NATIONAL TECHNICAL UNIVERSITY OF ATHENS
SCHOOL OF CHEMICAL ENGINEERING
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PhD Dissertation

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PECHAK OLENA

BSc, MSc in Ecology, National Technical University of Ukraine "KPI"

Scientific Committee:
G. Mavrotas
D. Diakoulaki
J. Psarras

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ΕΘΝΙΚΟ ΜΕΤΣΟΒΙΟ ΠΟΛΥΤΕΧΝΕΙΟ
ΣΧΟΛΗ ΧΗΜΙΚΩΝ ΜΗΧΑΝΙΚΩΝ
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Σύστημα Υποστήριξης Αποφάσεων για επιλογή επενδυτικών σχεδίων σε συνθήκες αβεβαιότητας με τη χρήση πολυκριτηριακής ανάλυσης και μαθηματικού προγραμματισμού

Διδακτορική Διατριβή

Pechak Olena

Συμβουλευτική Επιτροπή: Γ. Μαυρωτάς (επιβλέπων)
Δ. Διακουλάκη
Ι. Ψαρράς

ΑΘΗΝΑ 2017

Approved by

First reader

Dr. George Mavrotas

Associate Professor, School of Chemical Engineering, National Technical University of Athens, Greece

Second reader

Dr. Danae Diakoulaki

Professor, School of Chemical Engineering, National Technical University of Athens, Greece

Third reader

Dr. John Psarras

Professor, School of Electrical and Computer Engineering, National Technical University of Athens, Greece

Fourth reader

Dr. Yannis Caloghirou

Professor, School of Chemical Engineering, National Technical University of Athens, Greece

Fifth reader

Dr. Haris Doukas

Assistant Professor, School of Electrical and Computer Engineering, National Technical University of Athens, Greece

Sixth reader

Dr. Sevastianos Mirasgedis

Senior Researcher, National Observatory of Athens, Greece

Seventh reader

Dr. Carlos Henggeler Antunes

Professor, Department of Electrical Engineering and Computers, University of Coimbra, Portugal

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Extended Summary

Making a choice is an everyday activity, which in various professional domains often involves the search for additional information. However, abundance in input data requires special tools in order to perform a balanced selection. Over the last decades, numerous methods and decision support tools were developed, but unfortunately, possibly due to the lack of knowledge, decision makers may see these tools as black boxes. Ironically, systems developed to assist in decision making often seem to be too complex and unclear. In addition, within the selection process it is often necessary to make a subjective choice among objectively determined solutions.

This thesis addresses the so-called project portfolio selection problem, which aims at selecting a certain number from a wide set of proposed projects. Usually the projects are not independent, i.e., there are particular limitations that should be respected (segmentation constraints, mutually exclusive, precedence etc.) so that Multiple Criteria Decision Aid (MCDA) methods need to be combined with combinatorial optimization techniques. A popular way to deal with this problem is to use a two-step approach: (1) A multicriteria method to evaluate the projects, and (2) a mathematical programming model that incorporates constraints in which the objective function coefficients are the multicriteria scores.

This thesis develops a method that helps to perform a selection in a step-wise and transparent way. The core idea lies in the separation of project proposals into three separate sets. The approach is not totally new, but the rules of this separation are novel. The basic idea of the proposed Iterative Trichotomic Approach (ITA) is the classification of projects into three sets: the green projects (selected under all circumstances), the red projects (definitely excluded from the final portfolio) and the grey projects which are chosen in some (but not all) cases. The main focus is on building a balanced project portfolio from a wider set of proposals (a subset of projects is considered as a “portfolio of projects”), optimizing one or more criteria and satisfying specific constraints. In past, the usual solution was to rank projects using one or more criteria and choose the top ranking ones that cumulatively satisfy a budget limitation. However, in real world there are many circumstances that complicate the process of decision making. In other words, top ranking projects may only by chance satisfy imposed constraints. Unlike in financial problems (e.g., portfolio optimization problems), these projects are integer variables which are not divisible, and hence, Multiple Criteria Decision Analysis and mathematical programming are the most appropriate tools.

In this work, we are taking a step further, and we address the inherent uncertainty which can vary in nature, the most prominent type being the future project performance. While the financial world offers a great amount of data that help to build more or less robust forecasts, it is almost impossible to obtain historical

data for emerging technologies or pioneering solutions. The uncertainty may be present either in the project characteristics (e.g., costs, performances) or in the decision environment (e.g., criteria weights, total budget). In the proposed model, the uncertainties in various parameters or input data are modeled via stochastic approaches tackled with Monte Carlo simulation.

The method works iteratively, in decision rounds. In each decision round we use the obtained information or follow a predetermined rule in order to reduce the uncertainty. Gradually from round to round, the green and red sets increase while the grey set with the ambiguous projects is reduced. Eventually, the process ends with only green and red projects. In comparison to the conventional project selection approaches, with ITA we also obtain the "degree of certainty" for a project that is included or excluded from the final portfolio. The earlier (i.e., in the early decision rounds) a project is accepted or discarded from the process the more sure one can be about its incorporation or exclusion in the final portfolio, respectively.

Furthermore, the proposed method is also adequate when multiple decision makers are involved. When the selection process takes place within a group, the preferences of various experts are not the same and there must be a negotiation approach taking into account all points of view. The whole process can either have a predetermined number of decision rounds or continue until a convergence to the final portfolio is attained. Group-ITA provides a possibility to draw conclusions about the consensus over each individual project as well as on the final portfolio. Initially, a mathematical model is developed, where preferences of decision makers are expressed with appropriate weights of importance for the criteria, and a Delphi-like process is designed for the convergence of these preferences. Weights are updated from round to round, and in each round, the mathematical model is updated according to the new weights and solved. As the iterative process moves from round to round, the green and red sets are enriched whereas the grey set shrinks. The iterative process terminates when the grey set becomes empty. The final outcome is the consensus portfolio of projects, as well as the degree of consensus on each project and the consensus index for the whole portfolio according to the convergence path. The consensus index expresses the easiness to arrive at a final conclusion within a group. The more green projects that are identified from early rounds the greater is the degree of concordance among experts. This means that their preferences (expressed as weights) result in more or less the same outcome, or, in other words, the consensus is easily attained. On the contrary, if the majority of green projects is identified in the last rounds, this indicates the need to elaborate in the convergence process in order to agree at selected projects. In other words, the consensus is hardly attained. Apart from the consensus index, it is possible to extract the degree of consensus for each project according to the round that it enters or exits the final portfolio.

The membership of the projects in the final portfolio is also expanded to the membership of portfolios in the final Pareto set where more than one objective functions are considered. While the original ITA method was designed to implement a single objective mathematical programming problem, the latter version of ITA method is extended to multi-objective programming problems. The degree of certainty of the Pareto optimal portfolios that belong to the final Pareto set can also be measured.

ITA was applied to several real world problems that are presented in this thesis. The first topic that attracted our attention was the selection of projects in the telecommunications sector. Wide and fast spread of new technological developments requires effective tools to select options for expansion and meeting growing demand. The need for balanced introduction of new service offerings is a problem which involved different and conflicting aspects. The main feature of the proposed decision aid computational tool is the incorporation of several uncertainties in the selection process, and the gradual building of a project portfolio.

Other applications involve renewable energy projects both in national and worldwide levels. A case study with real data from the Clean Development Mechanism projects' database is elaborated, in order to build a balanced portfolio of "green" activities. The specific work is focused on the energy project portfolio selection problem, where the output of each project as well as other parameters may be uncertain. For this case study, we consider the implied uncertainty in the parameters as being of stochastic nature that is characterized by a probability distribution. Subsequently, a Monte Carlo simulation samples the values from these distributions and the mathematical programming models with the sampled values are solved. The process output is not only the final portfolio, but also the information about the certainty of participation or exclusion of each project in the final portfolio.

Another example deals with Greek renewable energy projects that seek support from financial institutions, where it is crucial for the donor organization to make a balanced selection and avoid the tactics of "all eggs in one basket". In this case, 133 Greek project proposals covering three renewable energy technologies (wind, small hydro, photovoltaic) were evaluated against 5 criteria. Since several experts with different preferences took part in the selection process, Group ITA is designed to gradually add projects to the portfolio according to the concordance within the team members until a final portfolio is reached.

The last example is an attempt to bridge the gap between business and public interests. Nowadays, increasing emphasis is put on the environmentally friendly activities that are considered to be among the key solutions in combating current financial and economic crisis. For this reason, we test the possibility to incorporate Energy and Environmental Corporate Responsibility (EECR) in decision making, supporting particularly the development of a new model for investment evaluation. A bi-objective programming model is introduced in order to provide Pareto optimal portfolios (Pareto set) based on the Net Present Value (NPV) of projects and the EECR score of firms. Moreover, a systematic decision making approach using Monte Carlo simulation is developed in order to deal with the inherent uncertainty in the objective functions' coefficients, namely the NPV of each project and the EECR score of each firm. In addition, the robustness of the Pareto set as a whole, as well as the robustness of the individual Pareto optimal portfolios can also be assessed. The proposed approach facilitates investment organizations and institutions to the selection of firms applying for financial support and credit granting, within the frame of their EECR.

Within all case studies it was more than evident how the ITA offered very fruitful information to the decision maker as it quantified the degree of certainty with which each project was treated in the final portfolio, a task that cannot be accomplished with the conventional methods using average and expected values in modeling the uncertainty.

Περίληψη

Η διαδικασία της επιλογής είναι μια καθημερινή δραστηριότητα, η οποία σε διάφορους επαγγελματικούς τομείς συχνά περιλαμβάνει την αναζήτηση πρόσθετων πληροφοριών. Ωστόσο, η αφθονία στα δεδομένα εισόδου απαιτεί ειδικά εργαλεία για την εκτέλεση μιας ισορροπημένης επιλογής. Τις τελευταίες δεκαετίες αναπτύχθηκαν πολλές μέθοδοι και εργαλεία υποστήριξης αποφάσεων, αλλά δυστυχώς, ενδεχομένως λόγω έλλειψης γνώσης, οι υπεύθυνοι για τη λήψη αποφάσεων βλέπουν τα εργαλεία αυτά ως μαύρα κουτιά. Κατά περίεργο τρόπο, τα ίδια τα συστήματα που αναπτύσσονται για να βοηθήσουν στη λήψη αποφάσεων συχνά φαίνονται πολύ περίπλοκα και ασαφή. Επιπρόσθετα, στο πλαίσιο της διαδικασίας επιλογής είναι συχνά απαραίτητο να γίνει μια υποκειμενική επιλογή ανάμεσα σε αντικειμενικά προσδιορισμένες λύσεις.

Η παρούσα διατριβή ασχολείται με το πρόβλημα επιλογής χαρτοφυλακίου έργων για την επιλογή ενός συγκεκριμένου αριθμού από ένα ευρύ σύνολο προτεινόμενων έργων. Συνήθως τα έργα δεν είναι ανεξάρτητα, δηλαδή υπάρχουν συγκεκριμένοι περιορισμοί που πρέπει να ικανοποιηθούν (περιορισμοί τμηματοποίησης, αμοιβαίου αποκλεισμού, προτεραιότητας, κ.λπ.), οπότε οι πολυκριτηριακές μέθοδοι υποστήριξης της απόφασης (Multiple Criteria Decision Aid - MCDA) δεν επαρκούν και πρέπει να συνδυαστούν με τεχνικές συνδυαστικής βελτιστοποίησης. Ένας δημοφιλής τρόπος αντιμετώπισης αυτού του προβλήματος είναι η χρήση μιας προσέγγισης δύο βημάτων: (1) Μιας πολυκριτηριακής μεθόδου για την αξιολόγηση των έργων και (2) ενός μοντέλου μαθηματικού προγραμματισμού που ενσωματώνει περιορισμούς με τους συντελεστές της αντικειμενικής συνάρτησης να είναι οι πολυκριτηριακές βαθμολογίες.

Η παρούσα διατριβή αναπτύσσει μια μέθοδο που βοηθά στην πραγματοποίηση μιας επιλογής βήμα-βήμα και με διαφάνεια. Η βασική ιδέα έγκειται στο διαχωρισμό των προτάσεων έργων σε τρία ξεχωριστά σύνολα. Η προσέγγιση δεν είναι εντελώς νέα, οι κανόνες όμως αυτού του διαχωρισμού είναι καινοτόμοι. Βασική ιδέα της επαναληπτικής τριχοτομικής προσέγγισης (Iterative Trichotomic Approach - ITA) είναι η ταξινόμηση των έργων σε τρία σύνολα: τα πράσινα έργα (που επιλέγονται υπό οποιεσδήποτε συνθήκες), τα κόκκινα έργα (οριστικά αποκλεισμένα από το τελικό χαρτοφυλάκιο) και τα γκρίζα έργα που επιλέγονται σε ορισμένες περιπτώσεις (αλλά όχι όλες). Ο κύριος στόχος είναι η δημιουργία ενός ισορροπημένου χαρτοφυλακίου έργων από ένα ευρύτερο σύνολο προτάσεων (ένα υποσύνολο έργων θεωρείται ως "χαρτοφυλάκιο έργων"), βελτιστοποιώντας ως προς ένα ή περισσότερα κριτήρια και ικανοποιώντας συγκεκριμένους περιορισμούς. Στο παρελθόν, η συνηθισμένη λύση ήταν η κατάταξη των έργων χρησιμοποιώντας ένα ή περισσότερα κριτήρια και η επιλογή των πρώτων κατά σειρά που ικανοποιούν αθροιστικά τον περιορισμό του προϋπολογισμού. Ωστόσο, στην πράξη η διαδικασία αυτή είναι αρκετά πιο

περίπλοκη. Τα πρώτα κατά σειρά κατάταξης έργα μπορούν μόνο κατά τύχη να ικανοποιήσουν τους επιβαλλόμενους περιορισμούς. Σε αντίθεση με τα οικονομικά προβλήματα (π.χ. προβλήματα βελτιστοποίησης χαρτοφυλακίου), τα έργα αυτά είναι ακέραιες μεταβλητές που δεν διαιρούνται, και κατά συνέπεια η πολυκριτηριακή ανάλυση της απόφασης και ο μαθηματικός προγραμματισμός αποτελούν τα πλέον κατάλληλα εργαλεία.

Στην παρούσα εργασία, προχωράμε ένα βήμα παρακάτω, εξετάζοντας την εγγενή αβεβαιότητα, η οποία μπορεί να ποικίλει στη φύση, με την πιο σημαντική να είναι η μελλοντική απόδοση του έργου. Ενώ στον χρηματοπιστωτικό κόσμο είναι διαθέσιμα πολλά δεδομένα που βοηθούν τις σχετικά ισχυρές προβλέψεις, είναι σχεδόν αδύνατο να αποκτηθούν ιστορικά δεδομένα για αναδυόμενες τεχνολογίες ή πρωτοποριακές λύσεις. Η αβεβαιότητα μπορεί να υπάρχει είτε στα χαρακτηριστικά του έργου (π.χ. κόστος, επιδόσεις) είτε στο περιβάλλον απόφασης (π.χ. σταθμίσεις κριτηρίων, συνολικός προϋπολογισμός). Στο προτεινόμενο μοντέλο οι αβεβαιότητες σε διάφορες παραμέτρους ή δεδομένα εισόδου διαμορφώνονται μέσω στοχαστικών προσεγγίσεων που χρησιμοποιούν προσομοίωση Monte Carlo.

Η μέθοδος λειτουργεί επαναληπτικά, σε γύρους αποφάσεων. Σε κάθε γύρο απόφασης χρησιμοποιούμε τις πληροφορίες που λαμβάνουμε ή ακολουθούμε έναν προκαθορισμένο κανόνα για να μειώσουμε την αβεβαιότητα. Σταδιακά από γύρο σε γύρο, τα πράσινα και κόκκινα σύνολα αυξάνονται ενώ το γκρίζο σύνολο με τα "ασαφή" έργα μειώνεται. Τελικά, η διαδικασία τελειώνει μόνο με πράσινα και κόκκινα έργα. Σε σύγκριση με τις συμβατικές προσεγγίσεις επιλογής έργων, με την ITA λαμβάνουμε επίσης το "βαθμό βεβαιότητας" για ένα έργο που περιλαμβάνεται ή αποκλείεται από το τελικό χαρτοφυλάκιο. Όσο νωρίτερα (δηλαδή στους πρώτους γύρους αποφάσεων) ένα έργο γίνεται δεκτό ή απορρίπτεται από τη διαδικασία, τόσο πιο σίγουρη μπορεί να είναι η ενσωμάτωση ή ο αποκλεισμός του από το τελικό χαρτοφυλάκιο, αντίστοιχα.

Επιπλέον, η προτεινόμενη μέθοδος είναι επίσης κατάλληλη όταν εμπλέκονται πολλοί υπεύθυνοι λήψης αποφάσεων. Όταν η διαδικασία επιλογής λαμβάνει χώρα μέσα σε μια ομάδα, οι προτιμήσεις διαφόρων εμπειρογνωμόνων δεν είναι οι ίδιες και πρέπει να υπάρχει μια προσέγγιση διαπραγματεύσεων που λαμβάνει υπόψη όλες τις απόψεις. Η όλη διαδικασία μπορεί είτε να έχει προκαθορισμένο αριθμό γύρων αποφάσεων είτε να συνεχίζεται μέχρι να επιτευχθεί σύγκλιση στο τελικό χαρτοφυλάκιο. Η ομαδική επαναληπτική τριχοτομική προσέγγιση (Group-ITA) παρέχει τη δυνατότητα εξαγωγής συμπερασμάτων σχετικά με τη συναίνεση για κάθε μεμονωμένο έργο καθώς και για το τελικό χαρτοφυλάκιο. Αρχικά, αναπτύσσεται ένα μαθηματικό μοντέλο όπου οι προτιμήσεις των υπευθύνων λήψης αποφάσεων εκφράζονται με τα κατάλληλα βάρη σπουδαιότητας για τα κριτήρια, και σχεδιάζεται μια διαδικασία τύπου Delphi για τη σύγκλιση αυτών των προτιμήσεων. Τα βάρη ενημερώνονται από γύρο σε γύρο και, κάθε φορά, το μαθηματικό μοντέλο επικαιροποιείται με τα νέα βάρη και επιλύεται. Καθώς η επαναληπτική διαδικασία εκτελείται, από γύρο σε γύρο, τα πράσινα και κόκκινα σύνολα εμπλουτίζονται, ενώ το γκρι σύνολο συρρικνώνεται. Η επαναληπτική διαδικασία λήγει όταν το γκρίζο σύνολο γίνει κενό. Το τελικό αποτέλεσμα είναι το συναινετικό χαρτοφυλάκιο έργων, καθώς και ο βαθμός συναίνεσης για κάθε έργο και ο δείκτης συναίνεσης για το σύνολο του χαρτοφυλακίου σύμφωνα με τη διαδρομή σύγκλισης. Ο δείκτης συναίνεσης (consensus index) εκφράζει την ευκολία με την οποία μια ομάδα καταλήγει σε ένα τελικό συμπέρασμα. Όσο περισσότερα πράσινα έργα εντοπίζονται από τους πρώτους γύρους, τόσο μεγαλύτερος είναι ο βαθμός συμφωνίας μεταξύ των εμπειρογνωμόνων. Αυτό σημαίνει ότι οι προτιμήσεις τους (εκφραζόμενες ως βάρη) οδηγούν σχεδόν στο ίδιο αποτέλεσμα ή, με άλλα λόγια, επιτυγχάνεται εύκολα η συναίνεση. Αντίθετα, αν η πλειοψηφία των πράσινων έργων εντοπιστεί στους τελευταίους γύρους, αυτό είναι ενδεικτικό της ανάγκης για περαιτέρω ανάπτυξη της διαδικασίας σύγκλισης προκειμένου να επιτευχθεί συμφωνία στα επιλεγμένα έργα. Με άλλα λόγια, η συναίνεση επιτυγχάνεται δύσκολα. Εκτός από τον δείκτη συναίνεσης, είναι δυνατό να εξαχθεί ο βαθμός συναίνεσης για κάθε έργο σύμφωνα με τον γύρο κατά τον οποίο εισέρχεται ή εξέρχεται από το τελικό χαρτοφυλάκιο.

Η συμμετοχή των έργων στο τελικό χαρτοφυλάκιο επεκτείνεται και στην συμμετοχή των χαρτοφυλακίων στο τελικό σύνολο Pareto όπου εξετάζονται περισσότερες από μία αντικειμενικές συναρτήσεις. Ενώ η αρχική μέθοδος ITA σχεδιάστηκε για να εφαρμόσει ένα πρόβλημα μαθηματικού προγραμματισμού με μία αντικειμενική συνάρτηση, η ομαδική έκδοση της μεθόδου ITA επεκτείνεται σε προβλήματα με περισσότερες αντικειμενικές συναρτήσεις. Ο βαθμός βεβαιότητας των βέλτιστων χαρτοφυλακίων Pareto που ανήκουν στο τελικό σύνολο Pareto μπορεί επίσης να μετρηθεί.

Η μέθοδος ITA εφαρμόστηκε σε αρκετά πραγματικά προβλήματα που παρουσιάζονται στην παρούσα διατριβή. Το πρώτο θέμα που προσέλκυσε την προσοχή μας ήταν η επιλογή έργων στον τομέα των τηλεπικοινωνιών. Η ευρεία και γρήγορη διάδοση των νέων τεχνολογικών εξελίξεων απαιτεί αποτελεσματικά εργαλεία για την επιλογή εναλλακτικών επέκτασης και κάλυψης της αυξανόμενης ζήτησης. Η ανάγκη για μια ισορροπημένη προσφορά νέων υπηρεσιών είναι ένα πρόβλημα με διαφορετικές και αλληλοσυγκρουόμενες πτυχές. Το κύριο χαρακτηριστικό του προτεινόμενου υπολογιστικού εργαλείου υποστήριξης της απόφασης είναι η ενσωμάτωση πολλών αβεβαιότητων στη διαδικασία επιλογής και η σταδιακή "οικοδόμηση" του χαρτοφυλακίου έργων.

Άλλες εφαρμογές ασχολούνται με έργα ανανεώσιμης ενέργειας τόσο σε εθνικό όσο και σε παγκόσμιο επίπεδο. Εξετάσθηκε μελέτη περίπτωσης με πραγματικά στοιχεία από τη βάση δεδομένων του Μηχανισμού Καθαρής Ανάπτυξης, προκειμένου να δημιουργηθεί ένα ισορροπημένο χαρτοφυλάκιο "πράσινων" δραστηριοτήτων. Η συγκεκριμένη εργασία επικεντρώνεται στο πρόβλημα επιλογής χαρτοφυλακίου ενεργειακών έργων όπου η ενεργειακή παραγωγή κάθε έργου καθώς και άλλες παράμετροι μπορεί να είναι αβέβαιες. Για την τρέχουσα μελέτη περίπτωσης θεωρούμε ότι η αβεβαιότητα στις παραμέτρους είναι στοχαστική και χαρακτηρίζεται από μία κατανομή πιθανότητας. Στη συνέχεια, με προσομοίωση Monte

Carlo λαμβάνονται οι τιμές από αυτές τις κατανομές και επιλύονται τα μοντέλα μαθηματικού προγραμματισμού με τις τιμές δειγματοληψίας. Το αποτέλεσμα της διαδικασίας δεν είναι μόνο το τελικό χαρτοφυλάκιο, αλλά και οι πληροφορίες σχετικά με τη βεβαιότητα συμμετοχής ή αποκλεισμού κάθε έργου από το τελικό χαρτοφυλάκιο.

Το επόμενο παράδειγμα αφορά έργα Ανανεώσιμης Πηγών Ενέργειας (ΑΠΕ) στην Ελλάδα, που επιδιώκουν οικονομική υποστήριξη από χρηματοπιστωτικά ιδρύματα, όπου είναι πολύ σημαντικό να γίνει μια ισορροπημένη επιλογή και να αποφευχθεί η τακτική "όλα τα αυγά σε ένα καλάθι". Στην εξετασθείσα περίπτωση, αξιολογήθηκαν 133 προτάσεις έργων που καλύπτουν τρεις τεχνολογίες ΑΠΕ (αιολικά έργα, μικρά υδροηλεκτρικά, φωτοβολταϊκά) με 5 κριτήρια. Δεδομένου ότι στη διαδικασία επιλογής συμμετείχαν αρκετοί ειδικοί με διαφορετικές προτιμήσεις, η ομαδική ΙΤΑ σχεδιάστηκε για να προσθέτει σταδιακά τα έργα στο χαρτοφυλάκιο ανάλογα με τη συμφωνία που επιτυγχάνεται στα μέλη της ομάδας μέχρι να φτάσει ένα τελικό χαρτοφυλάκιο.

Το τελευταίο παράδειγμα είναι μια προσπάθεια να γεφυρωθεί το χάσμα μεταξύ επιχειρηματικών και δημόσιων συμφερόντων. Σήμερα, δίδεται ολοένα και μεγαλύτερη έμφαση στις φιλικές προς το περιβάλλον δραστηριότητες, οι οποίες θεωρούνται μία από τις βασικές λύσεις για την καταπολέμηση της τρέχουσας χρηματοπιστωτικής και οικονομικής κρίσης. Αυτός είναι ο λόγος που δοκιμάζουμε τη δυνατότητα ενσωμάτωσης της ενεργειακής και περιβαλλοντικής εταιρικής ευθύνης (Energy and Environmental Corporate Responsibility - EECR) στη διαδικασία λήψης αποφάσεων, υποστηρίζοντας ιδιαίτερα την ανάπτυξη ενός νέου μοντέλου για την αξιολόγηση των επενδύσεων. Εφαρμόζεται ένα μοντέλο μαθηματικού προγραμματισμού δύο αντικειμενικών συναρτήσεων, προκειμένου να βρεθούν τα βέλτιστα κατά Pareto χαρτοφυλάκια (σύνολο Pareto) με βάση την Καθαρή Παρούσα Αξία (Net Present Value - NPV) των έργων και την βαθμολογία των επιχειρήσεων ως προς την EECR. Επιπρόσθετα, αναπτύσσεται μια συστηματική προσέγγιση λήψης αποφάσεων χρησιμοποιώντας προσομοίωση Monte Carlo, προκειμένου να αντιμετωπιστεί η εγγενής αβεβαιότητα των συντελεστών των αντικειμενικών συναρτήσεων, δηλαδή η NPV κάθε έργου και η βαθμολογία EECR κάθε επιχείρησης. Επιπλέον, μπορεί να αξιολογηθεί η ευρωστία του ίδιου του συνόλου Pareto, καθώς και η ευρωστία των επιμέρους βέλτιστων χαρτοφυλακίων του Pareto. Η προτεινόμενη προσέγγιση διευκολύνει τους επενδυτικούς οργανισμούς και ιδρύματα στην επιλογή των επιχειρήσεων που υποβάλλουν αίτηση για οικονομική υποστήριξη και χορήγηση πιστώσεων στο πλαίσιο της EECR.

Σε όλες τις περιπτωσιολογικές μελέτες ήταν περισσότερο από εμφανές ότι η μέθοδος ΙΤΑ προσέφερε πολύ χρήσιμη πληροφόρηση στον υπεύθυνο λήψης αποφάσεων, ποσοτικοποιώντας το βαθμό βεβαιότητας με τον οποίο κάθε έργο βρέθηκε στο τελικό χαρτοφυλάκιο, γεγονός το οποίο δεν μπορεί να επιτευχθεί με τις συμβατικές μεθόδους και τη χρήση μέσων και αναμενόμενων τιμές στη μοντελοποίηση της αβεβαιότητας.

Abbreviations

CDM – Clean Development Mechanism

CERs – Certified emission reduction (s)

CHPP – Combined heat power plant

CSR – Corporate social responsibility

DEA – Data Envelopment Analysis

DM – decision maker

EE – Energy Efficiency

EECR – Energy and Environment Corporate Responsibility

GAMS – General Algebraic Modeling System

GHG – Greenhouse gas emissions

GWP – Global Warming Potential

GS – Gild Standard certification of activities

HPP – Hydro Power Plant

ILP - Integer Linear Programming

IP – Integer Programming

IPCC – Intergovernmental Panel on Climate Change

IRR – Internal Rate of Return

IT – Information Technology industry

ITA – Iterative Trichotomic Approach

KP – Kyoto Protocol to the UNFCCC

MAUT – Multi-Attribuye Utility Theory

MAVT – Multi-Attribute Value Theory

MC – Monte Carlo simulation

MCDA/MCDM – Multi Criteria Decision Analysis/Making

MP – Mathematical Programming

NPV – Net Present Value

OR – Operations Research

OWA – Ordered Weighted Average Operator

PDD – project Desigh Document

RES – renewable energy sources

SD – Sustainable development

SMAA – Stochastic Multiobjective Acceptability Analysis

TPP – Thermal Power Plant

UNFCCC – United Nations Framework Convention on Climate Change

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1. Introduction

1.1. Aims and scope

The problems of choice surround us every day and everywhere. They may be complicated requiring more time and elaboration in order to pick up the best solution in comparison with simple ones. Moreover, with the growth of information necessary for the choice problem, the need for sophisticated assistance tools increases.

One of common examples is the need of a university to make balanced selection from applicants. Students, considering limited time, have to choose subjects and extra activities, such as additional research programs or outdoor activities. Further, both public and private sectors are engaged in research and development programs that were chosen from a plethora of proposals. The list of examples may continue for eternity as making choices among alternative courses of action is a recurring activity.

Initially, every choice problem seems to be different and unique. However, after a thorough analysis and structuring of the problem numerous similarities appear. First of all, these problems involve one or more decision makers (DM), who needs to work with a given set of alternatives. There may be more than one objective set to achieve, depending on preferences of one person or of a group of involved experts. Thorough assessment of available and necessary resources should be performed too. Unfortunately, it is impossible to be totally sure about the outcomes of a decision. Uncertainties in input data or preference information are almost always present and need to be taken into account. The environment, in which the decision is elaborated, is an open system and there is always a chance that something forgotten or discarded may significantly influence or even alternate final results (Salo et al. 2011).

Different multi-cirteria decision analysis (MCDA) methods aim at supporting complex planning and decision process by providing a framework for collecting, storing and elaboration of all relevant information. The core of any MCDA method is the decision model, which is a formal specification of how different kinds of information are combined together to reach a solution. These methods are helpful for the development of planning processes, to avoid numerous distortions, and to manage all the information, criteria, uncertainties, and importance of the criteria. With their assistance it is possible to alleviate the problems caused by limited human computational power. Intuitive and adaptive choices are replaced by a justified and jointly accepted model (Lahdelma et al. 2000). Numerous authors (Goicoechea et al. 1982, Hobbs 1984, Hobbs et al. 1992, Simpson 1996, Lahdelma et al. 2000) point attention on the difficulty of picking a certain MCDA tool due to the fact that distinctive methods may provide different results with the same data, and there is usually no means to objectively identify the best alternative or method. Therefore, the choice of the method should be well justified in real applications, although this is rarely done. When the problems are solved in close cooperation with experts, some requirements are applied for the MCDA method. First of all, the method should be well defined and easy to understand, particularly regarding its central elements, such as modeling of criteria and definition of weights. Next, the technique must be able to support the necessary number of DMs as well as to manage the necessary number of alternatives and criteria. Since the available time and financial support are usually limited, the need of preference information from the experts should be as small as possible. In addition, the ability to handle the inaccurate or uncertain criteria information should not be overlooked too. As a rule, these requirements cover the typical factors through which the practical relevance of decision support methods is usually evaluated. This is especially true, for example, in planning decisions in the domains of energy production and climate change abatement.

Most of problems can be attributed to several categories or typologies. Roy (1996) identifies four different problematiques for which MCDA may be useful:

- **The choice problematique** for making a simple choice from a set of alternatives;
- **The sorting problematique** for allocation of options into classes or categories;
- **The ranking problematique** for placing actions in some form of preference ordering which might not be necessarily complete;
- **The description problematique** summarizes actions and their consequences in a formalized and systematic manner so that decision makers can evaluate these actions. In core this is a way to gain better understanding of what may and may not be achievable.

To these four main groups Belton and Stewart (2002) add two more problematiques, namely:

- **The design problematique** to search for, identify or create new decision alternatives to meet the goals and aspirations through the MCDA process;
- **The portfolio problematique** to choose a subset of alternatives from a larger initial set, taking into account not only the characteristics of the individual alternatives, but also of the manner in which they interact and of positive or negative synergies.

In practice, the path of project selection combines several problematiques. Moreover, in most situations it is at least as much of a problem to identify suitable alternatives and to establish appropriate criteria, as it is to make a selection from the available alternatives. Consideration of numerous criteria and objectives leads to multi-objective design problems.

While problem structuring and analysis take a number of different forms, lack of knowledge leads to various uncertainties. The lack of knowledge influences the modeling process, the use of models for exploring trends and options, and the interpretation of results. For the purposes of multicriteria decision aid, Belton and Stewart (2002) differentiate between internal uncertainty, relating to the process of problem structuring and analysis, and external uncertainty, regarding the nature of the environment and thereby the consequences of a particular course of action. Uncertainty about the environment represents concern about issues outside the control of the decision maker.

Several approaches to integrate external uncertainty have been developed. Some of the most used are:

- **Scenario planning**, which usually requires decision makers to identify a number of scenarios relevant to the decision context (for instance: pessimistic, neutral, optimistic);
- **Decision Theory** to use probability to describe the likelihood of uncertain events;
- **Risk as criterion** in a multiple criteria analysis which implies that certain level of risk is acceptable in return for increased benefits of reduced costs in terms of other criteria.

Unfortunately, for such complex problems traditional sensitivity analysis that is usually performed on certain criteria within defined ranges is not enough. As it is well observed in Antunes and Climaco (1992), sensitivity analysis (also called post-optimal analysis) in single objective linear programming deals with computing ranges on the variation of some initial data such that the optimal basis remains optimal for the perturbed problem. The concept of optimal solution (in general unique) gives place in Multi-Objective Programming to the concept of efficient solution (in general many, even if only extreme points are considered). Moreover, changes in the underlying DM's preference structure as a result of the information gathered throughout an interactive process must be taken into account. For a complex problem which has its optimum at an extreme point of the feasible region, the simultaneous consideration of constraints, which may be nonlinear, makes the problem more intricate.This makes even more difficult to define sensitivity analysis in an MCDM context, and in fact this issue is not uniformly addressed in the literature.

On the other hand, scenario building also rarely reflects fine details and uncertainty in future performance of project proposals. As a rule, scenarios are developed for optimistic, business-as-usual and pessimistic conditions, which reveal certain trends, and then experts need to make a choice based in the inner feeling. Another approach is to model risk seeking, neutral, and risk avert behavior for the parameters that depend on human factor. Years of practice confirmed that almost always risk avert behavior is adopted by decision makers. Hence, all these approaches alone are imcomplete and need more search and modeling to deliver robust results.

That is why a combination of approaches and tools, fitted for certain problem, are better than a single one. The whole process needs more time and knowledge becomes more cumbersome, but the obtained results can lead towards balanced decisions. A typical list of tools starts with the identification of alternatives. In some occasions, alternatives to be evaluated may appear to be clearly defined. In other occasions, the definition or discovery of alternatives may be an integral part of a study. In certain circumstances, it may seem impossible to handle the overwhelming complexity of options.

The following step is to evaluate available options whether they are few or a large number. Most multicriteria evaluation methods are designed for the evaluation of independently defined alternatives. Sometimes screening techniques are applied for large number of proposals when certain targets are already defined and should be met. DEA (Data Envelopment Analysis) might be used as a way to identify alternatives from a long list of promising options. In addition, it has been suggested that an outranking method, like ELECTRE, could be used to draw up a short list of suggestion for a more thorough evaluation. These screening approaches should be carefully used because a degree of non-compliance on one criterion may be compensated for by exceptional performance elsewhere.

Simultaneously, a set of criteria should be decided upon. In a wide sense, criteria are seen as a certain standard by which one particular choice or course of action could be judged to be more desirable than another. For every separate problem a set of criteria is unique and needs to be well balanced in order to reflect project behavior in the future.

Taking all the aforementioned into account, in this work we incorporate already developed tools and address known weaknesses with a new approach that helps to build a balanced project portfolio. Project portfolio selection problem is defined as the problem of selecting a subset of projects usually based on one or more criteria that have to fulfill specific constraints. In the presence of the imposed constraints (e.g. policy, segmentation constraints) a simple MCDA method does not suffice. The combinatorial character of the problem implies the use of optimization methods aiming at the portfolio of projects that satisfy constraints and achieves the “best” performance. A combination of projects is defined as project portfolio. Usually the “best” performance is expressed emphasizing on economic and financial criteria while other criteria related with the promotion of sustainable practices, environmental issues, fostering green growth, were not taken into consideration in traditional models (Hobbs and Meier 2000).

The aim of the specific dissertation is to propose a method that effectively deals with decisions regarding the selection of a subset of projects from a wider set. This selection is driven not only by the performance of the projects (objectively of subjectively estimated) but also from various constraints and conditions among them that should be fullfiled. In addition, uncertainty is present either in a stochastic manner or in the subjective views of different decision makers and is treated carefully in the modeling process.

1.2. Thesis outline

This thesis focuses on building a balanced project portfolio with great consideration of performance uncertainty, which cannot be adequately captured via traditional tools of forecasts and sensitivity analysis. The proposed methodology helps to capture incomplete information both in objective function(s) as well as in model parameter values. Further, the influence and implications on project and portfolio decisions are studied closer. That is why gradual portfolio building reveals inner dynamics and provides the possibility to review and update initial assumptions and constraints.

The dissertation is structured as follows.

Chapter 2 describes historical background and summarizes current ways to address project portfolio selection problems.

Chapter 3 presents methodologies used along with crucial initial assumptions and concepts. The description moves from basic foundations towards more complicated ones. First, the types of tools applied for modeling are listed with short explanations of their use. Then, particularities of mathematical programming are discussed. Finally, assumptions about handling incomplete information in the current work are explained.

Chapter 4 is devoted to the main contribution of the dissertation, the Iterative Trichotomic Approach (ITA) and its versions. Initially, a two-phase approach is developed to perform a relatively quick project selection which has to meet certain constraints. The concept is further developed to handle large number of projects with more complicated constraints. For a certain case study it has been necessary to adopt the approach for the group decision making in order to handle experts' divergent points of view.

Within Chapter 5 different applications are demonstrated. The first case study deals with the selection of activities for expansion of services for a telecommunication company, since drastic developments in the area required well thought future steps. The need for balanced introduction of new service offerings is a problem which involves different and conflicting aspects. The main feature of the proposed decision aid computational tool is the incorporation of several uncertainties in the selection process and the gradual building of the project portfolio.

Among next examples there are a portfolio selection of climate related activities to be chosen for financial support, a more complicated case of group portfolio building and an example of a bi-objective problem among others. Most of these applications are focused on renewable energy projects' selection. The specific focus is on the energy project portfolio selection problem where the output of each project as well as other parameters may be uncertain. On the other hand, for the donor organization it is crucial to make a balanced selection and avoid the tactics of “all eggs in one basket”. The process output is not only the final portfolio, but also information about the certainty of participation or exclusion of every project in the final portfolio.

In all case studies it is very visible how ITA offers more fruitful information to the decision maker as it quantifies the degree of certainty with which each project is treated in the final portfolio, a task that cannot be accomplished with the conventional methods using average and expected values in the modeling of uncertainty.

In Chapter 6, the contribution of this thesis is summarized and some plans for future work are suggested. Also, the final chapter compiles conclusions and observations from case studies and about the whole framework of methodology.

The Appendix provides some general information about GAMS modeling language. In addition, the coding of models for the case studies is provided.

Overall, this thesis expands the material which has been published, submitted, or is under preparation, in various journals and conferences.

2. The problem: Project portfolio selection

Project portfolio selection is defined as the problem of selecting a subset from a wide set of proposals. Portfolio selection is a step further after simple ranking of projects. Usually the projects are not independent, i.e., there are particular limitations that should be respected (segmentation constraints, mutually exclusive, precedence etc.) so that Multiple Criteria Decision Aid (MCDA) methods do not suffice but they must be combined with combinatorial optimization techniques. A popular way to deal with this problem is to use a two step approach: (1) A multicriteria method to evaluate the projects and (2) a mathematical programming model that incorporates the constraints while the objective function coefficients are the multicriteria scores. Generally speaking, according to Vetschera and Almeida (2012) project portfolio selection involves:

- Selection of subset from a wider set of project proposals;
- Projects are indivisible and can be chosen as whole;
- Constraints are applied, so that not all available proposals can be selected;
- Outcomes are determined by some aggregation of properties of selected projects.

2.1. History and current status of portfolio selection

In project portfolio selection the intuitive approach is to rank projects using one or more criteria and select the top ranked ones that cumulatively satisfy the budget limitation, as shown in **Figure 2-1**. Often this straightforward approach is sufficient. However, this may result in the budget cutting off midway through an expensive project. Also, in real world decision making, there are two concepts that complicate the decision situation: (a) the existence of constraints and limitations imposed by the decision maker; (b) the uncertainty that accompanies the project evaluation, i.e., the output uncertainty. Moreover, projects are rarely independent (with most common logical constraints where alternatives A and B are mutually exclusive) and numerous interactions may take place. Among common examples are interactions in cost (e.g., the cost of C and D together is less than the cost of C by itself plus the cost of D by itself), interactions in the values (e.g., the value of E and F together is different from the value of E by itself plus the value of F by itself), or probabilistic covariance in outcome. All these problems can be addressed by formulating a suitable binary optimization program, which can be solved by using Excel's Solver or other commercial software. However, this approach should be used with caution. The math program can quickly become too large to be really understood by or explained to senior decision makers and stakeholders. The resulting optimum portfolios can be fragile, in the sense that they can change drastically with only a small change in data (for instance, a little additional budget can result in an alternative being deleted from the portfolio, which is very hard to explain to that alternative's proponent). Finally, if the problem is very large (hundreds of alternatives), it can take a long time (hours or days) to solve (Burk and Parnell, 2011).

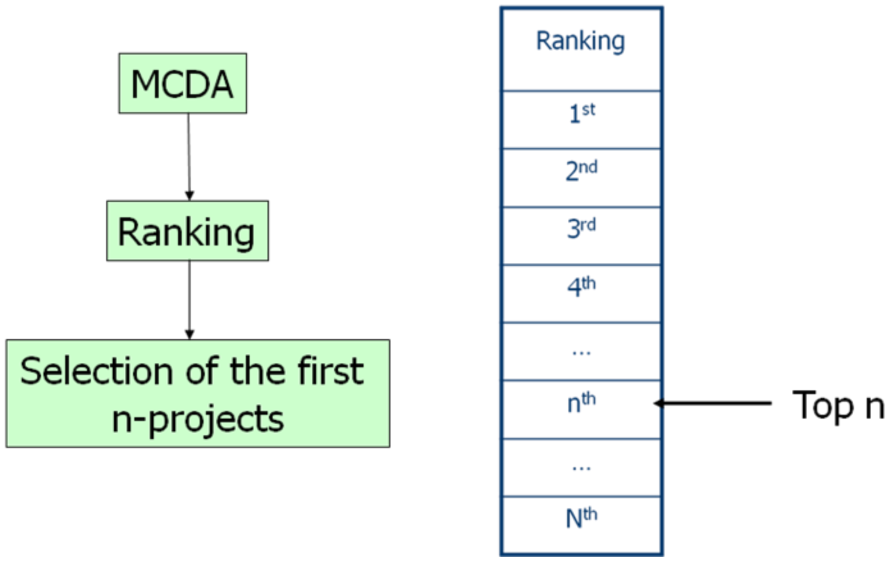

Figure 2-1. Selection of n top ranking projects.

The earliest contributions were published under the title of capital budgeting (see e.g. Lorie and Savage, 1955), using strictly financial measures to quantify the value of projects and portfolios, giving emphasis to the budget constraint. From early sixties, the so called capital budgeting problem was recognized as equivalent to the popular in Operational Research (OR) knapsack paradigm. The incorporation of multiple criteria can also be found in the literature within Goal Programming (see e.g. for a review Zanakis et al., 1995; for applications in Information Systems Badri et al., 2001; Santhanam et al., 1989; Santhanam and Kyparisis, 1996; for university resource allocation Albright, 1975; Kwak and Lee, 1998; Fandel and Gal, 2001; for an industrial application Mukherjee and Bera, 1995), combinations of MCDA with IP (see e.g. Golabi et al., 1981; Abu Taleb and Mareschal, 1995; Mavrotas et al., 2003; Mavrotas et al., 2006; Mavrotas et al., 2008), and Data Envelopment Analysis (Cook and Green, 2000; Oral et al., 1991; Oral et al., 2001) among others. Ghasemzadeh and Archer (2000) proposed the Project Analysis and Selection System (PASS) based on MCDA and Integer Programming. Hunt et al. (2013) proposed OUTDO for energy projects. Lourenco et al. (2012) proposed PROBE (Portfolio Robustness Evaluation) introducing the concept of robustness in project portfolio selection.

Project scoring methods do not necessarily ensure the quality of portfolio selection, because they do not explicitly take into account portfolio level considerations, such as multiple resource constraints, portfolio balance requirements and other project interactions. Sophisticated project portfolio models, on the other hand, seek to combine project portfolio optimization with explicit consideration of multiple value criteria (Golabi et al., 1981; Golabi, 1987). These models build on the well established Multi-Attribute Value Theory (MAVT; see, e.g., Keeney and Raiffa, 1976) to aggregate the multi-criteria project values into a portfolio overall value and use integer linear programming to determine the optimal composition of the project portfolio subject to resource and other constraints. Several high impact applications of multi-criteria portfolio models have been reported in the fields of military resource allocation (Ewing et al., 2006), R&D portfolio selection (Golabi et al., 1981), product release planning (Ruhe and Saliu, 2005) and healthcare capital allocation (Kleinmuntz, 2007), among others (Liesio 2008).

Based on the aforementioned studies, a project portfolio decision support framework needs to strike a balance between the following challenges:

Generality. The decision support model should be flexible enough so that it is applicable in various problems contexts. Most importantly it should allow consideration of multiple criteria and resources. Moreover, portfolio balance requirements and project interactions are common in applications. Finally, the model should support benefit-cost analyses, as the budget, for instance, is not always a fixed constraint but can be adjusted to some extent.

Modest data requirements. Even if a model could capture all aspects of project portfolio selection, the use of such a model would require large amounts of data and/or subjective evaluations to estimate the model parameters. Such data is often unavailable, whereas expert evaluations are costly to obtain and may contain considerable uncertainties. Therefore, models that offer approximate or inconclusive results with modest data requirements and explicitly take into account the incomplete or imprecise nature of the data, are more useful than models that require accurate data before offering any results.

Transparency. For a model to be accepted by practitioners, the key assumptions and concepts of the model have to be understood by the DMs. Empirical research supports this claim as practitioners often use simple scoring models to support project evaluation (Cooper et al., 1999). Also, from the aspect of decision support, models intelligible to non-experts are more readily applicable, as difficulties in elicitation of preferences and communication of results are likely to be avoided (Liesio et al. 2007).

In his seminal work for portfolio optimization Markowitz (1952) proposed the Modern Portfolio Theory (MPT) that incorporated portfolio risk in the decision making process. There, risk was quantified by the co-variance matrix of the returns (outputs) as calculated by historical data. The MPT was designed for securities where historical data is not a problem (Xidonas et al. 2012). In relation to projects the MPT cannot be easily applied as the decision variables are binary and historical data are scarce. While security prices can be correlated, most investments into securities are not logically dependent on each other. But in project portfolio selection there can be many forms of interdependencies due to logical relationships. For a more realistic modeling, the uncertainty characterizing the output of projects should be taken into account. In the literature, this is done either with the use of scenarios (see e.g. Georgopoulou et al., 1998) or with fuzzy parameters (see e.g. Damghani et al., 2011; Cavallaro, 2010) or with stochastic parameters (Liesio et al., 2008; Shakhsi-Niaei et al., 2011). An appropriate tool for dealing with stochastic uncertainty is Monte Carlo simulation, where sampling from certain probability distributions is performed for the inputs and the outputs with all the relevant obtained information. A great number of iterations is necessary in order to obtain reliable results from the outputs (distribution of output values etc.). Another feature to remember is the fact that projects are treated as binary variables which are either selected or rejected. This differs from financial portfolio optimization where essentially any fractional amount of resources can be invested into any security (Vilkkumaa et al. 2014).

We note that finding examples of project portfolio selection problems is not an easy task, because very often they may be called in another way. Research is spread between numerous specialized journals and books.

3. Methodology

Complex problems need elaborated models. Certain simplification is necessary, but oversimplification may lead to wrong results. In order to capture the complex nature of a problem it is worth to apply different tools and approaches. A short description of tools that were used in the current work is provided in the sub-chapters below.

3.1. Initial elaboration of projects

All proposals for activities may be called projects, items or alternatives and are subject of evaluation in terms of multiple criteria in order to make them comparable between each other. The criteria provide numerical measures for all relevant behaviors of different alternatives. The relevance of various impacts depends on experts' points of view. Also, it is necessary to define precisely how each criterion is measured. Usually criteria are aggregate values computed from a much larger amount of so-called primary factors, which form the lowest level of information, also known as the assessment level (Lahdelma et al. 2000). Within many years numerous researchers addressed this issue. Mainly either outranking or value and utility methods are used for that. A well done description of preference elaboration methods basic principles' is done by Stewart and Belton (2002). Further elaboration may be made by straightforward picking of one project representing each group or after a prioritization stage. Ranking is usually performed on the basis of one most important criterion, such as cost/benefit ratio, required resource or something else. There is no formal way of constructing a list of possible alternatives and no concrete way of knowing when the set of experts is complete enough, other than relying on experience, intuition, and on the vague concept of diminishing marginal return of satisfaction (Banville et al. 1998).

One of classical examples is the knapsack problem which focuses on selecting projects until the main resource (such as budget) is exhausted. Such an approach would produce the highest benefit for the money spent, but would not necessarily deliver the maximum benefit for the available budget (Lourenco, Morton and Bana e Costa 2012). Because of this, the concept of constraints becomes a vital part of the selection problem, which in turn destroys one of the main assumptions in ranking method - the independence of the projects (see e.g. Belton and Stewart, 2002). In other words, the top ranked projects only by chance may satisfy imposed constraints. A strong and useful tool to cope with such problems is Mathematical Programming that optimizes under specific constraints. More specifically, in case of project selection, the combinatorial character of the problem implies the use of Integer Programming (IP) with 0-1 (binary) variables expressing incorporation (X_i=1) or exclusion (X_i=0) of respective project in final selection.

In addition, numerous approaches were developed in order to capture a complicated nature of interactions between projects. When the cumulative effect of implementation of several projects is greater than the simple sum of their values – then synergy effects take place. In some cases, there may be opposite results with the cumulative sum smaller than the straightforward addition. It may be caused by overlaps in projects performance and output. Moreover, some projects may be mutually exclusive.

Within the current work we used different methods for initial evaluation of available options.

3.2. Tools for projects' assessment

The field of MCDA has developed rapidly over the past decades and in the process a number of divergent schools of thought have emerged. For a balanced presentation of approaches the book of Belton and Stewart (2002) is a good starting point. Here we will mention only briefly some major schools.

Among the oldest are **value measurement models**, in which numerical scores are constructed in order to represent the degree to which one decision option may be preferred to another. Such scores are developed for each individual criterion, and are then synthesized in order to affect aggregation into higher level preference models. Among widely used approaches of this school are Multiattribute Utility Theory (MAUT), Multiattribute Value Theory (MAVT) and Analytic Hierarchy Process (AHP). They differ primarily in terms of the underlying assumptions about preference measuring, the methods used to elicit preference judgements from experts involved, and the manner of transforming these into quantitative scores.

Other family is represented by **goal, aspiration or reference level models**, in which desirable or satisfactory levels of achievements are established for each of the criteria. The process then seeks to discover options which are in some sense closest to achieving these desirable goals or aspirations. In these models much depends on the framing of the problem, reference points and perception of what constitutes "gain" or "loss". Care thus needs to be taken in ensuring that decision makers understand and are satisfied with the implied reference points used in the model.

Wide popularity gained **outranking models**, in which alternative courses of actions are compared pairwise, initially in terms of each criterion, in order to identify the extent to which a preference of one over the other can be asserted. In aggregating such preference information across all relevant criterion, the model seeks to establish the strength of evidence favouring selection of one alternative over another.

In what follows, we have a closer look on some of most popular assessment methods.

Value function methods synthesize projects' performance assessment against certain criteria, together with inter-criteria information reflecting the relative importance of different criteria, to give an overall evaluation of each alternative indicative of the decision makers' preferences. However, it is worth to remember that learning and understanding which results from engaging in the whole analysis process is far more important than numerical results. That is why evaluation should incorporate extensive sensitivity analysis and robustness analysis.

Within the value measurement approach, the first step is to develop a hierarchy of criteria (so called "value tree"). Further, the components of preference modeling are achieved by constructing "marginal" / "partial" value functions (v_i *(a)*) for each criterion. It should be remembered that the properties of the partial value functions and the form of aggregation used are critically interrelated. Usually an additive aggregation is adopted, while multiplicative aggregation may be adopted in some MCDA approaches.

Utility theory can be viewed as an extension of value measurement, relating to the use of probabilities and expectations to deal with uncertainty. Here it is assumed that each criterion is directly associated with a quantitative attribute measured on a cardinal scale, which may also be influenced by unknown external factors. The consequences of each alternative are thus described in terms of a probability distribution on certain attribute vector. For a more detailed description of the method it is advised to read the work of Keeney and Raiffa (1976).

As for the AHP, its main difference from MAVT is in the use of pairwise comparisons between alternatives with respect to criteria and criteria within families, as well as the use of ratio scales for all judgements. The method was initially developed by Saaty (1980), it was elaborated through years and became widely used in practical applications.

In outranking methods, specially acclaimed became the variations of ELECTRE and PROMETHEE methods. The family of ELECTRE methods was developed through years by Roy B. and associates and differs according to the degree of complexity or richness of the information required or according to the nature of the underlying problem.

Roy was critical of the utility and value function methods on the grounds that they require all options to be comparable. In collaboration first with his associates at LAMSADE, University of Paris Dauphine, he started to develop ELECTRE outranking method. One of the major features of this new approach was the provision of weaker, poorer models than value function, built with less effort and fewer hypotheses, but not always allowing conclusion to be drawn. The family of ELECTRE methods differ according to the degree of initial information complexity and the nature of the underlying problem.

The earliest and simplest outranking method was ELECTRE I which is good for understanding of underlying concepts. The methods are based on the evaluation of two indices, namely the *concordance* and the *disconcordance indexes*, defined for each pair of options under consideration. The concordance index, *C(a,b)*, measures the strength of support in the information given, for the hypothesis that *a* is at least as good as *b*. The disconcordance index, *D(a,b)*, measures the strength of evidence against this hypothesis.

In general, the concordance index is the proportion of criteria weights allocated to these criteria for which *a* is equal or preferred to *b*. The index takes values between 0 and 1 where higher values indicate stronger preference of *a* over *b*. The disconcordance is expressed as a proportion of the maximum weighted difference between any two alternatives on any criterion. It ranges from 0 to 1 as the previous index and its high value indicates that on at least one criterion *b* performs substantially better than *a*. Still, the form of this index makes it appropriate only for evaluations that were made on a cardinal scale and the weights render scales comparable across criteria. These assumptions are not easy to meet and may be quite restrictive.

The concordance and disconcordance indices for each pair of options can be used to build an outranking relation. Also, simultaneously respective *thresholds* should be specified carefully. If the outranking relation is too severe, then almost all pairs of alternatives will be deemed to be incomparable, while a light relation will lead to a situation where too many options outrank too many others. Fortunately, an outranking relation can be represented visually by a graph with arrows showing the direction of outranking relation.

Having built the outranking relation, the final step is the exploration of that relation in the decision process. The procedure may have several shapes depending on the initial cause of the process. It can be either a determination of the "best" option, or an options ranking, or a separation into certain classes or groups of alternatives. Also, sensitivity and robustness analysis may be performed to support final decisions.

ELECTRE II was developed shortly after ELECTRE I and aims at the production of alternatives' ranking rather than simple search of the most preferred ones. This is reached via different pairs of concordance and disconcordance thresholds. These are referred to as the strong and weak outranking relations, the former having a higher concordance threshold and a lower discordance one. Another small change was the introduction of an additional constraint in the test for outranking in order to reduce the possibility of two alternatives each outranking the other.

Later developments put a greater emphasis on detailed preference modeling since not all alternatives perform identically on a given criterion. In ELECTRE III the notions of indifference and preference thresholds were introduced. However, this requires more work in modeling preferences with respect to each individual criterion before progressing to the building and exploitation of the outranking relation. In order to handle situations when it is impossible to specify criteria weights ELECTRE IV was developed. Outranking relations, of different strength, are then defined by direct reference to the performance levelsof alternatives.

ELECTRE TRI is for use in classification problems. The original procedure was designed to allocate alternatives to one of three categories: acceptable, unacceptable and indeterminate. Later this has been extended for use in classification problems with greater number of categories. In certain way it became one of filtering methods.

Another family of prominent outranking methods is represented by the PROMETHEE methods. The initial PROMETHEE method, developed by Brans and co-workers, uses preference function for each criterion. The next step determines a preference index for one option over another and defines a valued outranking relation, which is exploited to determine an ordering of the alternatives. Then, two other indices, the positive outranking flow and the negative outranking flow, are defined where the sums are taken over all alternatives under consideration. The positive outranking flow expresses the extent to which certain option outranks all others. The negative outranking flow expresses the level to which that option is outranked by all other options. Each of these indices defines a complete preorder of alternatives.

It should be remembered that the values of both positive and negative outranking flows depend on the complete set of alternatives under consideration. Hence, inclusion or exclusion of another option may influence strongly already obtained preorders.

The primary appeal of all outranking methods is in the avoidance of what are perceived to be overly restrictive assumptions of value or utility based approaches. All outranking methods focus on pairwise comparisons of alternatives, and are thus generally applied to discrete choice problems. Another advantage of these methods is the use of less precise inputs.

3.3. Mathematical programming tools

In operations research, mathematical programming, also alternatively named mathematical optimization or simply optimization, is the selection of a best alternative with regard to some criterion from an initial set of available options usually expressed by specific constraints.

The simplest example of an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function. The generalization of optimization theory and techniques to other formulations comprises a large area of applied mathematics. More generally, optimization includes finding "best available" values of some objective function given a defined domain (or input), including a variety of different types of objective functions and different types of domains. Many real-world and theoretical problems may be modeled in this general framework.

In the operations research domain has a wide array of methods and approaches is available to solve problems. One of the largest families is the Convex programming problems where the objective function is either convex (minimization) or concave (maximization) and the constraint set is convex. This can be viewed as a particular case of nonlinear programming or as a generalization of linear or convex quadratic programming.

Linear programming (LP) is a mathematical technique which tries to satisfy initial demands by assigning some amounts of resources so that a certain goal is elaborated in an optimal way while other limitations are also satisfied. LP addresses problems where the objective function *f(x)* is linear and the constraints are specified using only linear equalities and inequalities. Its feasible region is a convex polytope, which is a set defined as the intersection of finitely many half spaces, each of which is defined by a linear inequality. Its objective function is a real-valued affine (linear) function defined on this polyhedron. A linear programming algorithm finds a point in the polyhedron where this function has the smallest (or largest) value if such a point exists.

Methods of **Integer Programming** (IP) study linear programs in which some or all variables are constrained to take on integer values. This is not convex, and in general much more difficult than regular linear programming. In many settings the term refers to Integer Linear Programming (ILP), in which the objective function and the constraints (other than the integer constraints) are linear. There are two main reasons for using integer variables when modeling problems as a linear program:

- The integer variables represent quantities that can only be integer. For example, it is not possible to schedule 2.5 buses.
- The integer (binary) variables represent decisions and so should only take on the value 0 or 1.

These considerations occur frequently in practice and so integer linear programming can be used in many applications areas. Among typical examples are the number of trucks in a fleet, number of electricity generators for energy production etc.

One of typical problems that we have already mentioned earlier is the knapsack problem which is a relatively simple integer program. Furthermore, the coefficients of this constraint and the objective are all non-negative. Initial information covers a knapsack with certain capacity and a number of items, each with a size and a value. The objective is to maximize the total value of the items in the knapsack. To solve the associated linear program, it is simply a matter of determining which variable gives the most ‘bang for the buck". In other words, after finding the ratio between the objective coefficient and constraint coefficient for each variable, the one with the highest ratio is the best item to place in the knapsack. Then the item with the second highest ratio is put in and so on until we reach an item that cannot fit. At this point, a fractional amount of that item is placed in the knapsack to completely fill it. In certain way, a ranking is performed until the main resource is used. For more detailed descriptions see e.g. H. P. Williams (1999), G.L. Nemhauser and L.A. Wolsey (1999).

Much like linear programming problems, **Mixed Integer Linear Programming** (MILP) problems are very important when solving decision-making models. MILP involves problems in which only some of the variables are constrained to be integers, while other variables are allowed to be continuous. Efficient algorithms for solving complex problems of this type are known and are available in the form of solvers such as CPLEX or Gurobi. Winston (1994) made one of earliest attempts to gather and explain some of most widespread problems in one book. An extended review of models and solving methods can be found in Taha H.A. (2003), Hillier and Lieberman (2001).

Goal programming may be viewed as the bridge between single objective and multi-objective programming, namely concerning reference points approaches. The aim is to minimize the function of the deviations regarding targets established by DMs for the objective functions. These targets established by DMs may lead to a dominated solution to the problem under study if the DM is not sufficiently ambitious in specifying his goals. In this case, goal programming model leads to a satisfactory solution but may not belong to the nondominated solution set. More details may be found in Steuer (1986).

3.4. Multi-objective mathematical programming

Multi-objective Mathematical Programming (MOMP) is an extension of traditional mathematical programming theory dealing with mathematical optimization problems involving more than one objective function to be optimized simultaneously. The family of these methods can be also called multi-objective programming, vector optimization, multi-criteria optimization, multiattribute optimization or Pareto optimization. Adding more than one objective to an optimization problem adds complexity. Multi-objective optimization has been applied in many fields of science, including engineering, economics and logistics where optimal decisions need to be taken in the presence of tradeoffs between two or more conflicting objectives. A general formulation of a MOMP problem is as follows:

$Max\ or\ Min\ \{f_1(x), f_2(x),..., f_n(x)\}$

st

$x \in S$

where x is the vector of decision variables; $f_1, f_2, ..., f_n$; are the objective functions (linear or nonlinear) to be optimized; and S is the set of feasible solutions.

In contrast to traditional mathematical programming theory, within MOMP framework the usual concept of an optimal solution is no longer applicable. This is because objective functions are of conflicting nature (the opposite is rarely the case). Therefore, it is not possible to find a solution that optimizes simultaneously all the objective functions. In this regard, within the MOMP framework, the major point of interest is to search for an appropriate "compromise" solution. When searching for such a solution, only the efficient set is considered. The efficient set consists of solutions, which are not dominated by any other solution on the prespecified objectives. A solution is called nondominated, Pareto optimal, Pareto efficient or noninferior, if none of the objective functions can be improved in value without degrading some of the other objective values. Without additional subjective preference information, all Pareto optimal solutions are considered equally good as vectors cannot be ordered completely. That is why the involvement of DM in results' elaboration is welcomed. Most multiple objective programming procedures are interactive and a review of such interactive procedures is contained in Gardiner and Steuer (1994). One of the earliest examples is of Lawrence and Steuer (1981) who applied an interactive multiple objective programming procedure to capital budgeting to enable a decision maker to gain improved appreciations of how the objectives tradeoff against one another.

Several appropriate procedures have been developed to solve MOMP problems. These procedures are interactive and iterative. The general framework within which these procedures operate is a two-stage process. In the first stage, an initial efficient solution or group of solutions is presented to the DM. If this solution is found to be acceptable (i.e., if it satisfies expectations on the given objectives), the solution procedure stops. If it is not acceptable, the expert is asked to provide information regarding his preferences on the prespecified objectives. This information involves objectives that need to be improved and tradeoffs that he is willing to undertake to achieve these improvements. The purpose of defining such information is to specify a new search direction for the development of new, improved solutions. This process is repeated until a solution is obtained that is in accordance with the DM's preferences or until no further improvement of the current solution is possible (see e.g. Steuer, 1989, Mavrotas, 2000).

The set of all efficient points is called the *efficient set*. While the efficient set is normally a portion of the surface of the feasible region, the efficient set has the tendency to grow rapidly as problem size increases. For special kind of MOMP problems (mostly linear problems) of small and medium size, there are also methods that produce the entire efficient set (Mavrotas, 2009). In general, the most widely used generation methods are the weighting methodand the ε-constraint method. These methods can provide a representative subset of the Pareto set which in most cases is adequate. In this context, Mavrotas (2009) proposes the use of the augmented ε-constraint method (AUGMECON) which is a novel version of the conventional ε-constraint method that provides remedies for its well-known pitfalls. AUGMECON has been implemented in the widely used modeling language GAMS.

The advantage of multiple objective programming is that it provides the possibility to sample neighborhoods on any multi-dimensional efficient surface to any degree of resolution. A disadvantage is the CPU run time required.

3.5. Modeling incomplete information

In real world decision making there are two concepts that complicate the evaluation: (a) the limits of expert's knowledge; (b) the uncertainty that accompanies project's evaluation, i.e., its future performance (output) uncertainty (Mavrotas & Pechak 2013b). In the first case, the uncertainty is essentially a lack of information; complete ignorance represents one end of the spectrum and perfect information (i.e., certainty) the other. At a most fundamental level, uncertainty relates to a state of the human mind, i.e., lack of complete knowledge about something.

Moreover, before incorporating data into the model, the notions of **uncertainty** and **risk** should be cleared. Their definitions vary from one case study to another where the meanings range from being totally independent concepts to being synonyms. Numerous definitions, found in the literature, are very dependent on the context and field of a problem. The only thing that no one can argue against is the fact that these terms are closely related. The abundance of research focused on uncertainty and risk makes it impossible to cover all assumptions within a short review. Needless to say that development and understanding of risk and uncertainty concepts are heavily influenced by economy and finance theory, as well as of the portfolio. In early 20-th century, Knight (1921) noted that there are two types of uncertainty. The first, measureable probability, Knight labeled as 'risk', and the second, unquantifiable ambiguity, or uncertainty. In project management, risk can be assigned a probability value, whereas uncertainty is completely immeasurable (Regan, 2011). It is critical to note this distinction, as risk is concerned with objective probabilities, whereas uncertainty requires consideration of subjective probabilities (Rutherford, 1995; Koleczko 2012). Another dual classification is proposed by Roy and Oberkampf (2011) where uncertainty is classified as either **aleatory** – the inherent variation in a quantity that, given sufficient samples of the stochastic process, can be characterized via a probability density distribution, or **epistemic** – uncertainty due to lack of knowledge by the modelers, analysts conducting the analysis, or experimentalists involved in validation. Aleatory uncertainty is also referred to in the literature as variability, irreducible uncertainty, inherent uncertainty and stochastic uncertainty. This term is used to describe the inherent variation associated with the physical system or the environment under consideration. Epistemic uncertainty derives from some level of ignorance, or incomplete information, of the system or the surrounding environment and is also termed reducible uncertainty, subjective uncertainty and model form uncertainty. The lack of knowledge can pertain to, for example, modeling of the system of interest or its surroundings, simulation aspects such as numerical solution error and computer round-off error, and lack of experimental data. In scientific computing, there are many sources of uncertainty including the model inputs, the form of the model, and poorly-characterized numerical approximation errors. All of these sources of uncertainty can be classified as either purely aleatory, purely epistemic, or a mixture of the two.

In Operations Research the definition of uncertainty also distinguishes uncertainty as objective and subjective uncertainties. He is more concerned about subjective uncertainty and the following definition refers to it. “Uncertainty implies that in a certain situation a person does not dispose about information which quantitatively and qualitatively is appropriate to describe, prescribe or predict deterministically and numerically a system, its behaviour or the characteristics.” His list of causes for uncertainty includes lack or abundance of information, conflicting evidence, ambiguity measurement and belief. He also strongly believes that uncertainty should not be modelled context free and that there exists no “single method which is able to model all types of uncertainty equally well.” (Samson et al. 2009).

Basically, definitions are split in 3 areas: Operations Research, Economics and Finance, and Engineering. For a comprehensive review check Samson et al. (2009) and Stewart (2005). As a rule, people define “uncertain” as something not definitely known or decided; subject to doubt or question. In the context of practical applications in multi-criteria decision analysis, the definition given by Zimmermann is particularly appropriate. With minor editing, this is as follows: “Uncertainty implies that in a certain situation a person does not possess the information, which quantitatively and qualitatively is appropriate to describe, prescribe or predict deterministically and numerically a system, its behavior or other characteristics”.

On the other hand, the term “risk” is usually applied to situations in which probabilities on outcomes are (to a large extent) known objectively. More recently, the concept of risk has come to refer primarily to the desirability or otherwise of uncertain outcomes, in addition to simple lack of knowledge. Thus, for example, Fishburn (1984) refers to risk as “a chance of something bad happening”, and in fact separates uncertainty (alternatives with several possible outcome values) from the fundamental concept of risk as a bad outcome. Due to the fact that insurance industry widely uses this interpretation of the risk (with negative connotations), one understands and feels better this term in comparison with “uncertainty” in general. Further in thesis, the value-neutral term “uncertainty” will be used.

Moreover, modern views of uncertainty assert that it is based not only on randomness, but also on beliefs and behavior. Cultural norms and other informal institutions of society have an observable effect on decision makers (Rutherford, 1995). Bounded rationality recognizes that it is impossible to comprehend and analyze all of the possibly relevant information while making choices. It proposes an idea that in decision-making, rationality of individuals is limited by their formal training, experience, skill, the cognitive limitation of their minds, and the finite amount of time they have to make a decision (Elster, 1983). A further component is peer group pressures and the decision making that takes place in a group context, as opposed to individual (Flyvbjerg et al., 2006). Furthermore, behavioral studies indicate that when people are faced with prediction tasks, they tend to underestimate prior information about the “base rate” of the event which being predicted. Instead, they tend to make decisions based on most recent evidence, which can lead to errors in predicting rare events and extreme realizations (Kahneman and Tversky 1979, Vilkkumaa 2014). Specifically, in an attempt to maximize the value by choosing one out of many alternatives based on ex ante assessments that reflect recent evidence, the DM will choose the alternative with the highest estimate. Unfortunately, there is a high chance that this assessment is higher than the real value of the alternative and, consequently, the DM will be disappointed when the actual alternative’s value is realized. One of the ways to eliminate this post-decision disappointment is the Bayesian revision of value estimates defined formally as the expected negative gap between the realized and estimated value of the selected alternative (Brown 1974, Harrison and March 1984, Smith and Winkler 2006). Numerous studies conclude that the value of information varies in unexpected, ambiguous and sometimes counterintuitive ways (Mavrotas, 2000), but Delquie (2008) demonstrated that under general assumptions, the indifferent DM provides the most correct project evaluation, while the one with strong preference toward certain alternative provides lower quality of information.

Even more types of uncertainty are described by Kangas and Kangas (2004). For instance, they offer the generalized categories of metrical (measurement variability/imprecision), structural (system complexity), temporal (past/future states of nature), and translational (explaining results) uncertainty. Mendoza and Martins (2006) identify randomness, imprecision, and unknown preferences as factors contributing to uncertainty in multi-criteria decision analysis. Leskinen et al. (2006) point to errors in inventory and measurement, projections of future market conditions, projections of forest development over time in response to management intervention, and unknown preferences as sources of uncertainty in forest plans. Another example from Thompson and Calkin (2011) is a common situation when no one can predict, i.e. estimate the expected value of the amount of snow on the runway for any given day in the future, the amount of snow is random but non-quantifiable and therefore uncertain. This non-quantifiable randomness can be modeled as an interval representing uncertainty. Regardless of the specific typology ultimately chosen, using a coherent framework informs management by facilitating the identification of potential sources of uncertainty and the quantification of their impact.

Almost all of these definitions are problem sensitive, i.e., they may not perform as well if applied to a new problem area. Some of these scholars suggest that uncertainty can be modeled as an interval even though there is no consensus on whether it is quantifiable or not. Other researchers define risk using the variance concept. However, there is no common modeling method that they all agree upon (Samson et al., 2009).

3.6. Monte Carlo simulation

There is no consensus on how Monte Carlo (MC) should be defined. Very often scholars distinguish between a simulation (a fictitious representation of reality), a Monte Carlo method (a technique that can be used to solve a mathematical or statistical problem) and a Monte Carlo simulation which uses repeated sampling to determine the properties of some phenomenon or behavior.

Generally speaking, Monte Carlo methods (or MC experiments) are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. They are often used in physical and mathematical problems and are most useful when it is difficult or impossible to use other mathematical methods. MC methods are mainly used in three distinct problem classes: optimization, numerical integration, and generating draws from a probability distribution (Kroese et al. 2014).

Monte Carlo methods vary, but tend to follow a particular pattern:

a) Define a domain of possible inputs.

b) Generate inputs randomly from a probability distribution over the domain.

c) Perform a deterministic computation on the inputs.

d) Aggregate the results.

Monte Carlo simulation methods do not always require truly random numbers. Many of the most useful techniques use deterministic, pseudorandom sequences, making it easy to test and re-run simulations. The only quality usually necessary to make good simulations is for the pseudo-random sequence to appear "random enough" in a certain sense. This need for large amounts of random numbers spurred the development of pseudorandom number generators, which were far quicker to use than the tables of random numbers that had been previously used for statistical sampling.

There are several reasons for a large number of Monte Carlo simulations. Firstly, if random grains are not uniformly distributed, then the resulting approximation will be unreliable. The approximation is generally poor if only a few seeds (grains) are randomly dropped into the whole interval of interest. On average, the approximation improves as more grains are dropped.

In the current work, uniform and normal distributions are used within MC simulations and are indicated in respective cases.

3.7. Chapter summary

We briefly described tools that are used for the problem of projects selection where it is necessary to fulfill specific constraints based on one or more available criteria. Since the problems that we try to solve are complex, combinations of approaches need to be adopted. For different problems the same tools can hardly be applicable. Moreover, a single MCDA method does not suffice in the presence of the imposed constraints. Even problem formulation can result in different framing. As it was mentioned before, the problem can be shaped as a single or multi-objective one. The principal aim on initial stage is to help experts learn about the problem situation, about their own and others values and preferences with appropriate presentation of available information.

Uncertainty plays a significant role, especially for technologies that evolve considerably year after year or for pioneering solutions where historical performance data are not available. Here, family of goal programming methods should be treated with special care because a strong inclination towards overestimation of results is observed between project developers. The same stands for scenario building. Hence, value measurement methods are more suitable for initial projects' evaluation. Then, in the optimization process, performance or assessment uncertainties can be handled through Monte Carlo simulation or some other tools.

Further, the selection process leads to better considered, justifiable and explainable decisions. Process transparency is of crucial importance. As a rule, the decision cycle involves 3 stages: problem identification and structuring; model building and use; development of action plans. The combinatorial character of the problem implies the use of optimization methods aiming at a portfolio of projects that satisfies constraints and achieves"best" performance. With the tools described earlier, we move towards the development of a selection method that helps to build a balanced portfolio, which respects performance uncertainties. Further actions are still to be made by decision makers, nevertheless they are provided with additional information about the path of project selection.

4. The Iterative Trichotomic Approach

The trichotomic approach (trichotomy is separation of initial set into three parts) is based on the fact that projects can be assigned to three classes depending on the information available: Projects that are present in the final selection under all circumstances are labeled green, red projects are those to be excluded under all circumstances, and grey projects are the ones that need some additional elaboration before being included in the final set under certain conditions.

At the very beginning of the process isfound the evaluation of project proposals. The Decision Maker (DM) may select the MCDA method of his choice, either utility function based or outranking (e.g. PROMETHEE, ELECTRE). All MCDA methods have specific decision parameters (weights, thresholds etc) that can be considered stochastic with their values taken from appropriate distributions. This is implemented to counter balance the subjectivity in selecting these parameters that may eventually lead to specific results. Initial performance overestimation may damage the final selection on several ways. First of all, a seemingly high performing project may take the place of a duly estimated and better performing one. That is why the lack of exact input information due to various reasons is addressed with notion of uncertainty which is expressed through the probability distributions for the projects' output. Moreover, criteria weights or any other necessary parameters and thresholds can be also described by appropriate probability distributions. Then a Monte Carlo simulation is performed using sampling from these distributions. Finally, an optimization process with the Integer Programming (IP) model provides optimal portfolio. This pair of sampling & optimization is the core of calculations. For example, if the number of Monte Carlo simulations is set to T, then sampling & optimization rounds will be performed T times. The output will be T optimal portfolios based on sampling of model's parameters. Eventually, the initial set of projects is divided into three subsets (classes): The green projects that are present in the final portfolio under all circumstances (i.e., in all T Monte Carlo simulations), the red projects that are absent from the final portfolio under all circumstances and the grey projects that are present in some of the final portfolios. The classification in three subsets is not new in the literature. Liesio et al. (2007) used a similar approach in the framework of robust programming. However, the way the projects are assigned to each set is different. In addition, Mavrotas and Rozakis (2009) used similar concepts in a student selection problem for a post graduate program.

The concept behind trichotomic approach is that the DM can focus on the projects that are really at stake. Unlike "short list" approach (where k projects with the highest expected values are re-evaluated), the attention is only on the "ambiguous" ones (e.g. the grey set) while sure projects (either in or out of the portfolio) are determined. The method provides quantitative and qualitative information that cannot be acquired using e.g. expected values of distributions. In the latter case, the DM is provided with a unique optimal portfolio or, in other words, which are the “go” and the “no go” projects, without any discrimination about the degree of certainty for each of them.

On the contrary, the trichotomic approach provides extended information about the degree of certainty of every entrance in the final selection. In other words, the method gives a whole picture with multiple candidate projects and portfolios and provides the opportunity to fully control the process of selection. In case of “close winners” the expert is informed about the more or less equivalent solutions. In this way additional criteria for further discrimination of “close winners” can be used. Hence, the DM is aware of the prioritization of projects given that the round in which a project enters the green set is known. The earlier a project gets in the green set, the stronger are its chances to be included in the final portfolio. The illustrative examples from case studies in next sections demonstrate in practice the above mentioned concepts.

4.1. Initial two-phase ITA

The two-phase approach combines several techniques such as MCDA, Monte Carlo simulation and optimization through Mathematical Programming (MP) specially tailored to the project portfolio selection problem. In the first phase, a session of Monte Carlo simulation – MCDA – MP optimization is performed since **performance** of each project in each criterion is given by a probability distribution (project uncertainty). Moreover, criteria weights or any other necessary parameters and thresholds may also be represented by appropriate probability distributions. The output of first phase are multicriteria scores of each project, which are used to drive further optimization process. Namely, scores are used as objective function coefficients in the MP model of the next phase. Besides objective function’s coefficients, MP model may have additional stochastic parameters, i.e., in the body of constraints that form the feasible region. Values for uncertain parameters are sampled from specific probability distributions and resulting mathematical programming model is solved (optimized) providing the optimal portfolio.

On the **first phase**, using Monte Carlo simulation the previously described process is repeated N times and T optimal portfolios expressing all the possible states of nature (some of these optimal portfolios may be identical) are obtained. The first phase is depicted in **Figure 4-1**.

The MP model on *t*-th Monte Carlo iteration is identical to the one of iterative process.

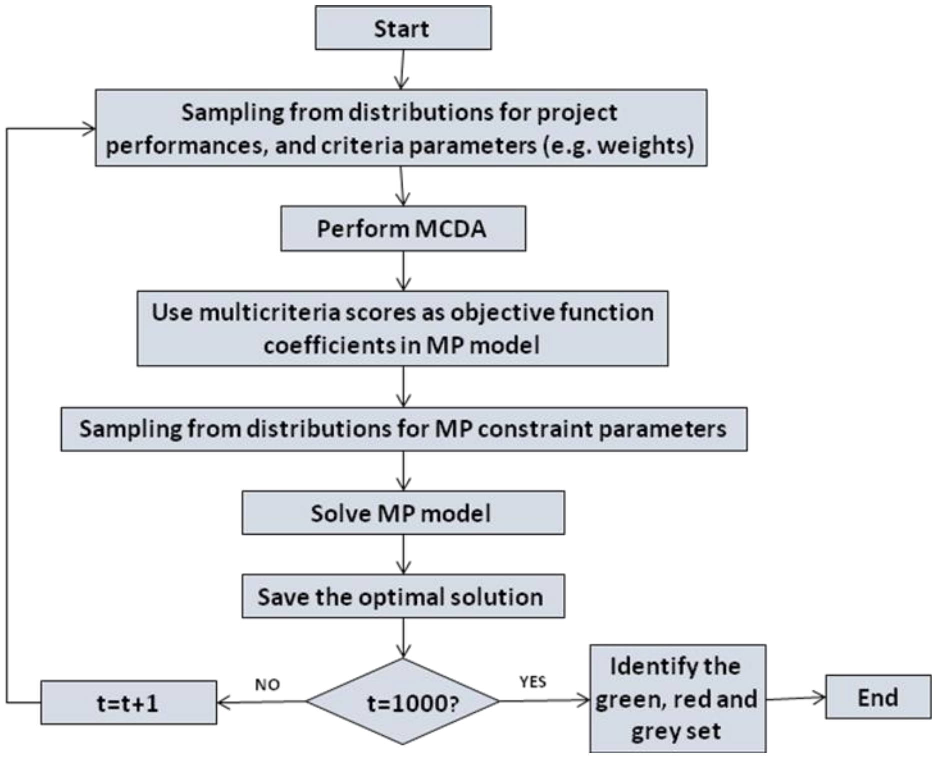

Figure 4-1. Monte Carlo simulation-optimization approach of phase 1.

As it was mentioned before, obtained portfolios are rarely same across initial *T* iterations. It is feasible to work further with project proposals. Hence, after completion of the cycle, on the basis of obtained *T* optimal portfolios projects are distributed between green, red and grey sets. In order to facilitate the selection process, membership thresholds for the green and the red sets may be introduced in order to relax membership requirements. The membership threshold can be used whenever the discrimination ability of the first phase needs to be increased, e.g. where the green and the red sets are almost empty.

In the **second phase** the main focus is on items from grey set while those in the green set are considered as already selected and those in the red set are considered as discarded ones. In case when grey set contains just a few projects (say 2-3), a direct comparison of them can be performed easily and probably suffices to determine the final selection. However, when more projects are present in the grey set, selection becomes a complex task that needs a systematic approach (given also that the MP model's constraints must be respected). The critical point of the second phase is that objective function coefficients of new model are no longer multicriteria scores but participation frequencies of the grey project in *N* optimal portfolios of the first phase. This means that objective function coefficients of the second phase are not stochastic but crisp numbers, hence reducing the variability of results. In **Figure 4-2** the unified process of first and second phase is shown schematically.

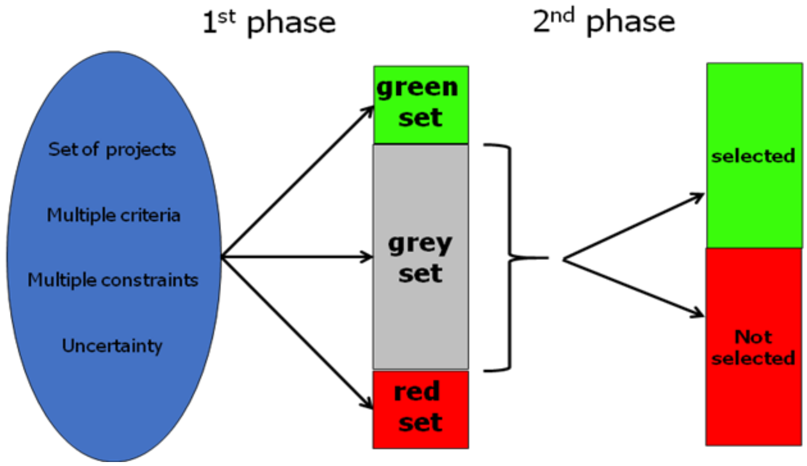

Figure 4-2. Unified process of two-phase approach.

Further in calculations two cases should be distinguished. The first case with no uncertainty related to the feasible region, meaning that there are no stochastic parameters in constraints, is depicted in a following way:

$$\begin{aligned} &\max F = \sum_{i \in grey} f_i X_i \\ &st \\ &\mathbf{X} \in S \\ &X_i \in \{0,1\} \\ &X_i = 1 \quad i \in green \\ &X_i = 0 \quad i \in red \end{aligned} \tag{4.1}$$

where *grey*, *green* and *red* denote the grey, green and red sets respectively, f_i is the frequency of the *i-th* project in T optimal portfolios from first phase. The objective function of the 2nd phase actually expresses the majority principle, i.e., the more times a project is present in optimal portfolios of the first phase, the greater the chance to be eventually selected. It is obvious from the formulation that optimization takes place among projects from grey set while green and red projects are already fixed to 1 and 0, respectively. The optimal solution of equation (4.1) is a project portfolio that has the greatest acceptance given existing uncertainty.

Another case still contains uncertainty related to the feasible region which means that there are stochastic parameters in constraints (but not in objective function). In this case, the Monte Carlo simulation – optimization scheme is used again only for the models' stochastic parameters sampling that exist in constraints. The MP model that is iteratively solved in the second case is described below:

$$
\begin{aligned}
&\max F^{(t)} = \sum_{i \in grey} f_i X_i \\
&st \\
&\mathbf{X} \in S^{(t)} \qquad (4.2) \\
&X_i \in \{0,1\} \\
&X_i = 1 \quad i \in green \\
&X_i = 0 \quad i \in red
\end{aligned}
$$

The Monte Carlo simulation – optimization process is repeated for $t = 1\ldots T$ times and result is T optimal portfolios (as it was in the first phase). However, now the variability is considerably reduced given the presence of crisp coefficients in the objective function. A project portfolio with the greatest acceptance is the one that appears more times within T iterations. If there are two or more portfolios with high frequency of appearances the DM is asked to select among them. Usually the choice is between two or three projects that alternate in obtained optimal portfolios.

Therefore, with the trichotomic approach projects are selected based on the notion of unanimity in the first case (green projects) and based on the notion of "majority" (among the iterations, i.e. the most frequent) in the second phase.

4.2. Simple iterative version

The term "iterative" indicates that a process develops in a series of decision rounds or cycles. A predetermined number of decision rounds may be defined from the beginning and every round feeds its subsequent until a convergence to the final portfolio is attained. From round to round the uncertainty is reduced for the grey projects forcing some of them to become either green or red. The uncertainty reduction can be performed by getting more information or by an automatic uniform narrowing of the grey projects' probability distributions. The whole process is depicted in **Figure 4-3**.

Monte Carlo simulation and optimization with Mathematical Programming is a rather recent development that becomes plausible with vast improvement in computer power during the last years. Although it is a computational demanding task it is worthwhile as it provides fruitful information regarding the uncertainty of the final solution.

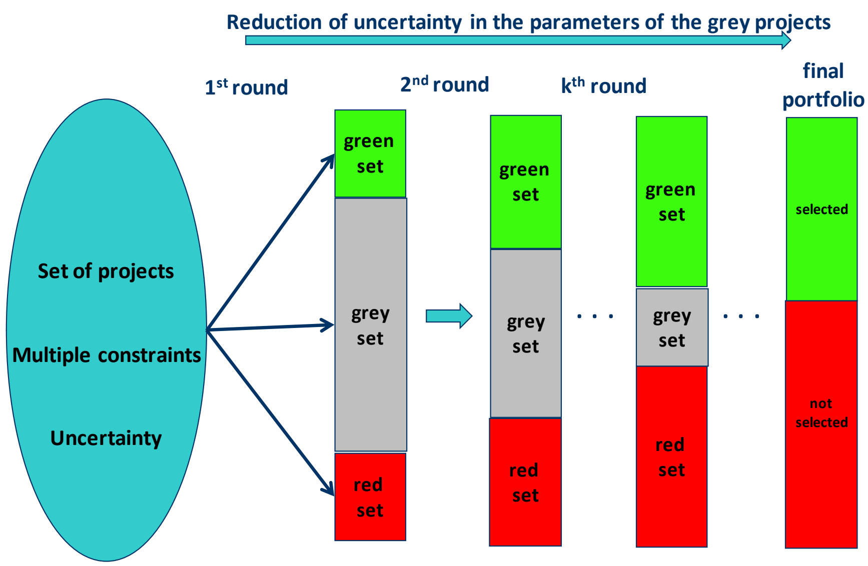

Figure 4-3. Graphical illustration of iterative process

Various probability distributions for uncertain parameters can be tested through Monte Carlo simulation (see e.g. Vose, 1996; 2006). By sampling from selected distributions, values of parameters are obtained from a Mathematical Programming model that is subsequently optimized. This process is repeated *T* times (*T* is a great number, e.g. *T*=1000) and *T* optimal portfolios are received expressing all possible states of nature (some of these optimal portfolios may be identical).

The MP model on the *t*-th Monte Carlo iteration is as follows:

$$\begin{aligned} &\max\ Z^{(t)} = \sum_{i=1}^{P} c_i^{(t)} X_i \\ &st \\ &\mathbf{X} \in S \\ &X_i \in \{0,1\} \end{aligned} \qquad (4.3)$$

where $c_i^{(t)}$ is the objective function's coefficient (some kind of output) of *i-th* project in the *t-th* Monte Carlo iteration. The value of $c_i^{(t)}$ is drawn from sampling of the corresponding distribution. X_i is the binary decision variable indicating if *i-th* project from initial set is either selected (X_i =1) or discarded (X_i =0) and S represents a feasible region formulated by all imposed constraints. It is prohibited to select a share or parts of one project, that is why the modeling is done with binary variables and not continuous ones as it is usually the case in the original portfolio selection problem which involves shares. Besides usual budget constraints, segmentation and policy constraints as well as interactions and interdependencies among projects can also be taken into account in the formulation of decision space S (Mavrotas et al., 2003; Liesio 2007).

The output of model (4.3) is an optimal portfolio *X(t)* with *Z(t)* as the value for the objective function. Exploiting information from T optimal portfolios the projects are distributed between three sets:

- The green set that holds projects that are present in all T portfolios
- The red set that contains projects that are excluded from all T portfolios
- The grey set that holds projects that are present in some of the T portfolios

Table 4-1 shows an example of green, red and grey projects in a problem with P projects and T=5 iterations. The rows contain values of iteration's decision variables while the columns contain values of the decision variables across Monte Carlo iterations.

Table 4-1. Example of results from initial round with 5 iterations.

Iteration	x_1	x_2	x_3	x_4	…	x_P
1	1	0	0	1	…	1
2	0	0	1	1	…	1
3	0	0	0	1	…	0
4	1	0	1	1	…	0
5	1	0	0	1	…	1
	grey	red	grey	green	…	grey

One thing to remember is the fact that especially in initial rounds it is almost impossible to draw conclusions about a portfolio that appears most frequently among *T* iterations, which means that obtained optimal portfolios are rarely the same across these *T* runs. Since conclusions cannot be drawn for the most frequent portfolios it is feasible to analyze the most frequently appearing projects in portfolios. Exactly this kind of information is exploited in the method where the main focus is on the grey set, i.e. the projects that require deeper attention.

As it was mentioned earlier, ITA incorporates decision rounds (or cycles). In every round of ITA a simulation - optimization process takes place providing the corresponding green, red and grey sets of projects. The process is quite flexible and can be implemented either with a predetermined, fixed number of rounds or until sufficient convergence is obtained in a less formal way.

4.2.1. Predetermined number of rounds

The number *R* of rounds may be set from the very beginning of the process. In the first round Monte Carlo sampling is performed with initial probability distributions of uncertain parameters and obtained results define *green(1)*, *red(1)* and *grey(1)* sets (the number in parenthesis indicates the round from which corresponding set emerges). In the second round projects from *green(1)* set are considered as given, those from the *red(1)* set as discarded and the variance (quantitative measure of the uncertainty) of the *grey(1)* projects' parameters is reduced by *1/R*. This reduction depends on the form of distribution. For example, for normal distribution the standard deviation is reduced by *1/R*, or, for the uniform distribution the range is cut by *1/2R* from both edges. It must be noted that this is done only for the grey projects while the sampling for green and red projects maintain the previous round's probability parameters. The model for the second cycle is as follows:

$$\begin{aligned}
&\max\ Z^{(t)} = \sum_{i=1}^{P} c_i^{(t)} X_i \\
&st \\
&\mathbf{X} \in S^{(t)} \qquad\qquad (4.4)\\
&X_i \in \{0,1\} \\
&X_i = 1 \quad i \in green(1) \\
&X_i = 0 \quad i \in red(1)
\end{aligned}$$

After the second round of simulation-optimization the process' output is elaborated. More specifically, green and red sets are enriched by new projects and new grey projects are identified. Subsequently, for the third round the variance of grey projects' performance is reduced even more and new green and red sets are considered as given. The flowchart of the decision making process is depicted in **Figure 4-4**.

The reduction in variance usually follows a uniform pattern across rounds. For example in the case of normal distribution the standard deviation is reduced by *1/R* after every round. This means that after round *r* the reduction of standard deviation is *sd* × *r/R*. Thus, in the final round grey projects' parameters are considered as deterministic (have no variance at all). The output of the final round is a unique portfolio as all Monte Carlo simulation-optimization iterations produce the same solution.

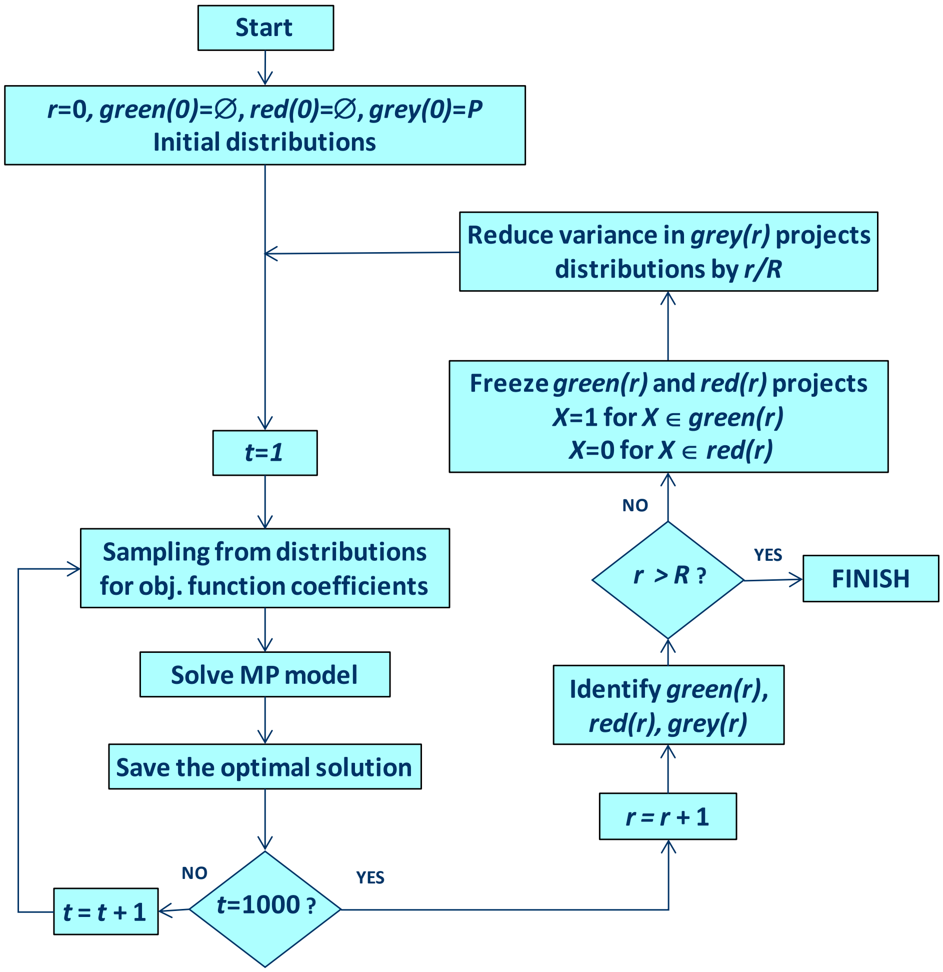

Figure 4-4. Flowchart of Iterative Trichotomic Approach (predetermined number of rounds).

4.2.2. Undetermined number of rounds

Another option is to avoid the determination of rounds and finish the decision making process when adequate convergence for the final portfolio has been attained. The whole process is less formal than the previous case one. After the simulation-optimization approach, the DM identifies grey projects (projects in doubt, gathers more information for these projects which is translated in variance reduction of their parameters' distribution). It must be noted that the narrowing of the probability distributions in grey projects' attributes at every cycle *r* can be done either uniformly or based on obtained information. In each round the grey set obviously shrinks and DM checks the frequency of each obtained optimal portfolio in the output of simulation. If, for example, a specific portfolio occurs in 567 out of 1000 iterations it actually has 56.7% probability to be the optimal portfolio under the given uncertainty level. If the DM finds a stochastic dominant portfolio then the selection process may be stopped. The term "dominant" is flexible. For instance, the DM can exit the loops of decision rounds as soon as a portfolio with 60% or 70% probability emerges.

The exit threshold (i.e., the probability of occurrence over which a portfolio is considered as selected) is determined by a DM according to a specific decision situation. The flowchart of the decision making process is depicted in **Figure 4-5**. The steps with darker shading indicate the alterations from the ITA with a predetermined number of rounds.

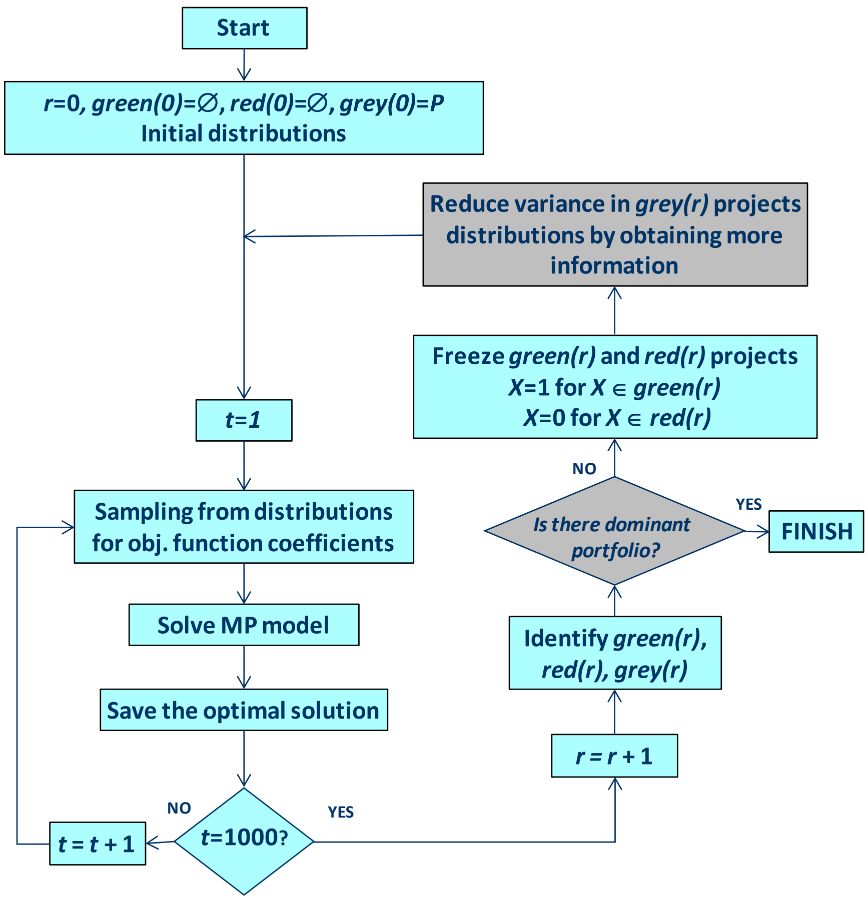

Figure 4-5. Flowchart of Iterative Trichotomic Approach (not a priori determined number of rounds).

4.3. Membership threshold

One of first observations within applications on illustrative examples is that on early iterations there is no dominant portfolio. When Monte Carlo simulation of uncertainties is used, among obtained optimal portfolios only few were the same and frequency of their appearance was found to be very low (less than 1-5%). Also, the number of projects in optimal portfolios considerably varied. Hence, the focus shifted to the most frequently appearing projects across portfolios, since it was hard to draw conclusions for portfolios as a whole.

In order to facilitate and speed up the decision process, it is worth to introduce membership thresholds for green and red sets in order to relax the membership requirements. It can be expressed through a "green" threshold of α% which means that if a project is present in optimal portfolio in α% of iterations, it is considered to be member of the green set. These thresholds are usually symmetric which means that a green threshold of α% implies a red threshold of 1-α %. For example, a "green" threshold of 95% means that if a project is present in optimal portfolio in 95% of iterations, it should be considered as a member of green set. Similarly, a "red" threshold of 5% means that a project which is present in the optimal portfolio in less than 5% of iterations is sent into the red set. The membership threshold can be used whenever the discrimination ability of previous rounds needs to be increased, e.g. where green and red sets are almost empty.

4.4. Group of decision makers

Project portfolio selection is initially a multi-objective problem where different points of view should be taken into account. A team of experts working on certain problem is a common practice in today's world, especially in large organizations where the aggregation of opinions is necessary or whenever consensus is sought among various stakeholders like e.g. when several levels of public policy are involved (Macharis et al., 2012; Vandaele and Decouttere, 2013). General agreement becomes crucial in situations when collaboration between individuals is required to build and implement shared goals with available resources. Within the process of development it is necessary to deal with various, sometimes conflicting, objectives represented by non homogenous groups of professionals (decision makers, experts, stakeholders etc.). Even if the final decision is to be taken by a single individual, the engagement of relevant experts is beneficial, as they can provide valuable information which can be otherwise overlooked or neglected due to countless reasons (Vilkkumaa et al. 2014). In general, Group Decision Making in multi-criteria analysis has been used in many applications such as water management (Morais and de Almeida, 2007; 2012, Morais et al. 2012), energy-environment issues (Turcksin et al., 2011; Hobbs and Meier, 2000), transportation issues (Macharis et al. 2010; 2012) etc. However these applications usually deal with a discrete number of given alternatives and not with a project portfolio problem.

One approach is to aggregate these points of view to a single metric through multi-criteria analysis and subsequently optimize the resulting single objective problem where coefficients of objective function are multi-criteria scores (Mavrotas et al., 2008). Alternatively, one can use a goal programming approach aggregating objective functions based on their distance from individual goals (see e.g. Zanakis et al., 1995;

Santhanam & Kyparisis, 1996). Furthermore, active use of MCDA methods may help not only to identify the areas of disagreement, but also to clarify possible alternatives (Salo, 1995; Salo and Hamalainen, 2010; Vilkkumaa et al., 2014). Decision support tools are useful on different stages. Initially, they help to describe the problem in details and to capture the preferences of each group member. Later, they highlight points of agreement and disagreement within the group. In addition, their skillful application can foster the formulation of innovative decision alternatives (Salo et al., 2003; Rios and Rios Insua, 2008) even in presence of important obstacles such as incomplete input information. Finally, good breakdown of preferences and possible options may help to discover and agree upon portfolio outside the initial set of options.

In all above mentioned approaches, the decision maker has to define criteria or goals and to assign them weights in order to aggregate them to a single objective function. Another way is to keep individual criteria as separate objective functions and proceed to a multi-objective optimization generating the Pareto set of the problem (or a Pareto front in criteria space) which comprises Pareto optimal solutions or portfolios. Then, the decision maker examines the obtained Pareto front before reaching his final choice. These methods are called "a posteriori" or "generation" methods in the popular Hwang and Masud (1979) terminology for multi-objective optimization methods (first generate Pareto front, examine it, and then select the most preferred Pareto portfolio). Their aim is not just to provide the most preferred solution but also to generate the Pareto set, either exactly or its approximation.

For the current case a combination of MCDA – IP is adopted in order to determine the optimal portfolio. Initially, one of MCDA methods is used in order to assign scores to projects based on their multi-criteria evaluation. Then, these scores are introduced as objective function coefficients in the IP model that incorporates constraints of the project selection problem. In the presence of multiple experts it is natural to assume that their preferences are expressed by assigning weights to the criteria of project evaluation, which means that each of them usually has an objective function that differs from the others. Hence, the obtained optimal portfolios are usually different among participants. In such case the membership of each project in green, red or grey set is determined according to the concordance between decision makers. Namely, the green set includes projects that are present in the final portfolio of every decision maker, the red set those projects that are absent from the final portfolio according to all experts, and projects that are picked by some group members form the grey set. The developed method is named "Group ITA" and is based on previous works of G.Mavrotas and O.Pechak. Here a Delphi-like approach is used to deal with the problem of providing decision support to multiple experts in project selection problems (see e.g. Wang et al. 2013; Lee and Kim, 2001; Juan et al., 2010). Delphi works in an iterative manner aiming at convergence of multiple opinions in a systematic way. Specifically, the iterative character is used and a converging process is performed in order to achieve a final consensus on project portfolio selection.

Assume that there are N projects, P DMs and K criteria of evaluation. Therefore the weight of importance that decision maker p assigns to criterion k is w_{pk} with p=1..P and k=1..K. For each DM p=1..P, multi-criteria scores ms_{pi} for every project i=1..N are calculated. The objective function of the IP problem for the p-th DM is then:

$$\max \sum_{i=1}^{N} ms_{pi} \times X_i \qquad (4.5)$$

where X_i is the binary variable that indicates if the i-th project is selected (X_i=1) or rejected (X_i=0). Solving P integer programming problems at most P different optimal portfolios (some of them may be identical) are obtained. Subsequently, the members of green, red and grey sets are identified. Members of the green set are projects that are present in all P optimal portfolios. Accordingly, the members of the red set are projects that are absent from all P optimal portfolios and the grey projects are those that are included in some of the P optimal portfolios.

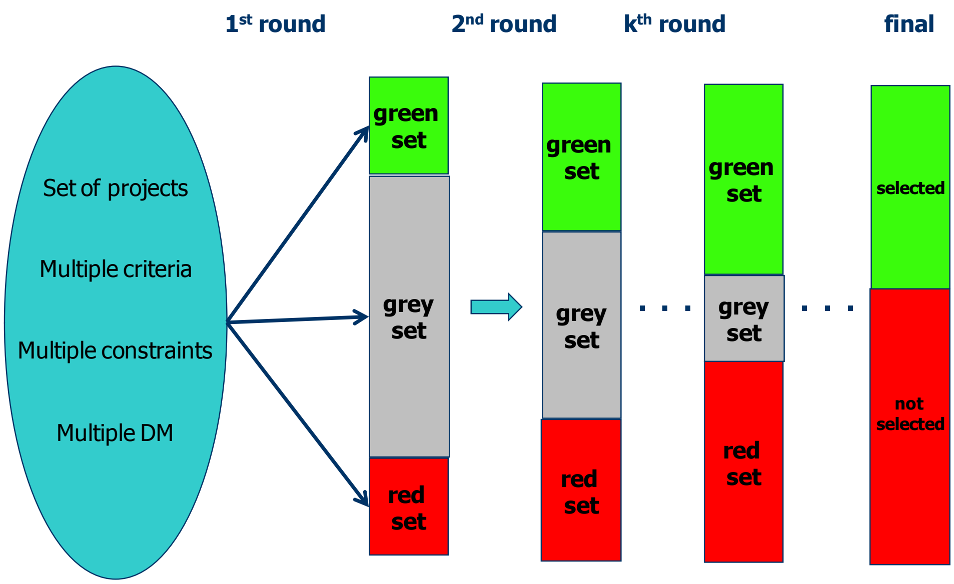

Figure 4-6. Illustration of Group ITA method.

If the grey set is not empty the process moves to the next round. The already found green and red projects are kept in their status by fixing the value of corresponding decision variable to X_i=1 for green projects and X_i=0 for red ones. This is done for all P models for the next round. In addition, necessary modifications are introduced in the objective function coefficients of the P models following the convergence process described in next paragraph.

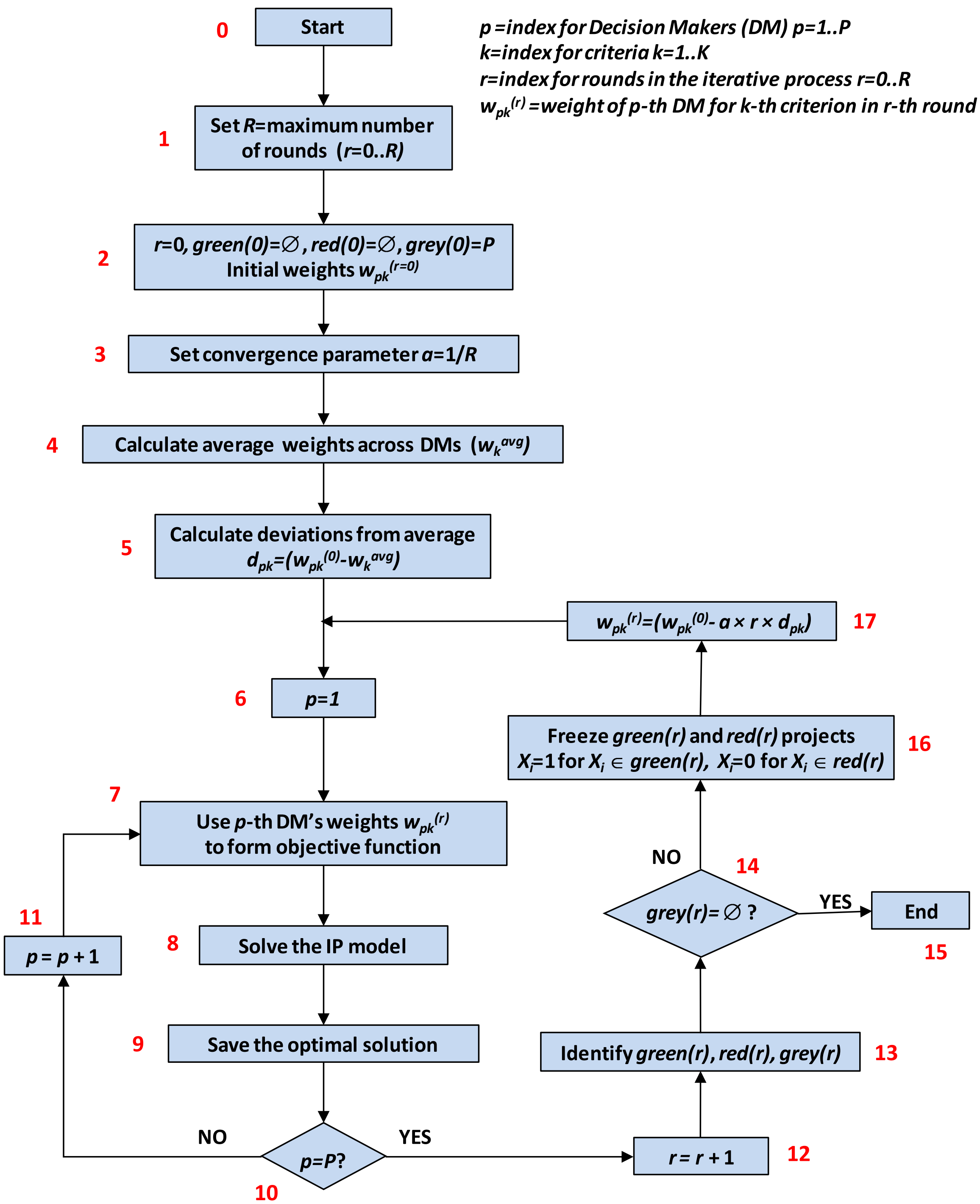

Figure 4-7. Flowchart of Group ITA method.

In the current case the convergence process deals with the weights of criteria and it is necessary to assure that the iterative process terminates with a unique portfolio as output. The following illustrative **Figure 4-6** depicts the concept of Group-ITA method.

Maximum number of rounds R in the Group-ITA method can be determined from the beginning. However, the method may converge earlier. The indication for convergence is an empty grey set. As it will be shown, in the next subsection the weights of importance are modified from round to round. Therefore $w_{pk}^{(r)}$ indicates the weight of importance for k-criterion of the p-th DM in round r=0...R. The methods' flowchart is shown in **Figure 4-7**. The step of calculation of next round's weights, which is the essence of convergence process, is described in detail in the next subsection.

4.4.1. Convergence process

The aim of convergence process is to provide an algorithm that gradually smoothes the divergence of criteria weights across decision makers. In other words, the weights of importance are adjusted from round to round in order to converge to a common solution after completion of iterative process.

Assume that original criteria weights for each DM are defined as $w_{pk}^{(0)}$. The maximum number of rounds in the iteration process (R) is agreed upon in advance and the convergence parameter α is accordingly determined as $\alpha=1/R$. Then, the deviation of each weight from their average (w_k^{avg}) across decision makers is calculated from the equation:

$$d_{pk}=w_{pk}^{(0)}-w_k^{avg} \quad (4.6)$$

The iterative process includes steps 6-17 in the flowchart of **Figure 4-7**. The adjustment of weights from round to round is performed in step 17 using the following equation:

$$w_{pk}^{(r)}=(w_{pk}^{(0)}- a \times r \times d_{pk}) \quad (4.7)$$

Actually, on every round the weights from each decision maker are moved towards averages and then P optimizations are performed again. Respective multi-criteria scores are updated and used further as objective function coefficients in the IP model. After P models are solved new green, red and grey sets that correspond to the r-th round (denoted as *green(r)*, *red(r)* and *grey(r)*) are identified. Binary variables that correspond to green projects are fixed to "1" and those of red projects are fixed to "0".

$$X_i = 1 \quad i \in green(r)$$
$$X_i = 0 \quad i \in red(r)$$

Once a project enters the green or the red sets it remains there for all subsequent iterations. It is obvious that from round to round the green and the red sets grow while the grey set shrinks:

$$|green(r)| >= |green(r-1)|$$

$|red(r)| >= |red(r-1)|$

$|grey(r)| <= |grey(r-1)|$

As iterations proceed more green and red projects are added into corresponding sets as the views of DMs are getting closer. Iterations are performed until the set *grey(r)* becomes empty what may happen before reaching round *R* $(r < R)$. In any case, convergence process implies that in *R*-th round all decision makers have common weights so that only one portfolio is finally obtained from *P* optimizations. Hence in the *R*-th round the grey set is by definition empty.

It must be noted that during convergence process the weights of each decision maker automatically satisfy the condition of summing to unity in every round *r* as it is proved below. Given the original weights $w_{pk}^{(0)}$ the initial equation is as follows:

$$\sum_{k=1}^{K} w_{pk}^{(0)} = 1 \quad for\ p = 1..P \tag{4.8}$$

The average across decision makers is calculated as:

$$w_k^{avg} = \frac{\sum_{p=1}^{P} w_{pk}^{(0)}}{P} \tag{4.9}$$

and the sum of w_k^{avg} equals to unity as it is shown below:

$$\sum_{k=1}^{K} w_k^{avg} = \sum_{k=1}^{K} \frac{\sum_{p=1}^{P} w_{pk}^{(0)}}{P} = \frac{\sum_{k=1}^{K}\sum_{p=1}^{P} w_{pk}^{(0)}}{P} = \frac{\sum_{p=1}^{P}\sum_{k=1}^{K} w_{pk}^{(0)}}{P} = \frac{(1+1+...+1)}{P} = 1 \tag{4.10}$$

Hence, for the weights of round *r*, i.e. $w_{pk}^{(r)}$ the expression is:

$$\begin{aligned}
\sum_{k=1}^{K} w_{pk}^{(r)} &= \sum_{k=1}^{K} (w_{pk}^{(0)} - a \times r \times d_{pk}) \\
&= \sum_{k=1}^{K} [w_{pk}^{(0)} - a \times r \times (w_{pk}^{(0)} - w_k^{avg})] \\
&= \sum_{k=1}^{K} [(1 - a \times r) \times w_{pk}^{(0)}] + a \times r \times \sum_{k=1}^{K} w_k^{avg} \\
&= (1 - a \times r) \sum_{k=1}^{K} w_{pk}^{(0)} + a \times r \times \sum_{k=1}^{K} w_k^{avg} \\
&= (1 - a \times r) + a \times r = 1
\end{aligned} \tag{4.11}$$

Therefore the weights do not need any normalization as it is applied automatically from their calculation.

An example of weight calculation from round to round is shown next. Assume that there is a team of five decision makers, whose initial weights for 4 criteria are shown in **Table 4-2**.

Table 4-2. Initial weights of group members.

	C_1	C_2	C_3	C_4	sum
DM1	0.25	0.25	0.25	0.25	1.00
DM2	0.8	0.05	0.1	0.05	1.00
DM3	0.1	0.1	0.7	0.1	1.00
DM4	0.2	0.6	0.1	0.1	1.00
DM5	0.05	0.15	0.1	0.7	1.00
Average	0.28	0.23	0.25	0.24	1.00

In **Table 4-3** the deviations from the average of each column (across DMs) are presented after calculations using equation (4.6).

Table 4-3. Deviation from column's average.

	C_1	C_2	C_3	C_4
DM1	-0.03	0.02	0.00	0.01
DM2	0.52	-0.18	-0.15	-0.19
DM3	-0.18	-0.13	0.45	-0.14
DM4	-0.08	0.37	-0.15	-0.14
DM5	-0.23	-0.08	-0.15	0.46

Table 4-4 presents new weights of next round according to equation (4.7) and using convergence parameter α=0.2 (i.e. maximum rounds R=5).

For example, the new weight for DM2 in the 3rd criterion is calculated as:

$w_{23}^{(1)}$=0.1 - 0.2×1×(-0.15) = 0.130

In the same way all cells are calculated and the sum of weights for each DM remains unity. By comparing **Table 4-2** and **Table 4-4** one can observe the movement towards average weights. On last iteration (when R=5) the convergence process ends with all weights of every column becoming the same, equal to the average (last row of **Table 4-2**).

Table 4-4. New weights of group members.

	C_1	C_2	C_3	C_4	sum
DM1	0.256	0.246	0.250	0.248	1.00
DM2	0.696	0.086	0.130	0.088	1.00
DM3	0.136	0.126	0.610	0.128	1.00
DM4	0.216	0.526	0.130	0.128	1.00
DM5	0.096	0.166	0.130	0.608	1.00

4.4.2. Consensus index

Within this work an approach to measure the level of consensus over the final portfolio according to the degree of concordance between DMs was developed. The consensus index expresses how easy or hard it is to arrive at a consensus among experts. The more green projects are obtained from early rounds the greater the degree of concordance among parties involved is. Specifically, their preferences (expressed as weights) result in more or less the same outcome without forcing their weights to converge or, in other words, the consensus is easily attained. On the contrary, if the majority of green projects is identified in last rounds it means that further elaboration of the convergence process is needed to reach agreement upon the selected projects. This means that the consensus is attained with great difficulties.

The index is found through a consensus chart where the percentages of green projects that are available in *r-th* round are plotted as a function of the respective decision round. The resulting curve is called *consensus curve*. In **Figure 4-8** one can observe that from round 2 to round 3 there are no new projects added in the green set. This may happen especially when the maximum number of rounds (R) is relatively high.

The Consensus Index (CI) is calculated as the area below the consensus curve divided by the rectangle area denoted by a dashed rectangular in **Figure 4-8**. The dashed rectangular actually expresses the maximum consensus (CI=1) that occurs when from round 0 already, all projects are allocated either to green or red sets (i.e., the grey set is empty). The minimum consensus occurs when all green projects are added in the final portfolio on the last round (CI=0). CI takes values between 0 and 1 and it is calculated using the trapezoid rule for piecewise linear functions according to the following equations:

$$CI = (\frac{a_0 + a_1}{2} + \frac{a_1 + a_2}{2} + \ldots + \frac{a_{R-1} + a_R}{2}) / R$$

$$CI = [\frac{a_0}{2} + \sum_{r=1}^{R-1} a_r + \frac{a_R}{2}] / R \qquad (4.12)$$

$$CI = [\frac{a_0}{2} + \sum_{r=1}^{R-1} a_r + \frac{1}{2}] / R$$

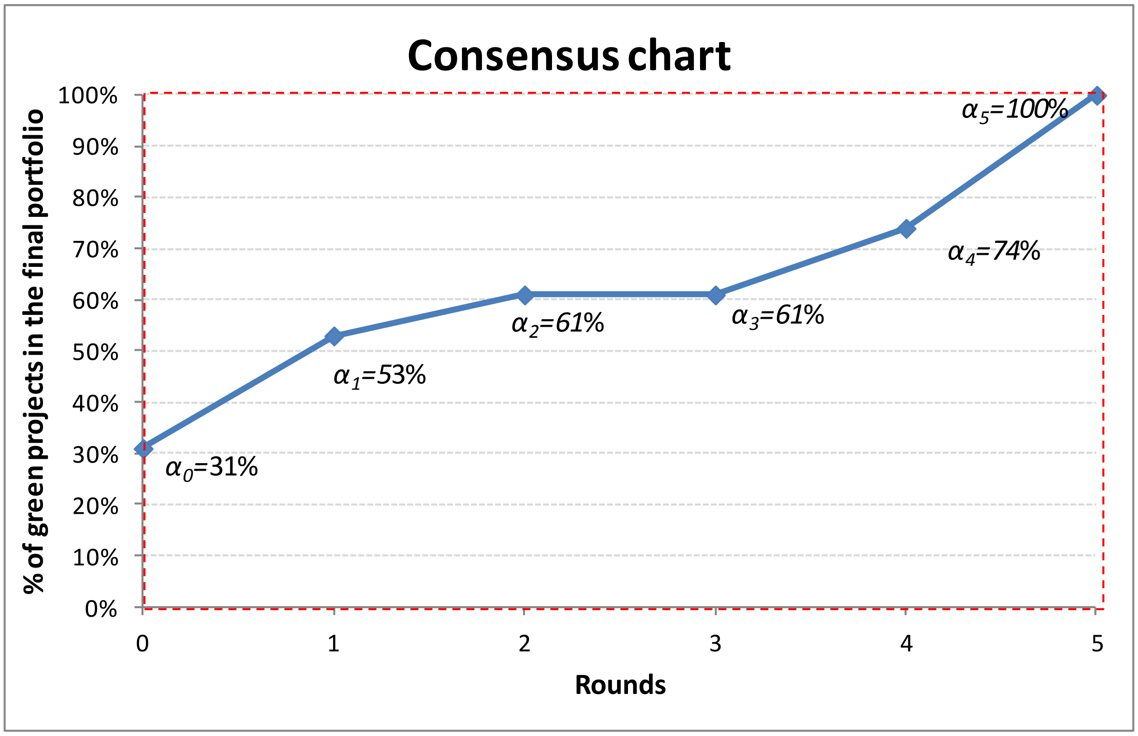

Figure 4-8. Example of consensus chart with R=5.

For example, from **Figure 4-8** the corresponding *CI* is:

$$CI = [\frac{0.31}{2} + 0.53 + 0.61 + 0.61 + 0.74 + \frac{1}{2}] / 5 = 62.9\%$$

Apart from the Consensus Index that characterizes the final portfolio it is possible to extract the degree of consensus for each project according to the round that it enters or exits the final portfolio. The Consensus Degree of the *i*-th project can vary in [0,1] and can be quantified by the following formula:

$$CD_i = \frac{R - r_i}{R} \qquad (4.13)$$

where r_i is the round that i-th project enters or exits the final portfolio or in other words the round that the respective project leaves the grey set.

4.5. Multi-objective project portfolio selection

After addressing the case of group project portfolio selection, usually the next issue that attracts attention is the multi-objective project selection. In the current section the applicability of Iterative Trichotomic Approach (ITA) is extended to the case of multi-objective optimization. Initially, ITA was focused on a single objective function problem structuring reflecting the optimization criterion. While the original approach provides the certainty degree of a specific project within the optimal portfolio given underlying uncertainty, multi-objective ITA provides certainty degree for a specific project portfolio within the Pareto set. A schematic representation of the multi-objective ITA is shown in **Figure 4-9**.

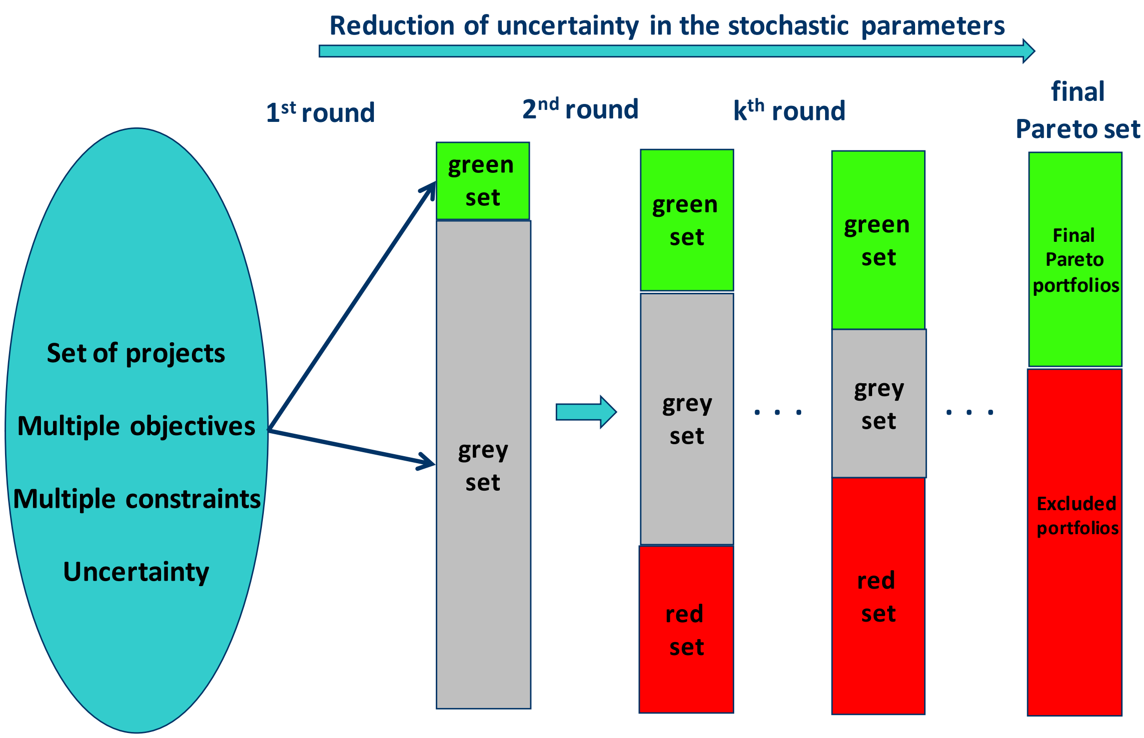

Figure 4-9. Graphical illustration of multi-objective ITA.

Unlike original ITA, the first iteration in multi-objective ITA has no red set as there are no portfolios to be excluded. The initial iteration provides the maximum number of generated portfolios as candidate final Pareto optimal portfolios. In subsequent iterations some of these portfolios are not present anymore in any Pareto set so they are labeled as red. With the movement from round to round, the uncertainty of parameters (objective functions' coefficients) is reduced (e.g. by reducing the standard deviation of a normal probability distribution or shrinking the interval of a uniform probability distribution). With diminishing uncertainty, portfolios gradually move from grey set into green (appear in all Pareto sets). The red set is implied indirectly by the initially generated portfolios that are not present in any current Pareto set.

The methodology is developed for the case of two objective functions. It can be easily extended to a greater number of objective functions, but with increasing number the elaboration of results may become too cumbersome. The Pareto Optimal Portfolios (POPs) of projects are actually the Pareto Optimal Solutions of the multi-objective integer problem with binary variables:

$$\begin{aligned}
&\max\ Z_1 = \sum_{i=1}^{\mathrm{N}} c_{i1} X_i \\
&\ldots \\
&\max\ Z_K = \sum_{i=1}^{\mathrm{N}} c_{iK} X_i \\
&st \\
&\mathbf{X} \in S \\
&X_i \in \{0,1\}
\end{aligned} \tag{4.14}$$

where N is the number of candidate projects, c_{ik} is the objective function coefficient of i-th project in k-th objective function, X_i is a binary decision variable indicating if the i-th project from initial set is selected (X_i=1) or not (X_i=0), and S represents the feasible region formulated by all imposed constraints. Apart from the usual budget constraints, segmentation and policy constraints, interactions and interdependencies among projects can be also taken into account in the formulation of decision space S. Eventually, a Pareto optimal Portfolio is represented by a vector of "0" and "1" of size N. According to the multi-objective version of ITA method, each portfolio from the initial set of Pareto Optimal Portfolios is eventually characterized as red or green with gradual decrease of uncertainty in model's parameters, which is performed in *computation rounds*.

In each computation round a great number (t=1..T with e.g. T=1000) of problems such as model (4.14) is solved, with different model parameters, specifically different objective function coefficients using a Monte Carlo simulation approach:

$$\max Z_1^{(t)} = \sum_{i=1}^{N} c_{i1}^{(t)} X_i$$

$$...$$

$$\max Z_K^{(t)} = \sum_{i=1}^{N} c_{iK}^{(t)} X_i \qquad (4.15)$$

$$st$$

$$\mathbf{X} \in S$$

$$X_i \in \{0,1\}$$

where $c^{(t)}_{ik}$ is the objective function coefficient of *i*-th project in *k*-th objective function during *t*-th Monte Carlo iteration. The values of $c^{(t)}_{ik}$ are sampled from the selected probability distributions (uniform, normal, triangular etc). Therefore, in each computation round *T* Pareto sets (PS_t, $t=1..T$) are produced. The generation of each Pareto set is performed using the AUGMECON2 method (Mavrotas and Florios, 2013). AUGMECON2 is an improved version of the well known ε-constraint method, especially appropriate for MOIP problems like model (4.14). It must be noted that AUGMECON2 is capable of generating the exact Pareto set in MOIP problems which means that no Pareto Optimal Solution is left undiscovered.

Like in original ITA, in each computation round there are three sets where all the Pareto Optimal Portfolios *p* are allocated: The green set (*G*), the red set (*R*) and the grey set (*Y*). The membership relations for each portfolio *p* in *G*, *R* and *Y* are shown below.

$$\begin{aligned} & p \in G : \forall t \in 1..T \;\; , p \in PS_t \\ & p \in R : \forall t \in 1..T \;\; , p \notin PS_t \\ & p \in Y : \exists t \in 1..T \;\; , p \in PS_t \end{aligned} \qquad (4.16)$$

In other words, the green set includes portfolios *p* that are present in all Pareto sets ($PS_1 \ldots PS_T$) of the computation round, the red set includes portfolios that were produced in the initial computational round but are not present in any of *T* Pareto sets in the current computational round, and the grey set includes portfolios that are present in some of *T* Pareto sets. In order to be more specific about the round *r* that a green, red and grey set refers to, the notation G_r, R_r and Y_r is used.

As it was mentioned earlier, the results of the first round define green and grey sets denoted as G_1 and Y_1. On the second round, the variance of Y_1 projects' parameters is reduced proportionally to the number of total rounds *R*. This reduction depends on the form of distribution. For instance, for a normal distribution the standard deviation is reduced by 1/(*R*-1), or, for a uniform distribution, it is cut by 1/(2(*R*-1)) from both edges of the range.

The variance reduction follows a uniform pattern across rounds. In the case of normal distribution, the standard deviation (*sd*) is reduced by 1/(*R*-1) after each round. This means that after round *r*, the reduction of standard deviation is $sd\times(r-1)/(R-1)$. Thus, in the final round projects' parameters (objective function coefficients) are considered as deterministic (have no variance at all). Therefore, the final round produces only one Pareto set which is the final Pareto set that comprises the final Pareto portfolios. The flowchart of the decision making process is depicted in **Figure 4-10**.

To facilitate and speed up the selection process, membership thresholds for the green set by relaxing membership requirements can be introduced. For example, a "green" threshold of 95% would mean that a portfolio is considered to be a member of green set if it is present in at least 95% of Pareto sets.

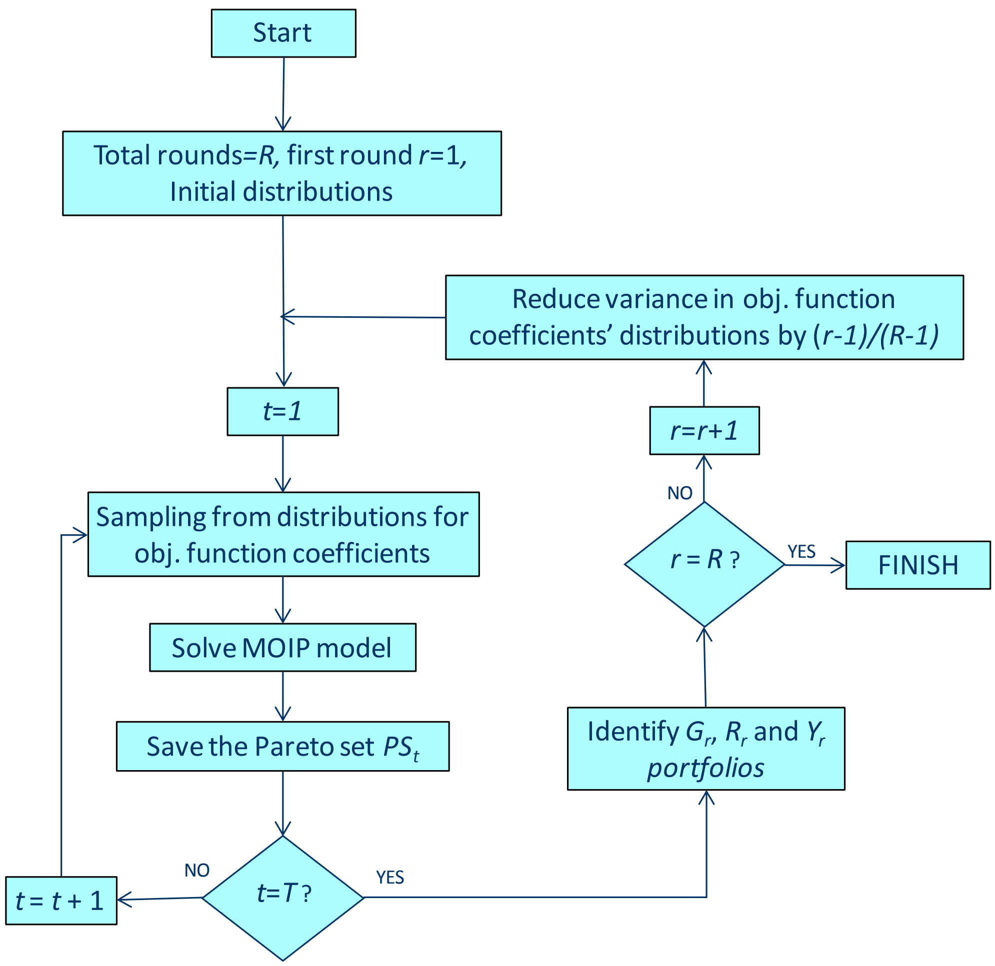

Figure 4-10. Flowchart for multi-objective ITA.

On the basis of the obtained information by the end of the multi-objective – ITA optimization process it is possible to compute the Robustness Degree of each Pareto Optimal Portfolio, to build the Robustness chart and find the Robustness Index of the Pareto set. In addition, the decision maker(s) is/are provided with informative charts that illustrate the Pareto front with additional information about the robustness of each Pareto Optimal Portfolio.

4.6. Robustness measuring

Robustness of the Pareto Optimal Portfolios in multi-objective ITA is associated with how sure one can be about the membership of a specific portfolio in the final (definitive) Pareto set, which is obtained in the last computation round. As uncertainty is reduced going from one computation round to the next, the sooner a Pareto Optimal Portfolio enters the green set, the more "secure" is its place in the final portfolio. Therefore, for the Pareto Optimal Portfolios, the measure of robustness can be quantified with the *Robustness Degree* for each Pareto Optimal Portfolio (RD_p) which is defined as follows:

$$RD_p = \frac{R - r_p}{R} \qquad (4.17)$$

where r_p is the computation round that p-th portfolio enters the green set (i.e. becomes member of the final Pareto set) and R the total number of computation rounds. As it is obvious from equation (4.17) Robustness Degree of p-th portfolio varies in [0, (R-1)/ R] and the closer it is to 1 the more robust is the specific portfolio.

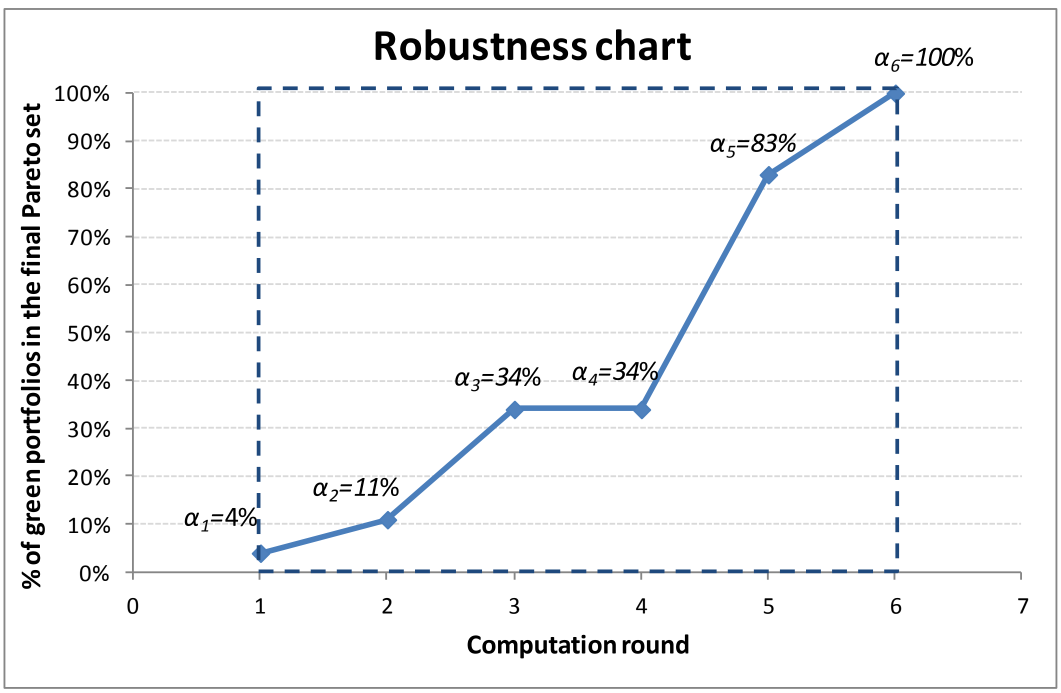

Figure 4-11. Example of Robustness Chart with R=6.

Also, according to the information about how early in the decision process the final Pareto optimal portfolios entered the green set, it is possible to measure the robustness of the final Pareto set. The more green portfolios are discovered from early rounds (i.e., with wider uncertainty range), the more robust the final Pareto set is. On the contrary, if the majority of green portfolios is identified in the last rounds, it means that the final Pareto set is not so stable.

For the assessment of robustness of the final Pareto set the *Robustness Index (RI)* is employed which is similar to the one used in the previous section for group decision making. In order to calculate the Robustness Index the so called *Robustness Chart* is drawn where the percentages of green portfolios that are available on ***r***-th round (denoted as a_r) are plotted as a function of the computation round. The resulting curve is called *Robustness Curve*. In **Figure 4-11** an example of a Robustness Chart with the corresponding Robustness Curve is presented. It is easy to observe that from round 2 to round 3 there are no new portfolios added in the green set. This may happen especially when the maximum number of rounds (*R*) is relatively high.

The Robustness Index of final Pareto set is calculated as the area below the robustness curve, divided by the rectangle area denoted by dashed rectangular in **Figure 4-11**. The dashed rectangular actually expresses the maximum robustness (*RI*=1) that occurs when already from the first computation round (i.e., when the uncertainty is on maximum) only one Pareto set is produced from all Monte Carlo iterations. The minimum robustness occurs when all green portfolios are added in the final Pareto set on the last round (*RI*=0). *RI* takes values between 0 and 1 and it is calculated using the trapezoid rule for piecewise linear functions according to the following equations:

$$RI = (\frac{a_1 + a_2}{2} + \frac{a_2 + a_3}{2} + ... + \frac{a_{R-1} + a_R}{2}) / (R-1)$$

$$RI = [\frac{a_1}{2} + \sum_{r=2}^{R-1} a_r + \frac{a_R}{2}] / (R-1) \quad (4.18)$$

$$RI = [\frac{a_1}{2} + \sum_{r=2}^{R-1} a_r + \frac{1}{2}] / (R-1)$$

For example, from **Figure 4-11** the corresponding *RI* is:

$$RI = [\frac{0.04}{2} + 0.11 + 0.34 + 0.34 + 0.83 + \frac{1}{2}] / 5 = 42.8\%$$

5. Applications

Methods of portfolio selection are widely employed to support decision procedures both in public administration and industrial firms. That is why here and further, after detailed description of theoretical concepts, the focus is on real case studies. Since the whole ITA method was developed on observations, examples will help to understand better all suggestions and concepts.

Extraordinary development of telecommunications technologies and tremendous possibilities they provide to handle information about the state of the environment made it interesting to study the selection problem in this domain. A case study from the literature made it possible to observe and compare the results within different decision support systems.

A second case study covers the problem of selecting projects for financing in the framework of Clean Development Mechanism (CDM), which comprised numerous uncertainties due to its novelty. The mechanism gained momentum in 2005 after the entry into force of the Kyoto Protocol to UNFCCC and was in full operation in the period of 2008 – 2012. Before the Protocol entered into force, investors considered this a key risk factor. The initial years of operation yielded fewer CDM credits than supporters had hoped for. Later, it turned out that the purchases were made mainly within European Union Emission Trading Scheme and it led to oversupply of emission allowances and to the crash of prices. The economic crisis within EU made the future of CDM even more uncertain. Still, these activities are maintained by industries from developed countries, which care about the environment and think about diversification of their activities.

The next sub-section focuses on local renewable energy projects. The energy sector has been a fertile ground for the application of operational research (OR) models and methods (Antunes and Martins, 2003).Greece is a mountain country meaning that there is almost always some wind blowing from or to the sea. The number of sunny days is also among the highest within European countries and hence, all the technologies aimed at capturing the energy from renewable sources are attractive from a long term perspective. The case study is focused on projects seeking for initial financial support from a development bank. Such decisions are usually taken by a board of experts from different fields of expertise that is why the situation of group decision making was being tested.

The last case study incorporates Energy and Environmental Corporate Responsibility (EECR) in decision making procedure in addition to the already widespread Net Present Value (NPV) of projects proposals. A bi-objective programming model is introduced in order to provide the Pareto optimal portfolios (Pareto set) based on the Net Present Value (NPV) of projects and the EECR score of firms. A systematic decision making approach using Monte Carlo simulation and multi-objective programming is also developed in order to deal with the inherent uncertainty in the objective functions' coefficients. The proposed approach facilitates banking organizations and institutions to the selection of firms applying for financial support and credit granting, within the frame of their environmental obligations.

5.1. Selection of telecommunications projects

As it was mentioned in previous chapters, various sectors of economy face problems of choice. Wide and fast spread of new telecommunications technologies required effective tools to select options for expansion and meeting growing demand. Technological advances made possible new ways of using telecommunications which could be only part of science fiction decades ago. Images and data, transferred via satellites, help to monitor the state of environment, prevent natural catastrophes or send the rescue teams in case of natural or industrial disasters.

During XX-th century there was relatively stable business environment in the telecommunications industry. Due to recent advancements in technologies and changes in markets it became necessary to reconsider long-term business goals (Lindstedt et al., 2008). One of the earliest applications that dealed with these new challenges was developed by Antunes and Craveirinha (1993). The need for balanced introduction of new service offerings was a problem which involved different and conflicting aspects. Both public and private companies were forced to reconsider their vision, mission and strategies. The achievement of these revised strategic objectives called for changes in their product portfolio, whereby companies were facing with the problem of choosing which products would effectively contribute to the achievement of their long-term goals.

The modernization planning of telecommunications networks, namely as far as the evolution towards new supporting technologies and service offerings are concerned, is a problem which involves different aspects, some of which are not directly quantifiable by an economic indicator. On a preliminary stage, project proposals should be grouped accordingly. Further, within assessments varying degrees of uncertainties, driven by the maturity of technologies and products, pace of technological advancements, developments in market prices, changes in competitive situation should not be overlooked.

5.1.1. The model for telecommunications project selection

An example from the literature is chosen for illustrative purposes. Namely a project portfolio optimization problem under uncertainty that refers to telecommunication projects (Niaei et al., 2011). The 40 candidate projects are classified in three types: Basic, Developing and Applied. Initial 40 projects are evaluated against five criteria:

- Cost: Total project cost including all expenses required for project completion (in million toomans which is the Iranian monetary unit).
- Proposed methodology: Degree of being step-by-step, well planned, scientifically-proven, disciplined, and proper for organization current status in the proposed methodology (qualitative, 0-10).
- The abilities of personnel: Work experience of project team related to concerned project (qualitative, 0-10).
- Scientific and actual capability: Scientific degree and educational certificates of project's team (qualitative, 0-10).
- Technical capability: Ability of providing technical facilities and infrastructures (qualitative, 0-10).

The performance of each project in each criterion is expressed as a uniform distribution with minimal and maximal probable values (see Niaei et al., 2011, for the exact data). The weights of criteria in the original paper were determined from expert judgment and fit appropriate distributions. In the current case, this information is simplified by using for all of them triangular distribution with the parameters shown in **Table 5-1**.

Table 5-1. Parameters for criteria weights' triangular distributions.

	Min	Mid	Max
Cost	0.17	0.21	0.23
Methodology	0.12	0.13	0.14
Personnel	0.12	0.14	0.16
Scientific ability	0.11	0.13	0.15
Technical ability	0.36	0.40	0.43

In addition to the original paper, some variability in the total budget is added, which it supposed to follow a normal distribution with mean 6 billion toomans and standard deviation of 0.3 billion toomans. There are also segmentation constraints that are expressed with upper bounds to each type of project. Namely, the sum of basic, developing and applied projects should not exceed the 20%, 70% and 40% of the total projects in the portfolio.

In the current subchapter a combination of three techniques is presented. MCDA, Mathematical Programming and Monte Carlo simulation are chosen in order to deal with project portfolio optimization with consideration of multiple criteria of projects' evaluation, multiple constraints and inherent uncertainty associated (a) with projects' characteristics and (b) with the decision situation. The uncertainty in the decision and the project parameters is represented with probability distributions (a stochastic nature is assumed) as it is also done in the multicriteria method SMAA (Ladhelma et al., 1998; Tervonen and Ladhelma, 2007) as well as other approaches (see e.g. Hyde et al., 2003). The proposed method presents a special case of ITA: the two-phase approach and compares results obtained with classic ITA.

The required models and the whole solution process was developed in the General Algebraic Modeling System (GAMS, see e.g. Brooke et al., 1988) using MIP solver CPLEX 11.1 for Mixed Integer Programming models optimization. The solution time was approximately about 3 minutes on Intel Pentium i5 at 2.53 GHz for the 1000 Monte Carlo simulations – optimizations.

5.1.2. Results and discussion on two-phase ITA

The theoretical basics for this unit are described in Chapter 2.3. The number of iterations of the first round was set to 1000. During the second phase the principle of majority for the projects' coefficients was in force, while there was still some flexibility on model's constraints.

The first observation after execution of phase 1 was the absence of any dominant portfolio. Among the 1000 optimal portfolios at most two were the same. So, it is obvious that it was too early to draw conclusions about the most widely accepted portfolio just from the first phase. Moreover, the number of projects in optimal portfolios varied from 21 to 27.

Subsequently, some membership thresholds ("green" and "red" thresholds as described in Chapter 2.2) were tested. The symmetric case, meaning that if the green threshold is α% and the red threshold is 1-α%, was adopted for calculations. As it is obvious, with growth of the membership threshold, proposals are easier attributed either to green or red sets as shown in **Table 5-2**.

Afterwards, different seeds for the random number generation in Monte Carlo simulation in order to check the results' robustness were considered. The outcomes for 15 different seeds were very similar, meaning that 15 different Monte Carlo simulation – optimization sessions were performed. In the first phase in 3 out of 15 runs the red set had one project less (2 instead of 3). However, in the second phase, 14 out of 15 runs provided exactly the same optimal portfolio.

Table 5-2. Influence of membership threshold on population of green and red sets.

Membership threshold	Green	Red	Grey
100%	6	0	34
99.5 %	7	1	32
99%	7	3	30
98%	7	3	30
95%	8	3	29
90%	10	5	25

Table 5-3. Frequency of appearance for projects in optimal portfolios.

#	**Freq**	**#**	**Freq**
1	944	**21**	882
2	*4*	**22**	249
3	674	**23**	150
4	76	**24**	548
5	**1000**	**25**	453
6	738	**26**	503
7	129	**27**	986
8	**1000**	**28**	732
9	*1*	**29**	**1000**
10	386	**30**	920
11	386	**31**	854
12	129	**32**	809
13	619	**33**	331
14	66	**34**	**1000**
15	**1000**	**35**	889
16	606	**36**	732
17	*6*	**37**	323
18	235	**38**	845
19	**1000**	**39**	623
20	711	**40**	**1000**

* **Bold** are projects from the green set, *Italic* are the ones from the red set.

The run of phase one, with membership threshold of 99% provided following results:

Green set - 7 projects (5, 8, 15, 19, 29, 34, 40)

Red set - 3 projects (2, 9, 17)

Grey set - 30 projects (the rest)

The frequency of projects' appearance in optimal portfolio is shown in **Table 5-3**.

Therefore, from the first phase it is safe to conclude that projects 5, 8, 15, 19, 29, 34 and 40 are in the final portfolio under any circumstances while there is no chance for projects 2, 9 and 17 to enter the final portfolio. Subsequently, on next phase under careful focus are the remaining projects of the grey set.

In the second phase only 30 projects from grey set participated as the values of the decision variables for green and red projects were fixed to "1" and "0" respectively. The objective function coefficients are the frequencies from **Table 5-3**. Due to the fact that there are still stochastic parameters in the constraints (the cost of each projects and the total budget) it is necessary to perform a Monte Carlo simulation – optimization session with 1000 iterations, according to the equation (2.3).

Even in the second phase a clearly dominating portfolio is not appearing. The optimal portfolio of highest frequency (portfolio A) is obtained in 22.6% of iterations (226/1000) while the next most frequent (portfolio B) is obtained in the 19% of the iterations (190/1000). These two, most frequent portfolios have 23 and 22 projects, respectively. The difference is only one project, namely project 16 which is present in portfolio A and not in portfolio B probably due to budget violation in respective runs.

It is interesting to compare the results of the two-phase approach with the results from a "conventional" approach, considering only expected values for uncertain parameters. Further in the unit is clearly shown that a significant part of information is left out of the analysis and the DM is losing essential information. In this case multicriteria scores and, hence, the objective function coefficients would be crisp numbers as well as all parameters of constraints in the MP model. The whole process would be similar to the approaches described among others by Abu Taleb et al. (1995), Mavrotas et al. (2003; 2006; 2008) where the uncertainty was not addressed. The difference with the trichotomic approach is on the results themselves as well as the information conveyed by these results. Results from both methods are shown in **Table 5-4**.

It can be seen that the obtained results are almost identical. Only projects #16 and #24 are interchanged, which are both in group of "applied" projects and have similar characteristics. **Table 5-5** reveals that in some criteria #24 performs weaker but it is characterized by less variation meaning more narrow distributions. The final decision (to violate the available budget constraint and if yes, which of 2 projects to choose) is still to be made by a person according to the main goals of the whole process.

Table 5-4. Optimal portfolio from "conventional" and two-phase ITA approaches.

Project #	Conventional (expected values)	Trichotomic (two-phase approach)	**Project #**	Conventional (expected values)	Trichotomic (two-phase approach)
1	1	1	**21**	1	1
2	0	*0*	**22**	0	0
3	1	1	**23**	0	0
4	0	0	**24***	1	0
5	1	**1**	**25**	0	0
6	1	1	**26**	0	0
7	0	0	**27**	1	1
8	1	**1**	**28**	1	1
9	0	*0*	**29**	1	**1**
10	0	0	**30**	1	1
11	0	0	**31**	1	1
12	0	0	**32**	1	1
13	1	1	**33**	0	0
14	0	0	**34**	1	**1**
15	1	**1**	**35**	1	1
16*	0	1	**36**	1	1
17	0	*0*	**37**	0	0
18	0	0	**38**	1	1
19	1	**1**	**39**	1	1
20	1	1	**40**	1	**1**

Table 5-5. Characteristics of borderline projects #16 and #24.

	Cost		Methodology		Personnel		Scientific ability		Technical ability	
	min	max	min	max	min	max	min	max	min	max
Project 16	374	486	2	6	4	8	1	3	2	6
Project 24	385	416	1	4	1	4	1	5	3	5

Seeing similar results one may wonder what is the contribution of the trichotomic approach. The real contribution is that it provides the DM with extra information. In the conventional approach the DM is not aware of the certainty degree for each project that is selected (with "1" in the corresponding column). By contrast, in the case of ITA the expert is aware of the degree of certainty for each project. This is fruitful information that may lead to better decisions (e.g. further adjustment of the total budget, identification of vulnerable and stable projects etc). Here, the fact of two similar projects provides the chance to perform a direct comparison before deciding about the final selection and maybe to reconsider initial assumptions and requirements.

5.1.3. Results and discussion on iterative ITA

For the comparison of outcomes the iterative version of the trichotomic approach by gradually reducing the uncertainty of grey projects in each cycle was applied. The reduction of uncertainty was done by the symmetric narrowing of their range of performances as expressed in the corresponding distributions provided in **Table 5-1**. A reduction step of 25% of the range was applied meaning that new min and max of the uniform distribution were calculated by the following formula:

$$\begin{aligned} \min^{(k)} &= \min + k \times \frac{25\%}{2} \times (\max - \min) \\ \max^{(k)} &= \max - k \times \frac{25\%}{2} \times (\max - \min) \end{aligned} \tag{5.1}$$

Therefore, sampling for Monte Carlo simulation was performed by all the more narrow ranges of the uniform distributions for grey projects. The midrange was reached on fourth iteration which meant there was no sampling but the midrange as the one and only representative value.

From **Table 5-6** it is obvious that the uncertainty reduction within grey projects drives gradually in more populated red and green sets. For example, it can be concluded that a DM is more confident about e.g. the inclusion of project 38 than 35, because it enters the green set in an earlier iteration. Similarly, one can be more sure about the exclusion of project 17 (excluded from the first round) than of project 14 (excluded in the third round).

Table 5-6. Results for iterative version of ITA.

Uncertainty reduction	Red set	Project id	Green set	Project id
0%	3	2,9,17	7	5,8,15,19,29,34,40
25%	3	2,9,17	7	5,8,15,19,29,34,40
50%	8	2,4,7,9,12,14,17,23	11	1,5,8,15,19,21,27,29,34,38,40
75%	8	2,4,7,9,12,14,17,23	14	1,5,8,15,19,21,27,28,29,30,34,35,38,40
100%	11	2,4,7,9,10,12,14,17,22,23,37	18	1,5,8,13,15,19,20,21,27,28,29,30,32,34,35,38,39,40

When all uncertainty is removed from grey projects (row of 100%) the unique optimal portfolio is still not reached because some uncertainty is related to the weights of criteria and the total budget remains. However, 18 green projects from the eventually 23 are identified. It should be remembered that once a project enters green or red sets in iteration *k* it remains there for all subsequent iterations, i.e., the green and red projects of iteration *k* are nested in respective sets of the consecutive iterations *k+1, k+2,* .... It must be noted that narrowing of uncertainty intervals refers only to grey projects of a specific iteration. For example, when the uncertainty is reduced from 50% to 75%, this reduction is not applied in the 8 red and 11 green projects of the second iteration but only for the remaining 21 grey projects. The concept is that on every iteration increased amount of information is obtained only for the currently grey projects in order to reduce their performance's variability. When the whole cycle of calculations is finished, the final portfolio turns out to be the same as with the two-phase approach. The main assistance here lies in the fact of gradual selection of projects, which is longer and covers more uncertain parameters then the previous one. Here, again, two projects are "close winners" and there is room for the expert to make or modify final portfolio according to the assigned task.

5.1.4. Conclusions for classic and two-phase ITA

An illustrative example from the literature was used to demonstrate and compare two approaches of the ITA method. One of the most useful advantages of the method is the additional information delivered to the DM and the direct control she/he has over the final solution (the disclosure of the borderline projects being a significant hint).

The two-phase approach may be considered as a short version of iterative ITA which suits better for relatively small set of project proposals. While the first part is the same for both approaches, the second part represents the majority principle where the variability of the results is reduced and the portfolio(s) of greater acceptance is(are) easily recognized. Robustness of results for the selection of telecommunications projects was additionally tested through different pseudo-random seeds of Monte Carlo simulation and there were no significant differences between them.

The iterative version has the advantage of gradual separation of projects between green and red sets giving information to the DM about the reliability projects' inclusion in the final portfolio or exclusion from it (according to the cycle that each project is included in the green or the red set). Such a procedure is more suitable for problems with large number of proposals seeking for support. Within the modeling procedure uncertain future outcomes may be modeled through different probability distributions. While in the current case study final portfolios from both approaches were the same, it would not be true in case of complicated probability distributions of several parameters. Still, two projects with similar characteristics leave some space for interpretation of results and possible review of some constraints. If closer pair wise comparison reveals increased importance of both of them, it is not prohibited to include them into the final selection. Still, it is easier to reconsider the future of one or two items instead of the whole portfolio.

5.2. Selecting a portfolio of CDM projects

In the last two centuries energy became one of the most critical resources for mankind's survival and development. Especially now, when the scarcity of fossil fuels and the impact of energy production and consumption to climate change were realized, the issue of energy is high in the global agenda. Energy projects are characterized by a variety of technologies and they are spread all around the world as they are related with indigenous sources. A special case of energy projects are those emerged recently in order to deal with the Climate Change issue. The international effort against the global phenomenon of global warming found its expression in early '90s with the establishment of the Intergovernmental Panel on Climate Change (IPCC) and United Nation Framework Convention for Climate Change (UNFCCC). Kyoto Protocol to the UNFCCC provided several options in order to reduce greenhouse gas (GHG) emissions. One of them was the Clean Development Mechanism (CDM) which gave the possibility to offset carbon emissions in the shape of environmentally friendly activities which turned out to be mostly energy related projects. Broadly speaking they are projects implemented in developing countries using technology and financing from developed countries. The benefit for the funders is that they get the "environmental" benefits quantified as Certified Emission Reduction units (CERs) in order to reduce their "emission balance". The case study presented further refers to this kind of projects and it is essentially a project portfolio selection problem.

The subject of specific case study refers to climate related projects which are mainly related to energy either from the supply side or from the side of energy efficiency. It is a growing domain of activities with many parties involved. Among main players are governments, who plan and introduce different climate friendly policies and address complex objectives of local development and employment as well as financial institutions and developers, searching for perspective ways for investments. In addition, private companies (both big and small) who care about public perception may also finance and support green activities. Even individual people interested in sustainable future, can buy carbon credits to offset their everyday GHG emissions.

Investors always face the problem of choice. Usually, the possibilities and options to invest money are greater than the available budget. One of the main tasks for a DM is to perform a balanced selection with consideration of technology, budget, policies, geographical distribution and other constraints that may be imposed by him/her. Moreover, the output of the projects is rarely known with certainty at the decision level (a priori). Therefore, in the current case the problem is stated as: which portfolio of climate related projects should be selected by an entity, given information about the total budget, policy and technical conditions that must be met as well as the inherent uncertainty in projects' output. The "universe" of available options is constituted from projects under the CDM and the relevant data are drawn from the CDM database.

Within the CDM projects' selection two techniques are combined, namely, Mathematical Programming and Monte Carlo simulation that helps to take into account numerous constraints and the inherent uncertainty associated with the projects' performance. The uncertainty is represented with probability distributions (a stochastic nature is assumed) as it is also assumed in other similar research works (Ladhelma et al., 1998; Tervonen and Ladhelma, 2007; Hyde et al., 2003). The problem is solved in iterative way using decision rounds. In each round a series of Monte Carlo simulations – IP optimizations is performed providing information about the membership of every project in resulting portfolios. This information is aggregated in order to separate projects into green, red and grey sets. From round to round the variation (measure of uncertainty) of grey projects is reduced so that the whole process converges to a final portfolio. The output of the process incorporates important information of certainty degree associated with every project which is included in the final portfolio.

5.2.1. Creating the "universe of projects" from CDM database

In the current case study a hypothetic set of projects, based on real data, is used. The main information source is CDM database, elaborated by UNEP Risoe Centre. Every activity, in order to be registered, submits a project design document (PDD) where its basic features are described and calculated. Subsequently, during their operation, registered projects are subject to performance monitoring and verification according to an adopted schedule.

The majority of CDM activities are renewable energy projects, which are represented by the following technologies:

- Wind energy,
- Hydro power plant (HPP),
- Biomass,
- Landfill gas,
- Methane avoidance,
- Energy efficiency in industry (EE).

As wind and hydro electricity generation are dominant technologies a great number of projects fall in this category. In order to refine the decision process further split of these projects into small scale (up to 15 MW of installed capacity) and large scale (more than 15 MW) ones is performed. Small scale projects are labeled with “S” at the beginning (SWind, SHydro), and large scale – “L” (LWind, LHydro). There is no need to create sub-groups for other technologies since the remaining projects are not that numerous.

Wind electricity generation is the largest set of projects and most installations are located in China and India. Technology success may be attributed to strong incentives that these hosting countries created during previous years (Pechak et al., 2011). Within hydro power generation projects there are ones that are focused on modernization of already existing, and those which started from zero (which in some cases means construction of a new dam). Hydro power plants bring together several issues, mainly environmental, both on local and international levels. In case of international rivers, active construction of dams and hydro power plants in one country may cause water shortages during dry seasons or other related problems in the countries, which are subsequent in the river flow. This is a complicated issue especially in South – East Asia (WWDR4, 2012). Biomass covers many sub technologies, mainly related to agricultural wastes of different kinds. Most of these projects are small scale and possess strong environmental potential, which makes them similar to power generation from landfill gas and methane avoidance on waste water treatment facilities. The objective of landfill gas projects is to install a highly efficient collection system to capture and destroy methane by flaring at high temperatures and use the generated heat for the needs of communities. Generally, the avoidance and reduction of methane emissions is very important not only from the public health point view. Methane is characterized by the global warming potential (GWP) 21 times greater of CO_2 and on the planetary scale makes a considerable input to the overall greenhouse effect. The biggest variety is found within the energy efficiency (EE) projects for own electricity generation from waste heat on such industrial facilities as cement plants, iron and steel production, non-ferrous metal production and others.

Geographical distribution covers 17 countries: Argentina, Brazil, Chile, China, Ecuador, Egypt, Honduras, India, Indonesia, Malaysia, Mexico, Peru, Philippines, South Africa, South Korea, Thailand, and Vietnam. According to the Kyoto Protocol classification, all these countries are considered to be developing. But each of them has many specific characteristics which should be taken into account before the selection process starts. For instance, the state support for wind energy projects led China to become a major player in this field and within few years it helped to develop a new industry from scratch. On the other side, for many other developing countries, last technology developments are still not accessible due to lack of financial resources and knowledge. Without technology transfers, they may follow the historic polluting trends of industrialised countries. Instead, CDM demonstrates an effective way to move quickly to environmentally sound and sustainable practices, institutions and technologies (Karakosta et al., 2010).

Within evaluation, strong emphasis is put on environmental performance. Actually, sustainability compound was supposed to be very strong on the stage of CDM development. But reality turned out to be not as "green" as expected. These criteria were very vague and led to strong critics of CDM. As a result, external companies began to perform sustainability check of the projects, both existing and under development. That is how demand for premium CERs occurred and the best known is Gold Standard (GS) labelling. It certifies renewable energy and energy efficiency carbon offset projects to ensure that they all demonstrate real and permanent greenhouse gas reductions and sustainable development benefits in local communities that are measured, reported and verified.

Table 5-7. Input data for CDM projects by countries and technologies.

	SWind	Lwind	SHydro	LHydro	Biomass	Landfill gas	Methane avoidance	EE own generation	*GS*	*Budget MUS$*	*kCERs/ year*	**Total projects**
China	5	53	21	27	2	6	4	10	*40*	*6733*	*2588*	**128**
India	36	4	10	5	15	1	2	6	*10*	*979*	*17050*	**79**
Argentina	0	0	0	0	1	1	0	0	*0*	*42*	*305*	**2**
Brazil	0	1	4	4	0	2	0	1	*0*	*541*	*885*	**12**
Chile	0	1	2	3	2	0	0	0	*1*	*490*	*1346*	**8**
Ecuador	1	0	0	2	0	0	0	0	*0*	*62*	*210*	**3**
Egypt	0	1	0	0	0	0	0	1	*0*	*135*	*359*	**2**
Honduras	0	0	1	0	0	0	1	0	*1*	*10*	*54*	**2**
Indonesia	0	0	0	0	2	1	3	0	*3*	*52*	*361*	**6**
Malaysia	0	0	0	0	5	1	4	0	*0*	*44*	*686*	**10**
Mexico	0	4	0	1	0	3	1	0	*0*	*1396*	*2101*	**9**
Peru	0	0	3	3	0	0	0	0	*0*	*360*	*879*	**6**
Philippines	0	1	0	0	1	0	1	1	*0*	*104*	*191*	**4**
South Africa	0	0	0	0	1	2	1	0	*0*	*30*	*133*	**4**
South Korea	1	1	2	1	0	0	0	0	*0*	*243*	*501*	**5**
Thailand	0	0	0	0	2	1	10	1	*6*	*161*	*958*	**14**
Vietnam	0	1	2	3	0	0	0	0	*2*	*119*	*198*	**6**
Gold Standard	*3*	*33*	*2*	*2*	*8*	*2*	*12*	*1*	*63*			
Budget MUS$	*436*	*6861*	*400*	*2555*	*389*	*165*	*105*	*593*	2846			
kCERs/year	639	11059	1257	6898	1794	3075	1439	2644	6242			
Totals	**43**	**67**	**45**	**49**	**31**	**18**	**27**	**20**	**63**	**11501**	**28805**	**300**

As it was mentioned before, the candidate projects are taken from UNEP Risoe Centre database. Only registered projects are under consideration as they have more rich information. A summary of the input data is presented in **Table 5-7**. 300 representative projects with specific technology and geographical characteristics in order to illustrate ITA method were taken as input. Solar, geothermal, tidal and several other types of energy efficiency projects are excluded from selection due to the lack of initial information (e.g. no investment costs).

The portfolio selection has a strong emphasis on environmental performance with respect to current situation on CDM map. Since already existing projects are the input data, for environmental criteria the GS labelling is used. In the model, availability of GS certification is represented by “1”, and “0” if not.

Within the model, projects were coded according to technology, i.e. Small scale wind: 1-43, Large scale wind: 44-110, Small scale hydro: 111-155, Large scale hydro: 156-204, Biomass: 205-235, EE own generation: 236-255, Landfill gas: 256-273, Methane avoidance: 274-300.

5.2.2. The model for CDM project selection

If not the most significant, one of the most critical criteria in specific decision situation is the amount of issued CERs. When a project is submitted the expected amount of CERs is declared. However, past experience from previous projects shows that declared amount usually differs from delivered CERs after implementation of the activity. An attempt to quantify this uncertainty by examining earlier projects' issuance success according to their technology was made. The issuance success was defined as the ratio between initially expected and actual CERs and it is calculated in the CDM database for projects that have one or more years of implementation. Since projects may vary by their duration, 10 years or 7 years (renewable) crediting period, it was feasible to consider the annual amount of CERs as a common basis. With consideration of available historical data, **Table 5-8** presents the levels of CERs issuance in comparison with expected amounts from PDDs.

Table 5-8. Distribution characteristics of CERs issuance success.

	Total projects	Average level of issuance success (*avis*)	Standard deviation of issuance success (*sdis*)
Wind	370	89%	24%
Hydro	465	85%	39%
Biomass	174	84%	35%
EE own generation	97	77%	25%
Landfill	90	52%	36%
Methane avoidance	122	61%	38%

In current model actual CERs of the portfolio constitute the objective function for maximization. Given the uncertainty characterizing issuance success of each project according to its technology, these values are drawn from the corresponding normal distributions with characteristics from **Table 5-8**. Therefore, objective function coefficients are random parameters sampled from the normal distribution with following characteristics:

$c^{(t)}_i = expcer_i \times \text{normal}(avis_j, sdis_j)$ (5.2)

where $c^{(t)}_i$ is the objective function coefficient declaring actual CERs for the *i*-th project according to the *t*-th sampling, $expcer_i$ is expected CERs declared during submission of the project, $avis_j$ is average issuance success for technology *j* that characterizes project *i* and $sdis_j$ is standard deviation of the issuance success of technology *j*. The two latter parameters are taken from **Table 5-8**. The second term of the product indicates that the parameter is sampled from a normal distribution with specific characteristics. Therefore, the objective function of the problem is following and is based on (4.3):

$$\max Z^{(t)} = \sum_{i=1}^{P} c_i^{(t)} X_i \quad (5.3)$$

where $Z^{(t)}$ is the total number of kCERs achived by the portfolio $P^{(t)}$ in the iteration *t* of the Monte Carlo simulation, $c_i^{(t)}$ is the number of kCERs from the *i*-th project as it is sampled in the *t*-th iteration and X_i is a binary variable indicating if the *i*-th project is included (X_i=1) or excluded (X_i=0) from the optimal portfolio.

Constraints of the problem express policy limitations imposed by the decision maker. They have to do with the desired technology mixture as well as the geographical distribution of the projects in final portfolio. In present case the imposed constraints are:

(a) Budget constraint

The total investment budget for the selected projects must be less than 2 billion US$ (all 300 projects accumulate to 11.5 billion US$)

$$\sum_{i=1}^{P} budg_i X_i \leq 2000 \quad (5.4)$$

where $budg_i$ is the budget of the *i*-th project in million US$

(b) Geographical distribution

Certain conditions about the geographical distribution of projects are incorporated in the model as it is usually the case in real investment problems. The following conditions are some examples just to illustrate modeling capabilities.

b1) At most 40% of allocated funds should be in projects in China

$$\sum_{i \in China} budg_i X_i \leq 0.4 \sum_{i=1}^{P} budg_i X_i \quad (5.5)$$

b2) At most 30% of allocated funds should be in projects in India

$$\sum_{i \in India} budg_i X_i \leq 0.3 \sum_{i=1}^{P} budg_i X_i \tag{5.6}$$

b3) At least 30% of the selected projects must be located in Latin America

$$\sum_{i \in LatAm} X_i \geq 0.3 \sum_{i=1}^{P} X_i \tag{5.7}$$

(c) Technology mix

There are conditions that can be imposed to affect technology mix of the final portfolio. This is often required in order to obtain a more or less balanced portfolio avoiding the “all eggs in one basket” policy. After several initial trial and error runs of the spontaneous model (without technology mix constraints) and it becomes obvious that a minimum or a maximum degree of representation of each technology in the final portfolio should be maintained. In the current model these additional constraints are:

c1) At least 40% of allocated funds should be in wind power installations (small and large scale)

$$\sum_{i \in Wind} budg_i X_i \geq 0.4 \sum_{i=1}^{P} budg_i X_i \tag{5.8}$$

c2) At least 30% of allocated funds should go to hydro power installations (small and large scale)

$$\sum_{i \in Hydro} budg_i X_i \geq 0.3 \sum_{i=1}^{P} budg_i X_i \tag{5.9}$$

c3) Remaining four technologies should not have (separately) more than 10% of the allocated funds

$$\sum_{i \in Biomass} budg_i X_i \leq 0.1 \sum_{i=1}^{P} budg_i X_i \tag{5.10}$$

$$\sum_{i \in EEff} budg_i X_i \leq 0.1 \sum_{i=1}^{P} budg_i X_i \tag{5.11}$$

$$\sum_{i \in Landfill} budg_i X_i \leq 0.1 \sum_{i=1}^{P} budg_i X_i \tag{5.12}$$

$$\sum_{i \in MethAv} budg_i X_i \leq 0.1 \sum_{i=1}^{P} budg_i X_i \tag{5.13}$$

c4) The Gold Standard projects should be at least 30% of total projects in the final portfolio

$$\sum_{i \in GoldStd} X_i \geq 0.3 \sum_{i=1}^{P} X_i \tag{5.14}$$

The before mentioned constraints are examples of limitations that in a real case any decision maker may face. In case of need of even more constraints, such as mutually exclusive, precedent projects and other logical conditions can be incorporated into the model. Moreover, if annual cash flows are available, constraints on annual expenses can also be incorporated. In general, the modeling with Integer Programming in project portfolio selection is very flexible.

5.2.3. Results and discussion for classic ITA

The ITA method was applied to specific problem in a following way: Five rounds of the iterative process were defined a priori (denoting with "0" the initial round, hence *R*=4). From round to round the grey projects' performance was sampled from a corridor of corresponding issuance success' distributions. Particularly, the standard deviation of respective probability distribution was reduced by 25% in each subsequent round as shown in **Figure 5-1**. Consequently, in the final round the standard deviation of grey projects was considered to be zero so that for them deterministic values of issuance success were assumed.

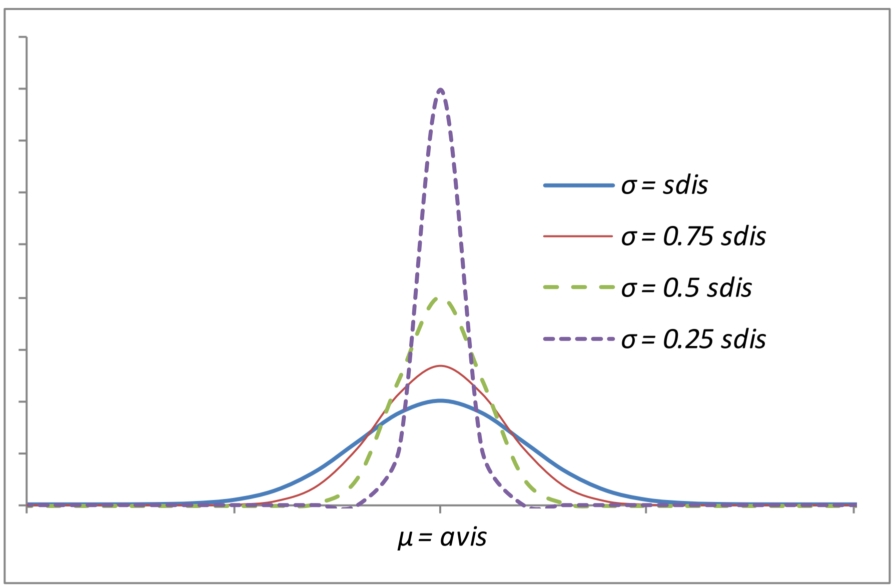

Figure 5-1. Variance reduction from round to round for the grey projects' probability distribution.

The model and the whole solution process were developed in the General Algebraic Modeling System (GAMS, see e.g. Brooke et al., 1988) using the MIP solver CPLEX 11.1 for optimizing the Mixed Integer Programming models. The number of iterations in Monte Carlo simulation was set to 1000. The solution time varied from 17 – 20 minutes across the five rounds in an Intel Pentium i5 at 2.53 GHz, which made the whole decision process not computationally prohibitive.

The membership threshold was set to 99% for the green set and 1% for the red set. This meant that projects that appeared in the final portfolio more than 990 times over the 1000 iterations were considered to be green projects, while those projects appearing less than 10 times in total were discarded.

Initially, the simulation optimization process was run with consideration of full uncertainty of projects' issuance success (σ = *sdis*). Specifically, for calculation of every objective function coefficient c_i the equation (4.3) used normal distributions' sampling from **Table 5-8**. Surprisingly, from the 1000 portfolios initially obtained none of them were the same. Therefore, no conclusions about a dominant portfolio could be extracted from the first round. The number of projects in portfolios varied from 70 to 103 across these iterations. Eventually, 10 projects were classified as green, 77 as red and the remaining 213 as grey.

In the second iteration, according to the equation (4.4), values of green projects' decision variables were fixed to be to 1 and those of the red projects to 0. The standard deviation of grey projects was reduced to 0.75 × *sdis* while for green and red projects it was left in the previous round's level. The output of the second round was 16 green, 100 red and 184 grey projects.

In the third round, the values of green projects' decision variables from previous round were set as 1 and those of red projects as 0 in the model. The standard deviation of grey projects was reduced to 0.5 × *sdis*. The output of the third round was 27 green projects, 117 red projects and 156 grey projects.

The output of the fourth round was 49 green projects, 151 red projects and the remaining 100 were grey for which the standard deviation was set to be 0.25 × *sdis*.

In the fifth and final round the standard deviation of remaining 100 grey projects was set to zero which meant their issuance success was considered as deterministic value taking the average value from **Table 5-8**. Then, all grey projects were fully allocated between green (51) and red sets (49). Conclusively, the whole process ended with 100 green and 200 red projects. In the final round the obtained CERs calculated from the final portfolio varied from 7089 to 8164 with a mean value of 7597 and a standard deviation of 190. The ID of projects as well as the decision round of their incorporation (for the green set) or their exclusion (for the red set) from the final portfolio is illustrated graphically in **Figure 5-2**. The darker the shading of a cell is, the earlier round it enters green or red sets, i.e., the sooner a conclusion about project's status ("go" or "no go") in the decision process is made. In other words, darker cells illustrate higher level of confidence about their inclusion (green set) or their exclusion (red set) from the final portfolio. Therefore, every project is accompanied not only with "go" or "no go" information, but also with the degree of certainty about this decision. It is a certain way to prioritize projects and is very useful for decision makers in the presence of underlying uncertainty on projects' performance. The analysis of the final portfolio is presented in **Table 5-9**.

13	23	30	36	40	48	56	61	92	94	113	115	116	124	127	128	130	136	137	138
144	154	155	156	158	161	165	167	168	170	177	179	180	182	188	192	196	199	204	206
208	210	211	214	215	216	219	221	222	224	227	228	229	231	233	234	235	236	237	238
239	244	245	247	250	252	256	257	261	262	263	264	265	266	267	269	270	271	272	273
275	276	277	278	282	284	285	286	288	289	290	291	292	293	294	296	297	298	299	300

(a) The green set

1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	17	18	19	20	21
22	24	25	26	27	28	29	31	32	33	34	35	37	38	39	41	42	43	44	45
46	47	49	50	51	52	53	54	55	57	58	59	60	62	63	64	65	66	67	68
69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88
89	90	91	93	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110
111	112	114	117	118	119	120	121	122	123	125	126	129	131	132	133	134	135	139	140
141	142	143	145	146	147	148	149	150	151	152	153	157	159	160	162	163	164	166	169
171	172	173	174	175	176	178	181	183	184	185	186	187	189	190	191	193	194	195	197
198	200	201	202	203	205	207	209	212	213	217	218	220	223	225	226	230	232	240	241
242	243	246	248	249	251	253	254	255	258	259	260	268	274	279	280	281	283	287	295

(b) The red set

Figure 5-2. Final green and red sets along with certainty degree for each project.

It is noteworthy to mention that a naïve approach of dealing with uncertainty is to use just the average (expected) values of issuance success and maximize the average CERs of the final portfolio, ignoring the variance associated with projects' performance. In this case, the final portfolio that is calculated from a single run (solution of an IP problem) is the same as in ITA approach. However, there is no information about performance variations of the final portfolio, as well as there is no supportive evidence about the degree of certainty for each project. In addition, if probability distributions were not symmetric the result of the two approaches may differ which means different final portfolios.

Table 5-9. Final selection by countries and technologies.

	SWind	Lwind	SHydro	LHydro	Biomass	Landfill gas	Methane avoidance	EE own generation	*GS*	*Budget MUS$*	*kCERs/ year*	**Total projects**
China	0	2	2	7	1	6	3	2	*8*	*799*	*3828*	**23**
India	4	1	1	0	10	0	1	5	*10*	*204*	*1063*	**22**
Argentina	0	0	0	0	1	1	0	0	*0*	*42*	*305*	**2**
Brazil	0	0	3	2	0	2	0	1	*0*	*106*	*1121*	**8**
Chile	0	0	2	2	1	0	0	0	*0*	*119*	*278*	**5**
Ecuador	1	0	0	2	0	0	0	0	*0*	*62*	*210*	**3**
Egypt	0	1	0	0	0	0	0	1	*0*	*135*	*359*	**2**
Honduras	0	0	1	0	0	0	1	0	*1*	*10*	*54*	**2**
Indonesia	0	0	0	0	1	0	3	0	*3*	*17*	*192*	**4**
Malaysia	0	0	0	0	3	0	4	0	*0*	*29*	*591*	**7**
Mexico	0	1	0	0	0	3	1	0	*0*	*220*	*751*	**5**
Peru	0	0	3	2	0	0	0	0	*0*	*182*	*485*	**5**
Philippines	0	0	0	0	0	0	0	0	*0*	*0*	*0*	**0**
South Africa	0	0	0	0	1	1	0	0	*0*	*16.5*	*399*	**2**
South Korea	0	0	0	0	0	0	0	0	*0*	*0*	*0*	**0**
Thailand	0	0	0	0	0	1	7	0	*6*	*33*	*638*	**8**
Vietnam	0	0	1	1	0	0	0	0	*2*	*24*	*43*	**2**
Gold Standard	*3*	*1*	*2*	*2*	*7*	*2*	*12*	*1*	*30*			
Budget MUS$	*33*	*767*	*111*	*595*	*163*	*88*	*68*	*173*	*241*			
kCERs/year	*54*	*1634*	*389*	*2255*	*1108*	*2713*	*1181*	*983*	*1287*			
Total	**5**	**5**	**13**	**16**	**18**	**14**	**20**	**9**	**30**	**1998.5**	**10317**	**100**

The geographical distribution is determined more or less by imposed constraints. It is easy to observe the fact that there are still countries that are not present in the final selection (Philippines and South Korea) as it is not explicitly required by the regional constraints. Moreover, it was found that projects from Latin America were entering the final portfolio from the first rounds. On the contrary, the majority of wind and hydro projects from China and India are excluded very early in the decision process. According to another requirement all available technologies are present in the mix of final portfolio. Because of restricted budget (2 billion US$), most of wind projects are excluded due to high initial investment costs. Thus, the share of Chinese projects dropped significantly although there were some projects with Gold Standard label among them. It was also observed that conditions for the HPPs were more favorable than those of the wind projects. In addition, the availability of already existing dam had a positive effect as it corresponded to lower investment cost.

Generally, consideration of minimal share of Gold Standard projects has a positive influence. In the final portfolio there are 30% of premium labeled projects while initially, in project universe, they had the share of 21%. The proportion of GS projects may be controlled by the decision maker through implied constraints. In the current case, all GS labeled projects for HPPs, Landfill gas, Methane avoidance and EE in industry are in the final selection.

It is not a surprise that the share of methane related projects is significant in the final portfolio (about 1/3). With modest investments they provide more emission reductions and thus CERs. One of the reasons is the higher Global Worming Potential (GWP) of methane towards CO_2. Secondly, these projects provide more of direct sustainability benefits such as improved air and water quality, and reduction of dangerous wastes within local communities.

Eventually, the final portfolio represents 17.4% of the investments in comparison with initial 11.5 billion US$ of 300 projects while it accounts for 35.8% of the project universe's total CERs (=28805 kCERs). In the current case study the aim was to maximize carbon credits, even though their final amount is not a certain fixed number. The final portfolio demonstrated how it is possible to make a balanced selection regarding financial as well as technological and geographical constraints. In this example the modeling of uncertainty in the most uncertain among project's parameters (CERs) was tested. Contrary to what was expected, the dominant technologies (wind and hydro) in the available project universe were not so favorable in the final portfolio, probably due to their increased investment cost. Because of the limited available total budget, lower investment cost projects were preferred even from the early rounds of the selection process.

5.3. RES projects in Hellas

The capability of reliable provision of energy to meet a vast range of needs and requirements in residential, services/commerce, agriculture, industrial and transportation sectors, is one of the most distinctive features of modern developed societies. From supplying power and heat to production systems to satisfying heating, cooling, lighting, and mobility needs, energy is pervasive in everyday life (Antunes and Henriques 2016).

The geographical position of Greece is extremely favorable for the operation of renewable energy installations. With more than 250 days of sunshine it is no surprise to have an excessive amount of proposals for photovoltaic power plants. While solar collectors are already a widespread technology for hot water supply in households, PVs are only gaining popularity. Currently, state support for new energy technologies is also of crucial importance since it is still cheaper to obtain electricity from fossil fuels. The same stands for wind installations too.

The basis of case study is 133 Greek project proposals covering three RES technologies (wind, small hydro, photovoltaic). These applications were evaluated against 5 criteria, namely: regional development, employment, economic performance (expressed with IRR), CO_2 emission reduction and land use. The data for this problem are available in Makryvelios (2011).

In this subchapter ITA is used for a case study with multiple decision makers. The preference of every expert is expressed by assigning their own weights of importance to the evaluation criteria. Hence, each decision maker has his/her own optimal portfolio of projects. Group ITA is designed to gradually add projects to the portfolio according to the concordance within the team members until a final portfolio is reached. A great advantage of Group ITA is that it also provides a measure of consensus for the final portfolio of projects (Consensus Index) as well as concordance indices for each project that is either selected or rejected.

5.3.1. Description of RES projects' proposals

In order to start elaboration of proposals, it is necessary to perform their evaluation. For current example, the MCDA method used for multi-criteria project evaluation is the value function method (von Winderfeldt and Edwards, 1986). The partial value function for each criterion has the following form:

$$y_{ik} = \frac{1-e^{c_k \cdot x_{ik}}}{1-e^{c_k}} \tag{5.15}$$

where y_{ik} is a score of i-th alternative in k-th criterion, x_{ik} is a linear score normalized to [0,1] of i-th alternative in k-th criterion and c_k is the value function coefficient for criterion k. Value function coefficients (c_k) are defined according to the dispersion of alternatives' performances by criteria. Specifically, an accumulation of performances in the upper half of criterion range indicates a convex value function while an accumulation in the lower half leads to a concave value function. In this way the discriminating ability of criteria is enhanced. In the present case the following values for value function coefficients are defined: c_1=0.001 (linear), c_2 = -1 (concave), c_3 =-3 (concave), c_4 =-5 (concave), c_5 =5 (convex).

The x_{ik} are obtained as follows:

(a) For maximization criteria:

$$x_{ik} = \frac{v_{ik} - v_k^{MIN}}{v_k^{MAX} - v_k^{MIN}} \tag{5.16}$$

(b) For minimization criteria:

$$x_{ik} = \frac{v_k^{MAX} - v_{ik}}{v_k^{MAX} - v_k^{MIN}} \quad (5.17)$$

where v_{ik} is raw value of the *i*-th alternative in the *k*-th criterion, v_k^{MAX} and v_k^{MIN} the maximum and minimum values across *k* criteria. Multi-criteria scores (ms_i) for each alternative are calculated using an additive value function:

$$ms_i = \sum_{k=1}^{5} y_{ik} \times w_k \quad (5.18)$$

In the present case 12 decision makers from different positions are assumed, with diverse points of view that provided weights of importance to the **Table 5-10**. These are actually initial weights $w_{pk}^{(0)}$.

Table 5-10. Importance weights for 12 decision makers.

	Criteria				
DM	Regional development	CO_2 emissions reduction	Economic performance (IRR)	Employment positions	Land use
1	0.14	0.13	0.46	0.13	0.14
2	0.25	0.37	0.15	0.08	0.15
3	0.41	0.21	0.03	0.14	0.21
4	0.07	0.41	0.35	0.16	0.01
5	0.02	0.02	0.50	0.33	0.13
6	0.20	0.20	0.20	0.20	0.20
7	0.15	0.25	0.40	0.02	0.18
8	0.08	0.28	0.35	0.17	0.12
9	0.22	0.25	0.28	0.17	0.08
10	0.15	0.35	0.25	0.20	0.05
11	0.21	0.30	0.15	0.15	0.19
12	0.20	0.20	0.30	0.25	0.05
Average	0.1750	0.2475	0.2850	0.1667	0.1258

The detailed classification of project proposals by technology and geographical distribution is presented below.

Table 5-11. Geographical and technological distributions of projects.

	Wind	Small hydro	PV	**Total**
EASTERN MACEDONIA-THRACE (EMD)	3		2	**5**
ATTICA (ATT)		1		**1**
NORTHERN AEGEAN (NAG)			6	**6**
WESTERN GREECE (WGR)			1	**1**
WESTERN MACEDONIA (WMD)	3		6	**9**
EPIRUS (EPR)		3	8	**11**
THESSALY (THE)	1	7	9	**17**
IONIAN ISLANDS (ION)	1			**1**
CENTRAL MACEDONIA (CMD)	3	5	6	**14**
CRETE (CRE)			4	**4**
SOUTHERN AEGEAN (SAG)	1			**1**
PELOPPONESE (PEL)	8	1	3	**12**
CENTRAL GREECE (STE)	33	13	5	**51**
Total	**53**	**30**	**50**	**133**

5.3.2. The model for Hellenic RES project selection

For solving the problem of project portfolio selection an IP model is developed with consideration of specific constraints that need to be satisfied. Technological and geographical distributions of proposals are shown in **Table 5-11**. In addition, it is necessary to meet such specific policy constraints as:

- Available budget of 150 M€ (the total cost of the 133 projects is 659 M€);
- Cost of projects in Central Greece should be less than 30% of the total cost;
- Cost of projects in Peloponnese should be less than 15% of the total cost;
- Cost of projects in East & West Macedonia, Northern & Southern Aegean, Epirus should be greater than 10% of the total cost;
- Number of projects by technology should be between 20% and 60% of selected projects;
- Total capacity of selected projects should be greater than 300 MW.

The decision variables of the corresponding IP model are binary and indicate acceptance (X_i=1) or rejection (X_i =0) of the *i*-th project in the final portfolio. The full model is:

$$
\begin{aligned}
& \max\ Z = \sum_{i=1}^{133} ms_i\, X_i \\
& st \\
& \sum_{i=1}^{133} \text{cost}_i X_i = C \\
& C \le 150 \\
& \sum\nolimits_{i \in STE} \text{cost}_i X_i \le 0.3 \times C \\
& \sum\nolimits_{i \in PEL} \text{cost}_i X_i \le 0.15 \times C \\
& \sum\nolimits_{i \in EMD, NAG, WMD, EPR, SAG} \text{cost}_i X_i \ge 0.1 \times C \\
& 0.2 \times \sum_{i=1}^{133} X_i \le \sum_{i \in W} X_i \le 0.6 \times \sum_{i=1}^{133} X_i \\
& 0.2 \times \sum_{i=1}^{133} X_i \le \sum_{i \in SH} X_i \le 0.6 \times \sum_{i=1}^{133} X_i \\
& 0.2 \times \sum_{i=1}^{133} X_i \le \sum_{i \in PV} X_i \le 0.6 \times \sum_{i=1}^{133} X_i \\
& \sum_{i=1}^{133} mw_i X_i \ge 300
\end{aligned}
\qquad (5.19)
$$

where C is the total cost of the portfolio, $cost_i$ is the cost of project i (in M€), mw_i is the installed capacity (in MW) and ms_i is the multi-criteria score of project i. The resulting model is an IP problem with 133 integer decision variables and 11 constraints.

For the problem Group-ITA method is applied with R=10 rounds (meaning that the convergence parameter α = 0.1). Required models and whole solution process is developed in General Algebraic Modeling System (GAMS, see e.g. Brooke et al., 1998) using the MIP solver CPLEX 11.1 for optimizing Integer Programming models. The solution time was a few seconds for each model in a core i5 64bit at 2.5 GHz.

5.3.3. Results and conclusions for Group ITA

Results obtained from round to round are depicted in **Figure 5-3** where color intensity expresses consensus degree on each project. The dark green projects were selected in early rounds and it means that there has been increased consensus for their selection. On the other side, the dark red projects were rejected in early rounds signifying increased consensus for their rejection. It is easy to observe that from round 0 to round 3 there are no additions in the green or red sets. The same is true also for rounds 4 and 5, 6 and 7, 8 and 9. That is why rounds 1, 2, 3, 5, 7 and 9 do not appear in **Figure 5-3**. It is noteworthy to highlight the fact of 73+40=113 projects with *CD*=1; 5+5=10 projects with *CD*=0 and the remaining 10 in between projects.

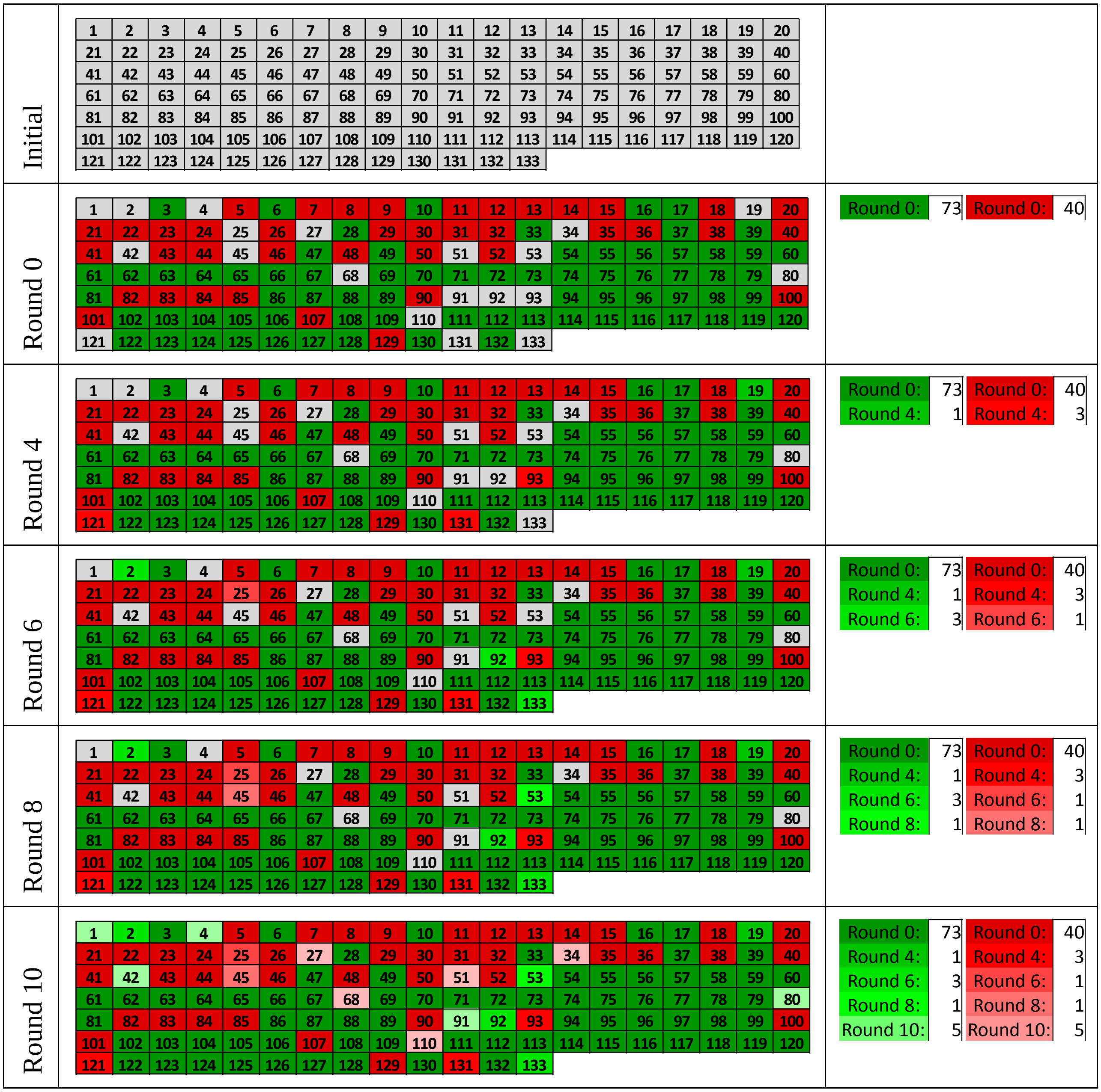

Figure 5-3. Results of iterative process for Hellenic RES projects.

Particular characteristics of the portfolio created by green projects in each round (consensus portfolio) are shown in **Table 5-12**. The violations of constraints are denoted with red, bold fonts. By studying **Table 5-12** decision makers may decide to select a consensus portfolio prematurely, i.e., before arriving to Round 10. This can be done having in mind that they accept the respective violations of constraints.

Table 5-12. Characteristics of consensus portfolio (green projects only).

	projects	MW (>=300)	Cost (<=150 M€)	STE (<=30%)	PEL (<=15%)	Other (>=10%)	W (20%-60%)	SH (20%-60%)	PV (20%-60%)
round 0	73	**185.5**	96.5	25.5%	3.2%	22.5%	**15.1%**	35.6%	49.3%
round 4	74	**202.6**	102.9	23.9%	9.2%	21.1%	**16.2%**	35.1%	48.6%
round 6	77	**222.6**	114.7	25.6%	8.2%	20.8%	**16.9%**	33.8%	49.4%
round 8	78	**235.2**	119.5	28.6%	7.9%	20.0%	**17.9%**	33.3%	48.7%
round 10	83	301.3	149.8	29.4%	10.3%	25.6%	20.5%	32.5%	47.0%

The consensus chart of the problem is depicted in **Figure 5-4** and is calculated using equation(4.12):

$$CI = [\frac{0.88}{2} + 3\times 0.88 + 2\times 0.892 + 2\times 0.928 + 2\times 0.94 + \frac{1}{2}]/10 = 91\%$$

While the same final portfolio is obtained with average weights from only one run, this naïve approach misses all information regarding the consensus degree for each project as well as the consensus index for the final portfolio.

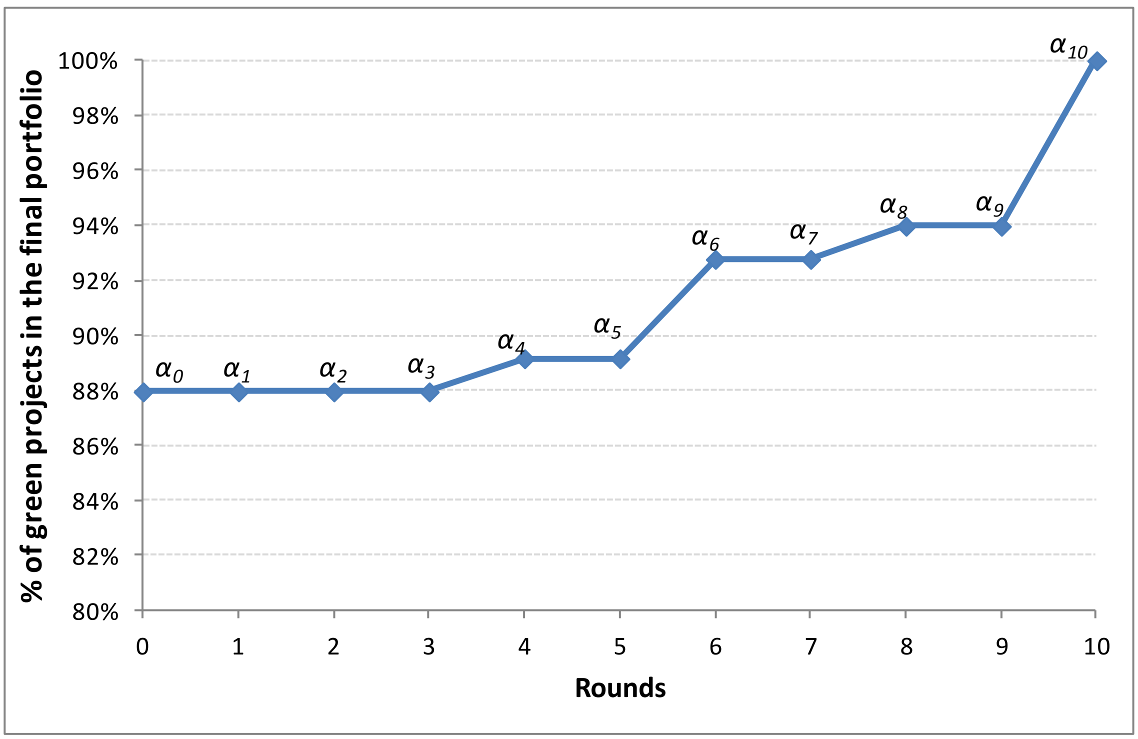

Figure 5-4. Consensus chart for RES project portfolio.

In other words, the current approach presents a systematic procedure towards convergence. The main advantage of proposed Group-ITA method is that not only helps to build the final portfolio, but also measures the degree of consensus over each project that is selected or rejected. Moreover, it provides a measure of consensus for the final portfolio. The outcome of Group-ITA is not merely the final portfolio, but also the “course” towards it that may provide fruitful information about project selection problem and may be used to reconsider some initial assumptions.

5.4. Incorporation of Energy and Environmental Corporate Responsibility into decision making procedure

One of the major reasons for economic crises is the irrational distribution and use of resources. This problem is one of the most common and oldest problems in Operations Research (OR). Financial organizations often face the issue of selection within a set of project proposals to fund. As a rule, several OR techniques are involved in this kind of problems such as Multiple Criteria Decision Analysis (MCDA) and Mathematical Programming (MP). These techniques are widely exploited in relevant decision problems, such as the portfolio selection, choice among alternative projects or investment opportunities, student selection, military applications, capacity expansion (see e.g. Golabi et al. 1981; Mavrotas & Rozakis 2009; Salo et al. 2011; Martinez-Costa et al., 2014). Usually the “best” performance is expressed emphasizing on economic and financial criteria. Other criteria related with the promotion of sustainable practices, fostering green growth, were not taken into consideration in traditional models (Hobbs and Meier 2000).

However, current financial and economic crisis, as well as growing socio-economic and environmental pressures, including climate change, put seriously under question traditional development patterns. The need to develop alternative models able to address current economic situation through the exploitation of sustainable patterns is of crucial importance (Hobbs and Meier 2000; Doukas et al. 2012). One of the most prominent examples comes from Oliveira and Antunes (2011), who developed a multi-objective model for interactions between economy, energy and environment for Portugal. The multi-sectoral model performs a prospective analysis of changes in the economic structure and the energy system, as well as assesses the corresponding environmental impacts, providing decision support in policy making. This model is a multi-objective linear programming model that allows for the explicit consideration of distinct axes of evaluation, generally conflicting and non-commensurate, of the merit of distinct policies. The policy recommendations obtained are subject to the inherent uncertainty associated with the model coefficients and, therefore, they may not be robust in face of changes of the input data.

Companies are at the heart of the Europe 2020 Strategy, taking into consideration their vital role towards national prosperity and Sustainable Development (SD). They have to integrate social and environmental concerns in their business operations and in their interaction with stakeholders on a voluntary basis, within the framework of the Corporate Social Responsibility (CSR) concept.

Enterprises with vision have to address problems in a long term plan, and become a driving force for adoption of relative initiatives towards “green” development and promotion of energy efficiency and environmentally friendly practices, within the CSR framework (Doukas et al. 2013). CSR has been incorporated recently in decision models using Data Envelopment Analysis (Lee & Farzipoor Saen, 2012), inventory policy (Barcos et al. 2013) and supply chain (Hsueh, 2014) among others. The interweaving of energy and environmental policies, as an aspect of CSR is definitely small and CSR does not appear to be a systematic activity in new conditions of European market, a conclusion further confirmed by Apostolakou and Jackson (2009) and Gjølberg (2009 a, b) studies. However, relevant works in various fields have been detected recently such as in supplier selection (Hashemi et al., 2014). In this context, new tools and methods are required to foster green entrepreneurship and green energy growth.

The innovation of the current study is the incorporation of Energy and Environmental Corporate Responsibility (EECR) in decision making, supporting particularly the development of a new model for investment evaluation. This model can assist financial institutions (with green loans applications) and governmental bodies funding energy - environmental friendly investments. The EECR performance of a firm is considered as an evaluation criterion of the submitted project. Therefore, in the current study the drivers of optimization are two objective functions: (1) The Net Present Value (NPV) representing the economic dimension that characterizes each project, and (2) the EECR index for the corporate social responsibility that characterizes each firm that submits the project. In this way, businesses with increased EECR are rewarded without ignoring the economic performance of relevant projects.

The resulting multi-objective model (specifically bi-objective) does not provide an optimal portfolio but a set of Pareto optimal portfolios among which the most preferred one is selected by the decision maker. In general, multi-objective optimization increases degrees of freedom within decision making process providing not an optimal solution (as in single objective optimization) but a set of candidate solutions among which the decision maker chooses. Therefore, the set of Pareto optimal solutions (Pareto set) is essential information in an integrated decision making approach. Worth to remember that Multi-Objective Integer Programming (MOIP) models help to produce the exact Pareto set (i.e., all the Pareto optimal solutions). Moreover, especially in the last years, the multi-objective character of project portfolio selection is addressed with multi-objective metaheuristic methods that produce an approximation of the Pareto set (see e.g. Yu et al. 2012; Tavana et al. 2013; Hassanzadeh et al. 2014a).

The current case study goes one step further, considering also the uncertainty characterizing basic parameters of the model, which are the coefficients of objective functions, namely NPV of each project and

EECR score of each firm. Given that these values are actually estimations, a systematic approach to deal with the inherent uncertainty is adopted. The latter is considered to be of stochastic nature, where a probability distribution is used instead of a crisp number for the values of objective functions' coefficients. It must be noted that a similar approach for project selection problems with multiple criteria that deals with stochastic uncertainty in projects' evaluation is Stochastic Multiobjective Acceptability Analysis (SMAA) introduced by Lahdelma et al. (1998). However, SMAA cannot handle the case of multiple constraints that are imposed to the constraints but is applied only with independent alternatives in an MCDM context.

Further in the subchapter an innovative approach that deals with parameters' uncertainty in a MOIP model and eventually converges to the final Pareto set is introduced. It uses the main idea of the Iterative Trichotomic Approach (ITA) (Mavrotas and Pechak 2013 a, b). The version of ITA described futher deals with multi-objective problems of project portfolio selection and provides information about the degree of certainty for inclusion of a specific portfolio in the final Pareto set, expanding thus its application area from project level to portfolio level. This kind of information is essential for the expert to be more confident to select project portfolios that have high degree of certainty regarding their Pareto optimality. In this respect, the decision maker has a sufficient tool to measure the robustness of the final Pareto set as well as the robustness of specific portfolios that appear in the final Pareto set. Robustness in project portfolio selection has also been addressed in a different way in the works of (Liesio et al., 2008; Hassanzadeh et al., 2014a, b).

5.4.1. Particularities of bi-objective programming

The basic idea of the current subchapter is to extend the applicability of Iterative Trichotomic Approach (ITA) to the case of multi-objective optimization, which was originally designed for single objective problems of project portfolio selection. It gives information about the degree of certainty for the inclusion or rejection of a specific project in the final portfolio. ITA was initially applied for project portfolio selection under the framework of Mathematical Programming and more specifically Integer Programming (IP). It was used with a single objective function reflecting the optimization criterion. The uncertainty associated with objective function coefficients has a stochastic nature (probability distributions instead of crisp numbers).

Project portfolio selection is by definition a multi-objective problem. Different points of view should be taken into account. One approach is to aggregate these points of view to a single metric through multicriteria analysis and subsequently optimize the resulting single objective problem where coefficients of objective function are multicriteria scores (Mavrotas et al. 2008). Alternatively, one can use a goal programming approach aggregating the objective functions based on their distance from individual goals (see e.g. Zanakis et al., 1995; Santhanam & Kyparisis, 1996).

In the above mentioned approaches, the decision maker has to assign weights to criteria or goals in order to aggregate them to a single objective function (scalarization). Another approach is to keep individual criteria as separate objective functions and proceed to a multi-objective optimization generating the Pareto set of the problem (or the Pareto front in criteria space). The Pareto set comprises Pareto optimal solutions (or Pareto portfolios in current case) which are examined before reaching the final choice. These methods are called "a posteriori" or "generation" methods in the popular Hwang and Masud (1979) terminology for multi-objective optimization methods (first generate Pareto front, examine it, and then select the most preferred Pareto portfolio). Their aim is not just to provide the most preferred solution but also to generate the Pareto set (either exactly or its approximation).

5.4.2. Description of the bi-objective model

The overall procedure that was adopted to address multi-objective project portfolio selection problem is graphically illustrated in **Figure 5-5**.

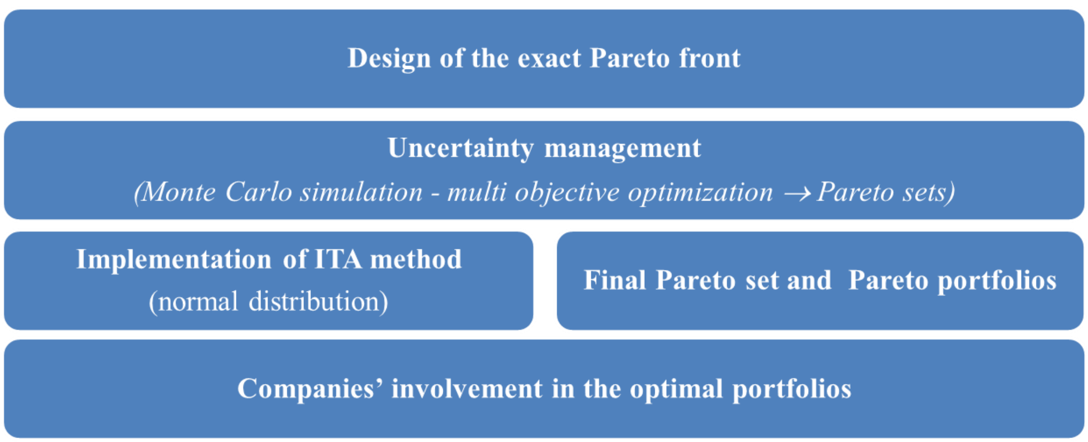

Figure 5-5. The adopted procedure for the portfolio building.

The idea of incorporating energy and environmental issues in Corporate Social Responsibility is rather recent (Doukas and Psarras, 2010; Doukas et al., 2012; 2014). In the present application a multi-criteria project portfolio selection problem is addressed taking into account both economic and environmental criteria. Given the uncertainty in quantifying economic as well as environmental performance of projects, multi-objective ITA method is an appropriate choice to extract results about the robustness of obtained project portfolios.

As it was mentioned before, the mathematical programming model that represents the optimization problem is a MOIP problem with several particular characteristics. In the specific case, firms' applications are expressed with 0-1 decision variables, with X_i denoting the i-th firm or application. More specifically:

if $X_i = 1$, then the corresponding application is approved.

otherwise, if $X_i = 0$, the corresponding application is rejected.

Two objective functions are considered in the model, namely the NPV of a portfolio and the EECR index of a portfolio. They are both additive functions of individual projects' relevant values.

$$\text{portfolio's } EECR: \quad max\ Z_1 = \sum_{i=1}^{N} eecr_i X_i$$
$$\text{portfolio's } NPV: \quad max\ Z_2 = \sum_{i=1}^{N} npv_i X_i \tag{5.20}$$

The parameters npv_i and $eecr_i$ are NPV of a specific project application and EECR score of a certain applied company.

The adopted procedure used for calculation of the EECR scoring was based upon the Ordered Weighted Average (OWA) operator, which had been introduced in 1988 by Yager. An aggregation operator is a function $F: I^n \rightarrow J$ where I and J are real intervals. I denotes the set of values to be aggregated and J denotes the corresponding result of aggregation. The set of aggregation operators is denoted as $A_n(I, J)$. An OWA operator is an aggregation operator from $A_n(I, J)$ with an associated vector of weights $w \in [0,1]^n$, such that:

$$Fw(x) = \sum_{i-1}^{n} w_i \times b_i \tag{5.21}$$

where: $\sum_{i-1}^{n} w_i = 1$ and b_i denoting the performance of an alternative in the criteria $x_1, \ldots, x_n$.

The criteria to be selected have to be operational, exhaustive in terms of containing all points of view, monotonic and non-redundant since each criterion should be countered only once, as pointed out by Bouyssou (1990). With respect to this, the research focuses on the provision of a small but clearly understood set of evaluation criteria, which can form a sound basis for the comparison of examined firms in terms of their systematic energy and environmental policy integration as a part of CSR and SD. Concisely, all six criteria are presented in **Table 5-13**. The data from these firms were mainly collected from the Global Reporting Initiative Disclosure Database (GRI, 2013).

Table 5-13. Criteria description for firms' evaluation.

Criteria	Description
C1: Management Commitment	Degree to which Management of a firm prioritizes actions related to the energy and environmental corporate policy, sets specific targets and corresponding time schedule for their accomplishment
C2: Monitoring Progress and Related Impact	Degree to which a firm adopts procedures and protocols for monitoring the set of targets, specific progress made in each related activity and the corresponding impact in companies operation and activation in the market
C3: Participation in Dissemination Activities	Reflects firms' participation in dissemination activities in broader community, including among others, educational and information activities regarding environmental practices, organization of workshops, conferences and other events, and sponsorships
C4: Promotion of Renewable Energy	Refers to the firms' involvement for investment in projects and initiatives related to renewable energy sources -wind power, solar power (thermal, photovoltaic and concentrated), hydro-electric power, tidal power, geothermal energy and biomass
C5: Promotion of Energy Efficiency	Extent to which a firm incorporates initiatives to provide energy-efficient products and services, to reduce direct and indirect energy consumption and other energy conservation practices and technological improvements.
C6: Waste and Water Management	Effort of firms in reducing total water use or discharge and the adoption of waste management activities.

The model includes constraints, imposed by each banking institution's specific credit policy. First of all, a budget constraint is used in order to secure that the cumulative cost of approved applications does not exceed the overall budget.

$$\sum_{i=1}^{N} \text{cost}_i \ \text{X}_i \leq \text{avb} \qquad (5.22)$$

where *avb* is the total available budget and $cost_i$ the cost of *i*-th project application. In the specific application the available budget is 3 M€ while the total cost of all 40 projects is 9.4 M€.

Specific bounds are imposed to control the distribution of projects according to their category, across various sectors. In particular, the non-dominance of a certain project category in portfolio can be expressed as "no sector or region is allowed to have more than half of the total approved applications". This condition is expressed with the following constraints:

$$\sum_{i \in S} X_i \leq 0.5 \times \sum_{i=1}^{N} X_i \quad \text{for } S = \textit{Sector 1,2,3,4} \qquad (5.23)$$

$$\sum_{i \in R} X_i \leq 0.5 \times \sum_{i=1}^{N} X_i \quad \text{for } R = \textit{Region 1,2,3,4} \qquad (5.24)$$

In order to assure that all sectors and regions will be present in final portfolios the following condition is added: “all sectors and areas will be funded with at least 10% of the total cost”. This condition is expressed with the following constraints:

$$\sum_{i \in S} \text{cost}_i X_i \geq 0.1 \times \sum_{i=1}^{N} \text{cost}_i X_i \quad \text{for } S = \textit{Sector 1,2,3,4} \tag{5.25}$$

$$\sum_{i \in R} \text{cost}_i X_i \geq 0.1 \times \sum_{i=1}^{N} \text{cost}_i X_i \quad \text{for } R = \textit{Region 1,2,3,4} \tag{5.26}$$

In the framework of ITA, the uncertainty characterizing the estimation of projects’ NPV as well as the calculation of firm’s EECR score is expressed with normal distributions for relevant projects’ values. Specifically, the mean value for the normal distributions the estimated value is presented in **Table 5-14** and as standard deviation of the initial round is the 5% of the mean. This is done for the NPV as well as the EECR values. From round to round the standard deviation of corresponding normal distributions is reduced to 4%, 3%, 2%, 1% and 0% in the final round. The whole process (model building, random sampling, Pareto set generation) is implemented within GAMS platform (GAMS, 2010).

Table 5-14. Input data for the projects.

	CSR	NPV (€)	Cost (€)	Sector	Region
1	12.97	2,500	5,930	S1	R3
2	14.66	49,800	50,830	S1	R3
3	9.76	8,300	5,000	S1	R2
4	6.23	63,600	33,860	S1	R3
5	6.99	244,600	191,870	S2	R1
6	14.64	36,700	37,500	S2	R1
7	7.10	14,100	6,070	S2	R1
8	11.92	22,500	23,030	S2	R4
9	11.81	261,300	190,000	S2	R1
10	21.59	455,000	422,670	S3	R2
11	13.64	696,800	415,000	S3	R1
12	13.59	53,900	39,330	S3	R1
13	3.86	238,900	95,330	S1	R4
14	9.62	3,400	5,630	S4	R1
15	40.00	600	7,370	S4	R1
16	2.95	74,600	37,670	S4	R2
17	25.87	4,900	30,100	S1	R4
18	5.25	12,500	5,700	S4	R2
19	11.39	389,900	909,310	S4	R3

20	11.67	378,100	160,300	S4	R4
21	15.39	53,100	26,190	S4	R2
22	17.13	51,400	161,010	S4	R3
23	5.76	460,100	353,420	S3	R1
24	8.93	422,800	184,410	S1	R3
25	16.12	146,900	87,910	S4	R2
26	12.38	477,100	614,620	S1	R2
27	7.19	431,600	277,040	S1	R3
28	21.95	208,500	158,790	S3	R3
29	4.70	324,400	1,410,180	S2	R1
30	18.07	324,100	533,640	S3	R1
31	7.75	603,200	529,130	S4	R2
32	4.54	648,800	396,670	S2	R4
33	19.18	179,600	123,640	S1	R3
34	15.85	220,000	149,770	S1	R1
35	22.01	204,300	93,050	S4	R2
36	4.04	352,100	311,780	S4	R3
37	19.39	223,000	772,970	S3	R2
38	17.81	228,800	117,580	S2	R3
39	12.86	428,500	190,870	S4	R4
40	5.85	516,100	262,030	S2	R1

The parameters' values of the model as well as the membership of projects in various sets (sectoral and geographical) are shown in **Table 5-14**. Still, more types of constraints may be considered in the mathematical programming framework such as the specific number (or range) of accepted applications (projects to be finally funded), or constraints for mutually exclusive projects etc.

5.4.3. Results and discussion for multi-objective ITA

The selection is based on the characteristics of 40 projects from 40 different firms, with a geographical, sectoral distribution as follows in **Table 5-15**:

Table 5-15. Characteristics for 40 projects.

Geographical regions	Sectors
11 southern European enterprises	11 energy enterprises
10 northern European enterprises	9 industrial enterprises
13 central European enterprises	7 electrical equipment enterprises
6 Greek enterprises	13 enterprises from other sectors

In each computation round 1000 Monte Carlo iterations were performed and the computation time varied between 7181 seconds and 9150 seconds from round to round in a core i-5 running at 2.5 GHz. For the specific application, the acceptance threshold for the green set was set at the level of 99% (if a portfolio was present in 99% of Pareto sets, i.e., in 990 out of 1000).

The results of multi-objective ITA are shown in **Table 5-16**. There are in total 398 Pareto optimal portfolios that participate in 1000 Pareto sets of the initial round. Among them only 4 were present in all Pareto sets. At subsequent iterations the standard deviation of sampling distributions as shown in the first column of **Table 5-16** was reduced. Eventually, in the last round, the final Pareto set that comprises 31 Pareto optimal portfolios of projects emerged. These portfolios contain from 18 to 28 projects.

Table 5-16. Results of multi-objective ITA from round to round.

		Computation time (sec)	Green	Red	Grey
σ =5%	Round 1	9178	4	0	394
σ =4%	Round 2	8247	4	109	285
σ =3%	Round 3	8592	5	215	178
σ =2%	Round 4	7811	9	275	114
σ =1%	Round 5	8685	16	324	54
σ =0%	Round 6	7.3*	31	367	0

* for just one iteration as there is no uncertainty quantified by standard deviation

After completion of modeling runs, a first brief look reveals which of these 31 portfolios can be considered more certain than others. The degree of certainty for each portfolio is directly related to the corresponding round that it enters the green set as shown in **Figure 5-6**. The darker the portfolio's background the more certain one can be about its Pareto optimality. **Figure 5-6** illustrates in a convenient way which portfolios are more robust given the uncertainty in the model's parameters. The decision maker can exploit this information in his final selection.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
17	**18**	**19**	**20**	**21**	**22**	**23**	**24**
25	**26**	**27**	**28**	**29**	**30**	**31**	

Figure 5-6. Coloring code for 31 portfolios.

A challenging task is to incorporate the robustness information in the Pareto front. As it is well known, the Pareto front of a problem with 2 or 3 objective functions is a relatively easy to draw graph of the Pareto set in predefined criteria space. The robustness of each portfolio can be expressed with a bubble chart, with the size of bubble being the portfolio's robustness degree.

The upper chart in **Figure 5-7** is a conventional Pareto front with 31 Pareto optimal solutions (different portfolios). The lower chart embodies also robustness information which is visualized with the size of the bubble. The greater the Robustness Degree of a Pareto Optimal Portfolio (i.e., the earlier it enters the green set), the greater the size of the bubble. This kind of information is essential for the decision maker to recognize regions of the Pareto front with higher or lower robustness.

From this chart the decision maker can draw conclusions about criteria values of each solution (and therefore assess the tradeoff) as well as about the robustness of solutions. In the specific case, it seems that the robust Pareto optimal solutions are in the region of high EECR (horizontal axis). This also means that the values of EECR have less uncertainty, and this is true, taking into consideration the detailed and precise way of their calculations.

As a rule, promising solutions are on the knee of the Pareto curve where the slope changes sharply meaning that with a little sacrifice in one objective function it is possible to achieve large improvement in the other. A promising solution (portfolio) in our case is the one pointed with an arrow. This means that a small compromise from the maximum EECR value leads to a great improvement in NPV. Besides, it is evident from the size of the respective bubble, which specific solutions are among the most robust. Conclusively, the robustness of Pareto optimal solutions which is visualized in **Figure 5-8** can be regarded as an additional characteristic that helps the decision maker to evaluate the attractiveness of the obtained Pareto optimal portfolios.

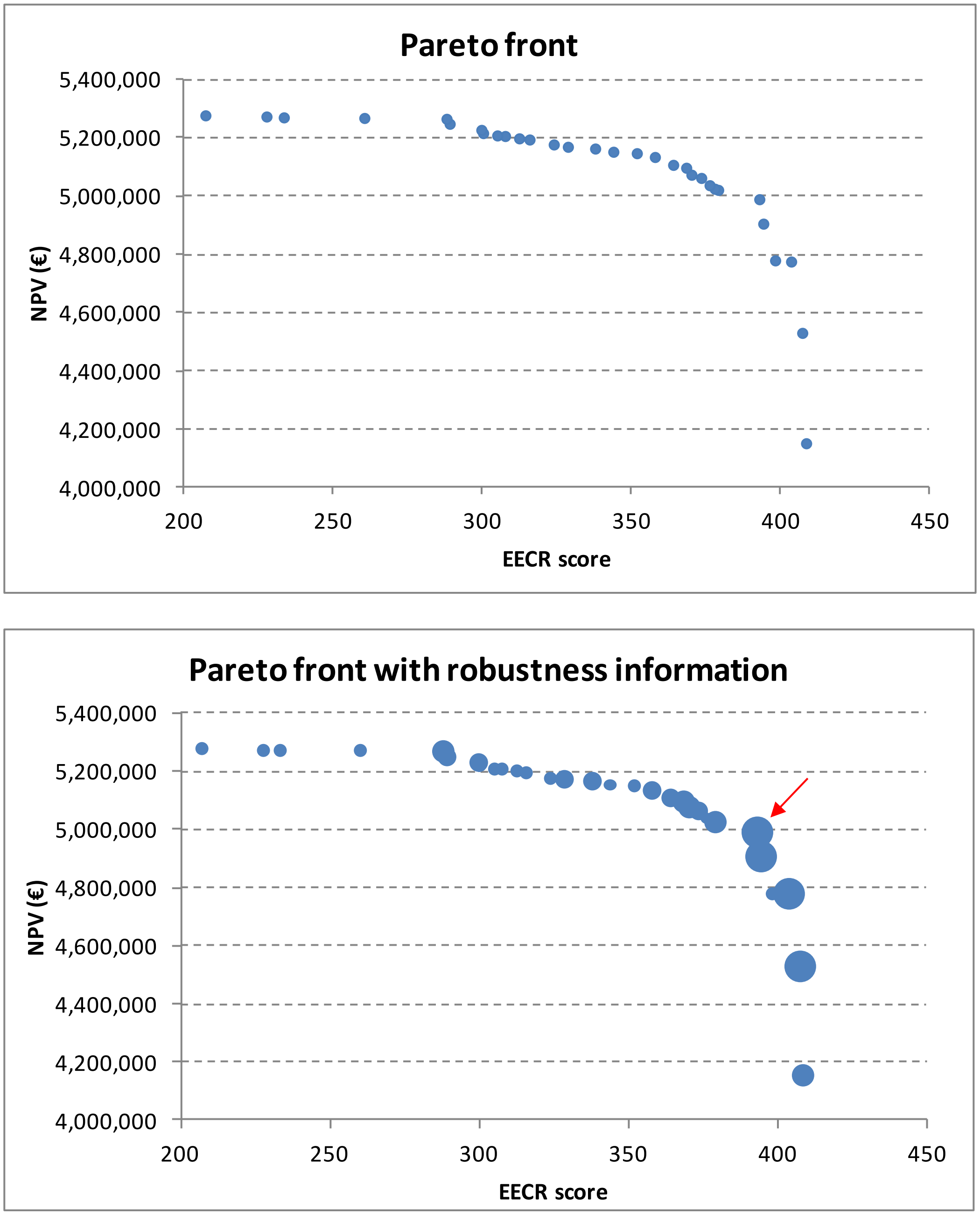

Figure 5-7. Visualizing the robustness with bubble charts.

The overall robustness of the final Pareto set can be measured using the Robustness Index. The Robustness chart and the Robustness Index for the current case is depicted in **Figure 5-8**. Applying equation (2.18), Robustness Index as the area underneath the Robustness Curve which is *RI*=0.33.

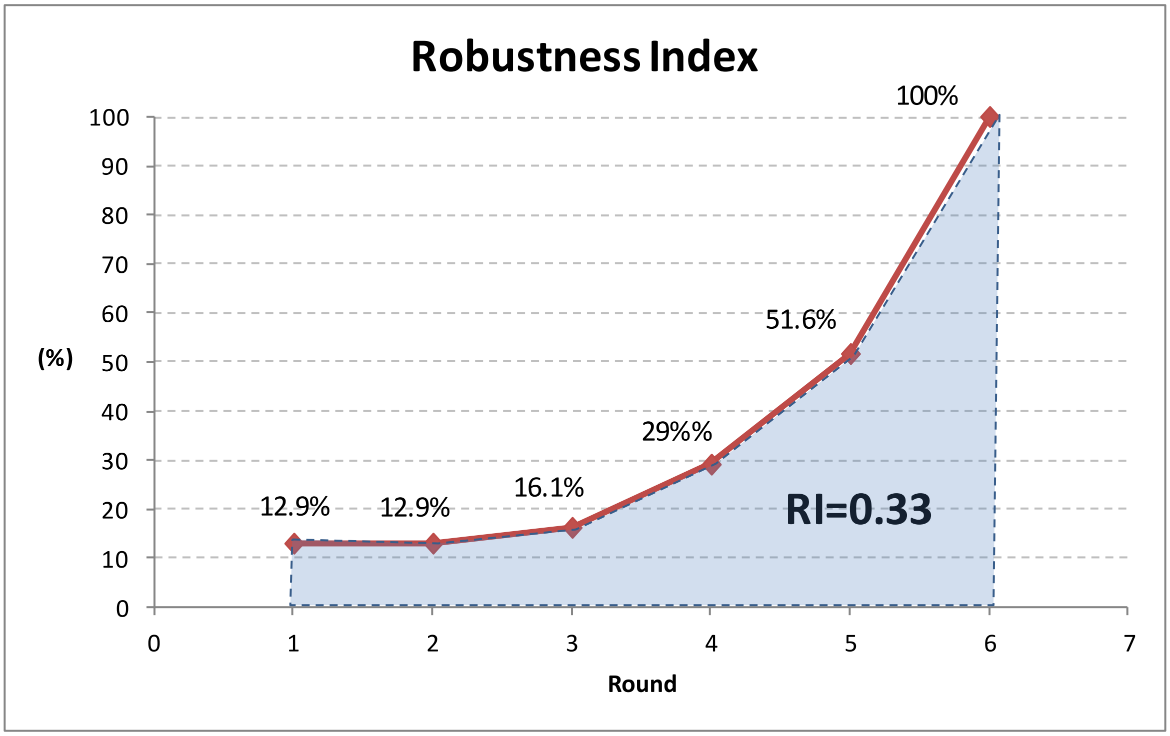

Figure 5-8. Robustness chart for the final Pareto set.

Regarding all 40 projects, it is possible to measure their presence in the Pareto front by counting how many times each of them appears in 398 initial Pareto portfolios and how many in times in 31 final Pareto portfolios as shown in **Figure 5-9**.

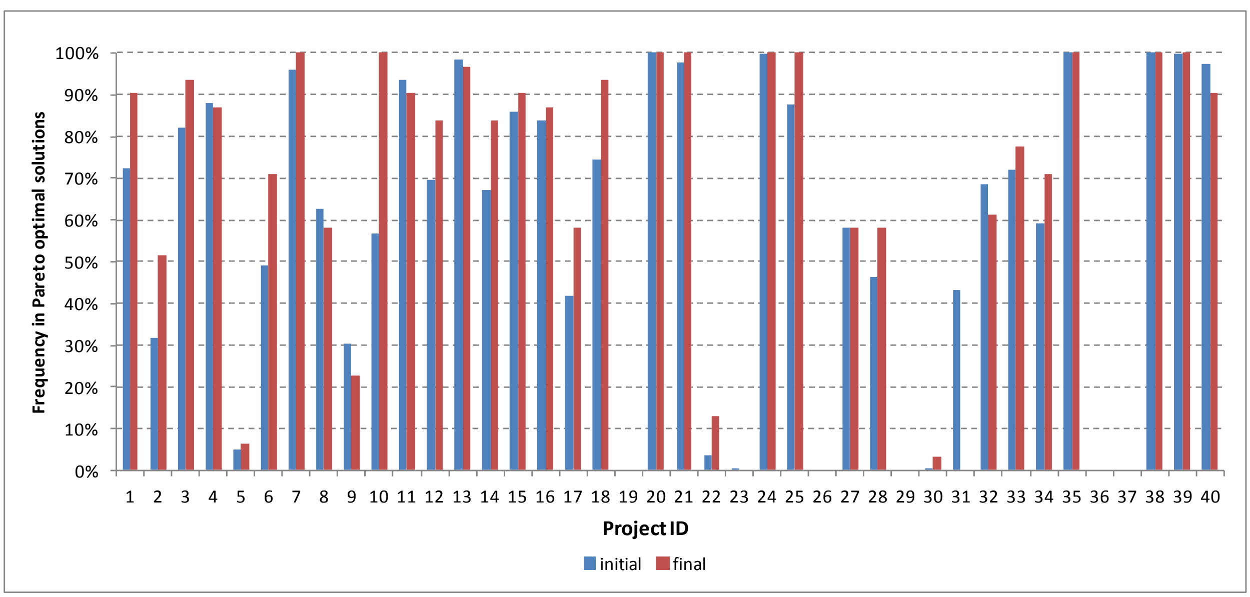

Figure 5-9. Frequency of projects in the initial and final Pareto portfolios.

The initial Pareto portfolios correspond to maximum uncertainty. From **Figure 5-9** it is possible to extract information about the robustness of individual projects. The closer the two frequency rates are (in the initial and in the final Pareto portfolios) for one project, the more robust the conclusions are for the participation frequency of the specific project. From **Figure 5-9** one can observe that there are projects included in more than 90% of Pareto portfolios (even when maximum uncertainty is considered, i.e., in the initial round) like projects 7, 11, 13, 20, 21, 24, 35, 38, 39, 40) and other projects that never appear in Pareto portfolios (19, 23, 26, 29, 36, 37).

Moreover, based on the results, it can be noted that companies requesting for larger loans, while having a low EECR index, tend to be rejected. On the other hand, companies asking for smaller loans and having a high NPV index, tend to be approved.

6. Concluding remarks

6.1. Conclusions about the method

ITA for project portfolio selection is an effective method that deals with uncertainty in a volatile decision environment. The aim is to provide the DM with as much as possible information to support his/her final choice, while the input information or future performance can be obtained with great difficulties. The existence of multiple limitations denoting projects' interactions and the underlying uncertainty expressed as probability distributions imply the use of a systematic approach. For this reason a hybrid method combining Multi-Criteria Analysis, Mathematical programming and Monte Carlo simulation was developed. Under these circumstances, the existence of a unique optimal portfolio is almost impossible, so that the trichotomic approach drives the DM to reach the portfolio with the greater acceptance. By doing this, the information burden decreases and the focus of an expert is moved towards ambiguous grey projects which are not that numerous. Due to its flexibility ITA can be easily adapted to various decision situations and DMs.

The term "trichotomy" refers to the separation of a set into three parts. Within the ITA procedure, projects are assigned to one of three groups based on their performance and current level of uncertainty. The latter is incorporated in various forms, depending on its nature. Stochastic parameters may be present either in the objective function or in the constraints of the model. Actually, incomplete information, expressed via probability distributions, may be present in all model's parameters simultaneously. Fruitful information is extracted not only about the projects that are eventually selected, but also about how sure an expert can be with respect to the selected or discarded ones. In contrast to the expected performance values of projects (naïve approach), ITA moves smoothly and prevents the feeling of a black-box. Gradual filling of green and red sets provides crucial information to the DM about the reliability of projects' inclusion or exclusion in the final portfolio according to the round that each project enters the respective set. Moreover, this level of certainty can be easily measured in a shape of index after the completion of Monte Carlo runs.

As the curiosity grew bigger with every case study, it was possible to test and compare the performance of several modifications of ITA. The first case study tested the influence of uncertainty in the future output of project proposals. Due to the novelty of climate related activities it was totally impossible to draw detailed forecasts. For initial approximations the information about similar installations that had been put in operation relatively recently was used. The classic version of ITA with the modeling of uncertainty described by a normal distribution led to a balanced portfolio. Especially striking was the fact of expensive projects exclusion which helped to build a selection with balanced distribution of projects between regions and technologies. The objective not to put all eggs in one basket was successfully met.

A simplified version of ITA also produced good results for a relatively small problem. Most probably, two-phase ITA should not be considered as a full scale tool since in its second part it uses the principle of majority according to the performance in the initial round. In a certain way, it helps to speed up the process of choice. It should be kept in mind that results may differ in the presence of uncertainties with particular distributions.

Perhaps, the most useful version of ITA is for groups of experts with divergent points of view. Increased transparency and gradual portfolio building are the main advantages of this version of ITA. Experts are provided with initial preferences of others as well as with the stepwise convergence of these preferences aimed at the building of a final selection.

It is extremely hard to avoid such highly subjective parameters as weights of importance, utilities etc. In this case, the sampling of Monte Carlo iterations was like a scenario building for every decision maker. According to the problem, several ways of search for a solution can be adopted. When a DM is aware that another expert may not insist that hard on his initial preferences in light of obtained information about projects in previous rounds, the others may follow the suit. Gradual building of the final portfolio also reveals the reasons behind rejection of certain proposals thus reducing the chances of being accused as manipulator for excluding some good projects. By the end of the process, the consensus index is calculated expressing how easy or hard it was to reach consensus among experts.

Since ITA is an interactive decision support tool, the DM(s) can control and adjust the process accordingly to newly obtained information. In the case that 2 or 3 similarly performing projects compete for the place in final selection, additional information can be asked directly from the projects' developers in order to perform deeper pair wise comparison. In case that all of them look attractive, basic constraints, such as budget, may be reconsidered too. It is not obligatory to increase the budget; it can be reduced too. All these details help to build the confidence about decisions under consideration without additional lengthy robustness checks.

While it is common to use a single objective model and put in constraints other desired outcomes, it was interesting to test the behavior of the ITA approach on a bi-objective problem. In this case, the feedback of calculations is a Pareto front consisting of various portfolios. For easier elaboration of this front, the robustness of each portfolio can be expressed with a bubble chart, whith the size of the bubble being the portfolio's robustness degree. As a rule, promising solutions are on the knee of the Pareto curve where the slope changes sharply meaning that with a little sacrifice in one objective function it is possible to achieve large improvement in the other. Moreover, robustness check for projects can be easily performed too.

The whole selection process may be accelerated with the adoption of a threshold for every iteration. The higher the threshold, the quicker the process of projects' separation between three sets.

Lastly, more complex probability distributions (triangular, uniform, normal, special cases) for uncertain parameters can be tested. The subject was barely touched by Group ITA, where each DM may have his own view about the shape of partial value functions or about the allocation of projects to regions and technologies. In addition, it is worth to test an interactive process of weights recalculation instead of their automatic recalculation.

6.2. Conclusions about case studies

The need to make a choice between countless suggestions is an everyday task. That is why for testing the ITA method it was decided to use real world applications. The data for case studies were taken from open sources, which actually helped to shape the process of modeling. Observations led to consideration of numerous criteria and plentiful constraints (budget, policy, allocation etc) that had to be satisfied. The combination of MCDA with optimization tools, such as Integer Programming, provided the chance to solve complex problems in limited time.

Nowadays, telecommunication technologies are an integral part of everyday life. Their appropriate work becomes crucial in case of extreme events, some of which can be caused by climate change. Telecommunications networks have been subject to continuing technical innovations and to constantly evolving multifaceted modes of communication. Due to recent advancements in technologies and changes in markets it became necessary to reconsider long-term business goals for the providers of these services. The achievement of new and revised strategic objectives called for changes in their product portfolio, whereby companies were facing with the problem of choosing which products would effectively contribute to the achievement of their long-term goals. Hence, many scientists approached these problems with MCDA tools. The resulting portfolio indicated exactly this direction – the projects from Developing group are the clear winners. We tested ITA method against a case study from the literature and obtained results similar to the initial paper. Actually, close performing proposals were identified and sometimes they were interchanging in the final portfolio when more performance uncertainty was added. In a certain way, portfolio confirms the conclusions of common sense that in a competitive environment it is necessary to continuously self-improve.

Energy and environmentally oriented activities attracted our attention as the issues to be addressed today in a large scale. Environmental crisis requires new ways to respond and adapt to the challenges of today.

Since fossil fuels are limited and harmful for the planet, more effort should be put in the deployment of renewable energy installations around the world. One of these support efforts is the Clean Development Mechanism (CDM), which permits to offset carbon emissions in the shape of environmentally friendly activities, with energy related projects being the most widespread ones. Unfortunately, financial resources are always limited, that is why there is a sharp need to choose effectively project proposals for CDM. In the current case study the aim was to maximize carbon credits, even though their final amount was not a certain fixed number. By the end of the selection process a final portfolio representing 17.4% of the investments in comparison with initial 11.5 billion US$ of 300 projects was built. Its main advantage lies in the fact that with modest financial support it represented 35.8% of the project universe's total CERs (=28805 kCERs). The final portfolio demonstrated how it is possible to make a balanced selection regarding financial as well as technological and geographical constraints.

Another case study focused on similar projects within only one country, namely Greece. It was necessary to make a portfolio with several different experts involved. Every decision maker expressed his/her preferences by assigning his/her own weights of importance to the evaluation criteria. Hence, every decision maker has his/her own optimal portfolio of projects.The final selection slightly violated imposed constraints with respect to the installed capacity and total budget. The portfolio was dominated by solar technologies (photovoltaic) with Central Greece being the winning region. Results are totally reasonable for a place with more than 250 days of sunshine. A great advantage of Group ITA is that it also provided a measure of consensus for the final portfolio of projects (Consensus Index) as well as concordance indices for each project that was either selected or rejected. It seems that experts were speaking the same language, because CI was 91%. Generally, the Index is rarely that high, especially in the presence of high uncertainty in the financial criteria.

The last case study highlighted the problem of shared responsibility. Not only is the government responsible for the environmental initiatives, but also private sector. The years of economic crisis put seriously under question traditional development patterns. It became clear that in the fight for survival in the market, enterprises have to integrate social and environmental concerns in their business operations and in their interaction with stakeholders on a voluntary basis, within the framework of the Corporate Social Responsibility (CSR) concept. In the case study equal emphasis was put both on economic and sustainability components. Based on the results of the model, it can be noted that companies requesting for larger loans, while having a low EECR index, tend to be rejected. On the other hand, companies asking for smaller loans and having a high NPV index, tend to be approved.

It is noteworthy that in today's organizations there is rarely a unique person that makes important decisions alone. Multiple experts from various positions, with different backgrounds and usually with conflicting views participate in the decision process and are expected to reach consensus over the final portfolio. Wide demand for decision support systems that deals with similar problems is nowadays undoubted. That is why we have tested the performance of ITA in different selection problems, that are based on real data and obtained balanced and reassuring results.

6.3. Directions for future research

The presented ITA method may be further improved along several directions.

Since the method is aimed at helping decision makers, it is feasible to develop a user friendly decision support system (DSS) platform. For the time being ITA is implemented as separated modules (one for multi-criteria analysis and another for Monte Carlo simulation along with mathematical programming). Our calculations were performed in GAMS and the model could be stored in GAMS library, which leaves the possibility to modify available models according to one's needs and introduce their own data. Still, for most of experts it would be more convenient to have a special platform with a more user-friendly interface that does not require explicit knowledge in programming.

Other fields still remain for exploitation. Special focus may be paid to projects whose description contains more qualitative than quantitative data. Because in such cases subjective judgments are involved, interpretation of results may be more challenging in comparison with results obtained by technical means of measurement. Such fields may touch the domains of education or healthcare where along with numerous performance criteria human factor plays one of leading roles.

Another direction of work is aimed at handling massive input data with thousands of projects. Immence data may be so large and complex that traditional methods of collection and analysis are no longer in position to handle them effectively. The amount and variety of big data has increased exponentially over the past decade. Tools to handle this issue would be especially useful for public agencies that assist applicants in need. Such examples could involve house retrofitting programs, improvement of energy efficiency in households or replacement of infrastructure in municipal districts.

In general, future research for ITA can be very fruitful as it can combine various OR techniques to address specific decision making problems that deal with the project portfolio selection problem (or can be modeled as such) as, for example, problems that involve group decision making with uncertain or vague data. The great advantage of ITA is that it measures the degree of consensus or certainty of the final choice, which is always meaningful in this kind of decision making situations.

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8. Annexes

8.1. Modeling language – GAMS, GAMS/CPLEX solver

The General Algebraic Modeling System (GAMS, see e.g. Brooke et al., 1988) is designed for high-level modeling and solving linear, nonlinear, and mixed-integer optimization problems. The system is tailored for complex, large-scale modeling applications and allows the user to build large maintainable models that can be adapted to new situations. The system is available for use on various computer platforms. GAMS contains an integrated development environment (IDE) and is connected to a group of third-party optimization solvers among which are BARON, COIN-OR solvers, CONOPT, CPLEX, DICOPT, Gurobi, MOSEK, SNOPT, SULUM, and XPRESS. GAMS allows users to implement a sort of hybrid algorithm combining different solvers. Models are described in concise, human-readable algebraic statements. The GAMS software was originally developed by a group of economists from the World Bank in order to facilitate the resolution of large and complex non linear models on personal computer. Within the main advantages of GAMS are:

- Simplicity of implementation,
- Portability and transferability between users and systems and
- Easiness of technical update because of the constant inclusion of new algorithms.

As a matter of fact, GAMS allows solving simultaneous non linear equation system, with or without optimization of some objective function (Dumont and Robichaud 2000).

Typically, a model programmed in GAMS can be decomposed in three modules corresponding respectively to data entry, model specification and solve procedure (as it is schematically shown in **Figure 8-1**). It is important to note that assignment, declaration and definition, must be completed for every element in use in the model (i.e. sets, parameters, variables and equations). On the whole, it is necessary to proceed to the declaration of any element before using it. Specifically, sets must be declared at the very beginning of the program.

It is particularly convenient that GAMS allows for statement on several lines or several statements on the same line. This property can help to reduce the length of the code or facilitate printing. In addition, capital and small letters are not distinguished in GAMS.

The definition of sets is useful for multidimensional variables. It corresponds to the indexes in mathematical representations of models. Next, the parameters should be defined. Parameters are the elements in the equations that will not change after a simulation, such as elasticity, tax rates, distribution and scale coefficients. In addition to these parameters, benchmark variables are also defined for their value at the base year will not change after simulation. A common way to define these variables is to add an "O" after the variable name so it will not be confused with the „true" variable. Parameters and benchmark variables definition begins with the statement PARAMETER and end with a semicolon. Once again, it is useful to put a description after the parameter designation, as it is done in the example. When a parameter is subject to an index, like A_j, the set over which it is defined is put between parentheses, A(j).

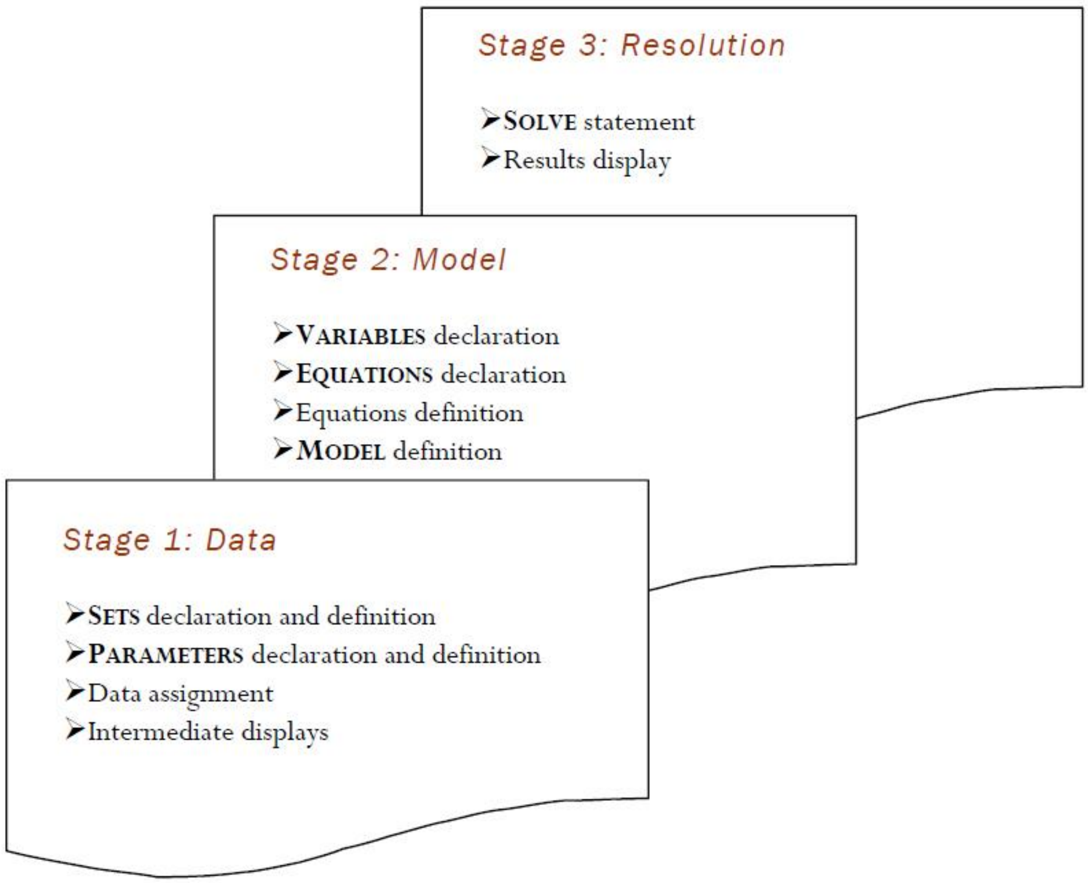

Figure 8-1. Organization chart of a typical GAMS code for a model.

Once the sets, parameters and benchmark variables are defined, data must be entered. This can be done using the TABLE command, which is useful for multidimensional variables.

All variables, endogenous or exogenous, appearing in the equations must first be declared. The statement VARIABLES begins this procedure and ends with a semicolon. Following the variable name, for example W, a brief description can be added.

As with the previous components of the model, the equation must be declared and defined. This step begins with the EQUATIONS statement followed by the symbols for which a description can be added. For example, the equation named EQ1(j) is followed by a short description. When all equations are declared, a semicolon indicates the end.

The list of main mathematical functions is provided in **Table 8-1**.

Table 8-1. Main mathematical functions in GAMS.

Multiplication	*	Equality in an operation	=	Logarithm	*LOG(.)*
Subtraction	-	Summation	*SUM(set domain, element)*	Maximum	*MAX(.)*
Addition	+	Product	*Prod(set domain, element)*	Minimum	*MIN(.)*
Division	/	Absolute Value	*ABS(.)*		
Exponent	**	Exponential	*EXP(.)*		

Further step is to choose a procedure from list of solvers which are provided in a shape of a table. The rows are the list of solvers available in the GAMS system and the columns are the problem types that GAMS can solve. The procedure to be used is determined by the type of model to solve. For example, linear programs can be solved using the LP procedure, while non-linear programs can be solved using NLP. The table tells what solvers are available for what problem types and what the current selection or default is. Each procedure uses a different solver (MINOS, CONOPT, MILES, CPLEX, etc). These solvers use different algorithms that can be efficient in resolving some systems but less in other cases. Specifically, CPLEX optimizers are designed to solve large, difficult problems quickly and with minimal user intervention. Access is provided (subject to proper licensing) to Cplex solution algorithms for linear, quadratically constrained and mixed integer programming problems. While numerous solving options are available, GAMS/CPLEX automatically calculates and sets most options at the best values for specific problems. For problems with integer variables, CPLEX uses a branch and cut algorithm which solves a series of LP, subproblems. Because a single mixed integer problem generates many subproblems, even small mixed integer problems can be very compute intensive and require significant amounts of physical memory.

Since GAMS has been used already for decades for solving various problems it is reasonable to use this great amount of experience and knowledge. The system has incorporated library of models that have been selected not only because they collectively provide strong basis for new users to stand on, but also because they represent interesting and sometimes classic problems. For example the tradeoff between consumption and investment is richly illustrated in the Ramsey problem, which can be solved using nonlinear programming methods. Examples of other problems included in the library are production and shipment by firms, investment planning in time and space, cropping patterns in agriculture, operation of oil refineries and petrochemical plants, macroeconomics stabilization, applied general equilibrium, international trade in aluminum and in copper, water distribution networks, and relational databases. Among other reasons to include a problem to the library is specification of initial solutions as staring points in the search for the optimal solution of dynamic nonlinear optimization problems. Moreover, some models have been selected for inclusion because they have been used in other modeling systems and permit the user to compare how problems are set up and solved in different modeling systems.

Most of the models have been contributed by GAMS users. Within many problems in the library there are several developed by Prof. G. Mavrotas (Mavrotas 2009, Mavrotas and Florios 2013). First one is a GAMS implementation of the augmented ε-constraint method for generating the efficient (Pareto optimal, nondominated) solutions in multiobjective problems. The ε-constraint method optimizes one of the objective functions using the remaining objective functions as constraints, varying their right hand side. The method uses lexicographic optimization in the construction of the payoff table (in order to secure the Pareto optimality of the individual optima) and a slightly modified objective function in order to ensure the production of Pareto optimal (and not weakly Pareto optimal) solutions. In addition, it performs early exit from infeasible loops improving the performance of the algorithm in multi-objective problems with several objective functions.

Further work resulted into the Augmented Epsilon Constraint Method version 2 (AUGMECON2). The method is applied to a Multi-Objective Integer Programming problem (specifically a Multi-Objective Multi-Dimensional Knapsack Problem) with 50 binary variables X, 2 objective functions and 2 constraints. The AUGMECON2 can be used to generate the exact Pareto set (all the Pareto optimal solutions) if the step size (i.e.the interval between the grid points of the objective functions that are used as constraints) is appropriately chosen. For problems with integer objective function coefficients the step size should be at most equal to unity.

The required models and the whole solution process for the case studies within the current thesis were developed in the General Algebraic Modeling System using the MIP solver CPLEX 11.1 for optimizing the Mixed Integer Programming models.

8.2. GAMS codes

Model for two-phase approach (40 telecommunication projects (Niaei et al. 2011))

Phase 1

```
$TITLE project selection under uncertainty
$eolcom //
$ontext
Project selection problem
Data from Computers and Industrial Engineering 61 226-237 (2011)
$offtext

SETS
I     projects /1*40/
APPLIED(I) /1,3,4,6,10,11,16,18,20,21,24,25,27,30,31,32,33,35,36/
BASIC(I) /2,7,9,12,13,14,17,22,23,26,28,37,38,39/
DEVELOP(I) /5,8,15,19,29,34,40/
J  criteria  /cost,meth,pers,sci,tech /
TRIPLET the parameters of the triangular distribution /MIN,MID,MAX/
;

alias (I,I2);

parameter costmin(I) minimum cost for project(I)
/
1      341
2      31
3      316

39      32
40      145
/
parameter costmax(I) maximum cost for project(I)
/
1      447
2      42
3      493

39      49
40      158
/
parameter methmin(I) minimum score for methodology
/
1      4
2      5
3      3

39      3
40      0
/

parameter methmax(I) maximum score for methodology
/
1      8
2      8
3      5

39      5
40      3
/

parameter persmin(I) minimum score for personnel
/
1      3
```

```
2       2
3       4

39       3
40       4
/

parameter persmax(I) maximum score for personnel
/
1       6
2       4
3       6

39       6
40       8
/

parameter scimin(I) minimum score for scientific contribution
/
1       2
2       3
3       4

39       1
40       2
/

parameter scimax(I) maximum score for scientific contribution
/
1       5
2       5
3       6

39       6
40       6
/

parameter techmin(I) minimum score for technical capacity
/
1       3
2       0
3       2

39       4
40       2
/

parameter techmax(I) maximum score for technical capacity
/
1       7
2       1
3       4

39       6
40       4
/


parameter sc(I,J) random generated scores for alternative I in criterion J
parameter rcost(I) random generated cost for project I
parameter rmeth(I) random generated cost for project I
parameter rpers(I) random generated cost for project I
parameter rsci(I) random generated cost for project I
parameter rtech(I) random generated cost for project I
parameter rwght(J) random generated weight for criterion J
parameter tot_sc(I) total score for project I


table wght(J,TRIPLET) the triplet for the triangular distribution for the weights
          min        mid         max
```

```
cost      0.167      0.2051      0.231
meth       0.12       0.1356      0.141
pers      0.119      0.1364      0.157
sci      0.114      0.1274      0.149
tech      0.363      0.3955      0.434
;

parameter trigmid(J) normalized MID score for triangular distribution;
trigmid(J) = (wght(J,'MID')-wght(J,'MIN'))/(wght(J,'MAX')-wght(J,'MIN'));

scalar
elapsed_time elapsed time for payoff and e-constraint
start start time
finish finish time
iter  counter for iterations
sumwj sum of random generated weights
r auxiliary parameter
MCiter number of Monte Carlo iterations /1000/
maxbudg  maximum budget (million toomans) /6000/
appl_ub applied projects upper bound percentage in total /0.7/
basic_ub basic research projects upper bound percentage in total /0.2/
dev_ub developing projects upper bound percentage in total /0.4/

rhsbudget RHS of the budget constraint
g2       counter for e-constraint
numg2     number of grid points /10/
meancost  mean value for costs
stdevcost standard deviation for costs
meanmeth  mean value methodology
stdevmeth standard deviation for methodology
meanpers  mean value personnel
stdevpers standard deviation for personnel
meansci   mean value  science
stdevsci  standard deviation for science
meantech  mean value technology
stdevtech standard deviation for technology

;

BINARY VARIABLES
X(I)    binary variable indicating if project I is selected or not

Positive variables
TOTBUDG total budget (million toomans)
TOTPROJ total projects;

FREE VARIABLES
PORTF_SCORE     total NPV in thousand USD;

EQUATIONS
EQ_TOTBUDG   equation for total budget
EQ_TOTPROJ   equation for total projects
EQ_APPL      constraint for applied projects
EQ_BASIC     constraint for basic research projects
EQ_DEV      constraint for developing projects
EQ_OBJ      objective function --> maximization of portfolio's score
;

EQ_TOTBUDG.. sum(I, rcost(I)*X(I))=e= TOTBUDG;
EQ_TOTPROJ.. sum(I, X(I)) =e= TOTPROJ;
EQ_APPL.. sum(APPLIED(I),X(I)) =l= appl_ub*TOTPROJ ;
EQ_BASIC.. sum(BASIC(I),X(I)) =l= basic_ub*TOTPROJ ;
EQ_DEV.. sum(DEVELOP(I),X(I)) =l= dev_ub*TOTPROJ ;
EQ_OBJ..  sum(I, tot_sc(I)*X(I))=e= PORTF_SCORE;

rhsbudget = maxbudg;
TOTBUDG.up = rhsbudget;

MODEL CAIE_40_model /ALL/ ;
OPTION OPTCR = 0;
```

```
option seed=1513;

FILE cdmfile /c:\tp\caie_40_1b NORMAL.txt/ ;
cdmfile.pw=2000;
put cdmfile ;

start=jnow;
iter=0;

for(iter=1 to MCiter,
* random generation of criteria weights from triangular distribution
   loop(J,
        r=uniform(0,1);
        if (r<trigmid(J),
            rwght(J)=wght(J,'MIN') + sqrt(r*(wght(J,'MAX')-wght(J,'MIN'))*(wght(J,'MID')-wght(J,'MIN')));
        else
            rwght(J)=wght(J,'MAX') - sqrt((1-r)*(wght(J,'MAX')-wght(J,'MIN'))*(wght(J,'MAX')-wght(J,'MID')));
            );
       );
    sumwj=sum(J,rwght(J));
    loop(J, rwght(J)=rwght(j)/sumwj)
* random generation of project scores from normal distribution
    loop(I,
        meancost=(costmax(I)-costmin(I))/2;
        stdevcost=(costmax(I)-costmin(I))/6;
        rcost(I)=normal(meancost,stdevcost);
        meanmeth=(methmax(I)-methmin(I))/2;
        stdevmeth=(methmax(I)-methmin(I))/6;
        rmeth(I)=normal(meanmeth,stdevmeth);
        meanpers=(persmax(I)-persmin(I))/2;
        stdevpers=(persmax(I)-persmin(I))/6;
        rpers(I)=normal(meanpers,stdevpers);
        meansci=(scimax(I)-scimin(I))/2;
        stdevsci=(scimax(I)-scimin(I))/6;
        rsci(I)=normal(meansci,stdevsci);
        meantech=(techmax(I)-techmin(I))/2;
        stdevtech=(techmax(I)-techmin(I))/6;
        rtech(I)=normal(meantech,stdevtech);

*         sc(I,'cost')=10*(1-(rcost(I)/smax(I2,costmax(I2))));
*         sc(I,'meth')=round(normal(methmin(I)-0.5,methmax(I)+0.5));
*         sc(I,'pers')=round(normal(persmin(I)-0.5,persmax(I)+0.5));
*         sc(I,'sci')=round(normal(scimin(I)-0.5,scimax(I)+0.5));
*         sc(I,'tech')=round(normal(techmin(I)-0.5,techmax(I)+0.5));
*         tot_sc(I)=sum(J,rwght(J)*sc(I,J));
       );

* random budget ariund maxbudg
   rhsbudget=normal(maxbudg, maxbudg/20);
* random budget between max, 110%max
*   rhsbudget=uniform(maxbudg, 1.1*maxbudg);
   TOTBUDG.up = rhsbudget;
   SOLVE CAIE_40_model using MIP maximizing PORTF_SCORE;
   if (CAIE_40_model.modelstat<>1,
       put iter:6:0, ' INFEASIBLE'/;
   else
       put iter:6:0;
*   put g2:4:0
       put PORTF_SCORE.L:12:2
       put TOTPROJ.L:12:0;
       put TOTBUDG.L:12:2
       put rhsbudget:12:2
       loop(I, put X.L(I):3:0);
       put /;
      );
   );

finish=jnow;
elapsed_time=(finish-start)*86400;
put cdmfile 'Elapsed time: ',elapsed_time:12:2, ' seconds' / ;
```

```
putclose cdmfile ;
*$offtext
```

Phase 2 (normal distribution)

```
$TITLE project selection under uncertainty
$eolcom //
$ontext
Project selection problem
Data from Computers and Industrial Engineering 61 226-237 (2011)
$offtext

SETS
I     projects /1*40/
APPLIED(I) /1,3,4,6,10,11,16,18,20,21,24,25,27,30,31,32,33,35,36/
BASIC(I) /2,7,9,12,13,14,17,22,23,26,28,37,38,39/
DEVELOP(I) /5,8,15,19,29,34,40/
J  criteria  /cost,meth,pers,sci,tech /
TRIPLET the parameters of the triangular distribution /MIN,MID,MAX/
;

alias (I,I2);

parameter costmin(I) minimum cost for project(I)
/
1     341
2     31
3     316

39     32
40     145
/
parameter costmax(I) maximum cost for project(I)
/
1     447
2     42
3     493

39     49
40     158
/
parameter freq(I) frequency of participation in phase 1
/
1     1
2     0
3     0.74

39     0.68
40     1
/


parameter sc(I,J) random generated scores for alternative I in criterion J
parameter rcost(I) random generated cost for project I
*parameter rmeth(I) random generated cost for project I
*parameter rpers(I) random generated cost for project I
*parameter rsci(I) random generated cost for project I
*parameter rtech(I) random generated cost for project I
parameter rwght(J) random generated weight for criterion J
parameter tot_sc(I) total score for project I


table wght(J,TRIPLET) the triplet for the triangular distribution for the weights
        min       mid       max
cost     0.167     0.2051     0.231
meth      0.12      0.1356     0.141
pers     0.119     0.1364     0.157
sci      0.114     0.1274     0.149
tech     0.363     0.3955     0.434
;

parameter trigmid(J) normalized MID score for triangular distribution;
```

```
trigmid(J) = (wght(J,'MID')-wght(J,'MIN'))/(wght(J,'MAX')-wght(J,'MIN'));

scalar
elapsed_time elapsed time for payoff and e-constraint
start start time
finish finish time
iter  counter for iterations
sumwj sum of random generated weights
r auxiliary parameter
MCiter number of Monte Carlo iterations /100/
maxbudg  maximum budget (million toomans) /6000/
appl_ub applied projects upper bound percentage in total /0.7/
basic_ub basic research projects upper bound percentage in total /0.2/
dev_ub developing projects upper bound percentage in total /0.4/

rhsbudget RHS of the budget constraint
g2        counter for e-constraint
numg2     number of grid points /10/
meancost  mean value for costs
stdevcost standard deviation for costs
meanmeth  mean value methodology
stdevmeth standard deviation for methodology
meanpers  mean value personnel
stdevpers standard deviation for personnel
meansci   mean value  science
stdevsci  standard deviation for science
meantech  mean value technology
stdevtech standard deviation for technology

;

BINARY VARIABLES
X(I)    binary variable indicating if project I is selected or not

Positive variables
TOTBUDG total budget (million toomans)
TOTPROJ total projects;

FREE VARIABLES
PORTF_SCORE     total NPV in thousand USD;

EQUATIONS
EQ_TOTBUDG   equation for total budget
EQ_TOTPROJ   equation for total projects
EQ_APPL      constraint for applied projects
EQ_BASIC     constraint for basic research projects
EQ_DEV      constraint for developing projects
EQ_OBJ      objective function --> maximization of portfolio's score
;

EQ_TOTBUDG.. sum(I, rcost(I)*X(I))=e= TOTBUDG;
EQ_TOTPROJ.. sum(I, X(I)) =e= TOTPROJ;
EQ_APPL.. sum(APPLIED(I),X(I)) =l= appl_ub*TOTPROJ ;
EQ_BASIC.. sum(BASIC(I),X(I)) =l= basic_ub*TOTPROJ ;
EQ_DEV.. sum(DEVELOP(I),X(I)) =l= dev_ub*TOTPROJ ;
EQ_OBJ..  sum(I, freq(I)*X(I))=e= PORTF_SCORE;

rhsbudget = maxbudg;
TOTBUDG.up = rhsbudget;

* red and green projects from phase 1
X.FX('2')=0;
X.FX('7')=0;
X.FX('9')=0;
X.FX('12')=0;
X.FX('14')=0;
X.FX('17')=0;
X.FX('23')=0;
X.FX('1')=1;
X.FX('5')=1;
```

```
X.FX('8')=1;
X.FX('15')=1;
X.FX('19')=1;
X.FX('21')=1;
X.FX('27')=1;
X.FX('29')=1;
X.FX('30')=1;
X.FX('34')=1;
X.FX('38')=1;
X.FX('40')=1;

MODEL CAIE_40_model /ALL/ ;
OPTION OPTCR = 0;
option seed=1513;

FILE cdmfile /c:\tp\NORMAL ph2 100GR.txt/ ;
cdmfile.pw=2000;
put cdmfile ;

start=jnow;
iter=0;

for(iter=1 to MCiter,
* random generation of criteria weights from triangular distribution
$ontext
   loop(J,
        r=uniform(0,1);
        if (r<trigmid(J),
            rwght(J)=wght(J,'MIN') + sqrt(r*(wght(J,'MAX')-wght(J,'MIN'))*(wght(J,'MID')-wght(J,'MIN')));
        else
            rwght(J)=wght(J,'MAX') - sqrt((1-r)*(wght(J,'MAX')-wght(J,'MIN'))*(wght(J,'MAX')-wght(J,'MID')));
            );
       );
    sumwj=sum(J,rwght(J));
    loop(J, rwght(J)=rwght(j)/sumwj)
$offtext
* random generation of project scores from normal distribution
    loop(I,
        meancost=(costmax(I)+costmin(I))/2;
        stdevcost=(costmax(I)-costmin(I))/6;
        rcost(I)=normal(meancost,stdevcost);
        sc(I,'cost')=10*(1-(rcost(I)/smax(I2,costmax(I2))));

$ontext
        meanmeth=(methmax(I)+methmin(I))/2;
        stdevmeth=(methmax(I)-methmin(I))/6;
        sc(I,'meth')=round(normal(meanmeth,stdevmeth));

        meanpers=(persmax(I)+persmin(I))/2;
        stdevpers=(persmax(I)-persmin(I))/6;
        sc(I,'pers')=round(normal(meanpers,stdevpers));

        meansci=(scimax(I)+scimin(I))/2;
        stdevsci=(scimax(I)-scimin(I))/6;
        sc(I,'sci')=round(normal(meansci,stdevsci));

       meantech=(techmax(I)+techmin(I))/2;
        stdevtech=(techmax(I)-techmin(I))/6;
        sc(I,'tech')=round(normal(meantech,stdevtech));

*         sc(I,'cost')=10*(1-(rcost(I)/smax(I2,costmax(I2))));
*         sc(I,'meth')=round(normal(methmin(I)-0.5,methmax(I)+0.5));
*         sc(I,'pers')=round(normal(persmin(I)-0.5,persmax(I)+0.5));
*         sc(I,'sci')=round(normal(scimin(I)-0.5,scimax(I)+0.5));
*         sc(I,'tech')=round(normal(techmin(I)-0.5,techmax(I)+0.5));
        tot_sc(I)=sum(J,rwght(J)*sc(I,J));
$offtext
       );

* random budget ariund maxbudg
```

```
   rhsbudget=normal(maxbudg, maxbudg/20);
* random budget between max, 110%max
*    rhsbudget=uniform(maxbudg, 1.1*maxbudg);
   TOTBUDG.up = rhsbudget;
   SOLVE CAIE_40_model using MIP maximizing PORTF_SCORE;
   if (CAIE_40_model.modelstat<>1,
       put iter:6:0, ' INFEASIBLE'/;
   else
       put iter:6:0;
*    put g2:4:0
       put PORTF_SCORE.L:12:2
       put TOTPROJ.L:12:0;
       put TOTBUDG.L:12:2
       put rhsbudget:12:2
       loop(I, put X.L(I):3:0);
       put /;
      );
   );

finish=jnow;
elapsed_time=(finish-start)*86400;
put cdmfile 'Elapsed time: ',elapsed_time:12:2, ' seconds' / ;
putclose cdmfile ;
*$offtext
```

Model for iterative approach (40 telecommunication projects (Niaei et al. 2011))

```
$TITLE project selection under uncertainty
$eolcom //
$ontext
Project selection problem
Data from Computers and Industrial Engineering 61 226-237 (2011)

05.01.2013  e-constraint with Monte Carlo
the only intervention needed is to take out of the monte carlo loop all the declarations including text files
$offtext

SETS
I     projects /1*40/
SE(I) /1*2,4,19,22,24,27*28,33,36,38 /
NE(I) /3,10,16,18,21,25*26,31,35,37 /
CE(I) /5*7,9,11*12,14*15,23,29*30,34,40 /
GR(I) /8,13,17,20,32,39 /
EN(I) /1*4,13,17,24,26*27,33*34 /
IN(I) /5*9,29,32,38,40/
EE(I) /10*12,23,28,30,37 /
CG(I) /14*16,18*22,25,31,35*36,39 /
k objective functions /1*2/
;

Parameter dir(k) direction of the objective functions 1 for max and -1 for min
/ 1  1
  2  1
/

parameter cost(I) cost for project(I)
/
1       5930
2       50830
3       5000
4       33860
5       191870

35       93050
36       311780
37       772970
38       117580
39       190870
40       262030
/

parameter avgreturn(I) average NPV of project I
/
1       2500
2       49800
3       8300
4       63600
5       244600

35       204300
36       352100
37       223000
38       228800
39       428500
40       516100
/

parameter return(I) random return  ;
*return(I)=normal(avgreturn(I),(0.05*avgreturn(I)));

parameter avgcsr(I) average CSR index for project I
/
1       12.97
2       14.66
```

```
3       9.76
4       6.23
5       6.99

35       22.01
36       4.04
37       19.39
38       17.81
39       12.86
40       5.85
/
;
parameter csr(I) random csr  ;
*csr(I)=normal(avgcsr(I),(0.05*avgcsr(I)));

scalar
maxbudg  maximum budget (euros) /3000000/
;

BINARY VARIABLES
X(I)    binary variable indicating if project I is selected or not

Positive variables
TOTBUDG total budget (million toomans)
TOTPROJ total projects
Z(K)    onjective function values
;

EQUATIONS
EQ_TOTBUDG   equation for total budget
EQ_TOTPROJ   equation for total projects
EQ_SE      constraint for southern europe
EQ_NE      constraint for northern europe
EQ_CE      constraint for central europe
EQ_GR      constraint for greece
EQ_EN      constraint for energy sector
EQ_IN      constraint for industry sector
EQ_EE      constraint for electric equipment
EQ_CG      constraint for consumer goods
EQ_NPV      objective function --> maximization of portfolio's NPV
EQ_CSR      objective function --> maximization of portfolio's CSR
EQ_SE2      constraint for southern europe
EQ_NE2      constraint for northern europe
EQ_CE2      constraint for central europe
EQ_GR2      constraint for greece
EQ_EN2      constraint for energy sector
EQ_IN2      constraint for industry sector
EQ_EE2      constraint for electric equipment
EQ_CG2      constraint for consumer goods
;

EQ_TOTBUDG.. sum(I, cost(I)*X(I))=e= TOTBUDG;
EQ_TOTPROJ.. sum(I, X(I)) =e= TOTPROJ;
EQ_SE.. sum(SE(I),X(I)) =l= 0.5*TOTPROJ  ;    //7   11
EQ_NE.. sum(NE(I),X(I)) =l= 0.5*TOTPROJ ;    //6   10
EQ_CE.. sum(CE(I),X(I)) =l= 0.5*TOTPROJ ;    //9   13
EQ_GR.. sum(GR(I),X(I)) =l= 0.5*TOTPROJ ;    //4   6
EQ_EN.. sum(EN(I),X(I)) =l= 0.5*TOTPROJ ;    //7   11
EQ_IN.. sum(IN(I),X(I)) =l= 0.5*TOTPROJ ;    //6   10
EQ_EE.. sum(EE(I),X(I)) =l= 0.5*TOTPROJ ;    //4   7
EQ_CG.. sum(CG(I),X(I)) =l= 0.5*TOTPROJ ;    //9   13

EQ_SE2.. sum(SE(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
EQ_NE2.. sum(NE(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
EQ_CE2.. sum(CE(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
EQ_GR2.. sum(GR(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
EQ_EN2.. sum(EN(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
EQ_IN2.. sum(IN(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
EQ_EE2.. sum(EE(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
EQ_CG2.. sum(CG(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
```

```
EQ_CSR..  sum(I, csr(I)*X(I))=e= Z('1');
EQ_NPV..  sum(I, return(I)*X(I))=e= Z('2');

TOTBUDG.up = maxbudg;
TOTPROJ.lo = 5;

MODEL CSR_40_model /ALL/ ;

*---------------------------------------------------------------------------
Set k1(k) the first element of k, km1(k) all but the first elements of k;
k1(k)$(ord(k)=1) = yes; km1(k)=yes; km1(k1) = no;
Set kk(k)     active objective function in constraint allobj

Parameter
   rhs(k)       right hand side of the constrained obj functions in eps-constraint
   maxobj(k)    maximum value from the payoff table
   minobj(k)    minimum value from the payoff table
   intervals(k) number of intervals that we divide the k-1 objective functions
   bestobj(k)   the best objective function value (maxobj for dir=1 minobj for dir=-1)
   worstobj(k)  the worst objective function value (minobj for dir=1 maxobj for dir=-1)
   step(k)      the step obtained from range divided by intervals
   jump(k)      the jump for augmecon2

Scalar
iter   total number of iterations
infeas total number of infeasibilities
elapsed_time elapsed time for payoff and e-constraint
start start time
finish finish time
summax  auxiliary parameter
firstOffMax, lastZero, mciter some counters
mcitermax  monte carlo iterations /10/

Variables
   a_objval   auxiliary variable for the objective function
   obj        auxiliary variable during the construction of the payoff table
Positive Variables
   sl(k)      slack or surplus variables for the eps-constraints
Equations
   con_obj(k) constrained objective functions
   augm_obj   augmented objective function to avoid weakly efficient solutions
   allobj     all the objective functions in one expression;

con_obj(km1)..   z(km1) - dir(km1)*sl(km1) =e= rhs(km1);

* We optimize the first objective function and put the others as constraints
* the second term is for avoiding weakly efficient points
*augm_obj..
*  sum(k1,dir(k1)*z(k1))+1e-3*sum(km1,sl(km1)/(maxobj(km1)-minobj(km1))) =e= a_objval;
augm_obj..
  sum(k$(ord(k)=1),dir(k)*z(k))  +  1.0e-3*sum(k$(ord(k)>1),power(10,-(ord(k)-1))*sl(k)/(maxobj(k)-minobj(k))) =e= a_objval;

allobj..  sum(kk, dir(kk)*z(kk)) =e= obj;

Model mod_payoff    / CSR_40_model, allobj / ;
Model mod_epsmethod / CSR_40_model, con_obj, augm_obj / ;

Parameter
  payoff(k,k)  payoff tables entries;
Alias(k,kp);

option optcr=0.000;

loop(k, intervals(k)=20);

option limrow=0, limcol=0, solprint=off ;
*option limrow=3, limcol=3 ;
option seed=1515;
```

```
File fx  / c:\gams\CSR_40_2obj_MC_exact.txt /;
*******************************************************************************
start=jnow;
for (mciter=1 to mcitermax,

* random generation of return and profit from normal distributions
return(I)=normal(avgreturn(I),(0.05*avgreturn(I)));
return(I)=1000*round(return(I)/1000);
csr(I)=normal(avgcsr(I),(0.05*avgcsr(I)));

* Generate payoff table applying lexicographic optimization
loop(kp,
  kk(kp)=yes;
  repeat
    solve mod_payoff using mip maximizing obj;
    payoff(kp,kk) = z.l(kk);
    z.fx(kk) = z.l(kk); // freeze the value of the last objective optimized
    kk(k++1) = kk(k);   // cycle through the objective functions
  until kk(kp); kk(kp) = no;
* release the fixed values of the objective functions for the new iteration
  z.up(k) = inf; z.lo(k) =-inf;
);
if (mod_payoff.modelstat<>1 and mod_payoff.modelstat<>8, abort 'no optimal solution for mod_payoff');

loop (kp,
       loop(k, put fx payoff(kp,k):12:2);
       put /;
     );
*put fx /;

*display payoff;
minobj(k)=smin(kp,payoff(kp,k));
maxobj(k)=smax(kp,payoff(kp,k));

*-----------------------------------------------------------------------
*new 17.03.2013
*-----------------------------------------------------------------------
*loop(k, intervals(k)=(maxobj(k)-minobj(k))/1000);
loop(k, intervals(k)=20);

loop(k,
     if (dir(k)=1,
        bestobj(k)=maxobj(k);
        worstobj(k)=minobj(k);
     else
        bestobj(k)=minobj(k);
        worstobj(k)=maxobj(k)
        );
     step(k)=(maxobj(k)-minobj(k))/intervals(k)
     );

rhs(k)=worstobj(k);
iter=0;
infeas=0;
*start=jnow;

repeat
  solve mod_epsmethod maximizing a_objval using mip;
  iter=iter+1;
  if (mod_epsmethod.modelstat<>1 and mod_payoff.modelstat<>8,  // not optimal is in this case infeasible
    infeas=infeas+1;
    put fx iter:5:0, '  infeasible'/;
    lastZero = 0;
    loop(k$(ord(k)>1),
        if(abs(rhs(k)-worstobj(k))>0.001 and lastzero=0, lastzero=ord(k))
        );
    loop(k$(ord(k)>1 and ord(k)<=lastzero),rhs(k)=bestobj(k));
  else
    put fx mciter:5:0;   // for monte carlo counter
    put fx iter:5:0;
```

```
   loop(k, put fx z.l(k):12:2);
   put TOTPROJ.L:10:0;
   put TOTBUDG.L:12:0;
   loop(I, put fx X.L(I):4:0);
*   the jump is for AUGMECON2
   jump(k)=1;
*   The jump is calculated for the innermost objective function (ord(k)=2)
   jump(k)$(ord(k)=2) = 1+max(0,floor(sl.L(k)/step(k)));
   put rhs('2'):10:0,jump('2'):10:0,step('2'):10:0;
   loop(k$(jump(k)>1),put '   jump', jump(k):4:0);
*    put /;
   );
* Proceed forward in the grid
  firstOffMax = 0;
  loop(k$(ord(k)>1),
      if(abs(rhs(k)-bestobj(k))>0.001 and firstOffMax=0,
        if (dir(k)=1, rhs(k)=min((rhs(k)+jump(k)*step(k)),bestobj(k))
        else rhs(k)=max((rhs(k)-jump(k)*step(k)),bestobj(k))
           );
        firstOffMax=ord(k)
        );
      );
  put firstOffmax:5:0 /
  loop(k$(ord(k)>1),
      if(ord(k)< firstOffMax, rhs(k)=worstobj(k));
     );
  summax=0;
  loop(k$(ord(k)>1),
      if(abs(rhs(k)-bestobj(k))<=0.001, summax=summax+1);
      );

*until iter >= 100;
until (summax=card(k)-1 and firstOffMax=0) //or iter=100;

);   // for loop
*******************************************************************************


finish=jnow;
elapsed_time=(finish-start)*86400;


put 'Elapsed time: ',elapsed_time:10:2, ' seconds' / ;
putclose fx; // close the point file
```

Model for 300 CDM projects (5 iterations and average values, 1000 runs)

```
$TITLE project selection under uncertainty
$eolcom //
$ontext
Project selection problem
Data from CDM pipeline
$offtext

SETS
I      projects /1*300/

SWIND(I) /1*43/
LWIND(I) /44*110/
SHYDR(I) /111*155/
LHYDR(I) /156*204/
BIOMS(I) /205*235/
EEPGN(I) /236*255/
LANDF(I) /256*273/
CH4AV(I) /274*300/
GOLDST(I)
/23,30,36,51,52,57,58,59,60,62,68,70,72,73,74,76,82,83,87,88,89,90,91,92,93,95,96,97,98,99,100,101,104,106,107,110,154,155,188,204,219,227,22
8,229,231,232,233,234,250,272,273,285,286,288,289,290,291,294,296,297,298,299,300/

CHINA(I)
/4,8,24,31,37,46,47,49,50,51,52,54,57,58,59,60,62,63,64,65,66,68,70,71,72,73,74,75,77,78,79,80,81,82,83,87,88,89,90,91,92,93,94,95,96,97,98,99,1
00,101,103,104,105,106,107,108,109,110,117,119,121,122,123,129,131,132,134,135,136,137,139,140,141,143,145,147,148,151,152,160,163,166,16
7,168,169,171,172,174,178,179,181,182,183,185,186,188,190,191,193,194,196,197,199,201,202,203,221,232,240,242,243,247,249,250,251,253,254
,255,257,262,267,269,272,273,281,291,294,300/
INDIA(I)
/1,2,3,5,6,9,10,11,12,14,15,16,17,18,19,20,21,22,23,25,26,27,28,29,30,32,33,34,35,36,38,39,40,41,42,43,44,61,85,102,111,112,114,115,120,125,126,
133,150,153,162,164,173,176,184,205,206,207,211,212,213,219,220,224,227,228,229,233,234,237,238,239,244,246,252,268,274,275/
BRASIL(I)   /53,113,116,130,142,170,175,180,189,236,256,263/
SKOREA(I)   /7,86,118,146,198/
ECUAD(I)   /13,156,161/
PHILL(I)   /45,223,241,295/
MEXICO(I)   /48,55,67,69,159,261,265,266,284/
EGYPT(I)   /56,245/
CHILE(I)   /76,127,144,157,177,192,214/
VIETN(I)   /84,149,155,195,200,204/
PERU(I)   /124,128,138,158,165,187/
HOND(I)   /154,292/
MALAY(I)   /210,215,218,230,258,276,277,282,293/
INDONES(I)   /216,217,260,286,288,289/
SAFRIC(I)   /222,259,264,280/
THAI(I)   /225,226,248,270,278,279,283,285,287,290,296,297,298,299/
ARGEN(I)   /235,271/

T    type of project - technology /sw, lw, sh, lh, bi, eep, ldf, mav/
;

alias (I,I2);

parameter budg(I) budget for project(I) in million euros
/
1       8.4
2       3.5
3       4.4
4       14.0
5       7.7

295       10.1
296       5.1
297       1.6
298       3.9
299       2.6
300       4.9
/
```

```
parameter expcer(I) annual expected cers for project(I)
/
1       11
2       6.9
3       7.6
4       20
5       15

295     34
296     52
297     10
298     33
299     18
300     61
/

parameter avcer(I) average issuance success fot technology T
/
1       0.85
2       0.85
3       0.85
4       0.85
5       0.85

295     0.61
296     0.61
297     0.61
298     0.61
299     0.61
300     0.61
/

parameter sdcer(I) standard deviation of issuance success fot technology T
/
1       0.2
2       0.2
3       0.2
4       0.2
5       0.2

295     0.375
296     0.375
297     0.375
298     0.375
299     0.375
300     0.375
/
parameter rcer(I) random cers for project(I)

scalar
elapsed_time elapsed time for payoff and e-constraint
start start time
finish finish time
iter  counter for iterations
r auxiliary parameter
MCiter number of Monte Carlo iterations /1000/
maxbudg  maximum budget (million euros) /2000/

;

BINARY VARIABLES
X(I)    binary variable indicating if project I is selected or not

Positive variables
TOTBUDG total budget (million euros)
TOTPROJ total projects;

FREE VARIABLES
PORTF_CER     total CER from portfolio;
```

```
EQUATIONS
EQ_TOTBUDG   equation for total budget
EQ_TOTPROJ   equation for total projects
EQ_GS        constraint for GoldenStandard condition
EQ_GEO1      constraint for geographical condition 1
EQ_GEO2      constraint for geographical condition 2
EQ_GEO3      constraint for geographical condition 3
EQ_TECH1     constraint for technology condition 1
EQ_TECH2     constraint for technology condition 2
EQ_TECH3     constraint for technology condition 3
EQ_TECH4     constraint for technology condition 3
EQ_TECH5     constraint for technology condition 3
EQ_TECH6     constraint for technology condition 3

EQ_OBJ      objective function --> maximization of portfolio's CERs
;

EQ_TOTBUDG.. sum(I, budg(I)*X(I))=e= TOTBUDG;
EQ_TOTPROJ.. sum(I, X(I)) =e= TOTPROJ;
EQ_GS..      sum(GOLDST(I),X(I)) =g= 0.3*TOTPROJ ;

EQ_GEO1..    sum(CHINA(I),budg(I)*X(I)) =l= 0.4*TOTBUDG;
EQ_GEO2..    sum(INDIA(I),budg(I)*X(I)) =l= 0.3*TOTBUDG;
EQ_GEO3..    sum(BRASIL(I),X(I))+sum(ECUAD(I),X(I))+sum(MEXICO(I),X(I))+sum(CHILE(I),X(I))
           +sum(PERU(I),X(I))+sum(HOND(I),X(I))+sum(ARGEN(I),X(I))=g= 0.3*TOTPROJ;

EQ_TECH1..   sum(SWIND(I),budg(I)*X(I))+ sum(LWIND(I),budg(I)*X(I)) =g= 0.4*TOTBUDG;
EQ_TECH2..   sum(SHYDR(I),budg(I)*X(I))+ sum(LHYDR(I),budg(I)*X(I)) =g= 0.3*TOTBUDG;
EQ_TECH3..   sum(BIOMS(I),budg(I)*X(I)) =l= 0.1*TOTBUDG;
EQ_TECH4..   sum(EEPGN(I),budg(I)*X(I)) =l= 0.1*TOTBUDG;
EQ_TECH5..   sum(LANDF(I),budg(I)*X(I)) =l= 0.1*TOTBUDG;
EQ_TECH6..   sum(CH4AV(I),budg(I)*X(I)) =l= 0.1*TOTBUDG;

EQ_OBJ..  sum(I, rcer(I)*X(I))=e= PORTF_CER;

TOTBUDG.up = maxbudg;

MODEL CAIE_40_model /ALL/ ;
OPTION OPTCR = 0;
option seed=1513;

FILE cdmfile /c:\tp\pdd_results_1000.txt/ ;
cdmfile.pw=2000;
put cdmfile ;

start=jnow;

for(iter=1 to MCiter,
* random generation of project scores from normal distribution
   loop(I, rcer(I)=expcer(I)*normal(avcer(I),sdcer(I)));
    SOLVE CAIE_40_model using MIP maximizing PORTF_CER;
    if (CAIE_40_model.modelstat<>1,
        put iter:6:0, ' INFEASIBLE'/;
    else
        put iter:6:0;
*    put g2:4:0
        put PORTF_CER.L:12:2
        put TOTPROJ.L:12:0;
        put TOTBUDG.L:12:2
        loop(I, put X.L(I):3:0);
        put /;
       );
    );

finish=jnow;
elapsed_time=(finish-start)*86400;
put cdmfile 'Elapsed time: ',elapsed_time:12:2, ' seconds' / ;
putclose cdmfile ;
*$offtext
```

Model for Bi-objective problem with 300 CDM projects (6 iterations and average values)

```
$TITLE project selection under uncertainty
$eolcom //
$ontext
Project selection problem
Data from CDM pipeline
$offtext

SETS
I     projects /1*300/
K      criteria /1*2/

SWIND(I) /1*43/
LWIND(I) /44*110/
SHYDR(I) /111*155/
LHYDR(I) /156*204/
BIOMS(I) /205*235/
EEPGN(I) /236*255/
LANDF(I) /256*273/
CH4AV(I) /274*300/
GOLDST(I)
/23,30,36,51,52,57,58,59,60,62,68,70,72,73,74,76,82,83,87,88,89,90,91,92,93,95,96,97,98,99,100,101,104,106,107,110,154,155,188,204,219,227,22
8,229,231,232,233,234,250,272,273,285,286,288,289,290,291,294,296,297,298,299,300/

CHINA(I)
/4,8,24,31,37,46,47,49,50,51,52,54,57,58,59,60,62,63,64,65,66,68,70,71,72,73,74,75,77,78,79,80,81,82,83,87,88,89,90,91,92,93,94,95,96,97,98,99,1
00,101,103,104,105,106,107,108,109,110,117,119,121,122,123,129,131,132,134,135,136,137,139,140,141,143,145,147,148,151,152,160,163,166,16
7,168,169,171,172,174,178,179,181,182,183,185,186,188,190,191,193,194,196,197,199,201,202,203,221,232,240,242,243,247,249,250,251,253,254
,255,257,262,267,269,272,273,281,291,294,300/
INDIA(I)
/1,2,3,5,6,9,10,11,12,14,15,16,17,18,19,20,21,22,23,25,26,27,28,29,30,32,33,34,35,36,38,39,40,41,42,43,44,61,85,102,111,112,114,115,120,125,126,
133,150,153,162,164,173,176,184,205,206,207,211,212,213,219,220,224,227,228,229,233,234,237,238,239,244,246,252,268,274,275/
BRASIL(I)   /53,113,116,130,142,170,175,180,189,236,256,263/
SKOREA(I)   /7,86,118,146,198/
ECUAD(I)   /13,156,161/
PHILL(I)   /45,223,241,295/
MEXICO(I)   /48,55,67,69,159,261,265,266,284/
EGYPT(I)   /56,245/
CHILE(I)   /76,127,144,157,177,192,214/
VIETN(I)   /84,149,155,195,200,204/
PERU(I)   /124,128,138,158,165,187/
HOND(I)   /154,292/
MALAY(I)   /210,215,218,230,258,276,277,282,293/
INDONES(I)   /216,217,260,286,288,289/
SAFRIC(I)   /222,259,264,280/
THAI(I)   /225,226,248,270,278,279,283,285,287,290,296,297,298,299/
ARGEN(I)   /235,271/

T    type of project - technology /sw, lw, sh, lh, bi, eep, ldf, mav/
;

alias (I,I2);



parameter budg(I) budget for project(I) in million euros
/
1       8.4
2       3.5
3       4.4
4       14.0
5       7.7

295       10.1
296       5.1
```

```
297       1.6
298       3.9
299       2.6
300       4.9
/

parameter expcer(I) annual expected cers for project(I)
/
1       11
2       6.9
3       7.6
4       20
5       15

295       34
296       52
297       10
298       33
299       18
300       61
/


scalar
elapsed_time elapsed time for payoff and e-constraint
start start time
finish finish time
iter  counter for iterations
minbudg  minimum budget (million euros) /1800/

;

BINARY VARIABLES
X(I)    binary variable indicating if project I is selected or not

Positive variables
TOTBUDG total budget (million euros)
OVBUDG   budget overflow
TOTPROJ total projects
PORTF_CER     total CER from portfolio;

FREE VARIABLES
Z(K)    obgective function variables

EQUATIONS
EQ_TOTBUDG   equation for total budget
EQ_OVBUDG   equation for budget violation
EQ_TOTPROJ   equation for total projects
EQ_GS        constraint for GoldenStandard condition
EQ_GEO1      constraint for geographical condition 1
EQ_GEO2      constraint for geographical condition 2
EQ_GEO3      constraint for geographical condition 3
EQ_TECH1     constraint for technology condition 1
EQ_TECH2     constraint for technology condition 2
EQ_TECH3     constraint for technology condition 3
EQ_TECH4     constraint for technology condition 3
EQ_TECH5     constraint for technology condition 3
EQ_TECH6     constraint for technology condition 3

EQ_TOTCER     portfolio's CERs
EQ_OBJ1       first objective function maximization of portfolio's cers
EQ_OBJ2       second objective function minimization of budget violation
;

EQ_TOTBUDG.. sum(I, budg(I)*X(I))=e= TOTBUDG;
EQ_OVBUDG.. TOTBUDG=e= minbudg+OVBUDG;

EQ_TOTPROJ.. sum(I, X(I)) =e= TOTPROJ;
EQ_GS..      sum(GOLDST(I),X(I)) =g= 0.3*TOTPROJ ;
```

```
EQ_GEO1..    sum(CHINA(I),budg(I)*X(I)) =l= 0.4*TOTBUDG;
EQ_GEO2..    sum(INDIA(I),budg(I)*X(I)) =l= 0.3*TOTBUDG;
EQ_GEO3..    sum(BRASIL(I),X(I))+sum(ECUAD(I),X(I))+sum(MEXICO(I),X(I))+sum(CHILE(I),X(I))
           +sum(PERU(I),X(I))+sum(HOND(I),X(I))+sum(ARGEN(I),X(I))=g= 0.3*TOTPROJ;

EQ_TECH1..   sum(SWIND(I),budg(I)*X(I))+ sum(LWIND(I),budg(I)*X(I)) =g= 0.4*TOTBUDG;
EQ_TECH2..   sum(SHYDR(I),budg(I)*X(I))+ sum(LHYDR(I),budg(I)*X(I)) =g= 0.3*TOTBUDG;
EQ_TECH3..   sum(BIOMS(I),budg(I)*X(I)) =l= 0.1*TOTBUDG;
EQ_TECH4..   sum(EEPGN(I),budg(I)*X(I)) =l= 0.1*TOTBUDG;
EQ_TECH5..   sum(LANDF(I),budg(I)*X(I)) =l= 0.1*TOTBUDG;
EQ_TECH6..   sum(CH4AV(I),budg(I)*X(I)) =l= 0.1*TOTBUDG;

EQ_TOTCER..  sum(I, expcer(I)*X(I))=e= PORTF_CER;

EQ_OBJ1.. Z('1')=e=PORTF_CER;
EQ_OBJ2.. Z('2')=e=OVBUDG;

OVBUDG.up = 400;
OVBUDG.lo = 0;

model pps300_cdm /all/;

*---------------------------------------------------------------------------
$STitle eps-constraint method

Set k1(k) the first element of k, km1(k) all but the first elements of k;
k1(k)$(ord(k)=1) = yes; km1(k)=yes; km1(k1) = no;
Set kk(k)     active objective function in constraint allobj

Parameter
   rhs(k)     right hand side of the constrained obj functions in eps-constraint
   maxobj(k)  maximum value from the payoff table
   minobj(k)  minimum value from the payoff table

Parameter dir(k) direction of the objective functions
/
1  1
2  -1
/ ;


Scalar
iter   total number of iterations
infeas total number of infeasibilities
elapsed_time elapsed time for payoff and e-sonstraint
start start time
finish finish time

Variables
   a_objval   auxiliary variable for the objective function
   obj        auxiliary variable during the construction of the payoff table
Positive Variables
   sl(k)      slack or surplus variables for the eps-constraints
Equations
   con_obj(k) constrained objective functions
   augm_obj   augmented objective function to avoid weakly efficient solutions
   allobj     all the objective functions in one expression;

con_obj(km1)..   z(km1) - dir(km1)*sl(km1) =e= rhs(km1);

* We optimize the first objective function and put the others as constraints
* the second term is for avoiding weakly efficient points
augm_obj..
  sum(k1,dir(k1)*z(k1))+1e-3*sum(km1,sl(km1)/(maxobj(km1)-minobj(km1)+0.001)) =e= a_objval;

allobj..  sum(kk, dir(kk)*z(kk)) =e= obj;

Model mod_payoff    / pps300_cdm, allobj / ;
Model mod_epsmethod / pps300_cdm, con_obj, augm_obj / ;
```

```
Parameter
  payoff(k,k)  payoff tables entries;
Alias(k,kp);


File fx  / c:\gams\pdd300_out.txt /;
fx.pw = 2000;

option optcr=0.000;
start=jnow;

* Generate payoff table applying lexicographic optimization
loop(kp,
  kk(kp)=yes;
  repeat
    solve mod_payoff using mip maximizing obj;
    payoff(kp,kk) = z.l(kk);
    z.fx(kk) = z.l(kk); // freeze the value of the last objective optimized
    kk(k++1) = kk(k);   // cycle through the objective functions
  until kk(kp); kk(kp) = no;
* release the fixed values of the objective functions for the new iteration
  z.up(k) = inf; z.lo(k) =-inf;
);
if (mod_payoff.modelstat<>1 and mod_payoff.modelstat<>8, abort 'no optimal solution for mod_payoff');


PUT fx ' PAYOFF TABLE'/   ;
loop (kp,
        loop(k, put payoff(kp,k):12:2);
        put /;
     );
put fx /;

*display payoff;
minobj(k)=smin(kp,payoff(kp,k));
maxobj(k)=smax(kp,payoff(kp,k));


*$set fname h.%scrext.dat%


$if not set gridpoints $set gridpoints 20
Set g grid points /g0*g%gridpoints%/
    grid(k,g) grid
Parameter
    gridrhs(k,g) rhs of eps-constraint at grid point
    maxg(k) maximum point in grid for objective
    posg(k) grid position of objective
    firstOffMax, lastZero some counters
    first counter for printing output
    numk(k) ordinal value of k starting with 1
    numg(g) ordinal value of g starting with 0
    step(k) step of grid points in objective functions
    jump(k) jumps in the grid points' traversing
;
lastZero=1; loop(km1, numk(km1)=lastZero; lastZero=lastZero+1); numg(g) = ord(g)-1;

grid(km1,g) = yes; // Here we could define different grid intervals for different objectives
maxg(km1) = smax(grid(km1,g), numg(g));
step(km1)=(maxobj(km1)- minobj(km1))/maxg(km1);
gridrhs(grid(km1,g))$(dir(km1)=-1) = maxobj(km1) - numg(g)/maxg(km1)*(maxobj(km1)- minobj(km1));
gridrhs(grid(km1,g))$(dir(km1)=1) = minobj(km1) + numg(g)/maxg(km1)*(maxobj(km1)- minobj(km1));
*display gridrhs;

PUT fx ' Grid points'/   ;
loop (g,
        loop(km1, put gridrhs(km1,g):12:2);
        put /;
     );
put fx /;
```

```
put fx 'Pareto front with efficient solutions'/;

* Walk the grid points and take shortcuts if the model becomes infeasible
posg(km1) = 0;
iter=0;
infeas=0;

repeat
  rhs(km1) = sum(grid(km1,g)$(numg(g)=posg(km1)), gridrhs(km1,g));
  solve mod_epsmethod maximizing a_objval using mip;
  iter=iter+1;
  if (mod_epsmethod.modelstat<>1 and mod_epsmethod.modelstat<>8,  // not optimal is in this case infeasible
    infeas=infeas+1;
    put fx iter:5:0, '  infeasible'/;
    lastZero = 0; loop(km1$(posg(km1)>0 and lastZero=0), lastZero=numk(km1));
    posg(km1)$(numk(km1)<=lastZero) = maxg(km1); // skip all solves for more demanding values of rhs(km1)
  else
    put fx iter:5:0,' ';
    loop(k, put fx z.l(k):12:2);
    put TOTPROJ.L:12:0;
    put TOTBUDG.L:12:2
    loop(I, put X.L(I):3:0);
    put /;


    jump(km1)=1;
*   find last objective function in the sequence with slack less than step
    lastZero = 0; loop(km1$(floor(sl.L(km1)/step(km1))=0 and lastZero=0), lastZero=numk(km1));
    jump(km1)$(numk(km1)=1)=1+floor(sl.L(km1)/step(km1));
    loop(km1$(jump(km1)>1),put ' jump' /);
    );

* Proceed forward in the grid
  firstOffMax = 0;
  loop(km1$(posg(km1)<maxg(km1) and firstOffMax=0), posg(km1)=posg(km1)+jump(km1); firstOffMax=numk(km1));
  posg(km1)$(numk(km1)<firstOffMax) = 0;
until sum(km1$(posg(km1)=maxg(km1)),1)=card(km1) and firstOffMax=0;

finish=jnow;
elapsed_time=(finish-start)*86400;

put fx /;
put fx 'Infeasibilities = ', infeas:5:0 /;
put fx 'Elapsed time: ',elapsed_time:7:2, ' seconds' / ;

putclose fx; // close the point file
```

Model for group of 12 members working on 133 projects (6 iterations and average values)

```
*TITLE eps-Constraint Method for Multiobjective Optimization (EPSCM,SEQ=319)
$ontext
The eps-Constraint Method
$offtext
$inlinecom [ ]
$eolcom //
$STitle Example model definitions

sets
    p project /1*133/
    geo districts /1*13/
$ontext
 1          Eastern Macedonia Thrace
 2          Attica
 3          Northern Aegean
 4          Western Greece
 5          Western Macedonia
 6          Epirus
 7          Thessaly
 8          Éonian
 9          Central Macedonia
 10          Crete
 11          Southern Aegean
 12          Peloponnese
 13          Sterea Ellada
$offtext
    tech type of technology / WIND, SH, PV/
    dm decision makers /1*12/

*green  green projects /3,6/;
*red    red projects   /5/;

pgeo(geo,p) mapping of projects to districts
/
1.(10,22,48,90,108)
2.(60)
3.(86,87,97,98,118,120)
4.(101)
5.(4,7,41,88,89,129,130,131,132)
6.(57,68,75,96,100,104,106,113,119,123,133)
7.(37,58,64,65,66,72,77,81,95,109,111,112,115,122,124,125,126)
8.(39)
9.(15,43,50,56,62,67,73,79,92,105,110,114,116,117)
10.(99,102,127,128)
11.(47)
12.(11,16,19,26,27,29,42,44,61,84,85,107)
13.(1,2,3,5,6,8,9,12,13,14,17,18,20,21,23,24,25,28,30,31,32,33,34,35,36,38,40,45,46,49,51,52,53,54,55,59,63,69,70,71,74,76,78,80,82,83,91,93,94,1
03,121)
/

ptech(tech,p) mapping of projects to technologies
/
WIND.(1*53)
SH.(54*83)
PV.(84*133)
/

alias(dm, ddm);

table mcs(p,dm) multicriteria score for project p for decision maker dm
        1        2        3        4        5        6        7        8        9        10       11       12
1     0.6256   0.5353   0.3421   0.7158   0.6146   0.4868   0.6517   0.6473   0.5571   0.6047   0.5066   0.5456
2     0.5840   0.4699   0.2861   0.6477   0.5800   0.4352   0.6024   0.5917   0.5003   0.5391   0.4469   0.4912
3     0.6320   0.4401   0.2936   0.5746   0.6345   0.4543   0.6418   0.5844   0.4780   0.4786   0.4399   0.4642
4     0.6140   0.6427   0.5746   0.7176   0.6654   0.6391   0.6171   0.6822   0.6387   0.6921   0.6518   0.6503
5     0.6352   0.6332   0.4914   0.7658   0.6870   0.6183   0.6547   0.7165   0.6268   0.7003   0.6384   0.6307
6     0.5492   0.4992   0.4861   0.4447   0.4925   0.5014   0.5905   0.5013   0.4635   0.4314   0.4998   0.4276
```

7	0.5972	0.6360	0.5682	0.7446	0.6527	0.6165	0.5811	0.6716	0.6572	0.7172	0.6315	0.6823
8	0.6198	0.6002	0.4647	0.7177	0.6598	0.5893	0.6431	0.6833	0.5916	0.6528	0.6059	0.5897
9	0.5850	0.5558	0.4117	0.6929	0.6208	0.5368	0.6025	0.6441	0.5594	0.6206	0.5538	0.5604
10	0.5826	0.4155	0.3168	0.4709	0.5719	0.4452	0.6074	0.5246	0.4217	0.4041	0.4281	0.3958
11	0.5505	0.6048	0.4848	0.7150	0.5560	0.5465	0.5761	0.6321	0.5924	0.6603	0.5819	0.5918
12	0.5884	0.5382	0.4069	0.6423	0.6081	0.5291	0.6155	0.6234	0.5312	0.5742	0.5407	0.5214
13	0.5959	0.5437	0.4186	0.6337	0.6126	0.5393	0.6268	0.6262	0.5307	0.5690	0.5495	0.5175
14	0.4839	0.6613	0.7045	0.6224	0.4331	0.5797	0.4950	0.5413	0.6341	0.6565	0.6186	0.6365
15	0.3764	0.5066	0.4490	0.5804	0.4662	0.4956	0.3780	0.5158	0.4831	0.5764	0.5235	0.5090
16	0.5336	0.3926	0.2849	0.4766	0.5253	0.4025	0.5502	0.4942	0.4099	0.4062	0.3931	0.3928
17	0.5698	0.4118	0.3155	0.4649	0.5643	0.4431	0.5945	0.5186	0.4154	0.4012	0.4267	0.3908
18	0.5009	0.5447	0.4189	0.6972	0.5742	0.5195	0.4995	0.6106	0.5541	0.6443	0.5443	0.5768
19	0.5191	0.4746	0.3551	0.5573	0.4957	0.4420	0.5543	0.5319	0.4646	0.4906	0.4587	0.4449
20	0.5546	0.5549	0.4419	0.6572	0.6034	0.5492	0.5716	0.6243	0.5455	0.6068	0.5645	0.5487
21	0.4681	0.5076	0.4512	0.6039	0.5154	0.4870	0.4518	0.5343	0.5285	0.5814	0.5011	0.5521
22	0.5312	0.5780	0.4754	0.6642	0.5829	0.5652	0.5537	0.6245	0.5534	0.6254	0.5883	0.5573
23	0.5662	0.5228	0.4054	0.6105	0.5876	0.5193	0.5933	0.6004	0.5113	0.5509	0.5293	0.5012
24	0.4783	0.5434	0.4045	0.7282	0.5605	0.5032	0.4687	0.6088	0.5637	0.6700	0.5346	0.5958
25	0.4461	0.4639	0.4620	0.4121	0.3929	0.4436	0.4834	0.4330	0.4247	0.4112	0.4547	0.3977
26	0.5128	0.5794	0.5133	0.6439	0.5762	0.5761	0.5227	0.6078	0.5587	0.6264	0.5950	0.5709
27	0.5129	0.4950	0.3769	0.5805	0.4939	0.4551	0.5474	0.5421	0.4823	0.5177	0.4769	0.4661
28	0.4378	0.3756	0.3683	0.3229	0.4015	0.3955	0.4713	0.3905	0.3451	0.3135	0.3870	0.3145
29	0.4539	0.5948	0.5250	0.6437	0.5167	0.5729	0.4817	0.5925	0.5436	0.6346	0.6090	0.5522
30	0.5113	0.4901	0.4525	0.5030	0.5194	0.4993	0.5280	0.5194	0.4752	0.4850	0.5004	0.4663
31	0.5478	0.4790	0.3604	0.5658	0.5564	0.4757	0.5753	0.5621	0.4730	0.5011	0.4821	0.4589
32	0.4238	0.5714	0.5975	0.5484	0.3771	0.4966	0.4350	0.4734	0.5496	0.5710	0.5316	0.5507
33	0.5698	0.4118	0.3155	0.4649	0.5643	0.4431	0.5945	0.5186	0.4154	0.4012	0.4267	0.3908
34	0.5545	0.4824	0.3710	0.5542	0.5602	0.4846	0.5855	0.5629	0.4706	0.4933	0.4890	0.4531
35	0.4219	0.5821	0.6210	0.5421	0.3748	0.5098	0.4350	0.4735	0.5536	0.5732	0.5450	0.5537
36	0.4462	0.4751	0.4151	0.5757	0.4905	0.4543	0.4293	0.5065	0.4994	0.5496	0.4671	0.5222
37	0.5199	0.5630	0.4690	0.6235	0.5352	0.5368	0.5540	0.5906	0.5302	0.5852	0.5624	0.5220
38	0.4839	0.5314	0.5249	0.5249	0.5212	0.5451	0.4930	0.5320	0.5056	0.5339	0.5513	0.5090
39	0.4928	0.4948	0.4424	0.5808	0.5263	0.4817	0.4775	0.5298	0.5201	0.5549	0.4884	0.5375
40	0.5374	0.4712	0.3546	0.5582	0.5474	0.4677	0.5633	0.5529	0.4661	0.4948	0.4741	0.4532
41	0.3034	0.5052	0.4950	0.5416	0.3593	0.4600	0.3019	0.4472	0.4776	0.5640	0.4998	0.5055
42	0.4937	0.4327	0.3163	0.5417	0.5055	0.4187	0.5072	0.5121	0.4445	0.4768	0.4265	0.4402
43	0.3773	0.4992	0.4600	0.5631	0.4587	0.4887	0.3717	0.5003	0.4844	0.5655	0.5125	0.5117
44	0.4719	0.5365	0.4319	0.6296	0.4924	0.4923	0.4959	0.5605	0.5173	0.5865	0.5246	0.5190
45	0.3668	0.3714	0.3483	0.3700	0.3203	0.3386	0.3883	0.3569	0.3591	0.3542	0.3506	0.3431
46	0.4783	0.5434	0.4045	0.7282	0.5605	0.5032	0.4687	0.6088	0.5637	0.6700	0.5346	0.5958
47	0.5239	0.2721	0.2063	0.3389	0.5448	0.3463	0.5089	0.4188	0.3249	0.2797	0.2997	0.3162
48	0.5266	0.5954	0.5290	0.6098	0.5229	0.5664	0.5715	0.5922	0.5434	0.5880	0.5947	0.5269
49	0.5452	0.4230	0.3272	0.4724	0.5393	0.4434	0.5748	0.5138	0.4166	0.4132	0.4350	0.3925
50	0.3764	0.5066	0.4490	0.5804	0.4661	0.4956	0.3780	0.5158	0.4830	0.5763	0.5235	0.5089
51	0.5435	0.4599	0.3556	0.5212	0.5447	0.4672	0.5748	0.5405	0.4479	0.4625	0.4682	0.4280
52	0.4791	0.4665	0.3328	0.6189	0.5251	0.4415	0.4809	0.5471	0.4857	0.5525	0.4590	0.4985
53	0.5432	0.4530	0.3519	0.5087	0.5422	0.4634	0.5753	0.5343	0.4405	0.4509	0.4626	0.4190
54	0.5150	0.2981	0.2514	0.3191	0.5280	0.3747	0.5166	0.4192	0.3219	0.2756	0.3327	0.3039
55	0.5576	0.3105	0.2536	0.3499	0.5749	0.3927	0.5529	0.4507	0.3471	0.2975	0.3455	0.3313
56	0.4808	0.2642	0.2344	0.2721	0.4993	0.3496	0.4776	0.3809	0.2891	0.2378	0.3036	0.2734
57	0.5690	0.4192	0.3502	0.4398	0.5571	0.4564	0.5944	0.5085	0.4160	0.3909	0.4373	0.3890
58	0.5495	0.3148	0.2593	0.3487	0.5632	0.3925	0.5483	0.4474	0.3460	0.2983	0.3483	0.3288
59	0.5554	0.3063	0.2511	0.3451	0.5738	0.3901	0.5496	0.4472	0.3439	0.2933	0.3420	0.3286
60	0.5566	0.3056	0.2494	0.3457	0.5757	0.3899	0.5504	0.4479	0.3439	0.2933	0.3414	0.3288
61	0.5164	0.2758	0.2368	0.2990	0.5380	0.3651	0.5085	0.4079	0.3107	0.2570	0.3151	0.2966
62	0.5283	0.2867	0.2426	0.3142	0.5482	0.3735	0.5216	0.4205	0.3218	0.2693	0.3246	0.3070
63	0.3651	0.2278	0.2294	0.1837	0.3727	0.3002	0.3771	0.2927	0.2195	0.1753	0.2669	0.1986
64	0.5234	0.3279	0.2745	0.3470	0.5273	0.3919	0.5334	0.4382	0.3421	0.3020	0.3573	0.3206
65	0.5158	0.3158	0.2667	0.3329	0.5224	0.3838	0.5233	0.4275	0.3322	0.2898	0.3470	0.3117
66	0.4985	0.2939	0.2541	0.3052	0.5093	0.3686	0.5020	0.4058	0.3128	0.2666	0.3283	0.2941
67	0.5214	0.2892	0.2454	0.3132	0.5390	0.3727	0.5175	0.4178	0.3201	0.2695	0.3261	0.3043
68	0.5239	0.4803	0.4350	0.4709	0.5256	0.5032	0.5576	0.5205	0.4468	0.4489	0.5011	0.4248
69	0.5106	0.2960	0.2512	0.3147	0.5234	0.3726	0.5124	0.4152	0.3190	0.2724	0.3307	0.3009
70	0.3318	0.2371	0.2496	0.1695	0.3274	0.2965	0.3544	0.2737	0.2109	0.1711	0.2721	0.1856
71	0.5095	0.2931	0.2494	0.3115	0.5232	0.3709	0.5104	0.4130	0.3169	0.2697	0.3283	0.2993
72	0.4969	0.2926	0.2527	0.3039	0.5080	0.3675	0.5005	0.4046	0.3114	0.2654	0.3272	0.2927
73	0.4880	0.2692	0.2368	0.2798	0.5061	0.3539	0.4849	0.3877	0.2949	0.2439	0.3080	0.2791
74	0.3274	0.2322	0.2455	0.1642	0.3242	0.2927	0.3495	0.2692	0.2062	0.1663	0.2680	0.1812
75	0.4585	0.3494	0.3182	0.3251	0.4468	0.3911	0.4864	0.4061	0.3298	0.3012	0.3734	0.3024
76	0.3182	0.2530	0.2715	0.1705	0.3042	0.3020	0.3485	0.2700	0.2146	0.1781	0.2843	0.1860

77	0.4835	0.2863	0.2498	0.2925	0.4943	0.3606	0.4882	0.3937	0.3021	0.2567	0.3213	0.2832
78	0.4964	0.2811	0.2430	0.2950	0.5120	0.3617	0.4959	0.3992	0.3048	0.2563	0.3180	0.2879
79	0.4947	0.3244	0.2759	0.3310	0.4942	0.3820	0.5109	0.4198	0.3277	0.2915	0.3525	0.3040
80	0.5053	0.3494	0.2902	0.3600	0.4990	0.3968	0.5285	0.4403	0.3458	0.3164	0.3734	0.3194
81	0.4542	0.2668	0.2413	0.2611	0.4659	0.3433	0.4588	0.3661	0.2790	0.2324	0.3038	0.2603
82	0.2992	0.2949	0.3126	0.1965	0.2737	0.3222	0.3453	0.2806	0.2323	0.2106	0.3197	0.1988
83	0.4375	0.4133	0.4060	0.3617	0.4341	0.4431	0.4707	0.4275	0.3708	0.3614	0.4384	0.3476
84	0.3211	0.4174	0.4517	0.4283	0.4146	0.4444	0.2906	0.4055	0.4181	0.4658	0.4463	0.4539
85	0.3695	0.2466	0.1961	0.3062	0.3688	0.2637	0.3605	0.3198	0.2824	0.2651	0.2464	0.2795
86	0.3803	0.2284	0.2257	0.1928	0.3911	0.3045	0.3889	0.3028	0.2263	0.1806	0.2684	0.2067
87	0.3802	0.2283	0.2257	0.1928	0.3911	0.3045	0.3889	0.3028	0.2262	0.1806	0.2683	0.2067
88	0.3133	0.2340	0.2573	0.1575	0.3049	0.2888	0.3349	0.2575	0.2062	0.1657	0.2664	0.1818
89	0.3081	0.2158	0.2351	0.1440	0.3068	0.2782	0.3282	0.2500	0.1904	0.1496	0.2526	0.1666
90	0.3718	0.2512	0.2396	0.2380	0.3806	0.3054	0.3766	0.3139	0.2547	0.2228	0.2787	0.2412
91	0.3428	0.2340	0.2389	0.1934	0.3399	0.2875	0.3538	0.2798	0.2268	0.1875	0.2617	0.2082
92	0.3073	0.2517	0.2953	0.1997	0.3705	0.3370	0.2963	0.2898	0.2435	0.2268	0.3062	0.2478
93	0.3394	0.2304	0.2339	0.1931	0.3368	0.2827	0.3495	0.2769	0.2249	0.1863	0.2571	0.2070
94	0.3523	0.2221	0.2271	0.1738	0.3590	0.2932	0.3650	0.2821	0.2116	0.1680	0.2609	0.1907
95	0.4396	0.2561	0.2326	0.2507	0.4522	0.3316	0.4427	0.3531	0.2692	0.2234	0.2925	0.2517
96	0.3368	0.2210	0.2296	0.1657	0.3404	0.2880	0.3526	0.2720	0.2050	0.1632	0.2586	0.1830
97	0.3802	0.2284	0.2257	0.1928	0.3911	0.3045	0.3889	0.3028	0.2263	0.1806	0.2683	0.2067
98	0.3802	0.2283	0.2257	0.1928	0.3911	0.3045	0.3889	0.3028	0.2262	0.1806	0.2683	0.2067
99	0.4374	0.2476	0.2298	0.2368	0.4530	0.3295	0.4388	0.3466	0.2614	0.2121	0.2874	0.2442
100	0.2906	0.2401	0.2674	0.1603	0.2777	0.2831	0.3142	0.2472	0.2075	0.1730	0.2671	0.1840
101	0.3560	0.2505	0.2324	0.2378	0.3604	0.2966	0.3670	0.3073	0.2470	0.2208	0.2751	0.2313
102	0.4240	0.2430	0.2289	0.2264	0.4386	0.3237	0.4271	0.3362	0.2531	0.2046	0.2829	0.2354
103	0.3242	0.2277	0.2439	0.1612	0.3213	0.2880	0.3432	0.2640	0.2055	0.1643	0.2624	0.1821
104	0.3368	0.2210	0.2296	0.1657	0.3404	0.2880	0.3526	0.2720	0.2050	0.1632	0.2587	0.1830
105	0.3669	0.2273	0.2234	0.1931	0.3745	0.2963	0.3770	0.2953	0.2234	0.1811	0.2637	0.2039
106	0.3282	0.2151	0.2259	0.1566	0.3325	0.2830	0.3444	0.2644	0.1975	0.1558	0.2537	0.1753
107	0.3180	0.2515	0.2441	0.2483	0.3407	0.2923	0.3175	0.2951	0.2532	0.2417	0.2752	0.2497
108	0.3477	0.2098	0.2058	0.1795	0.3569	0.2775	0.3555	0.2776	0.2085	0.1675	0.2451	0.1909
109	0.4219	0.2504	0.2302	0.2394	0.4326	0.3229	0.4272	0.3401	0.2592	0.2150	0.2861	0.2411
110	0.3073	0.2517	0.2953	0.1997	0.3705	0.3370	0.2963	0.2898	0.2435	0.2268	0.3062	0.2478
111	0.4116	0.2437	0.2202	0.2394	0.4218	0.3117	0.4157	0.3327	0.2548	0.2132	0.2765	0.2381
112	0.3937	0.2245	0.1933	0.2381	0.4052	0.2861	0.3927	0.3174	0.2446	0.2068	0.2522	0.2317
113	0.3180	0.2117	0.2254	0.1483	0.3216	0.2788	0.3357	0.2566	0.1910	0.1499	0.2505	0.1684
114	0.3629	0.2248	0.2255	0.1830	0.3711	0.2970	0.3742	0.2907	0.2177	0.1739	0.2636	0.1973
115	0.3313	0.2439	0.2500	0.1889	0.3248	0.2939	0.3519	0.2785	0.2221	0.1870	0.2729	0.1991
116	0.3716	0.2270	0.2255	0.1888	0.3809	0.3006	0.3817	0.2970	0.2223	0.1779	0.2661	0.2024
117	0.3711	0.2265	0.2248	0.1887	0.3805	0.2999	0.3811	0.2967	0.2221	0.1777	0.2655	0.2022
118	0.3802	0.2283	0.2257	0.1928	0.3911	0.3045	0.3889	0.3028	0.2262	0.1806	0.2683	0.2067
119	0.2856	0.2027	0.2257	0.1242	0.2854	0.2655	0.3077	0.2320	0.1724	0.1333	0.2411	0.1482
120	0.3802	0.2283	0.2257	0.1928	0.3911	0.3045	0.3889	0.3028	0.2262	0.1806	0.2683	0.2067
121	0.4348	0.2624	0.2338	0.2635	0.4455	0.3302	0.4390	0.3557	0.2749	0.2342	0.2949	0.2581
122	0.3392	0.2284	0.2319	0.1790	0.3401	0.2893	0.3558	0.2778	0.2128	0.1739	0.2626	0.1909
123	0.3006	0.2129	0.2306	0.1426	0.2991	0.2730	0.3214	0.2457	0.1865	0.1479	0.2488	0.1630
124	0.3392	0.2284	0.2319	0.1790	0.3401	0.2893	0.3558	0.2778	0.2128	0.1739	0.2626	0.1909
125	0.3392	0.2284	0.2319	0.1790	0.3401	0.2893	0.3558	0.2778	0.2128	0.1739	0.2626	0.1909
126	0.3392	0.2284	0.2319	0.1790	0.3401	0.2893	0.3558	0.2778	0.2128	0.1739	0.2626	0.1909
127	0.4098	0.2386	0.2279	0.2159	0.4231	0.3175	0.4148	0.3256	0.2446	0.1972	0.2784	0.2263
128	0.3932	0.2342	0.2281	0.2038	0.4045	0.3109	0.4007	0.3133	0.2351	0.1889	0.2738	0.2159
129	0.2730	0.2241	0.2537	0.1407	0.2673	0.2732	0.2966	0.2331	0.1888	0.1559	0.2560	0.1660
130	0.3438	0.2211	0.2284	0.1683	0.3492	0.2906	0.3583	0.2764	0.2073	0.1645	0.2597	0.1857
131	0.2931	0.2126	0.2183	0.1596	0.2919	0.2631	0.3130	0.2469	0.1899	0.1589	0.2430	0.1686
132	0.2833	0.1960	0.2019	0.1464	0.2839	0.2487	0.3004	0.2340	0.1773	0.1449	0.2268	0.1573
133	0.2010	0.1622	0.1853	0.0914	0.1941	0.2016	0.2228	0.1685	0.1305	0.1035	0.1883	0.1102

;
parameter budg(p) budget for project p in keuro
/

1	5630
2	4800
3	3450
4	14450
5	11250

130	610
131	3540
132	2120
133	2120

```
/

parameter mw(p) power of project p (MW)
/
1       15
2       12.8
3       9.2
4       28.9
5       30

130       0.51
131       2.95
132       1.77
133       1.77
/

parameter w(dm) weight of decision maker dm (takes values 0 or 1);
w(dm)=1.0;   //initial value

scalar
start, finish start and finish time
elapsed_time  solution time

free Variables
   PORTFMCS  portfolio's multicriteria score
   TOTBUDG   total budget
   TOTMW     total MW
   NUMP      number of projects
Binary Variables
   X(p)      decision variables indicating if project p is selected if eq to 1
Equations
   objfun1    objective function for multicriteria score
   objfun2    function for budget
   objfun3    function for MW
   totnum     calculation of total number of projects
   geo1       geographical constraint 1
   geo2       geographical constraint 2
   geo3       geographical constraint 3
   tech1a     technological constraint 1a
   tech1b     technological constraint 1b
   tech2a     technological constraint 2a
   tech2b     technological constraint 2b
   tech3a     technological constraint 3a
   tech3b     technological constraint 3b
;
objfun1..    sum(p,sum(dm, mcs(p,dm)*w(dm))*X(p)) =e= PORTFMCS;
objfun2..    sum(p,budg(p)*X(p)) =e= TOTBUDG;
objfun3..    sum(p,mw(p)*X(p)) =e= TOTMW;
totnum..     sum(p,x(p))=e= NUMP;

geo1..       sum(pgeo('13',p),budg(p)*X(p))=l=0.3*TOTBUDG;
geo2..       sum(pgeo('12',p),budg(p)*X(p))=l=0.15*TOTBUDG;
geo3..       sum(pgeo('1',p),budg(p)*X(p)) + sum(pgeo('3',p),budg(p)*X(p))+
           sum(pgeo('5',p),budg(p)*X(p)) + sum(pgeo('6',p),budg(p)*X(p)) + sum(pgeo('11',p),budg(p)*X(p))=g=0.10*TOTBUDG;
tech1a..     sum(ptech('WIND',p),X(p)) =g= 0.2*NUMP;
tech1b..     sum(ptech('WIND',p),X(p)) =l= 0.6*NUMP;
tech2a..     sum(ptech('SH',p),X(p)) =g= 0.2*NUMP;
tech2b..     sum(ptech('SH',p),X(p)) =l= 0.6*NUMP;
tech3a..     sum(ptech('PV',p),X(p)) =g= 0.2*NUMP;
tech3b..     sum(ptech('PV',p),X(p)) =l= 0.6*NUMP;

TOTMW.lo = 300;
TOTBUDG.UP = 120000;

*X.FX(green)=1;
*X.FX(red)=0;

model project133 / all/ ;
start=jnow;
```

```
FILE fx /c:\gams\pdd\proj_133_group12_results.txt/ ;
fx.pw=2000;
put fx ;
option optcr=0.000;

put '       MSCORE     BUDGET    NUMBER  MW ' / ;
loop(ddm,
    loop(dm, w(dm)=0);
    w(dm)$(ord(dm)=ord(ddm))=1;
    solve project133 maximizing PORTFMCS using mip;
    put fx ddm.tl:5:0;
    put PORTFMCS.L:10:4;
    put TOTBUDG.L:10:0;
    put NUMP.L:10:0;
    put TOTMW.L:10:2;
    loop(p, put X.l(p):3:0);
    put /;
*   the jump is for AUGMECON2
   );

finish=jnow;
elapsed_time=(finish-start)*86400;

put /;
putclose fx; // close the point file
```

Model for bi-objective EECR and NPV

```
$TITLE project selection under uncertainty
$eolcom //
$ontext
Project selection problem
Data from Computers and Industrial Engineering 61 226-237 (2011)

05.01.2013  e-constraint with Monte Carlo
the only intervention needed is to take out of the monte carlo loop all the declarations including text files
$offtext

SETS
I     projects /1*40/
SE(I) /1*2,4,19,22,24,27*28,33,36,38 /
NE(I) /3,10,16,18,21,25*26,31,35,37 /
CE(I) /5*7,9,11*12,14*15,23,29*30,34,40 /
GR(I) /8,13,17,20,32,39 /
EN(I) /1*4,13,17,24,26*27,33*34 /
IN(I) /5*9,29,32,38,40/
EE(I) /10*12,23,28,30,37 /
CG(I) /14*16,18*22,25,31,35*36,39 /
k objective functions /1*2/
;

Parameter dir(k) direction of the objective functions 1 for max and -1 for min
/ 1  1
  2  1
/

parameter cost(I) cost for project(I)
/
1       5930
2       50830
3       5000

39       190870
40       262030
/

parameter avgreturn(I) average NPV of project I
/
1       2500
2       49800
3       8300

39       428500
40       516100
/

parameter return(I) random return  ;
*return(I)=normal(avgreturn(I),(0.05*avgreturn(I)));

parameter avgcsr(I) average CSR index for project I
/
1       12.97
2       14.66
3       9.76

39       12.86
40       5.85
/
;
parameter csr(I) random csr  ;
*csr(I)=normal(avgcsr(I),(0.05*avgcsr(I)));

scalar
maxbudg  maximum budget (euros) /3000000/
;
```

```
BINARY VARIABLES
X(I)    binary variable indicating if project I is selected or not

Positive variables
TOTBUDG total budget (million toomans)
TOTPROJ total projects
Z(K)    onjective function values
;

EQUATIONS
EQ_TOTBUDG   equation for total budget
EQ_TOTPROJ   equation for total projects
EQ_SE      constraint for southern europe
EQ_NE      constraint for northern europe
EQ_CE      constraint for central europe
EQ_GR      constraint for greece
EQ_EN      constraint for energy sector
EQ_IN      constraint for industry sector
EQ_EE      constraint for electric equipment
EQ_CG      constraint for consumer goods
EQ_NPV      objective function --> maximization of portfolio's NPV
EQ_CSR      objective function --> maximization of portfolio's CSR
EQ_SE2      constraint for southern europe
EQ_NE2      constraint for northern europe
EQ_CE2      constraint for central europe
EQ_GR2      constraint for greece
EQ_EN2      constraint for energy sector
EQ_IN2      constraint for industry sector
EQ_EE2      constraint for electric equipment
EQ_CG2      constraint for consumer goods
;

EQ_TOTBUDG.. sum(I, cost(I)*X(I))=e= TOTBUDG;
EQ_TOTPROJ.. sum(I, X(I)) =e= TOTPROJ;
EQ_SE.. sum(SE(I),X(I)) =l= 0.5*TOTPROJ  ;    //7   11
EQ_NE.. sum(NE(I),X(I)) =l= 0.5*TOTPROJ ;    //6   10
EQ_CE.. sum(CE(I),X(I)) =l= 0.5*TOTPROJ ;    //9   13
EQ_GR.. sum(GR(I),X(I)) =l= 0.5*TOTPROJ ;    //4   6
EQ_EN.. sum(EN(I),X(I)) =l= 0.5*TOTPROJ ;    //7   11
EQ_IN.. sum(IN(I),X(I)) =l= 0.5*TOTPROJ ;    //6   10
EQ_EE.. sum(EE(I),X(I)) =l= 0.5*TOTPROJ ;    //4   7
EQ_CG.. sum(CG(I),X(I)) =l= 0.5*TOTPROJ ;    //9   13

EQ_SE2.. sum(SE(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
EQ_NE2.. sum(NE(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
EQ_CE2.. sum(CE(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
EQ_GR2.. sum(GR(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
EQ_EN2.. sum(EN(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
EQ_IN2.. sum(IN(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
EQ_EE2.. sum(EE(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;
EQ_CG2.. sum(CG(I),cost(I)*X(I)) =g= 0.1*TOTBUDG  ;


EQ_CSR..  sum(I, csr(I)*X(I))=e= Z('1');
EQ_NPV..  sum(I, return(I)*X(I))=e= Z('2');

TOTBUDG.up = maxbudg;
TOTPROJ.lo = 5;

MODEL CSR_40_model /ALL/ ;

*---------------------------------------------------------------------------
Set k1(k) the first element of k, km1(k) all but the first elements of k;
k1(k)$(ord(k)=1) = yes; km1(k)=yes; km1(k1) = no;
Set kk(k)     active objective function in constraint allobj

Parameter
   rhs(k)       right hand side of the constrained obj functions in eps-constraint
   maxobj(k)    maximum value from the payoff table
```

```
   minobj(k)    minimum value from the payoff table
   intervals(k) number of intervals that we divide the k-1 objective functions
   bestobj(k)   the best objective function value (maxobj for dir=1 minobj for dir=-1)
   worstobj(k)  the worst objective function value (minobj for dir=1 maxobj for dir=-1)
   step(k)      the step obtained from range divided by intervals
   jump(k)      the jump for augmecon2

Scalar
iter   total number of iterations
infeas total number of infeasibilities
elapsed_time elapsed time for payoff and e-constraint
start start time
finish finish time
summax  auxiliary parameter
firstOffMax, lastZero, mciter some counters
mcitermax  monte carlo iterations /10/

Variables
   a_objval   auxiliary variable for the objective function
   obj        auxiliary variable during the construction of the payoff table
Positive Variables
   sl(k)      slack or surplus variables for the eps-constraints
Equations
   con_obj(k) constrained objective functions
   augm_obj   augmented objective function to avoid weakly efficient solutions
   allobj     all the objective functions in one expression;

con_obj(km1)..   z(km1) - dir(km1)*sl(km1) =e= rhs(km1);

* We optimize the first objective function and put the others as constraints
* the second term is for avoiding weakly efficient points
*augm_obj..
*  sum(k1,dir(k1)*z(k1))+1e-3*sum(km1,sl(km1)/(maxobj(km1)-minobj(km1))) =e= a_objval;
augm_obj..
  sum(k$(ord(k)=1),dir(k)*z(k))  +  1.0e-3*sum(k$(ord(k)>1),power(10,-(ord(k)-1))*sl(k)/(maxobj(k)-minobj(k))) =e= a_objval;

allobj..  sum(kk, dir(kk)*z(kk)) =e= obj;

Model mod_payoff    / CSR_40_model, allobj / ;
Model mod_epsmethod / CSR_40_model, con_obj, augm_obj / ;

Parameter
  payoff(k,k)  payoff tables entries;
Alias(k,kp);

option optcr=0.000;

loop(k, intervals(k)=20);

option limrow=0, limcol=0, solprint=off ;
*option limrow=3, limcol=3 ;
option seed=1515;
File fx  / c:\gams\CSR_40_2obj_MC_exact.txt /;
*******************************************************************************
start=jnow;
for (mciter=1 to mcitermax,

* random generation of return and profit from normal distributions
return(I)=normal(avgreturn(I),(0.05*avgreturn(I)));
return(I)=1000*round(return(I)/1000);
csr(I)=normal(avgcsr(I),(0.05*avgcsr(I)));

* Generate payoff table applying lexicographic optimization
loop(kp,
  kk(kp)=yes;
  repeat
    solve mod_payoff using mip maximizing obj;
    payoff(kp,kk) = z.l(kk);
    z.fx(kk) = z.l(kk); // freeze the value of the last objective optimized
    kk(k++1) = kk(k);   // cycle through the objective functions
```

```
  until kk(kp); kk(kp) = no;
* release the fixed values of the objective functions for the new iteration
  z.up(k) = inf; z.lo(k) =-inf;
);
if (mod_payoff.modelstat<>1 and mod_payoff.modelstat<>8, abort 'no optimal solution for mod_payoff');

loop (kp,
        loop(k, put fx payoff(kp,k):12:2);
        put /;
     );
*put fx /;

*display payoff;
minobj(k)=smin(kp,payoff(kp,k));
maxobj(k)=smax(kp,payoff(kp,k));

*-----------------------------------------------------------------------
*new 17.03.2013
*-----------------------------------------------------------------------
*loop(k, intervals(k)=(maxobj(k)-minobj(k))/1000);
loop(k, intervals(k)=20);

loop(k,
     if (dir(k)=1,
         bestobj(k)=maxobj(k);
         worstobj(k)=minobj(k);
     else
         bestobj(k)=minobj(k);
         worstobj(k)=maxobj(k)
         );
     step(k)=(maxobj(k)-minobj(k))/intervals(k)
     );

rhs(k)=worstobj(k);
iter=0;
infeas=0;
*start=jnow;

repeat
  solve mod_epsmethod maximizing a_objval using mip;
  iter=iter+1;
  if (mod_epsmethod.modelstat<>1 and mod_payoff.modelstat<>8,  // not optimal is in this case infeasible
    infeas=infeas+1;
    put fx iter:5:0, '  infeasible'/;
    lastZero = 0;
    loop(k$(ord(k)>1),
         if(abs(rhs(k)-worstobj(k))>0.001 and lastzero=0, lastzero=ord(k))
         );
    loop(k$(ord(k)>1 and ord(k)<=lastzero),rhs(k)=bestobj(k));
  else
    put fx mciter:5:0;   // for monte carlo counter
    put fx iter:5:0;
    loop(k, put fx z.l(k):12:2);
    put TOTPROJ.L:10:0;
    put TOTBUDG.L:12:0;
    loop(I, put fx X.L(I):4:0);
*   the jump is for AUGMECON2
    jump(k)=1;
*   The jump is calculated for the innermost objective function (ord(k)=2)
    jump(k)$(ord(k)=2) = 1+max(0,floor(sl.L(k)/step(k)));
    put rhs('2'):10:0,jump('2'):10:0,step('2'):10:0;
    loop(k$(jump(k)>1),put '   jump', jump(k):4:0);
*    put /;
    );
* Proceed forward in the grid
  firstOffMax = 0;
  loop(k$(ord(k)>1),
       if(abs(rhs(k)-bestobj(k))>0.001 and firstOffMax=0,
          if (dir(k)=1, rhs(k)=min((rhs(k)+jump(k)*step(k)),bestobj(k))
          else rhs(k)=max((rhs(k)-jump(k)*step(k)),bestobj(k))
```

```
          );
        firstOffMax=ord(k)
        );
      );
  put firstOffmax:5:0 /
  loop(k$(ord(k)>1),
      if(ord(k)< firstOffMax, rhs(k)=worstobj(k));
     );
  summax=0;
  loop(k$(ord(k)>1),
      if(abs(rhs(k)-bestobj(k))<=0.001, summax=summax+1);
      );
*until iter >= 100;
until (summax=card(k)-1 and firstOffMax=0) //or iter=100;

);   // for loop
********************************************************************************


finish=jnow;
elapsed_time=(finish-start)*86400;


put 'Elapsed time: ',elapsed_time:10:2, ' seconds' / ;
putclose fx; // close the point file
```

LIST OF PUBLICATIONS

Thesis:

[1] O. Pechak, *Definition of atmospheric pollutants and their amounts in exhaust fumes from incineration of municipal wastes* (in Ukrainian), Master Thesis, Engineering ecology department, National Technical University of Ukraine, Kiev, Ukraine, July 2007.

Handbooks:

[2] O. Kofanova, O. Pechak, "Basics of ecology. Glossary". *Handbook for students of National Technical University of Ukraine*, Kiev, Ukraine, Polytechnica, 2014.

Book Chapters:

[3] G. Mavrotas, O. Pechak, "Combining Mathematical Programming and Monte Carlo simulation to deal with uncertainty in energy project portfolio selection" (Chapter 16) in *Assessment and Simulation Tools for Sustainable Energy Systems*. F. Cavallaro (ed.). Springer, 2013.

Refereed Journals:

[4] O. Pechak, "Climate policy achievements in Eastern European countries", *Reporter of National Technical University of Ukraine "KPI"* (in English), Mining series, Vol. 19, 2010, pp. 221 - 229, Kiev, Ukraine.

[5] O. Pechak, G. Mavrotas and Diakoulaki D., "Role and contribution of the clean development mechanism to the development of wind energy", *Renewable and Sustainable Energy Reviews*, Vol. 15, 2011, pp. 3380 - 3387.

[6] G. Mavrotas, O. Pechak, "The trichotomic approach for dealing with uncertainty in project portfolio selection: Combining MCDA, mathematical programming and Monte Carlo simulation", *International Journal of Multicriteria Decision Making*, vol. 3, no. 1, 2013, pp. 79 – 96.

[7] G. Mavrotas, O. Pechak, E. Siskos, H. Doukas, J. Psarras,. "Robustness analysis in Multi-Objective Mathematical Programming using Monte Carlo simulation", *European Journal of Operations Research*, Vol. 240, 2015, pp. 193 – 201.

[8] G. Mavrotas, H. Doukas, O. Pechak, P. Xidonas, "Environmental Corporate Responsibility for Investments' evaluation: An Alternative Multi-objective Programming Model", *Annals of Operations Research*, Vol. 247, 2016, pp.395–413.