Abstract

The main result of this doctoral thesis, is Theorem 3.6 . For the understanding of this Theorem, it is necessary to describe the difference between the variable exponent and the constant exponent which can be achieved by giving to the reader some appropriate mathematic tools in the second chapter.

More specifically, in paragraph 2.1 we give the definition of the variable exponent Lebesgue space, which we denote by $L^{p(\cdot)}(\Omega)$. The Theorems we prove here, help us directly or indirectly to understand the proof of Theorem 3.6 and in the same time familiarise the reader with the space $L^{p(\cdot)}(\Omega)$. In paragraph 2.2, we define the variable exponent Sobolev space, wich we denote by $W^{1,p(\cdot)}(\Omega)$. Also we present the log-Hölder continuity condition. A necessary condition discovered independently by S. Samko and L. Diening for density to be hold. At the end of the paragraph, we give the definition of the Riesz potential.

In chapter 3, the main Theorem (Theorem 3.6), relates the density of smooth functions to the boundness of the $L^{p(\cdot)}$-norm of the convolution

$$f * |x - y|^{1-n},$$

where n is the dimension of the space.

In chapter 4, we prove via Theorem 3.6 two important Theorems (Theorems 4.1 and 4.2) that point us a crucial relation between the dimension of the space and the density of smooth functions. Theorem 4.1 states that if the variable exponent is bigger than n, where n is the dimension of the space, density holds. Theorem 4.2 states that if the variable exponent is smaller than n, where n is the dimension of the space and another condition holds, then density holds.

In chapter 5, we prove a Theorem (Theorem 5.5), that certify a relation between the local boundness of the maximal operator and the density of smooth functions.

In chapter 6, we generalize these results to $W^{\kappa,p(\cdot)}(\mathbb{R}^n)$ spaces.