Εθνικό Μετσόβιο Πολυτεχνείο
Σχολή Ηλεκτρολόγων Μηχανικών
και Μηχανικών Υπολογιστών
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Ψηφιακές επικοινωνίες σε κανάλια γενικευμένων διαλείψεων

ΔΙΠΛΩΜΑΤΙΚΗ ΕΡΓΑΣΙΑ

Λυδία, Νικολάου Πολύζου

Επιβλέπων : **Αθανάσιος Δ.Παναγόπουλος**
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Αθήνα, Δεκέμβριος 2013

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Διπλωματούχος Ηλεκτρολόγος Μηχανικός και Μηχανικός Υπολογιστών Ε.Μ.Π.

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Απαγορεύεται η αντιγραφή, αποθήκευση και διανομή της παρούσας εργασίας, εξ ολοκλήρου ή τμήματος αυτής, για εμπορικό σκοπό. Επιτρέπεται η ανατύπωση, αποθήκευση και διανομή για σκοπό μη κερδοσκοπικό, εκπαιδευτικής ή ερευνητικής φύσης, υπό την προϋπόθεση να αναφέρεται η πηγή προέλευσης και να διατηρείται το παρόν μήνυμα. Ερωτήματα που αφορούν τη χρήση της εργασίας για κερδοσκοπικό σκοπό πρέπει να απευθύνονται προς τον συγγραφέα.

Οι απόψεις και τα συμπεράσματα που περιέχονται σε αυτό το έγγραφο εκφράζουν τον συγγραφέα και δεν πρέπει να ερμηνευθεί ότι αντιπροσωπεύουν τις επίσημες θέσεις του Εθνικού Μετσόβιου Πολυτεχνείου.

Digital Communications over Generalized Fading Channels

Lydia Polyzou

February 13, 2014

Περίληψη

Σκοπός της παρούσας διπλωματικής εργασίας είναι η παρουσίαση των φαινομένων των διαλείψεων και των πολλαπλών λήψεων στις ασύρματες κινητές επικοινωνίες. Οι διαλείψεις αποτελούν ένα από τα μεγαλύτερα εμπόδια στη σχεδίαση αποδοτικών συστημάτων ενώ οι πολλαπλές λήψεις αποτελούν έναν από τους πιο απλούς και ταυτόχρονα αποδοτικούς τρόπους αντιμετώπισης του προβλήματος των διαλείψεων.

Η παρούσα διπλωματική διαρθώνεται ως εξής: Αρχικά γίνεται μια περιληπτική αναφορά στις ασύρματες κινητές επικοινωνίες, προκειμένου να γίνει κατανοητή η σημασία τους και να υπογραμμιστεί η προβληματική συμπεριφορά που προκύπτει από τις διαλείψεις. Στη συνέχεια γίνεται μια εκτενής αναφορά στις διαλείψεις, στην οποία παρουσιάζονται διάφορα μοντέλα που τις εξηγούν. Σχετικά με τις πολλαπλές λήψεις παρουσιάζονται διάφορες τεχνικές, στις οποίες οι διάφορες συνιστώσες του σήματο συνδυάζονται προκειμένου να παράξουν ένα αποδοτικό αποτέλεσμα.

Για την υποστήριξη της θεωρητικής ανάλυσης πραγματοποιήθηκε και προσομοίωση του συστήματος και για την περίπτωση των διαλείψεων και των πολλαπλών λήψεων. Επίσης γίνεται εκτενής συζήτηση επί των αποτελεσμάτων που παρουσιάζονται.

Καθόλη τη διάρκεια της παρούσας εργασίας δίδεται ιδαίτερη έμφαση στη γενικευμένη-K κατανομή διαλείψεων, μια κατανομή που είναι ιδαίτερα χρήσιμη, καθώς αποτελεί μια από τις λίγες κατανομές που βρίσκουν εφαρμογή σε πραγματικά περιβάλλοντα μετάδοσης, ενώ παράλληλα περιέχει άλλες κατανομές σαν ιδιαίτερες περιπτώσεις.

Λέξεις Κλειδιά

ασύρματες κινητές επικοινωνίες, διαλείψεις, σκίαση, μοντέλο απωλειών διαδρομής, μοντέλο πολλαπλών διαδρομών, γενικευμένες κατανομές διαλείψεων, γενικευμένη-K κατανομή, πολλαπλές λήψεις, συνδυαστική μεγίστου λόγου, συνδυαστική σταθερού κέρδους, επιλεκτικός συνδυασμός, συνδυαστική μεταγωγής και παραμονής

Abstract

The purpose of this thesis is to present fading and diversity in mobile wireless communication systems. Fading is a great obstacle in the design of efficient systems and diversity is a powerful mean, used to overcome it.

In this thesis we will make a brief overview of mobile wireless communication systems, in order to denote the importance of those systems and outline the problems caused by fading. As far as fading is concerned, we make a detailed presentation of various models used to describe fading. As far as diversity is concerned, we present several techniques, used to combine the signal and output the most efficient result.

In order to support the theoretical analysis made in both fading and diversity, simulations of the systems under consideration are made. We discuss various results and compare diversity combining methods and fading channel models.

Throughout the whole thesis, special focus is given to the generalized-K fading distribution, a distribution that is very useful, as it includes other distributions as special cases and combines different types of fading, which means that it can apply in a realistic propagation environment, something that is not the case for numerous fading distributions.

Key words

wireless mobile communications, fading, shadowing, path loss, multipath fading, generalized fading distributions, generalized-K fading, diversity, maximal ratio combining, equal gain combining, selection combining, switch and stay combining

Contents

Introduction

Outline of the Thesis

This thesis is organized as follows:

In the introductory part a brief overview of wireless mobile communication systems is made, with the presentation of specific systems, i.e. cellular systems and mobile satellite communication systems.

In the first part fading is analyzed,where various models are presented that are used to describe fading for different propagation environments. In the end of the part a detailed performance analysis of fading in wireless mobile communication systems is made.

In the second part, where diversity is explained, several combining techniques are presented and in the end of the part, a detailed analysis of their performance is made.

The third part consists of the simulation of single receiver and multi receivers systems. In this part the MATLAB environment is used, where various combining techniques are simulated for the case of Rayleigh and generalized-K fading channels.

1. Wireless Mobile Communication Systems

Wireless mobile communication systems have become part of our everyday life with the use of cell phones, lap top computers and PDAs. In this introductory chapter we present cellular and mobile satellite systems and discuss some technical challenges that arise in the system design. In general, the design of robust and cost effective systems is a matter of major importance and great difficulty. Challenges are faced both in the transmitter and the receiver, but our focus will be on the channel, a difficult medium to predict and the main subject of the present thesis.

1.1. Cellular Systems

In cellular systems, a geographic region is divided into non overlapping land areas called cells, in order to achieve frequency reuse,which can be achieved between spatially separated cells in the following way: neighbor cells use different set of frequencies to avoid interference and provide guaranteed bandwidth in each cell. Thus spatially separated cells reuse frequency, time slots and codes. In this way the frequency spectrum needed is reduced but simultaneously there is co-channel interference between the spatially separated cells. Luckily this interference is within an acceptable minimum,thus not affecting the performance of the system. As far of the structure of the cells is concerned, each cell is served by its own base station consisting of a transmitter, receivers and a control unit. In order to determine the base station placement and an efficient distance between two spatially separated cells, an accurate characterization of signal propagation within the cells is needed, which can be quite difficult since both the transmitting and interfering signals experience random variations due to the nature of the wireless signal propagation [1].

Initially the size of the cells was quite large. Due to the high cost of base stations, each cell covered a large area of land. In this case the hexagonal shape for the cell was found more convenient. Such a cellular system is depicted in 1.1. The numbers from A-F denote the set of frequencies that each cell uses. As it can be seen neighbor cells do not use the same set of frequencies, while spatially separated cells use the same set of frequencies. The typical hexagonal shape consists of 7 cells and therefore 7 different frequency sets. Nowadays, smaller cells are used permitting a higher number of users per unit area. In such systems the hexagonal shape is in general not a good approximation[1].

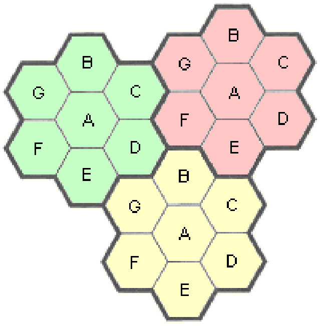

Figure 1.1.: Typical Cellular Network [2]

Figure 1.2.: Iridium 9505 SAT Phone [3]

1.2. Mobile Satellite Systems

Mobile satellite systems bare a great similarity to cellular systems, except that the base stations are now moving. The base stations are the satellites, which are very attractive since they can cover areas of world not well served by the existing terrestial infrastructure, such as ocean areas.

Satellite systems offer a variety of services including communications to ships, vehicles, planes and hand-held terminals, TV and radio broadcasting. There is also a type of mobile phone, a satellite phone, that connects to orbiting satellites instead of terrestrial cell sites. Many companies are active in this field, such as Inmarsat [4], a British satellite telecommunications company offering global mobile services through eleven geostationary telecommunication satellites or Iridium [5], a USA company that offers worldwide voice and data communication from hand-held satellite phones and other transceiver units with a system of 66 active satellites. A typical satellite phone is shown in 1.2, a solution very attractive due to its wide coverage but not affordable for everyone.

Another satellite technology widely used is DVB-SH, a standard use for delivering IP based media content and data to mobile terminals supporting such protocols. It was introduced in 2007, as an improvement to the existing DVB-H physical layer. DVB-SH uses both terrestial and satellite technology, which enables a coverage of regions much larger than DVB-H. It is designed to use frequencies under 3GHz, around 2.2GHz [6]. According to the transmission method used in the satellite, we distinguish two architectures: DVB-SH-A and DVB-SH-B, where in the first OFDM is used in both the terrestial and satellite link, while in the second TDM is used in the satellite link.

1.3. Technical Issues

Technical challenges are faced across all the aspects of the system design. In terms of the transmitter and the receiver, power consumption is one of the main issues. Specifically, existing batteries and other energy storage technologies are a critical factor in the production of new services, such as sensor networks[7]. Also, in order to support user mobility, complex software is needed, so the processing burden must be placed if possible in fixed sites with large power resources or distributed among the terminals, which is the case of Ad Hoc wireless networks.

In terms of the channel, technical challenges arise in many cases. Due to the wide appeal of wireless mobile communications, the number of frequencies used becomes higher, but the spectrum cannot be used completely and the systems are designed to operate under specific frequency interval. Therefore a new necessity is the design of higher frequency systems. Due to the nature of wireless communications, as a signal propagates through space it experiences various fluctuations, therefore the channel changes randomly over time, thus the design of reliable systems with guaranteed performance becomes difficult [1]. The above phenomena are described as fading, which will be thoroughly described in the following section. Fading is an important drawback in the design of an efficient system. that is combined with a need of higher frequency systems, a need caused by the scarcity of spectrum.

Part I.

Fading

2. Overview of Fading Channel Models

This chapter presents an overview of the most important fading channel models. Fading is defined as the deviation of the attenuation, affecting a signal as it propagates through a certain medium. The fading channel models depict the effects of electromagnetic transmission of information over the air in cellular networks and broadcast communication as well as the distortion caused by the water in underwater acoustic communications.Fading is more severe in coherent modulation schemes where phase coordination is needed. Therefore most of the times an ideal coherent modulation scheme is assumed, so only the fading envelope is statistically modeled. The deviation can be categorized according to the distance between the transmitter and the receiver into short and long term fading, due to the fact that some variations affect the system in relatively large distances while others in very short.In short term fading we will examine multipath fading and in long term fading path loss and shadowing. The effects of multipath, path loss and shadowing are depicted in Fig.2.1. As we can see path loss is a straight line with an attenuation factor that can be easily predicted, while shadowing and even more multipath are harder to predict.

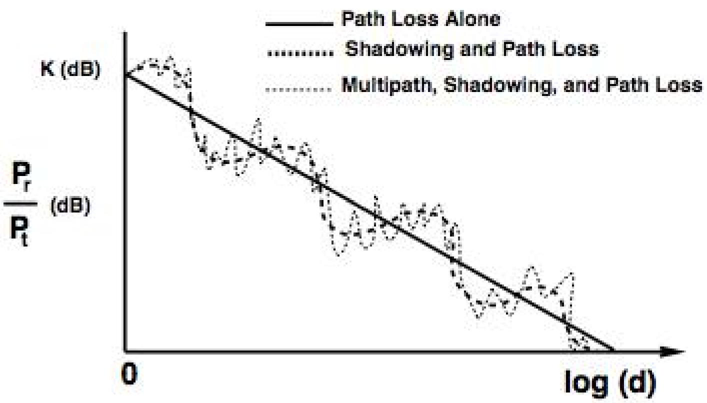

Figure 2.1.: Path loss, Shadowing and Multipath according to distance (logarithmic scale)[1]

2.1. Short Term Fading

Short term fading consists of multipath fading, which occurs in distances in the order of the wavelength. Multipath is the propagation phenomenon that results in radio signals reaching the receiving antenna by two or more paths. In particular, electromagnetic waves interact with physical objects through scattering, reflection, diffraction and absorption. This leads to a continuous distribution of partial waves [8, 9], where each has an amplitude, phase and delay defined by the physical properties of the surface. These partial waves are added in the receiver, having a constructive or destructive impact on the receiving signal. Causes of multipath include atmospheric ducting, ionospheric reflection, and refraction and reflection from water bodies and terrestrial objects. Multipath fading models are used to model the effects of electromagnetic transmission over the air in cellular networks and broadcast communication and the distortion caused by the water, which is the case of underwater communications. Multipath fading can be classified in slow and fast and frequency flat and frequency selective fading. As the latter indicates this classification is done according to frequency criteria while the first according to time criteria. The classification according to both time and frequency will be analyzed in the sections 2.1.1 and 2.1.2 respectively.

2.1.1. Slow and Fast Fading

According to time, we can categorize fading into slow and fast fading. This distinction is important for the mathematical modeling of fading channels and for the performance evaluation of communication systems operating over those channels [10]. Due to the movement of the transmitter or the receiver the Doppler effect occurs, in which the phase of the signal changes significantly at intervals of the inverse of the Doppler shift. When there are several paths, the Doppler shift varies in each path the magnitude of the resultant signal changes significantly at intervals of the Doppler spread f_d [11], which is defined as the largest difference between the Doppler shifts. We also define the coherence time T_c as

$$T_c \backsimeq 1/f_d \tag{2.1.1}$$

The coherence time is the time duration over which the channel impulse response is relatively constant and the fading process is correlated. The classification to slow and fast fading is closely related to the coherence time. A channel is called fast fading if T_c is much shorter than the delay requirement of the application, and slow if it is much larger [11]. Thus the specification of the channel as fast or slow depends not only on the environment but also on the application and the demanding bit rate [11]. In practice this specification depends strongly on the bit rate of the link. As data rates increase the channel is better described as a slow fading while as data decreases it is better described as a fast one [12].

2.1.2. Frequency Flat and Frequency Selective Fading

Due to multipath fading, the transmitted signal is received by several paths. The difference in propagation time between the shortest and the longest path is defined as the delay spread T_d [11],the inverse of which is connected to the significant change of the phase of the signal. We also define the coherence bandwidth as

$$f_c \backsimeq 1/T_d \tag{2.1.2}$$

The coherence bandwidth is the frequency range over which the fading process is correlated and is strongly related to the distinction between frequency flat and frequency selective fading. A channel is called frequency flat fading when the bandwidth of the input is much smaller than f_c, where all the spectral components are affected in a similar manner and is the case of narrowband systems. On the other hand when the bandwidth of the input is much larger than f_c, the channel is called frequency selective fading. In this case the spectral components of the transmitted signal are affected by different amplitude gains and phase shifts, which applies to wideband systems.

2.1.3. Modeling of Frequency Flat Channels

Depending on the particular propagation environment and the underlying communication scenario various models are proposed to describe the fading envelope's statistics. The simplest distributions assume a homogeneous diffuse scattering field, resulting from randomly distributed point scatterers. Throughout the presentation a narrowband digital receiver is assumed. The fading amplitude at the input of the receiver (α) is a random variable with mean-square value $\Omega=\overline{a^2}$ and PDF $p_a(a)$ determined by each model. The narrowband system can be modeled as $z = as+n$, where s is the transmitted symbol and n is the AWGN, which varies independently of a. The instantaneous SNR per symbol is $\gamma=a^2E_s/N_o$, where $E_s = E(|s^2|)$ is the energy per symbol. The corresponding average SNR per symbol $\overline{\gamma} = \Omega E_s/N_o$. From $p_a(a)$ we can the PDF of γ as

$$p_\gamma(\gamma) = \frac{p_a\left(\sqrt{\Omega\gamma/\overline{\gamma}}\right)}{2\sqrt{\overline{\gamma}\gamma/\Omega}} \tag{2.1.3}$$

Depending on whether there was a direct line between the transmitter and the receiver (LOS), the Rayleigh model and the Rice model were introduced. Later empirical models like the Nakagami-m were introduced that are presented below.

2.1.3.1. Rayleigh Fading Model

The Rayleigh distribution is often used to describe multipath fading with no LOS path, when there are many objects that scatter the signal before it arrives to the receiver. It was introduced by Lord Rayleigh in 1889, describing the resulting signal if many violinists in an orchestra play in unison[7, 13].It can be used to describe mobile communications [10], propagation through the troposphere and ionosphere as well as ship-to-ship radio links [14, 15, 16, 17]. If there is sufficient scatter, according to the central limit theorem, the channel impulse response can me modeled as a zero mean complex Gaussian process. The PDF of the fading amplitude a is given as

$$p_a(a) = \frac{2a}{\Omega} \exp\left(-\frac{a^2}{\Omega}\right),\ a \geq 0 \tag{2.1.4}$$

by substituting (2.1.4) in (2.1.3) we obtain the PDF of γ as

$$p_\gamma(\gamma) = \frac{1}{\overline{\gamma}} \exp\left(-\frac{\gamma}{\overline{\gamma}}\right),\ \gamma \geq 0 \tag{2.1.5}$$

2.1.3.2. Rice Fading Model

The Rice distribution is suitable to describe propagation paths consisting of one strong direct LOS component and many weaker components. It was named after Stephen O.Rice[18]. The model can be used to describe micro-cellular urban and suburban land-mobile communications, pico-cellular indoor and factory environments[19, 20, 21, 22]. It also applies to the dominant LOS path of satellite and ship-to-ship radio links [10].

The Rice distribution is related to the non-central Chi-square distribution with a PDF of the amplitude a as:

$$p_a(a) = \frac{2(1+n^2)e^{-n^2}a}{\Omega} \exp\left(-\frac{(1+n^2)a^2}{\Omega}\right) I_o\left(2na\sqrt{\frac{1+n^2}{\Omega}}\right),\ a \geq 0 \tag{2.1.6}$$

by substituting (2.1.6) in (2.1.3) we obtain the PDF of γ as

$$p_\gamma(\gamma) = \frac{2(1+n^2)e^{-n^2}}{\overline{\gamma}} \exp\left(-\frac{(1+n^2)\gamma}{\overline{\gamma}}\right) I_o\left(2n\sqrt{\frac{(1+n^2)\gamma}{\overline{\gamma}}}\right),\ \gamma \geq 0 \tag{2.1.7}$$

2.1.3.3. Nakagami-m Fading Model

The Nakagami-m fading model is not based on results derived from physical consideration of radio propagation. It was introduced by M.Nakagami in 1945 in [23]. The model gives often the best fit to land-mobile and indoor-mobile multipath propagation, as well as scintillating ionospheric radio links [24, 25]. The Nakagami-m fading model is also very important because it is mathematically tractable, leading to closed form analytical expressions. The PDF of the fading amplitude a is given as

$$p_a(a) = \frac{2m^m a^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{ma^2}{\Omega}\right),\ a \geq 0 \tag{2.1.8}$$

by substituting (2.1.8) in (2.1.3) we obtain the PDF of γ as

$$p_\gamma(\gamma) = \frac{2m^m \gamma^{m-1}}{\overline{\gamma}\Gamma(m)} \exp\left(-\frac{m\gamma}{\overline{\gamma}}\right),\ \gamma \geq 0 \tag{2.1.9}$$

2.2. Long Term Fading

Long term fading consists of path loss that occurs over distances (100-1000m) and shadowing that occurs over distances (10-100m), which are relatively large.

2.2.1. Path Loss

Path loss is the attenuation of the power of the electromagnetic wave as it propagates through space because of the large distance between the transmitter and the receiver.The model mostly used to describe this attenuation is free space path loss. In this model no obstacles that can cause reflection, diffraction, absorption or scattering of the signal are assumed. Thus a straight line, i.e. a line that is not blocked by obstacles, exists between the transmitter and the receiver. This line is called the line-of-sight (LOS) and it is the only path that exists between the transmitter and the receiver.

Free space path loss is used to describe satellite and microwave systems and millimeter wave systems, radio systems with high antenna directivity, and is a very useful model as it maps the signal's power attenuation according to the distance. Therefore it is a major component in the analysis and design of the link budget of a communication system.

2.2.2. Shadowing

Shadowing is the random variation of the signal power due to absorption,diffraction and scattering. It occurs when the main signal path between the transmitter and the receiver is obscured by obstacles causing a partial loss of the signal. When the attenuation is strong, the signal is blocked. Shadowing can be also viewed as the spatial mean of multipath fading [26].

Shadowing occurs in terrestrial and satellite land-mobile systems terrain, where buildings and trees affect the link quality by slow variation of the mean signal level. The log-normal distribution has been empirically confirmed to accurately model these variations in both indoor and outdoor environments and is usually used to describe shadowing [27].

Log Normal Fading Model

Except for terrestrial and satellite land-mobile systems the log normal fading model is used to describe wireless optical communication systems under weak atmospheric turbulence conditions [28, 29, 30, 31, 32, 33]. In general though, such variations affect the system's performance after the effects of multipath fading are eliminated.

In log-normal shadowing, the PDF of the instantaneous SNR per symbol γ is distributed according to:

$$p_{\gamma}(\gamma) = \frac{\xi}{\sqrt{2\pi}\sigma\gamma} \exp\left[-\frac{(10log_{10}\gamma - \mu)^2}{2\sigma^2}\right] \tag{2.2.1}$$

where $\xi = \frac{10}{ln10} = 4.3429$, $\mu(dB) = E[10log_{10}\gamma]$ and $\sigma(dB) = \sqrt{Var\,[10log_{10}\gamma]}$.

2.3. Generalized Fading Distributions

The above distributions assumed a homogeneous diffuse scattering field. This assumption simplifies the analysis but it cannot apply in a realistic propagation environment, where the surfaces are spatially correlated [8, 9]. To overcome this limitation more generic distributions have been proposed that apply to a non-homogeneous medium, such as generalized-Gamma,κ-μ and η-μ distributions. Furthermore in a realistic propagation environment multipath fading and shadowing co-exist, which gives rise to composite fading distributions, that take into consideration both multipath fading and shadowing, which is the case generalized K distribution. In this section we thoroughly describe these distributions.

2.3.1. Generalized Gamma Fading Model

The generalized Gamma (GG) distribution was introduced by Stacy in 1962 [34]. It includes as special cases the Weibull and the Nakagami-m fading model. The use of the latter one was questioned by some researchers [35] because its tail doesn't seem to be a good fitting to experimental data. Due to its generic form it can accurately describe the behavior of multipath and shadowing effects, which is the typical channel model observed in land mobile satellite communications [36].

The physical model considers multipath waves propagating in a non-homogeneous environment, which are grouped into clusters according to the delay times. In each cluster all the waves have similar delay times with no limitation opposed to the phases, and the delay-time spreads of different clusters are relatively large. The envelope is a non-linear function of the modulus of the sum of the multipath components [35]. This non-linearity is expressed in terms of a power parameter β, such that the resulting signal intensity is obtained not simply as the modulus of the sum of the multipath components but as this modulus to a certain given power.

The PDF of the fading envelope is given as

$$p_a(a) = \frac{\beta a^{m\beta-1}}{(\Omega\tau)^{\frac{m\beta}{2}}\Gamma(m)} \exp\left[\left(\frac{a^2}{\Omega\tau}\right)^{\beta/2}\right],\ a \geq 0 \tag{2.3.1}$$

where $\beta > 0, m > 1/2$ are the distribution's shaping parameters and

$$\tau = \frac{\Gamma(m)}{\Gamma(m + \frac{2}{\beta})} \tag{2.3.2}$$

For $\beta = 2, m = 1$ (2.3.2) reduces to the Rayleigh distribution, for $\beta = 2$ to the Nakagami-m distribution and for $m = 1$ to the Weibull model. Finally if $\beta = 0$ and $m \to \infty$ it approaches the log-normal distribution.

By substituting (2.3.1) in (2.1.3) we obtain the PDF of γ as

$$p_\gamma(\gamma) = \frac{\beta\gamma^{m\beta/2-1}}{2(\overline{\gamma}\tau)^{m\beta/2}\Gamma(m)} \exp\left[\left(\frac{\gamma}{\overline{\gamma}\tau}\right)^{\beta/2}\right] \tag{2.3.3}$$

2.3.2. Generalized K Fading Model

In several cases multipath fading and shadowing coexist. Such cases often occur in congested downtown areas with slow-moving pedestrians and vehicles and in land-mobile satellite systems subject to vegetative and/or urban shadowing [10]. In this environment the receiver cannot mitigate fading by averaging out the envelope due to multipath, because due to shadowing the average power of the fading amplitude is not anymore deterministic and becomes random.

Composite distributions are introduced to solve this problem but most of them model the average power using the log normal distribution, which leads to mathematically complicated distributions that are hard to apply in practice. The Rayleigh-log normal (R-L) fading is mathematically complicated, while the Nakagami-m log normal fading does not lead to closed form expression for the received signal power [10]. The use of the Gamma distribution instead of the log normal distribution led to mathematically more tractable distributions. The generalized-K (K_G) distribution, which is a mixture of Nakagami-m and Gamma distribution, has both theoretical and experimental support [37, 38]. It includes the K distribution as a special case and is more flexible than the former because it has two parameters available compared to the one that K distribution has. It does also include the Rician and the Double-Rayleigh fading model. The fading model can also accurately approximate the R-L distribution.

The PDF of the fading amplitude a can be obtained by superimposing the Nakagami-m and the Gamma distribution as follows:

$$p_a(a) = \int p_{a|\Omega}(a|\Omega) p_\Omega(\omega) d\omega \tag{2.3.4}$$

where $p_{a|\Omega}(\cdot)$ is the PDF of the fading amplitude a following the Nakagami-m distribution, i.e.

$$p_{a|\Omega}(a|\omega) = \frac{2m^m a^{2m-1}}{\omega^m \Gamma(m)} \exp(-\frac{ma^2}{\omega}),\ a \geq 0 \tag{2.3.5}$$

where ω is following the Gamma distribution, i.e.

$$p_\Omega(\omega) = \frac{k^k \omega^{2k-1}}{Y^k \Gamma(k)} \exp(-\frac{k\omega}{Y}),\ \omega \geq 0 \tag{2.3.6}$$

where $Y = E\{\Omega\}$. By averaging the Nakagami-m PDF (2.3.5) over the Gamma PDF(2.3.6), (2.3.4) becomes:

$$p_a(a) = \int p_{a|\Omega}(a|\Omega) p_\Omega(\omega) d\omega = \frac{4\Psi^{k+m}}{\Gamma(k)\Gamma(m)} a^{k+m-1} K_{k-m}(2\Psi a),\ a \geq 0 \tag{2.3.7}$$

where $\Psi = \sqrt{\frac{km}{\Omega}}$ and $K_{k-m}(\cdot)$ is the modified Bessel function of the second kind and order $k - m$.

By substituting (2.3.7) in (2.1.3) we obtain the PDF of γ as

$$p_\gamma(\gamma) = \frac{2\Xi^{\frac{\beta+1}{2}}}{\Gamma(k)\Gamma(m)} \gamma^{\frac{\beta-1}{2}} K_a(2\sqrt{\Xi\gamma}),\ \gamma \geq 0 \tag{2.3.8}$$

where $\beta = k + m - 1$, $a = k - m$ and $\Xi = \sqrt{\frac{km}{\bar{\gamma}}}$.

K_G is a two parameter (k, m) distribution, thus being able to describe various fading and shadowing models by using different combinations of k and m. For $k \to \infty$ it approximates the Nakagami-m distribution, for $m = 1$ it is the K distribution that approximate models the Rayleigh-log normal fading conditions. Also for $k \to \infty$ and $m \to \infty$ it approximates the AWGN channel, i.e. the channel without fading.

The K_G fading distribution can be employed to model the power statistics of cascaded multipath fading channels, i.e. channels that occur in propagation through keyholes or in serial relaying communications employing fixed gain relays [39, 40]. The squared K_G distribution can be used in the performance analysis of free-space optical (FSO) communications systems over atmospheric turbulence channels [41, 42, 43]. In FSO communications, the atmospheric turbulence results in rapid fluctuations at the received signal that severely degrade the optical link's performance. The Gamma-Gamma model in wireless optical communications theory is based on the assumption that small-scale irradiance fluctuations are modulated by large-scale irradiance fluctuations of the propagating wave, both modeled as independent Gamma distributions. This distribution has become the dominant fading channel model for FSO links due to its excellent agreement with measurement data for a wide range of turbulence conditions [41, 42].

2.4. Conclusions

In this chapter fading is analyzed in terms of short and long term fading with specific focus to the models used to describe flat fading channels. Fading is one of the most important and challenging obstacles to overcome in the design of wireless mobile communication system. These unpredictable fluctuations that the signal experiences while it propagates can lead to a malfunctioning system. Moreover the modeling of fading channels is a quite hard task and only the generalized fading models can apply to a more realistic environment. Therefore fading is still one inscrutable field that gathers a lot of interest.

3. Performance Evaluation of Digital Communication Systems over Fading Channels

In this chapter we will introduce the measures used to characterize the performance of fading channels. These include the channel capacity, the average bit error probability (ABEP), the outage probability and the amount of fading (AF). The channel capacity was pioneered by Shannon and is defined as the maximum rate of communication for which arbitrarily small error probability can be achieved,which is thoroughly presented in [1]. Channel capacity is the tightest upper bound on the rate of information that can be reliably transmitted over a communications channel. The average bit error probability (ABEP) is one of the most revealing metrics regarding the wireless system behavior and the one most often illustrated in technical documents containing performance evaluation of wireless communication systems [10]. The outage probability is the probability that a given rate will not be supported because of channel variations.Finally the amount of fading (AF) was first introduced by Charash in [44] and gives a unified measure about the severity of fading. In our analysis the use of the MGF, an alternative to the PDF of a variable, will be needed. A thorough analysis of it can be found in A.1.1.

3.1. Performance Measures

In this section we shall present the Channel Capacity,the Average Bit Error Probability, the Outage Probability and the Amount of Fading as measures used to evaluate the performance of fading channels.

3.1.1. Channel Capacity

For an AWGN channel with bandwidth B and power P the capacity C is found to be:

$$C = Blog_2(1+\gamma) \tag{3.1.1}$$

where $\gamma = P/(N_oB)$ is constant, with N_o the power spectral density of noise. For the case of flat fading channels γ is not constant. Moreover in the derivation of (3.1.1) no delay or complexity constraints are assumed. Therefore the Shannon capacity represents an optimistic upper bound that can serve as a benchmark against which spectral efficiency of practical transmission strategies can be compared [10]. In the case of flat fading channels and depending on the particular scenario and the ability of the transmitter or the receiver to adapt its strategy we have the following adaptive transmission schemes:

Optimal Rate Adaptation with Constant Transmit Power (ORA)

In this case Receiver CSI is assumed. The transmitter cannot adapt its transmission strategy, i.e. the transmit power is assumed constant. In this case the channel capacity in the Shannon's sense is given by (3.1.1) averaged by the PDF of the particular fading distribution, i.e.

$$C_{\langle ORA \rangle} = \int_{0}^{\infty} Blog_2(1+\gamma)p_\gamma(\gamma) \tag{3.1.2}$$

using the Jensen's inequality we obtain:

$$C_{\langle ORA \rangle} \leq log_2(1+\overline{\gamma}) \tag{3.1.3}$$

(3.1.3) implies that the capacity under the ORA policy is always worse than the Shannon capacity in an AWGN channel with the same average SNR, i.e. that fading reduces the capacity if the transmitter power is constant. Moreover if the Receiver CSI is not perfect the capacity can be significantly decreased.

Optimal Simultaneous Power and Rate Adaptation (OPRA)

In the case of simultaneous Transmitter and Receiver CSI the capacity can be found as

$$C_{\langle OPRA \rangle} = \int_{\gamma_o}^{\infty} Blog_2(\frac{\gamma}{\gamma_o})p_\gamma(\gamma)d\gamma \tag{3.1.4}$$

where γ_o is the optimal cutoff SNR below which the transmission is suspended. The optimal cutoff can be found by

$$\int_{\gamma_o}^{\infty} (\frac{1}{\gamma_o} - \frac{1}{\gamma})p_\gamma(\gamma)d\gamma = 1 \tag{3.1.5}$$

Channel Inversion with Fixed Rate (CIFR)

This scenario is a suboptimal adaptation scheme where the transmitter uses the CSI to maintain a constant received power. Because of that the channel appears at the encoder and decoder as an AWGN channel, i.e. the fading is inverted. The power adaptation that is called channel inversion is given in [1] as , where σ is the SNR of the equivalent AWGN channel. Under the constant received power constraint σ is found in [1] as

$$\sigma = \frac{1}{\int_0^\infty \frac{p_{\gamma(\gamma)}}{\gamma} d\gamma} \tag{3.1.6}$$

The channel capacity is obtained by substituting (3.1.6) in (3.1.1)

$$C_{\langle CIFR \rangle} = Blog_2(1+\sigma) = Blog_2 \left(1 + \frac{1}{\int_0^\infty \frac{p_{\gamma(\gamma)}}{\gamma} d\gamma} \right) \tag{3.1.7}$$

The variable $C_{\langle CIFR \rangle}$ is called zero-outage capacity since the data rate is fixed under all channel conditions. The zero-outage capacity can be also approximated in practice and many practical coding techniques are available in the open technical literature [1]. Channel inversion is also common in spread spectrum systems with near-far interference imbalances but it is not used in strong fading environments where zero-outage exhibits large data rate reduction and can be even zero as it is in the case of Rayleigh strong fading.

Truncated Channel Inversion (TCIFR)

Zero-outage capacity (3.1.7) is significantly smaller than the Shannon Capacity because it obtains channel inversion in all fading states. In truncated channel inversion the transmission is suspended in particularly bad channel states, so the data rates of the other states increase and so does the capacity. The outage capacity is defined as:

$$C_{\langle TCIFR \rangle} = max_{\gamma_o} Blog_2 \left(1 + \frac{1}{\int_0^\infty \frac{p_{\gamma(\gamma)}}{\gamma} d\gamma} \right) p(\gamma \geq \gamma_o) \tag{3.1.8}$$

3.1.2. Average Bit Error Probability (ABEP) - Average Symbol Error Probability (SEP)

The ABEP can be evaluated through the use of the PDF or the MGF of γ, leading to two approaches: the PDF and the MGF approach. A detailed presentation of both approaches can be found in [10]. For the generalized-K distribution the MGF approach is preferred [45] , which leads to a direct calculation of ABEP for non-coherent binary frequency-shift keying and differential binary phase-shift keying (DBPSK) modulation schemes or to calculation via numerical integration for binary phase-shift keying (BPSK), M-PSK,M-ary quadrature amplitude modulation (M-QAM) and M-DPSK since single integrals composed of elementary functions and with finite limits are obtained [45].

The average SEP, $\overline{P}_{se}$ can be evaluated directly by averaging the conditional symbol error probability, $P_e(\gamma)$, i.e. the symbol error probability for a fixed SNR γ, over the PDF of γ:

$$\overline{P}_{se} = \int_0^\infty P_e(\gamma) p_\gamma(\gamma) d\gamma \tag{3.1.9}$$

3.1.3. Outage Probability

The outage Probability P_{out} is defined as the probability that the received SNR does not exceed a threshold γ_{th}

$$P_{out} = P(\gamma < \gamma_o) = \int_0^{\gamma_o} p_\gamma(\gamma) d\gamma = P_\gamma(\gamma_o) \tag{3.1.10}$$

where $P_\gamma(\gamma_o)$ is the CDF of γ.

3.1.4. Amount of Fading

The AF is a measure easy to compute, since it contains the variance and the mean value of γ. Also it is typically independent of the average fading power Ω [10]. The amount of fading AF is defined as:

$$AF \triangleq \frac{Var(\gamma)}{E^2(\gamma)} = \frac{E(\gamma^2) - E^2(\gamma)}{E^2(\gamma)} = \frac{E(\gamma^2)}{E^2(\gamma)} - 1 = \frac{\mu_\gamma(2)}{\overline{\gamma}} - 1 \tag{3.1.11}$$

where $\mu_\gamma(n)$ is the n-th order moment,

$$\mu_\gamma(n) \triangleq \int_0^\infty \gamma^n p_\gamma(\gamma) d\gamma \tag{3.1.12}$$

3.2. Performance Evaluation over generalized-K Fading Channels

The performance of generalized-K fading channels has been thoroughly examined in [45, 46] . In this section we present briefly these results.

3.2.1. Channel Capacity

Optimal Rate Adaptation with Constant Transmit Power (ORA)

By substituting (2.3.8) in (3.1.2) the capacity under the ORA policy is found to be:

$$C_{\langle ORA \rangle} = \frac{2\Xi^{\frac{\beta+1}{2}}}{\Gamma(k)\Gamma(m)} \int_0^\infty log_2(1+\gamma)\gamma^{\frac{\beta-1}{2}} K_a(2\sqrt{\Xi\gamma}) d\gamma \tag{3.2.1}$$

For arbitrary values of a and b the channel capacity is expressed in [37] in terms of the Meijer's G-function [47, eq.(9.301)]

$$\langle C \rangle_{ORA} = \frac{BW\Xi^{(\beta+1)/2}}{ln2\Gamma(k)\Gamma(m)} G^{4,1}_{2,4}\left[\Xi| \begin{array}{c} -(\beta+1)/2, (1-\beta)/2 \\ a/2, -a/2, -(\beta+1)/2, -(\beta+1)/2 \end{array}\right] \tag{3.2.2}$$

and in [46]:

$$C_{\langle ORA \rangle} = \frac{2(\Xi\sqrt{\gamma_o})^{\beta+1}}{\Gamma(k)\Gamma(m)ln2} A_{a,m} \tag{3.2.3}$$

where

$$A_{a,m} = \int_0^\infty ln(1+x^2) x^{a+2m-1} K_a(2\sqrt{\Xi}x) dx \tag{3.2.4}$$

can be expressed in closed form in terms of the Lommel functions $S_{\mu,\nu}(\cdot)$ [47, eq.(8.570)]. When $k = n + \frac{1}{2},\ n\epsilon\mathbb{Z}$ a simpler form of (3.2.1) is found in [46]:

$$C_{\langle ORA\rangle} = \frac{4\Xi^{\beta+1}}{\Gamma(k)\Gamma(m)ln2}\sum_{l=0}^{n-m}\frac{\Gamma(n-m+1+l)}{\Gamma(n-m+1-l)}\times\frac{1}{\Gamma(l+1)(4\sqrt{\Xi})^{l}}\sum_{j=0}^{n+m-l-1}\frac{(n+m-l-j)_j}{a^{j+1}}\Upsilon_{j,l}(y) \tag{3.2.5}$$

where

$$\Upsilon_{j,l}(y) = (-1)^{q-1}[ci(y)cos(y)+(-1)^{v-1}si(y)sin(y)]+\frac{1}{y^{v-1}}\sum_{t=1}^{q}(2q-2t+1)!(-1)^{t-1}y^{2t-2} \tag{3.2.6}$$

and $v=n+m-l-j,\ q=\left\lfloor\frac{v}{2}\right\rfloor$ and $si(\cdot), co(\cdot)$ are the sine and cosine integrals.

Optimal Simultaneous Power and Rate Adaptation (OPRA)

By substituting (2.3.8) in (3.1.4) the capacity under the OPRA policy is found to be:

$$C_{\langle OPRA\rangle} = \frac{2\Xi^{\frac{\beta+1}{2}}}{\Gamma(k)\Gamma(m)}\int_{\gamma_o}^{\infty} log_2\left(\frac{\gamma}{\gamma_o}\right)\gamma^{\frac{\beta-1}{2}}K_a(2\sqrt{\Xi\gamma})d\gamma \tag{3.2.7}$$

which for the general case can be rewritten in [46] as:

$$C_{\langle OPRA\rangle} = \frac{2\left(\sqrt{\Xi\gamma_o}\right)^{\beta+1}}{\Gamma(k)\Gamma(m)ln(2)}I_{a,m} \tag{3.2.8}$$

where

$$\begin{aligned} I_{a,m} &= \int_{1}^{\infty}\frac{\int x^{2\sqrt{\Xi}+2m-1}K_a\left(2\sqrt{\Xi\gamma_o}x\right)dx}{x}dx \\ &= \frac{(m-1)!}{2\left(\sqrt{\Xi\gamma_o}\right)^{m}}I_k+\sum_{l=1}^{m-1}\frac{(m-l+1)_{l-1}}{2\left(\sqrt{\Xi\gamma_o}\right)^{l}}\times\sum_{j=1}^{m-1}\frac{(m-l-j+1)_{j-1}}{2\left(\sqrt{\Xi\gamma_o}\right)^{j}}K_{a+l+j}\left(2\sqrt{\Xi\gamma_o}\right) \end{aligned} \tag{3.2.9}$$

Depending on k I_k is found in [46] as:

1. $k\in\mathbb{R}$
   In the case where $k\in\mathbb{R}$, I_k can be written in terms of the Meijer G-function

$$I_k = 2^{-2}\left(\sqrt{\Xi\gamma_o}\right)^{-k}G_{1,3}^{3,0}\left(\frac{\left(2\sqrt{\Xi\gamma_o}\right)^2}{4}\ \Big|\ \begin{matrix}1\\0,k,0\end{matrix}\right) \tag{3.2.10}$$

2. $k \in \mathbb{Z}$

$$I_k = (k-1)! \sum_{j=0}^{k-1} \frac{1}{j!} \left(\frac{1}{\sqrt{\Xi \gamma_o}} \right)^{k-j-1} \frac{K_j \left(2\sqrt{\Xi \gamma_o} \right)}{2\sqrt{\Xi \gamma_o}} \tag{3.2.11}$$

3. $k = n + \frac{1}{2}$, $n \in \mathbb{Z}$

$$I_k = \sqrt{\frac{\pi}{4\sqrt{\Xi \gamma_o}}} \frac{1}{\left(2\sqrt{\Xi \gamma_o} \right)^n} \sum_{l=0}^{n} \frac{\Gamma(n+1+l)}{\Gamma(n+1-l)\Gamma(l+1)2^l} \Gamma \left(n-l, 2\sqrt{\Xi \gamma_o} \right) \tag{3.2.12}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function [47, eq.(8.35)]. Also, in [46] the optimal cutoff γ_o can be computed from the following equation:

$$\frac{2\sqrt{\Xi \gamma_o}}{\Gamma(k)\Gamma(m)} \left(-J_{a,m}(1) + J_{a,m-1}(1) \right) - \gamma_o = 0 \tag{3.2.13}$$

where $J_{a,m-1}(1)$:

- $m \geq 2$

$$J_{\xi,p}(x) = -\sum_{l=1}^{p} \frac{2^{l-1}(p-l+1)_{l-1}}{b^l} x^{\xi+2p-l} K_{\xi+l}(bx) \tag{3.2.14}$$

- $m = 1$

$$J_{a,0}(1) = I_a \tag{3.2.15}$$

Channel Inversion with Fixed Rate (CIFR)

By substituting (2.3.8) in (3.1.7) and with a change of variables $x = \sqrt{\gamma}$ the capacity under the CIFR policy is found to be [46]:

$$C_{\langle CIFR \rangle} = log_2 \left(1 + \frac{\Gamma(k)\Gamma(m)}{2^2 \Xi^{\frac{\beta+1}{2}} \int_0^\infty x^{\beta-2} K_a \left(2\sqrt{\Xi x} \right) dx} \right) \tag{3.2.16}$$

If $m \leq \frac{3}{2}$ the capacity is found to be zero [46]. For $m \geq 2$ the capacity is [46]:

$$C_{\langle CIFR \rangle} = \frac{1}{ln2} ln \left(1 + \overline{\gamma} \frac{(m-1)(k-1)}{mk} \right) \tag{3.2.17}$$

Truncated Channel Inversion (TCIFR)

In [48] the capacity of TCIFR is found to be:

$$C_{\langle TCIFR\rangle} = log_2\left(1 - \frac{2^{\beta-1}\Gamma(m)\Gamma(k)\gamma_o}{b^{\beta+1}J_{a,m-1}(1)}\right)(1 - P_{out}) \tag{3.2.18}$$

where $J_{a,m-1}(1)$ can be found on the preceding page in (3.2.14) and (3.2.15).

3.2.2. MGF

By substituting (2.3.8) in (A.1.1) the MGF for generalized K fading channels is found in [37] as:

$$M_\gamma(s) = \left(\frac{\Xi}{s}\right)^{\beta/2} \exp\left(\frac{\Xi}{s}\right) W_{-\beta/2,a/2}\left(\frac{\Xi}{s}\right) \tag{3.2.19}$$

where $W_{\lambda,\mu}(\cdot)$ is the Whittaker function, [47, eq.(9.220)].

3.2.3. Average SEP

In [37] the average SEP has been computed according to (3.1.9) for the following modulation schemes:

- non-coherent BFSK and BDPSK

$$\overline{P}_{se} = \frac{A}{log_2M}\left(\frac{\Xi}{B}\right)^{\beta/2} \exp\left(\frac{\Xi}{2B}\right) W_{-\beta/2,a/2}\left(\frac{\Xi}{B}\right) \tag{3.2.20}$$

- BPSK, square M-QAM and for high values of the average input SNR

$$\overline{P}_{se} = \frac{A}{log_2M}\left(\frac{\Xi}{B}\right)^{\frac{k+m}{2}} G^{2,2}_{2,3}\left(\frac{\Xi}{B}\Big|\begin{smallmatrix}\frac{1-\beta}{2}, -\frac{\beta}{2}\\ \frac{a}{2}, -\frac{a}{2}, -\frac{\beta+1}{2}\end{smallmatrix}\right) \tag{3.2.21}$$

- Gray encoded M-PSK and M-DPSK

$$\overline{P}_{se} = \frac{A}{log_2M}\Xi^{\beta/2}\int_0^{\Lambda} \frac{\exp\left(\frac{\Xi}{2B(\theta)}\right)}{B(\theta)^{\beta/2}} W_{-\beta/2,a/2}\left(\frac{\Xi}{B(\theta)}\right) d\theta \tag{3.2.22}$$

where A, B, Λ are constants depending on the modulation scheme.

3.2.4. Outage Probability

As shown in (3.1.10) the outage probability is the CDF of γ which in [37] is found to be:

$$P_\gamma(\gamma) = \pi csc(\pi a) \left[\frac{(\Xi\gamma)_1^m F_2(m; 1-a, 1+m; \Xi\gamma)}{\Gamma(k)\Gamma(1-a)\Gamma(1+m)} - \frac{(\Xi\gamma)_1^k F_2(k; 1+a, 1+k; \Xi\gamma)}{\Gamma(m)\Gamma(1+a)\Gamma(1+k)} \right], \ \gamma \geq 0 \quad (3.2.23)$$

where ${}_pF_q(\cdot)$ is the generalized hypergeometric function [47, eq.(9.14/1)] and $p, q \in \mathbb{Z}$. Under the OPRA policy an alternative presentation of P_{out} can be found in [46]:

$$P_{out} = 1 - \frac{\left(2\sqrt{\Xi\gamma_o}\right)^{\beta+1} \sum_{l=1}^{m} \frac{(m-l+1)_l K_{a+l}\left(2\sqrt{\Xi\gamma_o}\right)}{\left(2\sqrt{\Xi\gamma_o}\right)^l}}{\Gamma(m)\Gamma(k)} \quad (3.2.24)$$

3.2.5. Amount of Fading

In [37] the n-th order moment is given by:

$$\mu_\gamma(n) = \Xi^{-n} \frac{\Gamma(k+n)\Gamma(m+n)}{\Gamma(m)\Gamma(k)} \quad (3.2.25)$$

3.3. Conclusions

In this chapter we described the performance measures used to evaluate the fading channels with focus on generalized-K fading conditions. The performance evaluation is the most important and simultaneously a difficult component in the design of a system, since it depicts the behavior of the channel according to different parameters. As seen in this chapter many performance measures can be quite hard to compute and some do not even exist in closed form expressions even for special cases. At the same time other measures easier to compute might not be a representative estimation of the channel. Therefore the performance evaluation can be a quite challenging and crucial task, since a bad estimation of the performance of the channel can lead to malfunctioning systems.

Part II.

Diversity

4. Overview of Diversity

In order to achieve high performance in fading channels, the signal energy needs to raise significantly, which can be a quite expensive solution. Another approach is to use diversity, one simple yet effective and low cost solution. In diversity two or more copies of the same information baring signal are skillfully combined in order to increase the overall SNR [10].

In this chapter we will analyze the most important form of diversity, which makes use of multiple antennas that receive the same signal from different paths,which are combined to obtain a resultant signal. For this scheme to be successful, the antennas must be sufficiently spaced, in order to receive independent signal paths. In this case, the probability that each signal path simultaneously experiences deep fade is low and the error probability is polynomially reduced according to the number of the receiving antennas. Various techniques can be used to combine the signal paths into one resultant signal, and each of them will be presented in the following sections. These techniques can be classified into pure and hybrid techniques, where the latter are a combination of pure diversity combining techniques [10].

4.1. System Model

In the first part we described a system where the channel was single link. Now we generalize this model into a multilink channel, in which the transmitted signal is received over L independent, slowly varying flat fading channels.

As depicted in Figure (4.1) each channel $l, l \in [1, L]$ is associated with a random amplitude a_l, a time delay τ_l and a random phase θ_l. The receiver is said to operate over independent fading conditions if the fading amplitudes a_l are statistically independent random variables, with a PDF described by any of the family of distributions (Rayleigh, Nakagami-m, Nakagami-n). Furthermore, if the PDF is the same for every channel the receiver is said to operate over independent identical distributed (i.i.d) fading conditions, otherwise if the PDF of every channel belongs to the same family of distributions it is said to operate over independent not identical distributed (i.n.d.) fading conditions.

Except of fading, additive white Gaussian noise (AWGN) $n_l(t)$ with a one-sided power spectral density $2N_l$ (W/Hz) also affects the signal. The AWGN is assumed to be independent of a_l and statistically independent from channel to channel. Thus the instantaneous SNR per symbol in each channel (branch) is

$$\gamma_l = a_l^2 E_s / n_l \tag{4.1.1}$$

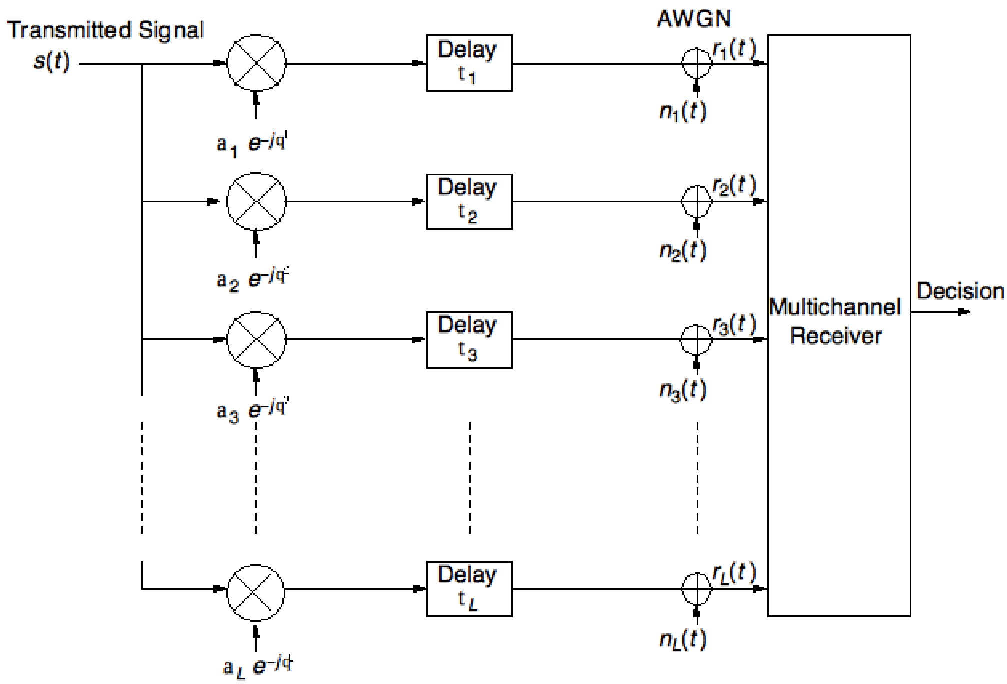

Figure 4.1.: multilink channel

where E_s is the energy per symbol. By averaging (4.1.1) the average SNR per symbol is given by

$$\overline{\gamma}_l = \Omega_l E_s / N_l \tag{4.1.2}$$

The purpose of the combining techniques is to determine the output signal that will increase the overall SNR and improve the radio link performance .

The receiver might use pre detection or post detection combining. In pre detection the combining is performed before the detection while in post detection it is performed after [1]. Pre detection combining is used in coherent detection while post detection can be used in both coherent and non-coherent detection.

4.2. Pure Combining Techniques

The most known pure combining techniques are the Maximal Ratio Combining (MRC), Equal Gain Combining (EGC), Selection Combining (SC) and Switch and Stay Combining (SSC).In MRC the amplitudes, phases and delays must be known, while in EGC the amplitude and in the case of non coherent EGC also the phases are not needed to be determined. Finally in the case of SC and SSC none of the above parameters needs to be described.

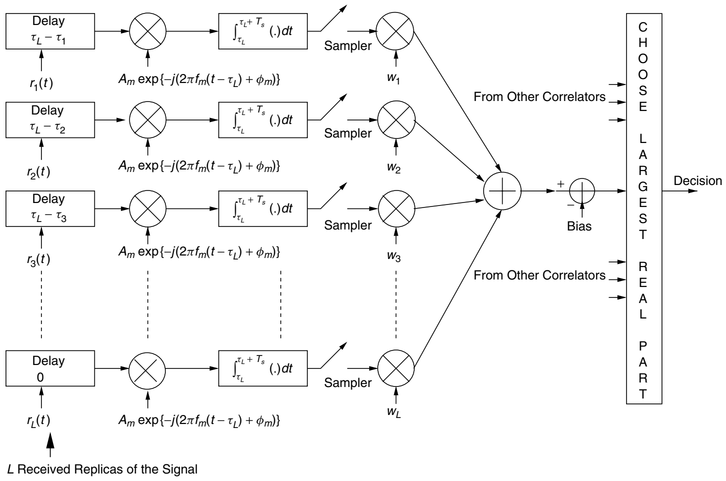

Figure 4.2.: MRC receiver structure [10]

4.2.1. Maximal Ratio Combining (MRC)

MRC receiver is the optimal multichannel receiver regardless of the fading statistics, because it results in a maximum likelihood receiver [10]. Its purpose is to maximize the total conditional SNR per symbol γ_t.

Figure (4.2) shows the structure of one branch of a MRC receiver. There is one correlator for every message (M correlators) and one branch for every signal path (L branches). Each signal path passes through a matched filter, after it is multiplied with a weight factor w_l, which needs to be determined. Before the final decision the resultant signal is reduced by a Bias factor, which also needs to be determined. The maximization of γ_t leads to the following results:

$$w_l = \frac{a_l}{N_l} \tag{4.2.1}$$

$$Bias = \sum_{l=1}^{L} \frac{a_l^2}{N_l} E_m \tag{4.2.2}$$

where E_m is the energy of the m-th symbol. The $\gamma_t = \gamma_{MRC}$ is found to be:

$$\gamma_{MRC} = \sum_{l=1}^{L} \gamma_l \tag{4.2.3}$$

Although MRC is the optimum combining technique regardless of the fading distribution, it is has its limitations. At first it requires coherent detection, thus making it an inappropriate technique for differentially coherent and non coherent detection schemes. Furthermore the knowledge of the CSI increases the complexity of the implementation [10].

4.2.2. Coherent Equal Gain Combining (EGC)

Although coherent EGC is suboptimal it is an attractive solution due to its simplicity, which comes the fact that the knowledge of the fading amplitude is not needed. The structure of the receiver is the same as the MRC receiver (Figure (4.2)) but in EGC each branch is weighted the same, i.e.

$$w_l = 1 \tag{4.2.4}$$

$$Bias = 0 \tag{4.2.5}$$

leading to a $\gamma_t = \gamma_{EGC}$:

$$\gamma_{EGC} = \frac{\left(\sum_{l=1}^{L} a_l\right)^2 E_s}{\sum_{l=1}^{L} N_l} \tag{4.2.6}$$

Although coherent EGC is simple to implement it is limited in practice to coherent modulations with equal-energy symbols (M-PSK), because of the absence of the fading amplitude's estimation [10].

4.2.3. Non-Coherent Equal Gain Combining (nEGC)

Non-coherent EGC or post detection EGC can be used in non-coherent or differentially coherent modulations. The structure of the receiver in both cases can be found in [10, p. 344,345]. For the non-coherent detection the square law or differentially coherent techniques are used. For equally likely transmitted signals $\gamma_t = \gamma_{nEGC}$ is found to be in [10, p. 294] :

$$\gamma_{nEGC} = \sum_{l=1}^{L} \gamma_l \tag{4.2.7}$$

4.2.4. Selection Combining (SC)

In Selection Combining the combiner outputs the signal on the branch with the higher SNR. Since only one branch is used at a time, often one receiver is required that is switched into the active antenna branch [1] and the coherent sum of individual branch signals is not longer necessary [10]. This makes the SC the simplest technique compared to MRC and EGC but there are still some limitations. The diversity gain of this technique depends on the number of independent fading channels. In practice, due to insufficient spacing between the antennas, fading might be correlated, thus the theoretical maximum diversity gain cannot be achieved. In general SC can be used with non coherent and differentially coherent modulation schemes. It can also be used with coherent schemes but MRC is in this case more effective and better fitting.

4.2.5. Switch and Stay Combining (SSC)

For systems that use uninterrupted transmission, such as frequency-division multiple-access systems, SC is impractical since it requires simultaneous and continuous monitoring of all diversity branches [10, p. 262]. Therefore another form of SC, SSC is introduced, in which the receiver selects a particular branch, as long as it doesn't fall below a predefined threshold. Because of this continuous monitoring of all the diversity branches is not longer necessary. If the SNR of the branch falls below the threshold then the combiner changes the branch. There are many algorithms that define which will be the next one, which can be found in [1, p. 196]. The simplest one is to pick the next branch randomly. For two-branch diversity this is equivalent to switch to the other branch, only in this case it does also stay with the other branch regardless of whether the SNR is above or below the threshold [10, p. 263]. As far as SSC's applications are concerned, it can be used in both coherent and non-coherent schemes as well as in differentially coherent schemes like SC. Also, since continuous monitoring is not necessary, SSC is the simplest technique to implement from those techniques presented.

4.3. Hybrid Combining Techniques

In order to achieve higher diversity gain or lower the complexity of a system, more sophisticated combining schemes are proposed. These schemes can be categorized in two groups: generalized diversity techniques and multidimensional diversity techniques. Multidimensional diversity techniques combine two or more conventional means of diversity (space,multipath, etc.), in order to achieve better performance [10].

4.3.1. Generalized Diversity Techniques

A gap exists between the pure diversity techniques. For MRC and EGC the complexity depends on the number of diversity paths available that can be quite high. This is the case for example in wideband CDMA signals [10]. On the other hand SC exploits only one of all the diversity paths available.

Generalized Selection Combining (GSC)

Generalized selection combining is a combination of SC with MRC or EGC, denoted by GSC(N, L), where $1 \leq N \leq L$. In particular in this scheme the first N branches with the largest SNR are chosen (SC) and then MRC or EGC is performed in the branches chosen. GSC has many advantages compared to the pure combining techniques it consists of. Compared to the MRC/EGC receivers it's complexity is lower, since only a fixed number of branches is taken into account, which is independent of the number of diversity paths. Nevertheless, GSC(SC/MRC) has been shown to approach the performance of MRC while GSC(SC/EGC) outperforms in certain cases conventional post detection EGC since paths with low SNR are most likely not chosen in the N branches [10].Compared to SC receivers it is certainly more efficient, since the resultant signal is a combination of at least one branch. Therefore GSC is said to bridge the gap between MRC/EGC and SC[10].

For a GSC(N, L) (SC/MRC) the output SNR γ_{GSC} is given by:

$$\gamma_{GSC} = \sum_{k=1}^{N} \gamma_{(k)} \tag{4.3.1}$$

where $\gamma_{(k)},\ 1 \leq k \leq L$ are the SNR of the branches γ_k in descending order, i.e. $\gamma_{(1)} \geq \gamma_{(2)} \geq \dots \geq \gamma_{(L)}$.

4.4. Conclusions

In this chapter an introduction to pure and hybrid combining techniques was made. As shown the optimal combining technique is MRC but since it is complex to implement the use of hybrid techniques like GSC seems to pave the way for the future. In particular generalized diversity techniques are a very attractive solution due to the low implementing complexity and the high performance that can be achieved.

5. Performance Evaluation of Diversity Combining Techniques

In this chapter we will introduce the measures used to characterize the performance of diversity combining techniques, i.e. the average output SNR (ASNR), the outage probability and the amount of fading (AF).The outage probability is the probability that a given rate will not be supported because of channel variations and was introduced in 3.1.3, the AF is a measure of the severity of the fading channel itself and was introduced in 3.1.4. In our analysis the MGF and the Padé approximants method will be needed. A detailed analysis of both can be found in A.1.2 and A.2 respectively.

5.1. Average Output SNR (ASNR)

The average output SNR is defined as the first statistical moment of the instantaneous output SNR $\mu_{\gamma_t}(1)$. It can be computed quite easily but it is not always a representative measure of performance of the system.

MRC

By using (4.2.3), the nth order moment is found in [27]:

$$\mu_{\gamma_{MRC}}(n) = \sum_{n_1=0}^{n} \sum_{n_2=0}^{n} \cdots \sum_{\substack{n_L=0 \\ n_1+n_2+\ldots+n_L=n}}^{n} \frac{n!}{n_1!n_2!\ldots n_L!} \prod_{l=1}^{L} \mu_{\gamma_l}(n_l) \tag{5.1.1}$$

where $\mu_{\gamma_t}(\cdot)$ is specified according to the fading distribution.

coherent EGC

In [27] $\mu_{\gamma_{EGC}}(\cdot)$ is found:

$$\mu_{\gamma_{EGC}}(n) = \frac{(2n)!}{L^n} \sum_{\substack{n_1, n_2, \ldots, n_L \\ n_1+n_2+\ldots+n_L=2n}}^{2n} \frac{\prod_{l=1}^{L} \mu_{\gamma_l}\left(\frac{n_l}{2}\right)}{n_1!n_2!\ldots n_L!} \tag{5.1.2}$$

where $\mu_{\gamma_t}(\cdot)$ is specified according to the fading distribution.

SC

As denoted in 4.2.4 SC outputs the signal with the highest instantaneous SNR. The CDF of γ_{SC}, $F_{\gamma_{SC}}(\gamma)$ can be expressed as:

$$F_{\gamma_{SC}}(\gamma) = \prod_{l=1}^{L} F_{\gamma_l}(\gamma) \tag{5.1.3}$$

where $F_{\gamma_l}(\gamma)$ is specified according to the fading distribution. From (5.1.3) $\mu_{\gamma_{SC}}(\cdot)$ can be expressed as:

$$\mu_{\gamma_{SC}}(n) = \int \gamma^n \sum_{l=1}^{L} \prod_{\substack{j=1 \\ j \neq l}}^{L} F_{\gamma_j}(\gamma) \frac{dF_{\gamma_l}(\gamma)}{d\gamma}\, d\gamma \tag{5.1.4}$$

which for $L \geq 3$ does not lead to closed form expressions.

Dual-Branch SSC

For Dual-Branch SSC the PDF of γ_{SSC}, $p_{\gamma_{SSC}}$ can be found in [27] as:

$$p_{\gamma_{SSC}}(\gamma) = \begin{cases} \frac{P_1 P_2}{P_1+P_2} \sum_{l=1}^{2} p_{\gamma_l}(\gamma), \ \gamma \leq \gamma_T \\ \frac{P_1 P_2}{P_1+P_2} \sum_{l=1}^{2} p_{\gamma_l}(\gamma)\,(1+1/P_l)\,, \ \gamma > \gamma_T \end{cases} \tag{5.1.5}$$

where $P_l = P_{\gamma_l}(\gamma_T)$. (5.1.5) leads to an $\mu_{\gamma_{SSC}}(n)$ of:

$$\mu_{\gamma_{SSC}}(n) = \frac{P_1 P_2}{P_1 + P_2} \sum_{l=1}^{2} \left[\mu_{\gamma_l}(n)\,(1+1/P_l) - \int_{-\infty}^{\gamma_T} \gamma^n p_{\gamma_{SSC}}(\gamma) d\gamma \right] \tag{5.1.6}$$

where the computation of the integral depends on the particular fading distribution.

GSC

From (4.3.1) and (5.1.1) we can compute $\mu_{\gamma_{GSC}}(\cdot)$ for the i.i.d. case as:

$$\mu_{\gamma_{GSC}}(n) = \sum_{\substack{n_1=0 \\ n_1+n_2+\ldots+n_N=n}}^{n} \sum_{n_2=0}^{n} \cdots \sum_{n_N=0}^{n} \frac{n!}{n_1! n_2! \ldots n_N!} \prod_{k=1}^{N} \mu_{\gamma_{(k)}}(n_k) \tag{5.1.7}$$

For the identical but not independent case the nth order moment of GSC$(2, L)$,i.e. with a dual-branch SC, can be written by using the binomial theorem [47, eq. (1.111)] as [49]:

$$\mu_{\gamma_{GSC}}(n) = \sum_{p=0}^{n} \binom{n}{p} \mathbb{E} \left\langle \gamma_{(1)}^{p} \gamma_{(2)}^{n-p} \right\rangle \tag{5.1.8}$$

5.2. Outage Probability

The outage probability defined in section 3.1.3 can be also expressed in terms of the MGF as

$$P_{out}(\gamma_T) = P_\gamma(\gamma_T) = L^{-1}\left[\frac{M_\gamma(s)}{s}; \gamma\right]\Bigg|_{\gamma=\gamma_T} \tag{5.2.1}$$

where $L^{-1}(\cdot)$ denotes the inverse Laplace transformation. In the cases that closed form expressions can be found for the MGF, (5.2.1) is a convenient solution. In the case that no closed form expressions the Padé approximants method can be used and P_{out} can be evaluated by numerical techniques [50, 51].

5.3. Amount of Fading

The Amount of fading for the output SNR is defined as:

$$AF = \frac{\mu_{\gamma_t}(2)}{\mu^2_{\gamma_t}(1)} - 1 \tag{5.3.1}$$

and in terms of the MGF [10]:

$$AF = \frac{\left.\frac{d^2 M_{\gamma_t}(s)}{ds^2}\right|_{s=0}}{\left(\left.\frac{dM_{\gamma_t}(s)}{ds}\right|_{s=0}\right)^2} - 1 \tag{5.3.2}$$

From (5.1.1), (5.1.2), (5.1.4), (5.1.6) and (5.1.7) we can derive by a simple substitution in (5.3.1) the AF for each diversity receiver.

The AF is more revealing than the average SNR, which although it is easy to compute it does not capture all the diversity benefits. To be more specific the average SNR is a representative measure of the average SNR gain, which could be also increased by increasing the transmitter power where diversity is not necessary. Therefore the combination of the second and first order moments of the output SNR as it depicts better the advantages of the diversity systems.

5.4. Performance Analysis over generalized-K Fading Channels

5.4.1. Average Output SNR (ASNR)

MRC, coherent EGC

The average output SNR can be obtained by substituting (3.2.25) in (5.1.1) and (5.1.2).

SC

In [27] $\mu_{\gamma_{SC}}(\cdot)$ is found as:

$$\mu_{\gamma_{SC}}(n) = \frac{\pi^2 csc\left[\pi\left(k-m_1\right)\right] csc\left[\pi\left(k-m_2\right)\right]}{\Gamma\left(m_1\right)\Gamma\left(m_2\right)\Gamma\left(k\right)^2} \times G\left(m_1,k,m_2,k,1\right) G(k,m_1,m_2,k,1) \times G(m_2,k,m_1,k,-1)G(k,m_2,m_1,k,-1) \quad (5.4.1)$$

where

$$G\left(x,y,z,w,l\right) = \frac{\Xi_1^x \Xi_2^{-x-n}}{x\left(x-y\right)!}\left\{\Gamma\left(x+z+n\right)\times\frac{{}_pF_q\left[x+z+n,x,w+x+n;1+x-y,1+x;\left(\frac{\Xi_1}{\Xi_2}\right)^l\right]}{\left(-1\right)^{z+x+n}\Gamma(1-w-x-n)} -\Gamma\left(x+w+n\right)\times\frac{{}_pF_q\left[x+w+n,x,z+x+n;1+x-y,1+x;\left(\frac{\Xi_1}{\Xi_2}\right)^l\right]}{\left(-1\right)^{w+x+n}\Gamma(1-z-x-n)}\right\} \quad (5.4.2)$$

Dual-Branch SSC

For Dual-Branch SSC $\mu_{\gamma_{SSC}}(n)$ can be found in [27] as:

$$\mu_{\gamma_{SSC}}(n) = \frac{P_1P_2}{P_1+P_2}\sum_{l=1}^{2}\left\{\mu_{\gamma_l}(n)\left(1+\frac{1}{P_l}\right) - \frac{\pi csc\left[\pi\left(k-m_l\right)\right]\gamma_T^n}{P_l\Gamma\left(k\right)\Gamma\left(m_l\right)} \left[\left(\Xi_l\gamma_T\right)^{m_l}\Gamma\left(m_l+n\right)\times{}_p\widetilde{F}_q\left(m_l+n;1-k+m_l,1+m_l;\Xi_l\gamma_T\right) - \left(\Xi_l\gamma_T\right)^k\Gamma\left(k+n\right){}_p\widetilde{F}_q\left(k+n;1+k-m_l,1+k+n;\Xi_l\gamma_T\right)\right]\right\} \quad (5.4.3)$$

GSC

In [38] the performance of GSC(2,3) over independent but not identically distributed K_G fading channels, where the absolute difference between the two distribution parameters (k_l, m_l) is a positive half-integer or half-integer values of the second fading parameter ,i.e $|k_l - m_l| = v_l + 1/2$, is analyzed. The nth order moment for this case is found as:

$$\mu_{\gamma_{GSC}}(n) = 2^{-2n+3}\pi^{3/2} \sum_{e_l \in S_3} \prod_{r=1}^{3} 2^{2m_{e_l[r]} - f(v_{e_l[r]})} \frac{\Xi_{e_l[r]}^{2m_{e_l[r]}+f(v_{e_l[r]})}}{\Gamma(m_{e_l[r]})\Gamma(m_{e_l[r]} + f(v_{e_l[r]}) + 1/2)} \sum_{q=0}^{n} \binom{n}{q} \Xi_{e_l[1]}^{-2q-2m_{e_l[r]}-f(v_{e_l[r]})}$$
$$\prod_{k=1}^{3} \sum_{p_{k=0}}^{v_{e_l[k]}} \sum_{i=0}^{2q+2m_{e_l[1]}+p_1-1+f(v_{e_l[1]})} \sum_{j=0}^{2q+2m_{e_l[1]}+p_2+-1+i+f(v_{e_l[2]})} \Xi_{e_l[1]}^{i}\Xi_{e_l[2]}^{-p_2}\Xi_{e_l[3]}^{-p_3} \prod_{r=1}^{3} \frac{(v_{e_l[r]}+p_r)!2^{-p_r}}{p_r!\left(v_{e_l[r]}-p_r\right)!}$$
$$\times \frac{\left(\Xi_{e_l[1]} + \Xi_{e_l[2]}\right)^{j-f(v_{e_l[2]})-2m_{e_l[2]}+p_2-i-2n+2q}}{\left(\Xi_{e_l[1]} + \Xi_{e_l[2]} + \Xi_{e_l[3]}\right)^{j+f(v_{e_l[3]})-p_3+2m_{e_l[3]}}} \left(f(v_{e_l[2]}) + 2m_{e_l[2]} - p_2 + i + 2n - 2q - 1\right)!$$
$$\times \left(f(v_{e_l[1]}) + 2m_{e_l[1]} - p_1 + 2q - 1\right)! \frac{\left(f(v_{e_l[3]}) + 2m_{e_l[3]} - p_3 + j - 1\right)!}{i!j!} \tag{5.4.4}$$

where $e_l = \{e_l[1], e_l[2], e_l[3]\}$ is one specific permutation of integers $l = 1, 2, 3$.

5.4.2. Moment Generating Function (MGF)

The definition of MGF can be found on page 65. According to the combining technique the MGF is found to be:

MRC

$M_{\gamma_{MRC}}(\cdot)$ can be found by substituting (3.2.19) in (A.1.2). Under the assumption of independent and identical distributed (i.i.d) fading conditions $M_{\gamma_{MRC}}(\cdot)$ is found in [27] as:

$$M_{\gamma_{MRC}}(s) = F(m, k, L)\left(\frac{\Xi}{s}\right)^{\tau} \exp\left(\frac{\Xi L}{s}\right) \tag{5.4.5}$$

where $F(m, k, L)$ and τ can be found in [27, eq. 10].

coherent EGC

By using the Padé approximants method the MGF can be directly computed from (5.1.2) and (3.2.25)[27] .

SC

By using the Padé approximants method the MGF can be directly computed from (5.4.1)[27] .

SSC

By substituting (2.3.8) and (3.2.19) in (A.1.3), we can obtain $M_{\gamma_{SSC}}(\cdot)$ in the case of generalized-K fading channel [27] .

GSC

In [38] the Padé approximants method is used in order to derive $M_{\gamma_{GSC}}(\cdot)$ from (5.4.4).

5.5. Conclusions

In this chapter we described the performance measures used to evaluate the diversity combining techniques over fading channels with focus to generalized-K fading conditions. As expected, MRC and GSC were the best combining techniques in terms of performance, while SC and SSC were the worst. However, when we consider complexity as a criterion, this order reverses. This trade-off relationship indicates that both low complexity and high performance are hard to achieve, and that compromises need to be made in order to achieve an overall optimal balance.

Part III.

Simulation

6. Simulation of Fading and Diversity

In this part we have implemented in software a wireless mobile communication system over a Rayleigh and a generalized-K fading channel as well as a wireless mobile communication system with multiple receivers and MRC,EGC,SC and SSC combining techniques.

6.1. Software Implementation using MATLAB

For the simulation of fading and diversity the MATLAB environment was used. MATLAB is a very powerful tool, specialized among others for signal processing. A detailed presentation of MATLAB and its features can be found in [52]. The programming style chosen was object oriented (OOP). An introduction and various examples about how to program in object-oriented style with MATLAB can be found in [53]. This choice was due to various reasons:

1. security
   OOP enables an explicit determination of the characteristics of a group of classes that are similar, e.g. both MRC and EGC are combining techniques and thus must implement some common methods. Thanks to this feature, a programmer is strictly instructed and restricted as far as the implementation of various classes is concerned.

2. modularity
   Thanks to the abstract classes , it is quite easy to expand the system to implement different modulation schemes, fading environments, combining techniques and performance measures.

3. structured programming
   In OOP each component of the system is represented by a different class.
   Therefore it is strictly specified which functionality each class shall implement.

4. debugging
   Since each class is an independent component it is quite easy to restrict possible errors and eliminate them. Moreover if there is an error in a class it won't affect other classes.

6.2. Implementation Outline

In this section we will present a brief outline about how where some variables implemented escorting our analysis with specific pieces of code.

Generating Rayleigh Random Variables

In order to generate Rayleigh random variables we used the fact that:

$$x = \sqrt{-2\sigma^2 ln(y)}$$

where x is the random variable following the Rayleigh distribution, σ is the variance of the Rayleigh distribution and y is the CDF of the Rayleigh distribution, which is a random variable uniformly distributed in $(0, 1)$. Therefore the code that generates Rayleigh random variables is found in 6.1.

Algorithm 6.1 the function that generates Rayleigh random variables

```
function signal=generate(n,sigma)
        y=rand(1,n);
        signal=sqrt((-2*sigma^2)*log(y));
end
```

Generating generalized-K Random Variables

In order to generate generalized-K random variables we used the fact that a generalized-K random variable (r.v.) is the product of a Nakagami-m and a Gamma random variable. The Nakagami-M r.v. implements multipath fading and the Gamma r.v. shadowing. The generalized-K is a two parameter (k, m) fading distribution as mentioned in 2.3.2, therefore m will be the fading parameter of the Nakagami-m r.v. and k will be the fading parameter of the Gamma r.v. .The code that generates generalized-K r.v. is found in 6.2.

Algorithm 6.2 the function that generates generalized-K random variables

```
function signal=generate(m,k,n,sigma)
        a=m;
        b=sigma^2/a;
        a1=k;
        b1=1/a1;
        fading=gamrnd(a,b,1,n);
        shadowing=gamrnd(a1,b1,1,n);
        signal=fading.*shadowing;
end
```

In order to generate the Nakagami-m and the Gamma r.v. we used the MATLAB function gamrnd which can be found in [54].

6.2.1. Implementing the Combining Techniques

In this simulation we implemented MRC,EGC,SC,SSC and hybrid SSC. In particular in SSC we chose first the branch with the maximum SNR and if this branch fell below a specified threshold we switched to the next branch regardless of its SNR. In the hybrid SSC if we had to switch, we chose the branch with the maximum SNR.

MRC

As shown in section 4.2.1 the output of the maximal ratio combiner is the normalized sum of the branches, which is implemented in 6.3.

Algorithm 6.3 Maximal Ratio Combining function

```
function out=combine(sig,fading)
        fadingSum=sum(fading.^2,1);
        out=sum(sig.*fading,1)./fadingSum;
end
```

EGC

As shown in section 4.2.2 the output of the equal gain combiner is the sum of the input branches,which is implemented in 6.4.

Algorithm 6.4 Equal Gain Combining function

```
function out=combine(sig)
        out=sum(sig,1);
end
```

SC

As shown in section 4.2.4 the output of the selection combiner is the branch with the maximum SNR, which is implemented in 6.5.

Algorithm 6.5 Selection Combining function

```
function out=combine(sig,fading)
    [~,I]=max(abs(sig));
    [~,n]=size(sig);
    for i=1:n
        out(i)=sig(I(i),i);
    end
end
```

SSC

As mentioned in the beginning of the section in SSC a random branch is firstly selected and then if this branch fell under a predetermined threshold the next branch was chosen and the same process was repeated. The function that implements SSC is found in 6.6.

Algorithm 6.6 Switch and Stay Combining function

```
function out=combine(threshold,sig)
    [nchann,r]=size(sig);
    out=zeros(1,r);
    j=randi([1,nchann]);
    out(1)=sig(j,1);
    for i=2:r
        if sig(j,i)< threshold
            if(j==nchann) j=1;
            else j=j+1;
            end
        end
        out(i)=sig(j,i);
    end
end
```

Hybrid SSC

As mentioned in the beginning of the section in hybrid SSC the branch with the maximum SNR is firstly selected and then if this branch fell under a predefined threshold the branch with the maximum instantaneous SNR was chosen. The function that implements hybrid SSC is found in 6.7.

Algorithm 6.7 hybrid Switch and Stay Combining function

```
function out=combine(threshold,sig,)
        [~,r]=size(sig);
        out=zeros(1,r);
        [out(1),j]=max(sig(:,1));
        for i=2:r
                if sig(j,i)< threshold
                        [out(i),j]=max(sig(:,i));
                else out(i)=sig(j,i);
                end
        end
end
```

6.2.2. Evaluation of BER

In the present simulation BER was chosen as the measure of performance of the systems, since it is a quite representative measure of the system. We implemented three techniques to evaluate the BER:

- theoretical
  With the use of known and preferable closed form expressions of measuring the BER, the BER could be easily calculated for several values of input SNR with no need of implementing the simulation of the system.

- simulation
  In this case after the simulation of the system, the BER was evaluated by counting the errors of the input and the output signal. This technique is the most time consuming but it doesn't require any knowledge of closed form expressions in order to compute the BER. For the results to be considered valid it was assumed that a minimum number of errors, 100 errors, or a maximum number of realizations,2500 realizations, should be reached.

- semi-analytic
  The semi-analytic technique lies in the middle of the theoretical and the simulation technique. In this case random variables of the SNR are generated according to each fading technique, which are used as input SNR values in the theoretical BER expressions. This technique is quite appealing since it takes as input simulated data and can be simultaneously really fast as it uses theoretical expressions for the BER.

All of the three techniques where applied whereas possible. In order to ensure the correctness of the system, all of the three plots must coincide.

7. Simulated Performance Evaluation Results

In this chapter a graphical presentation of fading and diversity schemes is done. The chapter is organized as follows: at first the single reception is compared to MRC,EGC,SC and SSC for the case of Rayleigh and generalized-K fading channels.Results are compared for several values of (k, m) and a comparison between Rayleigh and generalized-K fading channels will be made, though it is expected that the Rayleigh channel is always better than the generalized-K. Then the BER of MRC receivers is analyzed for different number of branches. At last a comparison is made between SC,SSC and hybrid SSC for the case of generalized-K fading channels. In each case a discussion of the results is made. Note that all three techniques, i.e. theoretical,semi-analytic,simulation, where used to derive these results, which in all cases coincided.

7.1. Rayleigh Fading Channel

As depicted in figure 7.1 MRC has the best response in comparison to all the other cases. EGC is the second best, followed by the SC and the SSC technique. Of course the case of no diversity (single receiver) has the worst performance. A quite interesting observation is that as the SNR increases, the difference in SNR between the MRC,EGC and SC stays the same while for the SSC reception it increases leading to worse performance.

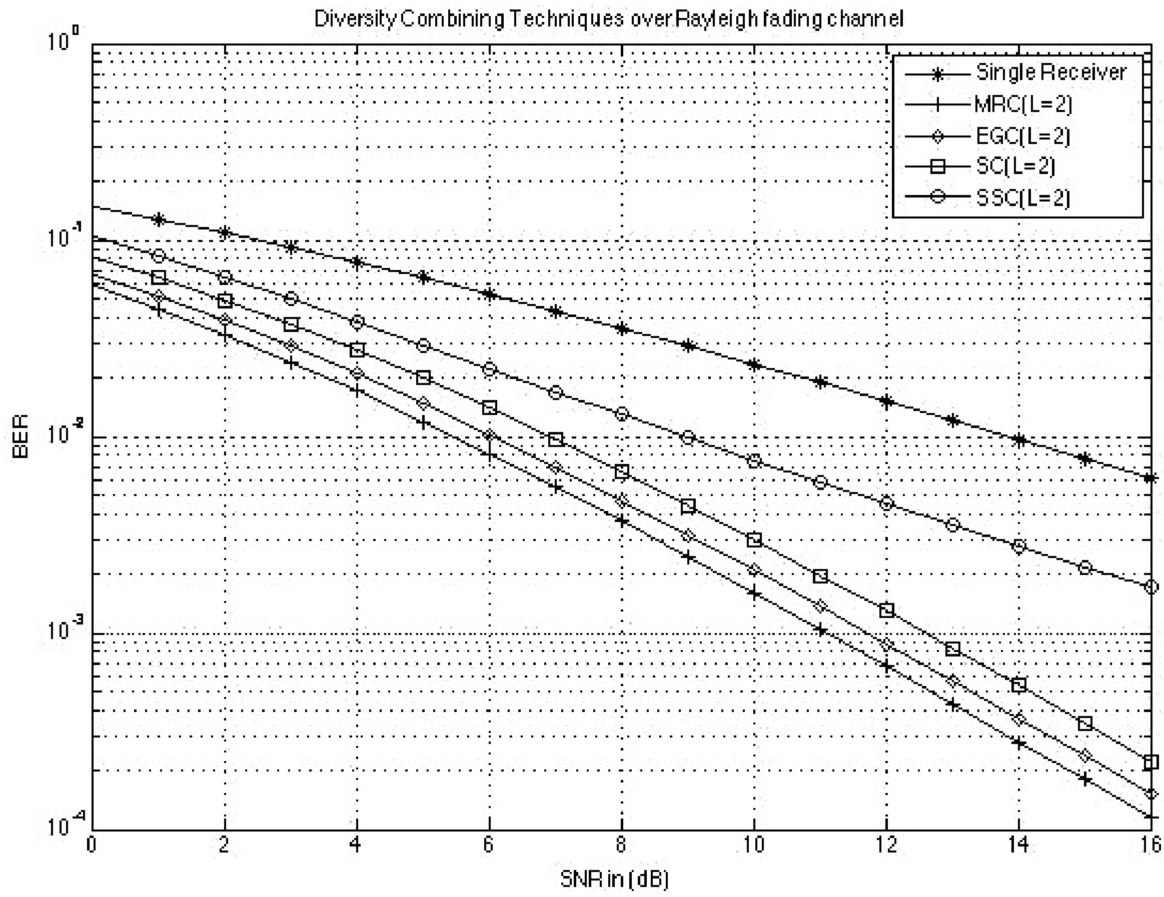

Figure 7.1.: BER for various combining techniques and a single receiver over Rayleigh channel

7.2. Generalized-K fading channel

As depicted in figure 7.2 the performance of generalized-K fading channels is overall worse than Rayleigh channels. In particular, SSC is even closer to the case of no diversity. Moreover as the SNR increases the BER doesn't decrease as rapid as in the Rayleigh case. In figure 7.3a comparison between the Rayleigh and the generalized-K fading channel is made.As depicted, Rayleigh overrides the generalized-K fading channel and the performance gap increases as the reception is improved(e.g. through diversity schemes). In figure 7.4 it is shown that as the parameters (k, m) increase the performance improves considerably, as it was expected.

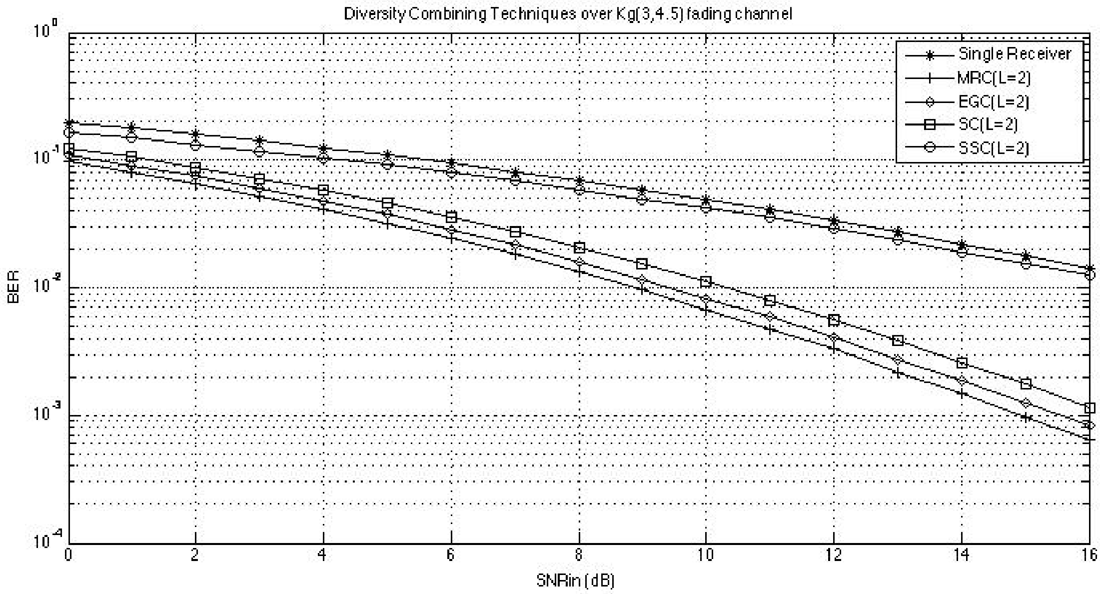

Figure 7.2.: BER for various combining techniques and a single receiver over generalized-K fading channel

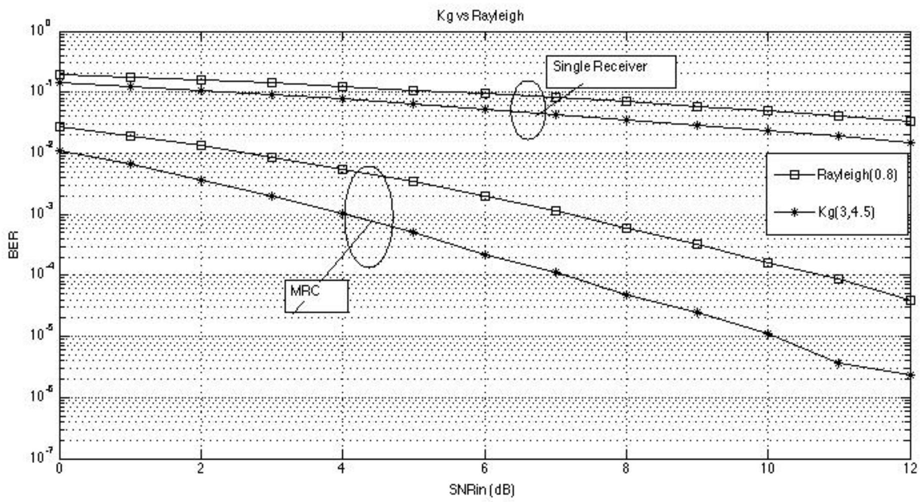

Figure 7.3.: Rayleigh and generalized-K fading channels for the single reception case

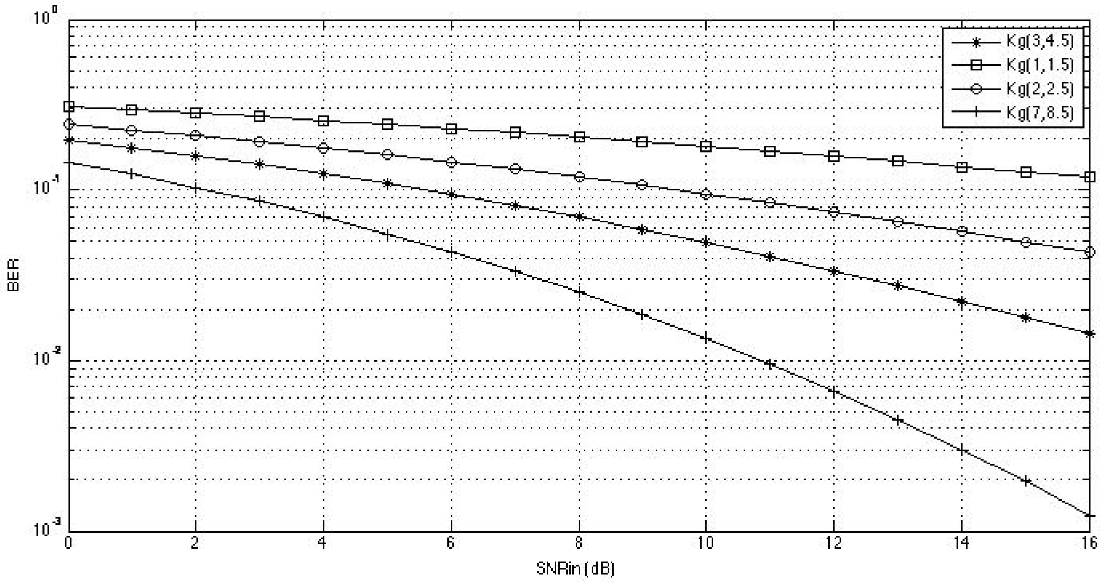

Figure 7.4.: Single reception for various values of generalized-K fading channels

7.3. MRC and EGC Receiver

As it is depicted in figure 7.5 as the number of branches increases, the performance is improved. For a small number of branches (e.g. 3) an increase will bring exponential improvement of the system, but as the number of branches grows higher, the improvement will not be so noticeable.

As depicted in figure 7.6 MRC is always better than EGC. Moreover, as the number of branches increases the difference between MRC and EGC increases.

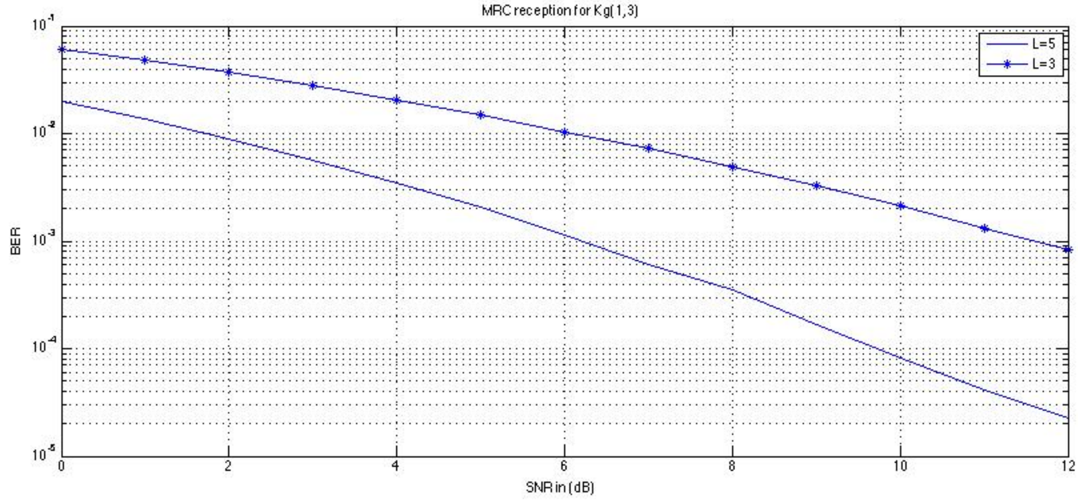

Figure 7.5.: BER of MRC receiver with L=3 and 5 receiving antennas

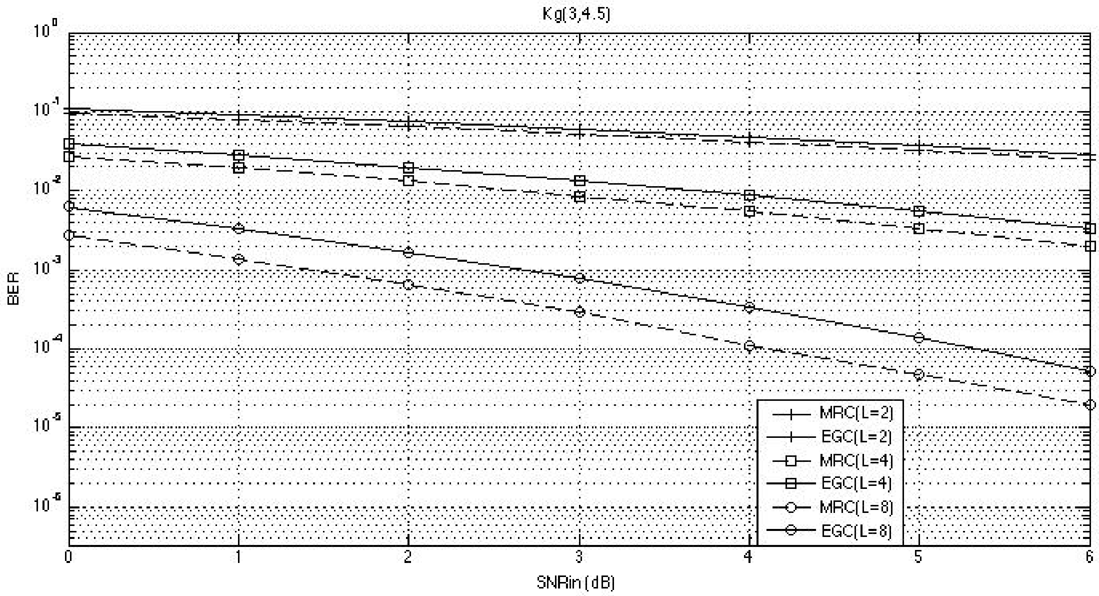

Figure 7.6.: Comparison between EGC,MRC for different number of branches

7.4. Comparison between SC,SSC,hybrid SSC

For this comparison we assumed 4 channels. In order to have a whole understanding of the combining techniques we compared them according to three different measures:

the BER, the number of channel switches, and the number of channel estimations. The number of switches refers to SSC and hybrid SSC and it is the times that the output branch changes because it falls under the threshold. In this measure the correct choice of a threshold is critical. As seen in figure 7.8 if the threshold is too low, then the number of switches will be quite small but the performance of the system (BER) will be quite bad, but if the threshold is too high the performance improves considerably but the number of switches increases. The number of channel estimations is the number of branches each technique is checking in every step of the algorithm. For the case of SC, since all the branches are checked each time, the number of channel estimations will be the number of branches, in SSC always one branch is checked, while in the case of hybrid SSC if the branch fells below the threshold it is all the branches while otherwise it is one branch.

As depicted in figure 7.7 as the threshold increases the hybrid SSC coincides with SC. But even for relatively small values of threshold hybrid SSC is closer to SC than SSC, which means that hybrid SSC is a good combining technique as it combines the simplicity of SSC and the better performance of SC.

As expected, in figure 7.8 the number of switches increases as the threshold increases. In this case SSC and hybrid SSC almost coincide, which is to be expected as both use the same criteria in order to stay or change a branch.

In figure 7.9 we can see that the number of channel estimations for the case of SSC and SC is constant and independent of the threshold. For the case of hybrid SSC as SNR increases it approaches the SC case, which is to be expected since the tighter the threshold is, the more frequent will the branches change, but these changes cause a great delay in the system.

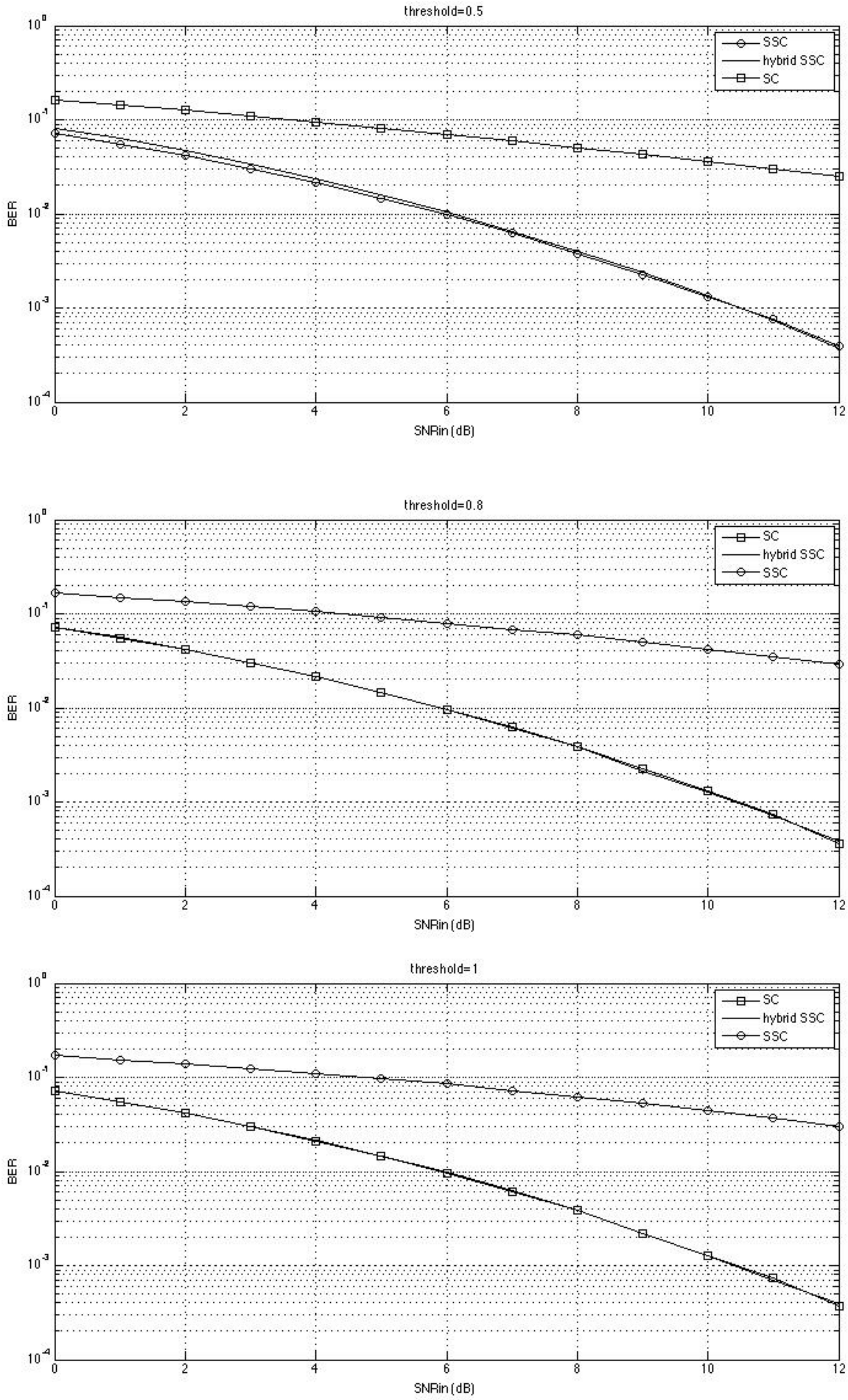

Figure 7.7.: BER of SC,SSC and hybrid SSC

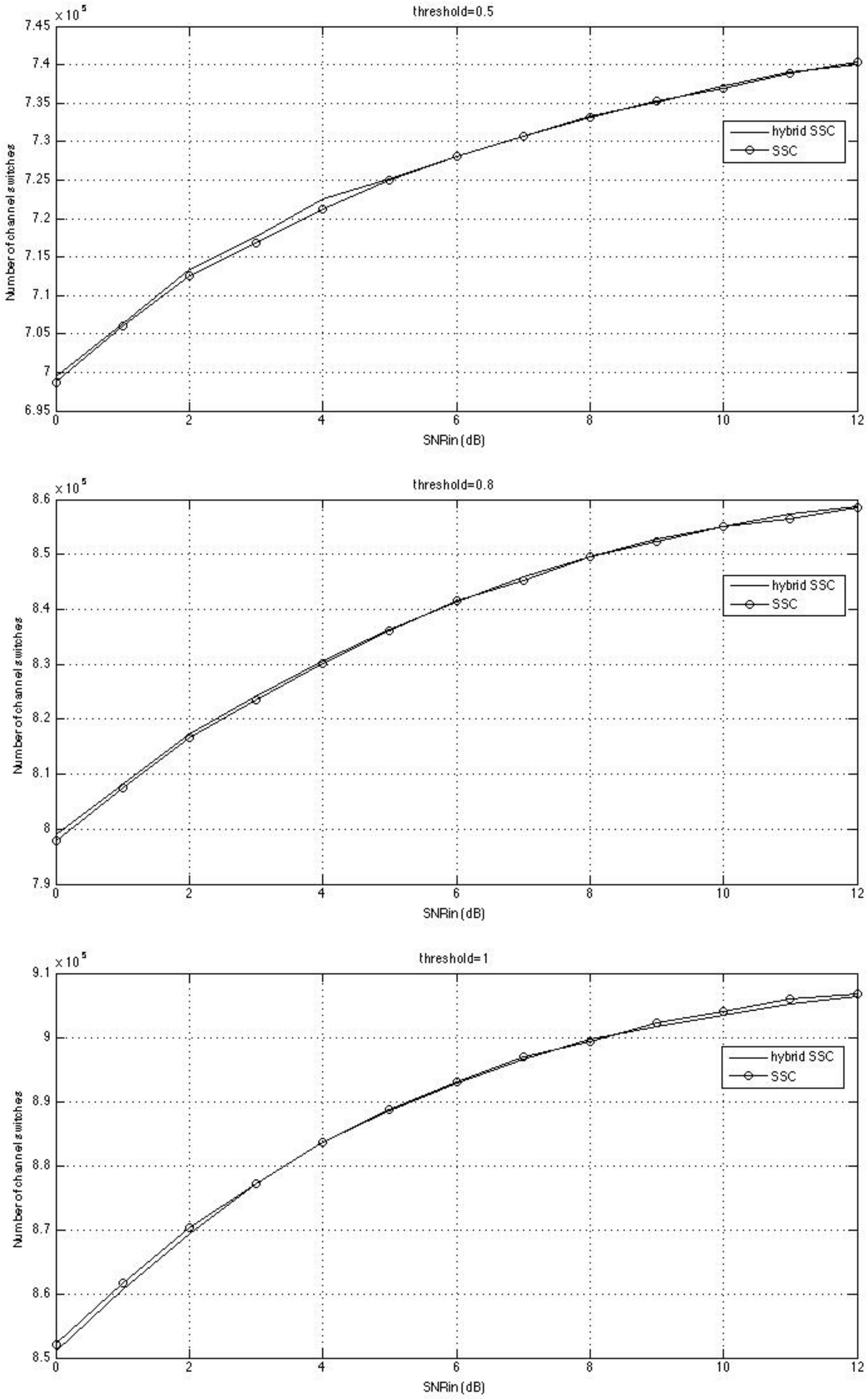

Figure 7.8.: Number of switches of SSC and hybrid SSC

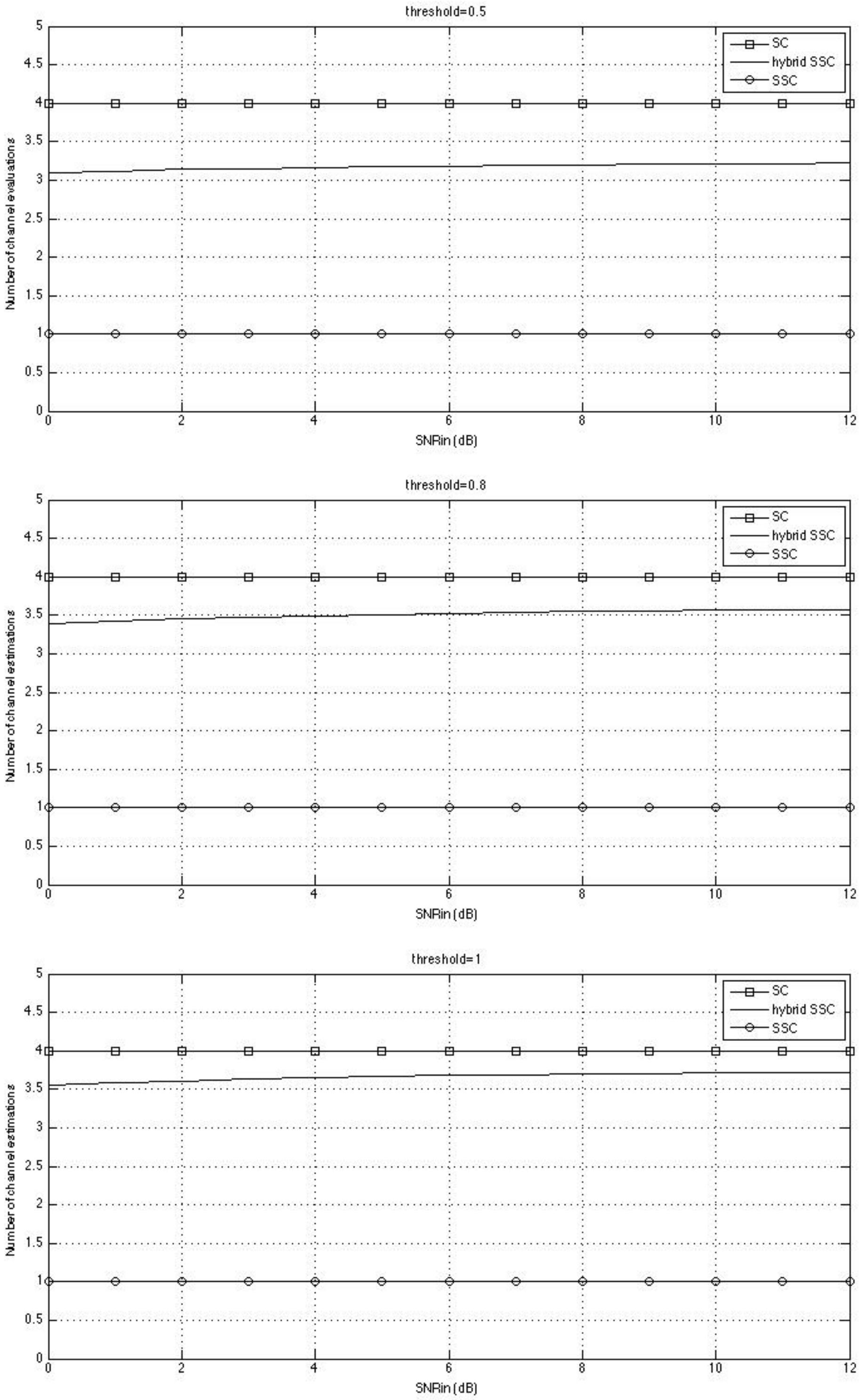

Figure 7.9.: Number of channel estimations for SC,SSC and hybrid SSC

8. Conclusions-Future Work

The purpose of this thesis was to make a bibliographical summary of fading in wireless mobile communications as well as a way to overcome it, i.e. diversity. At first, a theoretical analysis of both fading and diversity was made and then simulated results were presented.

As mentioned before fading is a great obstacle in the design of robust and efficient systems, which needs to be carefully modeled. With the find of generalized fading distributions a great step towards this task was made but there is still a long way to go. Many realistic environments have no distribution that applies to them or have distributions that are hard to manipulate. As far as the diversity is concerned, as the performance standards become more demanding, a need for more efficient and simultaneously simple combining techniques is implied. Finally a need, which applies to both the fading and the diversity case, is the derivation of closed form expressions. In the modeling of fading channels and diversity combining techniques closed form expressions play a vital role, since they can be easily and more important fast computed. Non closed form expressions are time consuming, a great drawback that increases the complexity of the system.

As far as this diploma thesis is concerned, there are a lot of aspects that can be covered in combination to the present one. In terms of the fading distributions, special focus could be also given in other generalized distributions such as the generalized Gamma distribution or the $\kappa - \mu$ and $\eta - \mu$ distributions. In terms of the diversity combining techniques, more techniques could be presented and compared such as selection and stay combining. There could be also an analysis of other diversity schemes, such as time, frequency and transmit diversity. It would be also of great interest to analyze the case where the fading distributions in the receiver are correlated, i.e. not independent in each path, a phenomenon that appears when the receiving antennas are ill-spaced. Finally the case of multi hop systems can be examined, which is the case when the signal passes from several stops before it reaches the final receiver.

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Appendix

A. Performance Evaluation Tools

A.1. Moment Generating Function (MGF)

A.1.1. Single Receiver

The MGF of a random variable is an alternative specification of its probability distribution. Thus, it provides the basis of an alternative route to analytical results compared with working directly with probability density functions or cumulative distribution functions. There are particularly simple results for the moment-generating functions of distributions defined by the weighted sums of random variables. However, not all random variables have moment-generating functions.The MGF is defined as:

$$M_\gamma(s) = \int\limits_0^\infty p_\gamma(\gamma) e^{-s\gamma} d\gamma \tag{A.1.1}$$

A.1.2. Multiple Receivers

The definition of MGF can be found on this page. According to the combining technique the MGF is found to be:

MRC

By combining (A.1.1) and (4.2.3), $M_{\gamma_{MRC}}(\cdot)$ is found to be in [27]:

$$M_{\gamma_{MRC}}(s) = \prod_{l=1}^{L} M_{\gamma_l}(s) \tag{A.1.2}$$

where $M_{\gamma_l}(\cdot)$ is the MGF of the fading distribution.

coherent EGC

For the EGC receiver it is very difficult, if not impossible, to derive a closed form expression for [27]. Thus the Padé approximants method can be used to obtain the best approximation using (5.1.2).

SC

In SC there is no general closed form of the MGF of the output SNR. Thus the Padé approximants method can be used. For special cases the MGF can be found in [10].

SSC

In [27] $M_{\gamma_{SSC}}(\cdot)$ is found:

$$M_{\gamma_{SSC}}(s) = \frac{P_1 P_2}{P_1 + P_2} \sum_{l=1}^{L} \left\{ M_{\gamma_l}(s) + \frac{M_{\gamma_l}(s)}{P_l} - \frac{1}{P_l} \int_0^{\gamma_T} \exp(-s\gamma)\, p_{\gamma_l}(\gamma)\, d\gamma \right\} \quad \text{(A.1.3)}$$

where $M_{\gamma_l}(\cdot), p_{\gamma_t}(\cdot)$ is the MGF and the PDF of the fading distribution.The infinite integral can be evaluated via numerical integration. From (A.1.3) we can also derive the optimum threshold , which is the result of

$$\left. \frac{dP_b(E)}{d\gamma_T} \right|_{\gamma_T = \gamma_T^*} = 0 \quad \text{(A.1.4)}$$

as found in [10, p. 428].

GSC

For the i.i.d. case from (A.1.2) $M_{\gamma_{GSC}}$ is found as:

$$M_{\gamma_{MRC}}(s) = \prod_{k=1}^{N} M_{\gamma_{(k)}}(s) \quad \text{(A.1.5)}$$

A.2. Padé Approximants Method

When $M_{\gamma_t}(\cdot)$ cannot be computed in closed form the Padé approximants method can be used to approximate it. $M_{\gamma_t}(\cdot)$ can be presented as formal power series as:

$$M_{\gamma_t}(s) = \sum_{n=0}^{\infty} \frac{\mu_{\gamma_t}(n)}{n!} s^n \quad \text{(A.2.1)}$$

where μ_{γ_t} is the nth order moment 3.1.12. In practice

$$M_{\gamma_t}(s) \cong \sum_{n=0}^{N} \frac{\mu_{\gamma_t}(n)}{n!} s^n + O\left(s^{N+1}\right) \quad \text{(A.2.2)}$$

where N is finite and $O\left(s^{N+1}\right)$ is the remainder after the truncation. Since it is not known whether and when (A.2.2) converges the Padé's approximants are the best approximation of (A.2.1). According to this method there is a rational function of given order, which power series agree with the power series of the function it is approximating, i.e. there is a rational function

$$R_{[A|B]}\left(s\right)=\frac{\sum_{i=0}^{A}a_{i}s^{i}}{\sum_{j=0}^{B}b_{j}s^{j}} \tag{A.2.3}$$

where

$$R_{[A|B]}\left(s\right)=\sum_{n=0}^{A+B}\frac{\mu_{\gamma_t}\left(n\right)}{n!}s^{n}+O\left(s^{N+1}\right) \tag{A.2.4}$$

Thus through the evaluation of μ_{γ_t} we can compute the MGF M_{γ_t}.