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POSTGRADUATE THESIS

Reinforced Concrete Shear Walls
A general view of their design using strut and ties

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ΕΥΧΑΡΙΣΤΙΕΣ

Θα ήθελα καταρχήν να ευχαριστήσω όλους όσους συνέβαλαν με οποιονδήποτε τρόπο στην επιτυχή εκπόνηση αυτής της μεταπτυχιακής εργασίας. Θα πρέπει να ευχαριστήσω θερμά τον κύριο Σπυλιόπουλο Κωνσταντίνο Καθηγητή του τμήματος Πολιτικών Μηχανικών του Εθνικού Μετσόβιου Πολυτεχνείου για την επίβλεψη αυτής της μεταπτυχιακής εργασίας. Ήταν πάντα διαθέσιμος να μου προσφέρει τις γνώσεις και την εμπειρία της για την βαθύτερη κατανόηση των βασικών εννοιών που διαπραγματεύεται η παρούσα μεταπτυχιακή διατριβή. Η επιμονή του, η καθοδήγηση του και κυρίως η ακεραιότητα του χαρακτήρα του αποτέλεσαν τους βασικούς πυλώνες για την ορθή διεκπεραίωση της εργασίας μου.

Επίσης, θα ήθελα να ευχαριστήσω θερμά τους καθηγητές της σχολής του Τμήματος Πολιτικών Μηχανικών του ΕΜΠ που με καθοδήγησαν τα δύο αυτά χρόνια του μεταπτυχιακού στο πολύ ενδιαφέρον και ευρύ αντικείμενο του πολιτικού μηχανικού.

Σε αυτό το σημείο θέλω να αναφέρω ανθρώπους, εκτός του στενού ακαδημαϊκού περιβάλλοντος, που υπήρξαν σημαντικοί πόλοι στη ζωή μου, προσδίδοντας την απαιτούμενη ισορροπία. Βέβαια, το μεγαλύτερο ευχαριστώ το οφείλω στους γονείς μου, των οποίων η πίστη στις δυνατότητες μου αποτέλεσε αρωγός σε όλους τους στόχους και τα όνειρά μου ως τώρα καθώς και στον σύζυγο μου που όλα αυτά τα χρόνια υπομένει και στηρίζει τις επιλογές μου.

Την παρούσα εργασία την αφιερώνω στο γιο μου Γιώργο.

Νοέμβριος, 2020

Νάνου Λαμπρινή

ΠΕΡΙΕΧΟΜΕΝΑ

CHAPTER 1 – WALLS AND SHEAR WALLS

1. DEFINITIONS—WALLS AND WALL LOADINGS

ACI Code Section 2.2 defines a wall as follows: “Wall—Member, usually vertical, used to enclose or separate spaces.”

This definition fails to consider the structural actions of walls. ACI Section 2.1 also defines the term “structural walls”: “Structural wall—Wall proportioned to resist combinations of shears, moments, and axial forces. A shear wall is a structural wall.”

Major factors that affect the design of structural walls include the following:

(a) The structural function of the wall relative to the rest of the structure.

• The way the wall is supported and braced by the rest of the structure.

• The way the wall supports and braces the rest of the structure.

(b) The types of loads the wall resists.

(c) The location and amount of reinforcement.

Two frequent characteristics of walls are their slenderness, height to thickness ratio, which is generally higher than for columns, and the reinforcement ratios, generally about a fifth to a tenth of those in columns.

Types of Walls

Structural walls can be classified as:

(**a**) **Bearing walls**—walls that are laterally supported and braced by the rest of the structure that resist primarily in-plane vertical loads acting downward on the top of the wall, as shown in Fig. 1a. The vertical load may act eccentrically with respect to the wall thickness, causing weak-axis bending.

(**b**) **Shear walls**—walls that primarily resist lateral loads due to wind or earthquakes acting on the building are called shear walls or structural walls. These walls provide lateral bracing for the rest of the structure, presented in Fig. 1b. They resist gravity loads transferred to the wall by the parts of the structure tributary to the wall, plus lateral-loads (lateral shears) and moments about the strong axis of the wall.

(**c**) **Nonbearing walls**—walls that do not support gravity in-plane loads other than their own weight. These walls may resist shears and moments due to pressures or loads acting on one or both sides of the wall. Examples are basement walls and retaining walls used to resist lateral soil pressures, Figs. 1c and 1d.

(**d**) **Tilt-up walls**—are very slender walls that are cast in a horizontal position adjacent to the structure. They are then tilted into their intended vertical position and fastened to the foundation, to the roof or floor diaphragm, and to the adjacent panels. They are designed to resist vertical and lateral loads.

(**e**) Although they are not walls as such, **plates that resist in-plane compression**, such as the compression flanges or the decks of box girder bridges, display some of the characteristics of walls, Fig. 1e.

1.1 ONE-WAY AND TWO-WAY WALLS

Walls may be supported and restrained against lateral deflections along one to four sides. Cantilever retaining walls are generally supported solely along the lower edge of the wall. Such walls act as vertical flexural cantilevers that resist lateral loads from the adjoining soil.

Bearing walls are generally laterally supported and restrained against deflection along two opposite sides, usually the top and bottom supports. Cantilever retaining walls and bearing walls transfer load in one direction: to supports at the top and bottom of the wall, for example, or to supports at the east and west edges. In the terminology used for one-way and two-way slabs, these are referred to as one-way walls. One-way walls may be designed as wide columns spanning between the top and bottom supports, using ACI Code Chapters 10 and 11, or they may be designed using ACI Code Chapter 14.

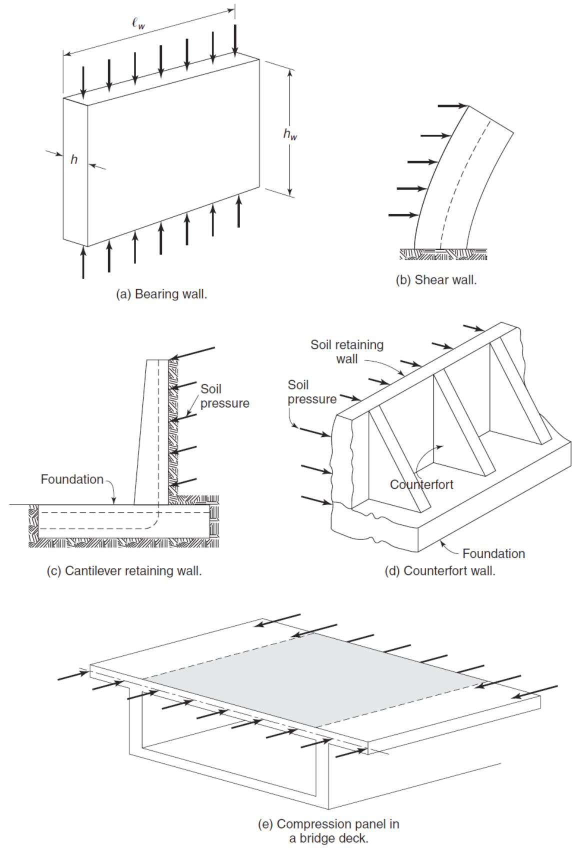


Fig. 1. Types of walls

Walls that transfer load in more than one direction are called two-way walls. Walls supported on three sides may occur in open-topped, rectangular tanks; in storage bins for bulk materials; or in counterfort retaining walls, Fig. 1d. Walls supported on four sides are used to resist forces or pressures applied perpendicular to the walls.

1.2 WALL ASSEMBLIES

Shear walls may be planar walls, standing in one vertical plane, or three-dimensional assemblies of planar walls or wall segments. The latter occur as elevator shafts in buildings where four or more vertical walls enclose a stairwell or a group of elevators. Figure 2 is a photograph of a wall assembly that will enclose an elevator shaft and will brace a steel frame building that will be built around it. The frame will be attached to the wall assembly by welding or otherwise fastening the frame to steel plates embedded in the wall concrete during construction of the walls.

In the design of a wall assembly, it is necessary to consider the transfer of shear forces from the wall segments serving as webs of the assemblies, to wall segments which act as flanges.

1.3 NOTATION

The orientation of the walls in a vertical direction leads to ambiguity in the notation for height and width. While most of the notation will be defined where it is first used, a few key symbols are defined here. Some of these are illustrated in Fig. 1a.
h_z - is the height of a location in a building
h_b - is the height of a beam

Fig. 2. Wall assembly in a building under construction

2. BEARING WALLS

Walls used primarily to support gravity loads in buildings are referred to as bearing walls. Design is by ACI Code Section 14.5, which was derived specifically to apply to walls subjected to axial loads and moments due to the axial loads acting at an eccentricity of one-sixth of the thickness of the wall from the midplane of the wall. The resulting moments are referred to as weak-axis bending moments. ACI Code Section 14.4 allows the design of bearing walls to be carried out either by:
1. Using the one-way column design and slenderness requirements in ACI Code Sections 10.2, 10.3, 10.10, 10.14, 14.2, and 14.3, or
2. The so-called empirical design method in ACI Code Section 14.5.

Walls with strong-axis moments and significant in-plane shear forces acting parallel to the wall, referred to as shear walls or structural walls, are not covered by ACI Chapter 14, although the code does not specifically state this. Shear walls will be discussed in Section 5.

Axial-Load Capacity—ACI Eq. (14-1) was based on the results of 54 wall tests reported by Oberlender and Everard [1]. Their test results showed no effect of the reinforcement ratios. Concentrically loaded test specimens with $h_w/h=28$ and eccentrically loaded specimens with $h_w/h = 16$ or more, exhibited buckling failures.

About the same time, Kripanarayanan [2] discussed the strength of slender walls based on analytical studies. He observed that reinforcement amounts of $\rho = 0.75$ to 1.0% were needed for the reinforcement to affect the failure loads of slender walls.

Equation (1) (ACI Eq. 14-1) was derived in a two-step procedure. First, the capacity of a short wall was derived. Then this was multiplied by a factor reflecting the effects of slenderness on the axial-load capacity.

The largest eccentricity at which a load can be applied to a plain concrete wall without developing tensile stresses is at one-sixth of the wall thickness from the mid-thickness of the wall (at the central core or kern point of the section). This load case can be approximated by a rectangular stress block extending from the compressed face of the wall for a distance of 2/3 of the thickness of the wall as shown in Fig. 3. The force per horizontal length l_w of wall, is:

$$P_{n,short} = 0.85f_c' \times \frac{2}{3} h \times l_w = 0.567 f_c' \times h \times l_w$$

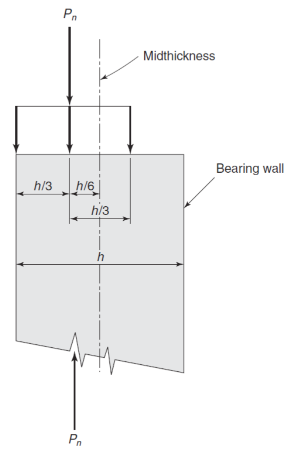


Fig. 3. Cross section through the top of a bearing wall

This was rounded off to $0.55f_c' \times h \times l_w$ and then multiplied by the term in the square brackets in Eq. (1) to account for the slenderness of the wall. The slenderness term was derived to give reasonable agreement with the slenderness effects in ACI Code Section 10.10. The equation for the axial-load capacity of a bearing wall is:

$$\phi P_n = 0.55 \phi f_c' A_g \left[1 - \left(\frac{k l_c}{32h} \right)^2 \right] \qquad (1)$$

(ACI Eq. 14-1)

where:
l_c – is the clear, vertical distance between lateral supports,
k – is the effective length factor for a wall, taken as:

0.8 if the wall is braced against translation at both ends and the top or bottom (or both) is restrained against rotation

1.0 if both ends are effectively hinged

2.0 for walls which are not effectively braced against lateral translation at the top, and therefore must be considered to be free-standing

h – is the overall thickness of the wall
ϕ – is the strength-reduction factor for compression-controlled sections, taken equal to 0.65.

Thickness, Reinforcement and Sustained Loads—Equation (1) is not affected by the amount of wall reinforcement and does not allow for creep under sustained axial loads.

This is in contrast to design by ACI Code Chapter 10. ACI Code Section 14.5.3.1 limits the minimum thicknesses of walls designed using the so-called empirical design method to 1/25 of the unsupported height or length of the wall, whichever is shorter, but not less than 4 in. ACI Code Chapter 14 does not require wall reinforcement to be designed for the loads on the wall. Instead, ACI Code Sections 14.3.2 and 14.3.3 give the minimum vertical and horizontal reinforcement ratios.

These reinforcement ratios can be written in terms of the maximum spacing of the bars. Thus, for No. 5 or smaller bars with f_y not less than 60 ksi, the maximum horizontal and vertical spacings are as follows:

Vertical steel:

$$s_{h,max} = A_v / (0.0012h) \tag{2a}$$

Horizontal steel:

$$s_{v,max} = A_h / (0.002h) \tag{2b}$$

If the reinforcement is in two layers, A_v is the total area of vertical bars within the spacing s_h, and similarly for the horizontal bars.

ACI Code Section 14.3 requires more reinforcement horizontally than vertically. This reflects the greater chance that vertical cracks in walls might form as a result of restrained horizontal shrinkage or temperature stresses, compared with a lower chance that horizontal cracks will form as a result of restrained vertical stresses. Generally, if shrinkage occurs in the vertical direction, the shrinkage stresses are dissipated by vertical compression stresses in the wall.

SOLVED EXAMPLE – Compute the Capacity of a Bearing Wall

A wall with a vertical height between lateral supports of 16 ft and a horizontal length of 25 ft between intersecting walls supports a uniformly distributed factored gravity load of 41 kips/ft, including the self-weight of the wall. The wall is supported on a strip footing that prevents lateral movement of the bottom of the wall. The wall supports a wooden-frame roof deck, which acts as a diaphragm to restrain lateral displacement of the top of the wall. Is an 8-in.-thick wall adequate if f_c' = 4000 psi? If so, select reinforcement for the wall. Use the load and resistance factors from ACI Code Sections 9.2 and 9.3.

***Step 1** – Check whether the wall thickness is enough:*

*ACI Code Section 14.5.3.1 limits the thickness of walls designed by the empirical design method to the larger of 4 in. and 1/25 of the shorter of the unsupported height or the length. Thus, the minimum thickness is (16×120/25 in. = 7.68 in. An 8-in.-thick wall satisfies the minimum thickness given in ACI Code Section 14.5.3.1. **Use an 8-in. wall**.*

***Step 2** – Compute the capacity of a 1 foot-wide strip of wall:*

$$\phi P_n = 0.55\phi f_c' A_g \left[1 - \left(\frac{kl_c}{32h}\right)^2\right]$$

ACI Code Section 14.5.2 gives k = 1.0 for the end restraints described in the statement of the problem. Walls will seldom have spiral reinforcement. As a result, the wall is an "other" type of member, and ACI Code Section 9.3.2.2 specifies ϕ = 0.65. Thus:

$$\phi P_n = 0.55 \times 0.65 \times 4000\text{psi} \times 8\text{in.} \times 12\text{in.} \left[1 - \left(\frac{1.0 \times 16 \times 12}{32 \times 8}\right)^2\right]$$

$$\phi P_n = 137000\text{lb} \times [0.438] \text{ per foot of wall length}$$

$\phi P_n = 60.1 \text{kips/ft}$

This value exceeds the applied factored gravity load of 41.0 kips per ft; thus, the wall has adequate capacity.

Step 3 *– Select the reinforcement*

ACI Code Sections 14.3.2 and 14.3.3 require minimum areas of 0.0012A_g and 0.0020A_g for vertical and horizontal reinforcement, respectively. ACI Code Section 14.3.4 allows the reinforcement to be placed in one layer, or "curtain," because the wall thickness is less than 10 in. ACI Code Section 14.3.5 gives the maximum bar spacing parallel to the wall as the smaller of 3 times the wall thickness—3 times 8 in. = 24 in. —and an upper limit of 18 in. The maximum spacing of the reinforcement is as follows:

Horizontal spacing of vertical reinforcement—from ACI Code Section 14.3.2: $s_{h,max} = A_v / (0.0012h)$

If the required vertical steel is placed in a single layer of vertical No. 4 bars, A_v = 0.2 in.^{2} and the spacing is:

$s_{h,max} = 0.2/(0.0012 \times 8) = 20.8$ *in. on centers. It is advisable to use 18 in. on centers.*

Vertical spacing of horizontal reinforcement—from ACI Code Section 14.3.3: $s_{v,max} = A_h / (0.002h)$

Using a single layer of No. 4 bars, the spacing of the minimum horizontal reinforcement is: $s_{v,max} = 0.2/(0.002 \times 8) = 12.5$ *in. on centers. It is advisable to use 12 in. on centers.*

Because the vertical reinforcement is not specifically used in the strength design, ACI Code Section 14.3.6 does not require the vertical bars to be tied in non-seismic regions provided that:
(a) *the area of vertical steel is less than 0.01 times the gross area of the wall, or*
(b) *the steel is not used as compression steel.*

The vertical steel provided has $A_s = 0.2\text{in.}^2 / (8 \times 18)\text{in.}^2 = 0.0014$ *times the gross area of the wall. It is good practice to provide a No. 5 bar vertically at each end of each curtain of wall steel.*

3. RETAINING WALLS

Reinforced concrete walls are used to resist the horizontal soil pressures when the surface of the ground is higher on one side of the wall than on the other. These are called retaining walls. The most common type is the cantilever retaining wall shown in Fig. 1c, which consists of a vertical cantilever wall that resists horizontal earth pressure and a footing that resists the moments at the base of the cantilever and transfers the forces to the ground. ACI Code Section 14.1.2 specifies that flexural design of retaining walls should be in accordance with the flexural design provisions of ACI Code Chapter 10, with minimum horizontal wall reinforcement in accordance with ACI Code Section 14.3.3.

The lateral soil pressure acting on the wall and the bearing capacity of the soil under the wall footing are obtained by consultation with a geotechnical engineer, especially if the wall resists surcharge loads acting on the upper ground surface. It is important to provide drainage through or around the wall to minimize hydrostatic pressure behind the wall.

4. TILT-UP WALLS

Industrial buildings and warehouses are frequently constructed from tall thin concrete walls that are cast in a horizontal position on a slab on grade that will become the floor slab for the completed building. After the walls have gained adequate strength, a crane is used to tilt them up to their final vertical position at which time they are connected together. Such walls and buildings are referred to as tilt-up walls and tilt-up buildings. These constitute a form of construction that is becoming common throughout North America. Most aspects of the design of tilt-up buildings is covered by the report of the ACI Committee 551, Tilt-up Concrete Construction [18-3]. ACI Code Section 14.8, Alternative Design of Slender Walls, presents design requirements for the very slender walls used in tilt-up construction.

Guidance is also found in the Concrete Design Manual [4]. Special consideration should be given to construction loads acting on these walls. Nathan [5] analyzed thin one-way walls and the design charts that he proposed are widely used in the design of tilt-up buildings.

5. SHEAR WALLS

The term shear wall is used to describe a wall that resists lateral wind or earthquake loads acting parallel to the plane of the wall in addition to the gravity loads from the floors and roof adjacent to the wall. Such walls are referred to as structural walls in ACI Code Chapter 21.

The strength and behavior of short, one- or two-story shear walls, as shown in Fig. 4a, are generally dominated by shear. These walls typically have a height-to length aspect ratio of less than or equal to 2 and are called short or squat walls.

Such walls can be designed by either the requirements given in ACI Code Chapter 11 or the strut-and-tie method given in ACI Code Appendix A. If the wall is more than three or four stories in height, lateral loads are resisted mainly by flexural action of the vertical cantilever wall (Fig. 4b) rather than shear action. Shear walls with h_w/l_w greater than or equal to 3 are referred to as slender or flexural walls. These walls typically are designed using the provisions of ACI Code Chapters 10 and 11. Shear walls with ratios h_w/l_w between 2 and 3 exhibit a combination of shear and flexural behavior and normally would be designed following the provisions of ACI Code Chapters 10 and 11.

Although shear walls may be simple planar walls, several wall segments are commonly connected together to act as a three-dimensional unit. Such wall assemblies have regular or irregular C, T, L, or H-shaped cross sections with webs and flanges that may enclose spaces in buildings, such as stair wells or elevator shafts.

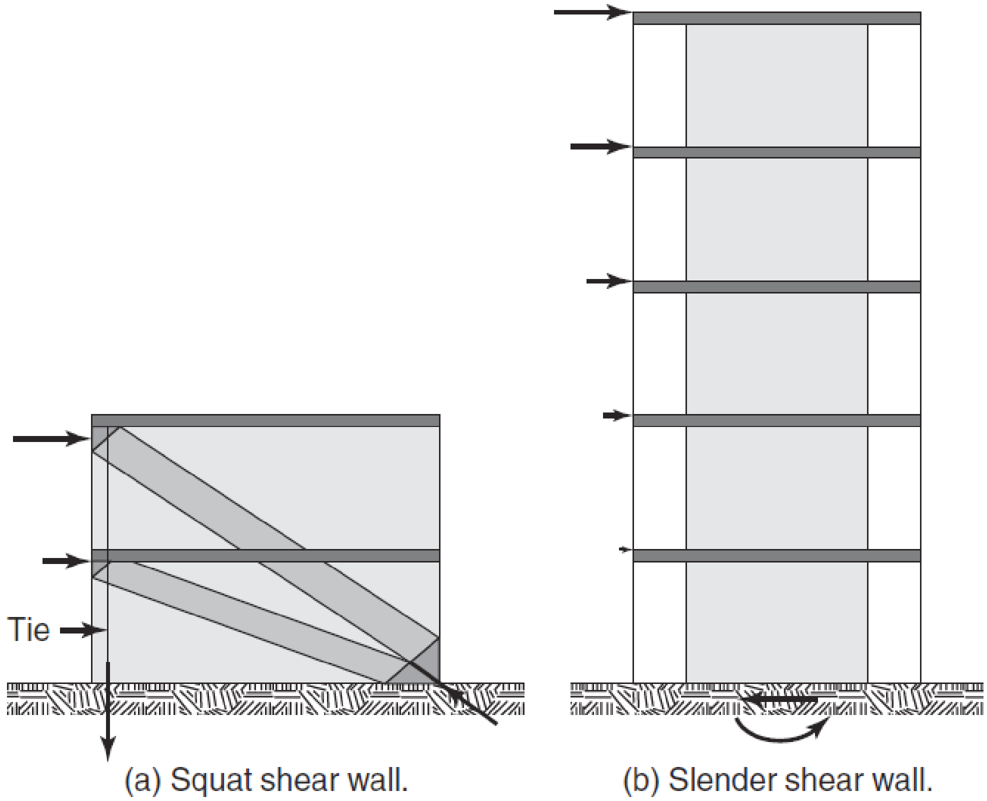


Fig. 4. Shear walls

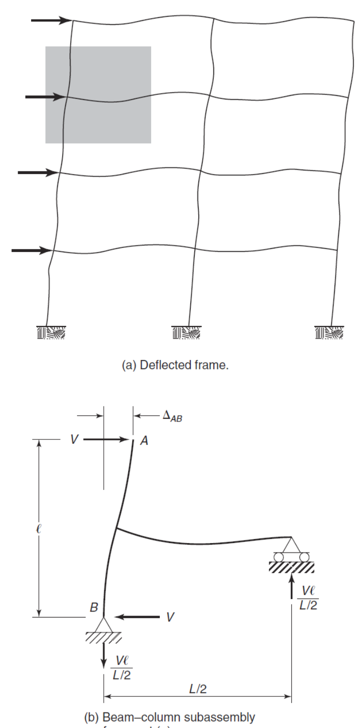


Fig. 5. Moment resisting frames

6. LATERAL LOAD-RESISTING SYSTEMS FOR BUILDINGS

Three common systems for resisting wind or earthquake lateral loads are presented in this section.

6.1 MOMENT-RESISTING FRAMES

Are made up of interconnected beams and columns. Lateral loads are resisted by bending of the beams and columns, shown in Fig. 5. Such frames undergo relatively large lateral deflections. Typically, the deflected shape of a moment-resisting frame is as shown in Fig. 5. If all stories have beams and columns with sizes proportional to the shear in the story, the lateral deflection of each story would be similar. To simplify construction, however, the sizes of the beams and columns selected for the lower stories are commonly used throughout, or are changed only every third or fourth story. Hence, the beams and columns in a building tend to be oversized in the upper stories. Moment-resisting frames are used for buildings up to 8 to 10 stories.

In a moment-resisting frame, deflections of the columns and beams both contribute to the sway deflections of the frame. If we isolate the shaded portion of the frame in Fig. 5, the deflection of A relative to B is given by:

$$\Delta_{AB} = \frac{Vl^3}{24E_cI_c} + \frac{Vl^2L}{24E_bI_b} \tag{3}$$

where L is the span of the beam, center to center of supports, and l is the height of the column from mid-height of the story above the beam to mid-height of the story below the beam. The first term on the right-hand side of Eq. (3) is the deflection due to the bending of the column whereas the second term is due to the bending of the beam. Shear deformations are not included in Eq. (3) because they are very small compared to bending deformations in a typical frame member.

Assuming $L = 2l$ and $E_cI_c = E_bI_b$ the portion of the relative horizontal deflection due to bending of the beam is 2/3 of the total deflection. Frequently in tall frames, it may be impossible to make the beams stiff enough to prevent large deflections. In such cases, walls or other stiffening elements are used to control lateral deflections.

6.2 BEARING-WALL SYSTEMS

Are used for apartment buildings or hotels. A bearing-wall building has a series of parallel transverse shear walls between rooms or apartments. The walls resist lateral loads by flexural action and deflect as vertical cantilevers.

6.3 SHEAR-WALL–FRAME BUILDINGS

Shown schematically in Fig. 6, are used in buildings ranging from about 8 to about 30 stories. The lateral load is resisted in part by the wall and in part by the frame. Some of the slenderest shear-wall–frame structures ever built are described by Grossman [6]. He presents some of the wind modeling rationale and two case histories of buildings with heights up to 10 times the least width at ground level.

6.4 VERY TALL CONCRETE BUILDINGS

Structural systems for very tall concrete buildings are discussed in [7]. One such systems, presented in [7], consists in closely spaced columns in the upper stories transfer vertical loads much like a continuous closed tube would. At the top of the second floor the vertical loads are transferred to a continuous deep beam called a transfer beam. It, in turn, transfers the vertical loads to the 10 large columns on the perimeter of the ground floor. In this region the more-or-less uniform compression in the tube is disrupted.

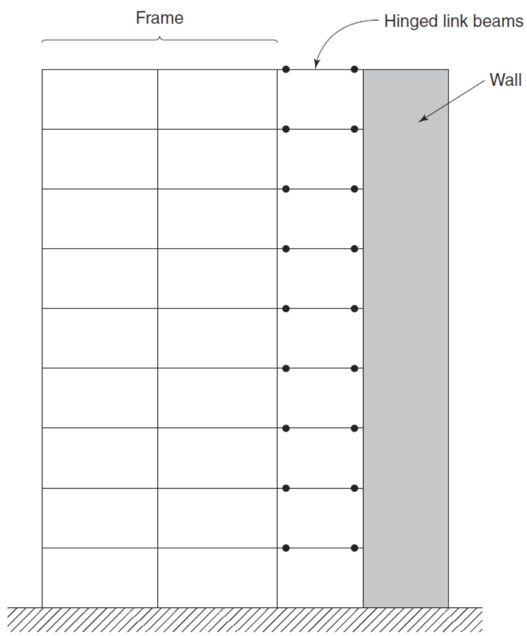


Fig. 6. Analytical model of a shear wall – frame building

7. SHEAR-WALL–FRAME INTERACTION

The division of lateral load between the wall and frame in a shear-wall–frame building can be analyzed by using a frame analysis and a model similar to that shown in Fig. 6. The frame members in the model represent the sums of the stiffnesses of the columns and beams in the building in the bays parallel to the plane of the wall. Similarly, the wall in the model represents the sum of the walls in the structure. The wall and frame are connected by axially stiff link beams at every floor. The link beams in the model shown in Fig. 6 may or may not be hinged. In computing internal forces and moments due to the factored loads, the flexural stiffnesses, EI, from ACI Code Section 10.10.4.1 could be used.

The model in Fig. 6 may be acceptable for buildings that are symmetrical in plan and have rigid floor diaphragms. A three-dimensional model is required for an unsymmetrical building and where a designer wants to account for diaphragm flexibility.

The lateral-force analysis of shear-wall–frame buildings must account for the different deformed shapes of the frame and the wall. Due to the incompatibility of the deflected shapes of the wall and the frame, the fractions of the total lateral load resisted by the wall and frame differ from story to story. Near the top of the building, the lateral deflection of the wall in a given story tends to be larger than that of the frame in the same story and the frame pushes back on the wall. This alters the forces acting on the frame in these stories. At some floors the forces change direction, as shown schematically by the range of possible moment diagrams in the wall presented in Fig. 7. As a result, the frame resists a larger fraction of the lateral loads in the upper stories than it does in the lower stories.

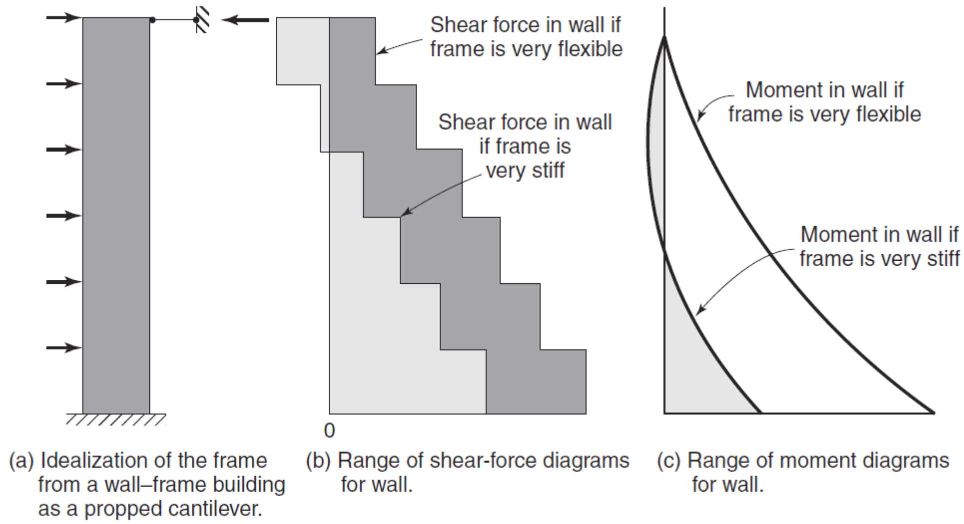


Fig. 7. Effect of frame stiffness on shear and moment in the shear wall

7.1 BOUNDS ON THE FORCES IN A SHEAR-WALL–FRAME BUILDING

The relative portions of the lateral loads resisted by the walls and frames in a shear wall–frame building can be estimated by considering the wall and the frame as two vertical cantilevers, fixed at the bottom and connected via a single extensionally rigid link beam joining the wall and the frame at the top, as shown in Fig. 7a [8]. If the frame is so stiff that it prevents horizontal deflection of the top of the wall, the reaction of the loaded frame at the top of the wall is $3/8wh_w$ where w is the lateral load per foot of height and h_w is the height of the wall. This is equivalent to the reaction at the pinned end of a uniformly loaded beam having a constant bending stiffness EI that is fixed at one end and pinned at the other.

As the lateral stiffness of the frame decreases relative to the lateral stiffness of the wall, the reaction at the top of the wall decreases, approaching zero for a very flexible frame combined with a stiff wall. As a result, the shear-force and bending-moment diagrams for the wall can vary between the approximate limits shown in Fig. 7b and 7c. However, the sum of the shear forces in the frame and the wall in a given story must equal the shear due to the applied loads.

8. COUPLED SHEAR WALLS

Two or more shear walls in the same plane (or two wall assemblies) are sometimes connected at floor levels by coupling beams, so that the walls act as a unit when resisting lateral loads, as shown in Fig. 8. The coupling beams frame into the edges of the walls, as shown in this figure.

The discussion will be limited to the case of two walls separated by a single vertical line of openings, which are spanned by reinforced concrete coupling beams. Walls with more than two lines of openings, as shown in Fig. 2, are handled similarly to what is discussed here for two coupled walls. Other coupled wall systems may need special attention, especially if the widths and heights of the line of openings are irregular.

When the coupled wall deflects, the axes of both parts of the wall at A and A' deflect laterally and rotate through an angle θ, as shown in Fig. 9. This results in shearing deflections of the coupling beams that join the two walls. Localized cracking of the beam-to-wall joint reduces the angle the coupling beam must go through at the connection point to the wall. It is customary to assume the effect of these localized deflections and reduced stiffness of the coupling beam can be represented by moving the assumed connection point in from the face of the wall by approximately $h_b/2$, where h_b is the height of the coupling beam [8]. Thus, it shall be assumed that the walls are joined by coupling beams spanning from B to B'. The downward deflection of point B is:

$$\Delta_B = \left(\frac{b_w}{2} - \frac{h_b}{2}\right)\tan\theta \tag{4}$$

where b_w is the width of the wall.

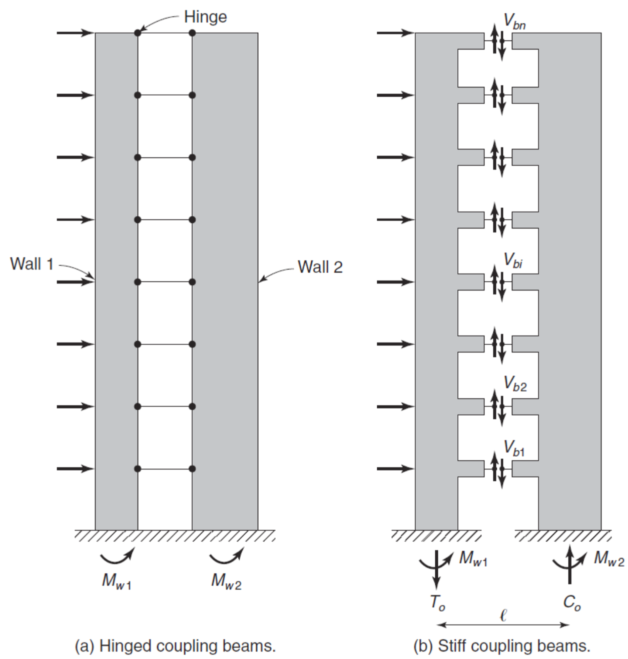


Fig. 8. Coupled Walls

The moments and axial forces in the two wall segments shown in Fig. 9 must be in equilibrium with the axial forces, shear forces, and moments in the entire coupled wall system. The signs of the moments and shear forces may change over the height of the building, similar to those shown by Figs. 7b and 7c.

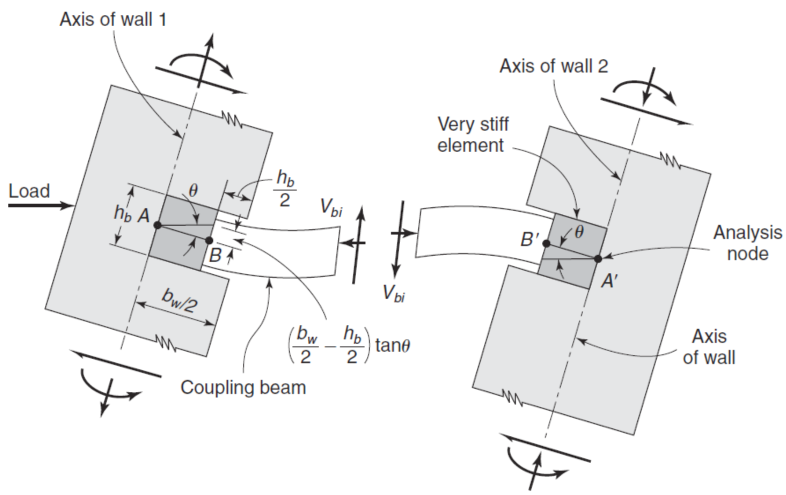


Fig. 9. Effect of shear wall deflections on the forces in a coupling beam

8.1 STATICAL SYSTEM

Figure 8a shows two prismatic walls, wall 1 and wall 2, connected at each floor level by link beams hinged at both ends. The moments at the bases of the two walls are equal to:

$$M_{w1} = M_0 \frac{I_{w1}}{I_{w1} + I_{w2}} \tag{5}$$

and

$$M_{w2} = M_0 \frac{I_{w2}}{I_{w1} + I_{w2}} \tag{6}$$

where I_{w1} and I_{w2} are the wall moments of inertia and M_0 is the moment at the base of the wall due to factored lateral loads. The total lateral load moments in the walls equal:

$$M_{w1} + M_{w2} = M_0 \tag{7}$$

Figure 8b shows the same two walls, except that the walls are now coupled by beams that are continuous with the walls at every floor level and have some flexural stiffness. As the walls deflect laterally, the coupling beams deflect as shown in Fig. 9 and shear forces and moments are generated in the coupling beams. A free-body diagram through the coupling beams halfway between the faces of the two walls has shear forces V_{bi} in each coupling beam, as shown in Fig. 8b and Fig. 9. There are also axial forces in the beams. For equilibrium of vertical forces, an axial tension T_0, must be added to the axial forces in the walls at the centroid of the bottom of wall 1 and an axial compression C_0, at the centroid of the bottom of wall 2, where

$$T_0 = \sum_{i=1}^{n} V_{bi} = C_0 \tag{8}$$

Taking the distance between the lines of action of the forces T_0 and C_0 as l, it can be found that the total moment at the base of the coupled wall system is:

$$M_0 = M_{w1} + M_{w2} + T_0 \times l \tag{9}$$

Equation 9 is plotted in Fig. 10 for a coupled wall consisting of two walls with $EI_{w1} = 2EI_{w2}$ [9]. The vertical axis is the slenderness of the beam, h_b/l_b, where h_b and l_b are the height and the adjusted span of the coupling beams, respectively. The beam slenderness is used here as a measure of the stiffness of the coupling beams. A coupling beam with h_b/l_b equal to zero has no flexural stiffness and as a result, the wall moments are divided in proportion to the ratio of the wall stiffnesses, as given by Eqs. (5) and (6). As the flexural stiffness of the coupling beams increases, the shear forces increase. As a result, the fraction of the overturning moment resisted by the axial force couple $T_0 \times l$ increases asymptotically. A major effect of the coupling beams is to reduce the moments M_{w1} and M_{w2} at the bases of the two walls. This makes it easier to transmit the wall reactions to the foundation.

The coupling beams also act to reduce the lateral deflections. If the beams are perfectly rigid, the two walls act as one wall. Figure 10 illustrates trends only. The actual division of M_0 into M_{w1}, M_{w2} and $T_0 \times l$ depends on more variables than just h_b/l_b.

Coupling beams may be rectangular beams, T beams, or portions of the floor slab [10, 11]. In seismic regions, short coupling beams may have diagonally placed reinforcement.

For seismic design, the Canadian concrete code [10] defines a coupled wall as one in which $T_0 \times l$ is at least 66% of M_0. In Fig. 10, this occurs when h_b/l_b is about 0.2. If $T_0 \times l$ is less than 66% of M_0 the wall is called a partially coupled wall.

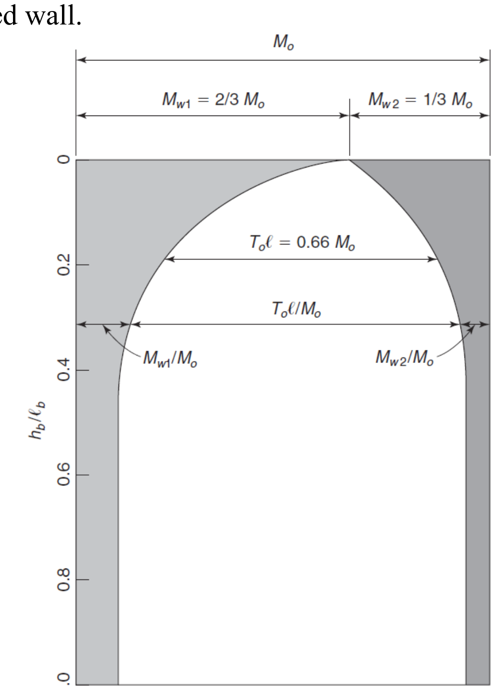


Fig. 10. Effect of the stiffness of the coupling beams

8.2 ANALYSIS OF COUPLED WALLS

Before modern structural analysis programs, the individual coupling beams shown in Figs. 8b and 9 were replaced for analysis by a series of closely spaced laminae, each having a unit height and stiffness I_b/h_s, where h_s is the story height. This allowed a closed-form solution of the forces in the laminae and the walls. However, such analyses were limited to a few uniform wall layouts [9]. Modern structural-analysis programs have made it possible to model coupled structural walls, as shown in Fig. 8b, and to compute the forces and moments in the coupling beams directly. As a result, laminar analyses are seldom used now.

In a frame analysis to determine factored moments for design, the member stiffnesses may be based on ACI Code Section 10.10.4.1. The coupling beams are joined to hypothetical members with high values of the moment of inertia, I, between the face of the wall and the centerline of the wall, as shown by the dark shaded regions in Fig. 9. Short, deep coupling beams develop both flexural and shear deflections. The shear deflections can be included by replacing the I_b of the coupling beam between the walls with

$$I_{eff} = \frac{I_b}{1 + 2.8\left(\frac{h_b}{l_b}\right)^2} \tag{10}$$

This equation comes from adding the moment deflections and shear deflections of the beam, where h_b and l_b are the depth and the span of the coupling beam from face to face of the walls. The second term in the denominator of Eq. (10) accounts for the shear deflections of the beam.

Floor slabs may serve as soft coupling beams. Their stiffness can be based on a slab with a width perpendicular to the wall equal to the wall thickness plus half of the width of the opening $l_b/2$, between the walls, added on each side of the opening [11-13]. In tests of shear walls coupled by slabs, the specimens failed by punching-shear failures in the slab around the ends of the walls. Under cyclic loads, the stiffness of slabs serving as coupling beams decreased rapidly.

Figures 11a, 11b, and 11c show the distributions of moments in the walls, the axial force couple, and the shear forces in the coupling beams for a typical coupled wall [4] where $I_{w1} = I_{w2}$. Typically, the maximum shear in the coupling beams occurs at about one-third of the height above the base. The sawtooth shape of Fig. 11a results from the end moments in the coupling beams.

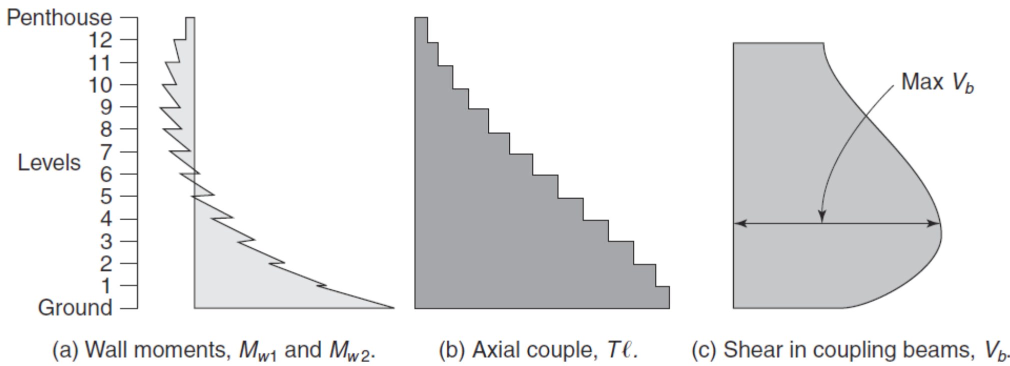


Fig. 11. Typical distribution of wall moments, axial-force couple, and shear in the coupling beams

9. DESIGN OF STRUCTURAL WALLS—GENERAL

9.1 LAYOUT OF BUILDING

Major considerations in selecting a structural system for a multistory building with structural walls are the following:

(**a**) The building must have enough rigidity to withstand the service loads without excessive deflections or vibrations.

(**b**) It is desirable that the wall be loaded with enough vertical load to resist any uplift of parts of the wall foundations due to lateral loads.

(**c**) The locations of frames and walls should minimize torsional deformations of the building about the vertical axis of the building.

(**d**) The walls must have adequate strength in shear and in combined flexure and axial loads.

(**e**) The wall thickness or cover on the reinforcement may be governed by the fire code.

9.2 DIAPHRAGMS

Lateral loads are transferred to the lateral-force resisting system by the floor and roof slabs serving as horizontal diaphragms that act as wide, flat beams in the plane of the floor or roof system. The diaphragms must have a tension flange, a compression flange and a web capable of transmitting the lateral loads. Because the direction of the lateral load changes back and forth during wind or earthquake cycles, the compression and tension flanges of the diaphragm must be able to accommodate changes in the sign of the loading. Notches or other discontinuities in the tension and compression flanges of the diaphragm should be avoided or reinforced to transmit the forces around the discontinuities.

9.3 DISTRIBUTION OF WALLS IN A BUILDING FLOOR PLAN

A common design recommendation is to minimize the separation, commonly referred to as the eccentricity, between the center of mass (geometric centroid of the floor plate) and the center of lateral resistance (CR) provided by the shear walls and moment resisting frames in the lateral-load system. Because lateral loads are assumed to act through the center of mass (CM), any eccentricity between the CM and CR will result in the generation of torsional moments. A central-core wall system, similar to that shown in Fig. 12, is commonly used to minimize eccentricity between the CM and CR.

When a building structure is subjected to large lateral displacements due to earthquake ground motions, the stiffnesses of the lateral-load resisting members are likely to change in a nonuniform fashion. As a result, the CR is likely to be relocated and the eccentricity between the CM and CR may increase. To account for this, the International Building Code [14] specifies a minimum eccentricity in the two principal directions that must be added to any calculated eccentricity. For structures where substantial torsional moments may be generated, a wide distribution of shear walls around the perimeter of the floor plan would be most efficient for resisting that torsion.

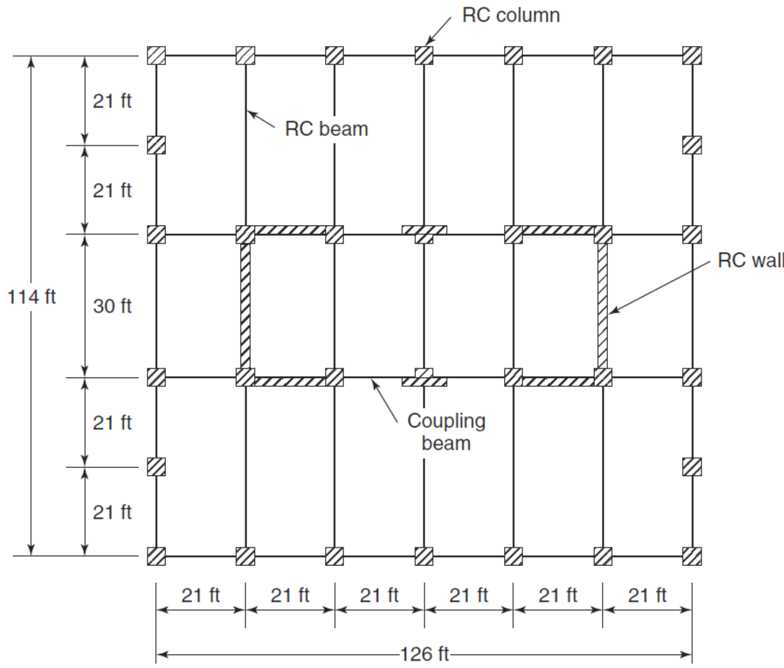


Fig. 12. General building floor plan

9.4 DISTRIBUTION OF STORY FORCES (STORY SHEARS) TO WALLS

In the following analysis of the distribution of lateral loads to structural members, only isolated shear walls will be considered as lateral-load resisting elements. It should be clear that this analysis could be expanded to include moment-resisting frames and wall assemblies, such as T-shapes, L-shapes, U-shapes, etc.

Consider the floor plan and isolated shear walls shown in Fig. 13. It will be assumed that the slab (diaphragm) connecting these walls is stiff in-plane but has a low flexural stiffness. Thus, the walls are not coupled, but they all should have the same lateral displacement under the acting lateral loads. Additionally, the walls have a very low stiffness when bent about their weak axis, and thus, only the stiffness of the walls when they are bent about their strong axes will be considered. Initially, the walls are assumed to be slender. Thus, only flexural stiffnesses will be considered. Modifications to account for shear deformations in short walls will be discussed at the end of this subsection.

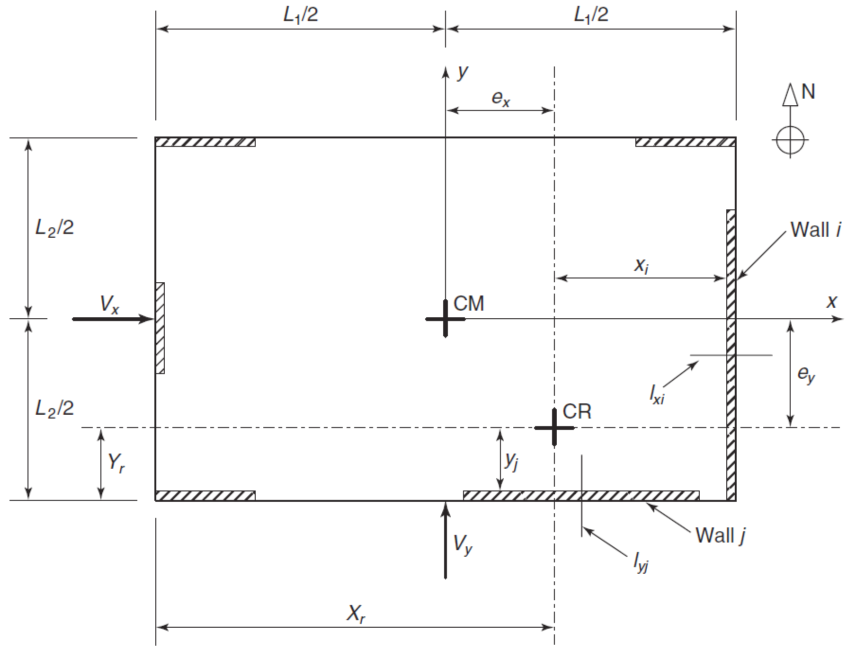


Fig. 13. Eccentricity of center of resistance (CR) with respect to center of mass (CM).

Finally, for this analysis of the distribution of lateral forces, it will be assumed that the walls are uncracked and thus will use the gross moment of inertia for the walls. The effect of flexural cracking can be accounted for in a second interaction of this analysis by assuming that the moment of inertia for a flexurally cracked wall is equal to one-half of the gross moment of inertia. If lateral displacements are to be calculated, the moment of inertia values should be reduced by 30% to correspond to those recommended in ACI Code Section 10.10.4.1:

Walls uncracked: $0.7 \times I_g$

Walls cracked: $0.35 \times I_g$

Returning to Fig. 13 and using the assumptions discussed here, the shear forces induced in each wall due to the lateral forces, V_x and V_y will be calculated. If there was no eccentricity between the CM and CR, the total lateral force V_x would be distributed to the four walls along the north and south edges of the floor plate in proportion to their moments of inertia about their strong axis (i.e., their y-axis), as shown in Fig. 13. Thus, the lateral force resisted by wall j would be

$$V'_{xj} = \left[\frac{I_{yi}}{\sum_n I_{yn}} \right] V_x \tag{18-11a}$$

where n is the number of walls resisting V_x in bending about their strong axis. Similarly, V_y would be resisted by the two walls along the east and west edges of the floor plate, and the lateral force resisted by wall i would be

$$V'_{yj} = \left[\frac{I_{xi}}{\sum_m I_{xm}} \right] V_y \tag{18-11b}$$

where m is the number of walls resisting V_y in bending about their strong axis (x-axis).

If there was an eccentricity between the CM and CR or a minimum eccentricity was specified by a design code, then the effects of torsion must be considered. To find the CR for the floor plate in Fig. 13, an origin at the southwest corner of the plate will be considered and measure distances from that origin in terms of X and Y. Only the walls resisting V_x through bending about their y-axis will be considered to find the location of the CR in the Y-direction. Following this procedure, the value for Y_r is

$$Y_r = \frac{\sum_j I_{yj} Y_j}{\sum_j I_{yj}} \tag{12a}$$

Similarly, to find the location of CR in the X-direction, only the walls resisting V_y by bending about their x-axis will be considered. Thus,

$$Y_r = \frac{\sum_i I_{xi} X_i}{\sum_i I_{xi}} \tag{12b}$$

The location of the CM is given in Fig. 13, so the eccentricities from CM to CR are

$$e_y = \frac{L_2}{2} - Y_r \tag{13a}$$

$$e_x = \frac{L_2}{2} - X_r \tag{13b}$$

These eccentricities or an increased value of eccentricity required to satisfy a governing building code requirement can be used to calculate the torsion caused by the lateral loads as follows:

$$T_x = V_x e_y \tag{14a}$$

$$T_y = V_y e_x \tag{14b}$$

The torsion resisted by each wall in the floor plan will be related to the lateral stiffness of the wall for bending about its strong axis multiplied by the distance in the x- or y-direction from the wall to the CR, as measured perpendicular to the weak axis of the wall. Thus, as stated previously, if a significant torsion is to be resisted, the use of widely distributed walls is more effective in resisting torsion. An equivalent torsional stiffness for all of the walls in the floor system (acting as a unit) can be calculated as the sum of the torsional resistance from each wall multiplied by their respective perpendicular distance to the CR. This torsional stiffness can be expressed as

$$K_t = \sum_i I_{xi} x_i^2 + \sum_j I_{yj} y_j^2 \tag{15}$$

With this equivalent torsional stiffness for the walls acting as a combined system, it can be determined how much shear is induced in each wall when resisting the torsional moments. However, there may be some questions regarding what torsional moments should be used when determining the total shear force in each wall. Typically, a structure is analyzed for lateral loads (wind or seismic) acting in only one principal direction and then reanalyzed for the lateral loads acting in the other principal direction. These two results normally are considered separately in the design. In this case, the torsion that is generated by either of the lateral loads acting in one principal direction is resisted by all of the walls—not just those with their principal axes perpendicular to the lateral load. Thus, it is not clear how much of the torsion generated due to lateral loading in the second principal direction should be included when considering the effect of lateral loading in the first principal direction. Assuming that some percentage of the effect of lateral loading in the second principal direction should be considered, the author presents the following expressions.

The first expression is for the shear induced in walls that have their strong axis perpendicular to the lateral force V_x:

$$V''_{xj} = \left[\frac{I_{yj} y_j}{K_t}\right]\left(T_x + \alpha T_y\right) \tag{16a}$$

Similarly, for the walls that have their strong axis perpendicular to the lateral force V_y.

$$V''_{yi} = \left[\frac{I_{xi} x_i}{K_t}\right]\left(T_y + \alpha T_x\right) \tag{16b}$$

For both expressions, the author recommends using $\alpha = 0.25$ to reflect the low probability of having the maximum lateral forces acting simultaneously in both principal directions.

We now can combine the results from Eqs. (11) and (16) to obtain the total shear resisted by walls with their strong axis perpendicular to the lateral force V_x as:

$$V_{xj} = V'_{xj} + V''_{xj} \tag{17a}$$

For walls with their strong axis perpendicular to the lateral force V_y is:

$$V_{yi} = V'_{yi} + V''_{yi} \tag{17b}$$

For all of the analysis results given up to this point, only flexural stiffnesses of the walls have been considered. For walls with aspect ratios h_w/l_w less than 3, the effect of shear deformations starts to become more significant. For such walls, it is recommended to use a modified moment of inertia to reflect the increased importance of shear deformations.

A value recommended in the PCA Design Handbook [15] is:

$$I_{mod} = \frac{I_e}{1 + \dfrac{24 I_e}{A_w h_w^2}} \tag{18}$$

where I_e is the effective flexural moment of inertia that has been selected based on the prior discussions, and A_w is the wall area, $h \times l_w$.

9.5 WALL FOUNDATIONS

The foundations at the base of the wall must be able to transfer the shear, moment, and axial force as required from the base of the wall to the supporting soil or rock. If the axial gravity forces P_u in the wall are small, the moments at the base of the wall may cause uplift under one side of the shear-wall footing, as shown in Fig. 14a. Both an axial load and a moment are resisted by the soil, and as a result, the soil pressure will vary across the width of the wall footing. For a vertical load acting at the central core of the footing area, or for a vertical load and moment acting on the footing, the soil pressure will range from smaller than average (as low as zero) on one side of the footing to higher than the average stress (up to twice as high) at the other side. Because tensile uplift stresses are difficult to resist, they should be avoided. If the footing size becomes excessive, possible solutions are:

(**a**) Replace the rectangular footing with an H-shaped footing, to increase the radius of gyration of the footing.

(**b**) Use pile or caisson foundations.

(**c**) Use coupled shear walls to widen the footprint of the wall and divide the moment to be transferred into the three components shown in Fig. 8b.

(**d**) Attach the base of the wall to horizontal outrigger beams in the basement which extend the foundation to receive the vertical loads from adjacent frame columns so that there is no uplift; or the uplift is counteracted (see Fig. 14b).

(**e**) Attach the base of the wall to the ground-floor diaphragm and basement floor to provide a horizontal force couple to react to the moment at the base of the wall.

(**f**) Use a mat foundation. Wyllie [16] and Paulay and Priestly [17] both discuss shear-wall foundations.

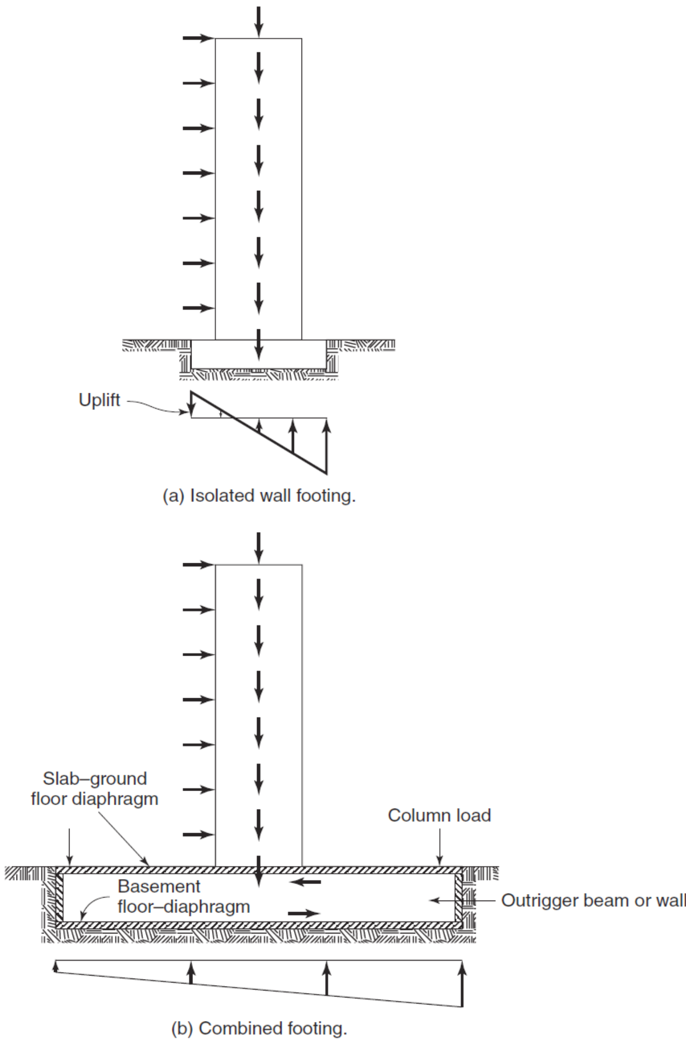


Fig. 14. Wall foundations

9.6 REQUIRED SIZE OF WALL

In choosing a structural wall section for a given building, the wall must: (**a**) have enough strength to resist the factored moments, shears, and axial loads acting on it; and (**b**) have enough stiffness to limit the lateral deflections.

There is no widely accepted way of doing this. A rough estimate of the minimum wall stiffness, EI, required to limit the lateral deflections to an acceptable value can be made by considering the walls as a vertical cantilever with a constant EI over the height, loaded with a constant wind load over the height, as shown in Fig. 14a. Wall thicknesses, and thus the structure's EI would generally decrease as the moments and shear forces decrease near the top of the structure, it shall be assumed it is constant over the building height and equal to the sum of the EI values of the walls in the bottom story. This cantilever will be loaded with a constant, uniform service wind load w_s of lb per foot of height. The wind load is the product of the length and height of the building and the wind pressure per foot of height, taken equal to the algebraic sum of the windward wind pressures and leeward wind suctions evaluated at the top of the building. It is customary to limit the relative horizontal deflection of the top and bottom of a story to a fraction of the height of the story, expressed as $\Delta = h_s/(200 \text{ to } 500)$, where Δ/h_s is called the story drift.

This limit is expressed in terms of the slope of the wall relative to the vertical in any story throughout the height of the building. The largest slope in a shear wall or shear-wall–frame building is generally at the top of the structure and is given by [6]

(slope at top) = (rotation of base of wall) + (area of M/EI diagram from base to top) (19)

If the base is fixed against rotation and assuming that EI = EI_{base} is constant over the height of the building, h_w, the following gives the slope at the top relative to the undeflected position:

$$\text{Slope at top} = \frac{w_s h_w^3}{6E_c I_g} \tag{20}$$

Here:

h_w – is the height of the wall

E_c – is the modulus of elasticity of concrete, psi (because all other units are in pounds and feet, the modulus of elasticity in psi is multiplied by (144 in^2/ft^2) to change the psi units to psf)

I_g – is the uncracked moment of inertia of the wall or walls at the bottom of the structure (an uncracked wall is taken to be representative of service-load levels, because codes limit service-load deflections and the walls are generally not severely cracked under service loads); ACI Code Section 10.10.4.1 gives I = 0.7×Ig ft^4 for an uncracked wall

w_s – is the unfactored wind load per vertical foot of wall, evaluated at the top of the wall, in lb/ft of height

9.7 LIMITS ON STORY DRIFT

ASCE/SEI Committee 7 [18] does not limit the lateral deflection or the story drift. The National Building Code of Canada [19] limits the maximum story drift under unfactored loads to less than 1/500 of the story height. The maximum acceptable drift depends on the ability for occupants of the building to perceive the motion of the building during a major windstorm. This storm may be chosen as a rare event, such as the maximum annual windstorm or the maximum windstorm in one tenancy of the building—say, every 3 to 8 years. Grossman discusses design for wind induced movement of slender concrete buildings [6].

If a wall having height h_w and constant wall stiffness EI is fixed at the base, the maximum drift will be in the top story. Setting the service-load story drift from Eq. (20) equal to 1/500, one can compute the minimum total I_g for the walls parallel to the direction of the wind load as

$$\sum\left(0.70 I_g\right) = \frac{500 w_s h_w^3}{6E_c} \tag{21}$$

Once the minimum has ΣI_g been estimated, walls can be selected to give ΣI_g equal to or greater than what is required by Eq. (21). Although this analysis is intentionally simplified, it gives an order-of-magnitude estimate of story drift. It does not consider torsional loadings on the structure, and it assumes that the wind pressure is constant over the height of the building. At the stage in design where the sizes of shear walls are initially chosen, these details are not yet known. A better estimate of Δ/h can be obtained by calculating the second term on the right-hand side of Eq. (19) more accurately based on a better estimate of the EI and moment.

9.8 MINIMUM WALL THICKNESS

The minimum thicknesses given in ACI Code Section 14.5.3 are intended to apply only to bearing walls designed via the empirical design method from ACI Code Section 14.5. The author would limit the thickness of walls with rectangular cross sections to the absolute minimum of 1/20 of the unsupported height of the wall. Preferably, the wall thickness should not be less than 1/15 of the unsupported height. The wall thickness must be large enough to allow the concrete to be placed without honeycombing.

9.9 REINFORCEMENT IN SHEAR WALLS

9.9.1 Distributed and Concentrated Reinforcement

The reinforcement in a shear wall is generally made up of:
(**a**) Distributed horizontal and vertical reinforcement spread uniformly over the length between the boundary elements and over the height of the wall.
(**b**) Concentrated vertical reinforcement is located in boundary elements at or near the edges of the wall and is tied in much the same way that column cages are.

9.9.2 Minimum Wall Reinforcement

In addition to the distributed reinforcement required by ACI Code Section 14.3, several other ACI sections require minimum amounts of distributed horizontal and vertical steel that may apply to walls. These are given in Table 1.

For deformed bars not larger than No. 5, ACI Code Section 14.3 requires minimum areas for distributed vertical reinforcement in walls to be equal to $0.0012A_g$ and the minimum area of distributed horizontal reinforcement in walls to be equal to $0.002A_g$. This steel can be placed in two layers or curtains of distributed vertical and horizontal reinforcement with the bars in the two curtains tied together at intervals, but not enclosed in ties. A single reinforcement curtain is allowed at the midplane of walls having thicknesses of 10 in. or less (ACI Code Section 14.3.4). This steel cannot be tied because there is only one curtain. Thicker walls require two curtains of reinforcement, each consisting of not less than half of the total minimum reinforcement required in each direction.

Each of these two layers is placed not more than one-third of the wall thickness from the surface. It is desirable to have the steel in two layers close to the sides of the wall because the internal lever arm, for weak-axis bending is much smaller if the reinforcement is at the center of the wall. ACI Code Section 14.3.5 requires that distributed vertical and horizontal bars be spaced not further apart than three times the wall thickness or 18 in., whichever is less. ACI Code Section 11.9.9.3 allows the maximum spacing of horizontal shear reinforcement to be the smallest of $l_w/5$, 3h and 18 in. ACI Section 11.9.9.5 allows the maximum spacing of vertical shear reinforcement to be the smallest of $l_w/3$, 3h, and 18 in.

TABLE 1. Minimum Reinforcement in Walls Compared to Other Members

Reason	ACI Code Section	Requirement	Maximum Spacing
Shrinkage and temperature	7.12.2.1	(b) Slabs where Grade-60 deformed bars or welded-wire fabric (plain or deformed) are used: 0.0018	Five times the slab thickness, no farther apart than 18 in.
Minimum flexural steel in	10.5.4	The minimum area of tensile reinforcement in the direction of the	Three times the slab thickness, no farther apart

one-way slabs		slab span is the same as by 7.12.2.1	than 18 in.
Deeps beams	11.7.4.1	The area of shear reinforcement perpendicular to the span shall not be less than $0.0025b_ws$	s shall not exceed d/5 or 12 in.
	11.7.4.2	The area of shear reinforcement perpendicular to the span shall not be less than $0.0025b_ws_2$	s_2 shall not exceed d/5 or 12 in.
Walls	11.9.9.2	Ratio ρ_t of horizontal shear reinforcement area to gross concrete area shall not be less than 0.0025	Spacing of horizontal shear reinforcement shall not exceed $l_w/5$, 3h or 18 in.
	11.9.9.4	Ratio ρ_l of vertical shear reinforcement shall not be less than $\rho_l=0.0025+0.5(2.5-h_w/l_w) \times (\rho_t-0.0025)$ nor 0.0025	Spacing of horizontal shear reinforcement shall not exceed $l_w/5$, 3h or 18 in.
Two-way slab reinforcement	13.3.1	Area of reinforcement in each direction shall be determined from moments at critical sections, but shall not be less than required by 7.12.2.1	Spacing of reinforcement at critical sections shall not exceed two times the slab thickness
Minimum reinforcement - walls	14.3.2	Minimum ratio of vertical reinforcement area to gross concrete area shall be: (a) 0.0012 for deformed bars not larger than No. 5 with a specified yield strength not less than 60,000 psi	14.3.5 Vertical and horizontal reinforcement shall not be spaced farther apart than three times the wall thickness, nor farther apart than 18 in.
	14.3.3	Minimum ratio of horizontal reinforcement area to gross concrete area shall be: (a) 0.0020 for deformed bars not larger than No. 5 with a specified yield strength not less than 60,000 psi	

Shear walls subjected to large moment reversals, as likely would occur during strong earthquake ground motions, may require larger minimum vertical-reinforcement areas than given by the previous requirements in order to prevent possible fracture of the vertical reinforcement [20].

9.9.3 Ties for Vertical Reinforcement

ACI Code Section 14.3.6 specifies that the distributed vertical reinforcement need not be enclosed by lateral ties if:
(**a**) the vertical reinforcement area is not greater than 0.01 times the gross concrete area or
(**b**) the vertical reinforcement is not required as compression reinforcement.

Many designers interpret part (b) of ACI Code Section 14.3.6 to mean that wall steel satisfying:
(a) in this list that is not enclosed in transverse ties should be ignored in strength calculations if
• it is stressed in compression under static loads, or
• it is stressed by cyclic loads that cause compression in the bars.

Distributed wall steel placed in two separate curtains can be tied through the thickness of the wall using stirrups or through-the-wall cross-ties engaging reinforcement on both faces of the wall. Although the ACI Code does not specify what fraction of the bars should be tied in this manner, it is customary for such ties to engage every second or third bar each way on both faces [21].

In many cases the distributed vertical reinforcement has enough moment capacity to resist the wind load moments. If not, concentrated reinforcement is provided in boundary elements at the ends of the walls or at the intersections of walls. These elements should contain vertical steel satisfying the minimum reinforcement requirements from ACI Code Section 10.9.1 based on the area of the boundary elements, rather than on the gross area of the wall. This steel is to be enclosed by ties satisfying ACI Code Section 7.10.5. As a result, the steel is assumed to be restrained against buckling, and thus able to resist compressive bar forces. The boundary elements may be the same thickness as the rest of the wall, or they may be thicker. Although ACI Code Chapter 14 does not require concentrated reinforcement at the ends of the walls, it is good practice to at least use larger bars at the extreme ends of the wall.

9.9.4 Transfer of Wall Load through Floor Systems

In tall buildings, the strength of the concrete in the walls may be higher than the strength of the concrete in the floor system. Walls may be of high-strength concrete to reduce the lateral deflections and the wind-induced sway vibrations of the building. On the basis of tests of column–floor joints, ACI Code Section 10.12 allows an increase in the effective strength of the floor concrete in locations where the floor concrete is confined on all sides. This effect is smaller in edge and corner column joints than in interior joints, because there is less confinement of the joint concrete at edge or corner columns. The lack of confinement is even more pronounced in wall–floor joints, because a greater length of joint concrete is unconfined. In the absence of tests, it is recommended that the strength of wall-to-floor joints be based on the lower of the wall and floor concrete strengths.

10. FLEXURAL STRENGTH OF SHEAR WALLS

10.1 FLEXURE—NOMINAL STRENGTH, FACTORED LOADS, AND RESISTANCE FACTORS

Cross sections in a wall are designed to satisfy:

$$\phi M_n \geq M_u \tag{22}$$

$$\phi N_n \geq N_u \tag{23}$$

$$\phi V_n \geq V_u \tag{24}$$

where M_n is the nominal resistance based on the specified material strengths, M_u is the required resistance computed from the factored loads, and so on. The strength-reduction factor, ϕ, comes from ACI Code Section 9.3.2 for flexure and axial loads and from ACI Code Section 9.3.2.3 for shear. The factored loads are from ACI Code Section 9.2.1.

10.2 STRENGTH-REDUCTION FACTORS FOR FLEXURE AND AXIAL LOAD—ACI CODE SECTION 9.3.2

The strength-reduction factors (ϕ) for combined flexure and axial loads for a shear wall vary, depending on the maximum steel strains anticipated at ultimate load. The calculation of strength-reduction factors, ϕ, is based on the strain, ε_t, in the layer of steel at the depth, d_t, which is located farthest from the extreme-compression fiber.

Tension-controlled limit for a rectangular wall. The strength-reduction factor, ϕ, can be computed directly from the computed strain in the extreme-tension layer of reinforcement, ε_t. ACI Code Section 10.3.4 defines the tension-controlled limit load as the load causing a strain distribution having a maximum strain of 0.003 in compression on the most compressed face of the member, when the steel strain in the extreme layer of tension reinforcement, ε_t reaches 0.005 in tension. From similar triangles, the neutral axis will be located at $c/d_t = 0.375d_t$ from the compressed face, where d_t is the distance from the extreme-compression fiber to the centroid of the layer of bars farthest from the compression face of the member. Thus, when c is less than or equal to 0.375 d_t, the wall is tension-controlled, and $\phi = 0.9$.

Compression-controlled limit for a rectangular wall. The compression-controlled limit corresponds to a strain distribution with the neutral axis at $0.6d_t$. So, when c is greater than or equal to d_t, the wall is compression-controlled, and $\phi = 0.65$.

10.3 FLEXURAL STRENGTH OF RECTANGULAR WALLS WITH UNIFORM CURTAINS OF VERTICAL DISTRIBUTED REINFORCEMENT

Code guidance on the use of vertical distributed reinforcement in walls loaded cyclically is ambiguous. In seismic regions, the loads resisted by vertical bars that are not tied, are ignored.

ACI Code Section 14.3.6 is more lenient. It requires that vertical bars be tied (**a**) if the total distributed vertical reinforcement A_s, exceeds $0.01A_g$ or (**b**) if the vertical reinforcement is included as compression reinforcement in the calculations. The following strength analysis applies to walls with two curtains of reinforcement with ties through the wall to the other curtain of bars. The equations in this subsection also apply if the area of steel is less than 0.01.

ACI Code Section 21.6.4.2 suggests that, in seismic regions, column reinforcement would be adequately tied if the center-to-center spacing of cross-ties or the legs of hoops did not exceed 14 in. Given this guidance, the author believes that the vertical reinforcement in a non-seismic wall can be taken as “tied” if at least every second bar in a curtain of reinforcement is tied through the wall to a bar in the other curtain of steel, near the opposite face.

Figure 15a shows a rectangular wall section with a uniform distribution of vertical steel. Assume that the wall is subjected to a factored axial load, N_u, and find the nominal flexural strength, M_n, for this wall using the strain distribution presented in Fig. 15b. A procedure developed by A. E. Cardenas and his colleagues [22] and [23] will be used. They made the following assumptions at nominal strength conditions for shear wall sections similar to that in Fig. 15a.

1. All steel in the tension zone yields in tension.
2. All steel in the compression zone yields in compression.
3. The tension force acts at mid-depth of the tension zone.
4. The total compression force (sum of steel and concrete contributions) acts at mid-depth of the compression zone.

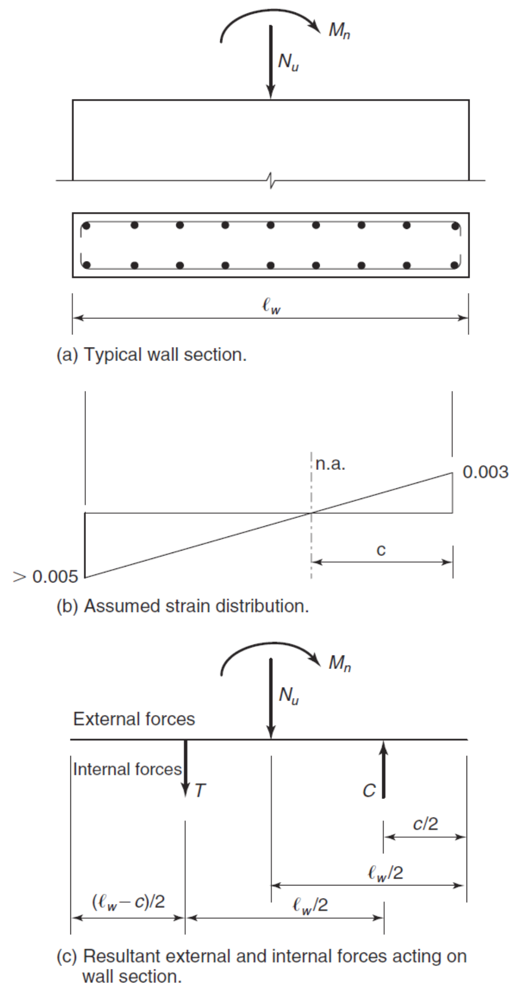


Fig. 18-15. Wall with uniform distribution of vertical reinforcement subjected to axial load and bending.

From those assumptions and using to represent the total area of longitudinal (vertical) reinforcement, the following expressions for the vector forces in Fig. 15c is obtained.

$$T = A_{st} f_y \left(\frac{l_w - c}{l_w} \right) \tag{25a}$$

$$C_s = A_{st} f_y \left(\frac{c}{l_w} \right) \tag{25b}$$

$$C_c = 0.85 f_c^{'} h \beta_1 c \tag{25c}$$

and

$$C = C_s + C_c \tag{25d}$$

The percentage of total longitudinal reinforcement is:

$$\rho_l = \frac{A_{st}}{hl_w} \tag{26a}$$

and the longitudinal reinforcement index is:

$$w = \rho_l \frac{f_y}{f_c'} \tag{26b}$$

Finally, Cardenas et al. defined an axial stress parameter as:

$$\alpha = \frac{N_u}{hl_w f_c'} \tag{27}$$

where N_u represents the factored axial load, positive in compression.

Enforcing section equilibrium leads to:

$$C_s + C_c - T = N_u$$

$$0.85f_c' h\beta_1 c + A_{st} f_y \left(\frac{c}{l_w}\right) - A_{st} f_y \left(\frac{l_w - c}{l_w}\right) = N_u$$

Combining some terms and dividing both sides of this force equilibrium expression by $hl_w f_c'$ results in:

$$0.85f_c'\beta_1 \frac{c}{w} - \left(1 - \frac{2c}{l_w}\right)\frac{A_{st}}{hl_w}\frac{f_y}{f_c'} = \frac{N_u}{hl_w f_c'}$$

Substituting the definitions from Eqs. (26) and (27), the distance to the neutral axis from the compression edge of the wall is obtained in the form:

$$c = \left(\frac{\alpha + w}{0.85\beta_1 + 2w}\right) l_w \tag{28}$$

With this value for c, we can use Eq. (18-25) to find all of the section forces. Then, summing moment about the compression force, C, in Fig. 18-15c, we get the following expression for the nominal moment strength of the wall section.

$$M_n = T\left(\frac{l_w}{2}\right) + N_u\left(\frac{l_w - c}{2}\right) \tag{18-29}$$

Cardenas and his colleagues were able to show good agreement between measured moment strengths from tests of shear walls and strengths calculated using Eq. (29). The critical load case for evaluating the nominal moment strength of a structural wall normally corresponds to ACI Eq. (9-6) or (9-7) in ACI Code Section 9.2.1.

U = 0.9D + 1.0W (ACI Eq. 9-6)

U = 0.9D + 1.0E (ACI Eq. 9-7)

If service-level wind forces are specified by the governing building code, use a load factor of 1.6 for W in ACI Eq. (9-6). Either of these load cases will minimize the factored wall axial load, N_u, and thus minimize the wall nominal moment strength. Also, it can be shown for essentially all structural walls that $c<0.375d$. Thus, the wall section is tension-controlled, and $\phi = 0.9$.

10.4 MOMENT RESISTANCE OF WALL ASSEMBLIES, WALLS WITH FLANGES, AND WALLS WITH BOUNDARY ELEMENTS

Frequently, shear walls have webs and flanges that act together to form H-, C-, T-, and L-shaped wall cross sections referred to as wall assemblies. The effective flange widths can be taken from ACI Code Sections 8.12.2 and 8.12.3. In regions subject to earthquakes, ACI Code Section 21.9.5.2 limits the flange widths to the smaller of

(**a**) half the distance to an adjacent web or

(**b**) 25 percent of the total height of the wall.

The limiting flange thicknesses from ACI Code Section 21.9.5.2 for both seismic and non-seismic walls shall be considered.

Frequently, the thickness is increased at the ends of a wall to give room for concentrated vertical reinforcement that is tied like a tied column (see ACI Code Section 7.10.5). The increased thickness also helps to prevent buckling of the flanges. Regions containing concentrated and tied reinforcement are known as boundary elements, regardless of whether or not they are thicker than the rest of the wall.

10.5 NOMINAL MOMENT STRENGTH OF WALLS WITH BOUNDARY ELEMENTS OR FLANGES

In this subsection, structural walls that have longitudinal reinforcement concentrated at the edges to increase their nominal moment strength will be discuss. A few typical examples of such walls are shown in Fig. 16. The longitudinal reinforcement in boundary elements similar to those in Fig. 16a and b will need to be tied with transverse reinforcement that satisfies ACI Code Section 7.10.5 if the walls were in regions of low or no seismic risk. The confinement requirements are more stringent for boundary elements of structural walls in regions of high seismic risk, as will be discussed in the next chapter.

When calculating the nominal moment strength, M_n, for walls similar to those in Fig. 16, the contribution from the vertical reinforcement in the web is usually ignored because its contribution to M_n is quite small compared to the contribution from the vertical reinforcement concentrated at the edges of the wall. For some flanged sections or wall assemblies, as shown in Fig. 16c, ignoring the vertical reinforcement in the web may be too conservative. An alternative procedure for analyzing such wall assemblies in flexure is given in the next subsection.

The model used to analyze the nominal moment strength of a structural wall with boundary elements is shown in Fig. 18-17. For the boundary element in tension, the tension force is:

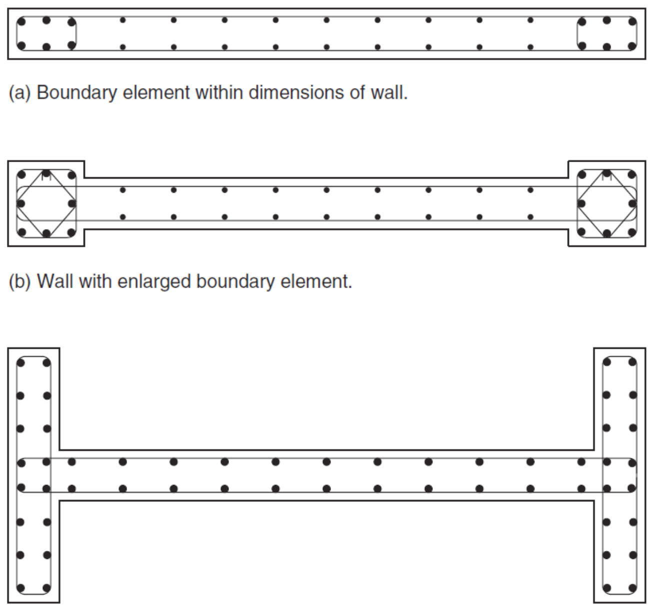
$$T = A_s f_y \tag{30}$$

where A_s is the total area of longitudinal steel in the boundary element. The longitudinal steel in the compression boundary element is ignored. Using the compression stress-block model the compression force is:

$$C = 0.85 f'_c ba \tag{31}$$

where b is the width of the boundary element. Enforcing section equilibrium for the vertical forces in Fig. 17 results in:

$$a = \frac{T + N_u}{0.85 f_c' b} \tag{32}$$



(a) Boundary element within dimensions of wall.

(b) Wall with enlarged boundary element.

(c) Wall with reinforcement concentrated in flanges.

Fig. 16. Structural walls with concentrated reinforcement at their edges.

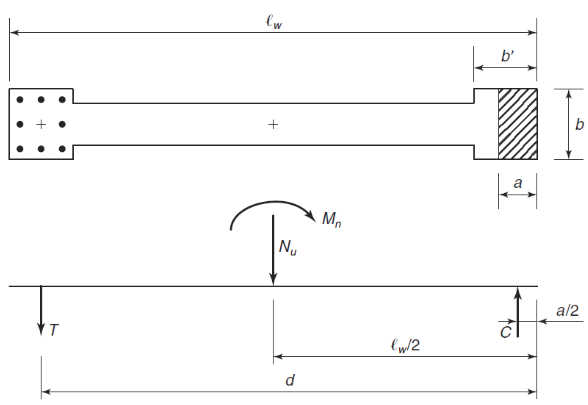


Fig. 17. Flexural strength model for wall with boundary elements

Normally, the compression stress block is contained within the boundary element, as shown in Fig. 17. If the compression force required for section equilibrium was large enough to cause the value of a to exceed the length of the boundary element shown as b' in Fig. 17, then a section analysis applied for a T shaped cross-section will be required.

The critical load case for evaluating the nominal moment strength of a structural wall normally corresponds to ACI Eq. (9-6) or (9-7) in ACI Code Section 9.2.1. Either of these load cases will minimize the factored axial load, N_u, and thus minimize the wall nominal moment strength. Summing the moment about the compression force in Fig. 18-17 leads to the following expression for M_n.

$$M_n = T\left(d - \frac{a}{2}\right) + N_u\left(\frac{l_w - a}{2}\right) \tag{33}$$

For essentially all structural walls, it can be shown that the neutral-axis depth, c, is well less than 0.375d, so the section is tension-controlled, and $\phi = 0.9$.

10.6 NOMINAL MOMENT STRENGTH OF WALL ASSEMBLIES

Paulay and Priestley [17] present the following method of computing the required reinforcement in a wall assembly. Figure 18 shows a plan of a wall consisting of a web and two flanges. The wall is loaded with a factored axial load, N_u, and a moment, M_{ua}, that causes compression in flange 1 and a shear V_u, parallel to the web. The axial load and the moment act through the centroid of the area of the wall. The moment can be replaced by an eccentric axial load located at:

$$e_a = \frac{M_{ua}}{N_u} \tag{34}$$

from the centroid, which is equivalent to it acting at $x_a = e_a - x_1$ from the centroid of the flange that is in compression.

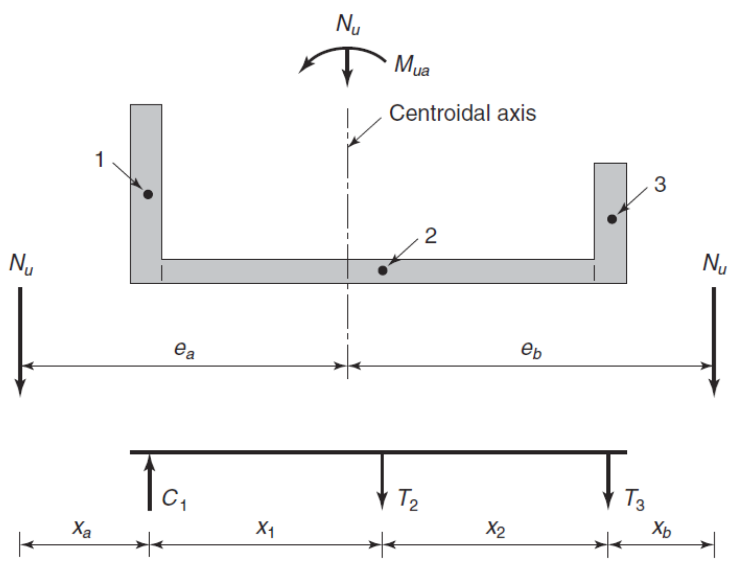


Fig. 18. Reinforcement forces acting on a wall segment.

Because of the shear V_u, the vertical reinforcement in the web will be assumed to be the minimum steel required by ACI Code Section 11.9.9.4. The calculations will be simplified by assuming that all of the web steel yields in tension. This assumption is valid because under cyclic loads, the neutral axis will alternately be close to the left and the right ends of the web. The steel in the web resists an axial force of:

$$T_2 = A_{s2} f_y \tag{35}$$

where A_{s2} is the area of distributed steel in wall element 2, the web. Summing moments about the centroid of the compression flange gives the following equation for the tension T_3, in the right-hand flange:

$$T_3 \approx \frac{N_u x_a - T_2 x_1}{x_1 + x_2} \tag{36}$$

In a similar manner, the force in the tension reinforcement, required in the left-hand flange for the axial load and the moment M_{ub} causing compression in the right-hand flange can be computed as:

$$T_1 \approx \frac{N_u x_b - T_2 x_2}{x_1 + x_2} \tag{37}$$

The required area of concentrated vertical reinforcement in the flanges can be computed by dividing the tension forces from Eqs. (36) and (37) by the product, $\phi \times f_y$.

10.7 BIAXIALLY LOADED WALLS.

A wall is said to be biaxially loaded if it resists axial load plus moments about two axes. One method of computing the strength of such walls is the equivalent eccentricity method. In this method, a fraction between 0.4 and 0.8 times the weak-axis moment is added to the strong-axis moment. The wall is then designed for the axial load and the combined biaxial moment treated as a case of uniaxial bending and compression.

Strictly speaking, the elastic moment resistance of an unsymmetrical wall should be computed allowing for moments about both principal axes of the cross section. This is not widely done in practice. It is generally assumed that cracking of the walls and the proportioning of the vertical wall reinforcement can be done considering moments about one orthogonal axis at a time.

10.8 SHEAR TRANSFER BETWEEN WALL SEGMENTS IN WALL ASSEMBLIES

For the flanges to work with the rest of the cross section of a wall assembly, so-called “vertical shear stresses” must exist on the interface between the flange and web, even when the wall and the wall segments are constructed monolithically. The stresses to be transferred are calculated in the same manner as for a composite beam. The reinforcement should satisfy ACI Code Section 11.6, Shear Friction.

11. SHEAR STRENGTH OF SHEAR WALLS

The design of structural walls for shear in non-seismic regions is covered in ACI Code Section 11.9, Provisions for Walls. The basis for this code section was a series of tests of one-third-size, planar, shear walls, reported in [22], [23], [24]. The test specimens were divided between flexural shear walls with ratios $M_u/(V_u l_w)$ of 1.0, 2.0, and higher and short shear walls with $M_u/(V_u l_w)$ of 0.5. The basic shear-design equations are similar to those for the shear design of prestressed concrete beams:

$$\phi V_n \geq V_u \tag{38}$$

(ACI Eq. 11-1)

$$V_n = V_c + V_s \tag{39}$$

(ACI Eq. 11-2)

$$V_s \geq \left(\frac{V_u}{\phi} - V_c \right) \tag{40}$$

ACI Code Section 11.9.3 limits V_n to a maximum value of $10\sqrt{f_c^{'}}hd$ where d shall be taken as $0.8l_w$ unless a strain–compatibility analysis is used to define the centroid of the tension force in bending. For walls with concentrated vertical reinforcement in boundary elements at the edges of the walls, d may be measured from the extreme compression edge to the centroid of the concentrated vertical reinforcement near the tension edge.

11.1 V_C FOR SHEAR WALLS

As with beam design, the concrete contribution to shear strength is set approximately equal to the value of shear that causes shear (inclined) cracking in a structural wall. For walls subjected to axial compression, a designer is permitted to use Eq. (41), unless a more detailed analysis is made, as will be discussed in the next paragraph,

$$V_c = 2\lambda\sqrt{f_c^{'}}hd \tag{41}$$

where λ is the factor for lightweight aggregate concrete. It is taken as 1.0 for normal weight concrete and shall be used as defined in ACI Code Section 8.6.1 for lightweight concrete. For walls subjected to axial tension, a designer must use ACI Code Eq. (11-8) with h substituted for b_w,

$$V_c = 2\left(1 + \frac{N_u}{500A_g}\right)\lambda\sqrt{f_c^{'}}hd \tag{42}$$

where N_u is negative for tension and N_u/A_g shall be expressed in psi units.

For structural walls subjected to axial compression, ACI Code Section 11.9.6 permits V_c to be taken as the smaller of:

$$V_c = 3.3\lambda\sqrt{f_c^{'}}hd + \frac{N_u d}{4l_w} \tag{43}$$

(ACI Eq. 11-27)

or,

$$V_c = \left[0.6\lambda\sqrt{f_c^{'}} + \frac{l_w\left(1.25\lambda\sqrt{f_c^{'}} + 0.2\frac{N_u d}{l_w h}\right)}{\frac{M_u}{V_u} - \frac{l_w}{2}} \right] hd \tag{44}$$

(ACI Eq. 11-28)

Equation (43) corresponds to the shear force at the initiation of web-shear cracking and normally will govern for short walls. Equation (44) normally will govern for slender walls and corresponds to the shear force at the initiation of flexural-shear cracking at a section approximately $l_w/2$ above the base of the wall. If the quantity $\left(\frac{M_u}{V_u} - \frac{l_w}{2}\right)$ in Eq. (44) is negative, then Eq. (44) does not apply to the wall being analyzed. The value for $\frac{M_u}{V_u}$ is to be evaluated at a section above the base of the wall, and the distance to that section is to be taken as the smallest of $l_w/2$, $h_w/2$ and one-story height (Fig. 19). The value of V_c computed at that section may be used throughout the height of the shear wall.

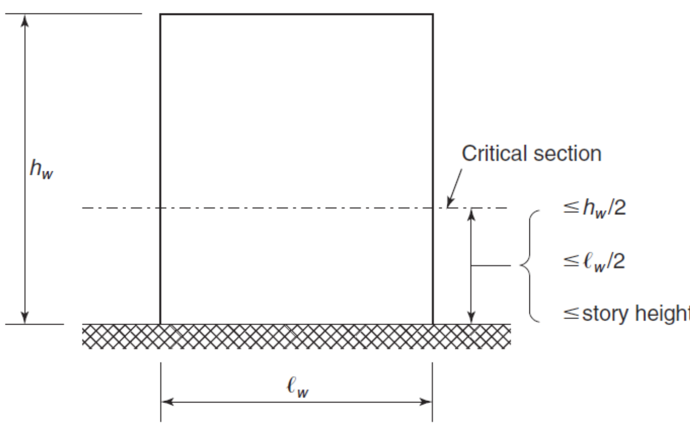


Fig. 19. Location of critical section for checking flexural-shear strength.

11.2 SHEAR REINFORCEMENT FOR STRUCTURAL WALLS

Shear reinforcement for structural walls always consists of evenly distributed vertical and horizontal reinforcement. In many cases, shear cracks in walls are relatively shallow (i.e., their inclination with respect to a horizontal line is less than 45°), so vertical reinforcement will be just as effective—if not more effective—as horizontal reinforcement in controlling the width and growth of such cracks. However, the shear-strength contribution from wall reinforcement is based on the size and spacing of the horizontal reinforcement.

In many cases, only minimum amounts of shear reinforcement are required in structural walls, and those minimum amounts are a function of the amount of shear being resisted by the structural wall. If the factored shear force, V_u, is less than $\phi V_c/2$ then the distributed vertical and horizontal wall reinforcement must satisfy the requirements of ACI Code Section 14.3, as summarized in Table 1. If a designer uses Grade-60 reinforcement and bar sizes not larger than No. 5, then the minimum percentage of vertical steel is 0.0012, and the minimum percentage of horizontal steel is 0.0020. Referring to Fig. 20 for notation definitions, the percentage of vertical (longitudinal) steel is:

$$\rho_l = \frac{A_{v,vert}}{hs_1} \qquad (45a)$$

And the percentage of horizontal (transverse) steel is:

$$\rho_t = \frac{A_{v,horiz}}{hs_2} \tag{45b}$$

For both the horizontal and vertical steel, ACI Code Section 14.3.5 limits the bar spacing to the smaller of 3h and 18 in.

For walls resisting a higher factored shear force (i.e., $\phi V_c/2 < V_u < \phi V_c$), the vertical and horizontal steel in the wall must satisfy the minimum percentages and the maximum spacing requirements of ACI Code Section 11.9.9, as summarized in Table 1. The minimum percentage of horizontal (transverse) reinforcement, ρ_t, is 0.0025 and is to be placed at a spacing that does not exceed the smallest of $l_w/5$, 3h and 18 in. Vertical reinforcement is to be placed at a spacing that does not exceed the smallest of and 18 in. The percentage of vertical (longitudinal) steel, ρ_l, shall not be less than the larger of 0.0025 and that calculated using ACI Code Eq. (11-30):

$$\rho_l = 0.0025 + 0.5\left(2.5 - \frac{h_w}{l_w}\right)(\rho_t - 0.0025) \tag{46}$$

(ACI Eq. 11-30)

For walls with $h_w/l_w \geq 2.5$ this equation will not govern. In shorter walls where the horizontal reinforcement percentage, ρ_t, exceeds 0.0025, the value of calculated in Eq. (46) does not need to exceed the amount (percentage) of horizontal reinforcement required for shear strength, as given next.

If walls are required to resist a factored shear force that exceeds ϕV_c, then horizontal reinforcement must be provided to satisfy the strength requirement expressed in Eq. (40). The shear strength, V_s, provided by the horizontal reinforcement is given by ACI Code Eq. (11-29):

$$V_s = \frac{A_v f_y d}{s} \tag{47}$$

(ACI Eq. 11-29)

In this equation, A_v is the same as $A_{v,horiz}$ shown in Fig. 20, and s is the same as s_2 in that figure. The designer has the option to select a bar size and spacing to satisfy the strength requirement in Eq. (40). In addition to satisfying the shear-strength requirement, a designer must check that both the horizontal and vertical wall reinforcement satisfy the minimum reinforcement percentages and maximum spacing limits given in ACI Code Section 11.9.9, which were discussed in the prior paragraph.

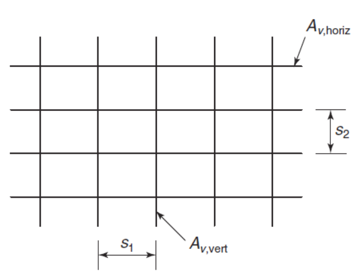


Fig. 20. Distribution of vertical and horizontal shear reinforcement in the web of a shear wall

11.3 SHEAR STRENGTH OF STRUCTURAL WALLS RESISTING SEISMIC LOADS

Even though the next chapter discusses the design for earthquake effects, the modified shear-strength requirements given in ACI Code Chapter 21 for structural walls is also included here for clearing up some aspects. Although the equation that defines the nominal shear strength in ACI Code Chapter 21 appears to be significantly different from that given by the equations in ACI Code Chapter 11, the final values for V_n are not substantially different. The nominal shear strength, V_n, for a wall designed to resist shear forces due to earthquake ground motions is given by ACI Code Eq. (21-7):

$$V_n = A_{cv}\left(\alpha_c \lambda \sqrt{f_c^{'}} + \rho_t f_y\right) \tag{48}$$

(ACI Eq. 21-7)

In this equation, A_{cv} is taken as the width of the web of the wall, h, multiplied by the total length of the wall, l_w. This area is larger than the effective shear area, h×d, used for the equations in ACI Code Chapter 11.

The first term inside the parenthesis of Eq. (48) represents the concrete contribution to shear strength, V_c. The coefficient α_c, represents the difference between the expected occurrence of flexure-shear cracking in slender walls and web-shear cracking in short walls. The value of α_c is taken as 2.0 for walls with $h_w/l_w \geq 2.5$ and as 3.0 for walls with $h_w/l_w \leq 1.5$. A linear variation for the value of α_c is to be used for walls with ratios between 1.5 and 2.0.

The second term inside the parenthesis of Eq. (48) represents the shear-strength contribution from the horizontal wall reinforcement, V_s. Multiplying the definition of ρ_t given in Eq. (45b) by the definition of $A_{cv}(hl_w)$ and the yield strength, f_y, results in the following equivalent value for

$$\begin{aligned} V_{s,equiv} &= \frac{A_{v,horiz}}{hs_2} f_y h l_w \\ V_{s,equiv} &= \frac{A_{v,horiz}}{s_2} f_y l_w \end{aligned} \tag{49}$$

This equation is similar to Eq. (47), which gives the value for V_s used in ACI Code Chapter 11. However, because $l_w > l$ the value of V_s used in ACI Code Chapter 21 exceeds the value of V_s used in ACI Code Chapter 11.

The maximum allowable value for the nominal shear strength, V_n, from Eq. (18-48) is limited to $8A_{cv}\sqrt{f_c^{'}}$ in ACI Code Section 21.9.4.4. Again, this value is similar, but not the same as the upper limit of $10\sqrt{f_c^{'}}hd$ given in ACI Code Section 11.9.3.

The requirements for minimum vertical and horizontal reinforcement percentages and for maximum permissible spacing are the same as those given in ACI Code Section 11.9.9, which were discussed previously. The only modification is given in ACI Code Section 21.9.4.3, which states that for walls with $h_w/l_w \geq 2.0$ the value of ρ_l (vertical steel) shall not be less that the value of ρ_t. This requirement reflects the fact that in short walls, the vertical reinforcement is equal to or more efficient than the horizontal reinforcement in controlling the width and growth of inclined shear cracks.

The discrepancies noted here between the shear-strength equations given in ACI Code Chapters 11 and 21 can create a dilemma for structural designers. The shear strength of walls designed to resist lateral wind forces should be determined from the equations in ACI Code Chapter 11. However, if the same wall is designed to also resist equivalent lateral forces due to earthquake ground motions, the shear strength must be checked using the equations in ACI Code Chapter 21. A unification of these different shear-strength equations is a major goal of the Code Committee as it works toward future editions of the ACI Code.

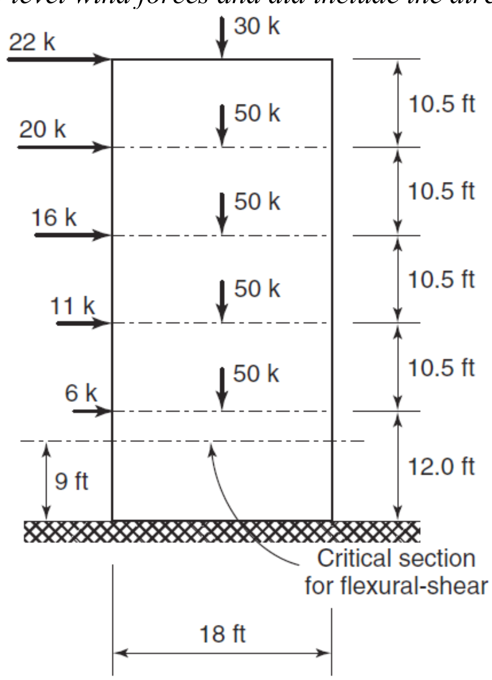
11.4 SHEAR TRANSFER ACROSS CONSTRUCTION JOINTS

The shear force transferred across construction joints can be designed using shear friction. The clamping force is the sum of the tensile reinforcement force components of all the tensile bar forces and permanent compressive forces acting on the joint. This includes all the reinforcement perpendicular to the joint, regardless of the primary use of this steel.

SOLVED EXAMPLE – Structural Wall Subjected to Lateral Wind-Loads

The moment and shear strengths of the structural wall shown in the following figure are to be evaluated for the combined gravity and lateral loads applied to the wall. The wall is 18 ft long and is 10 in. thick. A uniform distribution of vertical and horizontal reinforcement is used in two layers, one near the front face of the wall and the other near the back face. The vertical reinforcement consists of No. 5 bars at 18 in. on centers in each face, and the horizontal reinforcement consists of No. 4 bars at 16 in. on centers in each face. Normal weight concrete with a compressive strength of 4000 psi is used in the wall. All of the wall reinforcement is Grade-60 steel.

The gravity loads applied at each floor level, as shown in the figure, are due to dead load. The live loads are not shown but are assumed to be equal to approximately one-half of the dead loads. The lateral wind loads are based on service-level wind forces and did include the directionality factor.



Step 1 *– Make an Initial Check of Wall Reinforcement*

The percentage and spacing of the vertical and horizontal reinforcement will be checked before we do the strength calculations. The percentage of horizontal reinforcement is found using Eq. (45b):

$$\rho_t = \frac{A_{v,horiz}}{hs_2} = \frac{2 \times 0.2\text{in.}^2}{10\text{in.} \times 16\text{in.}} = 0.0025$$

This satisfies the minimum requirement in ACI Code Section 11.9.9.2, so it should be acceptable unless a larger amount is required to satisfy shear strength requirements. The maximum center-to-center spacing for the horizontal reinforcement is the smallest of $l_w/5$ (3.6 ft. = 43.2 in.), 3h (30 in.) an 18 in. Thus, the provided spacing for the horizontal reinforcement is enough.

Although it is good practice to use larger vertical bars at the edges of the wall, say No. 6 or No. 7 bars, we will calculate the percentage of vertical reinforcement assuming only No. 5 bars are used. From Eq. (45a):

$$\rho_l = \frac{A_{v,vert}}{hs_1} = \frac{2 \times 0.31\text{in.}^2}{10\text{in.} \times 18\text{in.}} = 0.00344$$

Because the wall has an aspect ratio, $h_w/l_w = 3.0 > 2.5$, Eq. (46) will not govern for the minimum percentage of vertical reinforcement. Thus, the minimum percentage of vertical reinforcement is 0.0025 (ACI Code Section 11.9.9.4), which is less than what is provided. The spacing limit for the vertical reinforcement is the smallest of $l_w/3$ (6 ft. = 72 in.), 3h (30 in.), and 18 in. Thus, the provided spacing of vertical reinforcement is enough.

***Step 2** – Check the Moment Strength*

The moment at the base of the wall is equal to the sum of the products of the lateral forces times their respective distances to the base of the wall:

$$M(\text{base}) = 22 \times 54 + 20 \times 43.5 + 16 \times 33 + 11 \times 22.5 + 6 \times 12 = 2910\text{kip} - \text{ft}$$

The appropriate load factor for service-level wind forces is 1.6. Thus, the factored moment at the base of the wall is:

$$M_u(\text{base}) = 1.6 \times 2910 = 4660\text{kip} - \text{ft}$$

The analysis given in [18-23] will be used to evaluate the moment strength of the wall. Using ACI Code Eq. (9-6), the factored axial load is:

$$N_u = 0.9N_D = 0.9 \times 230\text{kips} = 207\text{kips}$$

For 4000-psi concrete, $\beta_1 = 0.85$. Other required parameters are:

$$\omega = \rho_l \frac{f_y}{f_c^{'}} = 0.00344 \frac{60\text{ksi}}{4\text{ksi}} = 0.0516$$

and

$$\alpha = \frac{N_u}{hl_w f_c^{'}} = \frac{207\text{kip}}{10\text{in.} \times 216\text{in.} \times 4\text{ksi}} = 0.024$$

The depth of the neutral axis can be found as:

$$c = \left(\frac{\alpha + \omega}{0.85\beta_1 + 2\omega}\right) l_w = \left(\frac{0.024 + 0.0516}{0.85 \times 0.85 + 2 \times 0.0516}\right) 216\text{in.} = 19.8\text{in.}$$

If we assume that the effective flexural depth, d, is approximately equal to $0.8l_w$ = 173 in., it is clear that c is significantly less than 0.375d. Thus, this is a tension-controlled section, and we will use a strength reduction factor ϕ, equal to 0.9. To calculate the nominal moment strength, we first must determine the tension force at nominal strength conditions. The value of A_{st} for the vertical steel can be calculated as

$$A_{st} = 2A_b \frac{l_w}{s_1} = 2 \times 0.31\text{in.}^2 \times \frac{216\text{in.}}{18\text{in.}} = 7.44\text{in.}^2$$

$$T = A_{st} f_y \left(\frac{l_w - c}{l_w}\right) = 7.44\text{in.}^2 \times 60\text{ksi} \times \left(\frac{216\text{in.} - 19.8\text{in.}}{216\text{in.}}\right) = 405\text{kips}$$

$$M_n = T\left(\frac{l_w}{2}\right) + N_u\left(\frac{l_w - c}{2}\right) = 405\text{kips}\left(\frac{216\text{in.}}{2}\right) + 207\text{kips}\left(\frac{216\text{in.} - 19.8\text{in.}}{2}\right) = 5340\text{kip} - \text{ft}$$

By using the strength reduction factor ϕ

$$\phi M_n = 0.9 \times 5340\text{kip} - \text{ft} = 4800\text{kip} - \text{ft}$$

Becasue ϕM_n is larger than M_u, the wall has adequate flexural strength.

***Step 3** – Check for Shear*

The factored shear force at the base of the wall is:

$$V_u = 1.6(6 + 11 + 16 + 20 + 22) = 120\text{kips}$$

Because this wall is slender, h_w/l_w = 3.0, it can be safely assumed that Eq. (44) will govern for the shear-strength contribution from the concrete, However, both Eqs. (43) and (44) will be checked to demonstrate how each is used. For normal-weight concrete, λ = 1.0. Using d = $0.8l_w$ (173 in.) and putting all of the quantities into units of pounds and inches, the value of V_c from Eq. (43) is

$$V_c = 3.3\lambda\sqrt{f_c'}hd + \frac{N_u d}{4l_w} = 3.3 \times 1 \times \sqrt{4000} \times 10 \times 173 + \frac{207000 \times 173}{4 \times 216} = 402\text{kips}$$

For Eq. (44) one needs to evaluate M_u/V_u at the critical section above the base of the wall. The distance to that section is the smallest of $l_w/2$ (9 ft.), $h_w/2$ (27 ft.) and one story height (12ft.). In this case $l_w/2$ governs.

$$M_u(\text{crit.sec t.}) = M_u(\text{base}) - V_u(\text{base})\frac{l_w}{2} = 4660\text{kip} - \text{ft} - 120\text{kip} \times 9\text{ft} = 3580\text{kip} - \text{ft}$$

Thus, the ratio of M_u/V_u = 3580/120 = 29.8 ft. Using this value in Eq. (44) and expressing all of the quantities in pounds and inches, it leads to:

$$V_c = \left[37.9\text{psi} + \frac{216\text{in.}\left(79.1\text{psi} + 19.2\text{psi}\right)}{358\text{in.} - 108\text{in.}} \right] \times 10\text{in.} \times 173\text{in.} = 212\text{kips}$$

This value governs for V_c, as expected. Using ϕ = 0.75, as defined in ACI Code Section 9.3.2.3 for shear and torsion, ϕV_c = 0.75×212 = 159 kips. This exceeds V_u, so no calculation of the value of V_s or the provided horizontal (transverse) reinforcement is required. It is clear that the value of $0.5\phi V_c$ is less than V_u so the requirements for horizontal and vertical reinforcement in ACI Code Section 11.9.9 will govern for this wall. Those requirements were checked in step 1 and were found to be at or above the ACI Code required values.

12. CRITICAL LOADS FOR AXIALLY LOADED WALLS

12.1 BUCKLING OF COMPRESSED WALLS

The critical stress for buckling of a hinged column or a one-way wall, with a rectangular cross-sectional area (b × h) is:

$$\sigma_{cr} = \frac{P_{cr}}{bh} = \frac{\pi^2 EI}{(kl)^2}\left(\frac{1}{bh}\right) \qquad (50)$$

where b is the width of the wall, and kl is the effective length of the wall. The flexural stiffness of a wall section of width b and thickness h is:

$$EI = \frac{Ebh^3}{12}$$

The corresponding stiffness of a slab or a two-way wall, per unit of width of wall, is:

$$D = \frac{Eh^3}{12\left(1-\mu^2\right)} \qquad (51)$$

For an axially loaded and uncracked concrete column with little or no reinforcement the critical stress, σ_{cr}, is a function of the tangent modulus of elasticity, E_T, evaluated at the critical stress, σ_{cr}. Substituting E_T into Eq. (50), replacing l^2 with b^2, replacing EI with the plate stiffness, D, and including an appropriate edge restraint factor, K, gives the critical stress for a two-way wall:

$$\sigma_{cr} = \frac{P_{cr}}{bh} = K\frac{\pi^2 E_T}{12\left(1-\mu^2\right)}\left(\frac{b}{h}\right)^2 \qquad (52)$$

In this equation, b is the effective width of the panel, taken equal to the smaller of the width and height of the wall between lateral supports, h is the thickness of the wall, μ is Poisson's ratio, and K is an edge restraint factor that varies depending on the degree of fixity of the edges of the plate. It is similar to the effective length term k^2 in Eq. (50). The term K is plotted in Fig. 21. Values of K at buckling for other types of compressed plates are given in books on structural stability [27]. Equations (50) and (52) have been included to show the similarity between the critical loads for columns and walls.

12.2 PROPERTIES OF CONCRETE AFFECTING THE STABILITY OF CONCRETE WALLS

12.2.1 Compressive Stress–Strain Curve for the Concrete

The tangent modulus of the concrete in the wall will be used to estimate the tangent modulus buckling loads. A good approximation of the stress–strain curve for the wall is the second-degree parabola given by Eq. (53) and shown in Fig. 22a, with a peak stress of $f_{2,max} = 0.9f_c'$. The apex of the parabola will be taken at a strain of $\varepsilon_{c0} = 0.002$ and:

$$f_2 = f_{2,max}\left[2\left(\frac{\varepsilon_2}{\varepsilon_{c0}}\right)-\left(\frac{\varepsilon_2}{\varepsilon_{c0}}\right)^2\right] \tag{53}$$

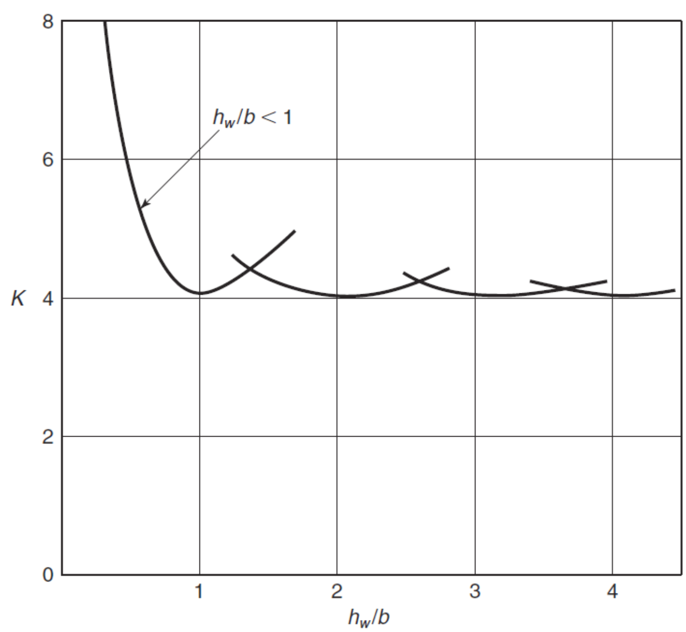


Fig. 21. Wall buckling factor, K, for buckling of a compressed plate analogous to a compressed wall in a hollow rectangular bridge pier.

where f_2 is the compressive stress in the most compressed wall of the box pier; ε_2 is the corresponding strain, assumed constant over the thickness of the wall. The strain at any level of stress in Eq. (53) is given by:

$$\varepsilon_c = \varepsilon_{c0}\left(1-\sqrt{1-\frac{f_2}{f_{2,max}}}\right) \tag{54}$$

The tangent modulus of elasticity, E_T, is the slope of the tangent to the stress–strain curve at some particular stress, in this case of point A in Fig. 18-24. It is computed using:

$$E_T = \frac{d\sigma}{d\varepsilon}$$

evaluated at the critical stress, σ_{cr}. For example, the tangent modulus at point A in Fig. 22b is calculated as

$$E_T = \frac{2\Delta f_2}{\Delta \varepsilon_2} \quad (55)$$

The 2 in Eq. (55) comes from the fact that the tangent to a second-order parabola intersects the vertical axis through ε_{c0} at 2 times the stress increment between f_2 and $f_{2,max}$, as shown in Fig. 22b. For example, the initial tangent modulus at $f_2 = 0$ for the stress–strain curve given by Eq. (55) is for $f_c^{'} = 4000$ psi:

$$E_T = \frac{2 \times 3600\text{psi}}{0.002} = 3600000\text{psi}$$

This is very close to the value obtained from ACI Code Section 8.5:

$$E_T = 57000\sqrt{f_c^{'}} = 3600000\text{psi}$$

The amount of reinforcement in a wall is generally so small that it can be ignored when determining E_T.

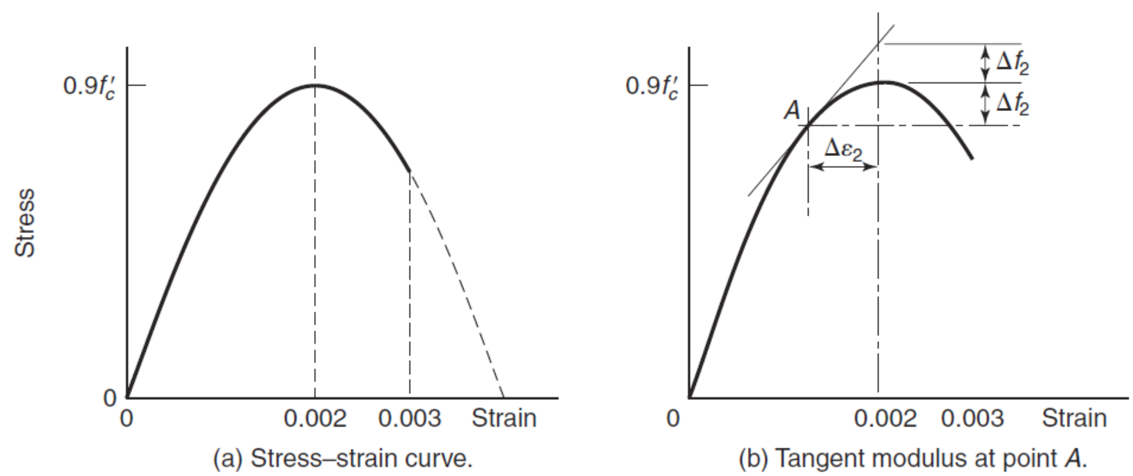


Fig. 22. Second-order parabolicstress–strain curve for compressed concrete

12.3 POST BUCKLING BEHAVIOR OF CONCRETE PLATES

After a uniaxially loaded wall panel hinged on four edges buckles, the strips of wall along the edges parallel to the load continue to resist a portion of the compressive in-plane load. At the same time, the load resisted by the center portion of the buckled plate decreases. Failure generally comes from excess bending of the buckled parts. The net effect is that the plate does not suddenly fail. Instead, strips along the edges of the wall continue to support a stress that may approach the buckling stress. This is offset by a drop in the stress resisted by the center of the plate.

CHAPTER 2 – DESIGN FOR EARTHQUAKE RESISTANCE

1 INTRODUCTION

Plate tectonics theory visualizes the earth as consisting of a viscous, molten magma core with a number of lower-density rock plates floating on it. The exposed surfaces of the plates form the continents and the bottoms of the oceans. As time goes by, the plates move relative to each other, breaking apart in some areas and jamming together in others.

Where the plates are moving apart, this movement causes cracks (or rifts) to form, generally in the ocean beds. In some cases, molten magma flows out of these rifts. The regions where the plates are either moving into each other or are sliding adjacent to each other are referred to as fault zones. Compression and shear stresses are generated in the plates and strain energy builds up in at the edges of the plates. At some point in time, the stresses and strain energy at a locked fault surpass the limiting resistance to rupture or slip along the fault. Once started, energy is released rapidly, causing intense vibrations to propagate out from the fault. Three main types of stress waves travel through the rock layers: primary (compression) waves, secondary (shear) waves, and surface waves—each at different speeds. As a result, the effects of these seismic waves and local soil conditions will lead to different ground motions at various sites. Earthquakes may involve regions of slip and/or offsets along surface faults.

Earthquake ground motions impart vertical and horizontal accelerations, a, to the base of a structure. If the structure was completely rigid, forces of magnitude $F = ma$ would be generated in it, where m is the mass of the structure. Because real structures are not rigid, the actual forces generated will differ from this value depending on the period of the building and the dominant periods of the earthquake ground motions. The determination of the seismic force, E, is made more complicated because recorded earthquake ground motions contain a wide range of frequencies and maximum values of base acceleration.

1.1 DEFINITIONS

Size of earthquake—The size of an earthquake typically is quantified in terms of total energy released.
Magnitude—The *magnitude* of an earthquake is an estimate of the total energy released during the earthquake event, often given by the *Richter magnitude, M* [29]. An increase in magnitude by one digit, from 6 to 7, for example, involves an increase in energy released of times.
Intensity—The intensity of an earthquake is a measure of the shaking at a given site, sometimes presented in terms of observed damage. A commonly used scale is called the *Modified Mercalli, MM Scale* [30]. Because these verbal descriptions are not quantifiable, a more precise engineering measure of intensity similar to the *Housner Spectral Intensity* [31] may be used.
Location of earthquake—The location where an earthquake is initiated is called the *hypocenter*. The location of the hypocenter is defined by the latitude, longitude and depth below the surface. The *epicenter* is the point on the surface of the earth directly over the hypocenter.

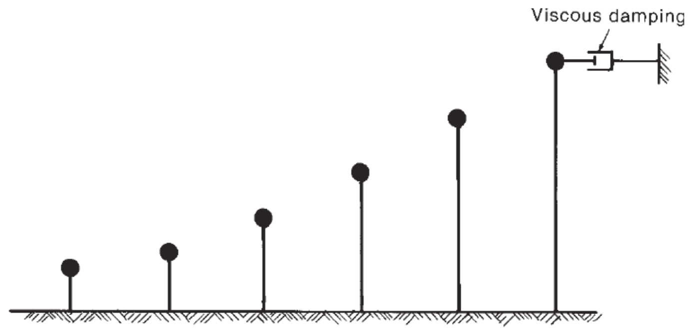
1.2 SEISMIC DESIGN REQUIREMENTS

Procedures for the analysis and design of structures to resist the effects of earthquake ground motions are in a continuous state of development. In addition to the work of the ACI Code Committee, several other regulatory bodies [32], [33] and research development groups [34], [35] constantly are evaluating and updating code-type analysis and design requirements. Therefore, significant changes to code requirements continue to appear at a rapid rate. For this reason, some of design requirements noted in this chapter will be modified within a few years. However, the design philosophy and general design procedures for reinforced-concrete members are well established and will not change significantly over time. This chapter concentrates on those general principles and gives the latest ACI Code requirements for the earthquake-resistant design of reinforced concrete members. A more in-depth discussion of the inelastic behavior of reinforced concrete members and earthquake-resistant design procedures for reinforced concrete buildings is given by Sozen and Garcia [36], [37].

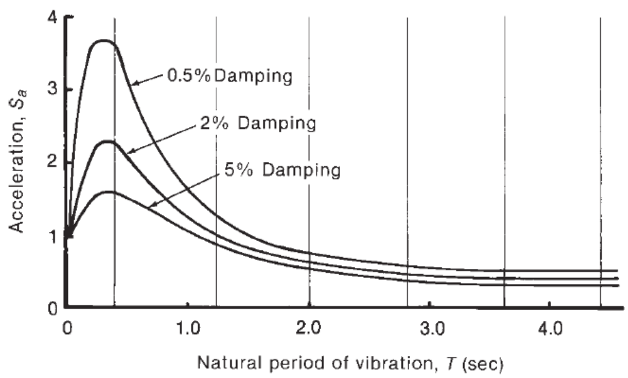
2 SEISMIC RESPONSE SPECTRA

The effect of the size and type of vibration waves released during a given earthquake can be organized so as to be more useful in design in terms of a *response spectrum* for a given earthquake or family of earthquakes. Figure 19-1a shows a family of inverted, damped pendulums, each of which has a different period of vibration, *T*. To derive a point on a response spectrum, one of these hypothetical pendulum structures is analytically subjected to the vibrations recorded during a particular earthquake. The largest acceleration of this pendulum structure during the entire record of a particular earthquake can be plotted as shown in Fig. 23b. Repeating this for each of the other pendulum structures shown in Fig. 23a and plotting the peak values for each of the pendulum structures produces an *acceleration response spectrum*.

Generally, the vertical axis of the spectrum is normalized by expressing the computed accelerations in terms of the acceleration due to gravity. If, for example, the ordinate of a point on the response spectrum is 2 for a given period *T*, it means that the peak acceleration of the pendulum structure for that value of *T* and for that earthquake was twice that due to gravity. The random wave content of an earthquake causes the derived acceleration response spectrum to plot as a jagged line, as shown in Fig. 24c. The spectra in Fig. 23b has been smoothed.



(a) Damped pendulums of varying natural frequencies.



(b) Acceleration response spectrum.

Fig. 23. Earthquake response spectrum

2.1 EARTHQUAKE RESPONSE SPECTRUM.

2.1.1 Velocity and Displacement Spectra

Following the procedure used to obtain an acceleration spectrum, but plotting the peak velocity relative to the ground during the entire earthquake against the periods of the family of pendulum structures, gives a velocity response spectrum. A plot of the maximum displacements of the structure relative to the ground during the entire earthquake is called a displacement response spectrum. These three spectra for a particular earthquake measured on rock or firm soil sites are shown in Fig. 24.

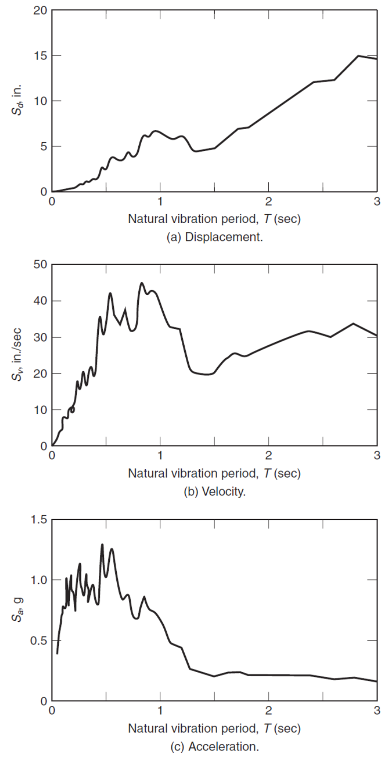


Fig. 24. Displacement, velocity, and acceleration spectra for a given earthquake

2.1.2 Factors Affecting Peak Response Spectra

Period of Building

Lateral seismic forces are closely related to the fundamental period of vibration of the building. At periods less than about 0.5 sec, the maximum effect for a structure on a firm soil site results from the magnification of the acceleration, as shown by the highest spikes in Fig. 24c. For structures with medium periods (from about 0.5 sec to about 2.0 sec), the largest structural response appears in Figure 24b, the velocity response spectrum. Finally, at long periods (above about 2.0 sec), the dominant structural response appears in the displacement spectrum.

The period of the first mode of vibration, referred to as the fundamental or natural period, can be estimated from empirical equations given in ASCE/SEI 7 [32], or from Rayleigh's method [38]. For concrete structures, the period, T, in seconds can be estimated from the formula

$$T = C_T h_n^{3/4} \tag{56a}$$

Where:
T – the period in seconds
h_n – the height of the building above exterior grade in feet for concrete buildings with moment-resisting frames providing 100 percent of the required lateral force resistance
$C_T = 0.020$ for all other concrete structures
$C_T = 0.030$

The exponent and coefficient were changed to variables in the most recent version of ASCE/SEI 7. Alternatively, the fundamental period of buildings not exceeding 12 stories and consisting entirely of concrete moment-resisting frames with a story height of at least 10 ft can be estimated from

$$T = 0.1N \tag{56b}$$

where N is the number of stories above the exterior grade.

From a series of studies following the 1985 Chilean Earthquake, Wight et al. [39] reported that for buildings where the lateral-force resisting system consisted of a high percentage of structural walls, the fundamental period of such buildings could be estimated as

$$T \cong 0.05N \tag{56c}$$

Effect of Damping on Response Spectrum

Each of the curves in Fig. 23b corresponds to a particular degree of damping. Damping is a measure of the dissipation of energy in the structure and is due to cracking, sliding friction on the cracks, and slip in connections to nonstructural elements. As the damping increases, the ordinates of the response spectrum decrease. Typically, a reinforced concrete building will have 1 to 2 percent of critical damping prior to the building being exposed to an earthquake.

As cracking and structural and nonstructural damage develop during the earthquake, the damping increases to about 5 percent. By definition, critical damping acts to quickly damp out structural vibrations.

Effect of Ductility on Seismic Forces

As an undamped elastic pendulum is deflected to the right, energy is stored in it in the form of strain energy. The stored energy is equal to the shaded area under the load-deflection diagram shown in Fig. 25a. When the pendulum is suddenly released, this energy reenters the system as velocity energy and helps drive the pendulum to the left. This pendulum will oscillate back and forth along the load-deflection diagram shown.

If the pendulum were to develop a plastic hinge at its base, the load-deflection diagram for the same lateral deflection would be as shown in Fig. 25b. When this pendulum is suddenly released, only the energy indicated by the triangle a–b–c reenters the system as velocity energy, the rest being dissipated primarily by crack development and reinforcement yielding.

Studies of hypothetical elastic and elastic–plastic buildings subjected to a number of different earthquake records suggest that the maximum lateral deflections of elastic and elastic–plastic structures are roughly the same for moderate to long period structures.

Figure 26 compares the load-deflection diagrams for an elastic structure and an elastic–plastic structure subjected to the same lateral deflection, Δ_u. The ratio of the maximum deflection, Δ_u, to the deflection at yielding, Δ_y, for the inelastic structure, is called the displacement ductility ratio, μ:

$$\mu = \frac{\Delta_u}{\Delta_y} \tag{57}$$

From Fig. 26, it can be seen that, for a ductility ratio of 4, the lateral load acting on the elastic–plastic structure would be of $\frac{1}{\mu} = \frac{1}{4}$ that on the elastic structure at the same maximum deflection. Thus, if a structure is ductile, it can be designed for lower seismic forces.

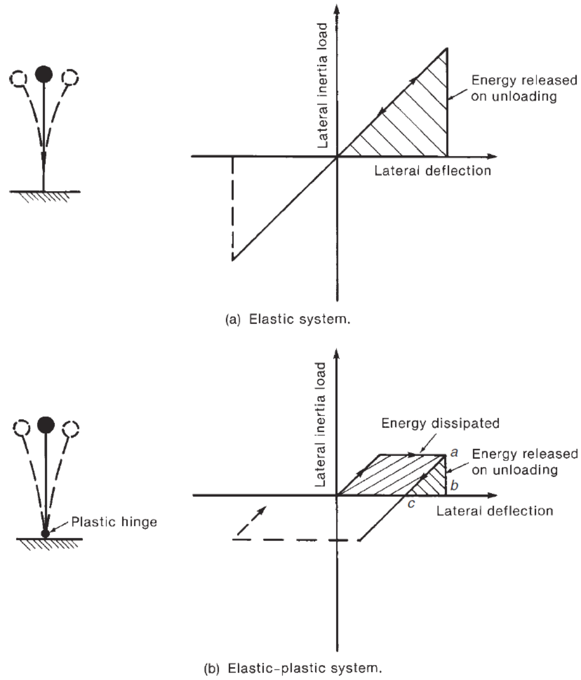


Fig. 25. Energy in vibrating pendulums

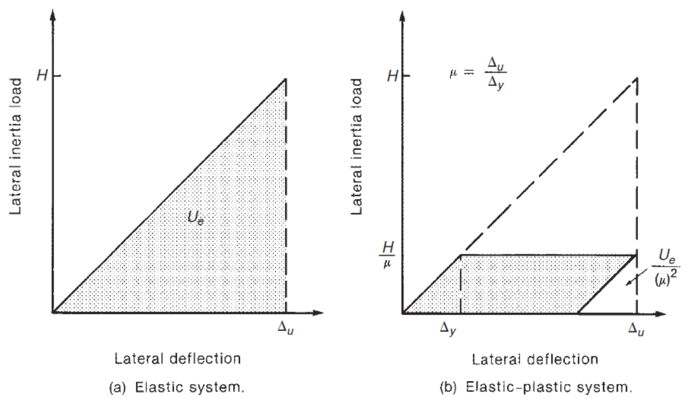


Fig. 26. Effect of ductility ratio, on lateral force and strain energy in structures deflected to the same Δ_u

Effect of Foundation Soil Stiffness on Response Spectrum

The response spectrum at bedrock normally plots below the response spectrum of a structure that is founded on layers of subsoil between the bedrock and the building footings. For structures on alluvial layers of soil, the increase in the ordinates of the response spectrum is a function of the amplification caused by the various soil layers. Since 1995, seismic design codes have required that designers recognize the effects of foundation soils on seismic response.

3. SEISMIC DESIGN REQUIREMENTS

3.1 SEISMIC DESIGN CATEGORIES

Seismic design codes [32], [33] require that all building structures be assigned to a particular seismic design category. This assignment is made on the basis of three key parameters, i.e., the expected intensity of the seismic ground motions, the site classification, and the building importance factor. The ASCE/SEI 7 Standard [32] gives several maps of spectral response accelerations at periods of 0.2 sec. and 1.0 sec. for all locations throughout the USA. The seismic response accelerations, along with the site classification, are used to establish the design response spectrum for a structure. Site classification is a function of the local soil properties where the structure is to be located. Classifications vary from Class A—hard rock, to Class E—soft clay and Class F—soil requiring a special site response analysis. The building importance factors are related to building occupancy categories, which are defined in [32]. These categories range from Category I – buildings and other structures that represent a low hazard to human life in the event of failure, to Category IV – buildings and other structures designated as essential facilities. Buildings in Occupancy Category IV have the highest importance factor, and vice-versa.

Based on the design response spectrum, which is a function of the expected seismic ground motions and the local site classification, and the building importance factor, ASCE/SEI 7 [32] assigns buildings to a specific seismic design category ranging from A to F.

Seismic design category A is for structures sited on firm soils where expected maximum ground motions are quite low. Seismic design categories B, C, D, E, and F represent structures where either larger ground motions are expected, or the site consists of softer soil conditions, or the building has a higher importance factor. ACI Code Chapter 21 now gives specific design requirements for concrete buildings based on their seismic design category, as will be discussed later.

3.2 LATERAL FORCE-RESISTING STRUCTURAL SYSTEMS

The magnitude of the lateral design force for concrete structures is a function of the design response spectrum for the building site and the type of structural system used to resist those forces. As discussed previously, more ductile structural systems can be safely designed for lower seismic forces than systems with limited ductility. This is handled in ASCE/SEI 7 [32] by defining a response modification coefficient, R, which is larger for more ductile structural systems. The general structural systems defined in [32] include bearing wall systems, moment-resisting frame systems, and dual systems consisting of a combination of shear walls and moment-resisting frames that work together to resist lateral loads. In general, bearing wall systems are less ductile than either moment-resisting frames or dual systems. Also, depending on the level of structural detailing, which ranges from ordinary to special, a variety of different R-factors are assigned to moment-resisting frames and dual systems. Buildings assigned to high seismic design categories will require special structural detailing, as is discussed in subsequent sections.

3.3 Effect of Building Configuration

One of the most important steps in the design of a building for seismic effects is the choice of the building configuration—that is, the distribution of masses and stiffnesses in the building and the choice of load paths by which lateral loads will eventually reach the ground. In recent years, seismic design codes [32], [33] have classified buildings as regular or irregular. Irregularities include many aspects of structural design that are conducive to seismic damage. Irregular buildings require a more detailed structural analysis, design provisions to reduce the impact of each irregularity, and more detailing requirements than do regular buildings. Irregularities are classified as plan irregularities or vertical irregularities, as summarized here.

3.3.1 Plan Irregularities

1. Torsional Irregularities.

Ideally, a building subjected to earthquakes should be symmetrical—or, at least, the distance between the center of mass (the point through which the seismic forces act on a given floor) and the center of resistance should be minimized.

If there is an eccentricity, as illustrated in Fig. 27a, the building will undergo torsional deflections, as shown. The column at A in Fig. 27a will then experience larger shears than the column at B. The location of the center of resistance is affected by the presence of both structural and "nonstructural" elements.

The computed relative deflection of the top and bottom of a story is referred to as the story drift, δ_{max}. A category 1a torsional irregularity exists when the maximum story drift, at one end of a story, is more than 1.2 times the average story drift in the same story [32].

This definition applies only to buildings with rigid or semirigid diaphragms.

A category 1b torsional irregularity exists when the ratio of maximum to average elastic computed drifts exceeds 1.4 [32].

Irregular buildings should have significant torsional resistances and stiffnesses. As discussed previously, because the individual walls in Fig. 27b are farther from the center of resistance than those in Fig. 27c, they provide more torsional resistance. The core of the building in Fig. 27c is almost a closed tube, which tends to be stiffer in torsion than disconnected walls. The plan in Fig. 27d, which has been used for corner buildings, is particularly unsuitable. It has a large eccentricity and very little torsional resistance.

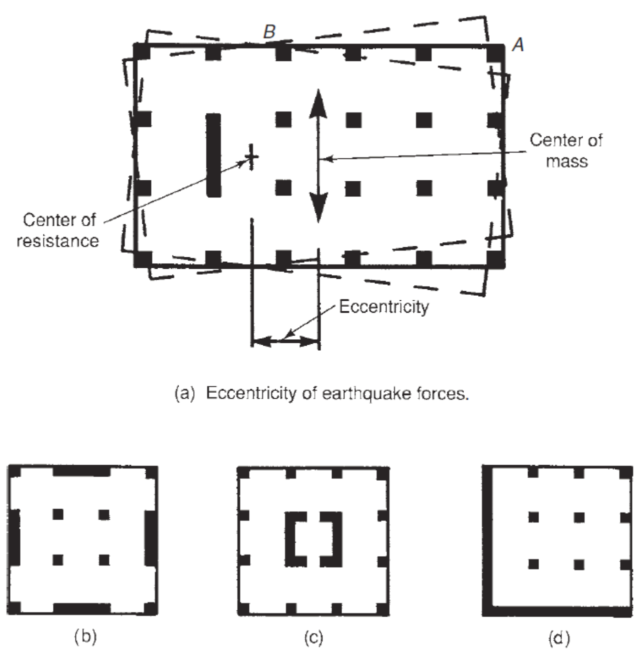


Fig. 27. Eccentricities and torsional deformations

2. Reentrant Corner Irregularity.

If the plan has reentrant corners and the floor system projects beyond the reentrant corner by more than 15% of the plan dimension of the building in the same direction, the building is said to have a reentrant corner irregularity.

For the building on the left in Fig. 28, one solution is to separate the two wings by a joint that is wide enough so that the wings can vibrate separately without banging together. If this is not practical, the region joining the two parts must be strengthened to resist the tendency to pull apart.

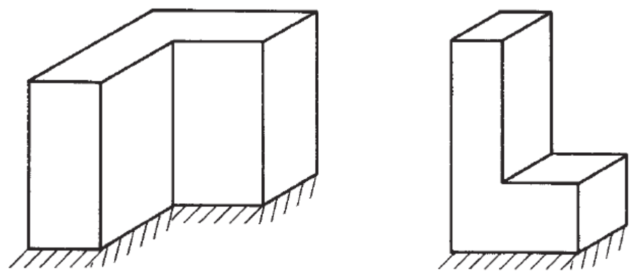


Fig. 28. Geometric irregularities

3. Diaphragm Discontinuity Irregularity.

Figure 29 shows a plan of a floor diaphragm transmitting seismic forces to shear walls at each end of a building. The diaphragm acts as a deep thin beam that develops tension and compression on its edges. Abrupt discontinuities or changes in the diaphragms, such as a notch in a flange, may lead to significant damage.

If there are abrupt changes in the stiffness of the diaphragms, including a cutout or open areas comprising more than 50% of the diaphragm or cross-sectional area, or 50% from one story to the next, the building is said to have a diaphragm discontinuity irregularity. The loss of cross-sectional area for the discontinuity shown in Fig. 29 is smaller than 50% and would not qualify as a diaphragm irregularity.

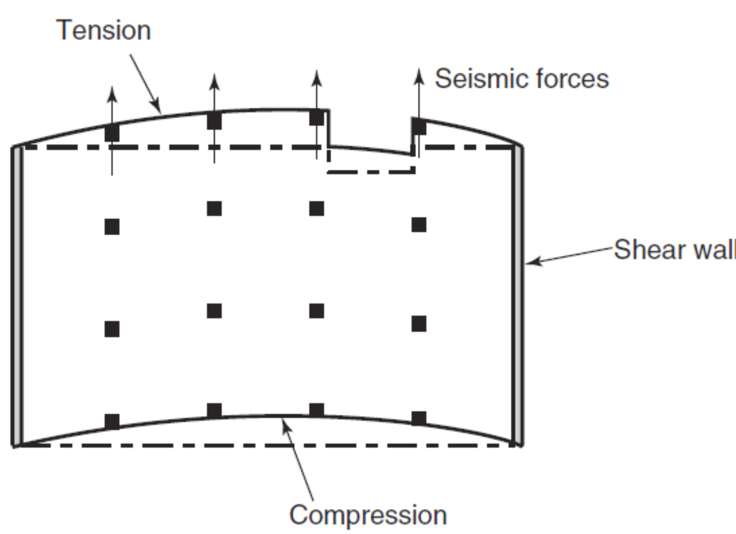


Fig. 29. Diaphragm discontinuties

3.3.2 Vertical Irregularities

Vertical irregularities are abrupt changes in the geometry, strength, or stiffness of a structure from floor to floor.

1. Stiffness Irregularity—Soft Story. A category (1a) soft story is one in which the lateral stiffness is between 70% and 80% of that of the story above or below it. This becomes an extreme soft-story irregularity (1b) if the lateral stiffnesses are from 60% to 70% of those of the adjacent stories [32]. The soft story created by terminating or greatly reducing the stiffness of the shear walls of the ground story, shown in Fig. 30, concentrates lateral deformations at that level.

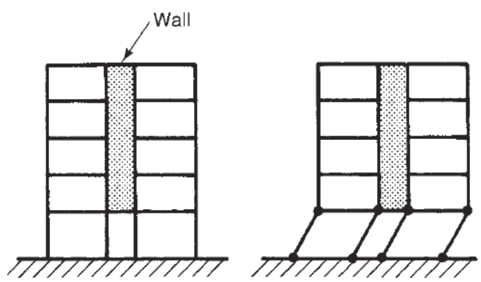


Fig. 30. Soft story due to discontinued shear walls

2. Weight (Mass) Irregularity. A mass irregularity exists where the effective mass of any story exceeds 150% of the effective mass of an adjacent story.
3. Vertical Geometric Discontinuity. This type of irregularity occurs when the horizontal dimension of the lateral-force-resisting system in any story is more than 130% of that in an adjacent story.
4. In-Plane Discontinuity in Vertical, Lateral-Force-Resisting Elements. An in-plane discontinuity is considered to exist where an in-plane offset of the lateral-force-resisting elements exists, as shown in Fig. 31a and 31c, or where the stiffness of the resisting element in the story below is smaller than that for the story in question, Fig. 31b.

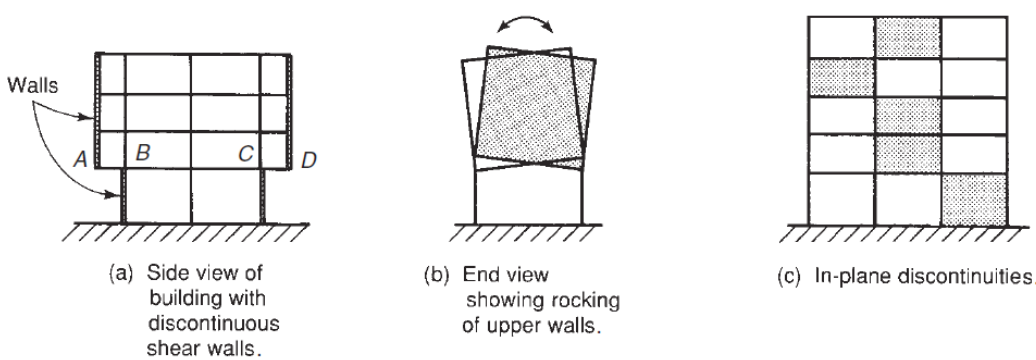


Fig. 31. Discontinuous shear walls

5. Discontinuity in Lateral Strength: Weak Story. A weak story exists if the lateral resistance of a story is less than 80% that of the story above. The lateral resistance of a story is the total strength of all lateral-force-resisting elements in the story.

Variations in the column stiffnesses attract forces to the stiffer columns. Because of their different free lengths, the lateral stiffness of column D in Fig. 32 would be four times that of column B for the same cross section. Initially, column D would be called upon to resist four times the shear force in column B. Frequently, such a column will fail in shear above the wall.

Sometimes the change in column stiffness is caused by the restriction of free movement caused by nonstructural elements, such as the masonry walls shown shaded in the figure.

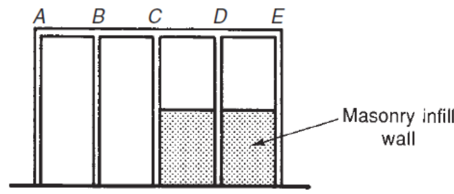


Fig. 32. Differences in column stiffnesses

4. SEISMIC FORCES ON STRUCTURES

The ACI Code does not specify earthquake ground motions for a given site or give details about how structures should be analyzed for seismic actions. These details are provided in the general building code for the area [33]. Currently, general building codes allow different levels of seismic analysis. The three analysis procedures permitted by ASCE/SEI 7 [32] are the equivalent lateral force procedure, the modal spectrum analysis procedure and an inelastic response history analysis procedure.

4.1 VERTICAL AND HORIZONTAL COMPONENTS OF E

American earthquake design codes [32], [33] published since 1997 have divided the earthquake load, E, into horizontal and vertical force components, and as follows:

$$E = E_h + E_v \tag{58}$$

$$E = Q_E + 0.25S_{DS}D \tag{59}$$

Q_E is the horizontal load effect caused by seismic forces. E_v is the vertical component of seismic forces, taken equal to $0.2S_{DS}D$, where S_{DS} the design spectral acceleration at a short period, such as 0.2 sec, and D is the dead load.

4.2 EQUIVALENT LATERAL FORCE METHOD FOR COMPUTING

4.2.1 Earthquake Forces

Typically, the equivalent lateral force method is permitted for regular buildings up to about 20 stories. Sometimes it can be used for irregular buildings if special attention is given to the types of irregularities. Geotechnical tests of the subsoil at the site help the designer estimate the degree to which soil–structure interaction will modify the seismic effects on the structure.

4.2.2 Seismic Base Shear

The seismic base shear is calculated as:

$$V = C_s W \tag{60}$$

where C_s is the seismic response coefficient for the building, and W is the effective seismic weight of the building.

4.2.3 Seismic Response Coefficient, C_s

Typically, the seismic response coefficient is given by

$$C_s = \frac{S_{DS}}{R/I} \tag{61}$$

where S_{DS} quantifies the response spectrum as a function of the period, T, the damping, and the foundation stiffness; R is a response-modification factor that accounts for the reduction in seismic loads caused by inelastic action and energy dissipation; and I is the earthquake importance factor for the building and its occupancy.

A typical plot of C_s as a function of the period, T, is given in Fig. 33. Sometimes, this curve is drawn with three branches, adding a steep rising section at low periods. The actual shape used depends on the factors incorporated in C_s.

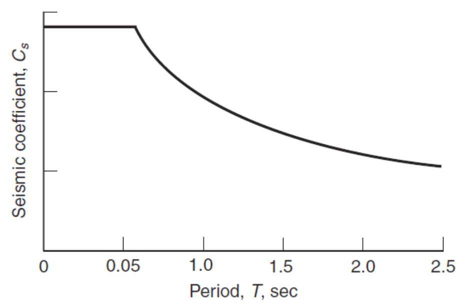


Fig. 33. Variation of seismic response coefficient C_s

4.2.4 Effective Seismic Weight of the Building, W

The weight, W, of the building used to compute V is intended to represent the gravity loads likely to be present when an earthquake occurs. In ASCE/SEI 7 [32], W is calculated as follows:

W = 100 percent of the unfactored dead load,
+ the partition load based on the weight of the partitions (or a minimum weight of 10 psf),
+ in areas used for storage, at least 25 percent of the unfactored live load,
+ where the design show load exceeds 30 psf, not less than 20 percent of the unfactored snow load on a roof,
+ the unfactored load from the full contents of any tanks, and
+ the weight of permanent equipment

4.2.5 Response-Modification Factor, R

The response-modification factor, R, reflects
(**a**) the ability of the structure to dissipate energy through inelastic action, as shown in Fig. 26b, and
(**b**) the redundancy of the structure.

It is assumed that the level of ductility governs the reduction in seismic forces for the various families of lateral-force-resisting systems. Typical values for reinforced concrete structures are given in Table 2.

4.2.6 Distribution of Lateral Forces over the Height of a Building

The base shear, V, from Eq. (60) is distributed as a series of lateral forces at each floor level and at the roof. In general, the distribution of the lateral forces is assumed to be similar to the deflected shape for the first mode of vibration, which corresponds to an inverted V, triangular distribution of lateral forces for short period structures.

The coefficient, k, is used to account for the higher modes of vibration. Thus, the lateral force at a given floor level, x, is:

$$F_x = VC_{vx} \tag{62}$$

where

$$C_{vx} = \frac{w_x h_x^k}{\sum_{i=1}^{n} w_i h_i^k} \tag{63}$$

TABLE 2. Response-Modification Coefficient, R, for Seismic Resistance

Basic Structural System (RC)	Seismic Force Resisting System	Response Modification Coefficient, R
Bearing wall system	Special reinforced concrete shear walls	5
Building frame system	Special reinforced concrete shear walls	6
Moment Resisting Frame System	Special moment frames (SMF)	8
	Intermediate moment frames (IMF)	5
	Ordinary moment frames (OMF)	3
Dual System with a SMF capable of resisting at least 25% of prescribed seismic force	Special reinforced concrete shear walls	7
	Ordinary reinforced concrete shear walls	6
Dual System with a IMF capable of resisting at least 25% of prescribed seismic force	Special reinforced concrete shear walls	6.5
	Ordinary reinforced-concrete shear walls	5.5

in which F_x, w_x and h_x are the lateral force, weight, and height, respectively, at level x above grade; i = 1 refers to the first level of the building above grade; and i = n refers to the top level (roof). The coefficient, k, is an exponent related to the structural period. It is taken equal to 1.0 for structural periods at or below 0.5 seconds, and it is equal to 2.0 for periods at or above 2.5 seconds. For structural periods between 0.5 and 2.5 seconds, k is determined by a linear interpolation between 1.0 and 2.0. Typical lateral load distributions are shown in Fig. 34.

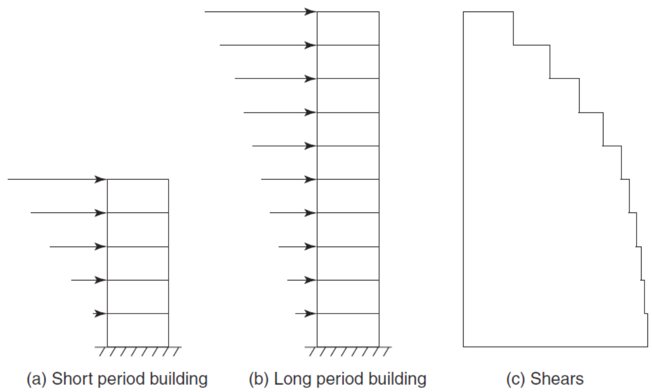


Fig. 34. Distribution of equivalent lateral forces and shears

4.2.7 Story Shear

The shear, V_x, in any story x is the sum of the lateral forces, F_i, acting above that story:

$$V_x = \sum_{i=x}^{n} F_i \tag{64}$$

4.2.8 Story Torsion

As discussed earlier, the story shear, V_x, in a story is assumed to act horizontally through the center of mass of the story, resulting in a torsional moment, T, equal to the product of the seismic forces times the horizontal eccentricity, e_x or e_y, between the center of mass and the center of resistance, measured perpendicular to the line of action of the seismic force (see Fig. 13.). To account for accidental torsion, a torsional moment, T_{ax}, equal to the story shear times a distance is added. ASCE/SEI 7 [32] takes e_{ay} equal to $\pm 0.05L_b$, where L_b is the horizontal length of the building perpendicular to the assumed direction of the applied forces. Thus,

$$T_x = V_x \times e_y + V_x \times e_{ay} \tag{65}$$

where T_x is the torsional moment due to earthquake forces in the x direction, e_y is the distance in the y direction between the center of mass and the center of resistance, and e_{ay} is 5% of the building length in the x direction. The second term represents an accidental increase in the torsional effects. Such an increase may occur if, for example, a corner column, such as A in Fig. 27a, cracks and loses some of its stiffness before the other columns crack. When this occurs, the center of resistance moves toward the stiffer columns (to the left in Fig. 27a), thereby increasing the torsional effects. Each element in the building is then designed for the most severe effects of the accidental torsions due to forces in the x direction and the y direction.

4.2.9 Analysis

The total lateral forces are used in a linearly elastic structural analysis of the frame. For regular structures, independent two-dimensional models may be used. For concrete buildings, cracked-section properties are assumed in the analysis. For irregular structures, three-dimensional analyses must be used. Where the diaphragms are flexible relative to the lateral-force-resisting members, that flexibility must be represented in the analysis.

In summary, two concepts are important here. First, the force developed in the structure does not have a fixed value, but instead results from the stiffness of the structure and its response to a ground vibration. Second, if a structure is detailed so that it can respond in a ductile fashion to the ground motion, the earthquake forces are reduced from the elastic values.

5. DUCTILITY OF REINFORCED CONCRETE MEMBERS

The flexural ductility of a beam increases as the tension–reinforcement ratio goes down and as the compression–reinforcement ratio, ρ' goes up. When a reinforced concrete member is subjected to load, flexural and shear cracks develop, as shown in Fig. 35a. When the load is reversed, these cracks close and new cracks form [41]. After several cycles of loading, the member will resemble Fig. 35b.

The left end of the beam is divided into a series of blocks of concrete held together by the reinforcing cage. If the beam cracks through its depth, as shown in Fig. 35b, shear is transferred across the crack at low rotations by dowel action of the longitudinal reinforcement and grinding friction along the crack. If the concrete cover crushes, the longitudinal bars will buckle unless restrained by closely spaced stirrups or hoops. The hoops also provide confinement of the core concrete, increasing the beam's ductility.

It should be noted that the beam's displacement ductility ratio discussed earlier, Δ_u/Δ_y, is defined in Eq. (57) in terms of the deflection Δ at the end of the beam, similar to that shown in Fig. 35. Because most of the deformation is concentrated in the cracked plastic hinging region, the rotational ductility, θ_u/θ_y, measured over the length of the plastic hinging zone and the curvature ductility, ϕ_u/ϕ_y, measured at the section of maximum moment for the beam are larger than the required deflection ductility, Δ_u/Δ_y.

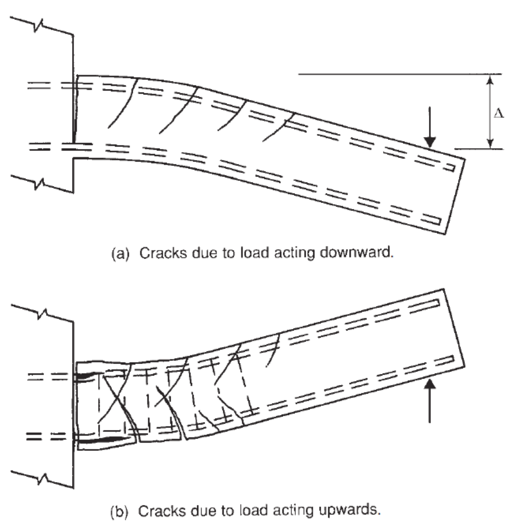


Fig. 35. Beam subjected to cyclic loads

It was shown that concrete subjected to triaxial compressive stresses increases in both strength and ductility. In a spiral column, the lateral expansion of the concrete inside the spiral stresses the spiral in tension, and this in turn causes a confining pressure on the core concrete leading to an increase in the strength and ductility of the core. ACI Code Chapter 21 requires that beams, columns, and the ends of shear walls have hoops in regions where the flexural reinforcement is expected to yield. Hoops are closely spaced closed ties or continuously wound ties or spirals, the ends of which have 135° hooks with six-bar-diameter (but not less than 3 in.) extensions into the confined core. The hoops must enclose the longitudinal reinforcement and give lateral support to those bars in the manner required for column ties in

ACI Code Section 7.10.5.3. Although hoops can be circular, they most often are rectangular, as shown in Fig. 36, because most beams and columns have rectangular cross sections. The core concrete shown shaded in Fig. 19-14a is confined by the hoop. As a result, it tends to be more ductile and a little stronger than the unconfined concrete. In addition to confining the core concrete, the hoops restrain the buckling of the longitudinal bars and act as shear reinforcement.

Special moment frame members designed using ACI Code Chapter 21 can achieve deflection ductilities in excess of 5 and well-detailed flexural walls can achieve about 4, compared to 1 or 2 for conventionally reinforced concrete frame members.

The response-modification coefficients, R, given in Table 2 are a measure of the deflection ductilities various types of structures can attain.

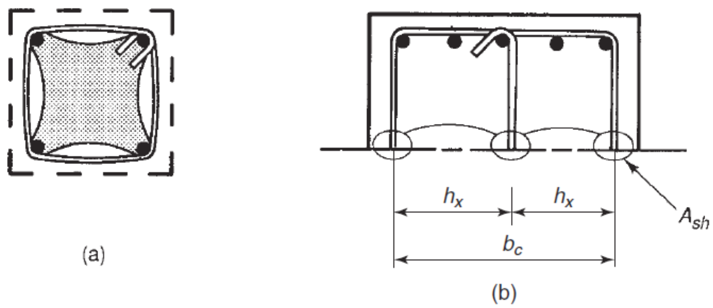


Fig. 36. Confinement by hoops

6. GENERAL ACI CODE PROVISIONS FOR SEISMIC DESIGN

6.1 APPLICABILITY

Seismic design provisions presented in ACI Code Chapter 21 were extended in the 2002 code to apply to cast-in-place and precast structures.

ACI Code Sections 21.1.1.2 through 21.1.1.6 give the design requirements for structures assigned to seismic design categories (SDC) B through F, which were discussed in Section 3 of this chapter. These requirements are summarized in Table 3.

ACI Code Chapter 21 refers to a moment-resisting frame designed by using Code Chapters 1 to 19 as an ordinary moment frame (OMF), and to a moment-resisting frame designed using Code Chapters 1 to 19 plus ACI Code Section 21.3, which requires special detailing, as an intermediate moment frame (IMF). A moment-resisting frame designed using Code Chapters 1 to 19 plus ACI Code Sections 21.1.3 through 21.1.7, and 21.5 through 21.8 is called a special moment frame (SMF).

TABLE 3. ACI Code Sections Applicable to Various Seismic Design Categories*

Seismic Design Cathegory (SDC)	Moment Resisting Frames	Beams, Columns and Joints	Structural Walls and Coupling Beams	Diaphragms	Foundations
A	None	None	None	None	None
B	21.2	None	None	None	None
C	21.3	None	None	None	None
D, E,F	21.5, 21.6, and 21.13 for frame members not part of lateral-force resisting system	21.7	21.9	21.11	21.12

*In addition to ACI Code Chapters 1 through 19

6.2 MATERIALS

The compressive strength of the concrete shall not be less than 3000 psi (ACI Code Section 21.1.4.2). Because some high-strength lightweight concretes display brittle crushing failures, the strength of lightweight concrete shall not exceed 5000 psi unless good behavior of the particular high-strength lightweight aggregate concrete is documented.

Reinforcement resisting earthquake-induced stresses in frame members and the boundary elements of walls shall comply with ASTM A 706, Standard Specification for Low-Alloy Steel Deformed and Plain Bars for Concrete Reinforcement. ASTM A 615 steel also may be used if ACI Code Section 21.1.5.2 parts (a) and (b) are satisfied.

6.3 LOAD FACTORS, LOAD COMBINATIONS, AND STRENGTH-REDUCTION FACTORS

Design will be based on

$$\phi R_n \geq R_u \tag{66}$$

where R_u is the sum of the factored load effects for an applicable load combination, U. R_n is the nominal resistance of the member and ϕ is the applicable strength-reduction factor from ACI Code Sections 9.2 and 9.3.

6.4 LOAD AND RESISTANCE FACTORS—ACI CODE SECTIONS 9.2 AND 9.3

ACI Code Section 9.2.1 presents seven load combinations, including two which include earthquake loads, E:

Load combination 9-5: $U = 1.2D + 1.0E + 1.0L + 0.2S$ (ACI Eq. 9-5)

and

Load combination 9-7: $U = 0.9D + 1.0E$ (ACI Eq. 9-7)

where D is an unfactored dead load effect, L is an unfactored live load effect, S is an unfactored snow load effect, E is an unfactored earthquake load effect. (An effect is the result of a force acting on a structure.). Load combination 9–7 is used when the dead load stabilizes a structure subjected to overturning loads or stress reversals.

ACI Code Section 9.3.4 defines special reduction factors, ϕ, for three types of shear-sensitive members encountered in seismic design. In part (a) for structural members with a nominal shear strength less than the shear corresponding to the development of the nominal flexural strength of the member, the factor ϕ shall be taken as 0.60. In part (b) for diaphragms, the ϕ factor for shear strength shall not exceed the lowest ϕ factor in shear used for the vertical components of the primary lateral-force-resisting system. In part (c), the ϕ factor for shear in joints and diagonally reinforced coupling beams is set at 0.85. The shear forces in these members are determined by a capacity design procedure and the ϕ factor was selected to be consistent with prior editions of the ACI Code.

6.5 CAPACITY DESIGN

The reinforcing details required to ensure adequate ductility of hinging regions in a laterally loaded structure tend to be tedious to design and expensive to place. This complexity can be reduced if the structure is designed so that only a few cross sections form hinges under the seismic loads, while the rest of the structure has enough reinforcement to remain elastic.

Consider, for example, a special structural wall with the shear and moment diagrams shown in Fig. 37. This wall is loaded by vertical loads totaling N_u and horizontal loads totaling E. In a non-seismic design, the size of the wall and the wall reinforcement would be chosen to have the desired stiffness plus a strength equal or greater than the effect of the factored loads.

Because this wall acts as a vertical cantilever, the first plastic hinge is expected to occur at the base of the wall. In capacity design this section is designed to hinge at a lateral load level that will allow the hinge to be detailed for ductile behavior under flexure and compressive axial loads. At the same time, however, the sections away from the hinging region are designed to remain elastic throughout the loading history, thereby avoiding the need for seismic detailing at those sections.

Because a shear failure of the hinge region would not be ductile, the shear strength at the base of the wall is chosen to exceed the shear expected at flexural hinging of the wall, as shown by the moment and shear envelopes in Fig. 37. Also, the moment strengths of all sections not chosen to be hinges exceed the moments from the assumed hinging mechanism in the structure. This process is called capacity design.

The structure is not merely designed to resist the applied load effects; instead it is proportioned so that the moment and shear strengths of all nonyielding elements in the structure exceed the loads corresponding to yielding of the critical elements that the designer has selected.

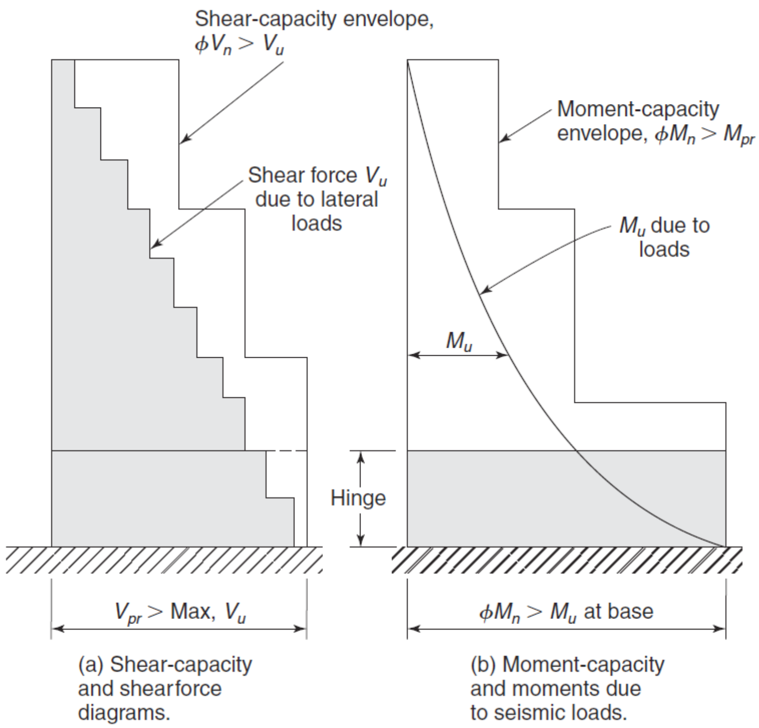


Fig. 37. Capacity design of a shear wall

6.6 STRONG COLUMN–WEAK BEAM DESIGN

If plastic hinges form in columns, the stability of the structure may be compromised, as shown in Fig. 30. As a result, the design of ductile moment-resisting frames attempts to force the structure to respond in what is referred to as strong column–weak beam action in which the plastic hinges induced by the seismic forces form at the ends of the beams, as shown in Fig. 38. The hinging regions are detailed to maintain their shear capacity while the plastic hinges undergo flexural yielding in both positive and negative bending.

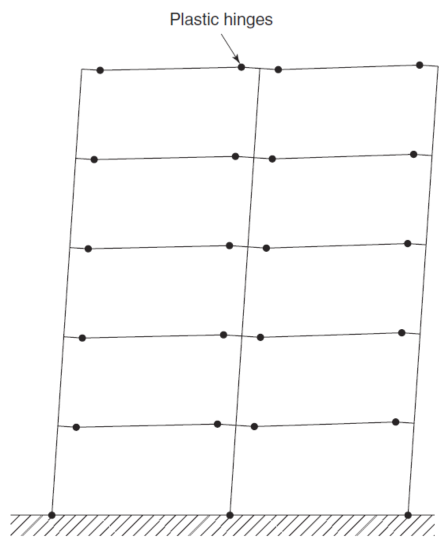


Fig. 38. Strong-column–weak-beam behavior

7. FLEXURAL MEMBERS IN SPECIAL MOMENT FRAMES

7.1 GEOMETRIC LIMITS ON BEAM CROSS SECTIONS

ACI Code Section 21.5 defines a flexural member as a member proportioned to resist primarily flexure and having either no axial load or a factored axial compressive force less than $\left(A_g f_c^{'} / 10\right)$. Geometric limitations are placed on the span-to-depth ratio ($l_n \geq 4h$) to avoid deep beam action, except that this limit does not apply to coupling beams in shear walls. The widths of flexural members in special moment frames shall not be less than (a) 0.3 times the depth of the beam, or (b) 10 in., or (c) not more than the width of the supporting member, c_2, plus a distance on each side of the supporting member equal to the smaller of the dimension, c_2, or 0.75 times the perpendicular dimension of the supporting member, c_1.

7.2 CLASSIFICATION OF RESISTING MOMENTS

Two levels of resisting moments are used in seismic design:

M_n = nominal moment strength, calculated using the specified yield strength, and the specified concrete strength, f_y, and the specific concrete strength $f_c^{'}$. The nominal moment strength is used in ACI Code Section 21.6.2.2 to ensure that the columns are stronger than the beams meeting at a joint.

M_{pr} = probable moment strength, calculated by using $1.25f_y$ because the average yield strength tends to be greater than f_y and because beam longitudinal reinforcement will likely go into strain hardening in plastic hinging zones. The probable moment strength is used in ACI Code Section 21.5.4.1 to ensure that the shear strengths of beams exceed the shears that equilibrate flexural hinging at the ends of the beams. It is also used in ACI Code Section 21.6.5.1 to compute shears in columns.

7.3 COMPUTATION OF MOMENT STRENGTH OF BEAM SECTIONS

In calculating the moment capacities of beams subjected to earthquake forces, it has been widespread practice to ignore compression reinforcement of the beam and any tension reinforcement in the flange (slab) for a beam in negative bending (tension at top). Ignoring the slab reinforcement results in the calculated strength of the beam being less than it would be if these effects were included in the calculations [42]. This overstrength in flexure, uses up some of the cushion of shear strength provided from a capacity-design procedure. To avoid an under-estimation of a beam's flexural strength, ACI Code Section 21.6.2.2 now requires that the steel in the beam tension flanges be considered during the computation of the required column strengths.

7.4 LONGITUDINAL (HORIZONTAL) REINFORCEMENT

Seismic loads cause the moment diagram shown by the solid line in Fig. 19-39b when the frame is swaying to the right and an opposite diagram shown by the dashed line when the frame is swaying to the left. To this must be added the dead- and live-load moments shown in Fig. 19-39c, giving the moment envelope in Fig. 19-39d. The maximum moments in the span normally occur at the face of a column. In addition to providing adequate moment strength, flexural reinforcement must satisfy the following detailing requirements from ACI Code Section 21.5.2 to provide adequate ductility:

1. At least two bars must be provided continuously top and bottom.

2. The areas of each of the top and bottom reinforcement at every section shall not be less than given by (ACI Eq. 10-3), nor less than $200b_wd/f_y$. The reinforcement ratio, $\rho=A_s/bd$ shall not exceed 0.025 for either the top or bottom reinforcement.

3. The positive-moment strength of the beam section at the face of the beam–column joint shall not be less than half the negative-moment strength as shown in Fig. 39e. This provides $\rho'\geq 0.5\rho$ which allows the beam to develop large curvatures at hinging regions and greatly improves the ductility of the ends of the beams.

4. At every section, the positive and negative moment capacity shall not be less than one-fourth the maximum moment capacity provided at the face of either joint. This is also plotted in Fig. 39e. The upper limit on ρ of 0.025 in item 2 is greater than the that would normally be used in a non-seismic beam with Grade-60 steel and most concrete strengths. It is set this high because there will always be confinement reinforcement and compression steel with equal to at least 0.25ρ.

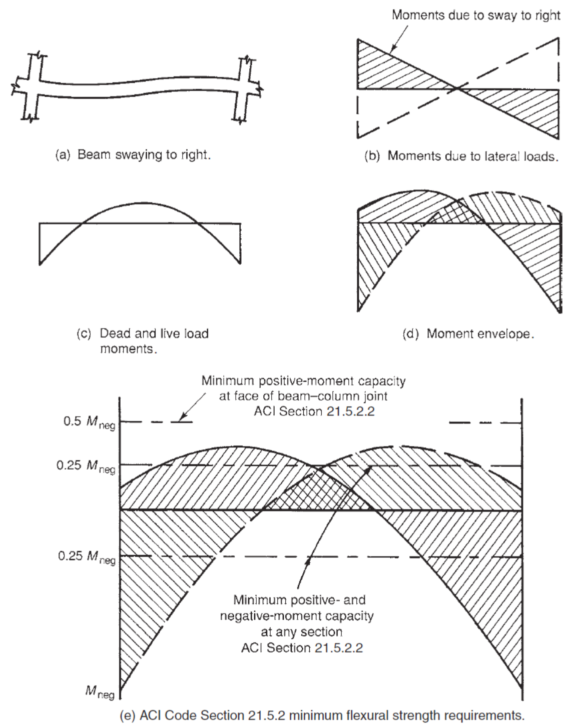


Fig. 39. Moment diagram due to gravity loads and seismic loads

7.5 DEVELOPMENT AND SPLICING OF FLEXURAL REINFORCEMENT

The development lengths and splice lengths specified in ACI Code Sections 12.2 and 12.15 apply to frames resisting seismic forces except as altered in ACI Code Section 21.7.5, which deals with development of bars in beam–column joints. ACI Code Section 21.5.2.3 prohibits lap splices

1. within joints;
2. within 2h from the faces of joints; and
3. in locations where flexural yielding can occur due to lateral deformations of the frame.

Lap splices must be enclosed by hoops or spirals at spacing equal to the smaller of 4 in. or d/4. Mechanical splices can be used as limited by ACI Code Section 21.5.2.4. Welding is not encouraged; tack welding of bars for assembly purposes embrittles the bars locally, and thus is not permitted.

7.6 TRANSVERSE REINFORCEMENT

Transverse reinforcement is required
1. to confine the concrete,
2. to prevent buckling of the compression bars in plastic hinging areas (ACI Section 21.5.3),
3. to provide adequate shear strength (ACI Code Section 21.5.4), and as mentioned in the preceding section, and
4. to confine lap splices.

7.7 CONFINEMENT REINFORCEMENT

Hoops for confinement and to control buckling of the longitudinal reinforcement are required:

1. over a length equal to 2h from the face of supports and
2. within 2h on each side of other locations where plastic hinging can result due to lateral deformations of the frame.

The spacing of the hoops is specified in ACI Code Section 21.5.3.2. In the rest of the beam, either stirrups or hoops are required at a maximum spacing of d/2.

ACI Code Section 2.2 defines a seismic hook as a hook on a stirrup, hoop, or crosstie having a bend not less than 135° with a six-diameter (but not less than 3 in.) extension that engages the longitudinal reinforcement and projects into the confined concrete in the interior of the stirrup or hoop.

A crosstie is defined as a continuous reinforcing bar having a seismic hook at one end and a hook not less than 90° with at least a six-diameter extension at the other end, as shown in Fig. 40a. Both hooks shall engage peripheral longitudinal bars. The 90° hooks of two successive crossties engaging the same longitudinal bars are alternated end for end, except as allowed in ACI Code Section 21.5.3.6. (b) (c) (d) (a).

A hoop is a closed tie, as shown in Fig. 40c, or a continuously wound tie. A closed tie can be made up of several reinforcing bars, each having seismic hooks at one or both ends. This allows the use of a number of bars or interlocking sheets of welded-wire fabric to make up a cage of hoops and longitudinal bars for a beam or column. In flexural members, ACI Code Section 21.5.3.6 allows hoops to be made up of a crosstie as shown in Fig. 40a plus a stirrup with seismic hooks at each end, as shown in Fig. 40b. If the longitudinal bars secured by the crossties are confined by a slab on only one side of the beam, as shown in Fig. 40d, the 90° hooks on the crossties must be placed on that side.

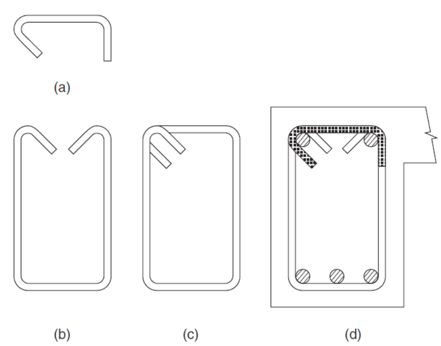


Fig. 40. Hoops and crossties

7.8 SHEAR REINFORCEMENT

When the frame is displaced laterally through the inelastic deformations required to develop the ductility of the structure, the reinforcement at the ends of the beam will yield unless the moment strength is several times the moment due to seismic loads. The yielding of the reinforcement sets an upper limit on the moments that can be developed at the ends of the beam. The design shear forces, V_e, are based on the shears due to factored dead and live loads (Fig. 41c) plus the shears due to hinging at the two ends of the beam for the frame swaying to the right or to the left, as shown in Fig. 41a. M_{pr} is the probable moment strength of the members, based on the dimensions and reinforcement at the joint and assuming a tensile strength of $1.25f_y$ and $\phi=1.0$. For a rectangular beam without axial loads, ACI Code Section 21.5.4.1 requires that beams be designed for the sum of:

$$V_{sway} = \frac{M_{pr1} + M_{pr2}}{l_n} \tag{67}$$

and

$$V_g = \frac{w_u l_n}{2} \tag{68}$$

Thus, the total design shear is: $V_e = V_g \pm V_{sway}$ where

$$M_{pr} = 1.25 f_y A_s \left(d - \frac{a}{2} \right) \tag{69}$$

where, $a = \frac{1.25 f_y A_s}{0.85 f_c^{'} b}$

The beam is then designed for the resulting shear force envelope with $V_u = V_e$ in the normal way, except that if

(a) the shear, V_{sway} due to the moments M_{pr1} and M_{pr2} is half or more of the total shear, V_e, within the span and

(b) the factored axial compressive force (if any) including earthquake effects is less than $A_g f_c^{'} / 20$,

Then V_c is taken equal to zero. The damage to the hinging area due to repeated load reversals greatly reduces the ability of the cross section to resist shear, requiring more transverse reinforcement [41]. Hoops and stirrups provided to satisfy ACI Code Section 21.5.3 also can serve as shear reinforcement.

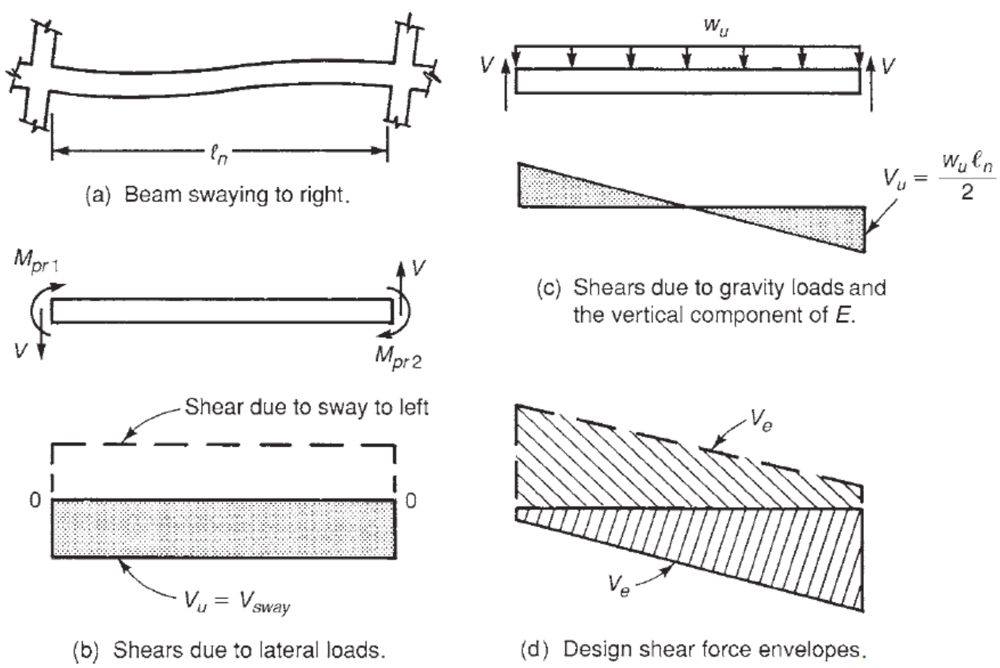


Fig. 41. Shear force diagrams due to gravity loads and seismic loads

8. COLUMNS IN SPECIAL MOMENT FRAMES

ACI Code Section 21.6 applies to columns in frames resisting earthquake forces and supporting a factored axial force exceeding $A_g f_c^{'}/10$. Columns in frames in regions of high seismic risk must satisfy two geometric requirements: the smallest dimension through the centroid of the column must be at least 12 in., and the ratio of the shortest to the longest cross-sectional dimension shall not be less than 0.4. These limits ensure a minimum robustness and produce a cross section that can be confined using practical hoop layouts, which might be difficult with highly rectangular columns.

8.1 REQUIRED CAPACITY AND LONGITUDINAL REINFORCEMENT

It is highly desirable that plastic hinges form in the beams rather than in the columns. Because the dead load must always be transferred down through the columns, damage to the columns should be minimized. ACI Code Section 21.6.2 requires the use of a strong column–weak beam design. In the event that this is not possible, the columns in question are disregarded in the structural analysis (i.e., assumed to have failed) if they add to the stiffness and strength of the building. (If inclusion of such columns in the analysis has a negative effect on the stiffness or strength, they should be included in the analysis.)

Strong column–weak beam behavior is made more likely by requiring (Fig. 42) that:

$$\sum M_{nc} \geq (6/5)\sum M_{nb} \qquad (70)$$

(ACI Eq. 21-1)

where M_{nc} is the nominal flexural capacity of the columns corresponding to the factored seismic load combination leading to the lowest axial load and hence the lowest flexural strength, and M_{nb} is the nominal flexural capacity of the girders at that joint.

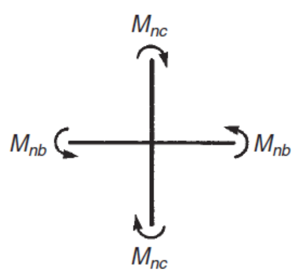


Fig. 42. Moments at a beam–column joint—general

Columns that do not satisfy Eq. (70) must have transverse reinforcement satisfying ACI Code Section 21.6.4.2 through 21.6.4.4 over their entire length.

Longitudinal reinforcement is designed for the axial loads and moments in the same way as in a non-seismic column. It may range from $\rho = 0.01$ to 0.06. Generally, it is difficult to place and splice much more than 2 to 3 percent reinforcement in a column.

Because the cover concrete will probably spall in plastic hinging regions, which may form near the ends of the column, longitudinal bars that are to be lap spliced must be spliced in the center half of the column height. Such splices must be designed as tension splices, because the alternating moments due to the cyclic loads alternately stress the bars on each side of the column in tension and compression.

Furthermore, there is frequently a possibility of uplift forces. The considerable length required for tension lap splices may require small diameter bars or mechanical splices.

8.2 TRANSVERSE REINFORCEMENT

8.2.1 Confinement Reinforcement

Transverse reinforcement in the form of spirals or hoops must be provided over a height of l_0 from each end of the column to confine the concrete and restrain the longitudinal bars from buckling. The height is the largest of (ACI Code Section 21.6.4.1)
(**a**) the depth of the column, h at the face of the joint,
(**b**) one-sixth of the height of the column, and
(**c**) 18 in.

Within the length l_0, ACI Code Section 21.6.4.3 requires that the spacing of the transverse reinforcement shall not exceed
(**a**) one-quarter of the minimum dimension, b or h, of the column cross section;
(**b**) six times the diameter of the longitudinal bar diameter; and
(**c**) the distance

$$s_0 = 4 + \left(\frac{14 - h_x}{3} \right) \tag{71}$$

(ACI Eq. 21-2)

where h_x = maximum horizontal spacing between hoop or crosstie legs on all faces of the column (Fig. 36b), but not less than 4 in. nor more than 6 in. Transverse reinforcement also serves as shear reinforcement and has to conform to minimum stirrup spacings.

If spirals are used, they are designed as outlined in ACI Eq. (10-5). An additional lower limit on the ratio of spiral reinforcement is given by the following equation.

$$\rho_s = 0.12 f_c' / f_y \tag{72}$$

(ACI Eq. 21-3)

This will govern if A_g/A_c is less than 1.27, which, for 1.1/2 in. cover, will occur for columns larger than 24 in. in diameter.

Because the pressure on the sides of the hoops causes the sides to deflect outward, hoops are less efficient than spirals at confining the core concrete (Fig. 36a). The equation for the required area of hoops, ACI Eq. (21-4), was based on the equation for spirals, ACI Eq. (10-5), but the constant was selected to give hoops with about one-third more cross-sectional area than would be required for spirals; that is:

$$A_{sh} = 0.3 \frac{s b_c f_c'}{f_{yt}} \left(\frac{A_g}{A_{ch}} - 1 \right) \tag{73}$$

(ACI Eq. 21-4)

but not less than:

$$A_{sh} = 0.09\frac{sb_c f_c^{'}}{f_{yt}} \tag{74}$$

(ACI Eq. 21-5)

where
A_{ch} – cross-sectional area of the core of the column measured out-to-out of the hoops
A_g – gross area of the section
A_{sh} – total cross-sectional area of all the legs of the hoops and crossties within a spacing s and perpendicular to the dimension b_c (Fig. 36b)
b_c – cross-sectional dimension of the column core, measured center to center of outer legs of the hoops
s – spacing of the hoops measured parallel to the axis of the column.
A_{sh} – is checked separately for each direction.

Figure 19-36b shows typical hoop arrangement for a column. The maximum distance between hoop or crosstie legs in the plane of the cross section is 14 in. The hoops must also satisfy ACI Code Section 7.10.5.3, which requires that every corner bar and alternate side bars be at the corner of a tie.

Columns supporting discontinued shear walls are extremely susceptible to seismic damage. ACI Code Section 21.6.4.6 requires hoops or spirals over the full height of such members. These hoops must extend into the wall from the face of the column and the footing or other member under the column.

8.2.2 Shear Reinforcement

The transverse reinforcement also must be designed for shear. The design shear force V_{sway} is computed by assuming inelastic action in either the columns or the beams.

(**a**) The shear corresponding to plastic hinges at each end of the column given by

$$V_{sway} = \frac{M_{prc,top} + M_{prc,btm}}{l_u} \tag{75}$$

where $M_{prc,top}$ and $M_{prc,btm}$ are the probable moment capacities at the top and bottom of the column and l_u is the clear height of the column. These are obtained from an interaction diagram for the probable strength, P_n-M_{pr} of the column, for the range of factored loads on the member for the load combination under consideration.

(**b**) It need not be more than

$$V_{sway} = \frac{\sum M_{prb,top} DF_{top} + \sum M_{prb,btm} DF_{btm}}{l_u} \tag{76}$$

where $\Sigma M_{prb,top}$ and $\Sigma M_{prb,btm}$ are the sum of the probable moment capacities of the beams framing into the joints at the top and bottom of the column for the frame swaying to the left or right, DF_{top} and DF_{btm} are the moment-distribution factors at the top and bottom of the column being designed. This reflects the strong-column–weak-beam philosophy and Eq. (70), which requires that the beams are weaker than the columns.

(**c**) but not less than the factored shear from a frame analysis.

Transverse reinforcement is designed for shear according to ACI Code Section 11.1.1, and V_c may be increased to allow for the effect of axial loads, except that, within the length l_0 defined in the discussion of confinement reinforcement, V_c shall be taken equal to zero when the earthquake-induced shear force makes up half or more of the maximum shear force in the lengths l_0 and if the factored compression force is less than $A_g f_c^{'} / 20$ (ACI Code Section 21.6.5.2). Columns with such low axial loads essentially behave like a beam. Thus, the concrete contribution to shear, V_c, is set equal to zero in potential plastic hinging zones at the ends of a column, just as was done for plastic hinging zones at the end of beams.

It should be noted that, although the axial load increases V_c, it also increases the rate of shear degradation [43]. For this reason, it may be prudent to ignore V_c when a major portion of the shear results from earthquake loads.

9. JOINTS OF SPECIAL MOMENT FRAMES

Code provisions for joints in special moment-resisting frames (SMFs) are given in ACI Code Section 21.7. A more extensive discussion of design recommendations for beam–column connections is given by ACI Committee 352 [44].

ACI Code Section 21.7.2.1 requires that joint forces be calculated by taking the stress in the flexural reinforcement in the beams as $1.25f_y$. This is analogous to using the probable strength in the calculations of shear in columns and beams in special-moment frames. ACI Code Section 21.7.2.3 limits the diameter of the longitudinal beam reinforcement that passes through a joint to 1/20 of the width of the joint parallel to the beam bars. When hinges form in the beams, the beam reinforcement is stressed to the actual yield strength of the bar on one side of the joint and is stressed in compression on the other side. This results in very large bond stresses in the joint, possibly leading to slipping of the bar in the joint. The minimum bonded length of such a bar in a joint is thus which is considerably less than is required by the development-length equations in ACI Code Chapter 12. The minimum bonded length was selected from test results of joints tested under cyclic loads to limit, but not entirely eliminate slip of the beam bar in the joint. Based on research by Leon [45] and others, better stiffness retention and energy dissipation is obtained for interior beam–column connections when the column dimension is increased to at least 24 times the diameter of beam bars passing through the joint.

ACI Committee 352 [44] uses the same limit for the diameter of column bars passing through a beam–column joint (i.e., the column–bar diameter should not exceed 1/20 of the overall depth of the shallowest beam framing into the joint). Although this design recommendation has not been adopted by the ACI Code Committee, based on research results [46], the author recommends that designers should attempt to satisfy this limit when selecting the size of column bars.

ACI Code Section 21.7.3.1 requires hoop reinforcement around the column reinforcement in all joints in special moment-resisting frames. In joints confined on all four sides by beams satisfying ACI Code Section 21.7.3.2, the amount of hoop reinforcement is reduced, and its spacing is less restrictive within the depth of the shallowest beam entering the joint.

ACI Code Section 21.7.4.1 gives upper limits on the shear strength of joints. As indicated in Section 17-12, these are lower than the joint shear strengths recommended in non-seismic joints. This reflects the possible damage to joints resulting from cyclic loads.

ACI Code Section 21.7.5 gives special development lengths for hooks and straight bars in joints. These are shorter than the development lengths given in ACI Code Chapter 12 because the effects of the joint confinement by hoops have already been included.

SOLVED EXAMPLE – Design an Interior Beam-Column Joint

Design the interior beam–column joint connecting the beams, which are 24 in. by 24 in. in section, and columns, 24-in.-by-24-in.

Step 1. Define the size of the joint. *The joint has width, depth, and vertical height of 24 in. The area of a horizontal section through the joint, A_j (ACI Code Section 21.7.4.1), is $A_j = 24 \times 24 = 576$ in.^{2}*

ACI Code Section 21.7.2.3 requires the length of the joint measured parallel to the flexural steel causing the joint shear to be at least 20 times the diameter of those bars (20 × 1 in.), —o.k.

From ACI Committee 352, the column–bar diameter should be less than 1/20 of the total depth of the shallowest beam framing into the joint, so 1/20 × 24 = 1.2 in. Thus, the selected No. 8 bars are o.k.

Step 2. Determine the transverse reinforcement for confinement. *ACI Code Section 21.7.3.1 requires confinement steel within the joint. Because the joint has beams on all four sides, ACI Code Section 21.7.3.2 sets the amount of confinement steel as half of the confinement steel required in the ends of the columns, given by Eqs. (73) and (74) (ACI Eqs. (21-4) and (21-5)).*

$$A_{sh} = 0.3\frac{sb_c f_c^{'}}{f_{yt}}\left(\frac{A_g}{A_{ch}} - 1\right)$$

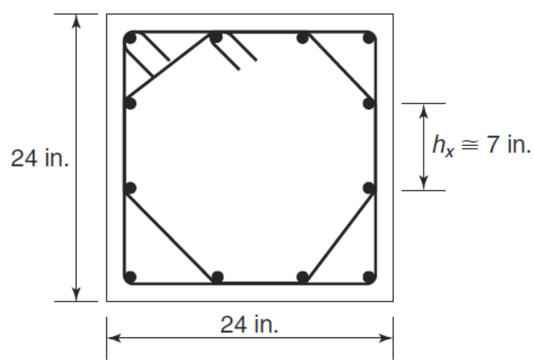
$$A_{sh} = 0.09\frac{sb_c f_c^{'}}{f_{yt}}$$

In the column, Eq. (73) (ACI Eq. (21-4)) governed and required that $A_{sh}/s = 0.129$ in.^{2}/in. Within the height of the joint, we require that:

$$\frac{A_{sh}}{s} = 0.5\times 0.129\text{in.}^2/\text{in.} = 0.065\text{in.}^2/\text{in.}$$

The vertical spacing of the hoops from ACI Code Section 21.7.3.2 is permitted to be 6 in.

The clear distance between the top and bottom beam steel is 18 in. Provide three sets of hoops, the first at 3 in. below the top steel. The required A_{sh} is 6×0.065 = 0.39 in.^{2}. Use No. 4 hoops in the arrangement shown below, so Ash = 0.68 in.^{2}.



Step 3. Compute the shear in the joint and check the shear strength. *The following figure presents a series of free-body diagrams of the joint for the frame swaying to the right. The beams entering the joint have probable moment capacities of -531 kip-ft and +286 kip-ft. At the joint, the stiffnesses of the columns above and below the joint are the same, giving distribution factors of DF = 0.5 for each column. Thus, the moment in the column above is:*

$$M_c = 0.5\times(531+286) = 409\text{kip}-\text{ft}$$

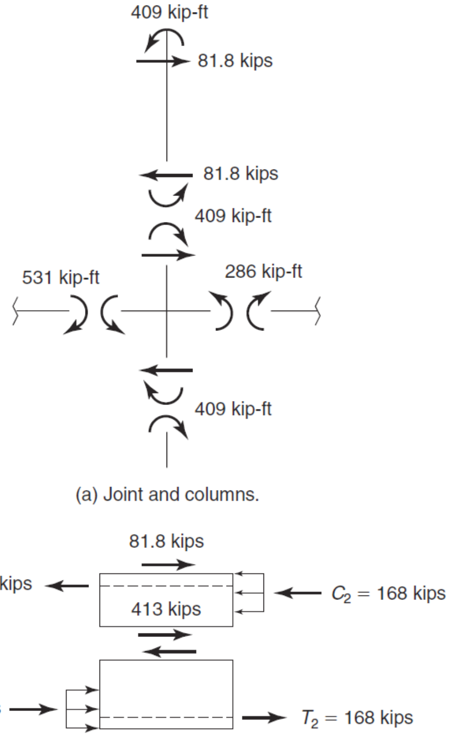
The shear in the column above is;

$$V_{sway} = \frac{409+409}{10\text{ft}} = 81.8\text{kips}$$

The area of the top steel at the interior support is four No. 8 bars plus two No. 7 bars, so A_s = 4.36 in.^{2}. The force in the steel in the beam on the left of the joint is:

$$T_1 = 1.25 A_s f_y = 1.25 \times 4.36 \text{in.}^2 \times 60 \text{ksi} = 327 \text{kips}$$

The compression force in the beam to the left is $C_1 = T_1 = 327$ kips. Similarly, T_2 and C_2 in the beam to the right of the joint are 1.25 × 2.24 × 60 = 168 kips.



(a) Joint and columns.

(b) Joint.

Summing horizontal forces gives the shear in the joint as:

$$V_j = V_{sway} - T_1 - C_2 = 81.8 \text{kips to the right - } 327 \text{kips to the left - } 168 \text{kips to the left}$$
$$V_j = 413 \text{kips to the left}$$

From ACI Code Section 21.7.4.1, the nominal shear strength of an interior joint confined on all four sides is:

$$V_n = 20 A_j \sqrt{f_c^{'}} = 20 \times 576 \text{in.}^2 \times \sqrt{4000 \text{psi}} = 729 \text{kips}$$

From ACI Code Section 9.3.4 (c), ϕ = 0.85 for seismic joints. So,

$$\phi V_n = 0.85 \times 729 \text{kips} = 619 \text{kips}$$

Therefore, the joint has adequate shear strength.

Provide three No. 4 hoops, as shown in the first figure of the solved example, at 6 in. on centers in the joint.

10. STRUCTURAL DIAPHRAGMS

Floor slabs, roof slabs, and cast-in-place toppings on precast concrete floors may all be used as diaphragms to transfer horizontal forces acting in the building to the lateral force-resisting system and, eventually, to the foundations, as shown in Fig. 29. In effect, they act as deep flexural members lying in horizontal planes. Cast-in-place toppings serving as diaphragms may or may not be composite with the floor or roof members. ACI Code Section 21.11.6 allows the use of 2-in.-thick composite topping slabs or 2.1/2 -in. non-composite toppings. ACI Code Section 21.11.5 requires that non-composite toppings be designed for the forces transferred as if they are acting alone.

10.1 FLEXURAL STRENGTH

Diaphragms are analyzed and designed in flexure using the same assumptions as used for walls, beams, and columns. Therefore, the design assumptions and general procedures in ACI Code Sections 10.2 and 10.3 shall apply for diaphragms, except that the nonlinear strain distributions discussed in ACI Code Section 10.2.2 for deep beams do not need to be applied to diaphragms. Designing diaphragms as a normal flexural member with distributed reinforcement represents a significant change in philosophy from earlier design practice that assumed moments were resisted entirely by tension and compression chord forces acting at opposite edges of the diaphragm. Now, all of the longitudinal reinforcements satisfying the requirements of ACI Code Section 21.11.7 is assumed to contribute to the flexural strength of the diaphragm.

Although the revised analysis and design procedure removes the need to use concentrated reinforcement in chords at the edges of a diaphragm, there may be some structural layouts where it is necessary to concentrate longitudinal reinforcement at the diaphragm edges to obtain adequate flexural strength. For such designs, ACI Code Section 21.11.7.5 requires that if the calculated concrete stress in the compression chord exceeds $0.2f_c^{'}$, confining transverse reinforcement that satisfies ACI Code Section 21.9.6.4(c) must be placed around the longitudinal steel. Furthermore, this transverse reinforcement must be used over the portion of the compression chord where the calculated concrete stress exceeds $0.15f_c^{'}$.

10.2 SHEAR STRENGTH

ACI Code Section 21.11.9 gives expressions for the nominal shear resistance of diaphragms. The nominal shear strength of monolithic structural diaphragms shall not exceed

$$V_n = A_{cv}\left(2\lambda\sqrt{f_c^{'}} + \rho_t f_y\right) \qquad (77)$$

(ACI Eq. 21-10)

This is equivalent to $V_n = V_c + V_s$, where V_c and V_s have been divided by the shear area, A_{cv}, defined as the area bounded by the thickness of the diaphragm and the length of the section in the direction of the shear force being considered.

For cast-in-place composite-topping-slab diaphragms and cast-in-place non-composite topping slabs on precast floors or roofs, V_c is taken equal to zero above the joints between the precast elements, to reflect the likelihood that the topping will be cracked by shrinkage or other effects during construction and service. The required slab steel in this case is computed using

$$V_n = A_{vf} f_y \mu \qquad (78)$$

(ACI Eq. 21-11)

where A_{vf} is the total area of shear friction reinforcement in the topping slab, including both the distributed reinforcement and concentrated reinforcement (if any) at the edges of the diaphragm that is oriented perpendicular to the joints in the precast elements. At least one-half of A_{vf} shall be distributed along the potential shear plane. The coefficient of friction, μ, is taken as 1.0λ where λ is the factor for lightweight concrete given in ACI Code Section 11.6.4.3. The value of V_n shall not exceed the limits given in ACI Code Section 11.6.5.

10.3 EFFECT OF DIAPHRAGM STIFFNESS ON LATERAL-LOAD DISTRIBUTION

Figure 43 illustrates the effect of diaphragm stiffness on the distribution of lateral loads to the lateral-load-resisting elements. The building shown in the figure has three walls of equal lateral stiffness. If the diaphragm is essentially rigid in plane and there is no torsion, all three walls will displace by the same amount, and each wall will resist one-third of the total lateral load, as shown in Fig. 43a. On the other hand, if the diaphragm is very flexible relative to the walls, the two end walls will each resist a quarter of the lateral shear and the center wall will resist half of it, as shown in Fig. 43b.

The following derivations are presented to give an idea of the factors affecting the relative stiffnesses of the walls and diaphragms. The lateral stiffness, K_l, of a cantilever of height l that is fixed at the base is

$$K_l = \frac{V}{\Delta} \tag{79}$$

where Δ is the lateral deflection on the top of the cantilever due to the load V at the top, equal to

$$\Delta = \frac{Vl^3}{3EI} + \frac{1.2Vl}{AG} \tag{80}$$

The first term represents the flexural deflections, the second the shear deflections. Substituting Eq. (80) into (79) and taking G = E/2 gives:

$$K_l = \frac{1}{\frac{l^3}{3EI} + \frac{2.4l}{AE}} \tag{81}$$

A similar expression can be derived for the lateral stiffness of a piece of diaphragm between two walls.

Benjamin [47] has shown that if the stiffness of the diaphragm exceeds about two times that of the walls, the diaphragm will act as a rigid diaphragm in transmitting loads to the walls.

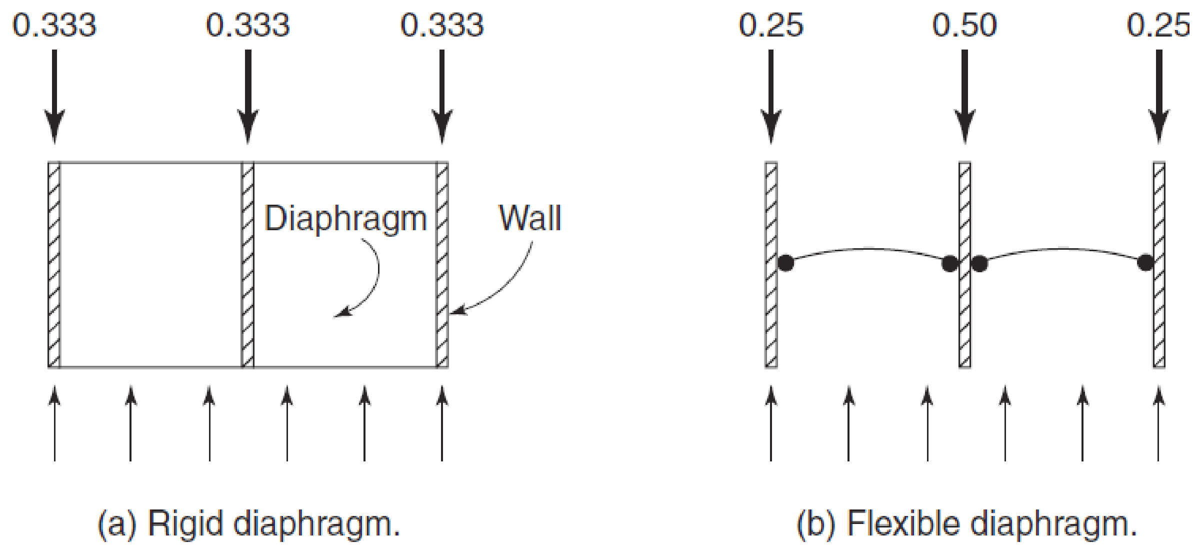


Fig. 43. Plan view of a building showing the effect of diaphragm stiffness on distribution of lateral loads to walls in a building.

11. STRUCTURAL WALLS

Structural walls or shear walls are frequently used to resist a major fraction of the design seismic shears. These are designed according to ACI Code Section 21.9. The factored shears, moments, and axial forces to be considered in the design of the wall are obtained from a frame analysis. Chapter 18 discussed the layout and design of structural walls.

11.1 DESIGN OF SHEAR WALLS

In previous chapter two types of shear walls were identified: slender shear walls, and short (or squat) shear walls. Slender shear walls are designed using the theory of flexure for members subjected to axial loads and bending. The design of short walls may be based, in part, on strut-and-tie models.

1. The first step in the design of a shear wall is to select the size and shape of the wall, using stiffness, building geometry, and the required moment and shear strengths. At this stage, the geometry of the wall probably is chosen.

2. The foundations for the walls may be required to transfer very large overturning moments to the soil or rock under the building. ACI Code Section 21.12 presents requirements for the design of foundations resisting earthquake forces. ACI Code Section 21.12.2 requires that special attention be given to the anchorage of the wall reinforcement into the foundations.

3. Once the concrete section is established, it is necessary to investigate the need for boundary elements. Boundary elements are regions at the ends of the cross section of the wall that are reinforced as columns, with the reinforcement enclosed by hoop reinforcement, as shown in Fig. 44. Boundary elements strengthen and confine the edges of the walls to resist stress reversals and prevent reinforcement buckling near the edges. They generally are thicker than the walls although ACI Code Section 2.2 allows them to have the same thickness as the wall. ACI Code Sections 21.9.6.2 and 21.9.6.3 give two methods of determining the need for special confinement reinforcement in boundary elements.

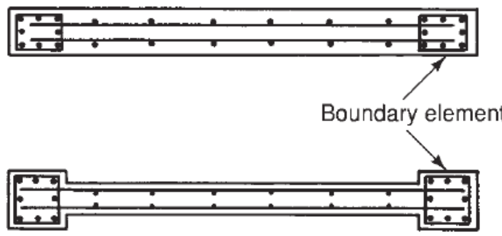


Fig. 19-44. Plan views of structural walls with boundary elements

(**a**) ACI Code Section 21.9.6.2 applies to walls that are effectively continuous from the base of the structure to the top of the wall and are designed to have a single critical section for axial loads and bending at the base of the wall. Boundary elements are required if

$$c > \frac{l_w}{600(\delta_u / h_w)} \tag{82}$$

(ACI Eq. 21-8)

where
c – depth from the neutral axis to the extreme compression fiber
l_w – horizontal length of entire wall or of a segment of wall considered in the direction of the shear force
h_w – height of the entire wall, or the segment of wall considered
δ_u – design displacement, defined as the total lateral displacement deflection of the top of the building for the design-basis earthquake

Equation (82) essentially represents a check on the maximum strain at the compression edge of the wall at the base of the structure [48]. The quantity δ_u/h_w referred to as the building drift, shall not be taken less than 0.007. Larger building drifts would lead to larger curvatures in the wall sections at the base of the structure and thus, to larger strains at the edges of the wall. Furthermore, for a given value of curvature, a larger value for the depth to the neutral axis, c, would result in a larger strain at the extreme compression fiber. Thus, by defining an upper limit for the neutral-axis depth as a function of the calculated building drift, Eq. (82) essentially is requiring that special confinement reinforcement must be used when the maximum strain experienced at the compression edge of the wall exceeds a certain value, which was suggested to be 0.004 in reference [48].

Where boundary elements are required by Eq. (82), the special boundary reinforcement shall extend upward from the base of the structure for a distance not less that the larger of l_w or $M_u/4V_u$. That dimension and the minimum required horizontal extension (ACI Code Section 21.9.6.4(a)) of the confined boundary zone from the compression edge of the wall are shown in Fig. 45. ACI Code Section 21.9.6.4(b) also requires that if a specially confined boundary zone is required for a wall with a compression flange, the boundary zone must extend at least 12 in. into the adjacent web, as shown in Fig. 46.

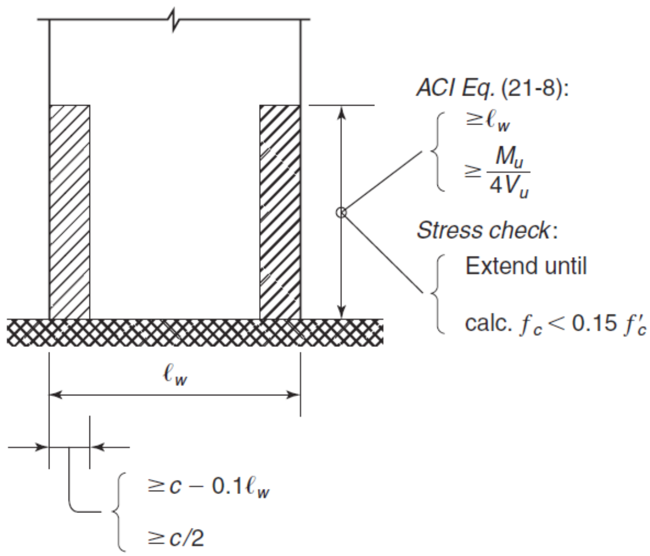


Fig. 45. Size of boundary region where special confinement steel is required

The transverse reinforcement in the specially confined boundary elements must satisfy the requirements for column confinement reinforcement given in ACI Code 21.6.4.4, except that Eq. (74) (ACI Eq. 21-4) does not need to be satisfied and the transverse reinforcement spacing limit of ACI Code Section 21.6.4.3(a) shall be one-third of the least dimension of the boundary element.

(**b**) In ACI Code Section 21.9.6.3, stresses in the uncracked wall are analyzed by using

$$f_c = \frac{N_u}{A} \pm \frac{M_u y}{I} \tag{83}$$

where A and I are based on the gross concrete section. If the computed maximum compressive stress in the extreme fiber exceeds $f_c = 0.2f_c'$ at any point, ACI Code Section 21.9.6.3 requires confined boundary elements over that portion of the height where the extreme-fiber stress exceeds $f_c = 0.15f_c'$. Figures 45 and 46 show the required vertical and horizontal dimensions of the specially confined boundary elements.

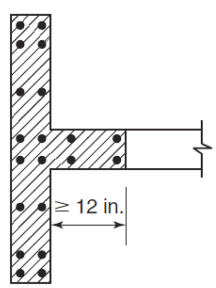


Fig. 46. Required extension of special confinement region into web of flanged wall

If specially confined boundary elements are not required when checking by either Eq. (82) or (83), then the reinforcement details at the edges of the wall must satisfy the requirements given in ACI Code Section 21.9.6.5.

Design for flexure should follow ACI Code Sections 10.2 and 10.3 except that the nonlinear strains mentioned in ACI Code Section 10.2.2 shall not apply. In addition, the axial-load capacity computed by ACI Code Section 10.3.6 does not apply.

11.2 DESIGN FOR SHEAR

As it was previously discussed, the design of structural walls for shear is given in ACI Code Section 21.9.4. The basic design equation is essentially the same as the $\phi(V_c + V_s)$ procedure used in beam design. The basic shear (v_c) stress carried by the concrete is $2\sqrt{f_c'}$ psi, except for short stubby walls, for which a higher stress is allowed. The horizontal reinforcement in the wall must be anchored in the boundary elements, as specified in ACI Code Section 21.9.6.4(e).

11.3 STRENGTH-REDUCTION FACTOR

Because walls generally have axial load ratios N_u/A_gF_c between 0.1 and 0.3, they are normally tension-controlled and the ϕ-factor for flexure is 0.9. The ϕ-factor for shear is 0.75 unless the nominal shear strength is less than the shear corresponding to development of the nominal flexural strength of the wall. In such a case, ϕ is taken as 0.6 (ACI Code Section 9.3.4(a)).

12. FRAME MEMBERS NOT PROPORTIONED TO RESIST FORCES INDUCED BY EARTHQUAKE MOTIONS

ACI Code Section 21.13 provides less stringent design requirements for members that are not part of the designated lateral-load-resisting system in structures subjected to severe earthquakes.

Such members must be able to resist the factored axial forces due to gravity loads and the moments and shears induced in them when the frame is deflected laterally through twice the elastically calculated lateral deflections under factored lateral loads. In some older structures building frame members were designed by assuming that the frame members supported only gravity loads, while shear walls resisted all of the lateral loads. In the 1994 Northridge earthquake, columns in a number of buildings of this type failed when subjected to the lateral displacements imposed by that earthquake [49]. After that earthquake, ACI Code Section 21.13 was made considerably more stringent.

13. SPECIAL PRECAST STRUCTURES

Since 2002, ACI Code Chapter 21 has included earthquake-resistant design requirements for precast frame members and precast structural walls. Requirements for special moment frames constructed with precast concrete elements are given in ACI Code Section 21.8. This section gives a designer the option of using ductile connections (Code Section 21.8.2) or strong connections (Code Section 21.8.3) between precast elements. The first option assumes that at least part of the inelastic behavior during the overall structural response to strong ground motions will occur in the connections between precast elements. The second option assumes that the connections will remain elastic and that plastic hinging zones will occur at other locations in the precast frame elements. Based on the probable member strengths at the plastic hinge locations, a capacity-based design procedure can be used to determine the required strength for the connections between the precast elements.

Special precast structural walls are required to satisfy all of the design provisions for special cast-in-place structural walls given in ACI Code Section 21.9 and discussed previously. Intermediate precast structural walls also are permitted and must satisfy the design requirements in ACI Code Section 21.4.

14. FOUNDATIONS

ACI Code Section 21.12 deals with foundations for seismic structures, including footings, mat foundations, pile caps, piles, piers, and caissons. The major emphasis is on the pull-out strength of reinforcement extending from the structure into the foundations. Minimum confinement reinforcement and shear reinforcement is required in piles, piers, and caissons.

CHAPTER 3 – DISCONTINUITY REGIONS AND STRUT-AND-TIE MODELS

1. DEFINITION OF DISCONTINUITY REGIONS

Structural members may be divided into portions called B-regions, in which beam theory applies, including linear strains and so on, and other portions called discontinuity regions, or D-regions, adjacent to discontinuities or disturbances, where beam theory does not apply. D-regions can be geometric discontinuities, adjacent to holes, abrupt changes in cross section, or direction, or statical discontinuities, which are regions near concentrated loads and reactions. Corbels, dapped ends, and joints are affected by both statical and geometric discontinuities.

For many years, D-region design has been by "good practice," by rule of thumb or empirical. Three landmark papers by Professor Schlaich of the University of Stuttgart and his coworkers [50-52] changed this. This chapter will present rules and guidance for the design of D-regions.

1.1 SAINT VENANT'S PRINCIPLE AND EXTENT OF D-REGION

St. Venant's principle suggests that the localized effect of a disturbance dies out by about one member-depth from the point of the disturbance. On this basis, D-regions are assumed to extend one member-depth each way from the discontinuity. This principle is conceptual and not precise. However, it serves as a quantitative guide in selecting the dimensions of D-regions.

Figure 47 shows D-regions in a number of structures, some of which have B-regions (bending regions) between two D-regions. Figure 48 shows examples of D-regions. The D-regions in Fig. 48b and c extend one member-width from the discontinuity as suggested by St. Venant's principle. Occasionally, D-regions are assumed to fill the overlapping region common to two members meeting at a joint. This definition is used in the traditional definition of a joint region.

1.2 BEHAVIOR OF D-REGIONS

Prior to any cracking, an elastic stress field exists, which can be quantified with an elastic analysis, such as a finite-element analysis. Cracking disrupts this stress field, causing a major reorientation of the internal forces. After cracking, the internal forces can be modeled via a strut-and-tie model consisting of concrete compression struts, steel tension ties, and joints referred to as nodal zones. If the compression struts are narrower at their ends than they are at midsection, the struts may, in turn, crack longitudinally. For struts without reinforcement crossing their longitudinal axis, this may lead to failure. On the other hand, struts with transverse reinforcement to restrain the cracking can carry additional load and may fail by crushing. Failure may also occur by yielding of the tension ties, failure of the bar anchorage, or failure of the nodal zones. As always, failure initiated by yield of the steel tension ties tends to be more ductile and is desirable. Strut-and-Tie Models

1.3 STRUT AND TIE MODELS

A strut-and-tie model for a deep beam is shown in Fig. 49. It consists of two concrete compressive struts, longitudinal reinforcement serving as a tension tie, and joints referred to as nodes. The concrete around a node is called a nodal zone. The nodal zones transfer the forces from the inclined struts to other struts, to ties and to the reactions.

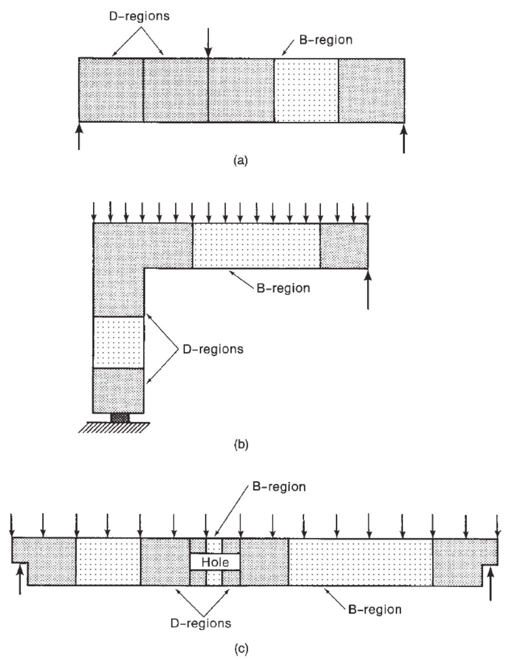


Fig. 47. B-regions and D-regions

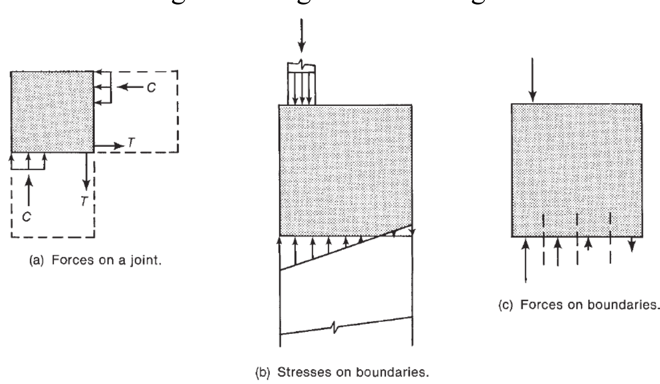


Fig. 48. Forces on boundaries of D-regions

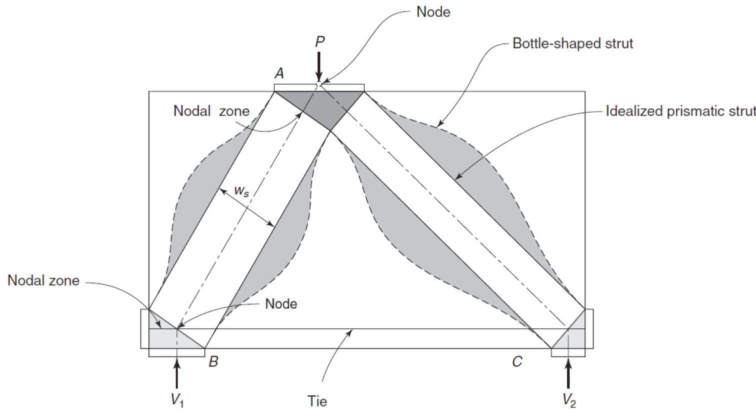


Fig. 49. Strut-and-tie model of a deep beam

ACI Code Section 11.1.1 allows D-regions to be designed using strut-and-tie models according to the requirements in ACI Appendix A, Strut-and-Tie Models. This Appendix was new in the 2002 ACI Code. The derivation of Appendix A is summarized in [53].

Examples in [54] and [55] were solved using the appendix as part of the internal check of Appendix A, by members of ACI Committee 318 Subcommittee E and ACI Committee 445. Additional strut-and-tie examples are available in a recent ACI publication [56]. A strut-and-tie model is a model of a portion of the structure that satisfies the following:

(**a**) it embodies a system of forces that is in equilibrium with a given set of loads, and

(**b**) the factored-member forces at every section in the struts, ties, and nodal zones do not exceed the corresponding design member strengths for the same sections. The lower-bound theorem of plasticity states that the capacity of a system of members, supports, and applied forces that satisfies both (a) and (b) is a lower bound on the strength of the actual structure. For the lower-bound theorem to apply,

(**c**) The structure must have sufficient ductility to make the transition from elastic behavior to plastic behavior that redistributes the factored internal forces into a set of forces that satisfy items (a) and (b).

The combination of factored loads acting on the structure and the distribution of factored internal forces is a lower bound on the strength of the structure, provided that no element is loaded or deformed beyond its capacity. Strut-and-tie models should be chosen so that the internal forces in the struts, ties, and nodal zones are somewhere between the elastic distribution and a fully plastic set of internal forces.

2. DESIGN EQUATION AND METHOD OF SOLUTION

Let's start by reviewing the strengths of struts, ties, and nodal zones, and factors affecting the layout of strut-and-tie models. In most applications of strut-and-tie models the internal forces, due to the factored loads, and the struts, ties, and nodal zones are proportioned using:

$$\phi F_n \geq F_u \tag{84a}$$

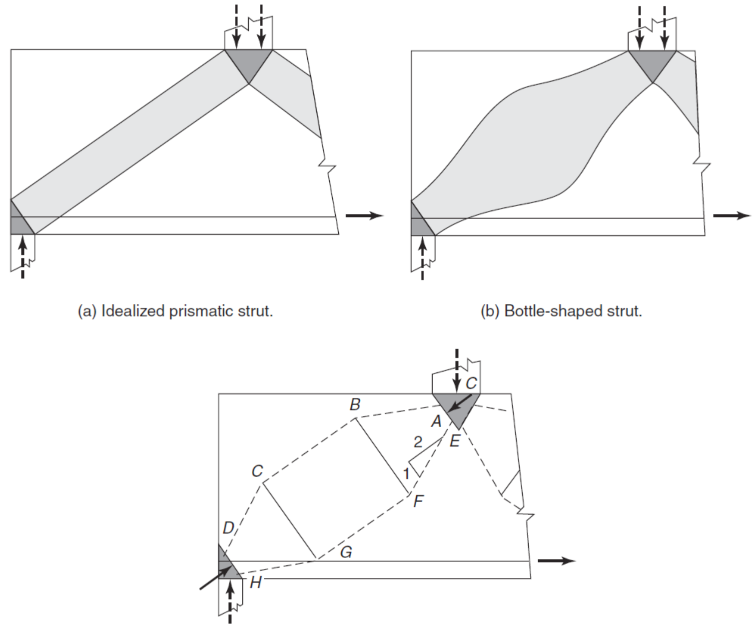
Alternatively, the structural analysis is carried out for loads equal to F_u/ϕ and the members are proportioned for F_n:

$$F_n \geq \frac{F_u}{\phi} \tag{84b}$$

Equations (84a) and (84b) are called the design equations. When design is based on a strut-and-tie model, the load and resistance factors in ACI Code Sections 9.2 and 9.3 will be used.

3. STRUTS

In a strut-and-tie model, the struts represent concrete compressive stress fields with the compression stresses acting parallel to the strut. Although they are frequently idealized as prismatic or uniformly tapering members, as shown in Fig. 50a, struts generally vary in cross section along their length, as shown in Fig. 50b and c. This is because the concrete stress fields are wider at mid-length of the strut than at the ends. Struts that change in width along the length of the member are sometimes called bottle-shaped due to their shape, as shown in Fig. 50b, or are idealized using local strut-and-tie models, as shown in Fig. 50c.



(a) Idealized prismatic strut.

(b) Bottle-shaped strut.

(c) Strut-and-tie model of a bottle-shaped strut.

Fig. 50. Bottle-shaped strut

The spreading of the compression forces gives rise to transverse tensions in the strut that may cause it to crack longitudinally. A strut without transverse reinforcement may fail after this cracking occurs. If adequate transverse reinforcement is provided, the strength of the strut will be governed by crushing.

3.1 STRUT FAILURE BY LONGITUDINAL CRACKING

Figure 51a shows one end of a bottle-shaped strut. The width of the bearing area is a, and the thickness of the strut is t. At mid-length the strut has an effective width b_{ef}. Reference [50] assumes that the bottle-shaped region at one end of a strut extends approximately 1.5b_{ef} from the end of the strut and in examples used $b_{ef} = l/3$ but not less than a, where l is the length of the strut from face to face of the nodes. For short struts, the limit that not be less than a often governs. It shall be assumed that in a strut with bottle-shaped regions at each end:

$$b_{ef} = a + \frac{1}{6} \quad \text{but no more than the available width} \qquad (85)$$

Figures 50c and 51b show local strut-and-tie models for the bottle-shaped region. It is based on the assumption made in [57], that the longitudinal projection of the inclined struts is equal to $b_{ef}/2$. The transverse tension force T at one end of the strut is

$$T = \frac{C}{2}\left(\frac{b_{ef}/4 - a/4}{b_{ef}/2}\right)$$

or

$$T = \frac{C}{4}\left(1 - \frac{a}{b_{ef}}\right) \tag{86}$$

The force T causes transverse stresses in the concrete, which may cause cracking. The transverse tensile stresses are distributed as shown by the curved line in Fig. 51c.

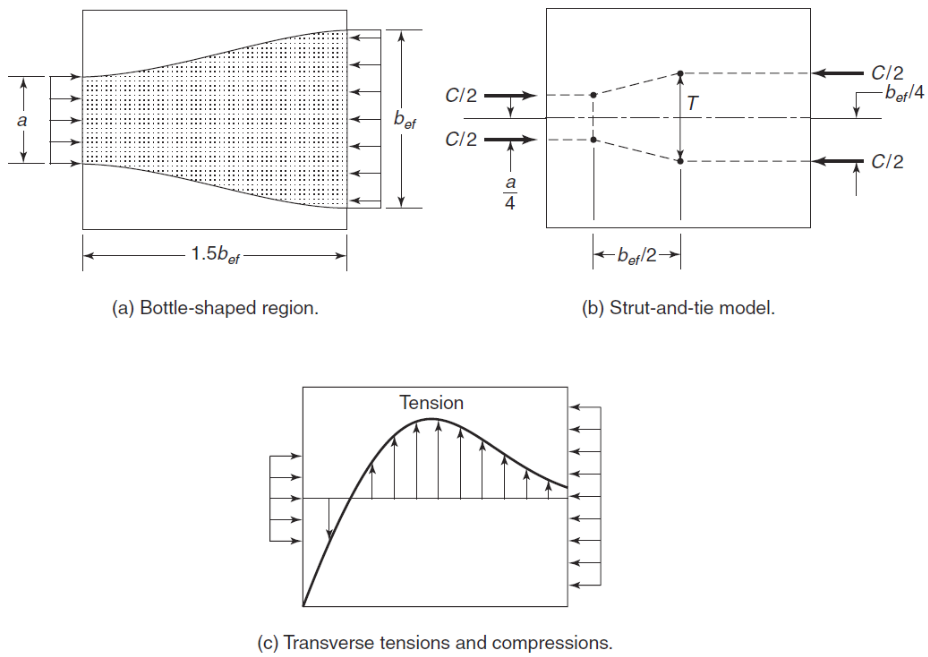


Fig. 51. Spread of stresses and transverse tensions in a strut

Analyses by Adebar and Zhou [58] suggest that the tensile stress distributions at the two ends of a strut are completely separate when exceeds about 3.5 and overlap completely when l/a is between 1.5 and 2. Assuming a parabolic distribution of transverse tensile stresses spread over a length of $1.6b_{ef}$ in a strut of length $2b_{ef}$ and equilibrating a tensile force of 2T indicates that the minimum load, C_n, at cracking is 0.51 to $0.57atf_c'$ for a strut with a/b_{ef} = ½. This analysis and [51] and [57] suggest that longitudinal cracking of the strut may be a problem if the bearing forces on the ends of the strut exceed:

$$C_n = 0.55atf_c' \tag{87}$$

where at is the loaded area at the end of the strut.

In tests of cylindrical specimens loaded axially through circular bearing plates with diameters less than that of the cylinders, failure occurred at 1.2 to 2 times the cracking loads [58].

The maximum load on an unreinforced strut in a wall-like member such as the deep beam in Fig. 49, if governed by cracking of the concrete in the strut, is given by Eq. (87). This assumes that the compression force spreads in only one direction. If the bearing area does not extend over the full thickness of the member, there will also be transverse tensile stresses through the thickness of the strut that will require reinforcement through the thickness shown in Fig. 52. This would require a reanalysis of the support region to design the transverse ties shown in Fig. 52a.

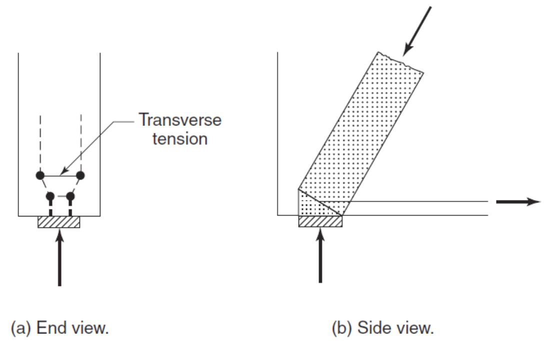


Fig. 52. Transverse spread of forces through the thickness of a strut.

3.2 COMPRESSION FAILURE OF STRUT

The crushing strength of the concrete in a strut is referred to as the effective strength,

$$f_{cu} = \nu f_c' \tag{88}$$

where ν is an efficiency factor having a value between 0 and 1.0. ACI Code Section A.3.2 replaces f_{cu} with the effective compressive strength, f_{ce}. Various sources give differing values of the efficiency factor [52] and [59-64]. The major factors affecting the effective compression strength are:

1. The concrete strength. Concrete becomes more brittle and tends to decrease as the concrete strength increases.

2. Load duration effects. The strength of concrete beams and columns tends to be less than the cylinder strength, f_c'. Various reasons are given for this lower strength, including the observed reduction in compressive strength under sustained load, the weaker concrete near the tops of members due to vertical migration of bleed water during the placing of the concrete, and the different shapes of compression zones and test cylinders. For flexural members, ACI Code Section 10.2.7.1 accounts for this, in part, by taking the maximum stress in the equivalent rectangular stress block as $0.85f_c'$. For struts, load duration effects are accounted for in the ACI Code by rewriting Eq. (88) as $f_{ce} = 0.85\beta_s f_c'$. Nodal zones are treated similarly except that is replaced by

3. Tensile strains transverse to the strut, which result from tensile forces in the reinforcement crossing the cracks [62-66]. In tests of uniformly strained concrete panels, such strains were found to reduce the compressive strength of the panels, as discussed in Section 3-2. The AASHTO Specification bases f_{ce} on this concept [66].

4. Cracked struts. Struts crossed by cracks inclined to the axis of the strut are weakened by the cracks. ACI Code Section A.3.1 presents the nominal compressive strength of a strut as:

$$F_{ns} = f_{ce} A_c \tag{89a}$$

(ACI Eq. A-2)

where subscript n = nominal, s = struct, A_c is the cross-sectional area at the end of the strut, and f_{ce} is:

$$f_{ce} = 0.85\beta_s f_c^{'} \quad (90a)$$

(ACI Eq. A-3)

TABLE 4. ACI Code Values of and for Struts and Nodal Zones

Struts, $f_{ce} = 0.85\beta_s f_c'$	
ACI Section A.3.2.1 For struts in which the area of the midsection cross section is the same as the area at the nodes, such as the compression zone of a beam ----------------------	$\beta_s = 1.0$
ACI Section A.3.2.2 For struts located such that the width of the midsection of the strut is larger than the width at the nodes (bottle-shaped struts):	
(a) with reinforcement satisfying A.3.3 --	$\beta_s = 0.75$
(b) without reinforcement satisfying A.3.3 --	$\beta_s = 0.60\lambda$
ACI Section A.3.2.3 For struts in tension members or the tension flanges of members ------------	$\beta_s = 0.40$
ACI Section A.3.2.4 For all other cases --	$\beta_s = 0.60\lambda$
Nodal zones, $f_{ce} = 0.85\beta_n f_c'$	
ACI Section A.5.2.1 In nodal zones bounded on all sides by struts or bearing areas, or both -----	$\beta_n = 1.0$
ACI Section A.5.2.2 In nodal zones anchoring a tie in one direction ---------------------------------	$\beta_n = 0.8$
ACI Section A.5.2.3 In nodal zones anchoring two or more ties --------------------------------------	$\beta_n = 0.6$

Values of β_s are given in Table 4. For nodal zones, Eqs. (89a) and (90a) become:

$$F_{nn} = f_{ce} A_n \quad (89b)$$

(ACI Eq. A-7)

and

$$f_{ce} = 0.85\beta_n f_c^{'} \quad (90b)$$

(ACI Eq. A-8)

Values of β_n are also given in Table 4. These were derived by ACI Committee 318E [53]. Examples of the use of ACI Appendix A are given in [54], [55], and [56].

3.3 EXPLANATION OF TYPES OF STRUTS DESCRIBED IN TABLE 4

A.3.2.1 applies to a strut equivalent to a rectangular stress block of depth, a, and thickness, b, as occurs in the compression zones of beams or eccentrically loaded columns. In this case β_s is equal to 1.0. The corresponding neutral axis depth is $c = a/\beta_1$. The strut is assumed to have a depth of a and the resultant compressive force in the rectangular stress block $C = f_{ce}ab$, acts at a/2 from the most compressed face of the beam or column as shown in Fig. 53.

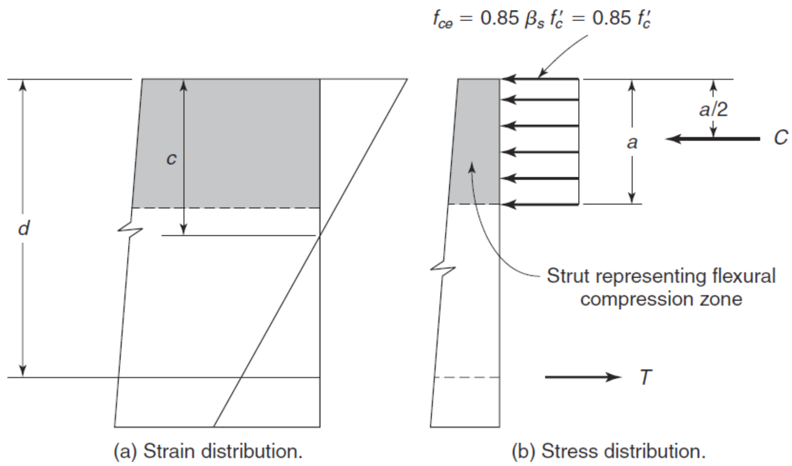


Fig. 53. Strut representing the compression stress block in a beam

A.3.2.2(a) applies to bottle-shaped struts similar to those in Fig. 50b which contain reinforcement crossing the potential splitting cracks. Although such struts tend to split longitudinally, the opening of a splitting crack is restrained by the reinforcement allowing the strut to carry additional load after the splitting cracks develop. For this case $\beta_s = 0.75$. If there is no reinforcement to restrain the opening of the crack, the strut is assumed to fail upon cracking, or shortly after, and a lower value of β_s is used.

The yield strength of the reinforcement required to restrain the crack is taken equal to the tension force that is lost when the concrete cracks. This is computed using a localized strut-and-tie model of the cracking in the strut as shown in Fig. 50c. As discussed earlier, the slope of the load-spreading struts is taken as a value slightly less than 2 to 1 (parallel to axis of strut, to perpendicular to axis):

$$T_n = \frac{C_n}{2}\left(\frac{b_{ef}/4 - a/4}{b_{ef}/2}\right) \tag{91}$$

Rearranging and setting T_n equal to $A_s f_y$ gives the transverse tension force T_n at the ends of the bottle-shaped strut at cracking as:

$$A_s f_y \geq \sum\left[\frac{C_n}{4}\left(1 - \frac{a}{b_{ef}}\right)\right] \tag{92}$$

where C_n is the nominal compressive force in the strut and a is the width of the bearing area at the end of the strut, as shown in Fig. 51a. The width of the bottle-shaped strut, b_{ef}, is computed from the distance between the longitudinal struts and the axis of the strut at mid-length of the strut-and-tie model of the strut, $b_{ef}/4$, also shown in Fig. 51b. The summation Σ implies the sum of the values at the two ends of the strut. If the reinforcement is at an angle θ to the axis of the strut, $A_s f_y$ should be multiplied by sin θ. This reinforcement will be referred to as crack-control reinforcement. In lieu of using a strut-and-tie model to compute the necessary amount of crack-control reinforcement, ACI Code Section A.3.3.1 allows the crack-control reinforcement to be determined using:

$$\sum \frac{A_{si}}{bs_i} \sin\gamma_i \geq 0.003 \tag{93}$$

(ACI Eq. A-4)

where A_{si} refers to the crack control reinforcement adjacent to the two faces of the member at an angle γ_i to the crack, as shown in Fig. 54. The arrangement of the crack-control reinforcement is specified in ACI Code Section A.3.3.2.

Equation (93) was written in terms of a reinforcement ratio rather than the tie force to simplify the presentation. This is acceptable for concrete strengths not exceeding 6000 psi. (See ACI Code Section A.3.3.) For higher concrete strengths the ACI Code Committee felt the load-spreading should be computed. A tensile strain in bar 1, ε_{s1}, in Fig. 17-8 results in a tensile strain of perpendicular to the axis of the strut. Similarly, for bar 2, the strain perpendicular to the axis of the strut is $\varepsilon_{s2}\sin\gamma_2$, where $\gamma_1+\gamma_2=90°$.

A.3.2.2(b) In mass concrete members such as pile caps for more than two piles, it may be difficult to place the crack control reinforcement. ACI Code Section A.3.2.2(b) specifies a lower value of f_{ce} in such cases. Because the struts are assumed to fail shortly after longitudinal cracking occurs, β_s is multiplied by the correction factor, for lightweight concrete when such concrete is used. Values of are defined in ACI Code Section 8.6.1. It is 1.0 for normal-weight concrete.

A.3.2.3 is used in proportioning struts in strut-and-tie models used to design the reinforcement for the tension flanges of ledger beams, box girders and the like. It accounts for the fact that flexural tension cracks will tend to be wider than cracks in beam webs.

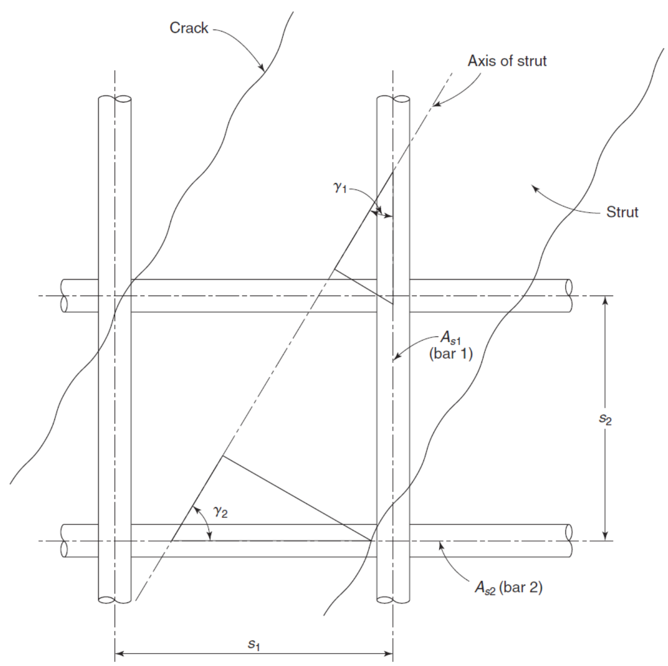


Fig. 54. Crack control reinforcement crossing a strut in a cracked web

A.3.2.4 applies to all other types of struts not covered in A.3.2.1, A.3.2.2, and A.3.2.3. This includes struts in the web of a beam where more or less parallel cracks divide the web into parallel struts. It also includes struts likely to be crossed by cracks at an angle to the struts.

4. TIES

The second major component of a strut-and-tie model is the tie. A tie represents one or several layers of reinforcement in the same direction. Design is based on:

$$\phi F_{nt} \geq F_{ut} \tag{94}$$

where the subscript t refers to "tie," and F_{nt} is the nominal strength of the tie, taken as:

$$F_{nt} = A_{ts} f_y + A_{tp} \left(f_{se} + \Delta f_p \right) \tag{95}$$

(ACI Eq. A-6)

The second term in the parentheses on the right-hand side of Eq. (95) is for prestressed ties. It drops out if the member or element does not contain prestressed reinforcement.

ACI Code Section A.4.2 requires that the axis of the reinforcement in a tie coincide with the axis of the tie. In the layout of a strut-and-tie model, ties consist of the reinforcement plus a prism of concrete concentric with the longitudinal reinforcement making up the tie. The width of the concrete prism surrounding the tie is referred to as the effective width of the tie, w_t. ACI Commentary Section R.A.4.2 gives limits for w_t. The lower limit is a width equal to twice the distance from the surface of the concrete to the centroid of the tie reinforcement. In a hydrostatic C–C–T nodal zone (defined in Section 5 of this chapter), the stresses on all faces of the nodal zone should be equal. As a result, the upper limit on the width of a tie is taken equal to:

$$w_{t,max} = F_{nt} / \left(f_{ce} b \right) \tag{96}$$

The concrete is included in the tie to establish the widths of the faces of the nodal zones acted on by ties. The concrete in a tie does not resist any load. It aids in the transfer of loads from struts to ties or to bearing areas through bond with the reinforcement. The concrete surrounding the tie steel increases the axial stiffness of the tie by tension stiffening. Tension stiffening may be used in modeling the axial stiffness of the ties in a serviceability analysis.

Ties may fail due to lack of end anchorage. The anchorage of the ties in the nodal zones is a critical part of the design of a D-region using a strut-and-tie model. Ties are normally shown as solid lines in strut-and-tie models.

5. NODES AND NODAL ZONES

The points at which the forces in struts-and-ties meet in a strut-and-tie model are referred to as nodes. Conceptually, they are idealized as pinned joints. The concrete in and surrounding a node is referred to as a nodal zone. In a planar structure, three or more forces must meet at a node for the node to be in equilibrium, as shown in Fig. 55. This requires that:

$$\sum F_x = 0; \sum F_y = 0; \sum M = 0 \tag{97}$$

The $\sum M = 0$ condition implies that the lines of action of the forces must pass through a common point, or must be able to be resolved into forces that act through a common point. The two compressive forces shown in Fig. 55a meet at an angle and are not in equilibrium unless a third force is added, as shown in Fig. 55b or c. Nodal zones are classified as C–C–C if three compressive forces meet, as in Fig. 55b, and as C–C–T if one of the forces is tensile as shown in Fig. 55c. C–T–T joints may also occur.

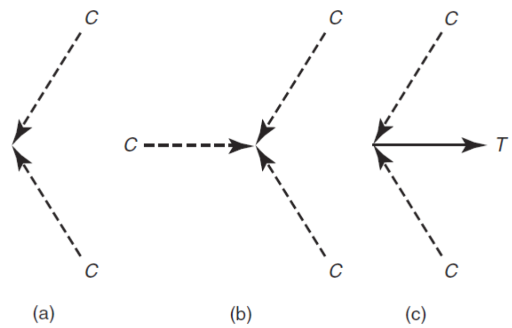


Fig. 55. Forces acting on nodes

5.1 HYDROSTATIC NODAL ZONES

Two common ways of laying out nodal zones are illustrated in Figs. 56 and 57. The prismatic compression struts in Figs. 49 and 50a are assumed to be stressed in uniaxial compression. A section perpendicular to the axis of a strut is acted on only by compression stresses, while sections at any other angle have combined compression and shear stresses. One way of laying out nodal zones is to orient the sides of the nodes at right angles to the axes of the struts or ties meeting at that node, as shown in Fig. 56, and to have the same bearing pressure on each side of the node. When this is done for a C–C–C node, the ratio of the lengths of the sides of the node, $w_1 : w_2 : w_3$, is the same as the ratio of the forces in the three members meeting at the node, $C_1 : C_2 : C_3$, as shown in Fig. 56a. Nodal zones laid out in this fashion are sometimes referred to as hydrostatic nodal zones because the in-plane stresses in the node are the same in all directions. In such a case, the Mohr's circle for the in-plane stresses reduces to a point.

If one of the forces is tensile, the width of that side of the node is calculated from a hypothetical bearing plate on the end of the tie, which is assumed to exert a bearing pressure on the node equal to the compressive stress in the struts at that node, as shown in Fig. 56b. Alternatively, the reinforcement may extend through the nodal zone to be anchored by bond, hooks, or mechanical anchorage before the reinforcement reaches point A on the right-hand side of the extended nodal zone, as shown by Fig. 56c. Such a nodal zone approaches being a hydrostatic C–C–C nodal zone. However, the strain incompatibility resulting from the tensile steel strain adjacent to the compressive concrete strain reduces the strength of the nodal zone. Thus, this type of joint should be designed as a C–C–T joint with $\beta_n = 0.80$.

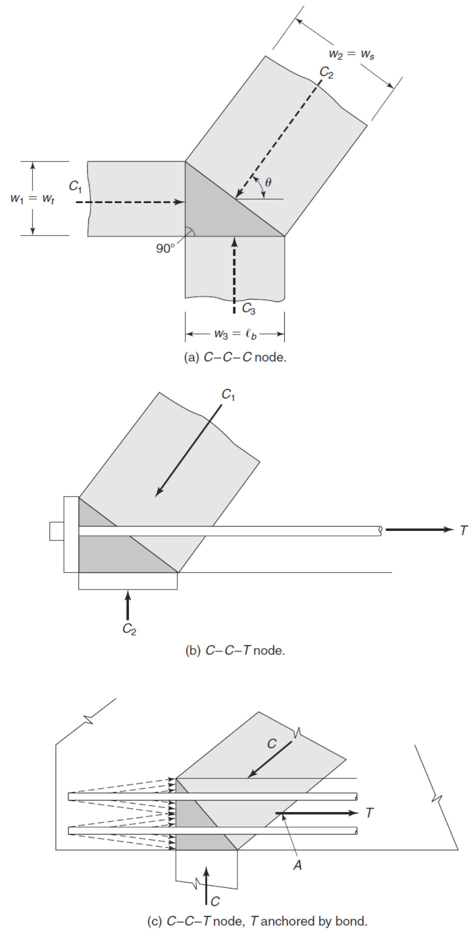


Fig. 56. Hydrostatic nodal zones in planar structures

5.2 GEOMETRY OF HYDROSTATIC NODAL ZONES

Because the stresses are equal or close to equal on all faces of a hydrostatic nodal zone that are perpendicular to the plane of the structure, equations can be derived relating the lengths of the sides of the nodal zone to the forces in each side of the nodal zone. Figure 56a shows a hydrostatic C–C–C node. For a nodal zone with a 90° corner, as shown, the horizontal width of the bearing area is $w_3 = l_b$. The height of the vertical side of the nodal zone is $w_1 = w_t$. The angle between the axis of the inclined strut and the horizontal is θ. The width of the third side, the strut, $w_2 = w_s$, can be computed as:

$$w_s = w_t \cos\theta + l_b \sin\theta \qquad (98)$$

This equation also can be applied to a C–C–T node, as shown in Fig. 56b. If the required width of the strut, w_s, computed from the strut force by using Eq. (89a), is larger than the width given by Eq. (98), it is necessary to increase either or w_t or l_b both until the width from Eq. (98) equals or exceeds the width calculated from the strut forces.

5.3 EXTENDED NODAL ZONES

The use of hydrostatic nodes can be tedious in design, except possibly for C–C–C nodes. More recently, the design of nodal zones has been simplified by considering the nodal zone to comprise that concrete lying within extensions of the members meeting at the joint as shown in Fig. 57 [52], [67]. This allows different stresses to be assumed in the struts and over bearing plates, for example. Two examples are given in Fig. 57. Figure 57a shows a C–C–T node. The bars must be anchored within the nodal zone or to the left of point A, which ACI Code Section A.4.3.2 describes as "the point where the centroid of the reinforcement in the tie leaves the extended nodal zone." The length, l_d, in which the bars of the tie must be developed is shown. The vertical face of the node is acted on by a stress equal to the tie force T divided by the area of the vertical face. The stresses on the three faces of the node can all be different, provided that

1. The resultants of the three forces coincide.
2. The stresses are within the limits given in Table 4.
3. The stress is constant on any one face.

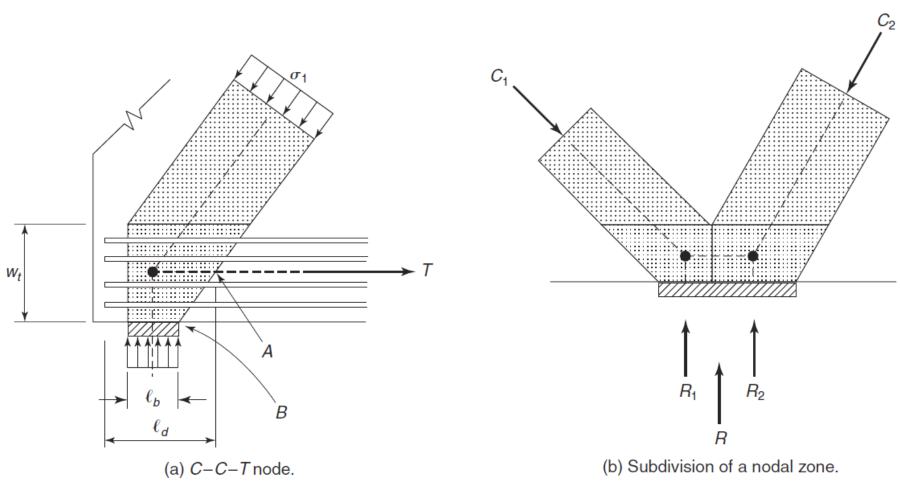


Fig. 57. Extended nodal zones

Equation (98) can be used to compute the widths perpendicular to the axis of the struts, as shown in Fig. 58, even though this equation was derived for hydrostatic nodal zones. This equation is useful in adjusting the width of an inclined strut if the original width is found to be inadequate.

An extended nodal zone consists of the node itself, plus the concrete in extensions of the struts, bearing areas, and ties that meet at a joint. Thus in Fig. 57a the darker shaded region indicates the nodal zone extends into the area occupied by the struts and ties at this node. This layout of a nodal zone contains much of the concrete stressed in compression over a reaction. An alternate and sometimes easier nodal zone to use is shown in Fig. 56b, where the assumed nodal zone is the smallest size possible for this node because it does not include any concrete that is not common to the struts, bearing areas, and ties at the node. The advantage of the nodal zones in Fig. 57a and b comes from the fact that ACI Section Code A.4.3.2 allows the length available for bar development to anchor the tie bars to be taken out to point A in Fig. 57a rather than point B at the edge of the bearing plate. This extended anchorage length recognizes the beneficial effect of the compression from the reaction and the struts for improving bond between the concrete and the tie reinforcement.

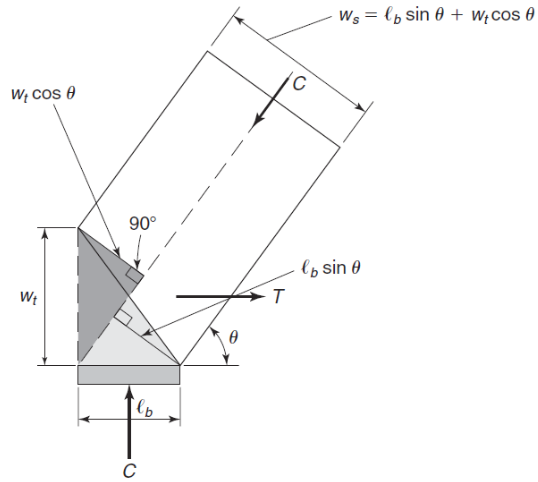


Fig. 58. Width of inclined strut at a C–C–T nodal zone

5.4 STRENGTH OF NODAL ZONES

Nodal zones are assumed to fail by crushing. Anchorage of the tension ties is a matter of design consideration. If a tension tie is anchored in a nodal zone there is a strain incompatibility between the tensile strains in the bars and the compressive strain in the concrete of the node. This tends to weaken the nodal zone. ACI Code Section A.5.1 limits the effective concrete strengths, f_{ce}, for nodal zones as:

$$F_{nn} = f_{ce} A_n \quad (89b)$$

(ACI Eq. A-7)

where A_n is the area of the face of the node that the strut or tie acts on, taken perpendicular to the axis of the strut or tie, or the area of a section through the nodal zone, and f_{ce} is the effective compression strength of the concrete:

$$f_{ce} = 0.85 \beta_n f_c' \quad (90b)$$

(ACI Eq. A-8)

ACI Code Section A.5.1 gives the following three values of for nodal zones. (See also Table 4.):
1. in C–C–C nodal zones bounded by compressive struts and bearing areas.
2. in C–C–T nodal zones anchoring a tension tie in only one direction.
3. in C–T–T nodal zones anchoring tension ties in more than one direction.

Tests of C–C–T and C–T–T nodes reported in [68] and [69] developed β_n = 0.95 in properly detailed nodal zones.

5.5 SUBDIVISION OF NODAL ZONES

Frequently, it is easier to lay out the size and location of nodal zones if they are subdivided into several parts, each of which is assumed to transfer a particular component of the load through the nodal zone. In Fig. 57b, the reaction R has been divided into two components R_1, which equilibrates the vertical component of C_1 and R_2, which equilibrates C_2.

Generally, this subdivision simplifies the layout of the struts and nodes. Subdivision is useful in dealing with the dead load of a beam which can be assumed to be applied as a series of equivalent concentrated loads, each of which is transferred to a reaction by an individual strut as shown in Fig. 59a. In this figure, the dead load is transferred to the support by four inclined struts, which are supported by the portion of the support nodal zone labeled as V_s. The horizontal width of the part of the nodal zone labeled V_s is the sum of the horizontal widths of the four struts that support the dead loads on this half of the beam. The vertical truss members represent the subdivided dead loads and stirrup forces.

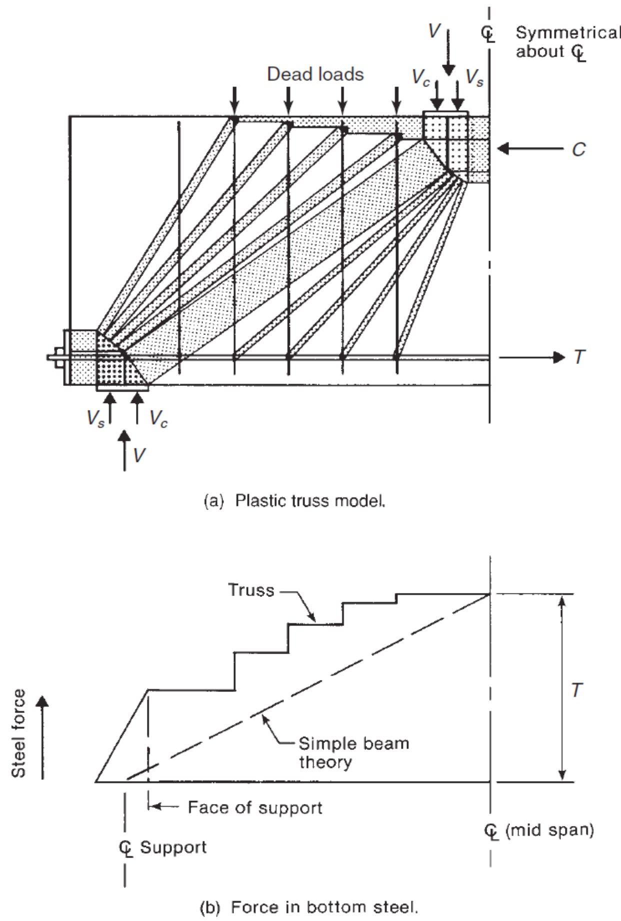


Fig. 59. Strut-and-tie model of a deep beam with dead load and stirrups

5.6 RESOLUTION OF FORCES ACTING ON A NODAL ZONE

If more than three forces act on a nodal zone in a planar strut-and-tie model, it is usually advantageous to subdivide the nodal zone so that only three forces remain on any part of the node. Figure 60a shows a hydrostatic nodal zone that is in equilibrium with four strut forces meeting at point D. The nodal zone for point D can be subdivided as shown in Fig. 60b. For sub-node E-F-G, the two forces acting on faces E-F and E-G can be resolved into a single inclined force (50.6 kips) acting between the two sub-nodes. That inter-nodal force must also be in equilibrium with the forces acting on faces A-B and B-C of sub-node A-B-C. The overall force equilibrium for node D is demonstrated in Fig. 60c.

Another example is shown in Fig. 57b, which shows two sub-nodes. It is necessary to ensure that the stresses in the members entering the node, the stress over the bearing plate, and the stress on any vertical line between the two sub-nodes are within the limits in Table 4.

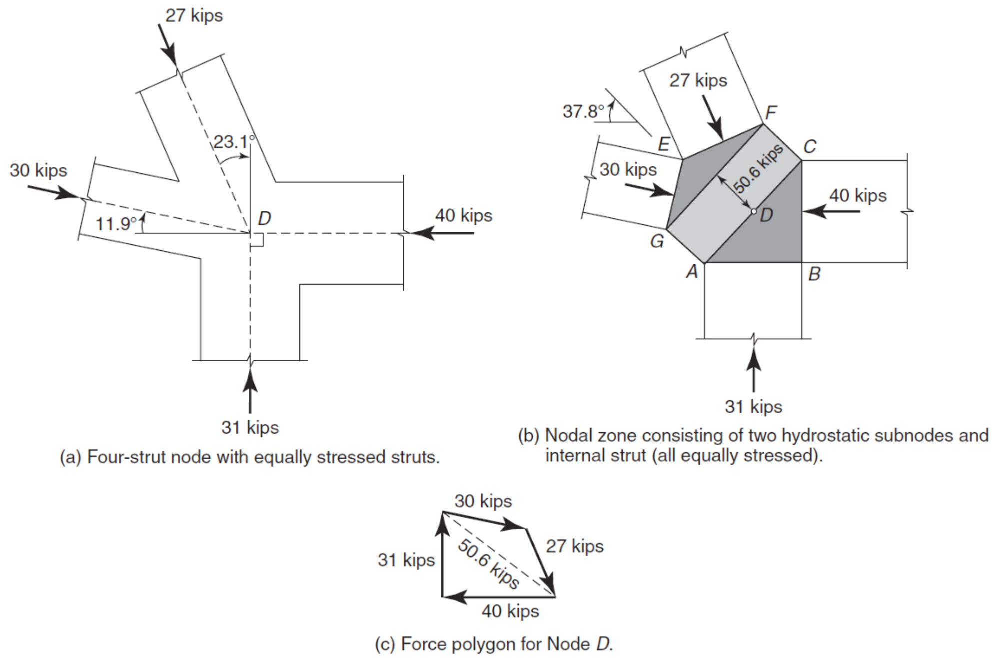


Fig. 60. Resolution of forces acting on a nodal zone

5.7 ANCHORAGE OF TIES IN NODAL ZONES

A challenge in design using strut-and-tie models is the anchorage of the tie forces at the nodal zones at the edges or ends of a strut-and-tie model. This problem is independent of the type of analysis used in design. It occurs equally in structures designed by elastic analyses or strut-and-tie models. In fact, one of the advantages of strut-and-tie models comes from the attention that the strut-and-tie model places on the anchorage of ties as described in ACI Code Section A.4.3. For nodal zones anchoring one tie, the tie must be developed by bond, by hooks, or by mechanical anchorage between the free end of the bar and the point at which the centroid of the tie reinforcement leaves the compressed extended portion of the nodal zone. This corresponds to point A in Fig. 57a. If the bars are anchored by hooks, the hooks should be confined within reinforcement extending into the member from the supporting column, if applicable.

European practice [67] sometimes uses lap splices between the tie bars and U bars lying horizontally. Typically, two layers of U bars are used to anchor one layer of tie bars. Each layer of U bars is designed to anchor one-third of the total bar force, leaving one-third to be anchored by bond stresses on the tie bars.

5.8 NODAL ZONE ANCHORED BY A BENT BAR

Sometimes the two tension ties in a C–T–T node are both provided by a bar bent through as shown in Fig. 61. The compressive force in the strut can be anchored by bearing and shear stresses transferred from the strut to the bent bar. Such a detail must satisfy the laws of statics and limits on the bearing stresses on the concrete inside the bent bar. A design procedure is given in a recent article by Klein [70].

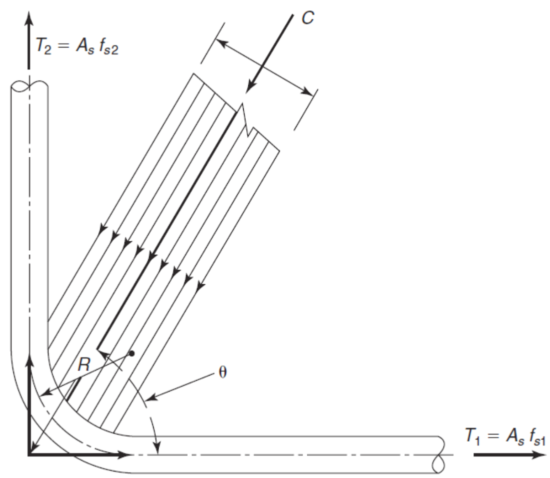


Fig. 61. C–T–T node anchored by a bent bar

5.9 STRUT ANCHORED BY REINFORCEMENT

Sometimes, diagonal struts in the web of a truss model of a flexural member are anchored by longitudinal reinforcement that, in turn, is supported by a stirrup, as shown in Fig. 62. Reference [72] recommends that the length of longitudinal bar able to support the strut be limited to six bar diameters each way from the center of the strut.

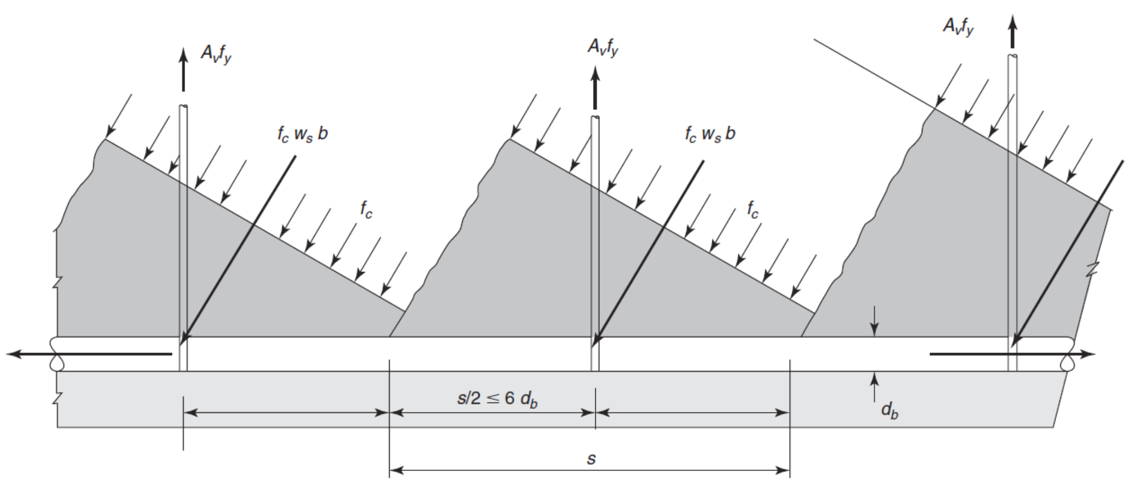


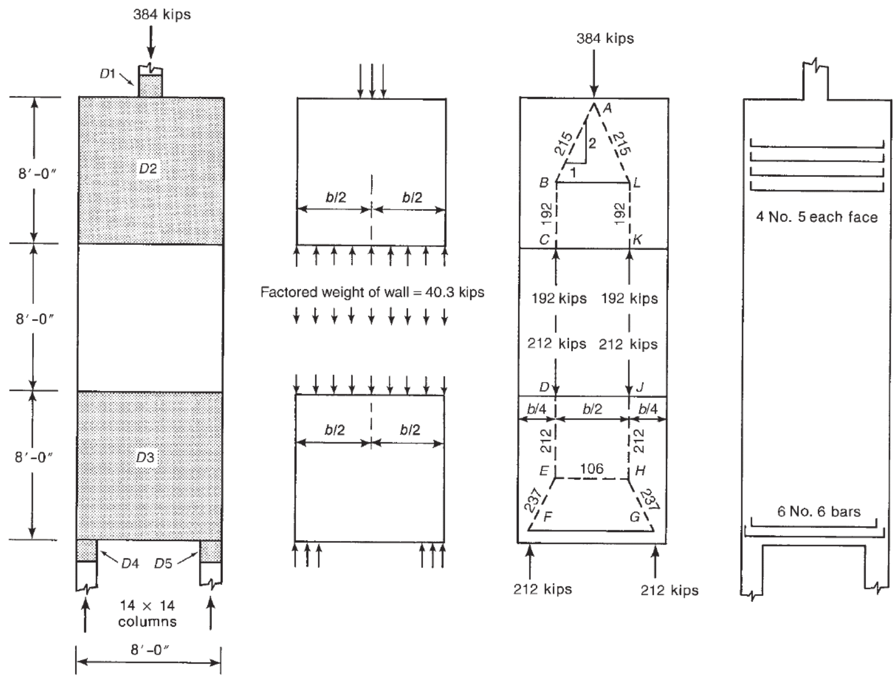
Fig. 62. Struts anchored by stirrups and longitudinal bars

5.10 THE USE OF A STRUT-AND-TIE MODEL IN DESIGN

The following solved example considers the design of a wall loaded and supported by columns. The purpose of the example is to illustrate the choice and use of a strut-and-tie model, to demonstrate the choice of D-regions, and to discuss reasons for making certain assumptions.

SOLVED EXAMPLE – Design of a D-Region in a wall

The 14-in. thick wall shown in Fig. 17-17 supports a 14 in. by 20 in. column carrying unfactored loads of 100 kips dead load and 165 kips live load. The wall in from the following figure, "a", supports this column and is supported on two other columns which are 14 by 14 in. The floor slabs (not shown) provide stiffness against out-of-plane buckling. Design the wall reinforcement. Use $f_c^{'}$ = 3000 psi and f_y = 60000 psi. The primary design equation is $\phi F_n \geq F_u$ where is F_n the nominal capacity of the element, and F_u is the force on the element due to the factored loads.



(a) Wall and *D*-regions. (b) Stresses on *D*-regions. (c) Truss model. (d) Reinforcement.

Step 1. Isolate the D-Regions. *The loading discontinuities due to the column load on the wall dissipate in a distance of approximately one member dimension from the location of the discontinuity. Based on this, the wall will be divided into two D-regions separated by a B-region as shown in figure "a". The wall has two statical discontinuities:*

(i) Under the column load at the top

(ii) Over the two columns supporting the bottom of the wall

Using St. Venant's principle, the D-regions are assumed to extend a distance equal to the width of the wall (8 ft) down from the top and the same distance up from the tops of the two columns that support the wall. There are three more D-regions at the ends of the columns, which have little effect on the wall and will not be considered in this example.

The self-weight of the wall is: $24\text{ft} \times 8\text{ft} \times 14/12\text{ft} \times 0.150\text{kips}/\text{ft}^3 = 33.6\text{kips}$

It shall be assumed that this acts as a uniformly distributed load acting on the structure at mid-height of the wall as shown in figures "b" and "c".

Step 2. Compute the Factored Loads. *Using the load factors in ACI Code Section 9.2, the factored load on the upper column is the larger of:*

$$\text{U} = 1.4 \times 100\text{kips} = 140\text{kips} \qquad \textit{(ACI Eq. 9-1)}$$

$$\text{U} = 1.2 \times 100\text{kips} + 1.6 \times 165\text{kips} = 384\text{kips} \qquad \textit{(ACI Eq. 9-2)}$$

By inspection, the rest of the ACI Load Combinations (ACI Eq. 9-3 to 9-7) do not govern the vertical loads on the wall. The factored weight of the wall is: 1.2 × 33.6 = 40.3 kips.

Step 3. Subdivide the Boundaries of the D-Regions and Compute the Force Resultants on the Boundaries of the D-Region. *For D2, we can represent the load on the top boundary by a single force of 384 kips at the center of the column, or as two forces of 384 kips / 2 = 192 kips acting at the quarter points of the width of the column at the interface with the wall. We shall draw the strut-and-tie model using one force. The bottom boundary of D-region D2 will be divided into two segments of equal lengths, b/2 each with its resultant force of 192 kips acting along the middle of the struts loaded by the column above. This gives uniform stress on the bottom of D2.*

4. Lay Out the Strut-and-Tie Models. *Two strut-and-tie models are needed, one in each of D2 and D3. The function of the upper strut-and-tie model of D2 is to transfer the column load from the center of the top of D2 to the bottom of D2, where the load is essentially uniformly distributed. Figure 66.a (discussed later) shows the stress trajectories from an elastic analysis of a vertical plate loaded with in-plane loads. The dashed lines in figure Fig. 66.a represent the flow of compression stresses, and the solid lines show the directions of the tensile stresses. Struts A–B and B–C in figure "c" replace the stress trajectories in the left half of the D-region in Fig. 66.a. The compression stresses fan out from the column, approaching a uniformly distributed stress at the height where the struts pass through the quarter points of the section. In D2 this occurs at level B–L. Below this level, struts B–C and L–K are vertical and pass through the quarter points of the width of the section. This gives uniform compression stresses over the width.*

For D-Region D3*, similarly, the load on the top of D-region D3 will be represented by struts at the quarter points of the top of the D-region. The strut-and-tie model in D3 transfers the uniformly distributed loads, including the dead load of the wall, from the top of D3 down to the two concentrated loads where the wall is supported by the columns.*

Step 5. Draw the Strut-and-Tie Models. *In drawing strut-and-tie models, compression struts will always be plotted using dashed lines and tensile members with solid lines. In Section 6, it is recommended that load-spreading strut-and-tie models with struts at a (2 to 1) slope relative to the axis of the applied load be used, i.e., struts at (2 units parallel to the force that is spreading) to (1 unit perpendicular to the force). These correspond to θ = arctan1/2 = 26.6° from the axis of the force. The strut-and-tie models are shown in figure "c". The forces in the struts and ties in the wall are listed in the following Table 5.*

Step 6. Compute the Forces and Strut Widths in Both Strut-and-Tie Models. *The calculations are given in Table 5.*

Step 7. D-region D2.

(a) Node A and struts A–B and A–L: *Treating node A as a hydrostatic node, either the node at A or one of the struts A–B and A–L will control.*

Node A*: Because this node is compressed on all in-plane faces, β_n = 1.0 and the effective compression stress for node A from Table 4 is:*

$$f_{ce} = 0.85 \times 1.0 \times 3000\text{psi} = 2550\text{psi}$$

Struts A–B and A–L: *Because the stresses in the concrete beside struts A–B and A–L are low, a portion of the stress in the struts is resisted by the concrete adjacent to the idealized prismatic struts, making these bottle-shaped struts. We will provide vertical and horizontal reinforcement satisfying ACI Code Section A.3.3, thereby allowing ACI Code Section A.3.2.2(a) to apply with β_s = 0.75. This allows f_{ce} in strut A–B or Strut A–C to be*

$$f_{ce} = 0.85 \times 0.75 \times 3000\text{psi} = 1910\text{psi}$$

Because this is less than 2110 psi for the node, 1910 psi governs. For the factored load in the column at A, P_u = 384 kips, and using ϕ = 0.75 from ACI Code Section 9.3.2.6, an area of

$$\frac{384\text{kips}\times 1000\text{lbs / kip}}{0.75\times 1910\text{psi}} = 268\text{in.}^2$$

is required. The column loading the wall is 14 × 20 = 280 in.^{2} and therefore the column is large enough.

(b) Minimum dimensions for nodes B and L: *These are C–C–T nodes, so the effective compressive stress from Table 4 is:*

$$f_{ce} = 0.85\times 0.8\times 3000\text{psi} = 2040\text{psi}$$

This will control the base dimension of nodes B and L because struts B–C and L–K are prismatic struts that can be designed by using β_s = 1.0. Thus, the minimum base dimension of node B and the width of strut B–C is:

$$w_s = \frac{192000\text{lbs}}{0.75\times 2040\text{psi}\times 14\text{in.}} = 8.96\text{in.}$$

This is much less than b/2 (4 ft), so the node easily fits within the dimensions of the wall. The minimum height of node B is of interest for tie B–L. So,

$$w_t = \frac{96000\text{lbs}}{0.75\times 2040\text{psi}\times 14\text{in.}} = 4.48\text{in.}$$

This is a very small dimension and the reinforcement for tie B–L will be spread over a larger distance. Essentially, the dimensions of nodes B and L will be much larger than the minimum values calculated here.

(c) Required area of reinforcement for tie B–L:

Tie force: $T_u = \dfrac{192\text{kips}}{\tan\theta} = 96.0\text{kips}$

Required $A_s = \dfrac{T_u}{\phi f_y} = \dfrac{96\text{kips}}{0.75\times 60\text{ksi}} = 2.13\text{in.}^2$

The type of steel will be chosen after the minimum reinforcement has been computed. Essentially, a band of transverse steel having this area should be provided across the full width of the wall extending about 25% of the width of the wall above and below the position of tie B–L so that the centroid of the areas of the bars is close to tie B–L. (See figure "d") Both ends of each bar should be hooked.

Table 5. Calculation of Forces in the Strut-and-Tie Models

D-Region	*Member*	*Vertical force component*	*Horizontal force component kips*	*Axial force kips*	*Effective concrete strength, f_{ce}*	*Min, width of strut or nodal zone, w_s*

		kips			psi	in.
1	*2*	*3*	*4*	*5*	*6*	*7*
D2	*Node A*	*384*	*0*	*384*	*1910*	*19.1*
	A-B	*192*	*96*	*215*	*1910*	*10.7*
	B-C	*192*	*0*	*192*	*2040*	*8.96*
	A-L	*192*	*96*	*215*	*1910*	*10.7*
	L-K	*192*	*0*	*192*	*2040*	*8.96*
	B-L	*0*	*96*	*96*	*2040*	*4.48*
D3						
	D-E	*212*	*0*	*212*	*2040*	*9.90*
	E-F	*212*	*106*	*237*	*1910*	*11.8*
	F-G	*0*	*106*	*106*	*2040*	*4.95*
	G-H	*212*	*106*	*237*	*1910*	*11.8*
	H-J	*212*	*0*	*212*	*2040*	*9.90*
	E-H	*0*	*106*	*106*	*2040*	*4.95*

8. D-region D3. *Nodes F and G are C–C–T nodes, similar to nodes B and L. We will use the effective compressive stress for these nodes (f_{ce} = 2040 psi) to determine the minimum dimensions for most of the struts, ties, and nodes in D-region D3, as given in column 7 of Table 5. For the inclined struts E–F and G–H, f_{ce} = 1910 psi. Clearly, all of these element dimensions easily fit within the dimensions of the wall and supporting columns.*

(a) Required area of reinforcement for tie F–G:

Tie force: $T_u = 106\text{kips}$

Required $A_s = \dfrac{T_u}{\phi f_y} = \dfrac{106\text{kips}}{0.75 \times 60\text{ksi}} = 2.36\text{in.}^2$

Thus, use six No. 6 bars, As = 2.64 in.^{2}, placed in two layers of three bars per layer. *This should put the centroid of these bars approximately at the mid-height of tie F–G, whose height (width) is given in Table 5. All of these bars must be hooked at the edges of the wall, as shown in figure "d".*

9. Minimum distributed wall reinforcement. *Minimum requirements for wall reinforcement are covered in detail in Chapter 18. For this problem, we will assume the requirements of ACI Code Section 14.3 govern. From ACI Code Section 14.3.3, the minimum percentage of Grade-60 horizontal reinforcement is 0.0020, with a maximum spacing not to exceed three times the wall thickness or 18 in. (ACI Code Section 14.3.5). For a wall width of 14 in., reinforcement will be required in each face. Thus, throughout the height of the wall, except for the locations of ties B–L and F–G, provide No. 4 bars in each face at a vertical spacing of 14 in. o.c. (horizontal reinforcement ratio = 0.00204).*

Reinforcement for tie F–G was selected in step 8. From step 7, the area of reinforcement required for tie B–L was 2.13 in.^{2}. ***Use eight No. 5 bars (As = 2.48 in.^{2}) at a vertical spacing of 12 in.—half in each face and hooked at both ends (figure "d").*** *This spacing provides a tie width of approximately 4ft, as recommended in step 6.*

6. COMMON STRUT-AND-TIE MODELS

6.1 COMPRESSION FANS

A compression fan is a series of compression struts that radiate out from a concentrated applied force to distribute that force to a series of localized tension ties, such as the stirrups. Fans are shown over the reaction and under the load in Fig. 63. The failure of a compression fan is shown in Fig. 6-22.

6.2 COMPRESSION FIELDS

A compression field is a series of parallel compression struts combined with appropriate tension ties and compression chords, as shown in Fig. 63. Compression fields are shown between the compression fans in Fig. 63.

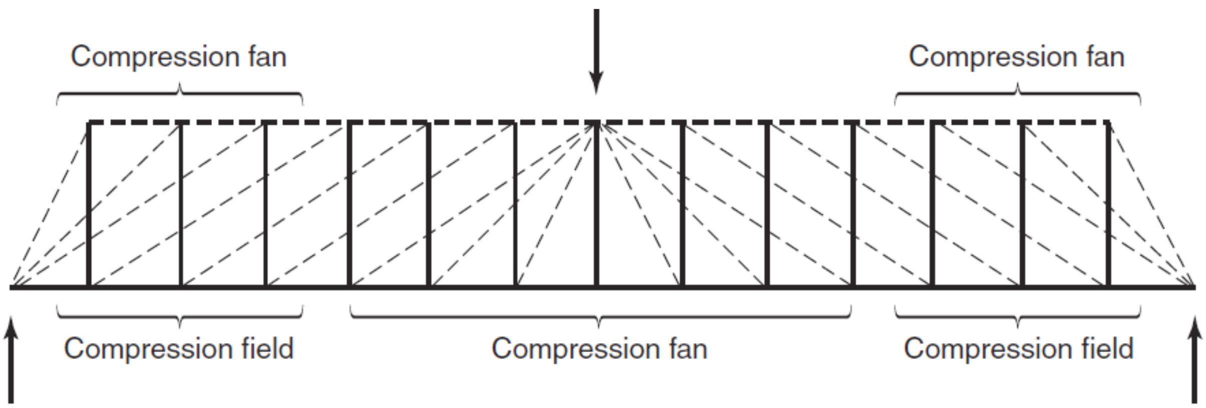


Fig. 63. Compression fans and compression fields

6.3 FORCE WHIRLS, U-TURNS

In Fig. 64 the column load causes the stresses shown by the shaded areas at the bottom of the D-region. The stresses on the bottom edge, A–I, have been computed with the use of the formula $\sigma = (P/A) + (M_y/I)$. The neutral axis (axis of zero strains) is at G, 26.7 in. from the right edge of the wall. The widths of E–G and G–I have both been chosen as 26.7 in. so that the upward force at F equals the downward force at H. The widths of the other two parts (A–C and C–E) were chosen so that the forces in them were equal. The right-hand two reactions are each 41.8 kips. They cause the force whirl made up of compression members F–O and O–P and tension member H–P. The reinforcement computed from the strut-and-tie model is shown in Fig. 65. The strut-and-tie model plotted in Figs. 64 and 65 is solved in [67].

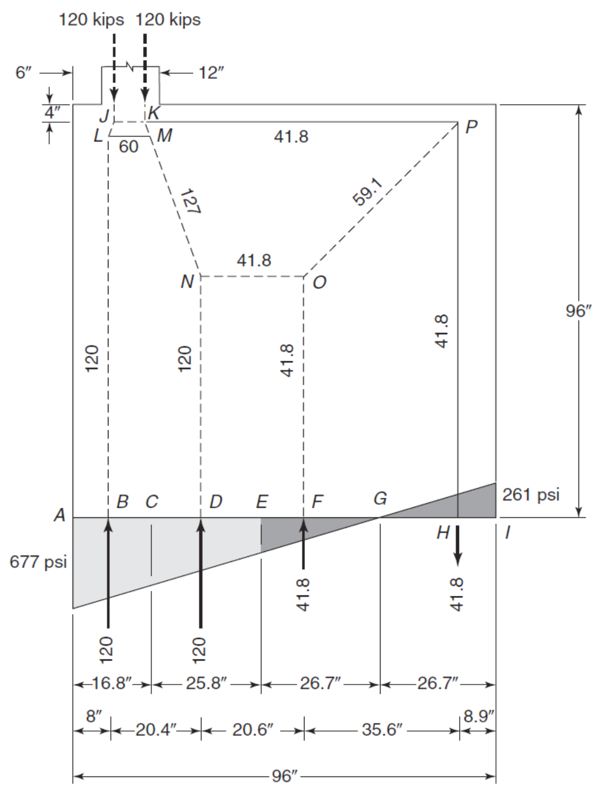


Fig. 64. Column supported near one end of a wall, showing a strut-and-tie model with a force whirl or U-turn.

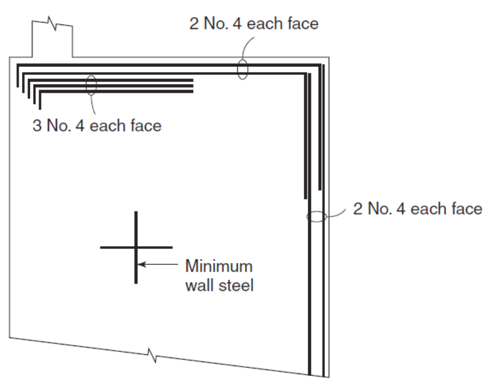


Fig. 65. Reinforcement in the wall in Fig. 64

6.4 LOAD-SPREADING REGIONS

Frequently, concentrated loads act on walls or other member. These loads spread out in the member, as shown by the dashed lines in Figs. 50c, 51b, and 66a. Transverse tension ties are required for equilibrium of the joints at, for example, points B and F in Fig. 50c. The magnitude of the tensile force in the ties depends on the slope of the load-spreading struts. In Figs. 50c and 51b, the slope of these struts has been assumed to be slightly less than 2 to 1, that is (2 parallel) to (1 perpendicular), relative to the axis of the force being spread. If half of the applied load C is resisted by each branch of the load-spreading strut-and-tie model, as shown in Figs. 50c, the tie force B–F will be C/4. On the other hand, if the slope were 1 to 1, the tie force would double, to C/2.

Elastic analyses of thin edge-loaded elastic members of width b subjected to inplane loads applied to one edge, as shown in Fig. 51, indicate that the load-spreading angle is primarily a function of the ratio of the width of the loading plate, a, to that of the loaded member, b. Using analyses, [17-1] shows that the angle between the load and the inclined struts varies from 28° for a concentric load with a/b=0.10, to 19° for a/b=0.20 and down to about 12° for a/b=0.50. As a result, the transverse tie in Fig. 51b would correspond to strut slopes from 1.9 to 1 for a/b=0.10, to 2.9 to 1 for a/b=0.20 and to 4.7 to 1 for a/b=0.50. Similar values are obtained for other cases of load spreading, such as concentrated loads acting near one edge of a member or multiple, concentrated loads.

A strut slope of 2:1 (longitudinal to transverse), as recommended by the ACI Codes, is conservative for a wide range of cases.

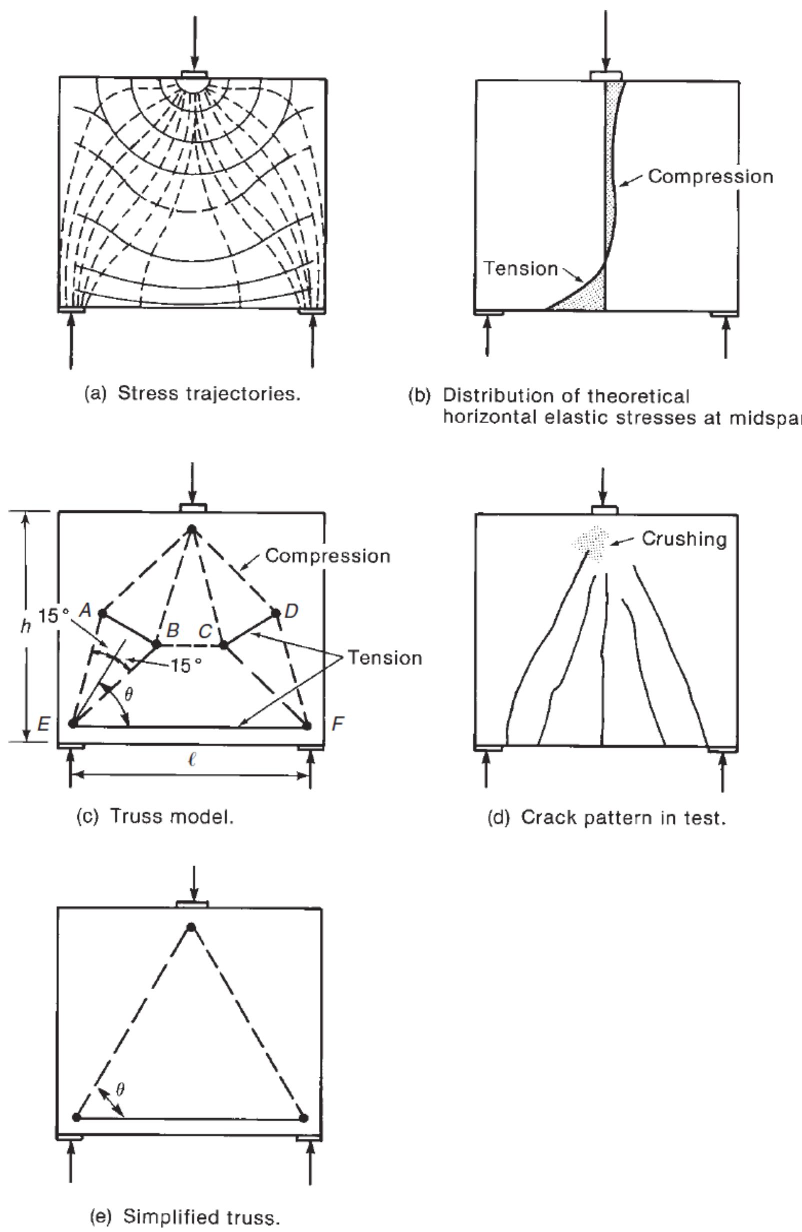


Fig. 66. Single-span deep beam supporting a concentrated load

7. LAYOUT OF STRUT-AND-TIE MODELS

7.1 FACTORS AFFECTING THE CHOICE OF STRUT-AND-TIE MODELS

A general procedure for laying out strut-and-tie models was presented in Section 2 and illustrated in the solved example. Additional guidelines for the choice of strut-and-tie models include the following areas.

7.1.1 Equilibrium

1. The strut-and-tie model must be in equilibrium with the loads. There must be a clearly laid out load path.

7.1.2 Direction of Struts and Ties

2. The strut-and-tie model for a simply supported beam with unsymmetrically applied, concentrated loads consist of an arch, made of straight-line segments or a hanging cable, that has the same shape as the bending-moment diagram for the loaded beam as shown in Fig. 67. This is also true for uniformly loaded beams, except that the moment diagram and the strut-and-tie model have parabolic sections.

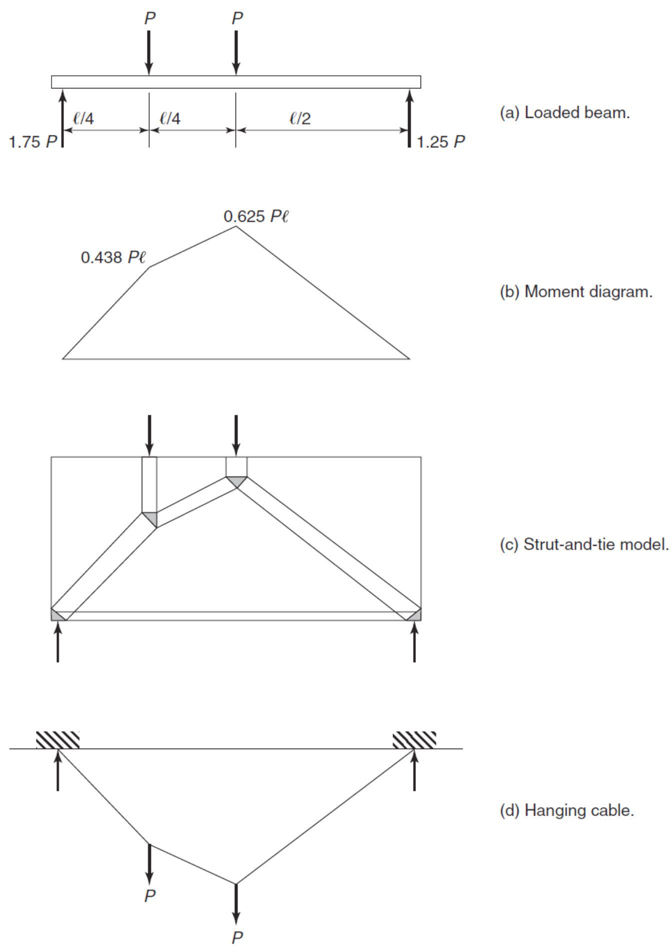


Fig. 67. Statical equivalence of various types of structures

3. The strut-and-tie model should represent a realistic flow of forces from the loads through the D-region to the reactions. Frequently this can be determined by observation. From an elastic stress analysis, such as a finite element analysis, it is possible to derive the stress trajectories in an uncracked D-region, as shown in Fig. 66a for a deep beam. Principal compression stresses act parallel to the dashed lines, which are known as compressive stress trajectories. Principal tensile stresses act parallel to the solid lines, which are called tensile stress trajectories. Such a diagram shows the flow of internal forces and is a useful, but by no means an essential step in laying out a strut-and-tie model. The compressive struts should roughly follow the direction of the compressive stress trajectories, as shown by the refined and simple strut-and-tie models in Fig. 66c and e. Generally, the strut direction should be within ±15° of the direction of the compressive stress trajectories [50].

Because a tie consists of a finite arrangement of reinforcing bars which usually are placed orthogonally in the member, there is less restriction on the conformance of ties with the tensile stress trajectories. However, they should be in the general direction of the tension stress trajectories.

4. Struts cannot cross or overlap, as shown in Fig. 68(b), because the width of the individual struts has been calculated assuming they are stressed to the maximum.

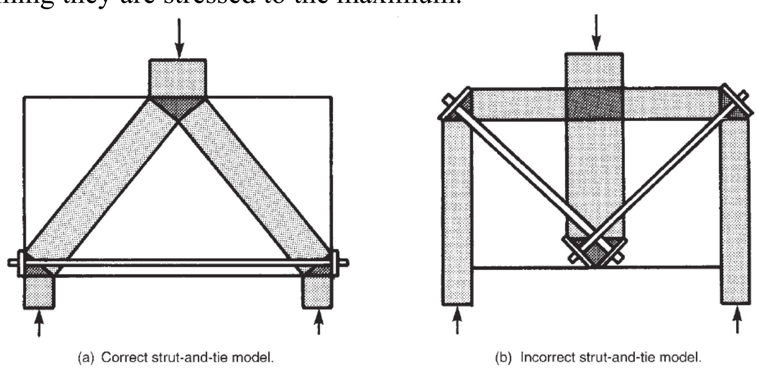


Fig. 68. Suitable and unsuitable strut-and-tie models

5. Ties can cross struts.

6. It generally is assumed that the structure will have enough plastic deformation capacity to adapt to the directions of the struts and ties chosen in design if they are within ±15° of the elastic stress trajectories. The crack-control reinforcement from ACI Code Section A.3.3 is intended to allow the load redistribution needed to accommodate this change in angles.

7.1.3 Ties

7. In addition to generally corresponding to the tensile stress trajectories, ties should be located to give a practical reinforcement layout. Wherever possible, the reinforcement should involve groups of orthogonal bars which are straight, except for hooks needed to anchor the bars.

8. If photographs of test specimens are available, the crack pattern may assist one in selecting the best strut-and-tie model.

7.1.4 Load-Spreading Regions

9. Elastic analyses of members of width b subjected to in-plane loads applied to one edge show that the load-spreading angle is primarily a function of the ratio of the width of the loading plate, a, to the width of the member, b. A strut slope of 2-to-1 (parallel to the axis of the load-to-perpendicular to the axis) is conservative for a wide range of cases. A slope of 2-to-1 will be used in all similar cases in this book.

10. Angles θ, between the struts and attached ties at a node, as shown in Fig. 66c, should be large (on the order of 45°) and never less than the 25° specified in ACI Code Section A.2.5. The size of compression struts is sensitive to the angle θ between the strut and the reinforcement in a tie. To illustrate this, consider a strut carrying a force with a vertical component of V_u = 200 kips in a deep beam made of 4000-psi concrete. The thickness of the beam is 12 in.

If θ=65°, the axial force in the strut needed to transfer this shear is 221 kips. Assuming a bottle-shaped strut, the effective strength of the concrete in the strut is fce = 0.85×0.75×4000 psi = 2550 psi, and the width of the strut must be $w_s = \frac{V_u}{\phi f_{ce} \times b} = \frac{221000}{0.75 \times 2550 \times 12} = 9.63\text{in.}$

For θ=45°, the axial force in the strut is 283 kips, and the width of the strut must be 12.3 in.

For θ=25°, the axial force in the strut is 473 kips, and the strut must be 20.6 in. wide. In some cases it will be difficult to fit a strut of this width within the space available. In the examples of deep beams θ has been limited arbitrarily to 40°, or larger, to keep the width of the strut within reasonable limits, even though this is not required by the ACI Code.

7.1.5 Minimum Steel Content

11. The loads will try to follow the path involving the least forces and deformations. Because the tensile ties are more deformable than the compression struts, the model with the least and shortest ties is the best. Thus, the strut-and-tie model in Fig. 68a is a better model than the one in Fig. 68b because Fig. 68a more closely approaches the elastic stress trajectories in Fig. 66a. Schlaich et al. [51] and [52] propose the following criterion for guidance in selecting a good model:

$$\sum F_i l_i \varepsilon_{mi} = \text{minimum}$$

where F_i, l_i and ε_{mi} are the force, length, and mean strain in strut or tie i, respectively. Because the strains in the concrete are small, the struts can be ignored in the summation.

7.1.6 Suitable Strut-and-Tie Layouts

12. The finite widths of struts and ties must be considered. The axis of a strut representing the compression zone in a deep flexural member should be located about a/2 from the compression face of the beam where a is the depth of the rectangular stress block, as shown in Fig. 53. Similarly, if hydrostatic nodal zones are used, the axis of a tension tie should be about a/2 from the tensile face of the beam. One of the first steps in modeling a beam-like member is to locate the nodes in the strut-and-tie model. This can be done by estimating values for a/2.

A possible strut-and-tie model for the beam shown in Fig. 69 consists of two trusses, one utilizing the lower steel as its tension tie, the other using the upper steel. For an ideally plastic material, the capacity would be the sum of the shears transmitted by the two trusses, V_1+V_2. Tests [59] have shown, however, that the upper layer of steel has little, if any, effect on the strength. In part this is due to the very low angle θ_2 in the upper truss. When this beam is loaded, the bottom tie yields first.

13. As the angle θ, between the struts and ties decreases, it is often desirable to include web reinforcement in addition to the confinement reinforcement required by ACI Code Section A.3.3. The following equation is used in European design standards [67] to require stirrup reinforcement in beams that approach the lower limit on the angle between struts and ties in ACI Code Section A.2.5:

$$F_{nw} \approx \frac{2a/jd - 1}{3 - N_n / F_n} F_n \tag{99}$$

Here F_{nw} is the yield force $\Sigma A_v f_y$ in the web reinforcement in the shear span; F_n is the concentrated load and also the reaction; N_n is the axial force acting on the beam, if any; a is the distance between the axes of the load and reaction; and jd is the lever arm between the resultant compression force and the resultant force in the longitudinal tie. For a beam with N_n=0 and having a/jd equal to 2, Eq. (99) requires that all the shear be carried by shear reinforcement. At a/jd=0.5, all the shear is resisted by the compression strut.

14. Subdividing a nodal zone, assuming each part of the nodal zone can be assigned to a particular force or reaction, makes the truss easier to lay out.

15. Sometimes a better representation of the real stress flow is obtained by adding two possible simple models, each of which is in equilibrium with a part of the applied load, provided the struts do not overlap or cross.

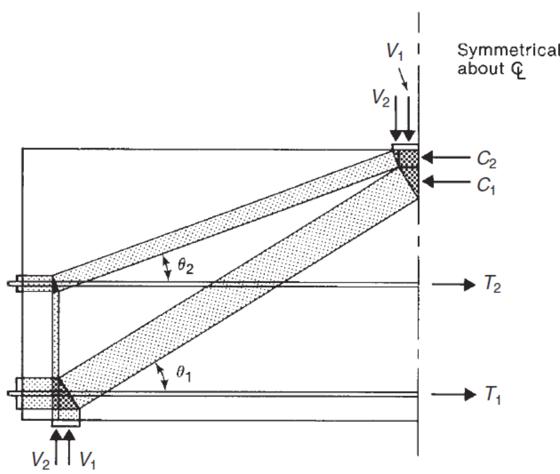


Fig. 69. Strut-and-tie model for beam with horizontal web reinforcement at mid-height

CONCLUSIONS

The term shear wall is used to describe a wall that resists lateral wind or earthquake loads acting parallel to the plane of the wall in addition to the gravity loads from the floors and roof adjacent to the wall. Such walls are referred to as structural walls in ACI Code Chapter 21. The strength and behavior of short, one- or two-story shear walls are generally dominated by shear. These walls typically have a height-to length aspect ratio of less than or equal to 2 and are called short or squat walls. Such walls can be designed by either the requirements given in ACI Code Chapter 11 or the strut-and-tie method given in ACI Code Appendix A. If the wall is more than three or four stories in height, lateral loads are resisted mainly by flexural action of the vertical cantilever wall rather than shear action. Although shear walls may be simple planar walls, several wall segments are commonly connected together to act as a three-dimensional unit. Such wall assemblies have regular or irregular C, T, L, or H-shaped cross sections with webs and flanges that may enclose spaces in buildings, such as stair wells or elevator shafts.

ACI Code Section 14.3 requires more reinforcement horizontally than vertically. This reflects the greater chance that vertical cracks in walls might form as a result of restrained horizontal shrinkage or temperature stresses, compared with a lower chance that horizontal cracks will form as a result of restrained vertical stresses. Generally, if shrinkage occurs in the vertical direction, the shrinkage stresses are dissipated by vertical compression stresses in the wall.

Three common systems for resisting wind or earthquake lateral loads are presented:

Moment resisting frames – are made up of interconnected beams and columns. Lateral loads are resisted by bending of the beams and columns. Such frames undergo relatively large lateral deflections. If all stories have beams and columns with sizes proportional to the shear in the story, the lateral deflection of each story would be similar. To simplify construction, however, the sizes of the beams and columns selected for the lower stories are commonly used throughout, or are changed only every third or fourth story. Hence, the beams and columns in a building tend to be oversized in the upper stories. Moment-resisting frames are used for buildings up to 8 to 10 stories. In a moment-resisting frame, deflections of the columns and beams both contribute to the sway deflections of the frame.

Bearing wall systems – are used for apartment buildings or hotels. A bearing-wall building has a series of parallel transverse shear walls between rooms or apartments. The walls resist lateral loads by flexural action and deflect as vertical cantilevers.

Shear wall-frame buildings – are used in buildings ranging from about 8 to about 30 stories. The lateral load is resisted in part by the wall and in part by the frame.

In the case of shear-wall-frame interaction, the division of lateral load between the wall and frame in a shear-wall–frame building can be analyzed by using frame analysis. The frame members in the model represent the sums of the stiffnesses of the columns and beams in the building in the bays parallel to the plane of the wall. Similarly, the wall in the model represents the sum of the walls in the structure. The wall and frame are connected by axially stiff link beams at every floor. In computing internal forces and moments due to the factored loads, the flexural stiffnesses, EI, from ACI Code Section 10.10.4.1 could be used. A two-dimensional model may be acceptable for buildings that are symmetrical in plan and have rigid floor diaphragms. A three-dimensional model is required for an unsymmetrical building and where a designer wants to account for diaphragm flexibility.

The lateral-force analysis of shear-wall–frame buildings must account for the different deformed shapes of the frame and the wall. Due to the incompatibility of the deflected shapes of the wall and the frame, the fractions of the total lateral load resisted by the wall and frame differ from story to story. Near the top of the building, the lateral deflection of the wall in a given story tends to be larger than that of the frame in the same story and the frame pushes back on the wall. This alters the forces acting on the frame in these stories. As a result, the frame resists a larger fraction of the lateral loads in the upper stories than it does in the lower stories. As the lateral stiffness of the frame decreases relative to the lateral stiffness of the wall, the reaction at the top of the wall decreases, approaching zero for a very flexible frame combined with a stiff wall. However, the sum of the shear forces in the frame and the wall in a given story must equal the shear due to the applied loads.

In a frame analysis to determine factored moments for design, the member stiffnesses may be based on ACI Code Section 10.10.4.1. The coupling beams are joined to hypothetical members with high values of the moment of inertia, I, between the face of the wall and the centerline of the wall. Short, deep coupling beams develop both flexural and shear deflections. Floor slabs may serve as soft coupling beams. Their stiffness can be based on a slab with a width perpendicular to the wall equal to the wall thickness plus half of the width of the opening, between the walls, added on each side of the opening. In tests of shear walls coupled by slabs, the specimens failed by punching-shear failures in the slab around the ends of the walls. Under cyclic loads, the stiffness of slabs serving as coupling beams decreased rapidly.

A common design recommendation is to minimize the separation, commonly referred to as the eccentricity, between the center of mass (geometric centroid of the floor plate) and the center of lateral resistance (CR) provided by the shear walls and moment resisting frames in the lateral-load system. Because lateral loads are assumed to act through the center of mass (CM), any eccentricity between the CM and CR will result in the generation of torsional moments. A central-core wall system is commonly used to minimize eccentricity between the CM and CR.

When a building structure is subjected to large lateral displacements due to earthquake ground motions, the stiffnesses of the lateral-load resisting members are likely to change in a nonuniform fashion. As a result, the CR is likely to be relocated and the eccentricity between the CM and CR may increase. To account for this, the International Building Code specifies a minimum eccentricity in the two principal directions that must be added to any calculated eccentricity. For structures where substantial torsional moments may be generated, a wide distribution of shear walls around the perimeter of the floor plan would be most efficient for resisting that torsion.

In choosing a structural wall section for a given building, the wall must: (a) have enough strength to resist the factored moments, shears, and axial loads acting on it; and (b) have enough stiffness to limit the lateral deflections. There is no widely accepted way of doing this. A rough estimate of the minimum wall stiffness, EI, required to limit the lateral deflections to an acceptable value can be made by considering the walls as a vertical cantilever with a constant EI over the height, loaded with a constant wind load over the height. Wall thicknesses, and thus the structure's EI would generally decrease as the moments and shear forces decrease near the top of the structure, it shall be assumed it is constant over the building height and equal to the sum of the EI values of the walls in the bottom story.

The minimum thicknesses given in ACI Code Section 14.5.3 are intended to apply only to bearing walls designed via the empirical design method from ACI Code Section 14.5.

Shear reinforcement for structural walls always consists of evenly distributed vertical and horizontal reinforcement. In many cases, shear cracks in walls are relatively shallow (i.e., their inclination with respect to a horizontal line is less than 45°), so vertical reinforcement will be just as effective—if not more effective—as horizontal reinforcement in controlling the width and growth of such cracks. However, the shear-strength contribution from wall reinforcement is based on the size and spacing of the horizontal reinforcement. In many cases, only minimum amounts of shear reinforcement are required in structural walls, and those minimum amounts are a function of the amount of shear being resisted by the structural wall.

The procedures for the analysis and design of structures to resist the effects of earthquake ground motions are in a continuous state of development. In addition to the work of the ACI Code Committee, several other regulatory bodies and research development groups constantly are evaluating and updating code-type analysis and design requirements. Therefore, significant changes to code requirements continue to appear at a rapid rate. However, the design philosophy and general design procedures for reinforced-concrete members are well established and will not change significantly over time.

The effect of the size and type of vibration waves released during a given earthquake can be organized so as to be more useful in design in terms of a response spectrum for a given earthquake or family of earthquakes.

Lateral seismic forces are closely related to the fundamental period of vibration of the building. At periods less than about 0.5 sec, the maximum effect for a structure on a firm soil site results from the magnification of the acceleration. For structures with medium periods (from about 0.5 sec to about 2.0 sec), the largest structural response appears in Figure 24b, the velocity response spectrum. Finally, at long periods (above about 2.0 sec), the dominant structural response appears in the displacement spectrum. The period of the first mode of vibration, referred to as the fundamental or natural period, can be estimated from empirical equations given in ASCE/SEI 7 or from Rayleigh's method.

Damping is a measure of the dissipation of energy in the structure and is due to cracking, sliding friction on the cracks, and slip in connections to nonstructural elements. As the damping increases, the ordinates of the response spectrum decrease. Typically, a reinforced concrete building will have 1 to 2 percent of critical damping prior to the building being exposed to an earthquake. As cracking and structural and nonstructural damage develop during the earthquake, the damping increases to about 5 percent. By definition, critical damping acts to quickly damp out structural vibrations.

Seismic design codes require that all building structures be assigned to a particular seismic design category. This assignment is made on the basis of three key parameters, i.e., the expected intensity of the seismic ground motions, the site classification, and the building importance factor. The ASCE/SEI 7 Standard gives several maps of spectral response accelerations at periods of 0.2 sec. and 1.0 sec. for all locations throughout the USA. The seismic response accelerations, along with the site classification, are used to establish the design response spectrum for a structure. Site classification is a function of the local soil properties where the structure is to be located. Classifications vary from Class A—hard rock, to Class E—soft clay and Class F—soil requiring a special site response analysis. The building importance factors are related to building occupancy categories, which are defined in. These categories range from Category I – buildings and other structures that represent a low hazard to human life in the event of failure, to Category IV – buildings and other structures designated as essential facilities. Buildings in Occupancy Category IV have the highest importance factor, and vice-versa.

The magnitude of the lateral design force for concrete structures is a function of the design response spectrum for the building site and the type of structural system used to resist those forces. As discussed previously, more ductile structural systems can be safely designed for lower seismic forces than systems with limited ductility. This is handled in ASCE/SEI 7 by defining a response modification coefficient, R, which is larger for more ductile structural systems. The general structural systems defined in include bearing wall systems, moment-resisting frame systems, and dual systems consisting of a combination of shear walls and moment-resisting frames that work together to resist lateral loads. In general, bearing wall systems are less ductile than either moment-resisting frames or dual systems. Also, depending on the level of structural detailing, which ranges from ordinary to special, a variety of different R-factors are assigned to moment-resisting frames and dual systems. Buildings assigned to high seismic design categories will require special structural detailing, as is discussed in subsequent sections.

Ideally, a building subjected to earthquakes should be symmetrical—or, at least, the distance between the center of mass (the point through which the seismic forces act on a given floor) and the center of resistance should be minimized. If there is an eccentricity the building will undergo torsional deflections. Irregular buildings should have significant torsional resistances and stiffnesses. Vertical irregularities are abrupt changes in the geometry, strength, or stiffness of a structure from floor to floor.

The ACI Code does not specify earthquake ground motions for a given site or give details about how structures should be analyzed for seismic actions. These details are provided in the general building code for the area. Currently, general building codes allow different levels of seismic analysis. The three analysis procedures permitted by ASCE/SEI 7 are the equivalent lateral force procedure, the modal spectrum analysis procedure and an inelastic response history analysis procedure.

The total lateral forces are used in a linearly elastic structural analysis of the frame. For regular structures, independent two-dimensional models may be used. For concrete buildings, cracked-section properties are assumed in the analysis. For irregular structures, three-dimensional analyses must be used. Where the diaphragms are flexible relative to the lateral-force-resisting members, that flexibility must be represented in the analysis.

In summary, two concepts are important here. First, the force developed in the structure does not have a fixed value, but instead results from the stiffness of the structure and its response to a ground vibration. Second, if a structure is detailed so that it can respond in a ductile fashion to the ground motion, the earthquake forces are reduced from the elastic values.

Seismic design provisions presented in ACI Code Chapter 21 were extended in the 2002 code to apply to cast-in-place and precast structures. ACI Code Sections 21.1.1.2 through 21.1.1.6 give the design requirements for structures assigned to seismic design categories (SDC) B through F, which were discussed in this report. ACI Code Chapter 21 refers to a moment-resisting frame designed by using Code Chapters 1 to 19 as an ordinary moment frame (OMF), and to a moment-resisting frame designed using Code Chapters 1 to 19 plus ACI Code Section 21.3, which requires special detailing, as an intermediate moment frame (IMF). A moment-resisting frame designed using Code Chapters 1 to 19 plus ACI Code Sections 21.1.3 through 21.1.7, and 21.5 through 21.8 is called a special moment frame (SMF).

Code provisions for joints in special moment-resisting frames (SMFs) are given in ACI Code Section 21.7. A more extensive discussion of design recommendations for beam–column connections is given by ACI Committee 352. ACI Code Section 21.7.2.1 requires that joint forces be calculated by taking the stress in the flexural reinforcement in the beams as 1.25fy. This is analogous to using the probable strength in the calculations of shear in columns and beams in special-moment frames. ACI Code Section 21.7.2.3 limits the diameter of the longitudinal beam reinforcement that passes through a joint to 1/20 of the width of the joint parallel to the beam bars. When hinges form in the beams, the beam reinforcement is stressed to the actual yield strength of the bar on one side of the joint and is stressed in compression on the other side. This results in very large bond stresses in the joint, possibly leading to slipping of the bar in the joint. The minimum bonded length of such a bar in a joint is thus which is considerably less than is required by the development-length equations in ACI Code Chapter 12. The minimum bonded length was selected from test results of joints tested under cyclic loads to limit, but not entirely eliminate slip of the beam bar in the joint.

ACI Committee 352 uses the same limit for the diameter of column bars passing through a beam–column joint (i.e., the column–bar diameter should not exceed 1/20 of the overall depth of the shallowest beam framing into the joint).

ACI Code Section 21.7.3.1 requires hoop reinforcement around the column reinforcement in all joints in special moment-resisting frames. In joints confined on all four sides by beams satisfying ACI Code Section 21.7.3.2, the amount of hoop reinforcement is reduced, and its spacing is less restrictive within the depth of the shallowest beam entering the joint.

ACI Code Section 21.7.4.1 gives upper limits on the shear strength of joints. As indicated in Section 17-12, these are lower than the joint shear strengths recommended in non-seismic joints. This reflects the possible damage to joints resulting from cyclic loads.

ACI Code Section 21.7.5 gives special development lengths for hooks and straight bars in joints. These are shorter than the development lengths given in ACI Code Chapter 12 because the effects of the joint confinement by hoops have already been included.

Since 2002, ACI Code Chapter 21 has included earthquake-resistant design requirements for precast frame members and precast structural walls. Requirements for special moment frames constructed with precast concrete elements are given in ACI Code Section 21.8. This section gives a designer the option of using ductile connections (Code Section 21.8.2) or strong connections (Code Section 21.8.3) between precast elements. The first option assumes that at least part of the inelastic behavior during the overall structural response to strong ground motions will occur in the connections between precast elements. The second option assumes that the connections will remain elastic and that plastic hinging zones will occur at other locations in the precast frame elements. Based on the probable member strengths at the plastic hinge locations, a capacity-based design procedure can be used to determine the required strength for the connections between the precast elements. Special precast structural walls are required to satisfy all of the design provisions for special cast-in-place structural walls given in ACI Code Section 21.9 and discussed previously. Intermediate precast structural walls also are permitted and must satisfy the design requirements in ACI Code Section 21.4.

Structural members may be divided into portions called B-regions, in which beam theory applies, including linear strains and so on, and other portions called discontinuity regions, or D-regions, adjacent to discontinuities or disturbances, where beam theory does not apply. D-regions can be geometric discontinuities, adjacent to holes, abrupt changes in cross section, or direction, or statical discontinuities, which are regions near concentrated loads and reactions. Corbels, dapped ends, and joints are affected by both statical and geometric discontinuities. Up to this point, most of this book has dealt with B-regions. For many years, D-region design has been by "good practice," by rule of thumb or empirical.

St. Venant's principle suggests that the localized effect of a disturbance dies out by about one member-depth from the point of the disturbance. On this basis, D-regions are assumed to extend one member-depth each way from the discontinuity. This principle is conceptual and not precise. However, it serves as a quantitative guide in selecting the dimensions of D-regions. Occasionally, D-regions are assumed to fill the overlapping region common to two members meeting at a joint. This definition is used in the traditional definition of a joint region.

Prior to any cracking, an elastic stress field exists, which can be quantified with an elastic analysis, such as a finite-element analysis. Cracking disrupts this stress field, causing a major reorientation of the internal forces. After cracking, the internal forces can be modeled via a strut-and-tie model consisting of concrete compression struts, steel tension ties, and joints referred to as nodal zones. If the compression struts are narrower at their ends than they are at midsection, the struts may, in turn, crack longitudinally. For struts without reinforcement crossing their longitudinal axis, this may lead to failure. On the other hand, struts with transverse reinforcement to restrain the cracking can carry additional load and may fail by crushing. Failure may also occur by yielding of the tension ties, failure of the bar anchorage, or failure of the nodal zones. As always, failure initiated by yield of the steel tension ties tends to be more ductile and is desirable. Strut-and-Tie Models

A strut-and-tie model for a deep beam consists of two concrete compressive struts, longitudinal reinforcement serving as a tension tie, and joints referred to as nodes. The concrete around a node is called a nodal zone. The nodal zones transfer the forces from the inclined struts to other struts, to ties and to the reactions. ACI Code Section 11.1.1 allows D-regions to be designed using strut-and-tie models according to the requirements in ACI Appendix A, Strut-and-Tie Models. This Appendix was new in the 2002 ACI Code. The combination of factored loads acting on the structure and the distribution of factored internal forces is a lower bound on the strength of the structure, provided that no element is loaded or deformed beyond its capacity. Strut-and-tie models should be chosen so that the internal forces in the struts, ties, and nodal zones are somewhere between the elastic distribution and a fully plastic set of internal forces.

In a strut-and-tie model, the struts represent concrete compressive stress fields with the compression stresses acting parallel to the strut. Although they are frequently idealized as prismatic or uniformly tapering members, struts generally vary in cross section along their length. This is because the concrete stress fields are wider at mid-length of the strut than at the ends. Struts that change in width along the length of the member are sometimes called bottle-shaped due to their shape or are idealized using local strut-and-tie models. The spreading of the compression forces gives rise to transverse tensions in the strut that may cause it to crack longitudinally. A strut without transverse reinforcement may fail after this cracking occurs. If adequate transverse reinforcement is provided, the strength of the strut will be governed by crushing.

The second major component of a strut-and-tie model is the tie. A tie represents one or several layers of reinforcement in the same direction. ACI Code Section A.4.2 requires that the axis of the reinforcement in a tie coincide with the axis of the tie. In the layout of a strut-and-tie model, ties consist of the reinforcement plus a prism of concrete concentric with the longitudinal reinforcement making up the tie. The width of the concrete prism surrounding the tie is referred to as the effective width of the tie. ACI Commentary Section R.A.4.2 gives limits for the effective width. The lower limit is a width equal to twice the distance from the surface of the concrete to the centroid of the tie reinforcement. In a hydrostatic C–C–T nodal zone (defined later in the report), the stresses on all faces of the nodal zone should be equal.

The concrete is included in the tie to establish the widths of the faces of the nodal zones acted on by ties. The concrete in a tie does not resist any load. It aids in the transfer of loads from struts to ties or to bearing areas through bond with the reinforcement. The concrete surrounding the tie steel increases the axial stiffness of the tie by tension stiffening. Tension stiffening may be used in modeling the axial stiffness of the ties in a serviceability analysis.

Ties may fail due to lack of end anchorage. The anchorage of the ties in the nodal zones is a critical part of the design of a D-region using a strut-and-tie model. Ties are normally shown as solid lines in strut-and-tie models.

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