, all quantities are evaluated at the nominal optimal condition, namely, $\hat{x}(k+j|k-1)$, $u^*(k-1)$. The analytic expression of $K^*(k+j)$ is derived in the Appendix and in particular in (5.54). From (5.7) we have

$$||du(k+j)|| \leq ||K^*(k+j)|| \cdot ||dx(k+j)|| \leq ||K^*(k+j)||e(k+j|k-1) \tag{5.8}$$

Suppose an upper bound on $||du(\cdot)||$. This is a because the system (5.1) is constrained in the inputs, so we must ensure that while using the neighbors extremals, the input constraints will be fulfilled. So,

$$||du(k+j)|| \leq \gamma^{\mu}. \tag{5.9}$$

5.3.2 NMPC Strategy

The general form of NMPC consists in solving on-line a finite-horizon, open-loop optimal control problem (abbr. OCP), based on the current state measurement. A cost function J_N is minimized with respect to a control sequence $u_F(k) \triangleq [u(k|k), u(k+1|k), \ldots, u(k+N-1|k)]$, thus, the OCP for the nominal system (5.1), can be formulated as follows

$$\min_{u_F(k)} J_N(x_k, u_F(k)) = \min_{u_F(k)} \sum_{i=0}^{i=N-1} F(\hat{x}(k+i|k), u(k+i|k)) + V(\hat{x}(k+N|k)) \tag{5.10a}$$

subject to

$$\hat{x}(k+j|k) \in X_j \qquad \forall j = 1, \ldots, N-1 \tag{5.10b}$$

$$u(k+j|k) \in U_j \qquad \forall j = 0, \ldots, N-1 \tag{5.10c}$$

$$\hat{x}(k+N|k) \in X_f \tag{5.10d}$$

where the positive integer $N \in \mathbb{Z}_{\geq 0}$, denotes the prediction horizon and X_f denotes the terminal constraint set.

The constraints on the state from (5.2) are being replaced by a restricted constraint set X_j while solving the OCP. It holds that $X_j = X \sim \mathscr{B}_j^x$ where $\mathscr{B}_j^x = \{x \in \mathbb{R}^n : ||x|| \leq L_{f_x}^j \gamma + L_m(j) L_{f_u} \gamma^u\}$. This state constraints' tightening for the nominal system with additive disturbances, while utilizing the correction term from the perturbation analysis, guarantees that the evolution of the real system will be admissible for all time. This is proven in Lemma 6 of the Appendix. Furthermore, the constrained set U_j is a restricted set in the same sense as in the state constraint tightening case. There is $U_j = U \sim \mathscr{B}^u$ where $\mathscr{B}^u = \{u \in \mathbb{R}^m : ||u|| \leq \gamma^u\}$ which guarantees the fulfillment of all input constraints. Notice that, the set operator "$\sim$" denotes the Pontryagin difference and that we denote

$$L_m(j) = \sum_{i=0}^{j-1} \{L_{f_x}^i\}$$

Similarly to the previous Chapters, the following assumptions for the stage cost $F(\cdot)$ and the terminal cost $V(\cdot)$ are stated:

Assumption 11 *The stage cost $F(x,u)$ is Lipschitz continuous with respect to x and u in $X \times U$, with Lipschitz constants denoted by L_{Fx} and L_{Fu}, respectively. Assume that $F(0,0) = 0$ and that there are positive constants $\alpha > 0$ and $\omega \geq 1$, such that $L(x,u) \geq \alpha||(x,u)||^{\omega}$.*

Assumption 12 *Let the terminal region X_f from* (5.10d) *be a subset of an admissible positively invariant set Φ of the nominal system. Assume that there is a local stabilizing controller $h(x_k)$ for the terminal state X_f. The associated Lyapunov function $V(\cdot)$ has the following properties $V(f(x_k,h(x_k)) - V(x_k) \leq -F(x_k,h(x_k)), \forall x_k \in \Phi$, and is Lipschitz in Φ, with Lipschitz constant L_V. The set Φ is given by $\Phi = \{x_k \in \mathbb{R}^n : V(x_k) \leq \alpha_\Phi\}$ such that $\Phi \subseteq X^h = \{x_k \in X_{N-1} : h(x_k) \in U\}$. The set $X_f = \{x_k \in \mathbb{R}^n : V(x_k) \leq \alpha_v\}$ is such that for all $x_k \in \Phi$, $f(x_k,h(x_k)) \in X_f$.*

Definition 1 *In the following, X^{MPC} will denote the set containing all the state vectors for which a feasible control sequence exists, i.e. a control sequence u that satisfies all the constraints of the MPC,* (5.10b) *through* (5.10d)*.*

Consider the control trajectories $u_F^n(k+m)$, for time steps $m = 0,\ldots,N-1$, based on the optimal solution in $k-1$, i.e. $u_F^*(k-1)$, in conjunction with a correction term from the perturbation solution of the MPC. The "neighboring" control trajectories can be denoted as

$$u^n(k+j|k+m) = \begin{cases} u^*(k+j|k-1) + du(k+j) & \text{for } j = m,\ldots,N-2 \\ h(x^n(k+N-1|k+m)) & \text{for } j = N-1 \end{cases} \tag{5.11}$$

Furthermore, the state of the system when the control law (5.11) is applied to the system, is given by

$$x^n(k+j+1|k+m) = f(x^n(k+j|k+m), u^n(k+j|k+m))$$

Definition 2 *A set $X^{MPC} \subseteq X$ is robust positively invariant (RPI) set for system* (5.3)*, if $x_k \in X^{MPC}$, $\forall x_{k-1} \in X^{MPC}$ and $\forall w_k \in W$.*

Next, the robust positively invariance of the set X^{MPC} of the closed-loop system will be shown.

Lemma 5 *Let the system described by* (5.3) *and is subject to* (5.2). *Under the Assumption 1, X^{MPC} is RPI for the closed-loop system if the uncertainties are bounded by $\gamma \leq (\alpha_\Phi - \alpha_v - L_V L_m (N-1) L_{f_u} \gamma^\mu)/L_V L_{f_x}^{N-1}$.*

For simplicity, we are going to treat the case $m = 0$. One can easily verify that $u^n(k+j|k) \in U_j$ for $j \in [0, N-2]$, and $h(x) \in U$, which yields that $u^n(\cdot)$ are feasible control trajectories. Also it must be shown that if $\hat{x}(k+N-1|k) \in \Phi$, then $x^n(k+N|k) \in X_f$. By applying Lemma in the Appendix, it holds that

$$\begin{aligned} V(x^n(k+N|k)) &\leq V(\hat{x}(k+N-1|k-1)) + L_V L_{f_x}^{N-1} \gamma + L_V L_m (N-1) L_{f_u} \gamma^\mu \\ &\leq \alpha_v + L_V L_{f_x}^{N-1} \gamma + L_V L_m (N-1) L_{f_u} \gamma^\mu \leq \alpha_\Phi \end{aligned}$$

Considering that $||x^n(k+j|k) - x^n(k+j|k-1)|| \leq L_{f_x}^j \gamma + L_m(j) L_{f_u} \gamma^\mu$, it can be concluded that $x^n(k+j|k) \in X_j$, and the proof is completed.

The next step is to prove convergence of the proposed scheme. In order to do so, an intermediate result is going to be stated first. The optimal cost at time step $k-1$ is $J_N^*(k-1)$ and the cost of the "neighboring" feasible sequence at a time step $j \in [0, N-1]$ is indicated by $J_N^n(k+j)$. Then the difference of these costs is

$$\Delta J_j = J_N^n(k+j) - J_N^*(k-1) \tag{5.12}$$

The next theorem can now be stated:

Theorem 6 *Consider the system* (5.3) *subject to* (5.2) *and assume that the previously presented Assumption 1 holds. Then, using the control law from* (5.11), *the difference between the cost of a feasible sequence at time step $k+j$ and the optimal cost of at time step $k-1$ is bounded by*

$$\Delta J_j \leq C_1^j e(k+j|k-1) - \alpha \sum_{i=0}^{j} \{||x_{k-i+j}||^\omega\} + C_2^j \tag{5.13}$$

where C_1^j is given by

$$C_1^j \triangleq L_{F_x} L_m(N-1-j) + L_V L_{f_x}^{N-1-j} \tag{5.14}$$

and C_2^j is given by

$$C_2^j \triangleq (L_V L_m(N-1-j) + L_{F_x} \sum_{i=0}^{N-2-j} \{L_m(i)\} + 1) L_{f_u} \gamma^\mu \tag{5.15}$$

First, the difference (5.12) is calculated for $j = 0$. Then the calculation will be repeated for $j = 1$, and finally the general rule for random j will be stated.
For $j = 0$ the difference (5.12) is given by

$$\begin{aligned}
\Delta J_0 = J_N^n(k) - J_N^*(k-1) = \sum_{i=0}^{N-1} \{F(x^n(k+i|k), u^n(k+i|k))\\
- F(\hat{x}(k+i-1|k-1), u^*(k+i-1|k-1))\} + V(x^n(k+N|k)) - V(\hat{x}(k+N-1|k-1))\\
= \sum_{i=0}^{N-2} \{F(x^n(k+i|k), u^n(k+i|k)) - F(\hat{x}(k+i|k-1), u^*(k+i|k-1))\}\\
+ F(x^n(k+N-1|k), h(x^n(k+N-1|k)) - F(x_{k-1}, u_{k-1}) + V(x^n(k+N|k))\\
- V(\hat{x}(k+N-1|k-1)) + V(x^n(k+N-1|k)) - V(x^n(k+N-1|k))
\end{aligned} \tag{5.16}$$

Recall from Assumption 11, that the stage cost is Lipschitz continuous in $X \times U$, so

$$\begin{aligned}
&F(x^n(k+i|k), u^n(k+i|k)) - F(\hat{x}(k+i|k-1), u^*(k+i|k-1))\\
&\leq L_{Fx}||x^n(k+i|k) - \hat{x}(k+i|k-1)|| + L_{Fu}||u^n(k+i|k) - u^*(k+i|k-1)||
\end{aligned} \tag{5.17}$$

From (5.56) of the Appendix, it can be concluded that

$$L_{F_x}||x^n(k+i|k) - \hat{x}(k+i|k-1)|| \leq L_{F_x} L_{f_x}^i e(k|k-1) + L_{F_x} L_m(i) L_{f_u} \gamma^\mu \tag{5.18}$$

Also, using the control law (5.11), we have

$$L_{Fu}||u^n(k+i|k) - u^*(k+i|k-1)|| =$$
$$L_{Fu}||u^*(k+i|k-1) + du(k+i) - u^*(k+i|k-1)|| = L_{Fu}||du(k+i)|| \leq L_{Fu}\gamma^{\mu} \quad (5.19)$$

The following inequality holds by Assumption 12,

$$V(x^n(k+N|k)) - V(x^n(k+N-1|k)) + F(x^n(k+N-1|k), h(x^n(k+N-1|k))) \leq 0 \quad (5.20)$$

Moreover, using (5.56) it follows that

$$V(x^n(k+N-1|k)) - V(\hat{x}(k+N-1|k-1)) \leq L_V L_{f_x}^{N-1} e(k|k-1) + L_V L_m(N-1) L_{f_u}\gamma^{\mu} \quad (5.21)$$

Let the stage cost to be

$$F(x,u) \geq \alpha||(x,u)||^{\omega} \geq \alpha||x||^{\omega}$$

Substituting (5.18)-(5.21) to (5.16), the following is derived

$$\begin{aligned}
\Delta J_0 \leq & \sum_{i=0}^{N-2} \{L_{F_x} L_{f_x}^i e(k|k-1) + L_{F_x} L_m(i) L_{f_u}\gamma^{\mu} + L_{f_u}\gamma^{\mu}\} \\
& + L_V L_{f_x}^{N-1} e(k|k-1) + L_V L_m(N-1) L_{f_u}\gamma^{\mu} \\
& \leq (L_{F_x} L_m(N-1) + L_V L_{f_x}^{N-1}) e(k|k-1) \\
& + (L_V L_m(N-1) + L_{F_x} \sum_{i=0}^{N-2} \{L_m(i)\} + 1) L_{f_u}\gamma^{\mu} - \alpha||x_{k-1}||^{\omega} \\
& \leq C_1^0 e(k|k-1) - \alpha||x_{k-1}||^{\omega} + C_2^0
\end{aligned} \quad (5.22)$$

where C_1^0, C_2^0 are constant terms from (5.14) and (5.15) for $j = 0$, respectively.

For $j = 1$ the difference (5.12) becomes

$$\Delta J_1 = J_N^n(k+1) - J_N^*(k-1) = \sum_{i=0}^{N-1} \{F(x^n(k+i+1|k+1), u^n(k+i+1|k+1))$$
$$-F(\hat{x}(k+i-1|k-1), u^*(k+i-1|k-1))\} + V(x^n(k+N+1|k+1)) - V(\hat{x}(k+N-1|k-1))$$
$$\leq (L_{F_x} L_m(N-2) + L_V L_{f_x}^{N-2}) e(k+1|k-1) + (L_V L_m(N-2) + L_{F_x} \sum_{i=0}^{N-3} \{L_m(i)\} + 1) L_{f_u} \gamma^{\mu}$$
$$-\alpha||x_{k-1}||^{\omega} - \alpha||x_k||^{\omega} \leq C_1^1 e(k+1|k-1) + C_2^1 - \alpha||x_{k-1}||^{\omega} - \alpha||x_k||^{\omega} \tag{5.23}$$

From the above it can be concluded using the same calculation, that for random $j \in [0, N-1]$ the difference $\Delta J_j = J_N^n(k+j) - J_N^*(k-1)$, is given from (5.13), and hence the proof is completed.

5.4 Triggering Condition

The proposed scheme must be convergent to a compact set where the system is ultimately bounded. In the following, a triggering condition for the OCP of the MPC that guarantees that the associate Lyapunov function is decaying at every time step and that all the constraints are fulfilled, is given.

Since $J_N^*(k)$ is the optimal cost at time step k, we have $J_N^*(k) - J_N^*(k-1) \leq \Delta J_0$, hence

$$J_N^*(k) - J_N^*(k-1) \leq C_1^0 e(k|k-1) - \alpha||x_{k-1}||^{\omega} + C_2^0 \tag{5.24}$$

The triggering condition is written in this case as

$$C_1^0 e(k|k-1) \leq \sigma(\alpha||x_{k-1}||^{\omega} - C_2^0) \tag{5.25}$$

Invoking this rule into (5.24), with $0 < \sigma < 1$, it can be concluded that $J_N^*(\cdot)$ is strictly decreasing. The triggering rule (5.25) is valid, though, only in the first step. In order to maintain stability we must ensure that ΔJ_j is strictly decreasing for all $j \in [0, N-1]$. The control law (5.11), is applied to the system as long as

$$\Delta J_{j+1} \leq \Delta J_j \tag{5.26}$$

In this case the convergence of the closed-loop system is guaranteed.

As we have already discussed, correcting the optimal control law with an approximation solution given by the neighboring extremals approach, may lead to control inputs that violate the constraints. To account for that, an upper bound on the norm of the correction term was assumed in (5.9). In order to assert that this is the case, we will impose an event-based condition that states that the control update will be triggered whenever the tracking error will exceed a specific limit. Using (5.7) and (5.9) the following condition can be derived

$$||K^*(k+j)||e(k+j) \leq \gamma^\mu$$

In a practical sense, this states that whenever the error is small enough we do not trigger a new MPC law, otherwise, we measure the state of the system and compute an appropriate control law.

Consequently, the triggering condition can be stated as

$$C_1^j e(k+j|k-1) \leq \sigma(\alpha \sum_{i=0}^{j} ||x_{k-i+j}||^\omega + C_2^j) \tag{5.27a}$$

and

$$\begin{aligned} &C_1^j e(k+j|k-1) - \sigma(\alpha \sum_{i=0}^{j} \{||x_{k-i+j}||^\omega\} + C_2^j) \leq \\ &C_1^{j-1} e(k+j-1|k-1) - \sigma(\alpha \sum_{i=0}^{j-1} \{||x_{k-i+j}||^\omega\} + C_2^{j-1}) \end{aligned} \tag{5.27b}$$

and

$$||K^*(k+j)||e(k+j) \leq \gamma^\mu \tag{5.27c}$$

The next OCP is triggered whenever condition (5.27a) or (5.27b) or (5.27c) is violated.

The previous analysis guarantees that the closed loop system will have the same convergence properties as in [MAC02]. However, the OCP in this paper is not calculated at each time instant, but only when the triggering condition is violated. The next theorem can be stated:

Theorem 7 *Consider the system* (5.3), *subject to* (5.2) *under an NMPC strategy and assume that the previously presented Assumption 1 holds. The NMPC control law provided by* (5.10a)-(5.10d) *is triggered whenever condition* (5.27a)-(5.27c) *is violated. Between inter-event times the control law* (5.11) *is applied to the system. The overall framework drives the closed loop system towards a compact set where it is ultimately bounded.*

The algorithm for the aperiodic NMPC can be summarized as follows

Algorithm 3 Aperiodic MPC with a Correction Term

1: **while** $x_k \notin X_f$ **do**
2:     Measure x_k.
3:     Calculate the MPC control law: $u_F^*(k)$
       Buffer: (Zero-based Indexing)
       $u_F^* = [u^*(k|k), u^*(k+1|k), \ldots, u^*(k+N-1|k)]$.
       $\hat{x}_F = [x_k, \hat{x}(k+1|k), \ldots, \hat{x}(k+N-1|k)]$.
       $K_F^* = [0, K^*(k+1), \ldots, K^*(k+N-1)]$.
4:     $k_0 \leftarrow k$.
5:     Error: $e(k|k_0) = ||x_k - \hat{x}_F(k-k_0)|| = 0$.
6:     **while** Error does not violate (5.27a) or (5.27b) or (5.27c) **do**
7:         Apply $u^*(k-k_0) + \delta u(k-k_0)$.
           At next step $k \leftarrow k+1$.
           Measure x_k.
           Calculate Error: $e(k|k_0)$.
8:     **end while**
9:     Continue from 3.
10: **end while**

5.5 Example

5.5.1 Event-Triggered MPC *vs.* Enhanced Event-Triggered MPC

In this section, a simulated example of the proposed event-based framework is presented. The system under consideration is a linear system under a quadratic MPC scheme. A comparison is made, between the event-based framework proposed in Chapter 3, and the proposed approach where the last computed control sequence is applied to the system along with a correction term given by a perturbation solution of the MPC. For illustrative purposes, the numerical values of the parameters of the system that was taken as

$$A \triangleq \begin{bmatrix} 1 & 0 & 0 & \delta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \delta & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \delta & 0 \\ 0 & 0 & 0 & 1-\delta\eta/m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-\delta\eta/m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1-\delta\eta/m \end{bmatrix}$$

$$B \triangleq \begin{bmatrix} 0.001 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.001 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta/m & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta/m & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta/J \end{bmatrix}$$

where $\delta = 0.1$, $m = 10$, $J = 8$, and $\eta = 15(m)^{2/3}$. Also the associated matrices for the cost function are taken as $P_N = P = diag\{100,\dots,100\}$ and $R = diag\{0.1,\dots,0.1\}$. The disturbance parameter and the length of the prediction horizon is set to $||w|| \leq 0.8rand$ and $N = 6$, respectively.

The simulation results are reported in Fig. 5-1 where the error (5.5) between the real state and the predicted state of the system given by the nominal model (5.1), is depicted. The error is zero when an event is triggered. From Fig.5-1 it can be witnessed that both approaches have comparable results and that with both approaches the inter-calculation times are strictly larger than one when the system is away from the equilibrium.

The next Fig.5-2, depicts a state sequence of the system under the event-based scheme given in Chapter 2 and is represented by the blue line. The state sequence of the system when the event-based scheme that exploits the neighboring extremal approach is applied to

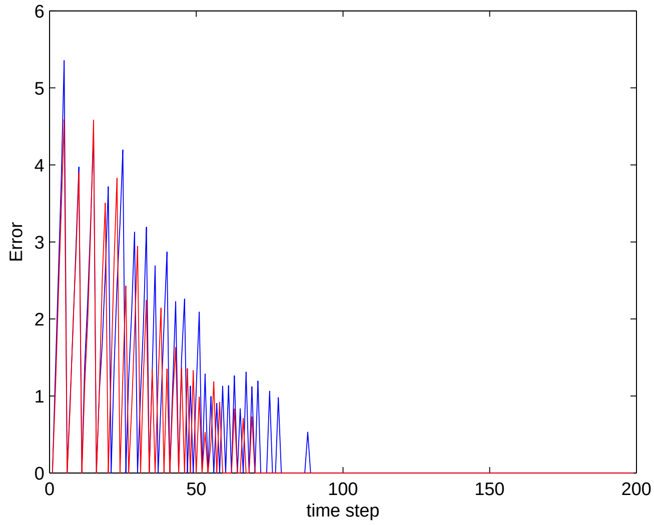

Figure 5-1: The error given from (5.5). The blue line represents the error when the last computed MPC law is applied to the system during the inter-event time-steps, while the red line represents the error when the last computed MPC law is applied to the system in conjunction with a correction term.

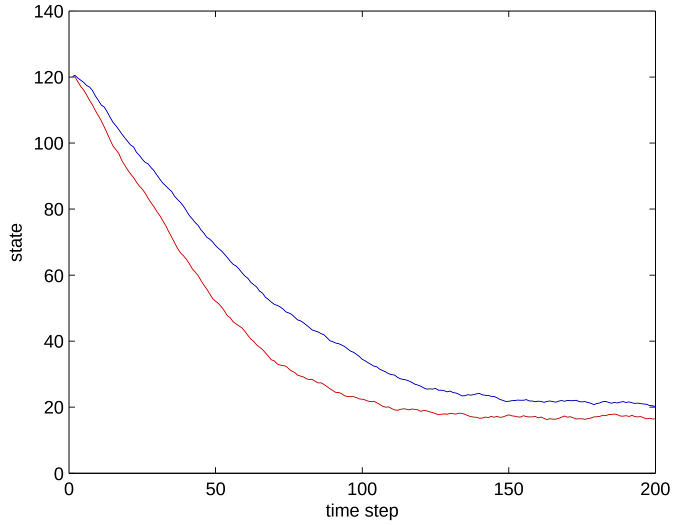

Figure 5-2: A state sequence of the system. The blue line represents the state sequence of the system when the last computed MPC law is applied to the system during inter-event time-steps, while the red line represents the state sequence of the system under the proposed approach.

the system is represented by the red line. As it can be seen in Fig.5-2 the system under the proposed event-based approach of this paper, has faster convergence properties.

5.6 Conclusions

In this Chapter, an event-based framework for the control of a general nonlinear constrained system under NMPC was proposed and analyzed. The event-based formulation consists in triggering the solution of the OCP of the NMPC, only when an event occurs. During the inter-event period the control sequence provided from the previous triggering event in conjunction with a correction term is used in an open-loop fashion. This correction term is provided by a perturbation analysis of the optimal trajectory, and it is used in order to account for the error between the predicted state of the nominal model of the system and the real evolution of the system. This event-based scheme is favorable in a number of occasions, because it is possible to reduce the number of times the control law should be computed. This results to the alleviation of the energy consumption.

Formulating the MPC control problem in a self-triggered scheme is one promising direction of research. This approach is presented in the subsequent Chapters, as with this approach the next control updates are decided at the previous ones, thus, the need for continuous monitoring of the measurement error can be relaxed.

5.7 Appendix

The perturbation solution for discrete-time MPC problems with state, control and terminal constraints is derived in this section. The cost function of the optimal control problem of MPC for the nominal system (5.1), is given by

$$J_N(x(0),u) = \sum_{k=0}^{N-1} \{F(x(k),u(k))\} + V(x(N))$$

Notice that a simpler notation of the cost function of the OCP, given by (5.10a) is used. The constraints (5.10b) through (5.10d) are assumed to have the form:

$$C(x(k), u(k)) \leq 0 \qquad \psi(x(N)) = 0$$

where $C : \mathbb{R}^{n+m} \to \mathbb{R}^l$ and $\psi : \mathbb{R}^n \to \mathbb{R}^q$. This assumption is helpful in the subsequent analysis, but not restrictive to the general constrained case. We treat the case $l = 1$ and $q \leq n$ for simplicity and that in the subsequent analysis, the subscripts x, u will denote the partial derivatives vector functions.

Next, the standard procedure of [BH75] and [GSK07] is followed. The augmented performance index, obtained by adjoining the constraints, is

$$\bar{J_N} = V^a(x(N)) + \sum_{k=0}^{N-1} \{H(x(k)) - \lambda(k+1)^\top x(k+1)\}$$

where

$$V^a = V(x(N)) + \nu^\top \psi(x(N))$$

and the Hamiltonian is given by

$$H(k) = F(x(k), u(k)) + \lambda(k+1)^\top f(x(k), u(k)) + \mu(k)^\top C(x(k), u(k))$$

where $\lambda(k)$ and $\nu^\top$ are multiplier sequence and a set of q multipliers, respectively. We will expand this augmented functional to the second order around a stationary solution x^*, u^* for which the first variation $d\bar{J_N}$ vanishes if the Lagrange multipliers are chosen as the accompanying multipliers of the stationary solution. Hence,

$$\begin{aligned} d^2\bar{J_N} &= 1/2 dx(N)^\top (V_{xx} + \nu^\top \psi_{xx}) dx(N) + \\ 1/2 \sum_{k=0}^{N-1} &\{ \begin{bmatrix} dx(k)^\top & du(k)^\top \end{bmatrix} \begin{bmatrix} H_{xx}(k) & H_{xu}(k) \\ H_{ux}(k) & H_{uu}(k) \end{bmatrix} \begin{bmatrix} dx(k)^\top \\ du(k)^\top \end{bmatrix} \} \end{aligned}$$

which is to be minimized subject to the linearized constraints:

$$dx(k+1) = f_x dx(k) + f_u du(k) \tag{5.28}$$

The terms $dx(0)$ and $d\psi(N) = \psi_x dx(N)$ are specified, and

$$C_x(k)dx(k) + C_u(k)du(k) = 0 \tag{5.29}$$

Let us suppose that an optimal control vector u^* has been determined, that meets all the first-order necessary conditions, or in other words satisfies the Euler-Lagrange equations

$$d\lambda(k) = H_{xx}dx(k) + f_x^\top d\lambda(k+1) + H_{xu}du(k) + C_x^\top d\mu(k) \tag{5.30}$$

also

$$0 = H_{uu}du(k) + H_{ux}dx(k) + f_u^\top d\lambda(k+1) + C_u^\top d\mu(k) \tag{5.31}$$

and

$$d\lambda^\top(N) = V_{xx}^a dx(N) + \psi_x^\top d\nu \tag{5.32}$$

From (5.28) and (5.30), we have

$$\begin{bmatrix} dx(k+1) \\ d\lambda(k) \end{bmatrix} = \begin{bmatrix} f_x(k) & 0 \\ H_{xx}(k) & f_x^\top(k) \end{bmatrix} \begin{bmatrix} dx(k) \\ d\lambda(k+1) \end{bmatrix} + \begin{bmatrix} f_u & 0 \\ H_{xu}(k) & C_x^\top(k) \end{bmatrix} \begin{bmatrix} du(k) \\ d\mu(k) \end{bmatrix} \tag{5.33}$$

Moreover, from (5.29) and (5.31) we have

$$\begin{bmatrix} du(k) \\ d\mu(k) \end{bmatrix} = -\begin{bmatrix} H_{uu}(k) & C_u^\top(k) \\ C_u(k) & 0 \end{bmatrix}^{-1} \begin{bmatrix} H_{ux}(k) & f_u^\top(k) \\ C_x(k) & 0 \end{bmatrix} \times \begin{bmatrix} dx(k) \\ d\lambda(k+1) \end{bmatrix} \tag{5.34}$$

From (5.33) and (5.34) we get

$$\begin{bmatrix} dx(k+1) \\ d\lambda(k) \end{bmatrix} = \begin{bmatrix} K_1 & K_2 \\ K_3 & K_1^\top \end{bmatrix} \begin{bmatrix} dx(k) \\ d\lambda(k+1) \end{bmatrix} \tag{5.35}$$

where

$$K_1 = f_x(k) - \begin{bmatrix} f_u(k) & 0 \end{bmatrix} \begin{bmatrix} H_{uu}(k) & C_u^\top(k) \\ C_u(k) & 0 \end{bmatrix}^{-1} \begin{bmatrix} H_{ux}(k) \\ C_x(k) \end{bmatrix} \tag{5.36}$$

$$K_2 = -\begin{bmatrix} f_u(k) & 0 \end{bmatrix} \begin{bmatrix} H_{uu}(k) & C_u^\top(k) \\ C_u(k) & 0 \end{bmatrix}^{-1} \begin{bmatrix} f_u^\top(k) \\ 0 \end{bmatrix} \tag{5.37}$$

$$K_3 = H_{xx}(k) - \begin{bmatrix} H_{xu}(k) & C_x^\top(k) \end{bmatrix} \begin{bmatrix} H_{uu}(k) & C_u^\top(k) \\ C_u(k) & 0 \end{bmatrix}^{-1} \begin{bmatrix} H_{ux}(k) \\ C_x(k) \end{bmatrix} \tag{5.38}$$

We seek solutions of the form

$$d\lambda(k) = S(k)dx(k) + R(k)d\nu \tag{5.39a}$$

$$d\psi = R^\top(k)dx(k) + Q(k)d\nu \tag{5.39b}$$

Also, we have

$$d\lambda(k+1) = S(k+1)dx(k+1) + R(k+1)d\nu \tag{5.40a}$$

$$d\psi = R^\top(k+1)dx(k+1) + Q(k+1)d\nu \tag{5.40b}$$

Given (5.28), it holds that

$$d\lambda(k+1) = S(k+1)\, f_x dx(k) + S(k+1)\, f_u du(k) + R(k+1)d\nu \tag{5.41a}$$

$$d\psi = R^\top(k+1)\, f_x dx(k) + R^\top(k+1)\, f_u du + Q(k+1)d\nu \tag{5.41b}$$

Plugging (5.41a) to (5.35), we get

$$d\lambda(k) = (K_3 + K_1^\top S(k+1)\, f_x)dx(k) + K_1^\top S(k+1)\, f_u du(k) + K_1^\top R(k+1)d\nu \tag{5.42a}$$

and

$$du(k) = Z_{uu} Z_{ux} dx(k) + Z_{uu} Z_{uv} d\nu \tag{5.42b}$$

where Z_{uu}, Z_{ux}, Z_{uv} are given by

$$Z_{uu} = [f_u + K_2 S(k+1) f_u]^{-1} \tag{5.43}$$

$$Z_{ux} = K_2 S(k+1) f_x - f_x + K_1 \tag{5.44}$$

$$Z_{uv} = K_2 R(k+1) \tag{5.45}$$

We use (5.42b) to eliminate $du(k)$ from (5.42a) and (5.41b), thus

$$d\lambda(k) = (K_3 + K_1^\top S(k+1) f_x + K_1^\top S(k+1) f_u Z_{uu} Z_{ux}) dx(k) + (K_1^\top S(k+1) f_u Z_{uu} Z_{uv}) d\nu \tag{5.46}$$

and

$$d\psi = (R^\top (k+1) f_x + R^\top (k+1) f_u Z_{uu} Z_{ux}) dx(k) + (R^\top (k+1) f_u Z_{uu} Z_{uv} + Q(k+1)) d\nu \tag{5.47}$$

For (5.46) and (5.47) to be equivalent to (5.39a) and (5.39b) the coefficients must be equal:

$$S(k) = K_3 + K_1^\top S(k+1) f_x + K_1^\top S(k+1) f_u Z_{uu} Z_{ux} \tag{5.48}$$

$$R(k) = K_1^\top S(k+1) f_u Z_{uu} Z_{uv} \tag{5.49}$$

$$Q(k) = R^\top (k+1) f_u Z_{uu} Z_{uv} + Q(k+1) \tag{5.50}$$

These recursive equations must satisfy the boundary conditions, namely $S(N) = V^a_{xx}$, $R(N) =$

$\psi_x(x(N))$ and $Q(N) = 0$. It holds that

$$d\nu = Q^{-1}(k)[d\psi - R^\top(k)dx(k)]$$

From that, given (5.42b), we get

$$du(k) = Z_{uu}(Z_{ux} + Z_{u\nu}Q^{-1}(k)R^\top(k))dx(k) + Z_{uu}Z_{u\nu}Q^{-1}(k)d\psi \tag{5.51}$$

In our case the desired $d\psi = 0$, so (5.51) becomes

$$du(k) = Z_{uu}(Z_{ux} + Z_{u\nu}Q^{-1}(k)R^\top(k))dx(k) \tag{5.52}$$

Equation (5.52) can be rewritten as

$$du(k) = K^*(k)dx(k) \tag{5.53}$$

with

$$K^*(k) \triangleq Z_{uu}(Z_{ux} + Z_{u\nu}Q^{-1}(k)R^\top(k)) \tag{5.54}$$

Hence, (5.53) is the neighboring optimum feedback law, when the present deviation from the optimal path $dx(k)$, is given, while minimizing the cost function.

Lemma 6 *The norm of the difference between the real evolution of the system when the control law* (5.11) *is applied to the system and the predicted evolution of the system at the same time step satisfies:*

$$||\hat{x}(k+j|k-1) - x_{k+j}|| \leq L_{f_x}^j \gamma + L_m(j) L_{f_u} \gamma^\mu$$

In order to prove the statement above, we shall make a slight violation in the notation.

The real state of the system at time step $k+j$ when an optimal control law $u^*(\cdot)$ is applied to the system will be denoted as $x_{k+j}(u^*)$, and the real state of the system at the same time step, when $u^*(\cdot)+du(\cdot)$ is applied to the system, will be denoted as $x_{k+j}(u^*+du)$.
For $j=0$:

$$||\hat{x}(k|k-1)-x_k(u^*)|| = e(k|k-1)$$

because there is $dx(k-1)=0 \Rightarrow du(k-1)=0$.
For $j=1$:

$$||\hat{x}(k+1|k-1)-x_{k+1}(u^*+du)|| \leq ||\hat{x}(k+1|k-1)-x_{k+1}(u^*)||$$
$$+||x_{k+1}(u^*+du)-x_{k+1}(u^*)|| \leq L_{f_x}e(k|k-1)+L_{f_u}||du(k+1)|| \leq L_{f_x}e(k|k-1)+L_{f_u}\gamma^{\mu}$$

In Lemma 1 of [MAC02], it has been proven that for a given sequence of inputs, the error between the nominal prediction of the state and the real state of the system is bounded by

$$e(k+j|k-1) \leq L_m(j+1)\gamma \tag{5.55}$$

Given (5.55), by induction we get:

$$||\hat{x}(k+j|k-1)-x_{k+j}(u^*+du)|| \leq L_{f_x}^{j}\gamma + L_m(j)L_{f_u}\gamma^{\mu}$$

Moreover, having this difference bounded it is easy to show that if $\hat{x}(k+j|k-1)\in X_j$ then the real state of the system will satisfy $x_{k+j}\in X$.

Lemma 7 *It holds that the norm of the difference between $x^n(k+j+m|k+m)$ and $\hat{x}(k+j+m|k-1)$ is bounded. Note, that $x^n(\cdot)$, as well as $\hat{x}(\cdot)$ are the "neighboring" state of the system and the predicted one, respectively, and that both are given by the nominal model* (5.1). *In particular,*

$$||x^n(k+j+m|k+m)-\hat{x}(k+j+m|k-1)|| \leq L_{f_x}^{j}e(k+m|k-1)+L_m(j)L_{f_u}\gamma^{\mu} \tag{5.56}$$

For the sake of simplicity, we are going to treat the case $m = 0$, but the results can be easily generalized to the cases for $m \geq 1$.
For $j = 1$:

$$||x^n(k+1|k) - \hat{x}(k+1|k-1)|| \leq L_{f_x}||x^n(k|k) - \hat{x}(k|k-1)|| + L_{f_u}||u^n(k|k) - u^*(k|k-1)||$$
$$\leq L_{f_x}e(k|k-1) + L_{f_u}||du(k)|| \leq L_{f_x}e(k|k-1) + L_{f_u}\gamma^\mu \quad (5.57)$$

By induction we get (5.56). Using (5.55), it can be concluded that

$$||x^n(k+1|k) - \hat{x}(k+1|k-1)|| \leq L_{f_x}^j\gamma + L_m(j)L_{f_u}\gamma^\mu \quad (5.58)$$

Chapter 6

Event-Based Strategies of Model Predictive Controllers for Continuous-Time Systems

This Chapter proposes novel event-triggered strategies for the control of uncertain nonlinear continuous-time systems with additive disturbances under robust Nonlinear Model Predictive Controllers (NMPC). The main idea behind the event-driven framework remains the same as in the previous Chapters, i.e. the main idea is to trigger the solution of the optimal control problem of the NMPC, only when it is needed. The updates of the control law depend on the error of the actual and the predicted trajectory of the system. Sufficient conditions for triggering are provided and is proven that the closed-loop system evolves to a compact set where it is ultimately bounded, under the proposed framework. The results are illustrated through a simulated example.

6.1 Introduction

In [VKFF09], an event-based NMPC approach for nonlinear continuous-time systems with nominal dynamics, is presented. The approach is used in order to overcome the bounded delays and information losses that often appear in networked control systems. Although the formulation is event-driven, a criterion for triggering was not provided. Moreover, most researchers have focused on the discrete-time frame, the ISS stability of a robust NMPC in continuous-time sampled-data systems was recently presented in [RRFM11].

In this work, the triggering condition of a continuous-time system under a robust NMPC control law is given, while a convergence analysis of an uncertain nonlinear system is also provided.

6.2 Problem Statement For Continuous-Time Systems

In the following a triggering condition for continuous-time nonlinear systems under NMPC control laws is going to be presented. The triggering condition is reached following the idea behind the analysis proposed in the previous Chapters for discrete-time systems, appropriately modified in this case, for continuous-time systems.

Consider a nonlinear continuous time system

$$\dot{x}(t) = f(x(t), u(t)), \qquad x(0) = x_0 \tag{6.1}$$

$$x(t) \in \mathscr{X} \subset \mathbb{R}^n, \qquad u(t) \in \mathscr{U} \subset \mathbb{R}^m \tag{6.2}$$

We also assume that $f(x,u)$ is locally Lipschitz in x, with Lipschitz constant L_f and that $f(0,0) = 0$. The whole state $x(t)$, is assumed to be available. Sets $\mathscr{X}$, $\mathscr{U}$ are assumed to be compact and connected, respectively, and $(0,0) \in \mathscr{X} \times \mathscr{U}$.

In a realistic formulation though, modeling errors, uncertainties and disturbances may exist. Thus, a perturbed version of (6.1) is going to be considered as well. The perturbed system can be described as

$$\dot{x}(t) = f(x(t), u(t)) + w(t), \qquad x(0) = x_0 \tag{6.3}$$

where the additive term $w(t) \in \mathscr{W} \subset \mathbb{R}^n$ is the disturbance at time $t \in \mathbb{R}_{\geq 0}$ and $\mathscr{W}$ is a compact set containing the origin as an interior point. Furthermore, note that $w(t)$ is bounded because it is defined in a compact set $w(t) \in \mathscr{W}$. Thus, there exists $\gamma^{\sup} \in \mathbb{R}_{\geq 0}$ such that $\sup_{t \geq 0} ||w(t)|| \leq \gamma^{\sup}$.

Given the system (6.1), the predicted state is denoted as $\hat{x}(t_i+\tau,u(\cdot),x(t_i))$. This notation will be equipped hereafter and it accounts for the predicted state at time $t_i+\tau$ with $\tau \geq 0$, based on the measurement of the real state at time t_i while using a control trajectory $u(\cdot;x(t_i))$ for time period t_i until $t_i+\tau$. It holds that $\hat{x}(t_i,u(\cdot),x(t_i)) \equiv x(t_i)$, i.e. the measured state at time t_i.

6.2.1 NMPC for Continuous-Time Systems

The main idea behind NMPC is to solve on-line a finite-horizon, open-loop optimal control problem, based on the measurement provided by the plant. At the recalculation time t_i, the actual state of the plant $x(t_i)$, is measured and the following Optimal Control Problem (OCP), is solved:

$$\min_{\tilde{u}(\cdot)} J(\tilde{u}(\cdot),x(t_i)) =$$

$$\min_{\tilde{u}(\cdot)} \int_{t_i}^{t_i+T_p} F(\tilde{x}(\tau),\tilde{u}(\tau))\, \mathrm{d}\tau + E(\tilde{x}(t_i+T_p)), \tag{6.4a}$$

subject to

$$\dot{\tilde{x}} = f(\tilde{x}(t),\tilde{u}(t)), \qquad \tilde{x}(t_i) = x(t_i), \tag{6.4b}$$

$$\tilde{u}(t) \in \mathscr{U}, \tag{6.4c}$$

$$\tilde{x}(t) \in \mathscr{X}_{t-t_i} \qquad t \in [t_i, t_i+T_p], \tag{6.4d}$$

$$\tilde{x}(t_i+T_p) \in \mathbb{E}_f, \tag{6.4e}$$

where $\tilde{\cdot}$ denotes the controller internal variables, corresponding to the nominal dynamics of the system. F and E are the running and terminal costs functions, respectively, with $E \in C^1$, $E(0) = 0$. The terminal constraint set $\mathbb{E}_f \subset \mathbb{R}^n$ is assumed to be closed and connected.

Assume, also, that the cost function F is quadratic of the form $F(x,u) = x^T Qx + u^T Ru$, with Q and R being positive definite matrices. Moreover we have $F(0,0) = 0$ and $F(x,u) \geq \lambda_{\min}(Q)||x||^2$, with $\lambda_{\min}(Q)$ being the smallest eigenvalue of Q. Since $\mathscr{X}$ and $\mathscr{U}$ are bounded, the stage cost is Lipschitz continuous in $\mathscr{X} \times \mathscr{U}$, with a Lipschitz constant L_F.

The state constraint set $\mathscr{X}$ of the standard MPC formulation, is being replaced by a restricted constraint set $\mathscr{X}_{t-t_i}$ in (6.4d). This state constraints' tightening for the nominal system with additive disturbance is a key ingredient of the robust NMPC controller and guarantees that the evolution of the real system will be admissible for all time.

Notice that the difference between the actual measurement at time $t_i + \tau$ and the predicted state at the same time under some control law $u(t_i + \tau, x(t_i))$, with $0 \leq \tau \leq T_p$, starting at the same initial state $x(t_i)$, can be shown [FIAF04] to be upper bounded by

$$||x(t_i + \tau) - \hat{x}(t_i + \tau, u(\cdot), x(t_i))|| \leq \frac{\gamma^{\sup}}{L_f}(e^{L_f \cdot \tau} - 1) \tag{6.5}$$

Set $\gamma(t) \triangleq \frac{\gamma^{\sup}}{L_f}(e^{L_f \cdot t} - 1) \qquad \forall t \in \mathbb{R}_{\geq 0}$.

The restricted constrained set is then defined as $\mathscr{X}_{t-t_i} = \mathscr{X} \sim \mathbb{B}_{t-t_i}$ where $\mathbb{B}_{t-t_i} = \{x \in \mathbb{R}^n : ||x|| \leq \gamma(t - t_i)\}$, with $t \in [t_i, t_i + T_p]$. The set operator "$\sim$" denotes the Pontryagin difference.

The solution of the OCP at time t_i provides an optimal control trajectory $u^*(t; x(t_i))$, for $t \in [t_i, t_i + T_p]$, where T_p represents the finite prediction horizon. A portion of the optimal control that corresponds to the time interval $[t_i, t_i + \delta_i)$, is then applied to the plant, i.e.,

$$u(t) = u^*(t; x(t_i)), \qquad t \in [t_i, t_i + \delta_i) \tag{6.6}$$

where δ_i represents the recalculation period that may not be equidistant for every t_i, $\delta_i = \delta(t_i) = t_{i+1} - t_i$. A time instant $t_i \in \mathbb{R}_{\geq 0}$ must be a proper recalculation time, in the sense defined in [VKFF09], i.e. a time instant $t_i \in \mathbb{R}_{\geq 0}$ is a proper recalculation time if there exists $\beta \in \mathbb{R}_{\geq 0}$, such that, $0 < \beta \leq t_{i+1} - t_i = \delta_i < T_p, \qquad \forall t_i, t_{i+1} \in \mathbb{R}_{\geq 0}$.

In order to assert that the NMPC strategy results in a robustly stabilizing controller, some stability conditions are stated for the nominal system. Thus, system (6.1) is supposed to fulfill the following assumptions

Assumption 13 *Let the terminal region $\mathbb{E}_f$ from* (6.4e) *be a subset of an admissible positively invariant set $\mathbb{E}$ of the nominal system, where $\mathbb{E} \subset \mathscr{X}$ is closed, connected and containing the origin.*

Assumption 14 *Assume that there is a local stabilizing controller $h(x(t))$ for the terminal set $\mathbb{E}_f$. The associated Lyapunov function $E(\cdot)$ has the following properties*

$$\frac{\partial E}{\partial x} f(x(\tau), h(x(\tau))) + F(x(\tau), h(x(\tau))) \leq 0 \qquad \forall x \in \mathbb{E}$$

and is Lipschitz in $\mathbb{E}$, with Lipschitz constant L_E.

Assumption 15 *The set $\mathbb{E}$ is given by $\mathbb{E} = \{x \in \mathbb{R}^n : E(x) \leq \alpha_{\mathbb{E}}\}$ such that $\mathbb{E} \subseteq \mathscr{X} = \{x \in \mathscr{X}_{T_p} : h(x) \in \mathscr{U}\}$. The set $\mathbb{E}_f = \{x \in \mathbb{R}^n : E(x) \leq \alpha_{\mathbb{E}_f}\}$ is such that for all $x \in \mathbb{E}$, $f(x, h(x)) \in \mathbb{E}_f$. Assume also that $\alpha_{\mathbb{E}}, \alpha_{\mathbb{E}_f} \in \mathbb{R}_{\geq 0}$ and is such that $\alpha_{\mathbb{E}} \geq \alpha_{\mathbb{E}_f}$.*

Assumption 16 $\exists\, T_p$, *such that* $0 < \beta \leq \delta(t) < T_p$, *for some* $\beta \in \mathbb{R}_{\geq 0}$.

Note that Assumptions 13 through 15 are standard assumptions for a NMPC system, see for example [RRFM11]. Assumption 16 can be verified either experimentally or theoretically for specific systems and it states that every recalculation time is a proper recalculation time.

The event-triggered strategy presented later in this paper, is used in order to enlarge, as much as possible, the inter-calculation period δ_i for the actual system (6.3). The enlargement of the inter-calculation period results in the overall reduction of the control updates which is desirable in numerous occasions, as for example energy consumption reasons. In an event-based framework the inter-calculation period is not equidistant but is "decided" *ex tempore*, based on the error between the actual state measurement of (6.3), and the state trajectory of the nominal system, (6.1). The triggering condition, i.e. how the next calculation time t_{i+1}, is chosen, is presented next.

6.3 Triggering Condition of NMPC for Continuous-Time Systems

In this section, the feasibility and the convergence of the closed loop system (6.3), (6.6) are provided first. Then, the event-triggering rule for sampling is reached.

6.3.1 Feasibility and Convergence

As usual in model predictive control, the proof of stability consists in two separate parts; the feasibility property is guaranteed first and then, based on the previous result, the convergence property is shown. Due to the fact that the system in consideration is perturbed, we only require “ultimate boundedness” results.

The first part will establish that initial feasibility implies feasibility afterwards. Consider two successive triggering events t_i and t_{i+1} and a feasible control trajectory $\bar{u}(\cdot, x(t_{i+1}))$, based on the solution of the OCP in t_i, $u^*(\cdot, x(t_i))$

$$\bar{u}(\tau, x(t_{i+1})) = \begin{cases} u^*(\tau, x(t_i)) & \forall \tau \in [t_{i+1}, t_i + T_p] \\ h(\hat{x}(t_i + T_p, u^*(\cdot), x(t_i))) & \forall \tau \in [t_i + T_p, t_{i+1} + T_p] \end{cases} \tag{6.7}$$

From feasibility of $u^*(\cdot, x(t_i))$ it follows that there is $\bar{u}(\tau, x(t_{i+1})) \in \mathscr{U}$, and similar to the procedure in [MAC02] $\hat{x}(t_{i+1} + T_p, \bar{u}(\tau, x(t_{i+1})), x(t_{i+1})) \in \mathbb{E}_f$ provided that the uncertainties are bounded by $\gamma^{\sup} \leq \frac{(\alpha_{\mathbb{E}} - \alpha_{\mathbb{E}_f}) \cdot L_f}{L_E \cdot (e^{L_f \cdot T_p} - 1)}$. Finally, the state constraints must be fulfilled. According to [MAC02] and [RRFM11] and considering that $||x(t) - \hat{x}(t, u(\cdot), x(t_i))|| \leq \gamma(t)$, for all $t \geq t_i$, it is verified that since the $\hat{x}(t, u^*(\cdot), x(t_i)) \in \mathscr{X}_{t-t_i}$, then $\hat{x}(t, \bar{u}(\cdot), x(t_{i+1})) \in \mathscr{X}_{t-t_{i+1}}$.

The second part involves proving convergence of the state and is being introduced now. In order to prove stability of the closed-loop system, it must be shown that a proper value function is decreasing starting from a sampling instant t_i. Consider the optimal cost $J^*(u^*(\cdot; x(t_i)), x(t_i)) \triangleq J^*(t_i)$ from (6.4a) as a Lyapunov function candidate. Then, consider the cost of the feasible trajectory, indicated by $\bar{J}(\bar{u}(\cdot; x(t_{i+1})), x(t_{i+1})) \triangleq \bar{J}(t_{i+1})$, where t_i, t_{i+1} are two successive triggering instants. Also, $\bar{x}(\tau, \bar{u}(\tau; x(t_{i+1})), x(t_{i+1}))$ is introduced, and it accounts for the predicted state at time τ, with $\tau \geq t_{i+1}$, based on the measurement of the real state at time t_{i+1}, while using the control trajectory $\bar{u}(\tau;x(t_{i+1}))$ from (6.7). Set

$$\begin{aligned}
x_1(\tau) &= \bar{x}(\tau,\bar{u}(\tau;x(t_{i+1})),x(t_{i+1})) \\
u_1(\tau) &= \bar{u}(\tau;x(t_{i+1})) \\
x_2(\tau) &= \hat{x}(\tau,u^*(\tau;x(t_i)),x(t_i)) \\
u_2(\tau) &= u^*(\tau;x(t_i))
\end{aligned}$$

The difference between the optimal cost and the feasible cost is

$$\begin{aligned}
&\bar{J}(t_{i+1}) - J^*(t_i) = \\
&\int_{t_{i+1}}^{t_{i+1}+T_p} F(x_1(\tau),u_1(\tau))\,\mathrm{d}\tau + E(x_1(t_{i+1}+T_p)) - \int_{t_i}^{t_i+T_p} F(x_2(\tau),u_2(\tau))\,\mathrm{d}\tau - E(x_2(t_i+T_p)) \\
&= \int_{t_{i+1}}^{t_i+T_p} F(x_1(\tau),u_1(\tau))\,\mathrm{d}\tau + E(x_1(t_{i+1}+T_p)) + \int_{t_i+T_p}^{t_{i+1}+T_p} F(x_1(\tau),u_1(\tau))\,\mathrm{d}\tau \\
&- \int_{t_i}^{t_{i+1}} F(x_2(\tau),u_2(\tau))\,\mathrm{d}\tau - \int_{t_{i+1}}^{t_i+T_p} F(x_2(\tau),u_2(\tau))\,\mathrm{d}\tau - E(x_2(t_i+T_p))
\end{aligned} \tag{6.8}$$

From (6.7), we have that $u_1(t) \equiv u_2(t) \equiv \bar{u}(t)$ for $t \in [t_{i+1}, t_i+T_p]$, so imposing this control law to the system (6.1), it yields

$$\begin{aligned}
&||x_1(t) - x_2(t)|| = \\
&||x(t_{i+1}) + \int_{t_{i+1}}^{t} f(\bar{x}(\tau),\bar{u}(\tau))\,\mathrm{d}\tau - x(t_i) - \int_{t_i}^{t_{i+1}} f(\hat{x}(\tau),u^*(\tau))\,\mathrm{d}\tau - \int_{t_{i+1}}^{t} f(\hat{x}(\tau),\bar{u}(\tau))\,\mathrm{d}\tau||
\end{aligned} \tag{6.9}$$

Note that for the nominal system (6.1), it holds that

$$\hat{x}(t_{i+1},u^*(\cdot),x(t_i)) = x(t_i) + \int_{t_i}^{t_{i+1}} f(\hat{x}(\tau),u^*(\tau))\,\mathrm{d}\tau$$

Also, we have

$$||\int_{t_{i+1}}^{t} f(\bar{x}(\tau),\bar{u}(\tau))\,\mathrm{d}\tau - \int_{t_{i+1}}^{t} f(\hat{x}(\tau),\bar{u}(\tau))\,\mathrm{d}\tau|| \le \gamma(t-t_{i+1}) \qquad \forall t \ge t_{i+1} \tag{6.10}$$

Define the error $e(t,x(t_i))$ as the difference between the actual state measurement at time $t \geq t_i$ and the predicted state measurement at the same time, i.e.,

$$e(t, x(t_i)) = ||x(t) - \hat{x}(t, u^*(\cdot), x(t_i))|| \tag{6.11}$$

Obviously we have $e(t_i, x(t_i)) = 0$. Then, (6.9) with the help of (6.10), (6.11) and $t \in [t_{i+1}, t_i + T_p]$ is

$$||x_1(t) - x_2(t)|| \leq e(t_{i+1}, x(t_i)) + \gamma(t - t_{i+1}) \tag{6.12}$$

The difference between the running costs, with the help of (6.12), is

$$\begin{aligned}
&\int_{t_{i+1}}^{t_i+T_p} F(x_1(\tau), u_1(\tau))\, \mathrm{d}\tau - \int_{t_{i+1}}^{t_i+T_p} F(x_2(\tau), u_2(\tau))\, \mathrm{d}\tau \\
&\leq \int_{t_{i+1}}^{t_i+T_p} ||F(x_1(\tau), \bar{u}(\cdot)) - F(x_2(\tau), \bar{u}(\cdot))||\, \mathrm{d}\tau \\
&\leq L_F \int_{t_{i+1}}^{t_i+T_p} ||x_1(\tau) - x_2(\tau)||\, \mathrm{d}\tau \\
&\leq L_F \cdot e(t_{i+1}, x(t_i)) \cdot (t_i + T_p - t_{i+1}) + L_F \cdot \mu(t_{i+1})
\end{aligned} \tag{6.13}$$

Where $\mu(t) \triangleq \frac{\gamma^{\sup}}{L_f}[\frac{1}{L_f}(e^{L_f\cdot(t_i+T_p)} - e^{L_f\cdot(t)}) - (t_i + T_p - t)]$. Integrating the inequality from Assumption 14 for $t \in [t_i + T_p, t_{i+1} + T_p]$, the following result can be obtained

$$\begin{aligned}
&\int_{t_i+T_p}^{t_{i+1}+T_p} F(x_1(\tau), u_1(\tau))\, \mathrm{d}\tau + E(x_1(t_{i+1} + T_p)) \\
&- E(x_2(t_i + T_p)) - E(x_1(t_i + T_p)) + E(x_1(t_i + T_p)) \\
&\leq E(x_1(t_i + T_p)) - E(x_2(t_i + T_p)) \\
&\leq L_E||x_1(t_i + T_p) - x_2(t_i + T_p)|| \\
&\leq L_E \cdot e(t_{i+1}, x(t_i)) + L_E \cdot \gamma(t_i + T_p - t_{i+1})
\end{aligned} \tag{6.14}$$

Relying on the fact that function F is positive definite, it can be concluded that

$$\int_{t_i}^{t_{i+1}} F(x_2(\tau), u_2(\tau))\, \mathrm{d}\tau \geq \lambda_{\min}(Q) \cdot L_Q(t_{i+1}) \geq 0 \tag{6.15}$$

with $L_Q(t) \triangleq \lambda_{\min}(Q) \cdot \int_{t_i}^{t} ||\hat{x}(\tau, u^*(\tau; x(t_i)), x(t_i))||^2 \, \mathrm{d}\tau$ for $t \geq t_i$. Substituting (6.13), (6.14), (6.15) to (6.8), the following is derived

$$\begin{aligned} &\bar{J}(t_{i+1}) - J^*(t_i) \\ &\leq (L_F(t_i + T_p - t_{i+1}) + L_E) \cdot e(t_{i+1}, x(t_i)) + L_F \cdot \mu(t_{i+1}) + L_E \cdot (t_i + T_p - t_{i+1}) - L_Q(t_{i+1}) \end{aligned} \tag{6.16}$$

The optimality of the solution results to

$$J^*(t_{i+1}) - J^*(t_i) \leq \bar{J}(t_{i+1}) - J^*(t_i) \tag{6.17}$$

Thus, it holds that the optimal cost $J^*(\cdot)$ is a Lyapunov function that has been proven to be decreasing, thus the closed-loop system converges to a compact set $\mathbb{E}_f$, where it is ultimately bounded.

6.3.2 Triggering Condition

In the following, the triggering condition will be provided. Consider that at time t_i an event is triggered. In order to achieve the desired convergence property, the Lyapunov function $J^*(\cdot)$ must be decreasing. For some triggering instant t_i and some time t, with $t \in [t_i, t_i + T_p]$, we have

$$\begin{aligned} &J^*(t) - J^*(t_i) \\ &\leq (L_F(t_i + T_p - t) + L_E) \cdot e(t, x(t_i)) + L_F \cdot \mu(t) + L_E \cdot (t_i + T_p - t) - L_Q(t) \end{aligned} \tag{6.18}$$

where $e(t, x(t_i))$ as in (6.11), and $x(t)$ is the state of the actual system, continuously measured. Suppose that the error is restricted to satisfy

$$(L_F(t_i + T_p - t) + L_E) \cdot e(t, x(t_i)) + L_F \cdot \mu(t) + L_E \cdot (t_i + T_p - t) \leq \sigma L_Q(t) \tag{6.19}$$

with $0 < \sigma < 1$. Plugging in (6.19) to (6.18) we get

$$J^*(t) - J^*(t_i) \leq (\sigma - 1) \cdot L_Q(t) \tag{6.20}$$

This suggests that provided $\sigma < 1$, the convergence property is still guaranteed.

This triggering rule states that when (6.19) is violated, the next event is triggered at time t_{i+1}, i.e., the OCP is solved again using the current measure of the state $x(t_{i+1})$ as the initial state. During the inter-event interval, the control trajectory $u(t) = u^*(t, x(t_i))$ with $t \in [t_i, t_{i+1}]$, is applied to the plant.

We are now ready to introduce the main stability result for the event-based NMPC controller.

Theorem 8 *Consider the system* (6.3), *subject to* (6.2) *under an NMPC strategy and assume that Assumption 1 holds. Then the NMPC control law provided by* (6.4a)-(6.4e) *is applied to the plant in an open-loop manner, until the rule* (6.19) *is violated and a new event is triggered. The overall event-based NMPC control scheme drives the closed loop system towards a compact set* $\mathbb{E}_f$ *where it is ultimately bounded.*

6.4 Conclusions

In Chapter, event-triggered strategies for control of both continuous and discrete-time systems under NMPC controllers, were proposed and analyzed. In both cases, uncertain nonlinear systems with additive disturbances, were considered. The main idea behind the event-triggered framework is to trigger the solution of the optimal control problem of the NMPC, only when it is needed. During the inter-event period the control law provided from the previous triggering event, is utilized in an open-loop fashion. This event-based approach is favorable in numerous occasions, because it is possible to reduce the number of times the control law should be computed, thus it can result to the alleviation of the energy consumption, or in the case of networks, it can result to amelioration of the network traffic.

Chapter 7

Self-Triggered Model Predictive Control for Nonholonomic Systems

This Chapter proposes a Model Predictive Control (MPC) framework combined with a self-triggering mechanism for constrained uncertain systems. Under the proposed scheme, the control input as well as the next control update time are provided at each triggering instant. Between two consecutive triggering instants, the control trajectory given by the MPC is applied to the plant in an open-loop fashion. This results to less frequent computations while preserving stability and convergence of the closed-loop system. A scenario for the stabilization of a nonholonomic robot, subject to constraints and disturbances, is considered with the aim of reaching a specific triggering mechanism. The robot under the proposed control framework is driven to a compact set where it is ultimately bounded. The efficiency of the proposed approach is illustrated through a simulated example.

7.1 Introduction

In this Chapter, a self-triggered MPC strategy is presented. We treat the case of constrained nonholonomic systems with additive disturbances under a NMPC law. The contribution relies in finding a framework that will provide control trajectories that lead to stable closed-loop responses and a mechanism that decides when the control updates should occur. In Chapter 6, a similar analysis was proposed for an event-based MPC framework. In the event-based set-up there is the need for continuously taking state measurements, in contrast to the proposed self-triggered set-up where this need is relaxed. Note, that in [YO09] a NMPC was applied to a nonholonomic vehicle under a discrete-time framework. However, the control horizon was decided ad-hoc and no triggering condition was given.

7.2 Problem Formulation

In this section the specifics for the stabilization scenario are presented. First, the mathematical model of the nonholonomic system is given along with the constraints that must be fulfilled. Next, the design and analysis of the proposed controller is provided along with some assumptions that are necessary in order to achieve stability of the closed-loop system.

7.2.1 Mathematical Modeling

Consider that the motion of the robot is governed by unicycle kinematics with respect to a global cartesian coordinate frame G. The kinematic model is given by

$$\dot{x} = f(x,u) \Rightarrow \begin{bmatrix} \dot{\chi} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \upsilon \\ \omega \end{bmatrix} \tag{7.1}$$

where $x = [\chi, y, \theta]^\top$ is the state vector comprised by the position of the robot (χ, y) and the orientation θ with respect to G. The vector $u = [\upsilon, \omega]^\top$ denotes the control inputs, and υ, ω are the linear and angular velocity of the robot, respectively, expressed in the body-fixed frame B.

The robot is equipped with an onboard camera with limited angle-of-view and laser pointers that provide the state vector x of the robot with respect to G. The requirements imposed by the sensors are: (i) The target should always be visible to the camera, i.e., $[-y_T, y_T] \subseteq [f_1, f_2]$, where $2y_T$ is the width of the target the distance $[f_1, f_2]$ is the camera field of view, (ii) the distance of the vehicle with respect to the target should not exceed a maximum range, $R_{\max}$, because the laser pointers have limited range, and (iii) there is a minimum turning radius of the vehicle, $r_{\min}$. These requirements along with a saturation bound in the velocity impose the constraints of the problem. Particularly, the requirements (i) and (ii) are captured by the connected state constraint set X, given by

$$x(t) \in X \subset \mathbb{R}^3 \tag{7.2}$$

which is formed by the following constraints

$$-y + \chi \tan(\theta - \frac{\alpha}{2}) - y_T \geq 0 \tag{7.3a}$$

$$y - \chi \tan(\theta + \frac{\alpha}{2}) - y_T \geq 0 \tag{7.3b}$$

$$R_{\max}^2 - \chi^2 - y^2 \geq 0 \tag{7.3c}$$

Figure 7-1 depicts the state constraints of the system. Note that the whole state $x(t)$ is assumed to be available, for all $t \in \mathbb{R}_{\geq 0}$. The control constraint set U is assumed to be compact and it is given by:

$$u(t) \triangleq [\upsilon(t), \omega(t)]^\top \in U \subset \mathbb{R}^2 \tag{7.4}$$

The constraints of the input are of the form $|\upsilon| \leq \bar{\upsilon}$ and $|\omega| \leq \bar{\omega}$. Therefor we get $||u|| \leq \bar{u}$, where $\bar{u} = \sqrt{\upsilon^2 + \omega^2}$. The minimum turning radius of the vehicle imposes another restriction: $|\frac{\upsilon}{\omega}| \geq r_{\min}$. We have $\bar{u}, \bar{\upsilon}, \bar{\omega} \in \mathbb{R}_{\geq 0}$. The nominal system (7.1) is Lipschitz continuous with Lipschitz constant $0 < L_f < \infty$. More specifically,

Lemma 8 *The nominal model $f(x,u)$, given the constraints* (7.3a)-(7.3c) *and* (7.4), *is locally Lipschitz in x for all $x \in X$, with a Lipschitz constant $L_f \triangleq \sqrt{2}\bar{\upsilon}$.*

Proof The Euclidean norm is used for the sake of simplicity. We have

$$||f(x_1,u)-f(x_2,u)||^2 = || \begin{pmatrix} \upsilon\cos\theta_1-\upsilon\cos\theta_2 \\ \upsilon\sin\theta_1-\upsilon\sin\theta_2 \\ \omega-\omega \end{pmatrix} ||^2$$

$$= |\upsilon|^2|\cos\theta_1-\cos\theta_2|^2+|\upsilon|^2|\sin\theta_1-\sin\theta_2|^2$$

$$\leq 2|\upsilon|^2|\theta_1-\theta_2|^2$$

where the mean value theorem is used. Thus, it can be concluded that $||f(x_1,u)-f(x_2,u)|| \leq \sqrt{2}\bar{\upsilon}||x_1-x_2||$ for all $x_1,x_2 \in X$.

We assume that the robot moves under the influence of an irrotational current w with respect to the global frame. The current has components with respect to the axes χ and y denoted by w_χ and w_y, respectively. The current is considered to be of constant or slowly-varying velocity w_c with direction β with respect to the global frame. We have

$$w_\chi(t) \triangleq w_c\cos\beta(t) \qquad w_y(t) \triangleq w_c\sin\beta(t) \tag{7.5}$$

Therefor we consider a perturbed system of the form:

$$\dot{x} = f(x,u)+w \tag{7.6}$$

with $w(t) = [w_\chi(t), w_y(t), 0]^\top \in W \subset \mathbb{R}^3$ and W to be a compact set. Since the uncertainty is assumed to be bounded we set $||w|| \leq \bar{w}$, with $\bar{w} = \sqrt{2}w_c$.

7.2.2 Control Design and Objective

The goal is to control the actual system (7.6) subject to $x(t) \in X$ and $u(t) \in U$, to a desired compact set that includes the desired state $x_d \triangleq [\chi_d, y_d, \theta_d]^T \in X$. A predictive controller is employed in order to achieve this task. With the NMPC law the state of the system is proven to converge to the desired set. Inside this set, a terminal controller is used to drive the system to the desired point. This terminal controller is a state feedback controller using

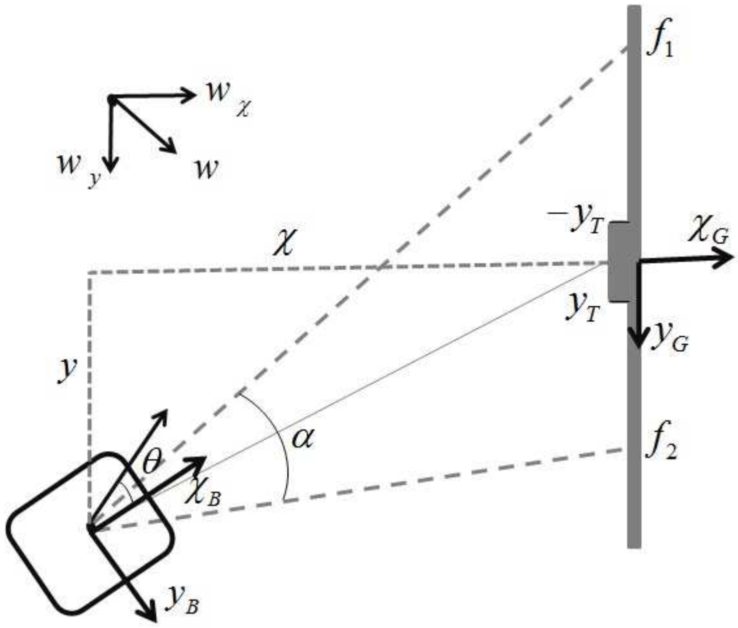

Figure 7-1: Modeling of the state constraints (7.3a)-(7.3c) imposed by the sensor system and the external disturbance (7.5).

dipolar vector fields [PTK11]. Note, that an analytic solution for that controller can be reached, thus, it is easily computable. The design of an ISS stable controller for system (7.6) is presented next.

The NMPC consists in solving a finite-horizon, open-loop optimal control problem, based on the actual state of the plant $x(t_i)$, at time t_i. The solution is a control trajectory $\tilde{u}(t)$, for $t \in [t_i, t_i + T_p]$, where T_p is the prediction horizon. The Optimal Control Problem (OCP) of the NMPC is given as

$$\min_{\tilde{u}(\cdot)} J(\tilde{u}(\cdot), x(t_i)) =$$

$$\min_{\tilde{u}(\cdot)} \int_{t_i}^{t_i+T_p} F(\tilde{x}(\tau), \tilde{u}(\tau))\, \mathrm{d}\tau + E(\tilde{x}(t_i + T_p)), \tag{7.7a}$$

subject to

$$\dot{\tilde{x}} = f(\tilde{x}(t), \tilde{u}(t)), \qquad \tilde{x}(t_i) = x(t_i), \tag{7.7b}$$

$$\tilde{u}(t) \in U, \tag{7.7c}$$

$$\tilde{x}(t) \in X_{t-t_i} \qquad t \in [t_i, t_i + T_p], \tag{7.7d}$$

$$\tilde{x}(t_i + T_p) \in \mathbb{E}_f, \tag{7.7e}$$

where $\tilde{\cdot}$ denotes the controller internal variables, corresponding to the nominal dynamics of the system. F and E are the running and terminal costs functions, respectively. The design parameters F and E, as well as the sets X_{t-t_i} and $\mathbb{E}_f$ are defined later in the text.

In order to proceed to the subsequent analysis a few definitions and some preliminary results are presented first.

The predicted state of the nominal system (7.1) at time $t_i + \tau$ with $\tau \geq 0$, is denoted as $\hat{x}(t_i + \tau, u(\cdot), x(t_i))$ and it is based on the measurement of the actual state $x(t_i)$ at time t_i, when a control trajectory $u(\cdot; x(t_i))$ is applied to the system for time period t_i until $t_i + \tau$. It holds that $\hat{x}(t_i, u(\cdot), x(t_i)) \equiv x(t_i)$. Moreover the following result is given:

Lemma 9 *The difference between the actual state $x(t_i + t)$ at time $t_i + t$ and the predicted state at the same time under the same control law $u(t_i + t, x(t_i))$, with $0 \leq t \leq T_p$, starting at the same initial state $x(t_i)$, can be shown to be upper bounded by*

$$||x(t_i + t) - \hat{x}(t_i + t, u(\cdot), x(t_i))|| \leq \gamma(t) \tag{7.8}$$

where $\gamma(t) \triangleq (2\sqrt{2}\bar{\upsilon} + \bar{w})t$ for all $t \in [0, T_p]$.

Proof Set the control trajectory $u(\cdot) \triangleq u(t_i + t, x(t_i))$ and $x(t) \triangleq x(t, u(\cdot), x(t_i))$ to be the state trajectory for system (7.6). Also we denote for the sake of simplicity, $\hat{x}(t) \triangleq \hat{x}(t, u(\cdot), x(t_i))$ for all $t \in \mathbb{R}_{\geq 0}$. Using the Euclidian norm and the triangular inequality for system (7.1) and system (7.6), we get

$$\begin{aligned}
&||x(t_i + t) - \hat{x}(t_i + t)|| = \\
&||x(t_i) + \int_{t_i}^{t_i + t} f(x(\tau), u(\cdot))\, \mathrm{d}\tau + \int_{t_i}^{t_i + t} w(\tau)\, \mathrm{d}\tau - x(t_i) - \int_{t_i}^{t_i + t} f(\hat{x}(\tau), u(\cdot))\, \mathrm{d}\tau|| \leq \\
&|| \int_{t_i}^{t_i + t} f((x(\tau), u(\cdot)) - f(\hat{x}(\tau), u(\cdot)))\, \mathrm{d}\tau|| + || \int_{t_i}^{t_i + t} w(\tau)\, \mathrm{d}\tau|| \leq \\
&\int_{t_i}^{t_i + t} ||[\upsilon(\cos\theta(\tau) - \cos\hat{\theta}(\tau)), \upsilon(\sin\theta(\tau) - \sin\hat{\theta}(\tau)), 0]^\top||\, \mathrm{d}\tau + \int_{t_i}^{t_i + t} ||w(\tau)||\, \mathrm{d}\tau \\
&\leq (2\sqrt{2}\bar{\upsilon} + \bar{w})t
\end{aligned}$$

To address to the divergence between the actual state trajectory of system (7.6) and the predicted state trajectory of the nominal system as given in Lemma 9, we replace the state constraint set X with the restricted constraint set X_{t-t_i} into (7.7d), with $X_{t-t_i} \subseteq X$. We resort to this constraint' tightening technique presented in [MAC02] and [PRMP09] since the control trajectory that results from (7.7a)-(7.7e) when it is applied to the system (7.6), results to a state trajectory that does not violate the state constraint set X. In particular, the restricted constraint set is defined as $\mathscr{X}_{t-t_i} = \mathscr{X} \sim \mathbb{B}_{t-t_i}$ where $\mathbb{B}_{t-t_i} = \{x \in \mathbb{R}^n : ||x|| \leq \gamma(t-t_i)\}$, with $t \in [t_i, t_i + T_p]$. The set operator "$\sim$" denotes the Pontryagin difference, i.e., given two sets $A, B \in \mathbb{R}^n$ the Pontryagin difference set C is defined as $C = A \sim B \triangleq \{x \in \mathbb{R}^n : x + \xi \in A, \forall \xi \in B\}$.

Assume now that the terminal cost $E(x)$ as well as the cost function $F(x,u)$, are quadratic of the form $E(x) = x^\top Px$ and $F(x,u) = x^\top Qx + u^\top Ru$, respectively, with P, Q and R being positive definite matrices. More specifically we set $P = diag\{p_1, p_2, p_3\}$, $Q = diag\{q_1, q_2, q_3\}$ and $R = diag\{r_1, r_2\}$. Moreover it can be shown that $F(0,0) = 0$ and that $F(x,u) \geq \min\{q_1, q_2, q_3, r_1, r_2\}||[x,u]^\top||^2 \geq \min\{q_1, q_2, q_3, r_1, r_2\}||x||^2$. Since X and U are bounded, it can be concluded that:

Lemma 10 *The stage cost $F(x,u)$ is Lipschitz continuous in $X \times U$, with a Lipschitz constant $L_F \triangleq 2(R_{\max}^2 + \pi^2)^{1/2}\sigma_{\max}(Q)$, where $\sigma_{\max}(Q)$ is the largest singular value of matrix Q.*

Proof We have

$$||F(x_1,u) - F(x_2,u)|| = ||x_1^\top Qx_1 - x_2^\top Qx_2|| = ||x_1^\top Qx_1 - x_1^\top Qx_2 + x_1^\top Qx_2 - x_2^\top Qx_2||$$
$$= ||x_1^\top Q(x_1 - x_2) + (x_1 - x_2)^\top Qx_2|| \leq (||x_1|| + ||x_2||)\sigma_{\max}(Q)||x_1 - x_2||$$

Notice though that $\forall x \in X$ we have $||x||^2 \leq |\chi|^2 + |y|^2 + |\theta|^2 \leq R_{\max}^2 + \pi^2$, which concludes the proof.

In order to assert that the NMPC strategy results in a robust stabilizing controller, some stability conditions are stated in the following:

Assumption 17 *Assume that a set $\mathbb{E} \subset X$ is an admissible positively invariant set for the nominal system* (7.1)*, and that $\mathbb{E}$ is such that $\mathbb{E} \triangleq \{x \in X : ||x|| \leq \varepsilon_0\}$, with ε_0 being a positive parameter.*

Assumption 18 *Assume that for the terminal set $\mathbb{E}_f$, there exists a local stabilizing controller $u_T(x(t)) \in U$, $\forall x \in \mathbb{E}$. The associated Lyapunov function $E(\cdot)$ has the following properties*

$$\frac{\partial E}{\partial x} f(x(\tau), u_T(x(\tau))) + F(x(\tau), u_T(x(\tau))) \leq 0 \qquad \forall x \in \mathbb{E}$$

and is Lipschitz in $\mathbb{E}$, with Lipschitz constant $L_E = 2\varepsilon_0 \sigma_{\max}\{P\}$ for all $x \in \mathbb{E}$. The proof for finding the Lipschitz constant L_E is the same as the proof of Lemma 3.

Assumption 19 *For the set $\mathbb{E}$ we have $E(x) = x^\top P x \leq \alpha_{\mathbb{E}}$ where $\alpha_{\mathbb{E}} = \max\{p_1, p_2, p_3\}\varepsilon_0{}^2 > 0$ and we assume that $\mathbb{E} = \{x \in X_{T_P} : u_T(x) \in U\}$. Take $\alpha_{\mathbb{E}_f} \in (0, \alpha_{\mathbb{E}})$ and assume that $\mathbb{E}_f = \{x \in \mathbb{R}^3 : E(x) \leq \alpha_{\mathbb{E}_f}\}$ is such that $\forall x \in \mathbb{E}$, $f(x, u_T) \in \mathbb{E}_f$.*

A state feedback controller using dipolar vector fields is assumed to be the terminal controller $u_T(\cdot)$. Notice that we assume that this controller fulfills the Assumptions previously stated for all $x \in \mathbb{E}$. In order to achieve that, the positive parameter ε_0 is determined off-line. Moreover we note that $x_d \in \mathbb{E}$. This dipolar-based controller guarantees the convergence of the system trajectories to the desired state [PTK11]. Define the 2-dimensional vector field $F(\cdot) = F_\chi \frac{\vartheta}{\vartheta \chi} + F_y \frac{\vartheta}{\vartheta y}$ where

$$F_\chi = 2(\chi - \chi_d)^2 - (y - y_d)^2 \tag{7.9a}$$

$$F_y = 3(\chi - \chi_d)(y - y_d) \tag{7.9b}$$

The vector field is nonsingular everywhere in $\mathbb{R}^2$ except for the point (χ_d, y_d) and has integral curves that all converge to the desired point with direction $\phi = \text{atan2}(F_y, F_\chi) \to 0$. The task is to design a feedback control law so that the unicycle aligns to the dipolar vector field and follows the integral curves until converging to $(0,0)$. This is achieved via a state feedback control law $u_T(x) \triangleq [\upsilon_T, \omega_T]^T : \mathbb{R}^3 \to \mathbb{R}^2$, given by:

$$\upsilon_T = k_1 \tanh((\chi - \chi_d)^2 + (y - y_d)^2) \tag{7.10a}$$

$$\omega_T = -k_2(\theta - \phi) + \dot{\phi} \tag{7.10b}$$

where $k_1 \in (0, \bar{\upsilon})$, $k_2 > 0$ and $\phi = \text{atan2}(F_y, F_\chi)$. A reason that the control using dipolar vector-fields is used only inside $\mathbb{E}$ as a terminal controller u_T, is because tracking the vector field (7.9a)-(7.9b) outside $\mathbb{E}$ could force the robot to loose visibility of the target, i.e., to violate the constraints.

7.2.3 Problem Statement

The solution of the OCP (7.7a)-(7.7e) at time t_i provides an optimal control trajectory denoted as $u^*(t; x(t_i))$, for $t \in [t_i, t_i + T_p]$. A portion this control trajectory, is then applied to the plant, i.e.,

$$u(t) = u^*(t; x(t_i)), \qquad t \in [t_i, t_{i+1}) \tag{7.11}$$

During the time interval $[t_i, t_{i+1})$ the control law is applied to the plant in an open-loop fashion. A question that naturally arises is how large this time interval can be? However, before we address this question we are going to make some necessary assumptions in order to make the whole problem feasible. A time instant $t_i \in \mathbb{R}_{\geq 0}$ must be a proper recalculation time, in the sense defined in [VKFF09], i.e. a time instant $t_i \in \mathbb{R}_{\geq 0}$ is a proper recalculation time if there exists $\beta \in \mathbb{R}_{\geq 0}$, such that, $0 < \beta \leq t_{i+1} - t_i \triangleq \delta_i < T_p$, $\forall t_i, t_{i+1} \in \mathbb{R}_{\geq 0}$.

Assumption 20 *$\exists\, T_p$, such that $0 < \beta \leq \delta_i < T_p$, for some $\beta \in \mathbb{R}_{\geq 0}$ and $\forall i \in \mathbb{Z}_{\geq 0}$.*

If this assumption cannot be fulfilled the whole problem would be infeasible.

The self-triggered strategy that will be presented later in this paper, provides sufficient conditions for finding the recalculation periods, or in other words sufficient conditions for triggering the computation of the NMPC law. In particular, the presented framework not only provides the control law to be applied to the actual system (7.6), but also provides the time of the next triggering instant, t_{i+1}. This leads us to the statement of the problem treated in this paper:

Problem Statement 3 *Consider the system* (7.6) *that is subject to constraints* (7.2) *and* (7.4)*. The objective is* (i) *to design a feedback control law provided by* (7.7a)-(7.7e) *such that the system* (7.6) *converges to the terminal constraint set and* (ii) *to find a mechanism to decide when the next control update should be.*

7.3 Stability Analysis of NMPC

In this section a stability analysis for the closed-loop system (7.6)-(7.11) is presented. Due to the fact that the system in consideration is perturbed, we only require "ultimate boundedness" results. Accordingly, it can be proven that the closed-loop scheme is Input to State stable (ISS) with respect to the disturbances, [Son08]. Moreover, through the ISS analysis it is possible to reach to a self-triggering mechanism which provides the triggering instants.

The proof of stability of a system under a predictive controller consists in guaranteeing (i) the feasibility property and (ii) the convergence property of the closed-loop system. We begin by showing that initial feasibility implies feasibility afterwards. Consider two successive triggering events t_i and t_{i+1}. A feasible control trajectory $\bar{u}(\cdot, x(t_{i+1}))$, at t_{i+1}, may be the following:

$$\bar{u}(\tau, x(t_{i+1})) = \begin{cases} u^*(\tau, x(t_i)) & \forall \tau \in [t_{i+1}, t_i + T_p] \\ u_T(\hat{x}(t_i + T_p, u^*(\cdot), x(t_i))) & \forall \tau \in [t_i + T_p, t_{i+1} + T_p] \end{cases} \tag{7.12}$$

where $u^*(\cdot, x(t_i))$ is the optimal solution of the OCP at t_i.

From feasibility of $u^*(\cdot, x(t_i))$ and the fact that $u_T(x) \in U$ for all $x \in \mathbb{E}$, it follows that $\bar{u}(\tau, x(t_{i+1})) \in \mathscr{U}$ for all $\tau \in [t_{i+1}, t_{i+1} + T_p]$. We continue by showing that $\hat{x}(t_{i+1} + T_p, \bar{u}(\tau, x(t_{i+1})), x(t_{i+1})) \in \mathbb{E}_f$. We have

$$\begin{aligned} &E(\hat{x}(t_i + T_p, u(\cdot), x(t_{i+1})) \leq E(\hat{x}(t_i + T_p, u(\cdot), x(t_i)) + L_E \gamma(T_p) \\ &\leq \alpha_{\mathbb{E}_f} + L_E (2\sqrt{2}\bar{\upsilon} + \bar{w}) T_p \leq \alpha_{\mathbb{E}} \end{aligned}$$

The uncertainties must then be bounded by $\bar{w} + 2\sqrt{2}\bar{\upsilon} \leq \frac{\alpha_{\mathbb{E}} - \alpha_{\mathbb{E}_f}}{L_E T_p}$. Moreover, the state constraints must be fulfilled: according to [MAC02] and [RRFM11] and considering that

$||x(t) - \hat{x}(t, u(\cdot), x(t_i))|| \leq \gamma(t)$, for all $t \geq t_i$, it can be verified that since $\hat{x}(t, u^*(\cdot), x(t_i)) \in \mathscr{X}_{t-t_i}$, then $\hat{x}(t, \bar{u}(\cdot), x(t_{i+1})) \in \mathscr{X}_{t-t_{i+1}}$.

The convergence of the state is discussed now. A proper value function must be shown to be decreasing in order to prove stability of the closed-loop system. Consider the optimal cost $J^*(u^*(\cdot; x(t_i)), x(t_i)) \triangleq J^*(t_i)$ from (7.7a) as a Lyapunov function candidate. Then, consider the cost of the feasible trajectory, indicated by $\bar{J}(\bar{u}(\cdot; x(t_{i+1})), x(t_{i+1})) \triangleq \bar{J}(t_{i+1})$. Note that t_i, t_{i+1} are two successive triggering instants. Also, we introduce the "feasible" state $\bar{x}(\tau, \bar{u}(\tau; x(t_{i+1})), x(t_{i+1}))$ which accounts for the predicted state at time τ, with $\tau \geq t_{i+1}$, based on the measurement of the real state at time t_{i+1}, while using the feasible control trajectory $\bar{u}(\tau; x(t_{i+1}))$ from (7.12). Set

$$\begin{aligned}
x_1(\tau) &= \bar{x}(\tau, \bar{u}(\tau; x(t_{i+1})), x(t_{i+1})) \\
u_1(\tau) &= \bar{u}(\tau; x(t_{i+1})) \\
x_2(\tau) &= \hat{x}(\tau, u^*(\tau; x(t_i)), x(t_i)) \\
u_2(\tau) &= u^*(\tau; x(t_i))
\end{aligned}$$

The difference between the optimal cost and the feasible cost is:

$$\begin{aligned}
&\bar{J}(t_{i+1}) - J^*(t_i) = \int_{t_{i+1}}^{t_{i+1}+T_p} F(x_1(\tau), u_1(\tau))\,\mathrm{d}\tau + E(x_1(t_{i+1}+T_p)) - \\
&\int_{t_i}^{t_i+T_p} F(x_2(\tau), u_2(\tau))\,\mathrm{d}\tau - E(x_2(t_i+T_p)) = \int_{t_{i+1}}^{t_i+T_p} F(x_1(\tau), u_1(\tau))\,\mathrm{d}\tau + E(x_1(t_{i+1}+T_p)) \\
&+ \int_{t_i+T_p}^{t_{i+1}+T_p} F(x_1(\tau), u_1(\tau))\,\mathrm{d}\tau - \int_{t_i}^{t_{i+1}} F(x_2(\tau), u_2(\tau))\,\mathrm{d}\tau \\
&- \int_{t_{i+1}}^{t_i+T_p} F(x_2(\tau), u_2(\tau))\,\mathrm{d}\tau - E(x_2(t_i+T_p))
\end{aligned} \tag{7.13}$$

From (7.12), we have that $u_1(t) \equiv u_2(t) \equiv \bar{u}(t)$ for $t \in [t_{i+1}, t_i + T_p]$. Imposing this control law to the system (7.1) we get:

$$\begin{aligned}
&||x_1(t)-x_2(t)|| = \\
&||x(t_{i+1})+\int_{t_{i+1}}^{t} f(\bar{x}(\tau),\bar{u}(\tau))\,\mathrm{d}\tau - x(t_i) - \int_{t_i}^{t_{i+1}} f(\hat{x}(\tau),u^*(\tau))\,\mathrm{d}\tau - \int_{t_{i+1}}^{t} f(\hat{x}(\tau),\bar{u}(\tau))\,\mathrm{d}\tau|| \\
&= ||x(t_{i+1})-\hat{x}(t_{i+1},u^*(\cdot),x(t_i))|| \le \gamma(t_{i+1}-t_i) \qquad (7.14)
\end{aligned}$$

The difference between the running costs, with the help of (7.14), becomes:

$$\begin{aligned}
&\int_{t_{i+1}}^{t_i+T_p} F(x_1(\tau),u_1(\tau))\,\mathrm{d}\tau - \int_{t_{i+1}}^{t_i+T_p} F(x_2(\tau),u_2(\tau))\,\mathrm{d}\tau \\
&\le \int_{t_{i+1}}^{t_i+T_p} ||F(x_1(\tau),\bar{u}(\cdot)) - F(x_2(\tau),\bar{u}(\cdot))||\,\mathrm{d}\tau \le L_F \int_{t_{i+1}}^{t_i+T_p} ||x_1(\tau)-x_2(\tau)||\,\mathrm{d}\tau \\
&\le L_F \int_{t_{i+1}}^{t_i+T_p} \gamma(t_{i+1}-t_i)\,\mathrm{d}\tau = L_F(2\sqrt{2}\bar{\upsilon}+\bar{w})(t_{i+1}-t_i)(t_i+T_p-t_{i+1}) \ge 0 \qquad (7.15)
\end{aligned}$$

Integrating the inequality from Assumption 18 for $t \in [t_i+T_p, t_{i+1}+T_p]$ results in the following:

$$\begin{aligned}
&\int_{t_i+T_p}^{t_{i+1}+T_p} F(x_1(\tau),u_1(\tau))\,\mathrm{d}\tau + E(x_1(t_{i+1}+T_p)) - E(x_2(t_i+T_p)) \\
&-E(x_1(t_i+T_p)) + E(x_1(t_i+T_p)) \le E(x_1(t_i+T_p)) - E(x_2(t_i+T_p)) \\
&\le L_E||x_1(t_i+T_p)-x_2(t_i+T_p)|| \le L_E(2\sqrt{2}\bar{\upsilon}+\bar{w})(t_{i+1}-t_i) \ge 0 \qquad (7.16)
\end{aligned}$$

Since function F is positive definite, it can be concluded that

$$\int_{t_i}^{t_{i+1}} F(x_2(\tau),u_2(\tau))\,\mathrm{d}\tau \ge L_Q(t_{i+1}) \ge 0 \qquad (7.17)$$

with $L_Q(t) \triangleq \min\{q_1,q_2,q_3,r_1,r_2\} \cdot \int_{t_i}^{t} ||\hat{x}(\tau,u^*(\tau;x(t_i)),x(t_i))||^2\,\mathrm{d}\tau$ for $t \ge t_i$. Substituting (7.15), (7.16), (7.17) to (7.13), the following is derived

$$\begin{aligned}
&\bar{J}(t_{i+1}) - J^*(t_i) \\
&\le L_F(2\sqrt{2}\bar{\upsilon}+\bar{w})(t_{i+1}-t_i)(t_i+T_p-t_{i+1}) + L_E(2\sqrt{2}\bar{\upsilon}+\bar{w})(t_{i+1}-t_i) - L_Q(t_{i+1}) \quad (7.18)
\end{aligned}$$

The optimality of the solution yields

$$J^*(t_{i+1}) - J^*(t_i) \leq \bar{J}(t_{i+1}) - J^*(t_i) \tag{7.19}$$

The Lyapunov function $J^*(\cdot)$ has been proven to be decreasing, thus the closed-loop system converges to a compact set $\mathbb{E}_f$, where it is ultimately bounded, due to Assumption 3.

7.3.1 Self-triggered Framework

In this section the self-triggering mechanism is going to be presented. Consider that at time t_i an event is triggered. The ISS of the NMPC was proven considering that the time t_{i+1}, i.e, the next triggering instant, was known. Here, the next control update time t_{i+1} is considered to be unknown and should be found. The next control update time t_{i+1} should be such that the closed-loop system does not lose any of its desired properties. Thus, we still need the Lyapunov function $J^*(\cdot)$ to be decreasing, which will preserve the convergence of the closed-loop system.

Given (7.18) and (7.19), then for some triggering instant t_i and some time t with $t \in [t_i, t_i + T_p]$ we get

$$\begin{aligned} &J^*(t) - J^*(t_i) \\ &\leq L_F(2\sqrt{2}\bar{\upsilon} + \bar{w})(t - t_i)(t_i + T_p - t) + L_E(2\sqrt{2}\bar{\upsilon} + \bar{w})(t - t_i) - L_Q(t) \end{aligned} \tag{7.20}$$

The time instant t should be such that

$$L_F(2\sqrt{2}\bar{\upsilon} + \bar{w})(t - t_i)(t_i + T_p - t) + L_E(2\sqrt{2}\bar{\upsilon} + \bar{w})(t - t_i) \leq \sigma L_Q(t) \tag{7.21}$$

with $0 < \sigma < 1$. Plugging in (7.21) to (7.20) we get

$$J^*(t) - J^*(t_i) \leq (\sigma - 1) \cdot L_Q(t) \tag{7.22}$$

This suggests that provided $\sigma < 1$, the convergence property is still guaranteed. Thus, the next control update time should be triggered when (7.21) is violated. This provides the triggering mechanism. Notice that the time t_{i+1} can be found beforehand at time t_i, i.e, this is a self-triggering mechanism. Moreover, it should be pointed out that the term $L_Q(t)$ includes only predictions of the nominal system that is forming a trajectory and that it can be found by forward integration of (7.1) for time $t \in [t_i, t_i + T_p]$.

Next we describe the self-triggering mechanism. At time t_i a control update is triggered and a control trajectory for $[t_i, t_i + T_p]$ is provided. With the help of (7.21) we get

$$(2\sqrt{2}\bar{\upsilon} + \bar{w})[L_F(t_i + T_p - t) + L_E](t - t_i) = \sigma L_Q(t) \tag{7.23}$$

The solution of (7.23) will provide the next update time t_{i+1}. During the time interval $t \in [t_i, t_{i+1})$ the control trajectory $u(t) = u^*(t, x(t_i))$ is applied to the plant in an open-loop fashion. Next, at time t_{i+1} the OCP is solved again using the current measure of the state $x(t_{i+1})$ as the initial state. The controller follows this procedure until the system converges to the terminal constraint set.

We are now ready to state the stability result for this self-triggered MPC framework:

Theorem 9 *Consider the system* (7.6) *that is subject to constraints* (7.2) *and* (7.4) *under the NMPC strategy and assume that Assumptions 17-20 hold. The control update times that are provided by* (7.23) *and the NMPC law provided by* (7.7a)-(7.7e) *which is applied to the system in an open-loop fashion during the inter-sampling periods, drive the closed-loop system towards a compact set* $\mathbb{E}_f$ *where it is ultimately bounded.*

7.4 Simulation Results

In this section, a simulated example of the proposed framework for a nonholonomic robot is presented. The objective is to control the robot through a NMPC law of the form (7.7a)-(7.7e) in order to reach a desired terminal constraint set. The nominal model of the non-holonomic system has the form (7.1). Furthermore we assume that disturbances exist and that they are bounded by $||w|| \leq 0.5$. Thus, the actual model is (7.6). The initial position of the robot is $x_{\text{initial}} = [-43, 11.5, -pi/6]^\top$ and the desired position is $x_d = [0, 0, 0]^\top$.

In order to evaluate the proposed self-triggered approach we are going to present some comparison results. The traditional time-triggered, periodical, scheme is given and the event-triggered MPC framework that was proposed in Chapter 3 given as well. The simulation shows that the actual system (7.6) under all three schemes, i.e., time, event and self-triggered NMPC, converges to the terminal set around the desired state, see Fig. 7-2.

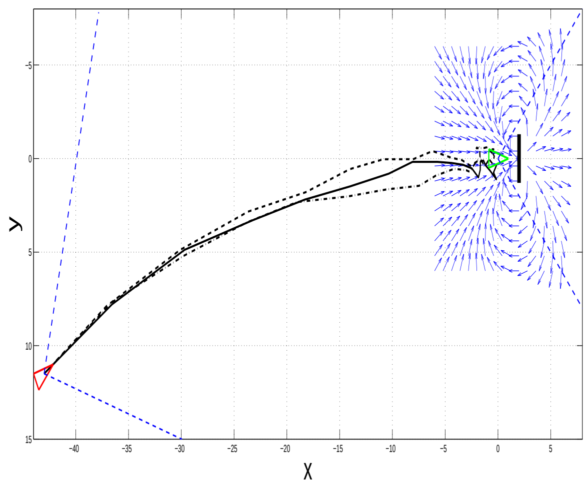

Figure 7-2: State trajectories of the nonholonomic robot under robust MPC. The solid line represents the trajectory of the robot under periodic MPC. The dashed line as well as the dash-dotted line represents the trajectories of the robot under the event-triggered MPC and the self-triggered MPC, respectively. The red triangle is the initial position of the robot, while the green is the desired state.

In Fig. 7-3, the evolution of the system trajectories under all three schemes is depicted. It is apparent that all three schemes have comparable results. Finally, Fig. 7-4, is capturing the triggering instants on both the event-triggered and the self-triggered frameworks. The time-triggered framework is not depicted because it is trivially triggered at each sampling period, i.e., the smallest triggering period $\beta = 0.1$sec.

7.5 Conclusions

We provided a self-triggered formulation for constrained nonholonomic systems under a model predictive controller. The main idea is to trigger the solution of the optimal control problem only when it is needed and not periodically as in the case of classic MPC schemes. This approach results to an improvement on the requirements on the computation resources.

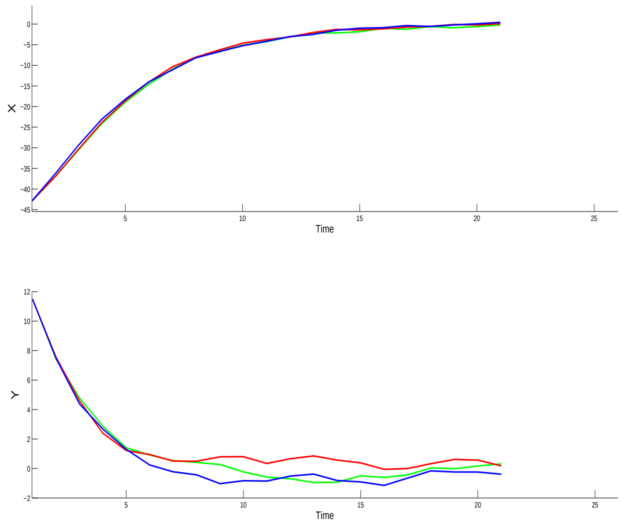

Figure 7-3: The evolution of the system trajectories in time. The green line represents the time-triggered case. The red and blue represent the event-triggered and self-triggered, respectively.

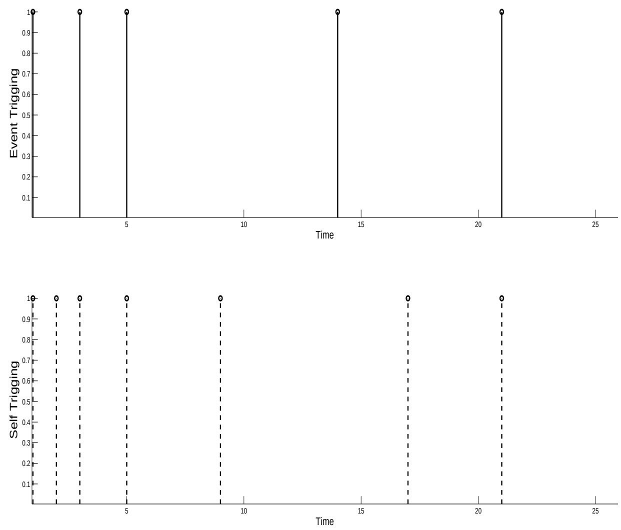

Figure 7-4: The triggering instants. When the vertical axis has the value 1, the NMPC is triggered. For value 0 the control law is implemented on the system in an open-loop fashion. (a) The event-triggered set-up, (b) The self-triggered set-up.

With the self-triggered approach both the control input and the next control update time are evaluated in order to avoid continuous supervision of the actual state of the system. During the inter-sampling times the control trajectory from the NMPC is applied to the system in an open-loop fashion. In this paper sufficient conditions for triggering were presented along with some simulation results that depict the validity of the overall framework.

Chapter 8

A Self-triggered Model Predictive Control Framework for the Cooperation of Distributed Nonholonomic Agents

In this Chapter we propose a decentralized Model Predictive Control (MPC) framework with a self-triggering mechanism, for a team of cooperating agents. The nonholonomic agents are controlled locally and exchange information with their neighbors. The aim at scheduling the control updates based on a self-triggering criterion is twofold: to reduce the updates of the control law for each agent and to reduce the communication effort between the agents. The input-to-state (ISS) stability of the agents is proven, the condition for triggering is provided and the theoretic results are then depicted by a simulated example.

8.1 Introduction

The event-based approaches, either it is event-triggered control or self-triggered control has a particular relevance in network systems and to distributed / decentralized frameworks. Both approaches are comprised, inter alia, by triggering mechanisms that determine when the new control update should be. Nevertheless, the event-triggered techniques require a constant measurement of the state of the plant, or in the case of distributed schemes, it requires the continuous monitoring of the state of the neighbors in order to decide when the control update must be triggered. In the case of self-triggered control only the latest state measurement needs to be known for determining the next triggering instant, which in fact can reduce the communication effort between the distributed agents. Related works on event/ self-triggered control in the distributed frameworks can be found in [DJ09c], [JT11], [MH11], [NC12], [PSW11] and the references quoted therein.

In this Chapter a distributed framework is considered for a team of cooperating agents governed by nonholonomic kinematic models. The agents run local predictive controllers and they are exchanging information with a set of neighboring agents only on their own triggering instants. The contribution of this work relies in finding sufficient conditions for triggering in the self-triggered control context: each one of the agents monitors its own triggering condition and between the intersampling periods applies in open-loop, the previously computed control sequence. This cooperative scenario has been introduced in the classic constant sampling framework, [FMP$^+$08] and in an event-based framework, see Chapter 4, for general nonlinear systems. However, with the self-triggered approach, the updates of the local control laws are reduced and additionally the communication effort between the agents is mitigated.

8.2 Problem Formulation

In this section, the cooperative scenario of multiple agents that work in the same environment is formulated. We consider a distributed framework, and for this reason, the model, the constraints and the design of the controllers for each of the agents, are given. In the subsequent sections the overall problem is stated rigorously.

8.2.1 Mathematical Modeling

Consider a general system which is composed by M local subsystems. The subsystems are all described by the same form of nonholonomic kinematic equations of the following form:

$$x^i(k+1) = f(x^i(k), u^i(k)) \Rightarrow \tag{8.1a}$$

$$\begin{bmatrix} \chi^i(k+1) \\ y^i(k+1) \\ \theta^i(k+1) \end{bmatrix} = \begin{bmatrix} \chi^i(k) + dt\cos\theta^i(k)\upsilon^i(k) \\ y^i(k) + dt\sin\theta^i(k)\upsilon^i(k) \\ \theta^i(k) + dt\omega^i(k) \end{bmatrix} \tag{8.1b}$$

with $k \in \mathbb{Z}_{\geq 0}$ and $i = 1, \ldots, M$.

The state of the subsystem i is denoted by $x^i(k) \triangleq [\chi^i(k), y^i(k), \theta^i(k)]^\top$, while $u^i(k) \triangleq [\upsilon^i(k), \omega^i(k)]^\top$ denotes the control variable. Suppose that the agents evolve on the same discrete-time space, i.e., they are synchronized. The state and the control vectors are required to fulfill the following constraints

$$x^i(k) \in X^i \qquad u^i(k) \in U \tag{8.2}$$

where $X^i \subseteq \mathbb{R}^3$ and $U^i \subseteq \mathbb{R}^2$ are compact sets containing the origin as an interior point. In particular, the constraints of the input are of the form $|\upsilon^i| \leq \bar{\upsilon}$ and $|\omega^i| \leq \bar{\omega}$. Therefor we get $||u^i|| \leq \bar{u}$, where $\bar{u} = \sqrt{\bar{\upsilon}^2 + \bar{\omega}^2}$ for all $i = 1, \ldots, M$.

The distributed system comprised of the M subsystems is decoupled, but in order to achieve some degree of cooperation it is assumed that each agent $\mathscr{A}^i$ for all $i = 1, \ldots, M$ exchanges information with a set of neighboring agents $\mathscr{G}^i \triangleq \{\mathscr{A}^j, j \in G^i\}$, where G^i denotes the set of indexes identifying the agents belonging to the set $\mathscr{G}^i$. Consider, now, a generic time-step k, then, for each $i = 1, \ldots, M$ the agent $\mathscr{A}^i$ receives from all neighboring agents $\mathscr{A}^j \in \mathscr{G}^i$ their state vectors $x^j(k)$ and their velocity vectors $u^j(k)$. More precisely, the information received by an agent $\mathscr{A}^i$ at time step k, can be written as

$$w^i(k) \triangleq col[x^j(k)] \forall j \in G^i \tag{8.3a}$$

$$w_u^i(k) \triangleq col[u^j(k)] \forall j \in G^i \tag{8.3b}$$

with $w^i(k) \in W^i \triangleq \Pi_{j \in G^i} X^j$ and $w_u^i(k) \in \Pi_{j \in G^i} U$. It is assumed that (i) this information is always available and accurate and (ii) can be exchanged without a delay. Notice however that we consider a self-triggering framework, so this exchange of information is not not taking place at each time-step, but whenever it is *necessary* as it will be explained later on.

8.2.2 Control Design and Objective

The goal for each generic agent $\mathscr{A}^i$, described by (8.1a) and is subject to (8.2), is to be driven to a desired state which is included in X^i. In order to achieve this task local NMPC controllers, for each of the agents, are employed. For all the subsystems, it can be proven that the closed-loop systems are ISS with respect to the information vectors received by their neighbors and more specifically that the state of each subsystem is converging to a desired terminal set. Inside this set, an auxiliary terminal controller is employed to drive the system to the desired point. The design of the local NMPC controllers for a generic subsystem (8.1a) is presented next.

For each agent $\mathscr{A}^i$ and at a time-step k, the local NMPC control law is computed by solving a finite-horizon, open-loop optimal control problem (OCP), based on its state measurement $x^i(k)$ and based on the information received from the neighbors; the state and the velocity vectors $w^i(k)$ and $w^i_u(k)$, respectively. The optimal problem consists in minimizing, with respect to a control sequence $\{u^i(k|k), u^i(k+1|k), \ldots, u^i(k+N^i-1|k)\}$, a cost function $J^i_N(x^i(k), w^i(k), w^i_u(k), N^i)$. The cost function for the OCP, is given by

$$J^i_N(x^i(k), w^i(k), w^i_u(k), N^i) = \tag{8.4a}$$
$$\sum_{t=0}^{N^i-1} \{L^i(\hat{x}^i(k+t|k), u^i(k+t|k)) + Q^i(\hat{x}^i(k+t|k), \hat{w}^i(k+t|k))\} + V^i(\hat{x}^i(k+N^i|k))$$

subject to

$$\hat{x}^i(k+t|k) \in X^i \qquad \forall t = 1, \ldots, N^i - 1 \tag{8.4b}$$
$$u^i(k+t|k) \in U \qquad \forall t = 0, \ldots, N^i - 1 \tag{8.4c}$$
$$\hat{w}^i(k+t|k) \in W^i \qquad \forall t = 0, \ldots, N^i - 1 \tag{8.4d}$$
$$\hat{x}^i(k+N^i|k) \in X^i_f \tag{8.4e}$$

where X_f^i denotes the terminal constraint set and $\hat{x}^i(k|k) = x^i(k)$. The positive integer $N^i \in \mathbb{Z}_{\geq 0}$, denotes the prediction horizon. The notation $\hat{x}^i(\cdot|\cdot)$ used in (8.4a), (8.4b) and (8.4e), denotes the predicted state of the agent $\mathscr{A}^i$ and is given as

$$\hat{x}^i(k+t+1|k) = f(\hat{x}^i(k+t|k), u^i(k+t|k)) \tag{8.5}$$

which accounts for the predicted state at time $k+t+1$ with $t \in \mathbb{Z}_{\geq 0}$, based on the measurement of the state at time k while using a control input u^i_{k+t} and the model of the system from (8.1a). In the same manner, the predicted states of the neighbors of the agent $\mathscr{A}^i$ are given as

$$\hat{w}^i(k+t+1|k) = f(\hat{w}^i(k+t|k), w_u^i(k+t|k)) \tag{8.6}$$

which is equivalent to

$$col[\hat{x}^j(k+t+1|k)] = col[f(\hat{x}^j(k+t|k), u^j(k+t|k))],\ j \in G^i$$

The vector $\hat{w}^i(k+t+1|k)$ for $t = 0, \ldots, N^i - 1$ denotes the prediction of the neighbors' states. However, at a generic time instant k, only $\hat{w}^i(k|k) \triangleq w^i(k)$ as well as $w_u^i(k)$ are known to the agent $\mathscr{A}^i$. In order to solve the OCP (8.4a)-(8.4e), the controller of the agent $\mathscr{A}^i$, assumes the following for the prediction horizon: the agents $\mathscr{A}^j$ for all $j \in G^i$, maintain the same velocity during the whole prediction horizon N^i, i.e., $w_u^i(k+t|k) = w_u^i(k), \forall t \in [0, N^i - 1]$. This assumption is utilized only for the prediction of the controller, and it is clear that the trajectories of the neighboring agents will diverge from the predicted ones due to individual dynamics. However, the closed-loop nature of the overall framework is able to overcome this limitation, as it will be shown in the stability analysis.

In order to proceed to the subsequent analysis, some standard stability conditions for the design of the local predictive controllers are introduced, in order to assert that the MPC strategy results in stabilizing local controllers for each of the subsystems.

Assumption 21 *The stage cost $L^i(x^i, u^i)$ is Lipschitz continuous in $X^i \times U$ and it holds that $L^i(0,0) = 0$. Moreover, there is a $\mathscr{K}_\infty$-function r^i, such that $L^i(x_k^i, u_k^i) \geq r^i(||x_k^i||)$.*

Assumption 22 *The running cost $Q^i(x^i, w^i)$ is such that $Q^i(x^i, w^i) \geq 0$. Moreover, Q^i is*

Lipschitz continuous in $X^i \times W^i$, with Lipschitz constants L^i_{qx} and L^i_{qw}, respectively.

Assumption 23 *Let the terminal set X^i_f be such that $X^i_f \subset X^i$, X^i_f to be closed, and $0 \in X^i_f$. Assume that there is a locally stabilizing controller $h^i(x_k)$ for the terminal set. The associated Lyapunov function $V^i(\cdot)$ has the following property, for all $x^i \in X^i_f$ and for all $w^i \in W^i$,*

$$V^i(f^i(x^i(k), h^i(x^i(k)))) - V^i(x^i(k)) \leq -L^i(x^i(k), h^i(x^i(k))) - Q^i(x^i(k), w^i(k))$$

8.2.3 Problem Statement

The solution of the OCP (8.4a)-(8.4e) at a time-step k provides an optimal control sequence. The classic framework of the MPC consists in applying to the system only the first control vector, i.e., $u^{*i}(k|k)$ and to discard all the remaining elements of the sequence. At the next time-step $k+1$, new state measurements are received and the whole procedure is repeated again. This is iteratively repeated until the system has reached to the desired terminal set. However, the self-triggering framework suggests that a portion of the computed control sequence may be applied to the system and not only the first vector. Suppose a triggering instant k_i. The control sequence that is applied to the plant is of the form

$$\{u^*(k_i|k_i), u^*(k_i+1|k_i), \ldots u^*(k_i+t|k_i)\} \tag{8.7}$$

for all $t \in [0, k_{i+1} - k_i - 1]$, where k_{i+1} is the next triggering instant. During the time interval $[k_i, k_{i+1})$ the control law is applied to the plant in an open-loop fashion, while no measurements from the neighboring agents are received. A question that naturally arises is how large this time interval can be? Notice, though, the smallest time interval is obviously 1, that is if $k_{i+1} = k_i + 1$. The self-triggered strategy that will be presented later in this paper, answers to this question and provides sufficient conditions for finding the recalculation periods, or in other words sufficient conditions for triggering the computation of the NMPC law. This leads us to the statement of the problem treated in this Chapter:

Problem Statement 4 *Consider a generic subsystem* (8.1a) *that is subject to constraints* (8.2), *while measuring* (8.3a)-(8.3b) *from the neighboring agents. The objective is* (i) *to design a feedback control law provided by* (8.4a)-(8.4e) *such that the subsystem* (8.1a) *converges to its terminal constraint set and* (ii) *to find a mechanism to decide when the recalculation instants of the local control law should be. This framework should result to the overall stability for the team of agents.*

8.3 Stability Analysis of NMPC

In this section the stability analysis of the closed-loop system (8.1a)-(8.7) for a generic agent $\mathscr{A}^i$, is presented. The analysis will be using the ISS notion for stability because even though no disturbances are considered, the influence of the neighboring agents requires ultimate boundedness results. An approach which considers a similar cooperative scenario was presented in [FMP$^+$08], but the analysis was performed under the classic MPC set-up which dictates the calculation of the MPC to be computed at each time step. Through a modification of the analysis proposed in [FMP$^+$08] it is possible to reach to a self-triggering mechanism which will provide the triggering instants of the local control laws.

Consider a time-step k when an event is triggered, then a new OCP (8.4a)-(8.4e) is solved which provides an optimal control sequence $\{u^{i*}(k|k),\ldots,u^{i*}(k+N^i-1|k)\}$. The optimal cost $J_N^{i*}(k)$, is the cost (8.4a) under this optimal control sequence. In order to prove stability of the closed-loop system, a sufficient condition is to find a proper value function that must be shown to be decreasing at every time-step. This value function will in fact be the optimal cost $J_N^{i*}(k)$. First we are going to evaluate the difference $J_N^{i*}(k+m)-J_N^{i*}(k)$ for all $m=[1,N^i-1]$ and then we are going to restrict the optimal cost to decrease at each time-step.

Consider now, control sequences $\vec{u}^i(\cdot)$ for time-steps $m=1,\ldots,N^i-1$, based on the optimal solution at the triggering instant k, given as

$$\vec{u}^i(k+t|k+m)=\begin{cases} u^{i*}(k+t|k) & \text{for } t=m,\ldots,N^i-1 \\ h^i(\hat{x}^i(k+t|k+m)) & \text{for } t=N^i,\ldots,N^i+m-1 \end{cases} \tag{8.8}$$

These control sequences are admissible and in general suboptimal. From the feasibility of the optimal control trajectory at time-step k it follows that for all $t,m = 1,\ldots,N^i-1$ we have $\bar{u}^i(k+t|k+m) \in U$ and $\hat{x}^i(k+N^i|k+m) \in X^i_f$. Now, let $\bar{J}^i_N(k+m)$ to be the "feasible" cost at a time step $k+m$, $\forall m \in [1,N^i-1]$. This cost is derived from (8.4a) for a control sequence (8.8). This "feasible" cost will help us to obtain the difference $J^{i*}_N(k+m) - J^{i*}_N(k)$. First we are going to evaluate this difference for $m=1$, then for $m=2$ and finally invoke the general formulation.

For $m=1$ we have

$$
\begin{aligned}
&\bar{J}^i_N(k+1) = J^{i*}_N(k) - L^i(x^i(k),u^i(k)) - Q^i(x^i(k),w^i(k)) \\
&+ \sum_{t=1}^{N^i-1} \{L^i(\bar{x}^i(k+t|k+1),\bar{u}^i(k+t|k+1)) + Q^i(\bar{x}^i(k+t|k+1),\hat{w}^i(k+t|k+1)) \\
&- L^i(\hat{x}^i(k+t|k),u^{i*}(k+t|k)) - Q^i(\hat{x}^i(k+t|k),\hat{w}^i(k+t|k))\} \\
&+ L^i(\bar{x}^i(k+N^i|k+1),h^i(\bar{x}^i(k+N^i|k+1)) + Q^i(\bar{x}^i(k+N^i|k+1),\hat{w}^i(k+N^i|k+1)) \\
&+ V^i(\bar{x}^i(k+N^i+1|k+1)) - V^i(\hat{x}^i(k+N^i|k))
\end{aligned} \tag{8.9}
$$

where $\bar{x}^i(\cdot)$ is the state of the agent $\mathscr{A}^i$ while a feasible control input from (8.8) is being applied. Notice that we consider nominal stability of the agents, thus, the predicted state $\hat{x}(\cdot)$ and the "feasible" state $\bar{x}(\cdot)$, computed at the same time-step are coinciding. Using Assumption 22, the following result can be obtained

$$
\begin{aligned}
&Q^i(\bar{x}^i(k+t|k+1),\hat{w}^i(k+t|k+1)) - Q^i(\hat{x}^i(k+t|k),\hat{w}^i(k+t|k)) \leq \\
&||Q^i(\cdot,\hat{w}^i(k+t|k+1)) - Q^i(\cdot,\hat{w}^i(k+t|k))|| \leq L^i_{qw}||\hat{w}^i(k+t|k+1) - \hat{w}^i(k+t|k)||
\end{aligned} \tag{8.10}
$$

From the Appendix and in particular from (8.21), it yields

$$
L^i_{qw}||\hat{w}^i(k+t|k+1) - \hat{w}^i(k+t|k)|| \leq L^i_{qw} dt(m-1) \sum_{j\in G^i} \{(2(\bar{\upsilon}+\upsilon^j_k)^2 + \bar{\omega}^2)^{1/2}\} \tag{8.11}
$$

Using the inequality from Assumption 23, and substituting (8.11) to (8.9), we obtain

$$\bar{J}_N^i(k+1) \leq J_N^{i*}(k) - L^i(x^i(k), u^i(k)) - Q^i(x^i(k), w^i(k)) \\ + L_{qw}^i dt(m-1) \sum_{j \in G^i} \{(2(\bar{\upsilon} + |\upsilon_k^j|)^2 + \bar{\omega}^2)^{1/2}\} \tag{8.12}$$

From the optimality of the solution that yields $J_N^{i*}(k+1) \leq \bar{J}_N^i(k+1)$ and with the help of the Assumption 1, the following is derived

$$J_N^{i*}(k+1) - J_N^{i*}(k) \leq -r^i(||x^i(k)||) + L_{qw}^i dt(m-1) \sum_{j \in G^i} \{(2(\bar{\upsilon} + |\upsilon_k^j|)^2 + \bar{\omega}^2)^{1/2}\} \tag{8.13}$$

For $m = 2$ we get

$$\bar{J}_N^i(k+2) \leq J_N^{i*}(k) - L^i(x^i(k), u^i(k)) \tag{8.14}$$
$$- Q^i(x^i(k), w^i(k)) - L^i(\hat{x}^i(k+1|k), u^i(k+1|k)) - Q^i(\hat{x}^i(k+1|k), \hat{w}^i(k+1|k))$$
$$+ \sum_{t=1}^{N^i-2} \{Q^i(\hat{x}(k+t+1|k+2), \hat{w}(k+t+1|k+2)) - Q^i(\hat{x}(k+t+1|k), \hat{w}(k+t+1|k))\}$$

Using similar arguments as before, we obtain the following

$$J_N^{i*}(k+2) - J_N^{i*}(k) \leq -r^i(||x^i(k)||) - r^i(||\hat{x}^i(k+1|k)||) \\ + L_{qw}^i dt(m-1) \sum_{j \in G^i} \{(2(\bar{\upsilon} + |\upsilon_k^j|)^2 + \bar{\omega}^2)^{1/2}\} \tag{8.15}$$

From the above it can be concluded using the same procedure that for random $m \in [1, N^i - 1]$, we get

$$J_N^{i*}(k+m) - J_N^{i*}(k) \leq \tag{8.16}$$
$$-r^i(||x^i(k)||) - \sum_{\rho=1}^{m-1} \{r^i(||\hat{x}^i(k+\rho|k)||)\} + L_{qw}^i dt(m-1) \sum_{j \in G^i} \{(2(\bar{\upsilon} + |\upsilon_k^j|)^2 + \bar{\omega}^2)^{1/2}\}$$

In (8.16), it is shown that the difference $J_N^{i*}(k+m) - J_N^{i*}(k)$ is bounded. In order to prove stability though, the optimal cost must be decreasing at each consecutive time-step. This restriction for a decreasing Lyapunov function will enable us to reach to the triggering conditions, thus, it will be discussed in the next subsection.

8.4 The self-triggered Framework

In this section the self-triggering mechanism is going to be presented. Consider that at time k_i, an event is triggered. Then (8.16) becomes

$$J_N^{i*}(k_i+m) - J_N^{i*}(k_i) \leq \tag{8.17}$$
$$-r^i(||x^i(k)||) - \sum_{\rho=1}^{m-1}\{r^i(||\hat{x}^i(k_i+\rho|k_i)||)\} + L_{qw}^i dt(m-1)\sum_{j\in G^i}\{(2(\bar{\upsilon}+|\upsilon^j(k_i)|)^2+\bar{\omega}^2)^{1/2}\}$$

For $m=[1,N^i-1]$, and if the following is valid

$$L_{qw}^i dt(m-1)\sum_{j\in G^i}\{(2(\bar{\upsilon}+|\upsilon^j(k_i)|)^2+\bar{\omega}^2)^{1/2}\} \leq \sigma(r^i(||x^i(k)||) + \sum_{\rho=1}^{m-1}\{r^i(||\hat{x}^i(k_i+\rho|k_i)||)\}) \tag{8.18}$$

for $0<\sigma<1$, then the Lyapunov function $J_N^{i*}(k)$ is decreasing and the ISS property of the system is guaranteed. For each agent, the condition (8.18) should be checked for each consecutive time-step, i.e., for $m=1, m=2, \dots$. The time-step that this condition does no longer holds, should be the next triggering instant k_{i+1}.

Next we describe the self-triggering mechanism for a generic agent $\mathscr{A}^i$. At time k_i a control update is triggered, the controller reads the local state measurement and receives the information from the neighboring agents and finally it provides a control sequence for $[k_i, k_i+N^i-1]$. The controller checks for how many steps inequality (8.18) is valid, and applies the optimal trajectory that was computed at time-step k_i for all those steps, in an open-loop fashion, until the next triggering instant k_{i+1}. The aforementioned procedure is repeated until the subsystem converges to the terminal constraint set.

We are now ready to state the stability result for this self-triggered MPC framework:

Theorem 10 *Consider the subsystem* (8.1a) *that is subject to constraints* (8.2) *under the*

NMPC strategy and assume that Assumptions 21-23 hold. The control update times that are provided by (8.18) *and the NMPC law provided by* (8.4a)-(8.4e) *which is applied to the system in an open-loop fashion during the inter-sampling periods, drive the closed-loop system towards a compact set* X_f^i *where it is ultimately bounded.*

Remark 3 *Let* X_{mpc}^i *be the set of states of the subsystem* $\mathscr{A}^i$*, where a solution of the OPC* (8.4a)-(8.4e) *exists. In [FMP+08], it has been shown that* $J_N^{i*}(\cdot)$ *is an ISS Lyapunov function of the system* (8.1a)*, relaying to similar assumptions as in this paper. Thus, the subsystem* (8.1a)*, subject to* (8.2)*, which satisfies Assumptions 21-23, is ISS stable inside* X_{mpc}^i*, with respect to the information received from the neighboring agents, under the NMPC strategy. Moreover, in the previous analysis it was shown that this property is guaranteed to be valid under the self-triggered framework. Furthermore, the proof for the stability of the team of agents is omitted due to space limitations and the reader is referred in [FMP+08].*

8.5 Simulation results

In this section, a simulated example of the proposed framework for a team of three non-holonomic agents moving in $\mathbb{R}^2$ is presented. The objective is to control each agent through a local NMPC law of the form (8.4a)-(8.4e) to reach the desired position, without colliding. The models of the subsystems are of the form (8.1b). The discretization time is $dt = 0.1$ and the cost functions are of quadratic form, i.e., $(x^i)^\top S^i x^i$, $(u^i)^\top R^i u^i$ and $(w^i - x^i + d^i)^\top Q^i (w^i - x^i + d^i)$, with $S^1 = S^2 = S^3 = diag[3, 5, 0.1]$, $R^1 = R^2 = R^3 = diag[1, 1]$ and $Q^1 = diag[8, 8, 0.1]$, $Q^2 = diag[6, 6, 0.1]$, $Q^3 = diag[5, 14, 0.1]$. The term $d^1 = d^2 = d^3 = [3, 3, 0]$ is the minimum desired distance between the agents. The initial and the desired position of agent $\mathscr{A}^1$ is $x_{initial}^1 = [-20, 7, \pi/4]$, $x_{desired}^1 = [6, -9, 0]$. For the agent $\mathscr{A}^2$ is $x_{initial}^2 = [-10, -7, \pi/3]$, $x_{desired}^2 = [14, 18, 0]$ and for the agent $\mathscr{A}^3$ is $x_{initial}^3 = [10, -7, \pi - \pi/3]$, $x_{desired}^3 = [-14, 18, \pi]$. Finally, the input is bounded by $\bar{u} = [10, 0.1]$ and the term σ is taken equal to 0.8.

In Fig.8-1, the trajectories of the agents are depicted. All three of them converge to a terminal constraint set that includes their desired states. It should be noted that the collision between the agents is avoided with the proposed framework. This is more apparent in Fig.8-2, where the χ^i and y^i positions are depicted. The agents are not coinciding at any sampling time. The coloring follows the same rule as in Fig., where the the red lines represent the

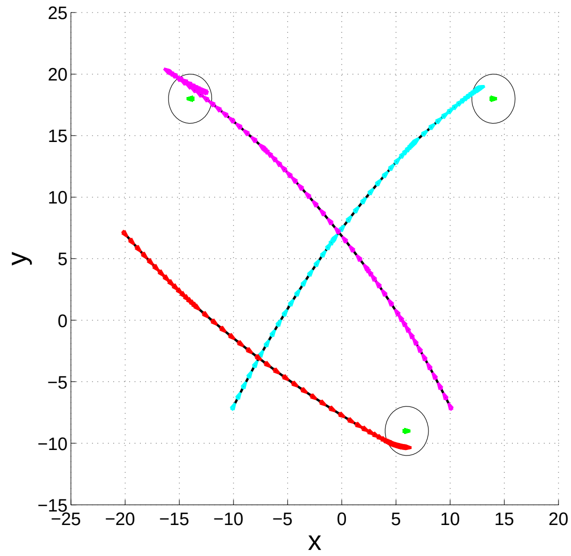

Figure 8-1: Trajectories of the team of agents. The red triangles represent the agent $\mathscr{A}^1$. The blue triangles represent the agent $\mathscr{A}^2$ and the magenta triangles represent agent $\mathscr{A}^3$.

agent $\mathscr{A}^1$, the blue lines represent the agent $\mathscr{A}^2$ and the magenta lines represent agent $\mathscr{A}^3$.

In the following the sampling times are depicted. Notice that when diagram has 1 value, there is a triggering instant, while when it has the value 0, the agents are controlled open-loop. Fig.8-3 depicts the triggering instants for agent $\mathscr{A}^1$ and Fig.8-4, Fig. 8-5 depict the triggering instants for agents $\mathscr{A}^2$ and $\mathscr{A}^3$, respectively.

It is apparent from the figures, that the updates of the control laws, as well as, the communication load between the agents is significantly reduced, while the systems have succeeded to converge to their desired states and to avoid collision.

8.6 Summary

In this Chapter, a cooperative framework for distributed nonholonomic agents under local model predictive controllers was considered. Also, for each subsystem a self-triggering

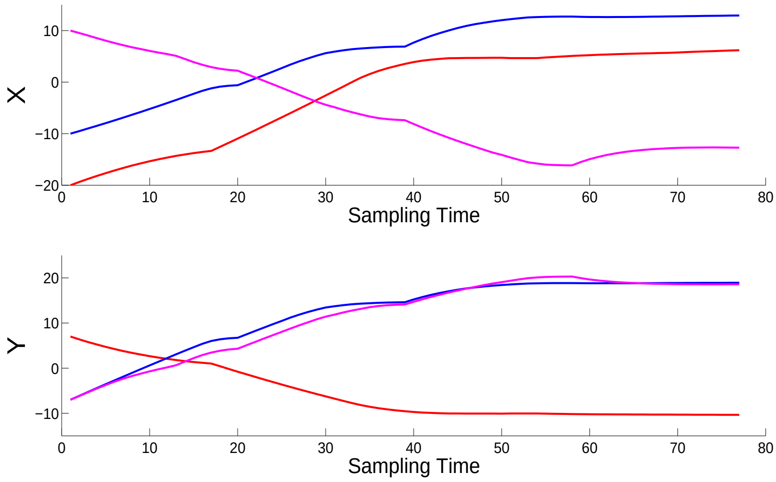

Figure 8-2: The χ^i and y^i positions of the agents with respect to sampling times, for $i = 1,2,3$.

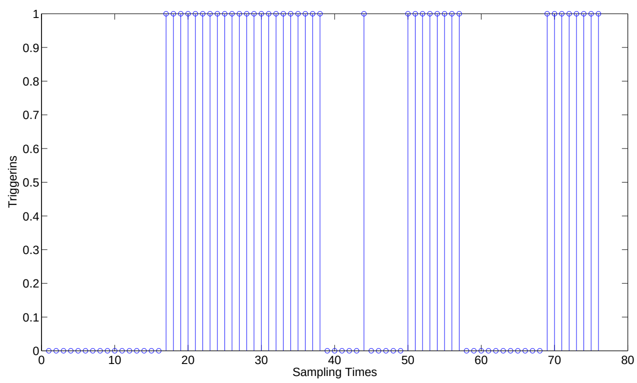

Figure 8-3: Triggering instants for agent $\mathscr{A}^1$.

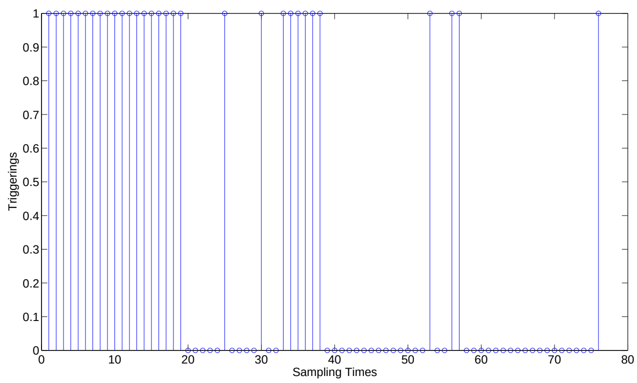

Figure 8-4: Triggering instants for agent $\mathscr{A}^2$.

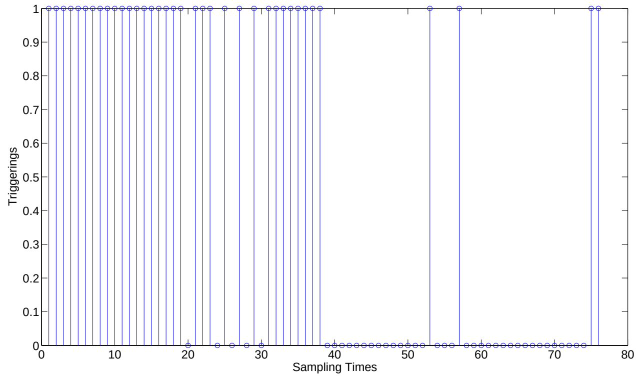

Figure 8-5: Triggering instants for agent $\mathscr{A}^3$.

condition was proposed. The main idea is to trigger the solution of the optimal control problem of the predictive controllers only when it is needed and not periodically as in the case of classic MPC schemes. During the inter-sampling times the control trajectory from the NMPC is applied to the system in an open-loop fashion. With the self-triggered approach both the control input and the next control update time are evaluated in order to avoid continuous supervision of the states of the neighboring agents. Thus, this approach results to a reduction of the updates of the control laws for each subsystem, as well to a reduction of the communication effort between the subsystems.

8.7 Appendix

In this section we are going to evaluate the inequality (8.10), which is crucial in order to reach to the triggering mechanism. First, the expression for the predicted states at a time-step $k+t$, with $t,m \in [1, N^i - 1]$, of the neighbors of the agent $\mathscr{A}^i$ measured from the the generic triggering instant k, i.e., $\hat{w}^i(k+t|k)$, is going to be given in Lemma 11 and then the predicted states $\hat{w}^i(k+t|k+m)$, measured from the time-step $k+m$, i.e., $\hat{w}^i(k+t|k+m)$ are going to be given in Lemma 12. Finally the expression for (8.10) will be provided.

Lemma 11 *The predicted states $\hat{w}^i(k+t|k)$ for $t \in [1, N^i - 1]$, are given as*

$$\hat{w}^i(k+t|k) \triangleq col[x^j(k+t|k)] = \\ col[\hat{\chi}^j(k+t|k), \hat{y}^j(k+t|k), \hat{\theta}^j(k+t|k)]^\top = \\ col\begin{cases} \chi_k^j + dt\cos\theta_k^j \upsilon_k^j + dt\upsilon_k^j \sum_{l=1}^{t-1} \cos(\theta_k^j + l dt \omega_k^j) \\ y_k^j + dt\sin\theta_k^j \upsilon_k^j + dt\upsilon_k^j \sum_{l=1}^{t-1} \sin(\theta_k^j + l dt \omega_k^j) \\ \theta_k^j + t dt \omega_k^j \end{cases} \tag{8.19}$$

with $j \in G^i$.

Proof At a triggering time-step k the vector $u_k = [\upsilon_k, \omega_k]^\top$ is measured and for the prediction horizon we assume that $[u_{k+1}, \ldots, u_{k+N-1}] = [u_k, \ldots, u_k]$. Having that, we get for $t = 2$,

$$\begin{cases} \hat{\chi}^j(k+2|k) \\ \hat{y}^j(k+2|k) \\ \hat{\theta}^j(k+2|k) \end{cases} = \begin{cases} \hat{\chi}^j(k+1|k) + dt\cos(\hat{\theta}^j(k+1|k))\upsilon_k^j \\ \hat{y}^j(k+1|k) + dt\sin(\hat{\theta}^j(k+1|k))\upsilon_k^j \\ \hat{\theta}^j(k+1|k) + dt\omega_k^j \end{cases}$$

Moving forward and by recursion we reach to the general rule (8.19).

Also we have that,

Lemma 12 *The predicted states $\hat{w}^i(k+t|k+m)$ for $t \in [1, N^i - 1]$ and for $m = [1, N^i - 1]$, are given as*

$$\hat{w}^i(k+t|k+m) \triangleq col[x^j(k+t|k+m)] = \\ col[\hat{\chi}^j(k+t|k+m), \hat{y}^j(k+t|k+m), \hat{\theta}^j(k+t|k+m)]^\top = \\ col\begin{cases} \chi_k^j + dt\cos\theta_k^j \upsilon_k^j \ldots \\ +dt\bar{\upsilon}\sum_{l=1}^{t-1} \cos(\theta_k^j + dt\omega_k^j + (l-1)dt\bar{\omega}) \\ y_k^j + dt\sin\theta_k^j \upsilon_k^j \ldots \\ +dt\bar{\upsilon}\sum_{l=1}^{t-1} \sin(\theta_k^j + dt\omega_k^j + (l-1)dt\bar{\omega}) \\ \theta_k^j + t dt\omega_k^j + dt(t-1)\bar{\omega} \end{cases} \tag{8.20}$$

with $j \in G^i$.

Proof Assume that $m = 1$, therefor, at time-step $k+1$ which follows the generic triggering time-step k, it is assumed that $[u_{k+1}, \ldots, u_{k+N-1}] = [\bar{u}, \ldots, \bar{u}]$ with $\bar{u} = [\bar{\upsilon}, \bar{\omega}]^\top$. The states of the neighbors of the agent $\mathscr{A}^i$, for a time-step $k+t$, with $t = 2$, are

From (8.1b) we get

$$
\begin{aligned}
&[\chi^j(k+2|k+1), y^j(k+2|k+1), \theta^j(k+2|k+1)]^\top = \\
&= \begin{cases} \chi^j(k+1|k) + dt\cos(\theta^j(k+1|k))\bar{\upsilon} \\ y^j(k+1|k) + dt\sin(\theta^j(k+1|k))\bar{\upsilon} \\ \theta^j(k+1|k) + dt\bar{\omega} \end{cases} \\
&= \begin{cases} \chi^i_k + dt\cos\theta^i_k\upsilon^i_k + dt\cos(\theta^i_k + dt\omega^i_k)\bar{\upsilon} \\ y^i_k + dt\sin\theta^i_k\upsilon^i_k + dt\sin(\theta^i_k + dt\omega^i_k)\bar{\upsilon} \\ \theta^i_k + dt\omega^i_k + dt\bar{\omega} \end{cases}
\end{aligned}
$$

which yields, by recursion for a $t \in [1, N^i - 1]$, the general form (8.20). The same applies for all $m \in [1, N^i - 1]$ as we consider nominal stability.

It should be noted that we used the abstraction $\sum_1^0 \equiv 0$.

From Lemma 11 and Lemma 12, while making some easy manipulations that is omitted due to space limitations, it can be concluded that for an agent $\mathscr{A}^i$, the predicted states of its neighbors at a time step $k+t$ are bounded by

$$
||\hat{w}^i(k+t|k+m) - w^i(k+t|k)|| \leq \sum_{j\in G^i} \{dt(m-1)(2(\bar{\upsilon} + |\upsilon^j_k|)^2 + \bar{\omega}^2)^{1/2}\} \tag{8.21}
$$

Chapter 9

Nonlinear Model Predictive Control for a Manipulator in Interaction with a Compliant Environment

In this Chapter, we present a Gradient-based Predictive Control methodology for stabilization and force control of a robotic manipulator that is in contact with a compliant environment. These two objectives are treated simultaneously in the context of parallel control. The use of the Nonlinear Model Predictive Controller renders the overall scheme more robust to disturbances due to model uncertainties, compared to classic interaction control schemes. The efficiency of the proposed framework as well as the advantage over traditional parallel control, is depicted through simulated examples.

9.1 Introduction

The goal is to stabilize a manipulator's end-effector in a desired position and apply a desired force on it, when in contact with the environment. We propose a novel parallel-like control scheme, using a Gradient Based Predictive Controller [LWH99]. The use of the Nonlinear Model Predictive Control (abbr. NMPC) approach renders the overall scheme more robust to disturbances due to gravity or model uncertainties. Moreover, using the proposed methodology, no impact effects, or/and instabilities at the transition phase from the no-contact case to the contact case occur.

Controlling the interaction of a robot manipulator with the environment is a topic that has been extensively studied during the past two decades. Many control schemes have been proposed in order to achieve a good system performance at steady state or to solve a tracking problem. Related surveys about the existing interaction control schemes can be found in [CSV99], as well as in [Yos00], where a summary of proposed control methodologies is presented. The main drawback of these classic control approaches is the fact that they seem inadequate for the task of interaction with a compliant environment in the case of scarce information about its model, or if there exist ambiguities in the model of the robot, as in most practical cases.

In this Chapter, we aim at enhancing the classic interaction controllers in order to tackle the problem of un-modelled disturbances that may cause unpredicted instabilities. The methodology of Model Predictive Control seems a suitable candidate for the control of such systems, since this control strategy has inherent virtues: It is arguably one of the best control strategies for handling severe nonlinearities and uncertainties, hard and soft constraints, as well as achieving near-optimal performance. Nevertheless, model predictive schemes involve finding the repeated on-line solution of constrained (possibly non-convex) optimization problems, that cause large computation periods. In order to overcome this problem we used an efficient method that have been proposed in [WJ03], whereas the stability and robustness of this gradient based algorithm as long as its applicability in real time experiments have been analyzed in [Yoo02].

There are a few papers that have used the NMPC framework to handle interaction control problems. In [BSC01a], the authors make use of an impedance controller integrated with a fuzzy predictive algorithm. The proposed scheme incorporates nonlinear model of the contact and uncertainties in the model of the robot, as well as inaccuracies in environment location and stiffness characteristics. By deploying this strategy, a considerable reduction on the force error compared to classic approaches has been achieved [BSC01b]. Notice though that the predictive algorithm computes the optimal trajectory for the impedance controller off-line.

Moreover, NMPC represents a good alternative for the control of systems with changing dynamics. In [CN95], the authors have shown that NMPC can solve the impact-contact motion control problem in a unified way. A single NMPC controller can be employed for the free motion control, the contact motion control and the impact control. Even though a nonlinear model of friction was used, no attention was given to the case of model uncertainties.

In this Chapter, we apply the NMPC framework and tackle the problem of driving a manipulator that does not interact with the environment to a desired position and apply a desired force on a planar surface. The transition from the no-contact case to the contact case is smooth and no impact effects occur. Computer simulations demonstrate the proposed approach. In particular, the proposed algorithm converges to the desired position and contacts the wall under the desired force for a number of cases, where the stiffness of the environment is not the expected. We show that under the same control constraints, the NMPC algorithm compared to a PD controller with gravity compensation, achieves the goal without impact phenomena.

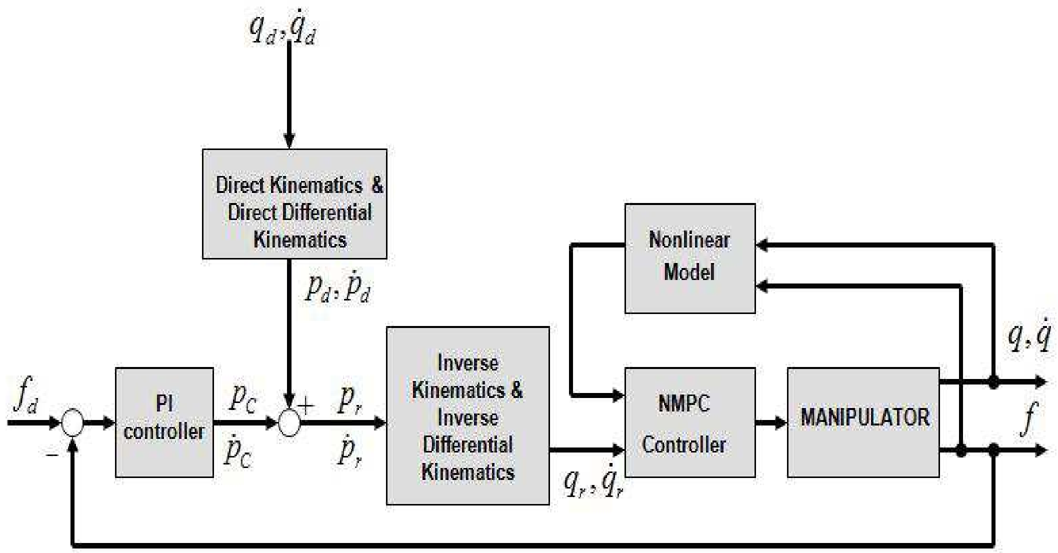

Figure 9-1: The integrated block scheme of Nonlinear Gradient Based Predictive Controller with force feedback loop.

9.2 Parallel Force/ Position NMPC Scheme - Adopted Models

In order to combine the features of stiffness control and force control, a parallel force/position regulator is designed, where PI force control action plus a desired force feed-forward is used in parallel to a NMPC position control action. In this parallel control scheme a position control action acts in parallel to a force control action. Along unconstrained directions the position reference $p_d \in \Omega$ must be reached by the end-effector's position p, where $\Omega \subseteq \mathbb{R}^2$ is the manipulator workspace, $p \in \Omega$ is the actual position of the end-effector in the Cartesian space and p_d is the desired position that the end-effector aims at reaching. On the other hand, along those directions where the motion of the manipulator's end-effector is constrained, p_d is treated as an additional disturbance. The position control scheme is a NMPC joint space controller. The integrated scheme is depicted in Fig. 9-1. Each component is specified next.

9.2.1 Environment Model

An interaction between the end effector and a frictionless, elastically compliant environment is assumed. Contact geometry is also assumed to be known, so that constrained and unconstrained directions can be clearly identified. The following equations will be used hereafter, to model the compliant environment:

$$f = K_f(p - p_e) \tag{9.1}$$

where $K_f \in \mathbb{R}^{2\times 2}$ is the positive semi-definite translational stiffness, which represents the elastic coefficient of the environment, p is the actual position of the end-effector in the Cartesian manipulator workspace, $p_e \in \Omega$ is the equilibrium position of the undeformed environment and finally, f is the force exerted by the end-effector on the environment during the interaction. In this work, the contact force f is assumed to be measurable and available to the controller.

9.2.2 PI Controller

Let f_d be the desired force that the manipulator should apply in the constrained direction. We introduce a frame Σ_c referred to as the compliant frame which is specified by a position vector $p_c \in \Omega$. The end-effector position should follow this frame during the interaction task. Accordingly, the actual end-effector linear velocity $\dot{p}$ is taken to follow the linear velocity of the compliant frame $\dot{p}_c$.

The force error is given by abstracting the desired force from the measured force:

$$\Delta f = f_d - f \tag{9.2}$$

The vector p_c can be chosen as a proportional-integral control on the force error, i.e.

$$p_c = K_F \Delta f + K_I \int_0^t \Delta f \, \mathrm{d}\varsigma \tag{9.3}$$

where $K_F, K_I \in \mathbb{R}^{2\times 2}$ are suitable positive definite matrix gains.

The idea of parallel control is to compose the compliant position p_c with the desired position p_d as

$$p_r = p_c + p_d \tag{9.4}$$

and use this reference position $p_r \in \Omega$ as input to the motion control scheme. The parallel composition can be extended to the velocity as well,

$$\dot{p}_r = \dot{p}_c + \dot{p}_d \tag{9.5}$$

where $\dot{p}_r$ is the velocity that must be followed by the end-effector.

9.2.3 Model Of the Manipulator

Consider a n degrees of freedom (abbr. d.o.f.) manipulator in a fixed reference frame. The joint-space dynamic model of the n d.o.f. manipulator in interaction with the environment is given by:

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + F\dot{q} + g(q) = \tau - J^T(q)h \tag{9.6}$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ are the vectors of position, velocity and acceleration of the joints, $B \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C \in \mathbb{R}^{n\times n}$ is the centrifugal and Coriolis matrix, $F\dot{q} \in \mathbb{R}^n$ are the viscous friction torques, $g(q) \in \mathbb{R}^n$ is the vector of the gravity torques, $\tau \in \mathbb{R}^n$ are the actuator torques and finally $h \in \mathbb{R}^6$ denotes the vector of force exerted by the end effector on the environment. The analytical Jacobian $J_A(q) \in \mathbb{R}^{6\times n}$ of the manipulator relates in a straightforward way the velocity of the end-effector with the velocity of the joints:

$$\dot{p} = J_A(q)\dot{q} \tag{9.7}$$

where $\dot{p}$ is the vector that describes the velocity of the end-effector in the operational space.

The inverse kinematics of this particular manipulator and the differential inverse kinematics that are used can be found in [SV99].

9.3 A Nonlinear Model Predictive Control Strategy

The parallel scheme is endowed with a position NMPC controller. This section is dedicated to reviewing the particular NMPC strategy. The general principle of model predictive control schemes is formulated so as to solve on-line a finite horizon open-loop optimal control problem subject to system's dynamics and constraints. At each step, the NMPC scheme generates an optimal control trajectory. This trajectory is applied, as the desired one, to the system, until the next system measurement is available. The NMPC framework of [WJ03], [Yoo02] that we are going to describe next, has the virtue of being applicable in systems with small sampling period, like torque controlled robotic manipulators. The computational burden is sufficiently reduced because the algorithm does not try to find the optimal solution, but only seeks to reduce the error at the end of the prediction horizon. Although the authors in [WJ03], [Yoo02] have used this strategy for point convergence, we are going to employ it for trajectory tracking.

Assume that the state of the manipulator is given as $x = [q, \dot{q}]^T \in \mathbb{R}^{2n}$ and consider the general discrete-time nonlinear system:

$$x(k+1) = f(x(k)) + g(x(k))u(k) \tag{9.8}$$

where the $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ represent the state and the control variables respectively. Also assume that the nonlinear system is subject to state and control constraints,

$$x(\cdot) \in \mathscr{X} \qquad u(\cdot) \in \mathscr{U}$$

where $\mathscr{U}$ is a convex, compact subset of $\mathbb{R}^m$ and $\mathscr{X}$ a convex, closed subset of $\mathbb{R}^n$, each containing the origin in its interior.

The prediction horizon is the time step ahead, that the algorithm predicts the system state, and is defined as M. The predictive control vector at time k and for prediction horizon M is denoted by $\underline{u}_{k,M}$:

$$\underline{u}_{k,M} = [u_1(k), \ldots, u_M(k)]^T, \qquad \underline{u}_{k,M} \in \mathbb{R}^{m \cdot M} \tag{9.9}$$

The current state of the system is always treated as the initial state at the next iteration. Let the end-point mapping for the system evolving from $x(k)$ to the terminal state $x(k+M) \in \mathbb{R}^n$ be denoted as $\phi_M(x(k), \underline{u}_{k,M})$. The predicted state error is:

$$e_M(k) = \phi_M(x(k), \underline{u}_{k,M}) - x^*(k) \tag{9.10}$$

where $x^*(k)$, is the desired state of the system at the time k. We can choose the Newton-step for updating the control signals:

$$\underline{v}_{k,M} = \underline{u}_{k,M} - \beta_k (\nabla \underline{u}_{k,M} \phi_M(x(k), \underline{u}_{k,M}))^\dagger e_M(k) \tag{9.11}$$

The Newton-step guarantees that the terminal state is strictly decreasing to the desired state if a singular control is not encountered, as we can see from [Son95]. In practice, for strongly accessible systems, it is “generically rare” to encounter singular control, [Yoo02].

The β_k factor is a constant that is computed in every iteration, by the rule of Armijo

[Pol97], which has been proved an efficient line search method. In the Armijo rule, the step size continues to be reduced in half until the prediction error is reduced. On the other hand, if a minimum of the step size is reached, only then the variation $\underline{v}_{k,M}$ is being recomputed.

As in all NMPC strategies, the first element of the control vector is applied into the system: $u_k = p_1 \underline{v}_{k,M}$ where $p_1 = \begin{bmatrix} I_m & 0_{m\times(M-1)m} \end{bmatrix}$.

The control vector $\underline{u}_{k,M}$ is then, updated forward by one step:

$$\underline{u}_{k+1,M} = \begin{bmatrix} u_1(k+1) \\ \vdots \\ u_{M-1}(k+1) \\ u_M(k+1) \end{bmatrix} = \begin{bmatrix} v_2(k) \\ \vdots \\ v_M(k) \\ u^*(k) \end{bmatrix} \tag{9.12}$$

where $u^*(k) \in \mathbb{R}^m$ is the equilibrium control

$$f(x^*(k)) + g(x^*(k))u^*(k) = x^*(k)$$

Equation (9.12) can be written in stack vector form as:

$$\underline{u}_{k+1,M} = G\underline{v}_{k,M} + Fu^*(k) \tag{9.13}$$

with $G \in \mathbb{R}^{mM\times mM}$, $F \in \mathbb{R}^{mM\times Mm}$ being defined as:

$$G = \begin{bmatrix} 0_{m(M-1)\times m} & I_{m(M-1)} \\ 0_{m\times m} & 0_{m\times m(M-1)} \end{bmatrix} \qquad F = \begin{bmatrix} 0_{m(M-1)\times m} \\ I_m \end{bmatrix}$$

Initial predictive control actions are chosen as zero vectors without loss of generality. Under the assumption that $(\nabla \underline{u}_{k,M} \phi_M(x(k), \underline{u}_{k,M}))^\dagger$ is of full rank for all k, it has been proved in [CN95], that the predicted state error will converge to zero as $k \to \infty$, although we have not an a priori time- specifiable convergence. It is also stated that the actual state $x(k)$ converges to $x^* = const.$ as $k \to \infty$. As far as the control signal $u(k)$ is concerned, it is uniformly bounded for all k and the elements of $\underline{u}_M(k)$ tend to $u^* = const.$ as $k \to \infty$.

The stability of this particular NMPC for point convergence has been proved using

Lyapunov theory for driftless, as well as control affine systems, [Yoo02]. The authors in [Wro04] have rendered it to a trajectory tracking controller that has shown sufficiently good results. A robust stability analysis of this NMPC strategy, for a general nonlinear system of the form (9.8), has also been conducted in [Yoo02]. The authors have analyzed the case of bounded noise and measurement noise. The dynamic equations of the perturbed system, are

$$x(k+1) = f(x(k)) + g(x(k))u(k) + w(k)$$

and $y(k)$ is the state measurement used for the feedback

$$y(k) = x(k) + s(k)$$

which is also perturbed by a bounded noise. The actual state evolving through this perturbed system is bounded provided that the bounds of the noise

$$w_{max} = \sup_k ||w(k)|| \qquad \text{and} \qquad s_{max} = \sup_k ||s(k)||$$

are sufficiently small. In the subsequent section, we use this property of the proposed scheme, because force signals can be quite noisy. Moreover the properties of the environment can also act as a disturbance to the model.

9.4 Simulation Setting & Results

9.4.1 Simulation Setting

A two-link, planar robotic manipulator, with $n = 2$, is considered in this section, as in Fig. 9-2. For illustrative purposes the numerical values of the parameters of the dynamic model of the manipulator are taken as in [LK97].

The end-effector geometric Jacobian $J^T(q) \in \mathbb{R}^{2\times 2}$ of the dynamic model of the 2 d.o.f.

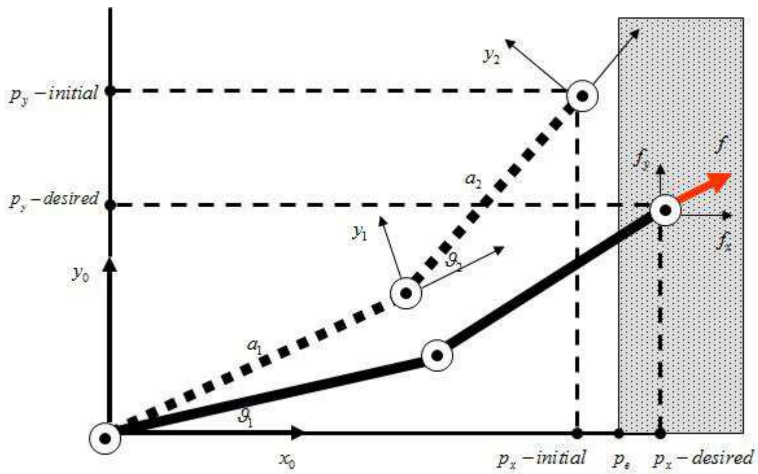

Figure 9-2: The 2 d.o.f. planar manipulator in interaction with the environment. The initial position of the end-effector is $(p_x, p_y)_{\text{initial}}$, final destination is $(p_x, p_y)_{\text{desired}}$, while position of the vertical wall is denoted by p_e.

manipulator is given by:

$$J(q) = \begin{bmatrix} -\alpha_1 s_1 - \alpha_2 s_{12} & -\alpha_2 s_{12} \\ \alpha_1 c_1 + \alpha_2 c_{12} & \alpha_2 c_{12} \end{bmatrix} \tag{9.14}$$

where only the rows that are relevant to the task appear; these refer to the two components of linear velocity along the Cartesian axes x_0, y_0.

The notations $s_{i\ldots j}$, $c_{i\ldots j}$ denote respectively $\sin(q_i + \cdots + q_j)$, $\cos(q_i + \cdots + q_j)$ and will be used also in the remainder of the paper.

The end-effector position is determined by the two coordinates $p_x, p_y \in \Omega$, while its orientation is determined by the angle ϕ formed by the end-effector with the axis x_0. The direct kinematics equation can be written in the form:

$$\tilde{p} = \begin{bmatrix} p_x \\ p_y \\ \phi \end{bmatrix} = k(q) = \begin{bmatrix} \alpha_1 c_1 + \alpha_2 c_{12} \\ \alpha_1 s_1 + \alpha_2 s_{12} \\ \vartheta_1 + \vartheta_2 \end{bmatrix} \tag{9.15}$$

Since there is no desired final orientation, we also define $p = \begin{bmatrix} p_x & p_y \end{bmatrix}$.

The main objective is to drive the end-effector of the robot which initially does not interact with the environment to touch a planar space in a specified position $p_d \in \Omega$ and a specified velocity $\dot{p_d}$ and push it with its end-effector until a desired force f_d is achieved.

The environment is assumed to be a vertical wall with a known elastic coefficient. K_f is defined as

$$K_f = \begin{bmatrix} k_f & 0 \\ 0 & 0 \end{bmatrix} \tag{9.16}$$

where the value of $k_f = 10^3 N/m$. So the interaction forces are parallel only to the x_0-axis.

The position reference is used for the x_0 and y_0 coordinates, i.e., $p = \begin{bmatrix} p_x & p_y \end{bmatrix}^T$ where p is the actual position of the end effector. The equilibrium position of the environment without deformation, along the x_0-axis, is equal to $p_e = 1.8m$ as in Fig. 9-2.

The initial position of the end effector in the Cartesian plane is considered to be $p_{\text{initial}} = \begin{bmatrix} 1.75 & 0.7 \end{bmatrix}^T$. It is apparent that the manipulator is not in contact with the environment. The final destination of the end-effector is the desired position that is considered to be at $p_{\text{desired}} = \begin{bmatrix} 1.85 & 0.5 \end{bmatrix}^T$. The end effector should push the environment with a specified force that is $f_{\text{desired}} = \begin{bmatrix} 60 & 0 \end{bmatrix}^T$. It must be pointed out that from (9.1), the desired position corresponds to a force $f_{\text{cor}} = \begin{bmatrix} 50 & 0 \end{bmatrix}^T$, which is $f_{\text{desired}} \neq f_{\text{cor}}$.

9.4.2 Results

We choose a prediction horizon of 5 steps, i.e., $M = 5$, and a fixed step size equal to $\delta t = 5 \times 10^{-3} sec$. Note, that larger prediction horizon would result to larger computation periods. The initial guess for the first control input of the NMPC position algorithm is chosen randomly, while the initial guess for the Armijo parameter is chosen equal to $\beta_0 = 0.1$.

We assume that the model does not have ambiguities and is an exact representation of the actual manipulator. In this nominal case, the end-effector reaches exponentially the desired position on y_0 axis, while a neighborhood of the desired position on the x_0-axis is reached, as we see in Fig. 9-3. This happens because, as it has already been mentioned, in constrained directions the desired position is treated as a disturbance.

Fig. 9-4, shows a *sui generis* property of the particular NMPC, that has been described

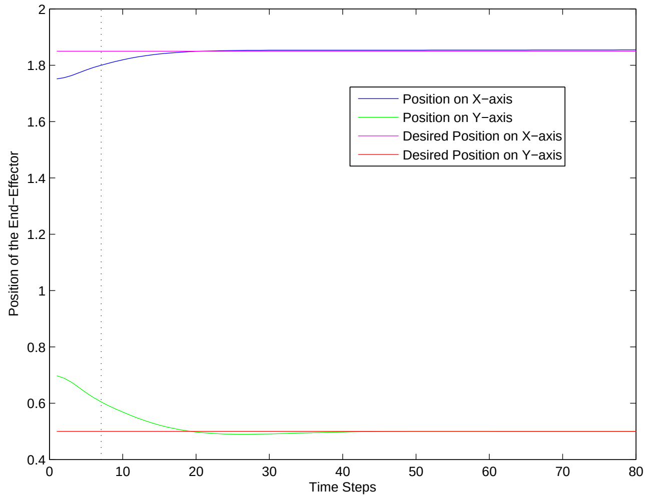

Figure 9-3: Nominal case. End-effector's position in x_0 and y_0 coordinates. Both reach a neighborhood of the desired position. The dotted line represents the time step when the end-effector reaches the vertical wall.

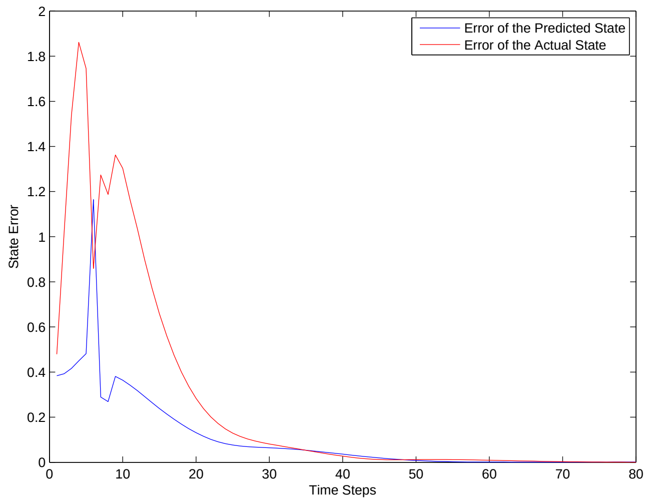

Figure 9-4: Nominal case. Error of the predicted state and error of the actual state of the nonlinear robotic system under the NMPC controller.

in Section ?. The predicted system state error, as well as the actual system state error, has an asymptotic convergence to zero.

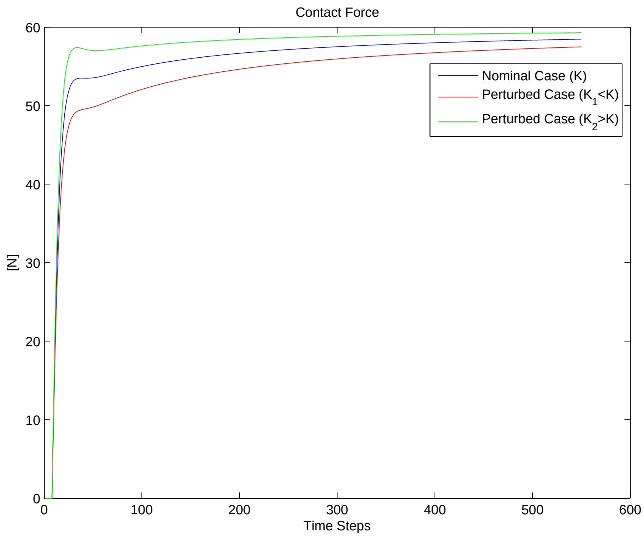

Figure 9-5: Force of the end-effector in the case of uncertainty of environment. The blue line represents the nominal case. The red line corresponds to the case where the actual environment has less stiffness than the modelled one, while the green line to the case where the actual environment stiffness is bigger than expected.

Fig. 9-5 depicts the contact force in three distinct cases. The first case is the nominal one, represented with the blue line. Using the preliminary results on the robustness of the proposed scheme we assume some uncertainty on the environment model. The environment compliance is modelled as $k_f = 1000N/m$ as in (9.16), which is used for the computation of the NMPC control law, while the actual stiffness matrix of the environment has a compliance of $\tilde{k}_{f_1} = 900N/m$ in the first perturbed case, and $\tilde{k}_{f_2} = 1100N/m$, in the second. The first perturbed case is represented with the red line, while the green line depicts the second perturbed case. All the other parameters are assumed to remain the same.

Notice that the force is zero in the unconstraint movement, i.e., when there is no interaction with the environment. The end-effector reaches the obstacle at time step $k = 9$ in all three cases. Simulations show that the algorithm still converges to the desired value. Actually in all cases, the contact force on the end-effector converges smoothly to the desired force as can be seen in Fig.9-5. It is evident that in a less compliant environment, the impact of the end-effector is bigger than in other cases.

9.4.3 Comparison

In the case of the common parallel position/force strategy where a PD controller with gravity compensation is used, under the constrained motion, the contact force reaches the desired value only after a transient period. The peak of this transient may appear due to the nonzero value of the end-effector velocity at the contact [CSV99]. If instead of the PD controller, we use a NMPC position controller; this peak in the force value during the impact is compensated.

Fig. 9-6 depicts the contact force of two parallel position/force controllers; the NMPC controller presented above, and a PD controller with gravity compensation. In order to compare the two cases, the NMPC controller, has been constrained in the control inputs with saturation limits, so that in both cases, torques never exceed the same threshold.

Simulations shows that in both cases, the actual force of the end-effector is obviously converging to the desired force smoothly. Nevertheless, the NMPC controller is compensating the external disturbances, i.e., the external force from the interaction of the manipulator with the environment. The transition from non-contact to contact at non-negligible end-effector speed under an NMPC controller shows a sufficiently good performance, while in the PD controller with gravity compensation case, a bigger impact force is apparent, something that might be undesirable in a number of situations.

9.5 Conclusion

In this Chapter, a parallel-like framework to stabilize a manipulator's end-effector to a desired position, and to apply a desired force when it interacts with the environment, has been presented. For the position control we have used an NMPC methodology. The motivation was that this control strategy is a natural candidate to compensate for gravity or other unmodeled uncertainties for which standard PD controllers seem unsatisfactory. Simulation results showed the effectiveness of the approach as well as its smooth behavior in the transition from the unconstrained to the constrained case.

Further research will exploit the virtues of this NMPC scheme, as the induction of unmodeled dynamics in the environment model like friction effects and changing payloads.

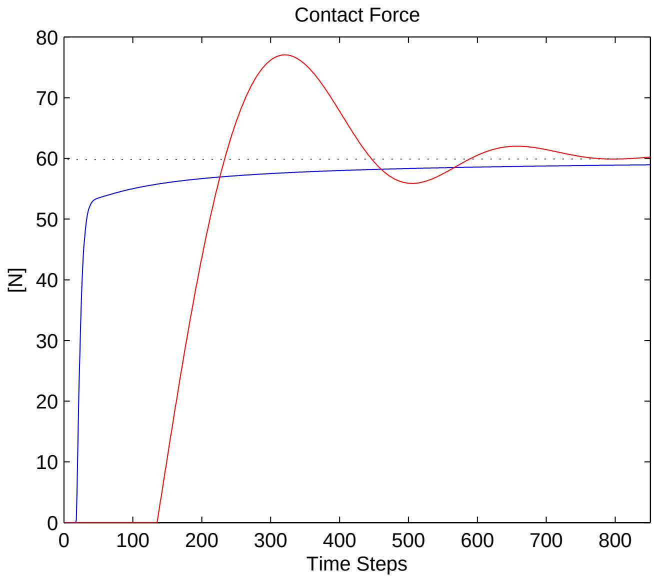

Figure 9-6: Contact force. The blue line represents the NMPC controller under the parallel scheme, with control constraints. The red line represents the PD controller with gravity compensation under the parallel scheme. The dotted line is the desired contact force.

A theoretical analysis of this phenomenon as well as of the general robustness of this algorithm is a topic of current research endeavors. Finally, the proposed real-time closed loop system will be tested in the experimental set up of the Control Systems Lab involving the Mitsubishi PA10 robotic manipulator [BAOK07].

Chapter 10

Contributions and Future Work

In this Chapter we are going to summarize the main contributions of this Dissertation. Also we are going to indicate some directions and open problems that are left for future research.

10.1 Contributions

The main contribution of this Dissertation is the event-based formulation of Model Predictive Controllers, as the title dictates. This scheme is first presented here and provides additional tools for the use of predictive controllers; in particular this formulation relaxes the time-triggered computation of the control law and provides sufficient conditions for triggering that can still guarantee the stability and the convergence of the closed-loop system.

More specifically the contributions are:

- The Event-Based Control framework particularly given for discrete-time general nonlinear systems was proposed in this Thesis. Moreover, this methodology was particularized for Linear Time Invariant systems. In addition, the proposed framework was further relaxed by the formulation of a Self-triggered framework, where no continuous monitoring of the actual state of the system is assumed.

- We formulated an Event-triggered scheme for Model Predictive Controllers for discrete-time systems affected by bounded and additive disturbances. This was conducted in a centralized manner for both discrete-time and continuous-time general nonlinear systems.

- Event-based formulation of MPC controllers for decentralized systems where the effect from the neighboring agents is considered as a perturbation of the nominal system. Also, this was extended to a distributed scheme where the formulation of the event-based MPC was regarding local controllers for agents that are cooperating in the same environment.

- Self-triggered formulation of MPC controllers for centralized and decentralized non-holonomic systems. The efficacy of the proposed scheme was evaluated by an experiment conducted in the Control Systems Laboratory, NTUA.

10.2 Future Work

Even though the Event-based formulation of Model Predictive Controllers was provided for a number of different scenarios, some extensions were not treated in this Thesis. In the sequel, we give the directions of our future research.

- In Chapter 2, the event-based formulation of general discrete-time systems was proposed. The simulation results in that Chapter, depict a periodicity that takes place in the event-triggered as well as in the self-triggered control for discrete-time systems. This behavior is very interesting but the formal analysis, is a topic of future research. Notice that some preliminary results have been presented by [VMB12] and [VML08b], where the notion of equilibrium sampling interval sequences is introduced. The authors suggest that the sampling intervals for event-driven control of continuous-time systems show different patterns, ranging from chaotic behaviors to periodic oscillatory patterns, named equilibrium sampling interval sequences (ESIS). This is an indication that similar behavior may be predicted for discrete-time systems, too.

- In this Thesis we assumed that the state measurement, either from the plant or the states of neighboring agents is always available and fully known. Thus, a straightforward extension would be to assume output-feedback controllers as well as to incorporate delays (imposed both by network traffic or computational reasons) in the measurement. This should result to more practical results.

- In Chapter 4, future work involves finding a triggering condition in a similar cooperative NMPC framework, however in this case the triggering condition should depend only on local information of the agent, the event-broadcasting state and the predicted state information of the neighboring agents. This event-based approach should be able to reduce the load on the communication medium in addition to agents' energy consumption.

- The self-triggered formulation in Chapter 7, was provided for the stabilization of a real nonholonomic vehicle. An extension to this approach will be to utilize the self-triggering set-up when the camera's field-of-view loses the target (maybe because of an unexpected disturbance). In this case there is no feedback and the ST-MPC could be proven to be helpful.

- Future work with respect to Chapter 8 involves an extension of the proposed distributed framework using less abstractions and having more realistic formulation. Namely, finding triggering conditions under the presence of disturbances and in the case where the information received by the neighbors is either delayed or not accurately known.

- The event-based MPC framework with guaranteed performance is an interesting direction that we will explore in the future.

- The parallel force control scheme via NMPC of Chapter 9, can be formulated as an event-based parallel scheme.

- Finally, an interesting idea that we currently explore is to use the ET-MPC in a hierarchical scheme (inner & outer controller) to control a UAV (helicopter).

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Curriculum Vitae

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Doctoral Thesis

Title Event-Based Model Predictive Controllers

Supervisors Professor K.J. Kyriakopoulos & Assistant Professor Dimos V. Dimarogonas

Description This thesis explores the idea of combining the well-known MPC scheme with the emerging field of event-based control for constrained, discrete & continuous nonlinear systems under disturbances.

Diploma Thesis

Title Hydraulic & Pneumatic Systems. Modeling of Transition State of an Open Pneumatic Cylinder.

Supervisor Professor Th.N. Kostopoulos

Description In this thesis a pneumatic cylinder was modeled as a system of nonlinear differential equations and two different approaches of finding the solution for this kind of systems were explored and compared.

Research Interests

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Publications

[1] Alina Eqtami, Dimos V. Dimarogonas and Kostas J. Kyriakopoulos, "Event-triggered Control for Discrete-Time Systems", 2010 American Control Conference, Baltimore, MD, USA, pp. 4719-4724, July 2010.

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