NATIONAL TECHNICAL UNIVERSITY OF ATHENS
SCHOOL OF NAVAL ARCHITECTURE AND MARINE ENGINEERING
DIVISION OF MARINE ENGINEERING

DIPLOMA THESIS

PAGONAS ATHANASIOS

Development of reverse engineering algorithms for automated generation of ship hulls from hydrostatic curves and general ship data

Thesis Committee :	
Supervisor :	Associate Prof.dr. NTUA C.I. Papadopoulos
Members :	Associate Prof.dr. NTUA A.A. Ginnis
	Prof.dr. NTUA L. Kaiktsis

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ABSTRACT

The present work aims at solving the reverse engineering problem of reconstructing the hull geometry of a ship utilising generally available information such as the hydrostatic table of the vessel. In general, the hull of a vessel is characterized by a complex topology which cannot be represented by analytical expressions. The present work investigates the possibility of recreating an approximation of the hull of an existing vessel using as few inputs as possible. The presented approach methodology builds on CAD modeling algorithms of the vessel geometry, combined with optimization algorithms, and benefits greatly by the available computational power offered by modern computational systems.

In order to keep a high level of flexibility, the majority of the construction algorithms that deal with hull parts, such as waterlines or sectors, are created with the help of the open source C++ library OpenCASCADE. At first, we initialise the shape of the hull using elementary information, such as the vessel's length, breadth and depth, along with fundamental lines that restrict its geometry. These lines are the transom, flat of bottom, flat of side and the hull profile. Then, by utilizing an optimization process, we compute the appropriate parameters of the hull CAD model that lead to hull geometries which have hydrostatic properties close to those reported in the hydrostatic table of the ship. Finally, the algorithm is applied for reconstructing (a) the hull of a simple test vessel, designed within our software, and (b) the hull of an actual cargo vessel. The results demonstrate that the presented algorithm can generate the hull of the simple test vessel with very high accuracy (mean relative error of the waterline geometry less than 0.1%), whereas promising results are obtained for the case of an actual vessel where the corresponding mean error has been below 2%.

ΣΥΝΟΨΗ

Η παρούσα εργασία στοχεύει στην επίλυση του προβλήματος αντίστροφου τεχνικού σχεδιασμού (reverse engineering) που αφορά στην ανακατασκευή της γεωμετρίας της γάστρας πλοίου, χρησιμοποιώντας γενικά διαθέσιμες πληροφορίες όπως το υδροστατικό διάγραμμα του πλοίου. Γενικά, η γάστρα του πλοίου χαρακτηρίζεται από περίπλοκη τοπολογία, η οποία δεν μπορεί να αναπαρασταθεί αναλυτικά. Η παρούσα εργασία διερευνά τη δυνατότητα αναδημιουργίας μιας προσέγγισης της γάστρας υφιστάμενων σκαφών, χρησιμοποιώντας όσο το δυνατόν λιγότερα δεδομένα εισόδου. Η μεθοδολογία προσέγγισης που παρουσιάζεται βασίζεται σε αλγόριθμους μοντελοποίησης CAD της γεωμετρίας του σκάφους, σε συνδυασμό με αλγόριθμους βελτιστοποίησης, και είναι εφικτή σε μεγάλο βαθμό με τη βοήθεια της διαθέσιμης υπολογιστικής ισχύος που προσφέρουν τα σύγχρονα υπολογιστικά συστήματα.

Προκειμένου να διατηρηθεί ένα υψηλό επίπεδο ευελιξίας, η πλειονότητα των αλγορίθμων κατασκευής που ασχολούνται με εξαρτήματα γάστρας, όπως οι ίσαλοι και οι εγκάρσιες τομές, δημιουργούνται με τη βοήθεια της βιβλιοθήκης ανοιχτού κώδικα C ++ OpenCASCADE. Αρχικά, προσεγγίζεται το περίγραμμα της γάστρας, χρησιμοποιώντας στοιχειώδεις πληροφορίες, όπως το μήκος, το πλάτος και το βάθος του σκάφους, μαζί με βασικές γραμμές που περιορίζουν τη γεωμετρία του. Αυτές οι γραμμές είναι τα transom, flat of bottom, flat of side, καθώς και το διάμηκες προφίλ του πλοίου. Στη συνέχεια, χρησιμοποιώντας μία διαδικασία βελτιστοποίησης, υπολογίζουμε τις κατάλληλες παραμέτρους του μοντέλου CAD της γάστρας, που οδηγούν σε γεωμετρίες οι οποίες έχουν υδροστατικές ιδιότητες κοντά σε αυτές που αναφέρονται στον υδροστατικό πίνακα του πλοίου. Τέλος, ο αλγόριθμος εφαρμόζεται για την ανακατασκευή (α) του κύτους ενός απλού δοκιμαστικού πλοίου, που έχει σχεδιαστεί εντός του λογισμικού της παρούσας εργασίας, και (β) της γάστρας ενός πραγματικού φορτηγού πλοίου. Τα αποτελέσματα καταδεικνύουν ότι ο παρουσιαζόμενος αλγόριθμος μπορεί να αναπαράξει τη γάστρα του απλού δοκιμαστικού πλοίου με πολύ υψηλή ακρίβεια (μέσο σχετικό σφάλμα της γεωμετρίας τωνισάλων μικρότερο από 0,1%), ενώ πολλύ υποσχόμενα αποτελέσματα λαμβάνονται για την περίπτωση ενός πραγματικού πλοίου, όπου το αντίστοιχο το μέσο σφάλμα είναι κάτω από 2%.

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1 INTRODUCTION

Reverse engineering is the deconstruction of an object to its original design. We usually apply reverse engineering techniques to complex objects to reveal their origin (designs or architecture) and retrieve all the appropriate information so as to generate them from the very beginning.

Reverse engineering is commonly utilized in all engineering fields. The present study tries to apply reverse engineering practices in the naval architect domain.

1.1 MOTIVATION

The construction of a vessel is a procedure divided in stages. There are two main categories, the design and the production stage. At the design stage, engineers are planing in details all the appropriate naval designs and set up the required computational simulations according to the these designs in order to estimate the vessel's stability and strength and also arrange the necessary changes to conform with the international rules. This procedure is very complicated and time consuming, as there are too many details that need to be settled for a vast number of components. At the end of this stage, there are detailed plans and 3D designs of the entire vessel including the hull, the superstructures and all the ship's accessories. The next phase is the production. The engineers at the shipyards make all the mandatory plans using the information acquired by the design stage in order to produce the vessel.

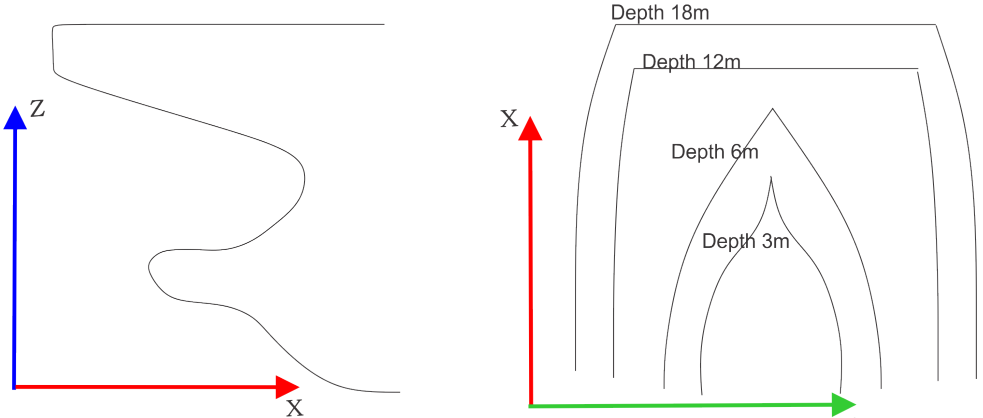
As mentioned above, in order to construct a vessel we need the detailed designs and especially the hull. By having the hull in such a form, engineers can run numerous simulations in order to inspect the behaviour of the hull (stability, strength) in many conditions and evaluate the design in order to improve it further. Also, in case of a damaged hull the lines of the hull are needed for the purpose of repairing and restoring it at its original state. Additionally, when refurbishment is required, it is essential to know the design of the hull in order to apply the appropriate changes.

In many cases, this digital model does not exist or it is damaged. For that reason applying a reverse engineering technique that will be able to recreate a digital hull design is very important and in some circumstances the only possible way.

1.2 PROBLEM STATEMENT

The purpose of this thesis is to develop a reverse engineering technique that will regenerate the hull of a vessel without the detailed designs, but by using information that every vessel holds regardless its age or its manufactured process (detailed 3D models are not available for every vessel) and with no need of travelling at its location.

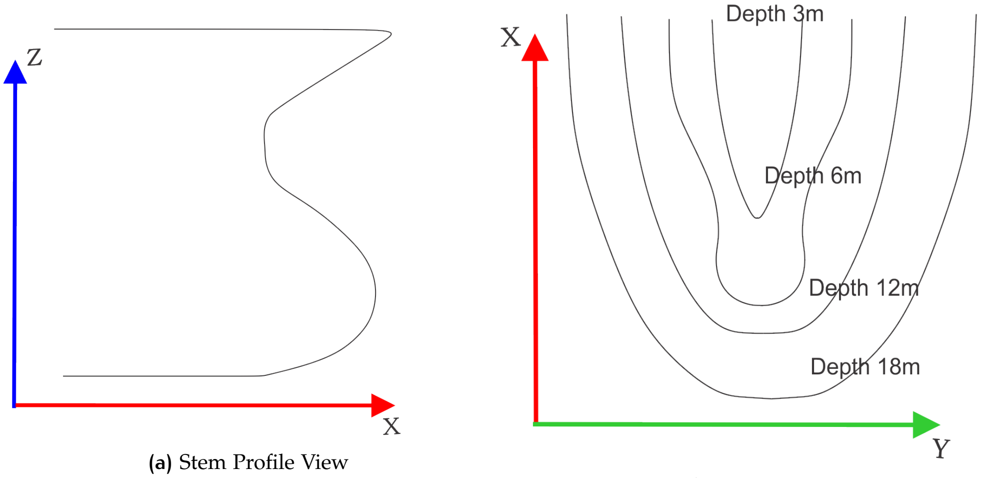
This is not an easy problem as hulls are very complicated shapes. The complexity of these shapes is mainly due to the parts of the hull at the aft (stern)1.1 and at the fore of the hull (stem)1.2.



(a) Stern Profile View

(b) Stern Top View

Figure 1.1: Stern of the hull



(a) Stem Profile View

(b) Stem Top View

Figure 1.2: Stem of the hull

1.3 LITERATURE REVIEW

1.3.1 Analysis

The idea of recreating a vessel using computational power is not something new. There are studies that show how to handle a collection of data and regenerate the hull or parts of a vessel.

Although, due to the ship's enormity and schedule limitations of the most vessels it is nearly impossible to apply the reverse engineering techniques that are being used in other industries.

For example, in automotive industry small scanners and coordinate measuring machines(CMM) are used to acquire the needed data from an automobile. Also, as described in [1] automobile companies create smaller models in order to speed up the process.

In aerospace industry, things are similar to the automotive industry. Laser trackers and 3D scanners are being used to create the appropriate data clouds[1]. The procedure is more complicated comparing to the automotive industry. Despite airplane's size, if we consider their working environment(airports) and the fact that an airplane is available for inspection more frequently compare to a vessel, a reverse engineering process can be applied without many problems.

In the ship industry there are not too many methods that can be used. According to H. J. Koelman[2] there are four categories.

The first category is the **standard manual measurement**. This method is the least efficient one. Even if a laser meter is being used, is really difficult to measure all the appropriate parameters in order to achieve a hull recreation. Also, there are technical limitations concerning vessel's length.

The second category is about devices like **contact scanners**. The problems with such devices are similar to the previous category and for that reason there are not being used at this kind of measure.

The following categories are the most important ones and they are being used globally in order to achieve an accurate result.

The **laser scanning technique** is the first category of great importance in the naval reverse engineering field. Vessels are digitized using large 3D scanners that are capable of retrieving enough data that allows the engineers to create 3D CAD models.

The most applicable technique of all is **photogrammetry** [3] [4]. It is a technique based on images captured from different angles and directions of the object we want to regenerate, in this case the hull.

After the appropriate data possession, what follows is the data pre-processing in order to avoid any inconsistencies, noise or human errors during the data collection. At the doctoral thesis of Desta Milkessa Edessa [5] there is a very detailed explanation of these methods such as Nearest Neighbourhood, Noise Filtering, Down-Sampling and Normal Estimation 1.1 .

Table 1.1: Data pre-processing methods

Nearest Neighbourhood	Optimization problem of finding the point in a given set that is closest to a given point
Noise Filtering	Removing unnecessary data from images by preserving the details of the same
Down-Sampling	Reduce the number of points to efficiently perform operations on point clouds
Normal Estimation	Estimation of the normal at each point in a point data cloud

Finally, these data must be constructed into an object; the surface of the hull. Again, there are numerous ways to recreate a surface but we will not analyse them as it is outside the scope of this work.

In the literature, many reverse engineering techniques are presented for parts of the vessel and not the whole hull. For example, the housing of the main propulsion propeller shaft [6]. In this study, a close range laser scanner was used to acquire the needed data and a special software to create the surface of the object based on the data cloud from the scanner. The average deviation of this model was 0.59mm while its dimensions are $\phi 290425mm$. The ship propeller blades [7] reconstructed using a 3D camera with high accuracy and a laser distance sensor. This study propose a fully automated procedure of a robotic scanning system that is able to reconstruct a propeller blade with resolution of 0.1 mm. The propeller diameter that used for this study is 3600 mm. Three photogrammetric procedures and one triangulation laser scanner used to reverse engineer the blades of a small highly skewed propeller[8]. These techniques along with the appropriate software post-processing produced a 3D model with the worst spatial residual at 0.065 mm.

In the study [9] a photogrammatry technique applied in reconstructing the hold of a bulk carrier with the help of a software to create the 3D model. The hold's dimensions are 30 x 25 x 15 m and for this study were used approximately 800 images with the final picture size computed at 1.2 mm.

Yet there are also some interesting studies where reverse engineering techniques that involve the whole hull or the even the entire vessel.

In Daniel Wujanz's research [10] terrestrial laser scanning was applied to determine the sensor positions on a ship and the outer shell of the vessel was captured by digital imagery and processed to a 3D point cloud by using multi view stereo software.

Anna Nora Tassetti, Michele Martelli and Gabriele Buglioni [11] used a combination of data acquisition methods, standard manual measurement and photogrammetry by using a laser distometer and a 18 Mega Pixels camera in order to obtain the appropriate data and recreate a fishing vessel. The the data that obtained from the distometer imported in a CAD software. In order to create the shape of the hull they used NURBS surfaces. As they mention, for this procedure they spent a lot of time. On the other hand, taking pictures of the vessel was pretty easy and with the help of the commercial photogrammetric software they regenerate the vessel's geometry.

F. Menna , E. Nocerino , A. Scamardella [12] recreate the hull of a torpedo-boat destroyer "Indomito" using photogrammetric techniques along with laser scanner technique. They used a 12 Mega Pixels camera where the distance of the model was 1m(average) and with the help of software they determine the circular coded target centers. Also, they used a 3D Scanner, to obtain a dense point cloud of the hull. Furthermore, they used some additional equipment to recreate the main deck and the superstructure.Their research contributed greatly in the maritime heritage. Two medium-scale prototype boats were scanned, a tsunami fisherman boat and a catamaran hull vessel and re-engineered using close range photogrammetry and a terrestrial laser scanner [13].

Pawel Burdziakowski and Pawel Tysiac [14] applied close-range photogrammetry and terrestrial laser scanning techniques to regenerate the hull of a decommissioned Polish Navy Ship Jastrzab(PNSJastrzab), ex-HNoMS "Kobben"(S-318). The mean error between their calculations and the actual hull is at 5cm with a standard deviation at 3cm.

In the study of Peter K.J. Tay, Collin H.H. Tang and H.E. Tang [15], a 40m anchor handling tug hull was reconstructed via a close range phogrammetric model with high accuracy of 4.1mm absolute mean of error. They used an digital SLR camera along with a photogrammetric software. Fabio Menna and Salvatore Troisi [16] re-engineer a 12m sailing boat with average distance of 1 millimetre of the actual vessel. The equipment they used to obtain the required data was a digital camera, 370 circular targets(161 of them were coded) and two plumb lines. Except of the equipment, they used different commercial software packages to edit the data and create the 3D model.

1.3.2 Comparison and summary of existing approaches

In 1.3.1 we mention a lot of studies relative to the field of reverse engineering in ship industry. All the researches are based on acquiring the needed data to reconstruct a vessel or a smaller part of it by taking pictures or use special equipment like CMM or laser scanners. These approaches require the natural presence of the researchers at the location of the vessel. This is a really demanding requirement as not only a group of people need to travel to a specific location but also the arrangement of such a procedure that will bind a vessel for so much time is very difficult to be found and may cost a lot of money to the owner of the vessel.

1.4 AIM OF THE WORK

In the present study we try to reverse engineer a vessel with as less inputs as possible. The innovation of this research is that we are trying to reconstruct the entire hull of a vessel with no actual photos or measurements. Although, we require a series of inputs. We need, the profile of the vessel, the transom curve, the flat of bottom and flat of sight curves along with the hydrostatic table. Moreover, we need the main characteristics of the ship, the length, the breadth, the depth, the length of the parallel mid section and also some coefficients related to the hull's shape 1.2. All of these data requirements exist for every operational vessel and can be easily obtained at any time as vessels are obligated to hold this information on board. Our approach to the problem is not based on image editing or data cloud processing. We use the original data of a hull that contains huge amounts of information that refers to its unique form(hydrostatic table) and shape limitations of its main characteristics. We provide these data to a mathematical model and then through optimization processes we try to reverse engineer the hull.

Table 1.2: Hull coefficients

C_B	Hull coefficient
C_M	Midship section coefficient
C_P	Prismatic coefficient
C_{WL}	Water line coefficient

1.5 METHODOLOGICAL APPROACH

In the following chapter 2 we analyse all the tools used, the calculations performed and the mathematical model we build along with the optimization process that was followed in order to create the results that are presented later 3 in this thesis.

2 | METHODOLOGY

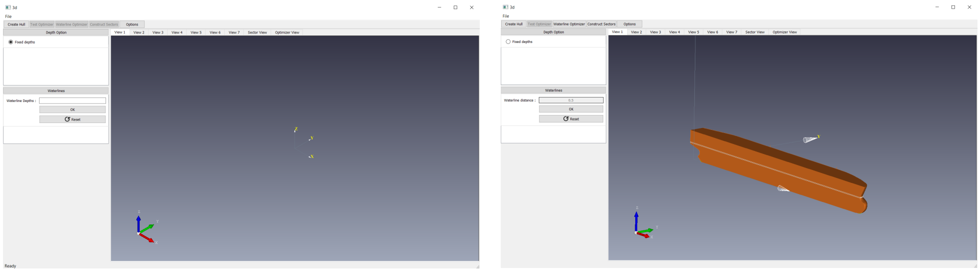
In this chapter, we will explain in detail our approach to the problem including the tools and the techniques that we utilised in order to obtain the results presented in the following chapter.

2.1 TOOLS

We created this project in the integrated development environment (IDE) Visual Studio 2019 using C++ as the programming language.

At this point, we will mention the core libraries of the project.

1. **Qt Software**
   Qt [17] is a free open-source library that we used in the current project in order to create a graphical user interface (GUI). Is one of the most popular free open-source libraries using c++. Creating GUI is really important and essential at this project because not only makes it easier for testing, but also the results can be displayed at a graphical environment and evaluated in a more efficient way. The pictures below 2.1 are an example of this GUI.



● Fixed depths

Waterline Depths :

○ Fixed depths

Waterline distance : 0.5

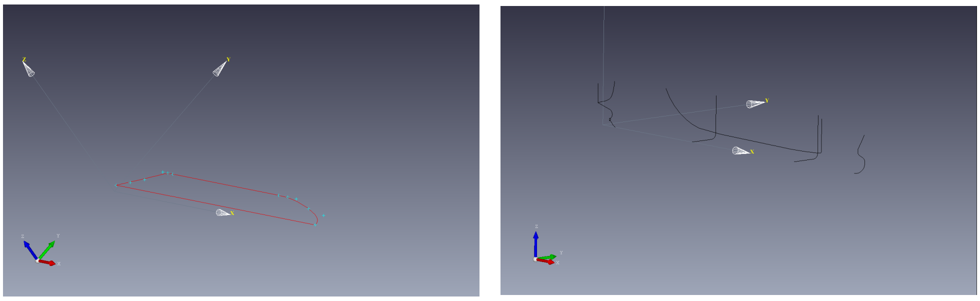
(a) GUI

(b) Presenting results

Figure 2.1: GUI using Qt library

2. **Open CASCADE Technology Overview**
   OCCT[18] is an open-source software development platform for 3D CAD, CAM and CAE developed by Open Cascade SAS. This project is a CAD software. For all the appropriate lines,surfaces and shapes we created, a 3D CAD was necessary. Although, we faced a problem related to the displayed graphical environment. We detect that the b-spline curve representation did not represent one hundred percent to the actual curves. OCCT simplifies the curve's detailed structure in order to display all the information.

   Also, most of the calculations that refer to geometrical properties are computed with the help of OCCT. The majority of the work is based on this library. The figures 2.2a and 2.2b are snapshots of this CAD.



(a) Create surfaces and points

(b) Boundary curves

Figure 2.2: OCCT library

3. **Dlib**
   Dlib [19] is an open-source, general purpose library written in C++. In this project we used it for its numerical optimization components.

4. **OpenGA**
   OpenGa [20] is a free C++ genetic algorithm library. We apply this library at some parts of the optimization process in order to obtain better results.

2.2 HYDROSTATIC & GEOMETRICAL CALCULATIONS

At this point, we will break down both the hydrostatic and the geometrical terms we use in this project along with the approach we chose to calculate them.

2.2.1 Water plane area (WPA)

WPA is the area of a longitudinal section(waterline) of the hull and differs for every depth. We calculated WPA with the help of the library OCCT. The following picture 2.3 displays three waterlines in different depths. The colored areas represent the WPA of each waterline.

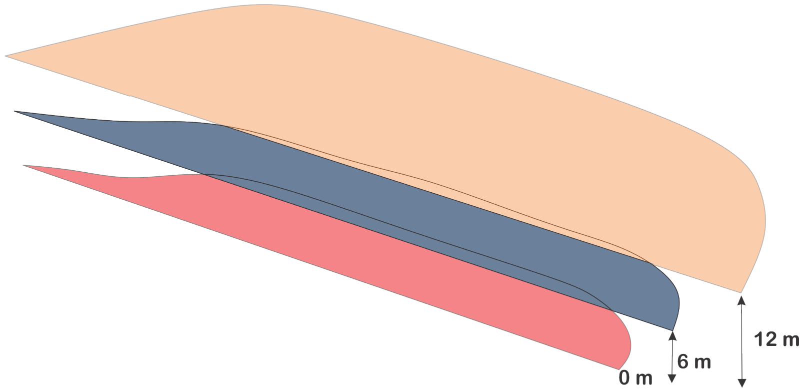


Figure 2.3: Longitudinal Sections

2.2.2 Second moment of area in Longitudinal Axis (Ixx)

Ixx is the geometrical property of the water plane area and reflects the points distribution of the waterline curve in x-axis. The mathematical equation of the second moment inertia is the following 2.1 .

$$I_{xx} = \iint_{A_v} y^2 dxdy \tag{2.1}$$

2.2.3 Longitudinal Center of Floatation (LCF)

Longitudinal center of floatation(LCF) is the center of the longitudinal section (waterline) of the hull and differs for every depth 2.4. It is calculated according to the following equation,

$$LCF = \frac{\iint_A x^2 dxdy}{WPA} \tag{2.2}$$

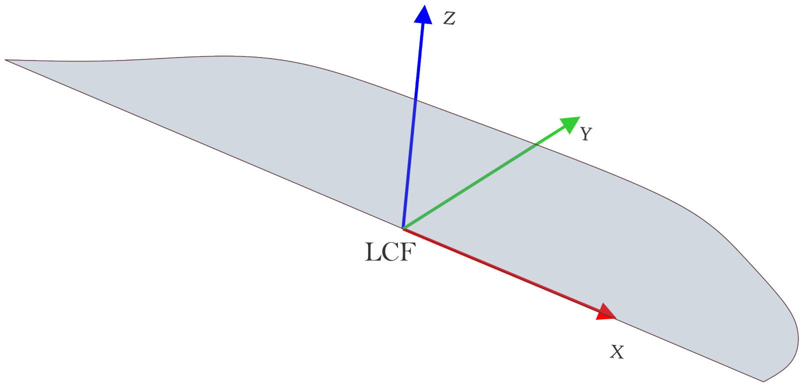


Figure 2.4: Longitudinal Center of Floatation

2.2.4 Volume (∇)

When we refer to the volume of the hull, is about the volume between two or more longitudinal sections 2.5.

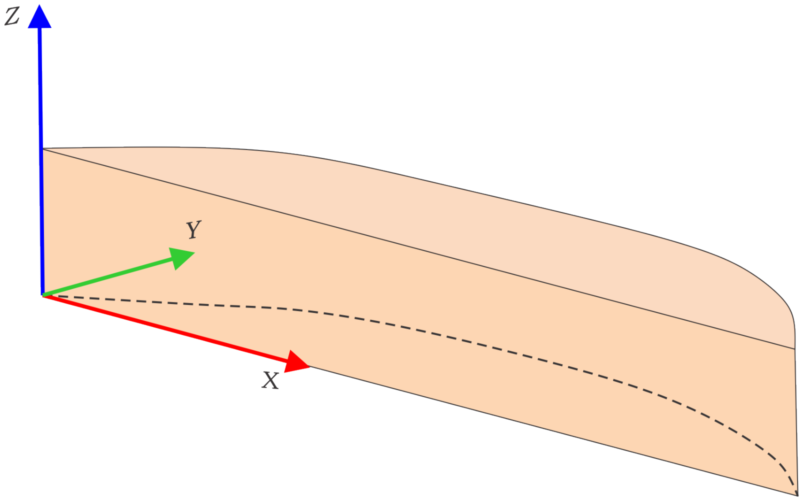
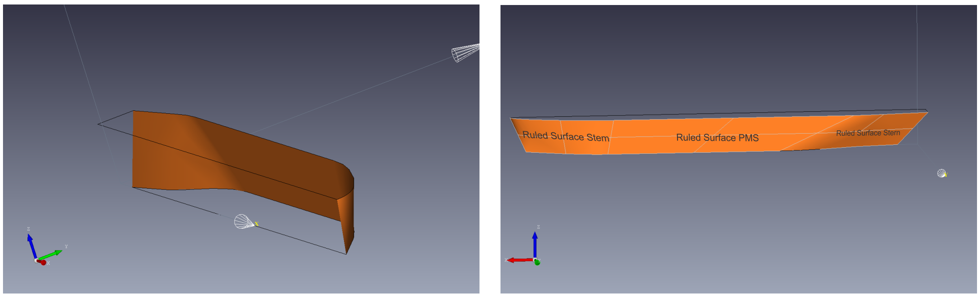


Figure 2.5: Volume between two longitudinal sections

2.2.5 Wetted Surface Area (WSA)

WSA stands for wetted surface area. It is the surface between two or more longitudinal sections. At the picture 2.6 we present an example with two waterlines. We create three ruled surfaces between every pair of waterlines, one for each region of these waterlines; the stern, the parallel mid section and the stem.



(a) WSA back view

(b) WSA front view

Figure 2.6: WSA

2.2.6 CB

CB stands for center of buoyancy. It is the center of volume between two or more longitudinal sections 2.7. It is separated in three coordinates as follows,
X-coordinate or Longitudinal Center of Buoyancy(LCB):

$$LCB = \frac{\iiint_{\nabla} x dV}{\iiint_{\nabla} dV} \tag{2.3}$$

Z-coordinate or Vertical Center of Buoyancy(VCB):

$$VCB = \frac{\iiint_{\nabla} z dV}{\iiint_{\nabla} dV} \tag{2.4}$$

The y-coordinate is always zero as the hull is symmetric on this axis.

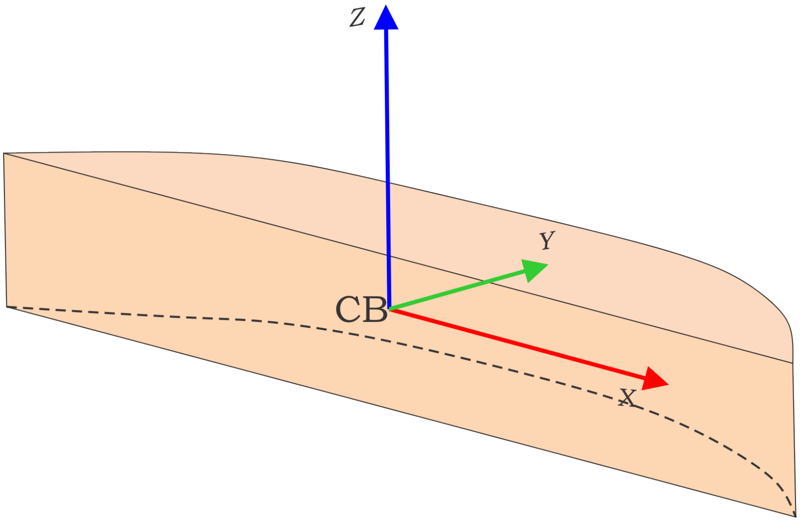
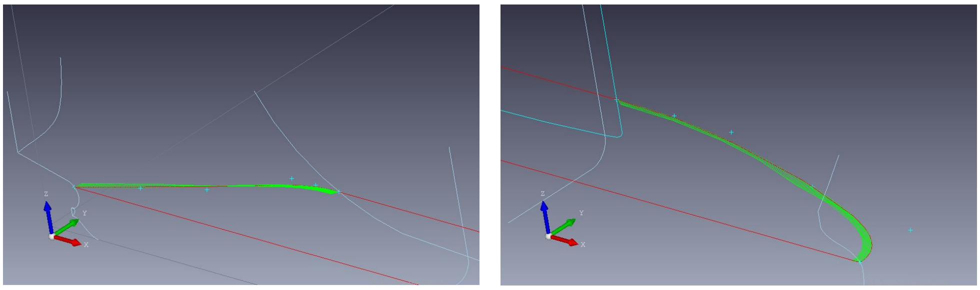


Figure 2.7: Center of Buoyancy

2.3 CURVATURE

Curvature is the amount from which a curve deviates from being a straight line. At first, we calculate the tangents of the surface, for a collection of x-positions at the curve and store these tangents as x and y coordinates. Then, we create points with these coordinates and normalized them by the y-coordinate to ensure a uniform display. Finally, we create two b-spline curves through these created points. One for the stern 2.8a and one for the stem 2.8b. These curves represent the curvature.



(a) Curvature on stern with OCCT

(b) Curvature on stem with OCCT

Figure 2.8: Curvature

2.4 CALCULATIONS EVALUATION

In this section we will present proof that our calculations are correct by comparing our results with the results that produced with the use of arithmetic methods with the help of the software package Matlab [21].

For the evaluation process we created a special two dimensional shape 2.9(waterline). This shape is separated in three parts. Each part is a polynomial curve. We use polynomials because the calculations will be much easier and more accurate than using splines or a complex function.

The first part is represented by the polynomial curve,

$$y(x) = -0.0037 \cdot x^2 - 0.4762 \cdot x + 0.0041, x \in [0, 60)$$

the second part is a straight line,

$$y(x) = 14.6, x \in [60, 130]$$

and the third part is represented by the polynomial curve,

$$y(x) = -4.8895 \cdot 10^{-8} \cdot x^5 + 3.6476 \cdot 10^{-5} \cdot x^4 - 0.0109 \cdot x^3 + 1.6387 \cdot x^2 - 123.06 \cdot x + 3.7126 \cdot 10^3, x \in (130, 190]$$

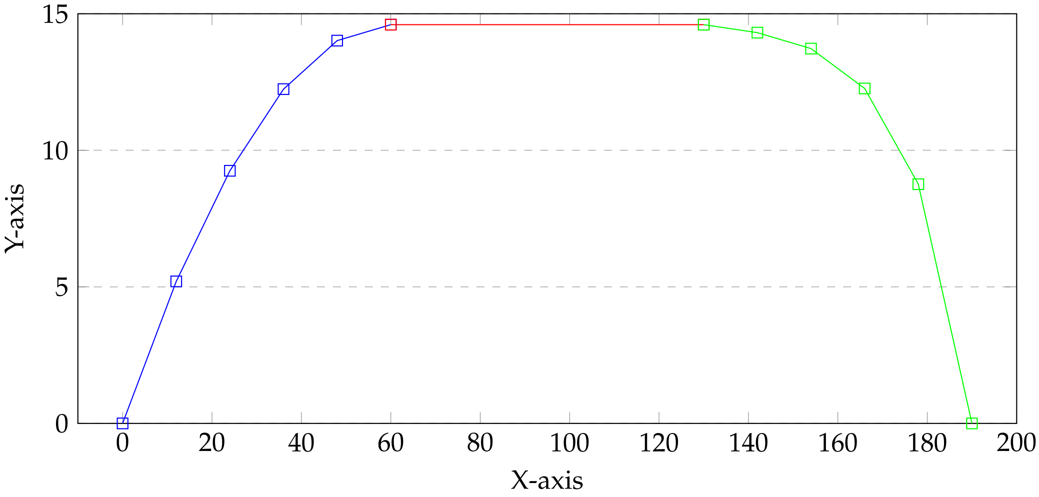


Figure 2.9: Polynomial Waterline

In the table below 2.1 we present the area attributes as calculated with our program and the software package Matlab.

Table 2.1: Area Attributes Comparison

	$WPA[m^2]$	$I_{xx}[m^4]$	$LCF[m]$
Our program	4586.859	278255	98.52
Matlab	4585.452	278240	98.494
Absolute difference	1.4068	15	0.026
Percentage Difference	0.03 %	0.005 %	0.026 %

In order to compare the volume attributes we create two identical waterlines as described above and form the 3d shape between them 2.10. The first waterline is created at zero depth and the second one three meters above.

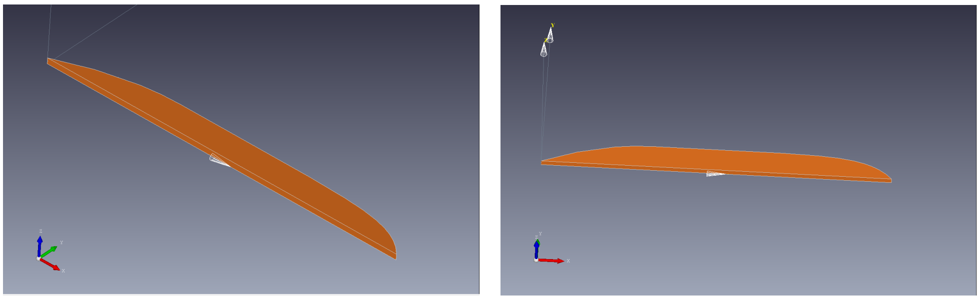


Figure 2.10: Polynomial waterline

In the table below 2.2 we present the volume attributes as calculated with our program and the software package Matlab.

Table 2.2: Volume Attributes Comparison

	Volume[m^3]	WSA[m^2]	Longitudinal Center of Volume[m]
Our program	13760.02	5762.305	98.51888
Matlab	13756	5760.859	98.4938
Absolute Difference	4.021435	1.446462	0.025081
Percentage Difference	0.029 %	0.025 %	0.025 %

From tables 2.1, 2.2 we can see that the differences of the two calculating methods are very close to zero.

2.5 GEOMETRY GENERATION

2.5.1 Waterlines

Waterlines are the longitudinal sections of the hull parallel to the XY plane. The common construction of a waterline includes b-spline curves passing through a large number of points creating smooth curves that meet the waterplane's area attributes. Also, these curves follow some boundary rules that are related to the vessel's limitations. In the following figure 2.11 we can see some waterlines at the stern part of a vessel.

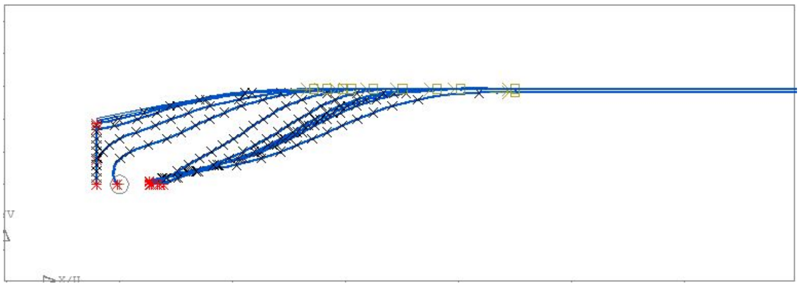


Figure 2.11: Waterlines at stern region-AVEVA Initial Design™

Every waterline meets the following structure :

1. at the stern region there is a b-spline that interpolates the points at the aft part of the ship for a specified depth
2. at parallel mid section of the vessel there is a straight line that unites the aft and the fore part of each waterline

3. at stem region there is also a b-spline curve interpolating the points at the fore part of the ship

In the figure 2.12 below we present the three parts of the waterline's structure.

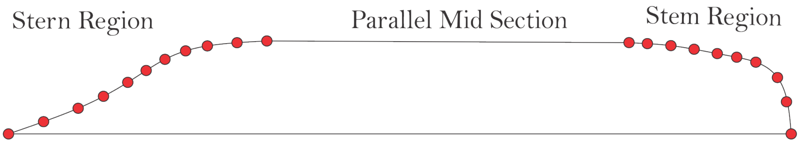


Figure 2.12: The form of a common waterline

We create waterlines in a similar way to a common construction, but with a few differences that determine their final form.

First of all, in order to create a waterline, we need the following input :

1. Desired depth
2. Profile
3. Flat of sight (FOS)
4. Flat of bottom (FOB)
5. Transom
6. Parallel mid section limits
7. Bilge radius

The mentioned inputs are the limitations that our vessel must follow. These limitations along with the vessel's hydrostatic table is the information we request from the user in order to start recreating the vessel's hull.

We use the input mentioned above to obtain four points (or five if the boundary curve transom is a part of this waterline), for each waterline.These points are part of the hull and are placed according to the initial input. The first point is a stern-point, the second and the third are the FOS-points and the fourth point is the stem-point (from now on, we will refer to these points as boundary points).

Subsequently, we create a b-spline between the boundary points 1 and 2, one straight line between boundary points 2 and 3 and one more b-spline between boundary points 3 and 4 2.13.

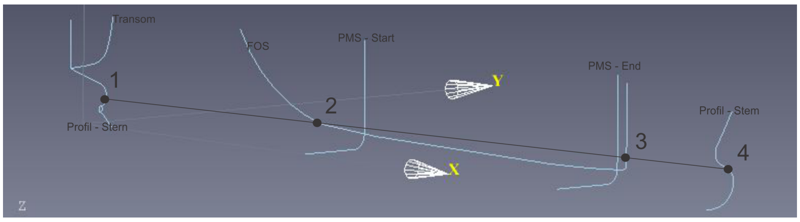


Figure 2.13: Example of boundary points at a random depth

If the waterline cuts the boundary curve transom, the line-point sequence is the following. First, we create a straight line between the boundary points 1 and 2, then a b-spline between the boundary points 2 and 3, one straight line between boundary points 3 and 4 and one more b-spline between boundary points 4 and 5 2.14.

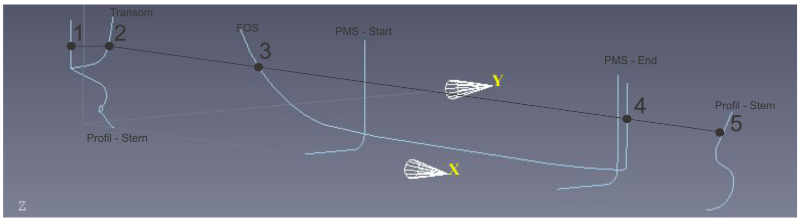


Figure 2.14: Example of boundary points at a random depth including transom

We will now clarify the way that b-splines are constructed.

The most important is that we do not pass the b-spline curves through points, but we use *control points*.

We use control points for two reasons:

(a) It is easier to reach the desired curve attributes such as continuity, smoothness, curvature.

(b) Control points are not necessary part of the curve. They are a set of points that determine the shape of the b-spline curve. As a result, we can move the control points more freely than the actual points of the curve.

We designed two types of curves for the stern (between boundary points 1 and 2). The distinction between these two types is one extra boundary point due to the transom curve.

2.5.1.1 *Stern Curve : Type 1*

The first type consists of a b-spline with $a \in (5, \infty)$ number of control points. The first and the last control point are the boundary points 1 and 2, so there are $a - 2$ control points now that need to be settled.

We set the $a - 2$ control points(middle control points) as follows :

The control points are separated in smaller groups. The name of each group refers to the number of control points. These groups are the $a - 3$ and the $a - 4$. All groups are part of the total control points and every smaller group is part of the next bigger group.

The x-coordinates of the $a - 3$ control points are equidistant between the boundary points. The last middle control point is placed between the last point of the group $a - 3$ and the boundary point 2. The y-coordinates of the $a - 4$ middle points are on the straight line that is created between the two boundary points 1 and 2. The last two middle control points are placed at the same y-coordinate as the boundary point 2.
We demonstrate this construction with an image-example of $a = 6$ 2.15.

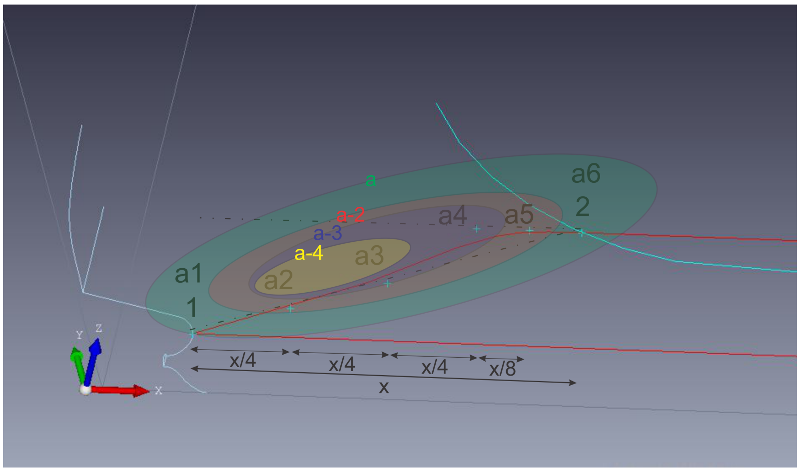


Figure 2.15: Example of b-spline curve type 1 with 6 control points

For our b-splines we need G_2 continuity in order to achieve curvature continuity. For that reason the degree of the b-splines must be at least 3 and as a result we require 4 control points minimum. Furthermore, for a b-spline curve of degree 3 we need two control points col-linearly in order to achieve tangential continuity with the parallel mid body, so the purpose of point $a5$ 2.15 is to create a smooth connection between our b-spline and the straight line at parallel mid section. That is why in our construction we need at least 5 control points. We place the x-coordinates of our control points equidistant to achieve uniformity. The y-coordinates of the $a - 4$ control points 2.15 are on a straight line between the end-boundary points. We choose to place these points in such way because of our assumption that regardless to the shape of the stern(narrow or wide) it is a safe and fairly rational starting positioning of the control points.

2.5.1.2 *Stern Curve : Type 2*

The second type is almost the same. But at this type of b-spline there is an extra boundary point due to the boundary curve transom 2.16.

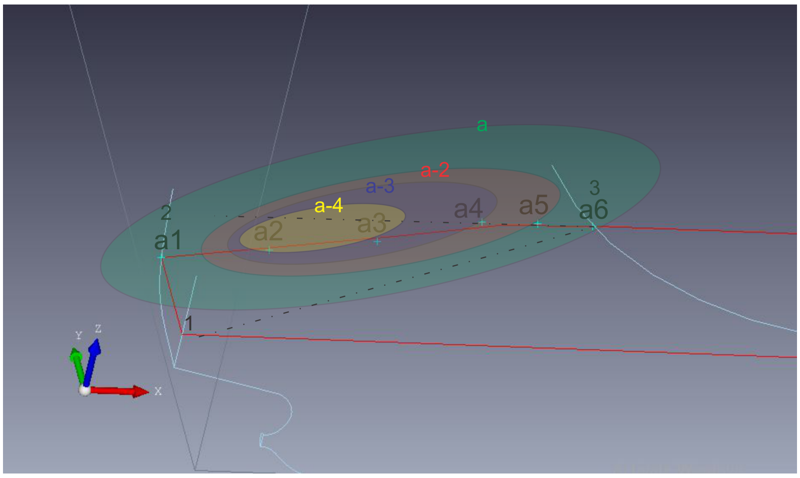


Figure 2.16: Example of b-spline curve type 2 with 6 control points

2.5.1.3 *Stem Curve*

For the stem we designed one type of b-spline curve(between boundary points 3 and 4). This b-spline consists of $a \in (5, \infty)$ number of control points. The first and the last points are the boundary points 3 and 4, so there are $a - 2$ control points now that need to be settled.

The $a - 2$ control points are setted (middle control points) as follow :

As we did for the stern b-splines, we separate the middle control points in smaller groups. These groups are the $a - 4$ and the $a - 5$. All groups are part of the total control points and every smaller group is part of the next bigger group.

The middle $a - 2$ control points are setted as follows :

The x-coordinate of the first middle point is the same as the the boundary point 4. The x-coordinates of the $a - 4$ points are equidistant at the space between boundary points. The last middle point is placed between the last point of the group $a - 4$ and the boundary point 3.

The y-coordinate of the first middle point is a percentage of the y-coordinate of the boundary point 3 and its value depends on ship's profile. The $a - 5$ points are on the straight line that is created between the first middle point and boundary point 3. The last two control points are placed at the same y-coordinate as the boundary point 3.
We will demonstrate this construction with an figure-example with $a = 6$.

The difference between stem b-splines and stern b-splines is the positioning of the point $a2$ 2.17. We choose to place this point at such way so the waterlines at fore end up to a vertical tangent. We choose this construction because of the vessel's shape at stem.

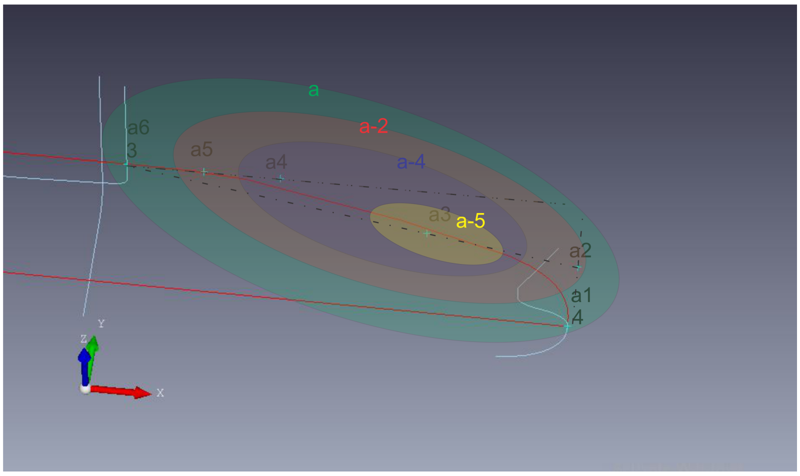


Figure 2.17: Example of b-spline curve at stem with 6 control points

2.5.2 Sectors

Sectors are the transverse sections of the hull. They are usually produced by the intersection of the waterlines. After sector's first initialisation, we can move some of the sector's points in order to reach the appropriate attributes like curvature or area. Then, we feed waterlines with these changes by doing the opposite procedure that described above. We repeat this process until both sectors and waterlines reach the necessary form.

We use the same method to initialise sectors. We create b-splines passing through the points that are intersected with each created waterline. We evaluate our results through the smoothness of the these curves and also it is one of the main output results of the program. In the figure 2.18 we have isolate two sectors.

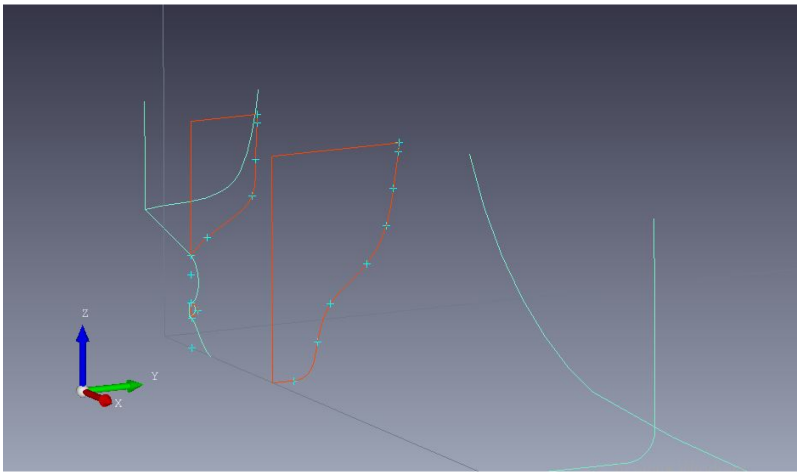


Figure 2.18: Example of two sectors created by seven waterlines

In the following picture 2.19, there is a grid of of sectors and waterlines which share the same points.

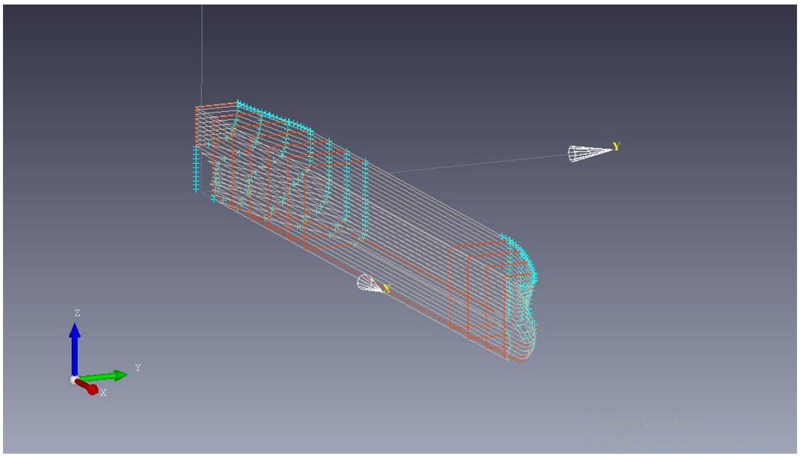
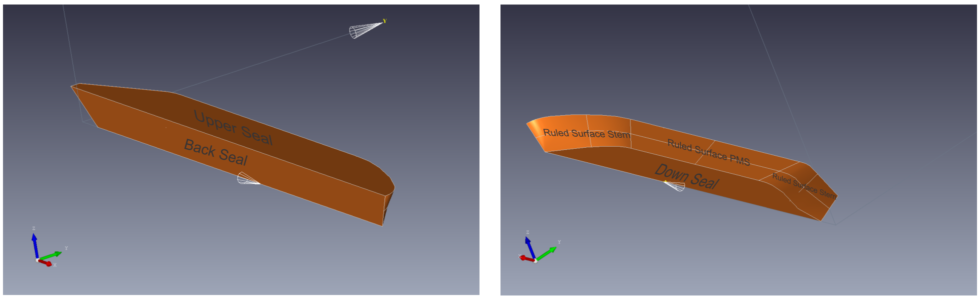


Figure 2.19: A grid of sections and waterlines

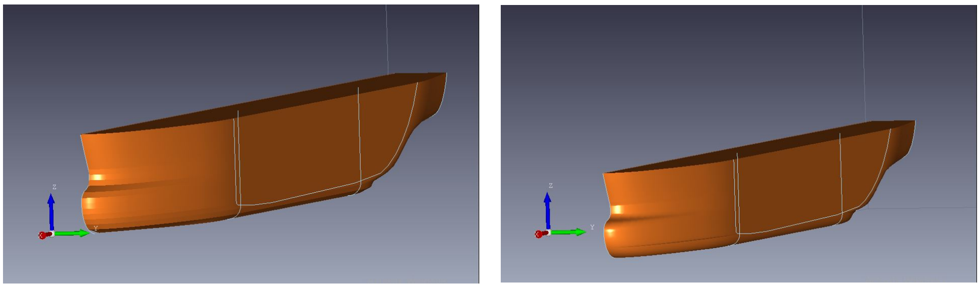
2.5.3 Hull

Hull is the part of the vessel most of which is underwater and is responsible for the vessel's behaviour underwater. We create hull using waterplanes. After the creation of multiple waterplanes we use OCCT library in order to create the shape of the hull. For each pair of waterplanes we create one shell. This shell consists of a pair of waterplanes (the up and down seals of the shell) , one trapeze surface from the waterlines's end points(the back seal of the shell) and three ruled surfaces(the front seal of the shell). We create one ruled surface, between the pair of waterlines, for each region of these waterlines; the stern, the parallel mid section and the stem. The unity of all these shells is the vessel's hull 2.20. As the quantity of well produced waterplanes is increasing the closer we get in recreating the desired hull. As mentioned in 2.5.1 in order to create a hull we need some initial information that includes, vessel's profile, flat of sight(FOS), flat of bottom(FOB), transom, parallel mid section limits and vessel's bilge radius.



(a) Back and Upper seals **(b)** Front and Down seals

Figure 2.20: Shell's compartments



(a) Hull created by 18 waterlines **(b)** Hull created by 90 waterlines

Figure 2.21: Ship Hull

2.6 OPTIMIZATION ALGORITHMS

In this section, we will clarify how we use the optimization algorithms. The main aspect of optimization algorithms is that they select and evaluate data more efficiently through iterations. The number of iterations depends on the stop strategy that the algorithm uses.

We apply all the following algorithms in order to optimize the *waterlines* of the hull.

First of all, there are some common characteristics in all the algorithms. They are based on single objective optimizations, which means that they evaluate the result of a single function. The initial data that every algorithm uses are in section 2.5. For every iteration the optimization algorithms determine the values of the variables that are to be setted through a given mathematical space (2.6.3, 2.6.1) and/or the evaluation process (2.6.2).
The values that change are the control points of both splines of each waterline that are not boundary points. We create two different approaches on how to change the coordinates of these points (the z-coordinate can not change because is one of main characteristics of each waterline, the depth).

1. Change One Coordinate
   After the initialization(2.5), the non boundary points can change only by the y-coordinate, except the ones that define the waterline design as explained in 2.5. This means that the control points that initially setted at the same y-coordinate 2.15, 2.16, 2.17 can change only by the x-coordinate. This is a simpler approach with few variables that need to be optimized.

2. Change both Coordinates
   This approach is much more complex and difficult as the non boundary points can change in both directions with the exception of the control points originally setted at the same y-coordinate.

We will now expound the mathematical spaces that we mention. These mathematical spaces are based on the initial design of the section 2.5 and limitations of the problem.
We build the spaces that refer to the x-coordinates of the control points according to the their initial positions. Every non boundary control point is enabled to move at the space between the second half of the distance of the initial position of the previous control points and its current initial position and the first half of the distance of the initial position of the following control points and its current initial position.
We set the mathematical spaces of the y-coordinates of the control points according to the hull's breadth. This means that the y-coordinates are taking values between 0.1 meters (we choose not to use zero due to very small values) and half of the hull's breadth as this is the waterline's structure limit.
The picture below, shows the mathematical spaces for a control point.

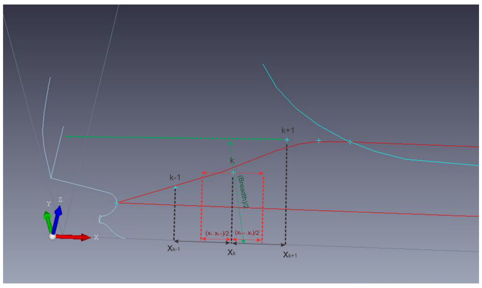


Figure 2.22: Example of the mathematical spaces for the control points

The only omission is for the waterlines of type 2 2.23. At this type of waterlines we set the first non boundary control point of the first spline (stern region) at a different mathematical space for the y-coordinate. We set the down limit to be equal to the y-coordinate of the second boundary point and the up limit to be the half of breadth (the up limit is the same as before). We set this control point in such way in order to avoid waterline shapes that do not apply in a vessel's hull 2.24 .

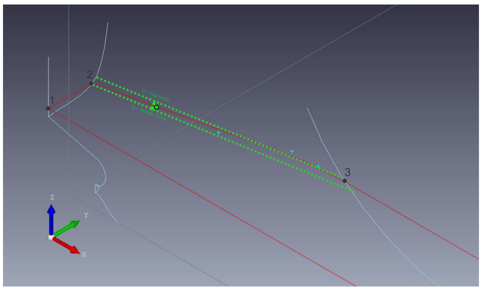


Figure 2.23: The y mathematical space for the first non boundary control point of waterline type2

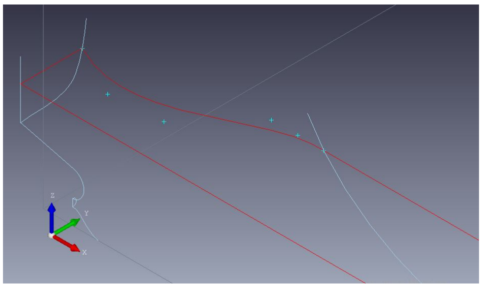


Figure 2.24: Waterline shapes that must be avoided

The first two optimization algoritms that we apply are part of the Dlib library.

2.6.1 DlibFindMinGlobal

2.6.1.1 *General Info*

This algorithm attempts to find the global minimum of a given objective function and not just a local minimum.We do not need to specify derivatives or starting positions for our control points but only the mathematical space for each variable. We also define the number of iterations for this algorithm (stop strategy).

2.6.1.2 *How it works*

The intention of this algorithm is to make as fewer function evaluations as possible and it manages to do so by preserving two different kinds of models, a global and a local model.

The global model is a non-parametric piecewise linear model that aims to construe an upper bound for the given objective function. This approach is build on the method described in lobal Optimization of Lipschitz Functions by Cédric Malherbe and Nicolas Vayatis in the 2017 International Conference on Machine Learning [22] .

The local model is a quadratic model fit around the best point until that moment and is analogous to what is proposed in: The NEWUOA software for unconstrained optimization without derivatives By M.J.D. Powell, 40th Workshop on Large Scale Nonlinear Optimization (Erice, Italy, 2004) [23].

There is a video in dlib's site that illustrates how this algorithm operates.

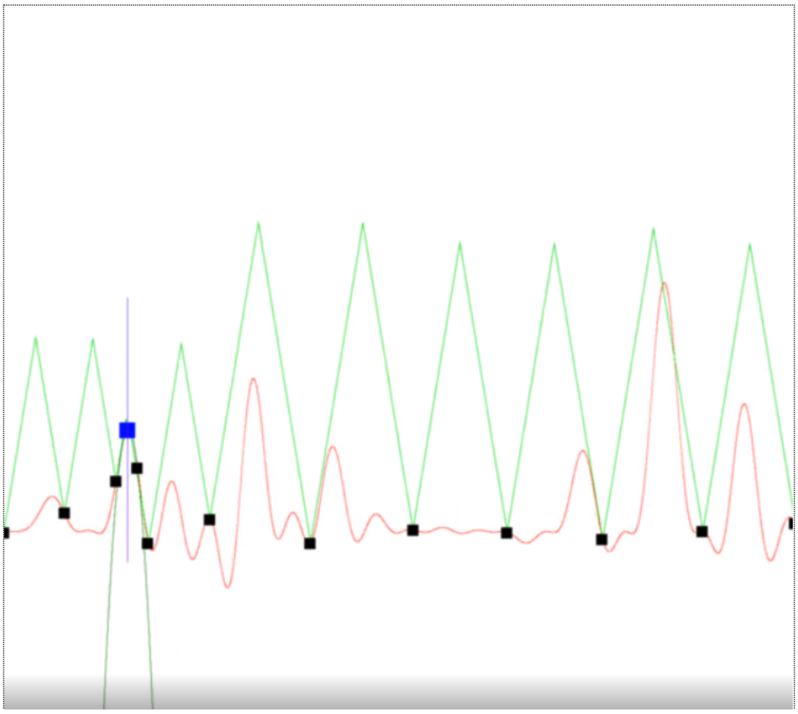


Figure 2.25: Dlib algorithm Find Min Global

Find GLobal Max

2.6.2 DlibFindMinUsingApproximateDerivatives

2.6.2.1 General Info

The find_min_using_approximate_derivatives algorithm is capable of finding the local minimum of a given objective function.

Thus, the effectiveness of find_min_using_approximate_derivatives is based on the starting values of the variables. The number of iterations of the algorithm, are determined by the stop strategy we use.

2.6.2.2 How it works

It uses an unconstrained minimization of a nonlinear function that uses a gradient based line search policy to make the objective function significantly smaller. This sort of practice is using the strong Wolfe conditions with a bracketing and then a sectioning phase, both using polynomial interpolation. The stop strategy we choose for this algorithm is the distinction between two consecutively results of the objective function. If this distinction is below a user-given value then the search ends.

2.6.3 OpenGA

2.6.3.1 General Info

This is a genetic optimizer. The approach of such algorithm differs from the aforementioned algorithms we use. The outcome of this algorithm is partly affected by the starting values and the number of iterations is depending on users threshold and distinction between the iterations.

2.6.3.2 How it works

All kind of genetic algorithms are trying to imitate the way that nature changes from generation to generation. In this genetic algorithm we choose the genes to be the position of the control points. So, in every generation, which consists of a number of different collection of genes (from now on we will call this collection of genes, population) that were produced randomly in given mathematical spaces, the algorithm evaluates every person of this population. After the evaluation, it preserves a collection of the best people in this population. In order for a person to be included in the population, the result of the objective function for this person must be less than 5000. The following step is the cross-over. The algorithm is pairing these selected people with each other in order to occur a better generation. The last phase is the mutation.The algorithm selects a random number of the produced genes and alter them at random in the given mathematical spaces. This procedure continues for as many generations as we decide. For our calculations we create populations consisting of 100 people for 30 generations. But there is an other way for the algorithm to stop. This alteration stoppage depends on how improved the result of the objective function is getting from generation to generation. If the result is not improved by a minimum selected value, the algorithm will stop.

2.6.4 Hybrid algorithms

Hybrid algorithms are the combination of the above algorithms in order to occur better results.

We used two kind of combinations.

(a) DlibFindMinGlobal - DlibFindMinUsingApproximateDerivatives

(b) OpenGA - DlibFindMinUsingApproximateDerivatives

The idea behind both combinations is the same. At first, we use an algorithm that will lead the results as close as possible to the global minimum of the objective function. Then, with well-collected variables and the use of DlibFindMinUsingApproximateDerivative algorithm we will get the desired results.

2.7 GEOMETRY EVALUATION

In order to rate the constructed geometry we created some tools. We built a series of functions that evaluate the geometrical characteristics of the hull and the constructed waterplane areas, by comparing the desired vessel's hydrostatic values acquired from the ship's hydrostatic table with the constructed geometry.

2.7.1 Objective Functions

We created two categories of objective functions. Objective functions that evaluate each waterline independently by ratting the area attributes and objective functions that rate not only the area attributes but also the volume attributes between two waterlines.

2.7.1.1 *Objective functions for Area attributes*

These kinds of objective functions rate the WPA, Ixx and LCF of each waterplane area (2.2). We provide these functions with the new positions of the waterline's control points. These new positions are a result of the optimization algorithms that we explained in the previous sector (2.6). At first, we alter the waterline according to the new control points and then calculate the area features of the waterplane area. Moreover, the actual/square differences between the attributes of the altered waterplane area and the actual waterplane area are calculated and every difference is normalized by dividing each result to the analogous attribute of the actual waterplane. Finally, the sum of these differences is the return value of the objective function.

Objective function for actual differences,

$$f(\vec{x}) = \frac{|WPA_{Computed} - WPA_{Real}|}{WPA_{Real}} + \frac{|Ixx_{Computed} - Ixx_{Real}|}{Ixx_{Real}} + \frac{|LCF_{Computed} - LCF_{Real}|}{LCF_{Real}} \tag{2.5}$$

Objective function for square differences,

$$f(\vec{x}) = \frac{(WPA_{Computed} - WPA_{Real})^2}{WPA_{Real}} + \frac{(Ixx_{Computed} - Ixx_{Real})^2}{Ixx_{Real}} + \frac{(LCF_{Computed} - LCF_{Real})^2}{LCF_{Real}} \tag{2.6}$$

where,
$\vec{x}$, is the vector with the new positions of the control points

2.7.1.2 *Objective functions for Area and volume attributes*

This kind of objective functions rate WPA, Ixx and LCB of the current waterplane area (area attributes), including ∇, CB and WSA that is created between the current waterplane area and the previous optimized waterplane area (if the current waterplane area is the first one, the volume attributes are calculated with the help of FOB 2.2). We provide these functions with the positions of the new waterline's control points. At first, we alter the waterline according to the new control points and then calculate the area features of the waterplane area and the volume attributes of the created shape. Moreover, the actual/square differences between the attributes of the altered shape and the actual hull at this depth are calculated and every difference is normalized by dividing each result with the analogous attribute of the actual waterplane area or shape. Finally, the sum of the differences is the return value of the objective function.

Objective function for actual differences,

$$\begin{aligned} f(\vec{x}) &= \frac{|WPA_{Computed} - WPA_{Real}|}{WPA_{Real}} + \frac{|I_{xx_{Computed}} - I_{xx_{Real}}|}{I_{xx_{Real}}} + \frac{|LCF_{Computed} - LCF_{Real}|}{LCF_{Real}} \\ &+ \frac{|Volume_{Computed} - Volume_{Real}|}{Volume_{Real}} + \frac{|WSA_{Computed} - WSA_{Real}|}{WSA_{Real}} + \frac{|LCB_{Computed} - LCB_{Real}|}{LCB_{Real}} \\ &+ \frac{|VCB_{Computed} - VCB_{Real}|}{VCB_{Real}} \end{aligned} \tag{2.7}$$

Objective function for square differences,

$$\begin{aligned} f(\vec{x}) &= \frac{(WPA_{Computed} - WPA_{Real})^2}{WPA_{Real}} + \frac{(I_{xx_{Computed}} - I_{xx_{Real}})^2}{I_{xx_{Real}}} + \frac{(LCF_{Computed} - LCF_{Real})^2}{LCF_{Real}} \\ &+ \frac{(Volume_{Computed} - Volume_{Real})^2}{Volume_{Real}} + \frac{(WSA_{Computed} - WSA_{Real})^2}{WSA_{Real}} + \frac{(LCB_{Computed} - LCB_{Real})^2}{LCB_{Real}} \\ &+ \frac{(VCB_{Computed} - VCB_{Real})^2}{VCB_{Real}} \end{aligned} \tag{2.8}$$

where,
$\vec{x}$, is the vector with the new positions of the control points

Additionally, we boost both of these objective function categories with some extra options so as to obtain more results in different ways and observe how these changes remodel our initial design.

2.7.1.3 *Extra options for the objective functions*

(a) **Curvature Changes**
For every waterline we have two b-spline curves. For each curve we calculate the changes of curvature. If the number of changes is bigger than 3 at the b-spline in stern region or the one at stem region then we add to the objective function's result a big number(10^8) as a punishment for these control points positioning. The number of curvature changes in most hulls is three for both stern and stem curves, so we will not accept a curve with more curvature changes. Moreover, in order to discourage curvature numbers with 2 or 3 changes we add to the result of the objective function the product of the initial result, the number of curvature changes of both curves and a normalization factor. This normalization factor is a number that its role is to not let this extra addition dominate the objective functions result, but only to affect it.

Altered objective function after curvature,

$$f(\vec{x})_{curvature} = f(\vec{x}) + NegativePunishment + k \cdot f(\vec{x}) \cdot (CurveChange1 + CurveChange2) \tag{2.9}$$

where,

NegativePunishment, is the number 10^8

$k \in (0, 1)$, is a normalization factor that shrinks this product in order to not dominate this function's results

CurveChange1 and CurveChange2 , are numbers that represent the changes in curvature for the B-splines at stern and stem regions.

(b) **Weights**

As mentioned before, the result of the objective functions is the sum of the actual/square differences between the waterplane area's actual attributes extracted by the hydrostatic table and the attributes we calculate from our design. We normalize these square differences because we want all the attributes to be at the same scale. But there is also a different approach to this problem. We can select different weights for each attribute in order to manipulate the curves.

For example, if we choose to use the objective function for the waterline attributes with the actual differences, the objective function's equation with the use of weights will be,

$$f(\vec{x}) = A \cdot \frac{|WPA_{Computed} - WPA_{Real}|}{WPA_{Real}} + B \cdot \frac{|Ixx_{Computed} - Ixx_{Real}|}{Ixx_{Real}} + C \cdot \frac{|LCF_{Computed} - LCF_{Real}|}{LCF_{Real}} \tag{2.10}$$

where,

$A, B, C \in [0, 100000]$

2.7.2 Modelling evaluation

We create the objective functions to evaluate the hydrostatic values. Although, in order to judge whether we are heading to the right direction we must also be able to evaluate the actual distances between the curves.

For that reason, we created a function to compare the calculated hull with a hull that we know its actual points a priori.

We use two metrics.

1. Square distances in waterlines
   We measure the y-distance in a range of x positions between the two waterlines, we deduct these distances and store the square of the result. Also, we add all these results for every x-position in order to obtain a total evaluation of the waterline.

2. Minimum distances in waterlines
   For the same x positions mentioned above we measure the minimum distance between the two curves and store the square of the result. For every longitudinal position we acquire a number that represents the minimum distance between these curves. Also, we store the minimum distance of its point.

The following figure 2.26 shows the difference between the two metrics.

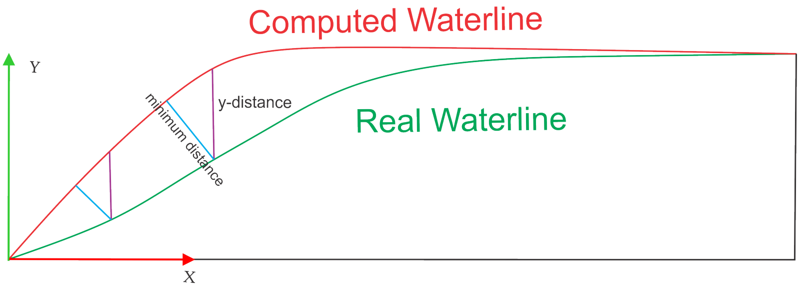


Figure 2.26: Example of the metrics

2.7.3 Selection of the proper objective function

The selection of the appropriate objective function is a really demanding process. We perform a series of tests in order to end up with one objective function. For all the following tests we use the optimization algorithm

find_min_using_approximate_derivatives as it is the only algorithm that we use in order to find local minimum.

2.7.3.1 Uniqueness

First of all, we prove that a waterline can not be fully described by setting its area, the second order inertia and the longitudinal center of its area. In other words, there are infinite waterlines that share these area attributes.

In order to prove this statement we enforce the following procedure.

Firstly, we use the waterline that created in 2.4 as the model that we want to reach its geometry. Afterwards, we create a waterline as described in 2.5.1 with one difference. Due to the special form of the polynomial waterline in the stem area, the spline we use for the stem area is the same type as the one at the stern region. The waterline's splines have degree 3 and 8 control points its spline.

Finally, in the optimization procedure we use the objective function for the area attributes as described in 2.7.1.1 with no use of either the curvature option or the weights option.

We repeat the above procedure three times by altering the initial position of the control points randomly.

The percentage difference between the waterlines attributes is less than $10^{-3}\%$ for each attribute (area, Ixx, LCF) and we consider these attributes equal to each other.

In following tables 2.3, 2.4 we can see that there are more than one waterlines with the same area attributes 2.5 and in the image 2.27 we observe that these waterlines have different shapes.

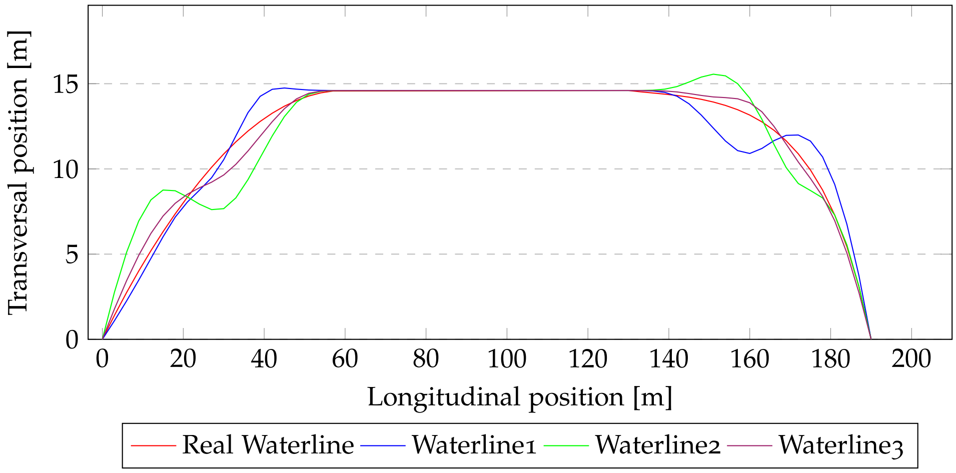


Figure 2.27: Waterlines with same area attributes

Table 2.3: Differences in area attributes - Waterline l

Waterline 1			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4586.858	4586.858	0.000
Inertia $[m^4]$	278255.218	278255.597	0.000
LCF [m]	98.520	98.521	0.000

Table 2.4: Differences in area attributes - Waterline 2

Waterline 2			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4586.858	4586.860	0.000
Inertia $[m^4]$	278255.218	278255.252	0.000
LCF [m]	98.520	98.519	0.000

Table 2.5: Differences in area attributes - Waterline 3

Waterline 3			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4586.858	4586.852	0.000
Inertia $[m^4]$	278255.218	278255.316	0.000
LCF [m]	98.520	98.520	0.000

2.7.3.2 *Weight&Initialization Testing*

After the conclusion of non uniqueness 2.7.3.1 we test the objective function for area attributes with the weight option 2.7.1.3. In order to test the impact of the weights we created particular sequences of the variables A,B and C that represent the weight for the waterplane area, the second order inertia and the longitudinal centre of area respectively. These sequences are all the possible combinations of the numbers 1, 10, 100, 1000, 10000, 100000 in trinities. The reason for this approach is to examine the relation between the attributes and how this could affect the final form of the waterlines. We use the metric of minimum distances 2.7.2 to evaluate the result of its combination.

For these tests we use the polynomial model as described in 2.7.2 with 6 control points of degree 5 for each spline. Also, we choose to use the objective function for actual differences. Finally, we perform these tests many times for different initial control points in order to examine the importance of the initialization.

In the two following tables 2.6, 2.7 we present some of the results as there is no point in displaying the results in total. In both tables we have included the trinity with the best result and highlighted. The difference between these tables is that the initial control points are different 2.28. In the first approach, the control points are closer to the desired design while in the second one they are not so close.

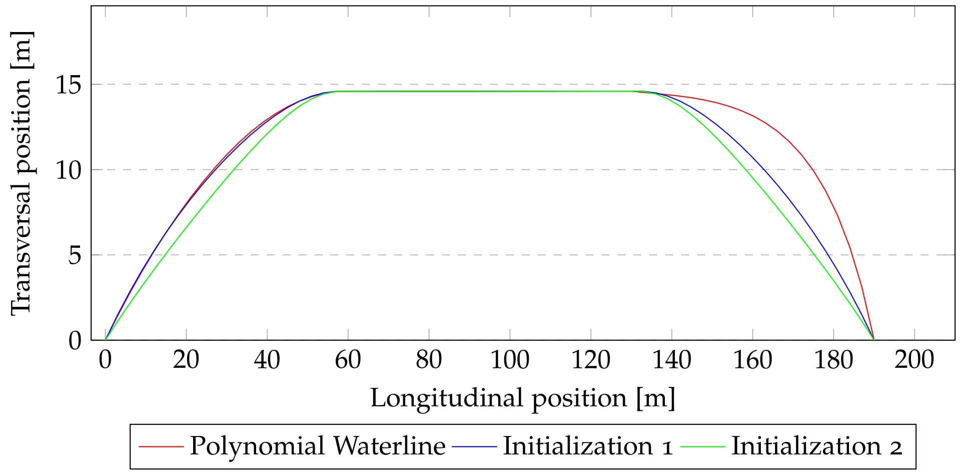


Figure 2.28: Comparison of different initialisations

Table 2.6: Objective function's behaviour with weights Initialization 1

A	**B**	**C**	**Sum of Square Minimum Distances [m^2]**
10000	100000	10000	0.533992
1	1000	10000	5.72001
10000	100	10000	2.70227
100000	1000	100000	2.62452
100000	1000	1000	1.48763
1	1000	1	46.9744
100	100000	10	44.8244
100000	1	100	2.60368
100	10	1	44.7921
10000	1	10000	2.99995
100000	10	1	43.6721
1000	10	10	44.6219
100000	10000	1000	0.357639
100000	1	1	45.2572
10	10000	10000	4.30862
100	1	10	38.9659
1000	100000	100000	5.68345
100	10	1000	37.9454
1	100000	100000	5.62299

Table 2.7: Objective function's behaviour with weights Initialization 2

A	B	C	Sum of Square Minimum Distances [m^2]
100000	100000	10000	26.501
1	1000	10000	105.738
10000	100	10000	2.25535
100000	1000	100000	2.35003
100	10000	100	0.8988
1	10	100000	28.7949
1	10000	10000	7.50583
1000	1000	1000	7.31867
100	10	100	2.1387
100000	10000	100	44.0368
1000	1000	100000	13.6209
1	10	1	39.6273
10	1000	10000	105.423
1000	10000	1000	0.359759
100000	10	100000	2.6798
10000	1000	1	45.1671
100000	1000	10	44.9854
100	10	1000	133.3
1000	100	10000	2.41391

As we can see in the tables, for each initialization we have different best trinities.

In the initialization where we are closer to the final design, the attribute that is boosted the most is the computed area attribute.

In the initialization where we are not so close to the desired form the attribute that is emphasized by the optimizer is the second order inertia.

We find the results pretty logical. When the initialization is closer to the solution the optimization process is focusing on the attributes that have the least impact to the shape of the curves. On the other hand, as we diverge from the solution the optimizer will make bigger and bigger changes to the the shape and the most important attribute that responds to these changes is the second order inertia.

To sum up, for this optimization process that we have chosen the weights can not help us solve this problem so we will not use them at all. Although, the initialization of the control points is really important and has a great impact to our optimization problem.

2.7.3.3 *Curvature&Stop Strategy Testing*

At this point we will examine the objective functions (actual/square differences) behaviour for the curvature option and how the stop strategy's accuracy that the optimization algorithm uses can affect the final design. We will perform these tests for the polynomial waterline 2.4 and for waterlines with more than one curvature changes by using waterlines from a real ship 2.29, 2.30. We choose to test waterlines of different shapes in order to gain better results. We use 8 waterlines at depths 0.1, 1, 3, 5, 7, 9, 12, 15.6 m.

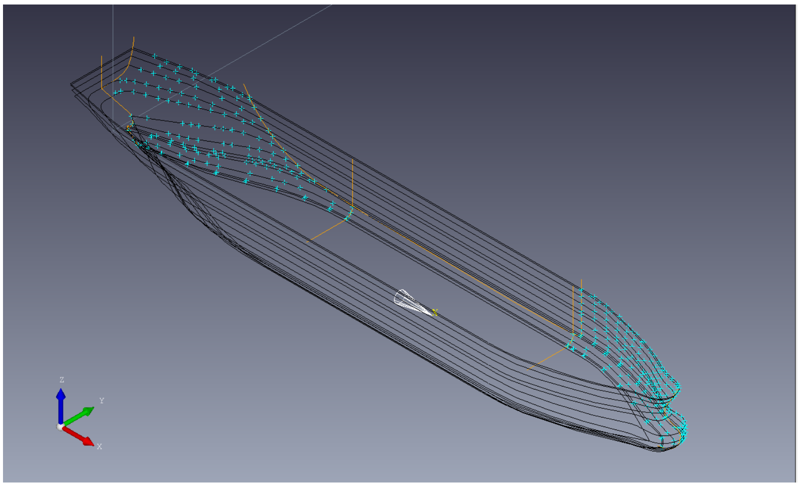
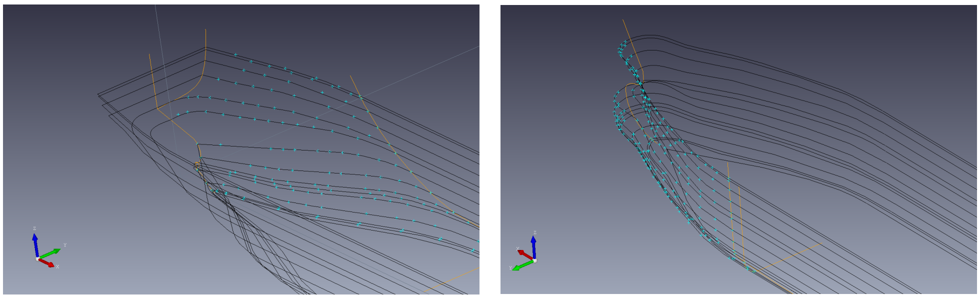


Figure 2.29: Aveva Model - 8 waterlines



(a) Stern View

(b) Bow View

Figure 2.30: fig:Aveva Model 8 waterlines -Views

In 2.7.1.3 we mentioned how we apply the curvature option in our objective function. In the testing process that follows, we try three different values for the normalization factor k. These values are 0, 0.01 and 0.1 .

As mentioned in 2.6.2 the stop strategy we choose for the algorithm is the distinction between two consecutively results of the objective function. If this distinction is below a user given value then the search ends. We will test seven values as user values, $10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}$.

For all the tests we create waterlines as described in 2.5.1. The waterlines splines have degree 5 and 6 control points.

Firstly, we present the results for the **polynomial model**.

Objective function for actual differences,

$$f(\vec{x}) = \frac{|WPA_{Computed} - WPA_{Real}|}{WPA_{Real}} + \frac{|I_{xx_{Computed}} - I_{xx_{Real}}|}{I_{xx_{Real}}} + \frac{|LCF_{Computed} - LCF_{Real}|}{LCF_{Real}} \tag{2.11}$$

Table 2.8: Polynomial Model - Objective function with actual differences - Curvature factor k = 0

Terminate Value	Sum of Square Minimum Distances $[m^2]$
10^{-1}	5.41434
10^{-2}	5.41434
10^{-3}	5.41434
10^{-4}	5.41434
10^{-5}	5.41434
10^{-6}	5.41434
10^{-7}	5.41434

Table 2.9: Polynomial Model - Objective function with actual differences - Curvature factor k = 0.01

Terminate Value	Sum of Square Minimum Distances $[m^2]$
10^{-1}	5.3601
10^{-2}	5.3601
10^{-3}	5.3601
10^{-4}	5.3601
10^{-5}	5.3601
10^{-6}	5.3601
10^{-7}	5.3601

Table 2.10: Polynomial Model - Objective function with actual differences - Curvature factor k = 0.1

Terminate Value	Sum of Square Minimum Distances $[m^2]$
10^{-1}	76.083
10^{-2}	76.0824
10^{-3}	76.0824
10^{-4}	76.0824
10^{-5}	76.0824
10^{-6}	76.0824
10^{-7}	76.0824

Objective function for square differences,

$$f(\vec{x}) = \frac{(WPA_{Computed} - WPA_{Real})^2}{WPA_{Real}} + \frac{(I_{xx_{Computed}} - I_{xx_{Real}})^2}{I_{xx_{Real}}} + \frac{(LCF_{Computed} - LCF_{Real})^2}{LCF_{Real}} \tag{2.12}$$

Table 2.11: Polynomial Model - Objective function with square differences - Curvature factor k = 0

Terminate Value	Sum of Square Minimum Distances $[m^2]$
10^{-1}	23.7249
10^{-2}	23.7155
10^{-3}	23.7155
10^{-4}	23.7155
10^{-5}	15.2759
10^{-6}	3.69427
10^{-7}	3.69813

Table 2.12: Polynomial Model - Objective function with square differences - Curvature factor k = 0.01

Terminate Value	Sum of Square Minimum Distances [m^2]
10^{-1}	23.7221
10^{-2}	23.7128
10^{-3}	23.7128
10^{-4}	23.7128
10^{-5}	15.269
10^{-6}	3.70068
10^{-7}	3.69789

Table 2.13: Polynomial Model - Objective function with square differences - Curvature factor k = 0.1

Terminate Value	Sum of Square Minimum Distances [m^2]
10^{-1}	23.6967
10^{-2}	23.6873
10^{-3}	23.6873
10^{-4}	23.6873
10^{-5}	15.276
10^{-6}	3.69769
10^{-7}	3.69917

From the tables above 2.8, 2.9, 2.10, 2.11, 2.12, 2.13 we can see that the best results for the polynomial model are from the objective function of square differences with a terminate value 10^{-6}. The curvature option in these tests do not have any impact as the polynomial model do not have any curvature changes.

Secondly, we present the results for the **Aveva model** for the 8 different waterlines.

Objective function for actual differences,

$$f(\vec{x}) = \frac{|WPA_{Computed} - WPA_{Real}|}{WPA_{Real}} + \frac{|Ixx_{Computed} - Ixx_{Real}|}{Ixx_{Real}} + \frac{|LCF_{Computed} - LCF_{Real}|}{LCF_{Real}} \tag{2.13}$$

Table 2.14: Aveva Model - Objective function with actual differences - Curvature factor k = 0

Terminate Value	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
Waterline Depth [m]	Square Y-Difference [m^2]						
0.1	11.161	8.244	2.519	2.382	2.382	2.381	2.381
1	24.527	24.527	22.599	7.76	7.76	7.76	7.76
3	13.877	10.961	10.958	10.958	10.958	10.958	10.958
5	6.679	3.689	3.689	3.689	3.689	3.689	3.689
7	49.1	48.858	45.304	4.696	4.696	4.696	4.696
9	126.23	8.032	8.032	6.26	6.268	6.268	6.268
12	36.832	36.832	2.531	2.531	2.531	2.531	2.531
15.6	17.7	1.479	1.479	0.936	0.928	0.928	0.929

Table 2.15: Aveva Model - Objective function with actual differences - Curvature factor k = 0.01

Terminate Value	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
Waterline Depth $[m]$	Sum of Square Minimum Distances$[m^2]$						
0.1	8.833	5.541	5.605	5.605	2.381	2.381	2.381
1	24.527	24.527	22.618	7.781	7.781	7.781	7.781
3	13.807	10.889	10.889	10.889	10.889	10.912	10.912
5	6.62	3.669	3.669	3.669	3.669	3.669	3.669
7	49.015	48.759	4.692	4.692	4.692	4.692	4.692
9	127.583	7.82	7.82	6.268	6.278	6.281	6.281
12	36.724	36.724	2.532	2.532	2.532	2.532	2.532
15.6	18.264	14.33	1.101	0.917	0.929	0.929	0.929

Table 2.16: Aveva Model - Objective function with actual differences - Curvature factor k = 0.1

Terminate Value	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
Waterline Depth $[m]$	Sum of Square Minimum Distances$[m^2]$						
0.1	7.916	4.702	4.702	4.702	4.702	4.702	4.702
1	24.527	24.527	22.603	7.754	7.754	7.754	7.754
3	13.211	10.349	10.437	10.482	10.482	10.482	10.482
5	6.161	6.298	6.298	3.882	3.882	3.882	3.882
7	48.276	47.798	4.717	4.717	4.717	4.717	4.717
9	160.91	160.91	129.7	129.994	129.994	129.994	129.994
12	36.609	36.609	2.843	1.702	1.702	1.702	1.702
15.6	17.181	1.197	1.197	0.945	0.945	0.929	0.929

Objective function for square differences,

$$f(\vec{x}) = \frac{(WPA_{Computed} - WPA_{Real})^2}{WPA_{Real}} + \frac{(I_{xx_{Computed}} - I_{xx_{Real}})^2}{I_{xx_{Real}}} + \frac{(LCF_{Computed} - LCF_{Real})^2}{LCF_{Real}} \tag{2.14}$$

Table 2.17: Aveva Model - Objective function with square differences - Curvature factor k = 0

Terminate Value	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
Waterline Depth $[m]$	Sum of Square Minimum Distances$[m^2]$						
0.1	14.846	14.846	21.79	21.83	2.358	2.358	2.358
1	27.377	27.377	27.377	35.393	3.602	3.598	3.598
3	17.018	17.013	17.013	22.04	22.04	5.709	5.71
5	9.202	9.202	9.207	10.334	3.116	3.116	3.116
7	54.647	54.631	54.631	54.631	4.502	4.502	4.503
9	161.234	161.234	42.416	39.179	6.535	6.535	6.827
12	37.824	37.685	0.694	0.676	0.676	0.676	0.676
15.6	23.178	23.162	23.162	1.464	1.464	1.465	1.465

Table 2.18: Aveva Model - Objective function with square differences - Curvature factor k = 0.01

Terminate Value	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
Waterline Depth [m]	Sum of Square Minimum Distances[m^2]						
0.1	14.846	14.846	21.79	21.83	2.358	2.358	2.358
1	27.418	27.418	35.406	35.441	3.598	3.598	3.599
3	17.019	17.013	17.013	22.04	22.04	5.709	5.71
5	9.202	9.202	9.207	10.334	3.117	3.117	3.117
7	54.647	54.632	54.632	54.632	4.497	4.503	4.503
9	161.376	161.376	42.925	40.168	6.561	6.802	6.813
12	37.822	37.685	0.693	0.676	0.676	0.676	0.676
15.6	23.178	23.162	23.162	1.47	1.465	1.465	1.465

Table 2.19: Aveva Model - Objective function with square differences - Curvature factor k = 0.1

Terminate Value	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
Waterline Depth [m]	Sum of Square Minimum Distances[m^2]						
0.1	14.846	14.846	21.79	21.831	2.358	2.358	2.358
1	27.377	27.377	27.377	35.395	3.595	3.598	3.598
3	17.019	17.013	17.013	22.041	22.041	5.709	5.709
5	9.202	9.202	9.207	10.334	3.117	3.117	3.117
7	54.648	54.633	54.633	54.633	4.495	4.501	4.503
9	162.067	160.695	41.359	40.721	6.442	6.797	6.806
12	37.915	37.915	37.915	0.678	0.677	0.677	0.677
15.6	23.178	23.162	23.162	1.469	1.465	1.465	1.465

From the tables above 2.14, 2.15, 2.16, 2.17, 2.18, 2.19 we can see that this problem is a bit more complex than the previous. Again, the best results are from the objective function of square differences. Furthermore, in all examined cases the terminate value does not need to be smaller than 10^{-6} to get the best results. But in order to be on the safe side we will choose as terminate value 10^{-7}. As for the curvature option, there is almost no difference between the results but there is a difference in the execution time. The faster results are produced by the objective function with the curvature k = 0.01.

Finally, the objective function we choose based on the previous analysis is the following equation,

$$f(\vec{x})_{curvature} = f(\vec{x}) + NegativePunishment + 0.01 \cdot f(\vec{x}) \cdot (CurveChange1 + CurveChange2) \tag{2.15}$$

where,

$$f(\vec{x}) = \frac{(WPA_{Computed} - WPA_{Real})^2}{WPA_{Real}} + \frac{(I_{xx_{Computed}} - I_{xx_{Real}})^2}{I_{xx_{Real}}} + \frac{(LCF_{Computed} - LCF_{Real})^2}{LCF_{Real}}$$

3 RESULTS

At this chapter, we present the results of a number of test cases. We created a categorization of testing in order to get as much trustworthy results as possible. The main categories we created are two. One category where we are trying to regenerate models created by our own construction and a second category with a vessel created by the program Aveva Marine. By this way, we escalate the adversity and leading to not treacherous results. The rest of the categories pertain to the objective functions and the optimization algorithms we already mentioned.

3.1 INITIAL MODEL

In order to create models through our program, we needed some initial data. So we used the boundary curves of a real ship. After that, we alter the initial control points that our program creates,by changing their y-coordinates, so as to build a different model. We calculate the hydrostatic values for that model and finally we try to regenerate this model through our optimization method.

We create a model where the original y-coordinates that our program created are decreased by 20%. We make an exception for the splines that are attached to the transom. In order to avoid creating a shape that will not comply with the common waterline shapes of a vessel, the points of the splines that are attached to the transom are reduced only by 4%.

3.1.1 Optimization Characteristics

For all the optimization procedures to reverse engineer this model we generate geometries with the following characteristics

1. Stern B-spline
   The degree of the spline is 5 with 6 control points
2. Stem B-spline
   The degree of the spline is 5 with 6 control points

This combination for the number of control points and the degree number for the splines was chosen after numerous of tests. For combinations with smaller number of control points and degrees the results were not as good as the selected and for bigger numbers of control points or degrees the optimization process was not only really slow but also the accuracy improvement was almost zero.

3.1.2 Vessel's Characteristics

We create the model based on the initial data bellow. We create 19 waterlines, which means one waterline every 1m and one extra in the maximum depth.

- Length : 181.45m
- Breadth : 29.2 m

- Depth : 15.6 m
- Radius : 2.523 m
- Parallel Midbody extent : 60.585 m - 129.825 m

We will present the main features of the vessel with the following figures 3.30, 3.31, 3.32, 3.33, 3.34.

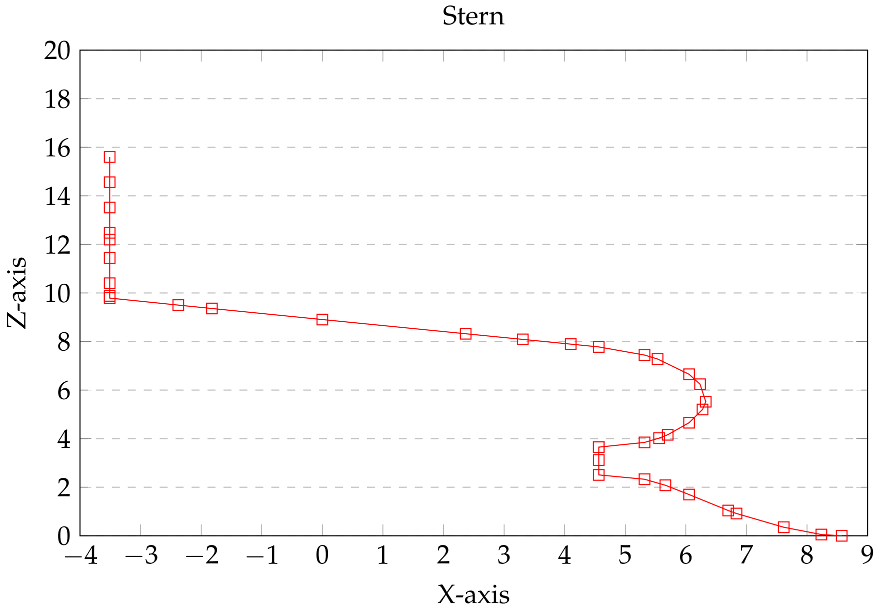


Figure 3.1: Profile - Stem

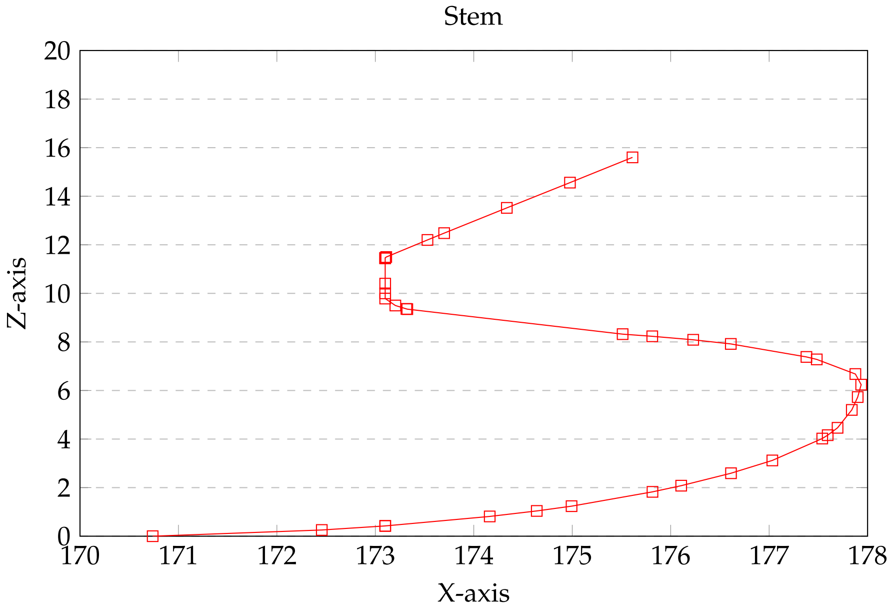


Figure 3.2: Profile - Stern

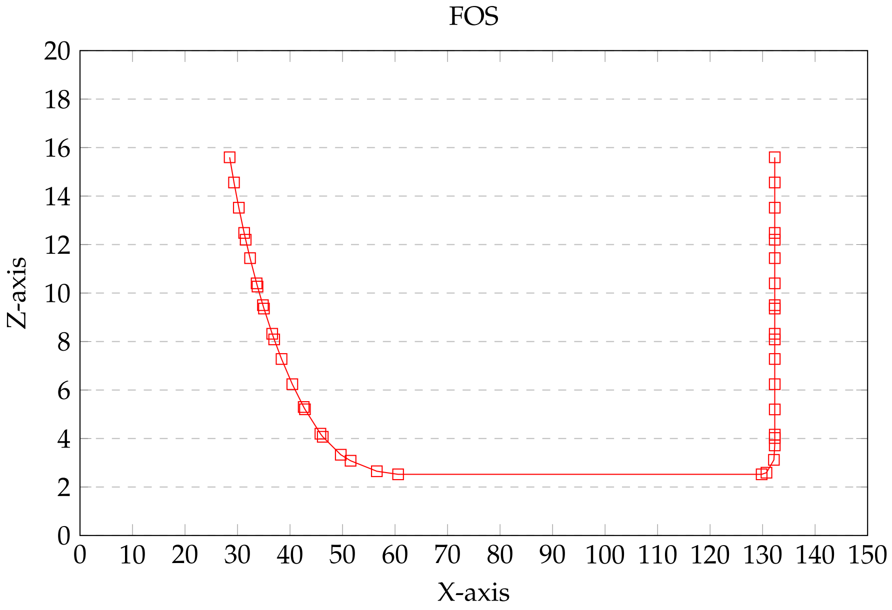


Figure 3.3: Flat of sight

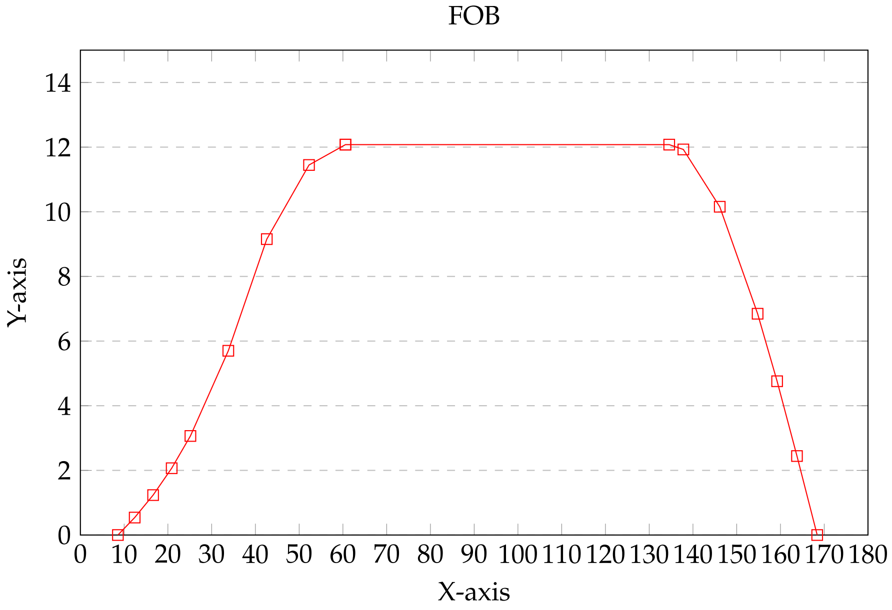


Figure 3.4: Flat of bottom

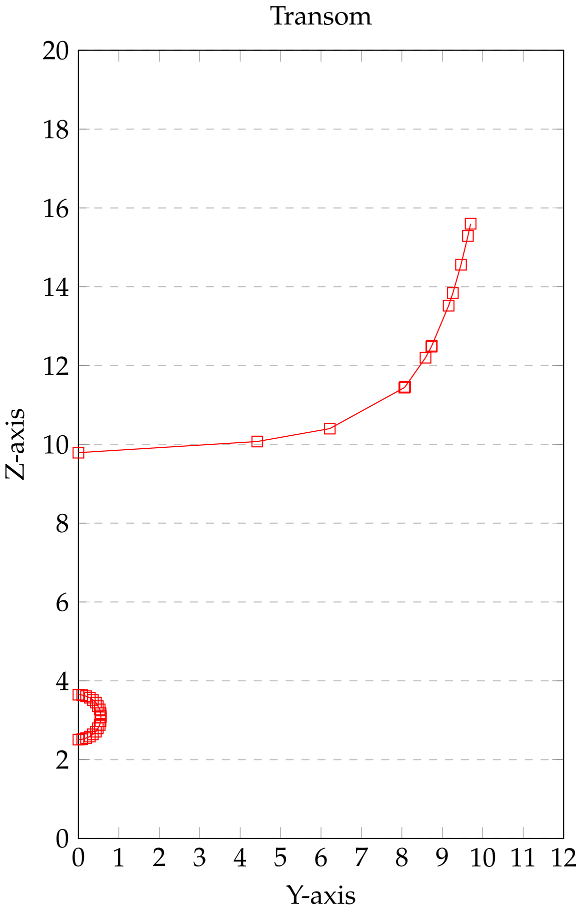


Figure 3.5: Transom

3.1.3 Results using DlibFindMinUsingApproximateDerivatives

This optimization algorithm produce good results if the initialization of the control points is close enough. Otherwise, the differences between the desired and the produced geometry will be very different in terms of shape but with similar area/volume attributes.

3.1.3.1 Objective function for area attributes

Firstly, we present the differences in the attributes between the waterlines that our optimizer calculated and the model's waterlines. We present here only part of the results 3.1. The entire data appears at Appendix A.

Table 3.1: Differences in attribute - Approximate.Derivatives - ObjFunc.Area - Model

Waterline 1.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	3638.650	3638.650	0.000
Inertia [m^4]	200695.437	200695.437	0.000
LCF [m]	96.473	96.473	0.000
Volume [m^3]	3286.598	3286.465	0.004
WSA [m^2]	3725.620	3725.822	0.005
LCB [m]	95.392	95.388	0.004
KB [m]	0.517	0.517	0.000

Waterline 3.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4040.104	4040.103	0.000
Inertia [m^4]	243814.578	243814.584	0.000
LCF [m]	94.297	94.297	0.000
Volume [m^3]	10913.262	10913.075	0.001
WSA [m^2]	4591.129	4591.100	0.000
LCB [m]	95.934	95.933	0.001
KB [m]	1.565	1.565	0.000

Waterline 5.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4103.510	4103.665	0.003
Inertia [m^4]	253854.937	253854.781	0.000
LCF [m]	94.149	94.300	0.160
Volume [m^3]	19034.183	19034.241	0.000
WSA [m^2]	5336.434	5335.612	0.015
LCB [m]	95.333	95.348	0.015
KB [m]	2.605	2.605	0.001

Waterline 10.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4403.614	4403.614	0.000
Inertia [m^4]	274006.906	274006.906	0.000
LCF [m]	85.496	85.496	0.000
Volume [m^3]	39943.433	39944.097	0.001
WSA [m^2]	7369.131	7372.643	0.047
LCB [m]	93.398	93.441	0.046
KB [m]	5.177	5.177	0.000

Waterline 14.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4665.364	4665.085	0.005
Inertia [m^4]	295876.375	295876.531	0.000
LCF [m]	82.221	82.319	0.119
Volume [m^3]	56619.175	56618.304	0.001
WSA [m^2]	8952.595	8951.434	0.012
LCB [m]	92.384	92.457	0.079
KB [m]	7.210	7.210	0.001

Waterline 15.600 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4702.847	4702.624	0.004
Inertia [m^4]	299220.500	299220.623	0.000
LCF [m]	82.311	82.390	0.096
Volume [m^3]	60792.464	60791.335	0.001
WSA [m^2]	9321.041	9319.928	0.011
LCB [m]	92.267	92.343	0.082
KB [m]	7.719	7.719	0.001

We observe that there is little to no actual difference between the real and the computed waterline attributes. At this point, we must highlight that the objective function we use is minimizing the attributes Area, Inertia and LCF.
Then, we present the differences between the actual points of the waterlines with the use of the two metrics, square y-distances 3.2 and minimum distances 3.3. For these metrics we use 40 points for each waterline. These points are equally distributed across the longitudinal axis, excluding the PMB, as these differences are always zero.

As, we can see in table 3.3 the maximum distances are between 3cm - 20cm for all the waterlines.
The mean error of minimum distances between the two curves is at 3.1 cm with the maximum error at 20 cm.

Table 3.2: Square y-Difference - Approximate.Derivatives - ObjFunc.Area - Model

Waterline.Depth [m]	Square.y-Difference $[m^2]$	Position.of.Max.Vertical.Distance [m]	Max.Vertical.Distance [m]
1.000	0.010	12.127	0.034
2.000	0.013	176.018	0.046
3.000	0.006	176.945	0.071
4.000	0.011	9.630	0.039
5.000	0.160	168.712	0.110
6.000	0.140	168.803	0.097
7.000	0.013	9.120	0.041
8.000	0.159	167.614	0.104
9.000	0.024	173.739	0.079
10.000	0.015	173.100	0.082
11.000	0.493	5.592	0.205
12.000	0.268	5.309	0.154
13.000	0.181	5.053	0.129
14.000	0.139	4.820	0.113
15.000	0.112	4.609	0.101
15.600	0.105	4.492	0.096

Table 3.3: Minimum Distances - Approximate.Derivatives - ObjFunc.Area - Model

Waterline.Depth [m]	Sum.Square.Min.Distance $[m^2]$	Position.of.Max.Distance [m]	Max.Distance [m]
1.000	0.009	12.134	0.033
2.000	0.010	11.242	0.035
3.000	0.001	174.692	0.015
4.000	0.009	9.641	0.038
5.000	0.136	168.749	0.103
6.000	0.114	168.836	0.091
7.000	0.010	9.133	0.039
8.000	0.132	167.649	0.097
9.000	0.009	3.224	0.038
10.000	0.001	160.869	0.013
11.000	0.447	5.633	0.200
12.000	0.246	5.337	0.152
13.000	0.171	5.075	0.127
14.000	0.130	4.839	0.111
15.000	0.106	4.626	0.100
15.600	0.095	4.507	0.095

Furthermore, as we can see in table 3.3 , there are two areas that the maximum distance appears. The first one is at the fore of the waterline between 4m - 13m and the second one is at the aft of the waterline between 160 m - 175 m(bulb area).The accuracy is extremely good and the parts with the biggest errors are the ones that are characterized by abrupt change in curvature.
In the following figures are presented the differences between the waterlines 3.43, 3.44 , we plot the computed waterlines and the model's waterlines in the same diagram. The two curves are very close to each other and only in waterlines at depths 14.000m and 15.000m in the stern region we spot two different curves.

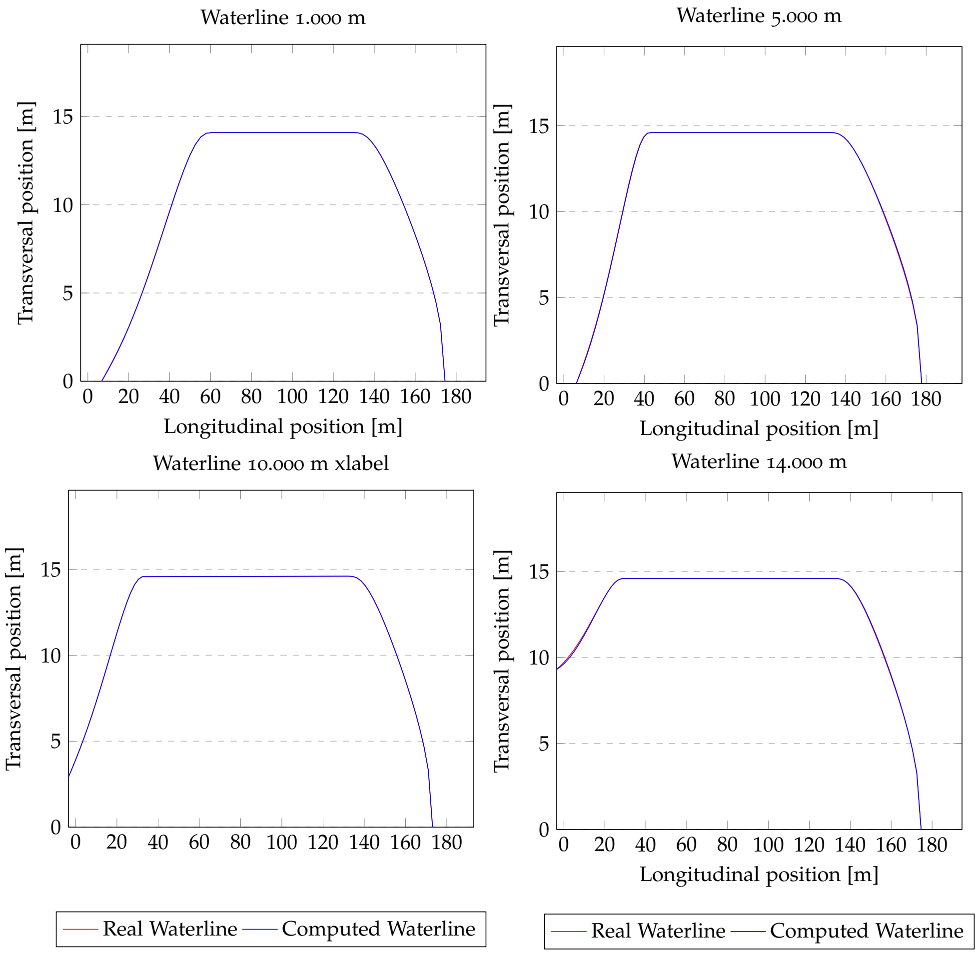


Figure 3.6: Waterlines- Computed vs Real - Approximate.Derivatives - ObjFunc.Area - Model

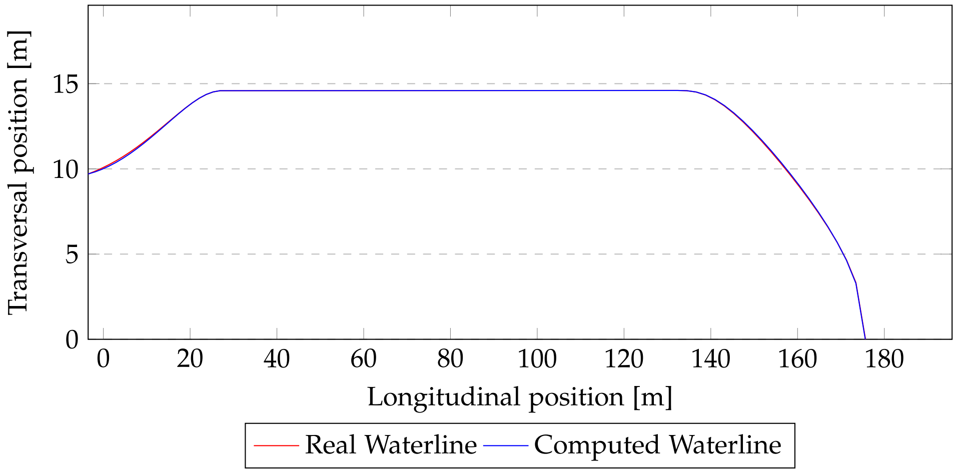


Figure 3.7: Waterline 15.600 m - Computed vs Real - Approximate.Derivatives - ObjFunc.Area - Model

Additionally, we plot the following sections 3.45. The approximation between the sections is also good but not as good as the waterlines. This occurs because in the optimization process we deal only with the waterlines and the sections are the produced transverse sections of the hull. So, their smoothness is depending on the waterlines approximation. As we mentioned before, there are some waterlines where the distance between the real and the computed curve reaches 20cm and this diversion is spotted clearly in the traversed sections.

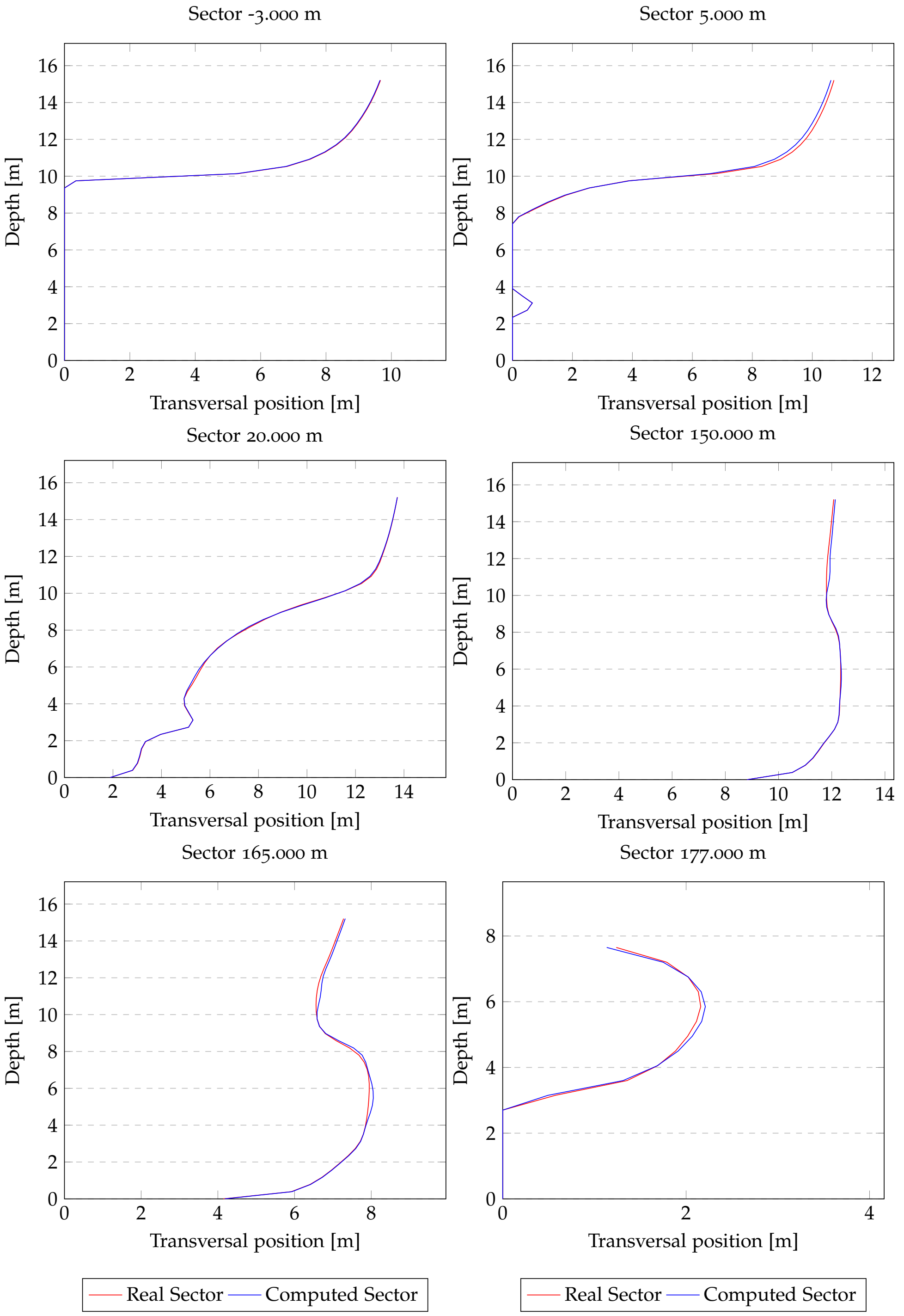


Figure 3.8: Sectors - Computed vs Real - Approximate.Derivatives - ObjFunc.Area - Model

3.1.3.2 Objective function for volume attributes

In the following table 3.4 we can see the differences in the attributes between the waterlines that our optimizer calculated for the objective function that includes the volume attributes and the model's waterlines. The difference between them is almost zero, although as the depth increases the differences are becoming relatively bigger. This result is only logical. Because of the volume attributes calculations, the error that may occur in any depth is added to the total error. So as the depth increases the error is transferred to the bigger waterlines. We present here only part of the results, the entire data appears at Appendix A.

Table 3.4: Differences in attribute - Approximate.Derivatives - ObjFunc.Volume - Model

Waterline 1.000 m

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3638.650	3638.629	0.000
Inertia $[m^4]$	200695.437	200695.402	0.000
LCF [m]	96.473	96.463	0.011
Volume $[m^3]$	3286.598	3286.464	0.004
WSA $[m^2]$	3725.620	3725.727	0.002
LCB [m]	95.392	95.382	0.010
KB [m]	0.517	0.517	0.000

Waterline 3.000 m

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4040.104	4040.142	0.000
Inertia $[m^4]$	243814.578	243814.640	0.000
LCF [m]	94.297	94.296	0.001
Volume $[m^3]$	10913.262	10913.078	0.001
WSA $[m^2]$	4591.129	4591.123	0.000
LCB [m]	95.934	95.926	0.008
KB [m]	1.565	1.565	0.000

Waterline 5.000 m

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4103.510	4103.510	0.000
Inertia $[m^4]$	253854.937	253854.794	0.000
LCF [m]	94.149	94.172	0.024
Volume $[m^3]$	19034.183	19034.166	0.000
WSA $[m^2]$	5336.434	5336.557	0.002
LCB [m]	95.333	95.340	0.007
KB [m]	2.605	2.605	0.000

Waterline 10.000 m

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4403.614	4403.591	0.000
Inertia $[m^4]$	274006.906	274023.388	0.006
LCF [m]	85.496	85.511	0.018
Volume $[m^3]$	39943.433	39943.666	0.000
WSA $[m^2]$	7369.131	7369.175	0.000
LCB [m]	93.398	93.410	0.013
KB [m]	5.177	5.177	0.000

Waterline 14.000 m

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4665.364	4665.656	0.006
Inertia $[m^4]$	295876.375	296364.808	0.164
LCF [m]	82.221	82.105	0.140
Volume $[m^3]$	56619.175	56619.677	0.000
WSA $[m^2]$	8952.595	8952.919	0.003
LCB [m]	92.384	92.384	0.000
KB [m]	7.210	7.210	0.000

Waterline 15.600 m

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4702.847	4703.142	0.006
Inertia $[m^4]$	299220.500	299774.985	0.184
LCF [m]	82.311	82.197	0.137
Volume $[m^3]$	60792.464	60793.299	0.001
WSA $[m^2]$	9321.041	9321.292	0.002
LCB [m]	92.267	92.258	0.010
KB [m]	7.719	7.719	0.001

In the next two tables we present the differences between the actual points of the waterlines with, square y-distances 3.5 and minimum distances 3.6.
In the table 3.6 the maximum distances are between 2cm - 15cm for the waterlines. The biggest distance appears at the last waterline(depth = 15.6 m) as the error at this waterline is affected by the all the other waterlines due to the objective function that includes the volume attributes.
The mean error of minimum distances between the two curves is at 2cm with the maximum error at 16cm.

Table 3.5: Square y-Difference - Approximate.Derivatives - ObjFunc.Volume - Model

Waterline.Depth [m]	Square.y-Difference $[m^2]$	Position.of.Max.Vertical.Distance [m]	Max.Vertical.Distance [m]
1.000	0.008	33.663	0.030
2.000	0.012	176.018	0.046
3.000	0.013	176.945	0.071
4.000	0.017	11.678	0.042
5.000	0.019	11.787	0.054
6.000	0.018	177.925	0.067
7.000	0.012	9.120	0.044
8.000	0.010	7.004	0.043
9.000	0.024	173.739	0.079
10.000	0.018	173.100	0.082
11.000	0.076	171.058	0.098
12.000	0.146	171.346	0.143
13.000	0.131	171.929	0.158
14.000	0.195	172.516	0.165
15.000	0.265	173.100	0.169
15.600	0.225	173.447	0.187

Table 3.6: Minimum Distances - Approximate.Derivatives - ObjFunc.Volume - Model

Waterline.Depth [m]	Sum.Square.Min.Distance $[m^2]$	Position.of.Max.Distance [m]	Max.Distance [m]
1.000	0.007	33.654	0.028
2.000	0.008	33.157	0.031
3.000	0.007	11.733	0.021
4.000	0.015	11.691	0.040
5.000	0.016	11.804	0.051
6.000	0.012	9.778	0.046
7.000	0.009	9.134	0.041
8.000	0.009	7.018	0.040
9.000	0.010	3.225	0.040
10.000	0.004	160.872	0.022
11.000	0.054	171.010	0.077
12.000	0.122	171.277	0.113
13.000	0.110	169.782	0.127
14.000	0.167	170.333	0.140
15.000	0.234	170.884	0.147
15.600	0.189	171.208	0.157

The following figures 3.43, 3.44 are some of the optimized waterlines along with the waterlines of the model. As we can see the results are very good and they seem to better that the optimization process where we did not use the volume attributes.

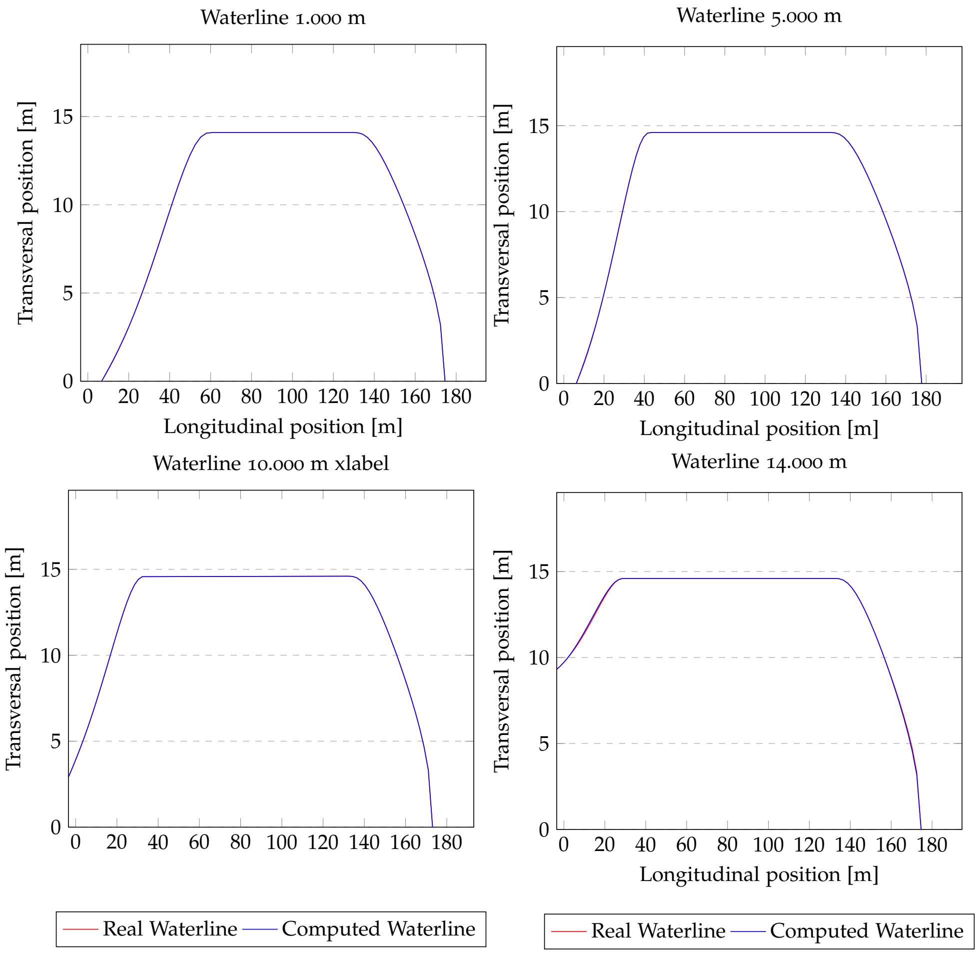


Figure 3.9: Waterlines- Computed vs Real - Approximate.Derivatives - ObjFunc.Volume - Model

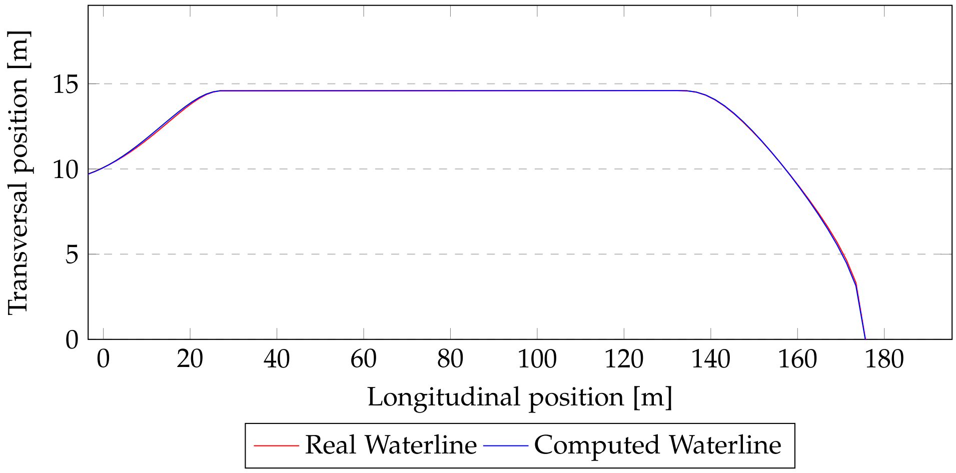


Figure 3.10: Waterline 15.600 m - Computed vs Real -Approximate.Derivatives - ObjFunc.Volume - Model

In the figures below we present some sectors of both the computed hull and the model's hull 3.45. Again, the sectors are better than the optimization process without the volume attributes with the only easy spotted difference at the sector 165.000 m.

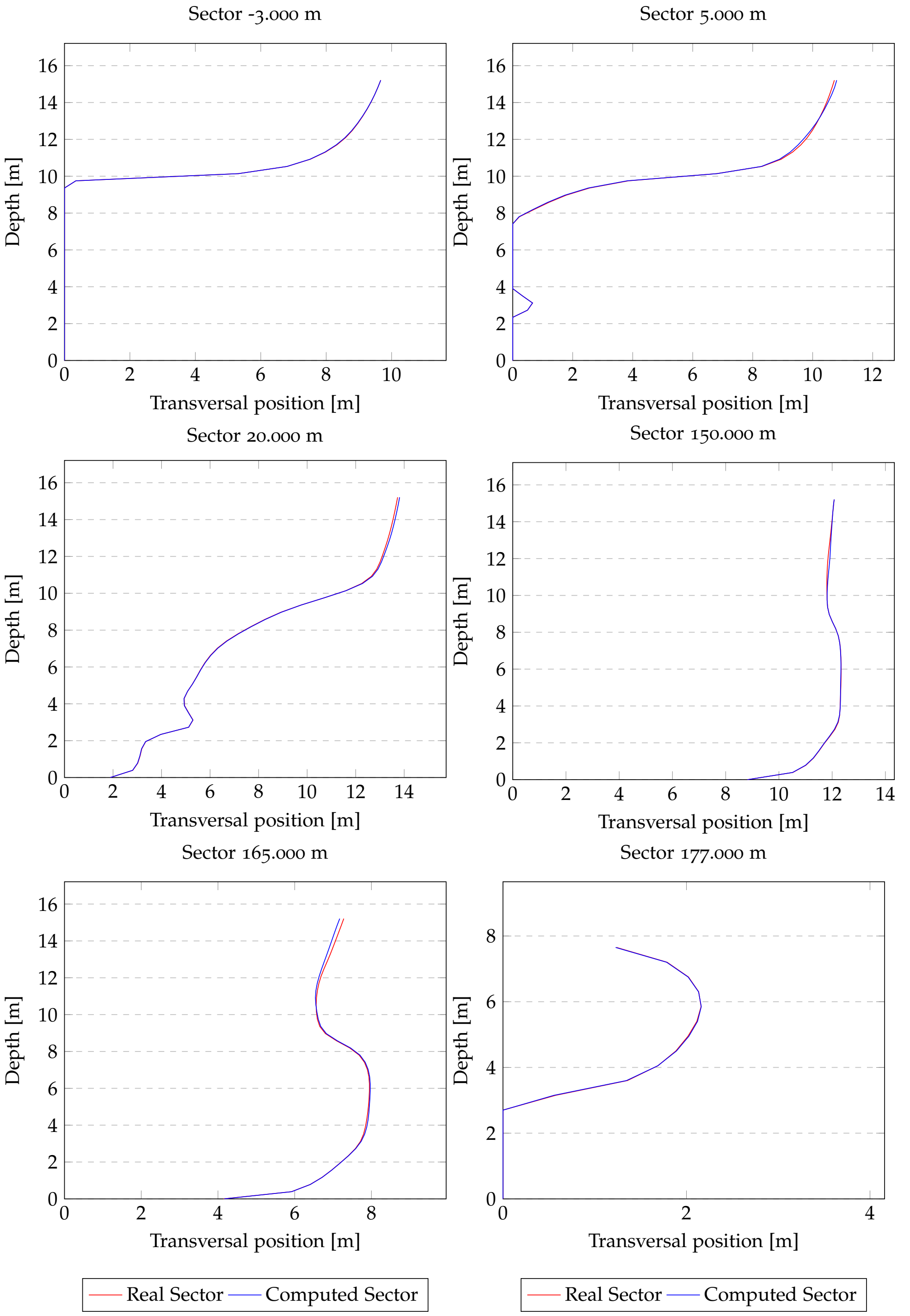


Figure 3.11: Sectors - Computed vs Real - Approximate.Derivatives - ObjFunc.Volume - Model

3.1.4 Results using DlibFindMinGlobal

With this optimizer we are trying to find the global minimum of the objective functions and the initialization of the control points is irrelevant to the produced results. For each waterline the algorithm stops after 2000 iterations.

3.1.4.1 Objective function for area attributes

The following results are presenting here only for reasons of completeness. The Global Minimum optimizer is not able to find a result on its own. Because of the complexity of the problem it is almost impossible to find a good approximation. The results are presenting in the following tables 3.7(the whole table appears at Appendix A), 3.8, 3.9. The mean error of minimum distances between the two curves is at 34cm with the maximum error at 166cm.

Table 3.7: Differences in attribute - Global.minimum - ObjFunc.Area - Model

Waterline 1.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	3638.650	3637.494	0.031
Inertia [m^4]	200695.437	200696.590	0.000
LCF [m]	96.473	95.644	0.859
Volume [m^3]	3286.598	3281.002	0.170
WSA [m^2]	3725.620	3722.870	0.073
LCB [m]	95.392	94.986	0.425
KB [m]	0.517	0.517	0.000

Waterline 3.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4040.104	4040.104	0.000
Inertia [m^4]	243814.578	243814.578	0.000
LCF [m]	94.297	94.297	0.000
Volume [m^3]	10913.262	10887.072	0.239
WSA [m^2]	4591.129	4650.952	1.286
LCB [m]	95.934	96.041	0.111
KB [m]	1.565	1.565	0.015

Waterline 5.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4103.510	4142.407	0.938
Inertia [m^4]	253854.937	253819.634	0.013
LCF [m]	94.149	93.356	0.842
Volume [m^3]	19034.183	19056.433	0.116
WSA [m^2]	5336.434	5392.777	1.044
LCB [m]	95.333	95.072	0.274
KB [m]	2.605	2.611	0.210

Waterline 10.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4403.614	4395.794	0.177
Inertia [m^4]	274006.906	274012.418	0.002
LCF [m]	85.496	85.898	0.468
Volume [m^3]	39943.433	40030.664	0.217
WSA [m^2]	7369.131	7402.476	0.450
LCB [m]	93.398	93.182	0.230
KB [m]	5.177	5.181	0.069

Waterline 14.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4665.364	4665.312	0.001
Inertia [m^4]	295876.375	295876.455	0.000
LCF [m]	82.221	82.235	0.017
Volume [m^3]	56619.175	56700.373	0.143
WSA [m^2]	8952.595	9113.651	1.767
LCB [m]	92.384	92.190	0.209
KB [m]	7.210	7.210	0.010

Waterline 15.600 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4702.847	4701.347	0.031
Inertia [m^4]	299220.500	299221.115	0.000
LCF [m]	82.311	82.763	0.547
Volume [m^3]	60792.464	60873.508	0.133
WSA [m^2]	9321.041	9482.340	1.701
LCB [m]	92.267	92.087	0.194
KB [m]	7.719	7.717	0.018

Table 3.8: Square y-Difference - Global.minimum - ObjFunc.Area - Model

Waterline.Depth [m]	Square.y-Difference [m^2]	Position.of.Max.Vertical.Distance [m]	Max.Vertical.Distance [m]
1.000	10.231	172.323	1.522
2.000	8.082	155.231	0.713
3.000	1.725	9.338	0.412
4.000	17.509	13.726	1.355
5.000	18.823	11.787	1.557
6.000	6.339	175.645	0.743
7.000	5.558	17.394	0.866
8.000	5.375	15.371	0.779
9.000	9.335	171.668	1.023
10.000	10.535	171.060	1.207
11.000	12.398	171.058	1.286
12.000	15.780	-1.746	1.662
13.000	15.488	10.191	1.014
14.000	10.396	-0.177	1.261
15.000	10.649	-0.262	1.178
15.600	5.648	173.447	0.819

Table 3.9: Minimum Distances - Global.minimum - ObjFunc.Area - Model

Waterline.Depth [m]	Sum.Square.Min.Distance [m^2]	Position.of.Max.Distance [m]	Max.Distance [m]
1.000	8.260	171.631	1.268
2.000	7.390	153.100	0.688
3.000	1.528	7.037	0.400
4.000	15.058	13.312	1.283
5.000	16.147	11.288	1.466
6.000	5.223	16.414	0.644
7.000	4.425	17.036	0.765
8.000	4.380	15.056	0.694
9.000	7.743	28.662	0.816
10.000	8.782	170.474	0.926
11.000	10.713	170.434	0.986
12.000	14.414	-1.776	1.662
13.000	14.425	9.991	0.994
14.000	9.469	-0.198	1.260
15.000	9.638	-0.347	1.176
15.600	4.956	156.323	0.669

As we can see in the figures below3.43, 3.44, the results are not good at all. The difference between the curves is easily spotted and appears at almost every longitudinal position.

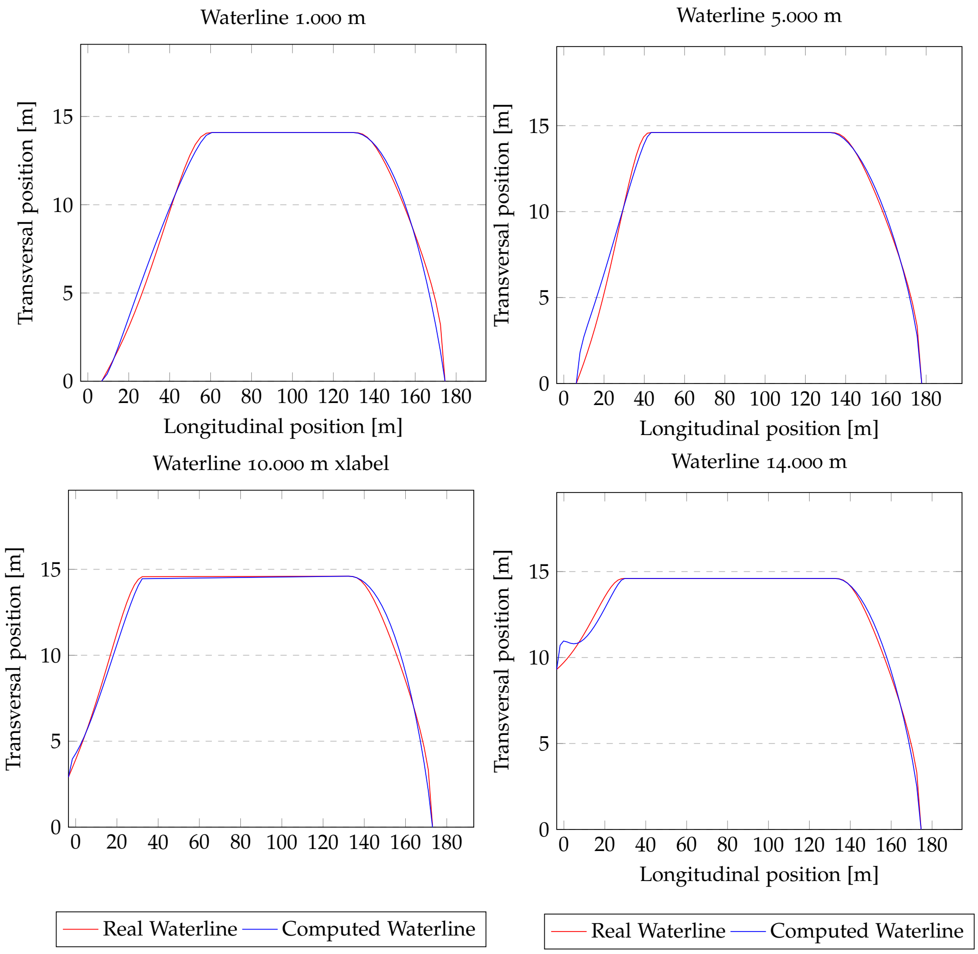


Figure 3.12: Waterlines- Computed vs Real - Global.minimum - ObjFunc.Area - Model

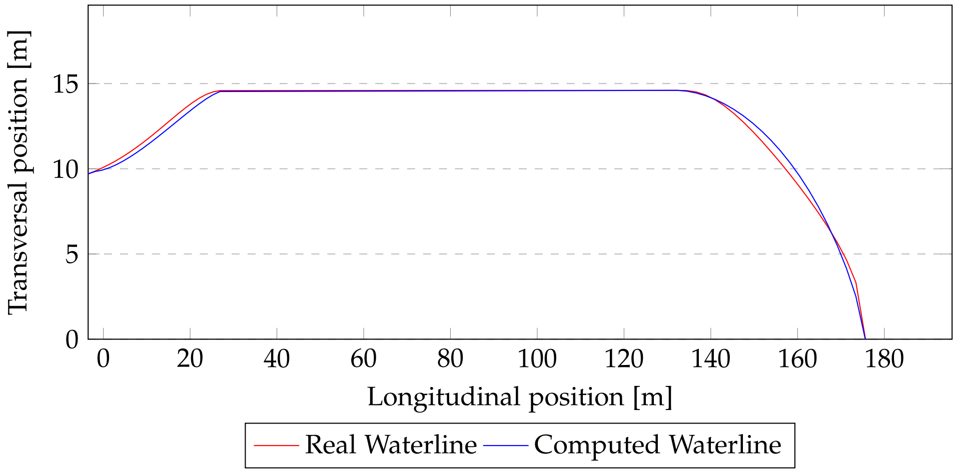


Figure 3.13: Waterline 15.600 m - Computed vs Real - Global.minimum - ObjFunc.Area - Model

We plot the sections 3.45 and the results are even worse. Not only the computed curves diverse from the original ones but also there is no smoothness in the sections. The only sections that appear to have a sort of similarity between the actual curves and the computed ones(at the early depths only) are the sections at -3.000m and 5.000m. These curves appear at the stern region and as we can see the curves geometry is more simpler than the rest of the sections.

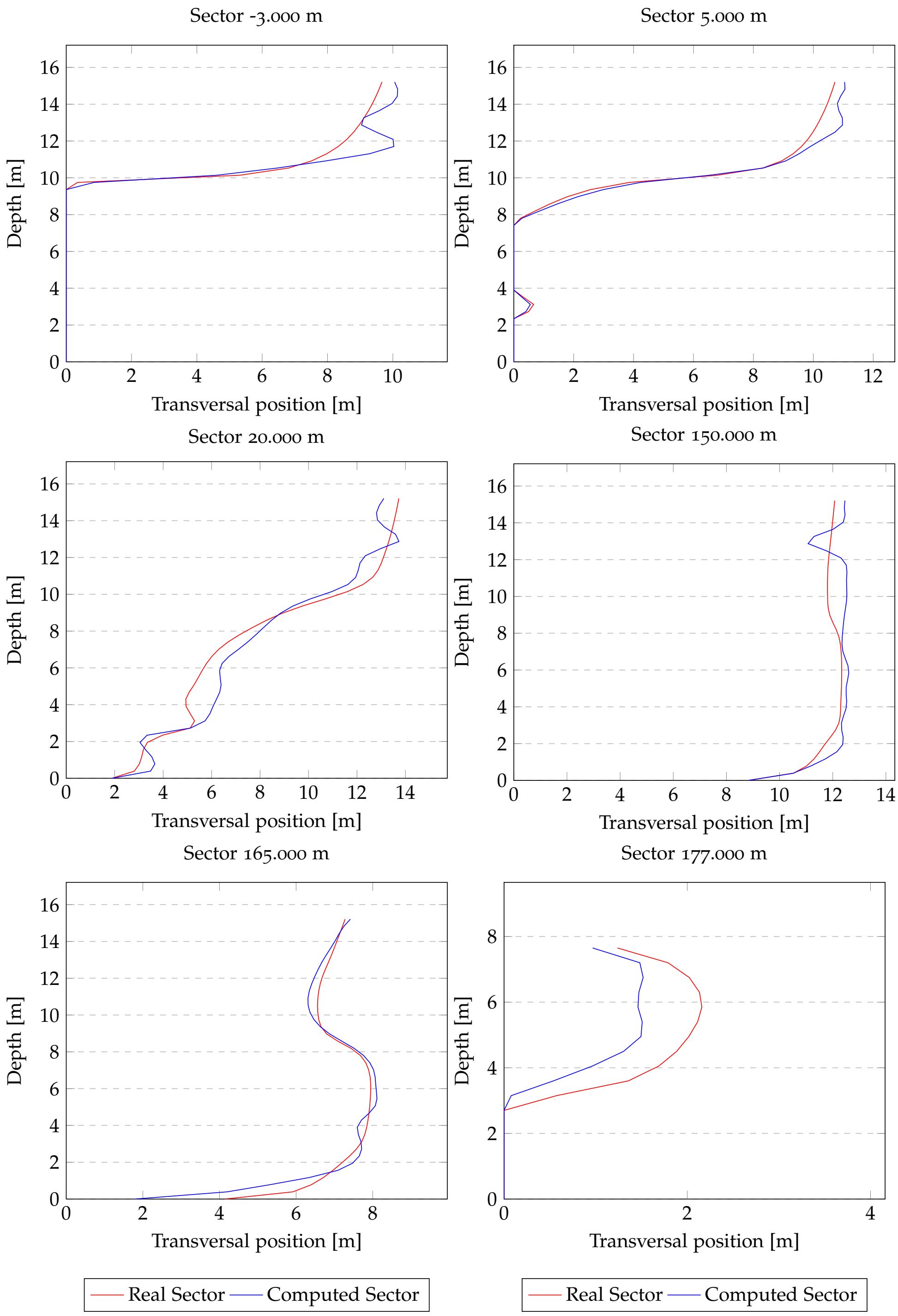


Figure 3.14: Sectors - Computed vs Real - Global.minimum - ObjFunc.Area - Model

3.1.4.2 Objective function for volume attributes

As in the previous process the following results are presenting here only for reasons of completeness. The following results appear to be even worse than the results of this algorithm with the use of the objective function for the area attributes only.
The results are presenting in the following tables 3.10(the whole table appears at Appendix A), 3.11, 3.12. In the following tables it is easy to observe that in the majority of the waterlines the maximum error overcome the 80 cm and in some case even the 200 cm.
The mean error of minimum distances between the two curves is at 50cm with the maximum error at 225cm.

Table 3.10: Differences in attribute - Global.Minimum - ObjFunc.Volume - Model

Waterline 1.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3638.650	3626.031	0.346
Inertia $[m^4]$	200695.437	200752.463	0.028
LCF [m]	96.473	95.972	0.519
Volume $[m^3]$	3286.598	3276.672	0.302
WSA $[m^2]$	3725.620	3712.842	0.342
LCB [m]	95.392	95.169	0.234
KB [m]	0.517	0.517	0.051
Waterline 3.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4040.104	4039.454	0.016
Inertia $[m^4]$	243814.578	243823.656	0.003
LCF [m]	94.297	95.594	1.356
Volume $[m^3]$	10913.262	10900.419	0.117
WSA $[m^2]$	4591.129	4577.804	0.290
LCB [m]	95.934	95.660	0.285
KB [m]	1.565	1.566	0.067
Waterline 5.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4103.510	4109.647	0.149
Inertia $[m^4]$	253854.937	253850.190	0.001
LCF [m]	94.149	94.494	0.365
Volume $[m^3]$	19034.183	19031.594	0.013
WSA $[m^2]$	5336.434	5336.926	0.009
LCB [m]	95.333	95.310	0.024
KB [m]	2.605	2.607	0.084
Waterline 10.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4403.614	4402.180	0.032
Inertia $[m^4]$	274006.906	274021.092	0.005
LCF [m]	85.496	85.825	0.383
Volume $[m^3]$	39943.433	39966.527	0.057
WSA $[m^2]$	7369.131	7369.961	0.011
LCB [m]	93.398	93.370	0.029
KB [m]	5.177	5.179	0.036
Waterline 14.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4665.364	4666.597	0.026
Inertia $[m^4]$	295876.375	296358.632	0.162
LCF [m]	82.221	84.117	2.254
Volume $[m^3]$	56619.175	56661.870	0.075
WSA $[m^2]$	8952.595	8975.835	0.258
LCB [m]	92.384	93.133	0.804
KB [m]	7.210	7.213	0.031
Waterline 15.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4702.847	4699.907	0.062
Inertia $[m^4]$	299220.500	299855.176	0.211
LCF [m]	82.311	82.449	0.167
Volume $[m^3]$	60792.464	60835.378	0.070
WSA $[m^2]$	9321.041	9346.425	0.271
LCB [m]	92.267	93.097	0.891
KB [m]	7.719	7.721	0.023

Table 3.11: Square y-Difference - Global.Minimum - ObjFunc.Volume - Model

Waterline.Depth [m]	Square.y-Difference $[m^2]$	Position.of.Max.Vertical.Distance [m]	Max.Vertical.Distance [m]
1.000	8.602	172.323	1.307
2.000	16.203	171.398	1.459
3.000	10.214	161.236	0.859
4.000	7.173	175.270	0.935
5.000	3.193	35.947	0.586
6.000	10.970	175.645	1.093
7.000	3.636	175.446	0.818
8.000	4.175	15.371	0.725
9.000	4.973	171.668	0.853
10.000	3.398	171.060	0.551
11.000	105.066	160.863	2.408
12.000	70.140	159.021	2.002
13.000	52.127	159.420	1.835
14.000	42.190	157.706	1.647
15.000	31.761	155.928	1.449
15.600	10.049	173.447	1.217

Table 3.12: Minimum Distances - Global.Minimum - ObjFunc.Volume - Model

Waterline.Depth [m]	Sum.Square.Min.Distance $[m^2]$	Position.of.Max.Distance [m]	Max.Distance [m]
1.000	7.094	169.551	1.134
2.000	13.891	170.797	1.288
3.000	9.195	161.485	0.819
4.000	6.105	174.834	0.760
5.000	2.820	36.142	0.547
6.000	8.841	16.300	0.906
7.000	2.868	175.059	0.655
8.000	3.394	15.073	0.642
9.000	4.081	171.255	0.657
10.000	2.954	154.903	0.501
11.000	94.148	161.675	2.254
12.000	64.250	7.441	1.911
13.000	47.803	157.861	1.732
14.000	38.961	156.039	1.562
15.000	29.239	156.341	1.385
15.600	8.739	172.868	0.967

In the following figures 3.43, 3.44, we plot the computed waterlines and the model's waterlines in the same diagram. Between the ploted waterline differences the only ones that seam to be fairly well generated are the waterlines at the depths at 5.000m and 10.000m .

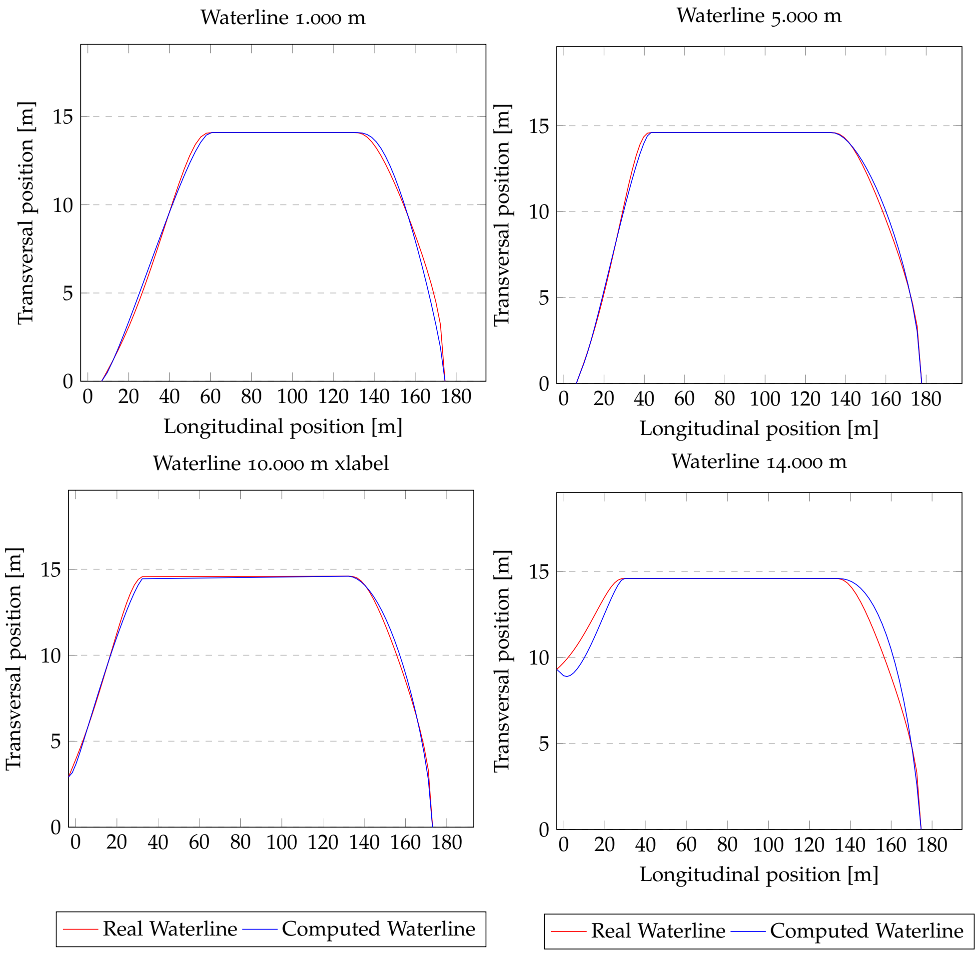


Figure 3.15: Waterlines- Computed vs Real - Global.Minimum - ObjFunc.Volume - Model

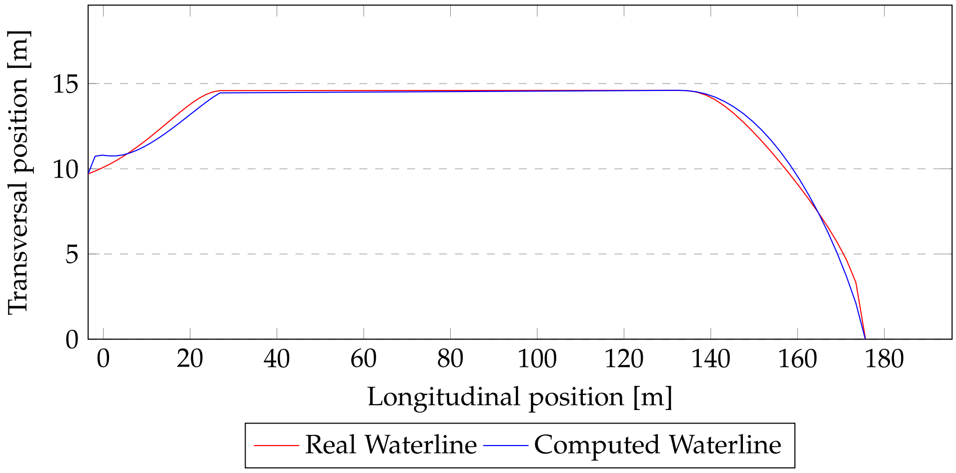
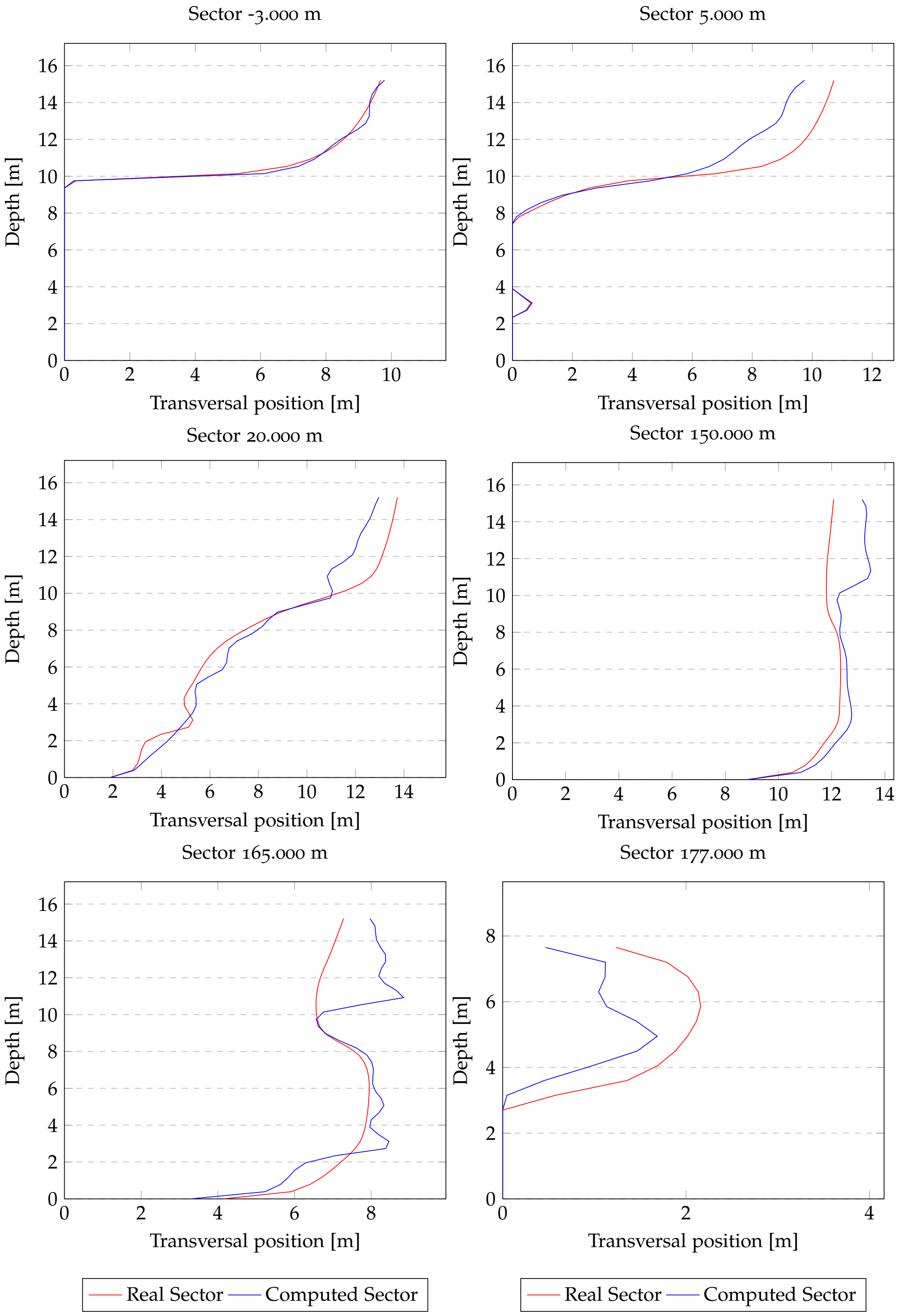


Figure 3.16: Waterline 15.600 m - Computed vs Real -Global.minimum - ObjFunc.Volume - Model

Finally, we plot the following sections 3.45. As we mentioned before, the produced sections are not good at all we the exception of the sections at -3.000m and 5.000m in the early depths.



Sector -3.000 m

Sector 5.000 m

Sector 20.000 m

Sector 150.000 m

Sector 165.000 m

Sector 177.000 m

Figure 3.17: Sectors - Computed vs Real - Global.Minimum - ObjFunc.Volume - Model

3.1.5 Results using GeneticOptimizer

At this optimization process we use the OpenGA algorithm which is a genetic optimizer. We created 30 generations consisting of a population of 100 people.

3.1.5.1 *Objective function for area attributes*

The results of this process are not good and we present the results in the following tables and figures.
In the table below 3.13 we can see the percentage difference at the attributes of the hull between the calculated geometry and the created model. At Appendix A there is the entire table of Differences in attribute. The differences between the curves are displayed in the upcoming tables 3.14, 3.15 .
In almost all the waterlines the maximum distance between the curves overcomes the 100 cm and at the waterlines with depth 3.000m, 4.000m and 5.000m the maximum distance is over 400cm. Furthermore, the position of the maximum distances in all the waterlines appears at the stern region.
The mean error of minimum distances between the two curves is at 86cm with the maximum error at 478cm.

Table 3.13: Differences in attribute - Genetic.Optimizer - ObjFunc.Area - Model

Waterline 1.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3638.650	3664.821	0.714
Inertia $[m^4]$	200695.437	200625.215	0.034
LCF [m]	96.473	95.366	1.147
Volume $[m^3]$	3286.598	3291.911	0.161
WSA $[m^2]$	3725.620	3746.872	0.567
LCB [m]	95.392	94.839	0.579
KB [m]	0.517	0.518	0.122
Waterline 3.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4040.104	4073.265	0.814
Inertia $[m^4]$	243814.578	243813.193	0.000
LCF [m]	94.297	87.309	7.410
Volume $[m^3]$	10913.262	11024.420	1.008
WSA $[m^2]$	4591.129	4855.347	5.441
LCB [m]	95.934	93.733	2.294
KB [m]	1.565	1.569	0.270
Waterline 5.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4103.510	4115.257	0.285
Inertia $[m^4]$	253854.937	253826.396	0.011
LCF [m]	94.149	87.486	7.077
Volume $[m^3]$	19034.183	19183.802	0.779
WSA $[m^2]$	5336.434	5614.472	4.952
LCB [m]	95.333	90.931	4.617
KB [m]	2.605	2.604	0.046
Waterline 10.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4403.614	4421.774	0.410
Inertia $[m^4]$	274006.906	273995.396	0.004
LCF [m]	85.496	84.026	1.719
Volume $[m^3]$	39943.433	40219.375	0.686
WSA $[m^2]$	7369.131	8036.846	8.308
LCB [m]	93.398	89.872	3.775
KB [m]	5.177	5.174	0.071
Waterline 14.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4665.364	4666.201	0.017
Inertia $[m^4]$	295876.375	295879.070	0.000
LCF [m]	82.221	81.581	0.777
Volume $[m^3]$	56619.175	56912.041	0.514
WSA $[m^2]$	8952.595	9634.730	7.079
LCB [m]	92.384	89.612	3.000
KB [m]	7.210	7.199	0.157
Waterline 15.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4702.847	4705.074	0.047
Inertia $[m^4]$	299220.500	299218.894	0.000
LCF [m]	82.311	81.208	1.339
Volume $[m^3]$	60792.464	61087.794	0.483
WSA $[m^2]$	9321.041	10010.259	6.885
LCB [m]	92.267	89.625	2.863
KB [m]	7.719	7.706	0.165

Table 3.14: Square y-Difference - Genetic.Optimizer - ObjFunc.Area - Model

Waterline.Depth [m]	Square.y-Difference $[m^2]$	Position.of.Max.Vertical.Distance [m]	Max.Vertical.Distance [m]
1.000	10.814	25.587	1.094
2.000	22.500	22.201	1.646
3.000	268.873	16.503	4.378
4.000	388.730	17.823	5.121
5.000	322.859	19.221	4.982
6.000	56.450	16.694	2.423
7.000	17.554	15.739	1.381
8.000	22.981	12.024	1.542
9.000	261.207	10.399	4.350
10.000	24.092	4.023	1.612
11.000	5.796	0.131	0.890
12.000	22.017	3.545	1.379
13.000	5.788	-0.084	0.931
14.000	7.710	-0.177	1.121
15.000	13.230	-0.262	1.364
15.600	18.892	6.093	1.160

Table 3.15: Minimum Distances - Genetic.Optimizer - ObjFunc.Area - Model

Waterline.Depth [m]	Sum.Square.Min.Distance $[m^2]$	Position.of.Max.Distance [m]	Max.Distance [m]
1.000	9.591	25.278	1.046
2.000	20.603	21.740	1.574
3.000	238.161	15.110	4.143
4.000	333.611	16.027	4.779
5.000	269.523	17.202	4.520
6.000	45.617	17.489	2.181
7.000	14.201	16.841	1.232
8.000	18.424	11.443	1.404
9.000	215.285	10.509	3.945
10.000	20.845	3.535	1.529
11.000	5.308	-0.029	0.876
12.000	20.905	3.314	1.359
13.000	5.440	-0.187	0.925
14.000	7.374	-0.277	1.116
15.000	12.753	-0.463	1.350
15.600	17.941	5.885	1.140

The difference between the waterlines is easily detected in the following figures 3.43, 3.44 and these results confirm the numbers that appear in the previous tables.

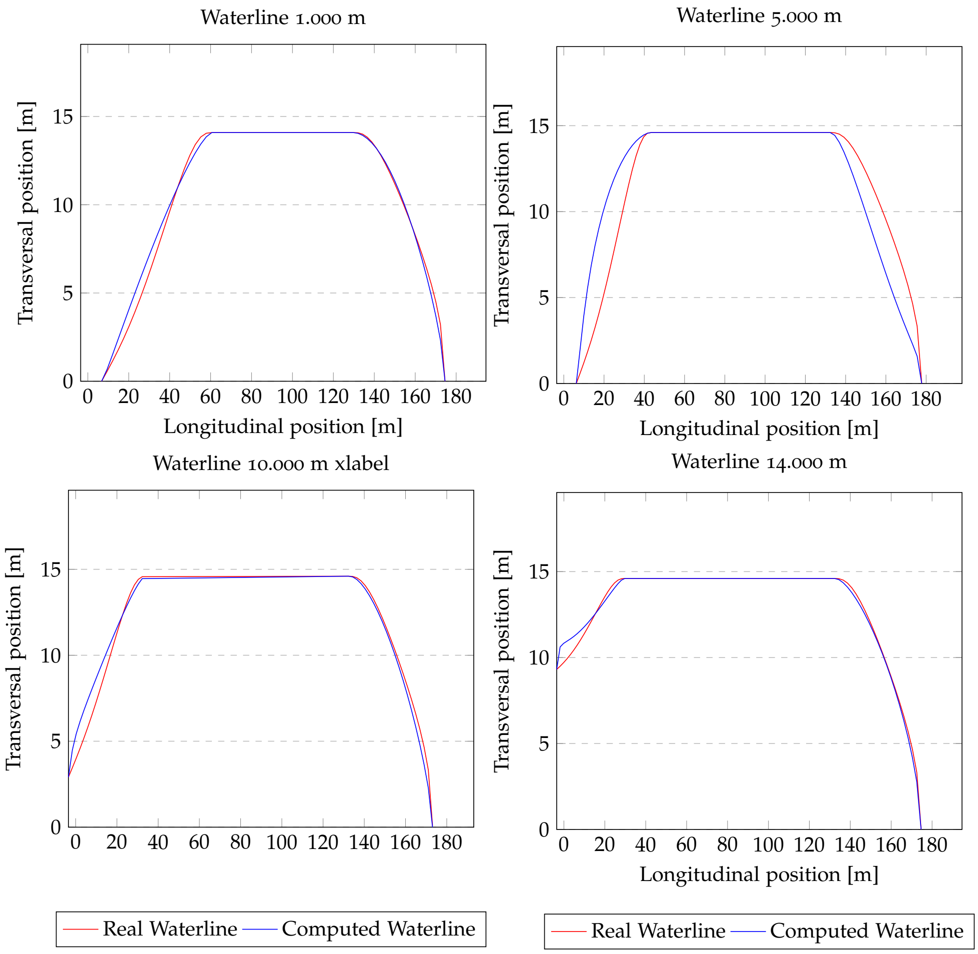


Figure 3.18: Waterlines- Computed vs Real - Genetic.Optimizer - ObjFunc.Area - Model

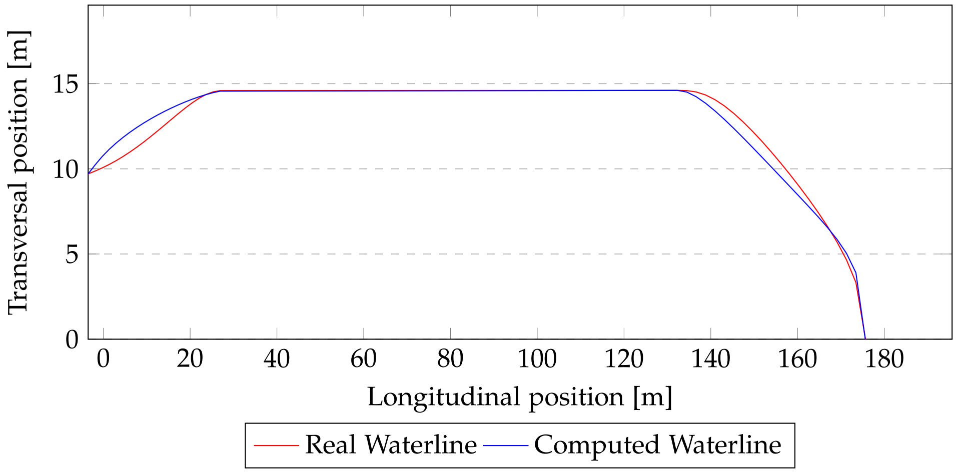


Figure 3.19: Waterline 15.600 m - Computed vs Real - Genetic.Optimizer - ObjFunc.Area - Model

Additionally, to the waterlines figures, the images of the sectors between the computed and the desired geometry is conspicuous 3.45 to the naked eye.

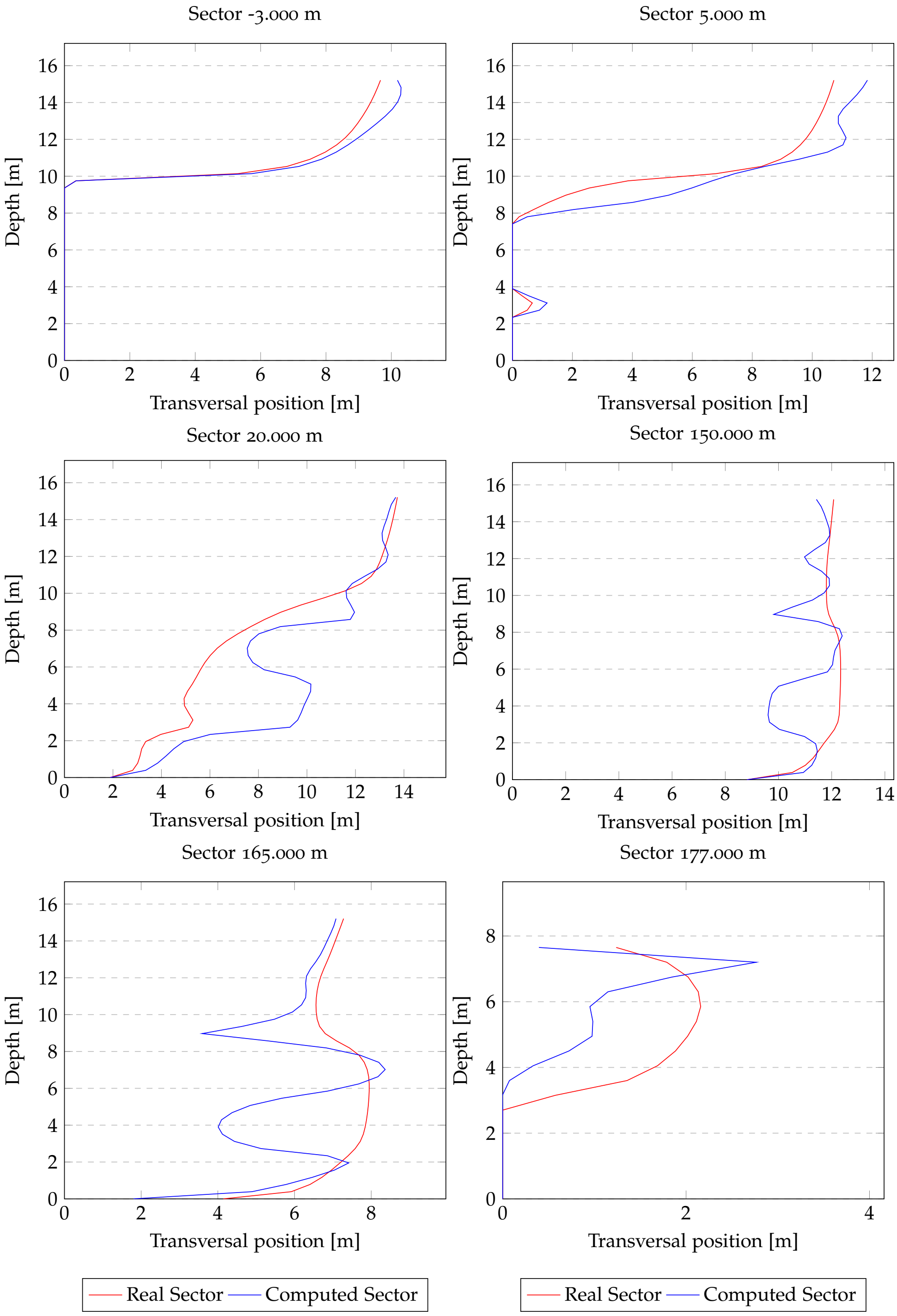


Figure 3.20: Sectors - Computed vs Real - Genetic.Optimizer - ObjFunc.Area - Model

3.1.5.2 *Objective function for volume attributes*

As presented in the process with the objective function only for the area attributes, the genetic algorithm can not produce better results by adding the volume attributes. In the table below 3.16 we can see the percentage difference at the attributes of the hull between the calculated geometry and the created model. In the Appendix A exists the whole table.

Table 3.16: Differences in attribute - Genetic.Optimizer - ObjFunc.Volume - Model

Waterline 1.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	3638.650	3672.422	0.919
Inertia [m^4]	200695.437	200719.114	0.011
LCF [m]	96.473	94.028	2.534
Volume [m^3]	3286.598	3296.419	0.297
WSA [m^2]	3725.620	3750.433	0.661
LCB [m]	95.392	94.157	1.294
KB [m]	0.517	0.518	0.154

Waterline 3.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4040.104	4069.784	0.729
Inertia [m^4]	243814.578	243774.891	0.016
LCF [m]	94.297	88.066	6.608
Volume [m^3]	10913.262	11026.496	1.026
WSA [m^2]	4591.129	4685.507	2.014
LCB [m]	95.934	92.312	3.775
KB [m]	1.565	1.568	0.211

Waterline 5.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4103.510	4111.908	0.204
Inertia [m^4]	253854.937	253856.812	0.000
LCF [m]	94.149	89.714	4.710
Volume [m^3]	19034.183	19209.978	0.915
WSA [m^2]	5336.434	5785.297	7.758
LCB [m]	95.333	92.022	3.473
KB [m]	2.605	2.605	0.003

Waterline 10.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4403.614	4404.598	0.022
Inertia [m^4]	274006.906	274034.612	0.010
LCF [m]	85.496	85.393	0.120
Volume [m^3]	39943.433	40283.881	0.845
WSA [m^2]	7369.131	8008.595	7.984
LCB [m]	93.398	90.308	3.308
KB [m]	5.177	5.175	0.041

Waterline 14.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4665.364	4668.387	0.064
Inertia [m^4]	295876.375	296414.228	0.181
LCF [m]	82.221	81.744	0.580
Volume [m^3]	56619.175	56984.353	0.640
WSA [m^2]	8952.595	9770.859	8.374
LCB [m]	92.384	89.984	2.597
KB [m]	7.210	7.199	0.163

Waterline 15.600 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4702.847	4706.089	0.068
Inertia [m^4]	299220.500	299815.554	0.198
LCF [m]	82.311	81.416	1.086
Volume [m^3]	60792.464	61160.630	0.601
WSA [m^2]	9321.041	10172.642	8.371
LCB [m]	92.267	89.959	2.501
KB [m]	7.719	7.705	0.177

These differences between the generated geometry and the actual geometry are displayed in the upcoming tables 3.17, 3.18.
Despite the fact that the results are slightly better than the previous process, they only confirm the fact that we can not approach the desired geometry with this algorithm.
The mean error of minimum distances between the two curves is at 79cm with the maximum error at 470cm.

Table 3.17: Square y-Difference - Genetic.Optimizer - ObjFunc.Volume - Model

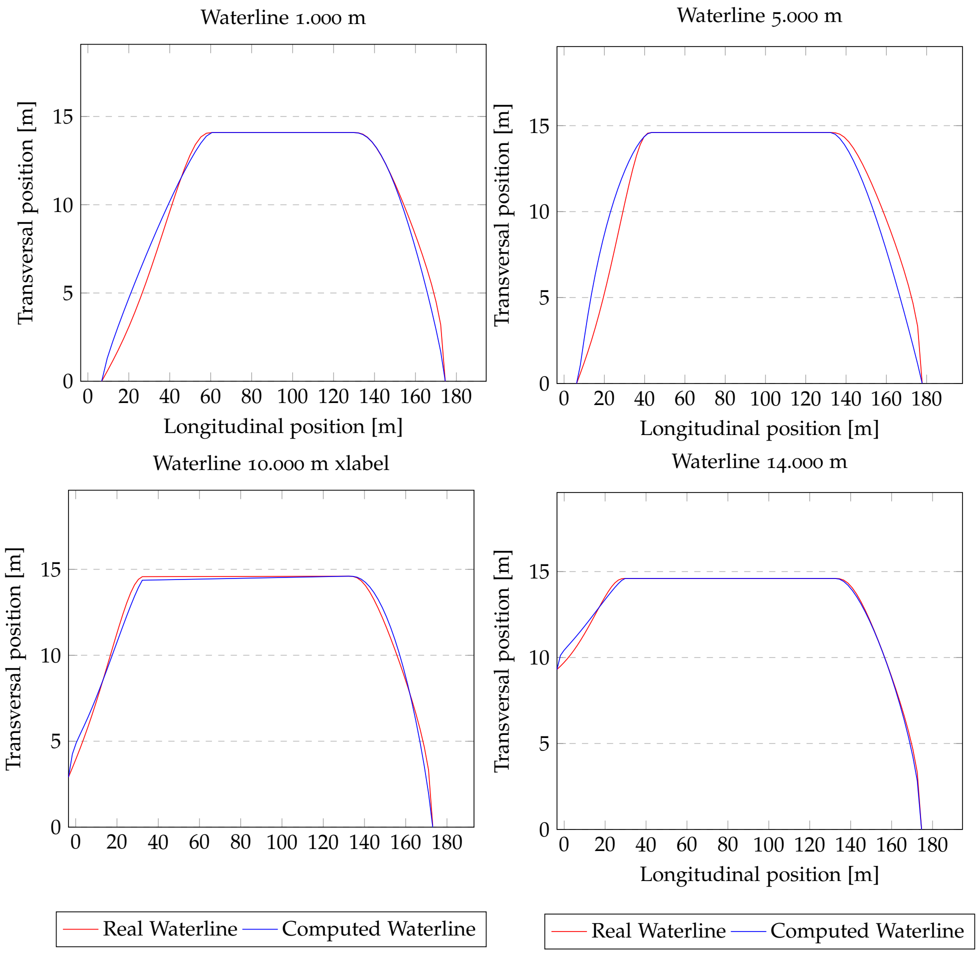
Waterline.Depth [m]	Square.y-Difference $[m^2]$	Position.of.Max.Vertical.Distance [m]	Max.Vertical.Distance [m]
1.000	32.843	22.895	1.666
2.000	106.003	22.201	3.009
3.000	215.098	18.891	3.894
4.000	11.190	17.823	1.146
5.000	145.851	19.221	3.444
6.000	303.865	14.961	5.170
7.000	52.227	15.739	2.448
8.000	137.368	13.698	3.635
9.000	15.428	6.805	1.303
10.000	12.535	171.060	1.358
11.000	6.283	0.131	1.039
12.000	60.982	0.017	2.388
13.000	2.138	171.929	0.701
14.000	4.342	1.488	0.710
15.000	26.668	7.857	1.367
15.600	10.043	2.892	0.996

As we can see in the following table 3.18 the reults of the waterlines are not good at all.

Table 3.18: Minimum Distances - Genetic.Optimizer - ObjFunc.Volume - Model

Waterline.Depth [m]	Sum.Square.Min.Distance $[m^2]$	Position.of.Max.Distance [m]	Max.Distance [m]
1.000	28.976	22.447	1.600
2.000	96.108	21.400	2.893
3.000	190.879	17.705	3.699
4.000	9.718	17.432	1.066
5.000	120.491	19.766	3.108
6.000	246.341	14.728	4.709
7.000	42.136	14.787	2.214
8.000	110.280	13.953	3.249
9.000	13.013	6.354	1.209
10.000	10.396	170.407	1.059
11.000	5.899	-0.001	1.031
12.000	57.116	1.477	2.361
13.000	1.750	171.591	0.546
14.000	4.087	1.394	0.704
15.000	25.250	7.626	1.347
15.600	9.667	2.746	0.985

The difference between the waterlines is easily detected in the following figures 3.43, 3.44.



Waterline 1.000 m

Waterline 5.000 m

Waterline 10.000 m xlabel

Waterline 14.000 m

Figure 3.21: Waterlines- Computed vs Real - Genetic.Optimizer - ObjFunc.Volume - Model

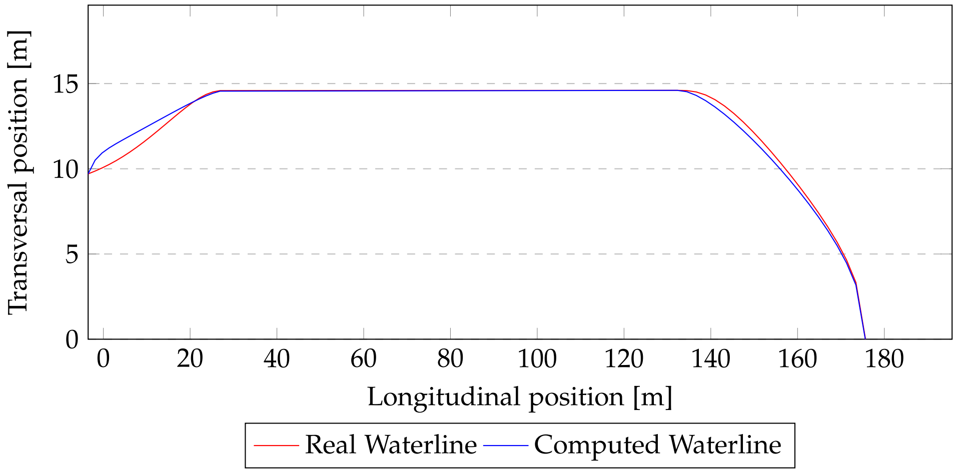


Figure 3.22: Waterline 15.600 m - Computed vs Real - Genetic.Optimizer - ObjFunc.Volume - Model

Additionally, to the waterlines figures, the images of the sectors between the computed and the desired geometry is conspicuous 3.45 to the naked eye.

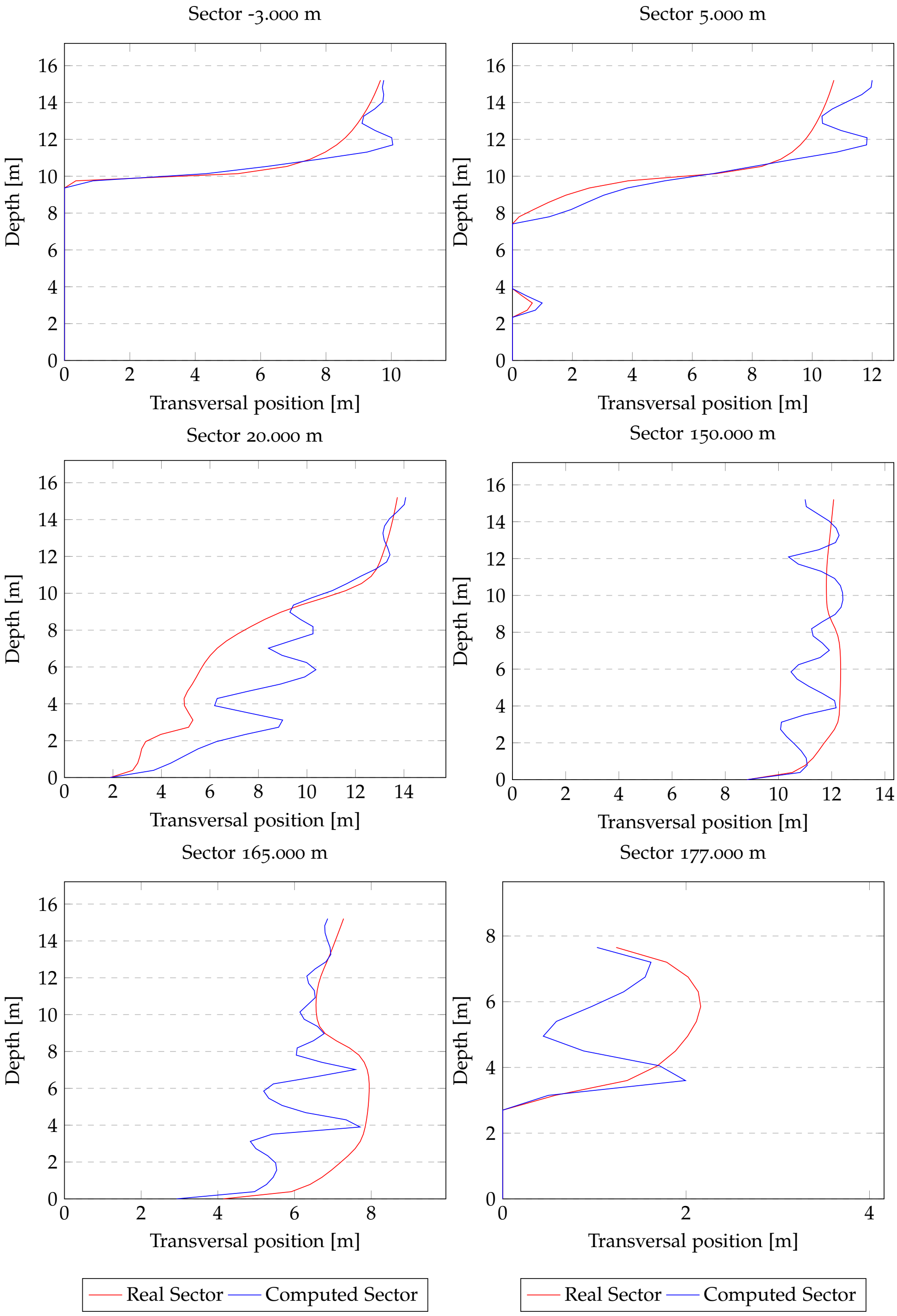


Figure 3.23: Sectors - Computed vs Real - Genetic.Optimizer - ObjFunc.Volume - Model

3.1.6 Results using Hybrid

At this section we present the results from the combination of two optimization algorithms. At first we use the genetic algorithm in order to find a global minimum that will be as close as possible to the waterplane area attributes. Then, we use the DlibFindMinUsingApproximateDerivatives algorithm to find the global minimum.

3.1.6.1 *Objective function for area attributes*

In this optimization process we combine the best two optimization algorithms. At first, we use the genetic algorithm in order to obtain a better approach and then we pass the results to the DlibFindMinUsingApproximateDerivatives to find the global minimum of the objective function that uses only the waterplane area attributes.

In the table below 3.19 we can see the percentage difference at the attributes of the hull between the calculated geometry and the created model(the entire data appears at Appendix A). The attribute differences are below 2 percent for all the waterlines. These results are not better than the DlibFindMinUsingApproximateDerivatives when used by itself but are better than the genetic algorithm. The fact the results are not better than DlibFindMinUsingApproximateDerivatives is because the approximate derivatives algorithm's success is based on the initial control points and in this case these initial control points are really bad.

The square-differences and the minimum distances between the waterlines are displayed in the upcoming tables 3.21, 3.20. The most of the maximum distances appear at the stern region of the waterlines. The mean error of minimum distances between the two curves is at 32cm with the maximum error at 182cm.

Table 3.19: Differences in attribute - Hybrid.Optimization - ObjFunc.Area - Model

Waterline 1.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	3638.650	3649.174	0.288
Inertia [m^4]	200695.437	200632.890	0.031
LCF [m]	96.473	96.244	0.237
Volume [m^3]	3286.598	3286.046	0.016
WSA [m^2]	3725.620	3736.304	0.285
LCB [m]	95.392	95.289	0.108
KB [m]	0.517	0.517	0.052

Waterline 3.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4040.104	4079.331	0.961
Inertia [m^4]	243814.578	243814.577	0.000
LCF [m]	94.297	94.302	0.004
Volume [m^3]	10913.262	10948.817	0.324
WSA [m^2]	4591.129	4627.659	0.789
LCB [m]	95.934	95.864	0.073
KB [m]	1.565	1.567	0.158

Waterline 5.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4103.510	4178.180	1.787
Inertia [m^4]	253854.937	253867.456	0.004
LCF [m]	94.149	94.144	0.005
Volume [m^3]	19034.183	19183.350	0.777
WSA [m^2]	5336.434	5386.594	0.931
LCB [m]	95.333	95.286	0.049
KB [m]	2.605	2.614	0.318

Waterline 10.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4403.614	4424.117	0.463
Inertia [m^4]	274006.906	273957.200	0.018
LCF [m]	85.496	85.494	0.001
Volume [m^3]	39943.433	40267.627	0.805
WSA [m^2]	7369.131	7474.190	1.405
LCB [m]	93.398	93.376	0.022
KB [m]	5.177	5.181	0.078

Waterline 14.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4665.364	4674.844	0.202
Inertia [m^4]	295876.375	295875.723	0.000
LCF [m]	82.221	82.221	0.000
Volume [m^3]	56619.175	56983.040	0.638
WSA [m^2]	8952.595	9060.505	1.190
LCB [m]	92.384	92.368	0.017
KB [m]	7.210	7.205	0.079

Waterline 15.600 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4702.847	4705.000	0.045
Inertia [m^4]	299220.500	299215.091	0.001
LCF [m]	82.311	82.310	0.000
Volume [m^3]	60792.464	61161.313	0.603
WSA [m^2]	9321.041	9428.094	1.135
LCB [m]	92.267	92.252	0.016
KB [m]	7.719	7.711	0.101

Table 3.20: Square y-Difference - Hybrid.Optimization - ObjFunc.Area - Model

Waterline.Depth [m]	Square.y-Difference $[m^2]$	Position.of.Max.Vertical.Distance [m]	Max.Vertical.Distance [m]
1.000	4.058	28.279	0.625
2.000	3.606	27.685	0.549
3.000	7.798	170.213	0.764
4.000	15.805	13.726	1.071
5.000	31.843	173.261	2.029
6.000	6.955	18.427	0.762
7.000	4.588	32.288	0.605
8.000	25.181	172.026	1.741
9.000	7.605	12.196	0.812
10.000	3.893	5.906	0.530
11.000	5.729	0.131	0.944
12.000	9.456	-1.746	1.368
13.000	5.697	-1.797	0.768
14.000	3.246	-1.843	0.701
15.000	4.949	173.100	0.739
15.600	4.284	-1.909	0.782

Table 3.21: Minimum Distances - Hybrid.Optimization - ObjFunc.Area - Model

Waterline.Depth [m]	Sum.Square.Min.Distance $[m^2]$	Position.of.Max.Distance [m]	Max.Distance [m]
1.000	3.592	28.096	0.595
2.000	3.285	52.470	0.523
3.000	6.830	21.042	0.714
4.000	13.672	13.393	1.011
5.000	26.169	174.126	1.825
6.000	5.879	18.129	0.686
7.000	3.810	32.501	0.559
8.000	20.308	172.765	1.559
9.000	6.413	11.883	0.735
10.000	3.468	5.742	0.501
11.000	5.433	0.007	0.936
12.000	8.965	-2.201	1.323
13.000	5.199	-1.955	0.753
14.000	3.123	-0.237	0.696
15.000	4.486	-2.024	0.720
15.600	3.953	-2.026	0.774

The difference between the waterlines is displayed in the following figures3.43, 3.44. In the pictures we confirm that the biggest differences appear in the stern region. In all waterline figures the maximum distance appear at the stern region while in waterline at the depth 5.000m appears at the bow region.

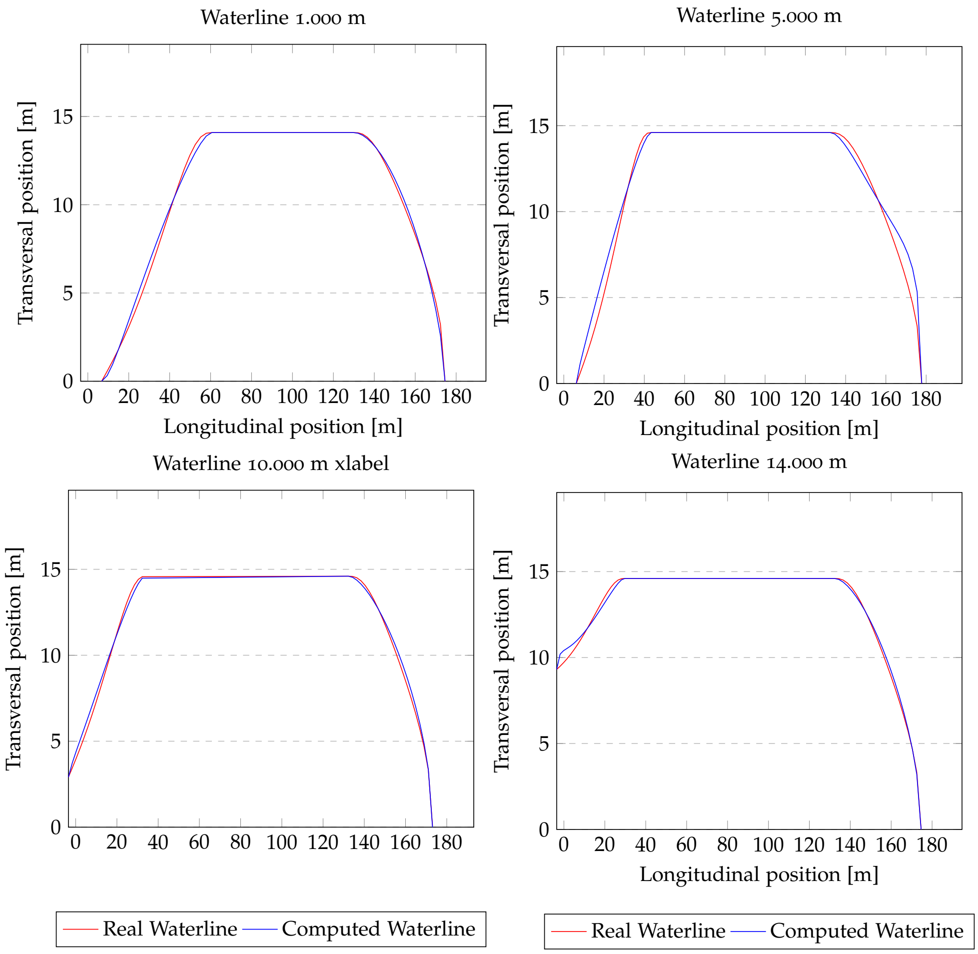


Figure 3.24: Waterlines- Computed vs Real - Hybrid.Optimizer - ObjFunc.Area - Model

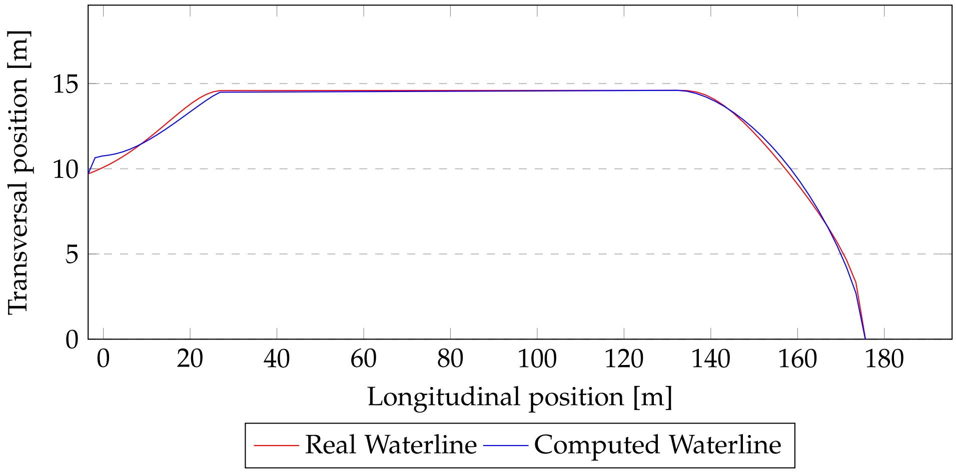
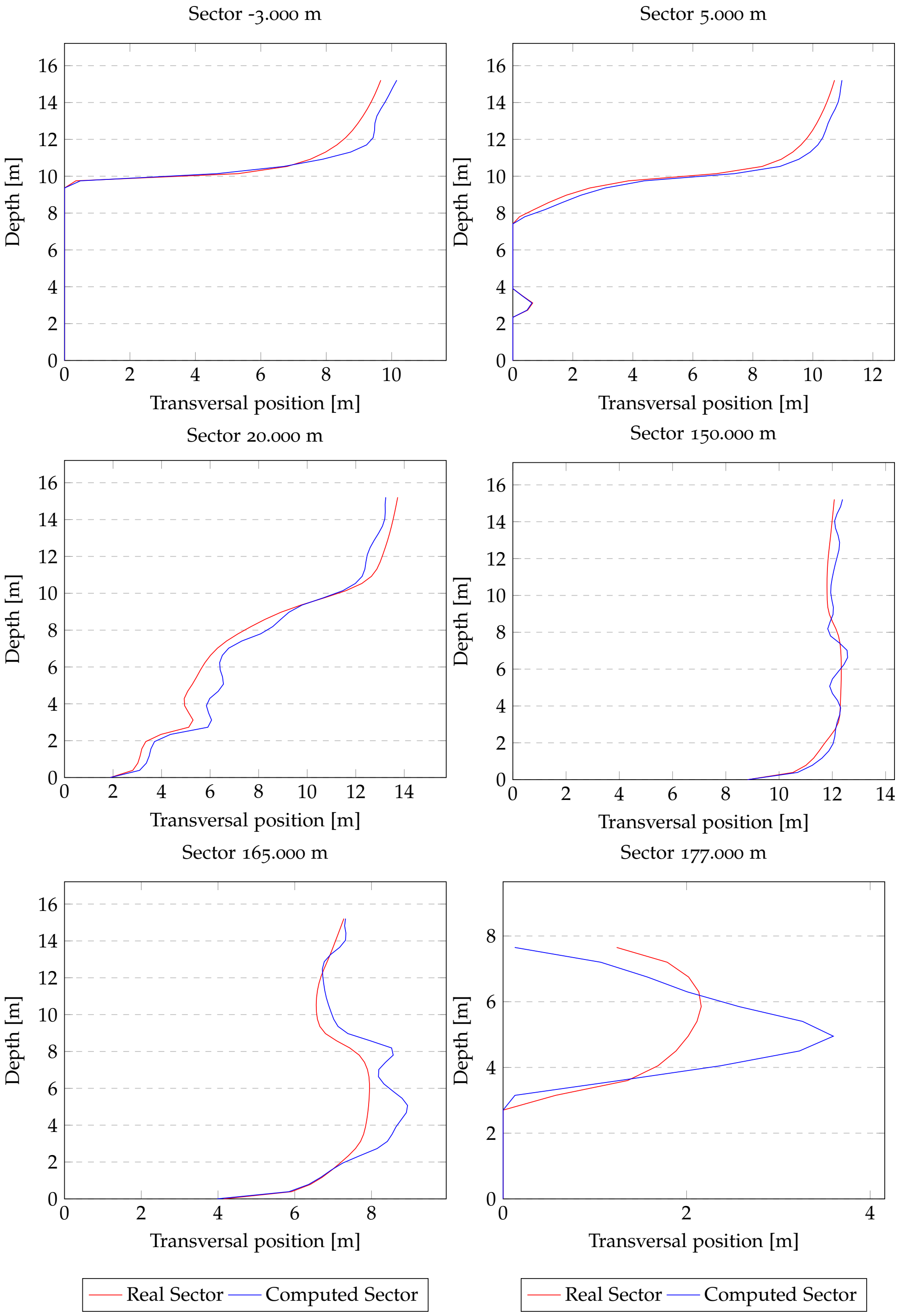


Figure 3.25: Waterline 15.600 m - Computed vs Real - Hybrid.Optimizer - ObjFunc.Area - Model

The figures of the sectors between the computed and the desired geometry are displayed3.45 below. On the contrary with the waterlines, the sections appear to be better in the stern region. This indicates that in the stern region the error seams not to change too much from waterline to waterline. Although, in the bow region the distance between the waterlines constantly changes.


Sector -3.000 m

Sector 5.000 m

Sector 20.000 m

Sector 150.000 m

Sector 165.000 m

Sector 177.000 m

Figure 3.26: Sectors - Computed vs Real - Hybrid.Optimizer - ObjFunc.Area - Model

3.1.6.2 Objective function for volume attributes

In the following table 3.22 we can see the percentage differences at the attributes between the calculated geometry and the created model. The entire data appears at Appendix A. The results with this hybrid process are similar to the previous one. Despite, the fact that we use both the volume and the area attributes the produced curves did not improve. Actually, the results are worse than the previous process. The mean error of minimum distances between the two curves is at 33cm with the maximum error at 255cm.

Table 3.22: Differences in attribute - Hybrid.Optimizer - ObjFunc.Volume - Model

Waterline 1.000 m

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3638.650	3714.858	2.051
Inertia $[m^4]$	200695.437	200656.219	0.019
LCF [m]	96.473	92.596	4.018
Volume $[m^3]$	3286.598	3321.051	1.037
WSA $[m^2]$	3725.620	3793.719	1.795
LCB [m]	95.392	93.430	2.057
KB [m]	0.517	0.519	0.333

Waterline 3.000 m

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4040.104	4054.545	0.356
Inertia $[m^4]$	243814.578	243822.726	0.003
LCF [m]	94.297	93.825	0.500
Volume $[m^3]$	10913.262	10997.145	0.762
WSA $[m^2]$	4591.129	4781.180	3.974
LCB [m]	95.934	94.581	1.409
KB [m]	1.565	1.562	0.166

Waterline 5.000 m

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4103.510	4118.884	0.373
Inertia $[m^4]$	253854.937	253781.369	0.028
LCF [m]	94.149	94.149	0.000
Volume $[m^3]$	19034.183	19131.741	0.509
WSA $[m^2]$	5336.434	5551.128	3.867
LCB [m]	95.333	94.508	0.865
KB [m]	2.605	2.600	0.193

Waterline 10.000 m

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4403.614	4411.900	0.187
Inertia $[m^4]$	274006.906	274058.419	0.018
LCF [m]	85.496	85.496	0.000
Volume $[m^3]$	39943.433	40121.768	0.444
WSA $[m^2]$	7369.131	7622.538	3.324
LCB [m]	93.398	92.793	0.647
KB [m]	5.177	5.174	0.072

Waterline 14.000 m

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4665.364	4670.124	0.101
Inertia $[m^4]$	295876.375	295864.265	0.004
LCF [m]	82.221	82.221	0.000
Volume $[m^3]$	56619.175	56811.847	0.339
WSA $[m^2]$	8952.595	9250.417	3.219
LCB [m]	92.384	91.863	0.563
KB [m]	7.210	7.202	0.110

Waterline 15.600 m

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4702.847	4706.331	0.074
Inertia $[m^4]$	299220.500	299214.896	0.001
LCF [m]	82.311	82.311	0.000
Volume $[m^3]$	60792.464	60990.956	0.325
WSA $[m^2]$	9321.041	9621.821	3.126
LCB [m]	92.267	91.781	0.526
KB [m]	7.719	7.710	0.108

The square-differences and the minimum distances between the them are displayed in the upcoming tables 3.24, 3.23. Te biggest difference in this process appears to be in the very first waterline with the maximum distance at 266 cm, a fact that affect the total results as in this process we include the volume attributes.

Table 3.23: Square y-Difference - Hybrid.Optimizer - ObjFunc.Volume - Model

Waterline.Depth [m]	Square.y-Difference $[m^2]$	Position.of.Max.Vertical.Distance [m]	Max.Vertical.Distance [m]
1.000	73.908	22.895	2.668
2.000	7.467	173.708	1.128
3.000	8.188	174.701	1.017
4.000	3.366	40.354	0.561
5.000	11.137	35.947	0.924
6.000	12.613	16.694	1.240
7.000	4.815	175.446	1.109
8.000	6.691	13.698	0.648
9.000	22.789	13.993	1.727
10.000	2.022	11.556	0.459
11.000	1.472	7.413	0.334
12.000	1.804	171.346	0.566
13.000	16.178	-0.084	1.367
14.000	1.968	-0.177	0.371
15.000	5.827	-1.886	1.038
15.600	3.760	-0.308	0.667

Table 3.24: Minimum Distances - Hybrid.Optimizer - ObjFunc.Volume - Model

Waterline.Depth [m]	Sum.Square.Min.Distance $[m^2]$	Position.of.Max.Distance [m]	Max.Distance [m]
1.000	67.760	22.159	2.557
2.000	6.379	173.176	0.906
3.000	7.029	174.227	0.828
4.000	2.961	40.506	0.538
5.000	9.646	36.256	0.862
6.000	10.360	16.212	1.120
7.000	3.750	174.921	0.885
8.000	5.634	13.444	0.583
9.000	18.175	13.299	1.546
10.000	1.782	11.399	0.427
11.000	1.369	7.340	0.325
12.000	1.530	171.073	0.441
13.000	15.327	-0.282	1.353
14.000	1.824	-0.226	0.368
15.000	5.589	-2.005	1.032
15.600	3.473	-0.363	0.665

In the following figures are presented the differences between the waterlines 3.43, 3.44. In the first figure(Waterline 1.000m), it is easily spotted the big difference between the computed and the actual waterline.

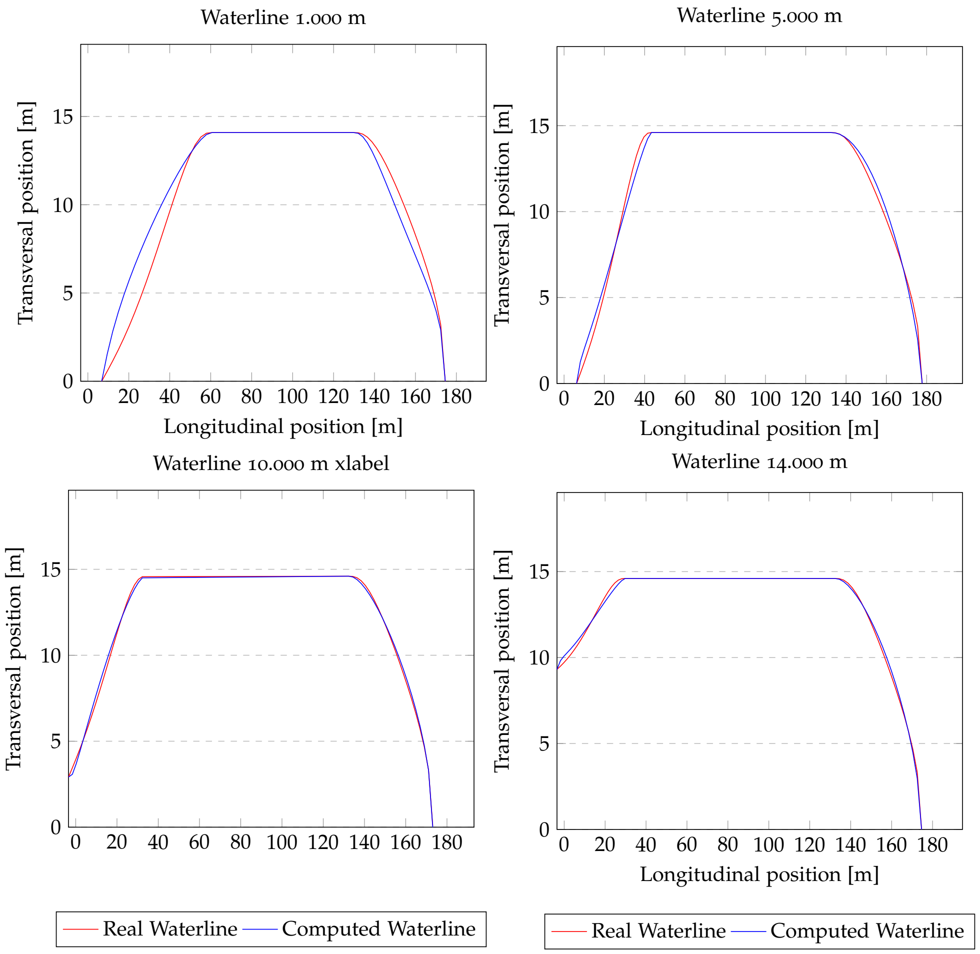


Figure 3.27: Waterlines- Computed vs Real - Hybrid.Optimizer - ObjFunc.Volume - Model

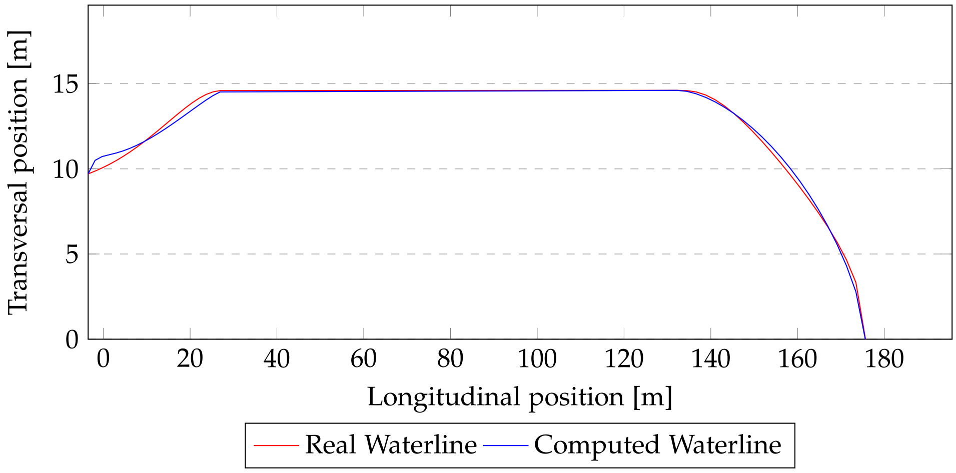
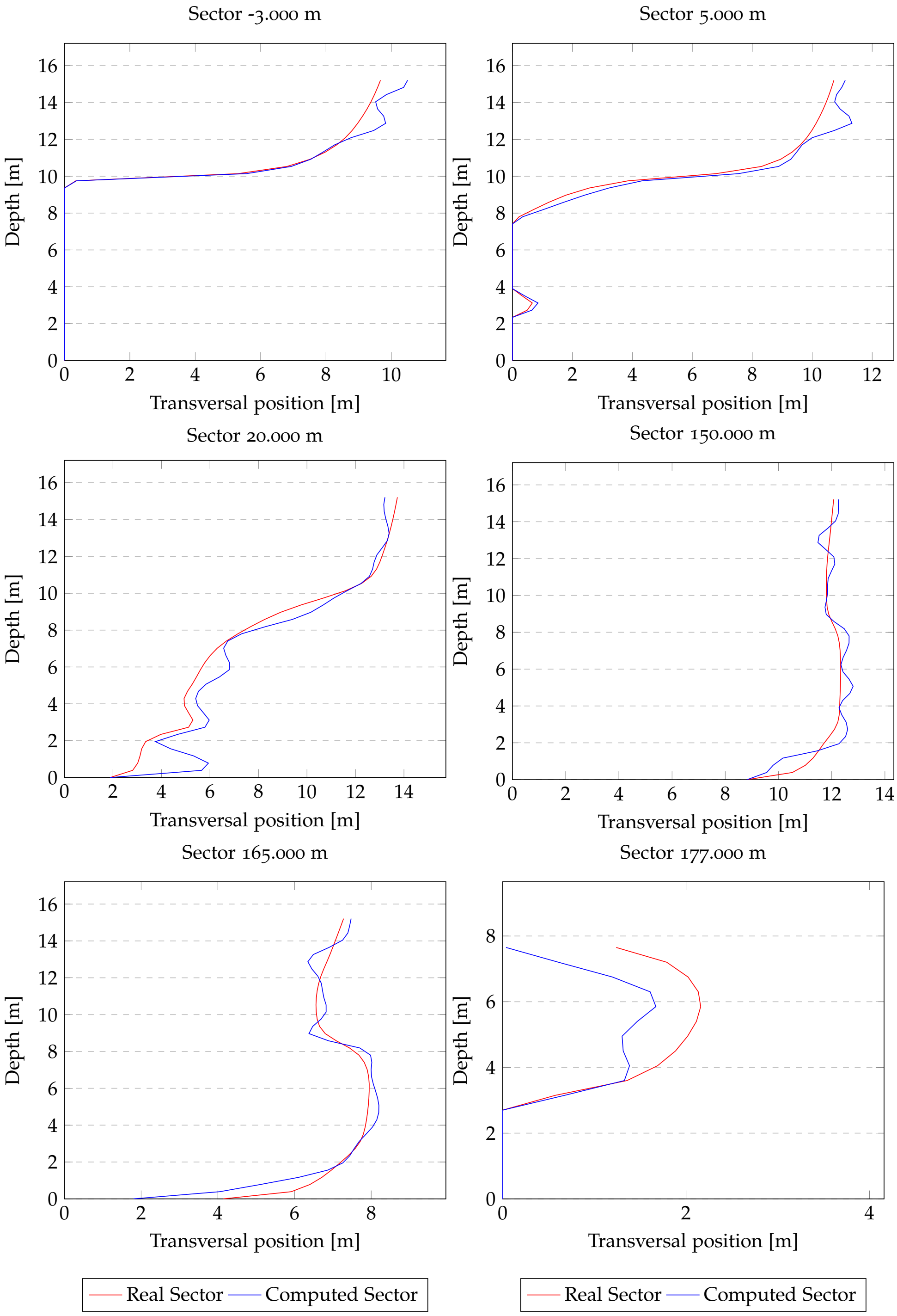


Figure 3.28: Waterline 15.600 m - Computed vs Real - Hybrid.Optimizer - ObjFunc.Volume - Model

The figures of the sectors between the computed and the desired geometry are displayed below 3.45. The produced sections are not smooth at all and appear to be similar with the hybrid process in which we used only the area attributes.


Sector -3.000 m

Sector 5.000 m

Sector 20.000 m

Sector 150.000 m

Sector 165.000 m

Sector 177.000 m

Figure 3.29: Sectors - Computed vs Real - Hybrid.Optimizer - ObjFunc.Volume - Model

3.2 VERIFICATION MODEL

After the evaluation of a model created especially for these tests we must test our optimization process on a real ship.

Firstly, we fed the program with the appropriate data taken from the AVEVA Initial Design™. The waterlines's points and the hydrostatic table of this ship were inserted in our program along with all the needed input 2.5.1 .

The main characteristics of both the model and the real ship are the same. We decide to use the same characteristics in order to compare the results between the created model and the real ship. If we did use a completely different vessel we would not be able to compare properly the results as the form of the hull can affect the quality of the results.

3.2.1 Characteristics

We create 10 waterlines with the data taken from the AVEVA Initial Design™. The depths are at 1m, 3m, 3.5m, 5m, 7m, 9m, 10m, 12m, 14m, 15.6m .

- Length : 181.45m
- Breadth : 29.2 m
- Depth : 15.6 m
- Radius : 2.523 m
- Parallel Midbody extent : 60.585 m - 129.825 m

We will present the main features of the vessel with the following figures 3.30,3.31,3.32,3.33,3.34.

3.2.2 Optimization Characteristics

For this model we did some modifications compare to the model we used in 3.1.

1. Stern B-spline
   The degree of the spline is 5 with 6 control points
2. Stem B-spline
   The degree of the spline is 5 with 6 control points

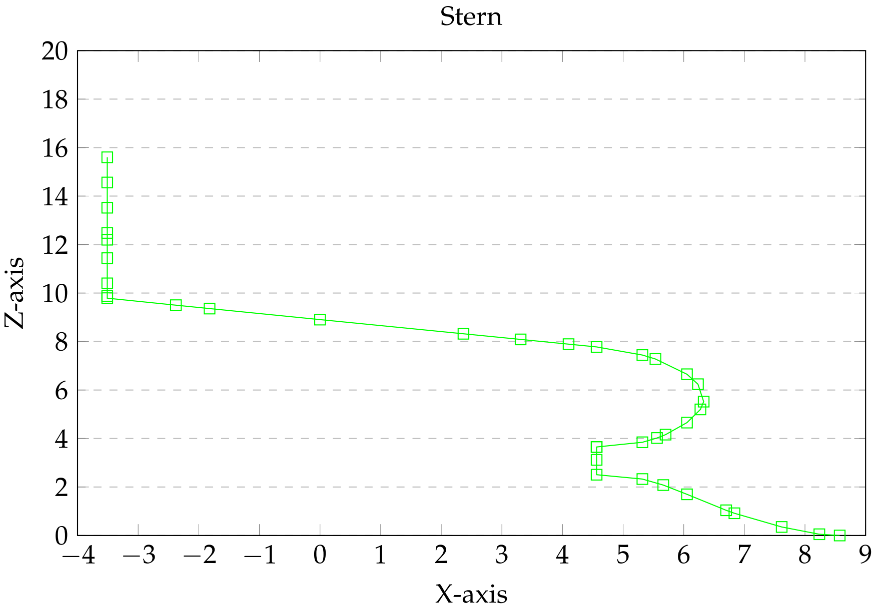


Figure 3.30: Profile - Stem

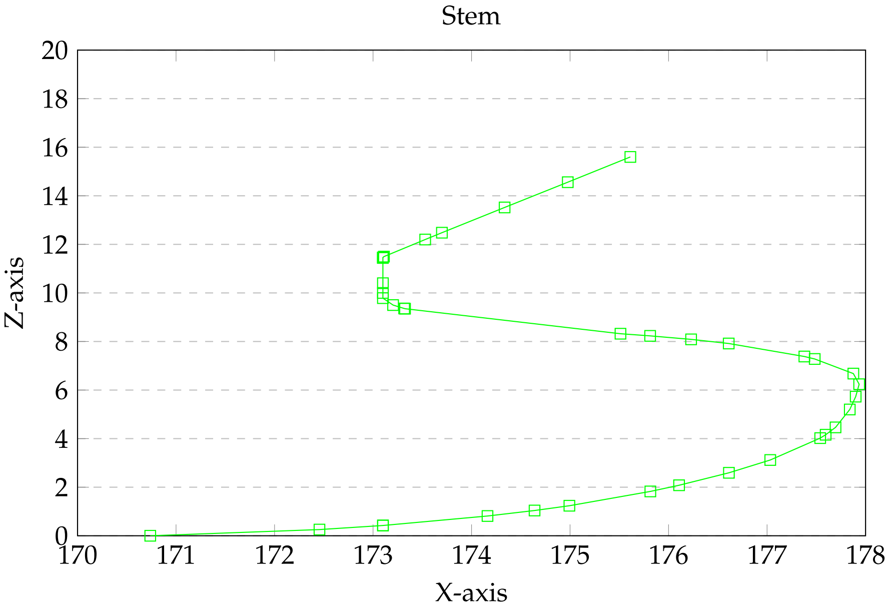


Figure 3.31: Profile - Stern

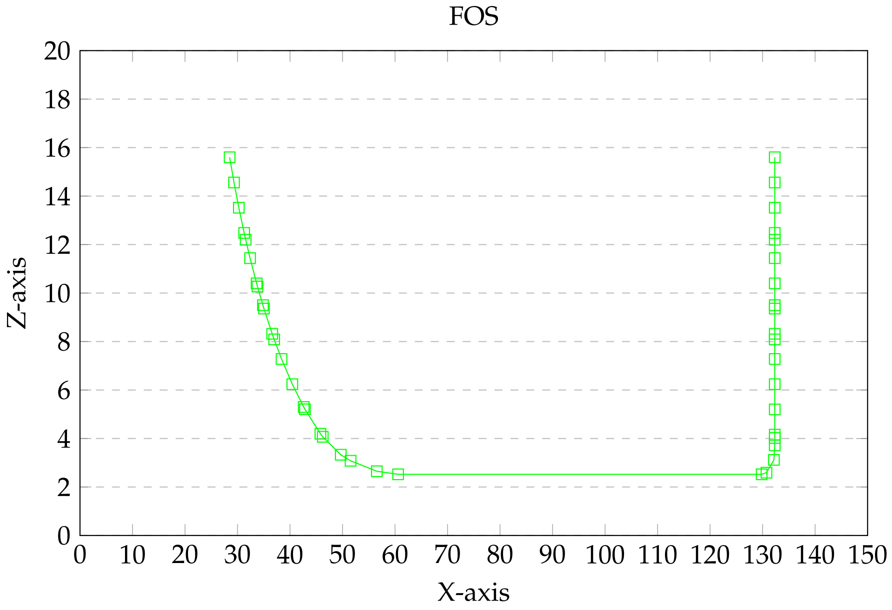


Figure 3.32: Flat of sight

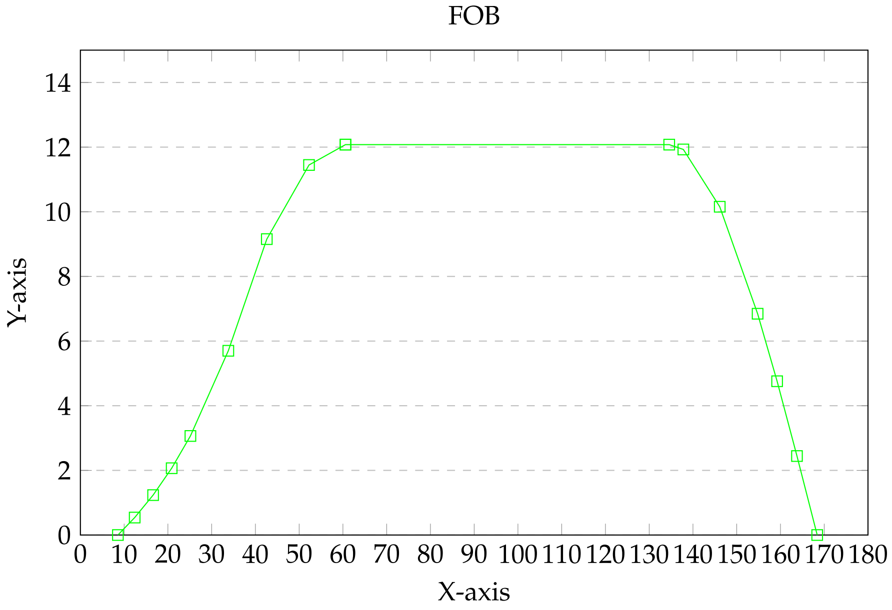


Figure 3.33: Flat of bottom

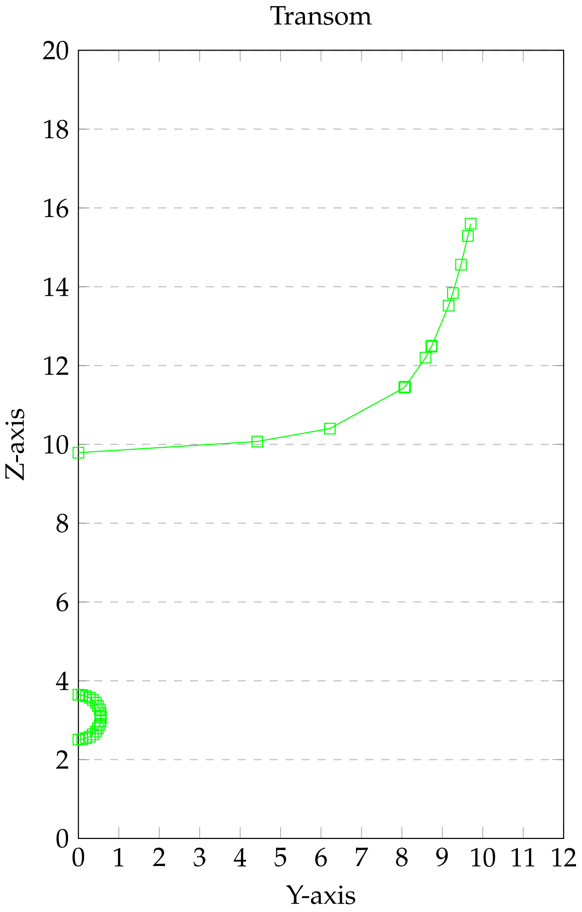


Figure 3.34: Transom

3.2.3 Results using Optimum Technique

In the previous section, we present the results of all the available optimization techniques that were developed for a model that we create for the needs of this study. For the real vessel we only use the optimization process that came up with the best results for the model that we create (DlibFindMinUsingApproximateDerivatives), as it is impossible for any of the optimization techniques to produce better results in much more difficult problem.

3.2.3.1 *Results with DlibFindMinUsingApproximateDerivatives - Objective function for area attributes*

In the table below 3.25 we can see the percentage difference at the attributes of the hull between the calculated geometry and the created model. The entire data appears at Appendix A. We observe that the differences between the two models for the area attributes are zero in all the waterlines. Although, for the volume attributes the differences are not zero and the volume diversity reaches 6.617 percent for the entire hull.

Table 3.25: Differences in attribute - Approximate.Derivatives - ObjFunc.Area - Aveva.Model

Waterline 1.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	3699.729	3699.729	0.000
Inertia [m^4]	206390.703	206390.710	0.000
LCF [m]	94.606	94.606	0.000
Volume [m^3]	3550.620	3316.972	6.580
WSA [m^2]	3803.479	3777.169	0.691
LCB [m]	94.737	94.400	0.355
KB [m]	0.515	0.518	0.758

Waterline 3.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4031.179	4038.177	0.173
Inertia [m^4]	245117.703	243686.623	0.583
LCF [m]	94.084	94.084	0.000
Volume [m^3]	11597.669	11021.927	4.964
WSA [m^2]	4621.299	4571.176	1.084
LCB [m]	94.456	94.550	0.099
KB [m]	1.555	1.564	0.594

Waterline 5.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4121.939	4121.936	0.000
Inertia [m^4]	255044.093	255043.955	0.000
LCF [m]	93.341	93.341	0.000
Volume [m^3]	19991.750	19165.671	4.132
WSA [m^2]	5375.649	5320.200	1.031
LCB [m]	94.200	94.323	0.130
KB [m]	2.583	2.601	0.692

Waterline 10.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4517.959	4529.251	0.249
Inertia [m^4]	285150.687	288204.298	1.059
LCF [m]	82.081	82.081	0.000
Volume [m^3]	42000.851	40551.944	3.449
WSA [m^2]	7556.029	7454.559	1.342
LCB [m]	91.328	91.351	0.025
KB [m]	5.180	5.202	0.440

Waterline 14.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4668.109	4668.108	0.000
Inertia [m^4]	298844.000	298843.859	0.000
LCF [m]	81.051	81.050	0.001
Volume [m^3]	60975.898	57461.214	5.764
WSA [m^2]	9126.330	8956.834	1.857
LCB [m]	88.148	90.271	2.352
KB [m]	7.305	7.227	1.066

Waterline 15.600 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4706.060	4706.060	0.000
Inertia [m^4]	301400.500	301400.466	0.000
LCF [m]	81.386	81.385	0.000
Volume [m^3]	68693.898	64148.377	6.617
WSA [m^2]	9752.259	9546.809	2.106
LCB [m]	87.370	90.196	3.133
KB [m]	8.147	8.030	1.424

The square-differences and the minimum distances between the them are displayed in the upcoming tables 3.32, 3.33. The mean error of minimum distances between the two curves is at 22cm with the maximum error at 129cm. Contrary to the produced model where the biggest differences where found in the stern region, in the Aveva model the biggest differences appear to be in the bow region. The aveva model we use has a complex bow region and the algorithm can not find a very good approach. We must mention that in AVEVA Initial Design™ for the creation of the bulb were used over one hundred points to reach the desired geometry.

Table 3.26: Square y-Difference - Approximate.Derivatives - ObjFunc.Area - Aveva.Model

Waterline.Depth [m]	Square.y-Difference $[m^2]$	Position.of.Max.Vertical.Distance [m]	Max.Vertical.Distance [m]
1.000	3.884	172.323	0.914
3.000	9.904	35.608	1.056
3.500	7.178	175.028	0.967
5.000	4.318	175.536	1.060
7.000	7.699	175.446	1.163
9.000	20.591	-0.381	2.650
10.000	6.342	-1.626	1.381
12.000	2.418	167.237	0.729
14.000	1.107	172.516	0.497
15.600	1.316	173.447	0.604

Table 3.27: Minimum Distances - Approximate.Derivatives - ObjFunc.Area - Aveva.Model

Waterline.Depth [m]	Sum.Square.Min.Distance $[m^2]$	Position.of.Max.Distance [m]	Max.Distance [m]
1.000	3.383	171.924	0.780
3.000	8.664	35.943	0.994
3.500	6.105	174.592	0.803
5.000	3.482	175.067	0.892
7.000	6.199	174.949	0.999
9.000	9.957	0.703	1.294
10.000	4.841	-0.961	1.076
12.000	2.097	167.519	0.660
14.000	0.935	168.466	0.439
15.600	1.100	167.144	0.521

The differences between the waterlines are displayed in the following figures 3.43, 3.44. It appears that the waterlines are close to the desired geometry but it also obvious the problem that we mentioned before for the bulb area.

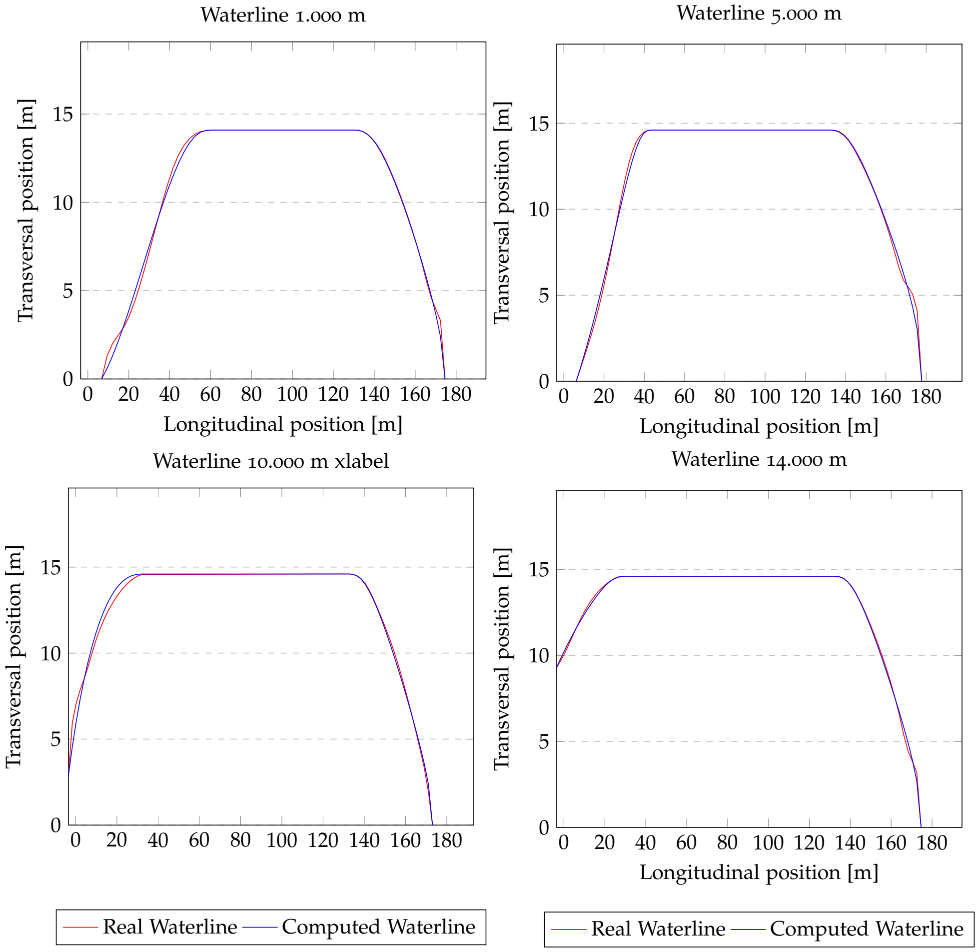


Figure 3.35: Waterlines- Computed vs Real - Approximate.Derivatives - ObjFunc.Area - Aveva.Model

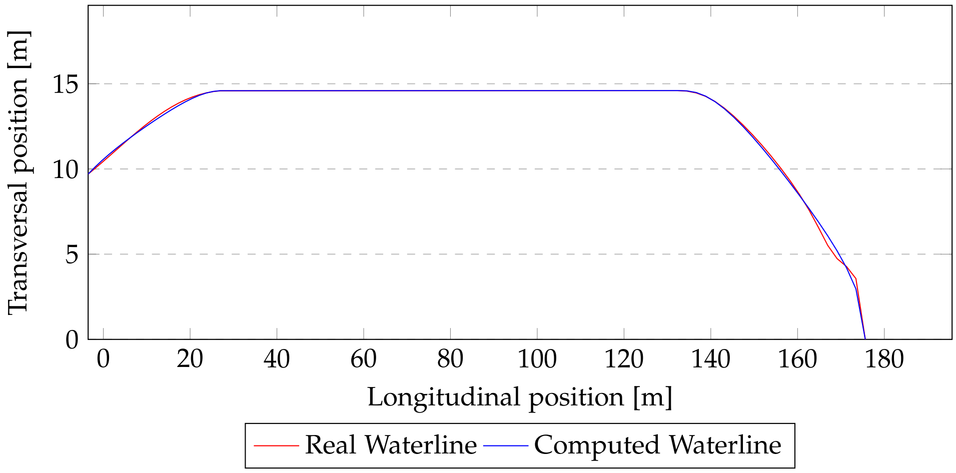
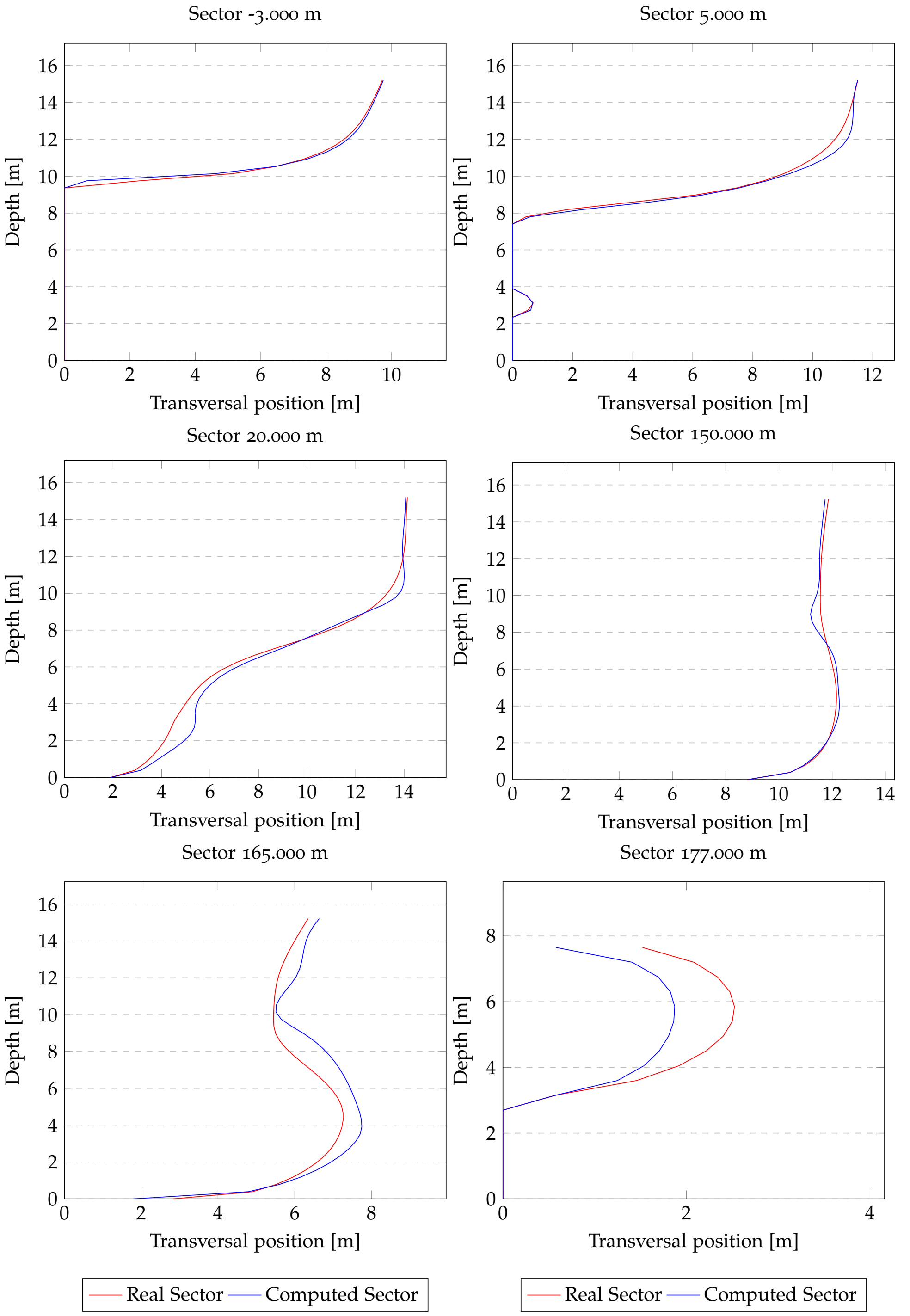


Figure 3.36: Waterline 15.600 m - Computed vs Real - Approximate.Derivatives - ObjFunc.Area - Aveva.Model

The figures of the sectors between the computed and the desired geometry are displayed 3.45 below. Despite the fact that the differences can easily be spotted the shape of the sections appear to be very close to the actual sections.



Sector -3.000 m

Sector 5.000 m

Sector 20.000 m

Sector 150.000 m

Sector 165.000 m

Sector 177.000 m

Figure 3.37: Sectors - Computed vs Real - Approximate.Derivatives - ObjFunc.Area - Aveva.Model

3.2.3.2 *Results with DlibFindMinUsingApproximateDerivatives - Objective function for volume attributes*

In the table below 3.28 are presented the percentage differences between the calculated geometry and the created model. The entire data appears at Appendix A. In the previous optimization process the area attributes were all zero, but in this process where we have included the volume attributes the area attributes along with the volume attributes are not zero and are increasing as the depth increases. This increase is due to the fact that the volume errors of every depth is transferred from depth to depth because calculation for each depth includes all the previous generated shapes.

Table 3.28: Differences in attribute - Approximate.Derivatives - ObjFunc.Volume - Aveva.Model

Waterline 1.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	3699.729	3699.729	0.000
Inertia [m^4]	206390.703	206390.705	0.000
LCF [m]	94.606	94.606	0.000
Volume [m^3]	3550.620	3316.936	6.581
WSA [m^2]	3803.479	3777.114	0.693
LCB [m]	94.737	94.399	0.356
KB [m]	0.515	0.518	0.758
Waterline 3.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4031.179	4031.179	0.000
Inertia [m^4]	245117.703	245117.797	0.000
LCF [m]	94.084	94.084	0.000
Volume [m^3]	11597.669	11014.419	5.029
WSA [m^2]	4621.299	4567.110	1.172
LCB [m]	94.456	94.554	0.104
KB [m]	1.555	1.563	0.562
Waterline 5.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4121.939	4121.939	0.000
Inertia [m^4]	255044.093	255044.017	0.000
LCF [m]	93.341	93.341	0.000
Volume [m^3]	19991.750	19144.636	4.237
WSA [m^2]	5375.649	5311.513	1.193
LCB [m]	94.200	94.326	0.133
KB [m]	2.583	2.600	0.665
Waterline 10.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4517.959	4517.959	0.000
Inertia [m^4]	285150.687	285174.743	0.008
LCF [m]	82.081	82.081	0.000
Volume [m^3]	42000.851	40504.764	3.562
WSA [m^2]	7556.029	7441.465	1.516
LCB [m]	91.328	91.327	0.000
KB [m]	5.180	5.202	0.438
Waterline 14.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4668.109	4668.109	0.000
Inertia [m^4]	298844.000	299341.397	0.166
LCF [m]	81.051	81.051	0.000
Volume [m^3]	60975.898	57364.463	5.922
WSA [m^2]	9126.330	8937.705	2.066
LCB [m]	88.148	90.262	2.342
KB [m]	7.305	7.224	1.097
Waterline 15.600 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4706.060	4706.060	0.000
Inertia [m^4]	301400.500	301968.439	0.188
LCF [m]	81.386	81.386	0.000
Volume [m^3]	68693.898	64051.634	6.757
WSA [m^2]	9752.259	9527.249	2.307
LCB [m]	87.370	90.187	3.123
KB [m]	8.147	8.030	1.434

The square-differences and the minimum distances between the them are displayed in the upcoming tables 3.29, 3.30. Like the previous optimization process the maximum distances appear to be in the bulbous area. The mean error of minimum distances between the two curves is at 20cm with the maximum error at 123cm.

Table 3.29: Square y-Difference - Approximate.Derivatives - ObjFunc.Volume - Aveva.Model

Waterline.Depth [m]	Square.y-Difference $[m^2]$	Position.of.Max.Vertical.Distance [m]	Max.Vertical.Distance [m]
1.000	3.874	172.323	0.924
3.000	7.129	174.701	1.325
3.500	5.717	175.028	1.198
5.000	4.325	175.536	1.061
7.000	6.114	175.446	1.342
9.000	24.714	-0.381	2.650
10.000	4.935	-1.626	1.068
12.000	0.769	171.346	0.383
14.000	0.935	172.516	0.688
15.600	1.228	173.447	0.827

Table 3.30: Minimum Distances - Approximate.Derivatives - ObjFunc.Volume - Aveva.Model

Waterline.Depth [m]	Sum.Square.Min.Distance $[m^2]$	Position.of.Max.Distance [m]	Max.Distance [m]
1.000	3.369	171.920	0.788
3.000	5.937	174.113	1.111
3.500	4.690	174.496	1.003
5.000	3.487	175.067	0.892
7.000	4.939	174.882	1.162
9.000	13.597	0.668	1.227
10.000	3.755	-1.106	0.800
12.000	0.666	171.171	0.320
14.000	0.739	172.207	0.578
15.600	0.951	173.077	0.693

The difference between the waterlines is displayed in the following figures 3.43, 3.44.

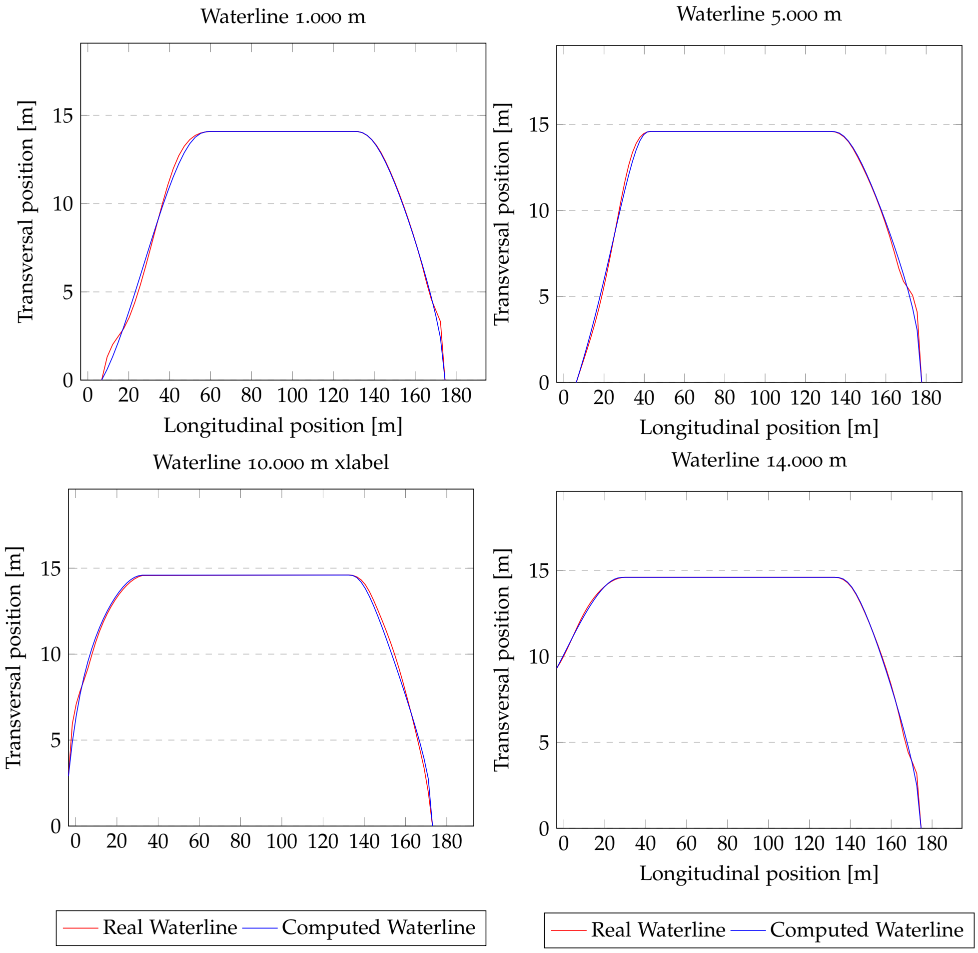


Figure 3.38: Waterlines- Computed vs Real - Approximate.Derivatives - ObjFunc.Volume - Aveva.Model

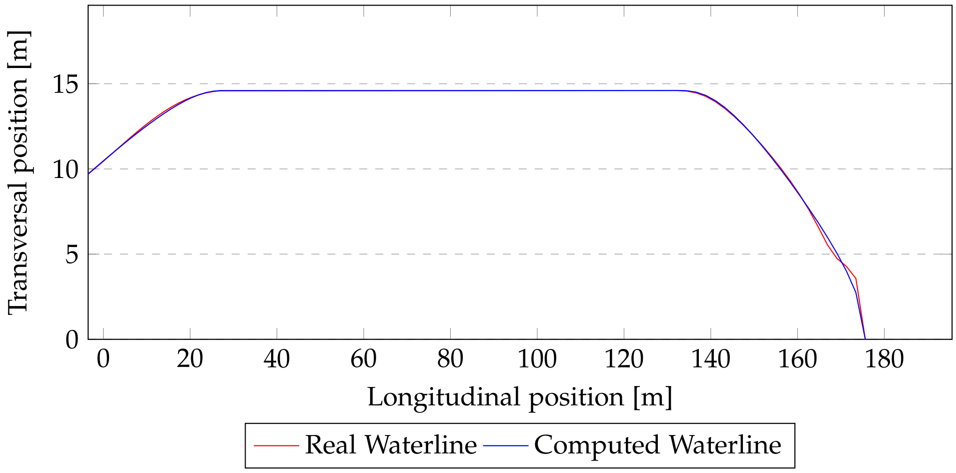


Figure 3.39: Waterline 15.600 m - Computed vs Real - Approximate.Derivatives - ObjFunc.Volume - Aveva.Model

The figures of the sectors between the computed and the desired geometry are displayed 3.45 below.

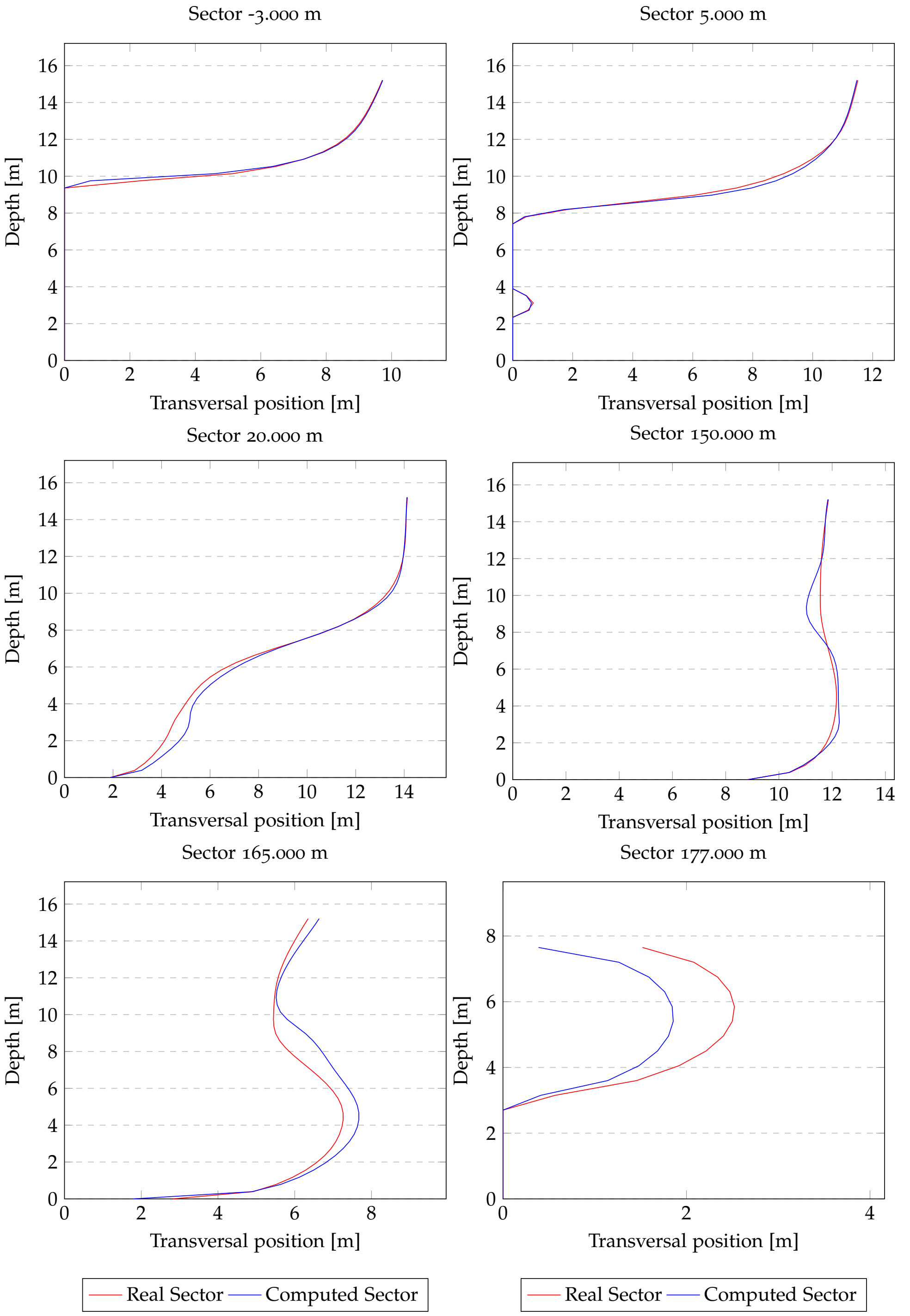


Figure 3.40: Sectors - Computed vs Real - Approximate.Derivatives - ObjFunc.Volume - Aveva.Model

3.2.4 Results using FOB

Because the previous results were not so close we tried a different approach. We used the flat of bottom, which is the waterline at depth zero as an initialization. In order to create the following waterlines we use the FOB's points as the next waterline's points after scaling them accordingly. And so on, for each waterline we use the previous waterline's points. In this approach, we use as many control points as needed in order to recreate the FOB's waterline as good as possible. In our tests we use,

1. Stern B-spline
   The degree of the spline is 9 with 11 control points
2. Stem B-spline
   The degree of the spline is 7 with 9 control points

For this approach we use 156 waterlines or one waterline every 0.1 m. This amount of waterlines is necessary because when we scale from one waterline to the other, it is easier to accomplish a better initialization. Because of the enormous number of waterlines, some of them are not perfectly smooth. This fact can lead to divergences. Also, in this approach there is a small difference in the spline's waterlines at the stem. As mentioned in 2.5.1.2 the first control point's y-coordinate is setted as a percentage of the vessel's breadth. In the previous tests we setted this percentage at 50 percent but in this approach is setted at 17 percent. The reason for this change is the shape of the waterlines at the low depths. The stem region is less inflate and for that reason we chose this initialisation.

In the following figures 3.41, 3.42 we present these waterlines.

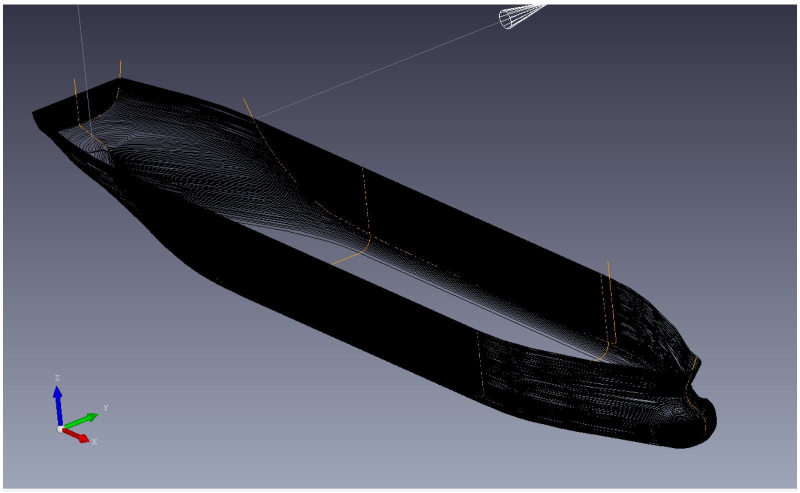
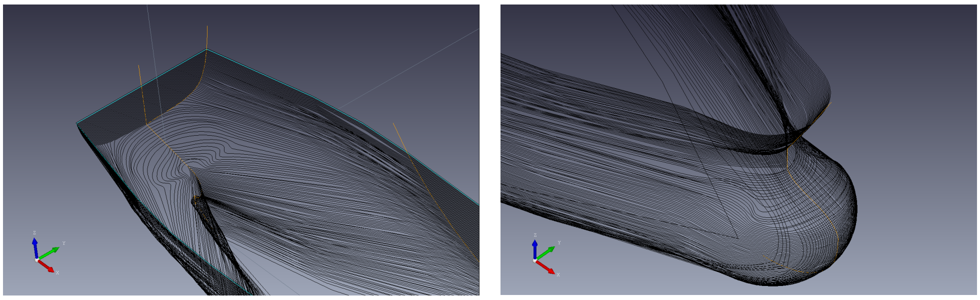


Figure 3.41: Aveva Model - 156 waterlines



(a) Stern View **(b)** Bow View

Figure 3.42: fig:Aveva Model 156 waterlines -Views

In the table below 3.31 we can see the percentage difference at the attributes of the hull between the calculated geometry and the created model. The square-differences and the minimum distances between the them are displayed in the upcoming tables 3.32, 3.33. Because the waterlines are too many we only present some of the results. The results are not better than the ones produced earlier but the mean error for all the waterlines is at 0.268 m . And if we consider that some waterlines do not have the appropriate form then the results are pretty good. As these tables are pretty big, we present only some of the results. The entire tables appear at Appendix A.

Table 3.31: Differences in attribute - Approximate.Derivatives - ObjFunc.Area - Aveva.model.156

Waterline 1.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	3716.822	3716.818	0.000
Inertia [m^4]	208015.812	208015.847	0.000
LCF [m]	94.607	94.611	0.004
Volume [m^3]	3467.250	3473.858	0.190
WSA [m^2]	3818.393	4055.314	5.842
LCB [m]	94.551	95.374	0.863
KB [m]	0.514	0.514	0.049

Waterline 3.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4037.486	4042.905	0.134
Inertia [m^4]	246213.375	246269.153	0.022
LCF [m]	94.052	96.400	2.435
Volume [m^3]	11297.814	11308.861	0.097
WSA [m^2]	4628.048	5697.869	18.775
LCB [m]	94.440	95.573	1.184
KB [m]	1.553	1.552	0.028

Waterline 5.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4122.475	4132.083	0.232
Inertia [m^4]	255708.750	255763.183	0.021
LCF [m]	93.387	96.004	2.725
Volume [m^3]	19457.250	19474.311	0.087
WSA [m^2]	5374.536	7232.698	25.691
LCB [m]	94.274	96.047	1.846
KB [m]	2.580	2.580	0.013

Waterline 10.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4523.910	4531.800	0.174
Inertia [m^4]	286013.531	283804.389	0.772
LCF [m]	82.157	84.624	2.915
Volume [m^3]	40739.292	40856.058	0.285
WSA [m^2]	7547.717	11086.540	31.919
LCB [m]	91.253	93.287	2.180
KB [m]	5.168	5.174	0.113

Waterline 14.000 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4672.187	4674.075	0.040
Inertia [m^4]	299591.718	299590.779	0.000
LCF [m]	81.065	79.883	1.457
Volume [m^3]	57439.718	57606.564	0.289
WSA [m^2]	9046.807	12881.799	29.770
LCB [m]	90.386	91.794	1.534
KB [m]	7.179	7.183	0.045

Waterline 15.600 m

Attribute	Real	Computed	Percentage difference
Area [m^2]	4709.800	4712.190	0.050
Inertia [m^4]	302136.375	302135.219	0.000
LCF [m]	81.390	79.978	1.734
Volume [m^3]	64123.421	63882.649	0.375
WSA [m^2]	9637.863	13444.703	28.314
LCB [m]	90.289	91.425	1.241
KB [m]	7.986	7.938	0.606

Table 3.32: Square y-Difference - Approximate.Derivatives - ObjFunc.Area - Aveva.Model.156

Waterline.Depth [m]	Square.y-Difference $[m^2]$	Position.of.Max.Vertical.Distance [m]	Max.Vertical.Distance [m]
0.100	2.539	167.527	0.685
1.800	6.065	173.486	0.961
6.800	5.341	175.553	1.372
9.000	16.113	1.414	2.172
10.100	8.987	0.243	1.048
12.400	4.930	167.451	0.658
14.200	3.585	166.267	0.593
15.600	2.675	173.447	0.663

Table 3.33: Minimum Distances - Approximate.Derivatives - ObjFunc.Area - Aveva.Model.156

Waterline.Depth [m]	Sum.Square.Min.Distance $[m^2]$	Position.of.Max.Distance [m]	Max.Distance [m]
0.100	2.297	167.304	0.643
1.800	4.466	173.062	0.813
6.800	4.334	174.984	1.195
9.000	10.729	2.472	1.429
10.100	7.873	0.642	0.954
12.400	4.596	22.574	0.627
14.200	3.316	166.479	0.547
15.600	2.374	167.166	0.555

The differences between the waterlines are displayed in the following figures 3.43, 3.44. The differences can easily be spotted in both the stern and the bow regions.

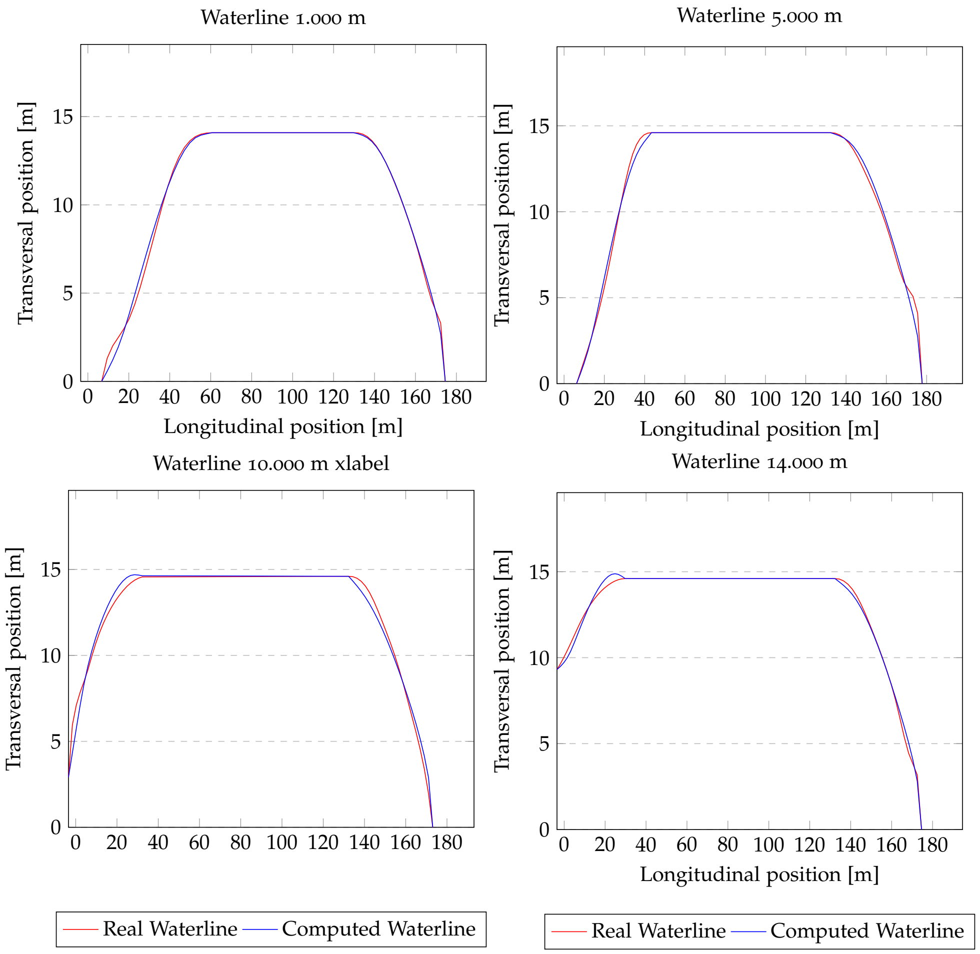


Figure 3.43: Waterlines- Computed vs Real - Approximate.Derivatives - ObjFunc.Area - Aveva.Model.156

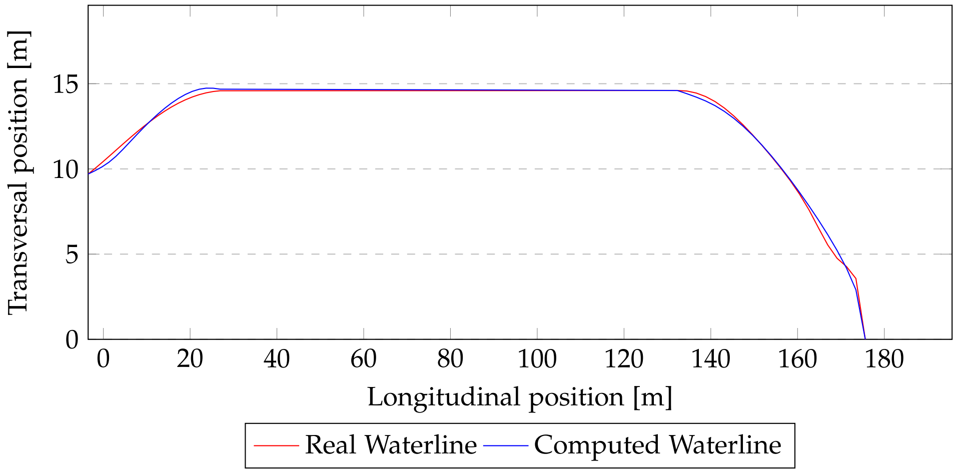


Figure 3.44: Waterline 15.600 m - Computed vs Real - Approximate.Derivatives - ObjFunc.Area - Aveva.model.156

The figures of the sectors between the computed and the desired geometry are displayed 3.45 below. The sections with some exceptions in between the sectors are pretty smooth, especially if we consider the fact that we optimise 156 waterlines. The depths where the sections are not so smooth are between 8.000m and 10.000m . We have mentioned earlier that because of the vast number of waterlines, they are not perfect relatively to smoothness so these results are expected. This remark is verified by the real sector at the longitudinal position 165 m.

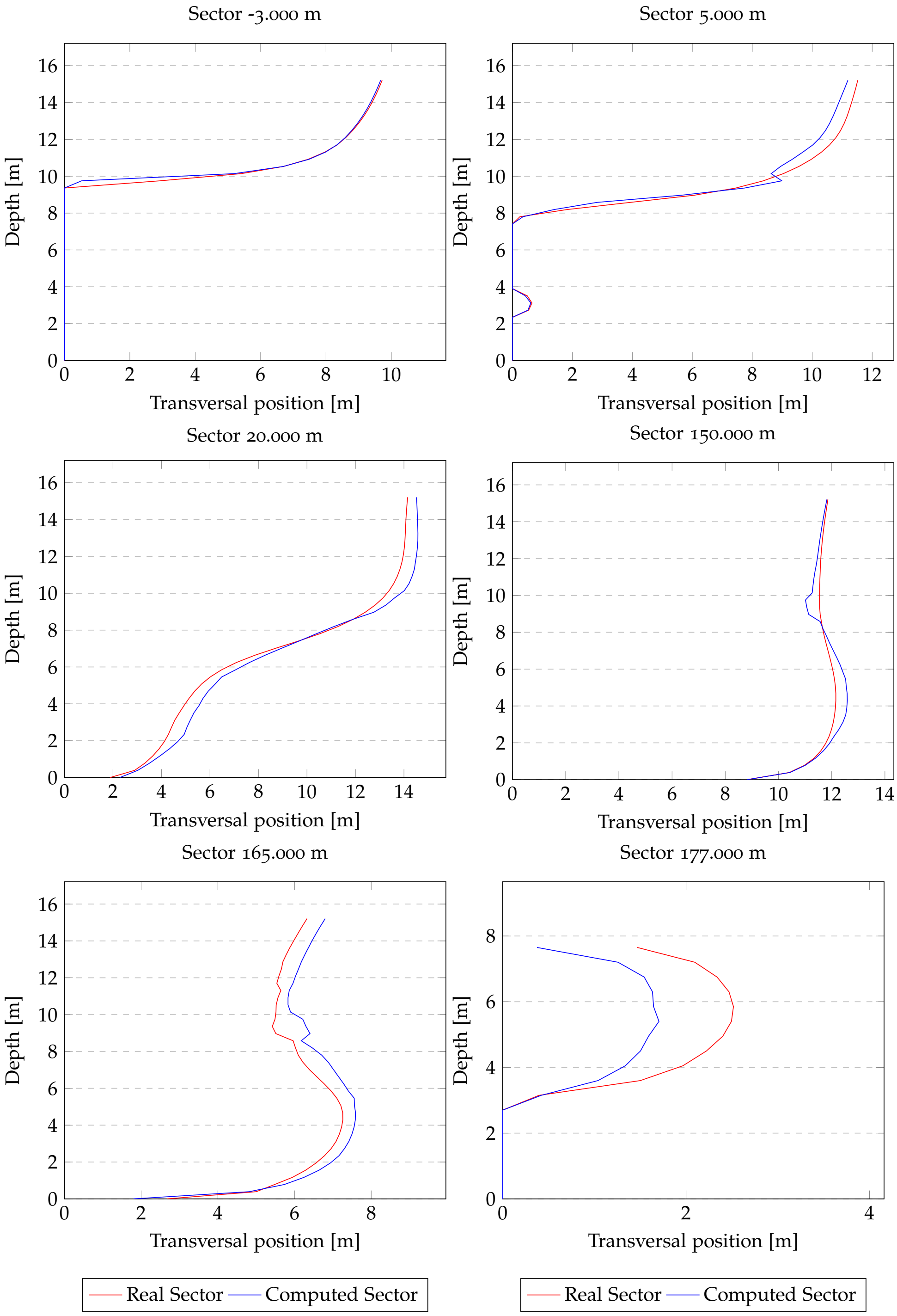


Figure 3.45: Sectors - Computed vs Real - Approximate.Derivatives - ObjFunc.Area - Aveva.Model.156

3.3 ANALYSIS OF THE RESULTS

As we can see from the produced results the best optimization procedure is when we use only the DlibFindMinUsingApproximateDerivatives algorithm. The algorithm's success is based on the starting position of our control points. In the case where we use the DlibFindMinUsingApproximateDerivatives algorithm along with the objective function both for the area attributes and volume attributes the results of the function are very close to zero. That means that the algorithm reaches an optimum local minimum based on its initialization. For the other two algorithms DlibFindMinGlobal and OpenGA, we do not expect from them to reach a global minimum by themselves. We expect from these algorithms to find positions of the control points as close as possible to the global minimum in order to use the DlibFindMinUsingApproximateDerivatives to find the minimum value. Although, none of these algorithms could find a solution better than our first initialisation. When we used a combination either of these algorithms along with the DlibFindMinUsingApproximateDerivatives algorithm could produce a good result. Finally, in the approach where we use the flat of bottom as a starting set up for our control points, the results are not better than by just using the DlibFindMinUsingApproximateDerivatives algorithm but both the produced waterlines and the sections are smooth curves, which is impressive if we consider that we use 156 waterlines.

3.3.1 Model a

For the model that we produced the results are pretty good. The average of the maximum distances between the computed waterlines and the model's waterlines that were produced by the DlibFindMinUsingApproximateDerivatives algorithm for the objective function for the area attributes only is 0.080 m, while the mean error of all points is 0.024 m. For the objective function that includes the volume attributes the average of the maximum distances between the computed waterlines and the model's waterlines is 0.07 cm and the mean error of all points is 0.023 cm. The reason why they are so good is because the model is created in a way to be very close to our first initialization. We include this example to prove that if our initialization is close enough to the desired design, it is possible to achieve the regeneration of an actual vessel.

3.3.2 AVEVA Model

The results of the real vessel are not so good. The average of the maximum distances between the computed waterlines and the model's waterlines that were produced by the DlibFindMinUsingApproximateDerivatives algorithm for the objective function for the area attributes only is 0.845 m, while the mean error of all points is 0.221 m. For the objective function that includes the volume attributes the average of the maximum distances between the computed waterlines and the model's waterlines is 0.857 cm and the mean error of all points is 0.205 cm.

In both cases, the biggest problem that we confront is reaching the desired geometry at the regions were the changes in curvature were very sharp at the stern and the stem regions.

4 CONCLUSIONS AND FUTURE WORK

4.1 CONCLUSIONS

The purpose of the present thesis is to introduce a novel approach for reverse engineering of the hull geometry of conventional cargo vessels. Today, reconstruction of hull geometry is made either manually, by reproducing the 3D geometry from several 2D drawings of the vessel, or with in-situ measurements, by utilizing emerging technologies such as the 3D Laser Scanning method or photogrammetry. Both methods are time consuming, costly and require great amounts of manual work. In the present thesis we developed and validated a computational procedure which, by utilizing very few input data, such as the hydrostatic table of the vessel, can lead to reconstruction of the vessel hull with acceptable accuracy. The results of the present study have demonstrated that the presented algorithm is capable of generating the hull of an actual cargo vessel with a mean geometry error of the order of 2%. This reconstruction cannot be considered sufficient for detailed analysis of the ship, but it forms a proof of concept of the proposed methodology. Additional work on the CAD modeling and fine tuning of the optimization algorithms will be required to attain hull geometry approximations with very small error tolerances. Furthermore, the application of the present algorithm has been limited to only two vessel models. More models with different hull shapes should be used in order to make the algorithm generic enough to be applicable to different ship types and hull designs (e.g. bulk carriers, tankers, containerships, etc.). However, the main advantage of the present study lies in the fact that the requirement of input data in minimal. The required data are generally available for most cargo vessels and it can be easily extracted for any vessel with no extra cost.

4.2 FUTURE WORK

The genetic algorithm 2.6.3 did not produce such good results. However, there are a lot of changes that could be done in order to make this algorithm more efficient. Namely, there are two important points at the algorithm's procedure that can change its behaviour. The first one has to do with the population; we accept a collection of genes (individuals) to be part of our population (we choose 100 individuals) if the objective function's result of genes collection is less than number x, in our case is 5000. We chose this limit because the algorithm had difficulties in finding better initial population with numbers smaller than that. Yet, more tests need to be done, examining the population and the down limit of a collection of genes to be accepted. The second point has to do with mutation; we choose to mutate a random number of genes by altering their values according to the vessel's initial attributes, like breadth or length. Perhaps, a different approach would be more beneficial and yield more accurate results. Two very important factors that affect the optimization algorithms FindMinGlobal and GA are the upper and lower bounds for the y-coordinate of the control points. If the variable mathematical spaces were smaller the algorithms might find better solutions faster. Still, this scenario must also be tested thoroughly. Another important factor is the number of control points. In chapter 3 we did some changes in order to gain better results by adding more control points. Although, as the number of control points is growing the waterline's shape is failing due to the vast number of variables. Thereby, further testing need to take place considering the number of control points and probably the starting position of those extra points, because in our approaches the control points are equally distributed in each curve. Moreover, there are different optimization algorithms that can be tested with the methodology that has already been created and will possibly produce better results than those produced. Additionally, a multi-objective function process that will focus in different attributes of the waterplane area and the volume's shape will create a pareto of solutions that can also lead to better results.

Furthermore, there is one more solution based on this methodology but is slightly different. We can create a data base of vessels by feeding our program with a lot of different vessels. In this way, we will possess the hydrostatic attributes for all of these ships and most importantly the attributes that have to do with the form of the hull, like the block coefficient (C_B) and the prismatic coefficient (C_M). Thereby, when we try to reverse engineer a vessel we will compare its attributes with the ones that exist in our database and then scale the database-vessel with the closest attributes accordingly in order to meet the attributes of the desired vessel. This way, we will be able to obtain a very good first approach, helping our optimization algorithms to generate better results.

Last but not least, all the above suggestions can be applied along parallel programming. If the processes are carried out simultaneously, the time reduction will be significant, allowing us to make more iterations of an algorithm and many more tests in shorter time.

BIBLIOGRAPHY

[1] R. Vinesh and J. F. Kiran. *Reverse Engineering, An Industrial Perspective*. London, England: Springer-Verlag, 2008. DOI: 10.1007/978-1-84628-856-2.

[2] H. J. Koelman. "Application of a photogrammetry-based system to measure and re-engineer ship hulls and ship parts: An industrial practices-based report". In: *Computer-Aided Design* 42 (2010), pp. 731–743. DOI: 10.1016/j.cad.2010.02.005.

[3] Bertil Hallert. "Photogrammetry: Basic Principles and General Survey". In: *Geological Magazine* 98 (1961), pp. 538–538. DOI: 10.1017/S0016756800062257.

[4] Aber J.S. et al. "Chapter 3 - Principles of Photogrammetry". In: *Small-Format Aerial Photography and UAS Imagery (Second Edition)*. Ed. by Aber J. S. et al. Academic Press, 2019, pp. 19–38. DOI: /10.1016/B978-0-12-812942-5.00003-3.

[5] D. M. Edess. "A Framework for Hull Form Reverse Engineering and Geometry Integration into Numerical Simulations". PhD thesis. University of Rostock, 2016.

[6] M. Deja, M. Dobrzyński, and M. Rymkiewicz. "Application of reverse engineering technology in part design for shipbuilding industry". In: *Polish Maritime Research* 26 (June 2019), pp. 126–133. DOI: 10.2478/pomr-2019-0032.

[7] E.B. Njaastad, N.H. Munthe-Kaas, and O. Egeland. "Robotic Autoscanning of Highly Skewed Ship Propeller Blades". In: *IFAC-PapersOnLine* 51 (2018), pp. 435–440. DOI: 10.1016/j.ifacol.2018.11.595.

[8] F. Menna and S. Troisi. "Low cost reverse engineering techniques for 3D modelling of propellers". In: *International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences* 38 (Jan. 2010), pp. 452–457.

[9] E. Kolyvas et al. "Application of photogrammetry techniques for the visual assessment of vessels' cargo hold". In: Sept. 2015, pp. 381–387. DOI: 10.1201/b18855-50.

[10] D. Wujanz et al. "Fusion of Point Clouds from TLS and MVS for the Generation of a 3D Ship Model". In: *Allgemeine Vermessungsnachrichten AVN* 03 (May 2013), pp. 83–89.

[11] A. N. Tassetti, M. Martelli, and G. Buglioni. "Reverse Engineering Techniques for Trawler Hull 3D Modeling and Energy Efficiency Evaluation". In: *18th International Conference on Ships and Shipping Research 2015* (June 24-26, 2015), pp. 1–12.

[12] F. Menna, E. Nocerino, and A. Scamardella. "Reverse Engineering and 3D Modeling for Digital Documentation of Maritime Heritage". In: *International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences* XXXVIII-5/W16 (Sept. 2012), pp. 245–252. DOI: 10.5194/isprsarchives-XXXVIII-5-W16-245-2011.

[13] M. A. Abbas et al. "Improvements to the accuracy of prototype ship models measurement method using terrestrial laser scanner". In: *Measurement* 100 (2017), pp. 301–310. DOI: 10.1016/j.measurement.2016.12.053.

[14] P. Burdziakowski and P. Tysiac. "Combined Close Range Photogrammetry and Terrestrial Laser Scanning for Ship Hull Modelling". In: *Geosciences* 9 (May 2019), p. 242. DOI: 10.3390/geosciences9050242.

[15] C.H.H. Tang, H.E. Tang, and P.K.J. Tay. "Low cost digital close range photogrammetric measurement of an as-built anchor handling tug hull". In: *Ocean Engineering* 119 (2016), pp. 67–74. DOI: 10.1016/j.oceaneng.2016.04.016.

[16] F. Menna and S. Troisi. “Photogrammetric 3D modelling of a boat’s hull”. In: pp. 347–354.

[17] Nasdaq Helsinki: QTCOM. *Qt Group*. https://www.qt.io/. May 1995.

[18] M. Datavision. *Open CASCADE Technology*. https://www.opencascade.com/. 1999.

[19] D.E. King. *Dlib*. http://dlib.net/. 2002.

[20] A. Mohammadi et al. “OpenGA, a C++ Genetic Algorithm Library”. In: *2017 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*. 2017, pp. 2051–2056.

[21] *MATLAB version 8.5.0.197613 (R2015a)*. Natick, Massachusetts: The Mathworks, Inc., 2015.

[22] C. Malherbe and N. Vayatis. “Global optimization of Lipschitz functions”. In: *Proceedings of the 34th International Conference on Machine Learning*. Ed. by D. Precup and Y.W. Teh. Vol. 70. Proceedings of Machine Learning Research. International Convention Centre, Sydney, Australia: JMLR.org, June 2017, pp. 2314–2323. DOI: 10.5555/3305890.3305920.

[23] M.J.D. Powell. “The NEWUOA software for unconstrained optimization without derivatives”. In: *Large-Scale Nonlinear Optimization*. Ed. by M. Di Pillo G.and Roma. Boston, MA: Springer US, 2006, pp. 255–297. DOI: 10.1007/0-387-30065-1_16.

A APPENDIX A

A.1 SOURCE CODE

A.2 DATA MODELS

In this section we present the full tables of the differences in the attributes for all the optimization processes.

A.2.1 Results using DlibFindMinUsingApproximateDerivatives

A.2.1.1 *Objective function for area attributes*

Waterline 1.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3638.650	3638.650	0.000
Inertia $[m^4]$	200695.437	200695.437	0.000
LCF [m]	96.473	96.473	0.000
Volume $[m^3]$	3286.598	3286.465	0.004
WSA $[m^2]$	3725.620	3725.822	0.005
LCB [m]	95.392	95.388	0.004
KB [m]	0.517	0.517	0.000
Waterline 2.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3801.612	3801.612	0.000
Inertia $[m^4]$	222918.328	222918.328	0.000
LCF [m]	96.733	96.733	0.000
Volume $[m^3]$	7006.495	7006.361	0.001
WSA $[m^2]$	4113.554	4113.767	0.005
LCB [m]	96.036	96.033	0.002
KB [m]	1.041	1.041	0.001
Waterline 3.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4040.104	4040.103	0.000
Inertia $[m^4]$	243814.578	243814.584	0.000
LCF [m]	94.297	94.297	0.000
Volume $[m^3]$	10913.262	10913.075	0.001
WSA $[m^2]$	4591.129	4591.100	0.000
LCB [m]	95.934	95.933	0.001
KB [m]	1.565	1.565	0.000
Waterline 4.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4065.494	4065.493	0.000
Inertia $[m^4]$	249646.921	249646.923	0.000
LCF [m]	94.537	94.537	0.000
Volume $[m^3]$	14949.680	14949.697	0.000
WSA $[m^2]$	4969.153	4969.520	0.007
LCB [m]	95.604	95.602	0.001

KB [m]	2.088	2.088	0.001
Waterline 5.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4103.510	4103.665	0.003
Inertia $[m^4]$	253854.937	253854.781	0.000
LCF [m]	94.149	94.300	0.160
Volume $[m^3]$	19034.183	19034.241	0.000
WSA $[m^2]$	5336.434	5335.612	0.015
LCB [m]	95.333	95.348	0.015
KB [m]	2.605	2.605	0.001
Waterline 6.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4137.835	4137.971	0.003
Inertia $[m^4]$	257324.015	257323.874	0.000
LCF [m]	93.679	93.814	0.143
Volume $[m^3]$	23154.855	23155.071	0.000
WSA $[m^2]$	5701.997	5701.429	0.009
LCB [m]	95.081	95.118	0.039
KB [m]	3.120	3.120	0.001
Waterline 7.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4169.185	4169.184	0.000
Inertia $[m^4]$	260325.875	260325.877	0.000
LCF [m]	93.012	93.012	0.000
Volume $[m^3]$	27308.365	27308.671	0.001
WSA $[m^2]$	6069.280	6071.053	0.029
LCB [m]	94.817	94.859	0.044
KB [m]	3.634	3.634	0.001
Waterline 8.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4200.771	4200.914	0.003
Inertia $[m^4]$	263016.375	263016.224	0.000
LCF [m]	91.542	91.683	0.153
Volume $[m^3]$	31493.345	31493.745	0.001
WSA $[m^2]$	6456.085	6454.035	0.031
LCB [m]	94.479	94.525	0.048
KB [m]	4.148	4.148	0.000
Waterline 9.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4229.549	4229.548	0.000
Inertia $[m^4]$	265277.375	265277.378	0.000
LCF [m]	89.067	89.067	0.000
Volume $[m^3]$	35708.503	35708.921	0.001
WSA $[m^2]$	6880.507	6884.066	0.051
LCB [m]	93.986	94.035	0.052
KB [m]	4.661	4.662	0.000
Waterline 10.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4403.614	4403.614	0.000
Inertia $[m^4]$	274006.906	274006.906	0.000
LCF [m]	85.496	85.496	0.000
Volume $[m^3]$	39943.433	39944.097	0.001
WSA $[m^2]$	7369.131	7372.643	0.047
LCB [m]	93.398	93.441	0.046
KB [m]	5.177	5.177	0.000

Waterline 11.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4562.796	4562.174	0.013
Inertia $[m^4]$	285926.312	285926.662	0.000
LCF [m]	82.786	82.999	0.256
Volume $[m^3]$	44146.347	44146.666	0.000
WSA $[m^2]$	7842.342	7839.784	0.032
LCB [m]	92.987	93.038	0.054
KB [m]	5.688	5.688	0.000
Waterline 12.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4604.721	4604.295	0.009
Inertia $[m^4]$	290190.062	290190.274	0.000
LCF [m]	82.324	82.471	0.178
Volume $[m^3]$	48301.007	48300.815	0.000
WSA $[m^2]$	8215.772	8214.250	0.018
LCB [m]	92.731	92.794	0.068
KB [m]	6.195	6.195	0.001
Waterline 13.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4637.713	4637.376	0.007
Inertia $[m^4]$	293313.968	293314.159	0.000
LCF [m]	82.228	82.345	0.141
Volume $[m^3]$	52455.910	52455.340	0.001
WSA $[m^2]$	8584.437	8583.167	0.014
LCB [m]	92.535	92.605	0.075
KB [m]	6.703	6.702	0.001
Waterline 14.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4665.364	4665.085	0.005
Inertia $[m^4]$	295876.375	295876.531	0.000
LCF [m]	82.221	82.319	0.119
Volume $[m^3]$	56619.175	56618.304	0.001
WSA $[m^2]$	8952.595	8951.434	0.012
LCB [m]	92.384	92.457	0.079
KB [m]	7.210	7.210	0.001
Waterline 15.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4689.464	4689.223	0.005
Inertia $[m^4]$	298042.687	298042.821	0.000
LCF [m]	82.268	82.353	0.104
Volume $[m^3]$	60792.464	60791.335	0.001
WSA $[m^2]$	9321.041	9319.928	0.011
LCB [m]	92.267	92.343	0.082
KB [m]	7.719	7.719	0.001
Waterline 15.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4702.847	4702.624	0.004
Inertia $[m^4]$	299220.500	299220.623	0.000
LCF [m]	82.311	82.390	0.096
Volume $[m^3]$	60792.464	60791.335	0.001
WSA $[m^2]$	9321.041	9319.928	0.011
LCB [m]	92.267	92.343	0.082
KB [m]	7.719	7.719	0.001

A.2.1.2 *Objective function for volume attributes*

Waterline 1.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	3638.650	3638.629	0.000
Inertia [m^4]	200695.437	200695.402	0.000
LCF [m]	96.473	96.463	0.011
Volume [m^3]	3286.598	3286.464	0.004
WSA [m^2]	3725.620	3725.727	0.002
LCB [m]	95.392	95.382	0.010
KB [m]	0.517	0.517	0.000
Waterline 2.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	3801.612	3801.608	0.000
Inertia [m^4]	222918.328	222923.164	0.002
LCF [m]	96.733	96.722	0.011
Volume [m^3]	7006.495	7006.347	0.002
WSA [m^2]	4113.554	4113.658	0.002
LCB [m]	96.036	96.025	0.010
KB [m]	1.041	1.041	0.000
Waterline 3.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4040.104	4040.142	0.000
Inertia [m^4]	243814.578	243814.640	0.000
LCF [m]	94.297	94.296	0.001
Volume [m^3]	10913.262	10913.078	0.001
WSA [m^2]	4591.129	4591.123	0.000
LCB [m]	95.934	95.926	0.008
KB [m]	1.565	1.565	0.000
Waterline 4.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4065.494	4065.429	0.001
Inertia [m^4]	249646.921	249652.222	0.002
LCF [m]	94.537	94.584	0.049
Volume [m^3]	14949.680	14949.677	0.000
WSA [m^2]	4969.153	4968.764	0.007
LCB [m]	95.604	95.603	0.000
KB [m]	2.088	2.088	0.001
Waterline 5.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4103.510	4103.510	0.000
Inertia [m^4]	253854.937	253854.794	0.000
LCF [m]	94.149	94.172	0.024
Volume [m^3]	19034.183	19034.166	0.000
WSA [m^2]	5336.434	5336.557	0.002
LCB [m]	95.333	95.340	0.007
KB [m]	2.605	2.605	0.000
Waterline 6.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4137.835	4137.830	0.000
Inertia [m^4]	257324.015	257329.253	0.002
LCF [m]	93.679	93.698	0.019
Volume [m^3]	23154.855	23154.846	0.000
WSA [m^2]	5701.997	5702.094	0.001
LCB [m]	95.081	95.090	0.009

KB [m]	3.120	3.120	0.000
Waterline 7.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4169.185	4169.181	0.000
Inertia $[m^4]$	260325.875	260325.835	0.000
LCF [m]	93.012	93.028	0.017
Volume $[m^3]$	27308.365	27308.356	0.000
WSA $[m^2]$	6069.280	6069.352	0.001
LCB [m]	94.817	94.827	0.011
KB [m]	3.634	3.634	0.000
Waterline 8.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4200.771	4200.769	0.000
Inertia $[m^4]$	263016.375	263021.783	0.002
LCF [m]	91.542	91.560	0.019
Volume $[m^3]$	31493.345	31493.333	0.000
WSA $[m^2]$	6456.085	6456.132	0.000
LCB [m]	94.479	94.490	0.012
KB [m]	4.148	4.148	0.000
Waterline 9.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4229.549	4229.547	0.000
Inertia $[m^4]$	265277.375	265277.219	0.000
LCF [m]	89.067	89.085	0.020
Volume $[m^3]$	35708.503	35708.506	0.000
WSA $[m^2]$	6880.507	6880.545	0.000
LCB [m]	93.986	93.998	0.012
KB [m]	4.661	4.662	0.000
Waterline 10.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4403.614	4403.591	0.000
Inertia $[m^4]$	274006.906	274023.388	0.006
LCF [m]	85.496	85.511	0.018
Volume $[m^3]$	39943.433	39943.666	0.000
WSA $[m^2]$	7369.131	7369.175	0.000
LCB [m]	93.398	93.410	0.013
KB [m]	5.177	5.177	0.000
Waterline 11.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4562.796	4562.776	0.000
Inertia $[m^4]$	285926.312	286186.093	0.090
LCF [m]	82.786	82.792	0.007
Volume $[m^3]$	44146.347	44146.617	0.000
WSA $[m^2]$	7842.342	7842.363	0.000
LCB [m]	92.987	93.000	0.013
KB [m]	5.688	5.688	0.000
Waterline 12.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4604.721	4604.620	0.002
Inertia $[m^4]$	290190.062	290555.615	0.125
LCF [m]	82.324	82.338	0.017
Volume $[m^3]$	48301.007	48301.218	0.000
WSA $[m^2]$	8215.772	8215.393	0.004
LCB [m]	92.731	92.743	0.013
KB [m]	6.195	6.195	0.000

Waterline 13.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4637.713	4637.880	0.003
Inertia $[m^4]$	293313.968	293748.035	0.147
LCF [m]	82.228	82.164	0.077
Volume $[m^3]$	52455.910	52456.169	0.000
WSA $[m^2]$	8584.437	8584.622	0.002
LCB [m]	92.535	92.544	0.009
KB [m]	6.703	6.703	0.000
Waterline 14.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4665.364	4665.656	0.006
Inertia $[m^4]$	295876.375	296364.808	0.164
LCF [m]	82.221	82.105	0.140
Volume $[m^3]$	56619.175	56619.677	0.000
WSA $[m^2]$	8952.595	8952.919	0.003
LCB [m]	92.384	92.384	0.000
KB [m]	7.210	7.210	0.000
Waterline 15.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4689.464	4689.826	0.007
Inertia $[m^4]$	298042.687	298574.868	0.178
LCF [m]	82.268	82.121	0.178
Volume $[m^3]$	60792.464	60793.299	0.001
WSA $[m^2]$	9321.041	9321.292	0.002
LCB [m]	92.267	92.258	0.010
KB [m]	7.719	7.719	0.001
Waterline 15.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4702.847	4703.142	0.006
Inertia $[m^4]$	299220.500	299774.985	0.184
LCF [m]	82.311	82.197	0.137
Volume $[m^3]$	60792.464	60793.299	0.001
WSA $[m^2]$	9321.041	9321.292	0.002
LCB [m]	92.267	92.258	0.010
KB [m]	7.719	7.719	0.001

A.2.2 Results using DlibFindMinGlobal

A.2.2.1 *Objective function for area attributes*

Waterline 1.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3638.650	3637.494	0.031
Inertia $[m^4]$	200695.437	200696.590	0.000
LCF [m]	96.473	95.644	0.859
Volume $[m^3]$	3286.598	3281.002	0.170
WSA $[m^2]$	3725.620	3722.870	0.073
LCB [m]	95.392	94.986	0.425
KB [m]	0.517	0.517	0.000
Waterline 2.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3801.612	3782.160	0.511
Inertia $[m^4]$	222918.328	222916.397	0.000

LCF [m]	96.733	97.808	1.099
Volume $[m^3]$	7006.495	6990.270	0.231
WSA $[m^2]$	4113.554	4139.648	0.630
LCB [m]	96.036	95.912	0.128
KB [m]	1.041	1.040	0.047
Waterline 3.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4040.104	4040.104	0.000
Inertia $[m^4]$	243814.578	243814.578	0.000
LCF [m]	94.297	94.297	0.000
Volume $[m^3]$	10913.262	10887.072	0.239
WSA $[m^2]$	4591.129	4650.952	1.286
LCB [m]	95.934	96.041	0.111
KB [m]	1.565	1.565	0.015
Waterline 4.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4065.494	4092.926	0.670
Inertia $[m^4]$	249646.921	249627.509	0.007
LCF [m]	94.537	93.477	1.121
Volume $[m^3]$	14949.680	14938.395	0.075
WSA $[m^2]$	4969.153	5024.057	1.092
LCB [m]	95.604	95.528	0.078
KB [m]	2.088	2.090	0.110
Waterline 5.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4103.510	4142.407	0.938
Inertia $[m^4]$	253854.937	253819.634	0.013
LCF [m]	94.149	93.356	0.842
Volume $[m^3]$	19034.183	19056.433	0.116
WSA $[m^2]$	5336.434	5392.777	1.044
LCB [m]	95.333	95.072	0.274
KB [m]	2.605	2.611	0.210
Waterline 6.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4137.835	4149.028	0.269
Inertia $[m^4]$	257324.015	257356.316	0.012
LCF [m]	93.679	93.454	0.240
Volume $[m^3]$	23154.855	23202.302	0.204
WSA $[m^2]$	5701.997	5761.166	1.027
LCB [m]	95.081	94.774	0.322
KB [m]	3.120	3.127	0.208
Waterline 7.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4169.185	4184.258	0.360
Inertia $[m^4]$	260325.875	260314.512	0.004
LCF [m]	93.012	92.677	0.359
Volume $[m^3]$	27308.365	27368.294	0.218
WSA $[m^2]$	6069.280	6132.380	1.028
LCB [m]	94.817	94.515	0.318
KB [m]	3.634	3.640	0.163
Waterline 8.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4200.771	4216.270	0.367
Inertia $[m^4]$	263016.375	263076.367	0.022
LCF [m]	91.542	91.287	0.277

Volume [m^3]	31493.345	31568.547	0.238
WSA [m^2]	6456.085	6516.471	0.926
LCB [m]	94.479	94.178	0.318
KB [m]	4.148	4.154	0.139
Waterline 9.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4229.549	4236.753	0.170
Inertia [m^4]	265277.375	265269.988	0.002
LCF [m]	89.067	89.209	0.159
Volume [m^3]	35708.503	35795.211	0.242
WSA [m^2]	6880.507	6929.976	0.713
LCB [m]	93.986	93.712	0.291
KB [m]	4.661	4.667	0.112
Waterline 10.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4403.614	4395.794	0.177
Inertia [m^4]	274006.906	274012.418	0.002
LCF [m]	85.496	85.898	0.468
Volume [m^3]	39943.433	40030.664	0.217
WSA [m^2]	7369.131	7402.476	0.450
LCB [m]	93.398	93.182	0.230
KB [m]	5.177	5.181	0.069
Waterline 11.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4562.796	4557.867	0.108
Inertia [m^4]	285926.312	285929.530	0.001
LCF [m]	82.786	83.050	0.317
Volume [m^3]	44146.347	44226.206	0.180
WSA [m^2]	7842.342	7879.443	0.470
LCB [m]	92.987	92.830	0.169
KB [m]	5.688	5.690	0.025
Waterline 12.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4604.721	4604.725	0.000
Inertia [m^4]	290190.062	290190.060	0.000
LCF [m]	82.324	82.323	0.001
Volume [m^3]	48301.007	48378.556	0.160
WSA [m^2]	8215.772	8261.169	0.549
LCB [m]	92.731	92.600	0.141
KB [m]	6.195	6.196	0.004
Waterline 13.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4637.713	4639.940	0.047
Inertia [m^4]	293313.968	293311.370	0.000
LCF [m]	82.228	81.274	1.160
Volume [m^3]	52455.910	52536.028	0.152
WSA [m^2]	8584.437	8699.213	1.319
LCB [m]	92.535	92.368	0.180
KB [m]	6.703	6.702	0.003
Waterline 14.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4665.364	4665.312	0.001
Inertia [m^4]	295876.375	295876.455	0.000
LCF [m]	82.221	82.235	0.017
Volume [m^3]	56619.175	56700.373	0.143

WSA $[m^2]$	8952.595	9113.651	1.767
LCB [m]	92.384	92.190	0.209
KB [m]	7.210	7.210	0.010
Waterline 15.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4689.464	4689.464	0.000
Inertia $[m^4]$	298042.687	298042.687	0.000
LCF [m]	82.268	82.268	0.000
Volume $[m^3]$	60792.464	60873.508	0.133
WSA $[m^2]$	9321.041	9482.340	1.701
LCB [m]	92.267	92.087	0.194
KB [m]	7.719	7.717	0.018
Waterline 15.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4702.847	4701.347	0.031
Inertia $[m^4]$	299220.500	299221.115	0.000
LCF [m]	82.311	82.763	0.547
Volume $[m^3]$	60792.464	60873.508	0.133
WSA $[m^2]$	9321.041	9482.340	1.701
LCB [m]	92.267	92.087	0.194
KB [m]	7.719	7.717	0.018

A.2.2.2 *Objective function for volume attributes*

Waterline 1.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3638.650	3626.031	0.346
Inertia $[m^4]$	200695.437	200752.463	0.028
LCF [m]	96.473	95.972	0.519
Volume $[m^3]$	3286.598	3276.672	0.302
WSA $[m^2]$	3725.620	3712.842	0.342
LCB [m]	95.392	95.169	0.234
KB [m]	0.517	0.517	0.051
Waterline 2.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3801.612	3807.176	0.146
Inertia $[m^4]$	222918.328	222919.976	0.000
LCF [m]	96.733	95.668	1.101
Volume $[m^3]$	7006.495	6992.863	0.194
WSA $[m^2]$	4113.554	4107.692	0.142
LCB [m]	96.036	95.514	0.543
KB [m]	1.041	1.041	0.056
Waterline 3.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4040.104	4039.454	0.016
Inertia $[m^4]$	243814.578	243823.656	0.003
LCF [m]	94.297	95.594	1.356
Volume $[m^3]$	10913.262	10900.419	0.117
WSA $[m^2]$	4591.129	4577.804	0.290
LCB [m]	95.934	95.660	0.285
KB [m]	1.565	1.566	0.067
Waterline 4.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4065.494	4073.820	0.204
Inertia $[m^4]$	249646.921	249643.347	0.001

LCF [m]	94.537	94.372	0.175
Volume [m^3]	14949.680	14940.131	0.063
WSA [m^2]	4969.153	4963.607	0.111
LCB [m]	95.604	95.548	0.058
KB [m]	2.088	2.089	0.075
Waterline 5.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4103.510	4109.647	0.149
Inertia [m^4]	253854.937	253850.190	0.001
LCF [m]	94.149	94.494	0.365
Volume [m^3]	19034.183	19031.594	0.013
WSA [m^2]	5336.434	5336.926	0.009
LCB [m]	95.333	95.310	0.024
KB [m]	2.605	2.607	0.084
Waterline 6.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4137.835	4152.365	0.349
Inertia [m^4]	257324.015	257321.934	0.000
LCF [m]	93.679	93.176	0.537
Volume [m^3]	23154.855	23162.631	0.033
WSA [m^2]	5701.997	5718.066	0.281
LCB [m]	95.081	95.046	0.036
KB [m]	3.120	3.123	0.094
Waterline 7.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4169.185	4169.621	0.010
Inertia [m^4]	260325.875	260323.744	0.000
LCF [m]	93.012	92.885	0.135
Volume [m^3]	27308.365	27323.807	0.056
WSA [m^2]	6069.280	6083.743	0.237
LCB [m]	94.817	94.739	0.082
KB [m]	3.634	3.637	0.085
Waterline 8.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4200.771	4215.490	0.349
Inertia [m^4]	263016.375	263011.475	0.001
LCF [m]	91.542	91.407	0.147
Volume [m^3]	31493.345	31516.335	0.072
WSA [m^2]	6456.085	6472.134	0.247
LCB [m]	94.479	94.393	0.090
KB [m]	4.148	4.151	0.079
Waterline 9.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4229.549	4225.270	0.101
Inertia [m^4]	265277.375	265279.372	0.000
LCF [m]	89.067	89.350	0.317
Volume [m^3]	35708.503	35736.236	0.077
WSA [m^2]	6880.507	6883.727	0.046
LCB [m]	93.986	93.918	0.071
KB [m]	4.661	4.665	0.065
Waterline 10.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4403.614	4402.180	0.032
Inertia [m^4]	274006.906	274021.092	0.005
LCF [m]	85.496	85.825	0.383

Volume $[m^3]$	39943.433	39966.527	0.057
WSA $[m^2]$	7369.131	7369.961	0.011
LCB [m]	93.398	93.370	0.029
KB [m]	5.177	5.179	0.036
Waterline 11.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4562.796	4568.946	0.134
Inertia $[m^4]$	285926.312	286190.177	0.092
LCF [m]	82.786	86.224	3.986
Volume $[m^3]$	44146.347	44174.210	0.063
WSA $[m^2]$	7842.342	7834.921	0.094
LCB [m]	92.987	93.155	0.180
KB [m]	5.688	5.690	0.034
Waterline 12.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4604.721	4604.301	0.009
Inertia $[m^4]$	290190.062	290546.385	0.122
LCF [m]	82.324	85.001	3.148
Volume $[m^3]$	48301.007	48330.519	0.061
WSA $[m^2]$	8215.772	8225.283	0.115
LCB [m]	92.731	93.172	0.473
KB [m]	6.195	6.197	0.027
Waterline 13.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4637.713	4650.567	0.276
Inertia $[m^4]$	293313.968	293773.006	0.156
LCF [m]	82.228	84.515	2.706
Volume $[m^3]$	52455.910	52491.815	0.068
WSA $[m^2]$	8584.437	8605.431	0.243
LCB [m]	92.535	93.159	0.669
KB [m]	6.703	6.705	0.029
Waterline 14.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4665.364	4666.597	0.026
Inertia $[m^4]$	295876.375	296358.632	0.162
LCF [m]	82.221	84.117	2.254
Volume $[m^3]$	56619.175	56661.870	0.075
WSA $[m^2]$	8952.595	8975.835	0.258
LCB [m]	92.384	93.133	0.804
KB [m]	7.210	7.213	0.031
Waterline 15.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4689.464	4688.494	0.020
Inertia $[m^4]$	298042.687	298577.885	0.179
LCF [m]	82.268	83.790	1.816
Volume $[m^3]$	60792.464	60835.378	0.070
WSA $[m^2]$	9321.041	9346.425	0.271
LCB [m]	92.267	93.097	0.891
KB [m]	7.719	7.721	0.023
Waterline 15.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4702.847	4699.907	0.062
Inertia $[m^4]$	299220.500	299855.176	0.211
LCF [m]	82.311	82.449	0.167
Volume $[m^3]$	60792.464	60835.378	0.070

WSA $[m^2]$	9321.041	9346.425	0.271
LCB [m]	92.267	93.097	0.891
KB [m]	7.719	7.721	0.023

A.2.3 Results using GeneticOptimizer

A.2.3.1 *Objective function for area attributes*

Waterline 1.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3638.650	3664.821	0.714
Inertia $[m^4]$	200695.437	200625.215	0.034
LCF [m]	96.473	95.366	1.147
Volume $[m^3]$	3286.598	3291.911	0.161
WSA $[m^2]$	3725.620	3746.872	0.567
LCB [m]	95.392	94.839	0.579
KB [m]	0.517	0.518	0.122
Waterline 2.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3801.612	3874.259	1.875
Inertia $[m^4]$	222918.328	222846.435	0.032
LCF [m]	96.733	95.184	1.600
Volume $[m^3]$	7006.495	7061.145	0.773
WSA $[m^2]$	4113.554	4166.255	1.264
LCB [m]	96.036	95.071	1.004
KB [m]	1.041	1.044	0.350
Waterline 3.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4040.104	4073.265	0.814
Inertia $[m^4]$	243814.578	243813.193	0.000
LCF [m]	94.297	87.309	7.410
Volume $[m^3]$	10913.262	11024.420	1.008
WSA $[m^2]$	4591.129	4855.347	5.441
LCB [m]	95.934	93.733	2.294
KB [m]	1.565	1.569	0.270
Waterline 4.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4065.494	4079.236	0.336
Inertia $[m^4]$	249646.921	249639.995	0.002
LCF [m]	94.537	86.631	8.363
Volume $[m^3]$	14949.680	15086.142	0.904
WSA $[m^2]$	4969.153	5230.564	4.997
LCB [m]	95.604	91.980	3.790
KB [m]	2.088	2.089	0.072
Waterline 5.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4103.510	4115.257	0.285
Inertia $[m^4]$	253854.937	253826.396	0.011
LCF [m]	94.149	87.486	7.077
Volume $[m^3]$	19034.183	19183.802	0.779
WSA $[m^2]$	5336.434	5614.472	4.952
LCB [m]	95.333	90.931	4.617
KB [m]	2.605	2.604	0.046
Waterline 6.000 m			
Attribute	Real	Computed	Percentage difference

Area $[m^2]$	4137.835	4170.104	0.773
Inertia $[m^4]$	257324.015	257291.093	0.012
LCF [m]	93.679	91.294	2.545
Volume $[m^3]$	23154.855	23324.773	0.728
WSA $[m^2]$	5701.997	6102.804	6.567
LCB [m]	95.081	90.660	4.649
KB [m]	3.120	3.118	0.068
Waterline 7.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4169.185	4218.981	1.180
Inertia $[m^4]$	260325.875	260346.950	0.008
LCF [m]	93.012	92.575	0.469
Volume $[m^3]$	27308.365	27519.009	0.765
WSA $[m^2]$	6069.280	6490.541	6.490
LCB [m]	94.817	90.856	4.177
KB [m]	3.634	3.634	0.019
Waterline 8.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4200.771	4222.602	0.516
Inertia $[m^4]$	263016.375	262979.670	0.013
LCF [m]	91.542	90.496	1.142
Volume $[m^3]$	31493.345	31740.163	0.777
WSA $[m^2]$	6456.085	6906.939	6.527
LCB [m]	94.479	90.948	3.737
KB [m]	4.148	4.148	0.006
Waterline 9.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4229.549	4240.772	0.264
Inertia $[m^4]$	265277.375	265266.329	0.004
LCF [m]	89.067	83.355	6.413
Volume $[m^3]$	35708.503	35968.535	0.722
WSA $[m^2]$	6880.507	7563.639	9.031
LCB [m]	93.986	90.472	3.738
KB [m]	4.661	4.659	0.050
Waterline 10.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4403.614	4421.774	0.410
Inertia $[m^4]$	274006.906	273995.396	0.004
LCF [m]	85.496	84.026	1.719
Volume $[m^3]$	39943.433	40219.375	0.686
WSA $[m^2]$	7369.131	8036.846	8.308
LCB [m]	93.398	89.872	3.775
KB [m]	5.177	5.174	0.071
Waterline 11.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4562.796	4566.914	0.090
Inertia $[m^4]$	285926.312	285925.899	0.000
LCF [m]	82.786	82.332	0.548
Volume $[m^3]$	44146.347	44432.834	0.644
WSA $[m^2]$	7842.342	8480.245	7.522
LCB [m]	92.987	89.698	3.537
KB [m]	5.688	5.683	0.095
Waterline 12.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4604.721	4607.985	0.070

Inertia $[m^4]$	290190.062	290187.648	0.000
LCF [m]	82.324	80.944	1.676
Volume $[m^3]$	48301.007	48591.777	0.598
WSA $[m^2]$	8215.772	8886.041	7.542
LCB [m]	92.731	89.635	3.339
KB [m]	6.195	6.188	0.120
Waterline 13.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4637.713	4637.815	0.002
Inertia $[m^4]$	293313.968	293317.967	0.001
LCF [m]	82.228	81.698	0.644
Volume $[m^3]$	52455.910	52748.364	0.554
WSA $[m^2]$	8584.437	9265.200	7.347
LCB [m]	92.535	89.597	3.174
KB [m]	6.703	6.693	0.141
Waterline 14.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4665.364	4666.201	0.017
Inertia $[m^4]$	295876.375	295879.070	0.000
LCF [m]	82.221	81.581	0.777
Volume $[m^3]$	56619.175	56912.041	0.514
WSA $[m^2]$	8952.595	9634.730	7.079
LCB [m]	92.384	89.612	3.000
KB [m]	7.210	7.199	0.157
Waterline 15.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4689.464	4693.476	0.085
Inertia $[m^4]$	298042.687	298038.162	0.001
LCF [m]	82.268	81.374	1.086
Volume $[m^3]$	60792.464	61087.794	0.483
WSA $[m^2]$	9321.041	10010.259	6.885
LCB [m]	92.267	89.625	2.863
KB [m]	7.719	7.706	0.165
Waterline 15.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4702.847	4705.074	0.047
Inertia $[m^4]$	299220.500	299218.894	0.000
LCF [m]	82.311	81.208	1.339
Volume $[m^3]$	60792.464	61087.794	0.483
WSA $[m^2]$	9321.041	10010.259	6.885
LCB [m]	92.267	89.625	2.863
KB [m]	7.719	7.706	0.165

A.2.3.2 *Objective function for volume attributes*

Waterline 1.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3638.650	3672.422	0.919
Inertia $[m^4]$	200695.437	200719.114	0.011
LCF [m]	96.473	94.028	2.534
Volume $[m^3]$	3286.598	3296.419	0.297
WSA $[m^2]$	3725.620	3750.433	0.661
LCB [m]	95.392	94.157	1.294
KB [m]	0.517	0.518	0.154
Waterline 2.000 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3801.612	3872.460	1.829
Inertia $[m^4]$	222918.328	222936.207	0.008
LCF [m]	96.733	91.906	4.990
Volume $[m^3]$	7006.495	7068.501	0.877
WSA $[m^2]$	4113.554	4185.996	1.730
LCB [m]	96.036	93.512	2.627
KB [m]	1.041	1.044	0.332
Waterline 3.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4040.104	4069.784	0.729
Inertia $[m^4]$	243814.578	243774.891	0.016
LCF [m]	94.297	88.066	6.608
Volume $[m^3]$	10913.262	11026.496	1.026
WSA $[m^2]$	4591.129	4685.507	2.014
LCB [m]	95.934	92.312	3.775
KB [m]	1.565	1.568	0.211
Waterline 4.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4065.494	4110.070	1.084
Inertia $[m^4]$	249646.921	249606.338	0.016
LCF [m]	94.537	94.040	0.525
Volume $[m^3]$	14949.680	15096.925	0.975
WSA $[m^2]$	4969.153	5274.127	5.782
LCB [m]	95.604	92.063	3.703
KB [m]	2.088	2.089	0.084
Waterline 5.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4103.510	4111.908	0.204
Inertia $[m^4]$	253854.937	253856.812	0.000
LCF [m]	94.149	89.714	4.710
Volume $[m^3]$	19034.183	19209.978	0.915
WSA $[m^2]$	5336.434	5785.297	7.758
LCB [m]	95.333	92.022	3.473
KB [m]	2.605	2.605	0.003
Waterline 6.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4137.835	4174.113	0.869
Inertia $[m^4]$	257324.015	257247.813	0.029
LCF [m]	93.679	87.688	6.395
Volume $[m^3]$	23154.855	23353.649	0.851
WSA $[m^2]$	5701.997	6202.620	8.071
LCB [m]	95.081	91.430	3.839
KB [m]	3.120	3.119	0.043
Waterline 7.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4169.185	4222.291	1.257
Inertia $[m^4]$	260325.875	260295.454	0.011
LCF [m]	93.012	91.093	2.062
Volume $[m^3]$	27308.365	27550.499	0.878
WSA $[m^2]$	6069.280	6674.198	9.063
LCB [m]	94.817	91.122	3.896
KB [m]	3.634	3.634	0.008
Waterline 8.000 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4200.771	4238.354	0.886
Inertia $[m^4]$	263016.375	262990.918	0.009
LCF [m]	91.542	87.772	4.117
Volume $[m^3]$	31493.345	31780.061	0.902
WSA $[m^2]$	6456.085	7131.831	9.475
LCB [m]	94.479	90.896	3.792
KB [m]	4.148	4.148	0.011
Waterline 9.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4229.549	4262.048	0.762
Inertia $[m^4]$	265277.375	265249.064	0.010
LCF [m]	89.067	88.474	0.665
Volume $[m^3]$	35708.503	36032.001	0.897
WSA $[m^2]$	6880.507	7544.055	8.795
LCB [m]	93.986	90.568	3.636
KB [m]	4.661	4.662	0.005
Waterline 10.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4403.614	4404.598	0.022
Inertia $[m^4]$	274006.906	274034.612	0.010
LCF [m]	85.496	85.393	0.120
Volume $[m^3]$	39943.433	40283.881	0.845
WSA $[m^2]$	7369.131	8008.595	7.984
LCB [m]	93.398	90.308	3.308
KB [m]	5.177	5.175	0.041
Waterline 11.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4562.796	4574.373	0.253
Inertia $[m^4]$	285926.312	286231.793	0.106
LCF [m]	82.786	82.616	0.206
Volume $[m^3]$	44146.347	44493.822	0.780
WSA $[m^2]$	7842.342	8493.220	7.663
LCB [m]	92.987	90.174	3.025
KB [m]	5.688	5.683	0.089
Waterline 12.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4604.721	4614.994	0.222
Inertia $[m^4]$	290190.062	290772.919	0.200
LCF [m]	82.324	80.147	2.645
Volume $[m^3]$	48301.007	48659.673	0.737
WSA $[m^2]$	8215.772	8954.658	8.251
LCB [m]	92.731	90.045	2.896
KB [m]	6.195	6.188	0.113
Waterline 13.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4637.713	4639.171	0.031
Inertia $[m^4]$	293313.968	293751.432	0.148
LCF [m]	82.228	82.275	0.057
Volume $[m^3]$	52455.910	52818.875	0.687
WSA $[m^2]$	8584.437	9393.822	8.616
LCB [m]	92.535	89.966	2.776
KB [m]	6.703	6.693	0.141
Waterline 14.000 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4665.364	4668.387	0.064
Inertia $[m^4]$	295876.375	296414.228	0.181
LCF [m]	82.221	81.744	0.580
Volume $[m^3]$	56619.175	56984.353	0.640
WSA $[m^2]$	8952.595	9770.859	8.374
LCB [m]	92.384	89.984	2.597
KB [m]	7.210	7.199	0.163
Waterline 15.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4689.464	4690.818	0.028
Inertia $[m^4]$	298042.687	298591.518	0.183
LCF [m]	82.268	80.887	1.678
Volume $[m^3]$	60792.464	61160.630	0.601
WSA $[m^2]$	9321.041	10172.642	8.371
LCB [m]	92.267	89.959	2.501
KB [m]	7.719	7.705	0.177
Waterline 15.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4702.847	4706.089	0.068
Inertia $[m^4]$	299220.500	299815.554	0.198
LCF [m]	82.311	81.416	1.086
Volume $[m^3]$	60792.464	61160.630	0.601
WSA $[m^2]$	9321.041	10172.642	8.371
LCB [m]	92.267	89.959	2.501
KB [m]	7.719	7.705	0.177

A.2.4 Results using Hybrid Optimizer

A.2.4.1 *Objective function for area*

Waterline 1.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3638.650	3649.174	0.288
Inertia $[m^4]$	200695.437	200632.890	0.031
LCF [m]	96.473	96.244	0.237
Volume $[m^3]$	3286.598	3286.046	0.016
WSA $[m^2]$	3725.620	3736.304	0.285
LCB [m]	95.392	95.289	0.108
KB [m]	0.517	0.517	0.052
Waterline 2.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3801.612	3812.542	0.286
Inertia $[m^4]$	222918.328	222918.328	0.000
LCF [m]	96.733	96.732	0.000
Volume $[m^3]$	7006.495	7016.372	0.140
WSA $[m^2]$	4113.554	4125.465	0.288
LCB [m]	96.036	95.925	0.115
KB [m]	1.041	1.041	0.082
Waterline 3.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4040.104	4079.331	0.961
Inertia $[m^4]$	243814.578	243814.577	0.000
LCF [m]	94.297	94.302	0.004
Volume $[m^3]$	10913.262	10948.817	0.324

WSA $[m^2]$	4591.129	4627.659	0.789
LCB [m]	95.934	95.864	0.073
KB [m]	1.565	1.567	0.158
Waterline 4.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4065.494	4122.507	1.382
Inertia $[m^4]$	249646.921	249646.921	0.000
LCF [m]	94.537	94.536	0.001
Volume $[m^3]$	14949.680	15033.831	0.559
WSA $[m^2]$	4969.153	5000.063	0.618
LCB [m]	95.604	95.546	0.059
KB [m]	2.088	2.093	0.250
Waterline 5.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4103.510	4178.180	1.787
Inertia $[m^4]$	253854.937	253867.456	0.004
LCF [m]	94.149	94.144	0.005
Volume $[m^3]$	19034.183	19183.350	0.777
WSA $[m^2]$	5336.434	5386.594	0.931
LCB [m]	95.333	95.286	0.049
KB [m]	2.605	2.614	0.318
Waterline 6.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4137.835	4162.832	0.600
Inertia $[m^4]$	257324.015	256446.013	0.341
LCF [m]	93.679	93.717	0.039
Volume $[m^3]$	23154.855	23353.028	0.848
WSA $[m^2]$	5701.997	5763.843	1.072
LCB [m]	95.081	95.043	0.039
KB [m]	3.120	3.129	0.267
Waterline 7.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4169.185	4178.942	0.233
Inertia $[m^4]$	260325.875	260324.456	0.000
LCF [m]	93.012	93.012	0.000
Volume $[m^3]$	27308.365	27523.625	0.782
WSA $[m^2]$	6069.280	6133.829	1.052
LCB [m]	94.817	94.788	0.030
KB [m]	3.634	3.639	0.141
Waterline 8.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4200.771	4262.960	1.458
Inertia $[m^4]$	263016.375	263016.374	0.000
LCF [m]	91.542	91.542	0.000
Volume $[m^3]$	31493.345	31743.810	0.789
WSA $[m^2]$	6456.085	6555.355	1.514
LCB [m]	94.479	94.452	0.028
KB [m]	4.148	4.153	0.116
Waterline 9.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4229.549	4261.526	0.750
Inertia $[m^4]$	265277.375	265267.443	0.003
LCF [m]	89.067	89.069	0.002
Volume $[m^3]$	35708.503	36006.295	0.827
WSA $[m^2]$	6880.507	6990.047	1.567

LCB [m]	93.986	93.961	0.025
KB [m]	4.661	4.667	0.121
Waterline 10.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4403.614	4424.117	0.463
Inertia $[m^4]$	274006.906	273957.200	0.018
LCF [m]	85.496	85.494	0.001
Volume $[m^3]$	39943.433	40267.627	0.805
WSA $[m^2]$	7369.131	7474.190	1.405
LCB [m]	93.398	93.376	0.022
KB [m]	5.177	5.181	0.078
Waterline 11.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4562.796	4577.103	0.312
Inertia $[m^4]$	285926.312	285917.646	0.003
LCF [m]	82.786	82.786	0.000
Volume $[m^3]$	44146.347	44488.099	0.768
WSA $[m^2]$	7842.342	7944.977	1.291
LCB [m]	92.987	92.967	0.021
KB [m]	5.688	5.690	0.032
Waterline 12.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4604.721	4613.102	0.181
Inertia $[m^4]$	290190.062	290190.062	0.000
LCF [m]	82.324	82.324	0.000
Volume $[m^3]$	48301.007	48654.277	0.726
WSA $[m^2]$	8215.772	8318.587	1.235
LCB [m]	92.731	92.711	0.020
KB [m]	6.195	6.195	0.010
Waterline 13.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4637.713	4639.122	0.030
Inertia $[m^4]$	293313.968	293314.002	0.000
LCF [m]	82.228	82.228	0.000
Volume $[m^3]$	52455.910	52814.028	0.678
WSA $[m^2]$	8584.437	8686.062	1.169
LCB [m]	92.535	92.517	0.019
KB [m]	6.703	6.699	0.051
Waterline 14.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4665.364	4674.844	0.202
Inertia $[m^4]$	295876.375	295875.723	0.000
LCF [m]	82.221	82.221	0.000
Volume $[m^3]$	56619.175	56983.040	0.638
WSA $[m^2]$	8952.595	9060.505	1.190
LCB [m]	92.384	92.368	0.017
KB [m]	7.210	7.205	0.079
Waterline 15.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4689.464	4689.893	0.009
Inertia $[m^4]$	298042.687	298042.637	0.000
LCF [m]	82.268	82.268	0.000
Volume $[m^3]$	60792.464	61161.313	0.603
WSA $[m^2]$	9321.041	9428.094	1.135
LCB [m]	92.267	92.252	0.016

KB [m]	7.719	7.711	0.101
Waterline 15.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4702.847	4705.000	0.045
Inertia $[m^4]$	299220.500	299215.091	0.001
LCF [m]	82.311	82.310	0.000
Volume $[m^3]$	60792.464	61161.313	0.603
WSA $[m^2]$	9321.041	9428.094	1.135
LCB [m]	92.267	92.252	0.016
KB [m]	7.719	7.711	0.101

A.2.4.2 *Objective function for volume attributes*

Waterline 1.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3638.650	3714.858	2.051
Inertia $[m^4]$	200695.437	200656.219	0.019
LCF [m]	96.473	92.596	4.018
Volume $[m^3]$	3286.598	3321.051	1.037
WSA $[m^2]$	3725.620	3793.719	1.795
LCB [m]	95.392	93.430	2.057
KB [m]	0.517	0.519	0.333
Waterline 2.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3801.612	3807.117	0.144
Inertia $[m^4]$	222918.328	222913.084	0.002
LCF [m]	96.733	96.733	0.000
Volume $[m^3]$	7006.495	7080.358	1.043
WSA $[m^2]$	4113.554	4293.619	4.193
LCB [m]	96.036	94.070	2.046
KB [m]	1.041	1.041	0.001
Waterline 3.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4040.104	4054.545	0.356
Inertia $[m^4]$	243814.578	243822.726	0.003
LCF [m]	94.297	93.825	0.500
Volume $[m^3]$	10913.262	10997.145	0.762
WSA $[m^2]$	4591.129	4781.180	3.974
LCB [m]	95.934	94.581	1.409
KB [m]	1.565	1.562	0.166
Waterline 4.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4065.494	4063.091	0.059
Inertia $[m^4]$	249646.921	247943.706	0.682
LCF [m]	94.537	94.537	0.000
Volume $[m^3]$	14949.680	15040.403	0.603
WSA $[m^2]$	4969.153	5180.679	4.082
LCB [m]	95.604	94.553	1.099
KB [m]	2.088	2.083	0.204
Waterline 5.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4103.510	4118.884	0.373
Inertia $[m^4]$	253854.937	253781.369	0.028
LCF [m]	94.149	94.149	0.000
Volume $[m^3]$	19034.183	19131.741	0.509

WSA [m^2]	5336.434	5551.128	3.867
LCB [m]	95.333	94.508	0.865
KB [m]	2.605	2.600	0.193
Waterline 6.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4137.835	4168.349	0.732
Inertia [m^4]	257324.015	257329.592	0.002
LCF [m]	93.679	92.973	0.754
Volume [m^3]	23154.855	23275.191	0.517
WSA [m^2]	5701.997	5938.266	3.978
LCB [m]	95.081	94.340	0.778
KB [m]	3.120	3.116	0.125
Waterline 7.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4169.185	4156.550	0.303
Inertia [m^4]	260325.875	259714.309	0.234
LCF [m]	93.012	92.990	0.022
Volume [m^3]	27308.365	27437.867	0.471
WSA [m^2]	6069.280	6304.816	3.735
LCB [m]	94.817	94.132	0.721
KB [m]	3.634	3.629	0.130
Waterline 8.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4200.771	4221.436	0.489
Inertia [m^4]	263016.375	263867.862	0.322
LCF [m]	91.542	91.542	0.000
Volume [m^3]	31493.345	31626.933	0.422
WSA [m^2]	6456.085	6696.588	3.591
LCB [m]	94.479	93.884	0.629
KB [m]	4.148	4.142	0.137
Waterline 9.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4229.549	4260.187	0.719
Inertia [m^4]	265277.375	266259.624	0.368
LCF [m]	89.067	87.803	1.418
Volume [m^3]	35708.503	35867.044	0.442
WSA [m^2]	6880.507	7176.248	4.121
LCB [m]	93.986	93.386	0.637
KB [m]	4.661	4.657	0.091
Waterline 10.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4403.614	4411.900	0.187
Inertia [m^4]	274006.906	274058.419	0.018
LCF [m]	85.496	85.496	0.000
Volume [m^3]	39943.433	40121.768	0.444
WSA [m^2]	7369.131	7622.538	3.324
LCB [m]	93.398	92.793	0.647
KB [m]	5.177	5.174	0.072
Waterline 11.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4562.796	4570.143	0.160
Inertia [m^4]	285926.312	285822.708	0.036
LCF [m]	82.786	82.787	0.000
Volume [m^3]	44146.347	44332.082	0.418
WSA [m^2]	7842.342	8095.584	3.128

LCB [m]	92.987	92.441	0.587
KB [m]	5.688	5.684	0.081
Waterline 12.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4604.721	4604.721	0.000
Inertia $[m^4]$	290190.062	290225.787	0.012
LCF [m]	82.324	82.387	0.076
Volume $[m^3]$	48301.007	48490.221	0.390
WSA $[m^2]$	8215.772	8466.641	2.963
LCB [m]	92.731	92.234	0.536
KB [m]	6.195	6.190	0.093
Waterline 13.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4637.713	4638.223	0.010
Inertia $[m^4]$	293313.968	293858.971	0.185
LCF [m]	82.228	81.022	1.466
Volume $[m^3]$	52455.910	52645.984	0.361
WSA $[m^2]$	8584.437	8865.358	3.168
LCB [m]	92.535	92.027	0.549
KB [m]	6.703	6.695	0.105
Waterline 14.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4665.364	4670.124	0.101
Inertia $[m^4]$	295876.375	295864.265	0.004
LCF [m]	82.221	82.221	0.000
Volume $[m^3]$	56619.175	56811.847	0.339
WSA $[m^2]$	8952.595	9250.417	3.219
LCB [m]	92.384	91.863	0.563
KB [m]	7.210	7.202	0.110
Waterline 15.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4689.464	4696.283	0.145
Inertia $[m^4]$	298042.687	298049.100	0.002
LCF [m]	82.268	82.268	0.000
Volume $[m^3]$	60792.464	60990.956	0.325
WSA $[m^2]$	9321.041	9621.821	3.126
LCB [m]	92.267	91.781	0.526
KB [m]	7.719	7.710	0.108
Waterline 15.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4702.847	4706.331	0.074
Inertia $[m^4]$	299220.500	299214.896	0.001
LCF [m]	82.311	82.311	0.000
Volume $[m^3]$	60792.464	60990.956	0.325
WSA $[m^2]$	9321.041	9621.821	3.126
LCB [m]	92.267	91.781	0.526
KB [m]	7.719	7.710	0.108

A.2.5 Results for the real vessel

A.2.5.1 *Objective function for area attributes*

Waterline 1.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3699.729	3755.698	1.490

Inertia $[m^4]$	206390.703	205811.629	0.280
LCF [m]	94.606	96.442	1.904
Volume $[m^3]$	3550.620	3340.625	5.914
WSA $[m^2]$	3803.479	3833.521	0.783
LCB [m]	94.737	95.337	0.629
KB [m]	0.515	0.520	0.999
Waterline 3.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4031.179	4101.393	1.711
Inertia $[m^4]$	245117.703	245333.287	0.087
LCF [m]	94.084	94.084	0.000
Volume $[m^3]$	11597.669	11166.402	3.718
WSA $[m^2]$	4621.299	4654.149	0.705
LCB [m]	94.456	95.449	1.041
KB [m]	1.555	1.567	0.790
Waterline 3.500 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4058.229	4058.229	0.000
Inertia $[m^4]$	248154.906	250117.608	0.784
LCF [m]	94.035	93.989	0.048
Volume $[m^3]$	13678.750	13193.609	3.546
WSA $[m^2]$	4810.540	4914.418	2.113
LCB [m]	94.397	95.301	0.949
KB [m]	1.813	1.825	0.698
Waterline 5.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4121.939	4121.939	0.000
Inertia $[m^4]$	255044.093	255044.044	0.000
LCF [m]	93.341	93.341	0.000
Volume $[m^3]$	19991.750	19313.952	3.390
WSA $[m^2]$	5375.649	5475.084	1.816
LCB [m]	94.200	94.837	0.671
KB [m]	2.583	2.594	0.452
Waterline 7.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4220.740	4257.951	0.873
Inertia $[m^4]$	263561.500	263557.907	0.001
LCF [m]	90.607	93.707	3.308
Volume $[m^3]$	28572.099	27694.461	3.071
WSA $[m^2]$	6162.500	6239.290	1.230
LCB [m]	93.578	94.443	0.916
KB [m]	3.610	3.626	0.456
Waterline 9.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4407.160	4419.680	0.283
Inertia $[m^4]$	276348.000	276347.999	0.000
LCF [m]	84.785	84.768	0.019
Volume $[m^3]$	37403.769	36368.151	2.768
WSA $[m^2]$	7090.120	7285.442	2.680
LCB [m]	92.318	93.190	0.936
KB [m]	4.648	4.670	0.488
Waterline 10.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4517.959	4524.534	0.145
Inertia $[m^4]$	285150.687	285809.068	0.230

LCF [m]	82.081	82.081	0.000
Volume [m^3]	42000.851	40758.523	2.957
WSA [m^2]	7556.029	7741.927	2.401
LCB [m]	91.328	92.249	0.999
KB [m]	5.180	5.193	0.263
Waterline 12.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4618.390	4673.725	1.183
Inertia [m^4]	295174.500	293411.159	0.597
LCF [m]	80.894	83.154	2.717
Volume [m^3]	51415.828	49343.970	4.029
WSA [m^2]	8350.240	8549.169	2.326
LCB [m]	89.487	91.468	2.166
KB [m]	6.246	6.214	0.502
Waterline 14.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4668.109	4703.678	0.756
Inertia [m^4]	298844.000	299744.674	0.300
LCF [m]	81.051	81.051	0.000
Volume [m^3]	60975.898	57772.807	5.253
WSA [m^2]	9126.330	9326.711	2.148
LCB [m]	88.148	91.262	3.412
KB [m]	7.305	7.222	1.122
Waterline 15.600 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4706.060	4707.313	0.026
Inertia [m^4]	301400.500	301400.500	0.000
LCF [m]	81.386	81.347	0.046
Volume [m^3]	68693.898	64489.720	6.120
WSA [m^2]	9752.259	9924.357	1.734
LCB [m]	87.370	91.078	4.072
KB [m]	8.147	8.026	1.480

A.2.5.2 Objective function for volume attributes

Waterline 1.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	3699.729	3699.729	0.000
Inertia [m^4]	206390.703	206390.705	0.000
LCF [m]	94.606	94.606	0.000
Volume [m^3]	3550.620	3316.936	6.581
WSA [m^2]	3803.479	3777.114	0.693
LCB [m]	94.737	94.399	0.356
KB [m]	0.515	0.518	0.758
Waterline 3.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4031.179	4031.179	0.000
Inertia [m^4]	245117.703	245117.797	0.000
LCF [m]	94.084	94.084	0.000
Volume [m^3]	11597.669	11014.419	5.029
WSA [m^2]	4621.299	4567.110	1.172
LCB [m]	94.456	94.554	0.104
KB [m]	1.555	1.563	0.562
Waterline 3.500 m			
Attribute	Real	Computed	Percentage difference

Area $[m^2]$	4058.229	4058.229	0.000
Inertia $[m^4]$	248154.906	248154.886	0.000
LCF [m]	94.035	94.035	0.000
Volume $[m^3]$	13678.750	13024.339	4.784
WSA $[m^2]$	4810.540	4754.938	1.155
LCB [m]	94.397	94.545	0.156
KB [m]	1.813	1.823	0.599
Waterline 5.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4121.939	4121.939	0.000
Inertia $[m^4]$	255044.093	255044.017	0.000
LCF [m]	93.341	93.341	0.000
Volume $[m^3]$	19991.750	19144.636	4.237
WSA $[m^2]$	5375.649	5311.513	1.193
LCB [m]	94.200	94.326	0.133
KB [m]	2.583	2.600	0.665
Waterline 7.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4220.740	4220.740	0.000
Inertia $[m^4]$	263561.500	263561.530	0.000
LCF [m]	90.607	90.607	0.000
Volume $[m^3]$	28572.099	27489.678	3.788
WSA $[m^2]$	6162.500	6083.364	1.284
LCB [m]	93.578	93.604	0.028
KB [m]	3.610	3.633	0.640
Waterline 9.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4407.160	4413.279	0.138
Inertia $[m^4]$	276348.000	276357.569	0.003
LCF [m]	84.785	84.602	0.215
Volume $[m^3]$	37403.769	36120.369	3.431
WSA $[m^2]$	7090.120	7000.356	1.266
LCB [m]	92.318	92.171	0.158
KB [m]	4.648	4.678	0.644
Waterline 10.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4517.959	4517.959	0.000
Inertia $[m^4]$	285150.687	285174.743	0.008
LCF [m]	82.081	82.081	0.000
Volume $[m^3]$	42000.851	40504.764	3.562
WSA $[m^2]$	7556.029	7441.465	1.516
LCB [m]	91.328	91.327	0.000
KB [m]	5.180	5.202	0.438
Waterline 12.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4618.390	4618.390	0.000
Inertia $[m^4]$	295174.500	295548.726	0.126
LCF [m]	80.894	80.894	0.000
Volume $[m^3]$	51415.828	49028.751	4.642
WSA $[m^2]$	8350.240	8204.468	1.745
LCB [m]	89.487	90.497	1.116
KB [m]	6.246	6.221	0.398
Waterline 14.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4668.109	4668.109	0.000

Inertia $[m^4]$	298844.000	299341.397	0.166
LCF [m]	81.051	81.051	0.000
Volume $[m^3]$	60975.898	57364.463	5.922
WSA $[m^2]$	9126.330	8937.705	2.066
LCB [m]	88.148	90.262	2.342
KB [m]	7.305	7.224	1.097
Waterline 15.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4706.060	4706.060	0.000
Inertia $[m^4]$	301400.500	301968.439	0.188
LCF [m]	81.386	81.386	0.000
Volume $[m^3]$	68693.898	64051.634	6.757
WSA $[m^2]$	9752.259	9527.249	2.307
LCB [m]	87.370	90.187	3.123
KB [m]	8.147	8.030	1.434

A.2.5.3 Objective function for area attributes using the FOB technique

Waterline 0.100 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3214.657	3216.874	0.068
Inertia $[m^4]$	148200.031	148201.932	0.001
LCF [m]	94.812	96.666	1.917
Volume $[m^3]$	308.108	308.229	0.039
WSA $[m^2]$	3216.919	3224.516	0.235
LCB [m]	94.481	95.565	1.134
KB [m]	0.050	0.050	0.008
Waterline 0.200 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3316.294	3318.276	0.059
Inertia $[m^4]$	159516.796	159530.914	0.008
LCF [m]	94.780	96.121	1.394
Volume $[m^3]$	635.098	635.224	0.019
WSA $[m^2]$	3324.318	3342.492	0.543
LCB [m]	94.559	95.943	1.442
KB [m]	0.101	0.101	0.007
Waterline 0.300 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3393.034	3396.238	0.094
Inertia $[m^4]$	168401.250	168400.071	0.000
LCF [m]	94.748	95.370	0.652
Volume $[m^3]$	970.862	970.948	0.008
WSA $[m^2]$	3408.476	3441.559	0.961
LCB [m]	94.577	95.873	1.351
KB [m]	0.153	0.153	0.008
Waterline 0.400 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3456.738	3456.831	0.002
Inertia $[m^4]$	175964.406	175963.694	0.000
LCF [m]	94.716	94.721	0.004
Volume $[m^3]$	1313.486	1313.600	0.008
WSA $[m^2]$	3480.821	3528.554	1.352
LCB [m]	94.587	95.657	1.118
KB [m]	0.204	0.204	0.008
Waterline 0.500 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3514.262	3514.549	0.008
Inertia $[m^4]$	182628.968	182628.989	0.000
LCF [m]	94.744	95.807	1.109
Volume $[m^3]$	1661.577	1662.169	0.035
WSA $[m^2]$	3549.457	3621.319	1.984
LCB [m]	94.584	95.575	1.036
KB [m]	0.256	0.256	0.007
Waterline 0.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3563.086	3563.086	0.000
Inertia $[m^4]$	188634.109	188634.104	0.000
LCF [m]	94.719	94.719	0.000
Volume $[m^3]$	2014.348	2016.053	0.084
WSA $[m^2]$	3609.540	3715.462	2.850
LCB [m]	94.573	95.520	0.991
KB [m]	0.307	0.307	0.032
Waterline 0.700 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3607.091	3607.406	0.008
Inertia $[m^4]$	194103.265	194103.150	0.000
LCF [m]	94.694	95.757	1.109
Volume $[m^3]$	2371.636	2374.578	0.123
WSA $[m^2]$	3665.811	3801.771	3.576
LCB [m]	94.567	95.478	0.954
KB [m]	0.359	0.359	0.045
Waterline 0.800 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3647.002	3646.983	0.000
Inertia $[m^4]$	199119.843	199121.544	0.000
LCF [m]	94.668	94.667	0.001
Volume $[m^3]$	2733.040	2737.300	0.155
WSA $[m^2]$	3719.008	3892.506	4.457
LCB [m]	94.562	95.442	0.922
KB [m]	0.410	0.411	0.052
Waterline 0.900 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3683.406	3683.642	0.006
Inertia $[m^4]$	203743.078	203742.894	0.000
LCF [m]	94.641	95.600	1.003
Volume $[m^3]$	3098.334	3103.833	0.177
WSA $[m^2]$	3769.749	3971.586	5.082
LCB [m]	94.555	95.406	0.892
KB [m]	0.462	0.462	0.052
Waterline 1.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3716.822	3716.818	0.000
Inertia $[m^4]$	208015.812	208015.847	0.000
LCF [m]	94.607	94.611	0.004
Volume $[m^3]$	3467.250	3473.858	0.190
WSA $[m^2]$	3818.393	4055.314	5.842
LCB [m]	94.551	95.374	0.863
KB [m]	0.514	0.514	0.049
Waterline 1.100 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3747.648	3747.897	0.006
Inertia $[m^4]$	211970.328	211970.126	0.000
LCF [m]	94.572	95.511	0.983
Volume $[m^3]$	3839.354	3847.095	0.201
WSA $[m^2]$	3865.432	4131.359	6.436
LCB [m]	94.550	95.344	0.832
KB [m]	0.566	0.566	0.047
Waterline 1.200 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3776.272	3776.155	0.003
Inertia $[m^4]$	215635.046	215637.692	0.001
LCF [m]	94.536	94.594	0.061
Volume $[m^3]$	4214.424	4223.300	0.210
WSA $[m^2]$	3911.079	4209.996	7.100
LCB [m]	94.547	95.318	0.808
KB [m]	0.618	0.618	0.044
Waterline 1.300 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3802.896	3803.107	0.005
Inertia $[m^4]$	219031.359	219031.670	0.000
LCF [m]	94.501	95.455	0.999
Volume $[m^3]$	4592.306	4602.264	0.216
WSA $[m^2]$	3955.541	4283.388	7.653
LCB [m]	94.541	95.294	0.789
KB [m]	0.670	0.670	0.040
Waterline 1.400 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3827.939	3827.988	0.001
Inertia $[m^4]$	222175.390	222175.345	0.000
LCF [m]	94.472	94.481	0.008
Volume $[m^3]$	4972.860	4983.821	0.219
WSA $[m^2]$	3999.285	4364.373	8.365
LCB [m]	94.532	95.269	0.773
KB [m]	0.722	0.722	0.037
Waterline 1.500 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3850.760	3850.989	0.005
Inertia $[m^4]$	225081.015	225081.038	0.000
LCF [m]	94.440	95.321	0.924
Volume $[m^3]$	5356.065	5367.772	0.218
WSA $[m^2]$	4041.951	4436.169	8.886
LCB [m]	94.521	95.242	0.757
KB [m]	0.774	0.774	0.030
Waterline 1.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3871.792	3872.266	0.012
Inertia $[m^4]$	227758.375	227758.423	0.000
LCF [m]	94.408	95.998	1.655
Volume $[m^3]$	5741.850	5753.933	0.210
WSA $[m^2]$	4083.801	4500.198	9.252
LCB [m]	94.510	95.270	0.798
KB [m]	0.826	0.826	0.020
Waterline 1.700 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3891.289	3891.248	0.001
Inertia $[m^4]$	230219.734	230219.666	0.000
LCF [m]	94.378	94.405	0.028
Volume $[m^3]$	6130.163	6142.113	0.194
WSA $[m^2]$	4124.916	4615.382	10.626
LCB [m]	94.496	95.266	0.808
KB [m]	0.878	0.878	0.006
Waterline 1.800 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3909.762	3909.992	0.005
Inertia $[m^4]$	232467.828	232467.939	0.000
LCF [m]	94.347	95.261	0.959
Volume $[m^3]$	6520.722	6532.178	0.175
WSA $[m^2]$	4165.439	4687.141	11.130
LCB [m]	94.482	95.240	0.796
KB [m]	0.930	0.930	0.007
Waterline 1.900 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3926.674	3927.072	0.010
Inertia $[m^4]$	234509.625	234509.473	0.000
LCF [m]	94.315	95.933	1.686
Volume $[m^3]$	6913.114	6924.030	0.157
WSA $[m^2]$	4205.400	4750.258	11.470
LCB [m]	94.469	95.260	0.831
KB [m]	0.983	0.982	0.018
Waterline 2.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3941.976	3941.981	0.000
Inertia $[m^4]$	236350.187	236350.197	0.000
LCF [m]	94.285	94.289	0.003
Volume $[m^3]$	7307.153	7317.489	0.141
WSA $[m^2]$	4244.929	4868.755	12.812
LCB [m]	94.457	95.252	0.835
KB [m]	1.035	1.034	0.027
Waterline 2.100 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3955.808	3955.929	0.003
Inertia $[m^4]$	237993.000	237993.310	0.000
LCF [m]	94.257	95.099	0.885
Volume $[m^3]$	7702.654	7712.388	0.126
WSA $[m^2]$	4284.065	4937.619	13.236
LCB [m]	94.446	95.224	0.816
KB [m]	1.087	1.086	0.035
Waterline 2.200 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	3968.428	3968.846	0.010
Inertia $[m^4]$	239442.328	239442.055	0.000
LCF [m]	94.230	95.766	1.603
Volume $[m^3]$	8099.448	8108.626	0.113
WSA $[m^2]$	4322.880	5000.273	13.547
LCB [m]	94.437	95.234	0.836
KB [m]	1.139	1.138	0.041
Waterline 2.300 m			

Attribute	Real	Computed	Percentage difference
Area [m^2]	3980.014	4001.986	0.549
Inertia [m^4]	240700.453	240883.012	0.075
LCF [m]	94.205	96.228	2.102
Volume [m^3]	8497.380	8507.167	0.115
WSA [m^2]	4361.439	5073.929	14.042
LCB [m]	94.429	95.270	0.882
KB [m]	1.191	1.190	0.033
Waterline 2.400 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	3990.299	4004.578	0.356
Inertia [m^4]	241769.750	239461.607	0.954
LCF [m]	94.174	96.623	2.534
Volume [m^3]	8896.170	8907.494	0.127
WSA [m^2]	4399.714	5137.047	14.353
LCB [m]	94.422	95.322	0.943
KB [m]	1.243	1.242	0.018
Waterline 2.500 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	3999.574	4018.029	0.459
Inertia [m^4]	242652.406	242611.670	0.016
LCF [m]	94.150	97.258	3.195
Volume [m^3]	9295.746	9308.627	0.138
WSA [m^2]	4438.051	5201.667	14.680
LCB [m]	94.415	95.392	1.023
KB [m]	1.295	1.294	0.005
Waterline 2.600 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4008.188	4009.533	0.033
Inertia [m^4]	243393.375	243400.533	0.002
LCF [m]	94.122	97.860	3.819
Volume [m^3]	9695.511	9709.077	0.139
WSA [m^2]	4476.573	5264.328	14.964
LCB [m]	94.414	95.488	1.124
KB [m]	1.346	1.346	0.002
Waterline 2.700 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4016.135	4016.172	0.000
Inertia [m^4]	244126.968	244134.002	0.002
LCF [m]	94.103	94.226	0.130
Volume [m^3]	10095.221	10108.437	0.130
WSA [m^2]	4514.643	5502.758	17.956
LCB [m]	94.420	95.524	1.155
KB [m]	1.398	1.398	0.008
Waterline 2.800 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4023.757	4024.101	0.008
Inertia [m^4]	244844.218	244843.905	0.000
LCF [m]	94.083	95.218	1.191
Volume [m^3]	10495.361	10508.010	0.120
WSA [m^2]	4552.533	5579.915	18.412
LCB [m]	94.427	95.511	1.134
KB [m]	1.449	1.449	0.015
Waterline 2.900 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4030.837	4031.525	0.017
Inertia $[m^4]$	245543.093	245542.324	0.000
LCF [m]	94.067	95.893	1.904
Volume $[m^3]$	10896.208	10908.048	0.108
WSA $[m^2]$	4590.323	5642.285	18.644
LCB [m]	94.434	95.531	1.148
KB [m]	1.501	1.501	0.022
Waterline 3.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4037.486	4042.905	0.134
Inertia $[m^4]$	246213.375	246269.153	0.022
LCF [m]	94.052	96.400	2.435
Volume $[m^3]$	11297.814	11308.861	0.097
WSA $[m^2]$	4628.048	5697.869	18.775
LCB [m]	94.440	95.573	1.184
KB [m]	1.553	1.552	0.028
Waterline 3.100 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4043.382	4064.713	0.524
Inertia $[m^4]$	246870.046	246902.098	0.012
LCF [m]	94.040	96.660	2.710
Volume $[m^3]$	11700.085	11711.290	0.095
WSA $[m^2]$	4665.695	5757.949	18.969
LCB [m]	94.445	95.625	1.233
KB [m]	1.604	1.604	0.026
Waterline 3.200 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4048.917	4058.525	0.236
Inertia $[m^4]$	247493.593	247483.103	0.004
LCF [m]	94.032	97.163	3.221
Volume $[m^3]$	12102.823	12114.514	0.096
WSA $[m^2]$	4703.411	5812.704	19.083
LCB [m]	94.450	95.686	1.291
KB [m]	1.655	1.655	0.021
Waterline 3.300 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4054.049	4064.546	0.258
Inertia $[m^4]$	248099.890	248100.260	0.000
LCF [m]	94.026	97.445	3.508
Volume $[m^3]$	12506.038	12517.847	0.094
WSA $[m^2]$	4741.146	5857.330	19.056
LCB [m]	94.454	95.755	1.359
KB [m]	1.707	1.707	0.020
Waterline 3.400 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4058.877	4072.430	0.332
Inertia $[m^4]$	248681.078	248742.559	0.024
LCF [m]	94.020	97.659	3.725
Volume $[m^3]$	12909.798	12922.100	0.095
WSA $[m^2]$	4778.848	5901.057	19.017
LCB [m]	94.456	95.827	1.429
KB [m]	1.758	1.758	0.017
Waterline 3.500 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4063.451	4071.419	0.195
Inertia $[m^4]$	249244.390	249305.676	0.024
LCF [m]	94.011	97.941	4.012
Volume $[m^3]$	13314.222	13327.072	0.096
WSA $[m^2]$	4816.504	5944.746	18.978
LCB [m]	94.457	95.899	1.503
KB [m]	1.810	1.809	0.014
Waterline 3.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4067.750	4083.488	0.385
Inertia $[m^4]$	249778.375	249771.705	0.002
LCF [m]	94.002	97.995	4.075
Volume $[m^3]$	13719.658	13733.252	0.098
WSA $[m^2]$	4854.108	5989.740	18.959
LCB [m]	94.454	95.969	1.578
KB [m]	1.861	1.861	0.010
Waterline 3.700 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4071.805	4071.770	0.000
Inertia $[m^4]$	250319.203	250319.836	0.000
LCF [m]	93.994	93.982	0.012
Volume $[m^3]$	14126.452	14140.596	0.100
WSA $[m^2]$	4891.841	6247.904	21.704
LCB [m]	94.446	95.973	1.591
KB [m]	1.913	1.912	0.008
Waterline 3.800 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4075.608	4076.072	0.011
Inertia $[m^4]$	250780.281	250779.790	0.000
LCF [m]	93.977	95.231	1.316
Volume $[m^3]$	14534.115	14548.022	0.095
WSA $[m^2]$	4929.249	6340.102	22.252
LCB [m]	94.435	95.935	1.563
KB [m]	1.964	1.964	0.011
Waterline 3.900 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4079.576	4088.371	0.215
Inertia $[m^4]$	251260.484	251260.817	0.000
LCF [m]	93.954	95.903	2.031
Volume $[m^3]$	14941.977	14956.243	0.095
WSA $[m^2]$	4966.502	6405.693	22.467
LCB [m]	94.425	95.925	1.563
KB [m]	2.016	2.015	0.011
Waterline 4.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4083.875	4097.103	0.322
Inertia $[m^4]$	251731.546	251806.111	0.029
LCF [m]	93.926	96.405	2.571
Volume $[m^3]$	15350.037	15365.514	0.100
WSA $[m^2]$	5003.673	6462.388	22.572
LCB [m]	94.416	95.931	1.579
KB [m]	2.067	2.067	0.006
Waterline 4.100 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4087.646	4101.142	0.329
Inertia $[m^4]$	252187.671	250963.192	0.485
LCF [m]	93.896	96.580	2.779
Volume $[m^3]$	15758.731	15775.426	0.105
WSA $[m^2]$	5040.867	6515.801	22.636
LCB [m]	94.404	95.946	1.606
KB [m]	2.118	2.118	0.001
Waterline 4.200 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4091.468	4095.761	0.104
Inertia $[m^4]$	252613.421	252663.842	0.019
LCF [m]	93.858	97.297	3.533
Volume $[m^3]$	16168.141	16185.265	0.105
WSA $[m^2]$	5077.917	6583.455	22.868
LCB [m]	94.392	95.971	1.645
KB [m]	2.170	2.170	0.001
Waterline 4.300 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4095.279	4101.819	0.159
Inertia $[m^4]$	253030.125	253012.634	0.006
LCF [m]	93.815	97.461	3.741
Volume $[m^3]$	16577.919	16595.144	0.103
WSA $[m^2]$	5114.930	6626.635	22.812
LCB [m]	94.379	96.006	1.694
KB [m]	2.221	2.221	0.002
Waterline 4.400 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4099.283	4106.046	0.164
Inertia $[m^4]$	253446.328	253478.507	0.012
LCF [m]	93.769	97.643	3.967
Volume $[m^3]$	16988.068	17005.535	0.102
WSA $[m^2]$	5151.936	6667.646	22.732
LCB [m]	94.366	96.043	1.745
KB [m]	2.273	2.272	0.003
Waterline 4.500 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4103.167	4105.905	0.066
Inertia $[m^4]$	253843.781	253840.488	0.001
LCF [m]	93.718	97.818	4.191
Volume $[m^3]$	17398.583	17416.131	0.100
WSA $[m^2]$	5188.941	6707.890	22.644
LCB [m]	94.353	96.083	1.800
KB [m]	2.324	2.324	0.004
Waterline 4.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4106.984	4111.796	0.117
Inertia $[m^4]$	254228.656	254222.887	0.002
LCF [m]	93.663	97.878	4.307
Volume $[m^3]$	17809.478	17827.016	0.098
WSA $[m^2]$	5225.961	6746.565	22.538
LCB [m]	94.339	96.123	1.856
KB [m]	2.375	2.375	0.006
Waterline 4.700 m			

Attribute	Real	Computed	Percentage difference
Area [m^2]	4110.729	4110.714	0.000
Inertia [m^4]	254607.750	254608.017	0.000
LCF [m]	93.603	93.601	0.002
Volume [m^3]	18220.822	18238.274	0.095
WSA [m^2]	5263.027	7017.152	24.997
LCB [m]	94.324	96.115	1.863
KB [m]	2.427	2.426	0.008
Waterline 4.800 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4114.627	4114.990	0.008
Inertia [m^4]	254983.812	254983.353	0.000
LCF [m]	93.537	94.813	1.346
Volume [m^3]	18632.607	18649.596	0.091
WSA [m^2]	5300.120	7108.358	25.438
LCB [m]	94.308	96.073	1.836
KB [m]	2.478	2.478	0.011
Waterline 4.900 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4118.527	4123.642	0.124
Inertia [m^4]	255348.390	255394.626	0.018
LCF [m]	93.465	95.525	2.156
Volume [m^3]	19044.750	19061.526	0.088
WSA [m^2]	5337.281	7176.044	25.623
LCB [m]	94.291	96.053	1.834
KB [m]	2.529	2.529	0.013
Waterline 5.000 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4122.475	4132.083	0.232
Inertia [m^4]	255708.750	255763.183	0.021
LCF [m]	93.387	96.004	2.725
Volume [m^3]	19457.250	19474.311	0.087
WSA [m^2]	5374.536	7232.698	25.691
LCB [m]	94.274	96.047	1.846
KB [m]	2.580	2.580	0.013
Waterline 5.100 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4126.516	4131.503	0.120
Inertia [m^4]	256072.671	256041.392	0.012
LCF [m]	93.304	96.403	3.215
Volume [m^3]	19870.107	19887.487	0.087
WSA [m^2]	5411.913	7283.889	25.700
LCB [m]	94.255	96.050	1.868
KB [m]	2.632	2.631	0.012
Waterline 5.200 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4130.643	4139.148	0.205
Inertia [m^4]	256439.562	256480.502	0.015
LCF [m]	93.214	96.663	3.568
Volume [m^3]	20283.322	20301.019	0.087
WSA [m^2]	5449.427	7330.656	25.662
LCB [m]	94.236	96.060	1.899
KB [m]	2.683	2.683	0.012
Waterline 5.300 m			

Attribute	Real	Computed	Percentage difference
Area [m^2]	4134.813	4142.185	0.177
Inertia [m^4]	256802.515	255906.825	0.348
LCF [m]	93.117	96.728	3.733
Volume [m^3]	20696.904	20715.086	0.087
WSA [m^2]	5487.092	7376.301	25.611
LCB [m]	94.215	96.073	1.933
KB [m]	2.734	2.734	0.011
Waterline 5.400 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4139.078	4143.845	0.115
Inertia [m^4]	257169.765	257197.177	0.010
LCF [m]	93.014	97.047	4.155
Volume [m^3]	21110.857	21129.386	0.087
WSA [m^2]	5524.917	7425.000	25.590
LCB [m]	94.193	96.089	1.972
KB [m]	2.786	2.785	0.010
Waterline 5.500 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4143.468	4139.579	0.093
Inertia [m^4]	257546.453	257566.320	0.007
LCF [m]	92.905	96.716	3.940
Volume [m^3]	21525.197	21543.557	0.085
WSA [m^2]	5562.912	7478.005	25.609
LCB [m]	94.170	96.104	2.012
KB [m]	2.837	2.836	0.012
Waterline 5.600 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4147.973	4142.958	0.120
Inertia [m^4]	257931.562	257989.000	0.022
LCF [m]	92.790	96.461	3.805
Volume [m^3]	21939.931	21957.680	0.080
WSA [m^2]	5601.079	7524.404	25.561
LCB [m]	94.145	96.113	2.047
KB [m]	2.888	2.888	0.016
Waterline 5.700 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4152.566	4153.227	0.015
Inertia [m^4]	258320.421	258355.652	0.013
LCF [m]	92.669	96.580	4.049
Volume [m^3]	22355.074	22372.491	0.077
WSA [m^2]	5639.424	7568.062	25.483
LCB [m]	94.119	96.121	2.082
KB [m]	2.939	2.939	0.018
Waterline 5.800 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4157.279	4159.927	0.063
Inertia [m^4]	258720.562	258744.663	0.009
LCF [m]	92.542	95.541	3.138
Volume [m^3]	22770.642	22788.158	0.076
WSA [m^2]	5677.947	7653.000	25.807
LCB [m]	94.092	96.120	2.109
KB [m]	2.991	2.990	0.019
Waterline 5.900 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4162.083	4162.119	0.000
Inertia $[m^4]$	259129.000	259129.042	0.000
LCF [m]	92.410	92.411	0.001
Volume $[m^3]$	23186.654	23204.290	0.076
WSA $[m^2]$	5716.645	7858.440	27.254
LCB [m]	94.063	96.081	2.100
KB [m]	3.042	3.041	0.019
Waterline 6.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4166.945	4167.019	0.001
Inertia $[m^4]$	259541.031	259540.759	0.000
LCF [m]	92.272	93.411	1.219
Volume $[m^3]$	23603.132	23620.772	0.074
WSA $[m^2]$	5755.517	7939.099	27.504
LCB [m]	94.033	96.025	2.074
KB [m]	3.093	3.093	0.020
Waterline 6.100 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4171.889	4172.100	0.005
Inertia $[m^4]$	259958.734	259957.996	0.000
LCF [m]	92.129	94.120	2.115
Volume $[m^3]$	24020.097	24037.727	0.073
WSA $[m^2]$	5794.561	8005.897	27.621
LCB [m]	94.001	95.986	2.067
KB [m]	3.145	3.144	0.021
Waterline 6.200 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4177.035	4178.933	0.045
Inertia $[m^4]$	260403.421	260440.457	0.014
LCF [m]	91.980	94.658	2.828
Volume $[m^3]$	24437.578	24455.277	0.072
WSA $[m^2]$	5833.788	8064.174	27.657
LCB [m]	93.969	95.959	2.073
KB [m]	3.196	3.195	0.021
Waterline 6.300 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4182.149	4186.031	0.092
Inertia $[m^4]$	260830.671	260865.495	0.013
LCF [m]	91.828	95.047	3.386
Volume $[m^3]$	24855.595	24873.524	0.072
WSA $[m^2]$	5873.172	8115.679	27.631
LCB [m]	93.934	95.940	2.090
KB [m]	3.247	3.247	0.021
Waterline 6.400 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4187.323	4193.613	0.149
Inertia $[m^4]$	261262.312	261312.776	0.019
LCF [m]	91.670	95.333	3.842
Volume $[m^3]$	25274.195	25292.506	0.072
WSA $[m^2]$	5912.722	8163.426	27.570
LCB [m]	93.899	95.928	2.115
KB [m]	3.299	3.298	0.020
Waterline 6.500 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4192.538	4206.159	0.323
Inertia $[m^4]$	261698.484	261692.164	0.002
LCF [m]	91.508	95.543	4.222
Volume $[m^3]$	25693.417	25712.495	0.074
WSA $[m^2]$	5952.440	8214.515	27.537
LCB [m]	93.862	95.920	2.145
KB [m]	3.350	3.349	0.018
Waterline 6.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4197.833	4207.465	0.228
Inertia $[m^4]$	262143.250	261183.492	0.366
LCF [m]	91.342	95.599	4.453
Volume $[m^3]$	26113.279	26133.176	0.076
WSA $[m^2]$	5992.326	8257.714	27.433
LCB [m]	93.823	95.914	2.179
KB [m]	3.401	3.401	0.016
Waterline 6.700 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4203.282	4204.626	0.031
Inertia $[m^4]$	262603.125	262596.055	0.002
LCF [m]	91.171	95.890	4.921
Volume $[m^3]$	26533.814	26553.780	0.075
WSA $[m^2]$	6032.384	8311.896	27.424
LCB [m]	93.784	95.911	2.218
KB [m]	3.453	3.452	0.016
Waterline 6.800 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4208.770	4208.645	0.002
Inertia $[m^4]$	263068.750	262971.718	0.036
LCF [m]	90.996	93.528	2.706
Volume $[m^3]$	26955.054	26974.475	0.071
WSA $[m^2]$	6072.596	8472.949	28.329
LCB [m]	93.743	95.893	2.241
KB [m]	3.504	3.504	0.018
Waterline 6.900 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4214.259	4224.785	0.249
Inertia $[m^4]$	263536.125	262131.184	0.533
LCF [m]	90.817	94.025	3.412
Volume $[m^3]$	27377.011	27396.157	0.069
WSA $[m^2]$	6112.959	8544.278	28.455
LCB [m]	93.702	95.860	2.251
KB [m]	3.556	3.555	0.018
Waterline 7.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4219.755	4250.014	0.711
Inertia $[m^4]$	264008.875	264012.066	0.001
LCF [m]	90.635	94.403	3.991
Volume $[m^3]$	27799.701	27819.898	0.072
WSA $[m^2]$	6153.467	8602.380	28.467
LCB [m]	93.659	95.835	2.270
KB [m]	3.607	3.607	0.014
Waterline 7.100 m			

Attribute	Real	Computed	Percentage difference
Area [m^2]	4225.304	4258.254	0.773
Inertia [m^4]	264491.625	264465.455	0.009
LCF [m]	90.450	94.564	4.350
Volume [m^3]	28223.136	28245.311	0.078
WSA [m^2]	6194.105	8648.253	28.377
LCB [m]	93.615	95.815	2.296
KB [m]	3.659	3.659	0.007
Waterline 7.200 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4230.931	4231.276	0.008
Inertia [m^4]	264985.125	264986.782	0.000
LCF [m]	90.262	90.515	0.280
Volume [m^3]	28647.318	28669.881	0.078
WSA [m^2]	6234.877	8907.241	30.002
LCB [m]	93.570	95.766	2.293
KB [m]	3.711	3.710	0.005
Waterline 7.300 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4236.565	4236.299	0.006
Inertia [m^4]	265482.125	265481.878	0.000
LCF [m]	90.071	91.395	1.448
Volume [m^3]	29072.257	29093.295	0.072
WSA [m^2]	6275.781	8983.505	30.141
LCB [m]	93.523	95.696	2.270
KB [m]	3.762	3.762	0.009
Waterline 7.400 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4242.163	4242.160	0.000
Inertia [m^4]	265983.625	265983.672	0.000
LCF [m]	89.877	89.879	0.002
Volume [m^3]	29497.955	29517.215	0.065
WSA [m^2]	6316.829	9097.349	30.564
LCB [m]	93.476	95.624	2.245
KB [m]	3.814	3.813	0.014
Waterline 7.500 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4247.721	4247.446	0.006
Inertia [m^4]	266496.062	266496.127	0.000
LCF [m]	89.677	90.531	0.943
Volume [m^3]	29924.380	29941.707	0.057
WSA [m^2]	6358.101	9162.258	30.605
LCB [m]	93.428	95.547	2.217
KB [m]	3.866	3.865	0.018
Waterline 7.600 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4253.343	4252.804	0.012
Inertia [m^4]	267010.375	267010.323	0.000
LCF [m]	89.474	91.084	1.768
Volume [m^3]	30351.531	30366.721	0.050
WSA [m^2]	6399.502	9222.825	30.612
LCB [m]	93.378	95.480	2.202
KB [m]	3.917	3.917	0.022
Waterline 7.700 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4258.920	4258.900	0.000
Inertia $[m^4]$	267534.906	267534.968	0.000
LCF [m]	89.264	89.268	0.004
Volume $[m^3]$	30779.638	30792.309	0.041
WSA $[m^2]$	6441.202	9352.787	31.130
LCB [m]	93.327	95.407	2.180
KB [m]	3.969	3.968	0.027
Waterline 7.800 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4264.040	4263.736	0.007
Inertia $[m^4]$	268066.968	268067.051	0.000
LCF [m]	89.037	89.852	0.907
Volume $[m^3]$	31208.761	31218.453	0.031
WSA $[m^2]$	6483.737	9414.318	31.128
LCB [m]	93.275	95.327	2.152
KB [m]	4.021	4.020	0.033
Waterline 7.900 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4269.081	4268.566	0.012
Inertia $[m^4]$	268604.187	268604.233	0.000
LCF [m]	88.796	90.357	1.727
Volume $[m^3]$	31638.144	31645.073	0.021
WSA $[m^2]$	6526.867	9473.142	31.101
LCB [m]	93.219	95.257	2.139
KB [m]	4.073	4.071	0.037
Waterline 8.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4277.905	4277.897	0.000
Inertia $[m^4]$	269163.781	269163.580	0.000
LCF [m]	88.532	88.534	0.001
Volume $[m^3]$	32071.103	32072.398	0.004
WSA $[m^2]$	6571.584	9603.098	31.568
LCB [m]	93.156	95.179	2.125
KB [m]	4.125	4.123	0.046
Waterline 8.100 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4284.176	4283.850	0.007
Inertia $[m^4]$	269729.968	269730.083	0.000
LCF [m]	88.222	89.065	0.946
Volume $[m^3]$	32501.851	32500.496	0.004
WSA $[m^2]$	6618.199	9663.013	31.509
LCB [m]	93.067	95.095	2.132
KB [m]	4.177	4.175	0.051
Waterline 8.200 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4289.008	4288.482	0.012
Inertia $[m^4]$	270304.906	270304.969	0.000
LCF [m]	87.975	89.491	1.694
Volume $[m^3]$	32916.476	32929.117	0.038
WSA $[m^2]$	6662.705	9720.193	31.455
LCB [m]	92.972	95.020	2.154
KB [m]	4.227	4.226	0.021
Waterline 8.300 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4298.019	4298.025	0.000
Inertia $[m^4]$	270908.125	270908.186	0.000
LCF [m]	87.684	87.686	0.002
Volume $[m^3]$	33313.367	33358.443	0.135
WSA $[m^2]$	6708.606	9848.967	31.885
LCB [m]	92.904	94.937	2.141
KB [m]	4.276	4.278	0.046
Waterline 8.400 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4308.932	4308.597	0.007
Inertia $[m^4]$	271552.812	271552.899	0.000
LCF [m]	87.359	88.172	0.922
Volume $[m^3]$	33706.929	33788.784	0.242
WSA $[m^2]$	6756.313	9908.469	31.812
LCB [m]	92.828	94.847	2.128
KB [m]	4.325	4.330	0.121
Waterline 8.500 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4321.772	4321.759	0.000
Inertia $[m^4]$	272273.500	272273.531	0.000
LCF [m]	86.977	86.978	0.000
Volume $[m^3]$	34117.230	34220.301	0.301
WSA $[m^2]$	6807.029	10001.845	31.942
LCB [m]	92.729	94.756	2.138
KB [m]	4.375	4.382	0.162
Waterline 8.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4320.947	4320.533	0.009
Inertia $[m^4]$	272869.468	272869.110	0.000
LCF [m]	86.571	87.349	0.891
Volume $[m^3]$	34546.984	34652.424	0.304
WSA $[m^2]$	6858.401	10056.444	31.800
LCB [m]	92.623	94.661	2.153
KB [m]	4.427	4.434	0.157
Waterline 8.700 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4346.504	4346.482	0.000
Inertia $[m^4]$	273838.687	273838.788	0.000
LCF [m]	86.173	86.175	0.002
Volume $[m^3]$	34982.492	35085.777	0.294
WSA $[m^2]$	6911.683	10149.551	31.901
LCB [m]	92.506	94.563	2.176
KB [m]	4.479	4.486	0.148
Waterline 8.800 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4364.386	4363.777	0.013
Inertia $[m^4]$	274754.781	274755.055	0.000
LCF [m]	85.741	86.631	1.026
Volume $[m^3]$	35419.199	35521.295	0.287
WSA $[m^2]$	6965.450	10207.952	31.764
LCB [m]	92.395	94.463	2.190
KB [m]	4.532	4.538	0.135
Waterline 8.900 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4384.299	4384.301	0.000
Inertia $[m^4]$	275789.281	275788.973	0.000
LCF [m]	85.254	85.258	0.005
Volume $[m^3]$	35858.234	35958.698	0.279
WSA $[m^2]$	7022.106	10312.544	31.907
LCB [m]	92.302	94.360	2.180
KB [m]	4.585	4.591	0.129
Waterline 9.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4401.180	4400.464	0.016
Inertia $[m^4]$	276805.250	276805.593	0.000
LCF [m]	84.822	85.700	1.024
Volume $[m^3]$	36297.210	36397.941	0.276
WSA $[m^2]$	7081.041	10369.198	31.710
LCB [m]	92.204	94.253	2.173
KB [m]	4.637	4.643	0.139
Waterline 9.100 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4417.137	4415.109	0.045
Inertia $[m^4]$	277879.625	277880.556	0.000
LCF [m]	84.447	85.976	1.778
Volume $[m^3]$	36738.730	36838.722	0.271
WSA $[m^2]$	7132.326	10420.029	31.551
LCB [m]	92.106	94.152	2.173
KB [m]	4.690	4.696	0.130
Waterline 9.200 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4432.436	4432.429	0.000
Inertia $[m^4]$	278940.812	278940.868	0.000
LCF [m]	84.109	84.113	0.005
Volume $[m^3]$	37182.328	37281.097	0.264
WSA $[m^2]$	7181.852	10554.188	31.952
LCB [m]	92.004	94.044	2.168
KB [m]	4.743	4.749	0.121
Waterline 9.300 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4447.402	4446.670	0.016
Inertia $[m^4]$	279952.156	279952.452	0.000
LCF [m]	83.767	84.572	0.952
Volume $[m^3]$	37625.144	37725.058	0.264
WSA $[m^2]$	7231.645	10610.779	31.846
LCB [m]	91.907	93.930	2.153
KB [m]	4.796	4.802	0.118
Waterline 9.400 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4459.192	4457.709	0.033
Inertia $[m^4]$	280889.625	280890.210	0.000
LCF [m]	83.502	84.863	1.604
Volume $[m^3]$	38067.214	38170.279	0.270
WSA $[m^2]$	7277.745	10660.737	31.733
LCB [m]	91.815	93.822	2.139
KB [m]	4.849	4.855	0.121
Waterline 9.500 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4469.463	4470.568	0.024
Inertia $[m^4]$	281789.000	281796.360	0.002
LCF [m]	83.274	85.163	2.218
Volume $[m^3]$	38509.878	38616.695	0.276
WSA $[m^2]$	7322.124	10711.374	31.641
LCB [m]	91.722	93.720	2.132
KB [m]	4.902	4.908	0.123
Waterline 9.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4482.515	4476.483	0.134
Inertia $[m^4]$	282700.343	281302.939	0.494
LCF [m]	82.997	85.568	3.004
Volume $[m^3]$	38954.753	39064.050	0.279
WSA $[m^2]$	7368.979	10768.888	31.571
LCB [m]	91.626	93.625	2.135
KB [m]	4.955	4.961	0.121
Waterline 9.700 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4496.876	4503.235	0.141
Inertia $[m^4]$	283656.625	283641.817	0.005
LCF [m]	82.726	85.953	3.754
Volume $[m^3]$	39401.585	39513.037	0.282
WSA $[m^2]$	7416.233	10828.240	31.510
LCB [m]	91.526	93.535	2.148
KB [m]	5.008	5.014	0.118
Waterline 9.800 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4506.744	4517.279	0.233
Inertia $[m^4]$	284482.000	284695.776	0.075
LCF [m]	82.478	86.244	4.365
Volume $[m^3]$	39849.976	39963.904	0.285
WSA $[m^2]$	7464.101	10880.765	31.400
LCB [m]	91.427	93.451	2.165
KB [m]	5.062	5.068	0.118
Waterline 9.900 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4515.141	4517.279	0.047
Inertia $[m^4]$	285253.312	285275.857	0.007
LCF [m]	82.316	84.406	2.476
Volume $[m^3]$	40297.960	40413.483	0.285
WSA $[m^2]$	7505.873	11015.716	31.862
LCB [m]	91.333	93.365	2.176
KB [m]	5.115	5.121	0.116
Waterline 10.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4523.910	4531.800	0.174
Inertia $[m^4]$	286013.531	283804.389	0.772
LCF [m]	82.157	84.624	2.915
Volume $[m^3]$	40739.292	40856.058	0.285
WSA $[m^2]$	7547.717	11086.540	31.919
LCB [m]	91.253	93.287	2.180
KB [m]	5.168	5.174	0.113
Waterline 10.100 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4532.262	4574.259	0.918
Inertia $[m^4]$	286740.531	286716.281	0.008
LCF [m]	82.004	84.726	3.212
Volume $[m^3]$	41171.062	41290.860	0.290
WSA $[m^2]$	7589.035	11157.866	31.984
LCB [m]	91.194	93.234	2.188
KB [m]	5.219	5.225	0.115
Waterline 10.200 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4539.867	4575.453	0.777
Inertia $[m^4]$	287450.437	285578.347	0.651
LCF [m]	81.870	84.800	3.454
Volume $[m^3]$	41597.015	41721.200	0.297
WSA $[m^2]$	7629.487	11209.686	31.938
LCB [m]	91.147	93.196	2.198
KB [m]	5.270	5.277	0.119
Waterline 10.300 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4546.540	4574.808	0.617
Inertia $[m^4]$	288116.468	284634.868	1.208
LCF [m]	81.757	84.834	3.626
Volume $[m^3]$	42020.457	42148.488	0.303
WSA $[m^2]$	7668.983	11256.342	31.869
LCB [m]	91.106	93.165	2.210
KB [m]	5.321	5.327	0.122
Waterline 10.400 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4552.261	4552.254	0.000
Inertia $[m^4]$	288732.343	288732.498	0.000
LCF [m]	81.664	81.667	0.003
Volume $[m^3]$	42442.429	42572.425	0.305
WSA $[m^2]$	7707.658	11474.796	32.829
LCB [m]	91.068	93.122	2.205
KB [m]	5.371	5.378	0.122
Waterline 10.500 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4558.157	4558.081	0.001
Inertia $[m^4]$	289345.312	289345.346	0.000
LCF [m]	81.573	81.641	0.083
Volume $[m^3]$	42863.335	42993.698	0.303
WSA $[m^2]$	7746.359	11530.398	32.817
LCB [m]	91.033	93.067	2.185
KB [m]	5.422	5.428	0.117
Waterline 10.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4563.744	4563.628	0.002
Inertia $[m^4]$	289931.062	289931.093	0.000
LCF [m]	81.489	81.588	0.121
Volume $[m^3]$	43283.296	43414.013	0.301
WSA $[m^2]$	7784.778	11573.214	32.734
LCB [m]	91.000	93.016	2.166
KB [m]	5.472	5.479	0.113
Waterline 10.700 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4569.086	4568.948	0.003
Inertia $[m^4]$	290498.812	290498.857	0.000
LCF [m]	81.413	81.528	0.141
Volume $[m^3]$	43702.503	43833.569	0.299
WSA $[m^2]$	7822.964	11613.527	32.639
LCB [m]	90.970	92.967	2.147
KB [m]	5.523	5.529	0.109
Waterline 10.800 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4574.181	4574.038	0.003
Inertia $[m^4]$	291042.281	291042.296	0.000
LCF [m]	81.343	81.462	0.145
Volume $[m^3]$	44121.105	44252.513	0.296
WSA $[m^2]$	7860.941	11652.927	32.541
LCB [m]	90.941	92.920	2.129
KB [m]	5.573	5.579	0.105
Waterline 10.900 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4579.054	4578.901	0.003
Inertia $[m^4]$	291564.187	291564.578	0.000
LCF [m]	81.279	81.389	0.134
Volume $[m^3]$	44539.222	44670.962	0.294
WSA $[m^2]$	7898.737	11691.945	32.442
LCB [m]	90.913	92.875	2.111
KB [m]	5.623	5.629	0.101
Waterline 11.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4583.719	4583.607	0.002
Inertia $[m^4]$	292064.718	292064.720	0.000
LCF [m]	81.221	81.311	0.110
Volume $[m^3]$	44956.960	45089.010	0.292
WSA $[m^2]$	7936.372	11730.807	32.345
LCB [m]	90.887	92.831	2.094
KB [m]	5.673	5.679	0.097
Waterline 11.100 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4588.195	4588.106	0.001
Inertia $[m^4]$	292545.250	292545.567	0.000
LCF [m]	81.169	81.229	0.074
Volume $[m^3]$	45374.394	45506.738	0.290
WSA $[m^2]$	7973.864	11769.607	32.250
LCB [m]	90.862	92.789	2.076
KB [m]	5.724	5.729	0.093
Waterline 11.200 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4592.487	4592.446	0.000
Inertia $[m^4]$	293005.468	293005.741	0.000
LCF [m]	81.121	81.144	0.029
Volume $[m^3]$	45791.585	45924.212	0.288
WSA $[m^2]$	8011.227	11808.391	32.156
LCB [m]	90.838	92.747	2.059
KB [m]	5.774	5.779	0.089
Waterline 11.300 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4596.604	4596.621	0.000
Inertia $[m^4]$	293446.062	293446.283	0.000
LCF [m]	81.077	81.058	0.023
Volume $[m^3]$	46208.593	46341.488	0.286
WSA $[m^2]$	8048.475	11847.173	32.064
LCB [m]	90.814	92.707	2.041
KB [m]	5.824	5.829	0.086
Waterline 11.400 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4600.548	4600.629	0.001
Inertia $[m^4]$	293866.750	293866.914	0.000
LCF [m]	81.038	80.971	0.082
Volume $[m^3]$	46625.457	46758.616	0.284
WSA $[m^2]$	8085.616	11885.949	31.973
LCB [m]	90.792	92.667	2.023
KB [m]	5.874	5.879	0.082
Waterline 11.500 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4605.714	4605.861	0.003
Inertia $[m^4]$	294259.875	294259.867	0.000
LCF [m]	81.026	80.920	0.131
Volume $[m^3]$	47042.039	47175.705	0.283
WSA $[m^2]$	8126.139	11924.894	31.855
LCB [m]	90.770	92.628	2.005
KB [m]	5.924	5.929	0.079
Waterline 11.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4609.578	4609.721	0.003
Inertia $[m^4]$	294638.250	294638.183	0.000
LCF [m]	81.001	80.886	0.141
Volume $[m^3]$	47458.320	47592.792	0.282
WSA $[m^2]$	8163.274	11964.830	31.772
LCB [m]	90.749	92.590	1.988
KB [m]	5.974	5.979	0.076
Waterline 11.700 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4613.323	4613.546	0.004
Inertia $[m^4]$	294997.500	294997.567	0.000
LCF [m]	80.980	80.814	0.205
Volume $[m^3]$	47874.457	48009.846	0.282
WSA $[m^2]$	8200.357	12003.964	31.686
LCB [m]	90.728	92.552	1.970
KB [m]	6.025	6.029	0.074
Waterline 11.800 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4616.928	4617.229	0.006
Inertia $[m^4]$	295338.093	295338.088	0.000
LCF [m]	80.962	80.742	0.271
Volume $[m^3]$	48290.484	48426.889	0.281
WSA $[m^2]$	8237.376	12042.702	31.598
LCB [m]	90.708	92.515	1.953
KB [m]	6.075	6.079	0.072
Waterline 11.900 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4620.390	4620.769	0.008
Inertia $[m^4]$	295659.625	295659.578	0.000
LCF [m]	80.947	80.673	0.338
Volume $[m^3]$	48706.410	48843.934	0.281
WSA $[m^2]$	8274.341	12081.412	31.511
LCB [m]	90.688	92.478	1.936
KB [m]	6.125	6.129	0.070
Waterline 12.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4623.706	4624.167	0.009
Inertia $[m^4]$	295961.812	295961.720	0.000
LCF [m]	80.935	80.607	0.406
Volume $[m^3]$	49122.269	49260.989	0.281
WSA $[m^2]$	8311.257	12120.106	31.425
LCB [m]	90.668	92.442	1.918
KB [m]	6.175	6.179	0.068
Waterline 12.100 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4626.869	4627.413	0.011
Inertia $[m^4]$	296244.156	296244.014	0.000
LCF [m]	80.926	80.542	0.474
Volume $[m^3]$	49538.066	49678.060	0.281
WSA $[m^2]$	8348.131	12158.766	31.340
LCB [m]	90.649	92.406	1.900
KB [m]	6.225	6.229	0.066
Waterline 12.200 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4629.899	4630.528	0.013
Inertia $[m^4]$	296509.562	296509.368	0.000
LCF [m]	80.920	80.480	0.542
Volume $[m^3]$	49953.808	50095.149	0.282
WSA $[m^2]$	8384.968	12197.374	31.255
LCB [m]	90.631	92.370	1.882
KB [m]	6.275	6.279	0.065
Waterline 12.300 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4632.804	4633.518	0.015
Inertia $[m^4]$	296758.687	296758.438	0.000
LCF [m]	80.916	80.421	0.610
Volume $[m^3]$	50369.523	50512.258	0.282
WSA $[m^2]$	8421.776	12235.914	31.171
LCB [m]	90.613	92.335	1.864
KB [m]	6.325	6.329	0.063
Waterline 12.400 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4635.594	4636.394	0.017
Inertia $[m^4]$	296992.656	296992.354	0.000
LCF [m]	80.913	80.365	0.677
Volume $[m^3]$	50785.210	50929.387	0.283
WSA $[m^2]$	8458.560	12274.382	31.087
LCB [m]	90.595	92.300	1.846
KB [m]	6.376	6.380	0.062
Waterline 12.500 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4638.284	4639.167	0.019
Inertia $[m^4]$	297212.843	297212.492	0.000
LCF [m]	80.913	80.312	0.743
Volume $[m^3]$	51200.890	51346.537	0.283
WSA $[m^2]$	8495.327	12312.781	31.003
LCB [m]	90.578	92.265	1.828
KB [m]	6.426	6.430	0.061
Waterline 12.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4640.880	4641.845	0.020
Inertia $[m^4]$	297419.718	297419.318	0.000
LCF [m]	80.915	80.262	0.806
Volume $[m^3]$	51616.566	51763.706	0.284
WSA $[m^2]$	8532.081	12351.111	30.920
LCB [m]	90.562	92.231	1.809
KB [m]	6.476	6.480	0.060
Waterline 12.700 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4643.082	4644.128	0.022
Inertia $[m^4]$	297611.593	297611.147	0.000
LCF [m]	80.912	80.208	0.870
Volume $[m^3]$	52032.203	52180.877	0.284
WSA $[m^2]$	8568.919	12389.403	30.836
LCB [m]	90.546	92.197	1.790
KB [m]	6.526	6.530	0.058
Waterline 12.800 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4645.510	4646.637	0.024
Inertia $[m^4]$	297796.781	297796.277	0.000
LCF [m]	80.917	80.164	0.930
Volume $[m^3]$	52447.796	52598.052	0.285
WSA $[m^2]$	8605.648	12427.618	30.753
LCB [m]	90.531	92.164	1.771
KB [m]	6.576	6.580	0.057
Waterline 12.900 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4647.885	4649.090	0.025
Inertia $[m^4]$	297972.968	297972.413	0.000
LCF [m]	80.923	80.124	0.987
Volume $[m^3]$	52863.371	53015.246	0.286
WSA $[m^2]$	8642.375	12465.758	30.671
LCB [m]	90.516	92.131	1.752
KB [m]	6.627	6.630	0.056
Waterline 13.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4650.212	4651.492	0.027
Inertia $[m^4]$	298140.812	298140.206	0.000
LCF [m]	80.931	80.088	1.042
Volume $[m^3]$	53278.964	53432.462	0.287
WSA $[m^2]$	8679.106	12503.833	30.588
LCB [m]	90.501	92.098	1.733
KB [m]	6.677	6.680	0.055
Waterline 13.100 m			

Attribute	Real	Computed	Percentage difference
Area [m^2]	4652.498	4653.850	0.029
Inertia [m^4]	298301.500	298300.841	0.000
LCF [m]	80.941	80.054	1.095
Volume [m^3]	53694.593	53849.702	0.288
WSA [m^2]	8715.840	12541.845	30.505
LCB [m]	90.487	92.065	1.714
KB [m]	6.727	6.731	0.055
Waterline 13.200 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4654.743	4656.163	0.030
Inertia [m^4]	298455.125	298454.422	0.000
LCF [m]	80.951	80.024	1.145
Volume [m^3]	54110.265	54266.969	0.288
WSA [m^2]	8752.581	12579.798	30.423
LCB [m]	90.474	92.034	1.694
KB [m]	6.777	6.781	0.054
Waterline 13.300 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4656.965	4658.457	0.032
Inertia [m^4]	298604.437	298603.753	0.000
LCF [m]	80.963	79.997	1.192
Volume [m^3]	54526.019	54684.267	0.289
WSA [m^2]	8789.330	12617.698	30.341
LCB [m]	90.461	92.002	1.675
KB [m]	6.827	6.831	0.053
Waterline 13.400 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4659.167	4660.724	0.033
Inertia [m^4]	298749.531	298748.787	0.000
LCF [m]	80.976	79.973	1.237
Volume [m^3]	54941.855	55101.600	0.289
WSA [m^2]	8826.088	12655.551	30.259
LCB [m]	90.448	91.971	1.655
KB [m]	6.878	6.881	0.052
Waterline 13.500 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4661.358	4662.976	0.034
Inertia [m^4]	298891.562	298890.784	0.000
LCF [m]	80.989	79.952	1.280
Volume [m^3]	55357.796	55518.971	0.290
WSA [m^2]	8862.858	12693.354	30.177
LCB [m]	90.436	91.940	1.635
KB [m]	6.928	6.931	0.051
Waterline 13.600 m			
Attribute	Real	Computed	Percentage difference
Area [m^2]	4663.532	4665.209	0.035
Inertia [m^4]	299029.531	299028.731	0.000
LCF [m]	81.004	79.935	1.319
Volume [m^3]	55773.855	55936.385	0.290
WSA [m^2]	8899.640	12731.108	30.095
LCB [m]	90.425	91.910	1.615
KB [m]	6.978	6.982	0.050
Waterline 13.700 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4665.719	4667.453	0.037
Inertia $[m^4]$	299168.968	299168.151	0.000
LCF [m]	81.019	79.919	1.357
Volume $[m^3]$	56190.027	56353.848	0.290
WSA $[m^2]$	8936.433	12768.827	30.013
LCB [m]	90.414	91.881	1.595
KB [m]	7.028	7.032	0.049
Waterline 13.800 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4667.809	4669.595	0.038
Inertia $[m^4]$	299308.437	299307.553	0.000
LCF [m]	81.033	79.904	1.393
Volume $[m^3]$	56606.414	56771.361	0.290
WSA $[m^2]$	8973.190	12806.506	29.932
LCB [m]	90.404	91.851	1.575
KB [m]	7.079	7.082	0.048
Waterline 13.900 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4669.990	4671.859	0.040
Inertia $[m^4]$	299448.968	299448.137	0.000
LCF [m]	81.049	79.893	1.425
Volume $[m^3]$	57023.003	57188.930	0.290
WSA $[m^2]$	9009.993	12844.171	29.851
LCB [m]	90.395	91.823	1.555
KB [m]	7.129	7.132	0.046
Waterline 14.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4672.187	4674.075	0.040
Inertia $[m^4]$	299591.718	299590.779	0.000
LCF [m]	81.065	79.883	1.457
Volume $[m^3]$	57439.718	57606.564	0.289
WSA $[m^2]$	9046.807	12881.799	29.770
LCB [m]	90.386	91.794	1.534
KB [m]	7.179	7.183	0.045
Waterline 14.100 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4674.410	4676.340	0.041
Inertia $[m^4]$	299737.500	299736.595	0.000
LCF [m]	81.082	79.876	1.486
Volume $[m^3]$	57856.542	58024.268	0.289
WSA $[m^2]$	9083.635	12919.406	29.689
LCB [m]	90.377	91.766	1.514
KB [m]	7.230	7.233	0.044
Waterline 14.200 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4676.651	4678.625	0.042
Inertia $[m^4]$	299886.312	299885.385	0.000
LCF [m]	81.098	79.871	1.512
Volume $[m^3]$	58273.414	58442.051	0.288
WSA $[m^2]$	9120.477	12956.992	29.609
LCB [m]	90.368	91.739	1.494
KB [m]	7.280	7.283	0.042
Waterline 14.300 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4678.913	4680.934	0.043
Inertia $[m^4]$	300037.500	300036.596	0.000
LCF [m]	81.116	79.869	1.537
Volume $[m^3]$	58690.343	58859.917	0.288
WSA $[m^2]$	9157.333	12994.562	29.529
LCB [m]	90.360	91.712	1.473
KB [m]	7.330	7.333	0.041
Waterline 14.400 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4681.197	4683.250	0.043
Inertia $[m^4]$	300191.281	300190.290	0.000
LCF [m]	81.133	79.867	1.560
Volume $[m^3]$	59107.375	59277.874	0.287
WSA $[m^2]$	9194.204	13032.115	29.449
LCB [m]	90.353	91.686	1.453
KB [m]	7.381	7.384	0.040
Waterline 14.500 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4683.497	4685.600	0.044
Inertia $[m^4]$	300347.062	300346.144	0.000
LCF [m]	81.151	79.868	1.581
Volume $[m^3]$	59524.515	59695.927	0.287
WSA $[m^2]$	9231.089	13069.657	29.370
LCB [m]	90.346	91.660	1.433
KB [m]	7.431	7.434	0.039
Waterline 14.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4685.819	4687.955	0.045
Inertia $[m^4]$	300505.125	300504.163	0.000
LCF [m]	81.170	79.870	1.600
Volume $[m^3]$	59941.773	60114.081	0.286
WSA $[m^2]$	9267.991	13107.187	29.290
LCB [m]	90.339	91.634	1.413
KB [m]	7.481	7.484	0.037
Waterline 14.700 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4688.159	4690.322	0.046
Inertia $[m^4]$	300664.437	300663.465	0.000
LCF [m]	81.189	79.874	1.619
Volume $[m^3]$	60359.144	60532.340	0.286
WSA $[m^2]$	9304.909	13144.706	29.211
LCB [m]	90.332	91.609	1.393
KB [m]	7.532	7.535	0.036
Waterline 14.800 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4690.513	4692.616	0.044
Inertia $[m^4]$	300824.843	300823.464	0.000
LCF [m]	81.208	79.878	1.638
Volume $[m^3]$	60776.652	60950.705	0.285
WSA $[m^2]$	9341.843	13182.200	29.132
LCB [m]	90.326	91.584	1.374
KB [m]	7.582	7.585	0.035
Waterline 14.900 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4692.854	4695.075	0.047
Inertia $[m^4]$	300987.250	300986.159	0.000
LCF [m]	81.228	79.886	1.651
Volume $[m^3]$	61194.308	61369.183	0.284
WSA $[m^2]$	9378.777	13219.718	29.054
LCB [m]	90.320	91.560	1.354
KB [m]	7.633	7.635	0.034
Waterline 15.000 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4695.237	4697.487	0.047
Inertia $[m^4]$	301150.843	301149.760	0.000
LCF [m]	81.249	79.895	1.666
Volume $[m^3]$	61612.156	61787.781	0.284
WSA $[m^2]$	9415.739	13257.218	28.976
LCB [m]	90.314	91.536	1.334
KB [m]	7.683	7.686	0.033
Waterline 15.100 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4697.631	4699.896	0.048
Inertia $[m^4]$	301314.437	301313.297	0.000
LCF [m]	81.270	79.905	1.680
Volume $[m^3]$	62030.183	62206.501	0.283
WSA $[m^2]$	9452.717	13294.711	28.898
LCB [m]	90.309	91.513	1.315
KB [m]	7.734	7.736	0.032
Waterline 15.200 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4700.041	4702.351	0.049
Inertia $[m^4]$	301478.687	301475.162	0.001
LCF [m]	81.293	79.917	1.691
Volume $[m^3]$	62448.398	62625.345	0.282
WSA $[m^2]$	9489.711	13332.213	28.821
LCB [m]	90.305	91.490	1.296
KB [m]	7.784	7.787	0.030
Waterline 15.300 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4702.465	4704.682	0.047
Inertia $[m^4]$	301643.125	301641.094	0.000
LCF [m]	81.316	79.927	1.707
Volume $[m^3]$	62866.824	63044.313	0.281
WSA $[m^2]$	9526.723	13369.686	28.743
LCB [m]	90.300	91.468	1.276
KB [m]	7.835	7.837	0.029
Waterline 15.400 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4704.904	4707.266	0.050
Inertia $[m^4]$	301808.250	301807.220	0.000
LCF [m]	81.340	79.945	1.714
Volume $[m^3]$	63285.460	63463.411	0.280
WSA $[m^2]$	9563.752	13407.209	28.667
LCB [m]	90.296	91.446	1.257
KB [m]	7.885	7.887	0.028
Waterline 15.500 m			

Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4707.351	4709.736	0.050
Inertia $[m^4]$	301973.062	301971.971	0.000
LCF [m]	81.364	79.961	1.724
Volume $[m^3]$	63704.324	63882.649	0.279
WSA $[m^2]$	9600.798	13444.703	28.590
LCB [m]	90.293	91.425	1.238
KB [m]	7.936	7.938	0.026
Waterline 15.600 m			
Attribute	Real	Computed	Percentage difference
Area $[m^2]$	4709.800	4712.190	0.050
Inertia $[m^4]$	302136.375	302135.219	0.000
LCF [m]	81.390	79.978	1.734
Volume $[m^3]$	64123.421	63882.649	0.375
WSA $[m^2]$	9637.863	13444.703	28.314
LCB [m]	90.289	91.425	1.241
KB [m]	7.986	7.938	0.606

B APPENDIX B

B.1 CONVERGENCE STUDY FOR DLIBFINDMINGLOBAL

B.1.1 Number of Iterations

For each waterline the number of iterations was 2000. We did some trails with the number of iterations reaching even the 40000 but the results were disappointing and for one waterline needs 3 days. After a series of testing on the number of iterations we ended up with the number of 2000. If we increase the iterations the results are not better or the improvement is very small compare to the time raise.

B.1.2 Time limits

The execution time for all the waterlines was 8087.12 seconds for the objective function that includes only the area attributes and 11008.22 seconds for the objective function that includes both area and volume attributes.

B.2 CONVERGENCE STUDY FOR DLIBFINDMINUSINGAPPROXIMATEDERIVATIVES

B.2.1 Number of Iterations

For this optimization algorithm the number of iterations are different from waterline to waterline. Because of the stop strategy we chose the algorithm will terminate if the difference between two consecutively results is smaller that 10^{-7}.

B.2.1.1 *Model*

The iteration's range for the created model is [355,753] and the average is at 515 iterations for the objective function for the area attributes and for the objective function that includes the volume attributes the iteration's range is [377,703] and the average is 573.

B.2.1.2 *Real Vessel*

The iteration's range for the real vessel is [235,1177] and the average is at 511 iterations for the objective function for the area attributes and for the objective function that includes the volume attributes the iteration's range is [1126,1728] and the average is 1350.

B.2.2 Time limits

B.2.2.1 *Model*

For the created model the total optimization time was 383.10 second for the objective function that includes only the area attributes and 1167.64 seconds for the objective function that includes both area and volume attributes.

B.2.2.2 *Real Vessel*

For the real vessel the total optimization time was 319.34 second for the objective function that includes only the area attributes and 2311.96 seconds for the objective function that includes both area and volume attributes.

B.3 CONVERGENCE STUDY FOR GENETICOPTIMIZER

B.3.1 Characteristics

The genetic algorithm's characteristics are the following.

1. Population : 100
2. Maximum generations : 30
3. Elite children : 5
4. Crossover fraction : 0.7
5. Stop strategy : if the difference of the objective function's best genes between two consecutively generations is less than 15 then the optimization process stops

B.3.2 Time limits

The execution time for all the waterlines was 7316.10 seconds for the objective function that includes only the area attributes and 11894.93 seconds for the objective function that includes both area and volume attributes.

B.4 CONVERGENCE STUDY FOR HYBRID

The hybrid model consists of the genetic algorithm that is being used to find a global minimum and the DlibFindMinUsingApproximateDerivatives algorithm in order to find the global minimum. Their characteristics are mentioned in the previous sections.

B.4.1 Number of Iterations

B.4.1.1 *Model*

The iteration's range for the created model is [62,433] and the average is at 280 iterations for the objective function for the area attributes and for the objective function that includes the volume attributes the iteration's range is [172,774] and the average is 414.

B.4.2 Time limits

The total optimization time was 8281.82 second for the objective function that includes only the area attributes and 12615.37 seconds for the objective function that includes both area and volume attributes.

B.5 CONVERGENCE STUDY FOR DLIBFINDMINUSINGAPPROXIMATEDERIVATIVES USING THE FLAT OF BOTTOM

B.5.1 Number of Iterations

For this optimization algorithm the number of iterations are different from waterline to waterline. Because of the stop strategy we chose the algorithm will terminate if the difference between two consecutively results is smaller that 10^{-7}.

B.5.1.1 Real Vessel

The iteration's range for the created model is [228,1329] and the average is at 1091 iterations for the objective function for the area attributes.

B.5.2 Time limits

B.5.2.1 Real Vessel

For the real vessel the total optimization time was 26881.19 second for the objective function that includes only the area attributes.