Εθνικό Μετσόβιο Πολυτεχνείο
Σχολή Ηλεκτρολόγων Μηχανικών
και Μηχανικών Υπολογιστών
Τομέας Τεχνολογίας Πληροφορικής και Υπολογιστών

Τεχνικές για Ανακατανομή του Φάσματος Ραδιοσυχνοτήτων

ΔΙΠΛΩΜΑΤΙΚΗ ΕΡΓΑΣΙΑ

ΠΕΡΙΚΛΗΣ ΑΘΑΝΑΣΑΚΗΣ

Επιβλέπων : Δημήτρης Φωτάκης
Αν. Καθηγητής Ε.Μ.Π.

Αθήνα, Νοέμβριος 2020

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Αθήνα, Νοέμβριος 2020

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Περικλής Αθανασάκης

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Περίληψη

Οι δημοπρασίες, είναι ένας μεγάλος τομέας οικονομίας και ένας τομέας έρευνας για πολλούς σπουδαίους οικονομολόγους και μηχανικούς. Συναντάμε δημοπρασίες σε απλές μορφές στον κόσμο εδώ και χιλιάδες χρόνια, ωστόσο πλέον είναι πολύ μεγάλο μέρος της οικονομίας καθώς "κυβερνούν" αποκλειστικά ορισμένες αγορές. Η αγορά ραδιοφάσματος είναι μία από αυτές τις αγορές. Από το 1994 το φάσμα κατανέμεται στις περισσότερες χώρες του κόσμου μέσω δημοπρασιών. Υπήρξαν πολλές δημοπρασίες φάσματος σε όλο τον κόσμο, διαφορετικών μορφών, κάποιες επιτυχημένες και άλλες αποτυχημένες. Η θεωρία των δημοπρασιών και ο σχεδιασμός μηχανισμών που τις υλοποιούν έχουν συνεισφέρει πολύ όσο αναφορά τις δημοπρασίες φάσματος, προκειμένου να κατανέμεται το φάσμα με τον "καλύτερο" δυνατό τρόπο. Η πιό συνηθισμένη μορφή δημοπρασίας μέχρι τώρα ήταν αυτή της δεύτερης-τιμής. Ωστόσο η τελευταία λέξη της τεχνολογίας σε ότι αφορά τις δημοπρασίες φάσματος, είναι της αναβαλλόμενης αποδοχής.

Επειδή ο χώρος του φάσματος είναι πεπερασμένος, οι χώρες πρέπει να βρουν έναν τρόπο για να "δημιουργήσουν" περισσότερο χώρο για τις κορυφαίες τεχνολογίες του σήμερα. Ένας προφανής τρόπος είναι να πάρουν πίσω το χώρο που είχε δοθεί στο παρελθόν σε τηλεοπτικούς σταθμούς και να τον εκχωρήσουν ξανά σε ασύρματες υπηρεσίες. Αυτό μπορεί να ακούγεται εύκολο, αλλά είναι ακριβώς το αντίθετο στην πράξη. Όπως αναφέραμε παραπάνω, έχουν πραγματοποιηθεί πολλές δημοπρασίες φάσματος από το 1994. αλλά δεν υπήρξε ποτέ δημοπρασία φάσματος για την ανακατανομή αυτού, έως το 2016. Η δημοπρασία, Incentive Auction του FCC το 2016 ήταν η πρώτη και ακόμη μόνο δημοπρασία που πέτυχε αυτό το δύσκολο έργο. Η αποστολή της δημοπρασίας ήταν να αγοράστουν πρώτα τα δικαιώματα από τηλεοπτικούς σταθμούς που θέλουν να συμμετάσχουν, στη συνέχεια να γίνει η ανακατανομή των υπόλοιπων τηλεοπτικών σταθμών σε ένα "μικρότερο" χώρο στο φάσμα και, τέλος, να δημιουργηθούν και να πωληθούν νέες άδειες χρήσης στον χώρο που δημιουργήθηκε σε επιχειρήσεις ασύρματων υπηρεσιών. Η ανακατανομή των εναπομείναντων τηλεοπτικών σταθμών σε "μικρότερο" χώρο στο φάσμα, είναι το Stations Repacking Problem, και είναι NP-Complete πρόβλημα, γενικεύοντας το Graph-Coloring που είναι επίσης NP-Complete. Έτσι, για να εφαρμοστεί επιτυχώς αυτή η δημοπρασία, υπήρχε επίσης ανάγκη να λυθεί αυτό το πρόβλημα βέλτιστα.

Σε αυτή τη διπλωματική εργασία, θα εισάγουμε πρώτα κάποιες βασικές αρχές στη θεωρία των δημοπρασιών και το σχεδιασμό των κατάλληλων μηχανισμών. Το κάνουμε αυτό με σκοπό να κατανοήσουμε σε βάθος την ιδέα και την ακριβή διαδικασία της δημοπρασίας αυτής και της σύνδεσής της με το πρόβλημα της ανακατανομής των τηλεοπτικών σταθμών. Η συμβολή μας σε αυτή την εργασία, είναι να προσπαθήσουμε να δημιουργήσουμε το δικό μας εργαλείο επίλυσης του προβλήματος επανακατανομής των σταθμών χρησιμοποιώντας ένα framework της Google - το OR-TOOLS. Θα συγκρίνουμε το εργαλείο επίλυσης με το αρχικό που χρησιμοποιήθηκε, καθώς είναι ένα εργαλείο επίλυσης ανοιχτού κώδικα, και θα μελετήσουμε τη βελτιστοποίηση του και την επίπτωση αυτού του εργαλείου επίλυσης σε όλη τη διαδικασία της δημοπρασίας.

Λέξεις Κλειδιά: δημοπρασίες, Incentive Auction, Station Repacking Problem, NP, OR-TOOLS, Graph-Coloring, βελτιστοποίηση, σχεδίαση μηχανισμών

Abstract

Auctions, are a big field of economics and a sector of research for many great economists and engineers. Auctions exist in simple forms in the world for thousands of years, but nowadays the are a very big part of economy as they "rule" exclusively some markets. Spectrum market is one of those markets. Since 1994 spectrum is allocated in most world countries via auctions. There has been many spectrum auctions throughout the world, of different formats and both of success and failure. Auction theory and mechanism design have contributed a lot when it comes to spectrum auctions, in order to allocate the spectrum in the "best" possible way. Vickrey (second-price) auctions have been the most common auctions when it comes to spectrum and in general in the past years. However, the state of the art *Deferred Acceptance* auctions seem to be more suitable for the spectrum auctions.

Because the spectrum space is finite, countries must find a way to "create" more space for the state of the art technologies. One obvious way is to take previously assigned to TV stations space and re-assign it to wireless services. This may sound easy in simple words, but it is the exact opposite in practice. As we mentioned above there has been many spectrum auctions in practice since 1994; but there has never been a spectrum auction to re-allocate the spectrum as we said, until 2016. The FCC's **Incentive Auction** of 2016, based on the deferred acceptance format, was the first and still only auction to achieve this difficult task. This auction's task was to first buy the rights from some TV stations who want to participate, then *repack* the remaining TV stations in a "smaller" space in spectrum, and finally sell new licences with the space that was made to wireless service businesses. To repack is the remaining TV stations in a "smaller" space in spectrum, is the **Stations Repacking Problem**, and it is NP-Hard as it generalizes the *graph-coloring* problem. So, in order for this auction to be implemented successfully, there was also a need for this problem to be solved "efficiently".

In this thesis, we will first establish some basic knowledge of auction theory and mechanism design. We do that in order to understand deeply the idea and the exact process of the Incentive Auction and also the connection of it with the station repacking problem. Our contribution in this thesis, is to try and create our own solver of the stations repacking problem using a google's framework - the OR-TOOLS. We will compare our solver with the original one that was used, as it is an open-source solver and observe the optimization and the importance of this solver in the whole auction's process.

Ευχαριστίες

Ολοκληρώνοντας αυτήν τη διπλωματική εργασία τελειώνω και τις προπτυχιακές μου σπουδές. Ένα αρκετά μεγάλο κεφάλαιο της ζωής μου κλείνει και νιώθω πως οφείλω να ευχαριστήσω κάποιους ανθρώπους. Αρχικά, θα ήθελα να ευχαριστήσω τον κ. Δημήτρη Φωτάκη, επιβλέποντα αυτής της διπλωματικής εργασίας. Είχαμε μια άψογη συνεργασία και επικοινωνία και σίγουρα έχω παρει πολλά πράγματα από εκείνον. Αισθάνομαι τυχερός που τον είχα αρχικά ως δάσκαλο στη σχολή και έπειτα που είχα την καθοδήγηση του σε αυτή τη διπλωματική εργασία.

Θέλω να ευχαριστήσω τη μητέρα μου για την αγάπη της όλα αυτά τα χρόνια, αλλά και γιατί μου έδειξε πως ποτέ δεν πρέπει να σταματήσουμε να παλεύουμε για το καλύτερο, ακόμα και στις πιό δύσκολες εποχές. Τον πατέρα μου, την αδερφή μου και τη μητέρα της, που ήταν δίπλα μου και με στηρίξανε, κυρίως την περίοδο της καραντίνας του 2020 που έτρεχα αυτήν την διπλωματική εργασία. Τον θείο μου Κώστα και τα ξαδέρφια μου Νικόλα και Ζώη γιατί πιστεύουν σε μένα και με στηρίζουν.

Φυσικά πρέπει να ευχαριστήσω και τους φίλους μου. Την παρέα μου από τη σχολή, που μαζί περάσαμε δυσκολίες και επιτυχίες μέχρι να φτάσουμε το στόχο του πτυχίου. Αλλά και την παρέα μου εκτός σχολής. Δεν θα αναφέρω ονόματα, αλλά πιστεύω ξέρουν αυτοί που είναι δίπλα μου τόσα χρόνια ότι πάει σε εκείνους.

Τέλος ευχαριστώ όσους μου έδωσαν ή μου δίνουν ακόμα κίνητρο να προσπαθώ να εξελίσσομαι και να γίνομαι καλύτερος μέρα με τη μέρα.

Περικλής Αθανασάκης,

Αθήνα, 4η Νοεμβρίου 2020

Contents

Chapter 1

Εκτενής Περίληψη στα Ελληνικά

Σε αυτό το κεφάλαιο θα παρουσιάσουμε εκτενώς την παρούσα διπλωματική εργασία μέσα από μία εκτεταμένη περίληψη στα Ελληνικά.

1.1 Το Πρόβλημα

Σε αυτή την εργασία θα ασχοληθούμε με την ανακατανομή του φάσματος των συχνοτήτων μέσω δημοπρασιών. Το εν λόγω φάσμα είναι πεπερασμένο και οι συχνότητες μέσα σε αυτό επεμβαίνουν μεταξύ τους. Έτσι, με το πέρασμα των χρόνων ο χώρος αυτός γέμισε. Οι καινούριες τεχνολογίες όμως, όπως οι ασύρματες υπηρεσίες, απαιτούν όλο και περισσότερο χώρο στο φάσμα.

Μία καλή λύση για αυτό είναι να παρθεί ο δοσμένος χώρος σε τηλεοπτικούς σταθμούς, ώστε να φτιαχτεί νέος χώρος για τις πιο σύγχρονες τεχνολογίες. Για να γίνει αυτό όμως πρέπει να δεχθούν οι τηλεοπτικοί σταθμοί να πουλήσουν τις άδειες τους, ώστε αυτές να μεταπωληθούν σε άλλες υπηρεσίες. Άρα πρέπει να τους δοθεί κίνητρο για να "αφήσουν" τα δικαιώματα τους επάνω στο φάσμα. Μια δημοπρασία κινήτρων ή Incentive Auction που πραγματοποίησε το ομοσπονδιακή επιτροπή επικοινωνιών (FCC) των ΗΠΑ. Σκοπός της διπλής αυτής δημοπρασίας ήταν ο παραπάνω : να αγοράσει "φθηνά" κάποιες άδειες από τηλεοπτικούς σταθμούς και να φτιάξει νέες άδειες που θα πουλήσει σε ασύρματες υπηρεσίες.

Εδώ όμως αναδύεται ένα ακόμη πρόβλημα. Καθώς αγοράζουμε άδειες από τους σταθμούς θα ελευθερώνεται χώρος στο φάσμα διάσπαρτα ανάμεσα στον χώρο που θα εκπέμπουν οι εναπομείναντες τηλεοπτικοί σταθμοί. Οι ασύρματες υπηρεσίες όμως χρειάζονται ενιαίο συνεχή χώρο στο φάσμα προκειμένου να εκπέμψουν. Εδώ δημιουργείται λοιπόν η ανάγκη για ανακατανομή. Να ανακατανεμηθούν δηλαδή οι εναπομείναντες τηλεοπτικοί σταθμοί σε μικρότερο χώρο στο φάσμα, ώστε να δημιουργηθούν νέες άδειες σε συνεχή και ενιαίο χώρο. Αυτό είναι το πρόβλημα της ανακατανομής των σταθμών και είναι ένα NP-Complete πρόβλημα, γενικεύοντας το graph-coloring πρόβλημα. Το πρόβλημα της ανακατανομής των σταθμών μαζί με το σχεδιασμό της δημοπρασίας ήταν οι προκλήσεις που είχε να αντιμετωπίσει η ομοσπονδιακή επιτροπή επικοινωνιών.

1.2 Συνεισφορά

Η κύρια συνεισφορά της παρούσας εργασίας εντοπίζεται στο πρόβλημα ανακατανομής των σταθμών και την επίπτωση αυτόυ στο σύνολο της δημοπρασίας. Αναλυτικά στην εργασία αυτή, εισάγουμε αρχικά το απαιτούμενο θεωρητικό υπόβαθρο. Έπειτα εξηγούμε σχολαστικά την όλη διαδικασία της δημοπρασίας, και τη σύνδεση της με το πρόβλημα ανακατανομής των σταθμών. Στη συνέχεια θα εξηγήσουμε αναλυτικά και επίσημα το πρόβλημα αυτό αλλά και την προσέγγιση και βελτιστοποίηση που έγινε από μεριάς της ομοσπονδιακής επιτροπής επικοινωνιών. Εμείς φτιάξαμε ένα εργαλείο επίλυσης αυτού του προβλήματος παρόμοιο με το αυθεντικό, που χρησιμοποιήθηκε για τη δημοπρασία. Για την δημιουργία αυτού του εργαλείου χρησιμοποιήθηκε ένα framework της Google, τα Or-Tools. Αφού καταφέραμε να λύσουμε επιτυχημένα το πρόβλημα αυτό με τον δικό μας λύτη, συγκρίναμε και την αποτελεσματικότητα των δύο. Και παρατηρήσαμε πόσο σημαντικό ρόλο παίζει η βελτιστοποίηση του. Με άλλα λόγια τι οικονομικό αντίκτυπο έχει το πρόβλημα της ανακατανομής των σταθμών στην δημοπρασία, ως προς την βελτιστοποίηση του.

1.3 Θεωρητικό Υπόβαθρο

1.3.1 Δημοπρασίες

Οι δημοπρασίες υπάρχουν στον κόσμο από τα αρχαία χρόνια. Λέγεται ότι στην αρχαία Βαβυλώνα, έθεταν τις γυναίκες σε δημοπρασία. Αργότερα στην αρχαία Ελλάδα συναντάμε δημοπρασίες σκλάβων, ενώ αργότερα στην αρχαία Ρώμη έγιναν πιο δημοφιλής αφού τις χρησιμοποιούσαν για την πώληση πολεμικού εξοπλισμού, μέχρι και την πώληση χρέους. Η θεωρία δημοπρασιών όμως είναι ένας κλάδος της μικροοικονομίας που ξεκίνησε ουσιαστικά τη δεκαετία του 1960 με την δουλειά του Ουίλλιαμ Βίκρυ, νικητής και του βραβείου Νόμπελ για αυτή του τη συνεισφορά το 1996.

Έχουμε τέσσερις βασικές κατηγορίες δημοπρασιών.

- **Αγγλικές Δημοπρασίες (Ανοιχτές ανοδικές δημοπρασίες)**
  Ίσως ο πιο κοινός τύπος δημοπρασίας, αφού αυτόν έχουμε συνηθίσει από ταινίες και βιβλία. Η χρήση τους αφορά κυρίως αντικείμενα αξίας που όμως δεν μπορεί να εκτιμηθεί εύκολα, όπως έργα τέχνης. Η τιμή ανεβαίνει συνεχώς και συνήθως ανακοινώνεται από τον δημοπράτη, και οι παίχτες πρέπει να αποσυρθούν από τη δημοπρασία όταν η τιμή φτάσει ένα όριο πέρα από το οποίο δεν θέλουν να πληρώσουν. Ο τελευταίος που μένει στην δημοπρασία κερδίζει.

- **Ολλανδικές Δημοπρασίες (Ανοιχτές καθοδικές δημοπρασίες)**
  Εδώ η χρήση τους συναντάτε κυρίως για μικρότερης αξίας αντικείμενα τα οποία πωλούνται σε μεγάλες ποσότητες. Πήραν το όνομα τους από της δημοπρασίες λουλουδιών που γινόντουσαν στην Ολλανδία, ενώ ακόμα ένα χαρακτηριστικό παράδειγμα είναι αυτό της πώλησης καπνού στον Καναδά. Δουλεύει με τον ακριβώς αντίθετο τρόπο από τις Αγγλικές δημοπρασίες. Δηλαδή, τιμή πέφτει συνεχώς και ο πρώτος που θα δηλώσει ότι θέλει το προϊόν, κερδίζει και πρέπει να

πληρώσει την τιμή που αναγράφεται εκείνη την στιγμή.

- **Κλειστές δημοπρασίες πρώτης τιμής**
  Σε αυτό το τύπο δημοπρασίας οι παίκτες καταθέτουν ταυτόχρονα κλειστές προτάσεις για το ποσό που διατίθενται να πληρώσουν για το αγαθό. Νικητής είναι αυτός που καταθέτει την υψηλότερη προσφορά και πληρώνει την προσφορά που έκανε. Τέτοιου τύπου δημοπρασίες συναντάμε στις ΗΠΑ για αδειοδότηση σε εξορύξεις αλλά και αρκετές φορές σε δημοπρασίες για διαφημίσεις στο διαδίκτυο.

- **Κλειστές δημοπρασίες δεύτερης τιμής**
  Ονομάζονται και δημοπρασίες Βίκρυ από τον Ουίλλιαμ Βίκρυ. Οι συμμετέχοντες καταθέτουν ταυτόχρονα κλειστές προτάσεις για το ποσό που διατίθενται να πληρώσουν για το αγαθό. Νικητής είναι αυτός που καταθέτει την υψηλότερη προσφορά αλλά πληρώνει ένα ποσό ίσο με την δεύτερη υψηλότερη προσφορά που έγινε. Η μόνη διαφορά τους με τις προηγούμενες είναι στην πληρωμή από τον νικητή της δημοπρασίας.

1.3.2 Ισορροπία και Στρατηγικές

Η Θεωρία Παιγνίων μελετά την αλληλεπίδραση στρατηγικών οντοτήτων. Πολύ σημαντικό ρόλο στην αλληλεπίδραση αυτή παίζουν τα λεγόμενα “σημεία ισορροπίας”. Διαισθητικά, λέμε ότι ένα παίγνιο είναι σε κάποιο σημείο ισορροπίας όταν οι συμμετέχοντες σε αυτό δεν έχουν λόγο να διαφοροποιήσουν τη στρατηγική τους, οπότε καθώς ο χρόνος περνά, οι κινήσεις που κάνουν δεν αλλάζουν. Έτσι, υπάρχει κατά κάποιο τρόπο ισορροπία. Για να δούμε βασικά παραδείγματα τέτοιων ισορροπιών είναι χρήσιμο να ορίσουμε ένας είδος παιγνίου.

Παίγνιο Ελαχιστοποίησης Κόστους

- Έχουμε n παίκτες.

- Ο πεπερασμένος χώρος των στρατηγικών S_i για τους παίκτες $i, i = 1, ..., n.$. Στις δημοπρασίες αυτός ο χώρος αποτελείται από διαφορετικές προτάσεις που μπορεί να κάνει ο κάθε παίκτης για το αγαθό. Έχουμε $s = (s_1, ..., s_n)$ τα διαφορετικά στρατηγικά προφίλ του παιχνιδιού.

- $C_i : S_1 \times ... \times S_n \to R$ μια συνάρτηση κόστους για κάθε παίκτη i. Υποθέτουμε ότι όλοι οι παίκτες αποσκοπούν στην ελαχιστοποίηση του κόστους τους.

Λέμε ότι *Κυρίαρχη Στρατηγική* s' για τον παίχτη i έχουμε όταν $\forall s \in S_i, \forall s_{-i} \in S1 \times ... \times S_{i-1} \times S_{i+1} \times ... \times S_n$ ισχύει: $C_i(s', s_{-i}) \leq C_i(s, s_{-i})$

Ορίζεται ως *Καθαρή Ισορροπία Nash*, όταν για κάθε παίκτη και στρατηγική ισχύει: $C_i(s) \leq C_i(s'_i, s_{-i})$

1.3.3 Σχεδιασμός Μηχανισμών

Ο Σχεδιασμος Μηχανισμών ασχολείται με τη δημιουργία κανόνων και συστημάτων που διέπουν τη λειτουργία τέτοιων στρατηγικών οντοτήτων. Στόχος είναι ο σχεδιασμός κανόνων που δίνουν στους παίκτες εύκολα ανιχνεύσιμες κυρίαρχες στρατηγικές. Με αυτό τον τρόπο είναι εύκολο για τους παίχτες να αποφασίσουν πώς θα παίξουν, αλλά και για τον δημιουργό να προβλέψει το αποτέλεσμα της αλληλεπίδρασης. Ένα παράδειγμα τέτοιου μηχανισμού είναι η δημοπρασία Δεύτερης Τιμής, στην οποία αναφερθήκαμε και νωρίτερα.

1.3.4 Δημοπρασία Δεύτερης Τιμής

Έστω ότι θέλουμε να πουλήσουμε ένα αντικείμενο σε ένα σύνολο από n αγοραστές. Ζητάμε από όλους να μας δώσουν μια προσφορά ιδιωτικά και δίνουμε το αντικείμενο σε αυτόν που έκανε τη μεγαλύτερη προσφορά και κατόπιν τον χρεώνουμε τη δεύτερη μεγαλύτερη προσφορά.

Έστω ότι πουλάμε ένα μόνο αγαθό σε n πιθανούς αγοραστές-παίκτες. Κάθε παίκτης έχει την δική του προσωπική και ιδιωτική εκτίμηση για το αγαθό αυτό, έστω v_i. Επίσης, κάθε παίκτης υποβάλει την πρόταση του για αυτό το αγαθό έστω b_i. Και τέλος έχουμε και την πληρωμή p που θα πληρώσει αυτός ο παίκτης που θα πάρει το αντικείμενο.

Ορίζουμε ώς $u_i = v_i - p$ την ωφέλεια του παίκτη.

Είπαμε ότι στην δημοπρασία δεύτερης τιμής, ο παίκτης που κερδίζει το αντικείμενη πληρώνει το ποσό της δεύτερης μεγαλύτερης πρότασης. Σε αυτόν τον τύπο δημοπρασίας βρίσκουμε ως κυρίαρχη στρατηγική για κάθε παίκτη να κάνει ως πρόταση την δική του προσωπική εκτίμηση για το αγαθό. Δηλαδή : Εάν ο παίκτης i κάνει πρόταση $b_i = v_i$ θα μεγιστοποιεί πάντα την ωφέλεια του u_i, ανεξάρτητα από τις προτάσεις των άλλων παικτών. Η παραπάνω ιδιότητα καθιστά τον μηχανισμό αυτό **αληθή** και είναι ιδιαίτερα σημαντική.

1.3.5 Δημοπρασίες Αναβαλλόμενης Αποδοχής

Αν και οι δημοπρασίες δεύτερης τιμής έχουν πολύ καλές θεωρητικές βάσεις, στην πράξη δεν απέδιδαν τόσο καλά για τις δημοπρασίας του φάσματος συχνοτήτων. Ο μηχανισμός αυτός σε δημοπρασίες φάσματος έχει γνωρίσει τραγικές αποτυχίες στο παρελθόν. Η ομοσπονδιακή επιτροπή επικοινωνιών ανέθεσε στον Paul Milgrom, το σχεδιασμό της δημοπρασίας. Εκείνος επιτυχημένα εκτέλεσε το σκοπό του βασιζόμενος στις Δημοπρασίες Αναβαλλόμενης Αποδοχής[45].

Ο τύπος αυτών των δημοπρασιών βασίζεται σε επαναλαμβανόμενες διαδικασίες αποδοχής ή απόρριψης. Υλοποιούνται μέσω αλγορίθμων αναβαλλόμενης αποδοχής οι οποίοι λειτουργούν ως εξής: Αρχικά οι παίκτες υποβάλλουν τις προτάσεις τους που θεωρούνται ενεργές. Σε κάθε μια επανάληψη οι ενεργές προτάσεις αξιολογούνται με ένα μη αρνητικό σκορ μέσω μιας επιλεγμένης συνάρτησης αξιολόγησης. Μια τέτοια συνάρτηση μπορεί να δεχθεί ως παραμέτρους: τις προτάσεις των παικτών, τις ταυτότητες των παικτών, τους εναπομείναντες παίκτες κ.α.. Σε κάθε επανάληψη ο αλγόριθμος απορρίπτει τον παίκτη που έχει το μεγαλύτερο σκορ, δηλαδή το λιγότερο ελκυστικό παίκτη ως προς τον δημοπράτη. Η διαδικασία τελειώνει όταν δεν υπάρχει κανένα θετικό σκορ, και οι νικητές είναι οι εναπομείναντες ενεργοί παίκτες.

Αυτός ο τύπος δημοπρασίας αρχικά είναι ένας αληθής μηχανισμός. Επιπλέον η συνάρτηση αξιολόγηση επιτρέπει στο δημοπράτη να λάβει υπ όψιν του πέρα από την "καλύτερη" πρόταση και τις τυχόν παρεμβολές που μπορεί να δημιουργεί κάποιος σταθμός. Για παράδειγμα, μπορεί ένας σταθμός να απαιτεί πολλά χρήματα για να αφήσει την άδεια του, αλλά μπορεί να επεμβαίνει με πολλούς άλλους, οπότε να είναι πιο σημαντικό να αποκτηθεί από κάποιον άλλο σταθμό που ζητά λιγότερα. Επιπλέον, θέτοντας το σκορ στο 0 έχουμε την δυνατότητα να "παγώσουμε" κάποιους σταθμούς τους οποίους στο συγκεκριμένο σημείο της δημοπρασίας δεν μπορούμε να επιστρέψουμε στον αέρα, επειδή δεν είναι εφικτό (δεν υπάρχει χώρος).

1.4 Η Δημοπρασία

Η δημοπρασία κινήτρων κατάφερε το σκοπό της. Διήρκεσε 13 μήνες την περίοδο 2016-2017 και πέρα από την επιθυμητή ανακατανομή, έφερε καθαρό κέρδος περισσότερο από επτά δισεκατομμύρια δολάρια στο αμερικανικό κράτος.

Αρχικά χρειάστηκε την υλοποίηση δύο δημοπρασιών. Μια αντίστροφη δημοπρασία, η οποία αποσκοπεί στην αγορά των αδειών από τους τηλεοπτικούς σταθμούς. Και μια προωθητική δημοπρασία για να πουληθούν οι νέες άδειες στις εταιρίες ασύρματων υπηρεσιών. Και οι δύο αυτές δημοπρασίες βασίζονται στις δημοπρασίες αναβαλλόμενης αποδοχής.

1.4.1 Στάδια

Πριν ξεκινήσουν οι δημοπρασίες αυτές, έπρεπε να είναι γνωστό το πόσος χώρος πρέπει να "καθαριστεί" στο φάσμα, προκειμένου να είναι εφικτή η ανακατανομή των εναπομεινάντων σταθμών. Αυτή η ερώτηση όμως δεν γίνεται να απαντηθεί χωρίς να γνωρίζουμε τη προσφορά και την ζήτηση στην αγορά, άρα δεν μπορούσε και να απαντηθεί πριν την δημοπρασία. Έτσι αποφασίστηκε να διεξαχθεί η δημοπρασία κινήτρων σε στάδια.

Κάθε στάδιο αποσκοπεί σε διαφορετικό στόχο όσο αφορά τον καθαρισμό του χώρου στο φάσμα, σε φθίνουσα σειρά. Δηλαδή, όσο περνούν τα στάδια τόσο μικρότερος ο "στόχος" για τον χώρο που θα καθαρίσει. Οι στόχοι αυτοί φυσικά είναι προκαθορισμένοι. Κάθε στάδιο αποτελείται από μια αντίστροφη δημοπρασία και μια προωθητική δημοπρασία, με αυτή τη σειρά. Εάν μετά το πέρας τους έχει επιτευχθεί ο *κανόνας του τελευταίου σταδίου* τότε το στάδιο αυτό είναι και το τερματικό. Ο κανόνας αυτός είναι ένας επιθυμητός οικονομικός στόχος.

1.4.2 Αντίστροφη Δημοπρασία

Η αντίστροφη δημοπρασία διεξάγεται σε γύρους. Σε κάθε γύρο όλοι οι διαγωνιζόμενοι καλούνται να αποδεχθούν ή να απορρίψουν τιμές. Οι τιμές αυτές μειώνονται για κάθε σταθμό σε κάθε γύρο. Προτού όμως μειωθεί για κάθε σταθμό η προσφερόμενη τιμή θα πρέπει πρώτα να λυθεί το πρόβλημα της ανακατανομής των σταθμών και να υπάρχει εφικτή ανακατανομή που να περιέχει και τον εκάστοτε σταθμό. Εάν βρεθεί πως δεν υπάρχει τέτοια ανακατανομή η τιμή του σταθμού "παγώνει" και δεν μπορεί να μειωθεί περαιτέρω για το στάδιο αυτό. Σε κάθε γύρο κάποιος σταθμός μπορεί να απορρίψει την προσφερόμενη τιμή και να αποχωρήσει από την δημοπρασία. Η αντίστροφη δημοπρασία τελειώνει όταν σε κάποιο γύρο δεν υπάρχουν πλέον "ενεργοί" παίκτες. Δηλαδή, όταν όλοι οι διαγωνιζόμενοι, είτε έχουν αποχωρήσει, είτε έχουν "παγώσει" οι τιμές τους.

1.4.3 Προωθητική Δημοπρασία

Στη συνέχεια η προωθητική δημοπρασία επίσης διεξάγεται σε γύρους. Εδώ, οι τιμές ξεκινούν χαμηλά, και όσο περνάνε οι γύροι ανεβαίνουν. Ωστόσο οι τιμές ανεβαίνουν για μία άδεια μόνον, όταν υπάρχει δραστηριότητα από τους συμμετέχοντες. Συγκεκριμένα, για μια άδεια η τιμή θα ανέβει όταν υπάρχει μεγαλύτερη ζήτηση από προσφορά. Όταν δηλαδή περισσότεροι από έναν παίκτη έχουν αποδεχθεί την τρέχουσα τιμή μιας άδειας, η τιμή της θα ανέβει στον αμέσως επόμενο γύρο. Η δημοπρασία αυτή τελειώνει, είτε όταν σταματήσει να υπάρχει δραστηριότητα από τους παίκτες για τις άδειες και εάν δεν έχει πιαστεί ο επιθυμητός στόχος των κερδών η διαδικασία προχωράει σε νέο στάδιο, είτε εάν πιαστεί αυτός ο στόχος κατά τη διάρκειά της και μπει σε διαδικασία τερματισμού.

1.5 Πόβλημα Ανακατανομής Σταθμών

Όπως αναφέραμε προηγουμένως, κατά την διάρκεια της αντίστροφης δημοπρασίας, προτού μειωθεί η προσφερόμενη τιμή για κάθε σταθμό, πρέπει πρώτα να λυθεί το πρόβλημα ανακατανομής και να αποφανθεί πως υπάρχει εφικτή ανάθεση. Αυτό είναι ένα δύσκολο NP-Complete πρόβλημα που γενικεύει το graph-coloring και απαιτεί ιδιαίτερη βελτιστοποίηση. Σε κάθε διεξαγωγή δημοπρασίας πρέπει να λυθούν περισσότερα από 100.000 τέτοια προβλήματα. Οπότε και μία ελάχιστα καλύτερη προσέγγιση στο πρόβλημα μπορεί να μεταφραστεί σε εκατομμύρια κέρδους.

Αναλυτικά αυτό το πρόβλημα είναι να βρεθεί μια ανακατανομή : $\gamma : S \rightarrow \bar{C}$ τέτοια ώστε, όλοι οι σταθμοί να ικανοποιούν τους περιορισμούς των παρεμβολών και όλοι οι σταθμοί να έχουν αντιστοιχηθεί σε κάποιο κανάλι. Δηλαδή εάν $I \subseteq (S \times C)^2$ είναι το σύνολο τών απαγορευμένων καναλιών-σταθμών $\{(s,c),(s',c')\}$, θέλουμε : $\gamma(s) = c \Rightarrow \gamma(s') \neq c' \; \forall \{(s,c),(s'.c')\} \in \; I.$

Η ομοσπονδιακή επιτροπή επικοινωνιών ανέθεσε την βελτιστοποίηση αυτού του προβλήματος στους Neil Newman, Alexandre Fréchette, και Kevin Leyton-Brown, οι οποίοι καταφέραν εκπληκτικά αποτελέσματα[41]. Δημιουργήσαν τον λύτη SATFC που είναι ανοικτής διανομής πρόγραμμα, το οποίο λύνει το εν λόγω πρόβλημα σχεδόν βέλτιστα, για τα στιγμιότυπα της δημοπρασίας αυτής. Αυτό το πρόγραμμα κωδικοποιεί το πρόβλημα μας σε SAT πρόβλημα και χρησιμοποιεί υπερσύγχρονους λύτες για αυτή τη κατηγορία προβλημάτων. Επιπλέον χρησιμοποιείται περαιτέρω βελτιστοποίηση πάνω στο πρόβλημα, όπως χρήση της προηγούμενης ανάθεσης, τεχνικές μνήμης κ.α..

1.6 Αποτελέσματα

Για να φτιάξουμε τον δικό μας λύτη για αυτό το πρόβλημα χρησιμοποιήσαμε ένα framework της google, τα Or-Tools. Αυτά περιέχουν μέσα υπερσύγχρονους λύτες προβλημάτων βελτιστοποίησης. Επιλέξαμε να προσεγγίσουμε το πρόβλημα με προγραμματισμό περιορισμών και το CP-SAT, ένα δυνατό εργαλείο επίλυσης περιορισμών.

Συνοπτικά, κωδικοποιήσαμε τις μεταβλητές μας σε έναν δισδιάστατος πίνακα ο οποίος μπορεί να λάβει τιμές 0 ή 1. Οι γραμμές του αφορούν τον κάθε σταθμό ενώ οι στήλες τα διαθέσιμα κανάλια. Όπου μπαίνει η τιμή ένα σημαίνει πως ο σταθμός της γραμμής εκπέμπει στο κανάλι της στήλης.

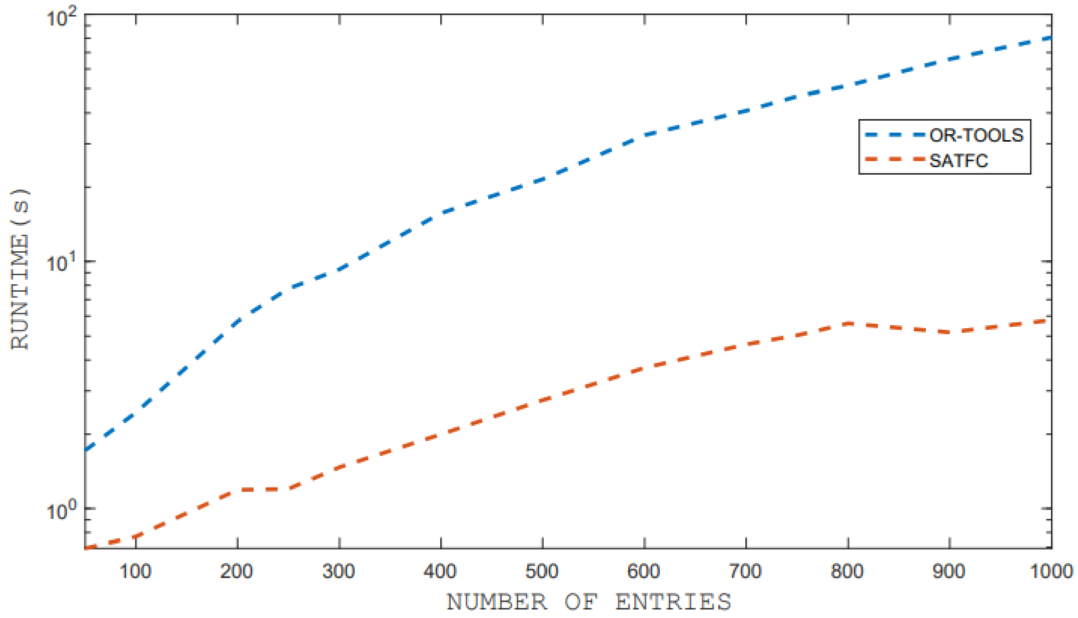
Για τους περιορισμούς εργαστήκαμε ως εξής :

- Λέμε πως για κάθε σετ απαγορευμένου σταθμού-καναλιού, έστω $(s, c), (s', c')$ θα έχουμε τον περιορισμό για τις μεταβλητές $[s, c] + [s', c'] \leq 1$

- Κάθε σταθμός πρέπει απαραίτητα να αντιστοιχηθεί σε κάποιο κανάλι: $\forall s, \sum [s, c] = 1$

Καταφέραμε να φτιάξουμε έναν λύτη ο οποίος δέχεται ακριβώς την ίδια είσοδο με το SATFC, αλλά και παράγει ακριβώς την ίδια έξοδο. Στις συγκρίσεις μας είχαμε πάντα ίδιο αποτέλεσμα στις εξόδους (εφικτό ή μη εφικτό). Όπως περιμέναμε το SATFC είναι κατά πολύ πιο αποδοτικό από τον δικό μας λύτη με τα Or-Tools.

Κάναμε πολλές συγκρίσεις ανάμεσα στα δύο προγράμματα με τα ίδια στιγμιότυπα. Κυρίως διαφοροποιώντας το μέγεθος της εισόδου, αφού δεν ήταν εφικτό να κάνουμε ακριβή προσομοίωση της δημοπρασίας, ώστε να δώσουμε πραγματικά δύσκολες εισόδους. Ένα από τα πιο σημαντικά διαγράμματα που βγάλαμε είναι αυτό που απεικονίζει τη διαφορά τους, για μέγεθος εισόδου που ανταποκρίνεται στα πραγματικά μεγέθη, όσο αυτό αυξάνεται.



Στο παραπάνω διάγραμμα συνοψίζεται και η μεγάλη διαφορά των δύο λυτών.

Για το μέλλον, μπορούμε να πούμε πως ο δικός μας λύτης, έχει χώρο για παραπάνω βελτιστοποίηση, ώστε να πάρουμε καλύτερα αποτελέσματα. Μια άλλη ιδέα, θα ήταν να προσεγγίσουμε το πρόβλημα αυτό με τεχνικές γραμμικού προγραμματισμού. Τέλος, θα ήταν πολύ καλό να γίνει μια ολική προσομοίωση μιας παρόμοιας δημοπρασίας, μια με τον ένα λύτη και μια με τον άλλο. Έτσι θα έχουμε ακριβής εικόνα της επίπτωσής τους στη διαδικασία της δημοπρασίας.

Chapter 2

Introduction

2.1 Motivation

We can say that the last 50 years technology have made huge steps, compared to the past. If we take a quick glance at the past decades, we will see that television was the main protagonist of technology during the 80's and the 90's. Every home in the "first world" countries had at least one TV and all the family was watching. It was we can say, the main protagonist of every day life technology at that time. After the new millennium and as we come closer to the present, we can see the "popularity" of TV declining little by little. Nowadays, the main protagonists of technology are the internet, mobile phones, and online streaming services (Netflix etc.). But what do all these have in common? They are services transmitted to us through the air. The same air. However, this air doesn't have infinite "space" for such transmission. Given that the air "space" is limited and that businesses related to these services are of multi million profits, allocating the rights for transmitting signals is not a trivial task.

What we call for simplicity "space in the air" is the so called *spectrum*; and the most common way to allocate the spectrum for almost 30 years now is through *auctions*. Welcome to *Spectrum Auctions*; mechanisms designed by governments to allocate spectrum rights to several broadcasting businesses, aiming to maximize revenues and social welfare. Spectrum auctions have been and still are the main object of research for a lot of economists and engineers.

2.2 Problem statement

In this thesis, we direct our focus into the Incentive Auction, that took place in the US five years ago. Why this particular auction? It carried through with a very difficult task; first of it's kind to succeed. As we said TV lost its place as the main "protagonist" in our houses, so naturally it should lose space in the spectrum as well. Nevertheless, rights are already allocated throughout the spectrum continuum. Thus, what is needed is a re-allocation. To shrink a little bit the "space" available for TV stations, in order to make some more for the wireless services (Internet etc.).

In order for this to be achieved, broadcasting stations $s \in S$ must be offered incentive prices, in return for giving up their broadcasting rights in the space C. This task is accomplished via an auction. Nevertheless, even if some of the broadcasters sell their rights the “space” cleared will not be contiguous; and wireless businesses require contiguous spaces in order to transmit. Thus, we have to *repack* the remaining on air broadcasting stations $S' \subseteq S$ to a smaller space $C' \subseteq C$, and make a contiguous clear space $C - C'$ for the new licences. Moreover, *repacking* the remaining on air broadcasting stations is an NP-Complete problem, generalizing the Graph-Coloring problem. What we described in simplicity, is what the incentive auction succeeded in the US territory, while at the same time making considerable profits.

The Incentive Auction started at 2016 in the US and concluded successfully in 2017. This 39 month process achieved the desired reallocation of spectrum and raised $19.8 billion in gross revenues; close to $10 billion profits for the US government. In this thesis, we are going to analyze in detail the mechanism of this auction and tackle with the difficult challenges that are billowing. One of these tasks we have great interest in, is the *station repacking problem*, and the need for optimization.

2.3 Related Work

Auction theory is a major field of economics. Auctions are very important in both theoretical and also piratical use. Practical, because especially nowadays some of the worlds most important markets are auction markets; and we have seen in the past that a good auction theory can make the difference between a successful auction and a disastrous one. We have many real examples of disastrous auctions, such as the New Zealand spectrum auction, several US auctions during the 90s and mane more[29]. Theoretical, because lessons learned through auction theory have led to important insights to other economic sectors.

Auctions exist in the world for many centuries. In fact, we can see in history books that in the ancient years, the Babylonians auctioned wives, later the Greeks auctioned mine concessions and held slave auctions; later the Romans began to auction everything from armor and weapons to debtors’ property. And as the years passed auctions were becoming more and more popular. But when did *auction theory* started to make a change in the history of auctions?

We see a critical result of auction theory in the *Revenue Equivalence Theorem*, which subject to some reasonable-sounding conditions, states that the seller can expect equal profits on average from all the standard (and many non-standard) types of auctions, and that buyers are also indifferent among them all. William Vickrey’s contribution through his papers [53, 52] which developed some special cases of the theorem, awarded him the Nobel Prize. Myerson (1981)[37] and Riley and Samuelson (1981)[47] provide more general treatments. More on the early literature, we have Griesmer, Levitan, and Shubik (1967)[12] analyze the equilibrium of a firs price auction in which contestants’ valuations are drawn from uniform distributions with different supports. Moreover, Wilson (1969)[55] introduced the (pure) common-value model and developed the first closed-form equilibrium analysis of the winner’s curse[1].

[1] Winner’s curse may occur in two ways : either the winning bid will exceed the value of the auctioned asset making the winner worse off in absolute terms, or the value of the asset will be less than the bidder anticipated, so the bidder may garner a net gain but will be worse off than anticipated

As we get closer to the recent literature of auction theory we have at the late 80's the McAfee and McMillan (1987)[31] and a helpful introductory article by Maskin and Riley (1985)[30] which manages to convey many of the key ideas in a few pages by focusing on the case with just two possible types of each of just two bidders.

Regarding the *Spectrum Auctions* we see Ronald Coase (1959)[21] first proposing to auction the spectrum, yet it was until 1994 that the Federal Communications Commission (FCC) began to auction the spectrum. A great analysis of the history that led to spectrum auctions in the US is provided by Hazlett (1998)[54]. After 1994, when auctions were established as the common way for governments to allocate the spectrum; there has been a lot of work and research in this field. One of the major contributors in the theory of auction theory and especially spectrum auctions is Paul Milgrom, who was awarded the 2020 Nobel Memorial Prize in Economic Sciences, together with Robert B. Wilson, "for improvements to auction theory and inventions of new auction formats". Paul Milgrom led the team that designed the broadcast *Incentive Auction*, we can see his work designing the auction mechanism in [45, 46]. Concerning the incentive auction, which is our subject in this thesis, we have the work of Neil Newman, Alexandre Fréchette and Kevin Leyton-Brown[41, 42] which analyzed and efficiently solved the *station repacking problem*, which emerges during the incentive auction.

2.4 Contribution

In this thesis first we try to understand the fundamentals of auction theory and mechanism design. Then we will direct our focus especially to spectrum auctions and the state of the art auction mechanisms that were used for the incentive auction; and are still utilized for same auctions. After we get a deep understanding on the theoretical context we will proceed into analyzing the Incentive Auction's process, as well as the station repacking problem.

The incentive auction of 2016 aimed to reallocate the spectrum. To put this in simple words, the government needed to buy back from TV participating in the auction their licences of transmission. After buying several of these licences and thus, had freed some "space" in the spectrum; new licences were going to be offered to wireless services. In the way the auctions work, in order to implement such a reverse auction, in which the government is acquiring licences from at the time broadcasting stations; a difficult problem requires to be solved. At any time during that auction, a TV station is able to walk out of the auction and continue transmitting. This means that, the auctioneer - the FCC - must at all times ensure that there is space available for the "walk out" station. The idea is that given that there is available space for a given station to go back on air, means that the FCC can keep lowering the offered price to it; but if there is not available space the FCC must not lower the price anymore, because if the station then rejects it there can not be a feasible way to go back on air. Weather there is "space" for a broadcasting station to go back on air is the **Station Repacking Problem**. This problem requires to be solved efficiently thousands of times during the Incentive Auction, as it has to be solved for each station many times. Due to interference of transmission between stations this problem has a huge amount of constraints that need to be satisfied. The stations repacking problem is NP-Hard, generalizing the graph-coloring problem[41].

Our main contribution with this thesis is regarding the station repacking problem. Once, we get familiar with the auction rules and the connection with the repacking problem, we try to implement our own solver for this problem. This solver will be implemented in order to run and interact with the auction process with the same exact way as the original one (the one that was actually used during the auction). We will utilize an open source software suite for optimization, which contains and uses many state of the art solvers; in order to create our own solver for the station repacking problem. This open source software is the Or-Tools created by Google. When we have created our solver, we can compare it with the original one, with real auction instances. Then we can observe the gap between the two and test the optimization done in practice. Moreover, we can try to see the impact that the two solvers, that solve the same problem but are optimized differently, have on the auction revenue's. How much money can a more efficient station repacking software can make?

2.5 Organization

- The **first** chapter is an Introduction. We provide a general overview of the problem domain that was investigated.
- In the **second** chapter Auctions and spectrum, we introduce to the reader some basic knowledge of auction theory. In addition we introduce, basic concepts of Mechanism Design, which aims to design systems that are to be used by strategic agents. We then focus on spectrum auctions, and the state of the art mechanisms that are used to implement them.
- In the **third** chapter we analyze the incentive auction's process in full detail. All of the context described in the previous chapter, being used in practice; we describe all of the rules and structure of the Incentive Auction.
- The **fourth** chapter is an analysis of the station repacking problem in theory but also in practice; as we introduce the solver of the FCC and it's usage.
- In the **fifth** chapter we introduce our own solver created for this thesis, utilizing the or-tools. Furthermore we present some comparisons between our solver and the FCC's solver.
- Finally, in the **sixth** chapter we discuss some further work that can be done after this thesis, and we take a glance at the Greece's spectrum auctions.

Chapter 3

Auctions and Spectrum

In this section we provide an introduction to auctions. Furthermore, we introduce basic auction sets and formats, and establish critical terminology, in respect of auction theory, that we will utilize throughout the paper.

3.1 What is an auction?

When referring to an auction, most people come up with a view that is formed by movies and television. They picture a hall, a crowded one, and one individual, who looks like a judge, that is presenting an item to be sold and announcing prices. People all over the room are bidding. They are shouting prices. Ascending prices. This goes on until nobody offers a bigger price, than the current one; the item is sold to the highest bidder who later pays the price that he announced. The auction continues with the next item on the list. This is not a misconception; it is actually how paintings of Picasso or Leonardo Da Vinci were sold in the past, or valuable antiques or books. If we go further in the past wives and slaves were also sold in auctions[49]. This indicates that people tend to use auction mechanisms because some products do not have standard values[27], such as artworks. Paintings and antiques were always common items to be sold in auctions over the years. That is because one cannot easily decide the price of an artifact, when it is unique. Interesting is the case of the American hip-hop group Wu-Tang-Clan. The group decided in 2014 to make a single copy of their album *Once Upon A Time In Shaolin* and sell it an online auction. Besides the fact that most music fans do not seem to purchase the music and that an album's price is significantly low, this album was sold at the record price of two million dollars. The winner of the auction had the right to keep the music for personal use or decide to release it to the world for free[20].

Nowadays auctions seem to drive the economy. Even in the past, auctions had many practical uses and the were different types of auctions. However, in the present years and due to the advance of the technology, auctions are thriving in the matter of selling goods and services. Worldwide their use is ranged for the allocation of radio spectrum necessary for mobile communication, to pollution permits and trading electricity, as well as internet advertising and many more.

An auction can be defined by its property, being a market clearing mechanism, and to equate demand

and supply. There are of course other market mechanisms, some including fixed price sales(as in retail sailing points), or bargaining(as in real estate, when the broker bargains the price of a house). The characteristic for which an auction stands out as a market mechanism that allocates scarce resources, is that the price formation process is explicit. That is, the rules that determine the final price are usually well-understood by all parties involved.[18]. Moreover, auctions are more flexible than a fixed price sale and often they demand less time than bargaining a price. For example, auctions are conducted for selling hundreds of goods such as used cars in a matter of several hours. Negotiating for each car independently and with each potential buyer would take days if not months. We must note however, that under certain conditions negotiation sale or even a fixed price can result in higher expected revenue for the seller. Otherwise these market sale techniques would not be used.

3.2 Auction Types

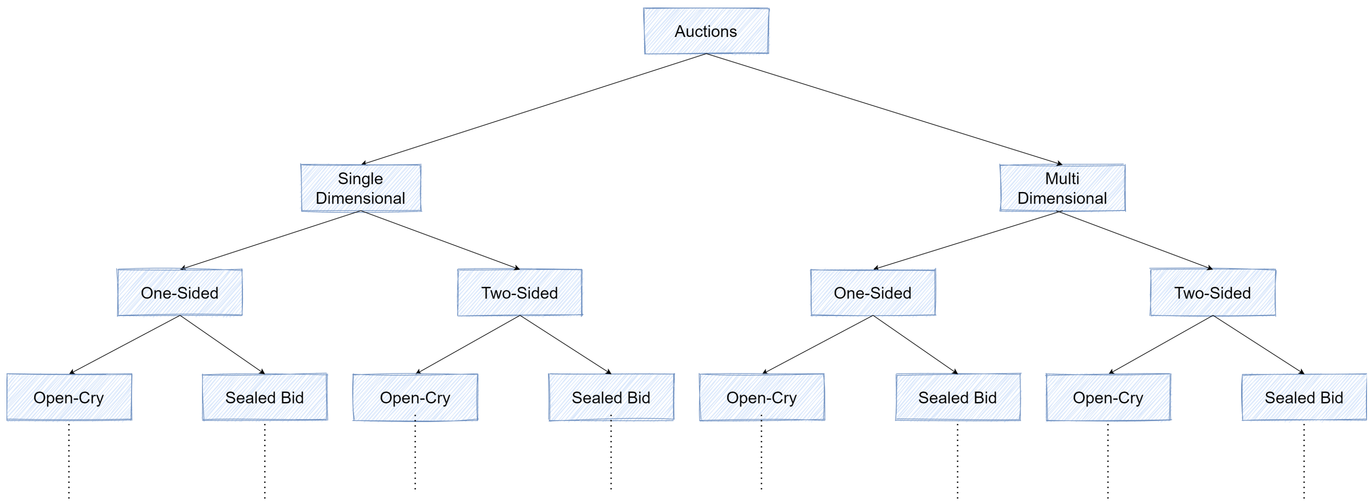
There are many criteria that can distinguish and classify an auction. We will present some features that can classify an auction according to the literature[50]. Auctions can be :

- Single dimensional, or multi-dimensional. In a single dimensional auction, the bid is only oriented by the price offered for a specific good. Whereas on a multi-dimensional auctions, other aspects affect the bid. Such aspects can be quality or delivery date.
- One sided or two sided (Forward, Reverse or Double) A *Forward* auction is an one sided auction, where buyers bid for the goods and the auctioneer has the job of deciding which is the winning bid. A *Reverse* auction is also an one sided auction , where sellers bid for the goods and the auctioneer has the job of deciding which is the winning bid. In a two sided auction both buyers and sellers submit bids and the job of the auctioneer is to match buyers to sellers.
- Open-cry or sealed bid. In open-cry auctions, all bids are publicly observable by all bidders. In a sealed bid auction, the bids are only known by the auctioneer.
- First price or kth price. In a first price auction the winning bidder pays the price of the winning bid, whereas in a kth price auction the winning bidder pays the price of the bid that is ranked kth. The most common kth price auction is the second price auction the bidder who bids highest pays the price bid by the second-highest bidder.
- Single-unit or Multi-unit. In a single unit auction, there can only be bids for a single good (although, a single good can be considered a bundle of items and not only a single item-for example, a pack of beers-). In a multi-unit auction, a bidder can bid for several units together. For example, in a multi-unit auction in which 100 pack of beers are being sold simultaneously, it is possible to bid for 10 packs only, at a price of $5 per pack)
- Single-item or Multi-item. Unlike the previous category, when we use the term multi-item; we refer to different goods. In the case, were a bundle of different goods may have a certain value for a bidder, but each item in the bundle has no value if it is purchased without the others; we

call it a *Combinatorial Auction*. In that auction, multiple, heterogeneous goods are auctioned simultaneously and bidders may bid for arbitrary combinations of goods. For instance, and not to be confused with the previous category, here a bidder may offer $30 for two packs of beer and a bottle of Gin; while another can bid $25 for just two packs of beers. Moreover, if we get combinatorial constraints for example, the first bidder may not have any value at all for the beers or the bottle of gin separately. We should note that a bidder's preference over a bundle of goods signals that a bidder's valuation for the bundle need not equal the sum of her valuations of the individual goods in the bundle

The above is a zoological categorization of auctions[43], according to a number of different possible features. These properties are independent, meaning that an auction can fall into more than one of the above. For instance, a combinatorial auction can be open-cry, one-sided and single dimensional.

The figure below illustrates the categorization described above.



Now we will present the most common and widely known kinds of auctions and place them properly in the categorization that was described above.

There are four basic types of auctions that are widely used and analyzed. The *English Auction*, the *Dutch Auction*, the *first-price sealed-bid auction* and the *second-price sealed-bid auction*.

The English auction is the auction form which is most commonly used for the selling of goods. In the English auction, the price is successively raised until only one bidder remains. That is why it is also called ascending-bid auction. This auction is usually conducted with an auctioneer who announces prices, or by having the bidders calling the bids themselves, or submitting bids electronically while the current best bid is posted. The essential feature of the English auction is that, at any point in time, each bidder knows the level of the current best bid. This type of auction is often used over the years for selling antiques and artwork.The word auction in fact derives from the Latin word auctus—the past participle of the verb au-gere, which means to augment or to raise. If you mention the word auction to someone, that person will probably think of an English auction; the whole idea that we described in the beginning of this section. It is an open-cry, one-sided and typically sell-side and it is first-price since the winner pays the winning price.

Second on the list is the Dutch auction, or descending-bid auction. It is probably the next most common format. The Dutch auction is the converse of the English Auction. This again is a single-dimensional, one-sided, open-cry, first-price auction. The descending auction works in exactly the opposite way from the ascending: the auctioneer starts at a very high price, and then lowers the price continuously. The first bidder who calls out that she will accept the current price wins the object at that price. This auction is commonly used for selling cut flowers in the Netherlands, fish in Israel and tobacco in Canada. Every bidder gets a buzzer connected to an electronic clock. The interior of the clock has information about what is being sold and the price at which the auction starts. As the time passes the clock displays the percentage that the price of the item has fallen. The fist one to hit the buzzer gets the item and pays the respective price.[29]. Furthermore, Dutch auctions have been used to sell shares of companies. For example, when Google went public in the summer of 2004, it was reported to have sold its shares using the Dutch auction[25].

With the first-price sealed-bid auction, potential buyers submit sealed bids and the highest bidder is warded the item for the price he bid. The basic difference between this and the English auction is that, with the English auction, bidders are able to observe their rival's bids and pick their strategies accordingly (and can even revise their bids, if the chose to). It is a sealed bid in contrast with the open-cry English auction, where each participant submits one and only bid. First price sealed-bid auctions have been used in the auctioning of mineral rights to U.S government-owned land; they are also sometimes used in the sales of artwork and real estate. It is also commonly used for online advertisements in the internet along with other types of auctions.

The second-price sealed bid auction, also known as Vickrey auction after William Vickrey[53] is like the first-price sealed bid auction. The difference here is that the price that is paid by the winner is the *second*-highest bidder's bid. This auction has very useful theoretical properties, but in practice it is not that often used.

Many variations upon these four basic auction forms are used. A common one is the *Japanese* auction. This variation is like a patchwork of English and Dutch auction. In this auction, ascending prices are announced by the auctioneer and bidders indicate that they are dropping out when the price gets too high for them. As bidders drop out of the auction, the last one in, is the winner; he obtains the good at the price at which they become the last bidder. A Japanese auction implementation is described by Milgrom and Weber[35], where bidders hold an electronic button, similar to the buzzer in the Dutch auction, and they release the button whenever they wish to drop out of the auction. This particular format is very similar to the *Reverse* auction that we are going to analyze later in this thesis . However, in the reverse auction items (licences) are sold, thus it is called reverse.

3.3 Auctions as Games

Game theory is as a collection of techniques, that enable the analysis of strategic situation between agents, who act in a selfish way in order to maximize their own objectives, and there are interdependencies among those decision makers. In these situations the payoffs are determined by the collective choices of all the strategic agents involved. At an auction, the winner and at times even the price he is going to pay, depends not on his bid, but how his bid relates to the bids of his rivals. Thus, we can understand that an auction is indeed a ”game”. However, if we want to be formal auctions are not games. The idea of allocating scarce resources to different strategic agents is. Auctions are the mechanisms that are designed with the proper allocation and bidding rules, in order to achieve the “best” outcome, in terms of efficiency, revenues etc.

3.3.1 Equilibrium and strategies

One of the most important notion in game theory, if not the most important, is that of an *equilibrium*. Equilibrium is a state were, in such a game, the players have no reason to unilaterally deviate from their current strategies. Thus, euilibria are steady states, in a manner that, when players reach those states we believe that they will not change their strategies.

Dominant Strategy is a strategy that, when followed by a decision maker, he gets the most out of the game, regardless of what strategies are followed by his rivals. It is a very important state, because no matter what, the agents are going to to follow this strategy. Thus when a game, or an auction, provides the players with dominant strategies it is easy to predict the outcome.

Mechanism design is a field in game theory, which aims to create the rules that regulate the interaction of strategic agents. In our case design the rules and the auction process.

Let us now define formally an auction as a game G :

- We have n players-bidders. Let the set of the bidders be $I = \{1, 2, .., n\}$
- Let $X_i = [0, \bar{u}]$ denote the set of possible types of bidders $i, i = 1, 2, ..., n$ and u_i the type that the bidder i receives.
- The finite space possible strategies-bids S_i for players $i, i = 1, ..., n.$. In the auctions setting, this strategy space consists of the different bids that each player is allowed to submit. We denote by $s = (s_1, ..., s_n)$ the strategy profile of the game.
- $C_i : S_1 \times ... \times S_n \to R$ a cost function for each player i (We assume that each bidder wants to minimize his cost)

We define as *Pure Nash Equilibrium-PNE* a strategy profile s if for every player i and every unilateral deviation $s'_i \in S_i$ we have that $C_i(s) \leq C_i(s'_i, s_{-i})$[39]

We define a *Dominant Strategy*, a strategy s' for a player i if $\forall s \in S_i, \forall s_{-i} \in S1 \times ... \times S_{i-1} \times S_{i+1} \times ... \times S_n$ we have that $C_i(s', s_{-i}) \leq C_i(s, s_{-i})$

If a mechanisms achieves to provide dominant strategy for every player, it is easy for the player to participate in it and for the designer to predict its outcome. That makes dominant strategy a very important asset for an auction. In addition we should note that, a pure Nash Equilibrium is a weaker notion than a dominant strategy, since it only requires that each player best responds to some specific strategies that the others follow, and not that he should behave in a specific way, no matter what the others are doing.

3.3.2 Single Item Auctions

We will now formally define a single object auction model. We will create this model in order to analyze the bidding strategies and the equilibriums that occur when we are facing auctions as the ones we described in this section.

We have n bidders, bidding for a single object. Each bidder $i, i = 1, ..., n$ has a private valuation for the object v_i. Every bidder has a positive valuation for the object, which can also be interpreted as the maximum amount of money he is willing to pay for it. The mechanism - auction format - auctioneer is the one to allocate this object along with the price. To decide who gets the item and how much he is charged for it, based on the players bids. We denote the payment with p. We then define the *utility* of each participant u_i when :

$$u_i = \begin{cases} v_i - p & \text{if } i\text{, receives the item} \\ 0 & \text{otherwise} \end{cases}$$

We see utility as the amount of "happiness" of a player i. He either gets the item and is "happy" of his private valuation of the object minus how much he actually paid for it, or he does not get the item at all.

We define the set B as the set of all possible bids and note that it is the same for each player. We also have a bid profile, $b = (b_1, b_2, ..., b_n)$ which includes the bids that were made. An allocation rule $x(b) : B^n \to \{0, 1\}^n$ such that $\sum_{i=1}^{n} x_i \leq 1$, is a mapping from the bid profile to the winner of the item, namely

$$x_i = \begin{cases} 0 & \text{if } i\text{, receives the item} \\ 1 & \text{otherwise} \end{cases}$$

Lastly, the payment rule, $p(b) : B^n \to R$, is a a mapping from the bid profile to the real numbers, that allocates the bids of the players with the prices they are charged. In our case only the winner is charged so this mapping is 0 for every player except the one who gets the object.

We say that *truthtelling* is a dominant strategy for a player i, if bidding at his own valuation v_i always maximizes his utility u_i, regardless of what the other agents bid. Mechanisms (auctions), that support as a dominant strategy telling the truth, are called **truthful**.

Two major indicators of how "good" a mechanism is, are the *Social Welfare*, and the *Revenue*; which measure the success of those designs. Revenue is $R(x) = \sum_{i=1}^{n} p * x_i$ and in simple terms is the money that the auctioneer made. And the Social Welfare of an allocation is $W(x) = \sum_{i=1}^{n} u_i * x_i$. We can view the Social Welfare as the total "happiness" of the players.

Despite the fact that we have the two above indicators of how good an auction is, typically the designers tend to maximize the Revenues and not care about the Social Welfare.

3.3.2.1 First-price Auctions

Now we will use the first-price sealed bid auction that was described earlier, as an lets say allocation rule for the game that was notated above. We give the item to the highest bidder and we charge him the amount that he decided to bid. Very simple rules, and can be very easily be implemented if we look from the auctioneer's perspective. However, it is not as easy to define the strategic that a player should adopt. Naturally, each player wishes to maximize his utility. To begin with, a player i does not have a reason to bid his own true valuation u_i for the object, because hit utility will always be 0. Same as not participating at all in the auction. Thus, the bidder will come up with a bid always less than his valuation (bidding higher than his valuation would end up with negative utility), which always results in two possible outcomes. Either the player will get the item but pay more than needed, or bid way off of his valuation and risk not getting the item at all. In order for someone to decide his best strategy, he must make assumptions on his competition, namely to find out the number of bidders that participate in the auction, their valuations' distribution and how these are correlated between them.[14]. This proves that the first-price auction is not a truthful mechanism. We should note that it was proven by Vasilis Syrgkanis et al.[17, 51] that this mechanism gives an $\frac{e-1}{e}$ approximation of the optimal Welfare, which is tight when the valuations are correlated. Later it was improved by Hoy et al.[24], that assuming furthermore that valuation distributions are independent, the Welfare of the First Price Auction is at least an $.743$ approximation of the optimal Welfare. However in practice, in the worst known example (from Hartline et al.[22]), the first price auction achieves $.869$-fraction of the optimal welfare, far better than the theoretical guarantee.

The first price auction is widely used but as we saw above, it is difficult for the participants to decide and pick their strategy. Furthermore, it is viable for a player to adopt "illegal" strategies metaphorically or literally, such as cooperate with other players etc. This is all because the mechanism lacks a dominant strategy.

3.3.2.2 Second Price Auction

In the second-price sealed-bid auction firstly introduced by Vickrey[53] and the named after him; that we have described in this section, players submit their bids simultaneously without observing the bids made by other players. It holds the same allocation rule as the first-price auction, but a different pricing rule. We will explain how in such auctions players always have a dominant strategy and in addition, this strategy is for a player to always bid his own private valuation. This is a remarkable property of the Vickrey auction, the also known as truth telling.

Lets say that bidder 1 has a valuation equal to v_1 for the object. And denote by $\hat{b}$ the highest bid among players $2, ..., n$. We assume that :

- Bidder 1 bids $b_1 < v_1$. If $b_1 > \hat{b}$ then bidder 1 wins the as he would have won with a bid equal to v_1. However, if $b_1 < \hat{b} < v_1$ then bidder 1 loses the auction. By bidding his valuation he would have won the auction and would have earned expected profits equal to $v_1 - \hat{b}$. Thus, bidder 1 does not gain by bidding less than his valuation and could possibly lose. That is, his expected profits decrease with a bid $b_1 < v_1$
- Bidder 1 bids $b_1 > v_1$. If $b_1 < \hat{b}$ then bidder 1 loses the auction as he would have lost if he had bid his valuation. However, if $v_1 < \hat{b} < b_1$, then Player 1 wins the object and pays more than his valuation. That is, he loses $\hat{b} - v_1$. Therefore, bidder 1 does not gain by bidding more than his valuation but could possibly lose. Thus, his expected profits decrease with a bid $b_1 > v_1$

This proves the state that the Vickrey auction has a dominant strategy for each player to bid at his own true valuation; thus, it is a **truthful** auction. It is also proven that the second-price auction also maximizes the social welfare.

3.4 Spectrum Auctions

What is the *spectrum*? And why governments decide to use auction systems in order to sell it? Or better, to allocate it. Questions that we need to answer first before we jump into studying the famous spectrum auctions.

3.4.1 Electromagnetic Spectrum

Spectrum in general, is a chart or a graph that shows the intensity of light being emitted over a range of energies.[5] The most common example of such thing, is a rainbow, something most people have come across in their lives. However, when we talk about spectrum auctions we are referring on the *Electromagnetic Spectrum*. The electromagnetic spectrum is the range of frequencies (the spectrum) of electromagnetic radiation and their respective wavelengths and photon energies. These frequencies range from below one hertz, to above of 10^{25} hertz.

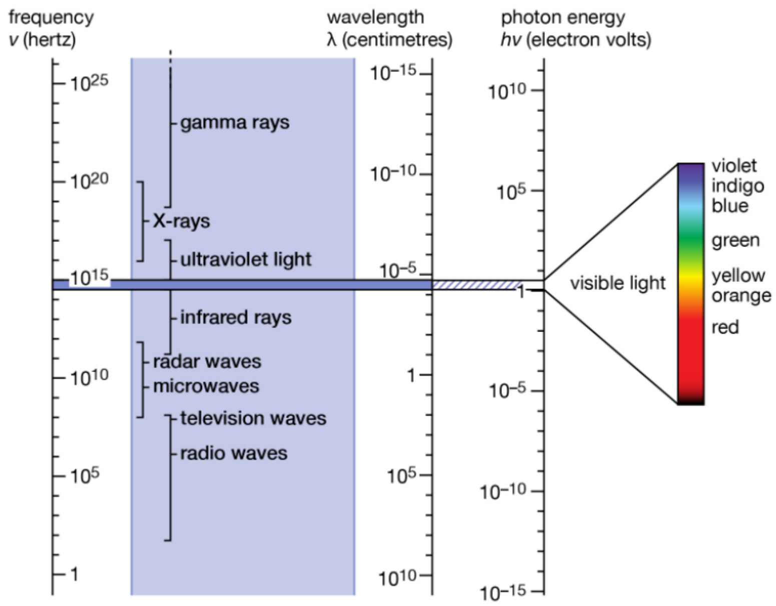


Figure taken from [3] displays the electromagnetic spectrum

Television and radio waves are the ones ”sold” in auctions. They include the very well known FM, VHF and UHF bands.

When we refer to selling electromagnetic spectrum, we actually mean selling the rights to companies to transmit at these frequency bands. Economists, hold a substantial agreement that the best way to allocate scarce spectrum resources, is through auctions[26]. Ronald Coase proposed in 1959[21], that auctions were the way to assign spectrum; however, it was not until 1994 that spectrum auctions were first established in the US. Deep analysis of the history of what led to spectrum auctions is provided in[54]. Other options to assign spectrum has been tested and rejected in the past, such as administrative process or lotteries.

One can now imagine, the importance of choosing the ”right” auction. To choose, design such a mechanism that maximizes revenue for the government and efficiency (getting the spectrum in the hands of those best able to use it). A mechanism able to ”discover” the best allocation there can be. If we just take into account the fact that, spectrum auctions have, in the past been a **multi-billion** dollar source of revenue for the US government, we can then understand the magnitude of the research done to design such mechanisms.

Over the course of time several auction types have been proposed and tested; some achieved their goals and other failed badly. We are not going to analyze properties and rules that have been established in these auction. Peter Cramton give an analysis in[11].

3.4.2 Deferred-Acceptance Auctions

We are going to focus on a particular type of auction, introduced by Paul Milgrom and Ilya Segal[45], as it is the one implemented in the Incentive Auction of 2015.

In 2015 the US government conducted the Incentive Auction. The main goal of this auction was to reallocate channels that was at the time allocated for television broadcasting to use for wireless broadband services. In order for this to be achieved, television broadcast rights should be first purchased from some of the TV stations and reassign or *repack* the remaining on air stations into a smaller set of channels. Then use the channels-spectrum that is cleared to create licences able for use in wireless broadband and then allocate them properly. Active broadcasting stations on the US vicinity at the time exceeded 2500. Repacking, say 2000 of those TV stations create a set of approximately 140000 interference constraints that must be satisfied. These constraints dictate which pairs of stations could not be assigned to the same or adjacent channels. Thus, a computationally challenging problem emerges, which is equivalent to the NP-hard graph coloring problem[9]. Therefore, the challenge for designing a strategy-proof auction was created. The idea as we view it is to make a double auction; that can reallocate the rights of TV broadcasting to the wireless services. At the same time it should repack the remaining on air TV stations to a smaller set of channels.

The *Deferred-Acceptance* auctions, generalize the idea of the matching algorithms which are introduced in [19] and [10], as well as related clock auction designs, such as the simultaneous multiple-round auctions used for radio spectrum sales. These auctions are iterative processes in which the highest scoring (and hence least attractive) bids are rejected in each round and the bids left standing at the end are accepted. Briefly a DA algorithm, that is used for the implementation of a DA auction, hold these properties: At first ever bid is considered to be active. In every iteration, a non-negative ”score” is assigned to each bid, according to a chosen scoring function. This scoring function can take as arguments the the bid amount, the bidder’s identity, the remaining set of active bidders, and the amounts and identities of the previously rejected bids. Termination of the algorithm occurs when no active bid has a positive score and the remaining active bids become winning. Otherwise, the algorithm rejects the active bid with the highest non-zero score, removing it from the active set, and iterates. Different scoring rules lead to different DA algorithms. By selecting appropriately a scoring rule, for a DA auction one can achieve a variety of different goals and can make the DA auction to take the shape of previously studied auctions. For instance, by using the ”standard” DA algorithm of Bikhchandani[16], where all active bids are scored by their bid values, an efficient allocation subject to constraints that have a matroid structure can be implemented. Another example, is the implementation for the ”knapsack problem” by Dantzig[13], in which there is a constraint on the total ”volume” of the rejected bids.

Deferred-acceptance auctions, are sealed-bid auctions. In these mechanisms the winning bids are selected with the use of DA algorithms and payments are made only to the winning bidders. A deferred acceptance threshold auctions which use a DA algorithm to select winners and pays each winner its threshold price is a strategy-proof, *truthful* auction[46].

Besides efficiency, there are other requirements for a design to have, that even some times are more important. For instance, as in the FCC’s repacking problem, an auctioneer may face hard budget problems constraints. With proper adjustments, DA threshold auctions can respect such constraints, nesting the budget-constrained mechanisms that were studied and analyzed by McAfee (1992)[32], Moulin (1999)[36], Mehta et al. (2007)[34] and Ensthaler and Giebe (2014)[15]. Furthermore, another target for a mechanism to achieve may be to minimize the procurement cost. If hypothetically the bidder valuations are drawn from known distributions independently, then the bids in DA auctions can be scored according to the corresponding virtual values of Mayerson[38]. In that case, and given that the constraints have a matroid structure, the upshot is an expected cost minimizing auction. On the other hand, id the bidders valuations are drawn i.i.d. from a single unknown distribution, then expected costs can be reduced by scoring the still-active bids using already-rejected bids[48].

We will provide a more formal definition of *Deferred-Acceptance Auctions*, in order to make more clear the rules and the scoring function.

Given a set of bidders N and possible bid profiles B, a deferred-acceptance algorithm (DA) is specified by a collection of *scoring functions* $(s_i^A)_{A \subseteq N, i \in A}$, where for each $A \subseteq N$ and each $i \in A$, the function $s_i^A : B_i \times B_{N \setminus A} \rightarrow R_+$ is non-decreasing in its first argument. The algorithm operates as follows. Let $A_t \subseteq N$ denote a set of "active bidders" at the beginning of iteration t. . We initialize the algorithm with $A_1 = N$. In each iteration $t \geq 1$, if $s_i^{A_t}(b_i, b_{N \setminus A_t}) = 0, \forall i \in A_t$ then stop and output $a(b) = A_t$. Else, let $A_{t+1} = A_t \setminus \arg\max_{i \in A_t} s_i^{A_t}(b_i, b_{N \setminus A_t})$ and proceed. In simple terms as we previously said, the algorithm in each iteration is rejecting the least "attractive" (highest-scoring) bids until only zero scores remain. We say that the DA algorithm computes allocation rule a if for every bid profile $b \in B$, B, when the algorithm stops the set of active bidders is exactly $a(b)$.

We can say that every DA algorithm is finite and computes a monotonic allocation rule. Therefore, we can define a *DA threshold auction* as a sealed-bid auction, in which the allocations are computed using a DA algorithm and the corresponding threshold payments are made to the winners. Like any threshold auction, each DA threshold auction is strategy-proof.

3.4.2.1 Descending Clock Auction

Finally, we arrive at the category of the *Descending Clock Auction*. It is actually the main auction format that was used for the FCC's incentive auction. This type of auctions are equivalent to the DA threshold auctions that we introduced before. Briefly, in a clock auction, same as in the DA threshold auction, iterative bids that are the least attractive to the auctioneer are rejected. Nevertheless, in the setting of the descending clock auctions the rejected bidders may remain active by improving their bids by some minimal bid increment; unlike the DA auction. It is shown by Milgrom[45] that for every sealed-bid DA threshold auction with finite bid spaces, there exists an equivalent descending clock auction that generates the same allocation and payments, and vice versa.

Such auctions are very efficient for many applications as they are themselves truthful and self-evident even for bidders who misunderstand or do not trust the auctioneers calculations[28]. In addition, they winning bidders are not required to reveal, not even to figure out their exact values. And lastly, these auction formats, permit information feedback so as to better aggregate common-value information. We should also highlight the fact that DA threshold auctions and of course clock auctions, are weakly group strategy-proof. This means, that should a ”group” of bidders be formed (a cooperation of bidders), there is no possible strategic that the group can have besides being truthful , that can be strictly profitable for all members of the coalition[34, 23]. We can understand that this is a very critical property in the real world auctions, were million dollar companies can cooperate ”against” the auctioneer, who in our case (spectrum auctions) is the government.

3.4.2.2 An Example

Before we start introducing the actual format that was developed for the FCC Incentive Auction, we will illustrate a simple example of a DA threshold mechanism compared to a Vickrey auction. This way, the reader can get an idea of how these auctions work, and on top of that show it's advantages over other previously adopted mechanisms.

Let there be three television stations, 1, 2 and 3. For the simplicity of this example, we can imagine three stations in a straight line, were station 3 is the one in the middle so it interferes with both of the others standing in the edges.All three of the stations are bidding to stop broadcasting in a reverse auction; to sell their rights of transmission. Suppose the respective bids b_1, b_2, b_3 Also, we assume that only a single channel is available to be assigned to the losing bidders (bidders who do not sell the rights). There is no interference constraint between stations 1 and 2; meaning that they can be assigned to the same channel no matter what. However, both of them interfere with station 3, and cannot be assigned on the same channel. In the case that we just described, the auctioneers has two options[1]. Either to acquire the licence of station 3 and assign stations 1 and 2 to the available channel, or acquire the licences of both 1 and 2 and assign station 3 to the available channel.

- A *Vickrey* auction mechanism, given the setting above, would behave in two possible ways:

  1. If $b_1 + b_2 \leq b_3$, licences of stations 1 and 2 are acquired, and the Vickrey payments for them are $b_3 - b_2$ and $b_3 - b_1$ respectively
  2. If $b_1 + b_2 > b_3$, licence of station 3 is acquired.

  (We assume that the ties are broken in favor of the lower-numbered bidders.)

  The first thing that we can note here, is that in the first case, bidders 1 and 2 have control over each others payment. By reducing their bids they can raise each other's payments. Thus, we can see that the Vickrey auction is not group strategy-proof. On top of that, we observe the fact that the total payment made to the bidders 1 and 2 equals $2 * b_3 - b_2 - b_1$ which exceed station's 3 value b_3. These properties can turn out to be disastrous in a real time auction, and there are a few examples to prove this point. A real world example of the terrible outcomes that the Vickrey auction can produce, is the one of New Zealand's 1990 spectrum auctions. In that particular auction, occurred cases where the winning bidder had to pay to the government just NZ$5.000

[1] It is also feasible, but clearly suboptimal, to acquire and clear station combinations $\{1, 3\}$, $\{2, 3\}$ and $\{1, 2, 3\}$; but it is obviously not optimal

while his original bid was NZ$7 million. In another case, a firm that bid $100.000, ended up paying just $6 since that was the second highest bid[33, 44].

- For the DA auction mechanism we first suppose that bidders 1 and 2 are given scores equal to their bids b_1, b_2 , while station 3 is given score b_3/w_3, so long as they are feasible. The factor $w_3 \leq 1$ is interpreted as the bidders "volume", to account for the greater interference it creates over the other two. With these scores, the DA algorithm, in the case were (i) $b_1, b_2 \leq b_3/w_3$, first rejects the station 3 which has the highest score among the three. Then stations 1 and 2 are acquired. To ensure strategyprofness each of them is paid b_3/w_3, the station's threshold price, which is its maximal bid that would have still been accepted given the bids of the others. In the case were (ii) $b_1, b_2 > b_3/w_3$, station 3 is acquired, it is paid its threshold price $w_3 \max(b_1, b_2)$

In the case of DA threshold mechanism, implemented as a descending-price clock auction in the above setting would perform considerably better. Analytically, at first all the bidders are considered active and are given a high price p given each volume respectively. This price will descend in every iteration , while the bidders are able to exit at any point in between iterations; if it is still feasible. Prices of bidders (who are still active) but not have a feasible exit, *freeze*. For instance, if station 3 (the center station) decides to exit the auction first, the others cannot be assigned to a channel, thus they do not have a feasible exit. Therefore, their prices are frozen at the "round" in which station 3 decided to drop out and they become winners. The process described achieves the optimality of truthful bidding self-evident. This means that a bidder can never benefit form not being truthful (stay at a price below his value or quit at a price above his value).

Chapter 4

The Incentive Auction

In this section we try to explain in detail the whole Incentive Auction of 2015 procedure. This multi-billion revenue spectrum auction had to accomplish the task of spectrum reallocation; which it actually did. This double auction is based on the Deferred-Acceptance Auctions format. It is critical to deeply understand the full structure of the auction first, in order to state the station repacking problem, which is also tackled by the auction mechanism with great success.

4.1 An Overview

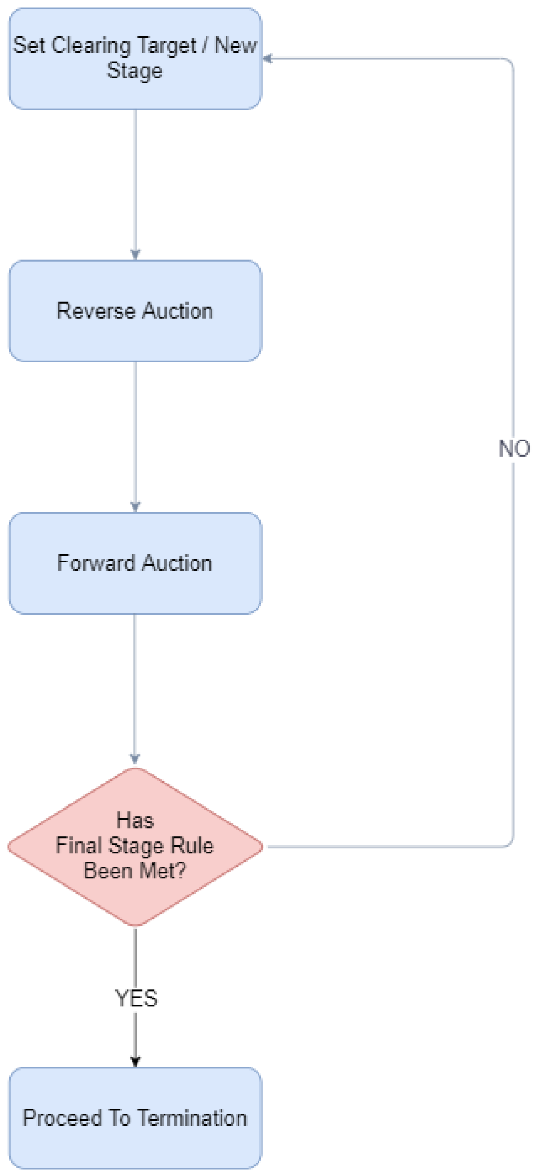
Before we dig into the full incentive auction's process we will try to give an overview on in, and try to make some comments.

As the years pass and technology advances with them, we notice that television and radio tend to "fade away". Of course, they are still a big part of every day life in the first world countries. Nevertheless, if we compare their standings back in the 90's with their standings after 2010 we can see this descending course. In contrast, we see wireless services, especially after 2010, having an ascending course. That created the need for a double auction. The government, in our case the US but it should be followed by other countries as well, needed to reallocate the spectrum. This reallocation, as we mentioned in the previous chapter, is enabled through a *double auction*. A process in which the auctioneer, is buying items from the sellers, that later allocates to the buyers. May sound simple, but is in reality a vary tricky process that demands high quality mechanism design. The 2016 *Incentive Auction* did actually manage to accomplish this challenging task and on top of that made revenues that came close to $10 billion.

The idea for the double auction at first, is to implement it through two different auctions. A *Reverse* auction, in which the FCC buys licences back from broadcasting stations; and immediately after a *Forward* auction, where companies of the wireless industry are are asked to buy the licences acquired. Both of these auctions are based on the deferred acceptance threshold auction format, as clock auctions. So the reverse and the forward auctions, are a descending and an ascending clock auction respectively.

Next we have to ask; how do we know how much of "space" to clear for the wireless services? The answer to that is the more the merrier, but it should be decided before the start of the auction, in order to know how much space is available for the remaining on air stations. To make it more clear, this so called “space” is the channels. Before the incentive auction, range of channels was 2-51; so we ask how many channels can we wipe? To answer this question in the most efficient way, there was set the *Clearing Target* and the idea of *Stages* was created. Each stage with a specific clearing target, and as the stages go on this clearing target descends. For instance, we begin with trying to clear 20 channels in the first stage; if we are not “happy” with the result we proceed to stage 2, were we aim to clear 15 channels and so forth.

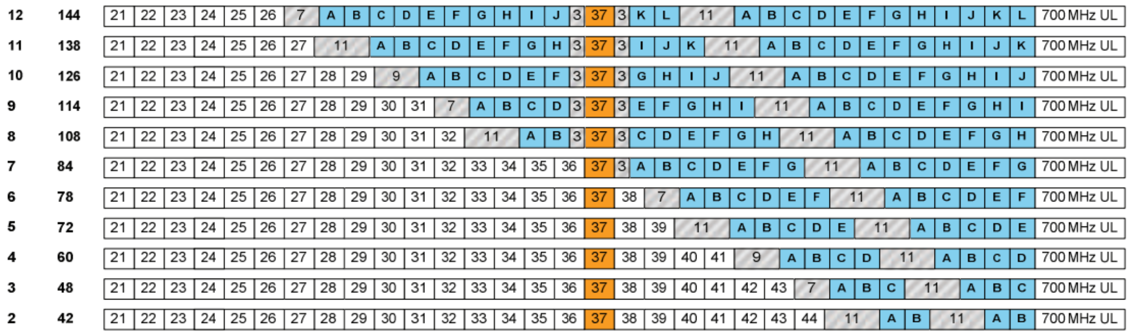
Lastly, we must know when to stop. At which time during the process, are we ”happy” with the result; and define that. That is the *Final Stage Rule*, a ”rule” that when is satisfied, means the termination of the auction. It makes the at the time current stage be the final stage of the auction.



Next in this chapter we are going to present in detail the exact rules and procedures that were followed, in order to implement these exact processes which constitute the *Incentive Auction*

4.2 The Spectrum Clearing Target

The 600 MHz Band Plan that was adopted in the Incentive Auction consists of an uplink band that begins at channel 51 (698 MHz), followed by a duplex gap, and then a downlink band.The Incentive Auction was conducted in stages. Each stage was to aim for a different “Spectrum Clearing Target”. A set of band plan scenarios based on the number of television channels cleared was associated with a specific potential clearing target as shown below.



Band Plan Scenarios : Figure 1 in[1]

The FCC had first, to determine an initial clearing target. The first stage of the forward auction would offer licenses corresponding to that target and any subsequent stages would offer licenses for scenarios corresponding to lower clearing targets—that is, moving down in the table shown above.

It is obvious that the FCC wanted to set the highest clearing target possible from among the available options given broadcaster participation in the reverse auction. We understand that if more broadcasters chose to participate, meant more space to be freed in the spectrum.

The auction system made use of mathematical optimization techniques to identify provisional TV channel assignments that protect the coverage area and population served of non-participating television stations as required by the Spectrum Act. Where necessary, non-participating stations were assigned to channels in the 600 MHz Band. In order to limit the extent of market variation in the provisional TV channel assignment plan, there was chosen to limit impairments on a nationwide aggregated basis to less than 20 percent of the total U.S. population (measured on a weighted basis). If a provisional channel plan did not exceed this limit, the auction system would apply any secondary objectives for TV channel assignments. If a provisional channel plan exceeded the less than 20 percent limit, the process was to start again with the next lower clearing target. We will not dig deeper on the Clearing Target Optimization Procedure, as it is not our mainly target on this paper.

As we have already quite indicated there was another challenge here for the FCC. There should be an initial channel assignment for all of the stations that did not chose to participate in the auction. That provisional assignment plan should satisfy a certain number of rules or constraints. Analytically, any assignment plan should include a permissible channel in its pre-auction band for every television station that is not participating in the reverse auction. Furthermore it should apply all of the repacking constraints, which we are going to explain in detail later in the paper, taking into account any fixed constraints specific to an area or a channel that would prevent an assignment of a station to a channel, as well as all other stations that cannot be located on a co- or adjacent channel.

We should highlight that such an assignment was conducted before the beginning of each stage of the auction, and not just before the first stage, as each on of the stages was aiming for a different clearing target. The assignment became possible through a serious of mixed-integer programs.Unfortunately, for our research, these programs and their results were never made public by the FCC, and as a result it is nearly impossible to make an exact simulation of the auction. Here we should mention that the auction concluded in the forth stage, and its the initial clearing target for the first stage was set to 126 MHz. That translates to 29 eligible channels for the UHF stations.

4.3 Stages of the Auction

As indicated before, the auction design consists of several stages. Each stage with a unique clearing target which is descending as the stages proceed.

The Incentive Auction begins with a Reverse Auction -which is our main point of interest-, and followed by a Forward Auction after its termination. If bidding in the forward auction does not satisfy the final stage rule, additional stages will be run with progressively lower spectrum clearing targets in the reverse auction and fewer licenses available in the forward auction, until the final stage rule is satisfied. Reverse and Forward auction will be analyzed in detail in this section.

4.3.1 Final Stage Rule

There was a need to determine whether or not, each stage should be the final, which would mean the termination of the auction.

The final stage rule is a set of conditions that must be met in order to close the auction at the current clearing target; failure to satisfy the rule would result in running a new phase at the next lowest clearing target.

The final stage rule is a reserve price with two components, both of which must be satisfied. The first component requires that the average price for low impairment licenses in the forward auction meets or exceed $1.25 per MHz-pop at a 70 MHz cleared benchmark. Alternatively, if the spectrum clearing target at a particular stage is greater than 70 MHz, then the first component will be met if the total proceeds of the forward auction exceed the product of $1.25 per MHZ/pop x 70 MHz x the total number of pops for the high-demand Partial Economic Areas (PEAs) with at least one Category 1 block in this stage. This alternative formulation will allow the auction to close if the incentive auction repurposes a relatively large amount of spectrum for wireless uses, even if the price per-MHz-pop is less than the benchmark price.

The second component of the final stage rule requires that the proceeds of the forward auction be sufficient to meet mandatory expenses set forth in the Spectrum Act. If the requirements of both components of the reserve price are met, then the final stage rule is satisfied.

4.4 Stage Structure

4.4.1 Reverse Auction

The procedure begins with the Reverse Auction. Its goal is to identify broadcasters who are willing to give up some or all of their spectrum usage rights, as well as the incentive payments required. Those rights that are bought by the FCC in the reverse auction, aim to clear and achieve a clearing target.

The Reverse Auction is a descending clock reverse auction, conducted in multiple rounds, with prices for a hierarchy of relinquishment options. The opening prices are determined before the beginning of the auction for the stations that are needed and wish to participate. A station that will be eligible in its pre-auction band after the termination of the auction in any case, is marked NOT NEEDED and does not get to participate in the auction.

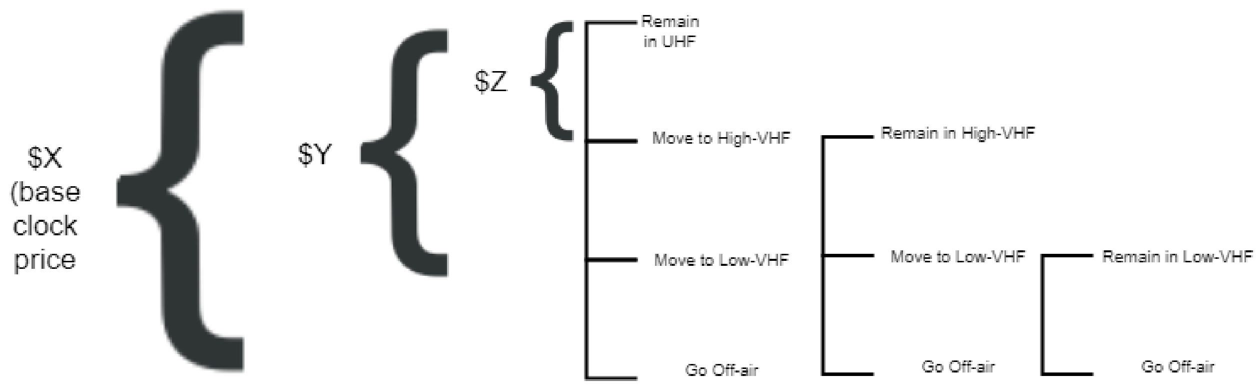
4.4.1.1 Relinquishment Options

There are three different auction bands that a station could reside in. The UHF (Ultra High Frequency) which corresponds to channels 14 to 51 (before the incentive auction). The High-VHF (High - Very High Frequency) band which corresponds to channels 7-13, and the Low-VHF (Low - Very High Frequency) which corresponds to channels 2-6. Channel 37 is not eligible for any station as it is used for radio astronomy across the country.

For a UHF station (participating stations whose pre-auction band is UHF) there are three clock prices. First is the price that is associated with the option of going off-air which is the highest among the three. Then the price that corresponds to moving to the Low-VHF band. And the lowest is the price for the moving to the High-VHF band option. If the UHF station is not satisfied with any of the three prices for their respective option, it can choose to drop out of bidding and keep broadcasting in the UHF, after the spectrum reallocation.

For High-VHF stations ((participating stations whose pre-auction band is High-VHF), there are two clock prices. One for going of air, which is the the higher priced option and one to move to Low-VHF. Low-VHF stations (participating stations whose pre-auction band is Low-VHF) only have the option to go off air, which translates to only one clock price.High and Low VHF station also can drop out of bidding any time in the auction if they are not satisfied by the clock prices that are presented to.

The figure below shows the prices for each pre-auction band relative to the UHF prices.



4.4.1.2 The Auction Prices

As mentioned above each station that was to participate at the auction, is presented with an opening price for each of the bidding options available. Opening prices for Low-VHF and High-VHF are a specific fraction of the opening price for going off air. Opening prices for going off air are calculated using a base clock price, which is the same for all the UHF stations and the station volume, which is different for every station. More specifically:

$$\text{Opening Price} = (\text{Base Clock Price}) * (\text{Station Volume}) \tag{4.1}$$

A station's volume is calculated using the number of people residing within the interference-free service area of station, and also the number and significance of co- and adjacent channel interference constraints that station would impose on repacking. That indicates in simple terms that the more interference a station causes, the higher the price it is going to be offered. Below we present the formula for the calculation of the volume as given by the FCC (Appendix D).

$$\text{Station volume} = (\text{Broadcast population})^1/2 * (\text{Interference})^1/2 \tag{4.2}$$

Station volume is scaled so that it does not exceed million. Low-VHF have an opening price at approximately 75 percent of going off-air. High-VHF have an opening price at approximately 40 percent of going off-air.

For the calculation of the base clock price the FCC, set the opening bid for the station with the highest volume at $900 million. Given that the highest volume could be 1 million, FCC ended up with an opening base clock price of $900 per unit of volume.

4.4.1.3 The Bidding Rounds

After the auction system has decided for each station the prices for every relinquishment option available to it, the bidding round launches. The auction collects bids and after each round the auction system processes the bids in order to determine the current relinquishment option to which each station is assigned, update the bidding status of all stations, and set clock prices for the next round. Before updating the bidding status of every station the auction mechanism must solve the *repacking problem*. According to the solution of each problem a respective status is assigned. For instance, if a station according to the repacking problem solver cannot be placed in each pre-auction band; the mechanism can not further decrease its price. In any round, a station may be assigned one of the following statuses.

1. **"BIDDING"**. When a stations holds that status, its bid at the clock prices is processed to determine its current relinquishment option, and its clock prices will be reduced for the next round.
2. **"FROZEN"**. The station status becomes frozen, in the case where the auction system cannot currently assign it a channel in its pre-auction band. In that case, the station's assigned option at the time that the system determined that the station was frozen remains its current relinquishment option and its current price is "frozen".
   (a) A frozen station can be **"FROZEN – Currently Infeasible"**. That status applies only to the VHF stations. In that case the system has determined that there is no feasible channel assignment for the station at that time, but it may be able to find a feasible assignment in the future rounds. (Such a case could occur if a UHF station that was holding the option to move to VHF drops out of bidding.)
   (b) A station can hold the status **"FROZEN – Pending Catch Up"**, only on stages 2 and later. It means that it became frozen on a previous stage and it may become unfrozen and asked to resume bidding in later rounds of the stage if the base clock drops below the station's "catch-up point," or the base clock price at the time that the station became provisionally winning in the previous stage and the station has a feasible channel in its pre-auction band.
   (c) **"FROZEN – Provisionally Winning"** is a station's status, once the auction system has determined that it cannot assign the station to its pre-auction band. As mentioned above that status can change in next stages.
3. **"EXITED"**. A station is assigned the status inactive in every other possible case. These cases may occur:
   (a) When a station has chosen to drop out of bidding, and is assigned with the status **"EXITED – Voluntarily"**.
   (b) When the system has determined that a feasible channel can always be found in a station's pre-auction band, the station will be designated to remain in its pre-auction band and be given the status **"EXITED – Not Needed"**.

The bidding statuses assigned to a station can change from one round to the next.

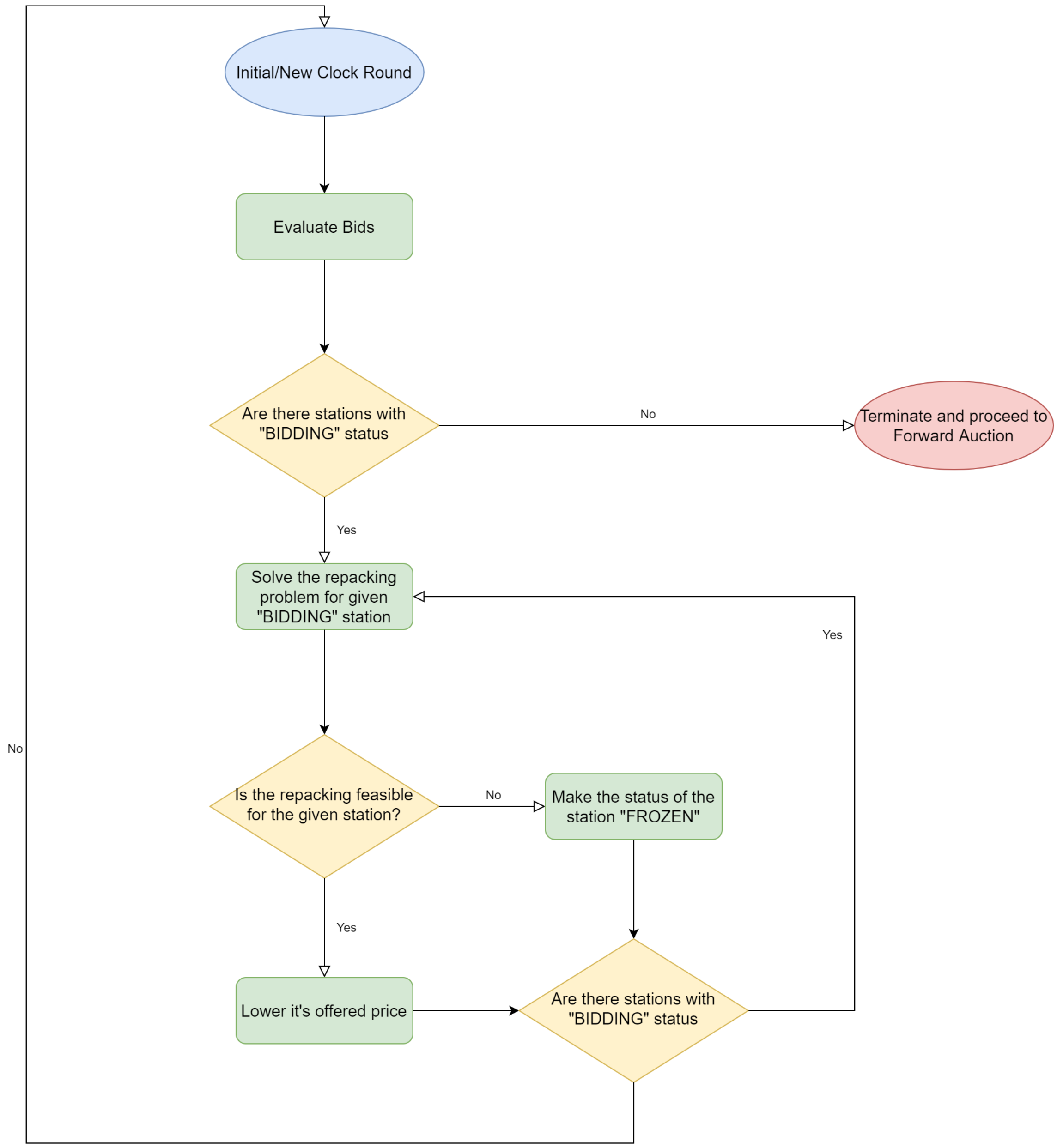
4.4.1.4 Termination And Winners

With the progression of the Reverse Auction and as the rounds proceed, as described, the prices are lowered and fewer and fewer stations are going to have the status “BIDDING”. The reverse auction of the first stage terminates when every participating station holds one of the following three statuses: “FROZEN – Provisionally Winning”, “EXITED – Voluntarily”, or “EXITED – Not Needed”.

The forward auction of stage 1 is succeeding the reverse auction of the same stage. IF the final stage rule is met during the forward auction, then stage 1 is the concluding stage of the incentive auction. That means that the provisionally winners of the reverse auction are declared winning stations. If the final stage rule is not met the auction will naturally proceed to stage 2 with a lower clearing target.

The process of the reverse auction as described applies to all stages that are necessary, until the final stage rule is met.

Regarding the reverse auction process and the bidding statuses we present a flow chart below.



4.4.1.5 Reverse Auction Example

We are going to present an example below, in order to make clear the process of the auction that we described above. The example is given by the Federal Communications Commission in appendix D.

There are two locations: A and B. Stations 1, 2, and 3 are in location A. Stations 4, 5, and 6 are in location B. The pre-auction band of all six stations is UHF. The volume of each of these stations is equal to 1.

The three stations of each location are interfering with each other in Low-VHF and High-VHF. Specifically, we assume that there is one channel available in Low-VHF in location A, and one in location B. That is, if one of the stations in location A is in Low-VHF, no other station in location A can be assigned there. We make the same assumption for the High-VHF band. All six stations are interfering in UHF. We assume that there is one available channel in UHF across both locations. That is, if one of the six stations is assigned in UHF, then no other station can be assigned there. The opening prices are: $1000 for going off-air, $700 for moving to Low-VHF and $400 for moving to High-VHF. In the application process, every station indicates going off-air as its preferred option. For the purposes of this example, we assume that the decrement is R = 5%, c = 0.5, VAC_FLOOR = 0.1, and that DRP is "OFF". In the calculations, we round prices to the nearest integer, and we round the values of Δ_s to two decimal points.

Round 1

- **Clock Price Calculation**

  In the initial assignment, the current relinquishment option of every station is to go off-air. For round 1, the vacancies are $V_{1,s,b}$ = 1 for all stations s and all bands b. This implies that the reduction coefficients are: $r_{1,s,HV}$ = 0.4 and $r_{1,s,LV}$ = 0.7 for all stations s. After computing the benchmark prices for round 1, we conclude that the clock prices are: $P_{1,s,OFF}$ = \$950, $P_{1,s,LV}$ = \$665, and $P_{1,s,HV}$ = \$380 for all stations s.

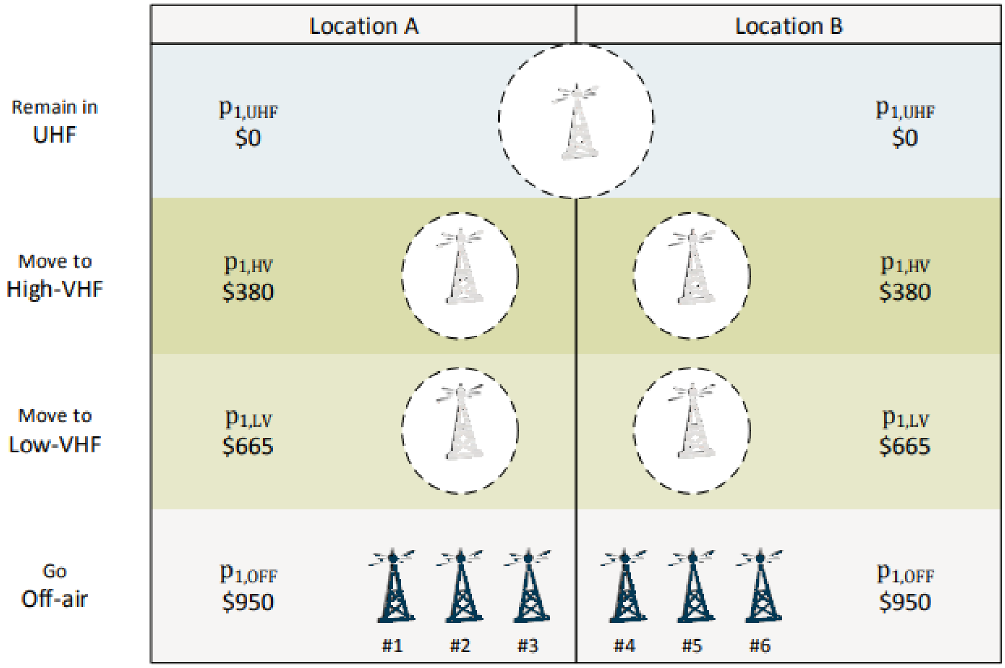
- **Bidding**

  We assume that in round 1, every station accepts the offer for going off-air at the clock price (that is, no station submits a bid to drop out of the auction), and no station submits a bid to switch to another option.

- **Bid Processing**

  Δ_s = 1 for all stations s, since no station submitted a bid to drop out. The stations are placed in the queue in a pseudo-random order. The result of bid processing is that $b_{2,s}$ = OFF and $\bar{P}_{2,s}$ = \$950 for all stations s. That is, for every station the current relinquishment option is going off-air and the current compensation is \$950.

This is illustrated in the figure bellow, taken from [1] .



Round 2

- **Clock Price Calculation**

  Since all stations are currently off-air, the vacancies are $V_{2,s,b} = 1$ for all stations s and all bands b. This implies that the reduction coefficients are: $r_{2,s,HV} = 0.4$ and $r_{2,s,LV} = 0.7$ for all stations s. After computing the benchmark prices for round 2, we conclude that the clock prices are: $P_{2,s,OFF} = \$903$, $P_{2,s,LV} = \$632$, and $P_{2,s,HV} = \$361$ for all stations s.

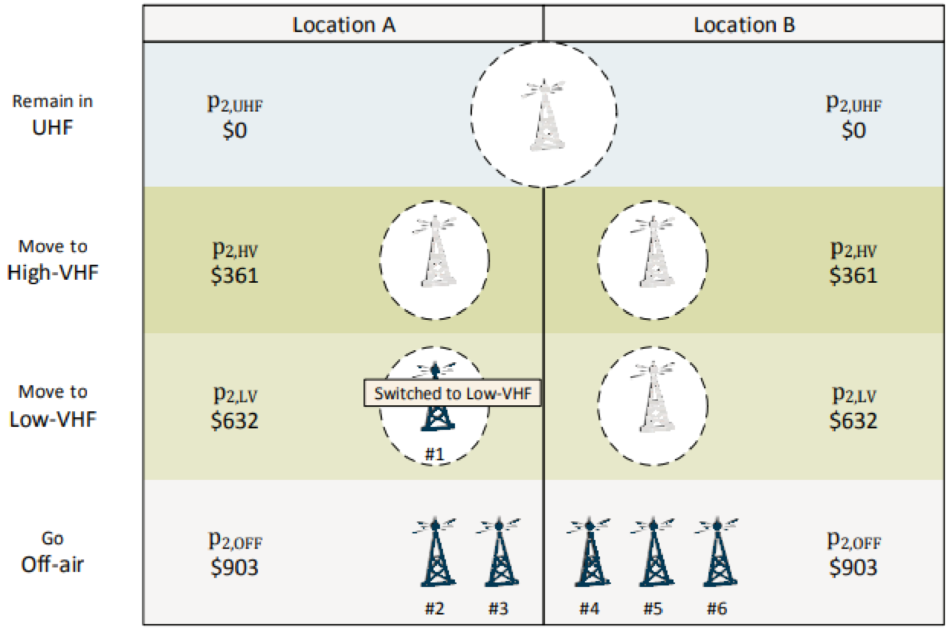
- **Bidding**

  We assume that in round 2, station 1 submits a bid to switch to move to Low-VHF, and no other station submits a bid to switch. Moreover, every station accepts the offer for going off-air at the clock price (that is, no station submits a bid to drop out of the auction).

- **Bid Processing**

  $\Delta_s = 1$ for all stations s, since no station submitted a bid to drop out. The stations are placed in the queue in a pseudo-random order. The result of bid processing is that $b_{3,1} = \text{LV}$ and $\bar{P}_{3,1} = \$632$; and $b_{3,s} = \text{OFF}$; and $\bar{P}_{3,s} = \$903$ for stations $s \in \{2, 3, 4, 5, 6\}$.

This is illustrated in the figure bellow, taken from [1].



Round 3

- **Clock Price Calculation**

  In location A, one of the stations (station 1) is currently in Low-VHF and the other two stations are off-air. In location B, all stations are currently off-air. The vacancies are $V_{3,s,b}$ = 1 for all stations s and bands $b \in \{HV, UHF\}$. Moreover, $V_{3,s,LV}$ = 0.5 for $s \in \{1, 2, 3\}$ and $V_{3,s,LV}$ = 1 for $s \in \{4, 5, 6\}$. That is, in location A the vacancy in Low-VHF is smaller than in location B, because a station is currently assigned in Low-VHF in location A. This implies that the reduction coefficients are: $r_{3,s,HV}$ = 0.4 for all stations s; and $r_{3,s,LV}$ = 0.8558 for $s \in \{1, 2, 3\}$ and $r_{3,s,LV}$ = 0.7 for $s \in \{4, 5, 6\}$. After computing the benchmark prices for round 3, we conclude that the clock prices are: $P_{3,s,OFF}$ = \$858, and $P_{1,s,HV}$ = \$343 for all stations s. Moreover, $P_{3,s,LV}$ = \$593 for $s \in \{1, 2, 3\}$ and $P_{3,s,LV}$ = \$600 for $s \in \{4, 5, 6\}$.

- **Bidding**

  We assume that in round 3

  – Station 1 accepts the clock price for Low-VHF. It submits neither a bid to switch nor a bid to drop out.

  – Station 2 submits a bid to switch to move to High-VHF, and a bid to drop out of the auction when the price of off-air drops to \$880.

  – Station 3 submits a bid to switch to move to High-VHF, and a bid to drop out of the auction when the price of off-air drops to \$870.

- Station 4 submits a bid to switch to move to Low-VHF, and a bid to drop out of the auction when the price of off-air drops to \$890.
- Station 5 submits a bid to drop out of the auction when the price of off-air drops to \$860.
- Station 6 accepts the clock price for going off-air. It submits neither a bid to switch nor a bid to drop out.

- **Bid Processing**

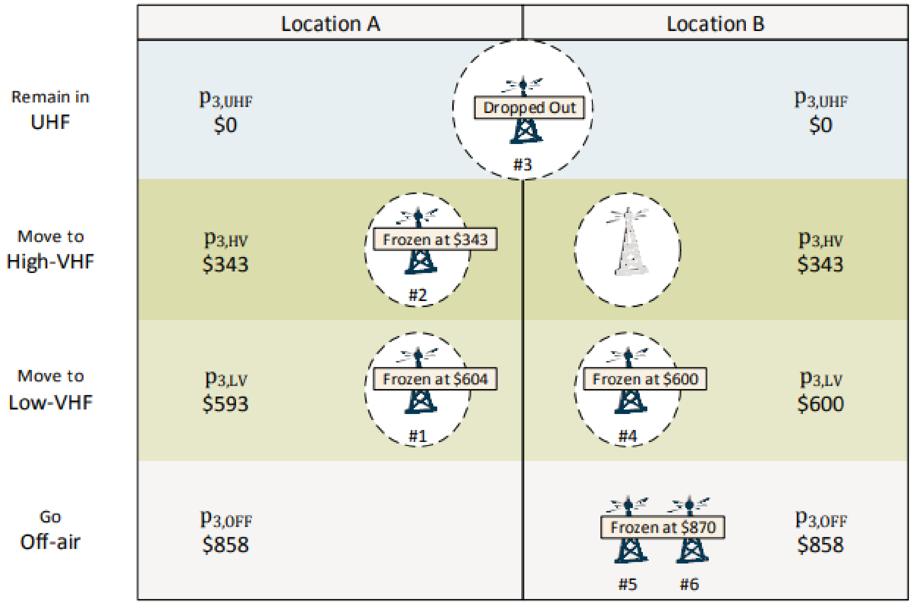
  Given the bids that were submitted in this round, the algorithm computes that $\Delta_1 = 1$, $\Delta_2 = 0.51$, $\Delta_3 = 0.73$, $\Delta_4 = 0.29$, $\Delta_5 = 0.95$, and $\Delta_6 = 1$. The station's compensations are initialized to $\bar{P}_{4,1}$ = \$632, and $\bar{P}_{4,s}$ = \$903 for stations $s \in \{2, 3, 4, 5, 6\}$. The stations are placed in the queue in the following order: 4, 2, 3, 5, 1, 6 (where the tie between stations 1 and 6 was broken pseudo-randomly).

  - In the first iteration, station 4 is considered and $\Delta_4 = 0.29$. Given that none of the stations are frozen, the current compensation of station 1 is updated to $\bar{P}_{4,1}$ = min($\bar{P}_{3,1}$(0.29),632) = \$621 and the current compensation of every other station $s \in \{2, 3, 4, 5, 6\}$ is updated to $\bar{P}_{4,s}$ = min($\bar{P}_{4,s}$(0.29),903) = \$890. Station 4 is then switched to Low-VHF and removed from the queue, and its current compensation is set to $\bar{P}_{4,s}$ = \$600.
  - In the second iteration, the current compensations of stations in the queue are updated as a function of $\Delta_2 = 0.51$. Station 2 is then switched to High-VHF, it is removed from the queue, and its current compensation is updated.
  - In the third iteration, station 3 is considered and the current compensations are updated as a function of $\Delta_3 = 0.73$. Station 3 is not feasible in High-VHF (because station 2 is currently assigned there), so station 3 drops out of the auction and is assigned a channel in UHF. This causes all other stations to be "frozen", since they are now infeasible in UHF. The current compensation of stations 2 and 4 is equal to the clock price of their current options, respectively. For stations 1, 5, and 6, the current compensation is determined by $\Delta_3 = 0.73$.

- **Outcome**

  At this point, station 3 is inactive (because it has dropped out of the auction) and all other stations are provisionally winning. As a result, this stage of the reverse auction ends. The provisionally winning options are: Low-VHF for station 1, High-VHF for station 2, Low-VHF for station 4, and off-air for stations 5 and 6. The provisionally winning prices are \$604 for station 1, \$343 for station 2, \$600 for station 4, and \$870 for stations 5 and 6.

This is illustrated in the figure below, taken from[1].



4.4.2 Forward Auction

The forward auction of the incentive auction that takes place immediately after the termination of the reverse auction, is a forward ascending clock auction. In this auction wireless service providers are bidding, in multiple rounds, for categories of generic license blocks in specific geographic areas. We will not analyze in depth the procedure of the forward auction as it is not our main point of interest in this paper.

4.4.2.1 Bidding Categories

In the forward auction there are two categories of generic blocks offered to the bidders.

- **Category 1** includes any block with potential impairments that affect t zero to 15 percent population of a PEA[1]
- **Category 2** includes any block with potential impairments that affect greater than 15 percent but less than or equal to 50 percent of the population of a PEA.

The FCC does not include in the forward auction any generic block that has potential impairments that affect more than 50 percent of the population.

[1] Partial Economic Area[7]

4.4.2.2 Bids

The FCC enables three types of bids that can be made by the participants in the forward auction. First there is the “simple” bids, the “switch bids” and the “all-or-nothing” bids.

1. **Simple Bids** A simple bid is placed by a participant and it indicates desired quantity of a product at a price. When there is not possible for the bid to be applied in its entirety the auction system may apply it partially.
2. **All-or-Nothing Bids** An all-or-Nothing bid, like the simple bid, also expresses desire for a quantity of blocks at a price. However, this type of bid can not be applied partially. That means that the bid will be applied fully if possible, or not at all.
3. **Switch Bids** The Switch Bid enables the bidder to be more flexible in range of a PEA. Specifically a bidder utilizing the switch bid, can bid for a desired quantity of blocks in a category at a given price point, and identify another category(in the same PEA) to switch the bid to, at a given price point.

4.4.2.3 Units

In order to calculate several needed quantities, the FCC came up with the bidding units. Each spectrum block available in the auction is applied with a specific number of bidding units.

Inside a particular PEA each block of spectrum has the same amount of bidding units, regardless of category. That enables the application of the switch bid that we discussed earlier.

4.4.2.4 Bidding Eligibility

To become a qualified bidder in the forward auction, each applicant was requested to make an **upfront payment**. The FCC established these upfront payments, in order to avoid any frivolous or insincere bidding and of course to raise funds for the execution of the Incentive Auction.

This rule ends up, with each bidder having a specific **Bidding Eligibility**. More precisely, the commission set the prices of upfront payments at $2,500 per bidding unit. Moreover, the bidders could not increase their bidding eligibility during the time that the auction took place. The eligibility of a bidder could only stay the same or decrease as the auction proceeded.

4.4.2.5 Activity Rule

The activity rule is a very important mechanism for the forward auction. It controls the rhythm of the auction as well as the pricing discovery process. This rule obligates every bidder, to maintain a minimum, high level of activity in each round of the auction. If bidder does not comply with the activity rule in a round, he is going to have reduced bidding eligibility in the net round. In concrete terms, the activity rule is satisfied when a bidder has bidding activity on blocks with bidding units that total at least 95 percent of its current eligibility in the round.

4.4.2.6 Rounds

1. **Initial Round** In the initial round, each bidder specifies the quantities it wants of each product at the minimum opening bid prices. All initial round bids are applied during bid processing, so a bidder's processed demand for a product is equal to the quantity specified in its bid. The posted prices for the initial round are the minimum opening bid prices of the forward auction.

2. **Regular Clock Round** In the regular clock rounds, participants bid to maintain processed demands at the clock price. One can also bid to change demand if possible.

   Reductions in demand can only be applied during the bid processing if there is excess in demand for the specific product. Similarly, increase in demand can only be applied during bid processing if the resulting processed activity would not exceed the bidder's eligibility for the round.

   After the demands have been determined, the posted prices of the products for a round are set as follows:

   - The posted price is set equal to the clock price for the round if aggregate demand exceeds supply.
   - The posted price is set at the highest price at which a reduction for the product was applied, when aggregate demand is exactly equal to supply.
   - In any other case the posted price stays the same as in the previous round.

   The auction system evaluates, after the termination of each round of the forward auction, if the final stage rule is met. If it indeed is met, it means that the incentive auction will end at the current stage and clearing target.

3. **Extended Round** In the case that the final stage rule is not yet met in the forward auction and the bidding activity has stopped for Category 1 blocks in the 40 high-demand markets[2], an extended round is triggered.

   The idea of the extended round is that the currently winning bidders are very likely to become winning bidders when the clock phase ends. That motivates those bidders to ensure that the final stage rules is met(these information are given to the bidders by the FCC), so they can be winners of the incentive auction.

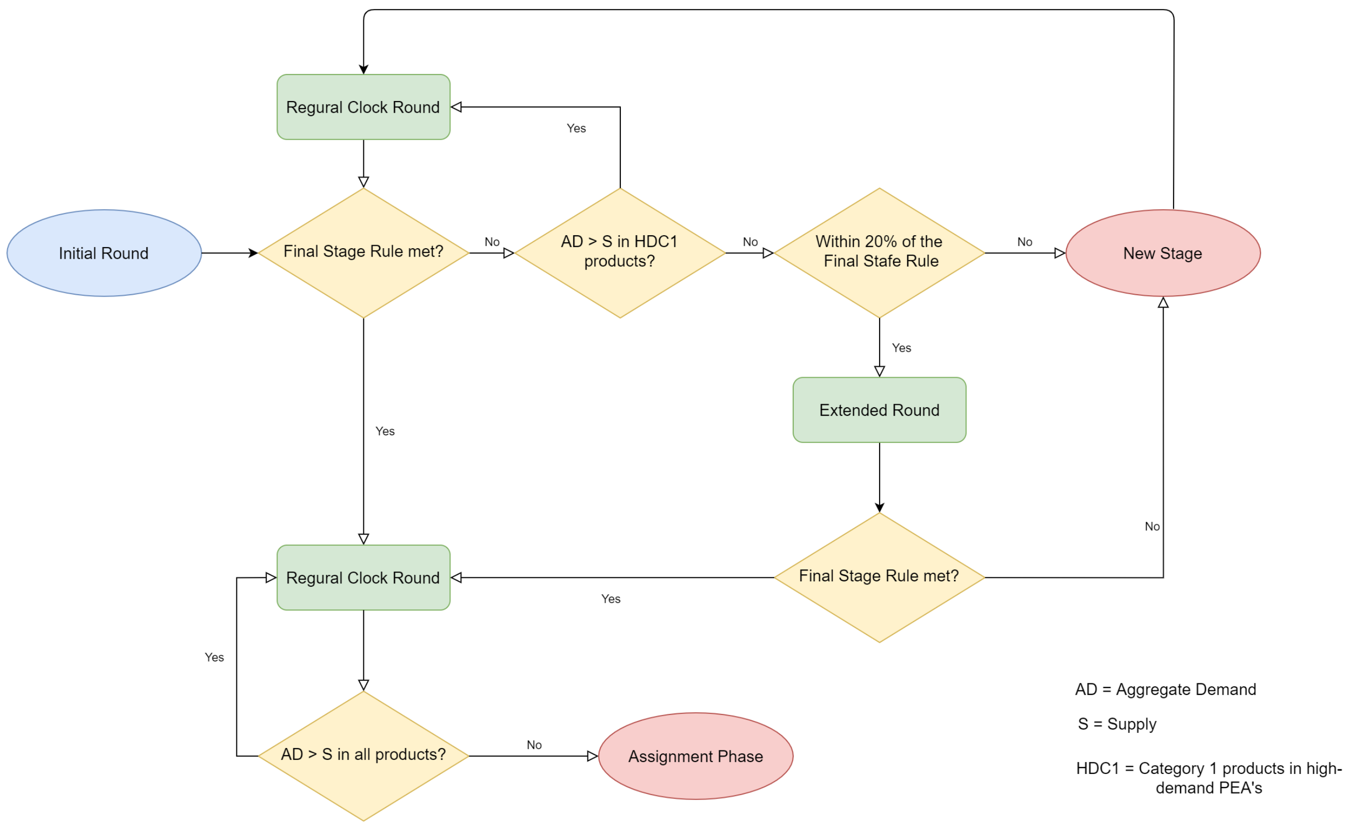
   If the final stage rule is not met in the extended round, the current stage of the incentive auction stops and a new stage with a decreased clearing target begins. We should note that, clock prices for the first round of a next stage will be bases on the extended round stopped prices.

[2] High-demand markets are PEAs 1–40

Stopping Rule

Similar to the reverse auction, there is also a stopping rule for the forward auction. In spite of there being an extended round or not, if the final stage rule is met during the forward auction, the clock phase of bidding ends for all categories of licenses following the first round in which there is no excess demand in any category in any PEA. The bidders that are still expressing demand for a category of a PEA at the time the stopping rule is met become the winning bidders, and are assigned specific frequencies in the **assignment phase**.

We present a flow chart below that gives an overview of the flow of the forward auction during the clock phase.



4.4.3 Assignment Phase

The incentive auction concludes with the assignment phase. Unlike the forward and the reverse auction the assignment place take place only once during the whole auction. Moreover, it is not mandatory for the bidders to participate in this part of the auction. All of the clock phase winners are assigned contiguous licenses within a category and PEA in spite of their bid amounts and whether they bid. It’s goal is to determine the optimal assignment of licenses within each PEA.

During the assignment phase, winners from the clock phase bid for frequency specific license assignments corresponding to the number of generic blocks they won in the clock phase for a category and PEA. The assignment phase progresses through a sequence of assignment rounds.In each one of these rounds, each clock phase winner have the opportunity to make sealed bids to one or more possible frequency assignments for which they wish to express a preference, that is conforming with the bids for generic blocks that were won in the clock phase.

This concludes a brief but complete summary of the whole process Incentive Auction. Out of the three parts the assignment phase is the one we have the less interest in. The reverse auction and the difficult tasks that had do be accomplished in order for it to take place, is our main point of interest.

Chapter 5

The Repacking Problem

In this section we analyze the repacking problem as it occurs in the incentive auction. There have been many studies considering the optimization of this particular problem. We are also introducing the tools that the FCC utilized in order to accomplish this challenging objective.

5.1 The challenge

The design of the reverse auction requires hundreds of thousands of repacking problems to be solved in a limited amount of time. The repacking problem is NP-complete, as it generalizes *Graph Coloring*, and it also falls in the category of *frequency assignment problems*[41, 9]; which makes it impossible to be solved deterministically in less than exponential time (Unless P = NP). Finding an algorithm that could solve this problem efficiently, was a great importance matter for the FCC. This is due to the fact that every failure to solve a single feasible repacking problem, correlate to a lost opportunity to lower a price offer. Given the enormous prices of the reverse auction and that there would be many such problems to be solved, even small differences on the performance of this algorithm could translate to millions of dollars for the US government. We understand that there is more than one way to approach this problem. It is shown that it can be solved with *Integer Programming* formulations, but the results are not that efficient[42]. In this section we will present the way that the FCC approached it; with *SAT* encoding, and we will introduce the open-source software *SATFC*, which was created as an official solver for the station repacking problem.

5.2 The problem in theory

Before the incentive auction took place in 2015, each television station $s \in S$ was assigned to a specific channel $c_s \in C \subseteq N$. This assignment was to ensure that there should be no interference between stations whatsoever. The FCC determined the harmful interference via a complex, grid-based physical simulation ("OET-69" (FCC 2013)). Furthermore, the commission processed the results of this simulation and obtained a CSP-style formulation listing all the forbidden pairs between the stations and channels. This formulation was made public by the FCC (FCC 2014e).

Let $I \subseteq (S \times C)^2$ denote the set of forbidden station-channel pairs $\{(s, c), (s', c')\}$, each representing the proposition that stations s and s' may not concurrently be assigned to channels c and c', respectively. The reverse auction moves some broadcasters (stations) completely off-air and at the same time reassigns to the remaining on-air stations, channels from a reduced channel set c'. This reduced channel set will is defined accordingly by the clearing target that was introduced earlier in this paper.

Clearing Target(MHz)	Channels	Stage
84	29	1
108	31	2
114	32	3
126	36	4

This table shows the reduced channel set for each stage that took place in the incentive auction, related to the clearing target that was set for the stage.

Analytically, some channel $\bar{c} \in C$ such that all stations are only eligible to be assigned channels from $\bar{C} = \{c \in C : c < \bar{c}\}$. The sets of channels that a station can be assigned to, regardless of the constraints, are given by a *domain* function $D : S \rightarrow 2^{\bar{C}}$ that maps from stations to these reduced sets. The **station repacking problem** is the task of finding a repacking $\gamma : S \rightarrow \bar{C}$ such that all of the stations are assigned to a channel from their domain and one of these assignments satisfy the interference constraints as given by the FCC. In mathematical terms : for which $\gamma \in D(s)\ \forall s \in S$, and $\gamma(s) = c \Rightarrow \gamma(s') \neq c'\ \forall \{(s, c), (s'.c')\} \in I$.

There are three different classes of channels ,as mentioned before.

- **LVHF** : channels 1-6
- **HVHF** : channels 7-13
- **UHF** : channels 14-$\bar{c}$ (where $\bar{c} \leq 51$ which was the highest UHF channel before the spectrum repacking). Furthermore channel 37 is excluded for every station.

The interference constraints are separated for each one of the three classes, which enables to decompose the problem in a straightforward way. An instance of the problem is to make an appropriate choice of stations $S \subset \mathbf{S}$ to pack into channels $C \subset \mathbf{C}$, with domains D and constraints I, being restricted to S and C; The resulting restrictions are noted as *D* and I.

The *interference graph* is defined as an undirected graph, which has the following properties: There is one vertex per station and an edge exists between two vertices s and s' if the corresponding stations participate together in any interference constraint: i.e., if there exist $c, c' \in C$ such that $\{(s, c), (s', c')\} \in I$.

The figure below shows the Incentive Auction interference graph.

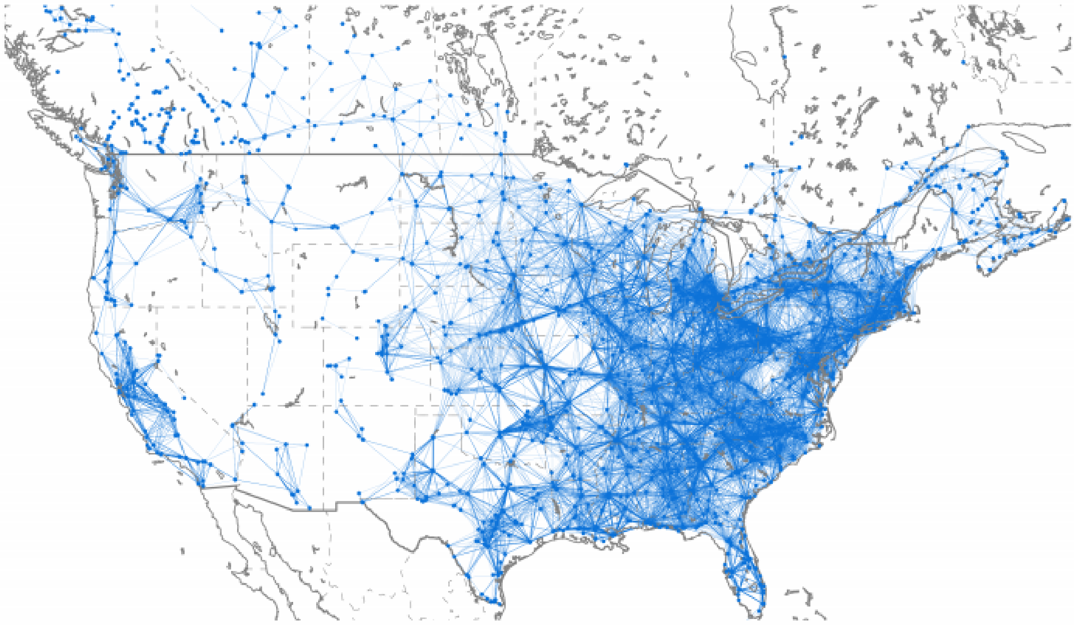


Figure taken from[42]

The domain file that was made public by the FCC includes more than 3000 stations and the amount of possible subsets is exponential on number. This makes it impractical, for an algorithm to exhaustively search over every possible subset.

There are two kinds of interference constraints:

- **Co-channel Constraints**, which specify that two stations cannot be assigned on the same channel.
- **Adjacent-channel Constraints**, which specify that two stations cannot be assigned to two nearby channels.

Thus, we come up with forbidden station-channel pairs that take the form $\{(s, c), (s', c')\}$ for some stations $s, s' \in S$, channel $c \in\ C$, and $i \in \{0, 1, 2\}$. These interference constraints as described may not involve channels in more than one band(LVHF, HVHF, UHF).

Naturally, worst case instances were not expected to come up in the real time reverse auction, so the work that needed to be done, was to optimize the performance on the sort of instances generated by actual reverse auctions.

The reverse auction repeatedly generates station repacking problems. To lower a price of a bidding station at a specific round the system first, has to make sure that there is a feasible assignment for that station in case it chooses to drop out of the auction. That is adding a single station s to a set S of provably repackable stations. This means that the problems that we are facing here, always come up with a partial assignment $\gamma^- : S^- \to C$ that we know is feasible on a restricted station set S^-. Each time we just want to add a single station s^+ to that restricted set for a specific set of channels C. This property is very significant and useful for the optimization of the problem. Furthermore, it explains the **initial assignment** process, that was explained in the previous chapter, and need to be done before the beginning of each stage. The initial assignment creates this partial assignment $\gamma^- : S^- \to C$ on the set of stations that did not took part in the incentive auction before the beginning of the first stage.

5.2.1 The Initial Assignment

As we have mentioned earlier, unfortunately none of the initial assignments were made ever pubic by the FCC. Thus, we cannot test the same instances that came up during the reverse auction. Although, FCC published the mathematical optimization problem (APPENDIX C) that was solved in order to find the initial assignment, and the later assignments of the clearing target of the next stages. Bellow we are going to appose this optimization problem in short.

5.2.1.1 Feasible Assignment

A feasible assignment of TV stations needs to meet the following conditions :

- All stations must be assigned to a channel or to go off-air.
- A station is only eligible to be assigned to a channel that is stated by the domain.csv file[3].
- Stations' assignments must not violate adjacent and co-channel pairwise interference restrictions as defined in the interference_paired.csv file.
- Every station that does not take part in the auction is assigned to a channel in its pre-auction band.
- All the stations participating in the reverse auction must be assigned either to a channel in a band associated the bidder's initial commitment(s) or to a channel in the bidder's pre-auction band or to go off-air.

Set Definitions:

S is the set of all stations (both participating and non-participating). C_s is the set of allowable channels for station s

For non-participating stations, the set C_s consists exclusively of allowable channels in their pre-auction band, which for UHF stations, includes their allowable channels in the 600 MHz Band. For

[3] domain.csv and interference_paired.csv are files given by the commission considering the Domain and feasibility constraints

participating stations, the set C_s consists of allowable channels in their pre-auction band as well as channels in the bands associated with their initial commitment(s) of relinquishment options. For participating stations that made an initial commitment to go off-air, the set C_s also consists of channel 0 which indicates an assignment of going off-air.

Variable Definitions:

$x_{s,c}$ is a binary decision variable which has a value of 1 if station s is assigned to channel c and 0 otherwise. Note $c = 0$ indicates the option to go off-air.

5.2.1.2 Constraints for Initial Commitments

The goal here is to assign as many participating stations as possible to their preferred relinquishment option, that they selected during the initial commitment process. However, there is limited capacity in the VHF bands, thus it is not possible to assign all of the stations to their preferred option. This optimization was made through a series of steps :

1. Determine the minimum number of UHF participating stations that must be assigned to their preauction band.
2. Add the outcome of (1) as a constraint on the optimization model, and then determine the minimum number of VHF participating stations that must be assigned to their pre-auction band.
3. Add the outcome of (2) as a constraint on the optimization model, and then determine the maximum number of participating stations that can be assigned to their preferred relinquishment option.
4. Add the outcome of (3) as a constraint on the optimization model, and then determine the maximum number of participating stations that can be assigned to go off the air.
5. Add the outcome of (4) as a constraint on the optimization model, and then minimize the sum of impaired weighted-pops across all licenses (i.e. solve the primary clearing target optimization model).

- For step (1) : Minimize the number of UHF stations assigned to their pre-auction band. We have:

  Subsets:

  - S_p is the set of participating stations
  - SP_U is the set of participating stations whose pre-auction band is UHF
  - C_s^H is the set of allowable pre-auction band channels for station s

Model Formulation:

$$\min Z_1 = \sum_{s \in S_{P_U}, c \in C_s^H} x_{s,c}$$

The objective function minimizes the number Z_1 1 of UHF participating bidders that are assigned to their pre-auction band. Thus, the optimization sums only the set of participating stations whose pre-auction band is UHF $(s \in SP_U)$ and determines the minimum number of such stations that must be assigned some channel in their pre-auction band $c \in C_s^H$ considering all stations and their allowable channels and relinquishment options. The value of Z_1 will be an integer greater than or equal to zero.

- For Step (2): Minimize VHF stations assigned to their pre-auction band. We have:

  Subsets:

  – SP_V is the set of participating stations whose pre-auction band is VHF.

  Model Formulation:

$$\min Z_2 = \sum_{s \in S_{P_V}, c \in C_s^H} x_{s,c}$$

  The objective function minimizes the number Z_2 of VHF participating stations that are assigned to their pre-auction band. Thus, the optimization sums only the set of participating stations whose pre-auction band is VHF $s \in S_{P_V}$ and determines the minimum number of such stations that must be assigned some channel in their pre-auction band $(c \in C_s^H)$ considering all stations and their allowable channels and relinquishment options and the minimum number Z_1 of UHF stations that must be assigned to their pre-auction band. The value of Z_2 will be an integer number greater than or equal to zero.

- For Step (3): Maximize the number of stations assigned to their preferred relinquishment option. We have:

  Subsets:

  – $C_s^p ref$ = the set of allowable channels for a participating station in its preferred option. While, For stations whose preferred option is to go off-air, the set $C_s^p ref$ consists solely of 0.

  Model Formulation:

$$\max Z_3 = \sum_{s \in S_P, c \in C_s^{\text{pref}}} x_{s,c}$$

  The objective function maximizes the total number Z_3 of participating stations that are assigned to their preferred option. The value of Z_3 will be an integer greater than or equal to zero.

- For Step (4): Maximize the number of stations assigned to their option of going off the air. We have:

  Model Formulation:

$$\max Z_4 = \sum_{s \in S_P} x_{s,0}$$

  The objective function maximizes the number Z_4 of participating stations that are assigned to go off the air. The value of Z_4 will be an integer number greater than or equal to zero.

Following these four steps we get the 4 Z constraints of the model.

5.2.1.3 Solving the problem

We are not going to analyze further each of the steps and their implementation, as we just want to give a simple overview of the in initial assignment problem.

Finding an initial assignment of participating and non-participating stations is to solve the primary clearing target optimization, which is described by a linear constraint problem. The optimization model that we are going to present, as was made public by the FCC, determines a feasible assignment of stations such that the sum of impaired weighted-pops across all licenses is minimized. If the near-nationwide threshold for impairments is not satisfied by the result of the optimization, it runs again with a lower clearing target.

$$\min \sum_{l \in L} w_l \rho_l$$

$$
\begin{aligned}
\text{s.t.} \quad & \sum_{c \in C_s} x_{s,c} = 1 && \forall s \in S && (5.1)\\
& x_{s,c} + x_{s',c} \leq 1 && \forall \{(s,c),(s',c)\} \in \text{ CoPairs} && (5.2)\\
& x_{s,c} + x_{s',c'} \leq 1 && \forall \{(s,c),(s',c')\} \in \text{ AdjPairs} && (5.3)\\
& x_{s,c} \in \{0,1\} && \forall s \in S, \forall c \in C_s && (5.4)\\
& \sum_{s \in S_{P_U}, c \in C_s^H} x_{s,c} \leq Z_1 && && (5.5)\\
& \sum_{s \in S_{P_V}, c \in C_s^H} x_{s,c} \leq Z_2 && && (5.6)\\
& \sum_{s \in S_P, c \in C_s^{\text{pref}}} x_{s,c} \geq Z_3 && && (5.7)\\
& \sum_{s \in S_P} x_{s,0} \geq Z_4 && && (5.8)\\
& \sum_{a \in A_l} pct_{a,l}^D y_{a,l}^D + \sum_{a \in A_l} pct_{a,l}^U y_{a,l}^U = 1 && \forall l \in L && (5.9)\\
& x_{s,c} \leq y_{a,l}^D && \forall (s,c) \in SC_{a,l}^D, a \in A_l, l \in L && (5.10)\\
& x_{s,c} \leq y_{a,l}^U && \forall (s,c) \in SC_{a,l}^D, a \in A_l, l \in L && (5.11)\\
& x_{s,c} \leq y_{a,l}^U && \forall (s,c) \in SC_{a,l}^U, a \in A_l, l \in L && (5.12)\\
& 0 \leq y_{a,l}^D \leq 1 && \forall a \in A_l, l \in L && (5.13)\\
& 0 \leq y_{a,l}^U \leq 1 && \forall a \in A_l, l \in L && (5.14)\\
& 0 \leq \rho_t \leq 1 && l \in L && (5.15)\\
& \rho_t \leq z + (1-z) N_l && l \in L && (5.16)\\
& \rho_t \geq N_l && l \in L && (5.17)\\
& N_l \in \{0,1\} && l \in L && (5.18)
\end{aligned}
$$

Explanation of the Constraints:

- **1-4 : Feasibility Constraints**
  - (1): Each station must be assigned.
  - (2): Station assignments must adhere to the co-channel interference restrictions.
  - (3): Station assignments must adhere to the adjacent channel restrictions.
  - (4): The decision variables can only take on the values zero or one.
- **5–8: Z Constraints**
  Constraints (5) - (8) are the constraints considering the Initial Commitment section.

- (5) : Constraint (5) states that the number of the UHF participating stations assigned to their pre-auction band must be less than or equal to the count obtained in the first optimization, namely the value Z_1.
- (6) : Constraint (6) states that the sum of the VHF participating stations assigned to their pre-auction band must be less than or equal to the result of the second optimization, namely the value Z_2.
- (7) : Constraint (7) requires that the number of participating stations assigned to their preferred option must be greater than or equal to the result of the third optimization, namely the value Z_3.
- (8) : The number of participating stations assigned off-air must be at least Z_4.

- **9-18: ISIX Constraints**
  Constraints (9) through (18) are ISIX constraints and are explained further in([1] APPENDIX B).

With Decision Variables:

- $x_{s,c}$ is a binary decision variable which has a value of 1 if station is assigned to channel and 0 otherwise. Were $c = 0$indicates the option to go off-air.
- $y^D_{a,l}$ and $y^U_{a,l}$ are decision variables which have a value of 1 if county-tile has impairment in the downlink or uplink portion of license respectively and 0 otherwise.
- ρ_l is the percentage of population in license l with predicted impairment.
- N_l is a binary variable which has a value of 1 if the license is more than $z * 100\%$ impaired.

Set Definitions

- S is the set of all stations (both participating and non-participating).
- C_s is the set of allowable channels for station s
- S_p is the set of participating stations
- S_{P_U} is the set of participating stations whose pre-auction band is UHF
- S_{P_V} is the set of participating stations whose pre-auction band is VHF
- C^H_s is the set of allowable pre-auction band channels station
- A_l is the set of county-tiles a covered by license which can be impaired partially or fully by at least one (facility, channel) pair
- L is the set of potentially impaired licenses; each license is defined by a clearing target, market id, block, and link types (uplink and downlink)

- $SC^D_{a,l}$ and $SC^U_{a,l}$ are the sets of impairing (facility, channel) pairs which impair county-tile a in license l for downlink and uplink respectively

And Constants:

- z is a threshold value between 0 and 1 that determines when a license l is considered too impaired to be auctioned
- w_l is the weighted-pops associated with license l
- $pct^D_{a,l}$ and $pct^U_{a,l}$ are the percent of license l's population in county-tile divided in half between the downlink and uplink respectively.

The FCC defined the initial clearing target by solving this model as well as the initial assignment. This process was called **Clearing Target Optimization**. The Clearing Target Optimization is also run between stages of the incentive auction, not for discovering a clearing target (it is already preset); but to take into account the UHF stations that have dropped the auction and the additional UHF channels available which come with the new target.

5.2.2 The SATFC

SATFC was the software developed exclusively for solving the repacking problem for the reverse auction. It was developed by Neil Newman, Alexandre Fréchette, Paul Cernek, Emily Chenn, Guillaume Saulnier-Comte, Nick Arnosti and Kevin Leyton-Brown and achieved excellent performances. SATFC is an open source solver. During the auction the feasibility checker needed to provide an answer within a minute for each instance, in order for the auction process to run smoothly. Out of many simulations run this software could solve the 96.03% within the one-minute cutoff time. An outstanding performance, given a NP-Complete problem.

5.2.2.1 SAT Encoding

The algorithm of the SATFC, encodes the station repacking problem as a propositional satisfiability (SAT) problem. It is a well suited encoding for this problem, as it is a pure feasibility problem with combinatorial constraints.

The SAT reduction:

Given a station repacking problem (S,C) (stations S, channels C), with domains D and Interference constraints I, we create a Boolean variable $x_{s,c} \in \{\top, \bot\}$ for every station–channel pair $(s, c) \in S \times C$, representing the proposition that station s is assigned to channel c. We then create three kinds of clauses:

1. $\vee_{d \in D(S)} x_{s,d} \forall s \in S$ (each station is assigned at least one channel)

2. $\neg x_{s,c} \vee \neg x_{s,c'} \forall s \in \ S, \forall c, c' \neq c \in \ D(s)$ (each station is assigned at most one channel)

3. $negx_{s,c} \vee \neg x_{s',c'} \forall \{(s,c),(s',c')\} \in \ I$ I (interference constraints are respected)

Number (2) is optional; as if a stations is assigned to multiple channels, a single one can be picked to assign it arbitrarily. To highlight the size of the instances, a sat encoding of a problem involving all stations at a clearing target of 36 involved 73187 variables and 2 917866 clauses.

SATFC utilizes two state of the art sat solvers that were best suited for the station repacking problem. First the clasp, an open-source solver based on conflict-driven nogood learning. And second the open-source SATenstein framework which allows arbitrary composition of design elements taken from a wide range of high-performance stochastic local search solvers.

5.2.2.2 Utilizing Previous Assignment.

As we have stated before, it is important to tell, when we have a partial assignment γ^- (previous solution), if we can add to it sation(s) s^+. In addition, such partial assignments always come up naturally during the reverse auction, as we have to continuously run instances of the problem in the same set of stations.

SATFC takes advantage of this property of the auction using two methods. The first one, checks if a simple transformation of γ^- is enough to satisfy the problem. Analytically, a small SAT problem is constructed in which the stations for repacking are : s^+ and all stations $\Gamma(s^+) \subseteq S$ that are neighboring s^+ in the interference graph, while all of the other stations $S \backslash \Gamma(s^+)$ remain in their assignment in γ^-. The second one, initializes local search solvers using γ^-. These solvers search a space of complete variable assignments. The seek to minimize an objective function, such as the number of violated constraints, by following gradients, with random steps.

At this point we are not going to go any further in analyzing the algorithm of SATFC and the optimization methods that were conducted.

5.2.2.3 SATFC usage

At this point, we need to introduce some basic usage of SATFC.

Firstly, we need the interference constraint data, created and published by the FCC, which consists of two files.

- **Domains (domains.csv)**. This csv file contains every station along with its eligible channels. For each row, the first entry is the DOMAIN keyword, the next is the station's ID and all the other entries are its eligible channels.
- **Interferences (Interference_Paired.csv)**. Each row in this csv file contains may pairwise interference constraints between a single subject station on a single subject channels; with

possible many target stations. The fist entry of each row is a **key** indicating the constraint type. Then the next two are the subject and the target channel. Immediately after is the subject station and finally a list with the target stations. The constraints key can either take one of the five values :

– 'CO'. Where the subject station cannot be assigned to the target/subject (it is the same here) channel if any of the target stations are assigned to it.
– 'ADJ+1'. This key indicates that the subject station cannot be assigned to the subject channel c when any of the target stations is assigned to the target channel $c + 1$.
– 'ADJ+2'. This key indicates that the subject station cannot be assigned to the subject channel c when any of the target stations is assigned to the target channel $c + 2$.
– 'ADJ-1'. This key indicates that the subject station cannot be assigned to the subject channel c when any of the target stations is assigned to the target channel $c - 1$.
– 'ADJ-2'. This key indicates that the subject station cannot be assigned to the subject channel c when any of the target stations is assigned to the target channel $c - 2$.

For instance, the following constraint "CO,5,5,1,2,3", means that station 1 cannot be assigned to channel 5 along with station 2 or station 3. The line "ADJ+1,3,4,10,12,15", means that station 10 cannot be assigned to channel 3 while stations 10 or 12 or 15 are assigned to channel 4.

Input for SATFC is given along with the path to the domains and constraints folders. And the input is as string list of a sat of stations to be packed, and for each station a set of eligible channels. This domains string consists of single station domains strings joined by ';', where each single domain string consists of a station with numerical ID, a ':', and a list of integer channels joined by ','.

For instance a problem o packing stations 1 and 2 into channels 1,2,3 and station 3 into channels 1,2,3,4,5,6 is given by the following string:

"1:1,2,3;2:1,2,3;3:1,2,3,4,5,6;"

SATFC solves the problem and returns as an output, a run result, runtime and a witness assignment. The run result can be either *SAT* (stands for satisfiable), *UNSAT* (stands for unsatisfiable), or TIMEOUT (when it runs out of given time. The default CUT-OFF time is 1 minute, as it was during the auction). The runtime is the wall time, in seconds, that SATFC has taken to solve the given problem. And the witness assignment is present only when the given problem is SAT and consists of a new valid channel assignment for the given stations. We took this information from the SATFC official manual[40], were everything regarding the SATFC usage can be found.

Chapter 6

The feasibility checker based on the or-tools of google

In this section, we are going to present a feasibility checker, that was created for the station repacking problem, using the or-tools by google. Furthermore, we are going to show some test results, from the comparison between SATFC and our feasibility checker.

6.1 The OR-TOOLS

OR-Tools is an open source software created by Google. It is well suited for combinatorial optimization. This software utilizes some of the best solvers, such as Gurobi, CPLEX, CP-SAT and more in its portfolio. Moreover, OR-Tools software includes solvers for :

- Linear and Mixed-Integer Programming
- Vehicle Routing
- Graph Algorithms
- **Constraint Programming**

Thus, we saw the opportunity to create a feasibility checker using this open-source software. We tried to model the station repacking problem into the Or-Tools solvers and compare its performance to the SATFC(Which was used by the FCC for the real time repacking problems).

Or-Tools support C++, Python, C#, or Java languages. For this task we chose to use Python, as it was more suited for the data handling that needed to be done for the real auction data.

6.2 Introducing the solver

Firstly, we will introduce the or-tools setting for python language and its features.

As we previously mentioned, this google software contains many solvers for different kinds of problems. Those problems are :

- Linear optimization
- **Constraint optimization**
- Bin packing
- Network flows
- Assignment
- Scheduling
- Routing

For this paper we approached the station repacking problem exclusively with the constraint optimization solvers of or-tools. We understand that it would be also reasonable to approach it with linear programming, however we chose to work only with the CP (constraint programming). Major difference between those two approaches is that CP focuses more on the constraints and variables of the problem rather than the objective functions. Nevertheless, we know that one can solve the other (like the initial assignment problem was solved with linear programming along with the clearing target optimization).

In the constraint programming section, we chose to work with the CP-SAT solver, that was available. This particular solver reduces the problem to SAT, thus we understand that the base of the solutions approach is similar to the one of the FCC.

We present a simple example of the solver along with some code, as given by the official google Or-tools site, in pursuance of getting a clearer view of its function.

In the example below we need to find a feasible solution when we have :

- Variables : $x, y, z \in \{0, 1, 2\}$
- Constraints : $x \neq y$

Coding of this problem comes down to some simple steps:

1. **Declare the model**
   The following code declares the CP-SAT model.

```
model = cp_model.CpModel()
```

2. **Create the variables**
   The following code creates the variables for the problem.

```
num_vals = 3
x = model.NewIntVar(0, num_vals - 1, ’x’)
y = model.NewIntVar(0, num_vals - 1, ’y’)
z = model.NewIntVar(0, num_vals - 1, ’z’)
```

3. **Create the constraint(s)**
   The following code creates the constraint x≠y.

```
model.Add(x != y)
```

4. **Call the solver**
   The following code calls the solver.

```
solver = cp\_model.CpSolver()
status = solver.Solve(model)
```

The CP-SAT solver can then have one of the following statuses:

- **OPTIMAL** An optimal feasible solution was found.
- **FEASIBLE** A feasible solution was found, but we don’t know if it’s optimal.
- **INFEASIBLE** The problem was proven infeasible.
- **MODEL_INVALID** The given CpModelProto didn’t pass the validation step.
- **UNKNOWN** The status of the model is unknown because no solution was found (or the problem was not proven INFEASIBLE) before something caused the solver to stop, such as a time limit, a memory limit, or a custom limit set by the user.

The libraries of the or-tools also contain a solution printer. Furthermore, we have the options to either (i) print all solutions found, (ii) print one solution and stop searching, (iii) set a time for our program to stop if it has not found a solution.

Our goal is to model the station repacking problem into the CP-SAT solver; and then utilize these features and the solver itself to get as similar as possible outputs to the SATFC software. Then we can run the same instances on both softwares and compare the outputs along with the performances.

6.2.1 Modeling the problem

As we presented above, in the or-tools trivial example, we need to break down our problem into these steps :

- Declare the model
- Create the variables
- Create the constraints
- Call the solver

Firstly, we are going to explain the idea behind our algorithm and then we will fully present our code.

We tried to make a similar input-output format with SATFC, and also use the same domain and interference constraint files.

Our variables are created with a two dimensional array that can take values 0 or 1. Where, each row stands for a station that is given as input, and each column for a channel (0 - 51). The idea is to have ”1” for each row (station), at one specific column (channel) for the station to be assigned to. We begin by setting all channels to 0; no station is assigned to a channel yet.

Then we give the input through a csv file ”Input.csv”, were the first entry of each line is a station's ID which we want to repack, and all the other entries are the channels we want to repack it to. For a given station s given at the input, we check the domain file and we enable the channels that are given (in the input) and also are in the station's domain to be either 0 or 1 (Vars[s][c] = (0, 1) for station s and channel c).

Then to create the constraints, we read the ”Intereference-Paired.csv” file, that was described in the end of the previous chapter, and for every row and then for each entry after the 4th which is the subject station we define that :

$$[subject\_station][subject\_channel] + [target\_station][target\_channel] \leq 1$$

The above line ensures that all of the constraints are respected. That a subject station cannot be assigned to the subject channel, while one or more of the target stations is/are assigned to the target channel.

Another constraint we need to take into account is to make sure that for each station in our problem (meaning for each row at our array), the sum of all entries are strictly 1. This ensures that every station, is assigned to a channel.

If we correctly create these variables with the respective constraints, the all we have to do is call the or-tools CP solver and me the right adjustments for the printed output. We chose to make a similar output with the SATFC: result, runtime, and one witness assignment (in our program, we chose to print the first solution found). If the given problem is satisfiable we return a witness assignment (the first solution found), and a runtime. If the probelm is unsat we return INFEASIBLE along with the runtime it took for our program to find it. We chose not to include a cut-off time, although we were able to do it, because our program is meant only to make some tests and thus we need to see all the run times. Therefore, there is no point in including a timeout.

6.3 Results

The original idea for this thesis, was to create a feasibility checker, which we did, and then run a simulation of the real incentive auction, in order to see the revenue difference that we were going to end up with. Unfortunately, as mentioned before in this thesis, it is not possible to run an exact simulation of the real auction not even for stage 1, because the initial assignments never made public. Therefore, we cannot know the ”repacking” done to the stations that did not make it to the auction, before the it took place. One could try to run the LP that was defined in this thesis, aiming to find that initial assignment, but probably it would still be different than the one that actually took place. Furthermore, even if we did get this assignment an exact simulation, requires a powerful computer, as the SATFC needs 8 cores per run and needs 32 GB of RAM to build a server.

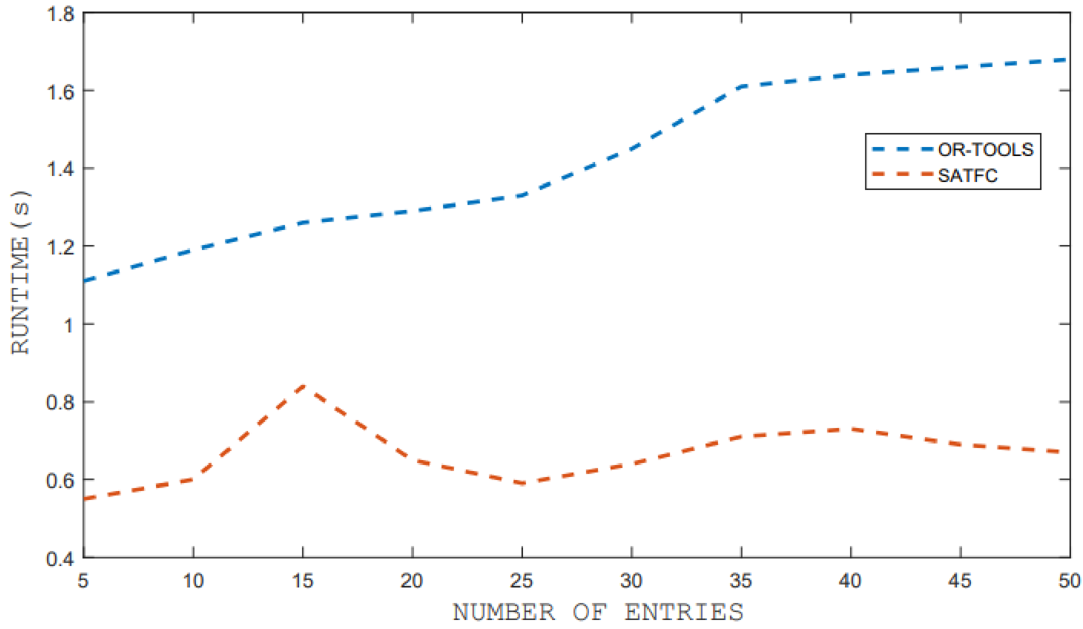
However, we still needed to somehow compare the two programs in terms off efficiency. We chose to run separate independent instances (same for both programs), and observe the results, and of course the runtime which is the most important aspect regarding efficiency.

We decided to run instances and as we proceed increase the size of the input. Meaning to repack more and more stations. Channels available are 2-29 as in the first stage of the auction, at first. We understand that the size of the input is not the only aspect which makes the problem difficult but it sure is one.

The first thing we checked, and was also the first in our list of goals is the output of the results. In every single instance both programs print out the same results (SAT, UNSAT), so we know that our program with the or-tools works correctly. At this point we should point out that we do not care if the result is SAT or UNSAT, but to determine the result fast. Then we analyzed the runtime output and put it into a graph. We present the graph bellow, which illustrates the runtime output as we increase the number of entries as input for both programs.

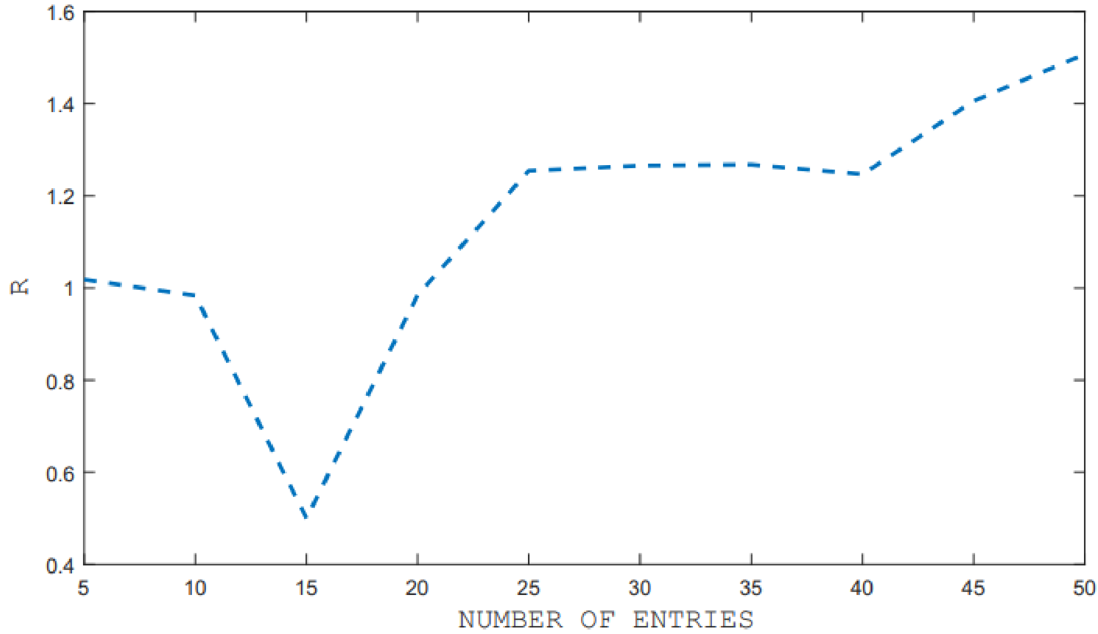
The runtime results were more or less what we expected to see. SATFC is by far better, compared to our feasibility checker in terms of speed. We see SATFC never exceeding 10 seconds to provide an answer even when the size of the input come close to one thousand station, while our program when the input size is very big takes even more than two minutes. SATFC is a software optimised specifically for this for the instances of the station repacking problem and achieves great efficiency in it. Or-tools feasibility checker would not be viable for this auction at this current state, and there is no point in running a simulation with this checker. This is given the fact that, the cut-off time of finding an answer for each instance was set at 60 seconds. We see or-tools by far exceeding this threshold in many instances; and on top of that, real auction instances can be far more challenging. As we continue we will present a few graphs that give us a good view, regarding the comparison between the solves.

We first show in the graph below the two solvers tested with small size inputs.



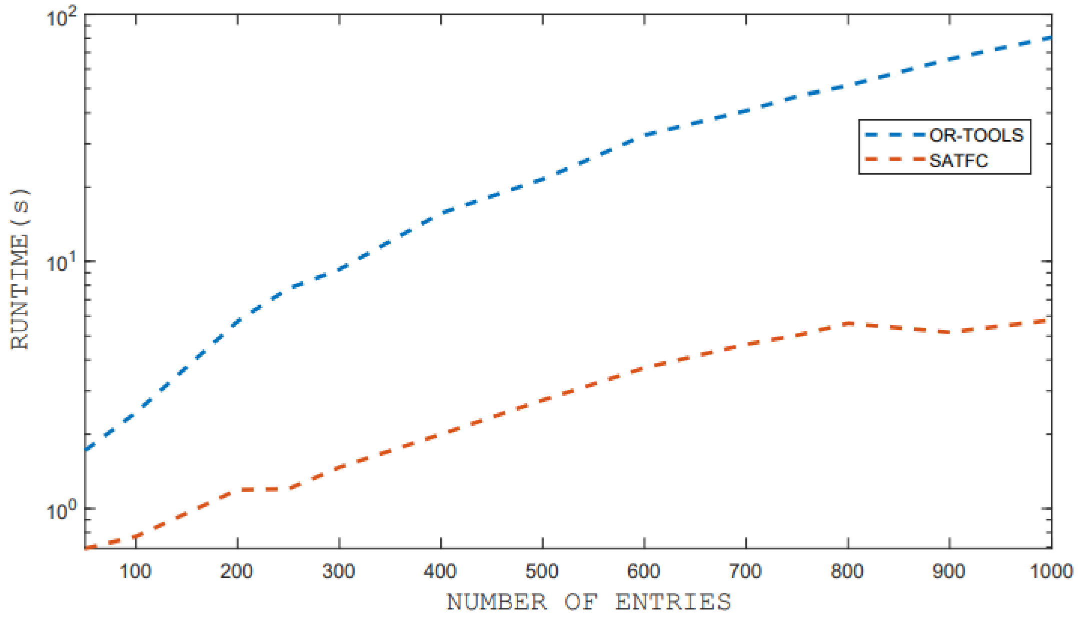
We show in the graph below the difference of the solvers, for small inputs, as

$$R = \frac{\text{SATFC RUNTIME} - \text{OR-TOOLS RUNTIME}}{\text{SATFC RUNTIME}}$$

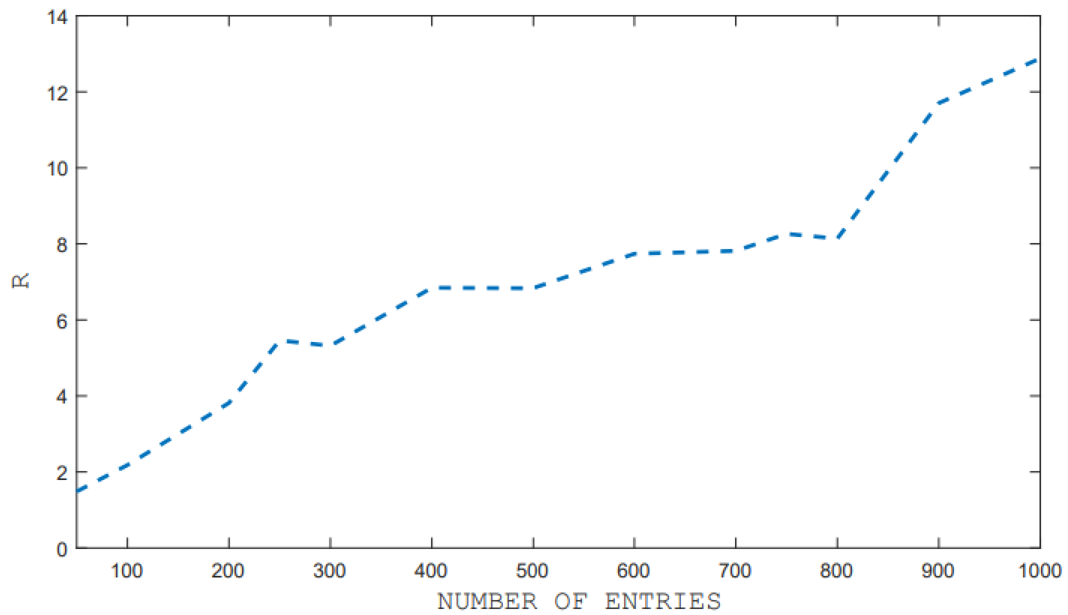


We see that for "small" size inputs the two solvers have hardly of a second difference, but we see that as the inputs grow the difference also grows. So we expect that this difference is going to get bigger as we test larger inputs.

Below we present the graph were we tested with “normal” size inputs. Input size that could occur in the auction.



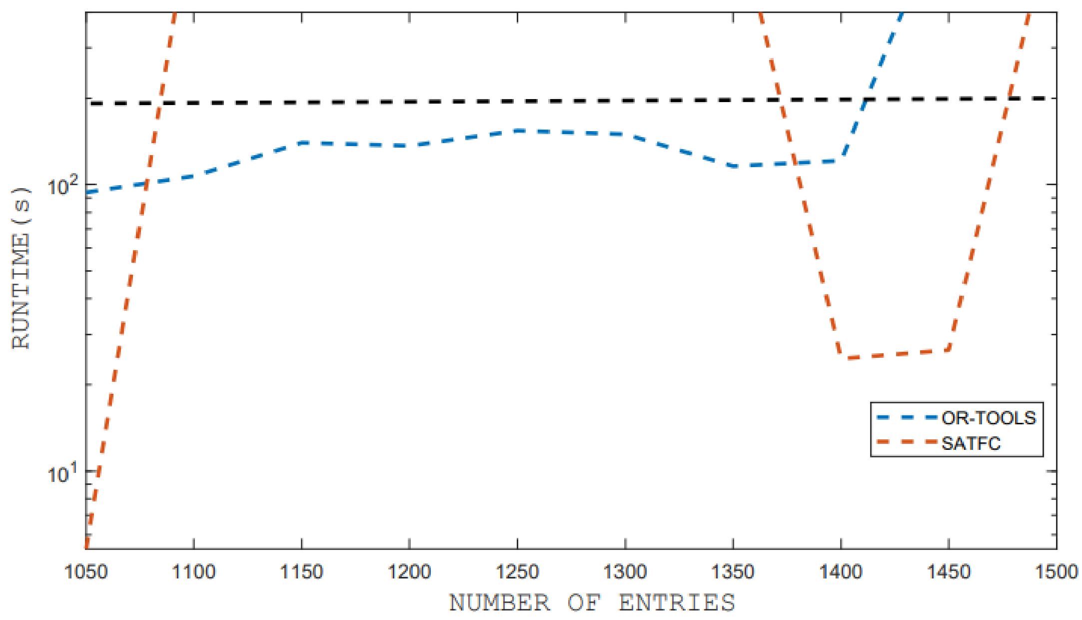
Again we show in the graph below the difference of the solvers, for normal inputs now, again as $R = \frac{\text{SATFC RUNTIME} - \text{OR-TOOLS RUNTIME}}{\text{SATFC RUNTIME}}$



This is probably the most important graphs as they concern instances that are actually tested during the auction process. They are the type of instances for which the SATFC is optimised. Therefore we see the huge gap between the two solvers, we see that SATFC does not take over 10 seconds even for the too large instance of 1000 entries, wile or-tools take as the inputs grow more than the cutoff time which is 60 seconds.

Or-tools CP solver is aimed to solve any constraint problem and also it is optimized in terms of worst-case instance and not any specific ones, unlike SATFC. We prove this point practically, in another comparison between the two programs. As shown in the previous graphs we compared the two with instances that had an input size ranging to 1000. In spite the fact that, SATFC ”destroyed” or-tools in this comparison, we notice that as the input size goes above 1000 the tables turn. Of course the or-tools take over two minutes to provide an answer, but we have many cases were SATFC can not provide an answer at all (we set the cutoff time to 180 seconds, but we know in theory that if it does not provide an answer within 60 seconds, probably it will never do). Bellow we illustrate with graphs how the two solvers “behace” in larger inputs.

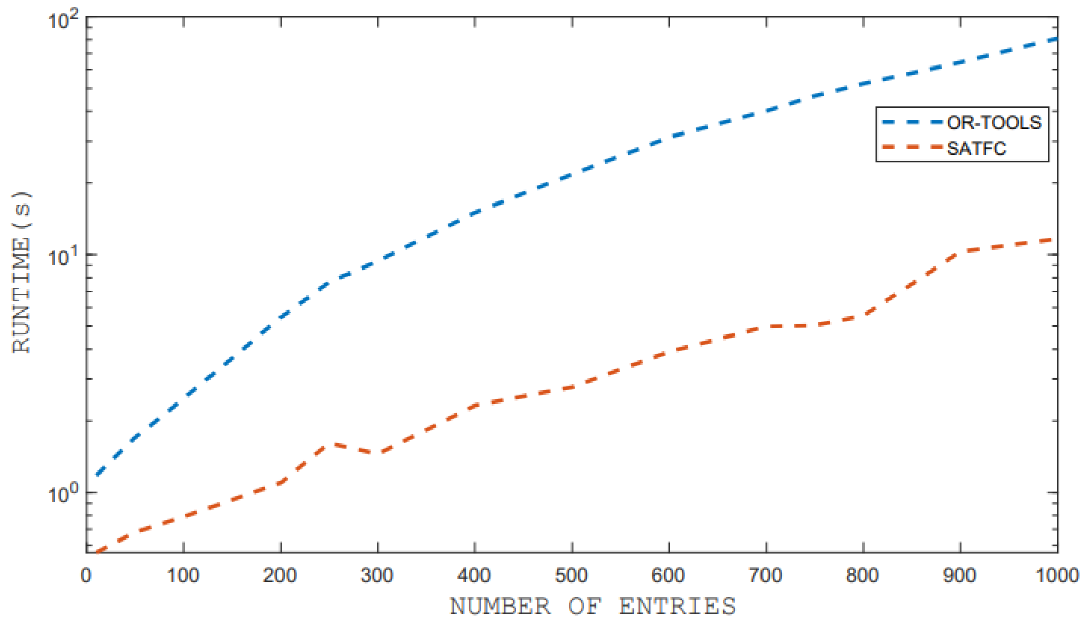
Below we present the graph were we tested with “high” size inputs. Input size that could **not** occur in the auction. The black doted line represents the timeout of 180 seconds.



We see that for the same exact instances SATFC times out in one hour and could not provide an answer while the or-tools takes bellow 180 seconds to print the answer. In fact, the SATFC probably would not provide an answer at all. We know in theory that if SATFC does not give an answer within the cutoff time of 60 seconds, probably it will not provide an answer at all. Why is that? As we described in chapter 4, SATFC was optimised not for the worst case instances but just for the instances of the auction, the ones that could occur. If we take a close look at the auction we see that the number of participating stations is 1030. This in theory makes the largest size of input that can be given to SATFC for the use it is created for 1030. Which in reality would never be that big. Therefore, we see that when we test the programs with inputs of size larger than 1000 (inputs that are unrealistic for the incentive auction's process) , or-tools are more efficient.

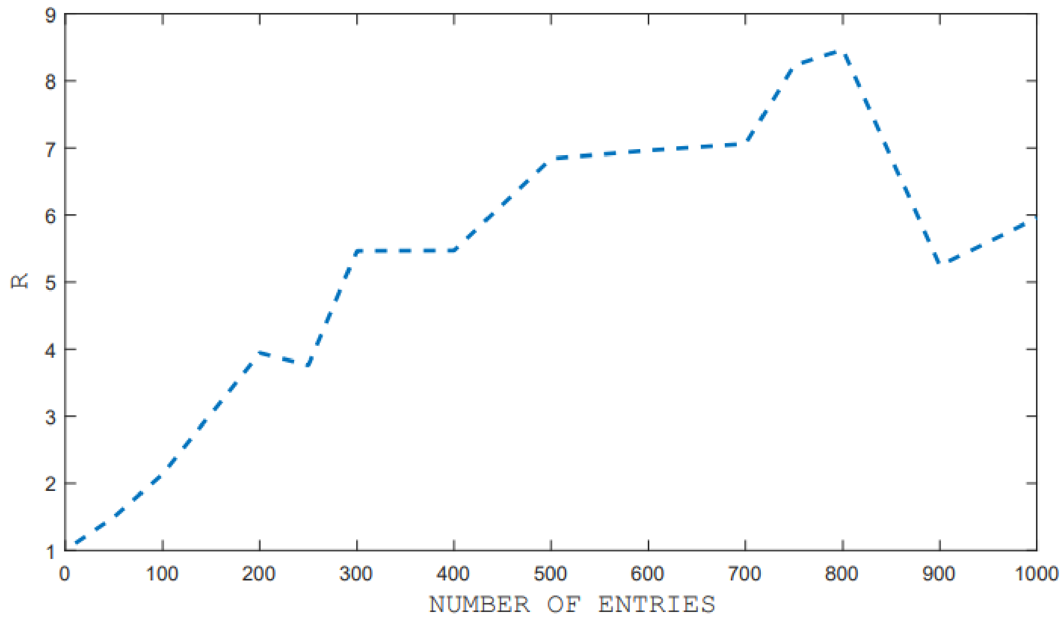
In order to get a better view we tested the same instances but now for available channels 2-31; channel space of the second stage of the auction.

Below we present the graph were we tested with "normal" size inputs, with available channels now 2-31.



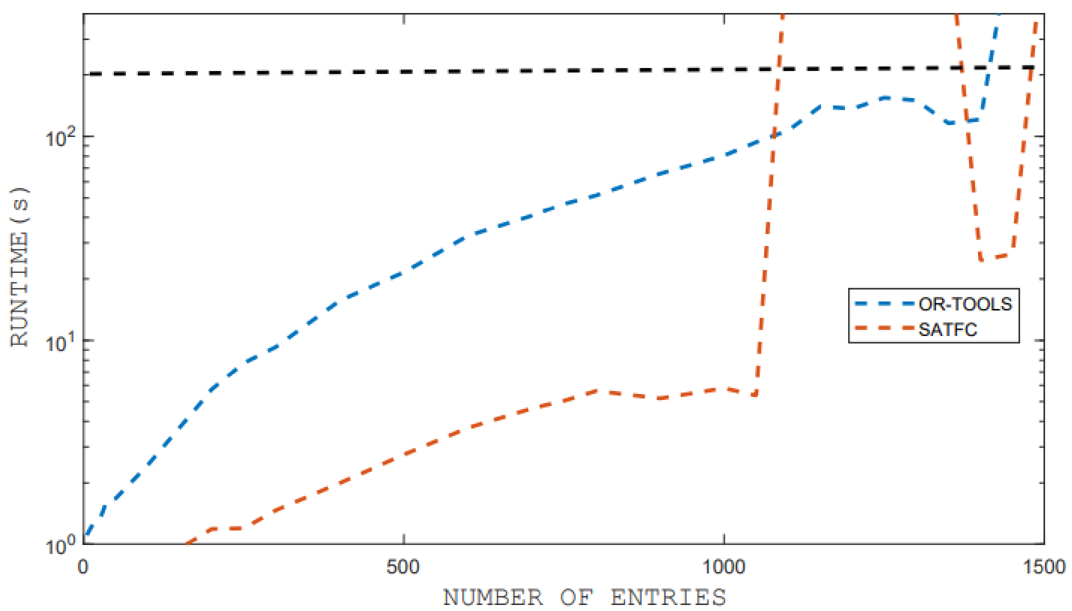
And the difference of the solvers, for normal inputs with channels available 2-32 (stage 2) again as

$$R = \frac{\text{SATFC RUNTIME} - \text{OR-TOOLS RUNTIME}}{\text{SATFC RUNTIME}}$$



We see that the outcome of the comparison of the two solvers for channel space of the second stage is similar to the first. The SATFC outperforms the OR-TOOLS for auction instances, refardless of the channel space.

Finally, we will present a graph of all “small”, “normall” and “big” size input instances given to both solvers to get a global view.



Chapter 7

Future Work

Undoubtedly, there can be continuation of the work done for this thesis, that can get more interesting research and results. There is space for optimizing further the feasibility checker of or-tools. Optimize it to perform better on the station repacking problem. For instance, being able to utilize a previous assignment is a critical property for the auction problem. Then, if after further optimizations, the or-tools based program achieves better efficiency (closer to the one minute cutoff) it would be interesting to run whole auction simulations this time, of the same incentive auction format but altering the repacking software. The revenue results after conducting this simulation can be very useful and is sure the best way to rate the repacking software, and also the auction's format.

But how can we simulate since we do not have the initial assignment. It is true that we can not simulate the incentive auction that took place in 2015. Although, there is a way to create stations and constraints and simulate a ”different” auction of the same format to get the results. FCC have produced the open-softwares TVStudy[6] and Constraint Generator[2]. The first one is used to create all the stations across the map with their interference; and the second to take these stations created and generate the pairwise interference constraints that are used by SATFC or any other similar feasibility checker.

7.1 Spectrum Auctions in Greece

Surprisingly, the first and only auction that was ever conducted in Greece for TV spectrum rights took place in 2016. That alone is little confusing, if we consider that private TV stations are transmitting in Greece since 1989. This fact, that it was just the first auction to take place; the lack of experience did result in several inconveniences. Nine stations participated for the total of four licences offered. Each licence sold separately. A very small size of participants and ”goods” offered, yet the procedure took over 70 hours; behind closed doors. Each licence was auctioned separately through two different stages. The first stage had an *ascending clock auction* format were the licences had an opening price of 3 million euros and they were ascending up to 500 thousand euros each round. When only two participants remained active stage two was triggered. Stage two appears to be a first-price sealed bid auction between the two remaining participants[8]. Due to the fact that the auction was conducted behind closed doors we can not, make any further comments. We can just highlight the fact that, offering 4 licences to just 9 stations took 3 consecutive days behind close doors.

As we close we should say that attention now for Greece's spectrum auctions is directed to 5G. The auction is set to take place in December of 2020. In this multiple round auction, spectrum licences in the 700MHz, 2GHz, 3.4GHz-3.8GHz and 26GHz bands will be offered[4]. We are still awaiting to see the format and corresponding rules under which this multi-round auction is going to be conducted.

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