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Modeling intertemporal trading and management of emission permits in greenhouse gas emission trading systems

ΔΙΠΛΩΜΑΤΙΚΗ ΕΡΓΑΣΙΑ

του

ΚΟΡΟΜΗΛΑ ΠΑΝΑΓΙΩΤΗ

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Διπλωματική Εργασία

του

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Αθήνα, Οκτώβριος 2020

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Ευχαριστίες

Η παρούσα εργασία πέρα από αρκετή δουλειά, χρειάστηκε και σωστή καθοδήγηση απο ανθρώπους με πολύχρονη εμπειρία στον χώρο της έρευνας. Στο κομμάτι αυτό στάθηκα τυχερός αφού την καθοδήγησή μου ανέλαβαν δύο εξαιρετικοί άνθρωποι και επιστήμονες. Ευχαριστώ λοιπόν τον επιβλέποντα Φωτάκη Δημήτριο αφενός για τον χρόνο που αφιέρωσε μελετώντας και κατευθύνοντας την δουλειά μου και αφετέρου για τις γνώσεις που μου μετέδωσε στα μαθήματα του προγράμματος σπουδών. Η πολύτιμη βοήθεια της Άντζελας Μαθιουδάκη είναι ένας απ΄ τους λόγους της άρτιας περάτωσης της διπλωματικής εργασίας. Η Άντελα αφιέρωσε πολύ χρόνο συζητώντας μαζί μου για προβλήματα που προέκυπταν, ενώ είναι και ο άνθρωπος που μου μετέδωσε τις αρχικές γνώσεις που χρειάστηκα για να ξεκινήσω την μελέτη μου. Για τους λόγους αυτούς την ευχαριστώ πραγματικά. Τέλος, θα ήθελα να ευχαριστήσω και τον Σωτήρη Δήμο για τις μέρες που χρειάστηκα επεξήγηση της βάσης δεδομένων που έχει φτιάξει.

Η παρούσα εργασία σηματοδοτεί το τέλος των προπτυχιακών σπουδών μου στο ΕΜΠ και για τον λόγο αυτό θα ήθελα να ευχαριστήσω την οικογένειά μου που, σε όλο αυτό το διάστημα, μου έδωσε την ελευθερία να ακολουθήσω τις δικές μου επιλογές. Φυσικά, ευχαριστώ και τους φίλους μου που πέρα από τις ωραίες εμπειρίες μου παρείχαν στήριξη.

Περίληψη

Τα συστήματα εμπορίας ρύπων αποτελούν, για τα κράτη, μία έξυπνη λύση για την μείωση της παραγωγής αερίων του θερμοκηπίου, αφού μπορούν να εγγυηθούν συγκεκριμένα επίπεδα ρύπανσης. Στην παρούσα εργασία ορίζουμε ένα μαθηματικό μοντέλο που περιγράφει σύνθετες συσχετίσεις μεταξύ των εμπλεκόμενων εταιρειών. Η μοντελοποίηση αυτή περιλαμβάνει όλους τους κανόνες του Ευρωπαϊκού Συστήματος Εμπορίας Ρύπων και θεωρεί μια ρεαλιστική μορφή ανταγωνισμού (εξάσκηση δύναμης απ΄ τις ηγετικές εταιρείες) μεταξύ των εταιρειών στην αγορά ρύπων, ενώ παράλληλα αφήνει τις εταιρείες αυτές να αλληλεπιδρούν και στην αγορά προϊόντων.

Μέσα απ΄ την διαδικασία επίλυσης αποδεικνύουμε ένα σύστημα εξισώσεων ισορροπίας και μία συνάρτηση τιμής. Αυτές οι εξισώσεις δείχνουν ότι οι μεγάλες εταιρείες (ηγέτες), παίζουν καθοριστικό ρόλο για το σύστημα. Συγκεκριμένα, χειριζόμενες την τιμή, εξαναγκάζουν τις μικρές (ακόλουθους) να αποφύγουν την μείωση ρύπων και να αγοράσουν όσες άδειες χρειάζονται, ενώ το περιβαλλοντικό αποτέλεσμα του συστήματος εξαρτάται πλήρως από την μείωση ρύπων των ηγετών. Μπορούν επιπλέον να μεταφέρουν την δύναμή τους μεταξύ των δύο αγορών (αγορά ρύπων και αγορά προϊόντων) ώστε να αποκτήσουν περισσότερο κέρδος. Όσο για την τιμή, δείχνουμε πως ακολουθεί την μορφή της συνάρτησης οριακού κόστους μείωσης ρύπων των μικρών εταιρειών, εξαρτάται απ΄ τους business as usual ρύπους τους, είναι θετικά συσχετισμένη με την συνολική ποσότητα αποθηκευμένων αδειών και αρχνητικά συσχετισμένη με την συνολική διανομή αδειών απ΄ τον ρυθμιστικό φορέα.

Στην συνέχεια κάνουμε κάποιες προσομοιώσεις πάνω στο μαθηματικό μοντέλο, οι οποίες μας δίνουν αρκετή πληροφορία. Συγκεκριμένα, δείχνουμε ότι μία αύξηση στην αποθήκευση αδειών οδηγεί το σύστημα σε καλύτερη απόδοση στο παρόν, κανονικοποιεί την τιμή στο μέλλον, αλλά κάνει το σύστημα πιο ασταθές σε μελλοντικές εξωγενείς αναταραχές. Επιπρόσθετα, υπό μία μείωση της διανομής αδειών από τον ρυθμιστικό φορέα, το σύστημα είναι πιο σταθερό όταν οι εταιρείες επιδίδονται σε μεγαλύτερες μειώσεις ρύπων. Βρίσκουμε επίσης πως για σταθερά επίπεδα μείωσης ρύπων, οι συνολικοί ρύποι μεταβάλλονται ανάλογα με την μεταβολή του παραγώμενου προϊόντος, ενώ για μιρκότερα επίπεδα μείωσης η τιμή είναι πιο επιρρεπής σε μεταβολές του παραγώμενου προϊόντος. Τελικά, ένα ενδιαφέρον αποτέλσμα είναι πως η ύπαρξη της δύναμης των μεγάλων εταιρειών αυξάνει τους συνολικούς ρύπους.

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Σύστημα Εμπορίας Εκπομπών, Μαθηματική μοντελοποίηση, Θεωρία Παιγνίων

Abstract

Emission trading systems are an efficient solution for governments to guarantee the output emissions level of industry sectors. In the present study we form a theoretical model to describe complex relations between participant firms. Our modeling includes all of the EU ETS rules and considers a realistic form of competition (excess of market power from leader companies) between firms on the permit market, while letting them to interact in the output market.

We come up with a system of equilibrium equations and a price function. These equations indicate that the leaders play a central role for the system. By manipulating the price, they force the followers to avoid abatement and buy all the permits they need, while the environmental outcome depends completely from their abatement level. They can also transfer their market power across the two markets (permit and output markets) and gain more profit. As for the price, we show that it is in the form of the marginal abatement cost of the followers, it depends from the followers' business as usual emissions, it is positively correlated with the total amount of banking and negatively correlated with the overall initial period allocation.

We then run some simulations on our mathematical model which provide us with great information. We show that an increase in banking lead to better system functionality in the present, normalizes the price in the future, but makes the system more unstable in future exogenous turmoils. Additionally, under a cap reduction the system is more stable for greater abatement levels, while for a cap rise it works better under small abatement levels. We also find that under specified abatement levels the emissions variate analogously to the output product variation, while for smaller abatement levels, the price is more prone to an output product change. Finally, an interesting result is that the excess of market power from the leaders raise the overall emissions.

Keywords

Emissions trading, Cap-and-Trade Systems, EU ETS, Mathematical Modeling

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Εκτεταμένη Περίληψη

0.1 Εμπορία Ρύπων

Η ρύπανση που προκαλούν τα εκπεμπόμενα αέρια του θερμοκηπίου τις τελευταίες δεκαετίες, αποτελεί παγκόσμιο πρόβλημα. Για τον λόγο αυτό έχουν υπάρξει διακρατικές συμφωνίες με σκοπό τον περιορισμό τους. Η περαιτέρω φορολόγηση των ρύπων έχει φανεί πως δεν βοηθά στην επίτευξη των περιβαλλοντικών στόχων που έχουν τεθεί. Για τον λόγω αυτό προτάθηκε από ερευνητές η λειτουργία ενός συστήματος εμπορίας δικαιωμάτων εκπομπής (Emissions Trading Scheme).

Τα συστήματα αυτά είναι μία λύση που εγγυάται την επίτευξη συγκεκριμένων περιβαλλοντικών στόχων, βοηθώντας τα κράτη να επιτύχουν τις δεσμεύσεις τους. Σε αυτού του είδους τα συστήματα, υπάρχει ένας ρυθμιστικός φορέας (regulator) ο οποίος θέτει ένα όριο στους ρύπους που μπορούν να παραχτούν, το οποίο εάν ξεπεραστεί φέρνει τις εταιρείες αντιμέτωπες με τεράστια πρόστιμα. Στην αρχή κάθε περιόδου, κάθε εταιρεία δέχεται μία ποσότητα αδειών η οποία αντιστοιχεί σε ένα επίπεδο ρύπων. Έπειτα τους δίνεται η δυνατότητα είτε να μειώσουν τους ρύπους, είτε να αγοράσουν - πουλήσουν άδειες, ώστε να προσαρμοστούν στο όριο που έχει τεθεί. Στο τέλος κάθε περιόδου, κάθε εταιρεία πρέπει να παραδώσει στον φορέα έναν αριθμό αδειών που να αντιστοιχεί στους πραγματικούς ρύπους που παρήγαγε. Με αυτόν τον τρόπο εξασφαλίζεται η επίτευξη του περιβαλλοντικού στόχου, αλλά εισάγεται μία αβεβαιότητα στην τιμή που θα πρέπει να πληρωθεί απ΄ τους εμπλεκόμενους.

Το Ευρωπαϊκό Σύστημα Εμπορίας Εκπομπών (EU ETS) αποτελεί την κεντική δράση της Ευρωπαϊκής Ένωσης έναντι στην κλιματική αλλαγή. Εξαιτίας του μεγάλου μεγέθους, αλλά και των πολλών διαφορετικών κρατών που παίρνουν μέρος σε αυτό, θεωρείται το σημαντικότερο σύστημα αυτού του είδους. Μέσα σε αυτό ουσιαστικά δημιουργείται μια καινούρια αγορά, η αγορά των αδειών, μέσω της οποίας οι εταιρείες πωλούν ή αγοράζουν άδειες ώστε να ακολουθήσουν τους κανόνες. Ο στόχος του συστήματος δεν είναι μόνο να επιτύχει περιβαλλοντικούς στόχους, αλλά αλλά και να προωθήσει επενδύσεις σε τεχνολογίες που προσφέρουν χαμηλή χρήση άνθρακα.

Παρά την μεγάλη του σημασία, η αρχική λειτουργία του έδειξε αρκετά μειονεκτήματα. Συγκεκριμένα, στο τέλος του 2007 (κατά την διάρκεια της φάσης I), η τιμή της άδειας κατέρρευσε και προσέγγισε το μηδέν. Η υπερ-διανομή αδειών και οι περιορισμοί στην δυνατότητα αποθύκευσης αδειών αποτελούν τους κύριους λόγους της αποτυχίας. Για τον λόγο αυτό, στην έναρξη της δεύτερης περιόδου (φάση II) επιτράπηκε η αποθύκευση αδειών για μελλοντική χρήση σε επόμενες περιόδους. Παρόλο που η δημιουργία αυτής της διασύνδεσης μεταξύ των περιόδων βοήθησε την τιμή να παραμείνει σε υψηλότερα επίπεδα, το άθροισμα αυτών των αδειών δημιούργησε ένα τεράστιο πλεόνασμα αδειών. Από το 2008 ο αριθμός των αδειών που δίνονταν στις εταιρείες ήταν μεγαλύτερος απ΄ αυτόν που πραγματικά χρειάζονταν, και με την οικονομική κρίση μεταξύ 2008 και 2012 η οποία επηρέασε τα επίπεδα παραγωγής, οδηγηθήκαμε στην εμφάνιση αυτόυ του μεγάλου πλεονάσματος.

Για να περιοριστούν οι συναίπειες του πλεονάσματος, η Ευρωπαϊκή Ένωση εισήγαγε την αναβολή (back-loading) της δημοπρασίας 900 εκατομμυρίων αδειών στην αρχή της φάσης III. Παράλληλα, το 2017 εισήχθηκε και ο μηχανισμός Market Stability Reserve με βάση τον οποίο εάν ο συνολικός αριθμός αδειών στο σύστημα ξεπερνά ένα συγκεκριμένο επίπεδο, γίνονται αναπροσαρμογές στην ετήσια διανομή αδειών από το σύστημα.

0.2 Σκοπός της διπλωματικής εργασίας

Η μαθηματική μοντελοποίηση ενός τέτοιου συστήματος θα μας βοηθήσει να μελετήσουμε και να αντιληφθούμε χρήσιμες συσχετίσεις μεταξύ κρίσιμων μεταβλητών του στυστήματος, ενώ θα μπορέσει να χρησιμοποιηθεί είτε απ΄ τον ρυθμιστικό φορέα για να βελτιώσει την απόδοση, είτε απ΄ τους εμπλεκόμενους για να επιλέξουν αποδοτικότερη στρατιγική. Για τον λόγο αυτό έχει καταβληθεί μεγάλη προσπάθεια στην βιβλιογραφία ώστε να υπάρξει μια σοβαρή μοντελοποίηση που να περιγράφει σωστά τις λειτουργείες ενός συστήματος εμπορίας ρύπων.

Υπάρχει μια ομάδα παραπμέτρων που καθορίζουν την απόδοση του συστημάτος. Μία έξυπνη διανομή αδειών απ τον ρυθμιστικό φορέα θα ωθήσει τις εταιρείες να μειώσουν παραπάνω τους ρύπους, ενώ μια κακή επιλογή θα οδηγήσει είτε σε αποτυχία των στόχων, είτε θα έχει αρνητικό αντίκτυπο στην οικονομία. Επιπρόσθετα, η αποθήκευση αδειών είναι η μεταβλητή που εξισώνει τα οριακά κόστη μείωσης ρύπων στον χρόνο, οδηγώντας σε ένα πιο υγειές σύστημα. Ένα ακόμη χαρακτηριστικό που χρειάζεται προσοχή κατά την μοντελοποίηση ενός οικονομικού συστήματος είναι η μορφή του ανταγωνισμού μεταξύ των εταιρειών. Πολλές μελέτες διατυπώνουν ότι σε τέτοιου είδους αγορές υπάρχει μια μορφή δύναμης αγοράς. Επίσης, οι εταιρείες που συμμετέχουν σε αυτά τα συστήματα έχουν να αντιμετωπίσουν ένα δυσκολότερο πρόβλημα απόφασης, αφού συμμετέχουν ταυτόχρονα σε δύο αγορές, την αγορά προϊόντος και την αγορά ρύπων. Υπάρχει μια διασύνδεση μεταξύ των δύο αυτών αγορών και ό,τι συμβαίνει στην μία επηρεάζει και την άλλη. Για τον λόγο αυτό δεν μπορεί να υπάρξει μοντελοποίηση της δεύτερης αγοράς χωρίς να συμπεριληφθεί και η πρώτη.

Εφόσον λοιπόν η διανομή αδειών, η αποθήκευση αυτών, ο ανταγωνισμός μεταξύ των εταιρειών και η διασύνδεση μεταξύ των αγορών αποτελούν σημαντικές παράμετρους του συστήματος, μια μοντελοποίηση δεν μπορεί να μην τις περιέχει. Δυστυχώς, δεν υπάρχει ακόμα μαθηματική μοντελοποίηση η οποία να λαμβάνει υπόψιν όλες τις παραπάνω μεταβλητές. Για τον λόγο αυτό, στην παρούσα εργασία επεκτείνουμε τα μοντέλα της βιβλιογραφίας ώστε να συμπεριλάβουμε όλα τα σημαντικά χαρακτηριστικά. Θα μοντελοποιήσουμε δυναμικά ένα σύστημα εμπορίας ρύπων υπό όλους τους κανόνες του EU ETS θεωρώντας μία ρεαλιστική μορφή ανταγωνισμού μεταξύ των εταιρειών, ενώ θα τους επιτρεπεί να αλληλεπιδρούν και στις δύο αγορές.

Ένα τέτοιο μοντέλο θα μπορεί έπειτα να χρησιμοποιηθεί για να μελετήσουμε, μέσω προσομοιώσεων, πως οι παραπάνω παράμετροι (διανομή αδειών κ.λπ.) συσχετίζονται με σημαντικές μεταβλητές του συστήματος, όπως οι συνολικές εκπομπές ρύπων, η τιμή κ.α.

0.3 Μαθηματική Μοντελοποίηση

Έστω ότι μία εταιρεία συμμετέχει σε μια αγορά προϊόντος παράγοντας q προϊόντα τα οποία φέρουν και τις αντίστοιχες εκπομπές ρύπων σαν παρα-προϊόντα. Για κάθε τόνο εκπομπών η εταιρεία θα πρέπει να παραδώσει στον ρυθμιστικό φορέα μία άδεια εκπομπής. Συνολικά λοιπόν, μέχρι το τέλος της περιόδου, χρειάζεται z άδειες, όπου z είναι η ποσότητα ρύπων της εταιρείας μετρημένη σε τόνους. Υπό φυσιολογικές συνθήκες, δηλαδή χωρίς την ύπαρξη του συστήματος, η εταιρεία θα χρησιμοποιούσε την βασική της τεχνολογία και καύσιμη ύλη τα οποία έχουν ένα συγκεκριμένο συντελεστή ρύπανσης, r. Σε μια τέτοια περίπτωση η εταιρεία θα απέδιδε τα bysiness as usual emissions (BAU), επομένως μπορούμε να γράψουμε $z_{BAU} = rq$.

Η εταιρεία θα πρέπει να πληρώσει ένα αντίτιμο για να μειώσει τους ρύπους. Αυτό το κόστος εκφράζεται απ΄ την συνάρτηση κόστους μείωσης (Abatement Cost Function) $C_A(\alpha)$, όπου α είναι η ποσότητα μείωσης των ρύπων. Αυτή η ποσότητα εκφράζεται όπως παρακάτω:

$$a = z_{BAU} - z \implies a = rq - z$$

Σύμφωνα με τους Phaneuf and Requate 2016, αυτή η συνάρτηση κόστους μείωσης έχει τις ακόλουθες ιδιότητες:

- $C_A(z_{BAU}) = 0$
- $C_A(a) > 0, \forall z < z_{BAU}$
- $\frac{\partial C_A(a)}{\partial z} < 0$
- $\frac{\partial^2 C_A(a)}{\partial z^2} > 0$
- $MAC = -\frac{\partial C_A(a)}{\partial z} > 0$

όπου MAC είναι το οριακό κόστος μείωσης (Marginal Abatement Cost €/ tCO_2). Επομένως η συνάρτηση κόστους μείωσης αποτελεί μία αυστηρά κυρτή συνάρτηση.

Στην αρχή κάθε περιόδου, κάθε εταιρεία δέχεται ένα ποσό z_{f_t} από την διανομή αδειών, IA. Έπειτα μπορούν να αποθηκεύσουν τις άδειες αυτές και να τις χρησιμοποιήσουν όπως επιθυμούν σε αυτή ή τις επόμενες περιόδους. Οι άδειες που μια εταιρεία αποθηκεύει κατά την περίοδο t, δηλώνονται με την μεταβλητή $\bar{z}_t$. Ένας άλλος τρόπος διαχείρησης των αδειών είναι να υπάρξουν αγοραπωλησίες για κάποιες από αυτές. Η ποσότητα αδειών που αγοράστηκε ($z_{b_t} > 0$) ή πωλήθηκε ($z_{b_t} < 0$), σε τιμή σ_t, από την εταιεία δίνονται από την σχέση $z_{b_t} = z_t - (z_{f_t} + \bar{z}_{t-1}) + \bar{z}_t$.

Μπορούμε πλέον να γράψουμε την συνάρτηση κόστους για κάθε εταιρεία, η οποία θα περιλαμβάνει το κόστος από την περίοδο t και έπειτα.

$$\begin{aligned} f_t &= C_{A_1}(z_{BAU_1} - z_t) + \sigma_t(z_{b_t}) - p_t q_t + C_{P_t}(q_t) + \beta_t f_{t+1} \\ \implies f_t(z_t, \bar{z}_t, \bar{z}_{t-1}, q_t) &= \\ & C_{A_1}(r_t q_t - z_t) + \sigma_t(z_t - z_{f_t} + \bar{z}_t - \bar{z}_{t-1}) - p_t q_t + C_{P_t}(q_t) + \beta_t f_{t+1}(z_{t+1}, \bar{z}_{t+1}, \bar{z}_t, q_{t+1}) \end{aligned} \tag{0.1}$$

Υποθέτοντας μόνο δύο περιόδους, έχουμε:

$$f_1 = C_{A_1}(r_1 q_1 - z_1) + \sigma_1(z_1 - z_{f_1} + \bar{z}_1) - p_1 q_1 + C_{P_1}(q_1) + \beta_1[C_{A_2}(r_2 q_2 - z_2) + \sigma_2(z_2 - z_{f_2} - \bar{z}_1) - p_2 q_2 + C_{P_2}(q_2) \tag{0.2}$$

όπου για να υπάρχει φυσική σημασία πρέπει να ισχύουν τα ακόλουθα:

- $0 \leq z_1$
- $z_1 \leq r_1 q_1$
- $0 \leq \bar{z}_1$
- $0 \leq z_2$
- $z_2 \leq r_2 q_2$

Για την επίλυση αυτού του προβλήματος θα θεωρήσουμε ότι μια εταιρεία μπορεί να είναι είτε ηγέτης (leader) είτε ακόλουθος (follower) στην αγορά ρύπων. Με άλλα λόγια δεχόμαστε ότι υπάρχουν εταιρείες οι οποίες διαθέτουν παραπάνω δύναμη αγοράς την οποία και εξασκούν.

Ακόλουθοι

Θα ασχοληθούμε τώρα με το υποπρόβλημα των ακολούθων. Θεωρώντας ότι η επιλογή ρύπανσης απ΄ τους ακόλουθους δεν επηρεάζει άμεσα την τιμή, παίρνουμε τα εξής βελτιστοποιώντας την συνάρτηση κόστους:

Εάν $\mathcal{F}_1(r_{1_i} q_{1_i} - z_{1_i}) = -\frac{\partial C_{A_1}}{\partial z_1}$, τότε

- $\sigma_1 = -\frac{\partial C^s_{A_1}}{\partial z_1} \implies z_1 = r_{1_i} q_{1_i} - \mathcal{F}_1^{s^{-1}}(\sigma_1)$
- $\sigma_1 = \beta_1 \sigma_2$
- $\sigma_2 = -\frac{\partial C^s_{A_2}}{\partial z_2} \implies z_2 = r_{2_i} q_{2_i} - \mathcal{F}_2^{s^{-1}}(\sigma_2)$
- $\frac{\partial C_{P_1}}{\partial q_1} + \frac{\partial C^s_{A_1}}{\partial q_1} = p_1(q_1) + \frac{\partial p_1}{\partial q_1} q_1$
- $\frac{\partial C_{P_2}}{\partial q_2} + \frac{\partial C^s_{A_2}}{\partial q_2} = p_2(q_2) + \frac{\partial p_2}{\partial q_2} q_2$

Κατά την διαδικασία επίλυσης, η οποία περιέχεται στο κεφάλαιο 3 προκύπτει η εξής συνάρτηση τιμής:

$$\implies \boxed{\sigma_1 = \mathcal{F}_1^s\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{1_i} + \sum_{i=1}^{m} r_{1_i}q_{1_i} - IA_1 + B_1\right]\right)} \tag{0.3}$$

Όπου $B_1 = \sum_{i=1}^{n} \bar{z}_{1_i}$ είναι ο συνολικός αριθμός αποθηκευμένων αδειών για την πρώτη περίοδο.

Η συνάρτηση αυτή μας δείχνει ότι η τιμή ακολουθεί την μορφή της οριακής συνάρτηση κόστους των μικρών εταιρειών, καθώς και ότι εξαρτάται από την ποσότητα ρύπων των μεγάλων. Επίσης συνδέεται με τους BAU ρύπους των μικρών. Τέλος είναι θετικά συσχετισμένη με τον συνολικό αριθμό αποθηκευμένων αδειών, ενώ είναι συσχετίζεται αρνητικά με την ποσότητα διανεμημένων αδειών απ΄ τον ρυθμιστικό φορέα.

Τελικά φτάνουμε στην λύση του προβλήματος των μικρών εταιρειών:

$$z_{1_i} = r_{1_i}q_{1_i} + \frac{1}{m}\left(\sum_{j=1,j\neq i}^{m} z_{1_j} - \sum_{j=1,j\neq i}^{m} r_{1_j}q_{1_j}\right) \tag{0.4}$$

$$z_{2_i} = r_{2_i}q_{2_i} + \frac{1}{m}\left(\sum_{j=1,j\neq i}^{m} z_{2_j} - \sum_{j=1,j\neq i}^{m} r_{2_j}q_{2_j}\right) \tag{0.5}$$

$$\mathcal{F}_1^s\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{1_i}q_{1_i} - \sum_{i=1}^{m} z_{1_i}\right]\right) = \beta_1\mathcal{F}_2^s\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{2_i}q_{2_i} - \sum_{i=1}^{m} z_{2_i}\right]\right) \tag{0.6}$$

$$\frac{\partial C_{P_{1_i}}}{\partial q_{1_i}} + \frac{\partial C^s_{A_{1_i}}}{\partial q_{1_i}} = p_{1_i}(q_{1_i}) + \frac{\partial p_{1_i}}{\partial q_{1_i}}q_{1_i} \tag{0.7}$$

$$\frac{\partial C_{P_{2_i}}}{\partial q_{2_i}} + \frac{\partial C^s_{A_{2_i}}}{\partial q_{2_i}} = p_{2_i}(q_{2_i}) + \frac{\partial p_{2_i}}{\partial q_{2_i}}q_{2_i} \tag{0.8}$$

Μία προφανής λύση για το σύστημα αυτό είναι η $z_1 = z_{BAU_1}$ και $z_2 = z_{BAU_2}$, το οποίο σημαίνει ότι οι μεγάλες εταιρείες ασκούν επιρροή στην τιμή και αναγκάζουν τις μικρές να μην μειώσουν τους ρύπους. Επίσης φαίνεται ότι η ελαστικότητα της τιμής για τις μικρές εταιρείες είναι αρνητική, το οποίο σημαίνει ότι θα αγοράσουν άδειες ανεξαρτήτου τιμής. Τέλος βλεπουμε πως εάν οι προμήθεια αδειών απ΄ τους ηγέτες είναι μικροί, οι ακόλουθοι θα πρέπει αναγκαστικά να μειώσουν τους BAU ρύπους τους μικραίνοντας την παραγωγή προϊόντος. Αυτό σημαίνει ότι οι μεγάλες εταιρείες μπορούν να μεταερουν την δύναμή μεταξύ των δύο αγορών ώστε να έχουν περισσότερο κέρδος.

Ηγέτες

Για τις μεγάλες εταιρείες θεωρούμε πως οι ρύποι που θα παράξουν επηρεάζουν άμεσα την τιμή. Βελτιστοποιώντας την συνάρτηση κόστους τους παίρνουμε:

- $\frac{\partial C^b_{A_1}}{\partial z_1} + \frac{\partial \sigma_1}{\partial z_1}(z_1 + \lambda) + \sigma_1 = 0$

- $\sigma_1 = \beta_1 \sigma_2$
- $\frac{\partial C^b_{A_2}}{\partial z_2} + \frac{\partial \sigma_2}{\partial z_2}(z_2 - \lambda \beta_1) + \sigma_2 = 0$
- $\frac{\partial C_{P_1}}{\partial q_1} + \frac{\partial C^b_{A_1}}{\partial q_1} = p_1(q_1) + \frac{\partial p_1}{\partial q_1} q_1$
- $\frac{\partial C_{P_2}}{\partial q_2} + \frac{\partial C^b_{A_2}}{\partial q_2} = p_2(q_2) + \frac{\partial p_2}{\partial q_2} q_2$

Λύνοντας το πρόβλημα των ακολούθων παίρνουμε τις ακόλουθες ποσότητες:

$$A_1 = \left(\sum_{j=1, j \neq i}^{m} z_{1_j} - \sum_{j=1, j \neq i}^{m} r_{1_j} q_{1_j} \right) \tag{0.9}$$

ανδ

$$A_2 = \left(\sum_{j=1, j \neq i}^{m} z_{2_j} - \sum_{j=1, j \neq i}^{m} r_{2_j} q_{2_j} \right) \tag{0.10}$$

Τις ποσότητες αυτές χρησιμοποιούμε και τελικά παίρνουμε την λύση των ηγετών:

$$\mathcal{F}_1^b(r_{1_i} q_{1_i} - z_{1_i}) = \frac{1}{m} \mathcal{F}_1^{s\prime} \left(\frac{1}{m} A_1 \right) (z_{1_i} + \lambda_i) + \mathcal{F}_1^s \left(\frac{1}{m} A_1 \right) \tag{0.11}$$

$$\mathcal{F}_2^b(r_{2_i} q_{2_i} - z_{2_i}) = \frac{1}{m} \mathcal{F}_2^{s\prime} \left(\frac{1}{m} A_2 \right) (z_{2_i} - \lambda_i \beta_1) + \mathcal{F}_2^s \left(\frac{1}{m} A_2 \right) \tag{0.12}$$

$$\begin{split} \mathcal{F}_1^s \left(\frac{1}{m} \left[\sum_{i=m+1}^{n} z_{1_i} + \sum_{i=1}^{m} r_{1_i} q_{1_i} - I A_1 + B_1 \right] \right) = \\ \beta_1 \mathcal{F}_2^s \left(\frac{1}{m} \left[\sum_{i=m+1}^{n} z_{2_i} + \sum_{i=1}^{m} r_{2_i} q_{2_i} - I A_2 - B_1 \right] \right) \end{split} \tag{0.13}$$

$$\frac{\partial C_{P_{1_i}}}{\partial q_{1_i}} + \frac{\partial C^b_{A_{1_i}}}{\partial q_{1_i}} = p_{1_i}(q_{1_i}) + \frac{\partial p_{1_i}}{\partial q_{1_i}} q_{1_i} \tag{0.14}$$

$$\frac{\partial C_{P_{2_i}}}{\partial q_{2_i}} + \frac{\partial C^b_{A_{2_i}}}{\partial q_{2_i}} = p_{2_i}(q_{2_i}) + \frac{\partial p_{2_i}}{\partial q_{2_i}} q_{2_i} \tag{0.15}$$

Βλέπουμε ότι σε αυτό το σύστημα εξισώσεων υπάρχουν περισσότερες μεταβλητές απο αξισώσεις, το οποίο σημαίνει ότι υπάρχουν άπειρες λύσεις. Οι πολλαπλασιαστές Λαγρανγε δείχνουν την στρατηγική που επιλέγει η κάθε εταιρεία. Έτσι το μοντέλο μας μπορεί να μοντελοποιήσει πολλές στρατηγικές ισορροπίας.

0.4 Προσομοιώσεις

Εφόσον το θεωρητικό μοντέλο είναι έτοιμο, θα τρέξουμε μερικές προσομοιώσεις ώστε να δούμε εάν μπορεί να ανιχνεύσει κάποιες συσχετίσεις μεταβλητών. Η εύρεση δεδομένων για το πρόβλημα ήταν αρκετά δύσκολη αφού χρειάζονται δεδομένα παραγωγής για επιμέρους εταιρείες, αλλά και συναρτήσεις κόστους κ.α. Για τον λόγο αυτό δημιουργήσαμε τα δικά μας δεδομένα βασιζόμενοι σε πραγματικά δεδομένα τα οποία βρέθηκαν στην έρευνα της εταιρείας Price Waterhouse Coopers 2008, αλλά και στην βάση δεδομένων που φτιάχτηκε για την μελέτη των Dimos et al. 2020.

Έτσι προκύπτουν τα παρακάτω δεδομένα:

Table 1: Δεδομένα εταιρειών για την πρώτη περίοδο

ID	Name	isLeader	Q	RQ	Free	Surrendered	Banking	Previous Banking
0	Leader1	True	651.999	88.02	105.624	61.614	6.049	0
1	Leader2	True	65.999	15.906	19.087	11.134	0.612	0
2	Leader3	True	71.999	23.759	28.512	16.632	0.668	0
3	Follower1	False	169.999	77.179	66.365	77.179	1.577	0
4	Follower2	False	41.999	20.034	17.227	20.034	0.389	0
5	Follower3	False	60.999	30.195	25.964	30.195	0.566	0
6	Follower4	False	64.999	36.985	31.802	36.985	0.603	0
7	Follower5	False	26.999	16.794	14.440	16.794	0.25	0
8	Follower6	False	168.999	133.848	115.093	133.848	1.568	0
9	Follower7	False	49.999	49.599	42.649	49.599	0.464	0

Table 2: Δεδομένα εταιρειών για την δεύτερη περίοδο

ID	Name	isLeader	Q	RQ	Free	Surrendered	Banking	Previous Banking
0	Leader1	True	650.102	87.764	105.316	81.638	0	6.049
1	Leader2	True	65.809	15.859	19.032	9.734	0	0.612
2	Leader3	True	71.790	23.691	28.429	17.565	0	0.668
3	Follower1	False	169.505	76.955	66.172	76.955	0	1.577
4	Follower2	False	41.877	19.976	17.177	19.976	0	0.389
5	Follower3	False	60.822	30.107	25.888	30.107	0	0.566
6	Follower4	False	64.811	36.877	31.709	36.877	0	0.603
7	Follower5	False	26.921	16.745	14.399	16.745	0	0.250
8	Follower6	False	168.508	133.458	114.758	133.458	0	1.568
9	Follower7	False	49.854	49.455	42.526	49.456	0	0.464

Συνάρτηση Οριακού Κόστους Μείωσης

Πέρα απ΄ τα δεδομένα θα πρέπει να διαλέξουμε ποια συνάρτηση οριακού κόστους μείωσης θα χρησιμοποιηθεί. Οι Ellerman and Decaux 1998 έφτιαξαν μια συνάρτηση αυτού του είδους χρησιμοποιώντας το μοντέλο MIT EPPA. Την συνάρτηση αυτή χρησιμοποίησε και ο Chevallier 2011 στις προσομοιώσεις του και έχει την ακόλουθη μορφή:

$$Y = aX^2 + bX + c$$

Επιλέξαμε την συγκεκριμένη συνάρτηση διότι μας δίνει μια πιο γενική προσέγγιση του συσχετισμού κόστους και μείωσης.

Έπειτα προσαρμόσαμε (fit) την συνάρτησή αυτή στα δεδομένα μας και βρήκαμε τους συντελεστές καταλήγοντας τελικά στην συνάρτηση:

$$Y = 0.1384923076923077X^2 + 0.0X + 18.0$$

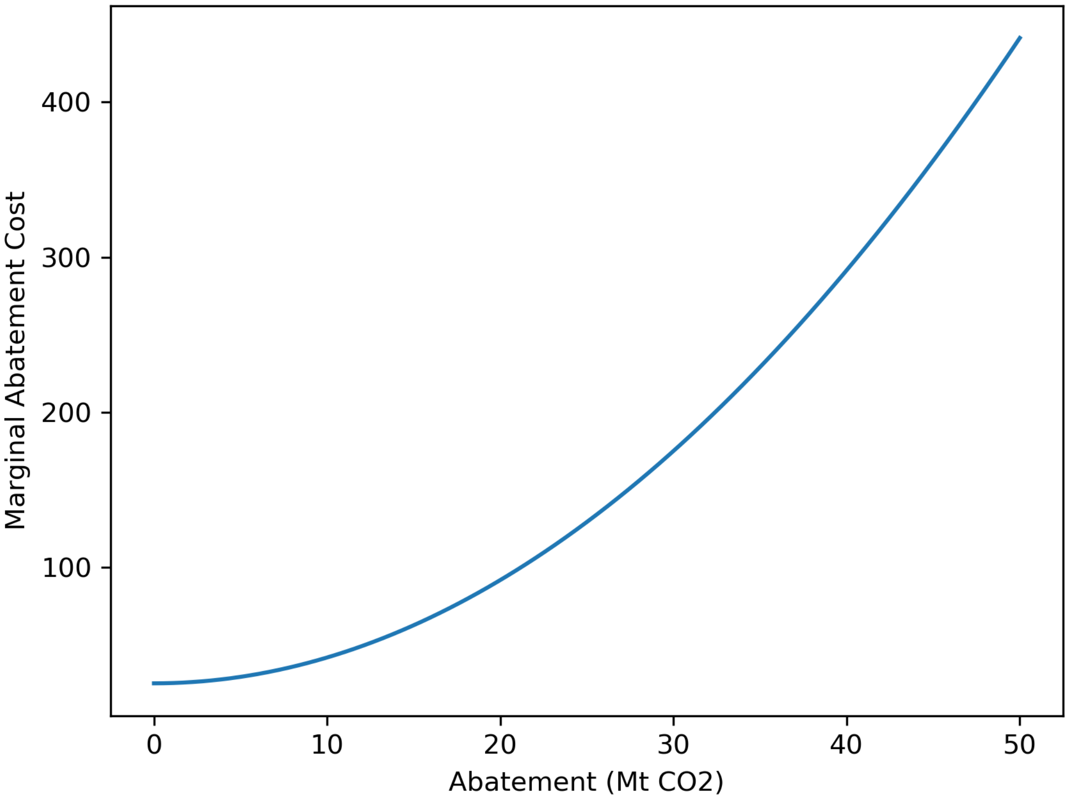


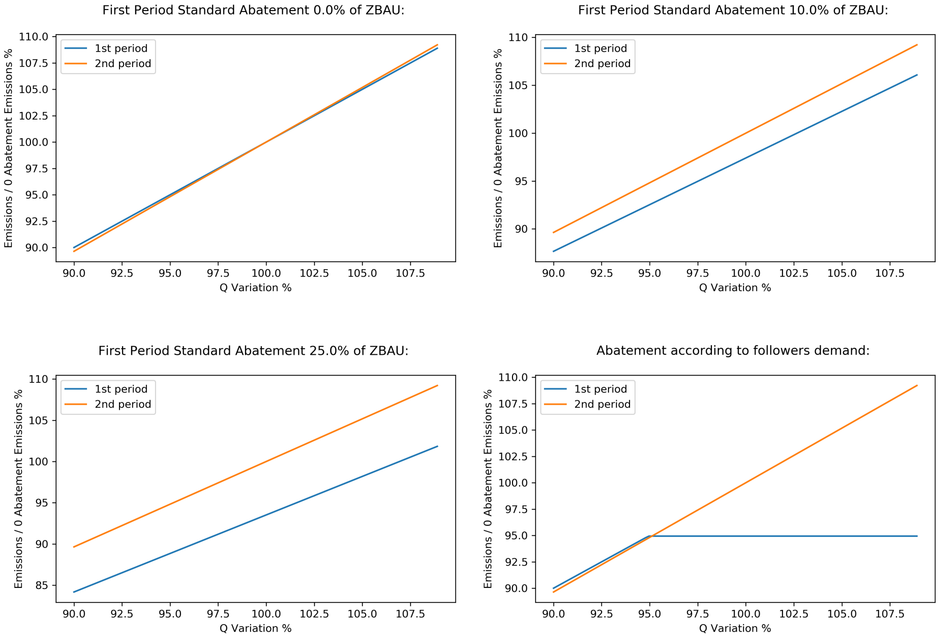
Figure 1: Συνάρτηση οριακού κόστους μείωσης βασισμένη στα παραγμένα δεδομένα

Γενική ιδέα

Θα κάνουμε δύο ειδών προσομοιώσεις. Στην πρώτη θα μεταβάλουμε το συνολικό προϊόν που παράγεται, ενώ στην δεύτερη την αρχική διανομή. Αυτές οι δύο μεταβλητές επιλέχτηκαν καθώς έχουν παίξει στο παρελθόν καθοριστικό ρόλο για την απόδοση του συστήματος. Χρησιμοποιώντας τους πολλαπλασιαστές Λαγρανγε θα επιλέξουμε αρκετές διαφορετικές στρατηγικές και θα συγκρίνουμε διαφορετικά σενάρια ισορροπίας. Δεδομένου ότι τα δεδομένα μας είναι τεχνητά δεν έχει νόημα να εξετάσουμε τις ακριβείς τιμές που θα παράγει σαν αποτέλεσμα το μοντέλο. Αντίθετα θα εξετάσουμε τις μεταβολές των τιμών αυτών σαν ποσοστά.

Προσομοίωση 1 - Διαφορετικά Επίπεδα Μείωσης

Στην προσομοίωση αυτή θα εξετάσουμε τα παρακάτω σενάρια:



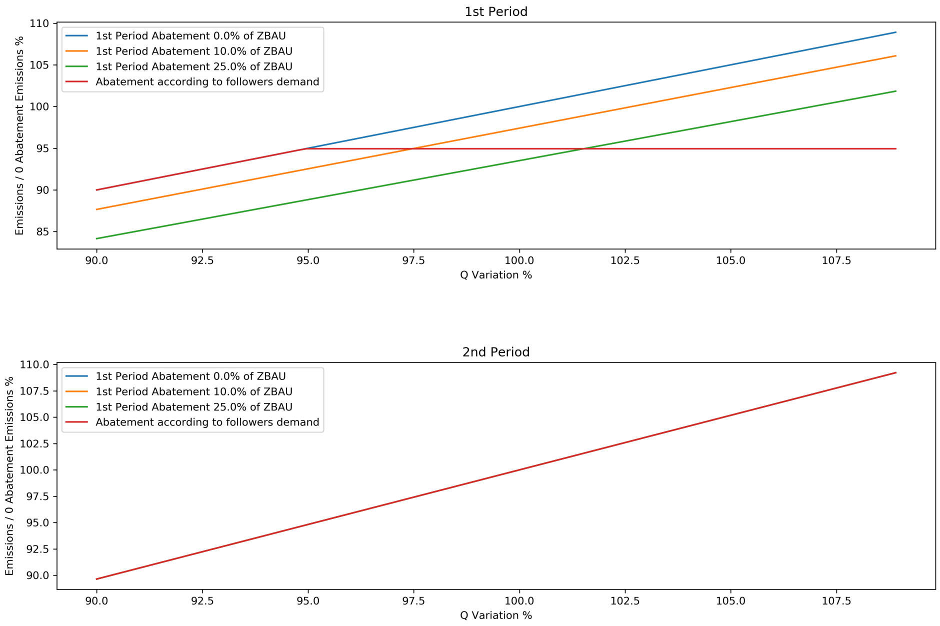
Σχήμα 2: Προσομοίωση 1 - Διαφορετικά επίπεδα μείωσης ρύπων: Ρύποι ως ποσοστό των αρχικών ρύπων, υπό μεταβολή του παραγόμενου προϊόντος

1. Μείωση κατά 0% απο τους BAU ρύπους.
2. Μείωση κατά 10% απο τους BAU ρύπους.
3. Μείωση κατά 25% απο τους BAU ρύπους.
4. Μείωση αναλογικά με την ζήτηση των μικρών εταιρειών. Αυτό σημαίνει ότι οι μεγάλες εταιρείες μειώνουν τους ρύπους ώστε να προμηθεύσουν τους ακολούθους ακριβώς με την ποσότητα αδειών που χρειάζονται.

Μεταβολή στο παραγόμενο προϊόν

Στα διαγράμματα 2 και 3 εξετάζουμε το σύστημα ως προς τους ρύπους. Βλέπουμε πως κάτω από συγκεκριμένα επίπεδα μείωσης οι ρύποι μεταβάλλονται ανάλογα με το παραγόμενο προϊόν. Επίσης όπως περιμέναμε, οι συνολικοί ρύποι ελλατώνονται για μεγαλύτερα ποσοστά μείωσης. Για την δεύτερη περίοδο οι ρύποι είναι ακριβώς ίδιοι. Αυτό συμβαίνει διότι οι μεγάλες εταιρείες εξαιτίας της υπερ-διανομής έχουν στην διάθεσή τους άδειες για να καλύψουν τις ανάγκες τους και επιλέγουν να παράξουν νέες καθώς υπάρχει ρίσκο στην πώλησή τους αφού βρισκόμαστε στην τελευταία περίοδο. Τέλος, για την τελευταία στρατηγική οι ρύποι σταθεροποιούνται στο 95%. Αυτό μας δείχνει ότι κάτω από τέλειο ανταγωνισμό θα είχαμε λιγότερους ρύπους, οι οποίοι μέσω της εξάσκησης δύναμης από τους ηγέτες αυξάνεται.

Προχωρώντας στην αποθήκευση αδειών και στο διάγραμμα 4 βλέπουμε πως όσο το προϊόν ελαττώνεται, τόσο περισσότερο η ανάγκη για άδειες καλύπτεται απ΄ την αρχική διανομή και

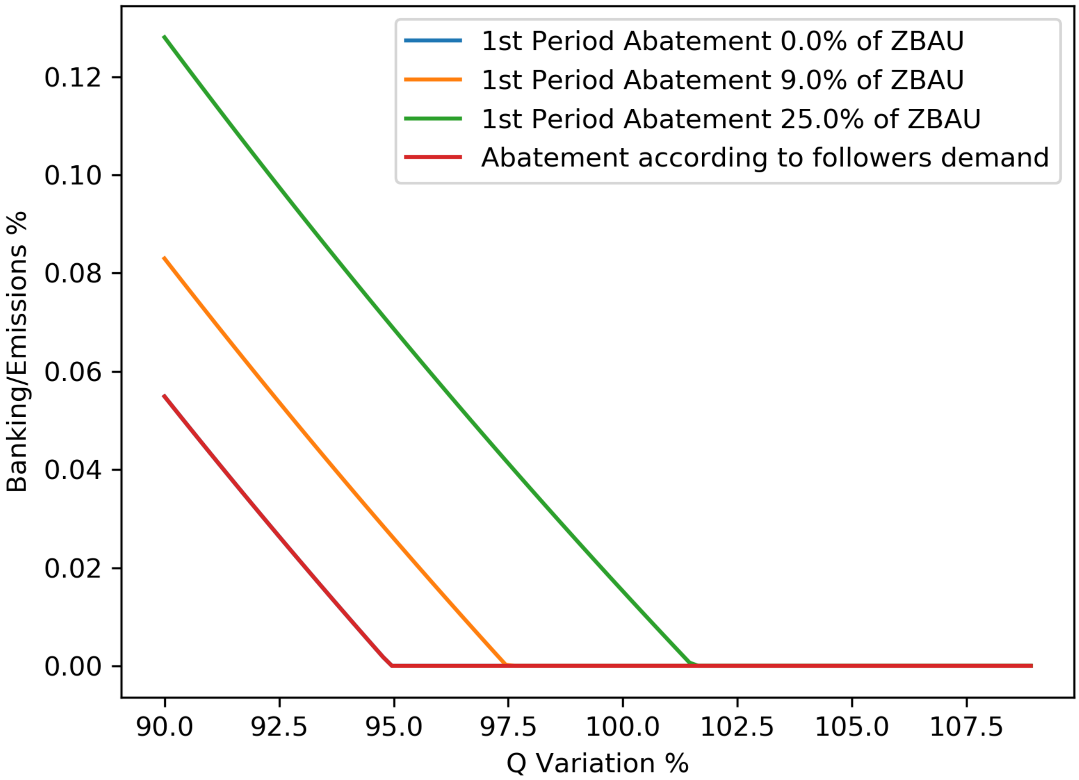


Σχήμα 3: Προσομοίωση 1 - Διαφορετικά επίπεδα μείωσης ρύπων: Σύγκριση ρύπων ανά περίοδο ως ποσοστό των αρχικών ρύπων, υπό μεταβολή του παραγόμενου προϊόντος

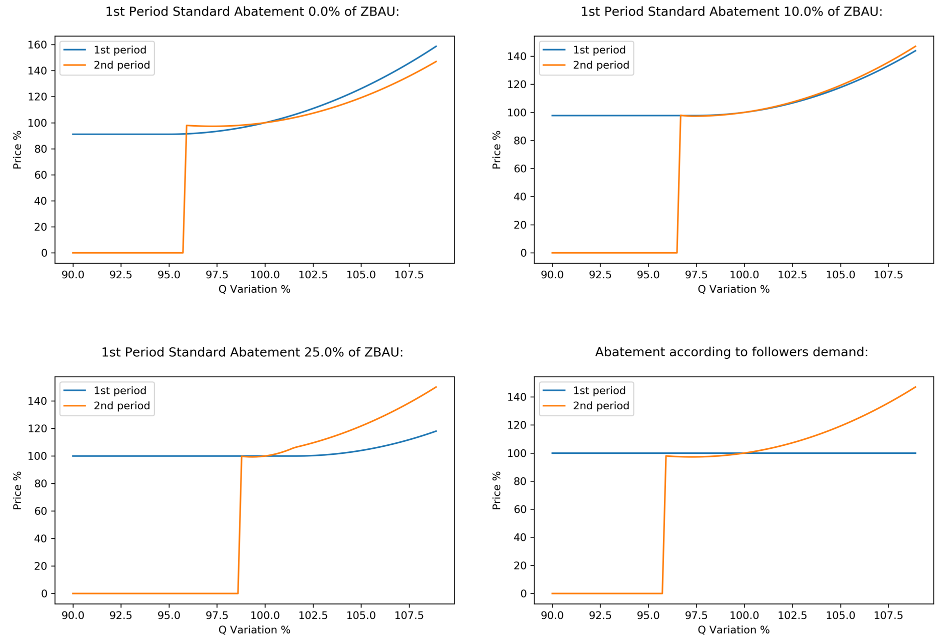
την μείωση, οδηγώντας σε αύξηση της αποθήκευσης. Επίσης για 0% μείωση, η καμπύλη είναι ακριβώς ίδια με αυτήν της τελευταίας στρατηγικής αφού για από το 95% και έπειτα, οι μεγάλες εταιρείες μειώνουν τους ρύπους με αποκλειστικό σκοπό να ικανοποιήσουν τις ανάγκες των μικρών με αποτέλεσμα να μην αποθηκεύουν άδειες.

Θα εξετάσουμε τώρα την, ίσως σημαντικότερη μεταβλητή του συστήματος, την τιμή. Στα διαγράμματα 5 και 6 βλέπουμε πως όσο περισσότερο είναι το παραγόμενο προϊόν, τόσο αυξάνεται η τιμή αφού μεγαλύτερη παραγωγή προϊόντος οδηγεί σε μεγαλύτερη ζήτηση αδειών. Επίσης, για μικρότερα επίπεδα μείωσης, η τιμή είναι πιο επιρρεπής σε μεταβολές του προϊόντος. Όπως φαίνεται η αποθήκευση αδειών κανονικοποιεί την τιμή στον χρόνο και συναίπειες από μεγάλες αλλαγές εμφανίζονται σε μελλοντικό χρόνο. Για τον λόγο αυτό από κάποιο σημείο και έπειτα η τιμή ισούται με 0.

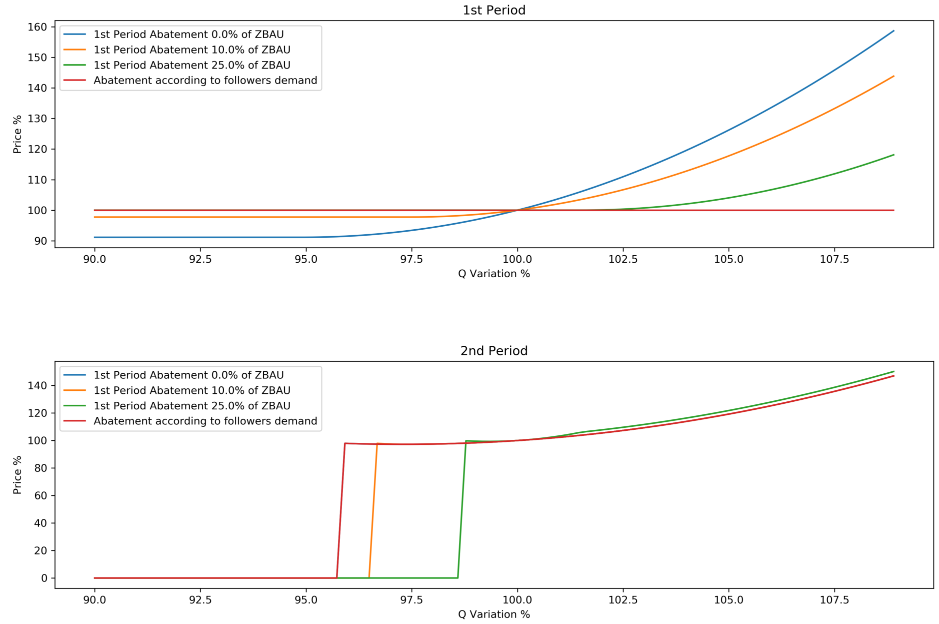
Παρατηρούμε επίσης πως μία αύξηση στην αποθήκευση αδειών οδηγεί σε καλύτερη λειτουργία του συστήματος στο παρόν, αλλά το κάνει πιο ασταθές στο μέλλον.



Σχήμα 4: Προσομοίωση 1 - Διαφορετικά επίπεδα μείωσης ρύπων: Αποθήκευση αδειών ως ποσοστό των ρύπων, υπό μεταβολή του παραγόμενου προϊόντος



Σχήμα 5: Προσομοίωση 1 - Διαφορετικά επίπεδα μείωσης ρύπων: Μεταβολη της τιμής, υπό μεταβολή του παραγόμενου προϊόντος



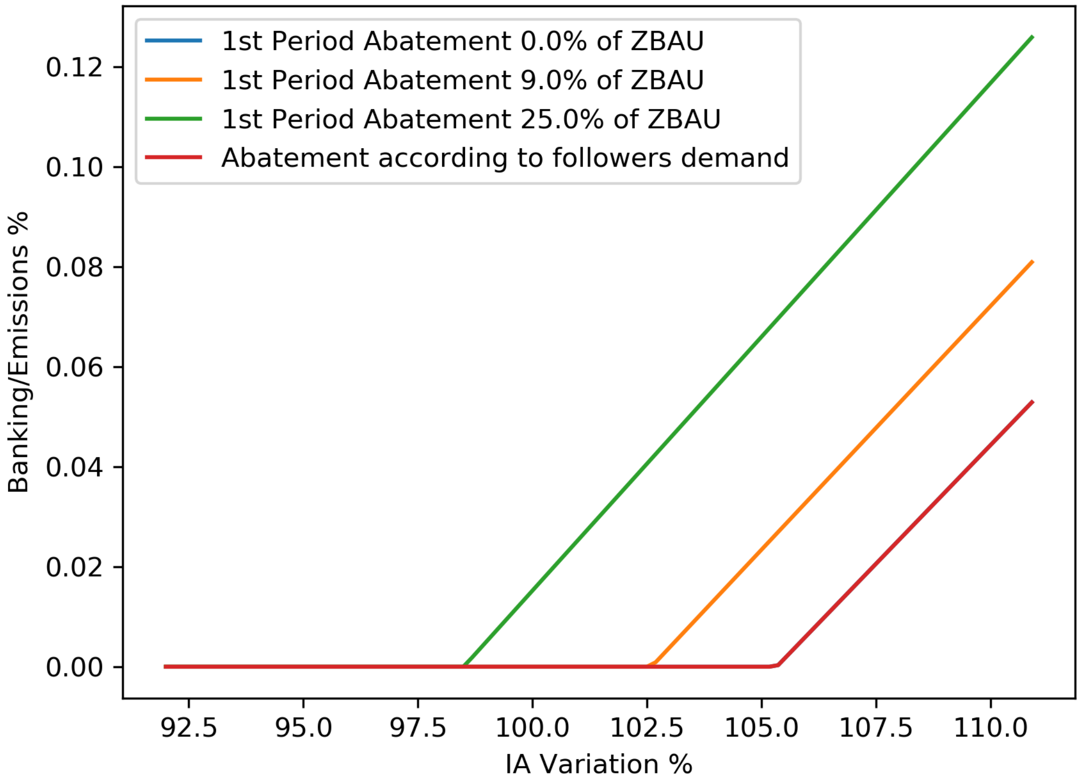
Σχήμα 6: Προσομοίωση 1 - Διαφορετικά επίπεδα μείωσης ρύπων: Μεταβολή της τιμής ανά περίοδο, υπό μεταβολή του παραγόμενου προϊόντος

Μεταβολή στην αρχική διανομή αδειών

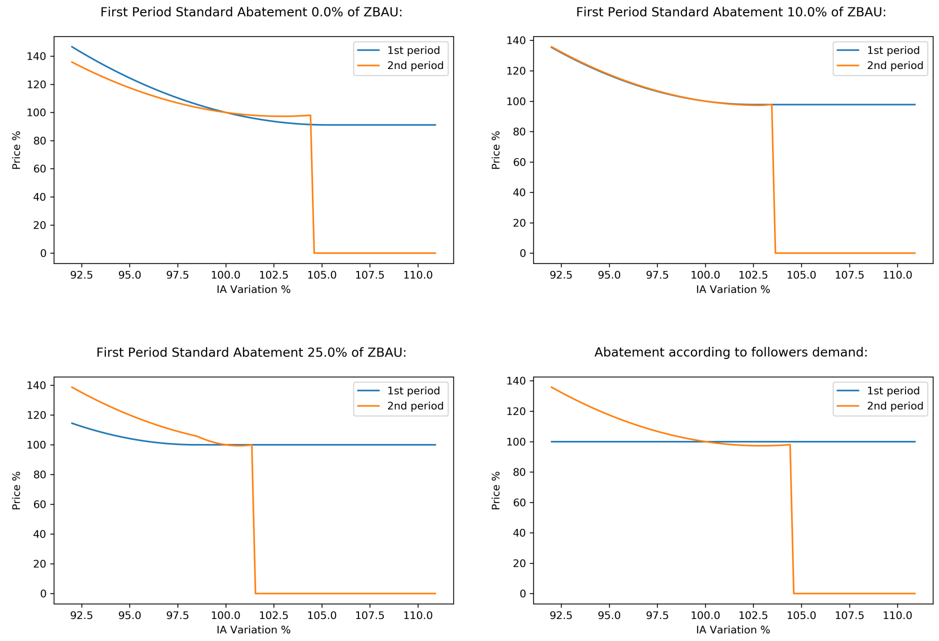
Προχωράμε τώρα στα πειράματα του δεύτερου είδους εισάγοντας μεταβολές στην αρχική διανομή αδειών.

Αρχικά θα μελετήσουμε την αποθήκευση αδειών από το διάγραμμα 7, όπου βλέπουμε πως οι μεταβολές στην αρχική διανομή επιφέρουν τις ακριβώς αντίστροφα αποτελέσματα στο σύστημα από αυτά που παρακολουθήσαμε για την μεταβολή του πραγόμενου προϊόντος. Παρατηρούμε επίσης πως για μεγαλύτερα επίπεδα μείωσης ρύπων οι εταιρείες μπορούν να ικανοποιήσουν τις ανάγκες τους ακόμα και για μικρότερα επίπεδα αρχικής διανομής. Τέλος, όπως περιμέναμε, όσο μεγαλύτερη είναι η μείωση ρύπων, τόσες περισσότερες άδειες αποθηκεύονται.

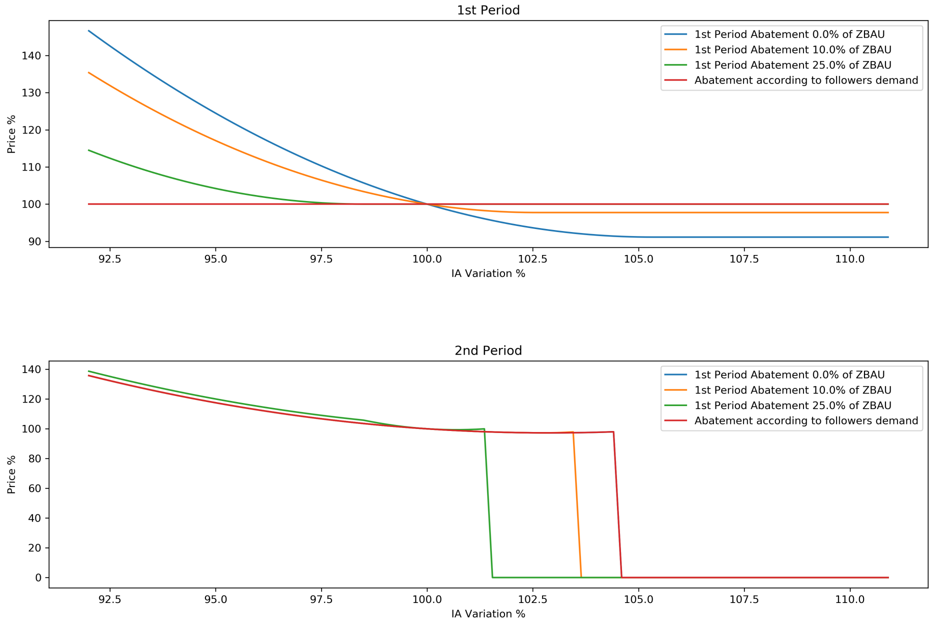
Σχολίαζοντας τώρα τα διαγράμματα της τιμής (8 και 9) παρατηρούμε πως όσο αυξάνεται η αρχική διανομή, τόσο η τιμή μειώνεται. Επίσης, οι εταιρείες που πραγματοποιούν μεγάλες μειώσεις ρύπων παραμένουν περισσότερο ανεπηέαστες από τυχόν μικρότερες διανομές, ενώ αποθηκεύουν περισσότερες άδειες σε περίπτωση αύξησης της διανομής αδειών. Η συμπεριφορά αυτή θα κρατήσει την τιμή σταθερή σε περίπτωση μιρκότερης διανομής, αλλά θα την ρίξει ραγδαία σε περίπτωση μεγαλύτερης διανομής.



Σχήμα 7: Προσομοίωση 1 - Διαφορετικά επίπεδα μείωσης ρύπων: Αποθήκευση αδειών ως ποσοστό των ρύπων, υπό μεταβολή της αρχικής διανομής αδειών



Σχήμα 8: Προσομοίωση 1 - Διαφορετικά επίπεδα μείωσης ρύπων: Μεταβολή στην τιμή, υπό μεταβολή του παραγόμενου προϊόντος



Σχήμα 9: Προσομοίωση 1 - Διαφορετικά επίπεδα μείωσης ρύπων: Σύγκριση της μεταβολής τιμής, υπό μεταβολή του παραγόμενου προϊόντος Σιμυλατιον 1 - Διφφερεντ Αβατεμεντ Λεελς: Πριςε αριατιον ςομπαρισον περ περιοδ, υνδερ αριατιον ιν ινιτιαλ αλλοςατιον

Περισσότερες προσομοιώσεις

Πραγματοποιήθηκαν δύο επιπλέον προσομοιώσεις οι οποίες όμως δεν προσθέτουν παραπάνω νόημα στις υπάρχουσες, γι΄ αυτό δεν δεν θα αναφερθουε σε αυτές. Παρ΄ όλα αυτά υπάρχουν στο κεφάλαιο 4 για περεταίρω μελέτη. Στην πρώτη εκ των δύο μελτήσαμε τι συμβαίνει για μιρκότερο ή μεγαλύτερο οριακό κόστος μείωσης ρύπων, ενώ στην δεύτερη επιβάλλαμε στις μεγάλες εταιρείες να αποθηκαύουν τουλάχιστον ένα ποσοστό των αδειών

0.5 Συμπεράσματα

Συνολικά μπρούμε να πούμε πως μοντελοποιήσαμε δυναμικά ένα σύστημα εμπορίας ρύπων, υπό όλους τους κανόνες του Ευρωπαϊκού Συστήματος Εμπορίας Ρύπων και θεωρώντας μία ρεαλιστική μορφή ανταγωνισμού. Επιτρέψαμε επίσης στις εταιρείες να αλληλεπιδρούν και στην αγορά του προϊόντος. Καταλήξαμε σε ένα σύστημα εξισώσεων ισορροπίας, ενώ αποδείξαμε και μία συνάρτηση τιμής. Τέλος εξετάσαμε μέσω προσομοιώσεων πως η διανομή αδειών απ΄ τον ρυθμιστικό φορέα και η ποσότητα του παραγόμενου προϊόντος απ΄ τις εταιρείες επηρεάζουν σημαντικές μεταβλητές του συστήματος. Όλα τα αποτελέσματα μας συνάδουν με την υπάρχουσα βιβλιογραφία.

Μερικές κατευθύνσεις για μελλοντική έρευνα θα ήταν να εισαχθεί στην μαθηματική μοντελοποίηση η έννοια της αβεβαιότητας, να πραγματοποιηθούν προσομοιώσεις σε πραγματικά δεδομένα και να λυθεί μαθηματικά το πρόβλημα για περισσότερες εξισώσεις.

Chapter 1

Introduction

1.1 Background

Green house Gas (GhG) emissions has been a global problem for the past decades. Many agreements have been made between governmental institutions in order to limit the emission levels. Taxing the emissions was not an effective solution since it does not guarantee that the environmental goals will be met. Researchers were summoned to help and they did by creating a new framework under which the industrial sectors should comply.

Cap-and-trade systems were a solution that ensures specific environmental outcomes and thus leading governments to achieve their obligations. In such a system a cap of emissions is set by the regulator, which indicates the limit of emissions that can be made. If the emissions exceed this limit, then heavy fines are imposed. At the beginning of each period, every firm receives an amount of allowances that correspond to a level of emissions. They can then abate their emissions or trade allowances with other participants. In this way a new market of permits, which introduces complex relations between firms, is created. At the end of the predetermined period each company has to concede to the regulator an amount of allowances that represent their emissions levels. In this way the environmental goals are guaranteed, but uncertainty is imposed on the permit price paid by the firms.

The EU ETS is the key instrument for Europe's climate and energy policy, which due to its large size and international market has become a globally observed field of interest for emissions abatement efficiency. The institutional framework of the scheme creates a market through which firms are compelled to buy or sell allowances in order to emit. In other words, the allowance market determines the carbon price. The main goal of the system is not only to reduce GhG emissions in a cost and environmental effective way, but also to promote investments in low carbon technology Hintermann, Peterson, and Rickels 2016.

Even though EU ETS is recognized as an innovative policy instrument, there have been several drawbacks, mainly concerning the first and the second Phase of its operation, which have been well criticized and studied. At the end of 2007 (during phase I), the EUA

(EU Allowance) price collapsed and almost reached zero. The overallocation of emission permits and the banking restrictions were the main reasons of this failure. Hence, at the beginning of the second phase, banking of permits between different phases was allowed, as a possible remedy to this drawback. Although the creation of this intertemporal link helped the EUA price to stay at higher levels at the end of the second phase, the accumulation of a substantial amount of stocked permits created a remarkable surplus. Since 2008, the amount of allowances issued every year was greater than the actual need and along with the economic recession between 2008 and 2012, that affected the European area and changed the production levels dramatically as well as the insertion of carbon reduction credits coming from Clean Development Mechanism were few of the reasons responsible for this surplus.

To mitigate the effects of this surplus and absorb some of the excess allowances, EU proposed to back-load the auctioning of 900 million allowances at the beginning of Phase III. Along with the Back-loading measure, in 2017 the Commission set in operation the Market Stability reserve Mechanism, under which whenever the total amount of allowances is above a predefined level, several adjustments in the annual allocation can be made.

1.2 Motivation

Mathematical modeling of emissions trading systems helps us understand the relations between crucial system variables and parameters and can be used either by the regulator to improve system performance or by the participants in order to construct a better reaction strategy in the permit market. That is the reason great effort has been given in various studies, in order to properly form a theoretical model that describes the functions of a cap-and-trade system.

There is a bunch of factors that directly affects the system's performance. An efficient initial allocation of allowances may force the firms to abate more, while an inefficient one will lead to either failure of environmental goals, or have a bad impact on the output market and overall economy. Concurrently, intertemporal trading is the variable that equalizes the marginal abatement costs across time leading to more healthy system functionality. One more characteristic that needs attention when modeling an economic system, is the form of competition between firms. Several studies have stated that in such markets a form of market power exists (e.g. Rico 1995, Hahn 1984). Additionally, the firms that operate under an emission trading system have a more difficult decision problem since they participate in two markets: the permit and the output market. That means, there is a link between these two markets and everything happens in one of them has great effects on both. Hence, we cannot model the former without including its interaction with the latter.

Since the initial allocation, banking, market power and linkage with the output market are very important system features a theoretical modeling that needs to be concise has to include all of them. Apparently none of the current studies, consider all of the above characteristics, hence there is a need for a modeling that does. In this work we extend the analysis of several theoretical models found in the literature in order to include all the important features. We will dynamically model an emissions trading system under all of the EU ETS rules and consider a realistic form of competition between firms on the permit market, while letting them to interact in the output market.

Such a model will then have great applicability. It is interesting to use it to examine how critical system parameters such as the free allocation, the banking, the output product or the marginal abatement cost function are related with important system variables such as the overall emissions and the price.

1.3 Literature Review

As mentioned in the previous section, great effort has been given in the literature for the purpose of properly construct a theoretical model that describes the functions of an emissions trading system.

Although banking (and borrowing) of allowances have been well established, theoretical analysis on this area remain scarce. As a first step Rubin 1996 shows that an intertemporal equilibrium exists and intertemporal trading allows firms to equalize marginal abatement cost and thus permit price grows at the discount rate (Hotelling's rule). One of the few works that form a mathematical model and give the firms the chance to bank their allowances is that of Chaton, Creti, and Peluchon 2015. By considering symmetric firms they form a static model of intertemporal trading and extend it to describe the backloading measure as a predefined mandatory banking. However they do not contemplate the freely allocated allowances.

In every economical system uncertainty is used to model complex information and lead to more robust results. This variable could not be absent of such a huge system that operates in the conjunction of many economical sectors. The work of Schennach 2000 is a first attempt to include output market uncertainty on firms' abatement decisions across time and suggests that the higher the expected electricity price, the lower the emissions in earlier periods. Zhang et al. 2013 explores, in a dynamic programming setting, the effect of uncertainty in an intertemporal emission trading system and suggests that increased output or input price uncertainty induces larger emission reductions and higher allowance prices.

A missing factor in the above settings is the strategic interactions between firms. Hahn 1984 introduced the idea of market power in emission trading systems, in his influential work. In a static context and only concerning the spatial exchange of permits, he demonstrated the effects of allocating permits to an agent able to exert market power. However his study did not include two main aspects of the system: the banking of permits, which directly affects the permit price, and the connection with the output market. Considering, he also included only one leader, which is unrealistic, his work was a great first approach to the concept of market power, but more adjustments need to be made in order to approximate the real systems. Liski and Montero 2005 study the effect of market power on the equilibrium of a permit market by introducing a large potentially dominant firm and a competitive fringe. Their analysis shows that the large agent can accumulate the entire stock of permits and thus manipulate the market. The analysis also reveals that large firm have incentive to exchange tradable permit with the fringe, in order to store allowances for the next period. Both banking and borrowing are allowed without restrictions in a continuous time setting. Chevallier 2011 develops a differential Stackelberg game with two types of non-cooperative agents: a large potentially dominant agent, and a competitive fringe the size of which are exogenously determined. Strategic interactions are modeled on an intra-industry permits market where agents can freely bank and borrow permits. This study is one of the few to use proper and realistic abatement cost functions. All of the above works include only one leader in their competition consideration. Instead, Phaneuf and Requate 2016 composes one of the few models that involve a greater number of leaders.

The so far presented works, do not consider the interaction with the output market, which is a main system factor. Hong et al. 2017 take under consideration the output product in each firm's production cost, but in the sense of competition, they suppose that firms are symmetrical and competitive equilibrium is found by a genetic algorithm.

A limited number of studies examines the interaction between the permit and the output market under market power and intertemporal trading. That is the case for Chen and Tanaka 2018, whose work, however, does not consider the important system factor of initial allocation. This absence together with use of a specific emitting rate coefficient instead of a proper abatement cost function, emphasizes the need for a more general theoretical model.

For a more extensive literature review on emissions trading systems please refer to section 2.3.

1.4 Thesis Statement

In the present study we achieved to model an emissions trading system by taking care of many aspects that were not combined in the literature. We made simulations on the theoretical model which provided us with outcomes that are in perfect shape with the literature, indicating that the model we made is powerful enough to capture complex relations between crucial system variables.

We dynamically modeled an emissions trading system under all of the EU ETS rules and considered a realistic form of competition between firms on the permit market, while letting them to interact in the output market. We examined how the cap level can affect crucial system variables, such as the price and showed how intertemporal trading can improve some of the system's functionalities. We also analyze the consequences of a possible variation of the output product.

In our theoretical model we considered a set of n firms, where m of them were the leaders and the rest $n-m$ the followers. Our analysis concluded to a system of equations that indicates system equilibrium. The solution of these equations denotes that the leaders force the followers to avoid abatement and buy all the permits they need, by manipulating the price. As a result, the price elasticity for the followers is negative, which is in perfect shape with what Chevallier 2011 has stated. Since the followers do not abate, every change arises at the emissions is due to abatement from the leaders. That means that the environmental outcome depends only from the abatement level of the leaders.

We also prove a rather undesired effect of the leaders. That is, if they happen to compete with the followers at the output market, they can become leaders at this market too. Hence the leaders can transfer their market power across the two markets and gain more profit.

In the procedure of determining the final equilibrium equations, we came up with a price equation (equation 3.8) which contains much information about the way the price changes and its dependencies. This equation confirms claims made in different studies across literature. That is, the price is in the form of the marginal abatement cost of the followers, it depends from the followers' business as usual emissions, it is positively correlated with the total amount of banking and negatively correlated with the overall initial period allocation.

Moving forward, we made some simulations on the proposed theoretical model, which provided us with great information. Specifically, we found that under specified abatement levels the emissions variate analogously to the output product variation. Of course, as we expected, the overall emissions get smaller for bigger abatement levels. We also show that for the last system's period the leaders choose to avoid abatement and sell their surplus in a greater price, since selling more permits might be a risk. The overall gain for them will be the same since we are in equilibrium. Additionally, for smaller abatement levels, the first period's price is more prone to an output product change. That means that if an incentive for abatement exists, the system will be more stable.

Furthermore, our model confirms the attributes that intertemporal trading gives to a cap-and-trade system. Specifically, we show that an increase in banking lead to better system functionality in the present, normalizes the price in the future, but makes the system more unstable in future exogenous turmoils.

From the figures produced by the simulations we also point out a rather unwelcome fact, that is the excess of market power from the leaders raise the overall emissions.

Adding on to the discussed results, we examined the influence of a variation on the initial allocation and found that it has the opposite outcomes than a variation in the output product. We conclude that under a cap reduction the system is more stable for greater abatement levels, while for a cap rise it works better under small abatement levels.

1.5 Outline

The remaining of the thesis is structured as follows. In chapter 2 the emissions trading systems are introduced, and emphases is given to the description of the EU ETS, while a extensive literature review for this system is included. In the next chapter (chapter 3) we form the theoretical model and come up with the output system of equilibrium equations. Within chapter 4, we use the produced mathematical model to run equations on data we made based on some real data. Finally, in chapter 5 we discuss the results of the present thesis and we suggest some directions for future work.

Chapter 2

Emissions Trading

Climate change has been a subject that concerns humanity for at least 50 years, leading to the creation of the *Intergovernmental Panel on Climate Change (IPCC)* n.d. in 1988. IPCC is the United Nations body which is dedicated to providing the world with scientific information about the risks of human-induced climate change. Its reports made clear how humanity has a negative impact on the planet showing that global actions must be taken. That is the reason why the *United Nations Framework Convention on Climate Change (UNFCCC)* 1992 established in 1992. The UNFCCC's objective is to "stabilize greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system". This framework outlines how specific international treaties (called "protocols" or "Agreements") may be negotiated to specify further actions towards the objective of the UNFCCC.

The international community decided to reduce the deterioration of global warming by keeping the temperature rise below 2^oC. The most important step in this direction was the signing of the Kyoto protocol[1], in Kyoto, Japan, on 11 December 1997 which entered into force on 16 February 2005. This protocol is an international treaty which operationalizes the UNFCCC by committing industrialized countries to limit and reduce greenhouse gas (GHG) emissions in accordance with agreed individual targets for the first commitment period (2008–2012). The Convention itself only asks those countries to adopt policies and measures on mitigation and to report periodically. The *Doha amendment to the Kyoto Protocol* n.d. was adopted for a second commitment period, starting in 2013 and lasting until 2020.

2.1 Cap and Trade Systems

As a consequence of the United Nations' actions, great attention has been given on institutional frameworks for market forces to determine the price of carbon emissions in energy-intensive industrial sectors. A first approach would be to tax all emissions. But in this way it would be uncertain if the emission reduction goals will be achieved, since some

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[1] *Kyoto Protocol to the United Nations Framework, Convention on Climate Change* 1998.

of the participating firms may choose to pollute regardless the tax. In this way under a tax, the price of emitting a unit of pollution is set, but the total quantity of emissions is not. Therefore a tax ensures everyone knows the price being paid (at least for the immediate future) for each unit of carbon dioxide emitted, but uncertainty remains about the actual quantity of emissions.

On the contrary, the purpose of the UNFCCC and its following protocols are to achieve a specific reduction on emissions. By using taxes, as discussed above, the total emissions are unknown leading to uncertainty on whether the reduction goals are met. It would be better if we could set a specific quantity of emissions to pollute, but of course, in an uncertain price. This is the reason *Emissions Trading Systems* were created.

Emissions Trading Systems are cap-and-trade systems. Their goal is to achieve a given emissions target at least cost by equalizing marginal abatement costs across firms (Montgomery 1972, T. H. Tietenberg and T. Tietenberg 1985). A government issues a limited number of annual permits that allow companies to emit a certain amount of carbon dioxide. The total amount permitted thus becomes the "cap" on emissions. Companies are taxed if they produce a higher level of emissions than their permits allow. Firms that reduce their emissions can sell, or "trade," unused permits to others. But the government lowers the number of permits each year, thereby lowering the total emissions cap. That makes the permits more expensive. Over time, companies have an incentive to invest in clean technology as it becomes cheaper than buying permits.

2.1.1 Existing systems

Cap and trade is considered one of the best ways to reduce emission rates across industries. For this exact reason, this concept has been applied across different countries and unions of countries.

United States

In the U.S.A., cap and trade systems are considered efficient for solving environmental problems and that's why they have been adopted for different situations.

Sulfur dioxide

One of the first examples of an emission trading system has been the sulfur dioxide (SO_2) trading system under the framework of the Acid Rain Program of the 1990 Clean Air Act in the U.S.A. Under the program, SO2 emissions were reduced by 50% from 1980 levels by 2007. Some experts argue that the cap-and-trade system of SO2 emissions reduction has reduced the cost of controlling acid rain by as much as 80% versus source-by-source reduction.[2]

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[2] *Acid Rain and Related Programs - Progress Report* 2007.

Nitrogen Oxides

In 2003 the NOx Budget Trading Program began to function. It was a market-based cap and trade program created to reduce emissions of nitrogen oxides (NOx) from power plants and other large combustion sources in the eastern United States. NOx is a prime ingredient in the formation of ground-level ozone (smog), a pervasive air pollution problem in many areas of the eastern United States. As a result of this program, ozone season's NOx emissions decreased by 43 percent between 2003 and 2008, even while energy demand remained essentially flat during the same period.[3]

Tokyo, Japan

Tokyo has greater energy consumption than many European countries. This consumption together with the high industrial engagement lead to great pollution. A scheme to limit carbon emissions launched in April 2010 and covers the top 1,400 emitters in Tokyo. In its fourth year, emissions were reduced by 23% compared to base-year emissions.[4] The aim is to cut Tokyo's carbon emissions by 25% from 2000 levels by 2020. These emission limits can be met by using technologies such as solar panels and advanced fuel-saving devices.

European Union

The European Union Emission Trading Scheme (or EU ETS) is the largest multi-national, greenhouse gas emissions trading scheme in the world. It is one of the EU's central policy instruments to meet their cap set in the Kyoto Protocol. According to the European Commission, in 2010 greenhouse gas emissions from big emitters covered by the EU ETS had decreased by an average of more than 17,000 tonnes per installation from 2005, a decrease of more than 8% since 2005. The goal is to reduce the emissions by 43% compared to 2005 until 2030. We will discuss more about the this system on the next section.

South Korea

South Korea's national emissions trading scheme officially launched on 1 January 2015, covering 525 entities from 23 sectors. With a three-year cap of 1.8687 billion tCO2e, it now forms the second largest carbon market in the world following the EU ETS. This amounts to roughly two-thirds of the country's emissions.

New Zealand

The New Zealand Emissions Trading Scheme (NZ ETS) is a partial-coverage all-free allocation uncapped highly internationally linked emissions trading scheme which legislated in 2008. The NZ ETS covers parts of forestry, energy, industry and waste but not

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[3] *Emissions Trading - Wikipedia* n.d.

[4] *Emissions Trading Worldwide - ICAP Status Report* 2015.

pastoral agriculture. However the absence of a cap for the emissions and the big import of cheaper international allowances have lead to great criticism against the NZ ETS.

China

China currently emits about 30% of global emission, and it became the largest emitter in the world. That is why the need of an emission reduction across this specific country is needed. Since 2011 China is running pilot tests on an emissions trading system and some of the pilot regions started trading at 2013/2014. When the market launches, it will be the largest carbon market in the world. The initial design of the system targets a scope of 3.5 billion tons of carbon dioxide emissions that come from 1700 installations. It has made a voluntary pledge under the UNFCCC to lower CO2 per unit of GDP by 40 to 45% in 2020 when comparing to the 2005 levels.

2.1.2 Cap and Trade Benefits

As understood from the above examples, cap and trade systems tend to dominate the emissions reduction measures. That is because the structure of this type of systems combine a variety of key benefits.

Predetermined environmental outcome

The most important feature of a cap-and-trade system is that the fixed aggregate emissions levels guarantee a certain and predetermined environmental outcome, while participants pay the market rate for the rights to pollute (Coase 1960). This also makes emissions trading more conducive to international environmental agreements, such as the Kyoto Protocol, because specific emissions reduction levels can be agreed upon more easily than tax rates or policy instruments, which may vary in appropriateness and applicability between states(Baumert 1998).

Cost effectiveness and participation

Trading promotes cost-effectiveness, broad participation, and equity in the international context, without the high-level coordination that a tax would require (Keohane 2009). A system of linked national cap and trade programs offers a promising way to achieve cost-effective reductions (Jaffe and R. Stavins 2007). Under such a system, individual countries or regions(such as the EU and the United States) would establish domestic allowance markets, and then let regulated firms purchase allowances from other systems for compliance with their own. In the absence of significant transactions costs (a reasonable assumption in a globalized world), such linkage would automatically equalize prices and hence marginal costs

Performance under uncertainty

Cap and trade systems perform better than taxes in the presence of uncertainty (R. N. Stavins 2019). Permits adjust automatically for inflation and external price shocks, while taxes do not(Baumert 1998). For example, the US has already experienced an extended period of stable greenhouse gas emissions levels from 1972 to 1985 because of high oil prices. Taxes would need to be designed to adjust for such external shocks.

Linkages across Jurisdictions

Initially separate cap-and-trade systems can be linked, and previously distinct carbon tax systems can be harmonized (that is, the rates can be set equal). Linkage and harmonization can yield cost savings (Goulder and Schein 2013). Linking separate emissions pricing programs yields greater abatement effort in the region with the initially lower emissions price and less abatement effort in the region with the initially higher emissions price, thus spurring equal abatement at overall lower costs. Linking once-separate cap-and-trade programs allows for further (cross-jurisdictional) reallocations of abatement effort and thereby yields further cost reductions beyond those generated by separate programs.

Empirical evidence

Cap and trade has long since proven to be environmentally effective and economically cost-effective (R. N. Stavins 2019). For example, it has been estimated that SO_2 trading (under the U.S. sulfur dioxide allowance trading system) reduced aggregate abatement costs by more than half compared with a commensurate, well-designed command-and-control approach (Metcalf 2018).

Also transaction costs have turned out to be low to trivial, particularly when compliance entities have been homogeneous. In particular, the SO_2 allowance trading system demonstrated that in properly designed systems, private markets can render transaction costs minimal (Schmalensee and R. N. Stavins 2013)

2.2 European Union Emissions Trading System (EU ETS)

The world's largest and most interesting emissions trading system, which launched in 2005 and continues to operate till now, is the *EU Emissions Trading System (EU ETS)* n.d. Approximately two billion tons of carbon dioxide (CO_2) and some other greenhouse gases (GHGs) are included in the system, together accounting for about 4 percent of global GHG emissions in 2014 (Olivier, Janssens-Maenhout, and Muntean 2014). Except from its size, another distinguishing feature is its function through a multinational framework, the European Union, rather than through the action of a single state or national government as has been the case for most of the existing cap-and-trade systems.

The EU, based on the *Green Paper on Greenhouse Gas Emissions Trading within the European Union* (European Commission 2000), decided to implement a cap-and-trade system in order to ensure achievement of the targets to which the EU and its member states had committed in the Kyoto Protocol (KP). The green paper defined the essential features of the system: a trial period from 2005 to 2007 (phase 1), followed by full implementation over the 5-year period corresponding to the First Commitment Period of the KP 2008–2012 (phase 2). Later on the EU Commission decided to introduce a third phase (2013-2020), while a forth phase (2021-2030) is about to take place.

The total amount of GhG that can be emitted by installations under EU ETS, is limited by a cap on the number of emission allowances and is reduced annually. By holding an allowance a company has the right to emit one tonne of carbon dioxide (CO_2), or the equivalent amount of two more powerful greenhouse gases, nitrous oxide (N_2O) and perfluorocarbons (PFCs). Within this cap, every installation receive for free or buy from auction, allowances to cover the total of its emissions, at the end of each year. They can sell/buy allowances to/from one another or buy limited number of credits from "green" projects around the world, in order to surrender enough allowances for compliance, otherwise heavy fines are imposed. They also have the possibility to keep the spare allowances for future use. Aircraft operators can surrender either aviation or general allowances for compliance, yet stationary installations can use aviation allowances for compliance from 2021 onwards. According to the European Union, the EU ETS is a policy instrument to achieve decarbonization of the European economy, it helps in generating green growth and hence potentiating longterm firms' competeviness. More specifically, the system gives incentives to invest in re-newable energy technology, and in energy efficiency means. However, by putting a price on carbon, pollution costs are increased and may become an obstacle to competitiveness of many industrial sectors, especially against corresponding sectors in countries where the pollution reduction legislative context is not as strict. Industry sectors at risk of carbon leakage are supported by additional free allowances. For further information see *EU Emissions Trading System (EU ETS)* n.d. and Commission 2015.

2.2.1 History

As already discussed, the system was launched in 2005 and initially divided in three periods-phases (Phase I (2005-2007), Phase II (2008-2012), Phase III (2013-2020)). The first, or pilot phase, was used to set the system's infrastructure, in terms of the price formation as well as the necessary tools for monitoring, reporting and verification of firms' emissions. Due to the absence of reliable data of the actual emissions, the cap was based on estimates. During this period, only emissions from power and energy intensive industries were covered and the largest amount of the allowances were given for free. Concerning the Linking Directive and the reduction units from Kyoto Protocol Mechanisms, in the pilot phase, companies had to surrender only credits from Clean Development Mechanism (CDM) for compliance.

In contrast, Phase II marked some improvements. Because of the documented annual verified emissions, EU had the proper data and managed to reduce the cap together with the free emissions to around 90%. From 2008, every firm could also surrender reduction units from Joint Implementation (JI) for compliance. A single Union Registry, the European Union Transaction Log (EUTL), was created in order to record any transaction under the EU ETS, while the national registries (CITL) were replaced. Additionally, a change that made the system more stable was that the fines for no compliance were raised twice as much, compared to the Phase I.

The third phase runs from 2013 to 2020. The European Commission learned from the mistakes of the past years and made significant changes. A major difference with the past phases was the percentage of auctioning, which in the third phase will reach the 40%. EU also limited the number of credits for compliance and kept aside millions of EUAs in the New Entrant Reserve (NER).

2.2.2 How it works

The level of the cap determines the number of allowances available in the whole system. The cap is designed to decrease annually from 2013, reducing the number of allowances available to companies covered by the EU ETS by 1.74% (linear reduction factor) per year. Each year, a proportion of the allowances are given to certain participants for free, while the rest are sold, mostly through auctions. At the end of a year the participants must return an allowance for every tonne of CO_2 they emit during that year. If a participant has insufficient allowances then it must either take measures to reduce its emissions or buy more allowances on the market. Participants can acquire allowances at auctions, or from each other or either bank some of them for later use. Significant fines are imposed if companies fail to comply by surrendering sufficient allowances in time, set at €$100/tCO_2$ and rising with the EU inflation from 2013 (Commission 2015).

2.2.3 Allowances allocation

In its early years the EU ETS was nothing more than a linkage to twenty five member-states systems, since each member state developed a National Allocation Plan (NAP) stating the total number of allowances to be created. Then, if the criteria of the ETS Directive were fulfilled, the EU had to confirm the NAP. This process proved to be long, laborious, and unrewarding for all concerned (Ellerman, Marcantonini, and Zaklan 2016). Hence, the member states agreed that from phase 3 and on, NAPs will be abandoned and a system-wide cap will instead be adopted. That is, free allocation would be implemented by applying new EU-wide allocation rules. However member states are still required to prepare an allocation plan known as National Implementation Measures (NIM).

The lack of an efficient and stable allocation procedure, introduced many anomalies to system in the first two phases. The two greatest criticisms of the first phase were the “windfall profits” from free allocation and the alleged competitive distortions resulting from different member-state rules for allocation (Ellerman, Marcantonini, and Zaklan

2016). The idea of significant auctioning of allowances was promising that windfall profits would be eliminated, as would the possibility of competitive distortions within the single EU market. Auctioning together with benchmarking was probably the concepts that most improved the functionality of the system.

A benchmark is a reference value relative to production activity, used to determine the level of free allocation that each installation within each sector will receive. Each benchmark corresponds to the average GHG emissions efficiency of the 10% best sector installations. For the best performing installations whose GHG emissions are lower than the benchmark, they will actually receive more free allowances than they need.

2.2.4 Performance across phases

As it turned out, the first 2 phases of the system were rather experimental. This poor performance reflected on the most important system's metric, the allowance price. The allowance price is determined by the balance of allowance supply against the demand of the permit market. Hence, it is the system variable that indicates best the overall performance. For example if the allowance allocation is greater than needed, the demand will decrease while the supply will increase leading to price reduction.

In the first phase prices were volatile. As described by Goulder and Schein 2013, about a year after its implementation, emissions allowance prices dropped dramatically with the release of information that indicated that the ETS Phase I permit allocations were very generous in the sense that they did not much constrain the covered sources. The December 2008 futures prices fell from €32.25 to €17.80 between April 19 and May 12, 2006. There was even greater volatility for the Phase I permit prices contained in December 2007 contracts. These prices dropped from €31.65 on April 19, 2006 to €11.95 on May 3, 2006. In the year 2008, the December 2009 futures prices ranged from €13 per ton (January) to €30 per ton (June).

Phase II of the ETS began in 2008, and while price instability has still been a problem, the swings have been less frequent and intense, including a stretch of almost two years (roughly mid-2009 to mid-2011) with very stable prices hovering between about €15 and €20. Since then, a surplus of permits equivalent to 1.5 billion to 2 billion tons of carbon has led to steady drop in prices, falling as low as €5 a tonne in early 2013 (*EEA ETS data* n.d.).

2.2.5 Intertemporal emission trading

One way to reduce potential price volatility is to allow for intertemporal banking and borrowing of allowances (Goulder and Schein 2013). With intertemporal borrowing, firms can apply toward present emissions the allowances allocated to them for future time periods. Similarly, with intertemporal banking, firms can apply to future periods the allowances they do not use in the current period. Such intertemporal flexibility makes the current supply of allowances more elastic and thereby can damp price volatility.

R. N. Stavins 2007 and Ellerman, Joskow, and Harrison 2003 point out that much of the allowance price volatility experienced was due to the absence of provisions for banking. And volatility in allowance prices for Phase I of the EU ETS has been attributed in part to the fact that the program prevented banking of allowances from the first phase to the second (California Market Advisory Committee et al. 2007). Nearly unlimited banking in the US SO_2 Trading Program is generally agreed to have been a successful design feature of that program, as it mitigated issues of price volatility and led firms to achieve SO_2 reductions faster than they would have without banking (Ellerman and Buchner 2008).

Going back to the advantages of cap-and-trade over taxes we discussed earlier, there are some things to notice about intertemporal trading. Murray, Newell, and Pizer 2009 address a different aspect of the uncertainty issue. They argue that a cap-and-trade system with intertemporal banking of allowances has more ability to adjust to new information in the presence of uncertainty than does the carbon tax. Their argument relies on the fact that under a carbon tax, current marginal abatement costs are largely determined by the carbon tax rate in place today. In contrast, under cap-and-trade system with intertemporal borrowing and banking, the current cap on allowances does not fully determine current marginal abatement costs: changes in expectations about future policy will lead to adjustments in current abatement decisions. This greater ability to respond to changing expectations gives cap and trade an advantage over the carbon tax in smoothing emissions prices over time.

2.3 Literature Review

As understood from the subjects discussed so far, the EU ETS is a system of great importance across different institutions, since it directly affects Europe's economy, environmental goals and of course politics. This impact together with it's big importance as the bigger functional cap and trade system to achieve the greatest emissions reductions across different countries is the reason big attention has been given to this very system in the literature.

Permit Price

The EUA (EU Allowance) price is probably the most important variable that indicates the good functionality of the system, since price dynamics directly affect the efficiency of the allowance market. That is the reason the permit price has been a huge field of study over the years. In theory, the EUA price is determined by business-as-usual (BAU) emissions and marginal abatement costs. In particular, given a predetermined emissions cap, the allowance price should be equal to the marginal abatement costs of the regulated entities (Montgomery 1972; Phaneuf and Requate 2016). Green house Gas (GhG) emission abatement can be achieved either by investing in cleaner technologies or by reducing production levels (E. D. Delarue, Ellerman, and D'HAESELEER 2010).

Preliminary studies on the drivers of the EUA price showed that energy and gas prices, weather variations, offset usage, industrial activity, and economic variations were significant EUA price determinants. As discussed by Christiansen et al. 2005 and Convery and Redmond 2007 natural gas and coal prices affect the demand for allowances and the EUA price, since they are strongly correlated with power production levels and price fuel switching which is considered a short-run investment for abatement. Of course, since brent prices (oil) are considered the main factor of natural gas prices, they directly affect the EUA price (Kanen 2006; Alberola, Chevallier, and Chèze 2008). Mansanet-Bataller, Pardo, and Valor 2007 and Alberola, Chevallier, and Chèze 2008 discuss about the connection between weather variations and the permit price. Considering the fact that energy demand is a U-shaped function of the average temperature, extremely high or low temperatures tend to increase power consumption, which should result in an increase in the allowance price. Klepper and Peterson 2006 showed that the usage of Kyoto offsets for compliance reduces the allowance price by one-third, hence this is another factor that influences the price.

Rickels, Görlich, Oberst, et al. 2010 confirmed the above price determinants based on data up to 2010. Specifically, their results show significant influences of gas, coal, and oil prices, of economic activity, and of some weather variations. The overall results suggest that the price dynamics are better explained by a fundamentals-based model than a purely autoregressive model, but time series characteristics are intrinsic to the problem and are needed to enhance forecasting.

As the system changed between phases I and III more permit price determinants where studied in the literature. In order to capture the effects of macroeconomics on the permit price, production and economic indices are also included in recent studies. The unusual price fall that occurred at the end of phase I is examined by Hintermann 2010 who also discusses that the EUA price was determined by market fundamentals related to aggregated marginal abatement costs. By constructing a model that is a function of important price determinants, he finds that the model provides a good fit well after the price crash, whereas the variables that explained the EUA price in the precrash period are lagged allowance price changes. He concludes that more searching is needed to identify the true allowance price drivers, and he proposes four future fields of interest for this issue, including market power and hedging by firms.

As for the 2008 economic crisis, experimental evidence suggests that reduced production levels led to lower GhG emissions and to a significant decrease in the EUA price (Declercq, E. Delarue, and D'haeseleer 2011; Grubb et al. 2012; Ellerman, Marcantonini, and Zaklan 2016).

Creti, Jouvet, and Mignon 2012 finds some key differences between the price determinants of phase I and phase II. They highlight that, while the oil price, equity price index, and the switching price between gas and coal were significant determinants of the EUA price in the second phase of the EU ETS, the switching price did not play a key role in the first phase.

Koch et al. 2014 consider fuel prices and economic indicators and use a Newey–West estimator for OLS regression on the EUA price during phase II of the EU ETS. They also introduce variables relating to wind and solar electricity production and find that these features are the second most important price determinant. Upon calculating the equilibrium prices, the permit price was found to be close to its equilibrium value, whereas it was overvalued at the beginning of phase II and undervalued at the end of 2009.

Using Granger causality Chung, Jeong, and Young 2018 shows that the EUA price has an one-sided causal effect on the electricity price and the natural gas price, while the causal relationship between CERs and EUAs disappears. Furthermore they show that during phase III, all variables have been positively correlated to the EUA price, except from the minimum temperature. Ultimately, they compare the correlations between the variables and the EUA price by using forecast error variance decomposition. The greatest influence on the current EUA price is the past EUA price, followed by the electricity price, and lastly the natural gas price.

While great research has been made on examining the main determinants of the permit price, there is a flicker of the existence of more factors that the price is affected from. For instance, Alberola, Chevallier, and Chèze 2008 discusses that institutional decisions concerning the overall cap (which determines the initial allocation) may have an impact on the EUA price. This presence is confirmed by the fact that the over-allocation of free allowances in phase I led to a dramatic price decrease at the end of that phase (Ellerman, Marcantonini, and Zaklan 2016). Therefore, the volume of free allowances was reduced in phase II, and the power sector stopped receiving free allowances in phase III (Commission 2015).

Intertemporal Trading

Another missing factors from the price determinants are the banking and borrowing. Intertemporal trading of allowances is considered a price stabilizing tool, as it limits the supply of allowances during a period and prices stay at higher levels, while borrowing has the opposite effect (Maeda 2004). In the absence of banking any economic variation has an impact on permit prices as showed by Hintermann 2010.

An important study is that from Ellerman, Marcantonini, and Zaklan 2016 since they highlight the importance of banking for establishing a floor on prices. They also comment upon the differences between the price drops of 2007 and 2012, and emphasize that the EUA price in 2012 did not reach zero because of banking.

From an investment point of view, Bredin and Parsons 2016 suggest that the only cost of banking an EUA is that associated to the opportunity cost of money when contrasted with other commodities. They suggest that the negative convenience yield for future prices after 2008 may imply that cash and carry arbitrage has been very popular in the market.

Neuhoff et al. 2012 attempt to identify the market actors (i.e. power, industry & financial sector) that are banking allowances for future use. They find that power generators bank allowances to hedge carbon for future use, while industry's actors' banking strategies vary across firms. Financial instruments, on the other hand, play a more speculative role in the market. Banks primarily buy and simultaneously sell forward or futures contracts, while other financial actors invest in carbon together with other assets.

Network Structure

he market structure can also be observed and understood through the actual level of trading. Various studies (Ellerman, Marcantonini, and Zaklan 2016; Hintermann, Peterson, and Rickels 2016; Martin, Muûls, and Wagner 2016) indicate that constructing the transaction network from transactions available through the EUTL may facilitate the discovery of more information on trading patterns.

Under this scope, Karpf, Mandel, and Battiston 2018 analyzed the EU ETS system as a network and found structural elements that induce market inefficiencies. Specifically, they identified a lack of easily accessible trading institutions, which led the industrial nodes of the network to use local connections and financial intermediaries. The resulting network structure induced increased bid-ask spread, among other issues.

Borghesi and Flori 2016 on the other hand, analyzed the system from a country-level perspective, and pointed out that person holding accounts (which approximated the intermediaries in the network) played a prominent role in the network when a variety of centrality measures such as PageRank, degree, average neighborhood in/out degree, and degree of centrality were used. Due to the size and the complexity of the market, a social network approach is quite helpful to understand participant behavior as well as the market structure in general.

Mathematical Modeling

Another subject of interest is mathematical modeling. This type of modeling helps us understand the relations between crucial system variables and parameters and can be used either by the regulator to improve system performance or by the participants in order to construct a better reaction strategy in the permit market. That is the reason great effort has been given in various studies, in order to properly form a theoretical model that describes the functions of an emissions trading system.

A mathematical description of such systems must take care of a range of important system variables. A crucial feauture of the system is intertemporal trading. A really important step in studying this feature made by Rubin 1996 who shows that in such systems, an intertemporal equilibrium exists and intertemporal trading allows firms to equalize marginal abatement cost and thus permit price grows at the discount rate (Hotelling's rule). By considering symmetric firms, Chaton, Creti, and Peluchon 2015 managed to form a static model of intertemporal trading and extend it to describe the backloading measure as a predefined mandatory banking. However they do not contemplate the freely allocated allowances.

An emission trading system operates under a conjuction of different sectors and sometimes (e.g. EU ETS) under different governments. Economists use uncertainty to model crucial variables like price. In such a complex framework uncertainty would model significant information about the system behavior. That is the reason great effort has been made to include this feature on a theoretical cap-and-trade model. The work of Schennach 2000 is a first attempt to include output market uncertainty on firms' abatement decisions across time and suggests that the higher the expected electricity price, the lower the emissions in earlier periods. In a dynamic programming setting under uncertainty, Zhang et al. 2013 explores the effect of uncertainty in an intertemporal emission trading system and suggests that increased output or input price uncertainty induces larger emission reductions and higher allowance prices.

A missing factor in the above settings is the strategic interactions between firms. Hahn 1984 was the first to introduce the idea of market power in emission trading systems. In a static context and only concerning the spatial exchange of permits, he demonstrated the effects of allocating permits to an agent able to exert market power. However his study did not include two main aspects of the system: the banking of permits, which directly affects the permit price, and the connection with the output market. Considering, he also included only one leader, which is unrealistic, his work was a great first approach to the concept of market power, but more adjustments need to be made in order to approximate the real systems. Liski and Montero 2005 study the effect of market power on the equilibrium of a permit market by introducing a large potentially dominant firm and a competitive fringe. Their analysis shows that the large agent can accumulate the entire stock of permits and thus manipulate the market. The analysis also reveals that the large firm has incentive to exchange tradable permit with the fringe, in order to store allowances for the next period. Both banking and borrowing are allowed without restrictions in a continuous time setting. Chevallier 2011 develops a differential Stackelberg game with two types of non-cooperative agents: a large potentially dominant agent, and a competitive fringe the size of which are exogenously determined. Strategic interactions are modeled on an intra-industry permits market where agents can freely bank and borrow permits. This study is one of the few to use proper and realistic abatement cost functions. All of the above works include only one leader in their competition consideration. Instead, Phaneuf and Requate 2016 composes one of the few models that involve a greater number of leaders.

The above works do not consider the interaction with the output market, which is a main system factor. Hong et al. 2017 take under consideration the output product in each firm's production cost, but in the sense of competition, they suppose that firms are symmetrical and competitive equilibrium is found by a genetic algorithm.

A limited number of studies examines the interaction between the permit and the output market under market power and intertemporal trading. That is the case for Chen and Tanaka 2018, whose work, however, does not consider the important system factor of initial allocation. This absence together with the use of a specific emitting rate coefficient instead of a proper abatement cost function, emphasizes the need for a more general theoretical model.

Chapter 3

Theoretical Model

Suppose there is a firm that is involved in an output market by producing a product q. This product comes with its emissions as a by-product. For every tone of emissions, the company should provide the regulator with one emission permit. In this way at the end of a specified period the firm will have to provide the regulator with z permits, where z is the amount of firm's emissions measured in tones. In normal conditions, i.e. without the existence of the compliance program, the firm would use its standard production technologies and fuel, which have a specific emitting rate, denoted by r, for producing the product. In such a situation the company would have its business as usual emissions, so we can write $z_{BAU} = rq$.

However, every company has to comply with the requirements of the regulator in three possible ways:

- Emissions reduction
- Purchase of emission permits
- Reduction of the output product

or by using a mix of these three compliance strategies.

Let's think of the first choice. The firm would have to pay a price to reduce its emissions. This cost is expressed by the *Abatement Cost Function*, $C_A(a)$, where a is the emissions' abatement quantity. This abatement is expressed as the business as usual minus the actual emissions, that is:

$$a = z_{BAU} - z \implies a = rq - z$$

According to Phaneuf and Requate 2016, this abatement function has the following specific identities:

- $C_A(\alpha) = 0, z = z_{BAU}$
- $C_A(\alpha) > 0, \forall z < z_{BAU}$

- $\frac{\partial C_A(\alpha)}{\partial z} < 0$
- $\frac{\partial^2 C_A(\alpha)}{\partial z^2} > 0$
- $MAC = -\frac{\partial C_A(\alpha)}{\partial z} > 0$

where MAC is the Marginal Abatement Cost(€/ tCO_2). As we can derive from the above identities, C_A constitutes a strictly convex function.

At the beginning of each period, every company receives a share z_{f_t} of this period's initial allocation, IA. The firms can store some permits and use them as they wish in the periods. The permits that a company banked during period t, are denoted by $\bar{z}_t$. Another possible way to handle the permits is to buy or sell some of them. The amount of permits bought($z_{b_t} > 0$) or sold ($z_{b_t} < 0$), at a permit price σ_t, by the firm are given by $z_{b_t} = z_t - (z_{f_t} + \bar{z}_{t-1}) + \bar{z}_t$. That means that the permits which have to be bought at period t are equivalent to the difference between the permits that the firm must have at period t (that is z_t for compliance and $\bar{z}_t$ for banking, $z_t + \bar{z}_t$ in total) from the permits that it has at the beginning of the period (that is $z_{f_t} + \bar{z}_{t-1}$).

We can now write the cost function of the firm, which will represent the cost from period t on-wards. This function will of course be dynamic and contain the future costs. So we have the following equation,

$$\begin{aligned} & f_t = C_{A_1}(z_{BAU_1} - z_t) + \sigma_t(z_{b_t}) - p_t q_t + C_{P_t}(q_t) + \beta_t f_{t+1} \\ \Longrightarrow\ & f_t(z_t, \bar{z}_t, \bar{z}_{t-1}, q_t) = \\ & \qquad C_{A_1}(r_t q_t - z_t) + \sigma_t(z_t - z_{f_t} + \bar{z}_t - \bar{z}_{t-1}) - p_t q_t + C_{P_t}(q_t) + \beta_t f_{t+1}(z_{t+1}, \bar{z}_{t+1}, \bar{z}_t, q_{t+1}) \end{aligned} \tag{3.1}$$

Assuming just two periods we get:

$$f_1 = C_{A_1}(r_1q_1 - z_1) + \sigma_1(z_1 - z_{f_1} + \bar{z}_1) - p_1q_1 + C_{P_1}(q_1) + \beta_1[C_{A_2}(r_2q_2 - z_2) + \sigma_2(z_2 - z_{f_2} - \bar{z}_1) - p_2q_2 + C_{P_2}(q_2) \tag{3.2}$$

Where in order to have a real meaning, the following constraints must hold:

- $0 \leq z_1$
- $z_1 \leq r_1 q_1$
- $0 \leq \bar{z}_1$
- $0 \leq z_2$
- $z_2 \leq r_2 q_2$

In order to solve this problem we will assume that a company can either be a follower or a leader in the permit market. In other words we accept that some companies can have and exert market power. This concept is not new at all. Rico (1995) provides empirical evidence that the early US sulfur dioxide market involved a few large and many small firms.

But which are those large companies and why are they the leaders? Based on econometric studies, the adoption of the advanced technology by electric plants (adoption of scrubbers technology) requires high initial capital investment and is positively correlated with prior experience, location and ownership. Thus, it is likely that firms which dominate the output market have adopted advanced abatement technologies (Keohane 2002). Such advanced technologies lead to less abatement costs and therefore more opportunities to manipulate the number of permits available on the market and consequently the permits' price.

On the other hand the rest of the companies posses limited financial and human resources preventing them from accessing knowledge-able brokers (Dasgupta and Heal 1979). These companies are considered to be the followers because they cannot effectively engage on permit trading.

Finally, it is interesting that, as stated by Hintermann 2011, the leaders can be created from some false decisions of the regulator, for instance if they receive a sufficiently large permit allocation.

3.1 Followers

We will now solve the problem for the followers. For ease of calculations, we consider that the abatement cost functions are the same among small companies, denoted by s, as among large companies, denoted by b. The peculiarity of the followers is that they are price takers, which means that the permit price is independent of their emissions.

Here we have to note that if we want to optimize the cost function based on its constraints we have to use the Karush–Kuhn–Tucker (KKT) conditions (Karush 1939, Khun and Tucker 1951) and by doing this we get a more general reaction function for every firm. In the current study we are trying to find the equilibrium of the system and of course the extreme cases where equality applies to the constraints will not be equilibrium choices for the firms. So from the multi-case solution given by the KKT conditions we choose only the equilibrium. So we will just force the partial derivatives of the function to equal zero.

By optimizing the cost function, we have:

If $\mathcal{F}_1(r_{1_i}q_{1_i} - z_{1_i}) = -\frac{\partial C_{A_1}}{\partial z_1}$

- $\sigma_1 = -\frac{\partial C^s_{A_1}}{\partial z_1} \implies z_1 = r_{1_i}q_{1_i} - \mathcal{F}_1^{s^{-1}}(\sigma_1)$
- $\sigma_1 = \beta_1\sigma_2$
- $\sigma_2 = -\frac{\partial C^s_{A_2}}{\partial z_2} \implies z_2 = r_{2_i}q_{2_i} - \mathcal{F}_2^{s^{-1}}(\sigma_2)$
- $\frac{\partial C_{P_1}}{\partial q_1} + \frac{\partial C^s_{A_1}}{\partial q_1} = p_1(q_1) + \frac{\partial p_1}{\partial q_1}q_1$

- $\frac{\partial C_{P_2}}{\partial q_2} + \frac{\partial C^s_{A_2}}{\partial q_2} = p_2(q_2) + \frac{\partial p_2}{\partial q_2} q_2$

The first thing to notice in the above condition is the price relation between periods:

$$\boxed{\sigma_1 = \beta_1 \sigma_2} \tag{3.3}$$

This equation is an outcome of the existence of intertemporal trading. It indicates that banking can normalize the price across periods leaving no room for arbitrage opportunities, as has greatly been emphasized in the literature.

By supposing that the first m companies are followers, while the rest $n-m$ are leaders, the following demand for permits is being formed.

$$D_1 = \sum_{i=1}^{m} (z_{1_i} - z_{f_{1_i}} + \bar{z}_{1_i}) \tag{3.4}$$

If, as mentioned before, $\frac{\partial C_{A_{1_i}}}{\partial z_{1_i}} = \frac{\partial C_{A_{1_j}}}{\partial z_{1_j}}$ then all companies act equivalently. So from (3.4) we get,

$$D_1 = \sum_{i=1}^{m} r_{1_i} q_{1_i} - m\mathcal{F}_1^{s^{-1}}(\sigma_1) - \sum_{i=1}^{m} z_{f_{1_i}} + \sum_{i=1}^{m} \bar{z}_{1_i} \tag{3.5}$$

In equilibrium the demand must equal supply for every separate period. But the supply is mostly produced by the leaders. That means,

$$S_1 = \sum_{i=m+1}^{n} (z_{f_{1_i}} - z_{1_i} - \bar{z}_{1_i}) \tag{3.6}$$

and

$$D_1 = S_1 \tag{3.7}$$

must hold.

Therefore, from equations (3.5), (3.6) and (3.7) we have,

$$\begin{aligned}
\sum_{i=1}^{m} r_{1_i} q_{1_i} - m\mathcal{F}_1^{s^{-1}}(\sigma_1) - \sum_{i=1}^{m} z_{f_{1_i}} + \sum_{i=1}^{m} \bar{z}_{1_i} &= \sum_{i=m+1}^{n} (z_{f_{1_i}} - z_{1_i} - \bar{z}_{1_i}) \\
\Longrightarrow \sum_{i=m+1}^{n} z_{1_i} &= IA_1 + m\mathcal{F}_1^{s^{-1}}(\sigma_1) - \sum_{i=1}^{m} r_{1_i} q_{1_i} - \sum_{i=1}^{n} \bar{z}_{1_i} \\
\Longrightarrow \mathcal{F}_1^{s^{-1}}(\sigma_1) &= \frac{1}{m}\left(\sum_{i=m+1}^{n} z_{1_i} + \sum_{i=1}^{m} r_{1_i} q_{1_i} - IA_1 + \sum_{i=1}^{n} \bar{z}_{1_i} \right) \\
\Longrightarrow & \boxed{\sigma_1 = \mathcal{F}_1^{s}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{1_i} + \sum_{i=1}^{m} r_{1_i} q_{1_i} - IA_1 + B_1 \right] \right)}
\end{aligned} \tag{3.8}$$

Where $B_1 = \sum_{i=1}^{n} \bar{z}_{1_i}$ is the total banking for the first period.

As we can see a permit price function is created, in the form of the marginal abatement cost of the followers. Firstly, this function depends on the amount of real emissions that the leaders are going to produce. This was something we expected when we thought of the followers as price takers. In fact it is shown that the large firms form the supply of permits and in this way they manipulate the permit price as they wish. It is also shown that the permit price will depend on the followers' business as usual emissions, which is reasonable since this quantity has an important role on the amount of permits the small firms are going to buy, i.e at the demand for permits.

From this function we can get some more dependencies that have been stated in the literature. Maeda 2004 showed that banking and price change in exactly the same directions. This can be easily seen in our equations, since it's obvious that σ and B are positive correlated. On the other hand, the negative relation between the permit price and the initial period allocation, is widely discussed in the literature (e.g. Ellerman, Marcantonini, and Zaklan 2016, Goulder and Schein 2013 etc) stated . Our function contains this information, since, σ and IA are negative correlated.

In the exact same fashion we get a permit price function for the second period:

$$\implies \sigma_2 = \mathcal{F}_2^s\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{2_i} + \sum_{i=1}^{m} r_{2_i}q_{2_i} - IA_2 - B_1\right]\right) \tag{3.9}$$

According to these permit price equations and the equilibrium condition 3.3, we can derive the following general system condition, which connects the variables between the two periods:

$$\mathcal{F}_1^s\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{1_i} + \sum_{i=1}^{m} r_{1_i}q_{1_i} - IA_1 + B_1\right]\right) = \beta_1\mathcal{F}_2^s\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{2_i} + \sum_{i=1}^{m} r_{2_i}q_{2_i} - IA_2 - B_1\right]\right) \tag{3.10}$$

We also get the solution of the z variables,for the small firms, which finally depend on the larges' emissions and consequently on the amount of permits that will be available on the market. So we have:

$$z_{1_i} = r_{1_i}q_{1_i} - \frac{1}{m}\left(\sum_{i=m+1}^{n} z_{1_i} + \sum_{i=1}^{m} r_{1_i}q_{1_i} - IA_1 + B_1\right) \tag{3.11}$$

$$z_{2_i} = r_{2_i}q_{2_i} - \frac{1}{m}\left(\sum_{i=m+1}^{n} z_{2_i} + \sum_{i=1}^{m} r_{2_i}q_{2_i} - IA_2 - B_1\right) \tag{3.12}$$

There is also one more condition that must hold. Specifically, the permit market equilibrium equation must hold, that is the demand must equal supply across all systems' periods, since every firm cannot own permits neither at the beginning nor at the end of the systems' duration. So we leave:

$\sum_{i=m+1}^{n}(z_{f_{1_i}} - z_{1_i} - \bar{z}_{1_i} + z_{f_{2_i}} - z_{2_i} - \bar{z}_{1_i}) = \sum_{i=1}^{i=m} z_{1_i} + \bar{z}_{1_i} - z_{f_{1_i}} + z_{2_i} - \bar{z}_{1_i} - z_{f_{2_i}}$

As we can find out, the above equation states that the intertemporal companies' demand must equal the intertemporal regulators' supply. That is,

$$\boxed{IA_1 + IA_2 = \sum_{i=1}^{m} z_{1_i} + \sum_{i=m+1}^{n} z_{1_i} + \sum_{i=1}^{m} z_{2_i} + \sum_{i=m+1}^{n} z_{2_i}} \tag{3.13}$$

Now in order to find the optimized emissions for the followers we need to use equation (3.12) on (3.13). We then get:

$IA_1 + IA_2 = \sum_{i=1}^{m} z_{1_i} + \sum_{i=m+1}^{n} z_{1_i} + \sum_{i=1}^{m} r_{2_i} q_{2_i} - \sum_{i=m+1}^{n} z_{2_i} - \sum_{i=1}^{m} r_{2_i} q_{2_i} + IA_2 + B_1 + \sum_{i=m+1}^{n} z_{2_i}$
$\implies IA_1 = \sum_{i=1}^{m} z_{1_i} + \sum_{i=m+1}^{n} z_{1_i} + B_1 \implies$

$$\sum_{i=m+1}^{n} z_{1_i} = IA_1 - \sum_{i=1}^{m} z_{1_i} - B_1 \tag{3.14}$$

By continuing the analysis we are going to use equation (3.14) on (3.11):

$z_{1_i} = r_{1_i} q_{1_i} - \frac{1}{m} \left(IA_1 - \sum_{i=1}^{m} z_{1_i} - B_1 + \sum_{i=1}^{m} r_{1_i} q_{1_i} - IA_1 + B_1 \right)$
$\implies z_{1_i} = r_{1_i} q_{1_i} - \frac{1}{m} \left(- \sum_{i=1}^{m} z_{1_i} + \sum_{i=1}^{m} r_{1_i} q_{1_i} \right) \implies$

$$\boxed{z_{1_i} = r_{1_i} q_{1_i} + \frac{1}{m} \left(\sum_{j=1, j \neq i}^{m} z_{1_j} - \sum_{j=1, j \neq i}^{m} r_{1_j} q_{1_j} \right), \forall i \in [1, m]} \tag{3.15}$$

Equation 3.15 provides a clear solution for the followers' emissions target. Working on the same fashion for the z_2 variables of the small firms, we derive:

$$\boxed{z_{2_i} = r_{2_i} q_{2_i} + \frac{1}{m} \left(\sum_{j=1, j \neq i}^{m} z_{2_j} - \sum_{j=1, j \neq i}^{m} r_{2_j} q_{2_j} \right), \forall i \in [1, m]} \tag{3.16}$$

Therefore, according to (3.3):

$$\mathcal{F}_1^s \left(\frac{1}{m} \left[\sum_{i=1}^{m} r_{1_i} q_{1_i} - \sum_{i=1}^{m} z_{1_i} \right] \right) = \beta_1 \mathcal{F}_2^s \left(\frac{1}{m} \left[\sum_{i=1}^{m} r_{2_i} q_{2_i} - \sum_{i=1}^{m} z_{2_i} \right] \right) \tag{3.17}$$

Finally based on (3.15), (3.16) and (3.17) we can write a system of equations for all small firms and get their emissions for the two periods by solving it.

3.2 Leaders

The leaders presume the followers' equilibrium conditions and based on the functions derived from them they solve their optimization problem, using the cost function (3.2). So they have,

$$minimize\{f_1\}$$

s.t:

- $\sigma_1 = \beta_1 \sigma_2$
- $0 \leq z_1$
- $z_1 \leq r_1 q_1$
- $0 \leq \bar{z}_1$
- $0 \leq z_2$
- $z_2 \leq r_2 q_2$

Now trying to solve this problem, we see that by choosing the equilibrium one of the KKT conditions, the only Lagrange multiplier left is the one corresponding to the $\sigma_1 = \beta_1 \sigma_2$ condition. That is because, in contrast with the followers' problem, the prices σ_1 and σ_2 are dependent from the emissions made by the leaders (i.e. z_1 and z_2). It is important to remind that since the followers are price-takers, the price is independent of their individual actions. Supposing now that λ is the Lagrange multiplier for the $\sigma_1 = \beta_1 \sigma_2$ condition, we have the following conditions:

- $\frac{\partial C^b_{A_1}}{\partial z_1} + \frac{\partial \sigma_1}{\partial z_1}(z_1 + \lambda) + \sigma_1 = 0$
- $\sigma_1 = \beta_1 \sigma_2$
- $\frac{\partial C^b_{A_2}}{\partial z_2} + \frac{\partial \sigma_2}{\partial z_2}(z_2 - \lambda \beta_1) + \sigma_2 = 0$
- $\frac{\partial C_{P_1}}{\partial q_1} + \frac{\partial C^b_{A_1}}{\partial q_1} = p_1(q_1) + \frac{\partial p_1}{\partial q_1} q_1$
- $\frac{\partial C_{P_2}}{\partial q_2} + \frac{\partial C^b_{A_2}}{\partial q_2} = p_2(q_2) + \frac{\partial p_2}{\partial q_2} q_2$

We can now use 3.8 and 3.14 to analyse the first equation of the above system.

$$
\begin{aligned}
-F^b_1(r_1 q_1 - z_1) + \frac{\partial \mathcal{F}^s_1\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{1_i} + \sum_{i=1}^{m} r_{1_i} q_{1_i} - IA_1 - B_1\right]\right)}{\partial z_1}(z_1 + \lambda) \\
+ \mathcal{F}^s_1\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{1_i} + \sum_{i=1}^{m} r_{1_i} q_{1_i} - IA_1 - B_1\right]\right) = 0
\end{aligned}
$$

$$
\begin{aligned}
\implies \mathcal{F}^b_1(r_1 q_1 - z_1) = \frac{\partial \mathcal{F}^s_1\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{1_i} + \sum_{i=1}^{m} r_{1_i} q_{1_i} - IA_1 - B_1\right]\right)}{\partial z_1}(z_1 + \lambda) \\
+ \mathcal{F}^s_1\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{1_i} q_{1_i} - \sum_{i=1}^{m} z_{1_i}\right]\right)
\end{aligned}
$$

$$\Longrightarrow \mathcal{F}_1^b(r_1q_1 - z_1) = \frac{1}{m}\mathcal{F}_1^{s\prime}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{1_i} + \sum_{i=1}^{m} r_{1_i}q_{1_i} - IA_1 - B_1\right]\right)(z_1 + \lambda)$$

$$+ \mathrm{F}_1^s\left(\tfrac{1}{m}\left[\textstyle\sum_{i=1}^{m} r_{1_i}q_{1_i} - \sum_{i=1}^{m} z_{1_i}\right]\right)$$

$$\Longrightarrow \mathcal{F}_1^b(r_1q_1 - z_1) = \frac{1}{m}\mathcal{F}_1^{s\prime}\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{1_i}q_{1_i} - \sum_{i=1}^{m} z_{1_i}\right]\right)(z_1 + \lambda)$$

$$+ \mathcal{F}_1^s\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{1_i}q_{1_i} - \sum_{i=1}^{m} z_{1_i}\right]\right) \quad (3.18)$$

In the same way, we analyse the third system equation:

$$- \mathcal{F}_2^b(r_{2_i}q_{2_i} - z_{2_i}) + \frac{\partial\mathcal{F}_2^s\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{2_i} + \sum_{i=1}^{m} r_{2_i}q_{2_i} - IA_2 + B_1\right]\right)}{\partial z_2}(z_2 - \lambda\beta_1)$$

$$+ \mathcal{F}_2^s\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{2_i} + \sum_{i=1}^{m} r_{2_i}q_{2_i} - IA_2 + B_1\right]\right) = 0$$

$$\Longrightarrow \mathcal{F}_2^b(r_{2_i}q_{2_i} - z_{2_i}) = \frac{\partial\mathcal{F}_2^s\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{2_i} + \sum_{i=1}^{m} r_{2_i}q_{2_i} - IA_2 + B_1\right]\right)}{\partial z_2}(z_2 - \lambda\beta_1)$$

$$+ \mathcal{F}_2^s\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{2_i}q_{2_i} - \sum_{i=1}^{m} z_{2_i}\right]\right)$$

$$\Longrightarrow \mathcal{F}_2^b(r_{2_i}q_{2_i} - z_{2_i}) = \frac{1}{m}\mathcal{F}_2^{s\prime}\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{2_i} + \sum_{i=1}^{m} r_{2_i}q_{2_i} - IA_2 + B_1\right]\right)(z_{2_i} - \lambda\beta_1)$$

$$+ \mathcal{F}_2^s\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{2_i}q_{2_i} - \sum_{i=1}^{m} z_{2_i}\right]\right)$$

$$\Longrightarrow \mathcal{F}_2^b(r_{2_i}q_{2_i} - z_{2_i}) = \frac{1}{m}\mathcal{F}_2^{s\prime}\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{2_i}q_{2_i} - \sum_{i=1}^{m} z_{2_i}\right]\right)(z_{2_i} - \lambda\beta_1)$$

$$+ \mathcal{F}_2^s\left(\frac{1}{m}\left[\sum_{i=1}^{m} r_{2_i}q_{2_i} - \sum_{i=1}^{m} z_{2_i}\right]\right) \quad (3.19)$$

Finally, equations 3.18 and 3.19, as we will see shortly, can solve the leaders' problem using the information gained from the followers optimization problem in their favor.

3.3 System of Equations

We can finally construct an overall system of equations and follow a specific way of work to solve it. Firstly we solve $\forall i \in [1, m]$ the system of equations for the followers:

$$z_{1_i} = r_{1_i} q_{1_i} + \frac{1}{m} \left(\sum_{j=1, j\neq i}^{m} z_{1_j} - \sum_{j=1, j\neq i}^{m} r_{1_j} q_{1_j} \right) \tag{3.20}$$

$$z_{2_i} = r_{2_i} q_{2_i} + \frac{1}{m} \left(\sum_{j=1, j\neq i}^{m} z_{2_j} - \sum_{j=1, j\neq i}^{m} r_{2_j} q_{2_j} \right) \tag{3.21}$$

$$\mathcal{F}_1^s \left(\frac{1}{m} \left[\sum_{i=1}^{m} r_{1_i} q_{1_i} - \sum_{i=1}^{m} z_{1_i} \right] \right) = \beta_1 \mathcal{F}_2^s \left(\frac{1}{m} \left[\sum_{i=1}^{m} r_{2_i} q_{2_i} - \sum_{i=1}^{m} z_{2_i} \right] \right) \tag{3.22}$$

$$\frac{\partial C_{P_{1_i}}}{\partial q_{1_i}} + \frac{\partial C^s_{A_{1_i}}}{\partial q_{1_i}} = p_{1_i}(q_{1_i}) + \frac{\partial p_{1_i}}{\partial q_{1_i}} q_{1_i} \tag{3.23}$$

$$\frac{\partial C_{P_{2_i}}}{\partial q_{2_i}} + \frac{\partial C^s_{A_{2_i}}}{\partial q_{2_i}} = p_{2_i}(q_{2_i}) + \frac{\partial p_{2_i}}{\partial q_{2_i}} q_{2_i} \tag{3.24}$$

The equations(3.20 - 3.24) define a system of equations which is easy to solve since it doesn't have any complex dependencies between the variables. By solving this system we have one free variable for the sub-system of the first period (equations 3.20, 3.23) and one other for the sub-system of the second period (equations 3.21, 3.24). Equation (3.22), which exists due to the existence of banking, combines these two free variables leading to one and only free variable for the whole system of the followers.

emissions = zbau + average abatement of the rest followers

From this solution we now know the next quantities,

$$A_1 = \left(\sum_{j=1, j\neq i}^{m} z_{1_j} - \sum_{j=1, j\neq i}^{m} r_{1_j} q_{1_j} \right) \tag{3.25}$$

and

$$A_2 = \left(\sum_{j=1, j\neq i}^{m} z_{2_j} - \sum_{j=1, j\neq i}^{m} r_{2_j} q_{2_j} \right) \tag{3.26}$$

These variables represent the behavior of the followers and can be adopted by the leaders in order to solve their optimization problem and reaction to the followers' strategy.

The system of the leaders consists of the equations below:

$$\mathcal{F}_1^b(r_{1_i}q_{1_i} - z_{1_i}) = \frac{1}{m}\mathcal{F}_1^{s\prime}\left(\frac{1}{m}A_1\right)(z_{1_i} + \lambda_i) + \mathcal{F}_1^s\left(\frac{1}{m}A_1\right) \tag{3.27}$$

$$\mathcal{F}_2^b(r_{2_i}q_{2_i} - z_{2_i}) = \frac{1}{m}\mathcal{F}_2^{s\prime}\left(\frac{1}{m}A_2\right)(z_{2_i} - \lambda_i\beta_1) + \mathcal{F}_2^s\left(\frac{1}{m}A_2\right) \tag{3.28}$$

$$\begin{aligned}\mathcal{F}_1^s\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{1_i} + \sum_{i=1}^{m} r_{1_i}q_{1_i} - IA_1 + B_1\right]\right) &= \\ \beta_1\mathcal{F}_2^s\left(\frac{1}{m}\left[\sum_{i=m+1}^{n} z_{2_i} + \sum_{i=1}^{m} r_{2_i}q_{2_i} - IA_2 - B_1\right]\right)&\end{aligned} \tag{3.29}$$

$$\frac{\partial C_{P_{1_i}}}{\partial q_{1_i}} + \frac{\partial C^b_{A_{1_i}}}{\partial q_{1_i}} = p_{1_i}(q_{1_i}) + \frac{\partial p_{1_i}}{\partial q_{1_i}}q_{1_i} \tag{3.30}$$

$$\frac{\partial C_{P_{2_i}}}{\partial q_{2_i}} + \frac{\partial C^b_{A_{2_i}}}{\partial q_{2_i}} = p_{2_i}(q_{2_i}) + \frac{\partial p_{2_i}}{\partial q_{2_i}}q_{2_i} \tag{3.31}$$

In contrast with the followers system which is linear, the above one is more difficult to solve since the form of the equations on z_1 and z_2 is unclear, provided that it depends on the form of the marginal abatement cost function.

3.4 Discussion on Equations

Except from the price equation 3.8, which is a great outcome since it contains much information about the way the price changes and its dependencies, we can get some valuable theoretical results from the system of equations that was constructed.

From the followers sub-system, we see that a clear solution is

$$z_{1_i} = z_{BAU_1}$$

and

$$z_{2_i} = z_{BAU_2}$$

That means, the leaders force the followers to avoid abatement and buy all the permits they need, by manipulating the price. As a result, the price elasticity for the followers is negative, meaning that whatever the price is the followers are going to buy the permits, which is in perfect shape with what Chevallier 2011 has stated.

If the leaders choose to sell a small amount of permits that means that the followers will have to decrease their z_{BAU} emissions. This is possible only by decreasing their output product. That is an interesting part, because it means that if the leaders happen to compete with the followers at the output market, they can become leaders at this market too. Hence the leaders can transfer their market power across the two markets and gain more profit. That outcome is in perfect shape with what Hintermann 2011 and Chen and

Tanaka 2018 have stated. That is, the leaders excess market power to both permit and output market.

From the leaders sub-system, we can find another significant result. Specifically, this system has more variables than equations because of the existence of the Lagrangian multipliers, which means that the system has a degree of freedom related to the number of leading companies. We can think of these multipliers as strategy variables, meaning that a different λ value can denote a specific strategy for the company, since this is the variable that denotes the relation of the emissions (z) with the output product (q). This relation has great effect on the abatement considering that it is defined as $rq - z$. That means that the proposed model can represent different equilibrium strategies across companies, making the modeling more relevant to real life situations.

Chapter 4

Simulations

Since the theoretical model has been set and described it's time to examine how it corresponds to input data, by performing some characteristic simulations. In order to achieve this, we have to use some acceptable data in the right scale.

Due to the absence of data for the output product produced by each company, it was decided to focus on the electricity market since it was easier to find the right scale for the data. Price Waterhouse Coopers 2008 provides a list of European power companies by carbon intensity per year until 2009. In this list, data for the output product and emissions, as well as the emitting rate for a large group of companies are given. This fits perfectly to our model since it also considers the emitting rate i.e. r for the model.

Beyond the output product and the emissions, we need some system information, such as free allocation. We can get a sense of the scale of these data by using databases such as the *European Union Transaction Log (EUTL) - Union Registry* n.d., the *EEA ETS data* n.d. made by *The European Environment Agency (EEA)* n.d. and the one developed by Dimos et al. 2020. In this way we can clearly assort some leaders. For example, in 2009 Edison S.p.A. got 8.202.117 initial allowances and surrendered 6.991.207 without making a significant technological upgrade, as shown in the PWC study. The regulator, hence, supplied this company with 1.17 times more allowances than actually needed. This is in perfect line with what Hintermann 2011 and Chevallier 2011 stated about the leaders, i.e. they receive a sufficiently large permit allocation. Adding that the same year, according to PWC, Edison S.p.A.'s emitting rate r was 495 CO_2/MWh (which is considered small), we see that our criteria for the leaders, as described in chapter 3 and in Rico (1995), are fulfilled. That is, the leaders have more advanced technology, which is denoted by the emitting rate and also enjoy a generous initial allocation from the regulator.

After understanding the scale of the data, we chose ten companies from the PWC study and kept their emitting rates and output product produced. Three of them, who fulfilled the criteria, were named the leaders and the rest seven were the followers. We then had to construct some realistic data for the rest of the system variables. Trying to model the differences between allocations provided to leaders and followers we made a way of sharing the allocation. Specifically, for the first period, leaders got 120% of their z_{BAU}

Company	Production2008^1	Emission2008^2	CO2/MWh2008^3	Production2009^1	Emission2009^2	CO2/MWh2009^3
EDF Group	704	103.79	147	652	88.09	135
EDF Energy	27	21.9	805	72	23.8	330
Edison S.p.A	63	32.4	514	61	29.9	495
RWE Group	194	144.46	747	169	133.7	792
EnBW	67	17.0	254	66	15.9	241
NPower	38	25	665	27	16.6	622
E.ON Group	239	100.07	418	216	84.7	393
Enel Group	186	83	447	170	77.29	454
EDP Group	40	19.78	500	42	20.01	477
Vattenfall	178	81.72	459	175	79.05	452
CEZ	68	40.38	597	65	37.2	569
DEI	52	52.2	996	50	49.7	992
Fortum	53	2.16	41	49	2.02	41
Statkraft	53	1.6	30	57	1.6	28
Union Fenosa	18	7.26	398	29	9.48	330
Verbund	29	2.89	101	30	2.21	74
Drax	27	22.3	818	24	19.85	815
Dong	19	7.43	401	18	6.93	383
PVO	22	2.92	131	22	2.88	131

^{1}Production measured in TWh ^{2}Emissions in $MtCO_2$ $^3CO_2/MWh$ in Kg

Table 4.1: Part of Price Waterhouse Coopers study's data

emissions, while followers got 86%. Of course, as happens in the EU ETS, the second periods' overall allocation was a bit smaller (by 1.5%)

Then, it was time to create the actual emissions data. Followers were easy to handle since, as shown on chapter 3, they will always emit their z_{BAU} emissions. As for the leaders, we considered the case of them abating some of their emissions and that's why an abatement of 30% of their z_{BAU} was chosen.

As for the banking, we found what was left from the allocations balance and gave a percentage of them to every company. This percentage was related to the percentage of their output product on the total product produced.

This procedure gave the 2-period data illustrated in tables 4.2 and 4.3.

Table 4.2: Companies data produced for period 1

ID	Name	isLeader	Q	RQ	Free	Surrendered	Banking	Previous Banking
0	Leader1	True	651.999	88.02	105.624	61.614	6.049	0
1	Leader2	True	65.999	15.906	19.087	11.134	0.612	0
2	Leader3	True	71.999	23.759	28.512	16.632	0.668	0
3	Follower1	False	169.999	77.179	66.365	77.179	1.577	0
4	Follower2	False	41.999	20.034	17.227	20.034	0.389	0
5	Follower3	False	60.999	30.195	25.964	30.195	0.566	0
6	Follower4	False	64.999	36.985	31.802	36.985	0.603	0
7	Follower5	False	26.999	16.794	14.440	16.794	0.25	0
8	Follower6	False	168.999	133.848	115.093	133.848	1.568	0
9	Follower7	False	49.999	49.599	42.649	49.599	0.464	0

On the database developed by Dimos et al. 2020, and as Goulder and Schein 2013

Table 4.3: Companies data produced for period 2

ID	Name	isLeader	Q	RQ	Free	Surrendered	Banking	Previous Banking
0	Leader1	True	650.102	87.764	105.316	81.638	0	6.049
1	Leader2	True	65.809	15.859	19.032	9.734	0	0.612
2	Leader3	True	71.790	23.691	28.429	17.565	0	0.668
3	Follower1	False	169.505	76.955	66.172	76.955	0	1.577
4	Follower2	False	41.877	19.976	17.177	19.976	0	0.389
5	Follower3	False	60.822	30.107	25.888	30.107	0	0.566
6	Follower4	False	64.811	36.877	31.709	36.877	0	0.603
7	Follower5	False	26.921	16.745	14.399	16.745	0	0.250
8	Follower6	False	168.508	133.458	114.758	133.458	0	1.568
9	Follower7	False	49.854	49.455	42.526	49.456	0	0.464

has stated, we can see that a price that occurs when the system is stable and works as expected, is between 15€and 20€. So we set the price of the first period to 18€and we consider a realistic 3% discount rate across periods, which means that β equals 0.971.

4.1 Marginal Abatement Cost function

Now it is time for a more interesting part. We are going to find which Marginal Abatement Cost function will be used in the simulations. This was a challenging part since the mathematical form of the MAC function is not considered into many studies. We finally decided to use a MAC function of the type that Ellerman and Decaux 1998 developed using the MIT EPPA model, because it gives a more general approach to the dependence between abatement of emissions and its cost. This function, as used by Chevallier 2011 at his numerical simulations, has the following functional form:

$$Y = aX^2 + bX + c$$

where Y is the marginal abatement cost and X is the extent of abatement in million metric tons of carbon (Mton).

This function might have the correct form but we need to find the coefficients that correspond to our data.

If we apply the data to the equation 3.8 we take that

$$\sigma_1 = \mathcal{F}_1^s(0) \implies \sigma_1 = c \implies c = 18$$

To find the remaining coefficients, i.e. a and b, we try some combinations of them in the range of the ones used by Chevallier 2011($a \in [0.0001, 0.2]$ and $b \in [0.0, 2]$). For each combination we form a MAC function and provide our models' equations with this function and the data we generated. If the equations hold, then our data, together with the MAC function, respect the EU ETS rules and our modeling. So we can tell that we are going to use data that were generated from our equations.

Here we need to note that in order to simulate the claim of Keohane 2002 that the leaders have adopted advanced abatement technologies, as described in chapter 3, we set their MAC function (i.e. $\mathcal{F}^b$) to be 20% less than the followers' one. Remember that, as stated at chapter 3, $\mathcal{F}^b < \mathcal{F}^s$ must hold.

By following the above procedure we found more than 150 possible functions. By choosing one of them, we conclude that we are going to use the following MAC function:

$$Y = 0.1384923076923077X^2 + 0.0X + 18.0 \tag{4.1}$$

which is shown below in figure 4.1.

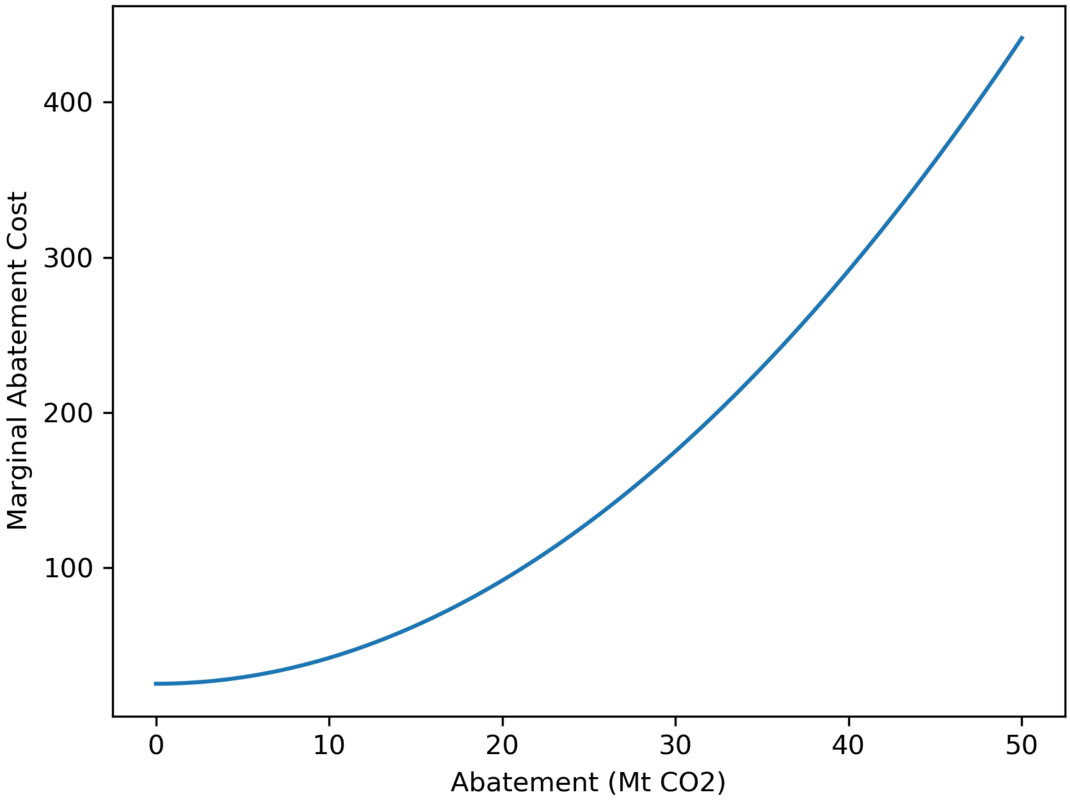


Figure 4.1: Marginal Abatement Cost function based on our data

4.2 Concept

Now that everything is set we are ready to run some simulations for the theoretical model on the data we described. But since our data is not exactly the real, the numerical outputs of the simulations will not considered significant. Therefore we will output the change rate of the variables instead of their actual value.

Specifically, our simulations are divided into two categories: variation at the initial allocation and variation at the output product. That is, we will specify a variable of interest, for example the price of the permits, and for the first category of simulations we will run the model for different initial allocation levels, while for the second one for different output product levels. After taking the value of the variable of interest we will plot it as a percentage of its value in our data. In this way we will get some insight on how our variable of interest reacts on variations of the initial allocation and of the product produced.

The choice of the two different categories was not random. It is proven that these two variables play a central role on the appropriate function of the system. Hintermann 2011 discussed how the level of initial allocation can change the balance of the system leading to non optimal system functions. On the other hand, Ellerman, Marcantonini, and Zaklan 2016 and Kettner, Kletzan-Slamanig, and Köppl 2011 discuss how the economic crisis of 2008 and the overall reduction of the output product produced by the companies, led the emissions to plump and hence the cap became not binding. Thinking of that, it would be interesting to simulate how the system reacts to changes on these important variables. This study is, as far as we know, the first that tests the stability of the system across abrupt changes in the output product.

One more innovation we made is that we examine the system under different 'strategies' for the leaders. Equations 3.27 and 3.28 contain the Lagrangian multiplier λ_i, which is a free variable for the system and ranges across all real numbers. That means, the system of equations we produced in chapter 3, has infinite solutions and each of them can be indicated by a λ_i value.

Since every solution of the equations puts the system under equilibrium, we can choose more than one strategies and compare different equilibrium scenarios. As understood, every companies' strategy has its unique λ_i value. If we want a company to adopt a specific abatement level, for example 10% of z_{BAU}, we can give as an input to equation 3.27 the value of emissions, i.e. $z_{1_i} = 0.9 z_{BAU}$, and solve for λ_i. We then pass that Lagrangian multiplier to equation 3.28 and get the emissions for the second period. In this way we get the overall emissions and product produced and we can then find the price and overall banking, which are two very important variables.

We can run the above procedure under different values of q (i.e. output product) and measure the change that a variation in product brings. That is how we are going to run the second category of simulations. As for the first one, we will keep the actual value for q, solve the system and just change the initial allocation.

Expected Outcomes

Before the simulations are made, we expect that a rise in the overall output product will, first of all, increase the business as usual emissions since, as stated, $z_{BAU} = rq$. Of course this change will lead to an increase of the output emissions, while the demand of permits will increase and the supply will be reduced leading to a rise in price. The individual surplus of permits for every company will thus decrease making them to store less permits for intertemporal trading.

On the other hand a grow of the initial allocation will have the opposite results. Specifically, the more free permits are given to the firms, the more their individual surplus will get leading to less abatement and more emissions. Also this will decrease the demand while increasing the supply causing a price drop. The individual surplus will of course escalate the banking.

4.3 Simulation 1 - Different Abatement Levels

Reason of Interest

The goal of an emissions trading system is to convince the participants to increase their abatement level. For this reason, the system will have to function better under greater abatement levels from the firms. Is this presupposition fulfilled? We are going to use our model to answer this question. So, the scope of the first simulation is to examine how different first periods' abatement levels for the leaders affect significant system variables, such as the banking, the price and the emissions.

Scenarios

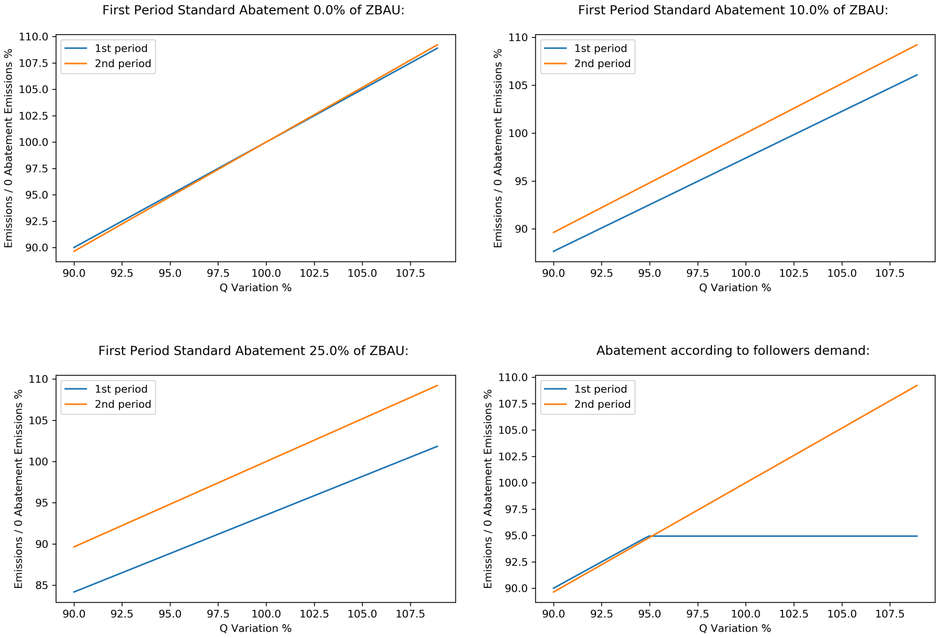
We set four different abatement levels for the leaders:

- Abatement 0% of business as usual emissions.
- Abatement 10% of business as usual emissions.
- Abatement 25% of business as usual emissions.
- Abatement according to followers demand. That is, the leaders abate in order to provide the followers with the exact amount of permits they need.

As described above for every abatement level we will find the appropriate Lagrangian multiplier for every company and solve the equations.

Expected Outcomes

Before running the simulation, we have some expectations from our experience with cap and trade systems. An instant consequence in a possible abatement level rise is, of course, a drop of the emissions, since this is the definition of abatement. This rise will increase the supply from the leaders, forcing the price to fall. This new abatement level will, in the same time, increase the individual surplus for every leader company making the number of stored permits to rise. So, overall, an increased abatement, will help reach the system's environmental goal, while ensure a good intertemporal system function by the increase of banking.


First Period Standard Abatement 0.0% of ZBAU:

First Period Standard Abatement 10.0% of ZBAU:

First Period Standard Abatement 25.0% of ZBAU:

Abatement according to followers demand:

Figure 4.2: Simulation 1 - Different Abatement Levels: Emissions as a percentage of 0 Abatement Emissions, under variation in output product

4.3.1 Variation on the output product produced(q)

Primarily we are going to examine how the variables of interest correspond to changes in the total product produced.

In order to clearly picture how different abatement strategies affect the emissions, we can plot, for every different abatement level, the overall emissions as a percentage of the emissions made under no abatement (i.e. 0%).

In figures 4.2 and 4.3 we can gain the insight we wanted. The first thing to notice is that for the three fixed abatement levels (i.e. 0%, 10% and 25%) the emissions grow as the product produced grows in both periods. This makes absolute sense since the more product is produced, the greater the $z_{BAU} = rq$ emissions are and under a fixed level of abatement, let us say l, the firm will emit $z = (1 - l)rq$ tons. That concludes that, under specified abatement levels the emissions variate analogously to the output product variation. Of course, as we expected, the overall emissions get smaller for bigger abatement levels.

Another important thing to observe is the differences between periods. As we can find out, for the second period the emissions are exactly the same for all situations. This comes as an outcome from the overallocation of allowances to the leaders. As we have discussed in chapter 3, the leaders force the followers to not make significant abatement. So every change arises at the emissions is due to abatement from the leaders. Because of the overallocation, the leaders will still have plenty of allowances left even if the outcome

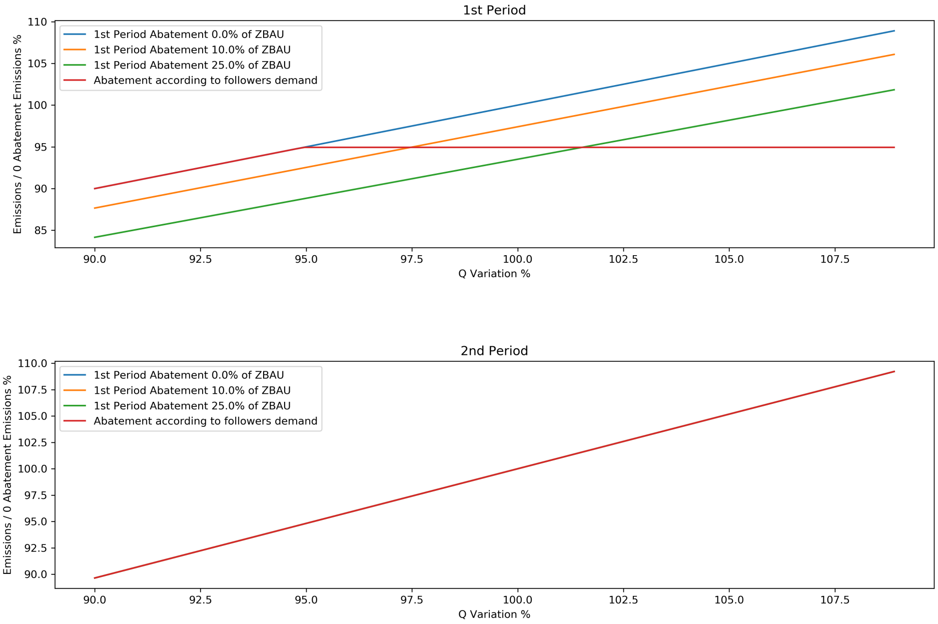


Figure 4.3: Simulation 1 - Different Abatement Levels: Comparison per period on emissions as a percentage of 0 Abatement Emissions, under variation in output product

product gets 10% greater. So since the second period is the last one and selling more permits might be a risk, they chose to not abate and sell their surplus in a greater price. The overall gain for them will be the same since we are in equilibrium.

It's important to note that while for the second period we have no abatement for all situations, the first and second periods curves are not identical in the case of 0% abatement. This is due to the reduction of the cap in the second period and the difference between the two periods' overall products.

But the biggest differences in the diagrams occur for the forth case, i.e. when the leaders abate according to the followers demand. As it is easily perceived, for less than 95% of the output product, leaders can handle all of the followers demand by only using their surplus. After that point, the followers start to need a greater number of permits (i.e. $rq - z_f$ grows), and the leaders ensure to abate as much as needed to fulfill their demand. That means that while the emissions of the followers grow, the emissions of the leaders get smaller in the same rate. The outcome is a fixed level of emissions independent on the variation in the output product. This strategy stabilizes the systems emissions to 95% of the level occurs for business as usual emissions under 100% of the product.

From the above analysis we can point out that under perfect competition the systems' emission goals would be fulfilled. On the other hand the excess of market power from the leaders raise the overall emissions.

Moving forward to banking and figure 4.4 we first have to note that the only quantity

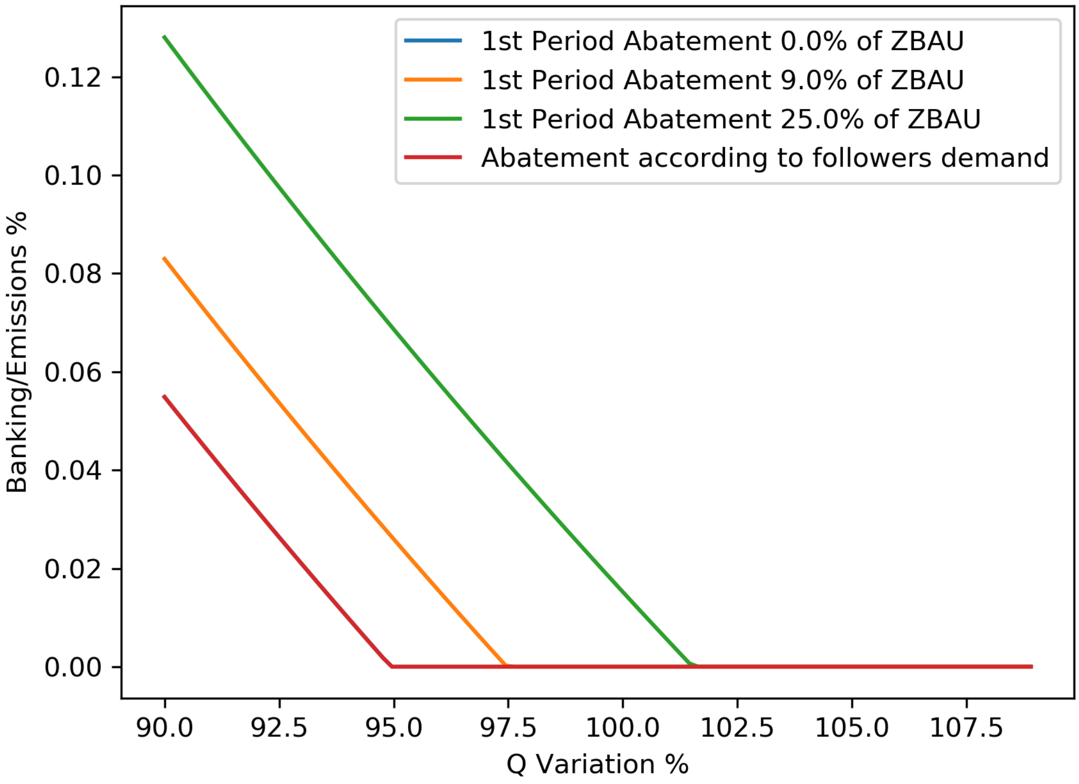
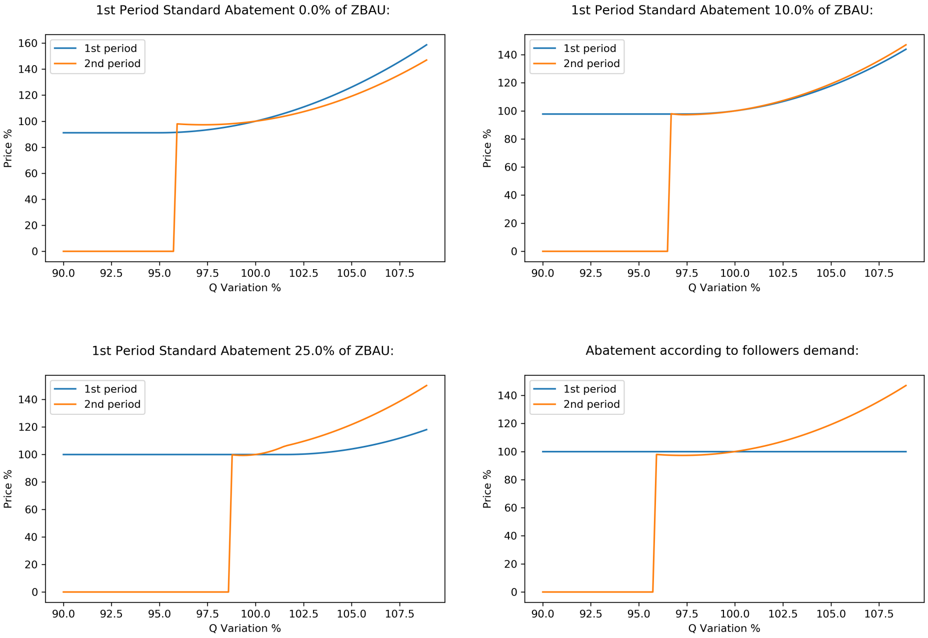


Figure 4.4: Simulation 1 - Different Abatement Levels: Banking as a percentage of Emissions, under variation in output product


1st Period Standard Abatement 0.0% of ZBAU:

1st Period Standard Abatement 10.0% of ZBAU:

1st Period Standard Abatement 25.0% of ZBAU:

Abatement according to followers demand:

Figure 4.5: Simulation 1 - Different Abatement Levels: Price variation, under variation in output product

that concerns intertemporal trading and makes sense to examine under variation in the product produced is the the percentage of banking over emissions, since the emissions change with q.

In this figure we can clearly see that while the output product gets lesser, the percentage of banking over emissions rises. That is because, while the product drops the need for permits is more and more fulfilled by the initial allocation, arriving to a point where a surplus of permits shows up, which the companies choose to bank for future use. We can say that this point occurs when the sum of the leaders overallocation's surplus with the permits saved due to abatement, equals the demand of the followers. This point is, of course, different for separate abatement levels, since the greater the abatement is the greater the surplus gets.

One last thing to notice about this figure, is the absence of the curve that corresponds to 0% abatement. In the sense of banking/emissions this case is exactly the same with the last one, i.e. when leaders abate according to the followers' demand. This is because, as stated earlier for less than 95% of the product, the leaders don't need to abate and store all of their surplus. For 95% and beyond the leaders abate in order to fullfil the demand, but they do not store the permits. That is the case for 0% too and that is why these two curves are identical.

Proceeding to figures 4.5 and 4.6 we can extract information about the system's most important aspect, the price.

Firstly, we can easily see that the greater the product produced is the greater the

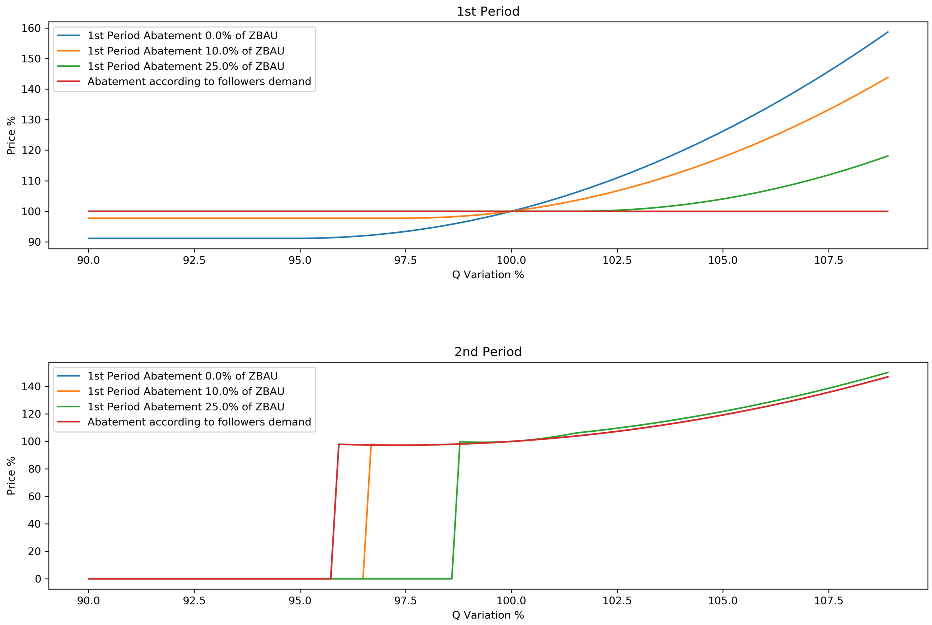


Figure 4.6: Simulation 1 - Different Abatement Levels: Price variation comparison per period, under variation in output product

price gets. That is because the allocation cap is the same, and while the product rises the demand for permits grows and pushes the price upwards. We can also see that for smaller abatement levels, the first period's price is more prone to an output product change. That means that if an incentive for abatement exists, the system will be more stable.

It was reasonable to expect that for a great drop of the amount of output product the price would drop equivalently. But this is not the case on our simulations and this is due to intertemporal trading. Banking normalizes the price across time and the consequences of big changes in system variables will be reflected to the price in the future. That is why from some point and below the second period's price equals zero. But, as we can see, this point depends on the abatement level. For bigger abatement, a greater amount of permits is banked and used in the second period, increasing the supply to the market and leading to earlier price drop. So we can say that an increase in banking lead to better system functionality in the present, but makes the system more unstable in the future.

As stated before, the leaders decide to avoid abatement in the second period. That means that from a point on, the supply of permits from the leaders will be the same regardless the abatement level at the first period, giving the same price across all different cases. Before this point we notice that the price drops abruptly to zero. Of course, as mentioned, the greater the banking the more unstable the system will be in the future. So for bigger abatement level, the price drops to zero for smaller product variances.

A rather smoother price fall to zero was expected. This abrupt change is due to the marginal abatement cost function we used, which is defined at 4.1. Since its second coefficient is zero, it doesn't take values between 0 and its third coefficient, meaning that it's not defined to take these values. If we used a different MAC function then this transition would have been smoother.However, the point still remains: the price drops aggressively to zero.

4.3.2 Variation on the initial allocation (IA)

Let us now consider variation on the free allocated permits.

By observing figure 4.7, we find out that it is almost the same with figure 4.4 but rotated by 180 degrees. That tells us that a variation in the initial allocation has exactly the opposite outcomes than a variation in the output product. This means that if there is a greater allocation, the surplus will grow, making the firms to store the extra permits and thus rising intertemporal trading.

Of course the greater the abatement is the more permits are stored. Additionally, it is easy to notice that for bigger abatement levels, the firms can fulfill their obligations given an even smaller amount of free permits.

Moving forward to a more interesting factor, let us discuss about the price, represented in figures 4.8 and 4.9. Firstly we need to mark that while the cap grows the price drops. This is because a greater allocation will grow the supply of permits from the leaders while shrink the demand from the followers. On the opposite direction, a cap reduction will downhill the supply and boost the demand causing a price rise.

We can also note that a bigger abatement level can stabilize the price, so it can become neutral in cap changes. The firms that made great abatement are more immune to smaller allocations, while they will store more permits in case of a cap rise. That behavior will keep the price stable across time in case of a cap reduction, but drop it rapidly in a cap rise.

As an overall conclusion we can point out that under a cap reduction the system is more stable for greater abatement levels, while for a cap rise it works better under small abatement levels. That is great information that can be used by the regulator. Past abatement data from the system could be applied to this type of modeling and find a good allocation level under specific criteria in order to achieve better system performance.

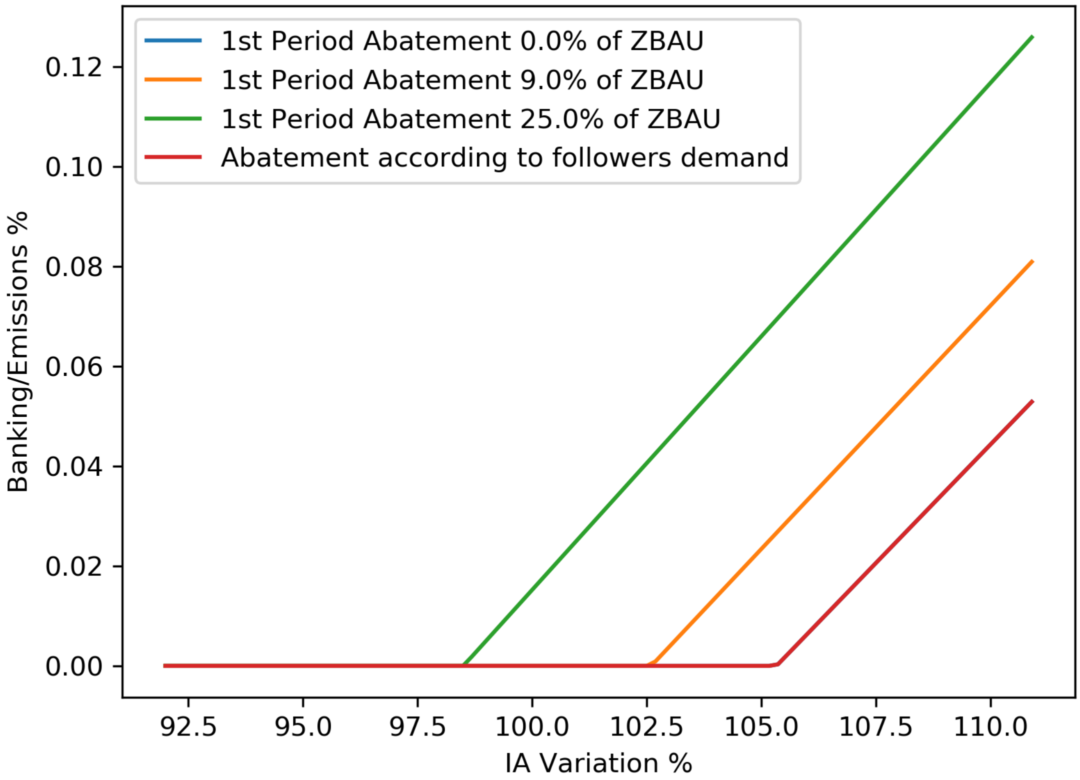
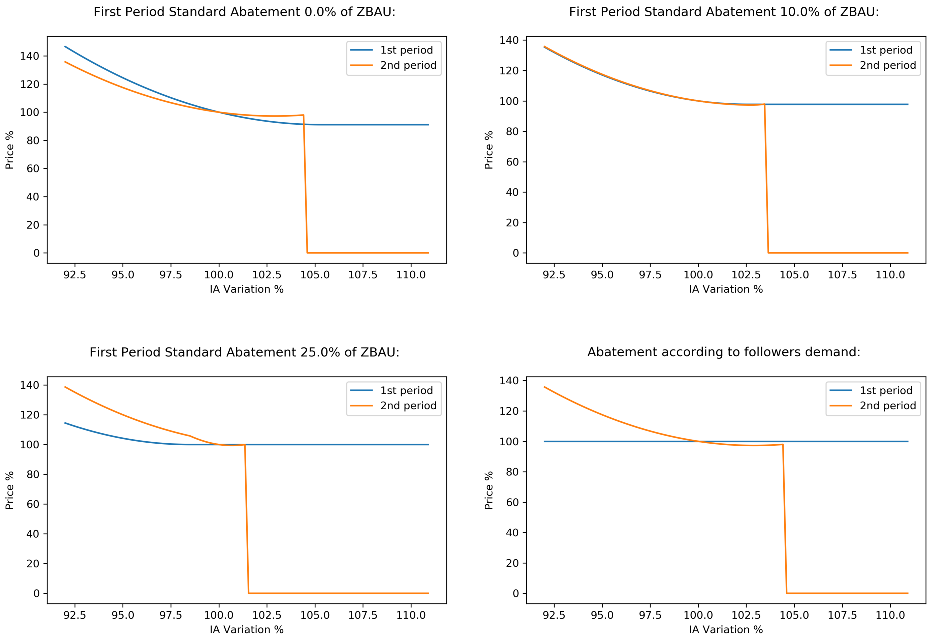


Figure 4.7: Simulation 1 - Different Abatement Levels: Banking as a percentage of Emissions, under variation in initial allocation



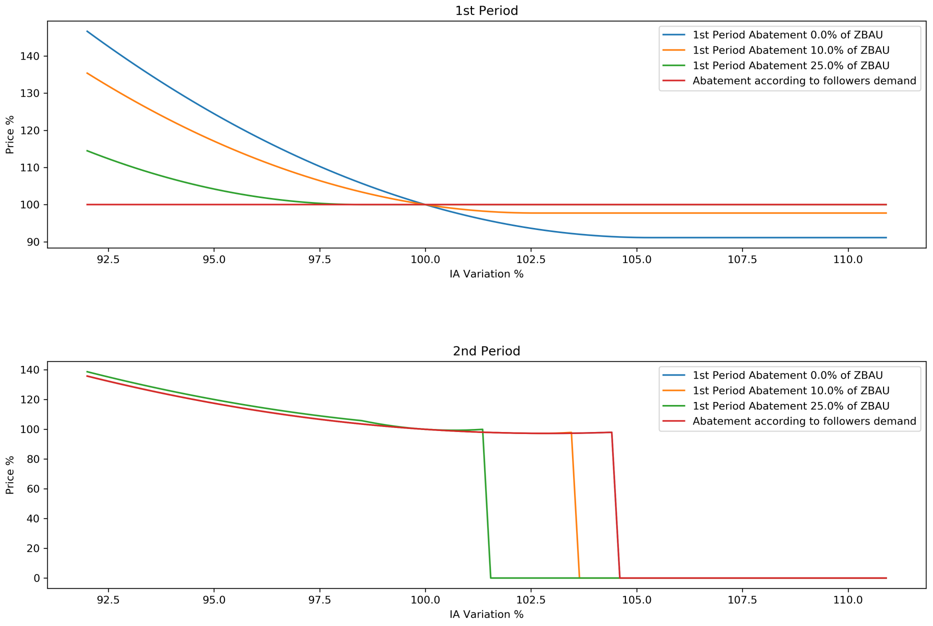
First Period Standard Abatement 0.0% of ZBAU:

First Period Standard Abatement 10.0% of ZBAU:

First Period Standard Abatement 25.0% of ZBAU:

Abatement according to followers demand:

Figure 4.8: Simulation 1 - Different Abatement Levels: Price variation, under variation in initial allocation



1st Period

2nd Period

Figure 4.9: Simulation 1 - Different Abatement Levels: Price variation comparison per period, under variation in initial allocation

4.4 Simulation 2 - Different MAC functions for fixed first period's abatement (10% of z_{BAU})

Reason of Interest

A great debate has been held about how an emissions trading system raises the costs of a firm leading to a dysfunctional output market. The cost of participating in a cap and trade system is closely related to the marginal abatement cost since this is the measure that the firms use to decide whether they are going to abate. Changing the MAC function will help us gain an insight about how it practically affects the functionality of both the permit and the output market.

In this simulation we want to examine this relationship. Of course a change in MAC affects directly the amount of emissions the companies will choose to abate, leading to closely related results on system variables.

Scenarios

In order to highlight the actual differences caused by the MAC we will fix the abatement to 10% of z_{BAU}, since , as shown above, the price changes almost with the same rate across the two periods for this level of abatement.

The four scenarios we examine are the following:

- Marginal Abatement Cost function is 80% of the original.
- Marginal Abatement Cost function is 90% of the original.
- Marginal Abatement Cost function is 100% of the original.
- Marginal Abatement Cost function is 110% of the original.

Expected Outcomes

It is proven in theory that the price of permits follows the form of the marginal abatement cost function. Having that in mind we expect that the form of the price function will not change for different MAC function levels, while the more the MAC will drop, the less the price will become.

4.4.1 Variation on the output product produced(q)

We are going to examine only the price behavior, since the emissions are expected to be the same due to the fixed level of abatement.

As we can see the price relations between the two periods is the same despite the change on the marginal abatement cost. That means that the relations are strongly connected to the abatement options.

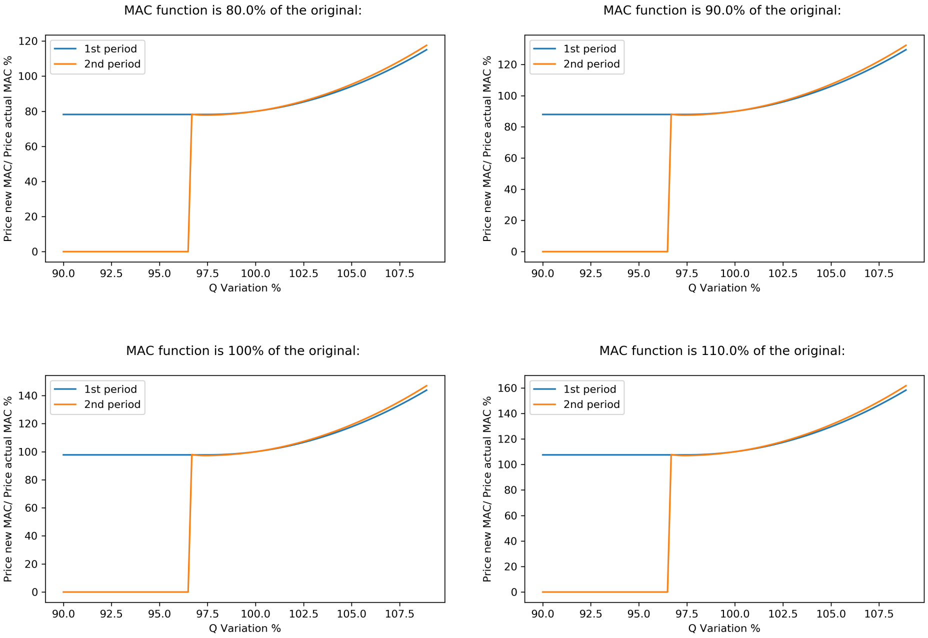


Figure 4.10: Simulation 2 - Different MAC functions for fixed first period's abatement (10% of z_{BAU}): Price under the new MAC as a percentage of the actual MAC, under variation in output product

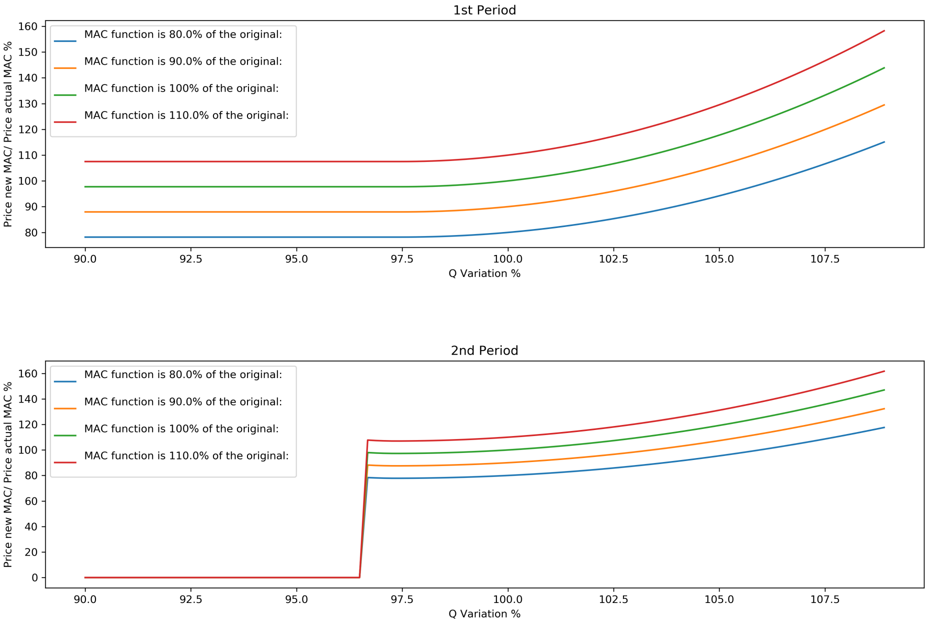


Figure 4.11: Simulation 2 - Different MAC functions for fixed first period's abatement (10% of z_{BAU}): Period comparison for price under the new MAC as a percentage of the actual MAC, under variation in output product

The main inference from the figures is that after an increase in the marginal abatement cost, the price increases equivalently. Also, as it is discussed in chapter 3, the permit price is in the form of the marginal abatement cost function.

4.4.2 Variation on the initial allocation (IA)

The simulations gave us exactly what we expected. The exact same remarks about the relation between price and the marginal abatement cost function can be made from figures 4.12 and 4.13. Additionally, as noted at 4.3.2, for smaller cap the price rises since while the supply of permits grow, the demand drops. Of course, for a greater allocation, prices are stabilized in the first period due to banking, but drop dramatically in the second one because of a drop down in demand.

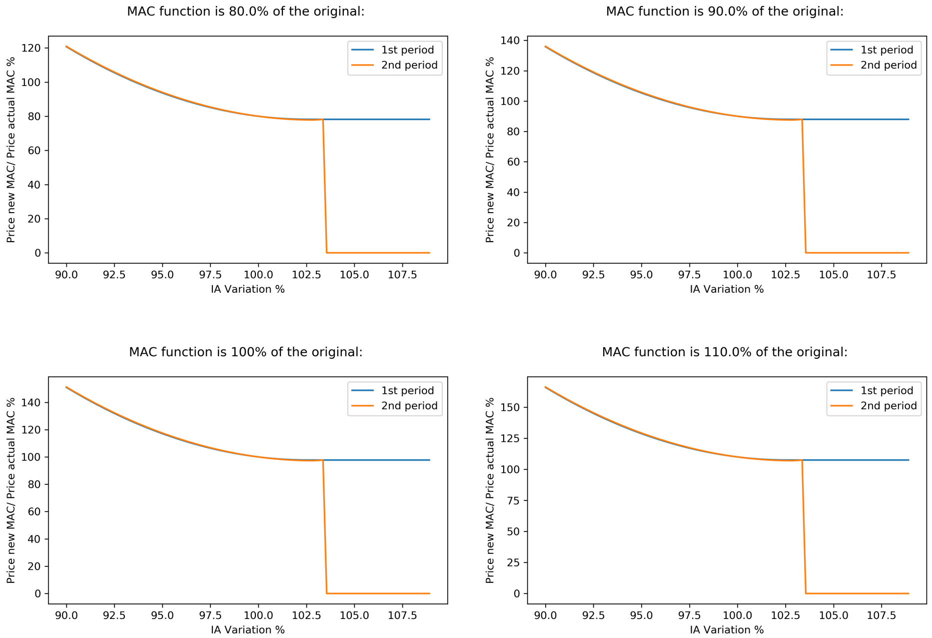


Figure 4.12: Simulation 2 - Different MAC functions for fixed first period's abatement (10% of z_{BAU}): Price under the new MAC as a percentage of the actual MAC, under variation in initial allocation

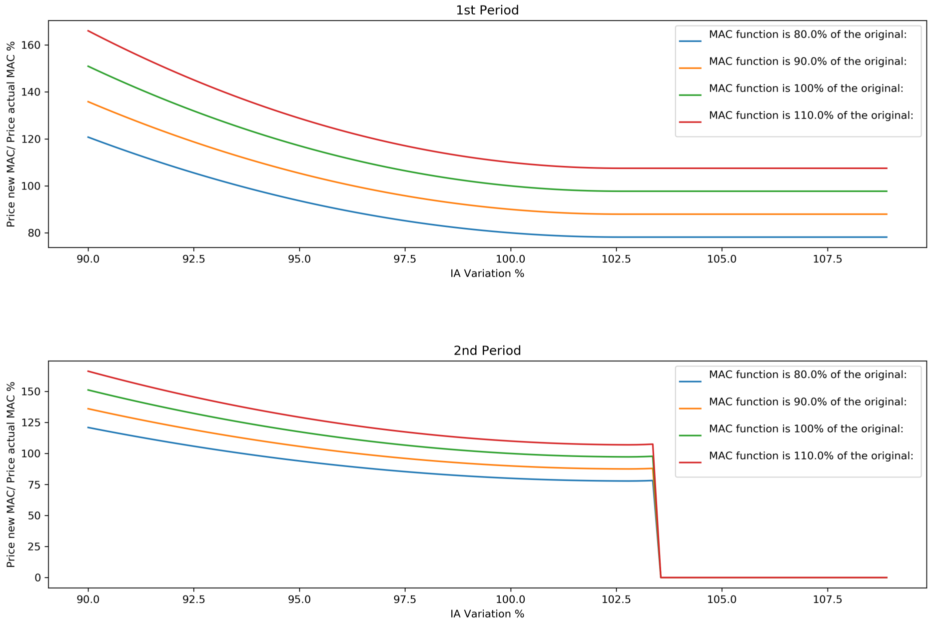


Figure 4.13: Simulation 2 - Different MAC functions for fixed first period's abatement (10% of z_{BAU}): Period comparison for price under the new MAC as a percentage of the actual MAC, under variation in initial allocation

4.5 Simulation 3 - Leaders bank at least a percentage of their free allocation for fixed first period's abatement (10% of z_{BAU})

Reason of Interest

Banking is one of the most important system variables which helps to stabilize the system against temporal rapid changes (i.e economic crisis etc). But how directly does banking affect the price over time and how much it secures a good system functionality under unpredicted events? We know that banking helps the system but we need to examine which levels make efficient changes and whether an imposed banking level to the firms would be a good idea.

Scenarios

Under the same reasoning as in the second simulation, we will fix the abatement level for the first period to 10% of z_{BAU}. The scenarios we chose to run was considering that:

- The leaders bank at least 0% of their initial allocation.
- The leaders bank at least 3% of their initial allocation.
- The leaders bank at least 5% of their initial allocation.
- The leaders bank at least 8% of their initial allocation.

Expected Outcomes

From our experience with the system we expect that a rise in banking will stabilize the price through time and help the system function better. That means that if an unpredicted event occurs, for example a drop in the output product, the banking will keep the first periods' price to a reasonable level. We, on the other hand think that for a fixed product and allocation level, a greater banking in the first period will result to a small price drop in the second.

4.5.1 Variation on the output product produced(q)

In such a simulation we expect to see constant banking levels for every different case.

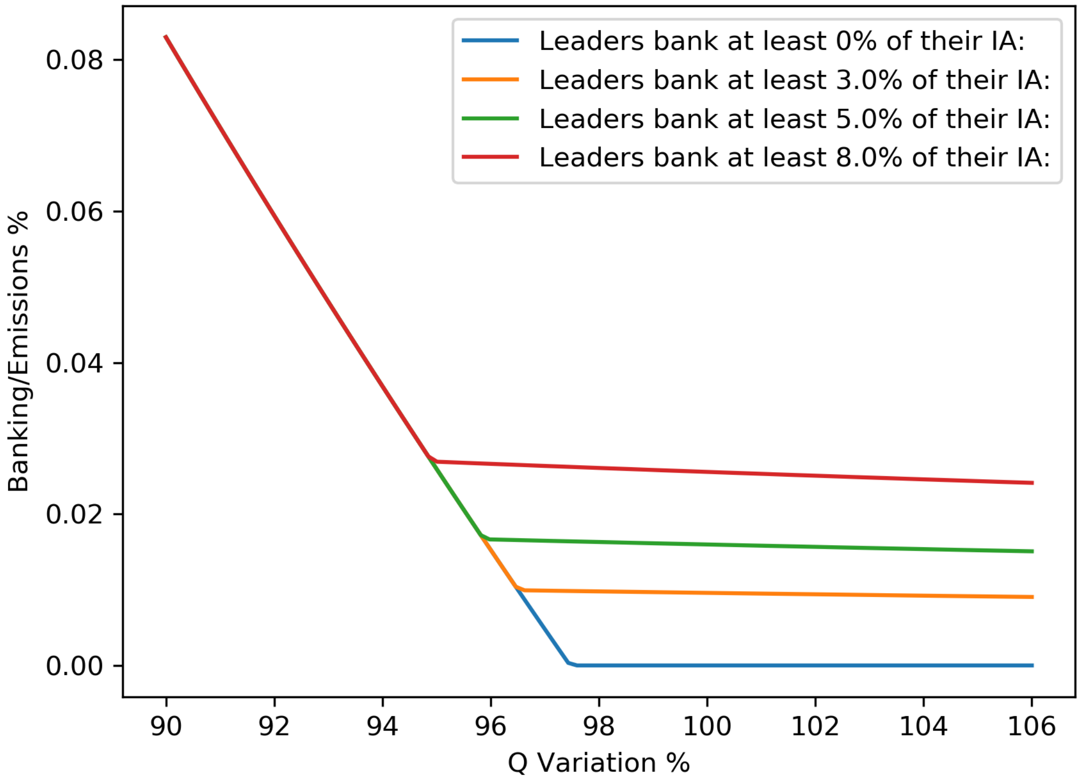


Figure 4.14: Simulation 3 - Leaders bank at least a presentage of their free allocation for fixed first period's abatement (10% of z_{BAU}): Banking as a percentage of Emissions, under variation in output product

As shown in figure 4.14 our expectations are met. Specifically, when the product is quite small, some of the overallocation's surplus is stored leading the banking to rise. After a significant advance in the product, the surplus cannot continue to fulfill the banking requirements so the leaders become to abate and sell less permits. Under this mechanism the overall banking is kept at the desired levels, no matter the increase in the product.

Let's now explore how the price reacts to different banking levels, under variation in the output product.

From 4.15 and 4.16 diagrams we can observe an interesting behavior. That is, that if there is a greater need for storing permits in the first period, the price rises with the demand. On the other hand, while the stored permits increase, the demand in the second period falls, making the price to drop. Hence is confirmed what we discussed in section 4.3.1, that is that an increase in banking lead to better system functionality in the present, normalizes the price in the future, but makes the system more unstable in future exogenous turmoils.

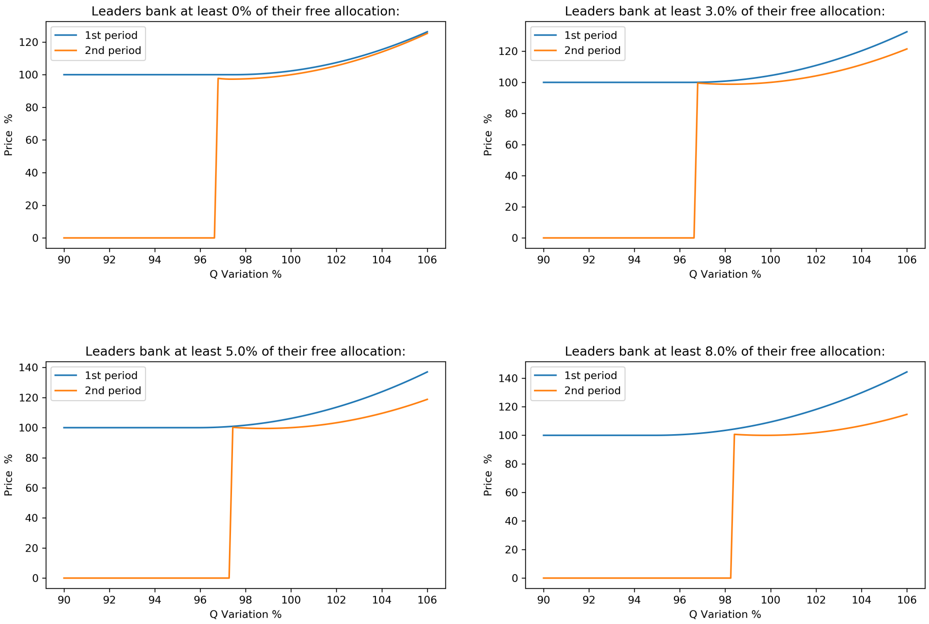


Figure 4.15: Simulation 3 - Leaders bank at least a presentage of their free allocation for fixed first period's abatement (10% of z_{BAU}): Price variation, under variation in output product

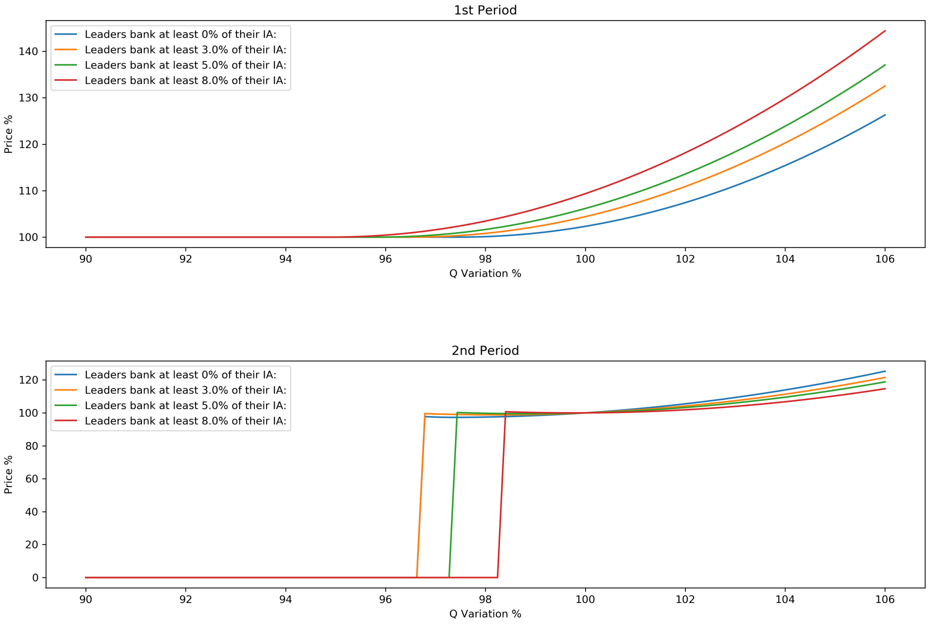


Figure 4.16: Simulation 3 - Leaders bank at least a presentage of their free allocation for fixed first period's abatement (10% of z_{BAU}): Price variation comparison per period, under variation in output product

4.5.2 Variation on the initial allocation (IA)

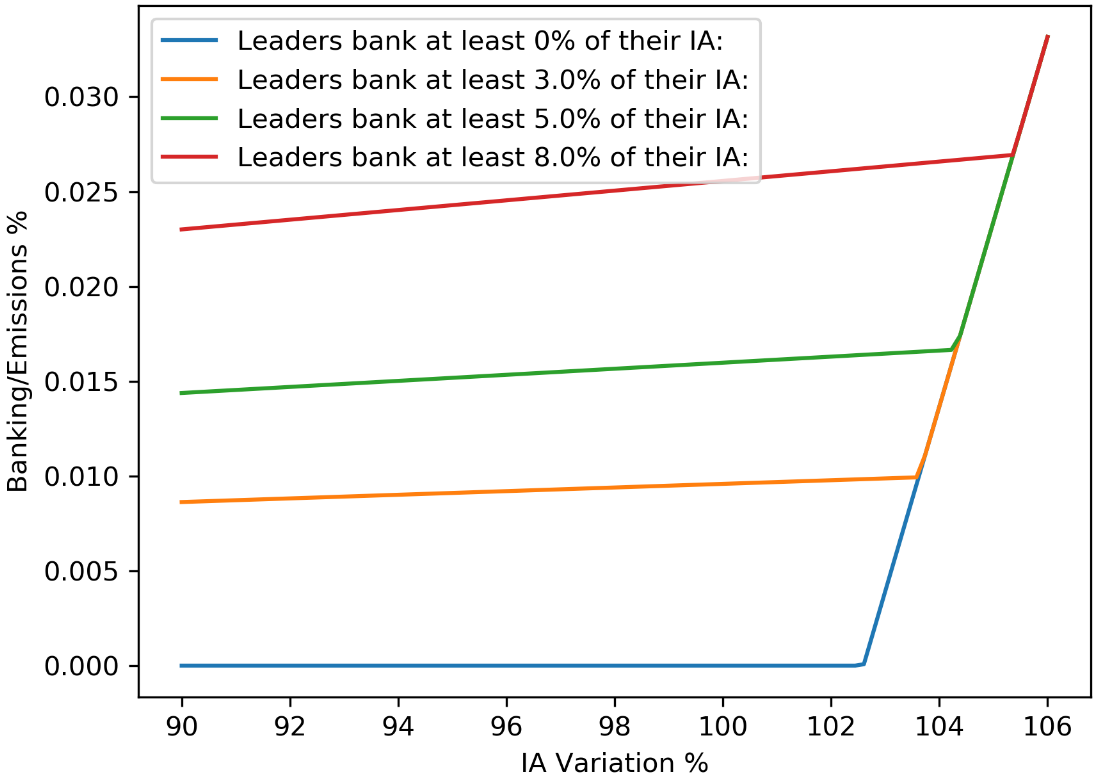


Figure 4.17: Simulation 3 - Leaders bank at least a percentage of their free allocation for fixed first period's abatement (10% of z_{BAU}): Banking as a percentage of Emissions, under variation in initial allocation

Again, in sense of intertemporal trading, we get what we expected from this simulation. As seen in figure 4.17 banking levels are the ones desired. We can also note that for a great allocation, a big surplus of permits is created forcing the firms to store the remaining allowances leading the banking to increase.

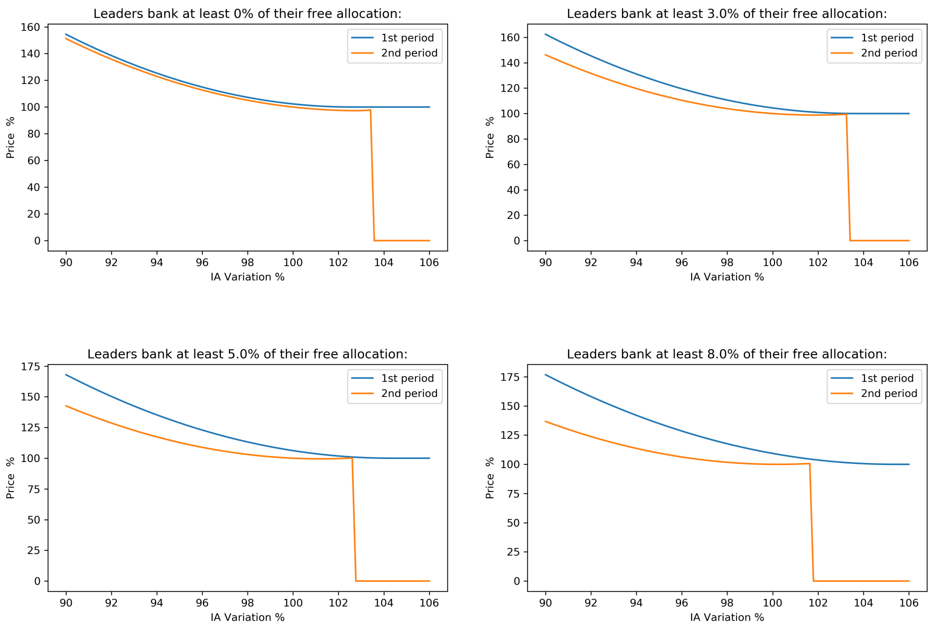


Figure 4.18: Simulation 3 - Leaders bank at least a percentage of their free allocation for fixed first period's abatement (10% of z_{BAU}): Price variation, under variation in initial allocation

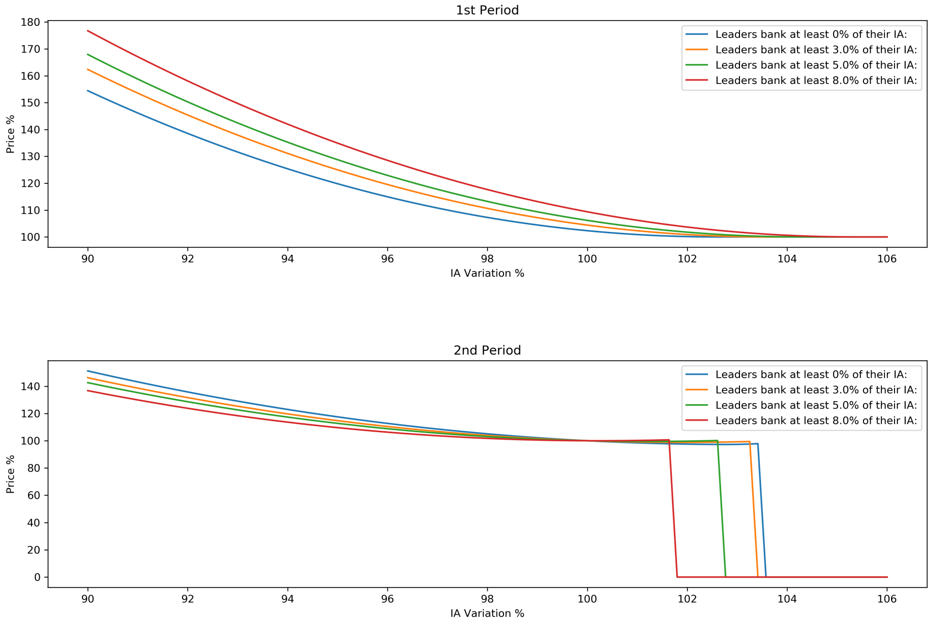


Figure 4.19: Simulation 3 - Leaders bank at least a percentage of their free allocation for fixed first period's abatement (10% of z_{BAU}): Price variation comparison per period, under variation in initial allocation

Moving on to discuss the price, as seen on figures 4.18 and 4.19 we can point out that while the banking level is increasing the demand for permits grows too, resulting in a rise in price. Another important result is that for greater allocation, a smaller banking level is needed to keep the price stable for both periods. On the other hand, the more the stored allowances are, the more stable the system becomes under in future cap reductions.

Chapter 5

Conclusions

5.1 Concluding Remarks

In this work we achieved to model an emissions trading system by taking care of many aspects that were not combined in the literature. We made simulations on the theoretical model which provided us with outcomes that are in perfect shape with the literature, indicating that the model we made is powerful enough to capture complex relations between crucial system variables.

Specifically, we dynamically modeled an emissions trading system under all of the EU ETS rules and considered the existence and excess of market power between firms on the permit market. We let the participants to interact in the output market, since this is the case in a real system and examined how the cap and the output product level can affect crucial system variables.

On the theoretical part, we found a system of equations that indicates system equilibrium. The leaders and followers have different sub-systems of equations, which combined can solve the overall problem. The free variables for each participant are the emissions and output product for each period.

In the solving process we came up with a price equation (equation 3.8), which is in the form of the marginal abatement cost of the followers and contains much information about the way the price changes and its dependencies. Specifically this function indicates that the large firms form the supply of permits and in this way they manipulate the permit price as they wish. It is also shown that the permit price will depend on the followers' business as usual emissions, the level of banking and the cap.

From the solution of the equation's system we can derive that the leaders force the followers, by manipulating the price, to avoid abatement and buy all the permits they need. As a result, the price elasticity for the followers is negative. Thus the small firms don't make significant abatement and every change arises at the emissions is due to abatement from the leaders. That means, the environmental outcome depends only from the abatement level of the leaders.

One more result indicated by the solution of the followers sub-system, is that if the leaders happen to compete with the followers at the output market, they can become leaders at this market too. Hence the leaders can transfer their market power across the two markets and gain more profit.

The next steps of our research provided us with great results, since the simulations we made were rich in information. We, first of all, saw that under specified abatement levels the emissions variate analogously to the output product variation. Of course, as we expected, the overall emissions get smaller for bigger abatement levels. For smaller levels, the first period's price is more prone to an output product change. That means that if an incentive for abatement exists, the system will be more stable.

An interesting aspect of our modeling is the existence of market power. The greatest example of excess of this market power from the leaders showed on our simulations is that at the last system's period they choose to not abate and sell their surplus in a greater price, since selling more permits might be a risk. The overall gain for them will be the same since we are in equilibrium.

We also show that when the leaders supply the followers with exactly the number of permits they need for their not-decreased output product, the system's emissions goals are fulfilled. Thus, under perfect competition the system would function better. On the other hand the excess of market power from the leaders raise the overall emissions.

Our research and simulations attach great importance on intertemporal trading. For this aspect we conclude that an increase in banking lead to better system functionality in the present, normalizes the price in the future, but makes the system more unstable in future exogenous turmoils.

We also examined the influence of different cap levels on the system performance. We found out that a cap change has the exact opposite result than an output product change. That is, when the cap increases, the system reacts in the same way as it does when the output product decreases. We also confirmed that, as expected, the more the cap grows the more the price drops. As for the relation between abatement and the free allocation, we point out that under a cap reduction the system is more stable for greater abatement levels, while for a cap rise it works better under small abatement levels.

Finally, by examining the marginal abatement cost, we confirmed that while it increases, the price increases equivalently and the permit price is in the form of the marginal abatement cost function.

5.2 Future Work

The proposed theoretical model was a first approach to model most of the important system rules and variables at once. But yet there are some improvements to make it more powerful and realistic. Specifically, the most important feature that is absent in our modeling is uncertainty. Of course the introduction of a concept like that will change the theoretical analysis, but a possible model that contains all the aspects we modeled together with uncertainty will give new boundaries to the modeling of cap and trade systems.

Rather than this demanding change, a simpler improvement can be made. Specifically, we wrote a dynamic cost function for the firms that gives their cost across many periods. However, we solved the mathematical problem for only two periods. This is not a problem since we can derive the generic outcomes by observing the two periods results, but it would be really interesting to see the form of a mathematical solution for more periods and derive more accurate observations.

Finally, it would be more than useful, but yet really challenging, to apply this type of modeling to real data and compare the resulted system behaviour with the actual one. This could help improve more the mathematical modeling of cap and trade systems and see whether these models are close to real life applications.

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