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Διατμηματικό Πρόγραμμα Μεταπτυχιακών Σπουδών
Φυσική και Τεχνολογικές Εφαρμογές

Μελέτη των resistive layers και προσομοίωση ανιχνευτων Micromegas

ΜΕΤΑΠΤΥΧΙΑΚΗ ΔΙΠΛΩΜΑΤΙΚΗ ΕΡΓΑΣΙΑ

του

Ιωάννη Δρίβα Κουλούρη

Επιβλέπων:
Θεόδωρος Αλεξόπουλος
Καθηγητής Ε.Μ.Π.

ΑΘΗΝΑ
Ιούνιος 2019

Εθνικο Μετσοβιο Πολυτεχνειο
ΣΧΟΛΗ ΕΦΑΡΜΟΣΜΕΝΩΝ ΜΑΘΗΜΑΤΙΚΩΝ & ΦΥΣΙΚΩΝ ΕΠΙΣΤΗΜΩΝ
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Εγκρίθηκε από την τριμελή εξεταστική επιτροπή στις .

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Αθήνα, Ιούνιος 2019

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Ιωάννης Δρίβας Κουλούρης
Φυσικός Εφαρμογών

Περίληψη

Η παρούσα διπλωματική εργασία εκπονήθηκε στα πλαίσια της ολοκλήρωσης των μεταπτυχιακών μου σπουδών και αποτελεί μια εισαγωγή της μελέτης των ανιχνευτών Micromegas.

Στο πρώτο κεφάλαιο γίνεται μια αναφορά στους ανιχνευτές αερίου και κυρίως στους ανιχνευτές MicroMegas.Ο ανιχνευτής MicroMegas ανήκει στην κατηγορία των ανιχνευτών αερίου, έχει μικρό κόστος, παρουσιάζει αντοχή σε περιβάλλον υψηλής ακτινοβολίας και παρέχει πολύ καλή χωρική και ενεργειακή διακριτική ικανότητα. Έχει χρησιμοποιηθεί σε πολλά πειραματα στο CERN έχει επιλεχθεί για την αναβάθμιση του New Small Wheel θαλάμου των μιονίων του πειράματος ATLAS.

Στο δεύτερο κεφάλαιο εργαζόμαστε πάνω στα στατικά και χρονοεξαρτημένα ηλεκτρικά πεδία κάποιων ανιχνευτών με παράλληλα στρώματα με δοσμένη ορατότητα και αδύναμη αγωγιμότητα. Δουλεύουμε πάνω στο πεδίο ενός σημειακού φορτίου, καθώς και στα weighting fields για readout pads και readout strips αυτές τις γεωμετρίες. Μελετάμε πως η διάδοση ενός φορτίου επηρεάζει τα στρώματα με αντίσταση. Επίσης προσπαθούμε να διερευνήσουμε την επίδραση της ¨ογκώδους¨ αντίστασης στα ηλεκτρικά πεδία και τα σήματα. Εφαρμόζουμε τα αποτελέσματα για να εξάγουμε πεδία και επαγόμενα σήματα σε Resistive Plate Chambers, ανιχνευτές MicroMegas που περιλαμβάνουν στρώματα αντίστασης για διασπορά φορτίου και προστασία από την εκφόρτιση. Αναλύουμε επίσης λεπτομερώς πως επηρεάζουν τα Resistive layers τα σχήματα των σημάτων και αυξάνουν το crosstalk μεταξύ των readout electrodes.

Στο τρίτο κεφάλαιο πραγματοποιείται η προσομοίωση διαφόρων μοντέλων ανιχνευτών MicroMegas τη χρήση του ANSYS Maxwell με στόχο την παρατήρηση και τον υπολογισμό της χωρητικότητας μεταξύ των strips.

Στο τέταρτο κεφάλαιο πραγματοποιείται η προσομοίωση διάφορων μοντέλων ανιχνετών MicroMegas μέσω του LT-spice και παρατηρήθηκε το σήμα εξόδου του κεντρικού strip και των γειτονικών του. Επίσης παρατηρήθηκε πως αλλάζει το σήμα με το να μεταβάλλουμε την χωρητικότητα μεταξύ των resistive strips, των readout strips και τη χωρητικότητα μεταξύ readout-resistive strips.

Abstract

The present diploma thesis is an introduction of the study of MicroMegas.

In section 1 we give a report to Gaseous detectors and specially to MicroMegas detectors. MicroMegas is a gaseous detector, with a low construction cost, a tolerance at high radiation environment and a very good spatial and energy resolution. This technology has been used so far in many experiments at CERN and is chosen for the New Small Wheel upgrade at the ATLAS experiment.

In section 2 we work on the static and time dependent electric fields in some detectors geometries with parallel layers of a given perimittivity and weak conductivity. We work on the field of a point charge,as well as the weighting fields for Readout pads and Readout strips in these geometries.We investigate how the spreading of the charge effect the Resistive layers.We also try to investigate the effect of "bulk" Resistivity on electric fields and signals.We apply the results to derive fields and induced signals in Resistive Plate Chambers,MicroMegas detectors including Resistive layers for charge spreading and discharge protection. We also discuss in details how the Resistive layers affect signal shapes and increase the crosstalk between readout electrodes.

In section 3 takes place the simulation of a variety of Modules of MicroMegas detectors with the help of ANSYS Maxwell,to observe and calculate the capacitance between the strips.

In section 4 takes place the simulation of a variety of Modules of Micromegas detector with LTspice. With spice were taken the figures of the output signal for the central and the neighbor strips. It also observed how this signal changes if we change the capacitance between the Resistive strips, the Readout strips and the capacitance between Readout-Resistive strips.

Ευχαριστίες

Με την ολοκλήρωση της παρούσας διπλωματικής εργασίας θα ήθελα να εκφράσω τις ειλικρινείς μου ευχαριστίες σε όλους όσους με βοήθησαν και στήριξαν σε όλη την διάρκεια της.

Αρχικα θα ήθελα να ευχαριστήσω την επιβλέποντα καθηγητή μου, κ. Θεόδωρο Αλεξόπουλο για την εμπιστοσύνη που μου έδωσε, καθώς και για την καθοδήγηση και τη βοήθεια του.

Θα ήθελα να ευχαριστήσω τον Αν. Καθηγητή Σταύρο Μαλτέζο για τις γνώσεις που προσφέρει πάντα με χαρά στους φοιτητές πάνω στη φυσική και στα ηλεκτρονικά.

Θα ήθελα να ευχαριστήσω τη διδάκτορα Χαρά Κιτσάκη για τη πολύτιμη βοήθεια που μου προσέφερε στην αρχή της εργασίας, αλλα και τις συμβουλές της μέχρι την ολοκλήρωση της, καθώς και για τις συζητήσεις που είχαμε κατά τη διάρκεια των ωρών που βρισκόμασταν στο γραφείο.

Θα ήθελα να ευχαριστήσω επίσης το διδάκτορα Αιμίλιο Κουλούρη για τη βοήθεια του πανω στα προγράγραμμα προσομοίωσης, αλλα και τον μεταπτυχιακό φοιτητή Πολυνείκη Τζανή για τη βοήθεια του πάνω σε τεχνικά θέματα της εργασίας.

Τέλος θα ήθελα να ευχαριστήσω τους γονείς μου Κωνσταντίνο και Κωνσταντίνα για την υποστήριξη και αγάπη τους.

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1 Gaseous Detectors

1.1 Avalanche

1.1.1 Gas Multiplication

One of the most important phenomenon that happens on the gaseous detectors is the Avalanche.

The primary electrons, so called the first ionisation products, drift to the electrodes of the anode through the gas. When their energy exceeds a sufficient value due to the affection of the field, they can create further ionizations in the gas. Secondary electrons, if the energy is sufficient, will do the same and this process goes on. So we have a production of ion-electron pairs and the formation of the avalanche. The electrons drifts faster and so the avalanche has the shape of a drop (figure 1). In the front we have the side electrons are moving along and on the back we have the slower electrons. At the end there is a large number of electrons in the cathode, that can easily be detected by electronic devices

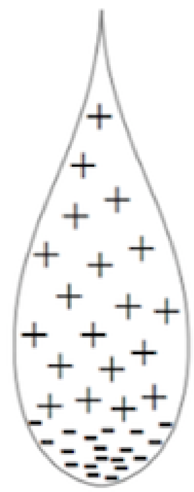


Figure 1: Avalanche in the shape of drop

Avalanche depends on electric field that is applied on gas chamber and the pressure of the gas, which affects the free path of the electrons.

If we consider λ as the mean free path of the electrons between two collisions then the coefficient a = $1/\lambda$ is the ionization probability per length. For a number of n electrons a path dx we will have further dn electrons :

$$\mathrm{d}n = na\mathrm{d}x \tag{1.1}$$

So if we derive the number n of electrons, in the path x is :

$$n = n_0 e^{ax} \tag{1.2}$$

with a the ionization probability :

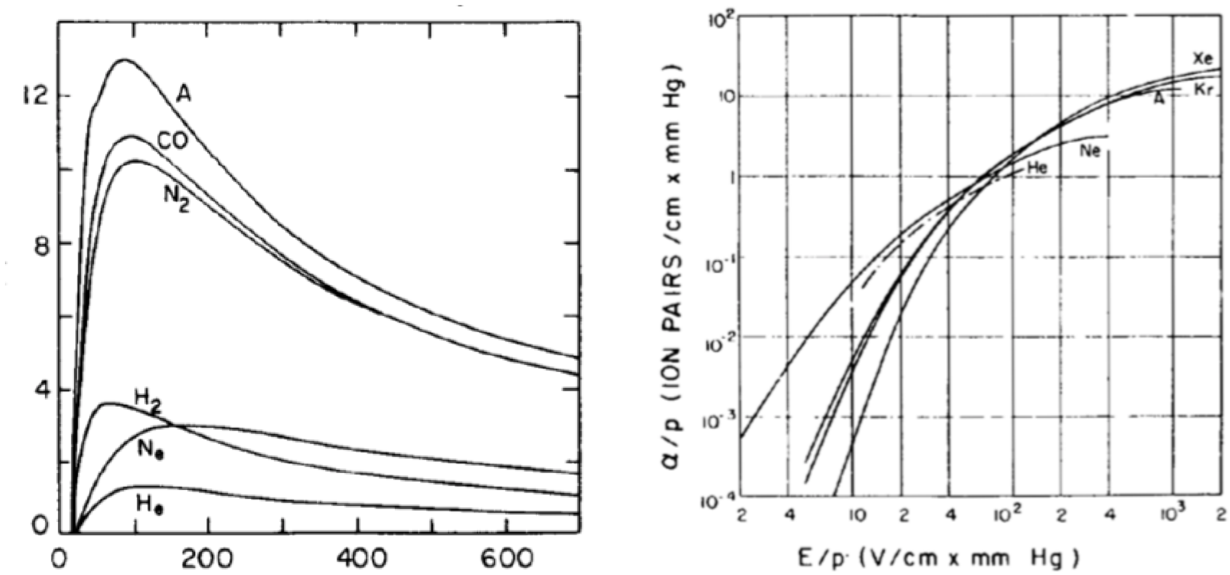
$$a = PAe^{\frac{BP}{E}} \tag{1.3}$$

with E the intensity of the electric field and A,B two constants depending on the gas in unit of $cm^{-1}Torr^{-1}$ and $Vcm^{-1}Torr^{-1}$ respectively.
Back to the number of electrons we can define M such the multiplier factor :

$$M = \frac{n}{n_0} = e^{ax} \tag{1.4}$$

The M factor is help to find a good approximation for the number of electrons that reach the anode, where we read the signal. The multiplier factor M has a limit of $M = 10^8$.

In figure 2 are represent for a variety of gases the ionization probability depending on energy and the dependence of the factor a/P due to E/P.



(a) Ionization Probability **(b)** Townsend Coefficient

Figure 2

1.1.2 The role of photons

During the multiplication process some electrons, that gained enough energy, instead of ionizing further the gas molecules, may bring them in an excited state. These excited molecules do not contribute directly to the avalanche but decay their ground state through the emission of a visible ultraviolet photon. Under some those photons can create ionizations in the gas with the limit of $M = 10^8$. These poly-atomic gases called the **quench gases**.

1.1.3 Signal Formation

During the avalanche formation, a number of electron-ions pairs is created. The electrons and the positive ions, separated by the electric field, drift towards different directions. Their motion within the gas volume induce charge on electrons. Electrons drift fast and within a few nanoseconds reach the anode. Therefore, the current flowing on the electrodes is on the order of the nanoseconds and usually ignored by the detector electronics. On the other hand the positive ions drift with a velocity two to three orders of magnitude less than this of electrons. Hence they induce charge on the electrodes with hundred nanoseconds of duration.
The method used to calculate the charge induced on an electrode is by using the Shockley-Ramo theorem and the concept of the weighting field. In the case of a charge q moving with a drift velocity u_drift the instantaneous current induced at a given electrode will be :

$$i(t) = qu_{drift}E_w \tag{1.5}$$

where E_w is the weighting field.

1.2 Gaseous Detectors

The gaseous detectors have been employed and operated in various applications and experiments with every successful results over the last century.
The differences between various types of gas counters with respect to their operation voltage is illustrated in figure 3. The number of ions pairs, equivalent to the detected pulse amplitude, is plotted as a function of the electric field for two different types of radiation.

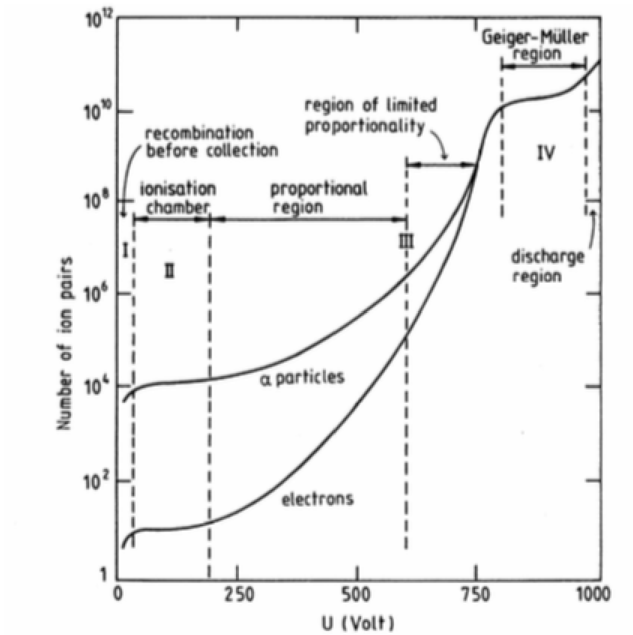


Figure 3: The different regions of gaseous detectors operation with respect to the applied voltage.

1. **Recombination Region** : in this region the electric fields are very low so the separation of the primary electron-ions pairs is not reliable and a fraction of the charge-pairs recombine with result we dont get any current.

2. **Ion Chamber Region** : in this region the voltage is enough to get electrons but cannot observe Avalanche and so the multiplication of the electrons.

3. **Proportional Counting Region** : in this region we can get the multiplication of $10^3 - 10^4$ proportional of applied voltage.

4. **Limited of proportionality** : at higher amplification the amount of charged ion in the vicinity of the anode increases and their space charge reduces the electric field by following electrons. We can see that the curve goes up so it does don't use by the detector.

5. **Geiger Region** : due to high voltage we have new Avalanches until the electric field to decreased so that to stop the multiplication and so we observe a constant value of multiplication.

6. **Discharge Region** : in this region the voltage is so high so we can observe a transition in the electric field with or without ionization and so it cannot be used by the detector.

1.3 Micromegas

Micromegas or Micro Mesh Gaseous Structure is a very assymetric double structure two stage parallel plate detector. Like the others gaseous detectors can detect charged and neutral particles. The difference with the others detector is that its two distinguished regions are no longer separated by a plane of wires but by a micromesh.

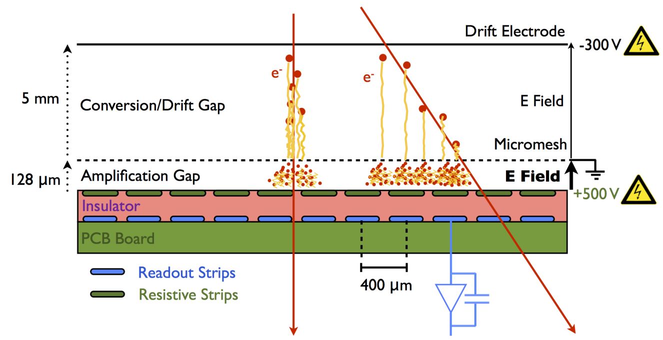


Figure 4: Micromegas

A MM consists of the following components :

1. anode electrode. Anode strips of gold-coated copper of 150 µm, with 200 µm pitch, are printed on a 1 mm substrate. The thickness of the copper strip was 5 µm. Thinner strips were obtained by vacuum deposition. These allow a substantial reduction of the inter strip capacitance. Both metal-deposition techniques can be applied on a 50 µm thick Kapton substrate, whenever a reduction of the material of the detector is required. The strips were grounded through low-noise charge pre-amplifiers of high gain (4V/pC).

2. quartz fibres of 75 µm, with 2 mm pitch, were stretched and glued on a G10 frame. The quartz frame was then mounted on the strip surface, defining a precise(2%) gap. Thicker(140 and 230 µm) quartz spacers were also utilized during our tests.

3. the micromesh. In figure is a photograph of the micromesh obtained with a microscope. It is a metallic grid, 3 µm thick, with 17 µm openings every 25 µm. It is made of nickel, using the electroforming technique, which is flexible and exhibits as high degree of fidelity of the electroposited layer.

4. the conversion-drift electric field was defined by applying negative voltages on the micromsh (HV2) and a slightly higher voltage on a second electrode (HV1), spaced by 3 mm in order to define a conversion-drift space. It was made by a standard nickel mesh, 100 µm thick, having 80% transparency, in order to allow a efficient penetration of the various radioactive sources used for the test and fixed on the top of the gross mesh. For the final detector, thin aluminized mylar can be used to define electrode HV1 and ensure at the same time the required gas tightness of the chamber.

5. the gas volume. The various elements of the parallel-plate structure were placed in a tight stainless steel vessel flushed by a standard gas mixture of Ar + 10% CH4 at atmospheric pressure. A metallic holder was mounted on top of the parallel plate chamber to support the radioactive source and a stainless steel collimator 1 mm thick with a 2 mm hole. The metallic source holder can move horizontally and allow a rough scan of the active surface of the detector.

The Micromegas detector can be separated on two region by the electric field.

- drift region
- multiplication region

In the drift region we have the first ionization and due to not too strong electric field ($1 - 5$ kV/cm) the electrons are heading to the multiplication region without a a strong Avalanche.
In the multiplication region the electric field is more stronger ($20 - 100$ kV/cm) and the result is a strong Avalanche and there is a strong signal on anode.

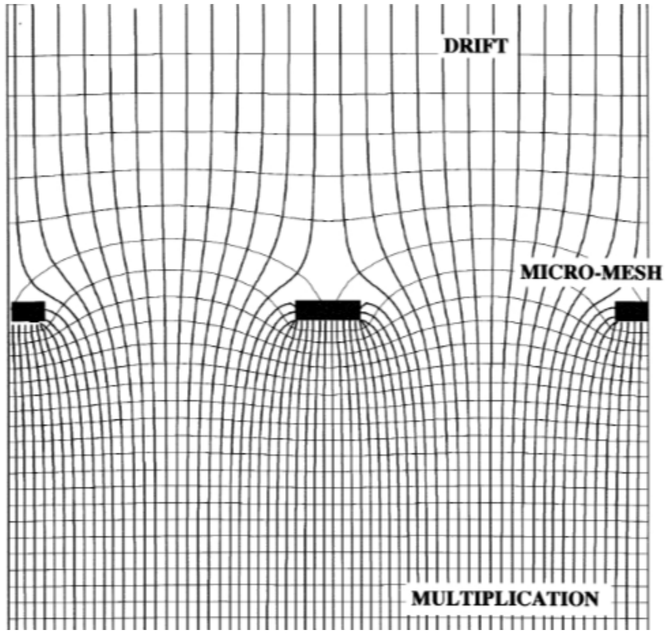


Figure 5: Two areas of electric field

1.3.1 Signal Formation

When charges are moving in front of a conductor, a proportional charge is induced on the conductor. The Micromegas geometry, as shown in figure 4, contains the amplification gap, where the electric field is up to $50\,\mathrm{kV/cm}$. This gives rise to an avalanche effect of ion-electron pairs being created due to ionizations.
To simplify for this geometry for a charge q, the induced current is :

$$I_a = -q\frac{E_A u}{V_a} \tag{1.6}$$

with E_A the electric field at position of the charge, u the velocity of the charge and V_a the potential of the strips. Taking all the charges into account as a current density J(x,t) the induced current take the form :

$$I_a = -\frac{1}{V_a}\int J(x,t)\cdot E(x,t)\mathrm{d}^3x \tag{1.7}$$

To find the total signal formation, we need to find the evolution of charge over time. The charge multiplication charge depends on the (first) Townsend coefficient a which in general is a function of the electric field. For n the number of one type of charged particles at a certain point,then their increase at a nearby point along the path of the moving charge will be $dn = andr$, where dr is the distance between the two points.
To find the increase of charge we have to integrate the above formula. So :

$$I(t) = -q_0\frac{a\beta e^{a\beta u_p t}u_p}{V_a}\int E_z(z)e^{a\beta z}\mathrm{d}z \tag{1.8}$$

where q_0 the initial charge (one e^-), u_p/n the velocity of the ions/electrons, E_z the z-component of the electric field. The derived current as a function of time is drawn in figure 6.

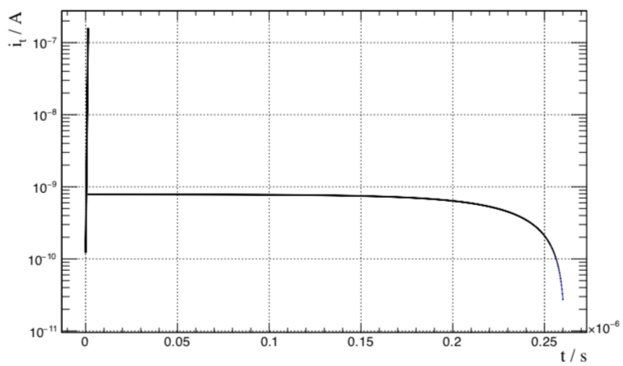


Figure 6: Total auxiliary current as a function of time. The peak at the origin lasts for less than 2 ns is due primarily to the electrons movement while after 2 ns the distribution id due to the ion drifting towards the mesh.

1.3.2 Resistive strips Micromegas

Despite the excellent characteristics of the Micromegas module and the promising industrial bulk fabrication procedure, the very thin amplification region along with the finely sculpted readout structure makes them particularly vulnerable to discharges(sparks).Sparks occur when the electron avalanche population goes beyond $\sim 10^6$.Sparks may damage the detector and readout electronics and/or lead to large dead times as result of HV breakdown. To find a solution for this problem we create bulk-micromegas chambers spark resistant while mainting their ability to measure with excellent minimum-ionizing particles in high-rated environments.

(a) view along the strip direction

(b) side view,orthogonal to the strip direction

Figure 7: Sketch of the detector principle(not the scale), illustrating the resistive protection theme

In the figures 7 we see two orthogonal side views of the chamber. It is a bulk-micromegas structure built on top of a printed circuit board with 18 µm thick Cu readout strips covered by a resistive protection layer.
The protection consists a thin layer of insulator on top of which strips of resistive paste (with a resistivity of a few MΩare deposited. Geometrically, the resistive strips match the pattern of the readout strips. They both are 150 µm wide and 80 µm long, their strip pitch is 250 µm. The resistive strips are 64 µm thick; the 100 µm wide gaps between neighboring strips are filled with insulator. The resistive strips are connected at one end to the detector ground through a $15 - 50\,\mathrm{M\Omega}$ resistor, see below. We opted for resistive strips rather than a continuous resistive layer for two reasons: i) to avoid charge spreading across several readout strips, and ii) to keep the area affected by a discharge as small as possible.

The Micromegas structure is built on top of the resistive strips. It employs a woven stainless steel mesh with 400 $lines/inch$ and a wire thickness of 18 µm. The mesh is kept at a distance of 128 µm from the resistive strips by means of small pillars (400 µm diameter) made of the same photoimageable coverlay material that is used for the insulation layer. The pillars are arranged in a regular matrix with a distance between neighbouring pillars of 2.5 mm in x and y. The mesh covers an area of $100\times\ 100\,\mathrm{mm}^2$.

Above the amplification mesh, at a distance of 4 or $5\,mm$, another stainless steel mesh ($350\,textrmlines/inch$, wire diameter: 22 µm served as drift electrode. Its lateral dimensions are the same as for the amplification mesh. The chamber comprises 360 readout strips. The readout strips are left floating at one end. At the other end they are connected in groups of 72 strips to five 80-pin connectors. The remaining eight pins of each connector serve as grounding points.

The detector housing consists of a $20\,\text{mm}$ high aluminum frame, mounted on top of the readout board and sealed by an O-ring, and a cover plate (again sealed by an O-ring) with some opening windows, made of $50\,\mu\text{m}$ thick Kapton foil.

2 Electric fields,weighting fields and signals in detectors including resistive materials

In this section we discuss the electric fields and the signals in detectors that represent parallel plate geometries with segmented readout like GEM's,Micromegas,RPC's. In these detectors, the charges generated inside the sensor volume act as a source of the signal.

2.1 Potential of a point charge centered at the origin

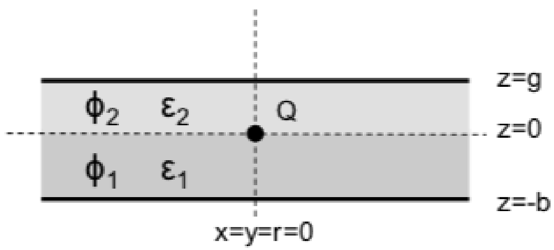


Figure 8: A point charge Q on the boundary between two dielectric layers.

We first investigate the electric field of a point charge in a two layer geometry. We assume that the two layers have thickness of b and g with constant dielectric permittivity of $\epsilon_1$and ϵ_2, surrounded by grounded metal plates. At r=0,z=0(the boundary between those two layers) we put a charge Q.
We will use cylindrical coordinates due to problem's rotational symmetry. Because we don't have charges on the two layers we will use the Laplace equation. For r=0 the coefficients $Y_0(kr)$ are zero so the general solution for the two areas will be :

$$\varphi_1(r,z) = \frac{1}{2\pi}\int_0^\infty J_0(kr)[A_1(k)e^{kz} + B_1(k)e^{-kz}]\mathrm{d}k \qquad -b<z<0 \tag{2.1}$$

$$\varphi_2(r,z) = \frac{1}{2\pi}\int_0^\infty J_0(kr)[A_2(k)e^{kz} + B_2(k)e^{-kz}]\mathrm{d}k \qquad 0<z<g \tag{2.2}$$

Boundary Conditions

Because we have ground at z=g, z=-b we have the conditions $\varphi_1(r,-b)=0 \quad \varphi_2(r,g)=0$:

$$A_1e^{-kb} + B_1e^{kb} = 0 \tag{2.3}$$

$$A_2e^{kg} + B_2e^{-kg} = 0 \tag{2.4}$$

At z=0, the barrier between the two layers, we assume a surface charge density q(r). From Gauss Law for a medium inhomogeneous perimittivity we derive that passing through an infinitely thin sheet of charge with a surface charge density q(r), the pontential is continuous so $\varphi_1(r,0)=\varphi_2(r,0)$ which gives :

$$A_1 + B_1 = A_2 + B_2 \tag{2.5}$$

and the ϵE component perpendicular to the sheet "jumps" by q(r) :

$$\epsilon_1\frac{\partial\varphi_1(r,z)}{\partial z}|_{z=0} - \epsilon_2\frac{\partial\varphi_2(r,z)}{\partial z}|_{z=0} = q(r)$$

with $q(r) = \frac{q\delta(r)}{2\pi}\frac{1}{r}$

$$\frac{1}{2\pi}\int_0^\infty J_0(kr)[(k\epsilon_1A_1 - k\epsilon_1B_1) - (k\epsilon_2A_2 - k\epsilon_2B_2)]\mathrm{d}k = \frac{q\delta(r)}{2\pi}\frac{1}{r}$$

by multiplying both sides with $rJ_0(k'r)$ and integrate from 0 to infinity :

$$\frac{1}{2\pi}\int_0^\infty rJ_0(kr)J_0(k'r)dr\int_0^\infty [(k\epsilon_1 A_1 - k\epsilon_1 B_1) - (k\epsilon_2 A_2 - k\epsilon_2 B_2)]\mathrm{d}k = \int_0^\infty rJ_0(k'r)\frac{q\delta(r)}{2\pi}\frac{1}{r}\mathrm{d}r$$

$$\frac{1}{2\pi}\int_0^\infty \delta(k-k')[\epsilon_1\left(A_1 - B_1\right) - \epsilon_2\left(A_2 - B_2\right)]\mathrm{d}k = \frac{Q}{2\pi}$$

$$\epsilon_1\left(A_1 - B_1\right) - \epsilon_2\left(A_2 - B_2\right) = Q \tag{2.6}$$

From (2.3),(2.4) :

$$B_1 = -A_1 e^{-2kb}$$
$$B_2 = -A_2 e^{2kg}$$

By using them on (2.5) :

$$A_1\left(1 - e^{-2kb}\right) = A_2\left(1 - e^{2kg}\right)$$

$$A_2 = \frac{1 - e^{-2kb}}{1 - e^{2kg}}$$

And so the (2.6) :

$$2\epsilon_1 A_1 e^{-kb}\cosh\left(kb\right) - \epsilon_2 A_1 \frac{e^{-kb}\left(e^{kb} - e^{-kb}\right)}{e^{kg}(e^{-kg} - e^{kg})} e^{kg}\left(e^{-kg} + e^{kg}\right) = Q$$

$$A_1 = \frac{Qsinh\left(kg\right)e^{kb}}{2\left(\epsilon_1\cosh\left(kb\right)\sinh\left(kg\right) + \epsilon_2 sinh\left(kb\right)\cosh\left(kg\right)\right)}$$

we set

$$D\left(k\right) = 4\left(\epsilon_1\cosh\left(kb\right)\sinh\left(kg\right) + \epsilon_2 sinh\left(kb\right)\cosh\left(kg\right)\right)$$

and so

$$A_1 = \frac{2Qsinh\left(kg\right)e^{kb}}{D\left(k\right)}$$
$$B_1 = \frac{-2Qsinh\left(kg\right)e^{-kb}}{D\left(k\right)}$$
$$A_2 = \frac{-2Qsinh\left(kb\right)e^{-kg}}{D\left(k\right)}$$
$$B_2 = \frac{2Qsinh\left(kb\right)e^{kg}}{D\left(k\right)}$$

And so the solution read as :

$$\phi_1(r,z) = \frac{Q}{2\pi}\int_0^\infty J_0\left(kr\right)\frac{4sinh\left(gk\right)\sinh\left(k\left(b+z\right)\right)}{D(k)}\mathrm{d}k \qquad -b < z < 0 \tag{2.7}$$

$$\phi_2(r,z) = \frac{Q}{2\pi}\int_0^\infty J_0(kr)\frac{4\sinh(bk)sinh((k(g-z))}{D(k)}\mathrm{d}k \qquad 0 < z < g \tag{2.8}$$

Those integrals cannot be expressed in closed form, so we have to find some techniques to express the result as an infinite series.
We will take the integral quantity for ϕ_1 : $\frac{4\sinh(gk)\sinh(k(b+z))}{D(k)}$ and add and remove the quantity $\frac{e^{kz}}{\epsilon_1+\epsilon_2}$:

$$\frac{e^{kz}}{\epsilon_1+\epsilon_2}+\frac{4\sinh(gk)\sinh(k(b+z))}{D(k)}-\frac{e^{kz}}{\epsilon_1+\epsilon_2}=\frac{e^{kz}}{\epsilon_1+\epsilon_2}+f_1(k,z)$$

We know that

$$\frac{1}{\sqrt{r^2+z^2}}=\int_0^\infty J_0(kr)e^{-kz}dk \tag{2.9}$$

By doing that ϕ_1 take the form :

$$\phi_1(r,z)=\frac{Q}{2\pi(\epsilon_1+\epsilon_2)}\frac{1}{\sqrt{r^2+z^2}}+\frac{Q}{2\pi}\int_0^\infty J_0(kr)f_1(k,z)\mathrm{d}k \tag{2.10}$$

Now the solution is the combination of the solution for the potential of a charge Q on the boundary of two infinite half-spaces of permittivity ϵ_1 and ϵ_2 with a correction term. For the correction term, for large values of k the quantity $f_1(k,z)$ take the form :

$$f_1(k,z)=\frac{e^{-k(2b+z)}}{\epsilon_1+\epsilon_2}-\frac{-2\epsilon_2 e^{-k(2g-z)}}{\epsilon_1+\epsilon_2} \tag{2.11}$$

As result the potential for $-b<z<0$:

$$\phi_1(r,z)=\frac{Q}{2\pi(\epsilon_1+\epsilon_2)}\frac{1}{\sqrt{r^2+z^2}}-\frac{Q}{2\pi(\epsilon_1+\epsilon_2)}\frac{Q}{\sqrt{r^2+(2b+z)^2}}-\frac{2Q\epsilon_2}{2\pi(\epsilon_1+\epsilon_2)}\frac{1}{\sqrt{r^2+(2g-z)^2}}+\frac{Q}{2\pi}\int_0^\infty J_0(kr)f_2(k,z)\mathrm{d}k \tag{2.12}$$

The two new terms correspond to two "mirror" charges, one with value $-Q$ at $z=-2b$ that is reflected at the grounded plate at $z=-b$ and one with value $-2\epsilon_2 Q$ at $z=2g$ that is reflected at the grounded plate at $z=g$. If we want to find the ϕ_2 is the same process and result is reversed.

2.2 Dicharges on a resistive MICROMEGA

From the previous subsection we know that the solution for a two layer problem with a charge Q on the boundary is :

$$\phi_1(r,z)=\frac{Q}{2\pi}\int_0^\infty J_0(kr)\frac{4\sinh(gk)\sinh(k(b+z))}{D(k)}\mathrm{d}k \qquad -b<z<0$$

$$\phi_2(r,z)=\frac{Q}{2\pi}\int_0^\infty J_0(kr)\frac{4\sinh(bk)\sinh(k(g-z))}{D(k)}\mathrm{d}k \qquad 0<z<g$$

The electric field E_z at $r=0$ and $z=g$ is then

$$E_z=\frac{Q}{2\pi}\int_0^\infty\frac{k\sinh(bk)}{D(k)}\mathrm{d}k \tag{2.13}$$

We consider the following parameter for the resistive MICROMEGA : The distance between the mesh and the resistive strips(amplification gap) is 128 µm. Insulator between the resistive strips and the readout strips is 64 μm. We assume the permittivity of the insulating layer to be $\epsilon_r=5$. The normal operation voltage is $500\,V$. Applying the 500 V between the resistive strips and the mesh gives a field of 500 V/128 µm = 39kV/cm.
In case of a 'discharge' there is a charge flowing from the mesh to the surface of the resistive layer over a short time. During this time the charge does not diffuse on the resistive layer, so we simply have a point charge accumulating at $r=0$ and $z=0$. We assume now that the discharge stop when the electric field E_z(due to the point charge) at the surface of the mesh equals the applied electric field.

With those parameters if we calculate the integral with the help of the Mathematica is gives as :

$$\int_0^\infty \frac{ksinh(bk)}{D(k)}dk = 0.000014631$$

so to find the charge Q:

$$E_z = \frac{Q}{2\pi\epsilon_0}\int_0^\infty \frac{k\,\sinh(bk)}{D(k)}\mathrm{d}k$$

$$Q = \frac{2\pi\epsilon_0 E_z}{0.000014631} \Rightarrow Q = 14.8\,\mathrm{pC}$$

This would be the approximate maximum charge in 'spark'.
There is no RC description of this situation. The charge has not yet started to diffuse and there is no equivalent capacitance anywhere. There is simply charge sitting on the surface of the resistive layer, producing and electric field that is counter-acting the applied field.
If we assume a MICROMEGA of $A = 10 \times 10\ cm^2$ size without metallic readout electrode and an avalance gap g, the total charge stored in the capacitor is

$$Q = \epsilon_0 \frac{A}{g} 500 \approx 346\,\mathrm{nC}$$

which is 2.3×10^4 larger compare to the resistive layer case and this amount of charge will enter the amplifier. Now we will try to change the applied voltage($500 - 600\,V$) to see how the charge Q will change.

Table 1: Charge on Mesh for different values of Voltage.

Applied Voltage(V)	Electric Field E_z (kV/cm)	Charge Q(pC)
500	39	14.8
520	41	15.6
540	42	16.0
560	44	16.7
580	45	17.1
600	47	17.9

With the results of the table we will create a figure with the Q and the applied voltage.

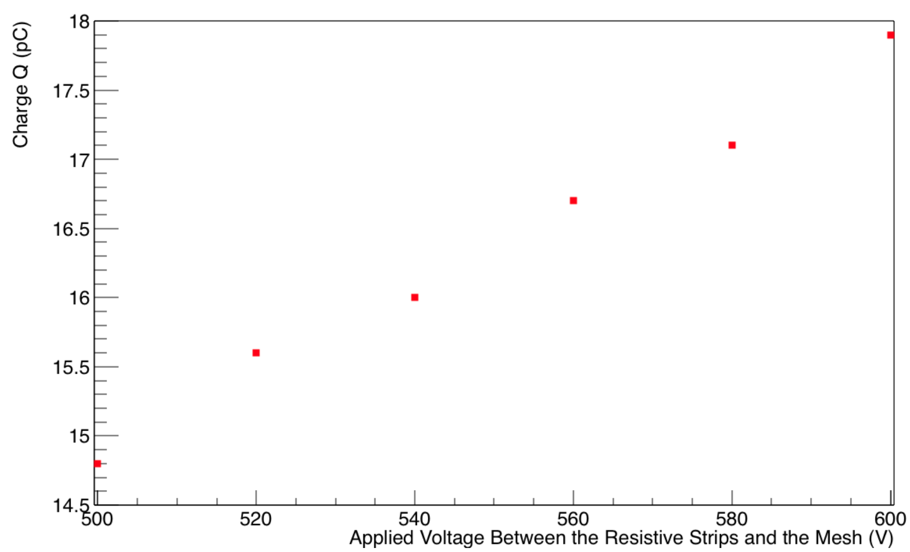


Figure 9: Charge Q for different values of voltage.

We see that there is a linear increase to the charge with the steadily increase of the voltage.

Table 2: Charge on Mesh for different values of g.

g(μm)	value of the integral	Charge(pc)
100	0.000026	11.9
110	0.000021	13.6
120	0.000017	15.4
130	0.000014	17.3

Now we will create the figure to see the relation between the g and and Q. We see again a linear increase to the charge with the steadily increase of the avalanche gap g.

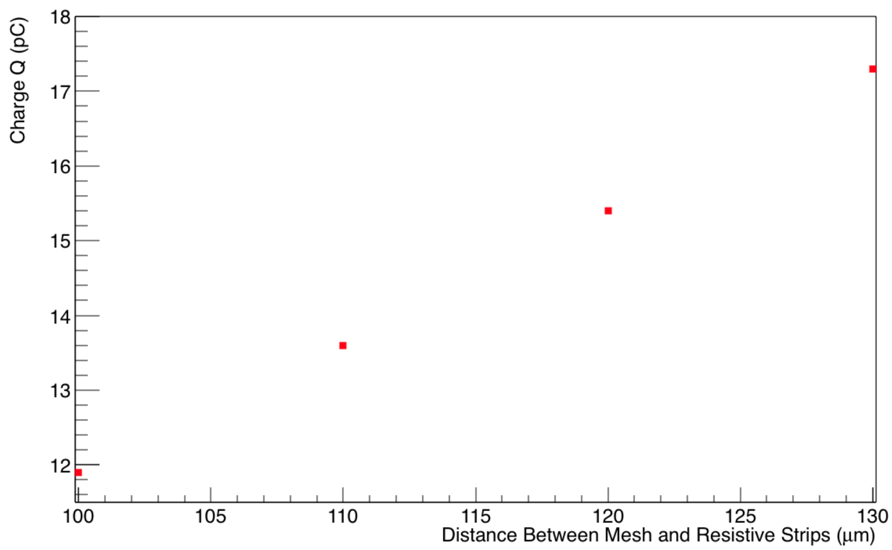


Figure 10: Values of charge for different values of distance between mesh and the resistive strips for applied voltage 570 V

2.3 Potential of a point charge in a geometry grounded on a rectangle

For this case the geometry is grounded at $x = 0, a$ and $y = 0, b$ and the charge is placed at position x_0, y_0.

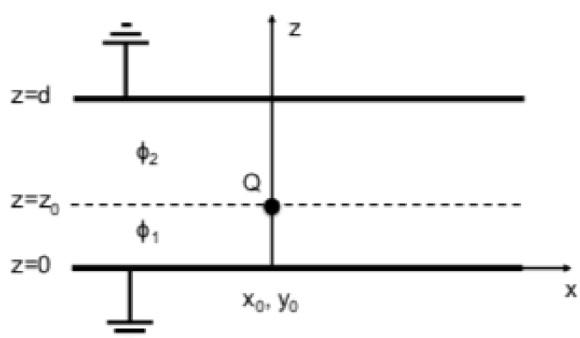


Figure 11: A point charge Q in an empty condenser.

We have to solve the Laplace equation in Cartesian coordinates.
The general solution is :

$$\Phi_1(x, y, z) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \sin\left(\frac{lx\pi}{a}\right) \sin\left(\frac{my\pi}{b}\right) [E_1(k_lm)e^{k_lmz} + F_1(k_lm)e^{-k_lmz}] \tag{2.14}$$

Boundary Conditions

$$\Phi_1 = \Phi_2 \Rightarrow E_1 + F_1 = E_2 + F_2 \qquad z = 0 \tag{2.15}$$

$$\Phi_1 = 0 \Rightarrow E_1 e^{-kb} + F_1 e^{kb} = 0 \qquad z = -b \tag{2.16}$$

$$\Phi_2 = 0 \Rightarrow E_2 e^{kg} + F_2 e^{-kg} = 0 \qquad z = g \tag{2.17}$$

$$\epsilon_1 \frac{\partial \Phi_1(r,z)}{\partial z}|_{z=0} - \epsilon_2 \frac{\partial \Phi_1(r,z)}{\partial z}|_{z=0} = Q\delta(x-x_0)\delta(y-y_0) \tag{2.18}$$

$$(2.18) \Rightarrow \epsilon_1[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \sin\left(\frac{lx\pi}{a}\right)\sin\left(\frac{my\pi}{b}\right)(kE_1 - kF_1)] - \epsilon_2[\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \sin\left(\frac{lx\pi}{a}\right)\sin\left(\frac{my\pi}{b}\right)(kE_2 - kF_2)] = Q\delta(x-x_0)\delta(y-y_0)$$

$$(2.18) \times \sin\left(\frac{l'x\pi}{a}\right)\sin\left(\frac{m'y\pi}{b}\right)$$

and by integrate :

$$\int_0^b\int_0^a \sin\left(\frac{l'x\pi}{a}\right)\sin\left(\frac{m'y\pi}{b}\right)\sin\left(\frac{lx\pi}{a}\right)\sin\left(\frac{my\pi}{b}\right)[kE_1 - kF_1) - (kE_2 - kF_2)]\mathrm{d}x\mathrm{d}y =$$

$$\int_0^b\int_0^a Q\delta(x-x_0)\delta(y-y_0)\sin\left(\frac{l'x\pi}{a}\right)\sin\left(\frac{m'y\pi}{b}\right)\mathrm{d}x\mathrm{d}y$$

for $l = l'$ and $m = m'$:

$\frac{a}{2}\frac{b}{2}[(kE_1 - kF_1) - (kE_2 - kF_2)] = Qsin\left(\frac{lx_0\pi}{a}\right)sin\left(\frac{my_0\pi}{b}\right)$

$$[kE_1 - kF_1) - (kE_2 - kF_2)] = 4Q\frac{Qsin\left(\left(\frac{lx_0\pi}{a}\right)sin\left(\frac{my_0\pi}{b}\right)\right.}{kab}$$

$$(2.16) \Rightarrow F_1 = -E_1 e^{-2kb} \qquad (2.17) \Rightarrow F_2 = -E_2 e^{2kg}$$

From (2.16) and (2.17) :

$$(2.15) \Rightarrow E_1 - E_1 e^{-2kb} = E_2 - E_2 e^{2kg}$$

$$E_1 e^{-kb}(e^{kb} - e^{-kb}) = E_2 e^{-kb}(e^{-kg} - e^{-kg})$$

$$E_2 = -E_1 e^{-kb} e^{-kg}\frac{sinh(kb)}{sinh(kg)}$$

So $F_2 = E_1 e^{-kb} e^{-kg}\frac{sinh(kb)}{sinh(kg)}$

$$(2.18) \Rightarrow \epsilon_1(E_1 + E_1 e^{-2kb}) - \epsilon_2(-E_1 e^{-kb}e^{-kg}\frac{\sinh(kb)}{\sinh(kg)} - E_1 e^{-kb}e^{kg})\frac{\sinh(kb)}{sinh(kg)}) = 4\frac{Q\sin\left(\frac{lx_0\pi}{a}\right)\sin\left(\frac{my_0\pi}{b}\right)}{kab}$$

$$\epsilon_1 e^{-kb}E_1(e^{kb} + e^{-kb}) + \epsilon_2 F_1\frac{\sinh(kb)}{sinh(kg)}(e^{-kg} + e^{kg}) = 4\frac{Q\sin\left(\frac{lx_0\pi}{a}\right)\sin\left(\frac{my_0\pi}{b}\right)}{kab}$$

$$2\epsilon_1 e^{-kb} E_1 \cosh(kb) + 2\epsilon_2 E_1 e^{-kb} \frac{\sinh(kb)}{\sinh(kg)} \cosh(kg) = 4\frac{Q \sin\left(\frac{lx_0\pi}{a}\right) \sin\left(\frac{my_0\pi}{b}\right)}{kab}$$

$$E_1 = 4\frac{Qsin\left(\frac{lx_0\pi}{a}\right) \sin\left(\frac{my_0\pi}{b}\right)}{kab} \frac{\sinh(kg)e^{kb}}{2(\epsilon_1 \cosh(kb)\sinh(kg) + \epsilon_2 \sinh(kb)\cosh(kg))}$$

From 2.1 :

$$E_1 = \frac{2Q\sinh(kg)e^{kb}}{D(k)} \times 4\frac{\sin(\frac{lx_0\pi}{a})\sin(\frac{my_0\pi}{b})}{kab} = \frac{4A_1 \sin(\frac{lx_0\pi}{a})\sin(\frac{my_0\pi}{b})}{kab} \tag{2.19}$$

$$F_1 = \frac{4A_1 sin\left(\frac{lx_0\pi}{a}\right) \sin\left(\frac{my_0\pi}{b}\right)}{kab} e^{-2kb} = -4\frac{2Q\sinh(kg)e^{kb}}{D(k)} \frac{4A_1 \sin\left(\frac{lx_0\pi}{a}\right) \sin\left(\frac{my_0\pi}{b}\right)}{kab} e^{-2kb}$$

$$F_1 = \frac{4B_1 \sin\left(\frac{lx_0\pi}{a}\right) \sin\left(\frac{my_0\pi}{b}\right)}{kab} \tag{2.20}$$

So :

$$\Phi_1(x,y,z) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \sin\left(\frac{lx\pi}{a}\right) \sin\left(\frac{my\pi}{b}\right) [\frac{4A_1 \sin\left(\frac{lx_0\pi}{a}\right) \sin\left(\frac{my_0\pi}{b}\right)}{kab} e^{kz} + \frac{4B_1 \sin\left(\frac{lx_0\pi}{a}\right) \sin\left(\frac{my_0\pi}{b}\right)}{kab} e^{-kz}]$$

$$\Phi_1(x,y,z) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} 4\sin\left(\frac{lx\pi}{a}\right) \sin\left(\frac{my\pi}{b}\right) \frac{sin\left(\frac{lx_0\pi}{a}\right) \sin\left(\frac{my_0\pi}{b}\right)}{kab} [A_1 e^{kz} + B_1 e^{-kz}]$$

$$k_x = \frac{l\pi}{a} \Rightarrow dk_x = \frac{\pi}{a} dl \Rightarrow dl = \frac{a}{\pi} dk_x, \quad dm = \frac{b}{\pi} dk_y$$

$$\Phi_1(x,y,z) = \frac{4}{\pi^2} \int_0^{\infty}\int_0^{\infty} [\cos[k_x(x-x_0)] - \cos[k_x(x+x_0)][\cos[k_y(y-y_0)] - \cos[k_y(y+y_0)] \frac{1}{k}[A_1 e^{kz} + B_1 e^{-kz}] \mathrm{d}k \tag{2.21}$$

2.4 Weighting Fields

In this part we want to calculate the weighting field of a rectangular pad centred at $x = y = 0$ with a width of w_x and w_y for the geometry of figure, which is infinitely extended and where the permittivity of both layers is equal to ϵ_0. We will use the previous solution and shift the coordinate system such there is a grounded plate at $z = 0$ and $z = g$ and the point charge at x_0,y_0,z_0.

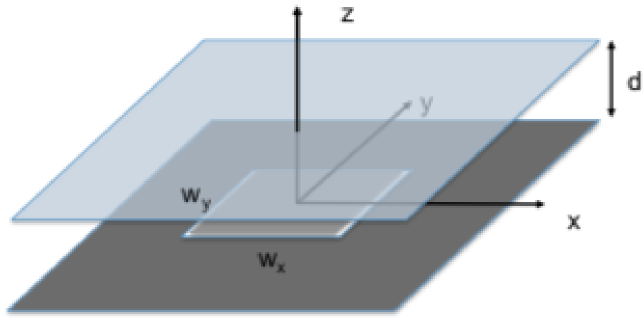


Figure 12: rectangular readout pad

We use the A_1 and B_1 from 3.1 and replace $g = d - z_0$,$b = z_0$.

$$D(k) = 4[\epsilon_1 \cosh(bk) \sinh(gk) + \epsilon_2 \sinh(bk) \cosh(gk)]$$

$$D(k) = 4[\epsilon_1 \frac{e^{bk} + e^{-bk}}{2} \frac{e^{gk} - e^{-gk}}{2} + \epsilon_2 \frac{e^{bk} - e^{-bk}}{2} \frac{e^{gk} + e^{-gk}}{2}] \quad \epsilon_1 = \epsilon_2 = \epsilon_0$$

$$\epsilon_0[(e^{bk} + e^{-bk})(e^{gk} + e^{-gk}) + (e^{bk} - e^{-bk})(e^{gk} + e^{-gk})] = a\epsilon_0[e^{k(b+g)} - e^{-k(b+g)}]$$

$$D(k) = 4\epsilon_0 \sinh[k(b+g)] = 4\epsilon_0 \sinh[k(z_0 + d - z_0)] = 4\epsilon_0 \sinh[kd] \tag{2.22}$$

$$A_1 = \frac{2Q \sinh(k(d - z_0))}{4\epsilon_0 \sinh(kd)} e^{kz_0} \tag{2.23}$$

$$B_1 = \frac{-2Q \sinh(k(d - z_0))}{4\epsilon_0 \sinh(kd)} e^{-kz_0} \tag{2.24}$$

So to find the potential to area 1 :

$$\Phi_1(x,y,z) = \frac{4}{\pi^2} \int_0^\infty \int_0^\infty [cosk_x(x-x_0) - cosk_x(x+x_0)][cosk_y(y-y_0) - cosk_y(y+y_0)] \frac{1}{k} \frac{2Q \sinh(k(d-z_0))}{4\epsilon_0 \sinh(kd)} [e^{kz_0} e^{kz} + e^{-kz_0} e^{-kz}] \mathrm{d}k_x \mathrm{d}k_y$$

$$\Phi_1(x,y,z) = \frac{Q}{\pi^2 \epsilon_0} \int_0^\infty \int_0^\infty [cosk_x(x-x_0) - cosk_x(x+x_0)][cosk_y(y-y_0) - cosk_y(y+y_0)] \frac{1}{k} \frac{sinh[k(d-z_0)] sinh[k(z+z_0)]}{sinh(kd)} \mathrm{d}k_x \mathrm{d}k_y$$

$$\Phi_1(x,y,z) = \frac{Q}{\pi^2 \epsilon_0} \int_0^\infty \int_0^\infty [\sin(k_x x) \sin(k_x x_0) sin(k_y y) \sin(k_y y_0)] \frac{1}{k} \frac{\sinh[k(d-z_0)] \sinh[k(z+z_0)]}{\sinh(kd)} \mathrm{d}k_x \mathrm{d}k_y \tag{2.25}$$

Φ_2 is given if we exchange z with z_0
In this rectangular pad an charge Q_ind is induced :

$$Q_ind(x_0, y_0, z_0) = \int_{\frac{-w_x}{2}}^{\frac{+w_x}{2}} \int_{\frac{-w_y}{2}}^{+\frac{w_y}{2}} -\epsilon_0 \frac{\partial \varphi_1}{\partial z}|_{z=0} \mathrm{d}x \mathrm{d}y \tag{2.26}$$

$$\frac{\partial\phi_1}{\partial z} = \frac{Q}{\pi^2\epsilon_0}\int_0^\infty\int_0^\infty[\sin(k_x x)\sin(k_x x_0)\sin(k_y y)\sin(k_y y_0)]\frac{1}{k}\frac{\sinh[k(d-z_0)](ke^{kz}e^{kz_0}+ke^{-kz}e^{-kz_0})}{2\sinh(kd)}\mathrm{d}k_x\mathrm{d}k_y$$

$$\frac{\partial\phi_1}{\partial z}|_{z=0} = \frac{Q}{\pi^2\epsilon_0}\int_0^\infty\int_0^\infty[\sin(k_x x)\sin(k_x x_0)\sin(k_y y)\sin(k_y y_0)]\frac{\sinh[k(d-z_0)]\cosh(kz_0)}{\sinh(kd)}\mathrm{d}k_x\mathrm{d}k_y$$

From the reciprocity theorem we know that $Q_ind = -Q/V_w\varphi_w(x_0,y_0,z_0)$ where φ_w is the potential at x_0,y_0,z_0 in case is removed and the pad is put to potential V_w. So :

$$\phi_w(x_0,y_0,z_0) = \frac{-V_wQ_ind}{Q}$$

$$\phi_w(x_0,y_0,z_0) = \frac{4V_w}{\pi^2}\int_0^\infty\int_0^\infty\frac{\cos(k_x x_0)\sin(\frac{k_x w_x}{2})\cos(k_y y_0)\sin(\frac{k_y w_y}{2})\cosh(kz_0)sinh[z(d-z_0)]}{k_xk_y\sinh(kd)}\mathrm{d}x\mathrm{d}y$$

$$\phi_w(x_0,y_0,z_0) = \frac{4V_w}{\pi^2}\int_0^\infty\int_0^\infty\frac{\cos(k_x x)\sin(\frac{k_x w_x}{2})\cos(k_y y)\sin(\frac{k_y w_y}{2})\cosh(kz)\sinh[k(d-z)]}{k_xk_y\sinh(kd)}\mathrm{d}x\mathrm{d}y \qquad (2.27)$$

2.5 N-Layer geometry

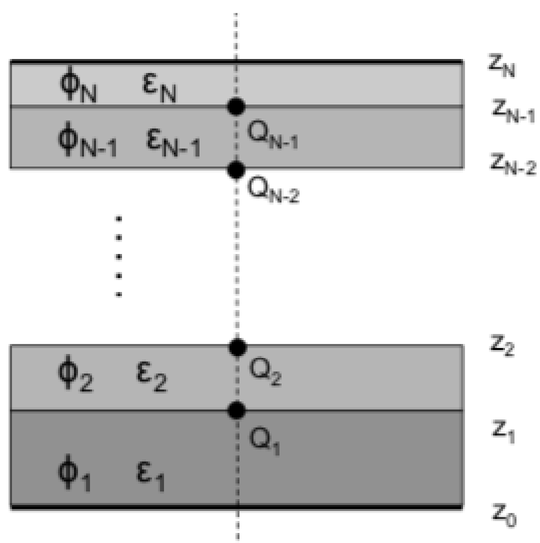


Figure 13: A geometry on N dielectric layers enclosed by grounded metallic plates. On the boundary between two layers at $r=0$ there are point charges Q.

We will approach the geometry in figure 13. We assume N dielectric layers ranging from $z_n-1<z<z_n$ of constant permittivity ϵ_n. On the boundaries at $z=z_n$, there are charges Q_n. At $z=z_0$ and $z=z_N$ there are grounded metal plates. We define a characteristic function $f_n(k,z)$ for each layer as

$$f_n(k,z) = A_ne^{kz}+B_ne{-kz} \qquad n = 1...N \qquad (2.28)$$

With this characteristic function we can find the solution for different geometries :

For an infinitely extended geometry with the chargers at position x_0,y_0 in Cartesian coordinates is given by

:

$$\Phi_n(x,y,z) = \frac{1}{\pi^2}\int_0^\infty\int_0^\infty \sin(k_x x)\sin(k_x x_0)\sin(k_y y)\sin(k_y y_0)\frac{f_n(k,z)}{k}dk_x dk_y \tag{2.29}$$

with $k = \sqrt{k_x^2 + k_y^2}$

For the case where the geometry is grounded on a rectangle at $x = 0, a$ and $y = 0, b$ the solution is :

$$\Phi_n(x,y,z) = \frac{4}{ab}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty} sin\left(\frac{lx\pi}{a}\right)\sin\left(\frac{lx_0\pi}{a}\right)\sin\left(\frac{my\pi}{b}\right)\sin\left(\frac{my_0\pi}{b}\right)\frac{f_n(k_{ml},z)}{k} \tag{2.30}$$

with $k = \pi\sqrt{\frac{l^2}{a^2} + \frac{m^2}{b^2}}$
For the case where the geometry is grounded on a rectangle at $x = 0, a$ and insulated at $y = 0, b$ the solution is :

$$\Phi_n(x,y,z) = \frac{4}{ab}\sum_{l=1}^{\infty}\sum_{m=0}^{\infty} \sin\left(\frac{lx\pi}{a}\right)\sin\left(\frac{lx_0\pi}{a}\right)\cos\left(\frac{my\pi}{b}\right)\cos\left(\frac{my_0\pi}{b}\right)(1-\delta_0 m/2)\frac{f_n(k_{ml},z)}{k} \tag{2.31}$$

The 2N coefficients $A_n(k)$ and $B_n(k)$ are defined by the two conditions at the grounded plated and at the 2(N-1) conditions at the N-1 dielectric interfaces :

$$A_1 e^{kz_0} + B_1 e^{-kz_0} = 0$$

$$A_N e^{kz_N} + B_N e^{-kz_N} = 0$$

$$A_n e^{kz_n} + B_n e^{-kz_n} = A_{n+1}e^{kz_n} + B_{n+1}e^{-kz_n}$$

$$\epsilon_n(A_n e^{kz_n} - B_n e^{-kz_n}) - \epsilon_{n+1}(A_{n+1}e^{kz_n} + B_{n+1}e^{-kz_n}) = Q_n$$

From these equations we will approach a general solution for the different forms of RPCS we are going to discuss in this section. We will start with a 3-layer geometry.

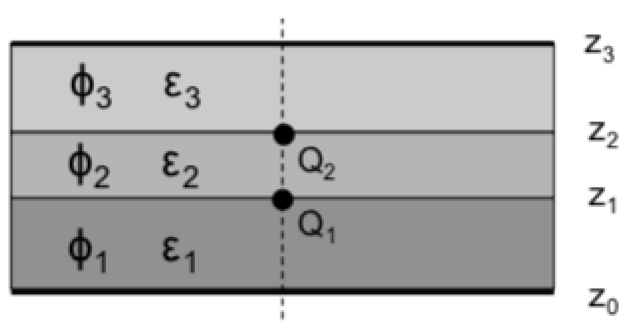


Figure 14: A geometry with 3 dielectric layers.

Boundary Conditions

$$A_1e^{kz_0} + B_1e^{-kz_0} = 0 \tag{2.32}$$
$$\tag{2.33}$$
$$A_1E^{kz_1} + B_1e^{-kz_1} = A_2e^{kz_1} + B_2e^{-kz_1} \tag{2.34}$$
$$\tag{2.35}$$
$$\epsilon_1\frac{\partial f_1}{\partial z} - \epsilon_2\frac{\partial f_2}{\partial z} = Q_1 \tag{2.36}$$
$$\tag{2.37}$$
$$\epsilon_1A_1e^{kz_1} - \epsilon_1B_1e^{-kz_1} - \epsilon_2A_2e^{kz_1} + \epsilon_2B_2e^{-kz_1} = Q_1 \tag{2.38}$$
$$\tag{2.39}$$
$$A_2E^{kz_2} + B_2e^{-kz_2} = A_3e^{kz_2} + B_3e^{-kz_2} \tag{2.40}$$
$$\tag{2.41}$$
$$\epsilon_2\frac{\partial f_2}{\partial z} - \epsilon_3\frac{\partial f_3}{\partial z} = Q_2 \tag{2.42}$$
$$\tag{2.43}$$
$$\epsilon_2A_2e^{kz_2} - \epsilon_2B_2e^{-kz_2} - \epsilon_3A_3e^{kz_2} + \epsilon_3B_3e^{-kz_2} = Q_2 \tag{2.44}$$
$$\tag{2.45}$$
$$A_3e^{kz_3} + B_3e^{-kz_3} = 0 \tag{2.46}$$

For these equation we create the M matrix for this 3-layer geometry. The equation to solve is then :

$$M\overrightarrow{a} = \overrightarrow{b} \tag{2.47}$$

$$\mathbf{M} = \begin{pmatrix} e^{kz_0} & e^{-kz_0} & 0 & 0 & 0 & 0 \\ e^{kz_1} & e^{-kz_1} & -e^{kz_1} & -e^{-kz_1} & 0 & 0 \\ \epsilon_1e^{kz_1} & -\epsilon_1e^{kz_1} & -\epsilon_2e^{kz_1} & \epsilon_2e^{-kz_1} & 0 & 0 \\ 0 & 0 & e^{kz_2} & e^{-kz_2} & -e^{kz_2} & -e^{-kz_2} \\ 0 & 0 & \epsilon_2e^{kz_2} & -\epsilon_2e^{-kz_2} & -\epsilon_3e^{kz_2} & \epsilon_3e^{-kz_2} \\ 0 & 0 & 0 & 1 & e^{kz_3} & e^{-kz_3} \end{pmatrix} \tag{2.48}$$

$\overrightarrow{a} = (A1, B1, A2, B2, A3, B3)^T \qquad \overrightarrow{b} = (0, 0, Q_1, 0, Q_2, 0)^T$

Last we will approach a generalization of (2.12) from 2.1.
We will use a example for the infinitely extended geometry in cylindrical coordinates with the charges centred at $r_0 = 0$ and have $n = 1, ...N$.

$$\phi_n(r,z) = \frac{Q_{n-1}}{2\pi(\epsilon_{n-1}+\epsilon_n)}\frac{1}{\sqrt{r^2+(z-z_{n-1})^2}} + \frac{Q_n}{2\pi(\epsilon_n+\epsilon_{n+1})}\frac{1}{\sqrt{r^2+(z-z_n)^2}}\frac{1}{2\pi}\int_0^\infty J_0(kr)[A_n(k)e^{kz} + B_n(k)e^{-kz} - \frac{Q_{n-1}}{\epsilon_{n-1}+\epsilon_n}e^{-k(z-z_{n-1})} - \frac{Q_n}{\epsilon_n+\epsilon_{n+1}}e^{-k(z_n-z)}\mathrm{d}k \tag{2.49}$$

2.6 Single Layer RPC

We will now approach a geometry with 3 layers(figure 15), a single gap RPC with one resistive layer. We will start by finding the coefficients.

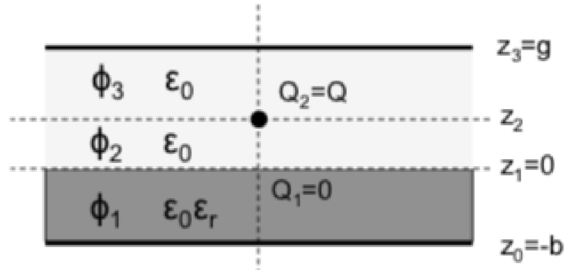


Figure 15: Geometry with three layers and one point charge representing a single gap RPC.

$$A_1 e^{-kb} + B_1 e^{kb} = 0 \tag{2.50}$$

$$A_1 + B_1 = A_2 + B_2 \tag{2.51}$$

$$\epsilon_1 k A_1 - \epsilon_1 k B_1 - \epsilon_2 k A_2 + \epsilon_2 k B_2 = 0 \tag{2.52}$$

$$A_2 e^{kz_2} + B_2 e^{-kz_2} = A_3 e^{kz_2} + B_3 e^{-kz_2} \tag{2.53}$$

$$\epsilon_2 k e^{kz_2} - \epsilon_2 k B_2 e^{-kz_2} - \epsilon_3 k A_3 e^{kz_2} + \epsilon_3 k B_3 e^{-kz_2} = Q \tag{2.54}$$

$$A_3 e^{kg} + B_3 e^{-kg} = 0 \tag{2.55}$$

From (2.51) :

$$B_1 = -A_1 e^{-2kb}$$

By using (2.51) on (2.52):

$$A_1 - A_1 e^{-2kb} = A_2 + B_2$$

We multiply (2.52) with $\epsilon_2 k$ and add with (2.53) and we have :

$$A_1 k(\epsilon_1 + \epsilon_2) + k B_1(\epsilon_2 - \epsilon_1) - 2\epsilon_2 k A_2 = 0$$

$$2\epsilon_0 k A_2 = A_1 k(\epsilon_0 + \epsilon_0\epsilon_r - A_1 k e^{-2kb}(\epsilon_0 - \epsilon_0\epsilon_r))$$

$$A_2 = \frac{A_1}{2}[(1+\epsilon_r) - e^{-2kb}(1-\epsilon_r)]$$

with $\epsilon_1 = \epsilon_0\epsilon_r, \epsilon_2 = \epsilon_0, \epsilon_3 = \epsilon_0$

By returning to (2.52) :

$$A_1 - A_1 e^{-2kb} = \frac{A_1}{2}[(1+\epsilon_r) - e^{-2kb}(1-\epsilon_r) + B_2]$$

$$B_2 = A_1[1 - e^{-2kb} - \frac{1}{2}(1+\epsilon_r) + \frac{e^{-2kb}}{2} - \frac{e^{-2kb}\epsilon_r}{2}]$$

$$B_2 = \frac{A_1}{2}[(1-\epsilon_r) - e^{-2kb}(1+\epsilon_r)]$$

From (2.55):

$$B_3 = -A_3 e^{2kg}$$

Now we add them together the relations (2.53) and (2.54) :

$$A_2 e^{kz_2} - B_2 e^{-kz_2} - A_3 e^{kz_2} + B_3 e^{-kz_2} = \frac{Q}{\epsilon_0 k}$$

By using B_3 from (2.55) and A_2 from above :

$$A_3 = \frac{A_1}{2}[(1+\epsilon_r) - e^{-2kb}(1-\epsilon_r)] - Q\frac{e^{-kz_2}}{2k\epsilon_0}$$

and so

$$B_3 = -\frac{A_1}{2}[(1+\epsilon_r) - e^{-2kb}(1-\epsilon_r)]e^{2kg} + Q\frac{e^{-kz_2}}{2k\epsilon_0}e^{2kg}$$

Now its important to find A_1. From (2.54):

$$\epsilon_0 k A_2 e^{kz_2} - \epsilon_0 k B_2 e^{-kz_2} - \epsilon_0 k A_3 e^{kz_2} + \epsilon_0 k B_3 e^{-kz_2} = Q$$

$$-\frac{A_1}{2}e^{-kz_2}[1-\epsilon_r) + (1+\epsilon_r)e^{2kg} - e^{-2kb}(1+\epsilon_r) - e^{-2kb}e^{2kg}(1-\epsilon_r) = \frac{Q}{2\epsilon_0 k} - \frac{Q}{2\epsilon_0}e^{-2kz_2}e^{2kg}$$

The relation inside the bracket can become :

$$e^{kg}(e{-}kg + ekg) - e^{-2kb}(1+e^{2kg}) + \epsilon_r(-1+e^{ekg}) + \epsilon_r(e^{-2kb}e^{2kg} - e{-}2kb)$$

$$(e^{kg} + e^{-kg})e^{kg}(e^{kb} - e^{-kb} + \epsilon_r e^{kg}e^{-kb}(e^{kg} - e^{-kg}(e^{kb} + e^{-kb}))$$

$$4\cosh(kg)\sinh(kb)e^{kg}e^{-kb} + 4\epsilon_r\sinh(kg)\cosh(kb)e^{kg}e^{-kb}$$

$4e^{kg}e^{-kb}D(k)$ with$D(k) = \cosh(kg)\sinh(kb)e^{kg}e^{-kb} + \epsilon_r\sinh(kg)\cosh(kb)$

Now returning to the previous relation to find A_1:

$$-\frac{A_1}{2}e^{-kz_2}4e^{kg}e^{-kb}D(k)$$

$$A_1 = \frac{Qe^{kb}}{4\epsilon_0 D(k)}(e^{k(g-z_2)} - e{-}k(g-z_2)$$

$$A_1 = \frac{Qe^{kb}}{2\epsilon_0 D(k)}\sinh(k(g-z2)) \tag{2.56}$$

And so to find the others variables :

$$A_2 = \frac{A_1}{2}[(1+\epsilon_r) - e^{-2kb}(1-\epsilon_r)]$$

$$A_2 = \frac{Q}{4\epsilon_0 D(k)}\sinh(k(g-z_2))[2sinh(kb) + 2\epsilon_r\cosh(kb)]$$

$$A_2 = \frac{Q}{2\epsilon_0 D(k)}\sinh(k(g-z_2))[\sinh(kb) + \epsilon_r\cosh(kb)] \tag{2.57}$$

$$B_2 = \frac{Q}{2\epsilon_0 D(k)}\sinh(k(g-z_2))[\sinh(kb) - \epsilon_r\cosh(kb)] \tag{2.58}$$

$$A_3 = \frac{Q\sinh(k(g-z_2))}{2\epsilon_0 D(k)}(\sinh(kb) + \epsilon_r\cosh(kb)) - \frac{Qe^{-kz_2}}{2\epsilon_0 k} \tag{2.59}$$

$$B_3 = \frac{Qe^{-kz_2}e^{2g}}{2\epsilon_0 k} - \frac{Q}{2\epsilon_0 kD(k)}\sinh(k(g-z_2))[\sinh(kb) + \epsilon_r\cosh(kb)e^{2kg}] \tag{2.60}$$

As we told before the solution on different areas are on form of : $f_i = A_i e^{kz} + B_i e^{-kz}$

We will show the potential on areas 2 and 3,that they have a charge Q between them.

$$f_2 = A_2 e^{kz} + B_2 e^{-kz}$$

$$f_2 = \frac{Q}{2\epsilon_0 D(k)}\sinh(k(g-z_2))[\sinh(kb)e^{kz} + e_r\cosh(kb)e^{kz} + \sinh(kb)e^{-kz} - e_r\cosh(kb)e^{-kz}]$$

$$f_2 = \frac{Q}{2\epsilon_0 D(k)}\sinh(k(g-z_2))[\sinh(kb)(e^{kz} + e^{-kz}) + e_r\cosh(kb)(e^{kz} - e^{-kz})]$$

$$f_2 = \frac{Q}{\epsilon_0 D(k)}\sinh(k(g-z_2))[\sinh(kb)\cosh(kz) + e_r\cosh(kb)\sinh(kz)] \tag{2.61}$$

$$f_3 = A_3 e^{kz} + B_3 e^{-kz}$$

$$f_3 = \frac{Q}{2\epsilon_0 D(k)}\sinh(k(g-z_2))[\sinh(kb) + e_r\cosh(kb)]e^{kz} - \frac{Q}{2\epsilon_0 k}e^{-kz_2}e^{kz} + \frac{Q}{2\epsilon_0 k}e^{-kz_2}e^{2kg}e^{-kz}]$$

$$f_3 = \frac{Q}{\epsilon_0 D(k)}\sinh(k(g-z))[\sinh(kb)\cosh(kz_2) + e_r\cosh(kb)\sinh(kz_2)) \tag{2.62}$$

If we assume that the charge are on $r_0 = 0$ the the voltages for these areas are in form of :

$\varphi_2(k,z) = \frac{1}{2\pi}\int_0^\infty J_0(kr)f_2(k,z)dk$ $\qquad$ $\varphi_3(k,z) = \frac{1}{2\pi}\int_0^\infty J_0(kr)f_3(k,z)dk$
From (2.49) :

$$\phi_2(r,z) = \frac{Q}{4\pi(\epsilon_0}\frac{1}{\sqrt{r^2+(z_2-z)^2}}\frac{1}{2\pi}\int_0^\infty J_0(kr)[f_2(k,z) - \frac{Q}{2\epsilon_0}e^{-k(z_2-z)}]dk \tag{2.63}$$

$$\phi_3(r,z) = \frac{Q}{4\pi(\epsilon_0}\frac{1}{\sqrt{r^2+(z-z_2)^2}}\frac{1}{2\pi}\int_0^\infty J_0(kr)[f_3(k,z) - \frac{Q}{2\epsilon_0}e^{-k(z-z_2)}]\mathrm{d}k \tag{2.64}$$

The expressions represent a point charge Q in free space together with a term that accounts for the presence of the dielectric layer and the grounded plate, which is more suited for numerical evaluation.

2.6.1 Weighting fields

To find the weighting potential for a readout pad,readout strip and the full electrode we use (2.29) :

$$\Phi_1(x,y,z) = \frac{1}{\pi^2}\int_0^\infty\int_0^\infty \sin(k_x x)\sin(k_x x_0)\sin(k_y y)\sin(k_y y_0)\frac{Q\sinh[k(g-z_2)]\sinh[k(z+b)]}{\epsilon_0 k D(k)}\mathrm{d}k_x\mathrm{d}k_y \tag{2.65}$$

The Q_{ind} for a readout pad is :

$$Q_{ind} = \int_{\frac{-w_x}{2}}^{\frac{+w_x}{2}}\int_{\frac{-w_y}{2}}^{+\frac{w_y}{2}}\int_0^\infty\int_0^\infty -\epsilon_r + \epsilon_0\frac{\partial\varphi_1}{\partial z}|_{z=0}\mathrm{d}x\mathrm{d}y$$

$$Q_{ind} = \frac{-4Q\epsilon_r}{\pi^2}\int_0^\infty\int_0^\infty \sin\left(k_x\frac{w_x}{2}\right)\sin\left(k_y\frac{w_y}{2}\right)\sin(k_x x_0)\sin(k_y y_0)\frac{\sinh[k(g-z_2)]}{k_x k_y D(k)}\mathrm{d}k_x\mathrm{d}k_y$$

We know that :

$$Q_{ind} = -\frac{Q\phi_w(x_0,y_0,z_0)}{V_w}$$

and by the reciprocity theorem :

$$\phi_w(x,y,z) = \frac{4V_w\epsilon_r}{\pi^2}\int_0^\infty\int_0^\infty \sin\left(k_x\frac{w_x}{2}\right)\sin\left(k_y\frac{w_y}{2}\right)\sin(k_x x_0)\sin(k_y y_0)\frac{\sinh[k(g-z)]}{k_x k_y D(k)}\mathrm{d}k_x\mathrm{d}k_y \tag{2.66}$$

For a readout strip $w_y \to \infty$

$$\phi_w(x,z) = \frac{4V_w\epsilon_r}{\pi^2}\int_0^\infty \sin\left(k_x\frac{w_x}{2}\right)\sin(k_x x_0)\frac{\sinh[k(g-z)]}{k_x k_y D(k)}\int_0^\infty\frac{\cos\left(\frac{2s_y y}{w_y}\right)\sin(sy)}{\frac{2sy}{w_y}}\frac{2}{w_y}\mathrm{d}s_y\mathrm{d}k_x$$

$$\phi_w(x,z) = \frac{4V_w\epsilon_r}{\pi^2}\int_0^\infty \sin\left(k_x\frac{w_x}{2}\right)\sin(k_x x_0)\frac{\sinh[k(g-z)]}{k_x k_y D(k)}\frac{\pi}{2}\mathrm{d}k_x$$

$$\phi_w(x_0,y_0,z_0) = \frac{2V_w\epsilon_r}{\pi}\int_0^\infty \sin\left(k_x\frac{w_x}{2}\right)\sin(k_x x_0)\frac{\sinh[k(g-z)]}{kD(k)}\mathrm{d}k_x \tag{2.67}$$

with $k_y = \frac{2s_y}{w_y}$

For the full electrode $w_x \to \infty \qquad w_y \to \infty \qquad k = 0$:

$$\frac{\sinh[k(g-z)]}{kD(k)} = \frac{k(g-z)}{kb + kg\epsilon_r}$$

because for small values of k $\sinh(x) \simeq x$ and $\cosh(x) \simeq 1$

And so for the weighting potential :

$$\phi_w(z) = \frac{\epsilon_r V_w(g-z)}{b+g\epsilon_r} \tag{2.68}$$

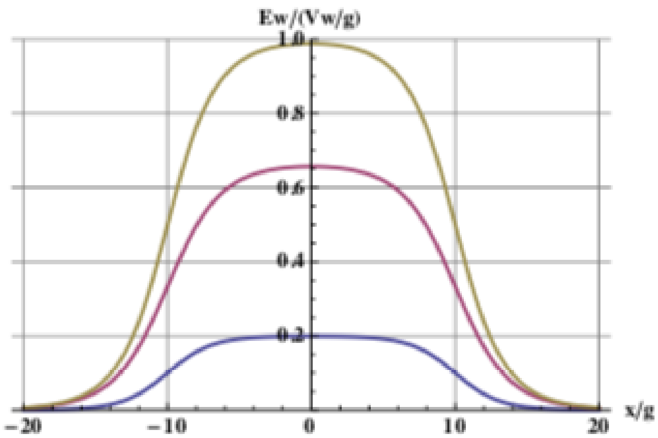


Figure 16: Weighting field E_z at position $z = g/2$ for $b = 4g$ and $w_x = 20g$. The three curves represent $\epsilon_r = 1$(bottom),$\epsilon_r = 8$(middle),$\epsilon_r = \infty$(top).

In figure 17 we can see the a weighting field for a strip electrode of width w_x and infinity extension

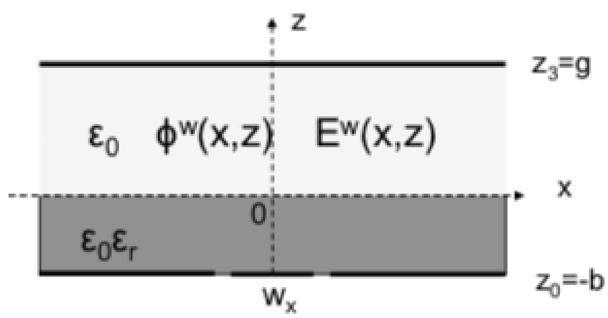


Figure 17: Weighting field for a strip electrode of width w_x and infinity extension.

We first assume the geometry to represent a single layer RPC with a gas gap of $g = 0.25$ mm and a resistive layer of dielectric permittivity ϵ_r and thickness $b =$ 1 mm. We assume a very wide readout strips width $w_x = 5\,\text{mm}$ and we find for the z-component of the weighting field in the center of the gas gap($z = 0.125\,\text{mm}$). as shown in figure 16. The three curves represent dielectric permittivities of $\epsilon_r = 1(bottom), 8(middile)\infty(top)$. The strip extends between $-10 < x/g < 10$ and the value at $x/g = 10$ is therefore half of the peak as required by symmetry for a wide readout strip. The value in the center of the strip is close to the one from 2.68 for the 'infinitely wide' strip and it is clear from this expression that a higher dielectric permittivity of the resistive plate will increase the weighting field and therefore the induced signal. The value $\epsilon_r = 8$ which is typical for glass and bakelite used in RPC's gives a shape that is already close to the one for an arbitrarily large permittivity.

2.6.2 Effect of Resistivity

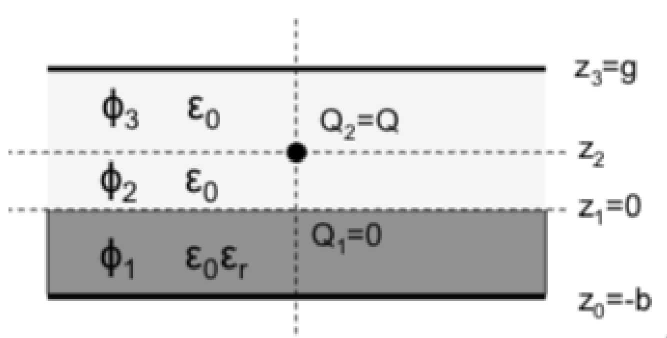


Figure 18: A geometry with three layers and one point charge.

Now we will calculate what happens when we put a point charge Q on the surface of the resistive plate at $t = 0$ as shown in figure 18 :

for $\epsilon_1 = \epsilon_0\epsilon_r + \sigma/s \quad \epsilon_2 = \epsilon + 0 \quad Q_1 = Q/s$

$$E_1(r,z,s) = -\frac{Q}{2s\pi}\int_0^\infty kJ_0(kr)\frac{\sinh(gk)\cosh(k(b+z))}{\epsilon_0[\sinh(bk)\cosh(gk)+(\epsilon_r+\sigma/(\epsilon_0 s)\cosh(bk)\sinh(gk)}\mathrm{d}k \tag{2.69}$$

$$E_2(r,z,s) = \frac{Q}{2s\pi}\int_0^\infty kJ_0(kr)\frac{\sinh(bk)\cosh(k(g-z))}{\epsilon_0[\sinh(bk)\cosh(gk)+(\epsilon_r+\sigma/(\epsilon_0 s)\cosh(bk)\sinh(gk)}\mathrm{d}k \tag{2.70}$$

We want to know the stationary situation so :

$$\lim_{s\to 0} sE(r,z,s) :$$

$$E_1(r,z) = -\frac{I_0}{2\sigma\pi}\int_0^\infty kJ_0(kr)\frac{\cosh(k(b+z))}{\cosh(bk)}\mathrm{d}k \tag{2.71}$$

$$E_2(r,z) = \frac{I_0}{2\sigma\pi}\int_0^\infty kJ_0(kr)\frac{\tanh(bk)cosh(k(g-z))}{\sinh(gk)}\mathrm{d}k \tag{2.72}$$

with $Q = I_0/S^2$

We see that E_1 does not depend on g but depends only on the thickness b of the resistive layer. This is evident from the fact that there is no DC current that can flow through the gas gap, so only the geometry of the resistive layer is relevant.
The current density $i_0(r)$ flowing into the grounded plate at $z = -b$ is related on the field on the surface of the grounded plate by

$$i_0(r) = -\sigma E_1(r, z=-b) = \frac{I_0}{b^2\pi}\int_0^\infty \frac{1}{2}J_0\left(\frac{yr}{b}\right)\frac{y}{\cos(y)}\mathrm{d}y \tag{2.73}$$

with $y = bk$

For small values of r we can insert the series expansion for $J_0(x)$ and evaluate the integrals gives :

$$\int_0^\infty \frac{1}{2}J_0\left(\frac{yr}{b}\right)\frac{y}{\cos(y)} \approx 0.916 - 1.483(\frac{r}{b})^2 + 1.873(\frac{r}{b})^4 - ... \tag{2.74}$$

For large values of r :

$$\int_0^\infty \frac{1}{2}J_0\left(\frac{yr}{b}\right)\frac{y}{\cos(y)} \approx \frac{\pi}{2\sqrt{r/b}}e^{-\frac{r\pi}{2b}} \qquad \frac{r}{b} \gg 1 \tag{2.75}$$

Both those approximation of current are plotted in figure 19 and we see that for $r/b > 2$ the exponential approximation describes the situation already to very high accuracy.

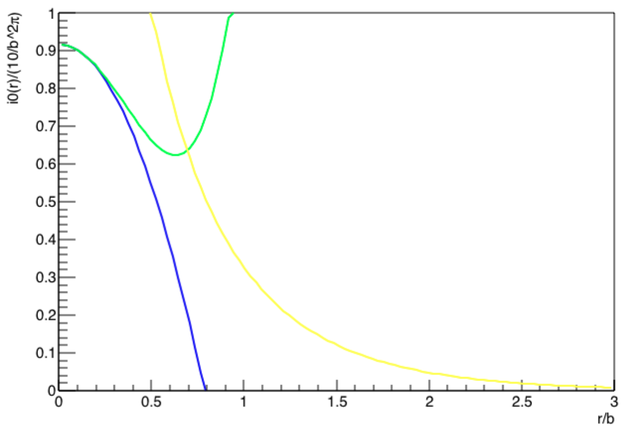
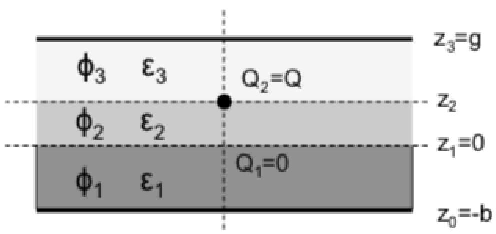
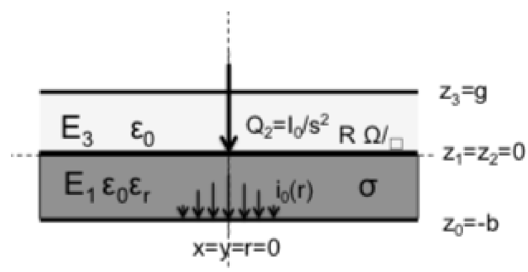


Figure 19: Current density $i_0(r)$ at $z = -b$. The blue curve represent the second order approximation of (2.74), the green curve the fourth order approximation of (2.74) and the yellow curve the approximation of (2.75).

2.6.3 Surface resistivity



(a) Genetal 3 layer geometry with a point charge Q_2.



(b) A resistive plate with conductivity σ together with an thin layer of surfave resistivity RΩ /square and impressed current I_0

Figure 20

The glass or Bakelite might develop a conductive surface once the electric field is applied. We employ the formalism for 3 layer geometry like before with :

$\epsilon_1 = \epsilon_0\epsilon_r + \sigma/s \quad \epsilon_2 = \epsilon_0 + \frac{1}{sz_2R} \quad \epsilon_3 = \epsilon_0 \quad Q_1 = 0 \;\; Q_2 = \frac{I_0}{s^2}$

with $z_0 = -b \quad z_1 = 0 \quad z2 \quad z_3 = g$

and we perform the $z_2 \to z_1 = 0$ With the previous process and by using $\lim_{s\to 0} sf(k,z,s)$:

$$f_1(k,z) = \frac{I_0}{\sigma[\cosh(kb) + k/(R\sigma)\sinh(bk)]}[\sinh[k(b+z)]] \tag{2.76}$$

$$f_3(k,z) = \frac{1}{\sigma}\frac{I_0}{\sinh(kg)[\cosh(kb) + k/(R\sigma)\sinh(bk)]}[\sinh(kb)\sinh[k(g-z)]] \tag{2.77}$$

$$E_1(r,z) = -\frac{I_0}{2\sigma\pi}\int_0^\infty kJ_0(kr)\frac{\cosh[k(b+z)]}{\cosh(kb) + k/(R\sigma)\sinh(bk)}\mathrm{d}k \tag{2.78}$$

$$E_3(r,z) = \frac{I_0}{2\sigma\pi}\int_0^\infty kJ_0(kr)\frac{\sinh(kb)\cosh[k(g-z)]}{\sinh(kb) + k/(R\sigma)\sinh(bk)}\mathrm{d}k \tag{2.79}$$

And the current $i_0(r)$:

$$i_0(r) = -\sigma E_1(r, z = -b) = \frac{I_0}{b^2\pi} \int_0^\infty \frac{1}{2} J_0\left(\frac{yr}{b}\right) \frac{y}{\cos(y) + \frac{y}{\beta^2}\sinh(y)} \mathrm{d}y \tag{2.80}$$

with $\beta^2 = R\sigma b$

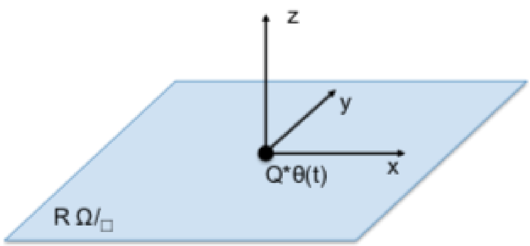
In the limit of high resistivity $R \to \infty$ we recuperate the expression from the previous section without any resistive surface layer.

$$i_0(r) = \frac{I_0}{b^2\pi} \frac{\beta^2}{2} \sqrt{\frac{\pi}{2}} \frac{e^{-\frac{\beta r}{b}}}{\sqrt{\frac{\beta r}{b}}} \tag{2.81}$$

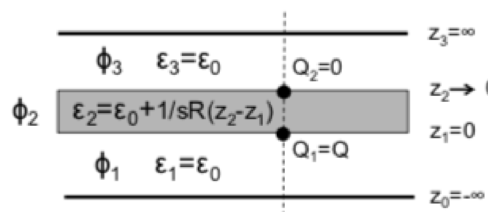
Comparing this with relation (2.75) we see that the radial exponential decay of the current is not any more governed ny the characteristic length $2b/\pi$ by b/β.

2.7 Single Thin Resistive Layer

Now we want to study the fields of a single layer of surface resistivity R at $z = 0$,where we place a charge Q at $r = 0$ at $t = 0$. We write $Q(t) = Q\Theta(t)$ with $\Theta(t)$ the Heavyside step function. If we use the Laplace domain in take the form $Q(s) = Q/s$.



(a) A resistive layer with surface resistance R.



(b) 3 layer geometry

In this case also we use a 3-layer geometry with variables :

$\epsilon_1 = \epsilon_0 \quad \epsilon_2 = \epsilon_0 + \frac{1}{sz_2R} \quad \epsilon_3 = \epsilon_0 \quad Q_1 = \frac{Q}{s} \;\; Q_2 = 0$

By taking the limits :

$z_0 \longrightarrow -\infty \quad z_1 = 0 \quad z_2 \longrightarrow 0 \quad z_3 \longrightarrow +\infty$
The solution for the 3 areas are :

$$f_1 = A_1 e^{kz} + B_1 e^{-kz} \Rightarrow f_1 = A_1 e^{kz}$$

$$f_2 = A_2 e^{kz} + B_2 e^{-kz}$$

$$f_3 = A_3 e^{kz} + B_3 e^{-kz} \Rightarrow f_3 = B_3 e^{-kz}$$

$B_1 = 0$ and $A_3 = 0$ because at the infinities the quantities f_1, f_2 must be zero.

Boundary conditions

$$f_1 = f_2 \Rightarrow A_1 = A_2 + B_2 \qquad z = z_1 = 0 \tag{2.82}$$

$$\epsilon_0 A_1 - \left(\epsilon_0 + \frac{1}{sRz_2}\right)(A_2 - B_2) = Q \Rightarrow \epsilon_0 A_1 - (\epsilon_0 + \frac{1}{sRz_2})A_2 + \left(\epsilon_0 + \frac{1}{sRz_2}\right) B_2 = Q \qquad z = z_1 = 0 \tag{2.83}$$

$$A_2 e^{kz_2} + B_2 e^{-kz_2} = B_3 e^{-kz_2} \qquad z = z_2 \tag{2.84}$$

$$\left(\epsilon_0 + \frac{1}{sRz_2}\right) A_2 e^{kz_2} - \left(\epsilon_0 + \frac{1}{sRz_2}\right) B_2 e^{-kz_2} + \epsilon_0 B_3 e^{-kz_3} = 0 \qquad z = z_2 \tag{2.85}$$

We multiply (2.82) with $(\epsilon_0 + \frac{1}{sRz_2})$ and add it with (2.83) :

$$\left(2\epsilon_0 + \frac{1}{sRz_2}\right) A_1 - 2\left(\epsilon_0 + \frac{1}{sRz_2}\right) A_2 = Q$$

$$A_2 = A_1 \frac{2\epsilon_0 sRz_2 + 1 - QsRz_2}{2(sRz_2\epsilon_0 + 1)} \tag{2.86}$$

$$B_2 = A_1 - A_2 = A_1 - A_1 \frac{2\epsilon_0 sRz_2 + 1 - QsRz_2}{2(sRz_2\epsilon_0 + 1)}$$

$$B_2 = \frac{A_1 + QsRz_2}{2(sRz_2\epsilon_0 + 1)} \tag{2.87}$$

from (2.84) by using A_2 and B_2 from above :

$$B_3 = \frac{A_1}{2(sRz_2\epsilon_0 + 1)}[2(sRz_2\epsilon_0 + 1)e^{2kz_2} + 1] - \frac{QsRz_2}{2(sRz_2\epsilon_0 + 1)}(e^{2kz_2 - 1}) \tag{2.88}$$

Now from all the above we have:

$$A_1 = Q\frac{sRz_2\epsilon_0 e^{kz_2} + \cosh(kz_2)}{k(sRz_2\epsilon_0 + 1)} \frac{sRz_2(\epsilon_0 sRz_2 + 1)}{2(\epsilon_0 sRz_2)^2 e^{kz_2} + 2sRz_2\epsilon_0 e^{kz_2} + \sinh(kz_2)}$$

$$A_1 = Q\frac{sRz_2\epsilon_0 e^{kz2} + \cosh(kz_2)sRz_2}{2\epsilon_0 sRz_2 e^{kz_2}(\epsilon_0 sRz_2 + 1) + \sinh(kz_2)}$$

$$A_1 = Q\frac{(sRz_2)^2\epsilon_0 e^{kz_2} + \frac{e^{kz_2} + e^{-kz_2}}{2} sRz_2}{2\epsilon_0 sRz_2 e^{kz_2}(\epsilon_0 sRz_2 + 1) + \frac{e^{kz_2} - e^{-kz_2}}{2}} \tag{2.89}$$

If we use $z_2 \longrightarrow 0$:

$$A_1 = \frac{QsR}{k(2\epsilon_0 sR + k)}$$

and if we use
$Q = \frac{Q}{s}$

$$A_1 = \frac{QR}{2\epsilon_0 sR + k} \tag{2.90}$$

From (2.86),(2.87) :

$$A_2 = \frac{QR}{2(2\epsilon_0 sR + k)} = B_2 \tag{2.91}$$

With the knowledge of the 3 of 4 coefficients we can go now to (2.88) and find the B_3 :

$$B_3 = \frac{QR}{2\epsilon_0 sR + k} \tag{2.92}$$

If we return to the solutions f_1,f_2,f_3 :

$$f_1 = A_1 e^{kz} \Rightarrow f_1 = \frac{QR}{2\epsilon_0 sR + k} e^{kz} \tag{2.93}$$

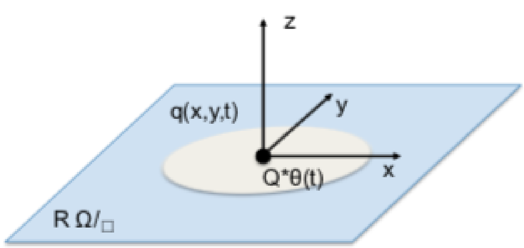
$$f_2 = A_2 e^{kz} + B_2 e^{-kz} \Rightarrow f_2 = \frac{QR}{2\epsilon_0 sR + k} cosh(kz) \quad for \;\; z = 0 \tag{2.94}$$

$$f_3 = B_3 e^{-kz} \Rightarrow f_3 = \frac{QR}{2\epsilon_0 sR + k} e^{-kz} \tag{2.95}$$

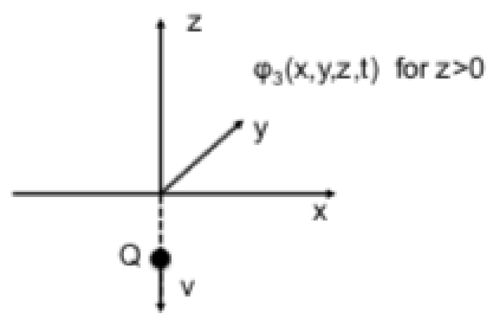
We will use the solution f_1 and f_3 for some cases of layers in the next parts. We prefer to use them in the time domain :

$f_1 = \frac{Q}{2\epsilon_0} e^{-k(vt-z)}$ $\qquad$ $f_3 = \frac{Q}{2\epsilon_0} e^{-k(vt+z)}$ $\qquad$ for $v = \frac{1}{2\epsilon_0 R}$

2.7.1 Infinitely extended resistive layer



(a) A point charge placed at an infinitely extended resistive layer at $t = 0$.



(b) The solution for the time dependent potential is equal to a point charge moving with velocity v along the z-axis.

Figure 21

We will start from an infinitely extended layer. The charge Q will cause currents to flow inside the resistive layer that are "destroying" it. The solution for the potential is :

$$\phi_1(k,z,t) = \frac{Q}{4\pi\epsilon_0} \int_0^\infty J_0(kr) e^{-k(vt-z)} \mathrm{d}k \tag{2.96}$$

$$\phi_3(k,z,t) = \frac{Q}{4\pi\epsilon_0} \int_0^\infty J_0(kr) e^{-k(vt+z)} \mathrm{d}k \tag{2.97}$$

and from (2.9) :

$$\phi_1(r,z,t) = \frac{Q}{4\pi(\epsilon_0} \frac{1}{\sqrt{r^2 + (-z+vt)^2}} \tag{2.98}$$

$$\phi_3(r,z,t) = \frac{Q}{4\pi(\epsilon_0} \frac{1}{\sqrt{r^2 + (z+vt)^2}} \tag{2.99}$$

The potential to the point charge placed on the infinitely extended resistive layer at $t = 0$ is equal to the potential of a charge Q that is moving with velocity $v = \frac{1}{2\epsilon_0 R}$ away from the layer along the z-axis.

We can calculate the charge density q(r,t) on the resistive layer through Gauss law :

$$q(r,t) = \epsilon_0 \frac{\partial \varphi_1}{\partial z}|_{z=0} - \epsilon_0 \frac{\partial \varphi_3}{\partial z}|_{z=0}$$

$$q(r,t) = \frac{Q}{2\pi} \frac{vt}{\sqrt{r^2 + (v^2+t^2)^3}} \tag{2.100}$$

Lastly we calculate the total current I(r) flowing radially through a circle of radius r :

$$I(r) = \frac{2r\pi}{R} E(r) = -\frac{2r\pi}{R} \frac{\partial \varphi_1}{\partial r}|_{r=0} = \frac{Qvr^2}{r^2 + (v^2 t^2)^{3/2}} \tag{2.101}$$

2.7.2 Resistive layer grounded on a rectangle

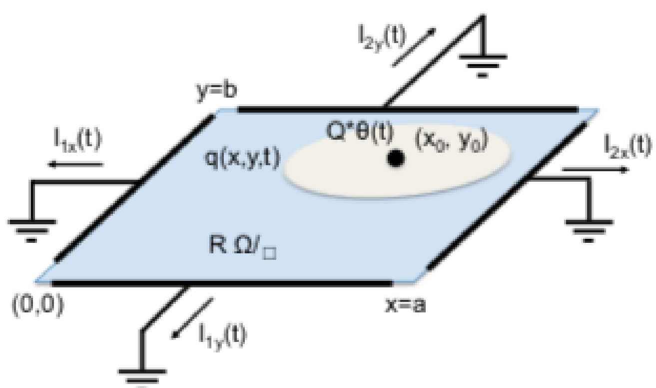


Figure 22: A point charge placed on a resistive layer that is grounded on a rectangle.

In this case the resistive layer a grounded boundary at $x = 0$, $x = a$ and $y = 0$, $y = b$ and place a charge Q at position x_0, y_0 at $t = 0$. The potential is given by (2.30). We assume that the currents pointing outside of the boundary, the currents flowing through 4 boundaries are :

$I_1 x(t) = -\frac{1}{R}\int_0^b -\frac{\partial\varphi_1}{\partial x}|_{x=0} dy$ $\qquad I_2 x(t) = \frac{1}{R}\int_0^b -\frac{\partial\varphi_1}{\partial x}|_{x=a}\mathrm{d}y$

$I_1 y(t) = -\frac{1}{R}\int_0^a -\frac{\partial\varphi_1}{\partial y}|_{y=0} dy$ $\qquad I_2 y(t) = \frac{1}{R}\int_0^a -\frac{\partial\varphi_1}{\partial y}|_{y=b}\mathrm{d}y$

with $\frac{1}{R} = 2\epsilon_0 v$

$$\phi_1 = \frac{4}{ab}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty} \sin\left(\frac{lx\pi}{a}\right)\sin\left(\frac{lx_0\pi}{b}\right)\sin\left(\frac{my\pi}{a}\right)\sin\left(\frac{my_0\pi}{b}\right)\frac{f_1}{k}$$

with $f_1 = \frac{Q}{2\epsilon_0}e^{-k(vt-z)}$

We will start with the $I_1 x(t)$

$$\phi_1 = \frac{Q}{2\epsilon_0}\frac{4}{ab}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sin\left(\frac{lx\pi}{a}\right)\sin\left(\frac{lx_0\pi}{b}\right)\sin\left(\frac{my\pi}{a}\right)\sin\left(\frac{my_0\pi}{b}\right)\frac{e^{-k(vt-z)}}{k}$$

$$-\frac{\partial\phi_1}{\partial x}|_{x=0} = \frac{Q}{2\epsilon_0}\frac{4}{ab}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{l\pi}{a}\sin\left(\frac{lx_0\pi}{b}\right)\sin\left(\frac{my\pi}{a}\right)\sin\left(\frac{my_0\pi}{b}\right)\frac{e^{-k(vt-z)}}{k}$$

$$I_1x(t) = -\frac{1}{R}\int_0^b -\frac{\partial\phi_1}{\partial x}|_{x=0}\mathrm{d}y =$$

$$2\epsilon_0 v\frac{Q}{2\epsilon_0}\frac{4}{ab}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{l\pi}{a}\sin(\frac{lx_0\pi}{b})\sin\left(\frac{my\pi}{a}\right)\sin\left(\frac{my_0\pi}{b}\right)\frac{e^{-k(vt-z)}}{k}\mathrm{d}y$$

$$I_{1x}(t) = \frac{4Qv}{a^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{l}{m}[1-(-1)^m]\sin\left(\frac{lx_0\pi}{b}\right)\sin\left(\frac{my_0\pi}{b}\right)\frac{e^{-k(vt-z)}}{k} \tag{2.102}$$

By doing the same we have for the other currents :

$$I_2x(t) = \frac{4Qv}{a^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{l}{m}(-1)^l[-1+(-1)^m]\sin\left(\frac{lx_0\pi}{b}\right)\sin\left(\frac{my_0\pi}{b}\right)\frac{e^{-k(vt-z)}}{k} \tag{2.103}$$

$$I_1y(t) = \frac{4Qv}{b^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{m}{l}[1-(-1)^l]\sin\left(\frac{lx_0\pi}{b}\right)\sin\left(\frac{my_0\pi}{b}\right)\frac{e^{-k(vt-z)}}{k} \tag{2.104}$$

$$I_2y(t) = \frac{4Qv}{b^2}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{m}{l}(-1)^m[-1+(-1)^l]\sin\left(\frac{lx_0\pi}{b}\right)\sin\left(\frac{my_0\pi}{b}\right)\frac{e^{-k(vt-z)}}{k} \tag{2.105}$$

2.7.3 Resistive layer grounded at $\pm a$ and insulated at $\pm b$

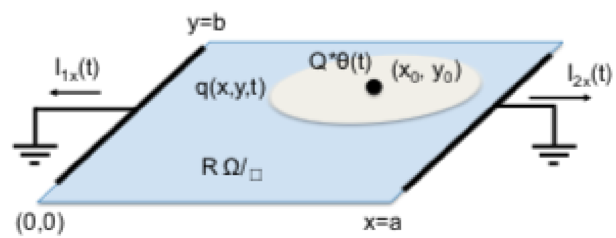


Figure 23: A point charge placed on a resistive layer that is grounded on at $x = 0$ and $x = a$ but insulated on the others border.

In this case the resistive layer is grounded at $x = 0$, $x = a$ and insulated at $y = 0$, $y = b$. The currents are only flowing into the grounded elements at $x = 0$, $x = a$. By using (2.31) we can have :

$$I_1x(t) = -\frac{1}{R}\int_0^{\infty} -\frac{\partial\varphi_1}{\partial x}|_{x=0}dy = -\frac{Q}{T\pi}\frac{\sin\left(\frac{x_0\pi}{a}\right)}{\cosh\left(\frac{t}{T}\right)-\cos((fracx_0\pi a)} \tag{2.106}$$

$$I_2x(t) = -\frac{1}{R}\int_0^{\infty} -\frac{\partial\varphi_1}{\partial x}|_{x=a}dy = -\frac{Q}{T\pi}\frac{\sin\left(\frac{x_0\pi}{a}\right)}{\cosh(\frac{t}{T})+\cos\left(\frac{x_0\pi}{a}\right)} \tag{2.107}$$

For big large times both the expressions :

$$I_1x(t) = I_2x(t) \simeq -\frac{2Q}{T\pi}\sin\left(\frac{x_0\pi}{a}\right)e^{\frac{-t}{T}} \tag{2.108}$$

Both those two currents and the approach for large values can be observe on the next figure. We assume that we have a charge deposit at position $x_0 = a/4$

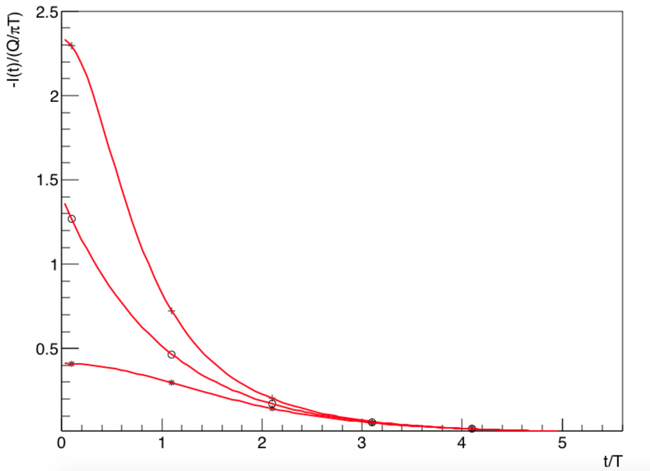
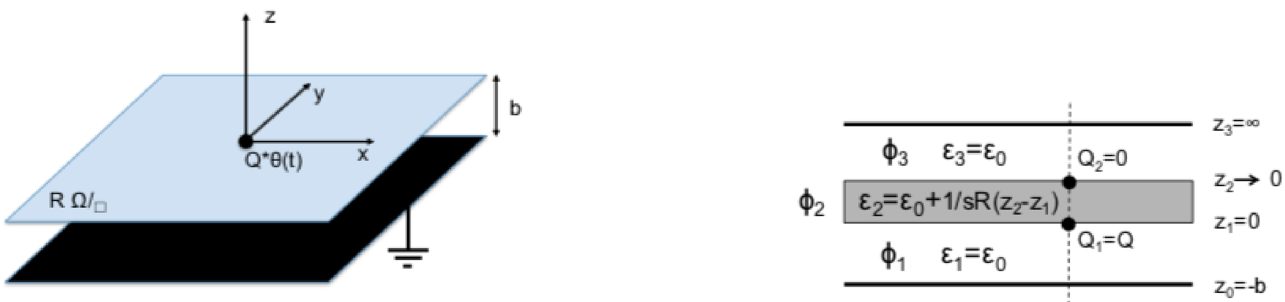


Figure 24: Currents $I_{1x}(t)$(top) and $I_{2x}(t)$(bottom) from the of figure 23 for $x_0 = a/4$. The straight line in the middle refers to the approximation from 2.108.

2.7.4 Resistive layer parallel to a grounded plane



(a) A resistive layer with surface resistance R in presence of a ground layer at distance b

(b) 3-layer geometry by performing the indicated limits of the expressions for z_2,z_3

Figure 25

In this part we want to study the fields and charges in a layer of surface resistivity R at $z = 0$ where we place a charge Q. AT $r = 0$ at $t = 0$ in presence of a grounded plane at $z = -b$ as shown in figure 25.
Like before we have $\epsilon_1 = \epsilon_0 \quad \epsilon_2 = \epsilon_0 + \frac{1}{sz_2 R} \quad \epsilon_3 = \epsilon_0 \quad Q_1 = \frac{Q}{s} \; Q_2 = 0$
By taking the limits :

$z_0 = -b \quad z_1 = 0 \quad z_2 \longrightarrow 0 \quad z_3 \longrightarrow +\infty$
The solution for the 3 areas are :

$$f_1 = A_1 e^{kz} + B_1 e^{-kz} \Rightarrow f_1 = A_1 e^{-bk} + A_2 e^{kb}$$

$$f_2 = A_2 e^{kz} + B_2 e^{-kz}$$

$$f_3 = A_3 e^{kz} + B_3 e^{-kz} \Rightarrow f_3 = B_3 e^{-kz}$$

$A_3 = 0$ because at the infinity the quantity f_2 must be zero.

By using the previous process or with the help of some programs like Mathematica we find :

$$A_1 = \frac{QRe^{kb}}{2D(k)} \quad B_1 = \frac{-QRe^{-kb}}{2D(k)} \quad A_3 = 0 \quad B_3 = \frac{-QR\sinh(kb)}{D(k)}$$

with

$$D(k) = k\sinh(kb) + e^{kb}\epsilon_0 Rs$$

In the Laplace domain :

$$A_1 = \frac{QRe^{kb}}{2D(k)} \quad B_1 = \frac{-QRe^{-kb}}{2D(k)} \quad A_3 = 0 \quad B_3 = \frac{-QR\sinh(kb)}{D(k)} \tag{2.109}$$

2.8 Uniform currents on thin resistive layers

In this part we discuss the potentials that are created on thin resistive layers from uniform charge deposition. In detectors like RPC's and Resistive Micromegas such resistive layers are used for application of the high voltage and for spark protection. The resistivity must be chosen small enough to ensure that potentials that are established on these layers due to charge-up are not influencing the applied electric fields responsible for the proper detector operation. If such detectors are in an environment of uniform particle irradiation the situation can be formulated by placing a uniform "externally impressed" current per unit area i_0 on the resistive layer.

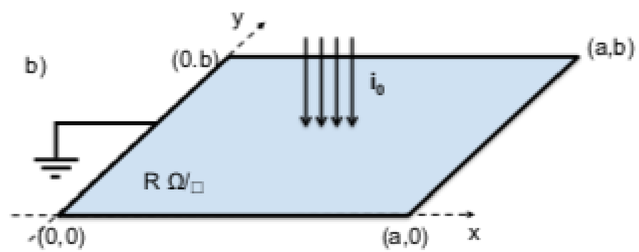


Figure 26: Uniform current "impressed" on the resistive layer will result in a potential distribution that depends strongly on the boundary conditions.

First we will investigate the geometry shown in figure 25. We have a charge dq at position x_0 and y_0 and after a time t is given by :

$dq(t) = i_0 r_0 dr_0 d\phi_0 t$ and in Laplace domain $dq(s) = i_0 r_0 dr_0 d\phi_0 \frac{1}{s^2}$

From 2.7 we replace Q/s by q(s) so :

$$f_1(k,z) = \frac{Ri_0 r_0 dr_0 d\phi_0}{k} e^{kz} \qquad f_2(k,z) = \frac{Ri_0 r_0 dr_0 d\phi_0}{k} e^{-kz} \tag{2.110}$$

In this geometry we have a rectangular. We replace $r_0 dr_0 d\phi_0$ by $dx_0 dy_0$ and perform integration $\int_0^a dx_0 \int_0^b dy_0$ on (2.30) and so :

$$\phi_1(x,y,z) = \phi_3(x,y,-z) = abRi_0 \frac{4}{\pi^4} \sum_{l=1}^{\infty} \sum_{m=1}^{\infty} \frac{[1-(-1)^l][1-(-1)^m]\sin(\frac{lx\pi}{a})\sin(\frac{my\pi}{b})}{l^3 mb/a + m^3 la/b} e^{k_{lm} z} \tag{2.111}$$

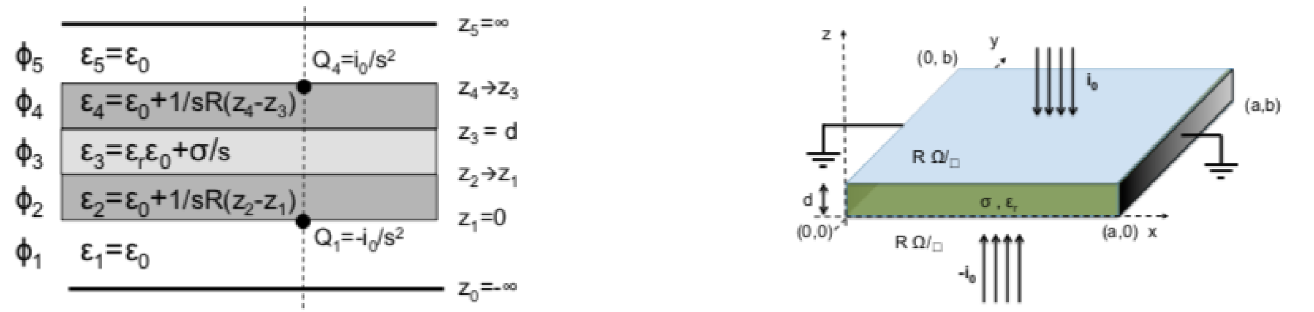
This expression cannot be written in closed form. The peak can be found by setting $d\phi_1/dx$=0,$d\phi_1/dy = 0$. It can be found at $x = a/2$ and $y = a/2$, which is also evident for symmetry on this geometry.
The maximum potential on the resistive layer is then :

$$\phi_{max}(a/2, b/2, z=0)\frac{1}{8}a^2b^2Ri_0\frac{128}{\pi^4}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^{l+m}}{b^2(2l-1)^3(2m-1)+a^2(2m-1)^3(2l-1)} \tag{2.112}$$

for square geometry ($a = b$) the sum evaluates to ≈ 0.59 so the peak voltage in the center is :

$$\phi_{max} \approx 0.074 R i_0 a^2 = 0.074 R I_{tot} \tag{2.113}$$

2.9 Uniform Currents on a resistive plate covered with a thin resistive layer



(a) Geometry to define a single resistive layer of thickness d covered by a resistive surface layer of resistance R Ω/ Square.

(b) Uniform currents of$+i_0$,$-i_0$ on the top and bottom of the surface

Figure 27

In this part we want to investigate the potential drop across a rectangular resistive plate that is covered by a thin resistive layer, that is grounded on two sides.
From before in this case we have :

$$f_1(k,z) = \frac{i_0}{\sigma} \frac{e^{kz} \sinh\left(\frac{kz}{2}\right)}{\cosh\left(\frac{kd}{2}\right) + \frac{k}{R\sigma}\sinh\left(\frac{kd}{2}\right)} \tag{2.114}$$

$$f_5(k,z) = -\frac{i_0}{\sigma} \frac{e^{k(d-z)} \sinh\left(\frac{kz}{2}\right)}{\cosh\left(\frac{kd}{2}\right) + \frac{k}{R\sigma}\sinh\left(\frac{kd}{2}\right)} \tag{2.115}$$

We can see that $f_5(k,z) = f_1(k, d-z)$, which is due to the symmetry of the problem. For uniform illumination we proceed as before by talking the solution a grounded geometry at $x = 0$ and $x = b$ and insulated at $y = 0$ and $y = b$ and we write $i_0 dx_0 dy_0$ and integrate over x_0 and y_0, which gives :

$$\phi_n(x,z) = 2a \sum_{l=1}^{\infty} \frac{[1-(-1)^l]\sin\left(\frac{l x \pi}{a}\right)}{l^2\pi^2} f_n(l\pi/a, z) \tag{2.116}$$

The potential difference between the top and the bottom of the plate, is given by :

$$\Delta V(x) = \phi_1(x, z=0) - \phi_5(x, z=d) = \frac{4 a i_0}{\sigma} \sum_{l=1}^{\infty} \frac{[1-(-1)^l]\sin\left(\frac{l x \pi}{a}\right)}{l^2\pi^2} \frac{\sinh\left(\frac{l d \pi}{2a}\right)}{\cosh\left(\frac{l d \pi}{2a}\right) + \frac{l\pi}{aR\sigma}\sinh\left(\frac{l d \pi}{2a}\right)} \tag{2.117}$$

The maximum potential found at $x = a/2$ and evaluates at :

$$\Delta V(a/2) = frac2ai_0\sigma \sum_{l=1}^{\infty} \frac{[1-(-1)^l]^2(-1)^{(l+1)/2}}{l^2\pi^2} \frac{\sinh(\frac{l d \pi}{2a})}{\cosh(\frac{l d \pi}{2a}) + \frac{l\pi}{aR\sigma}\sinh(\frac{l d \pi}{2a})} \tag{2.118}$$

For a infinitely long layer we expand the expression for small values of d/a :

$$\Delta V(a/2) \approx frac2ai_0\sigma \sum_{l=1}^{\infty} \frac{[1-(-1)^l]^2(-1)^{(l+1)/2}}{l^2\pi^2} \frac{l d \pi}{2a} = \frac{i_0}{\sigma} d = \Delta V_0 \tag{2.119}$$

which is the expected expression for the voltage drop across s resistive plate. The effective resistance of a small surface A id therefore given by :

$$\Delta V_0 = \frac{i_0}{\sigma} d = \frac{i_0 A}{\sigma A} d = I_0 \frac{d}{\sigma A} = I_0 R_0 \tag{2.120}$$

with $R_0 = \frac{d}{\sigma A}$

In case the plate resistivity is much larger than the surface resistivity we can neglect the first term in the denominator and the expression evaluates to :

$$\Delta V(a/2) = \frac{1}{4} a^2 R i_0 := \Delta V_1 \tag{2.121}$$

The effective resistance of a small surface A is :

$$\Delta V_1 = \frac{1}{4} a^2 R i_0 = \frac{1}{4A} a^2 R A i_0 = \frac{1}{4A} a^2 R I_0 = R_1 I_0 \tag{2.122}$$

with $R_1 = \frac{1}{4A} a^2 R$
The transition between the two cases pf surface resistivity only and bulk resistivity only is therefore given when $R_1 = R_2$

For

$$R_0 = R_1 \rightarrow R = \frac{4D}{\sigma a^2} := R_{eff} \tag{2.123}$$

The expression for the potential difference across the plate can be written :

$$\Delta V(x) = \frac{4 a i_0}{\sigma} \sum_{l=1}^{\infty} \frac{[1 - (-1)^l] \sin\left(\frac{l x \pi}{a}\right)}{l^2 \pi^2} \frac{\sinh\left(\frac{l d \pi}{2a}\right)}{\cosh\left(\frac{l d \pi}{2a}\right) + \frac{R_{eff}}{R} \frac{l \pi a}{4d} \sinh\left(\frac{l d \pi}{2a}\right)} \tag{2.124}$$

and defining $f = d/(2a)$, the maximum potential ΔV in the center of the resistive plate at $x = a/2$:
The expression for the potential difference across the plate can be written :

$$\Delta V(x) - \Delta V_0 = \sum_{l=1}^{\infty} \frac{[1 - (-1)^l]^2 (-1)^{(l+1)/2}}{l \pi} \frac{\sinh\left(\frac{l f \pi}{l \pi f}\right)}{\cosh(l \pi f) + \frac{R_{eff}}{R} \frac{l \pi}{8f} \sinh(l \pi f)} \tag{2.125}$$

As an approximate model one can assume that the current that is placed on the resistive plate will divide to the effective resistances R_1 and R_2, so the voltage drop should be given by

$$\Delta V \approx \frac{R_0 R_1}{R_0 + R_1} I_0 = \frac{\Delta V_0}{1 + \frac{R_{eff}}{R}} \tag{2.126}$$

This expression is very close to the first term of the sum in (2.125),($l = 1$) in face of $f \ll 1$.
For practical RPC's applications with glass RPC's, the glass thickness is typically 0.4 mm and the length a is about $a = 0.7cm$, so a typical value is $f = 0.01$. The figure 28 shows both those expressions and we see that the expression works quite well.

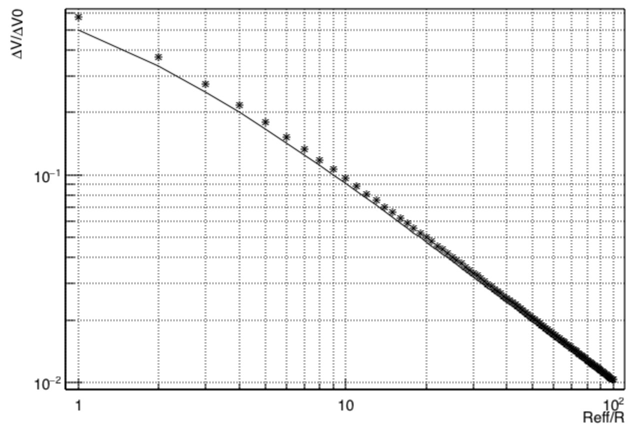


Figure 28: Voltage across the center of the resistive plate for a value of $f = d/(2a) = 0.01$. The dots refer to the exact formula (2.125), the curved line corresponds to the approximation from (2.126).

2.10 Signals and charge spread in detectors with resistive elements

In this subsection we calculate the signals induced on a readout pad or a readout strip in presence of a resistive layer, either as a bulk resistive layer touching the readout structure figure 29 or as a thin resistive layer that is insulated from readout pads(figure). Like we discussed before the time dependent weighting fields for a pad of

(a) Weighting field for a geometry with a resistive layer having a bulk resistivity of $\rho = \frac{1}{\sigma}$.

(b) Weighting field for a geometry with a thin resistive layer of value R.

dimension w_x and w_y centred at zero and a infinitely long strip of width w_x and w_y centred at zero, can be written :

$$E_w^z(x, y, z, t) = \frac{V_w}{g}\frac{4}{\pi^2}\int_0^\infty\int_0^\infty \cos\left(k_x\frac{x}{g}\right)\sin\left(k_x\frac{w_x}{2g}\right)\cos\left(k_y\frac{y}{g}\right)\sin\left(k_y\frac{w_y}{2g}\right)\frac{h(k,z,t)}{k_x k_y}\mathrm{d}k_x\mathrm{d}k_y \tag{2.127}$$

$$E_w^z(x, y, z, t) = \frac{V_w}{g}\frac{2}{\pi}\int_0^\infty\int_0^\infty \cos\left(k_x\frac{x}{g}\right)\sin\left(k_x\frac{w_x}{2g}\right)\frac{h(k,z,t)}{k_x k_y}\mathrm{d}k_x\mathrm{d}k_y \tag{2.128}$$

for both geometries.

2.10.1 Layer with bulk resistivity

For a layer with bulk resistivity of $\rho = 1/\sigma$ the expression of h(k,z) (for 0<z<g) is :

$$h(k, z, t) = kcosh\left(k\left(1 - \frac{z}{g}\right)\right)\left(\frac{\epsilon_r\delta(t)}{D(k)} + \frac{1}{\tau_0}b_1(k)e^{-\frac{tf_1(k)}{\tau_0}}\right) \tag{2.129}$$

$$D(k) = \sinh\left(k\frac{b}{g}\right)\cosh(k) + \epsilon_r\cosh\left(k\frac{b}{g}\right)\sinh(k) \tag{2.130}$$

$$b_1(k) = \frac{\sinh\left(k\frac{b}{g}\right)\cosh(k)}{D(k)^2} \qquad f_1(k) = \frac{\sinh(k)\cosh\left(k\frac{b}{g}\right)}{D(k)} \tag{2.131}$$

with $\tau_0 = \epsilon_0/\sigma = \epsilon_0\rho$

We investigate the geometry where the ground plate at $z = -b$ is segmented into infinitely long strips of width w_x. We assume a pair of charges Q,-Q produced at $t = 0$ at $z = 0$, the charge Q does not move and the charge -Q moves from $z = 0$ to $z = g$ with uniform velocity $z(t) = vt = gt/T$, 0<t<T,$T = g/v$(figure).
The current is :

$$I(t) = -\frac{-Q}{V_w}\int_0^\infty E_w(x, z(t'), t-t')z'(t')dt' = \frac{Q}{V_w}\int_0^\infty E_w(x, z(t'), t-t')g/T\mathrm{d}t' \quad t < T \tag{2.132}$$

$$I(t) = -\frac{-Q}{V_w}\int_0^T E_w(x, z(t'), t-t')z'(t')dt' = \frac{Q}{V_w}\int_0^T E_w(x, z(t'), t-t')g/T\mathrm{d}t' \quad t > T \tag{2.133}$$

And so :

$$I(t<T) = \frac{Q}{T}\int_0^\infty \frac{2}{\pi}\cos\left(k_x\frac{x}{g}\right)\sin\left(k_x\frac{w_x}{2g}\right) \times [\frac{\epsilon_r\cosh\left(k - kt/T\right)}{D(k)} +$$

$$+b_1\frac{e^{\frac{-tf1}{\tau_0}}(f_1\cosh(k) + \frac{\tau_0}{T}ksinh(k)) - f_1\cosh\left(k - k\frac{t}{T}\right) - K\frac{\tau_0}{T}\sinh\left(k - k\frac{t}{T}\right)}{k^2\frac{\tau_0^2}{T^2} - f_1^2}\mathrm{d}k \tag{2.134}$$

$$I(t>T) = \frac{Q}{T}\int_0^\infty \frac{2}{\pi}\cos\left(k_x\frac{x}{g}\right)\sin\left(k_x\frac{w_x}{2g}\right) b_1 e^{-\frac{t-T}{\tau_0}f1}\frac{e^{\frac{-Tf_1}{\tau_0}}(f_1\cosh(k) + k\frac{\tau_0}{T}sinh(k)) - f_1}{k^2\frac{\tau_0^2}{T^2} - f_1^2}\mathrm{d}k \tag{2.135}$$

For a very high resistivity limit $\tau_0 \to \infty$ the layer represents and insulator and :

$$\lim_{\tau_0\to\infty} I(t<T) = \frac{Q}{T}\int_0^\infty \frac{2}{\pi}\cos\left(k_x\frac{x}{g}\right)\sin\left(k_x\frac{w_x}{2g}\right)\frac{\epsilon_r\cosh\left(k - k\frac{t}{T}\right)}{D(k)}\mathrm{d}k$$

$$\lim_{\tau_0\to\infty} I(t>T) = 0 \tag{2.136}$$

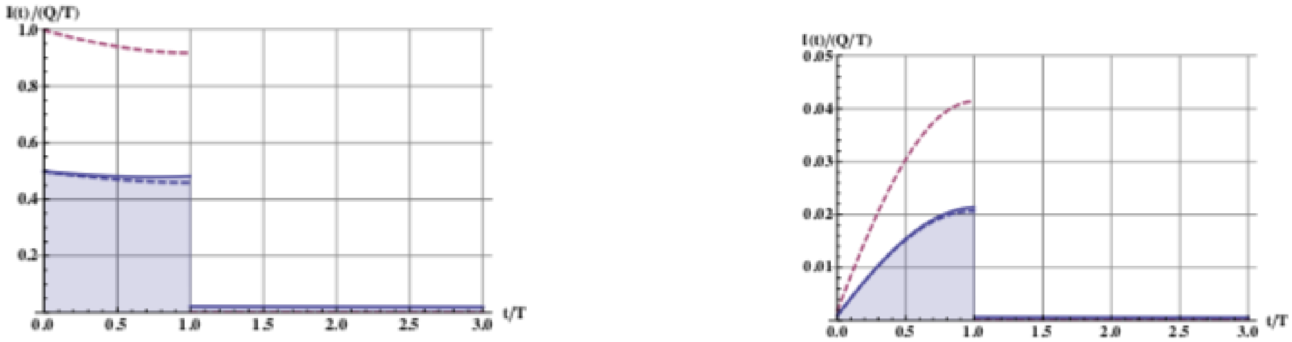
For the case where the layer represents a perfect conductor the expression becomes :

$$\lim_{\tau_0\to 0} I(t<T) = \frac{Q}{T}\int_0^\infty \frac{2}{\pi}\cos\left(k_x\frac{x}{g}\right)\sin\left(k_x\frac{w_x}{2g}\right)\frac{\cosh\left(k - k\frac{t}{T}\right)}{\sinh(k)cosh\left(k\frac{b}{g}\right)}\mathrm{d}k$$

$$\lim_{\tau_0\to 0} I(t>T) = 0 \tag{2.137}$$

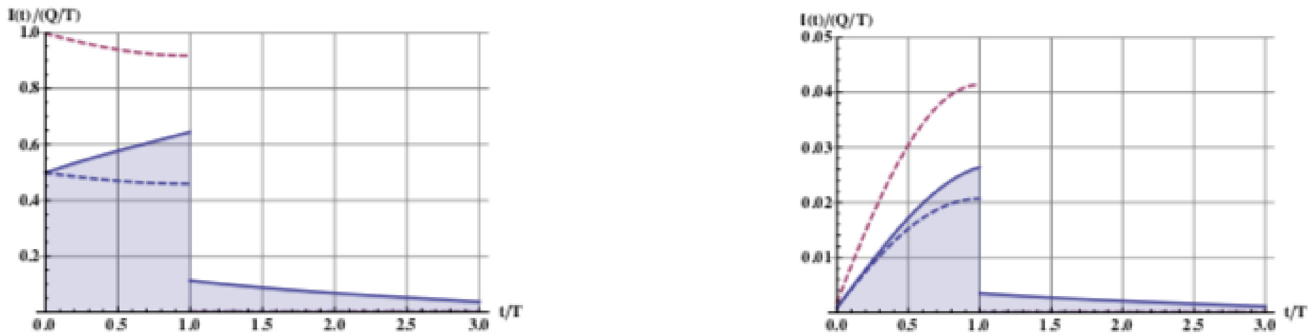
This last expression is correct if the strips are truly grounded.

For any realistic setup where the strips are connected to readout electronics and therefore have a finite resistance to ground, the signal will spread to all the strips together with the bulk behave as one single node. The result is therefore correct only to levels of conductivity σ where the impedance between the strips is significantly larger than the input resistance of the amplifier.
Figures 29,30,31 show the induced current signals given above on a central strips $w_x = 4g$ and the first neighbouring strip centred at $x = 4g$ for different values of conductivity, for a different time constants τ_0. The figures show in dashed lines also the limiting cases for a very large and very small values of τ_0.



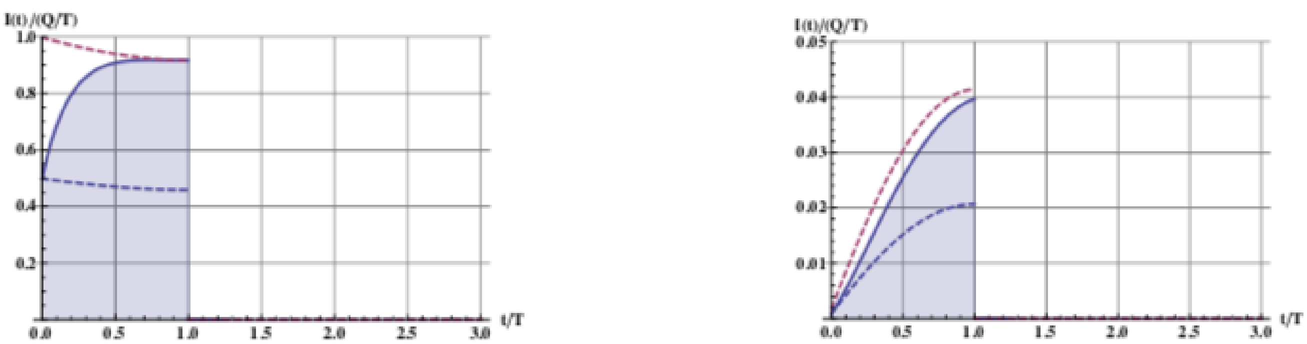
(a) $x = 0$ (b) $x = 4g$

Figure 29: Uniform charge movement from for $z = 0$ to $z = g$, with $\epsilon_r = 1$,$w_x = 4g$,$b = g$,$t_0 = 10T$.



(a) $x = 0$ (b) $x = 4g$

Figure 30: Uniform charge movement from for $z = 0$ to $z = g$, with $\epsilon_r = 1$,$w_x = 4g$,$b = g$,$t_0 = T$.



(a) $x = 0$ (b) $x = 4g$

Figure 31: Uniform charge movement from for $z = 0$ to $z = g$, with $\epsilon_r = 1$,$w_x = 4g$,$b = g$,$t_0 = 0.1T$.

First we observe that all the signals are unipolar, which is due to the fact that the charge that is flowing in the resistive bulk layer in order to compensate the charge $-Q$ sitting in the surface of the resistive plate. is truly coming out of the readout strips. For $\tau_0 = T$ the signal is significantly affected and develops a long tail for t>T due to the flow of charge compensating the point charge on the surface. The smaller the conductivity, the longer(smaller) is the tail of the signal, for $\tau_0 = 10T$. For short time constants of the resistive layer the signal on the central strip is large and has short tail and the crosstalk to the neighbor strips increases(for $\tau_0 = 0.1T$).
Next we will give the result for a pair of charges created at $z = g$ and the charge -Q moving from $z = g$ to $z = 0$ with $z(t) = g - \frac{gt}{T}$.

$$I(t<T)=\frac{Q}{T}\int_0^\infty \frac{2}{\pi}\cos\left(k_x\frac{x}{g}\right)\sin\left(k_x\frac{w_x}{2g}\right)\left(\frac{\epsilon_r\cosh(kt/T}{D(k))}+b_1f_1\frac{e^{\frac{-tf1}{\tau_0}}-f_1\cosh\left(k\frac{t}{T}\right)+k\frac{\tau_0}{T}\sinh\left(k\frac{t}{T}\right)}{k^2\frac{\tau_0^2}{T^2}-f_1^2}\right)\mathrm{d}k \tag{2.138}$$

$$I(t>T)=\frac{Q}{T}\int_0^\infty \frac{2}{\pi}\cos(k_x\frac{x}{g})\sin\left(k_x\frac{w_x}{2g}\right)b1e^{-\frac{t-T}{\tau_0}f1}f_1\frac{e^{\frac{-Tf_1}{\tau_0}}+k\frac{\tau_0}{T}sinh(k)-f_1\cosh(k)}{k^2\frac{\tau_0^2}{T^2}-f_1^2}dk \tag{2.139}$$

For a very high resistivity limit $\tau_0\to\infty$ the layer represents and insulator and :

$$\lim_{\tau_0\to\infty}I(t<T)=\frac{Q}{T}\int_0^\infty \frac{2}{\pi}\cos\left(k_x\frac{x}{g}\right)\sin\left(k_x\frac{w_x}{2g}\right)\frac{\epsilon_r\cosh\left(k\frac{t}{T}\right)}{D(k)}\mathrm{d}k$$

$$\lim_{\tau_0\to\infty}I(t>T)=0 \tag{2.140}$$

For the case where the layer represents a perfect conductor the expression becomes :

$$\lim_{\tau_0\to 0}I(t<T)=\frac{Q}{T}\int_0^\infty \frac{2}{\pi}\cos\left(k_x\frac{x}{g}\right)\sin\left(k_x\frac{w_x}{2g}\right)\frac{\cosh\left(k\frac{t}{T}\right)}{\sinh(k)cosh\left(k\frac{b}{g}\right)}\mathrm{d}k$$

$$\lim_{\tau_0\to 0}I(t>T)=0 \tag{2.141}$$

2.11 Layer with surface resistivity

Last we discuss the example for a thin layer of surface resistivity R on top of an insulating layer. The expression of h(k,z) :

$$h(k,z,t)=k\cosh\left(k(1-\frac{z}{g})\right)\left(\frac{\epsilon_r\delta(t)}{D(k)}-\frac{1}{T_0}b_2(k)e^{-\frac{tf_2(k)}{T_0}}\right) \tag{2.142}$$

$$D(k)=\sinh(k\left(\frac{b}{g}\right))\cosh(k)+\epsilon_r\cosh\left(k\frac{b}{g}\right)\sinh(k) \tag{2.143}$$

$$b_2(k)=k\frac{\epsilon_r\sinh\left(k\frac{b}{g}\right)\sinh(k)}{D^2(k)}\qquad f_2(k)=k\frac{\sinh(k)\sinh\left(k\frac{b}{g}\right)}{D(k)} \tag{2.144}$$

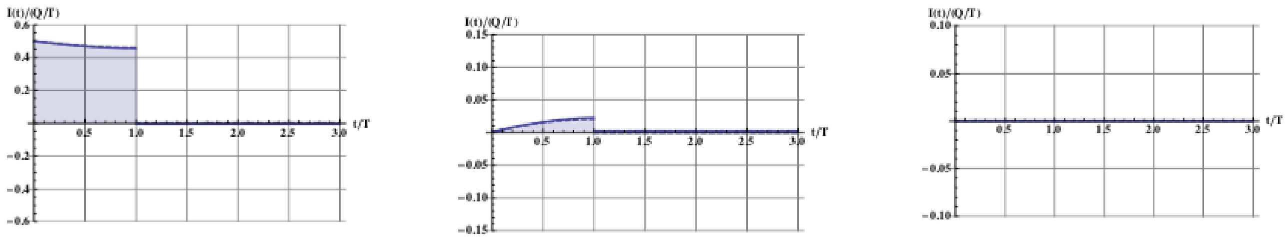
with $T_0=\epsilon_0 Rg$ is the 'time constant associated with the resistive layer' in the given geometry.
For a pair of charges Q,-Q produced at $t=0$ at $z=0$, the charge Q does not move and the charge -Q moves from $z=0$ to $z=g$ during a time T, we find :

$$I(t<T)=\frac{Q}{T}\int_0^\infty \frac{2}{\pi}\cos\left(k_x\frac{x}{g}\right)\sin\left(k_x\frac{w_x}{2g}\right)\times[\frac{\epsilon_r\cosh\left(k-kt/T\right)}{D(k)}+$$

$$-b_2\frac{e^{\frac{-tf2}{\tau_0}}(f_2\cosh(k)+\frac{\tau_0}{T}ksinh(k))-f_2\cosh\left(k-k\frac{t}{T}\right)-K\frac{\tau_0}{T}\sinh\left(k-k\frac{t}{T}\right)}{k^2\frac{\tau_0^2}{T^2}-f_2^2}\mathrm{d}k \tag{2.145}$$

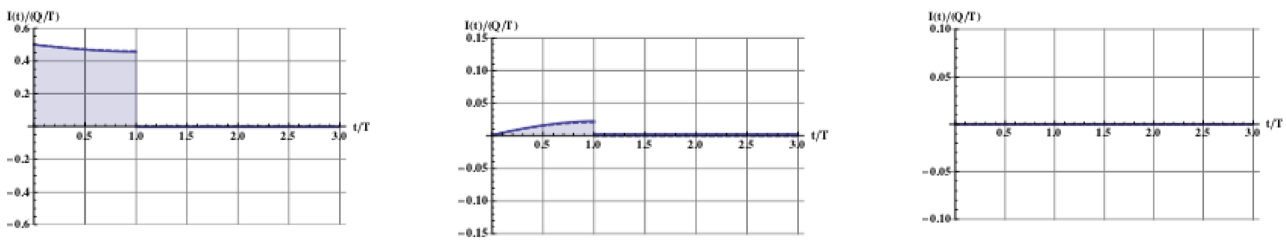
$$I(t>T)=\frac{Q}{T}\int_0^\infty \frac{2}{\pi}\cos\left(k_x\frac{x}{g}\right)\sin\left(k_x\frac{w_x}{2g}\right)b_2e^{-\frac{t-T}{\tau_0}f2}\frac{e^{\frac{-Tf_2}{\tau_0}}(f_2\cosh(k)+k\frac{\tau_0}{T}\sinh(k))-f_2}{k^2\frac{\tau_0^2}{T^2}-f_2^2}\mathrm{d}k \tag{2.146}$$

The limited case for high resistivity is equal to the previous subsection's where there is only an insulating layer. For the limited case for small resistance R, the I(t) becomes zero, since the resistive layer turns into a 'metal plate' that shields the strips from the charges $-Q$ and Q.
The signals for a central strip of width $w_x = 4g$ as well as the neighbouring strips at $x = 4g$ and $x = 8g$ as shown in figures 32-36 for different values of R and different time constant T_0.



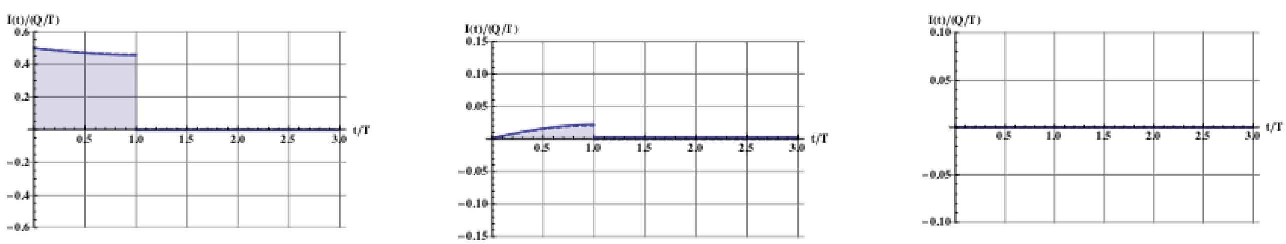
(a) $x = 0$ **(b)** $x = 4g$ **(c)** $x = 8g$

Figure 32: Uniform charge movement from for $z = 0$ to $z = g$, with $\epsilon_r = 1$,$w_x = 4g$,$b = g$,$t_0 = 10T$.



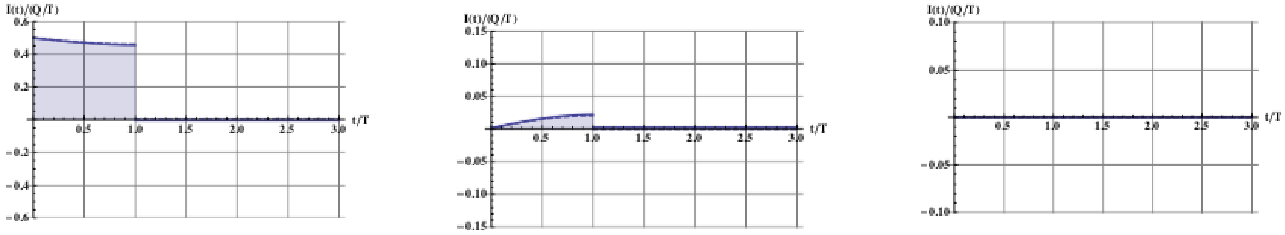
(a) $x = 0$ **(b)** $x = 4g$ **(c)** $x = 8g$

Figure 33: Uniform charge movement from for $z = 0$ to $z = g$, with $\epsilon_r = 1$,$w_x = 4g$,$b = g$,$t_0 = T$.



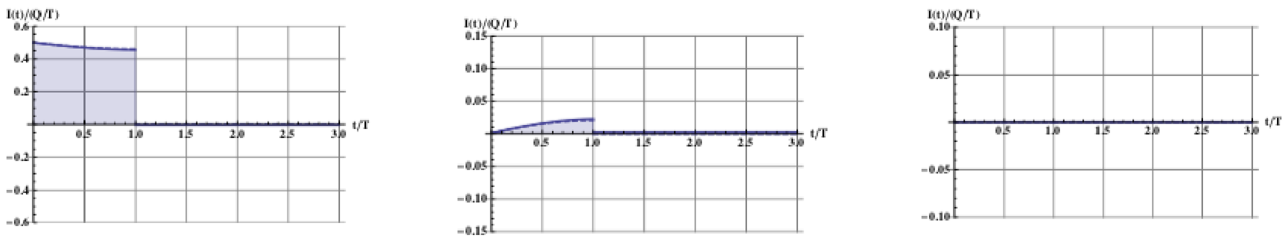
(a) $x = 0$ **(b)** $x = 4g$ **(c)** $x = 8g$

Figure 34: Uniform charge movement from for $z = 0$ to $z = g$, with $\epsilon_r = 1$,$w_x = 4g$,$b = g$,$t_0 = 0.1T$.



(a) $x = 0$ **(b)** $x = 4g$ **(c)** $x = 8g$

Figure 35: Uniform charge movement from for $z = 0$ to $z = g$, with $\epsilon_r = 1$,$w_x = 4g$,$b = g$,$t_0 = 0.01T$.



(a) $x = 0$ **(b)** $x = 4g$ **(c)** $x = 8g$

Figure 36: Uniform charge movement from for $z = 0$ to $z = g$, with $\epsilon_r = 1$,$w_x = 4g$,$b = g$,$t_0 = 0.001T$.

In case the time constant T_0 is large, the effect of resistivity disappears and the case of $T_0 = 10T$ shows signal shapes very close to the on from the previous section for large values of τ_0. For decreasing resistivity and so T_0, we see that the signal on the central strip starts to be "differentiated" and develops an undershoot and the crosstalk to the other strips increases.
The signal are strictly bipolar. This is due to the fact that the current compensating the point charge $-Q$ is entirely flowing inside the thin resistive layer and no net charge is taken from or is arriving at the strips.
Next we will give the result for a pair of charges created at $z = g$ and the charge -Q moving from $z = g$ to $z = 0$

$$I(t<T) = \frac{Q}{T}\int_0^\infty \frac{2}{\pi}\cos\left(k_x\frac{x}{g}\right)\sin\left(k_x\frac{w_x}{2g}\right)\frac{\epsilon_r\cosh(kt/T)}{D(k)} - b_2 f_2 \frac{e^{\frac{-tf2}{T_0}} - f_2\cosh(k\frac{t}{T}) + k\frac{T_0}{T}\sinh(k\frac{t}{T})}{k^2\frac{T_0^2}{T^2} - f_2^2}]\mathrm{d}k \quad (2.147)$$

$$I(t>T) = \frac{Q}{T}\int_0^\infty \frac{2}{\pi}\cos\left(k_x\frac{x}{g}\right)\sin\left(k_x\frac{w_x}{2g}\right) b2e^{-\frac{t-T}{T_0}f2} f_2 \frac{e^{\frac{-Tf_2}{T_0}} + k\frac{T_0}{T}\sinh(k) - f_2\cosh(k)}{k^2\frac{T_0^2}{T^2} - f_2^2}\mathrm{d}k \quad (2.148)$$

3 Micromegas Simulation with Ansys Maxwell

In this Section and the next one we will try to analyze some structures of Micromegas Modules. Specifically in this Section we will create some variety of MM, with the help of Ansys Maxwell 14.0. Our aim is to check how the capacitance between the strips are changing for every one of those modules. Our analysis will focus mostly for the values between the central strip and the two neighbor in all those cases.

Maxwell is a high-performance interactive software package that uses finite element analysis(FEA) to solve two-dimensional and three-dimensional (3D) electric,magnetostatic, eddy current and transient problems. In this section in order to create the modules we need, we solve two-dimensional problems.

3.1 Structure of the creating Modules by Maxwell

In the following figure that was taken by a microscope we can see a form of the Micromegas Module that we examine in our work.

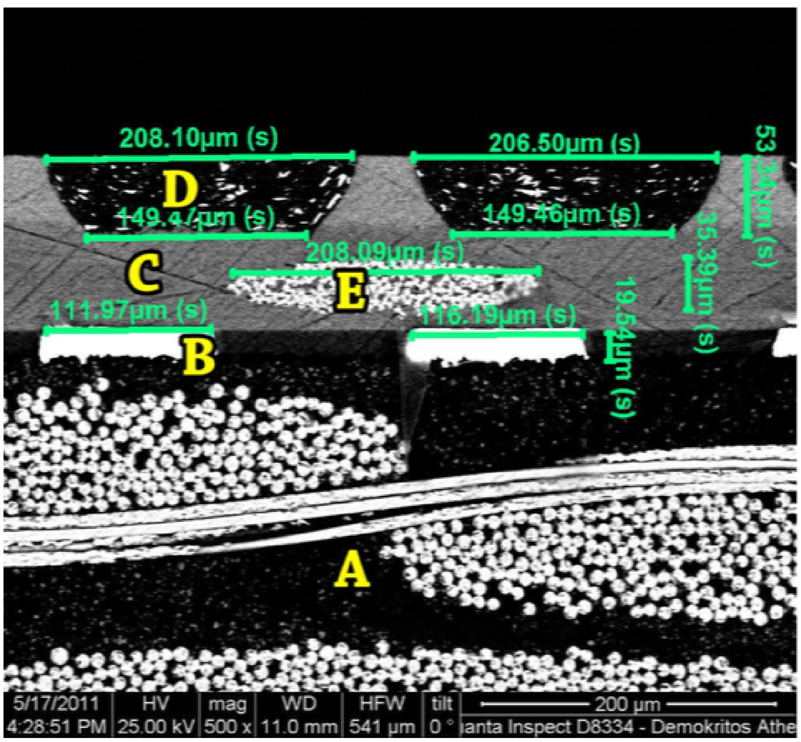


Figure 37: Micromegas Module by microscope

The Micromegas modules that we are creating in this section are consist of the following parts :

(A) the bottom is the element Fr4 with dielectric constant 4.4.

(B) the readout strips that they are important to read the signal and they are creating from copper.

(C) the area between the Resistive and the Readout strips consisting of insulating material PC1025 Dupont with dielectric constant 3.5.

(D) the Resistive strips that they consisting of Resistive material with conductance $0.059\,siemens/m$ and dielectric constant 1.

(E) a support area of the Module(we will not focus on this).

In the next subsection we will give more details about those parts and their dimensions.

3.2 Micromegas Modules

In the following figure is shown the general structure of the Micromegas module that was built by the Ansys Maxwell program. The following module has 3 Resistive strips and 3 read out strips.

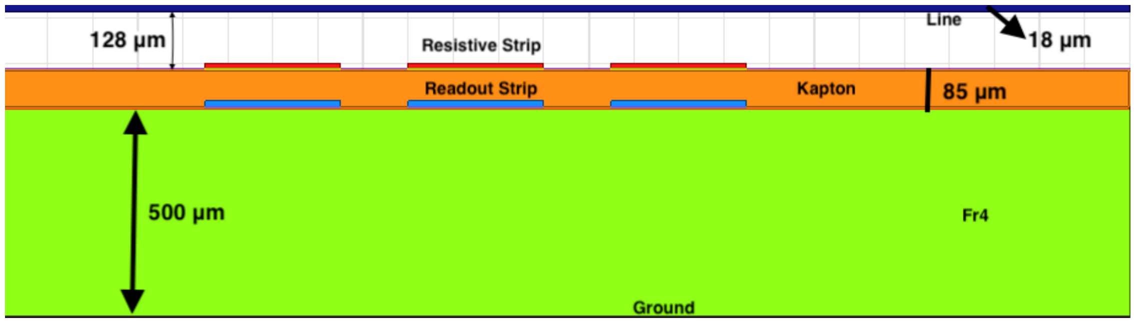


Figure 38: Micromegas Module with 3 Resistive and 3 readout strips.

As we discussed in the previous subsection all of our models are included by the fr4, the Dupont and a number of Resistive and Readout Strips. We will also add a Mesh(line) from Iron on a distance 18 µm from the Resistive strips and also we put the Ground on the down part of the Fr4.

As for the size of its components Ansys Maxwell give us a great variety of options to change the dimensions on the 3 axis and "play" with them. We will hold the 2-D option for our structure.

So to have a basis for all of the Modules we are going to discuss, we will give the the following dimensions to the elements :

- The resistive strips have width 300 µm and thickness 15 µm
- The Read Out strips have width300 µm and thickness 17 µm
- The Fr4 has 500 µm thickness.
- The kapton has 85 µm thickness .
- The line has 27 µm thickness.
- Between the kapton and the line we left a space of 128 µm.

This section is concerned with 5 strips and 9 strips Modules. Our Modules were slipped on two categories the **LM Modules** and the **SM Modules**. The different between those two Modules is the pitch. The LM Module has 450 µm pitch and the SM Module has 425 µm pitch.

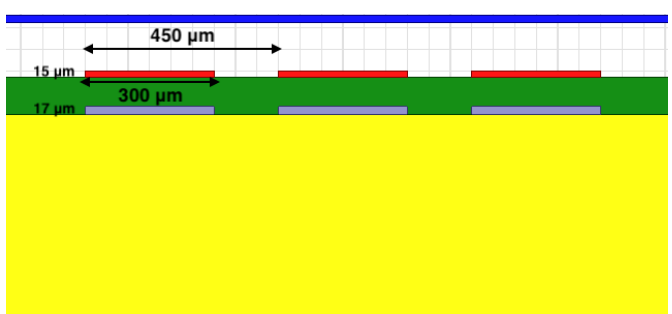


Figure 39: LM Module

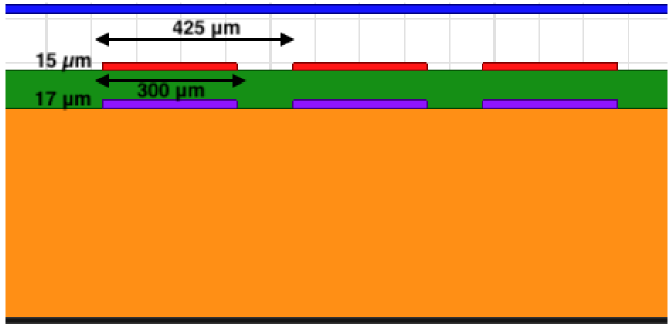


Figure 40: SM Module

For this section we built with Maxwell 7 Modules :

- two LM Modules with 5 and 9 strips each
- two SM Modules with 5 and 9 strips each
- one 5 strips Module with 100 μm distance between the strips
- and finally two Modules with 5 strips each for those two categories, but this time we added as new material glue.

By doing all these Modules we want to see how the capacitance between the strips change, by add more materials on the Module or increase the distance of some of the components.

3.2.1 Micromegas 5 strips LM Module

The first module is discussed is a 5 strips(Resistive and Readout) LM Module. In the end of the analysis with Maxwell, the program will give us a panel with the capacitance of all the strips.We focus on the central strips that in this case is Readout Strip 3 and the Resistive strip 3(Res-Ro) and the capacitance between this strip and Readout Strip 2(Ro-Ro), Resistive Strip 2 and the capacitance between Resistive strip 3 and 2(Res-Res). The values we got from this module are :

- **Res-Res : 2.6861**
- **Res-Ro : 163.3**
- **Ro-Ro : 30.39**

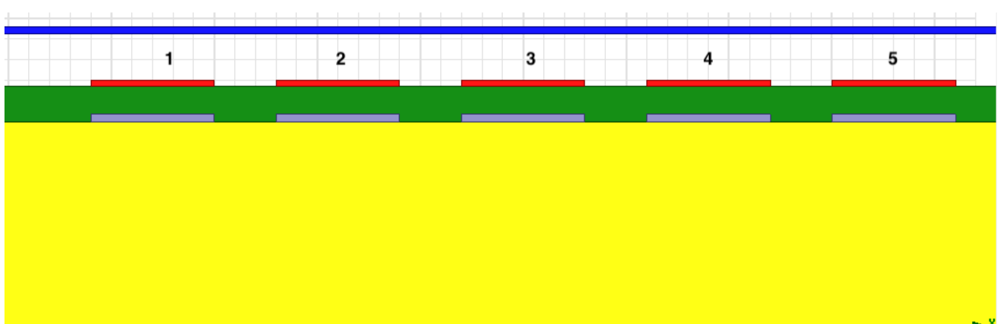


Figure 41: 5 strips LM Module

In the following table Maxwell gathered all the capacitance between the strips. The abbreviations Res and Ro are for Resistive and Readout strips. As for the the numbers of the strips were selected for the row that they were placed on the Module.

	ground	line	res1	res2	res3	res4	res5	st1	st2	st3	st4	st5
ground	288,25	-79,952	-2,296	-0,56658	-0,56401	-0,5644	-3,4541	-46,262	-34,409	-34,254	-34,444	-44,685
line	-79,952	281,12	-34,415	-31,947	-32,209	-31,942	-35,865	-11,017	-2,4006	-2,117	-2,4168	-9,9517
res1	-2,296	-34,415	220,68	-2,9729	-0,0002031	-1,32E-05	-1,74E-06	-175,07	-5,9091	-0,010851	-0,0008029	-6,77E-05
res2	-0,56658	-31,947	-2,9729	213,52	-2,6861	0,0017694	-1,70E-05	-6,0968	-163,02	-6,2232	-0,0085365	-0,0007374
res3	-0,56401	-32,209	-0,0002031	-2,6861	214,11	-2,6861	-0,0002529	-0,0099098	-6,3071	-163,33	-6,3071	-0,0097896
res4	-0,5644	-31,942	-1,32E-05	0,0017694	-2,6861	213,52	-2,9847	-0,0007465	-0,0085363	-6,2232	-163,02	-6,092
res5	-3,4541	-35,865	-1,74E-06	-1,70E-05	-0,0002529	-2,9847	217,72	-8,73E-05	-0,0010204	-0,013786	-5,9518	-169,45
st1	-46,262	-11,017	-175,07	-6,0968	-0,0099098	-0,0007465	-8,73E-05	269,24	-30,148	-0,58054	-0,043004	-0,0035961
st2	-34,409	-2,4006	-5,9091	-163,02	-6,3071	-0,0085363	-0,0010204	-30,148	273,15	-30,395	-0,50868	-0,042478
st3	-34,254	-2,117	-0,010851	-6,2232	-163,33	-6,2232	-0,013786	-0,58054	-30,395	274,11	-30,396	-0,57346
st4	-34,444	-2,4168	-0,0008029	-0,0085365	-6,3071	-163,02	-5,9518	-0,043004	-0,50868	-30,396	273,15	-30,05
st5	-44,685	-9,9517	-6,77E-05	-0,0007374	-0,0097896	-6,092	-169,45	-0,0035961	-0,042478	-0,57346	-30,05	260,86

Figure 42: Table of capacitance for 5 strips LM Module.

3.2.2 Micromegas 5 strips SM Module

The process in this module is the same with the difference that in the SM Module the distance between the strips is less than before.

- **Res-Res : 4.4941**
- **Res-Ro : 158.27**
- **Ro-Ro : 36.27**

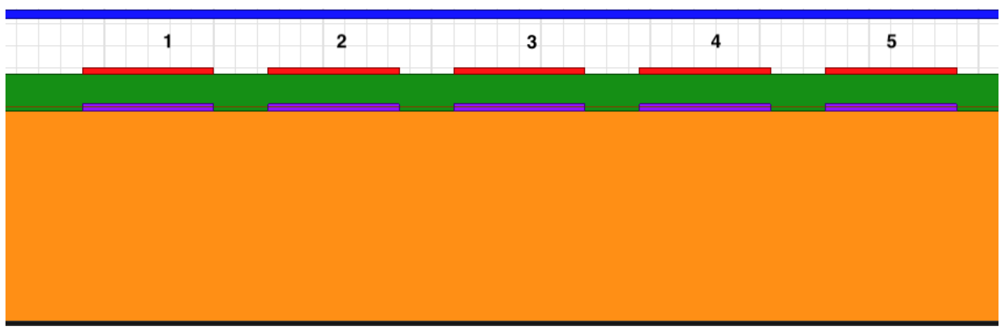


Figure 43: 5 strips SM Module

	ground	line	res1	res2	res3	res4	res5	st1	st2	st3	st4	st5
ground	284,27	-83,22	-2,6589	-0,37587	-0,36966	-0,39012	-2,4375	-44,079	-32,824	-32,609	-32,753	-45,759
line	-83,22	277,74	-33,217	-30,684	-30,784	-31,837	-35,918	-10,686	-1,8792	-1,5925	-1,3876	-9,6381
res1	-2,6589	-33,217	210,71	-4,4908	0,00051881	-1,33E-05	-1,29E-06	-163,89	-6,4382	-0,013044	-0,001135	-9,90E-05
res2	-0,37587	-30,684	-4,4908	211,01	-4,4941	-8,94E-05	-1,31E-05	-6,4376	-158,14	-6,3767	-0,0080412	-0,0006905
res3	-0,36966	-30,784	0,00051881	-4,4941	212,56	-5,9014	0,18861	-0,0085999	-6,5489	-158,27	-6,3782	0,004718
res4	-0,39012	-31,837	-1,33E-05	-8,94E-05	-5,9014	219,44	-7,9658	-0,0007658	-0,0080088	-6,8676	-159,33	-7,1376
res5	-2,4375	-35,918	-1,29E-06	-1,31E-05	0,18861	-7,9658	230,86	-7,20E-05	-0,0007447	0,019336	-6,7745	-177,97
st1	-44,079	-10,686	-163,89	-6,4376	-0,0085999	-0,0007658	-7,20E-05	262,18	-36,245	-0,75	-0,06522	-0,0056569
st2	-32,824	-1,8792	-6,4382	-158,14	-6,5489	-0,0080088	-0,0007447	-36,245	279,1	-36,277	-0,67634	-0,058614
st3	-32,609	-1,5925	-0,013044	-6,3767	-158,27	-6,8676	0,019336	-0,75	-36,277	280,03	-36,667	-0,62945
st4	-32,753	-1,3876	-0,001135	-0,0080412	-6,3782	-159,33	-6,7745	-0,06522	-0,67634	-36,667	281,59	-37,548
st5	-45,759	-9,6381	-9,90E-05	-0,0006905	0,004718	-7,1376	-177,97	-0,0056569	-0,058614	-0,62945	-37,548	278,75

Figure 44: Table of capacitance for 5 strips SM Module.

If we compare the 5 LM Module with this we can observe a different in the 3 values. The Res-Res and the Ro-Ro values are higher in the SM Module, but the Res-Ro value are lower.

3.2.3 Micromegas 9 strips LM Module

In this case we increase the number of strips from 5 to 9 in the strips and their distances from each other are the same with the 5 strips LM module.

- **Res-Res : 4.4941**
- **Res-Ro : 158.27**
- **Ro-Ro : 36.27**

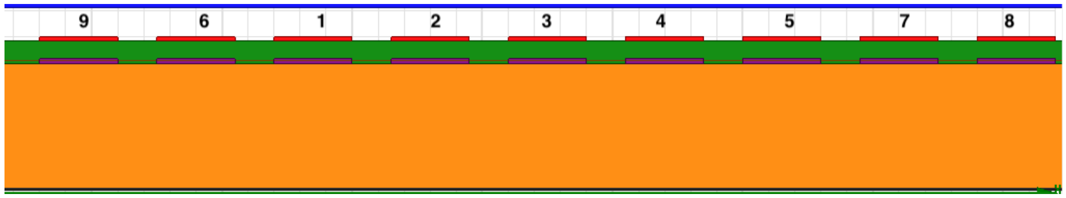


Figure 45: 9 strips LM Module

	ground	line	res1	res2	res3	res4	res5	res6
ground	361,17	-21,332	-0,55774	-0,56326	-0,56349	-0,56325	-0,5577	-0,56249
line	-21,332	359,83	-31,491	-31,945	-32,209	-31,945	-31,491	-31,494
res1	-0,55774	-31,491	212,4	-2,9729	-0,000187	-1,20E-05	-1,05E-06	-3,0375
res2	-0,56326	-31,945	-2,9729	213,52	-2,6861	0,001769	-1,20E-05	0,000165
res3	-0,56349	-32,209	-0,000187	-2,6861	214,11	-2,6861	-0,000187	-1,38E-05
res4	-0,56325	-31,945	-1,20E-05	0,001769	-2,6861	213,52	-2,9729	-1,04E-06
res5	-0,5577	-31,491	-1,05E-06	-1,20E-05	-0,000187	-2,9729	212,4	-8,64E-08
res6	-0,56249	-31,494	-3,0375	0,000165	-1,38E-05	-1,04E-06	-8,64E-08	212,4
res7	-0,55977	-31,493	-8,61E-08	-1,04E-06	-1,37E-05	0,000166	-3,0375	-7,95E-09
res8	-2,1169	-35,091	-3,05E-08	-1,00E-07	-1,30E-06	-1,83E-05	0,000118	-2,77E-07
res9	-2,1465	-35,109	0,000102	-1,96E-05	-1,39E-06	-1,07E-07	-2,98E-08	-2,9765
st1	-34,199	-2,4578	-162,51	-6,0912	-0,009512	-0,000716	-5,93E-05	-5,9166
st2	-34,211	-2,249	-5,8957	-163,02	-6,3071	-0,008536	-0,000731	-0,011207
st3	-34,223	-2,094	-0,009882	-6,2232	-163,33	-6,2232	-0,009882	-0,00081
st4	-34,21	-2,2489	-0,000731	-0,008536	-6,3071	-163,02	-5,8957	-6,00E-05
st5	-34,197	-2,4562	-5,93E-05	-0,000716	-0,009512	-6,0912	-162,51	-4,87E-06
st6	-34,334	-2,6264	-5,9163	-0,010597	-0,000762	-5,77E-05	-4,78E-06	-162,51
st7	-34,316	-2,6054	-4,78E-06	-5,76E-05	-0,000762	-0,010588	-5,9162	-4,43E-07
st8	-42,77	-11,474	-5,22E-07	-4,77E-06	-6,29E-05	-0,000881	-0,011578	-1,62E-06
st9	-42,613	-11,332	-0,011348	-0,000864	-6,16E-05	-4,67E-06	-5,06E-07	-5,8925

	res7	res8	res9	st1	st2	st3	st4	st5	st6	st7	st8	st9
ground	-0,55977	-2,1169	-2,1465	-34,199	-34,211	-34,223	-34,21	-34,197	-34,334	-34,316	-42,77	-42,613
line	-31,493	-35,091	-35,109	-2,4578	-2,249	-2,094	-2,2489	-2,4562	-2,6264	-2,6054	-11,474	-11,332
res1	-8,61E-08	-3,05E-08	0,000102	-162,51	-5,8957	-0,009882	-0,000731	-5,93E-05	-5,9163	-4,78E-06	-5,22E-07	-0,011348
res2	-1,04E-06	-1,00E-07	-1,96E-05	-6,0912	-163,02	-6,2232	-0,008536	-0,000716	-0,010597	-5,76E-05	-4,77E-06	-0,000864
res3	-1,37E-05	-1,30E-06	-1,39E-06	-0,009512	-6,3071	-163,33	-6,3071	-0,009512	-0,000762	-0,000762	-6,29E-05	-6,16E-05
res4	0,000166	-1,83E-05	-1,07E-07	-0,000716	-0,008536	-6,2232	-163,02	-6,0912	-5,77E-05	-0,010588	-0,000881	-4,67E-06
res5	-3,0375	0,000118	-2,98E-08	-5,93E-05	-0,000731	-0,009882	-5,8957	-162,51	-4,78E-06	-5,9162	-0,011578	-5,06E-07
res6	-7,95E-09	-2,77E-07	-2,9765	-5,9166	-0,011207	-0,00081	-6,00E-05	-4,87E-06	-162,51	-4,43E-07	-1,62E-06	-5,8925
res7	212,4	-2,9737	-2,56E-07	-4,86E-06	-5,98E-05	-0,000808	-0,011173	-5,9162	-4,41E-07	-162,51	-5,9006	-1,48E-06
res8	-2,9737	221,26	-7,92E-05	-1,70E-06	-5,78E-06	-7,67E-05	-0,001064	-0,013266	-1,56E-05	-6,1246	-174,91	-0,000451
res9	-2,56E-07	-7,92E-05	221,25	-0,014159	-0,001135	-8,19E-05	-6,16E-06	-1,66E-06	-6,1294	-1,46E-05	-0,000454	-174,84
st1	-4,86E-06	-1,70E-06	-0,014159	271,67	-29,824	-0,55713	-0,041269	-0,003348	-29,416	-0,00027	-2,93E-05	-0,63101
st2	-5,98E-05	-5,78E-06	-0,001135	-29,824	273,15	-30,396	-0,50867	-0,041268	-0,6218	-0,00332	-0,000275	-0,050269
st3	-0,000808	-7,67E-05	-8,19E-05	-0,55713	-30,396	274,11	-30,396	-0,55712	-0,044848	-0,04481	-0,003699	-0,003626
st4	-0,011173	-0,001064	-6,16E-06	-0,041269	-0,50867	-30,396	273,15	-29,824	-0,003322	-0,62128	-0,051284	-0,000269
st5	-5,9162	-0,013266	-1,66E-06	-0,003348	-0,041268	-0,55712	-29,824	271,67	-0,00027	-29,409	-0,64375	-2,84E-05
st6	-4,41E-07	-1,56E-05	-6,1294	-29,416	-0,6218	-0,044848	-0,003322	-0,00027	271,37	-2,45E-05	-9,13E-05	-29,757
st7	-162,51	-6,1246	-1,46E-05	-0,00027	-0,00332	-0,04481	-0,62128	-29,409	-2,45E-05	271,67	-30,103	-8,45E-05
st8	-5,9006	-174,91	-0,000454	-2,93E-05	-0,000275	-0,003699	-0,051284	-0,64375	-9,13E-05	-30,103	266,06	-0,002586
st9	-1,48E-06	-0,000451	-174,84	-0,63101	-0,050269	-0,003626	-0,000269	-2,84E-05	-29,757	-8,45E-05	-0,002586	265,31

Figure 46: Table of capacitance for 9 strips LM Module.

3.2.4 Micromegas 9 strips SM Module

This module also is the same with the 5 strip SM Module but with 9 strips instead of 5.

- **Res-Res : 2.6861**
- **Res-Ro : 163.3**
- **Ro-Ro : 30.39**

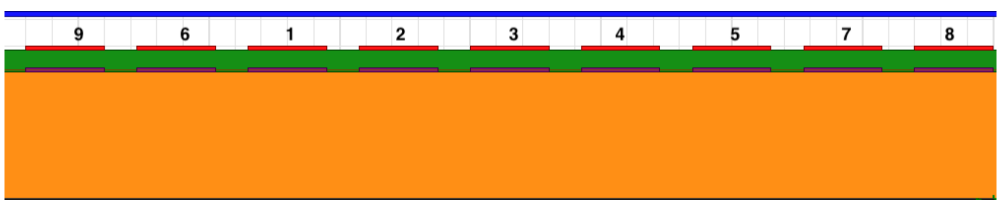


Figure 47: 9 strips SM Module

	ground	line	res1	res2	res3	res4	res5	res6
ground	354,98	-27,1	-0,37281	-0,37288	-0,37315	-0,37043	-0,36799	-0,37847
line	-27,1	353,46	-30,628	-30,681	-30,918	-31,23	-31,389	-30,787
res1	-0,37281	-30,628	210,83	-4,4903	0,000568	-8,71E-06	-7,14E-07	-4,4906
res2	-0,37288	-30,681	-4,4903	211,01	-4,4941	-0,000119	-7,44E-06	0,000226
res3	-0,37315	-30,918	0,000568	-4,4941	211,72	-4,4734	0,000504	-9,45E-06
res4	-0,37043	-31,23	-8,71E-06	-0,000119	-4,4734	212,63	-4,4192	-7,96E-07
res5	-0,36799	-31,389	-7,14E-07	-7,44E-06	0,000504	-4,4192	213,08	-6,54E-08
res6	-0,37847	-30,787	-4,4906	0,000226	-9,45E-06	-7,96E-07	-6,54E-08	211,32
res7	-0,37248	-31,232	-6,29E-08	-6,54E-07	-3,48E-06	0,002844	-4,4192	-5,99E-09
res8	-2,7137	-35,814	-1,65E-08	-8,72E-08	-9,24E-07	-6,36E-06	0,000471	-9,07E-08
res9	-2,0494	-34,719	0,000306	-1,38E-05	-1,25E-06	-1,07E-07	-1,65E-08	-4,4978
st1	-32,574	-1,7241	-158,08	-6,4343	-0,008305	-0,000715	-5,87E-05	-6,4935
st2	-32,564	-1,6882	-6,3922	-158,14	-6,5491	-0,007755	-0,000631	-0,008745
st3	-32,569	-1,5831	-0,008802	-6,3767	-158,39	-6,6574	-0,006775	-0,000806
st4	-32,584	-1,4042	-0,000766	-0,008038	-6,5124	-158,73	-6,7836	-7,01E-05
st5	-32,606	-1,3119	-6,34E-05	-0,000659	-0,007042	-6,7385	-158,9	-5,81E-06
st6	-32,721	-1,8413	-6,3644	-0,008317	-0,000758	-6,47E-05	-5,31E-06	-158,24
st7	-32,765	-1,5837	-5,45E-06	-5,66E-05	-0,000608	-0,006275	-6,7836	-5,20E-07
st8	-43,721	-10,164	-5,78E-07	-5,45E-06	-5,84E-05	-0,000622	-0,007064	-6,48E-07
st9	-41,782	-10,918	-0,008693	-0,000783	-7,15E-05	-6,10E-06	-5,45E-07	-6,4233

	res7	res8	res9	st1	st2	st3	st4	st5	st6	st7	st8	st9
ground	-0,37248	-2,7137	-2,0494	-32,574	-32,564	-32,569	-32,584	-32,606	-32,721	-32,765	-43,721	-41,782
line	-31,232	-35,814	-34,719	-1,7241	-1,6882	-1,5831	-1,4042	-1,3119	-1,8413	-1,5837	-10,164	-10,918
res1	-6,29E-08	-1,65E-08	0,000306	-158,08	-6,3922	-0,008802	-0,000766	-6,34E-05	-6,3644	-5,45E-06	-5,78E-07	-0,008693
res2	-6,54E-07	-8,72E-08	-1,38E-05	-6,4343	-158,14	-6,3767	-0,008038	-0,000659	-0,008317	-5,66E-05	-5,45E-06	-0,000783
res3	-3,48E-06	-9,24E-07	-1,25E-06	-0,008305	-6,5491	-158,39	-6,5124	-0,007042	-0,000758	-0,000608	-5,84E-05	-7,15E-05
res4	0,002844	-6,36E-06	-1,07E-07	-0,000715	-0,007755	-6,6574	-158,73	-6,7385	-6,47E-05	-0,006275	-0,000622	-6,10E-06
res5	-4,4192	0,000471	-1,65E-08	-5,87E-05	-0,000631	-0,006775	-6,7836	-158,9	-5,31E-06	-6,7836	-0,007064	-5,45E-07
res6	-5,99E-09	-9,07E-08	-4,4978	-6,4935	-0,008745	-0,000806	-7,01E-05	-5,81E-06	-158,24	-5,20E-07	-6,48E-07	-6,4233
res7	212,63	-4,4739	-9,50E-08	-5,17E-06	-5,55E-05	-0,000597	-0,006275	-6,7385	-4,87E-07	-158,73	-6,6609	-5,81E-07
res8	-4,4739	222,62	-3,55E-05	-1,35E-06	-7,40E-06	-7,90E-05	-0,000857	-0,009944	-7,54E-06	-6,5475	-173,05	-0,000202
res9	-9,50E-08	-3,55E-05	220,61	-0,012454	-0,00117	-0,000108	-9,44E-06	-1,47E-06	-6,6427	-8,43E-06	-0,000237	-172,65
st1	-5,17E-06	-1,35E-06	-0,012454	278,6	-35,957	-0,72342	-0,062911	-0,005209	-35,815	-0,000448	-4,75E-05	-0,71413
st2	-5,55E-05	-7,40E-06	-0,00117	-35,957	279,1	-36,277	-0,67635	-0,055994	-0,70987	-0,00481	-0,000462	-0,066922
st3	-0,000597	-7,90E-05	-0,000108	-0,72342	-36,277	280,01	-36,687	-0,60302	-0,065438	-0,05179	-0,004975	-0,00617
st4	-0,006275	-0,000857	-9,44E-06	-0,062911	-0,67635	-36,687	281,44	-37,368	-0,005692	-0,56192	-0,053968	-0,000537
st5	-6,7385	-0,009944	-1,47E-06	-0,005209	-0,055994	-0,60302	-37,368	282,34	-0,000471	-37,371	-0,62868	-4,84E-05
st6	-4,87E-07	-7,54E-06	-6,6427	-35,815	-0,70987	-0,065438	-0,005692	-0,000471	278,56	-4,22E-05	-5,38E-05	-36,138
st7	-158,73	-6,5475	-8,43E-06	-0,000448	-0,00481	-0,05179	-0,56192	-37,371	-4,22E-05	281,4	-36,997	-5,14E-05
st8	-6,6609	-173,05	-0,000237	-4,75E-05	-0,000462	-0,004975	-0,053968	-0,62868	-5,38E-05	-36,997	271,38	-0,00135
st9	-5,81E-07	-0,000202	-172,65	-0,71413	-0,066922	-0,00617	-0,000537	-4,84E-05	-36,138	-5,14E-05	-0,00135	268,89

Figure 48: Table of capacitance for 9 strips SM Module.

3.3 More cases of micromegas Modules

As the next step we examine some more cases of Micromegas Modules with different distances between the strips and what happens if we put more elements in one of these Modules.

3.3.1 Micromegas 5 strips LM Module with $100\,\mu$m

We will start with reducing the distance between the Resistive strips and the Readout Strips at $100\,\mu$m. In the next table we have the results for the central Strip.

- **Res-Res : 6.92**

- **Res-Ro : 153.3**
- **Ro-Ro : 45.92**

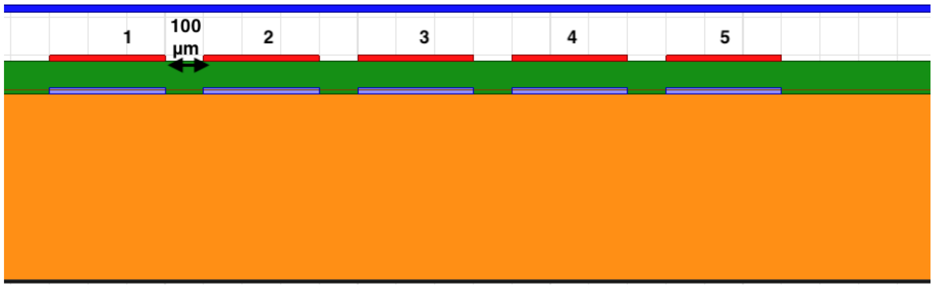


Figure 49: 5 strips 100 µm LM Module

	ground	line	res1	res2	res3	res4	res5	st1	st2	st3	st4	st5
ground	278,11	-84,068	-3,0689	-0,21262	-0,21024	-0,20906	-2,2021	-43,009	-31,141	-30,917	-31,067	-45,042
line	-84,068	271,92	-35,125	-29,825	-29,924	-30,036	-34,948	-8,6043	-1,1629	-0,95012	-0,99395	-9,7425
res1	-3,0689	-35,125	225,35	-6,8392	0,00077905	-8,97E-06	-1,32E-06	-173,32	-6,9746	-0,013659	-0,0013737	-0,0001461
res2	-0,21262	-29,825	-6,8392	210,95	-6,9205	-8,86E-05	-4,65E-06	-7,0205	-153,2	-6,9225	-0,0053028	-0,0005453
res3	-0,21024	-29,924	0,00077905	-6,9205	211,41	-7,0112	-0,0006611	-0,0054324	-7,0602	-153,3	-6,9754	-0,005275
res4	-0,20906	-30,036	-8,97E-06	-8,86E-05	-7,0112	211,92	-7,1087	-0,000541	-0,0050825	-7,098	-153,41	-7,048
res5	-2,2021	-34,948	-1,32E-06	-4,65E-06	-0,0006611	-7,1087	226,68	-7,53E-05	-0,0006864	-0,0067298	-7,1536	-175,26
st1	-43,009	-8,6043	-173,32	-7,0205	-0,0054324	-0,000541	-7,53E-05	278,81	-45,925	-0,82021	-0,083095	-0,0087692
st2	-31,141	-1,1629	-6,9746	-153,2	-7,0602	-0,0050825	-0,0006864	-45,925	292,23	-45,921	-0,76063	-0,080217
st3	-30,917	-0,95012	-0,013659	-6,9225	-153,3	-7,098	-0,0067298	-0,82021	-45,921	292,96	-46,244	-0,7655
st4	-31,067	-0,99395	-0,0013737	-0,0053028	-6,9754	-153,41	-7,1536	-0,083095	-0,76063	-46,244	293,79	-47,1
st5	-45,042	-9,7425	-0,0001461	-0,0005453	-0,005275	-7,048	-175,26	-0,0087692	-0,080217	-0,7655	-47,1	285,06

Figure 50: Table of capacitance for 5 strips 100 µm LM Module.

3.3.2 Micromegas 5 strips LM Module with Glue

In this last two cases we have the same 5 strips LM and SM modules but with an extra glue layer above the Dupont. The layer is actually Glue from teflon with Relative Permittivity 2.08.

- **Res-Res : 4.28**
- **Res-Ro : 136.38**
- **Ro-Ro : 28.42**

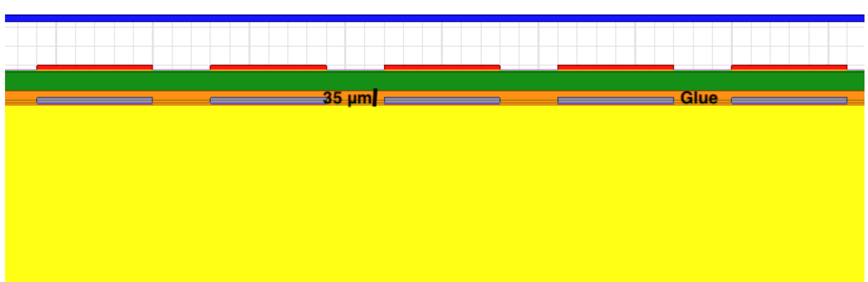


Figure 51: 5 strips LM Module with glue

	ground	line	res1	res2	res3	res4	res5	st1	st2	st3	st4	st5
ground	290,93	-71,437	-1,3917	-0,50377	-0,566	-0,5404	-4,5666	-53,543	-34,314	-34,246	-34,391	-48,318
line	-71,437	282,93	-41,722	-31,926	-32,218	-31,946	-40,206	-9,9855	-2,2253	-2,1068	-2,2671	-10,183
res1	-1,3917	-41,722	236,3	-4,4585	-9,14E-05	-1,14E-05	-3,74E-06	-184,04	-4,6723	-0,0098196	-0,0007376	-7,30E-05
res2	-0,50377	-31,926	-4,4585	186,71	-4,2824	0,0020047	-1,57E-05	-4,9392	-135,89	-4,708	-0,0077059	-0,0007034
res3	-0,566	-32,218	-9,14E-05	-4,2824	187,43	-4,2824	-0,000151	-0,010059	-4,844	-136,38	-4,8441	-0,009846
res4	-0,5404	-31,946	-1,14E-05	0,0020047	-4,2824	186,71	-4,5102	-0,000719	-0,007707	-4,7081	-135,89	-4,825
res5	-4,5666	-40,206	-3,74E-06	-1,57E-05	-0,000151	-4,5102	216,1	-8,64E-05	-0,0009989	-0,013361	-4,7891	-162,01
st1	-53,543	-9,9855	-184,04	-4,9392	-0,010059	-0,000719	-8,64E-05	281,39	-28,232	-0,59463	-0,044096	-0,0037395
st2	-34,314	-2,2253	-4,6723	-135,89	-4,844	-0,007707	-0,0009989	-28,232	239,16	-28,419	-0,50792	-0,043122
st3	-34,246	-2,1068	-0,0098196	-4,708	-136,38	-4,7081	-0,013361	-0,59463	-28,419	240,19	-28,425	-0,58206
st4	-34,391	-2,2671	-0,0007376	-0,0077059	-4,8441	-135,89	-4,7891	-0,044096	-0,50792	-28,425	239,06	-27,886
st5	-48,318	-10,183	-7,30E-05	-0,0007034	-0,009846	-4,825	-162,01	-0,0037395	-0,043122	-0,58206	-27,886	253,88

Figure 52: Table of capacitance for 5 strips SM Module with glue.

3.3.3 Micromegas 5 strips SM Module with Glue

In this last case we have the same 5 strips LM module but with an extra glue layer(thickness of 35 μm above the Dupont).

- **Res-Res : 6.07**
- **Res-Ro : 133.09**
- **Ro-Ro : 34.14**

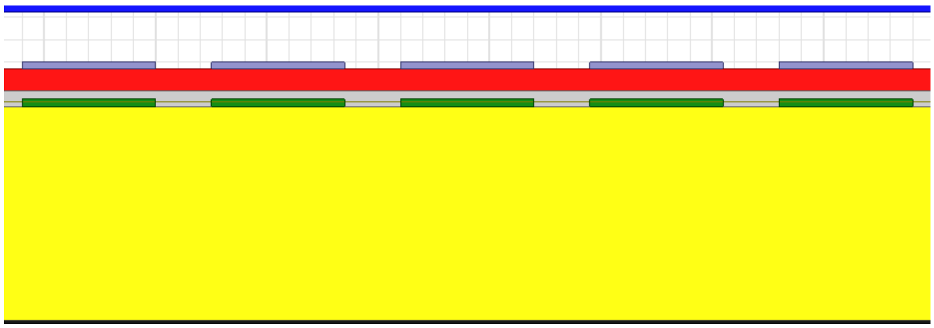


Figure 53: 5 strips SM Module with glue

	ground	line	res1	res2	res3	res4	res5	st1	st2	st3	st4	st5
ground	287,15	-74,427	-1,2915	-0,29945	-0,34971	-0,38073	-4,2656	-52,739	-32,704	-32,603	-32,736	-48,241
line	-74,427	279,85	-41,366	-30,962	-31,246	-31,449	-39,378	-9,5582	-1,5533	-1,3545	-1,3626	-10,491
res1	-1,2915	-41,366	236,49	-5,6523	0,00031599	-8,85E-06	-2,37E-06	-182,73	-5,4363	-0,0074068	-0,000697	-7,31E-05
res2	-0,29945	-30,962	-5,6523	187,06	-6,0699	0,00071048	-7,71E-06	-5,7072	-133,47	-4,8954	-0,0068146	-0,0006709
res3	-0,34971	-31,246	0,00031599	-6,0699	187,11	-5,9327	0,0032305	-0,0070555	-5,0588	-133,09	-5,3467	-0,0077416
res4	-0,38073	-31,449	-8,85E-06	0,00071048	-5,9327	187,98	-5,9407	-0,0008063	-0,0078945	-5,3385	-133,58	-5,3559
res5	-4,2656	-39,378	-2,37E-06	-7,71E-06	0,0032305	-5,9407	213,12	-0,0001008	-0,0010195	-0,011457	-5,3962	-158,13
st1	-52,739	-9,5582	-182,73	-5,7072	-0,0070555	-0,0008063	-0,0001008	285,21	-33,68	-0,71541	-0,063762	-0,0060667
st2	-32,704	-1,5533	-5,4363	-133,47	-5,0588	-0,0078945	-0,0010195	-33,68	246,76	-34,144	-0,64393	-0,061296
st3	-32,603	-1,3545	-0,0074068	-4,8954	-133,09	-5,3385	-0,011457	-0,71541	-34,144	245,77	-32,88	-0,72735
st4	-32,736	-1,3626	-0,000697	-0,0068146	-5,3467	-133,58	-5,3962	-0,063762	-0,64393	-32,88	245,28	-33,266
st5	-48,241	-10,491	-7,31E-05	-0,0006709	-0,0077416	-5,3559	-158,13	-0,0060667	-0,061296	-0,72735	-33,266	256,29

Figure 54: Table of capacitance for 5 strips SM Module with glue.

3.4 Coclusion

In the following table are included the values for the central strips for all the previous cases.

Table 3: Capacitances for different modules of Micromegas

	5 LM	5 Sm	9 LM	9 SM	5 LM with Glue	5 SM with Glue	100 μm
Res-Res	2.6861	4.4941	2.6861	4.4941	4.2824	6.07	6.9205
Ro-Ro	30.39	36.27	30.39	36.27	28.42	34.14	45.92
Res-Ro	163.3	158.4	163.3	158.4	136.4	133.09	153.3

From the table some conclusions have been drawn :

- For the 5 strips Modules(with Glue or without) the capacitance between the Resistive strips and the Readout Strips of 100 μm Module has higher values from the SM Module and this has higher values than LM Module but the values between the Resistive strips and the Readout strips are upside down. We can say that as long we reduce the distance between the Resistive and the Readout strips the values of the Capacitance increase and also the values between the Resistive and the Readout strips reduce.
- For 9 strips Module we have the same results with the 5 strips Modules and the same values. It seems that the most important thing is the central strips and so the results did not change.
- If we compare the 5 LM and SM Modules(with and without Glue) we can make a conclusion that we every layer we add the value between the Resistive and between the Readout strips starting to reduce.

4 Micromegas Simulation with LTspice

4.1 Introduction

In this Section we will continue the analysis with the help of LTspice. With LTspice we will create the circuits that correspond to the Maxwell modules of the previous Section. We will also observe the signal from the central strips and those around it and how it changes for different values of capacitance.
As a basis We use the circuit by Ludwig-Maximilians-Universitat Munchen - Lehrstuhl Schaile(figure 55).

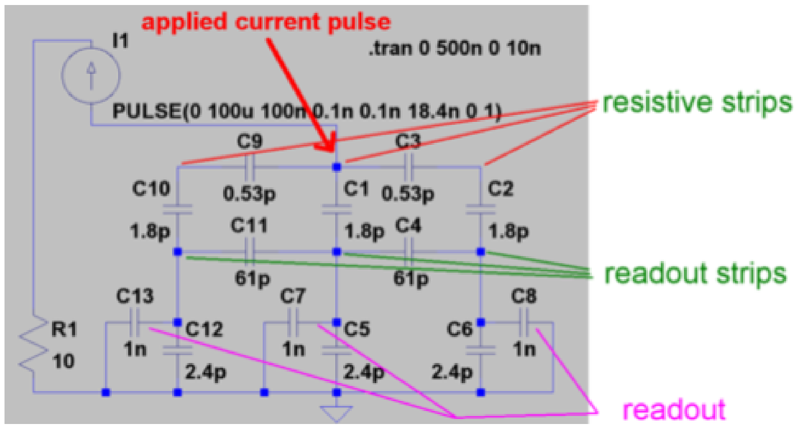


Figure 55: Ludwig-Maximilians-Universitat Munchen - Lehrstuhl Schaile Module.

As it clear from the figure we applied a current pulse.

Table 4: variables of current I

Current	Values(Amper)
I_1	0 A
I_2	100 µA
T_{dellay}	100 nA
T_{rise}	0.1 nA
T_{fall}	0.1 nA
T_{on}	18.4 nA
T_{period}	0.1 nA

The capacitors C9 and C3 correspond to the capacitances between the resistive strips. C11 and C4 correspond to the capacitances between the readout strips. C10,C11,C1 are the capacitances between resistive and readout strips.
The capacities C5,C7,C6,C8,C12,C13 correspond to the readout for our circuit and we use the values from our prototype model(1 nf and 2, 4 pf).

4.2 Spice Simulation with LM and SM Modules

As in the previous section we working on 5 and 9 strips modules LM and SM. And for every one of them we draw the figures for the output signal of the central and neighbor strips,output voltage and the output current. For those figures we will obtain the percentage of diffusion from the central to the neighbor strips.
In all those figures the line with the bigger depth represents the central strip and the others two are for the neighbor strips.

- **5 strips LM Module**

First we start with the 5 strips LM Module(Figure 56).

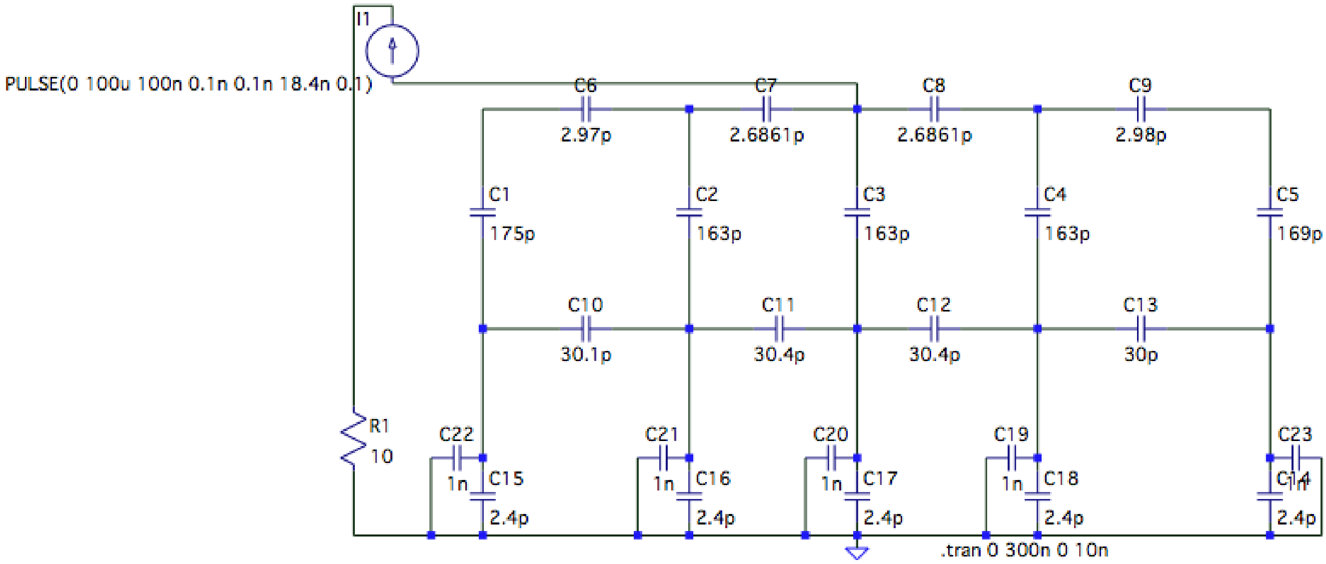


Figure 56: 5 strips LM Module with Spice

After we create the circuit for 5 strips with the help of spice we draw our two graphs(Figures 57-58)

The percentage of diffusion from the central to neighbor strips as we can observe from the two figures is **5%**

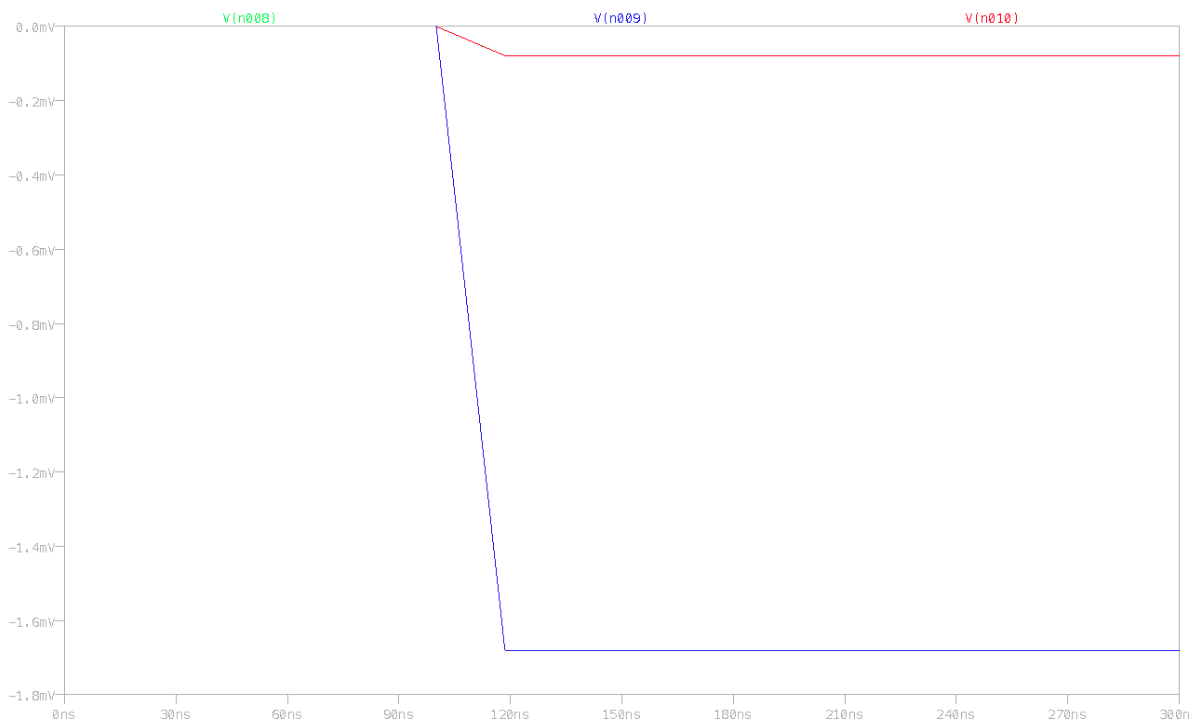


Figure 57: output voltage for central and neighbor strips

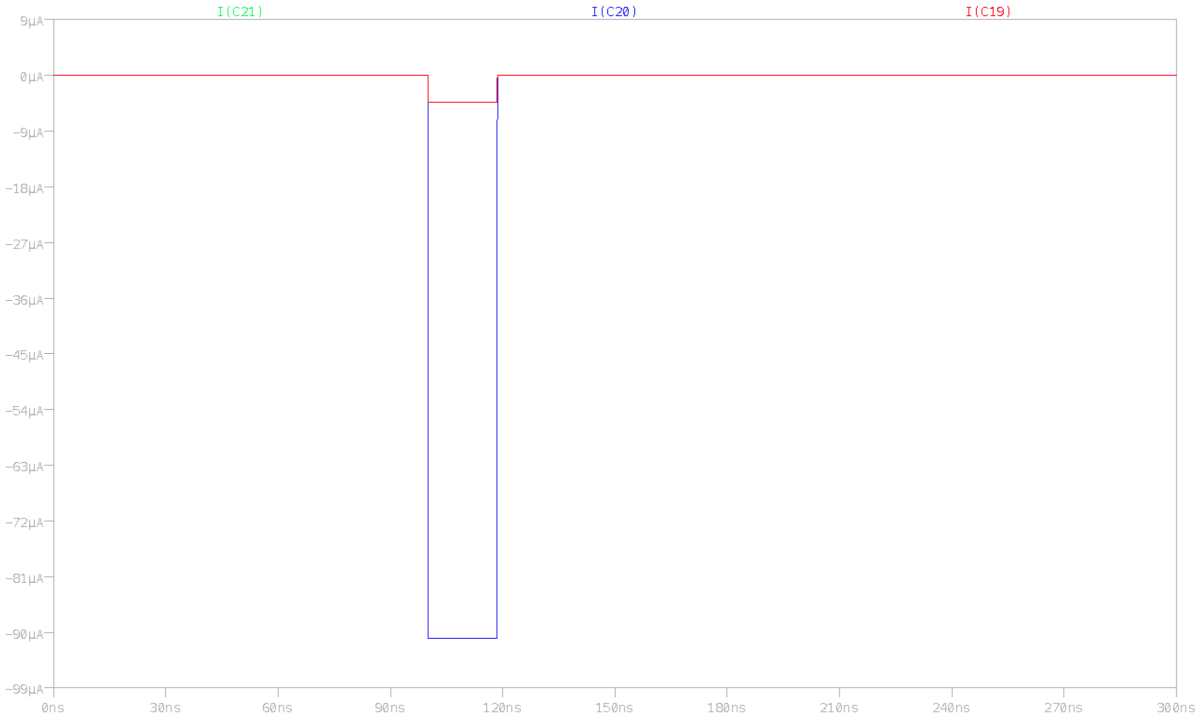


Figure 58: output current for central and neighbor strips

- **5 strips SM Module**

Next we continue for the 5 strips SM Module. We follow the same steps as in the LM Module for 5 strips but this time with the values of capacitance for the SM Module.
As we can clearly see for the figure no changes were made for the resistance, the current pulse or the capacities of the output. The only thing that changed are the capacitance between the resistive-readout strips and each other.

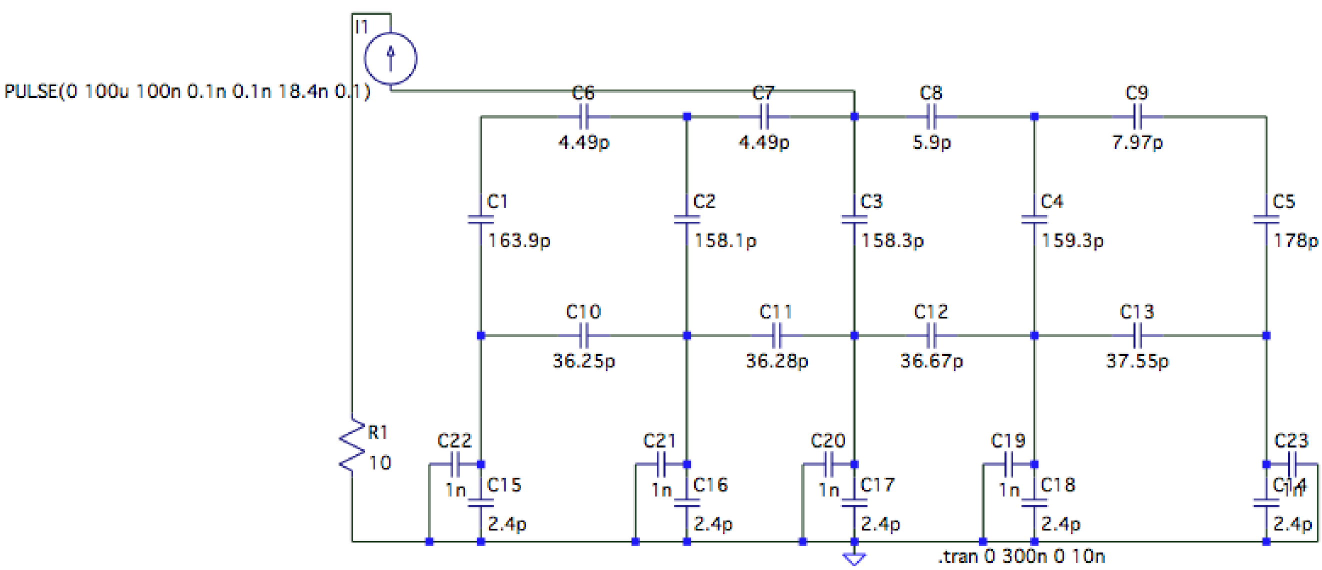


Figure 59: 5 strips SM Module with Spice

In the next page are represented the figures for the output Voltage and Current for the central and neighbor strips. The percentage of diffusion from the central to the neighbor strips as we can observe from the two figures is **7.5%**

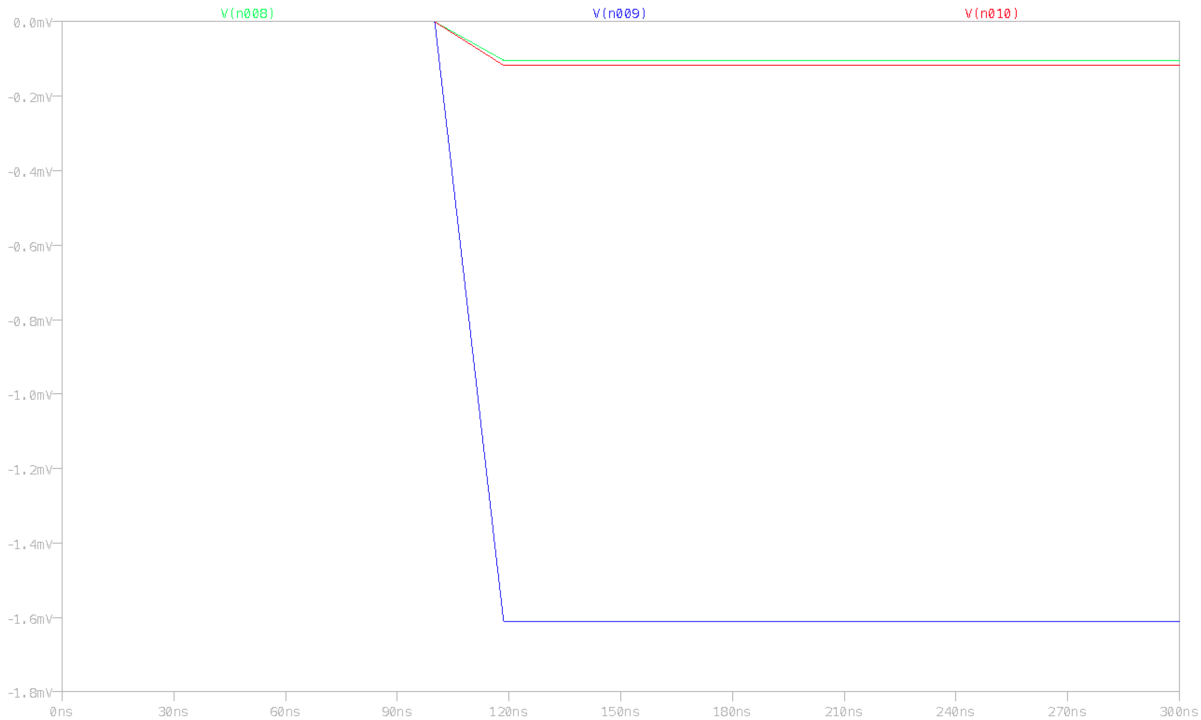


Figure 60: output voltage for central and neighbor strips

Figure 61: output current for central and neighbor strips

- **9 strips LM Module**

For this case we extended our module circuit from 3 strips Module to 9. As in 5 strips Modules we did not change the resistance, the current pulse or the capacities of the output, we just put 9 capacitors for the resistive strips and 9 capacitors for the readout strips.

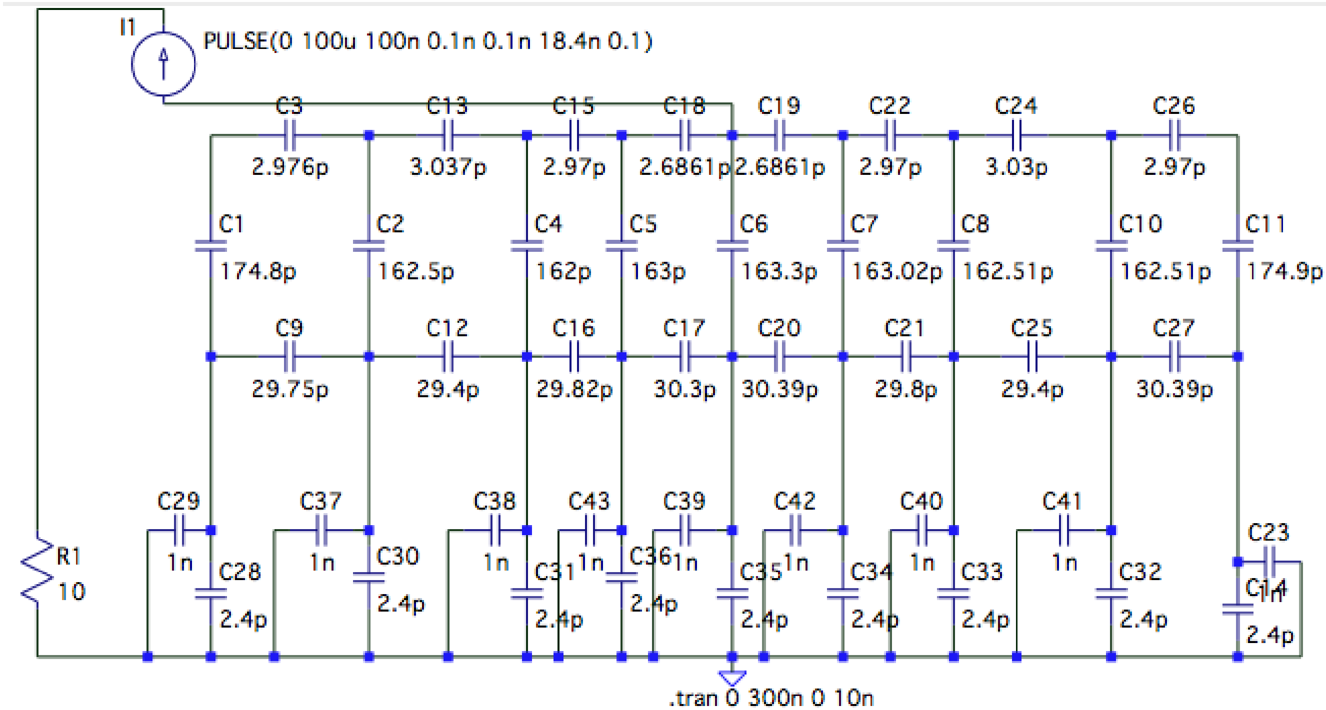


Figure 62: 9 strips LM Module with Spice

By observing the circuit of LM Modules we can see that the values between the central and the neighbor strips are almost the same, so we expected the percentage to be the same in this case.
Sure enough the percentage as we can observe from the two figures is also **5%**

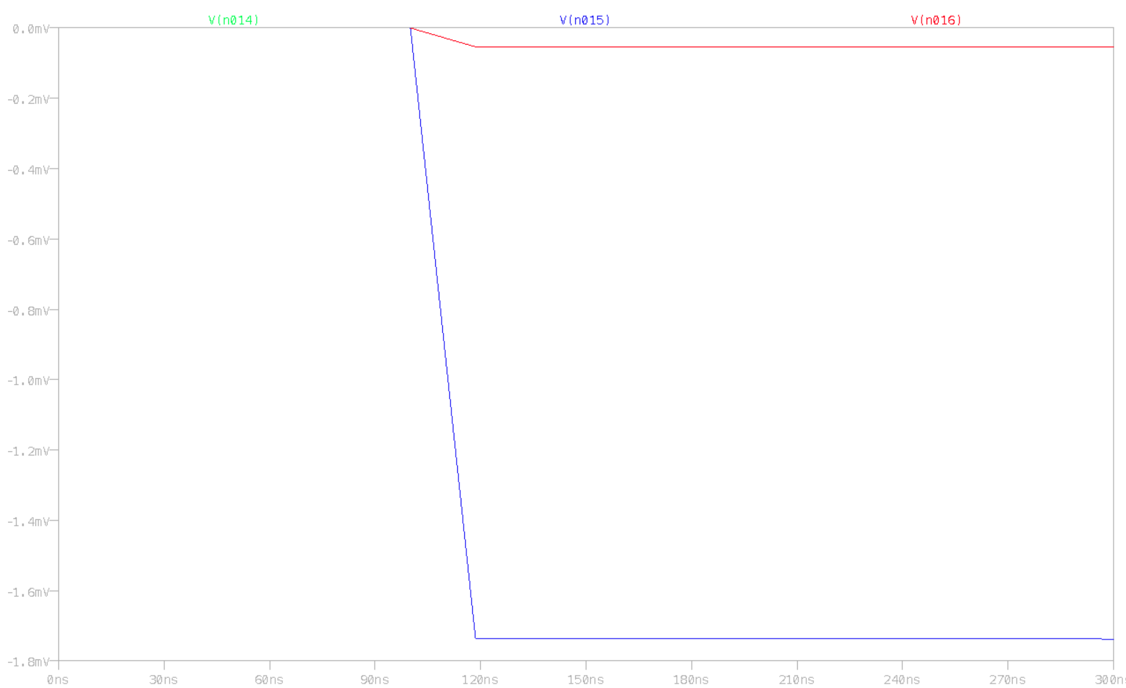


Figure 63: output voltage for central and neighbor strips

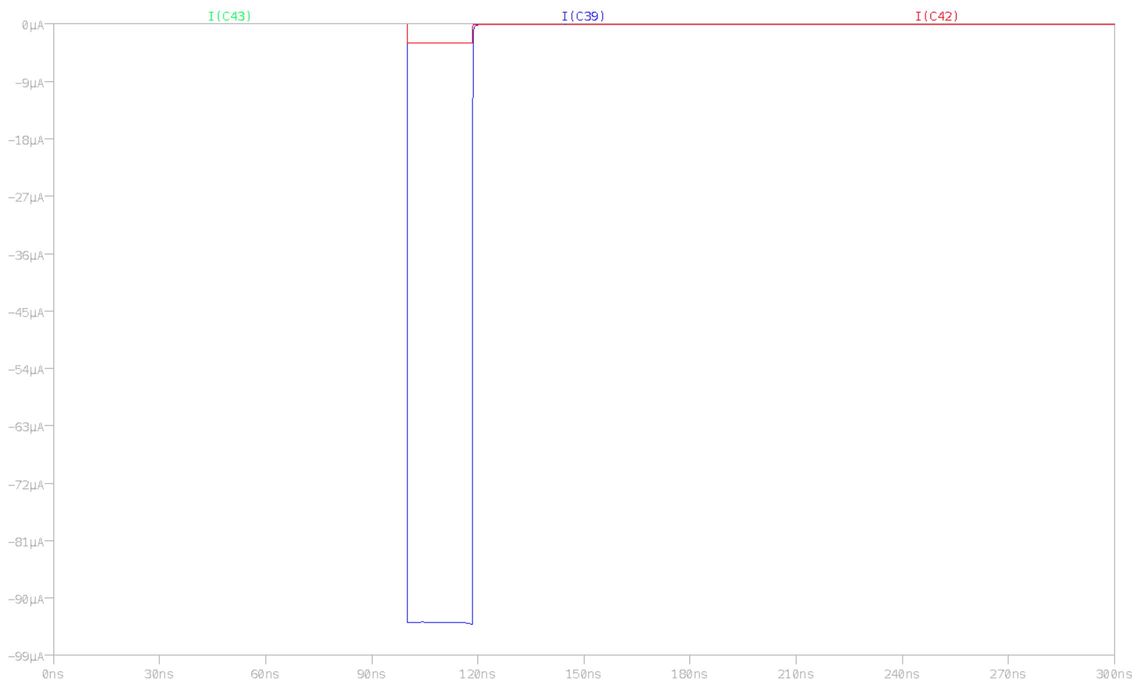


Figure 64: output current for central and neighbor strips

- **9 strips SM Module**

Lastly the same procedure was followed for the 9 strips SM Module.We still only change the values of the 3 kinds of strips that we gathered from Maxwell, that in this case we have higher values between the resistive strips and between the readout Strips and lower between resistive-readout strips unlike on LM Module.

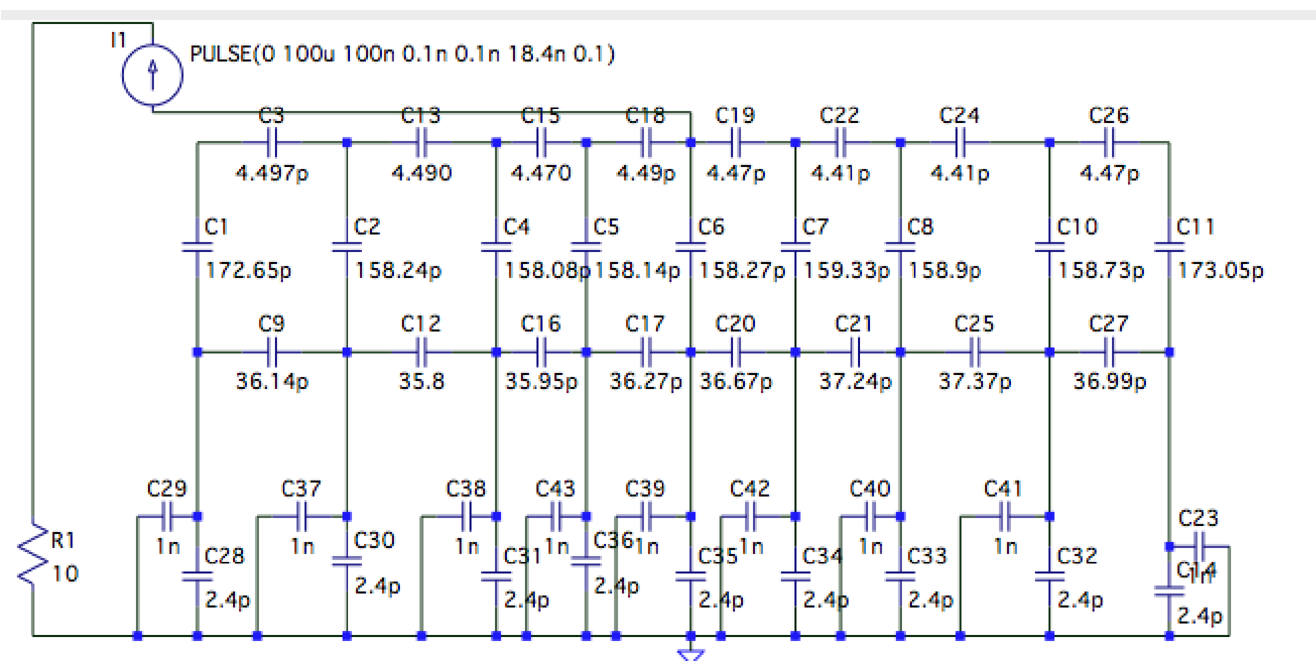


Figure 65: 9 strips SM Module with Spice

As we expectedthe percentage of diffusion from the central to the neighbor strips is also **7.5% such as the 5 strips SM Module.**

From those results we can confirm that the quotient of central and the neighbor strips is independent to the number of strips but it depends to the distance between the strips. As we put the strips closer to each other we observe that the percentage is growing and we expect that it will go even higher in other case for example for the 100 um Module we discussed in the previous Section.

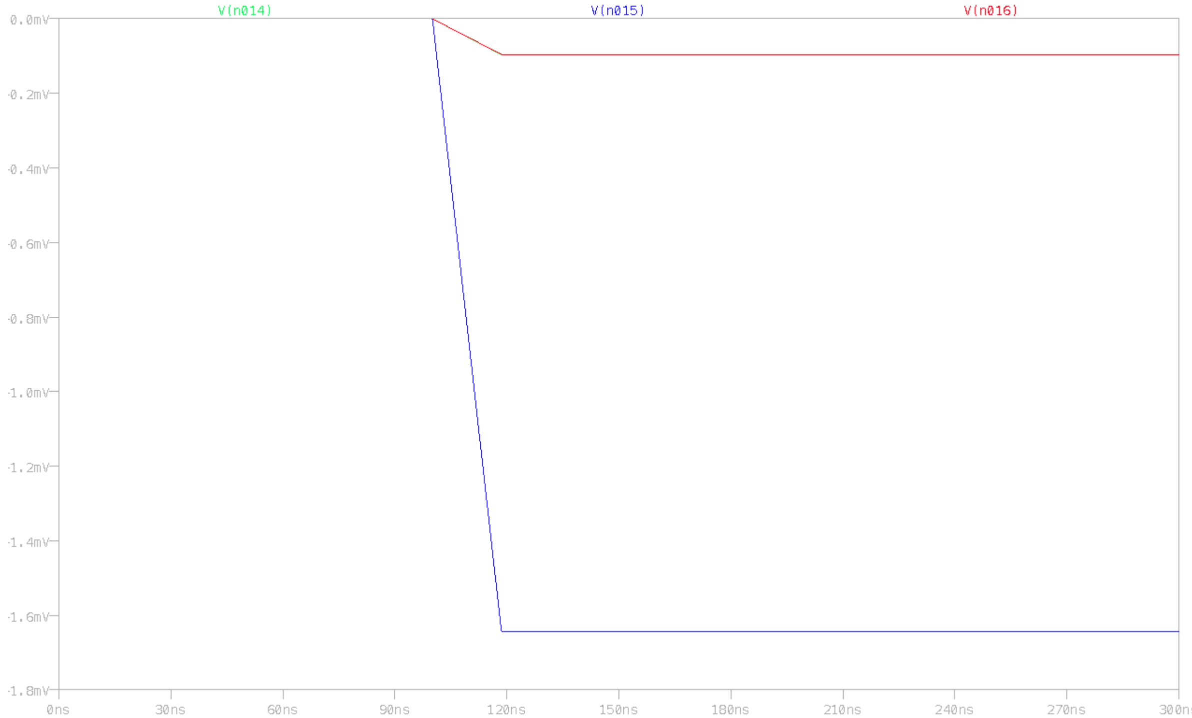


Figure 66: output voltage for central and neighbor strips

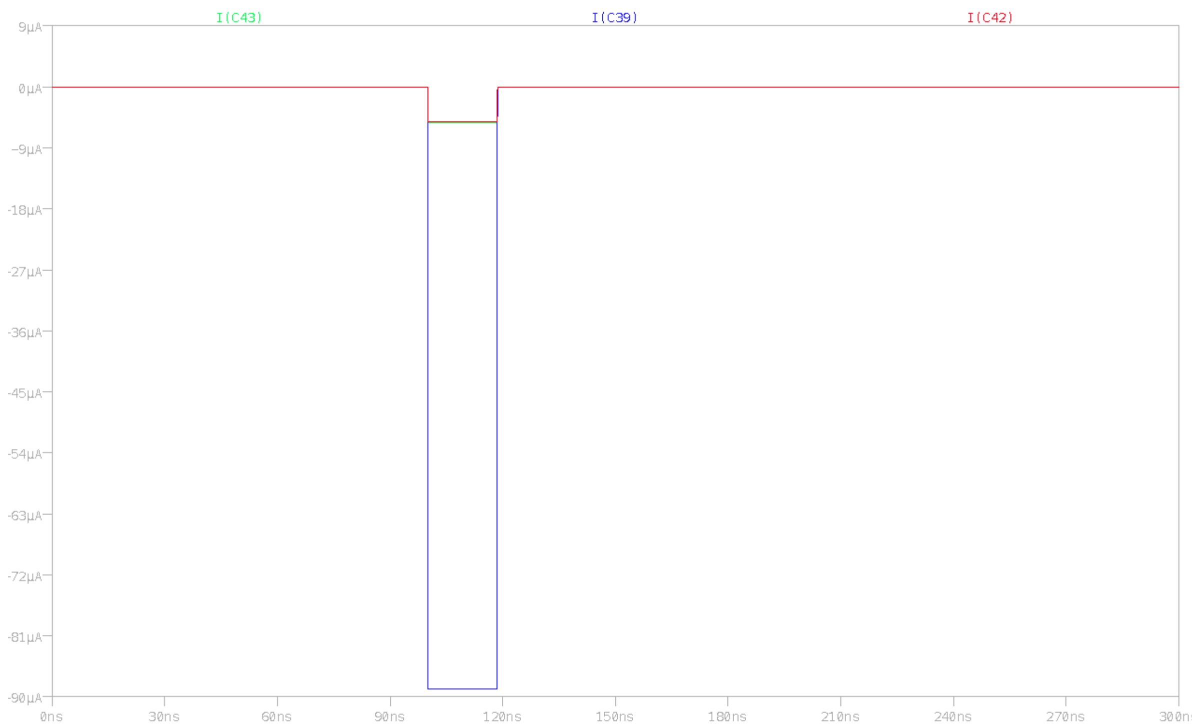


Figure 67: output current for central and neighbor strips

4.3 Spice with Mesh

In all those previous circuits we used Modules with just Resistive and Readout strips. In this subsection we will add a mesh with a resistor to have a case more close to our Maxwell Modules.
In figure 68 is shown the circuit with the mesh that was created. We want to see how the resistance in the mesh affect the previous graphs and the percentages.

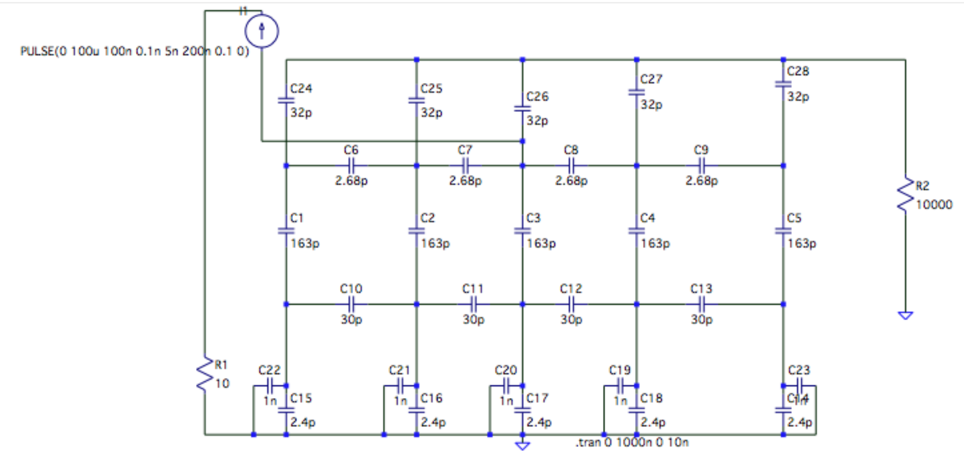
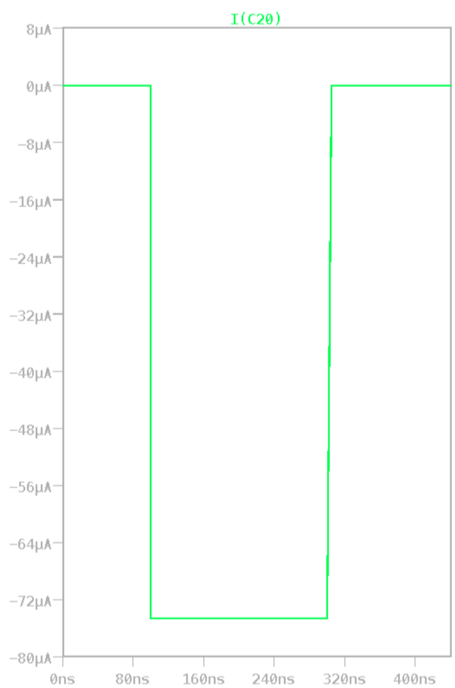


Figure 68: Circuit with a Mesh and a resistance

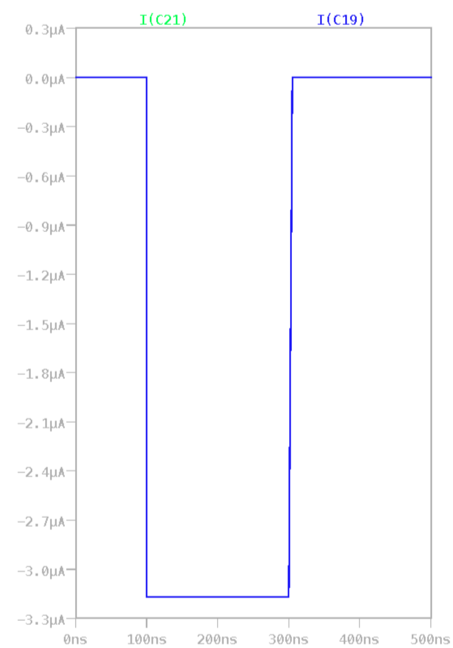
We will give to the resistance range of values from 0 Ohm to 10000 Ohm.

Table 5: Resistance on the Mesh and percentage of diffusion

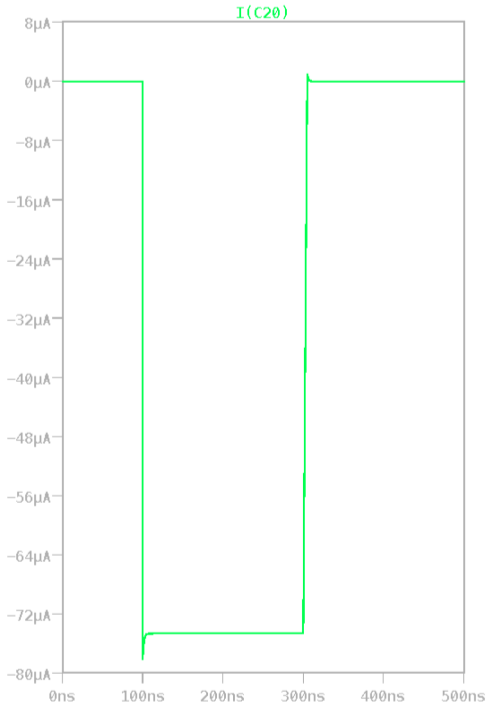
Resistance(Ohm)	percentage of diffusion
0	4.7
10	4.8
100	7
1000	8.5
10000	9



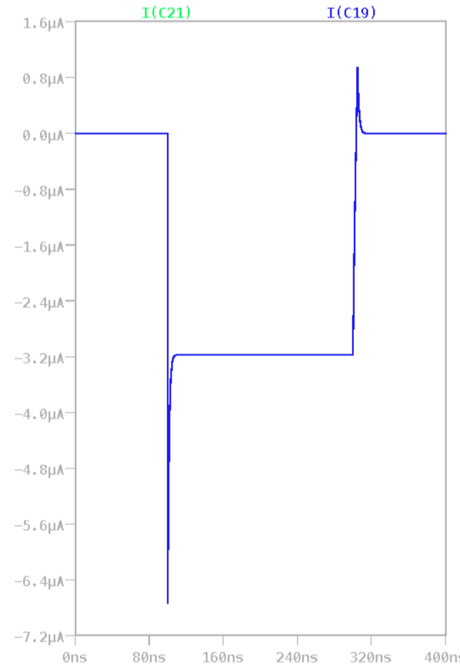
(a) Resistor 0 Ohm Central Strip



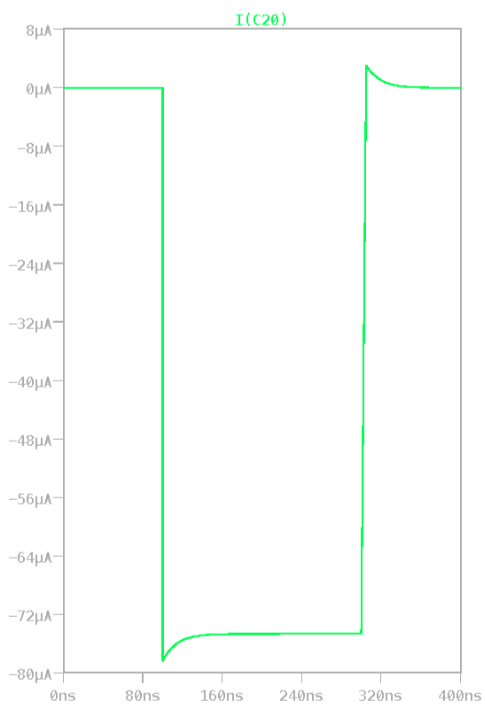
(b) Resistor 0 Ohm Neighbor Strips



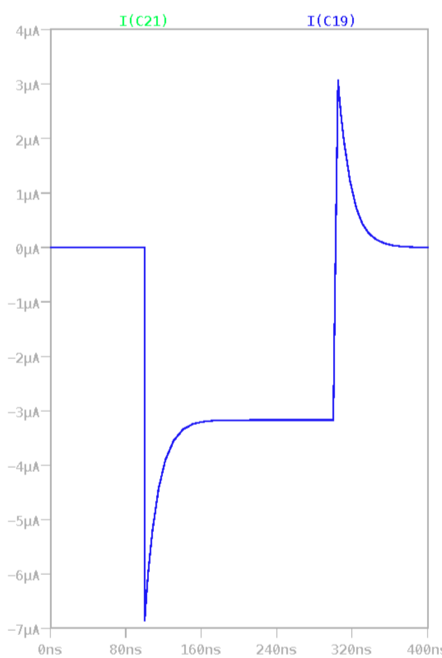
(a) Resistor 10 Ohm Central Strip



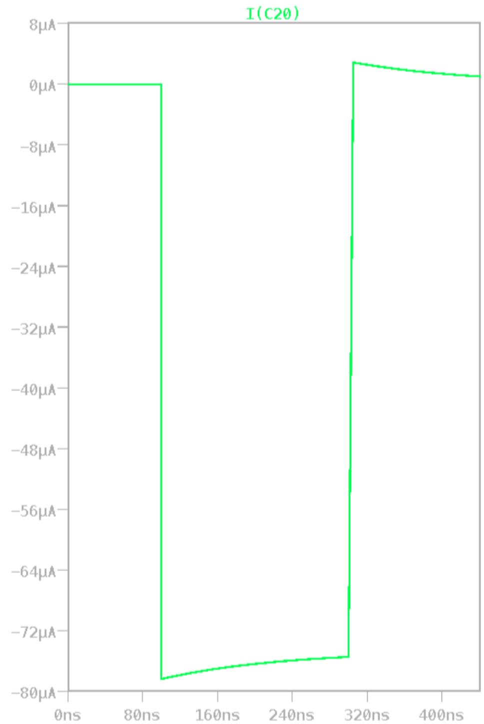
(b) Resistor 10 Ohm Neighbor Strips



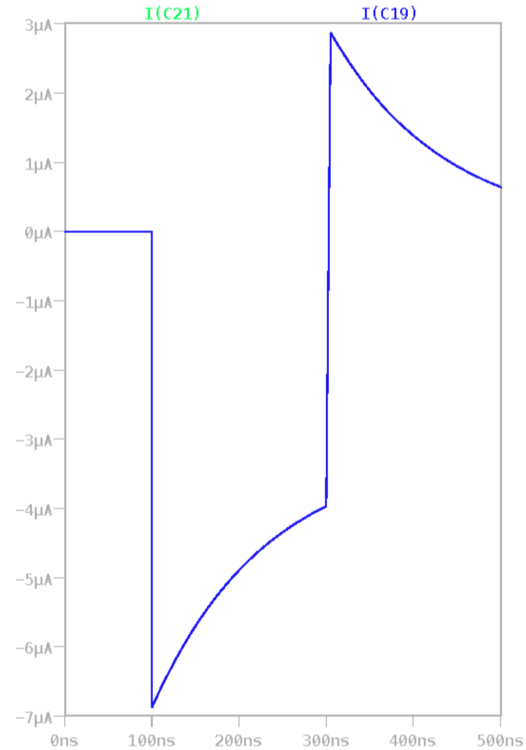
(a) Resistor 100 Ohm Central Strip



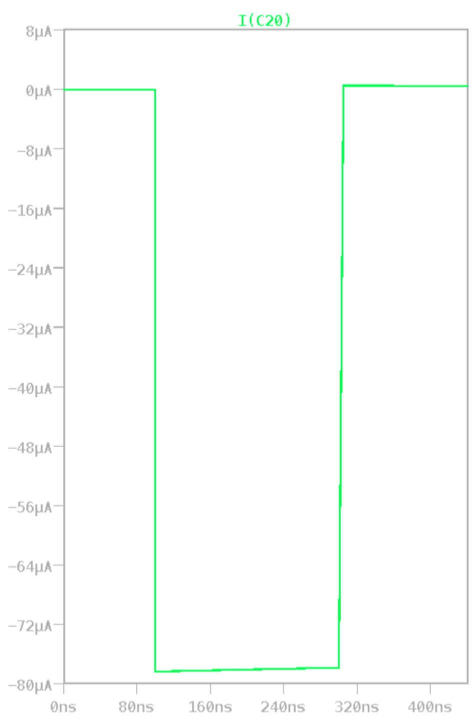
(b) Resistor 100 Ohm Neighbor Strips



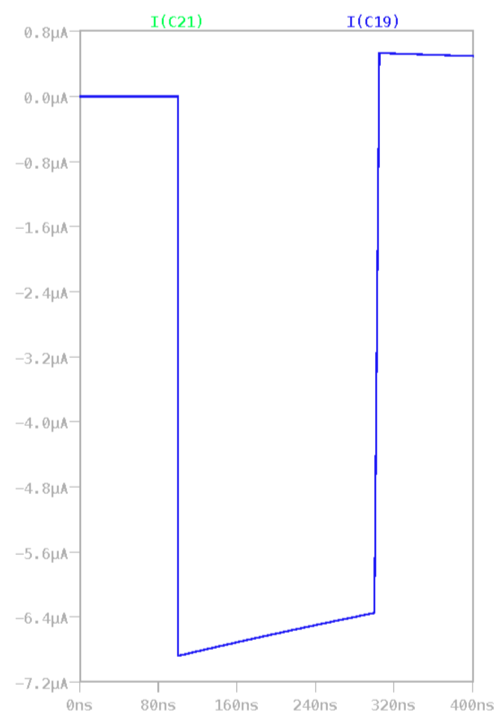
(a) Resistor 1000 Ohm Central Strip



(b) Resistor 1000 Ohm Neighbor Strips



(a) Resistor 10000 Ohm Central Strip



(b) Resistor 10000 Ohm Neighbor Strips

We forgot to add that in this circuit we make some small changes to Current Source I.

Table 6: variables of current I

Current	Values(Amper)
I_1	0 A
I_2	100 µA
T_{dellay}	100 nA
T_{rise}	0.1 nA
T_{fall}	5 nA
T_{on}	200 nA
T_{period}	0.1 nA

From the figures we can we conclude some results about the use of a mesh with a resistor :

- First of all by using a zero resistance the result is the same with the previous 5 strips LM Module we worked.
- With the increase of the value of the resistance the percentage between the central and the neighbor strips starting to increase too but without great observable increase.
- After we put a resistance with value we observe a change in the voltage of the current, but due to the complexity of the circuit is not easy to apprehend the reason for it.

4.4 Changes on Capacitances

The next step is to observe how the signal from the central strip and the neighbor strips change if we change the capacities between the strips for the LM and SM modules. We will focus on 3 cases :

1. values of capacitance between resistive strips
2. values of capacitance between readout strips
3. values of capacitance between resistive and readout strips

4.4.1 Changes on capacitance between Resistive strips

For the first case we will give to the capacitance between the Resistive strips values from 10 pf to 350 pf. We we will start with the LM module and after that we will do the same process with the SM module.

With the help of the table we create the figures of the above percentage with the values they respond.

Observation : By observing the graphs we can see that there is a non-linear increase of the percentage for those two modules for increase values of capacitance. The values of the percentage are slightly more in the SM module.

Table 7: Capacitance between Resistive strips and percentage of diffusion(%)

Capacitance(pf)	LM	SM
10	3.8	9.7
30	18.5	18.7
50	24.3	25
100	34.7	35.5
150	41.5	42.2
200	46.2	47.2
250	50	51.4
300	53	54.1
350	55.8	56.6

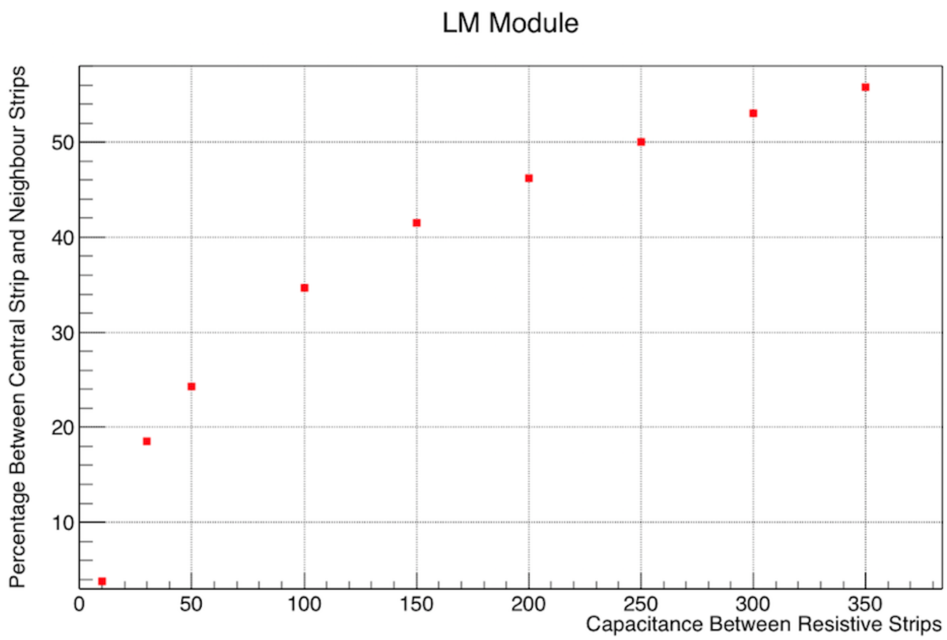


Figure 69: Value of percentage due to capacitance between Resistive strips.

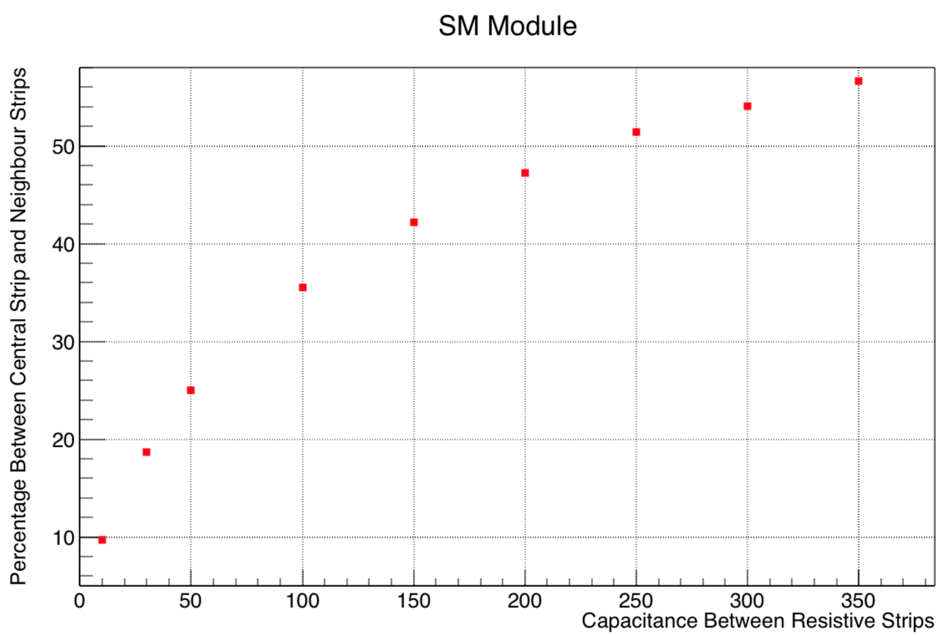


Figure 70: Value of percentage due to capacitance between Resistive strips.

4.4.2 Changes on capacitance between readout strips

We will continue like before but this time we will give to the capacitance between the readout strips values from 10 pf to 350 pf. We we will start with the LM module and after that we will do the same process with the SM module.

Table 8: Capacitance between readout strips and percentage of diffusion

Capacitance(pf)	LM	SM
30	5	7.5
50	6.8	8.5
100	10.5	12.3
150	13.5	15.5
200	16.3	18.3
250	19	21
300	21.3	23
350	23.2	25

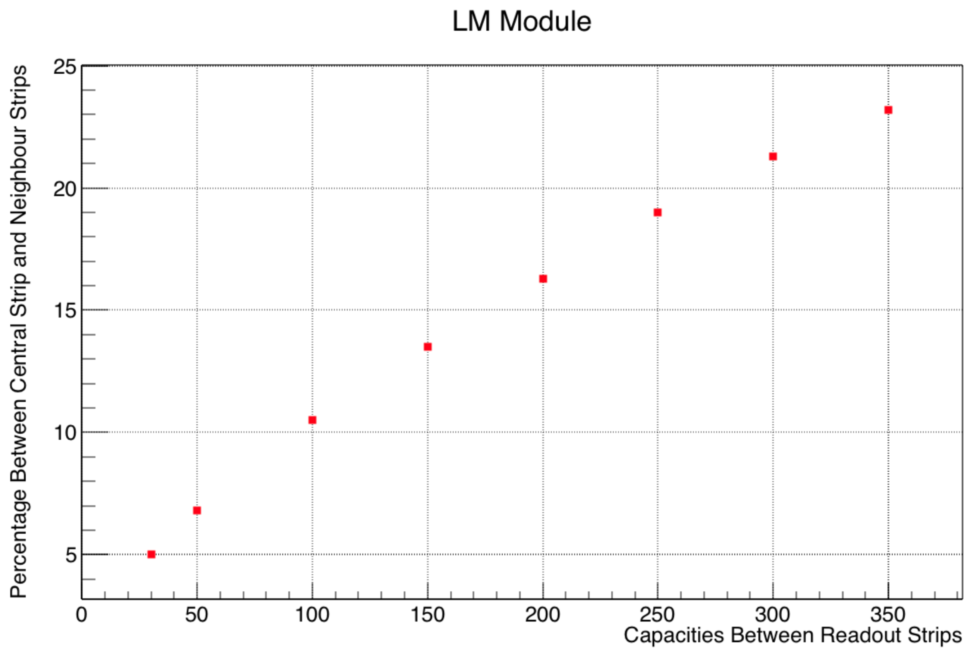


Figure 71: Value of percentage due to capacitance between readout strips.

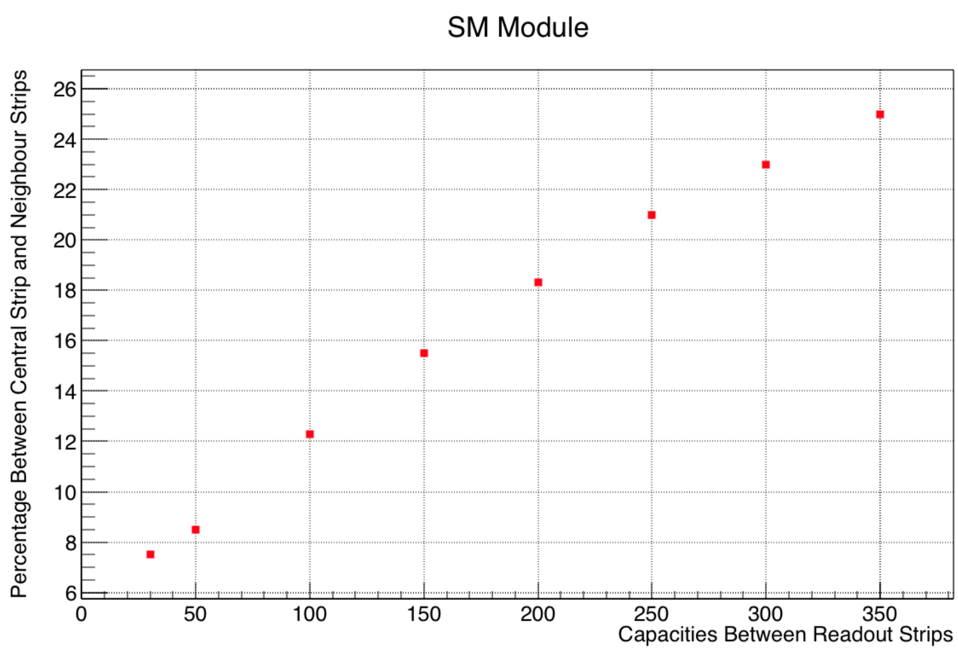


Figure 72: Value of percentage due to capacitance between readout strips.

Observation : By observing the graphs we can see that there is a linear increase of the percentage for those two modules for increase values of capacitance. The values of the percentage are slightly more in the SM module.

4.4.3 Changes on capacitance between Resistive-Readout strips

we continue with the capacities between Resistive-Readout strips with values from 10 pf to 350 pf.

Table 9: Capacitance between Resistive-Readout strips and percentage of diffusion

Capacitance(pf)	LM	SM
10	20.8	31.7
30	10.6	17.6
50	7.8	13.3
100	5.6	9.08
150	5	7.75
200	4.6	6.61
250	4.3	6.31
300	4.29	5.96
350	3.9	5.59

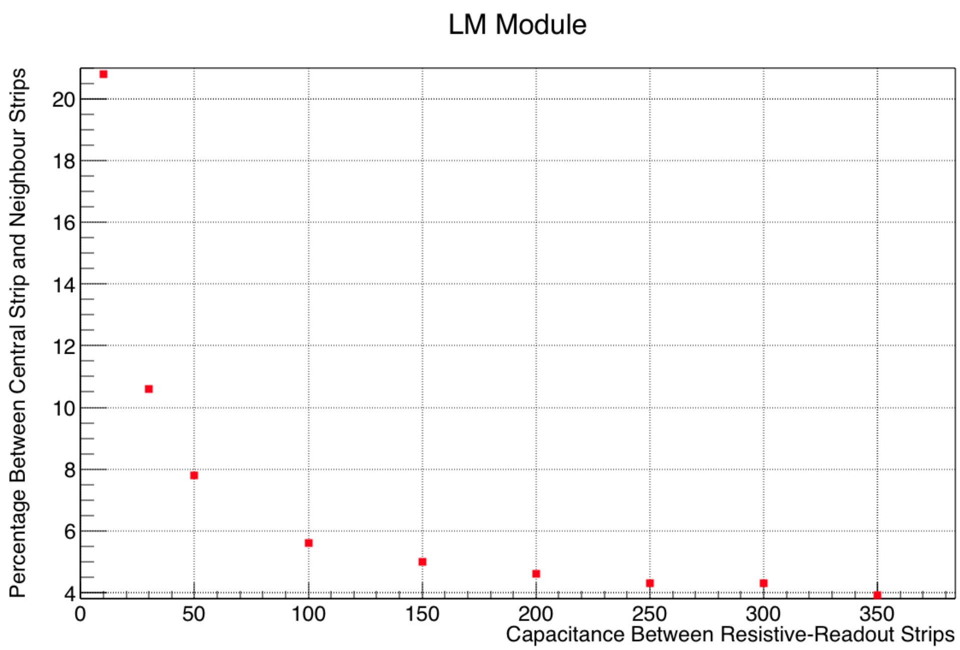


Figure 73: Value of percentage due to capacitance between Resistive-Readout strips.

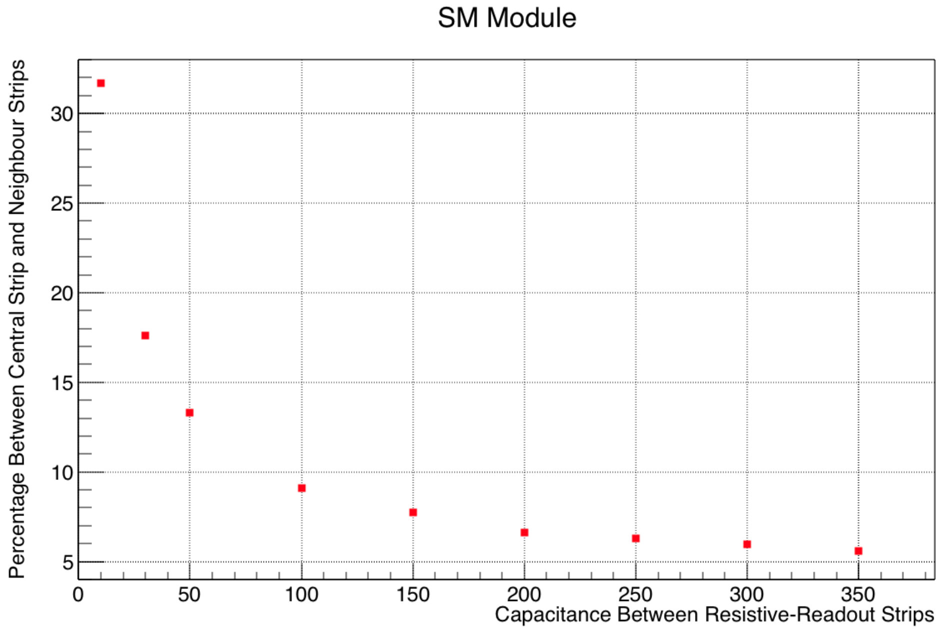


Figure 74: Value of percentage due to capacitance between Resistive-Readout strips.

Observation : By observing the graphs we can see that there is a drastic reduction of the percentage for those two modules for increase values of capacitance. The values of the percentage are slightly more in the SM module.

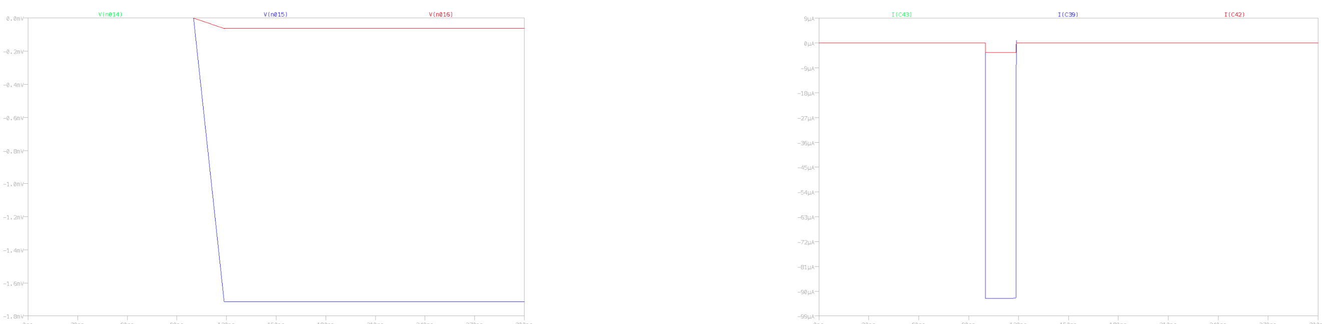
Appendices

Appendix A

Changes to Capacitances between Resistive Strips

LM Module

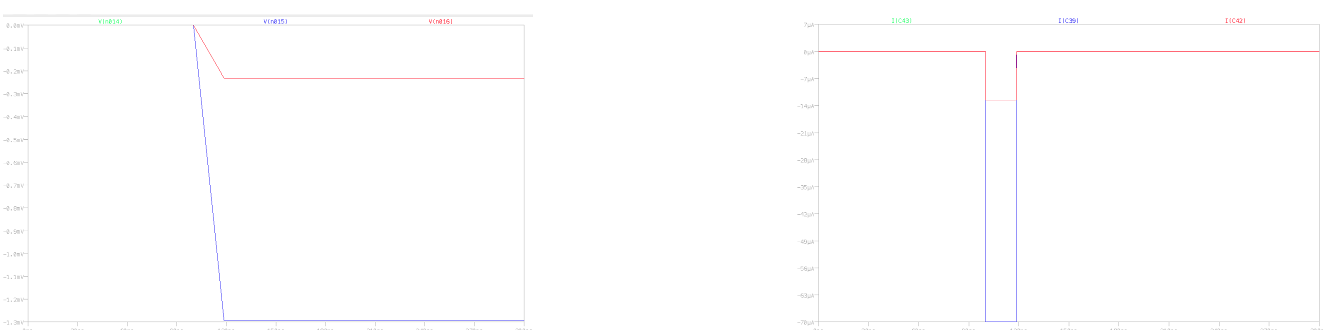
- **10 pf**



(a) output voltage for central and neighbor strips   **(b)** output current for central and neighbor strips

Figure A.1: 10 pf between Resistive strips

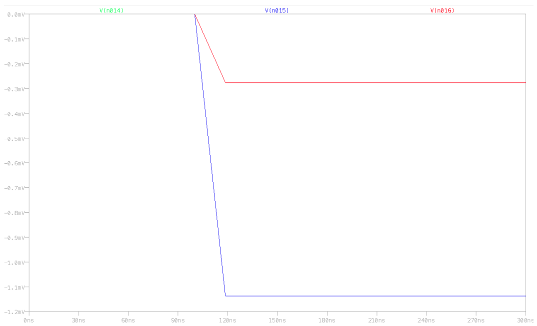
- **30 pf**



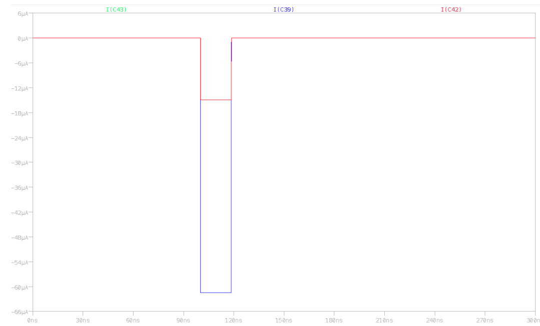
(a) output voltage for central and neighbor strips   **(b)** output current for central and neighbor strips

Figure A.2: 30 pf between Resistive strips

- **50 pf**



(a) output voltage for central and neighbor strips

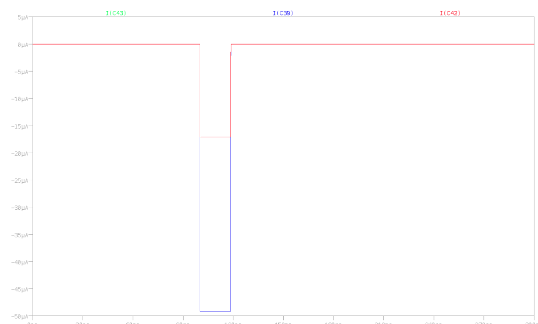


(b) output current for central and neighbor strips

Figure A.3: 50 pf between Resistive strips

- **100 pf**

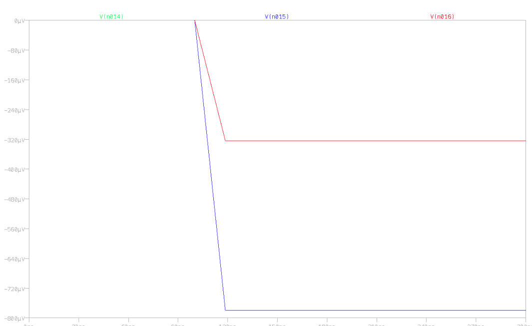
(a) output voltage for central and neighbor strips



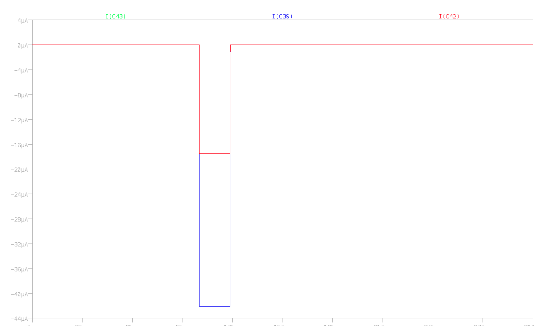
(b) output current for central and neighbor strips

Figure A.4: 100 pf between Resistive strips

- **150 pf**



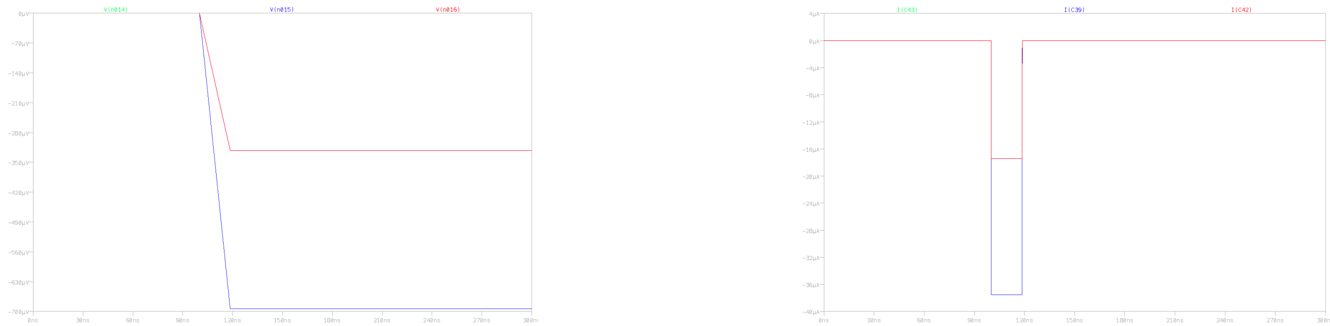
(a) output voltage for central and neighbor strips



(b) output current for central and neighbor strips

Figure A.5: 150 pf between Resistive strips

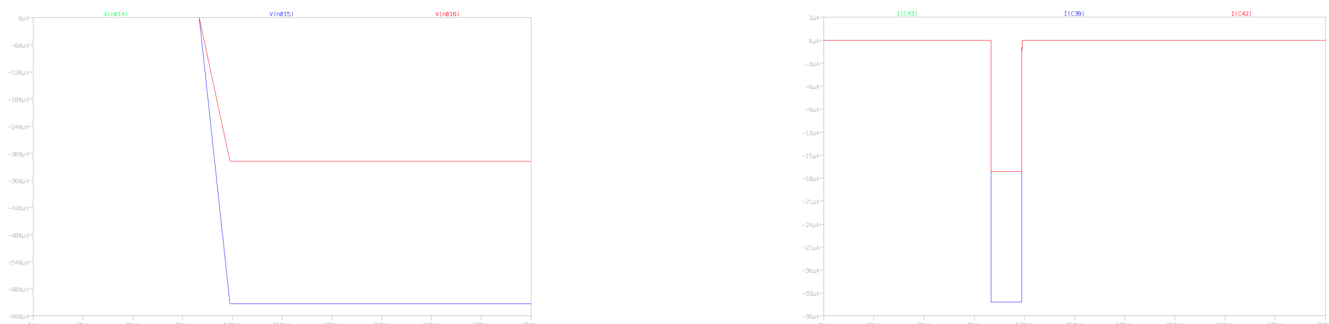
- **200 pf**



(a) output voltage for central and neighbor strips
(b) output current for central and neighbor strips

Figure A.6: 200 pf between Resistive strips

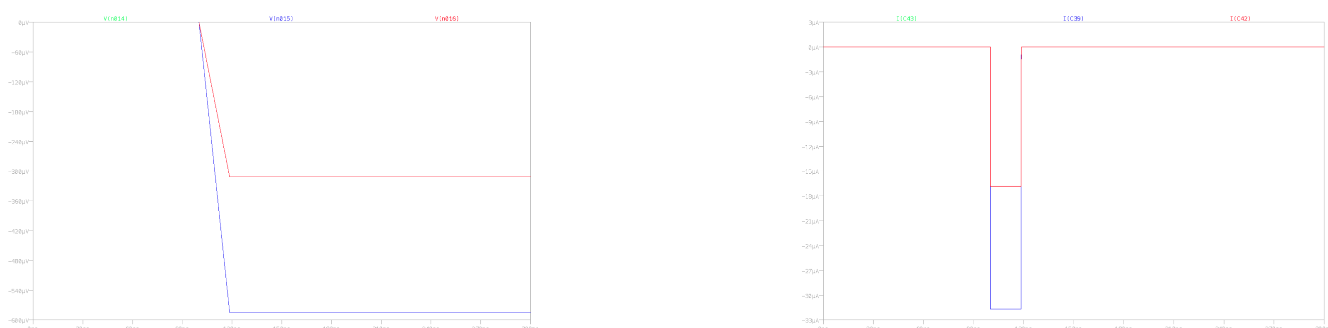
- **250 pf**



(a) output voltage for central and neighbor strips
(b) output current for central and neighbor strips

Figure A.7: 250 pf between Resistive strips

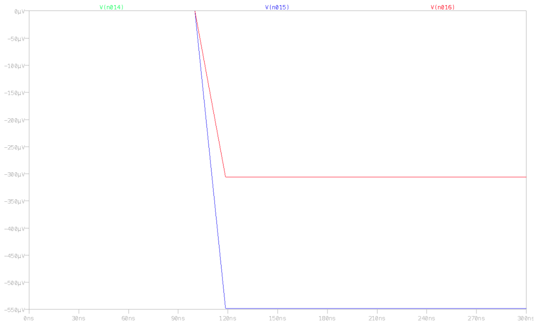
- **300 pf**



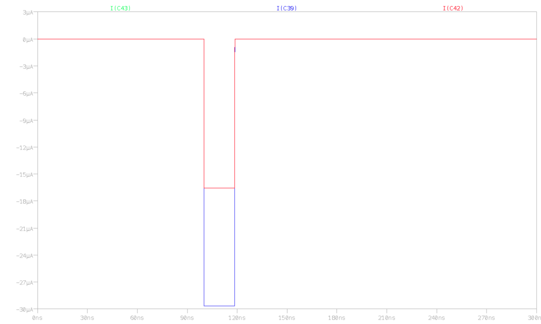
(a) output voltage for central and neighbor strips
(b) output current for central and neighbor strips

Figure A.8: 300 pf between Resistive strips

- **350 pf**



(a) output voltage for central and neighbor strips

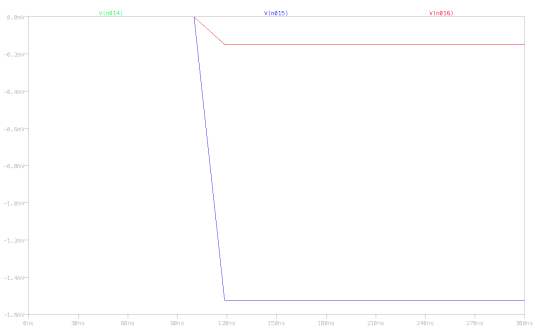


(b) output current for central and neighbor strips

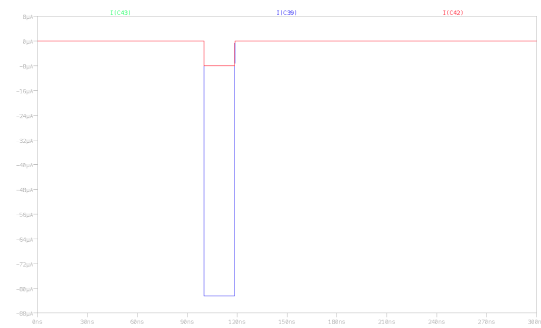
Figure A.9: 350 pf between Resistive strips

SM Module

- **10 pf**



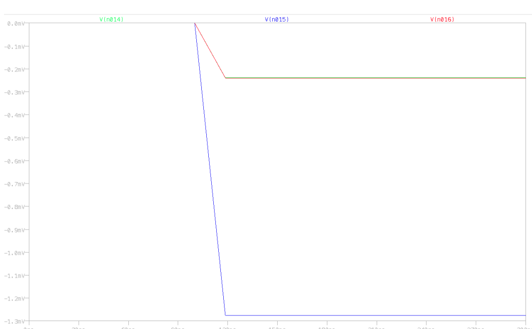
(a) output voltage for central and neighbor strips



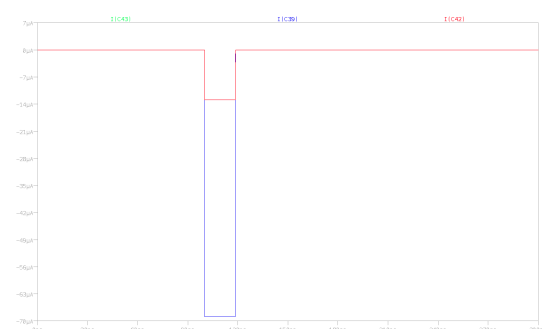
(b) output current for central and neighbor strips

Figure A.10: 10 pf between Resistive strips

- **30 pf**



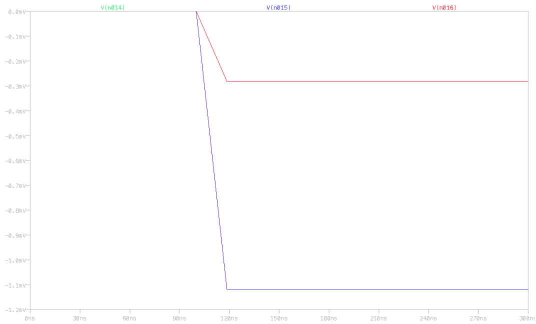
(a) output voltage for central and neighbor strips



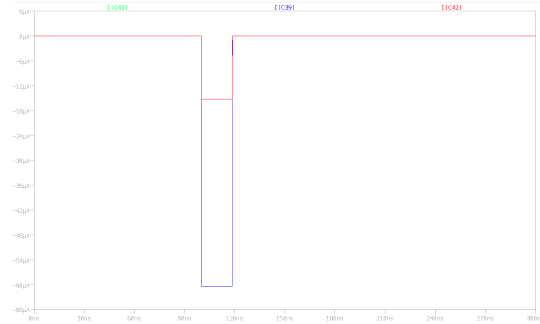
(b) output current for central and neighbor strips

Figure A.11: 30 pf between Resistive strips

- **50 pf**



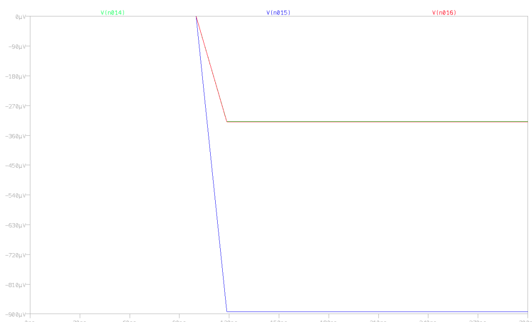
(a) output voltage for central and neighbor strips



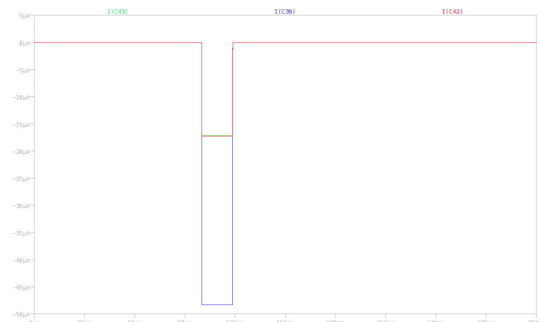
(b) output current for central and neighbor strips

Figure A.12: 50 pf between Resistive strips

- **100 pf**



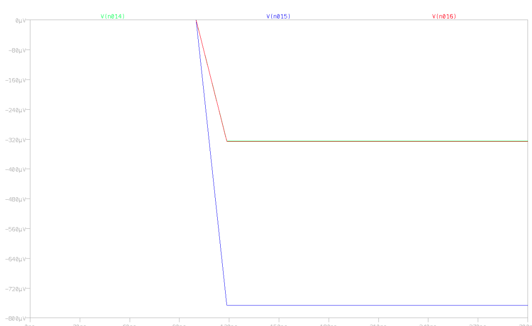
(a) output voltage for central and neighbor strips



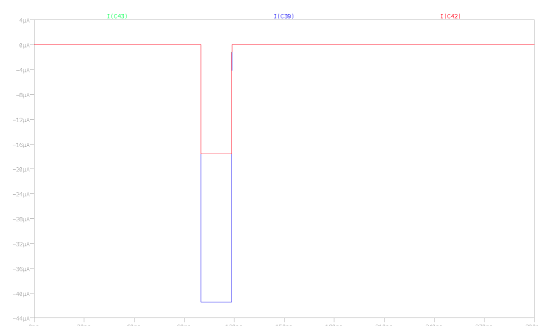
(b) output current for central and neighbor strips

Figure A.13: 100 pf between Resistive strips

- **150 pf**



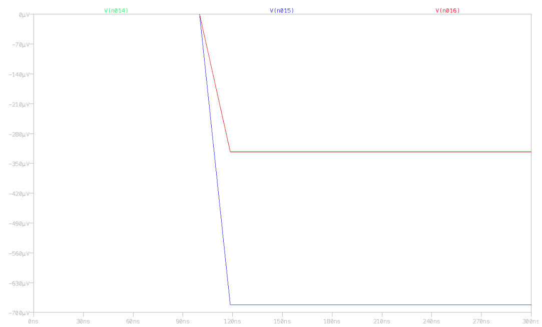
(a) output voltage for central and neighbor strips



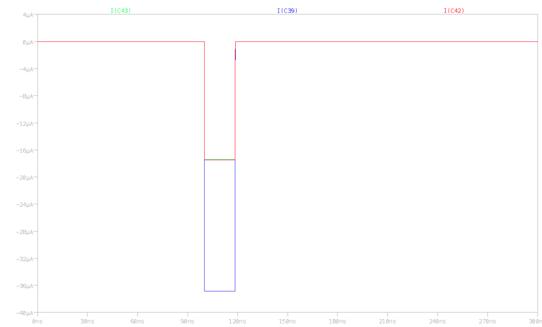
(b) output current for central and neighbor strips

Figure A.14: 150 pf between Resistive strips

- **200 pf**



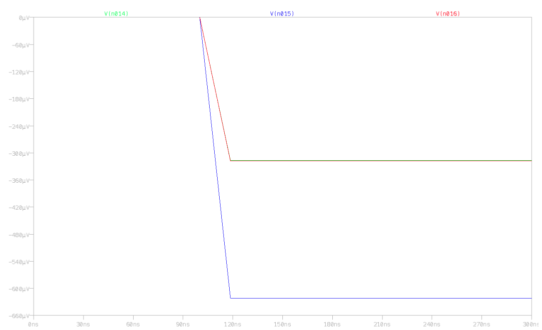
(a) output voltage for central and neighbor strips



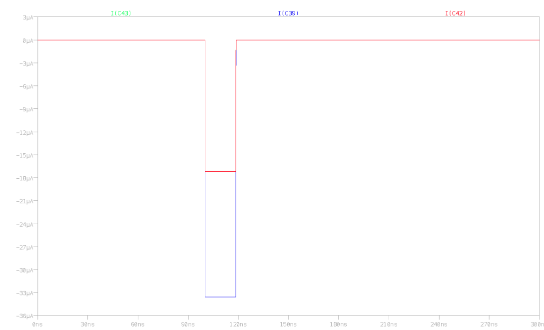
(b) output current for central and neighbor strips

Figure A.15: 200 pf between Resistive strips

- **250 pf**



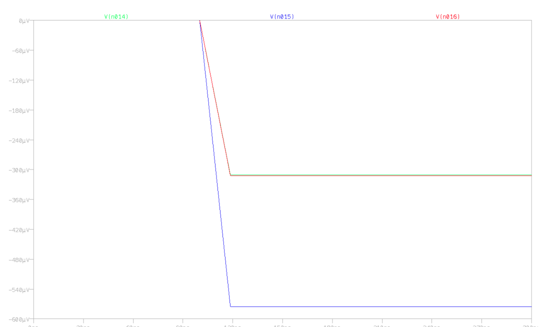
(a) output voltage for central and neighbor strips



(b) output current for central and neighbor strips

Figure A.16: 250 pf between Resistive strips

- **300 pf**

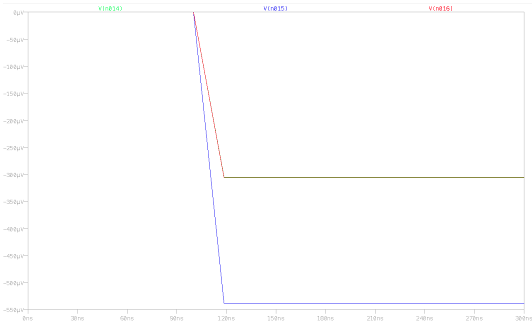


(a) output voltage for central and neighbor strips

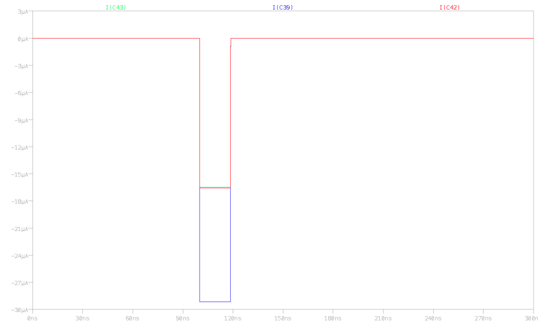
(b) output current for central and neighbor strips

Figure A.17: 300 pf between Resistive strips

- **350 pf**



(a) output current for central and neighbor strips



(b) Resistor 0 Ohm Neighbor Strips

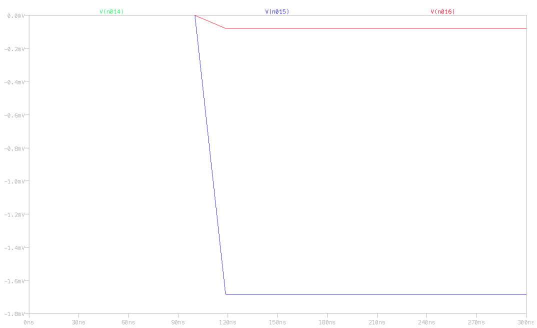
Figure A.18: 350 pf between Resistive strips

Appendix B

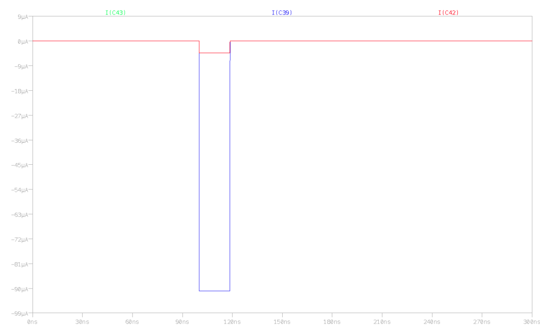
Changes to Capacitances between Readout Strips

LM Module

- **30 pf**



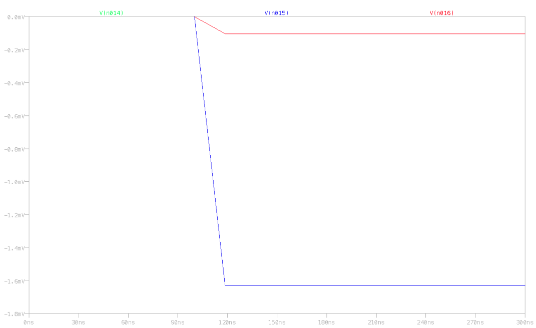
(a) output voltage for central and neighbor strips



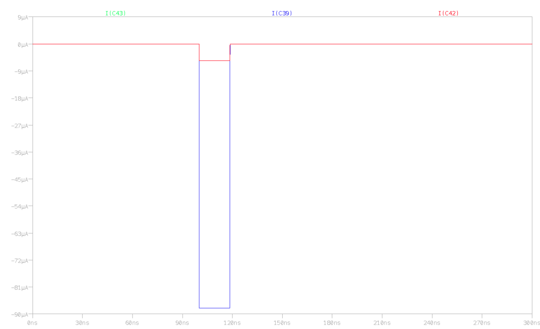
(b) output current for central and neighbor strips

Figure B.1: 30 pf between Readout strips

- **50 pf**



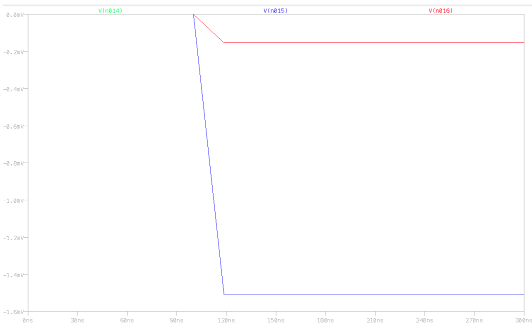
(a) output voltage for central and neighbor strips



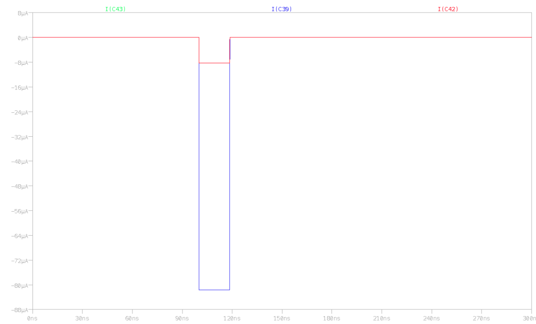
(b) output current for central and neighbor strips

Figure B.2: 50 pf between Readout strips

- **100 pf**



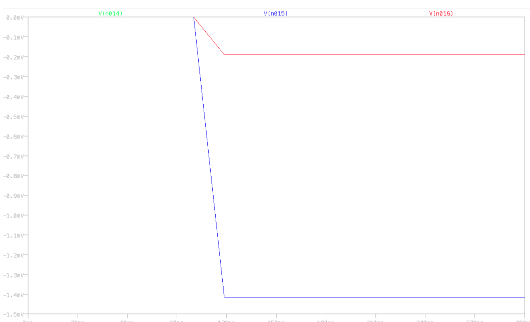
(a) output voltage for central and neighbor strips



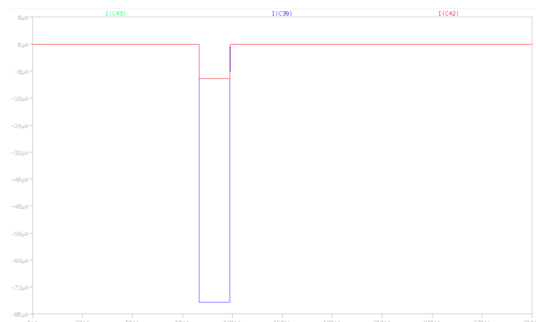
(b) output current for central and neighbor strips

Figure B.3: 100 pf between Readout strips

- **150 pf**



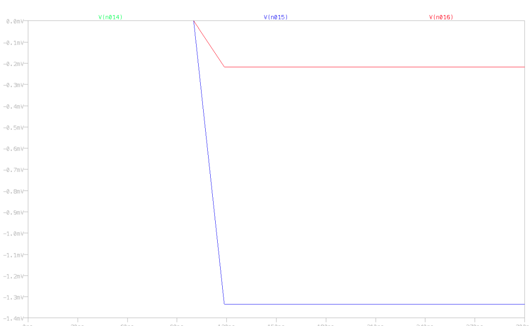
(a) output voltage for central and neighbor strips



(b) output current for central and neighbor strips

Figure B.4: 150 pf between Readout strips

- **200 pf**

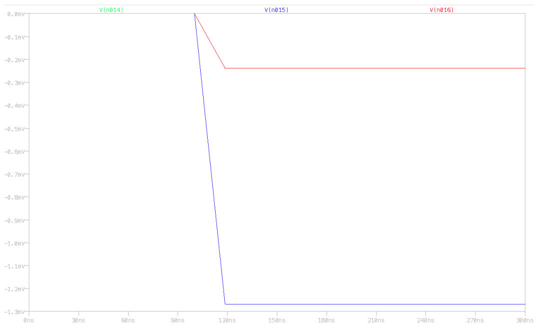


(a) output voltage for central and neighbor strips

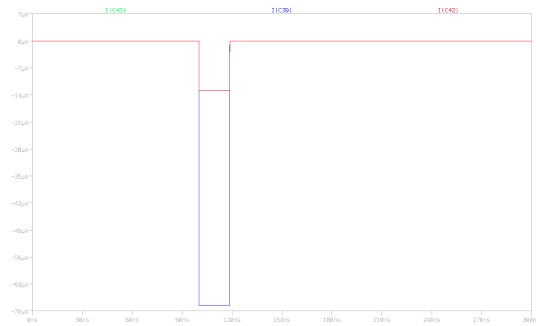
(b) output current for central and neighbor strips

Figure B.5: 200 pf between Readout strips

- **250 pf**



(a) output voltage for central and neighbor strips

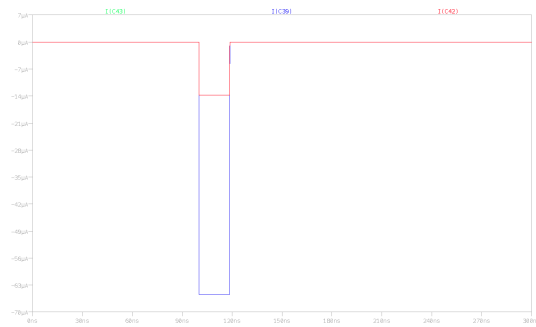


(b) output current for central and neighbor strips

Figure B.6: 250 pf between Readout strips

- **300 pf**

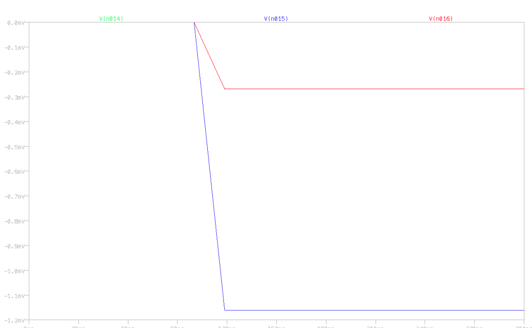
(a) output voltage for central and neighbor strips



(b) output current for central and neighbor strips

Figure B.7: 300 pf between Readout strips

- **350 pf**



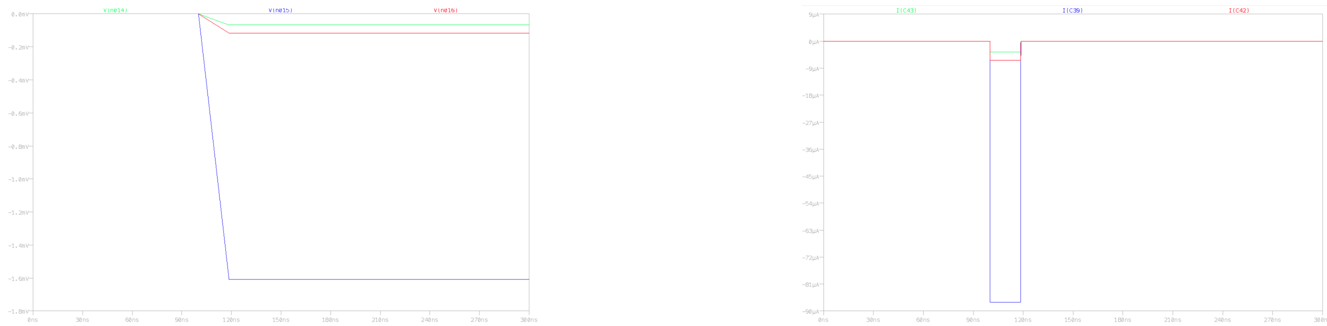
(a) output voltage for central and neighbor strips

(b) output current for central and neighbor strips

Figure B.8: 350 pf between Readout strips

SM Module

- **30 pf**



(a) output voltage for central and neighbor strips

(b) output current for central and neighbor strips

Figure B.9: 30 pf between Readout strips

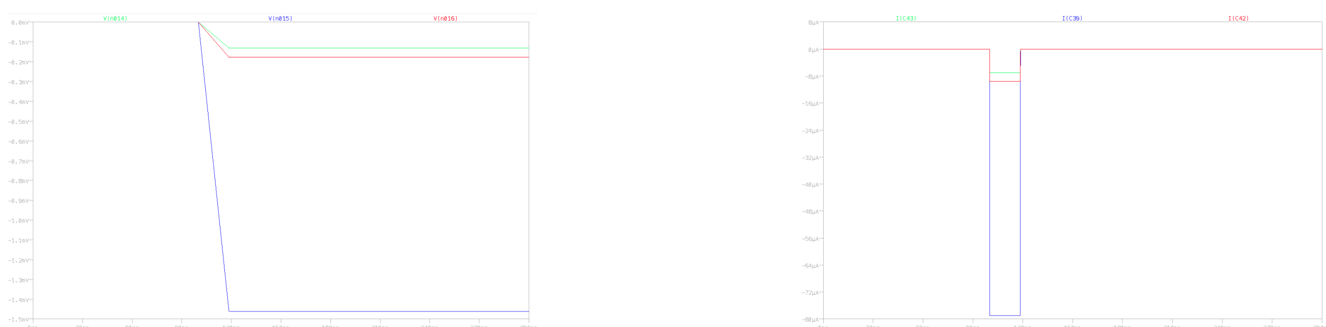
- **50 pf**

(a) output voltage for central and neighbor strips

(b) output current for central and neighbor strips

Figure B.10: 50 pf between Readout strips

- **100 pf**



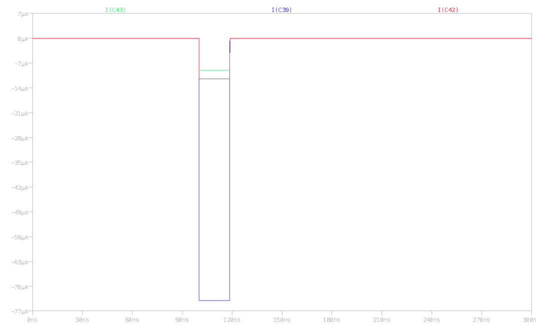
(a) output voltage for central and neighbor strips

(b) output current for central and neighbor strips

Figure B.11: 100 pf between Readout strips

- **150 pf**

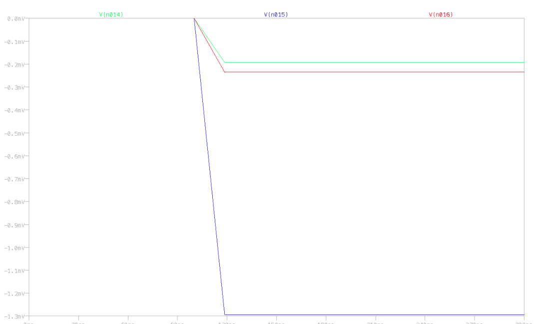
(a) output voltage for central and neighbor strips



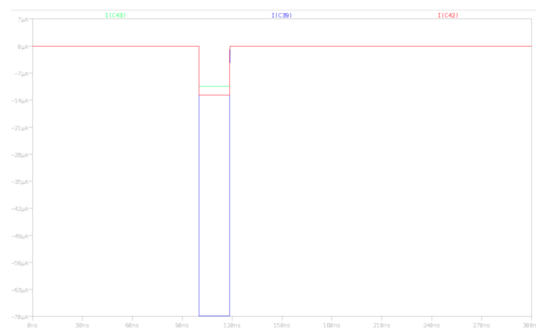
(b) output current for central and neighbor strips

Figure B.12: 150 pf between Readout strips

- **200 pf**



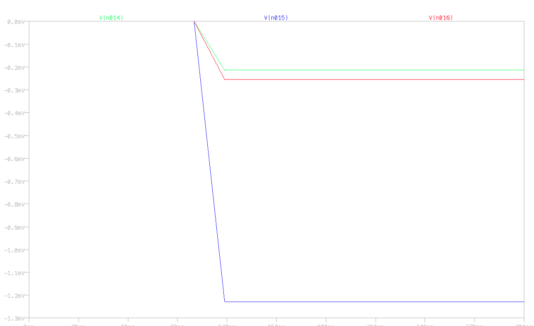
(a) output voltage for central and neighbor strips



(b) output current for central and neighbor strips

Figure B.13: 200 pf between Readout strips

- **250 pf**

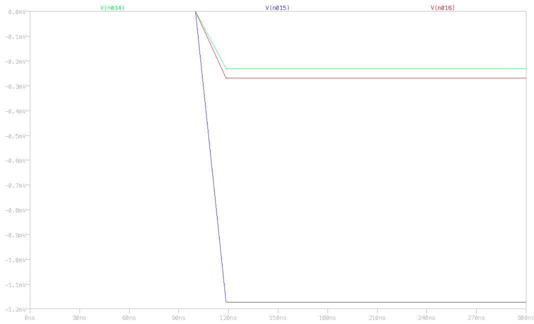


(a) output voltage for central and neighbor strips

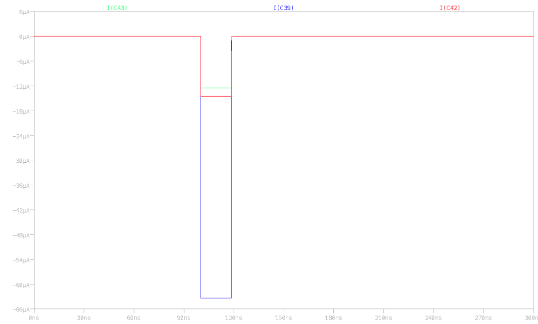
(b) output current for central and neighbor strips

Figure B.14: 250 pf between Readout strips

- **300 pf**



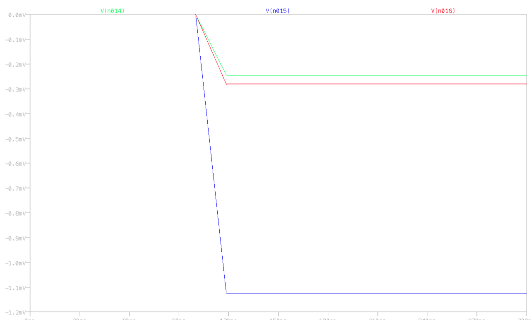
(a) output voltage for central and neighbor strips



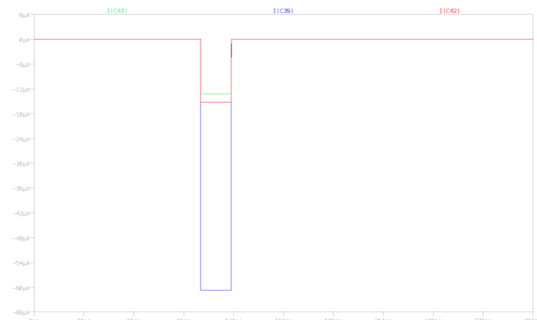
(b) output current for central and neighbor strips

Figure B.15: 300 pf between Readout strips

- **350 pf**



(a) output voltage for central and neighbor strips



(b) output current for central and neighbor strips

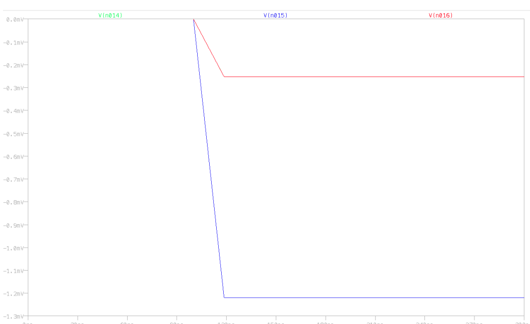
Figure B.16: 350 pf between Readout strips

Appendix C

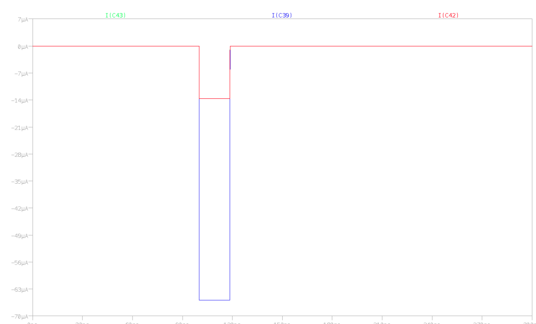
Changes to Capacitances between Resistive-Readout strips

LM Module

- **10 pf**



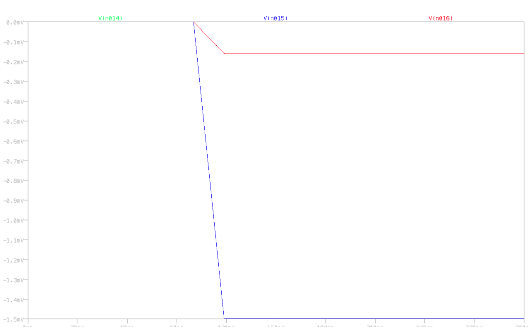
(a) output voltage for central and neighbor strips



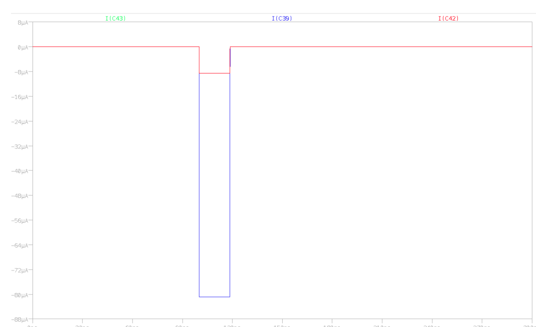
(b) output current for central and neighbor strips

Figure C.1: 10 pf between Resistive and Readout strips

- **30 pf**



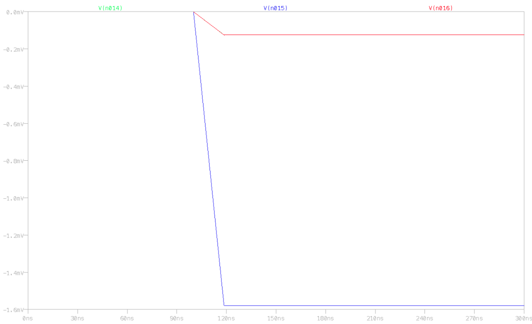
(a) output voltage for central and neighbor strips



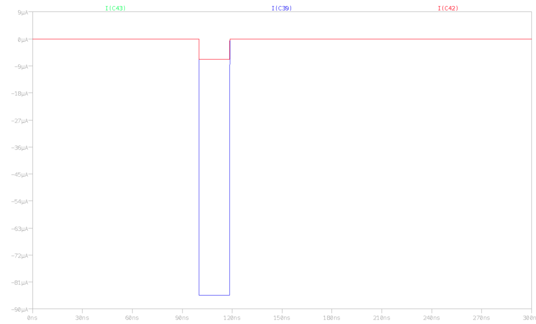
(b) output current for central and neighbor strips

Figure C.2: 30 pf between Resistive and Readout strips

- **50 pf**



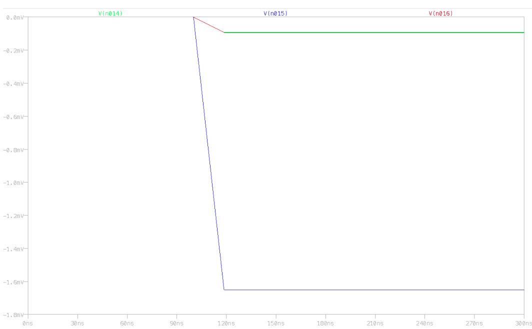
(a) output voltage for central and neighbor strips



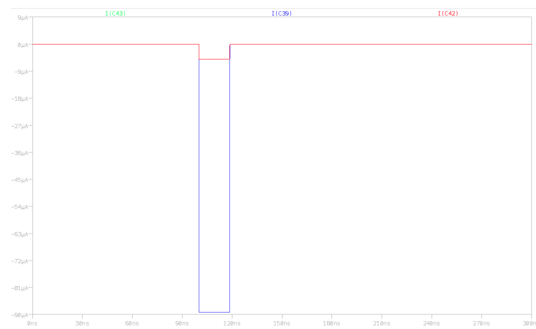
(b) output current for central and neighbor strips

Figure C.3: 50 pf between Resistive and Readout strips

- **100 pf**



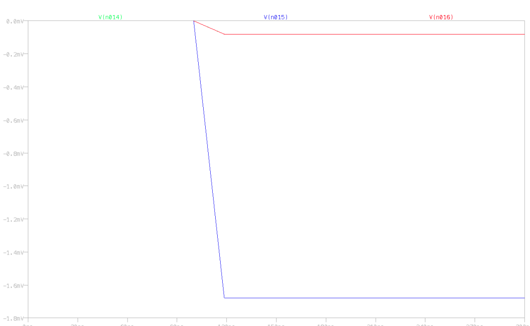
(a) output voltage for central and neighbor strips



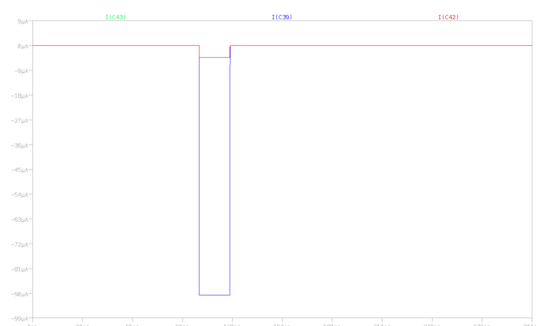
(b) output current for central and neighbor strips

Figure C.4: 100 pf between Resistive and Readout strips

- **150 pf**



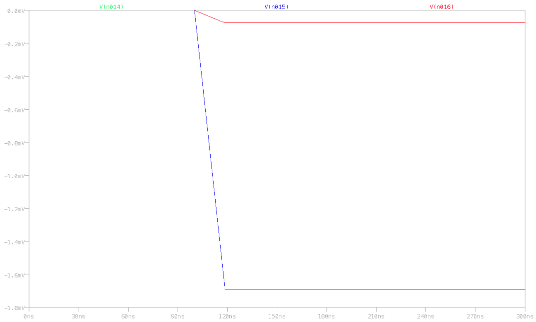
(a) output voltage for central and neighbor strips



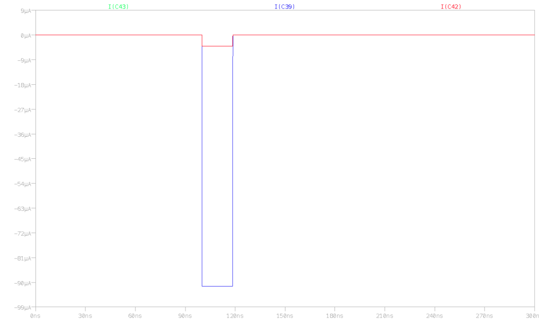
(b) output current for central and neighbor strips

Figure C.5: 150 pf between Resistive and Readout strips

- **200 pf**



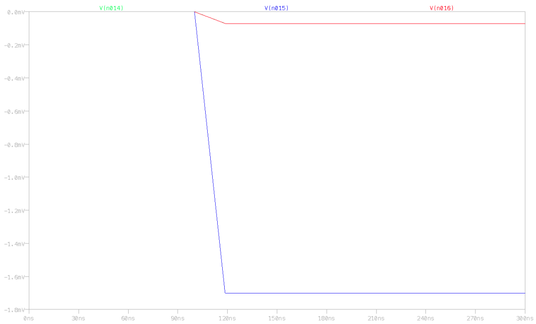
(a) output voltage for central and neighbor strips



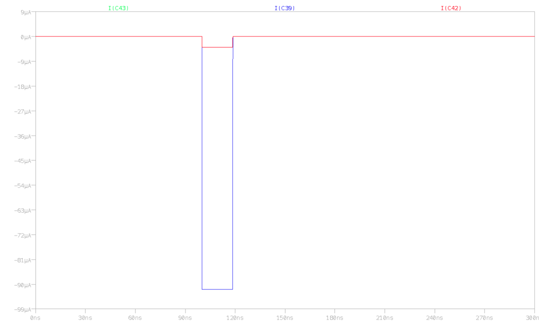
(b) output current for central and neighbor strips

Figure C.6: 200 pf between Resistive and Readout strips

- **250 pf**



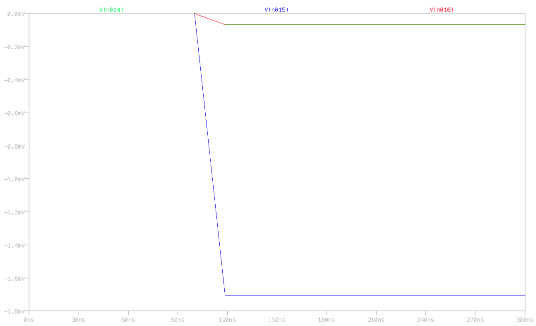
(a) output voltage for central and neighbor strips



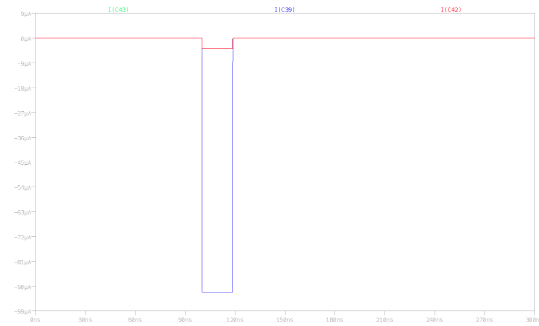
(b) output current for central and neighbor strips

Figure C.7: 250 pf between Resistive and Readout strips

- **300 pf**



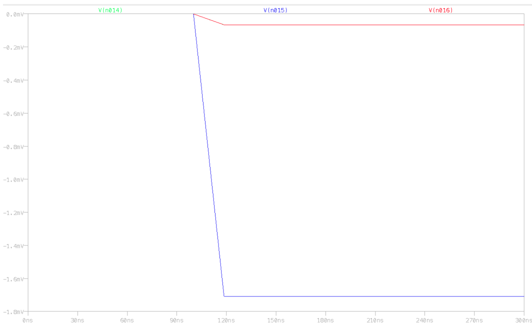
(a) output voltage for central and neighbor strips



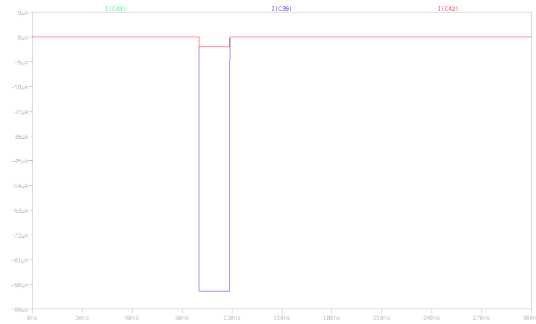
(b) output current for central and neighbor strips

Figure C.8: 300 pf between Resistive and Readout strips

- **350 pf**



(a) output voltage for central and neighbor strips

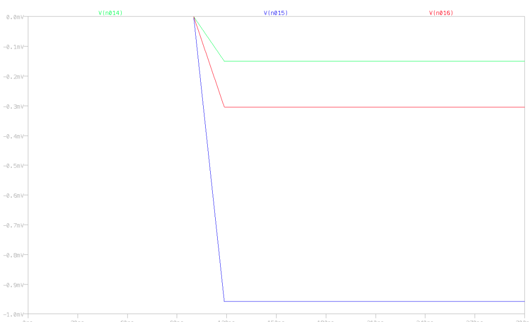


(b) output current for central and neighbor strips

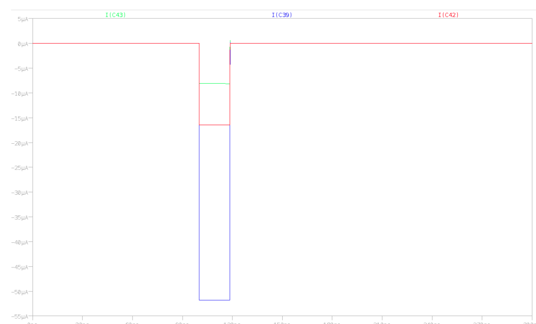
Figure C.9: 350 pf between Resistive and Readout strips

SM Module

- **10 pf**



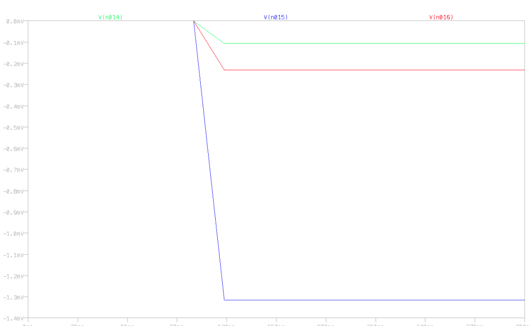
(a) output voltage for central and neighbor strips



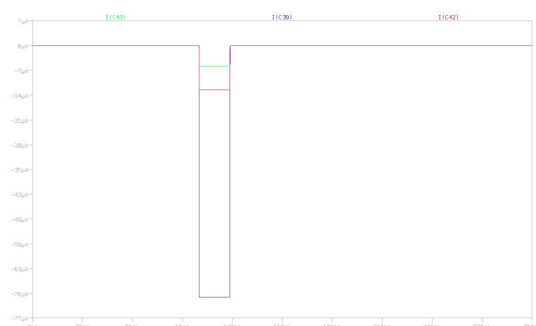
(b) output current for central and neighbor strips

Figure C.10: 10 pf between Resistive and Readout strips

- **30 pf**



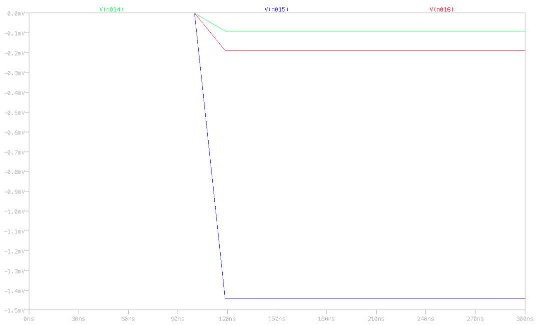
(a) output voltage for central and neighbor strips



(b) output current for central and neighbor strips

Figure C.11: 30 pf between Resistive and Readout strips

- **50 pf**

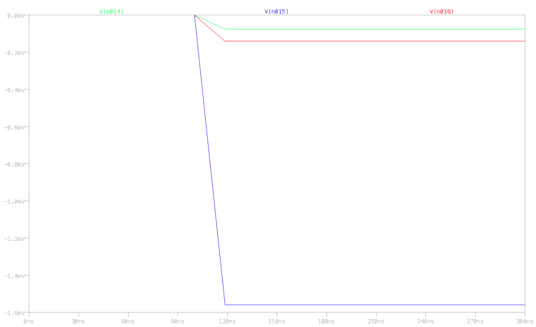


(a) output voltage for central and neighbor strips

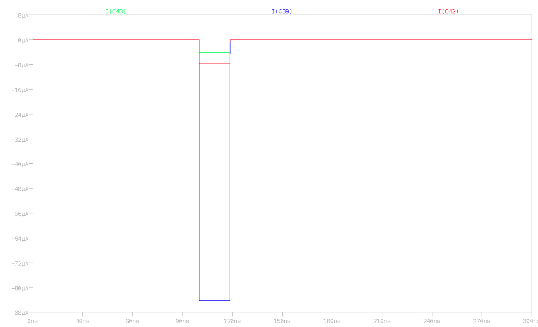
(b) output current for central and neighbor strips

Figure C.12: 50 pf between Resistive and Readout strips

- **100 pf**



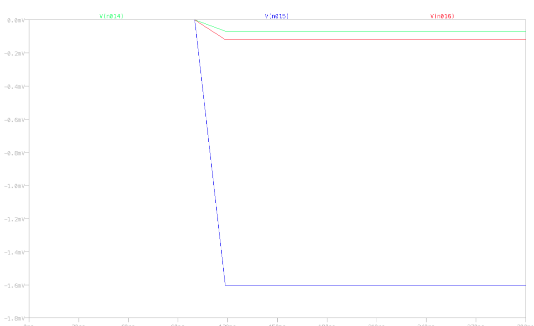
(a) output voltage for central and neighbor strips



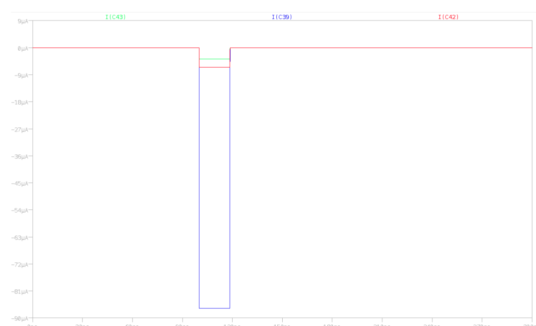
(b) output current for central and neighbor strips

Figure C.13: 100 pf between Resistive and Readout strips

- **150 pf**



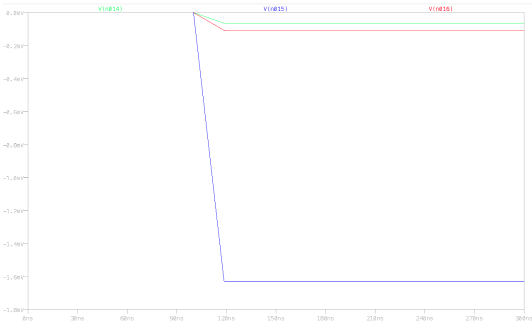
(a) output voltage for central and neighbor strips



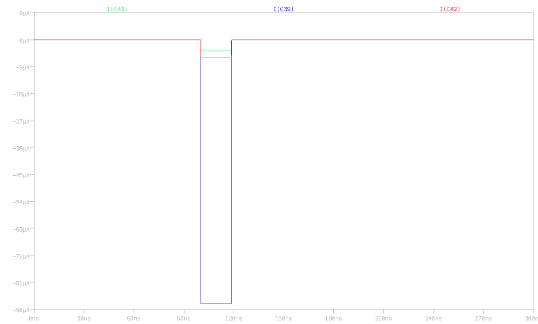
(b) output current for central and neighbor strips

Figure C.14: 150 pf between Resistive and Readout strips

- **200 pf**



(a) output voltage for central and neighbor strips

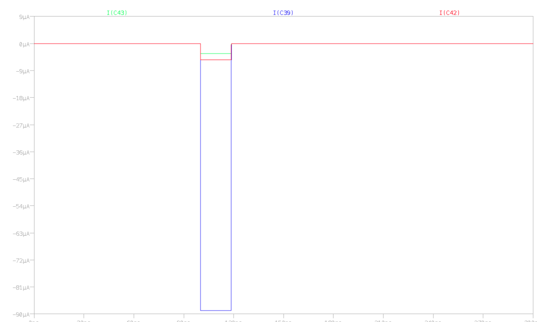


(b) output current for central and neighbor strips

Figure C.15: 200 pf between Resistive and Readout strips

- **250 pf**

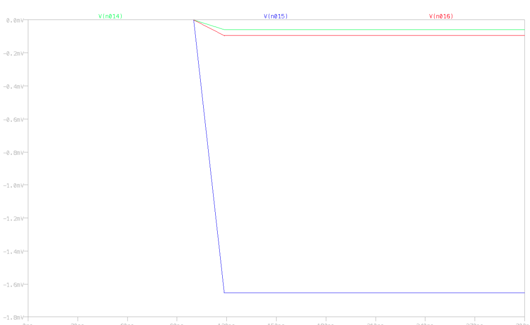
(a) output voltage for central and neighbor strips



(b) output current for central and neighbor strips

Figure C.16: 250 pf between Resistive and Readout strips

- **300 pf**

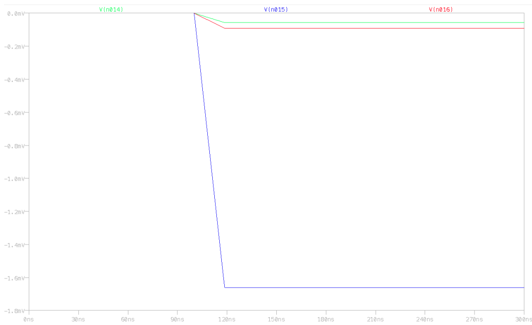


(a) output voltage for central and neighbor strips

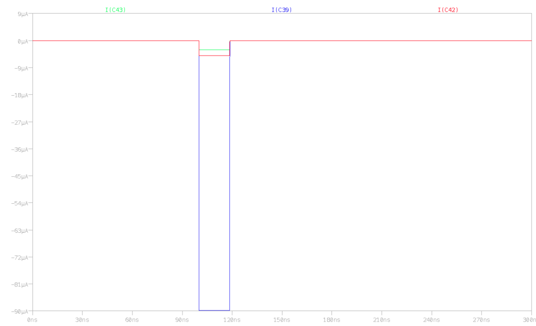
(b) output current for central and neighbor strips

Figure C.17: 300 pf between Resistive and Readout strips

- **350 pf**



(a) output voltage for central and neighbor strips



(b) output current for central and neighbor strips

Figure C.18: 350 pf between Resistive and Readout strips

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