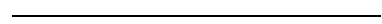




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Abstract. The present diploma thesis explores the problem of the possibility of mathematical explanations of natural phenomena. The problem is subsumed under the wider issue of the applicability of mathematics to the study of the non-mathematical universe and touches upon both philosophy of mathematics and philosophy of the (empirical) natural sciences. In the first part of the thesis, the problem is analyzed and arguments are expounded to the effect that there exist mathematical explanations of natural phenomena – arguments drawn from known examples in the relevant literature. The second part of the thesis reviews the theory of scientific explanation. The third part of the thesis reviews the major schools in the philosophy of mathematics focusing on which one can better approach the problem of applicability. Arguments are expounded toward the following conclusions. First, the causal-mechanical tradition in scientific explanation, conjoined with an ontological view of mathematical objects as abstract entities, does not allow for the possibility of explaining natural phenomena with the aid of mathematics. Second, Platonism, logicism and formalism put forth such ontological chasms between mathematical and natural objects that make the solution of the problem of applicability more difficult. By contrast, intuitionism conjoined with Kantian elements offers an interesting solution. Last, mathematical structuralism seems to offer the easiest and more reasonable solution.

ΠΕΡΙΕΧΟΜΕΝΑ

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3.5	μ	μ	62
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(indispensability arguments).⁴

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² . Polya (1954, 1977)

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. . . , Shapiro (2000, . 27).

⁴ « μ μ μ μ » .
. Colyvan (2008) Marcus (2016).

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μ Mark Steiner (Steiner, 1978a, 1978b).

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⁵ μ Hartry Field, , $\mu \mu$ *Science without Numbers* , “ μ (2) μ . Shapiro (2000, . 227-237). μ , “

(Steiner 1978b, p. 17-18).

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Steiner (1978b, p. 19):

⁶ $\det(A - \lambda I) = 0$.

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Fibonacci : 1,1,2,3,5,8,13,21,34,55,89,144, ...

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⁸ . Hales (2001) Hales (2000).

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⁹ . Baker (2005, . 229-236).

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(explanans-statements)

(explanans-facts).

1948 Carl Hempel Paul Oppenheim
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(requirement of maximal specificity, RMS).

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²¹ Scriven (1958, . 185).

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$a_0, a_1, a_2 \in \mathbb{R}$

$$a_0 r^2 + a_1 r + a_2 = 0,$$

$$r_1, r_2,$$

$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$

$$y = c_1 e^{r_1 x} + c_2 e^{r_2 x} \quad (c_1, c_2 \in \mathbb{R}).$$

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Colyvan (1999),

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$$y'' + y = 0 \quad ($$

$y'' - y = 0$

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μ μ μ μ (translation logicism).

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1910 David Hilbert.

«5+5=6+4»;

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