National Technical University of Athens

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Comparison between Theoretical and Statistical models for the estimation of Fuel Oil Consumption using data

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Diploma Thesis

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Abstract

The estimation of the main's engine fuel oil consumption (FOC) of a ship in random operational conditions as expressed by her loading condition, speed and weather conditions is an issue that constantly concerns ship's operations while it could be an essential element of a framework for the monitoring of ship performance.

In this thesis, theoretical/semi empirical and data driven models are employed for the estimation of FOC aiming at the examination of their prediction capabilities utilising operational data of a specific ship. The first model used is the modified Kwon method (Kwon, 2008) which is an approximate method that predicts the speed loss due to the added resistance in irregular waves and wind. The second theoretical model is the power correction procedure provided by the ITTC (2017), which uses a power correction formula to express the added resistance due to waves and wind. Moreover, a statistical approach is implemented based on an integrated dataset and after applying extended pre-processing analysis. The statistical model is a multiple linear regression model entailing operational variables, such as wind velocity and direction, speed through water, draft and trim, in order to calculate the fuel oil consumption for different loading and weather conditions. Our analysis concerns the operation of a containership for 18 months. The comparison study focuses on the direct examination of the FOC within a typical range of ship speeds as well as through selected key performance indicators. To sum up, the two theoretical models estimate the FOC with an average margin of 10% from the actual FOC, while the respective deviations of the statistical model are much lower.

Περίληψη

Η εκτίμηση της κατανάλωσης καυσίμου της κύριας μηχανής (FOC) ενός πλοίου σε τυχαίες συνθήκες λειτουργίας, όπως εκφράζεται από την κατάσταση φόρτωσης, την ταχύτητα και τις καιρικές συνθήκες είναι ένα ζήτημα που απασχολεί μόνο τον τρόπο λειτουργίας του πλοίου, ενώ θα μπορούσε να είναι ουσιαστικό στοιχείο για την απόδοση του πλοίου.

Σε αυτή την εργασία χρησιμοποιούνται θεωρητικά / ημι-εμπειρικά μοντέλα για την εκτίμηση της κατανάλωσης καυσίμου με σκοπό την εξέταση των δυνατοτήτων πρόβλεψής των μεθόδων, χρησιμοποιώντας τα δεδομένα ενός πραγματικού πλοίου. Το πρώτο μοντέλο που χρησιμοποιείται είναι η τροποποιημένη μέθοδος του Kwon (Kwon, 2008), η οποία είναι μια προσεγγιστική μέθοδος που προβλέπει την απώλεια ταχύτητας λόγω της προστιθέμενης αντίστασης σε τυχαίους κυματισμούς και τυχαίο άνεμο. Το δεύτερο θεωρητικό μοντέλο είναι η διαδικασία διόρθωσης ισχύος που προτείνεται από την ITTC (2017), η οποία χρησιμοποιεί την διόρθωση ισχύος για να εκφράσει την πρόσθετη αντίσταση λόγω κυματισμού και ανέμου. Επιπλέον, μια στατιστική προσέγγιση εφαρμόζεται σε ένα σύνολο δεδομένων, μετά την εφαρμογή φιλτραρίσματος, με σκοπό την αντιμετώπιση των ανεπιθύμητων δεδομένων. Το στατιστικό μοντέλο είναι ένα μοντέλο πολλαπλής γραμμικής παλινδρόμησης που περιλαμβάνει μεταβλητές, όπως η ταχύτητα και η κατεύθυνση του ανέμου, ταχύτητα του πλοίου, το βύθισμα και η διαγωγή, προκειμένου να υπολογιστεί η κατανάλωση καυσίμου για διαφορετικές συνθήκες φόρτωσης και καιρού. Η ανάλυσή αφορά τη λειτουργία ενός εμπορευματοκιβωτίου για 18 μήνες. Η μελέτη επικεντρώνεται στην εκτίμηση της κατανάλωση καυσίμου εντός ενός τυπικού εύρους ταχύτητας καθώς και με την χρήση δεικτών απόδοσης (KPI). Συνοψίζοντας, τα δύο θεωρητικά μοντέλα εκτιμούν την κατανάλωση καυσίμου με απόκλιση 10% από την πραγματική κατανάλωση, ενώ οι αντίστοιχες αποκλίσεις του στατιστικού μοντέλου είναι πολύ χαμηλότερες.

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1 Introduction

The continuous growth of the world population, as well as the depletion of local resources increase the dependency of the economy to international trade. Ship transport accounts for most of the world trade (90 %). Energy efficient shipping is required in order to reduce the Green House Gas (GHG) emissions.

In order to control and eventually reduce the CO_2 emissions a set of regulations have been implemented by the International Maritime Organisation (IMO, 2011). The most recent regulations are the implementation of the Energy Efficiency Existing Ship Index (EEXI) (MEPC 76), which essentially is the calculation of the Energy Efficiency Design Index (EEDI) for existing ships, as well as the adoption of the Carbon Intensity Indicator (CII), which is used in order to achieve a reduction in carbon intensity of 40% by 2030 compared to the 2008 level. The target is to reduce GHG emissions by improving vessels' energy efficiency as well as introducing new technologies and low or zero-carbon fuels. Moreover, unpredictability of the rising fuel prices increases the need for energy efficient shipping.

Energy efficient shipping can be achieved by the means of energy saving devices, such as Propeller ducts (Mewis Duct, Schneekluth Duct). Another proposal, in order to reduce the GHG, is the creation of an optimum route models. However, the optimum route is going to be evaluated under the spectrum of the decrease of fuel oil consumption. Voyage optimization has multiple objectives:

- Minimizing Costs
- Safety
- Passenger Comfort
- On Time Arrival

In order to improve one objective another may reduce its efficiency.

Voyage optimization is usually extracted and calculated using data from similar ships. However the performance of each ship in various voyage conditions (speed, fouling, loading condition, weather conditions) is different. The need of data – driven methods and ship specific modelling, in order to provide increased accuracy of the ships operational performance is essential.

The present study provides results for the estimation of the fuel oil consumption, of the main engine, under different conditions and takes into consideration the state of the sea. The first theoretical model, that is used, is the modified Kwon's method (2008), which is proposed by Lu (2015).

The Kwon method is an empirical method for the prediction of added resistance due to weather conditions, sea state and wave directions. After using the aforementioned formula the fuel oil consumption of a ship can be estimated for different operational profiles (waves, weather conditions, different speeds, drafts, and wave encounter angle). This can be used, in order to optimize a voyage, provided that the weather forecast is accurate enough.

Furthermore, a different theoretical method is introduced. The ITTC method (2017) uses power correction, due to added resistance, as a tool to estimate the actual fuel oil consumption of the ship, in different conditions. The ITTC method, uses the Fujiwara regression formula (2005) for the added resistance due to wind and the Stawave – 1 (Boom, 2013) for the added resistance due to waves.

Moreover a statistical model is fitted in the pre – existing dataset of the ship, in order to calculate the fuel oil consumption in different weather and loading conditions. The main purpose of the study is the calculation and comparison of the fuel oil consumption, while using methods that are not correlated with each other.

The implementation of a new set of rules and guidelines, in order to reduce the GHG emissions emphasizes the need to use theoretical and statistical models, in order to estimate the fuel oil consumption and optimize the operations of a vessel, by understanding the correlations between the fuel consumption in different weather and loading conditions.

1.1 Ship Energy & Ship Resistance

The calculation of the fuel oil consumption needs to be analysed, by firstly, it's necessary to describe the main flow of energy in ships. The estimation of the fuel oil consumption is heavily influenced by the total resistance of the ship (Still Water Resistance – Added Resistance) and it's a necessity to describe the basic terms and formulas. The ships energy is divided into two categories the energy that is provided to the ship and the energy that is used by the ship.

The provided energy is in the form of fuels (HFO, LSDO, VLSFO, and LNG) and is the only energy source the ship uses, with the exception of small ships that use solar power or use electric power (Nordic Sea). The main engine and the diesel generators are responsible for powering the ship. The energy loss of a diesel main engine is remarkable and can be up to 40%. The main reason for the aforementioned energy loss is due to heat loss, created by friction, radiation and cooling.

The remaining mechanical energy that is created due to the rotation of the ship's shaft, suffers additional heat loss, due to friction between the bearings, the shaft itself and the lubricants. The mechanical energy from the shaft is used by the propeller, in order to move the blades of the propeller and create the required thrust and speed through water. Energy losses appears, during the transformation of shaft's torque to thrust power. During the procedure of thrust generation energy losses occur, due to pressure difference in the two sides of the propeller disc.

Ideally, all the power transmitted by the shaft needs to be converted into thrust power, which accelerates the ship. However, this particular concept cannot be achieved, because of the relative efficiency due to the interaction of the hull type and the propeller. The hull, in front of the propeller, distorts the potential flow of the water (wake friction) and the rotation of the propeller increases the drag of the ship, by reducing the pressure in the stern area (thrust deduction). The two aforementioned phenomena affect the total efficiency of the propeller and more importantly deduct 30% of the produced energy.

In addition the ship efficiency is affected by the increase of the hull resistance, due to the added resistance of the appendages. Appendages may be added to the bare hull for manoeuvring, structural or stability reasons. The most commonly used equipment is the rudder. The aforementioned energy description of the ship, is used in order to understand that the brake power provided from the main engine, due to the consumption of fuel, isn't the same as the propeller shaft power that facilitates the movement of the ship.

A basic ship design procedure is the evaluation of the still water resistance and the added resistance due to wind and irregular waves, in order to derive the required power of the ship. Since, the consumption of fuel is the main powering form, it's necessary to correct the heating value of the fuel, by ISO (15 °C), and calculate the correct amount of fuel that is provided to the main engine. The energy efficiency of the ship can be reproduced in the following flow diagram (Figure 1).

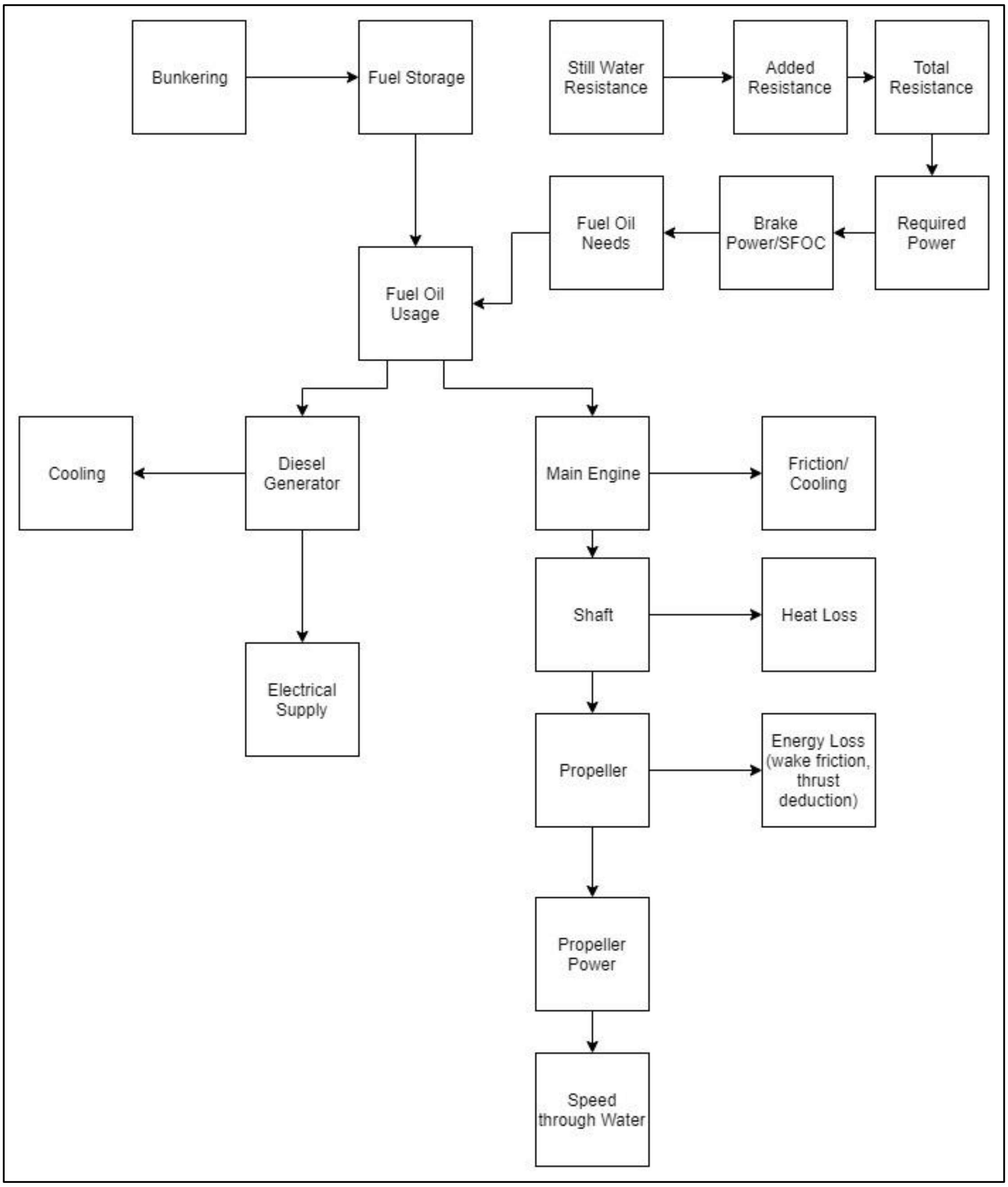


Figure 1. Energy Flow & Fuel Requirements

After analysing the procedure of the energy flow from the main engine to the propeller, that determines the speed through water, it is important to understand that the required power of the ship is highly correlated to the ship's total resistance. The total resistance is calculated by the use of the still water resistance and the added resistance due to waves. The still water resistance can be divided in six categories, as presented by Holtrop & Mennen (1982):

- ❖ Frictional Resistance
- ❖ Resistance of Appendages
- ❖ Wave – making and Wave – breaking resistance
- ❖ Additional pressure resistance due to immersed transom stern

- ❖ Additional pressure resistance of bulbous bow near the water surface
- ❖ Model – Ship correlation resistance

The added wave resistance can be calculated by (P.A. Wilson, 1985):

- ❖ Hull pressure methods, hydrodynamic solutions via the velocity of potential techniques of disturbing sources.
- ❖ Momentum and Energy Methods, the foundation of these approaches is to consider a control volume around the ship and then to derive either an energy or momentum balance.
- ❖ Radiated Energy Methods, the method equates added resistance and the work that is thus done to overcome it, to that of the energy contained in the radiated damping waves.
- ❖ Semi – Empirical Methods

The ship efficiency is optimized for specific conditions (design conditions, ballast conditions, scantling condition). The conditions are affected by the ships speed, the draft and the trim of the ship, the combination of the latter defines the loading condition of the ship. The aforementioned variables affect the total resistance of the ship and are the most crucial parameters in the calculation of baseline measurements for calm water.

The correlation between the total resistance, the required shaft power and the engine brake power is essential for the calculation and estimation of the actual fuel oil consumption.

1.2 Literature Review

The prediction model for voyage optimization is analysed by Lu (2015) and practically is correlated to the fuel oil consumption for different loading conditions and weather conditions. The aforementioned model is based upon the modified Kwon method (2008) for the calculation of the added resistance due to waves and transform it to absolute speed loss. Furthermore, the Holtrop & Mennen method (1982) is used for the calculation of calm water resistance and finally with the use of the key performance indicator ‘’Energy Efficiency of Operation’’ (EEO) the fuel oil consumption, the ship’s operational performance are correlated and results are provided for two oil tankers (Suezmax, Aframax) for wind speeds of 3 and 4 BN.

A different approach is provided by Kim & Roh (2020), in which the fuel oil consumption of the vessel is calculated using the ISO 15016:2015 method (ISO, 2015). In this particular study the guidelines of ISO 15016 are followed strictly. The purpose of the study is to improve the methodology of ISO 15016:2002 and compare the results of the estimated fuel consumption with real data from a grey box model based on operating data.

Another approach to the estimation of fuel oil consumption is the use of a statistical model. The aforementioned method is provided by Bialystocki & Konovessis (2016). This study suggests an operational approach for the estimation of fuel consumption and speed curves, by taking into account the most significant operational variables such as draft and displacement, weather speed and weather direction, hull and propeller roughness and wave encounter angle. A statistical analysis of noon reports is carried out and the influence of the above factors is calculated. The conclusion is that, stronger wind and head weather increases the fuel consumption, and a vast majority of estimated consumption could be quantified for several different weather conditions. The statistical model that depicts from the following study is a simple regression, which can be used easily by ship operators.

Moreover, although not related to the estimation of the estimation of the fuel oil consumption the study of Vitali (2020) provides a methodology for the coupling of water and voyage data. The speed loss is investigated for container ships with different main dimensions and the results are compared with existing methods for speed loss.

An approach for the propulsion coefficient is provided by the study of Kristensen (2015). The propulsion coefficient is provided with the use empirical and numerical methods as well as the use of figures provided by Andersen & Berslin (1994) and the methodology of Harvald (1983). This study estimates the key parameters of the propulsion coefficient and the total resistance with the use of numerical formulas.

The theory for the creation and the evaluation of the encounter angle of waves and wind is described in the paper of Varela & Soares (2011), in order to represent the ship motions at interactive frame rates. Although the goal of the paper is different from the estimation of the fuel oil consumption the explanation and the formula for the calculation of the encounter angle is vital for the implementation of the Kwon method (2008).

The guidelines and recommendations provided by the ITTC (2017) are implemented in order to correct the power using three added resistance variables. To be more specific the guidelines provide the exact procedure in order to correct the relative wind speed, wind direction at the anemometer height and to calculate the added resistance due to waves (Stawave – 1, Boom 2013), added resistance due to wind (Fujiwara, 2005) and the deviation of the temperature of the sea. The aforementioned added resistances are converted in added shaft power, hence a corrected power can be calculated.

1.3 Purpose and Structure of the Study

The development of the data collection systems, with the use of electronics, sensors and satellites can support the development of performance monitoring and data analytics using big data. The data, alone, provide valuable information about the ship condition and can provide trend lines and baselines for the need of either hull or engine maintenance. The combination of big data and the use of the statistical models helps evaluate the trustworthiness of the theoretical models and produce a highly accurate model, for a specific ship. With the existence of an accurate model, for the estimation of the fuel consumption, a lot of operational measures can be thoroughly and effectively investigated and provide feedback for the percentage of effective fuel consumption reduction, after the installation of an energy saving device or the use of an optimum route (weather routing).

The purpose of the current study is to use big data, corresponding to a sampling rate period of 18 months, for a container ship, in order to estimate the consumption of the ship in irregular waves (random speed, encounter angle) with the use of two theoretical models and one statistical model (multiple linear regression). The use and creation of KPI’s is essential, as well as the representation of the fuel consumption and its relationship with the speed through water, in different weather conditions. This particular application can be the first step for the creation of a model that is used for route optimization, speed optimization, weather routing, and performance monitoring.

An overview of the proposed procedure of the estimation of the fuel oil consumption and the comparison of the results is presented in Figure 2. Firstly, operational data are collected, by a variety of sensors and measuring devices. The data pre – processing procedure is the use of a set of filters that are meaningful (GPS speed < 0) for the function of the ship, described in the

Chapter 2 and are necessary, in order to exclude unusual and random data. Furthermore the most significant part of Chapter 2 is the exclusion of data points that don't comply to the aforementioned filters and the statistical outlying detection procedure. The excluded data points distort the relationships between the physical quantities, hence the reasoning of the exclusion and the creation of the final dataset, which will be used throughout the whole procedures, is finalized. Once the operational data are filtered, the two theoretical models are used in order to find a corrected propeller shaft power, which is the cornerstone for the estimation of the actual fuel oil consumption of the ship. The study of the Kwon method (2008) and the ITTC method (2017), conducted in Chapter 3, is a basic part of this study as it reveals the results of the two models and the actual fuel consumption is estimated, for different weather conditions. A regression model is produced in Chapter 4 to fit the available operational data and to provide predictions about the fuel oil consumption (FOC) for different operational scenarios. The regression analysis is a different approach for evaluating the actual fuel oil consumption of the vessel. Finally, in Chapter 5, a comparison analysis of the estimation of the fuel oil consumptions from the three methods is presented.

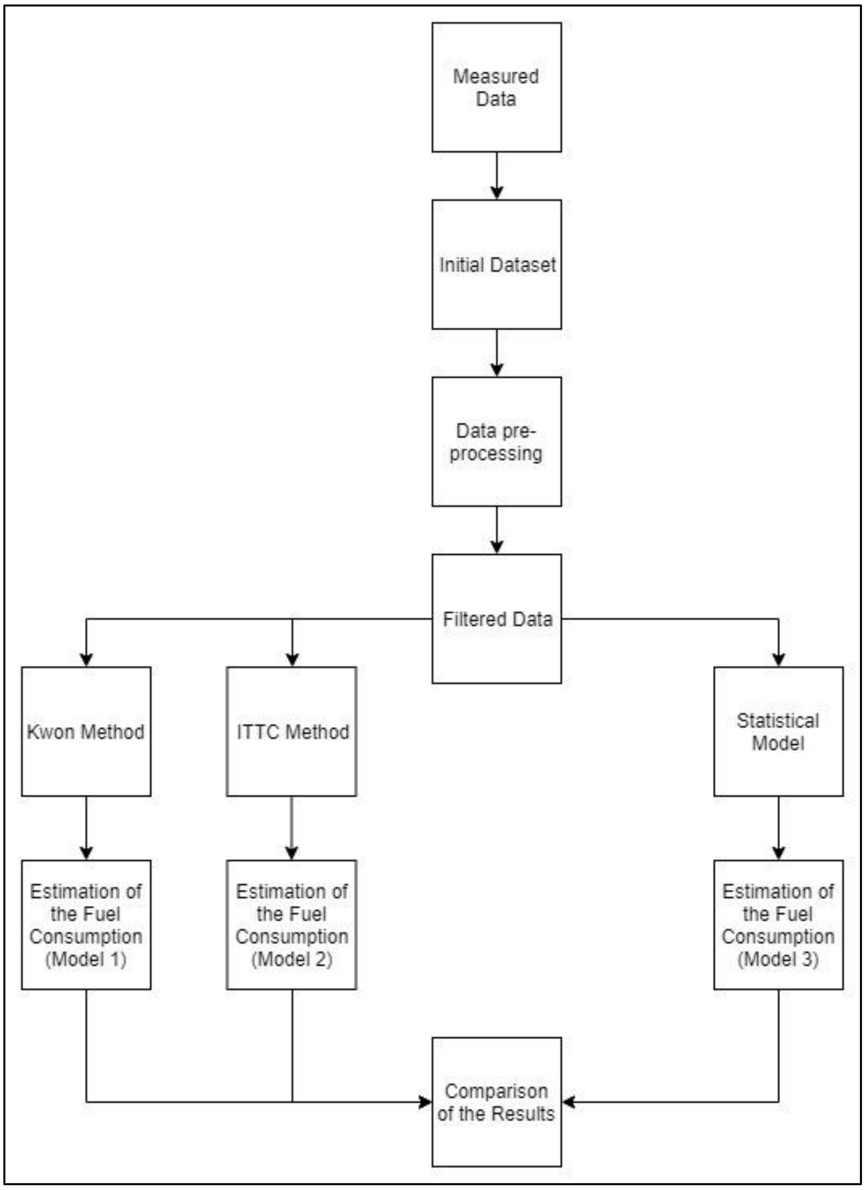


Figure 2. Structure of the thesis

2. Data Analysis

This thesis is strongly connected to the acquisition of large data from the operation of a ship. The aforementioned ship is a Containership and the main particulars are presented in Table 1.

Table 1. Main particulars of the Containership

Main Particulars	
Ship type	Container Ship (2550 TEU)
Length between Perpendiculars (L_{BP})	199 m
Depth (D_m)	16.7 m
Breadth (B)	30.2 m
Draft (T)	11.5 m
Ballast Draft ($T_{ballast}$)	6.07 m
Engine MCR	21560 kW
Engine rpm (MCR)	99 rpm
Engine type	Hyundai – Wärtsilä 7RTA72U-B
Propeller diameter	7.2 m
Blades	5

The sensors equipped to the ship monitor the propulsion parameters, the loading condition, the relative wind speed as well as the relative wind direction. The parameters are thoroughly presented in Table 2.

The parameters have a sampling period of 1 minute and are given a corresponding timestamp following the format: *MM/DD/YYYY HH: MM: SS.* The collected data correspond to a period of almost 1.5 years from December 2016 till May 2018. The data are provided in csv files and the total amount of data points, is equal to 531560. Because of the sampling frequency being one minute, one data point is equal to one minute in real time and by simply calculating the amount of minutes that consist of 17.5 months (the data begin from 16/12/2016 and end at 30/05/2018) the actual data points are 764637. Hence it's clear that with a simple division the recordings cover 70% of the total time. It's considered that the data correspond to sailing condition and the omitted data are the data that correspond to port related activities.

The data are in need of a filtering procedure, which is separated in two parts. In the first part a series of threshold values is implemented to the data points. All measurements below the thresholds are discarded. The filter aims to discard values with no physical meaning, such as negative ship speed values or zero Propeller shaft Power and Propeller Shaft rpm. Furthermore, low ship speed and low engine power are considered unfit to participate in the modelling processes, because they tend to happen in port operations (loading & unloading) and while the ship is approaching a harbour in which case they have to slow down and follow particular speed regulations that don't provide the desired results (manoeuvring).

The second part of the filtering procedure aims to further reduce the amount of data by identifying and excluding outlying data points. The procedure depends on the co-dependency of the variables. The highly correlated parameters are paired with each other and filtered through a statistical process that omits values of one parameter, based on the outliers of the other parameter and vice versa.

2.1 Data acquisition system

The recorded parameters that are used in the dataset are displayed in the following Table.

Table 2. Dataset Parameters

Parameters	*Units*
Time	*MM/DD/YYYY HH: MM: SS*
Speed through Water (STW)	Knots (kn)
Speed over Ground (SOG)	Knots (kn)
Mean Draft	Meters (m)
Trim	Meters (m)
Rudder Angle	Degrees (deg.)
Propeller shaft power	Kilo Watts (kW)
Propeller shaft torque	Kilo Newton · Meters (kN · m)
Propeller shaft rpm	Revolutions per minute (rpm)
Main Engine start air pressure	Bar (bar)
Main Engine Fuel Oil Consumption (FOC)	Tonnes per 24 hours (tn/24h)
Wind Speed	Meters per seconds (m/s)
Wind Direction	Degrees (deg.)
Wave Height	Meters (m)
Heading	Degrees (deg.)
Longitude	Degrees (deg.)
Latitude	Degrees (deg.)

The various sensors used to record the parameters of Table 2 are grouped in Table 3 and analysed.

Table 3. Measuring devices/sensors

Device	*Parameters*
Global Positioning System (GPS)	Speed over Ground, Longitude, Latitude
Speed Log	Speed through Water
Pressure Sensor	Draft, Trim
Shaft torque meter	Propeller shaft torque, Propeller shaft rpm, Propeller shaft Power
Mass flow meter	Main Engine FOC
Anemometer	Wind Speed, Wind Direction
Rudder angle indicator	Rudder Angle
Gyrocompass	Ship Heading

Global Positioning System (GPS)

The GPS retrieves information about the ship's position in global coordinates (longitude, latitude). The ship speed over ground (SOG) is obtained from the arithmetical derivation of the ships position. The GPS operation requires constant communication with a system of satellites, for the location of ships position, and has an accuracy of a few meters.

Speed Logs

Two sensors are used for the measurement of the ship's speed through water (STW):

I. Doppler Log: An acoustic speed log based on the Doppler Effect in which the wave lengths of moving objects appear to shift in relation to the observer. This shift can be converted to speed, thereby producing a very accurate result. The Dual Axis Doppler Speed Log utilizes the Doppler shifted returns from high frequency acoustic energy transmitted into water to provide precise speed data, distance travelled and water depth below the transducer. The transmitted signal is scattered back from the sea bottom and/or scatters in the water mass. The system amplifies the received signals and processes them to determine the Doppler shift.

II. Electromagnetic log: The electromagnetic log works by generating a small alternating current in a transducer producing an electromagnetic field in the adjacent water. As the vessel moves through the water, the voltage proportional to the speed is generated at 90 degrees to the direction of travel. This signal voltage is detected by the probes and transmitted to the master electronic unit where it is amplified and processed digitally before being passed to the speed and distance displays.

Pressure Sensor

The draft of the ship can be estimated by the hydrostatic pressure on the hulls bottom surface. Sensors that measure the pressure are placed on the outer surface of hull's bottom and can deduce the instantaneous draft of the hull at the position that they are installed. From the measurement of the draft on two different longitudinal positions of the hull, the ship's trim can be calculated.

Shaft torque meter

The shaft torque meter is a piece of equipment the measures the torque and the rotational speed of the shaft, and multiplies them to estimate the transmitted power's value. The instrument consists of strain gauges, arranged on a ring and mounted directly on the shaft for the continuous monitoring and logging the aforementioned values. The basic principal of operation is that any deformations of the strain gauges are transferred into voltages deviation which determine the strain of the shaft.

Mass flow meter

Mass flow meters, also known as Coriolis mass flow meters are the most reliable devices to measure the fuel oil consumption of a ship, because they eliminate the conversion of volumetric flow to mass flow, according to the fuel's density estimations. The reason is that the Coriolis acceleration induces oscillations to the tubes of the device that depend on the mass flow in them. As a result, the magnitude and the frequency of these oscillations help determine the fuel mass flow through the tubes.

Anemometer

The wind anemometer is a device that provides both, the relative speed and direction of the wind with respect to the ship's orientation. The devices that are used by the anemometer, are a helicoid propeller and a vane that measure the wind's speed and direction, respectively. The angular displacement of the vane helps estimate the wind's relative direction, while the rotational speed of the helicoid propeller helps estimate the wind speed.

Rudder angle indicator

The Rudder angle indicator is an electrical device that measures the actual angle of the rudder. It consists of two parts, the transmitter which is mounted on the steering system of the ship (steering gear room) and the receiver that is placed in the wheelhouse and displays the transmitter's signal. The measuring accuracy is usually below the range of ± 0.5° common angles and ± 1.5° hard over rudder.

Gyrocompass

The gyrocompass is a form of gyroscope (non-magnetic compass) that is used in ships for monitoring their heading orientation. It is based on a fast-spinning disc and the rotation of the Earth, to find geographical direction automatically. It has the ability to point always the true north, and so the ship's heading is accurately estimated with respect to this direction.

2.2 Initial Data

In this particular sub-section the dataset is unfiltered and a variety of figures is presented to pinpoint the problematic areas (Outliers) and the frequency with which the value of the significant variables appears (Histograms). The dataset is considered to be separated into two loading conditions:

- Loading Condition with Mean Draft: 9.3 m (6.65 m – 9.875 m)
- Loading Condition with Mean Draft: 10.5 m (9.875 m – 11.65 m)

The separation of the dataset is a necessity, because in order to form the Speed through Water – Propeller shaft Power figure or Speed through Water – Fuel Oil Consumption figure, we have to normalize the dataset. The normalization formula derives from the formula of the coefficient of admiralty, provided that the brake power is proportional to the 3rd degree of power of the velocity and is shown below:

$$V_{norm} = V \cdot \left(\frac{\Delta^{\frac{2}{3}}}{\Delta_{ref}^{\frac{2}{3}}} \right)^{\frac{1}{3}} \quad (2.1)$$

For each loading condition it's considered that the reference displacement is the one of the reference draft. The criteria in which we use the normalization formula are the following:

- $| 100 \cdot \frac{\Delta_{ref} - \Delta_{act}}{\Delta_{ref}} | > 5$
- $| trim_{ref} - trim_{act} | > 0.2 \cdot L_{BP}/100$

If a data point meets one of the two criteria then it's normalized, in order to have a reference loading condition. After the normalization of the data and the separation of the dataset, the figures of the most significant parameters are presented:

- Speed over Ground – Speed through Water

- ❖ Speed through Water (normalized) – Propeller shaft power
- ❖ Propeller shaft rpm – Propeller shaft Power
- ❖ Propeller shaft rpm – Fuel Oil Consumption
- ❖ Propeller shaft Power – Fuel Oil Consumption

The figures are separated in two categories, in histograms presenting the frequency of the data (Figure 3, Figure 5) and in Scatter plots (Figure 4, Figure 6).

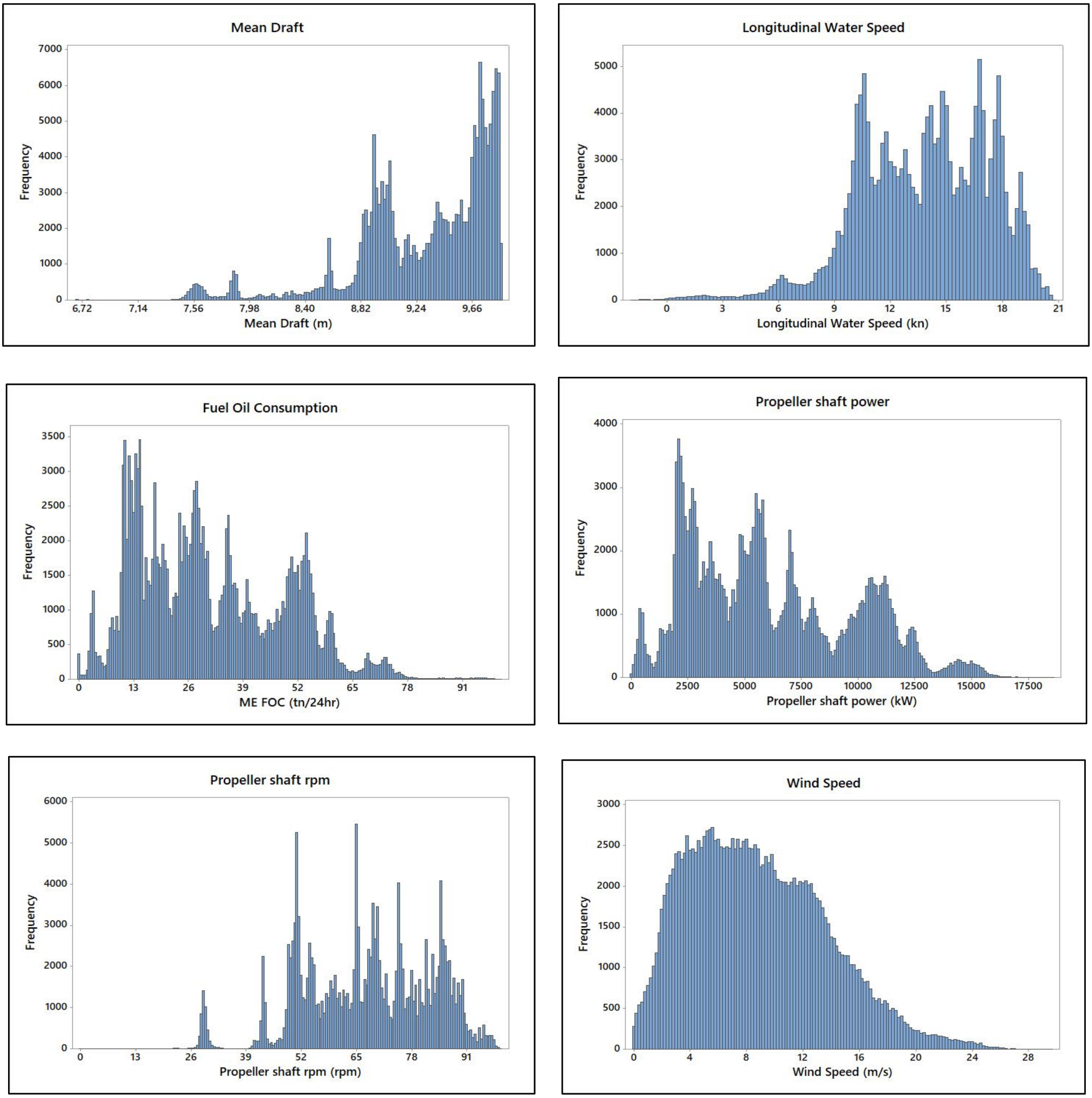


Figure 3. Histograms of the initial parameters for mean draft = 9.3 m

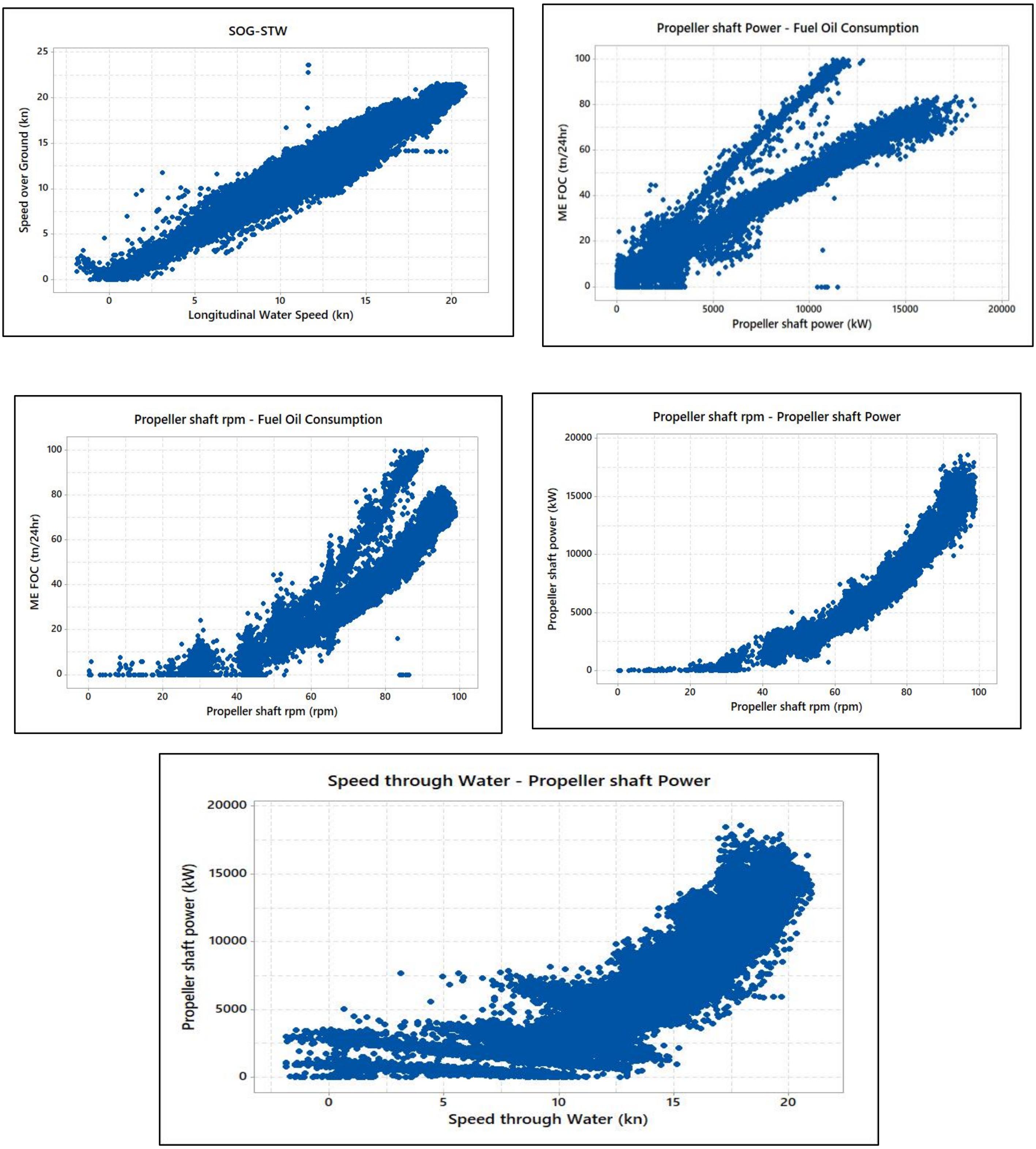


Figure 4. Scatter plot of the initial data for mean draft = 9.3 m

In addition the graphs for Mean Draft = 10.5 m are also illustrated in the following pages.

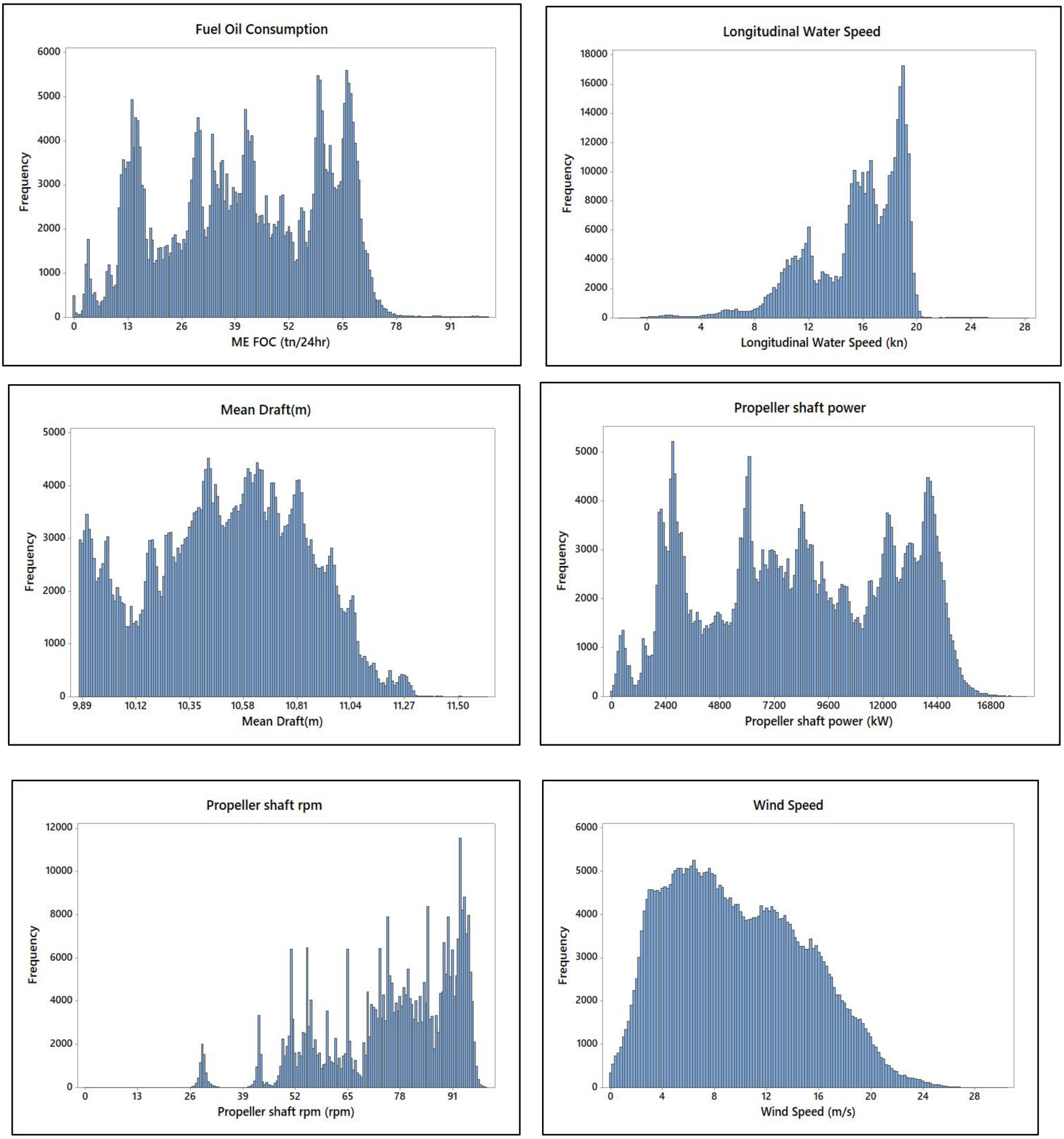


Figure 5. Histograms of the initial parameters for mean draft = 10.5 m

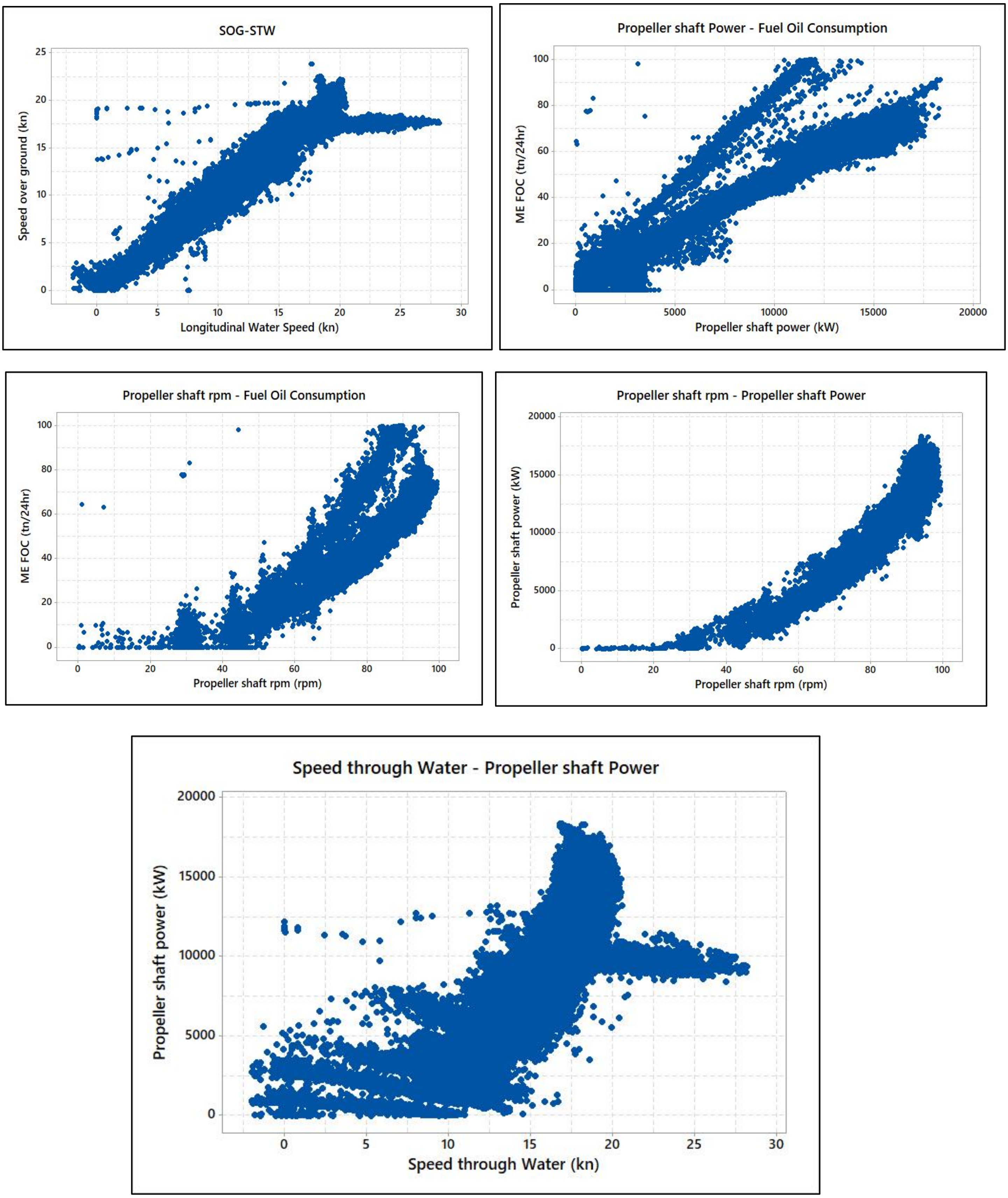


Figure 6. Scatter plot of the initial data for mean draft = 10.5 m

As it can be observed above, the SOG – STW graph depicts the linear relationship between the two speed variables. However there are points that are considered as outliers and need to be excluded from the dataset. The aforementioned points usually tend to be part of horizontal or vertical regions, which depict from the main scatter plot that respects the linear relationship of the two variables (SOG ≈ STW).

- SOG = 0 & STW > 0
- STW < 0
- SOG < 0
- SOG > 20 & STW < 15
- STW > 15 & SOG < 15
- STW > 20 & SOG < 18
- SOG = 18 & STW < 15

The Speed through Water – Propeller shaft Power curve appears to have a lot of outlying regions, especially for low speeds, such as STW < 5 knots and for Propeller shaft Power in the 6000 – 9000 kW region. Furthermore an outlying region is illustrated when STW > 15 kn and Propeller shaft Power < 12000 kW. All in all the horizontal areas that appear are considered problematic. Furthermore, negative values in Propeller shaft Power should not exist. The relationship of the graph is, empirically, expressed by the Propellers Law: $P = B \cdot V^3$.

The Propeller shaft rpm – Propeller shaft Power graph is problematic for low shaft revolutions per minute, 40 rpm and lower, and the data are considered as outliers. The Propeller shaft rpm – Fuel Oil Consumption is supposed to oblige to the propeller law: $P = C \cdot n^a$, a = 3.

The aforementioned theoretical formulas are derived from the fact that the water resistance is proportional to the 2nd degree of power of velocity ($R = \frac{1}{2} \cdot C_d \cdot \rho \cdot A \cdot V$) and the effective power of the ship follows the following formula ($P = R \cdot V$). Hence, the relationship of the power and the velocity is the following: $P = C \cdot n^a$, a = 3.

The Propeller shaft rpm – Fuel Oil Consumption is thought to have a linear relationship between the two parameters, however it appears to have two linear distribution points. This phenomenon is depicted again in the Propeller shaft rpm – Fuel Oil Consumption graph. Since its common knowledge that $FOC = P \cdot SFOC$, where SFOC is the specific fuel oil consumption. The relationship between the Propeller shaft rpm – Fuel Oil Consumption is expressed by the following formula: $FOC = D \cdot n^a$. The separation that is illustrated in both graphs is considered as an outlying region and will be eliminated from the dataset. Moreover negative values of Fuel Oil Consumption and zero values are considered as false data and are eliminated from the dataset.

In this particular study the Draft – Trim graph is not used, because it’s not necessary to separate the dataset into two different loading condition, ballast and laden. However the Relative Wind Speed and Relative Wind Direction is an intriguing variable that is going to be a vital for the theoretical models and statistical model. The Relative Wind Speed is also measured and used in Beaufort scale.

2.3 Flow Diagrams describing the filtering procedures

The following flow diagrams are used to summarize all the filtering procedures mentioned in the below sub – sections and to visualize the whole filtering method. Two flow diagrams are illustrated in the following page. The one describes the whole filtering procedure (Figure 7) and the second one describes the Statistical outlier detection, multifilter application procedure, (Figure 8).

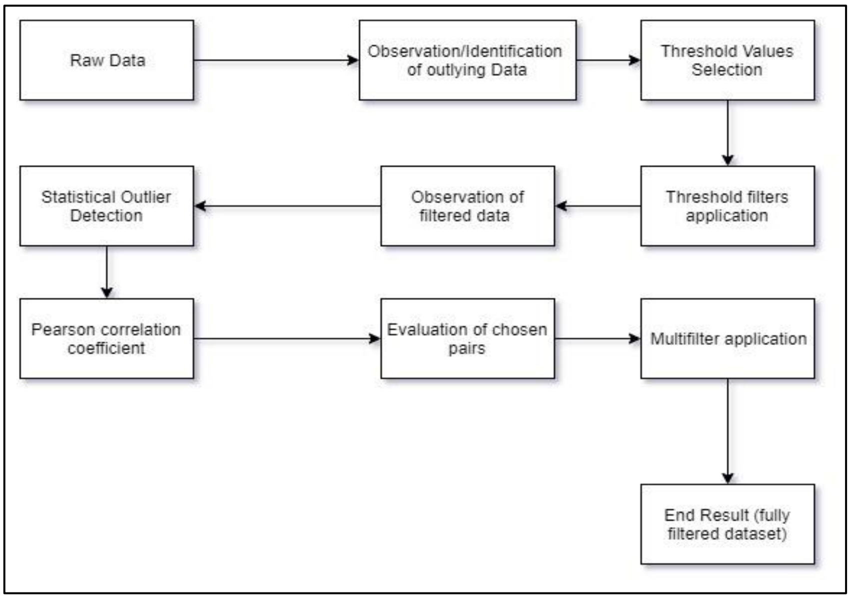


Figure 7. Flow Diagram for the filtering procedure

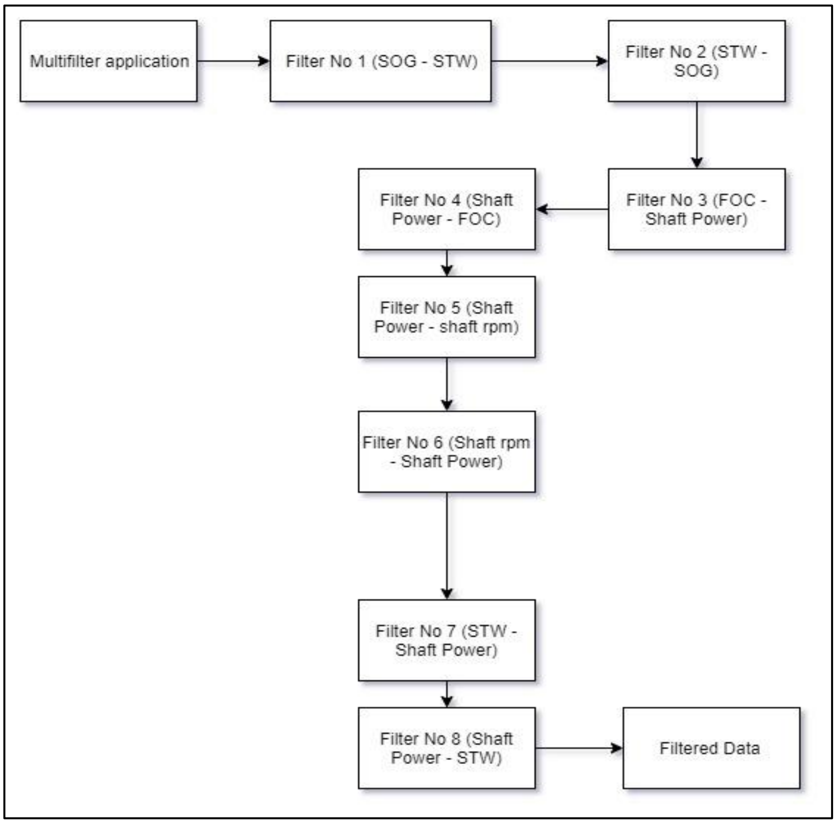


Figure 8. Flow Diagram for the multifilter function

2.4 Data Filtering based on Threshold Values

This sub-section purpose is to processes the data by applying filters based on the relationship of the examined parameters ($P = B \cdot V^3$, $FOC = P \cdot SFOC$) and the physical meaning of the parameters ($FOC < 0$, doesn't exist). Through the filtering procedure the dataset becomes reliable, realistic and corrected in order to comply with the empirical formulas.

Threshold values for speed

Speed is considered the most important variable in this particular study and is measured through Speed over Ground (SOG/GPS) and Speed through Water (STW/Speed Logs). The variables should always be of positive value, as the sensors calculate the speed's absolute values. Furthermore, low speed through water occurs at ports, where the ship is waiting for berths to become available or while the ship is manoeuvring through a port or a Canal. Hence, Speed through Water values are considered to be greater than 3 knots and anything below this threshold isn't used in the final dataset.

- ❖ SOG > 0 kn
- ❖ STW > 3 kn

Threshold values for Propeller shaft Power & Fuel Oil Consumption

Propeller shaft Power, shaft revolutions and main engine Fuel Oil Consumption are considered to be the most significant parameters that describe the operation of the main engine. The three parameters are highly correlated and as mentioned above its necessary to omit a set of data that illustrate port operation condition. To achieve that three threshold values are applied and they eliminate any data points that have a lower value than the following thresholds.

- ❖ Propeller shaft Power > 1000 kW
- ❖ Propeller shaft rpm > 20 rpm
- ❖ Main Engine Fuel Oil Consumption > 3 t/d

2.4.1 Threshold filters application

With the application of the above filters the dataset is partially corrected, because the data, that illustrate a port – related condition, are excluded and the values with no physical meaning (negative values) are omitted. The results are portrayed in the following graphs (Figure 9, Figure 10). The blue colored data points are the excluded data and the orange colored data points are the remaining data.

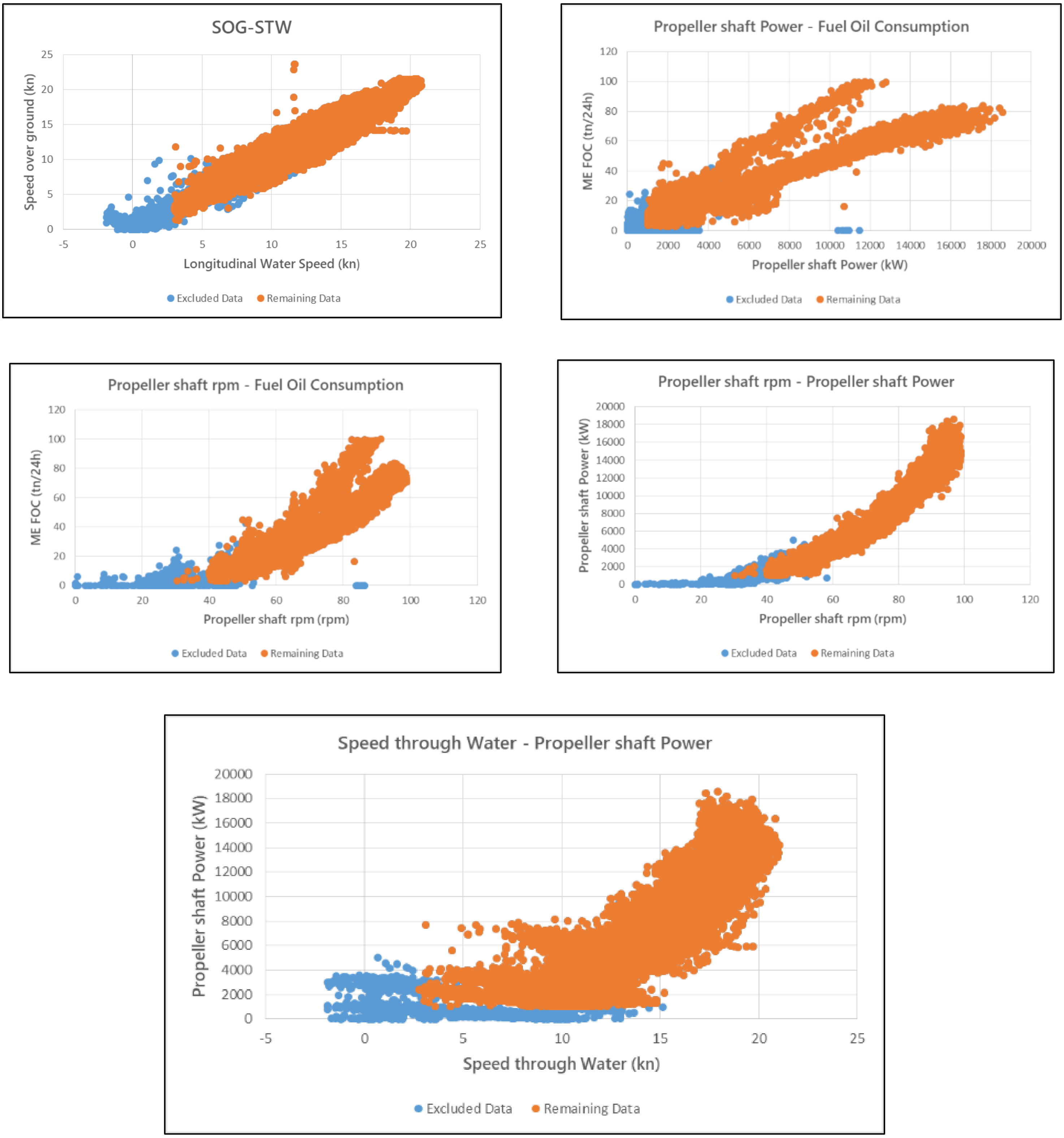
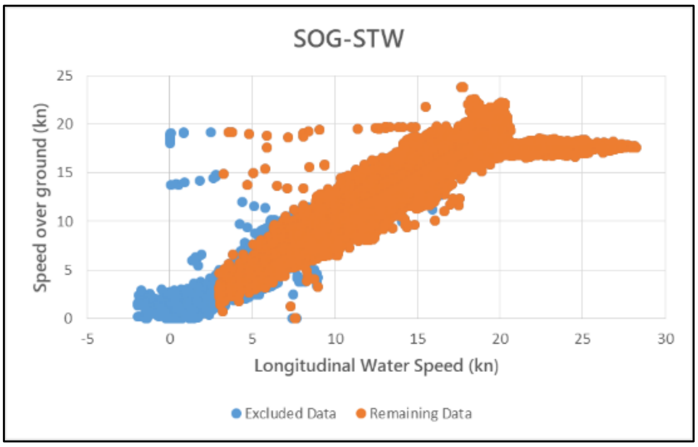
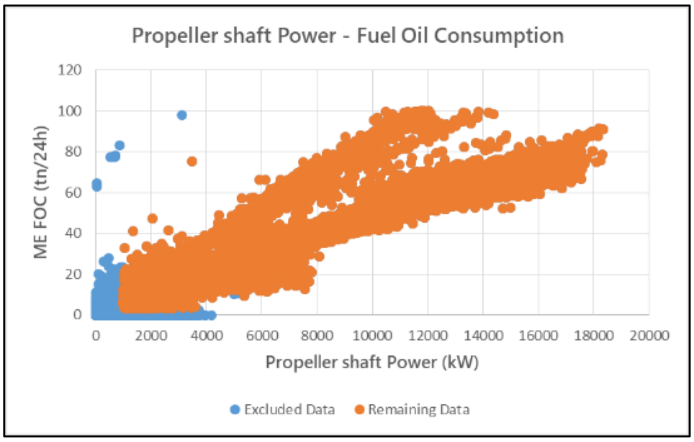
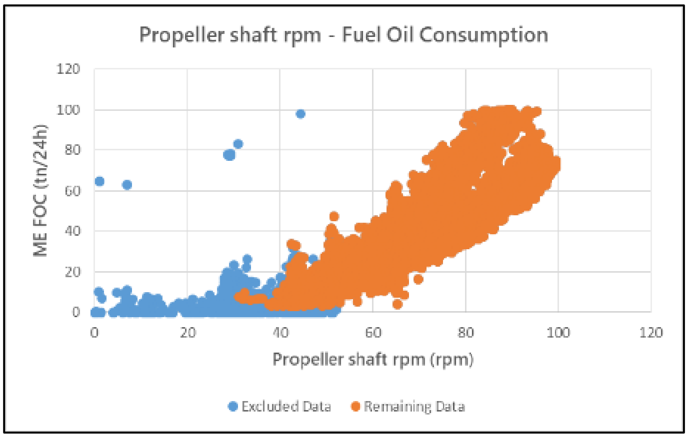


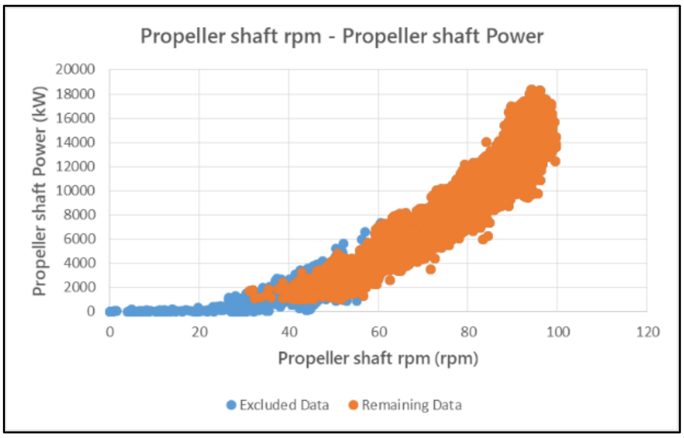
Figure 9. Scatter plots for the filtered data (Color = blue) and remaining data (Threshold Filter – Color = orange), Mean Draft = 9.3 m

In addition the graphs for Mean Draft = 10.5 m are also illustrated in the following page.









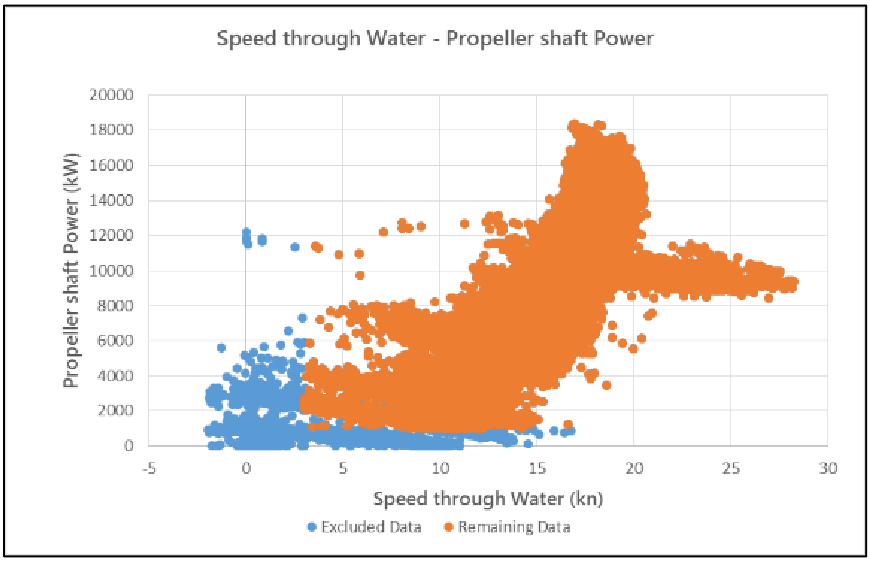


Figure 10. Scatter plots for the filtered data (Color = blue) and remaining data (Threshold Filter – Color = orange), Mean Draft = 10.5 m

The first part of the filtering procedure, in order to create a realistic dataset, manages to improve the dataset by omitting non – logical values and undesirable operations condition (Port operations).

The Speed over Ground – Speed through Water linear relationship is illustrated in the respective graph and the filter eliminates low speed values and negative values. However the filtering procedure doesn't eliminate data points that have high speed. Under this specific circumstances filtering the outlying regions with a different method is inevitable. The areas that need to be filtered are:

- STW > 20 & SOG < 18
- SOG > 18.5 & STW < 15
- STW < 17.5 & SOG > 21

This particular areas appear, mostly, at the graphs used for Mean Draft = 10.5 m, because this is the Draft we have the majority of the data points.

The Propeller shaft Power – Fuel Oil Consumption is not affected by the filtering procedure and only a small area of data points, bottom left corner, is excluded. The separation of the data points, into two distribution areas, needs to be eliminated, so that a linear relationship between the two parameters is achieved.

The Propeller shaft rpm – Propeller shaft Power graph is affected less by the preliminary filter, as only the low revolutions speeds and low shaft Power are filtered. The graph complies with the formula of the Propeller Law and appears to have a small amount of data that are detached from the main curve and are in need of elimination. The filtering procedure eliminates a small area of data points at the bottom left corner, similar to the data correction of the Propeller shaft Power – Fuel Oil Consumption graph.

The Propeller shaft rpm – Fuel Oil Consumption is affected by the filtering procedure and eliminates outlying data with FOC < 5 t/d and shaft revolutions < 30 rpm. The main problem in this graph is the separation of the data points into two areas. This separation is not ideal and needs to be filtered in order to coincide with the empirical formulas, mentioned above. Moreover, vertical areas are not filtered and are in need of a stricter and more reliable filtering method. The vertical areas are shown below:

- Shaft revolutions ≈ 43 rpm & 17 ≤ FOC ≤ 30 tn/d
- Shaft revolutions ≈ 50 rpm & 30 ≤ FOC ≤ 42 tn/d
- Shaft revolutions ≈ 63 rpm & 10 ≤ FOC ≤ 20 tn/d

The Speed through Water (normalized) – Propeller shaft Power graph is affected by the filtering procedure in a positive way. However, the filter only eliminates values for low speed and low shaft Power. Under this specific circumstances it's necessary to filter the outlying regions with a different method. The areas that need to be filtered are:

- STW > 17 & Propeller shaft Power < 12000
- STW < 10 & 6000 ≤ Propeller shaft Power ≤ 8000
- STW < 7.5 & 2000 ≤ Propeller shaft Power ≤ 4000
- STW > 12.5 & 2000 ≤ Propeller shaft Power ≤ 4000
- 11 ≤ STW ≤ 13 & Propeller shaft Power < 12000

After identifying the already existing outlying regions in the, Speed through Water – Speed Over Ground, Propeller shaft rpm – Fuel oil consumption, Propeller shaft Power and Propeller shaft Power – Fuel oil consumption curves, that the threshold values didn't filter (Above set of regions) it's necessary to implement a new set of filters, in order to get a robust dataset.

2.5 Statistical Outlier Detection

The purpose of this particular sub-section is to further process the data from the previous sub - sections. The filtering procedure is heavily influenced by the relationships between the parameters and is applied only in highly correlated variables. Firstly the correlations among the parameters is uncovered and used as a measure of co – dependency. Through the filtering procedure the effect of each filter is evaluated and an optimal combination of filters is determined. After the completion of this procedure an acceptable dataset is provided.

2.5.1 Correlation Calculation

The correlation of the examined variable is calculated using the Pearson correlation coefficient. Given two parameters the Pearson correlation coefficient (PCC) of the pair is calculated:

$$\text{PCC} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})\cdot(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2}\cdot\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2}} \quad (2.2)$$

Where,

- x_i: The data points of the x parameter
- $\bar{x}$: The mean value of the x dataset
- y_i: The data points of the y parameter
- $\bar{y}$: The mean value of the y dataset
- n: The number of data in the dataset

The calculated coefficients are presented in Table 4.

Table 4. Pearson correlation coefficient

	SOG	*STW*	*T*	*Trim*	*Rudder Angle*	*Shaft RPM*	*Shaft Power*	*Shaft Torque*	*FOC*	*Start Pressure*	*Wind Speed*	*Wind Direction*	*Heading*
SOG	1	0.975	0.248	0.47	-0.109	0.954	0.908	0.900	0.902	-0.024	0.048	-0.102	-0,1542
STW		1	0.246	0.478	-0.111	0.968	0.916	0.909	0.911	-0.023	0.058	-0.0994	-0,1535
T			1	0.519	-0.012	0.283	0.286	0.271	0.279	-0.005	0.1376	-0.0163	0,0391
Trim				1	0.003	0.424	0.381	0.359	0.378	-0.0101	-0.207	-0.0427	-0,0744
Rudder Angle					1	-0.11	-0.13	-0.142	-0.13	-0.0006	-0.1000	0.454	0,0521
Shaft RPM						1	0.973	0.985	0.966	-0.0275	0.1893	-0.0962	-0,1356
Shaft Power							1	0.994	0.983	-0.0326	0.2592	-0.1227	-0,1733
Shaft Torque								1	0.981	-0.0297	0.2939	-0.1330	-0,1531
FOC									1	-0.0294	0.2641	-0.1188	-0,1653
Start Pressure										1	0.0088	-0.0016	0,0212
Wind Speed											1	-0.1387	0,0273
Wind Direction												1	0,1170
Heading													1

Table 5. Color Correlations

Values	*Correlation*	*Colour*
| 0 – 0.2 |	Not correlated	
0.2 – 0.7	Slightly correlated	
0.7 – 0.99	Highly correlated	
1	Totally correlated	
Blanks	0	

The Pearson correlation coefficients have values between + 1 and – 1:

- + 1 indicates a total positive linear correlation.
- – 1 indicates a total negative linear correlation.
- 0 indicates no correlation

The pink colored cells represent a small absolute Pearson coefficient, indicating that the parameters are not correlated. The blue colored cells represent a slight correlation between the parameters. The green colored cells represent a high correlation coefficient between the variables. Also the orange colored cells represent the diagonal of the matrix and have a Pearson correlation coefficient + 1 because the same variables are obliged to be positively linear correlated with each other (Table 5).

The highly correlated parameters, are paired with each other and used in the next filtering procedure. In this particular moment the pairs are going to be evaluated, in order to determine the best combination of sub – filters that if combined, are going to give a reliable dataset, which will be used in the creation of the statistical model and the theoretical models.

The following pairs are chosen for evaluation:

- Speed through Water – Speed over Ground
- Speed through Water – Propeller shaft rpm
- Speed through Water – Propeller shaft Power
- Speed through Water – Propeller shaft Torque
- Speed through Water – Fuel Oil Consumption
- Propeller shaft rpm – Propeller shaft Power
- Propeller shaft rpm – Propeller shaft Torque
- Propeller shaft rpm – Fuel Oil Consumption
- Propeller shaft Power – Fuel Oil Consumption

From the nine pairs chosen not all of them are going to be used in the filtering procedure, because a lot of them provide the same results (Propeller shaft rpm – Propeller shaft Power, Propeller shaft rpm – Propeller shaft Torque).·

2.5.2 Filtering process

The proposed filtering method is described in the following page and consists of eight steps:

I. Choose a primary parameter X. (STW)
II. Divide the parameter values in groups with range r. (1 kn)
III. Pick a second parameter Y, which is highly correlated with the primary parameter X
IV. Group the data points G_i according to the division mentioned in Step 2.
V. Calculate the mean value m_{yi} and the standard deviation σ_{yi} for the secondary parameter in each group of data G_i.
VI. Pick a factor k, outlier threshold, to be multiplied with the standard deviation σ_{yi}
VII. For every value of Y in the G_i group test if the following inequality is fulfilled

$$| Y_{ij} - m_{yi} | < k \cdot \sigma_{yi} \quad (2.3)$$

VIII. If the inequality isn't fulfilled the data point is rejected

The outlier threshold receives values greater than $1.5 < k < 2.5$ since there can be multiple curves representing the relationships between two values, whose data points would be falsely discarded if a low threshold (k) value was applied. To be even more specific in the Propeller shaft rpm – Propeller shaft Power graph the curves follow an empirical formula:
$P = C \cdot n^{a}$, each curve represents a different weather condition (Wind Speed = 4 BN, Relative Wind Direction = Head Sea (0 – 30 deg.), Significant Wave Height = 2 m) and a different hull condition. The data points deviating from the main empirical figure shouldn't be omitted because they provide a series of valuable information for the condition of the ship and the handling by the crew members. These data points are vital for the performance analysis of a vessel.
The described method is applied for four pairs of parameters that are chosen below and the main factor, in order to choose a pair, is the correlation coefficient.
For each pair the process is applied twice by swapping the primary and the secondary variables.

- Speed through Water – Speed over Ground
- Speed through Water – Propeller shaft Power
- Propeller shaft rpm – Fuel Oil Consumption
- Propeller shaft Power – Fuel Oil Consumption

The particular pairs are used, because it's not advised to use highly correlated (0.95 – 0.99) parameters a lot of times (Propeller shaft Torque – Propeller shaft Power) and it's not mandatory to use Speed through Water in a lot of pairs that don't make logical sense, such as Speed through Water – Propeller shaft rpm. The goal was to use different parameters that cover the vast majority of remaining outlying regions, as shown in Figure 5, 6.

2.5.3 Multifilter process and results

The corrected dataset is achieved by the application of a multifilter that effectively combines eight single filters (sub – filters). The multifilter contains the following single filters (Table 6).

Table 6. Multifilter parameters

Filter Name	*Primary Parameter*	*Secondary Parameter*	*Outlier threshold k*	*Range r*
SOG – STW	SOG	STW	2	1 kn
STW – SOG	STW	SOG	2	1 kn
FOC – Shaft Power	FOC	Shaft Power	2	5 tn/d
Shaft Power – FOC	Shaft Power	FOC	2	750 kW
Shaft Power – Shaft rpm	Shaft Power	Shaft rpm	2	750 kW
Shaft rpm – Shaft Power	Shaft rpm	Shaft Power	2	4 rpm
STW – Shaft Power	STW	Shaft Power	2	1 kn
Shaft Power – STW	Shaft Power	STW	2	750 kW

The range of each Group is experimentally chosen so that a stricter Outlier Threshold (k) can be applied. It's easier to change the Outlier threshold (k), than change the range of the Groups for each parameter. Moreover the goal was to create the same number of Groups for all of the parameters (16 – 19), with the exception of the Speed over Ground parameter that is divided in 21 individual Groups. The range of the Groups is affected by the number of data, that each Groups, consists of. For example the number of data each Group consists of is about 15000 – 25000 data points. Furthermore, the amount of data each filter eliminates is shown in the following Table 7.

Table 7. Number of remaining data, unfiltered data and percentage of excluded data after each sub – filter is applied

Filter Name	*Number of unfiltered data*	*Number of remaining data after the application of a sub-filter*	*Percentage of excluded data (%)*
First Filtering Procedure (Threshold filters application)	531557	518502	2.54%
SOG – STW	518502	494687	4.51%
STW – SOG	494687	472154	4.54%
FOC – Shaft Power	472154	460112	2.55%
Shaft Power – FOC	460112	445867	3.096%
Shaft Power – Shaft rpm	445867	426461	4.352%
Shaft rpm – Shaft Power	426461	406100	4.774%
STW – Shaft Power	406100	390977	3.724%
Shaft Power – STW	390977	378108	3.291%
Final	531557	378108	28.8%

The percentage of the excluded data is 29% and every single filter eliminates approximately 4 % of the data, except the 2nd and 3rd set of filters, because the amount of outlying data is smaller, which is affected from the previous filter and is affected by the correlation coefficient that the parameters share. In the following pages the dataset is illustrated in a series of graphs for the two different loading conditions (mean Draft values). The percentage of excluded data is given by the following method:

$$\text{Percentage of Excluded Data (\%)} = \frac{Unfiltered\ data - remaining\ data\ after\ \ sub{-}filter}{Unfiltered\ data} \cdot 100 \quad (2.4)$$

Example:

- ❖ Unfiltered Data = 531557
- ❖ Remaining Data = 518502 (SOG – STW, sub – filter).
- ❖ Percentage of Excluded Data (%) = $\frac{531557 - 518502}{531557} \cdot 100 = 2.54\%$

In continuity of the above statistical filtering procedure a series of graphs of the reaming data, after the application of the statistical outlier detection, (green colour) compared to the dataset provided after the first filtering procedure of chapter 2.4 (blue colour) for mean Draft = 9.3 m is illustrated in the following pages (Figure 11, Figure 12):

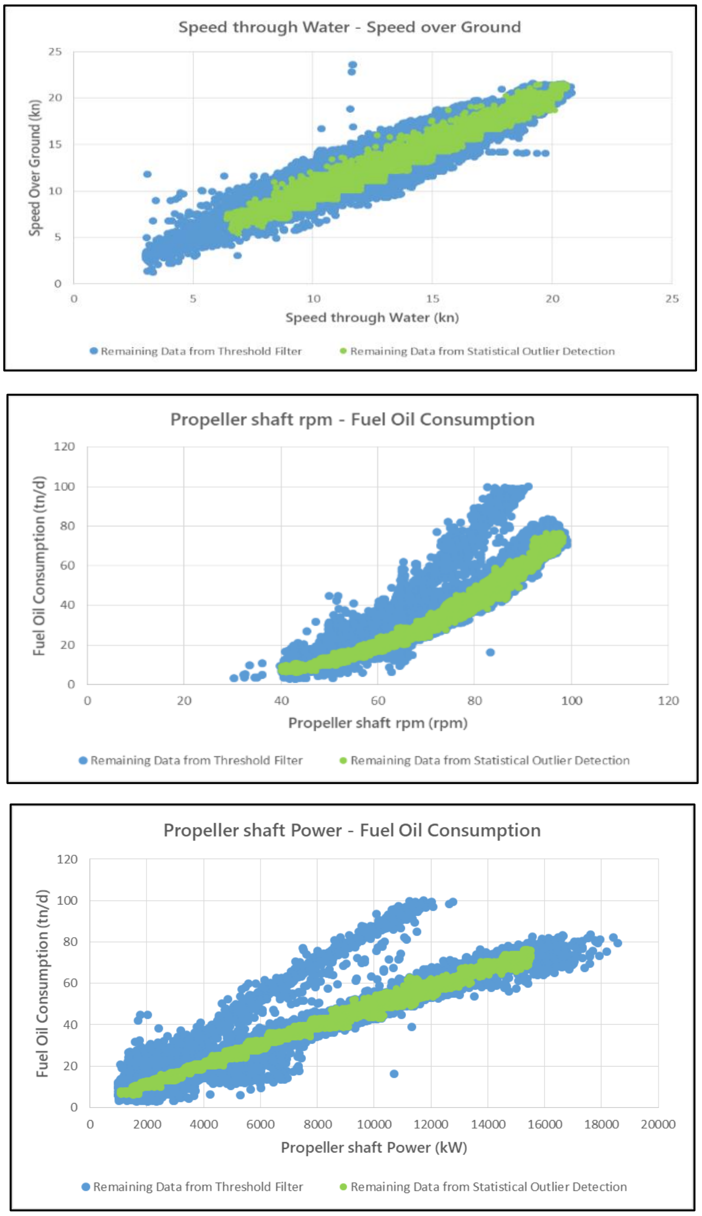
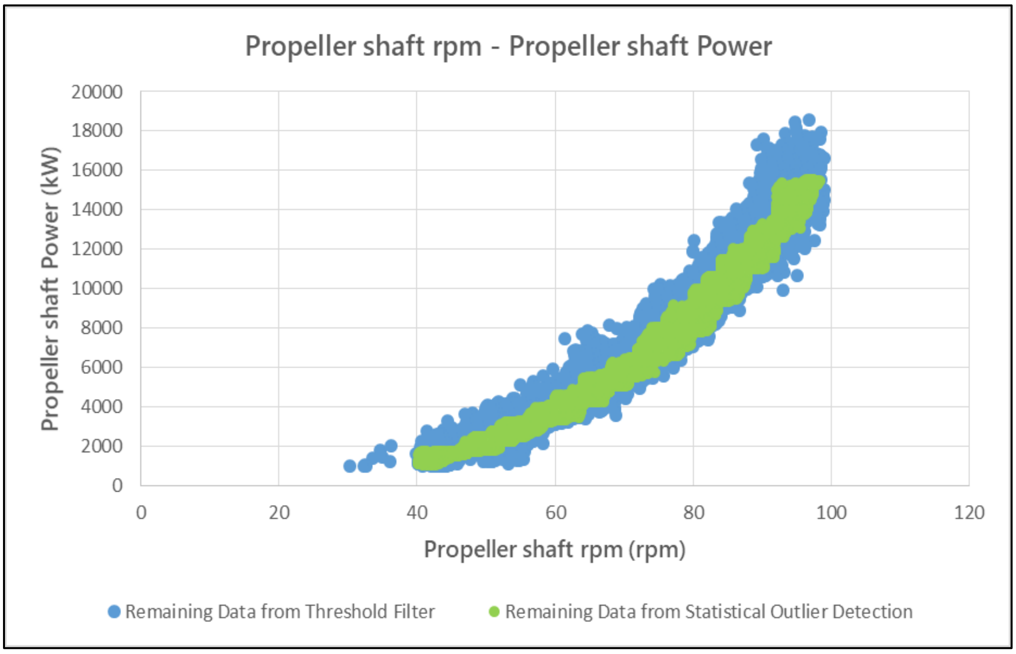


Figure 11. Comparison between the remaining data of the threshold filter (Color = blue) and the actual remaining data of the dataset (Color = green), mean draft = 9.3 m (1)



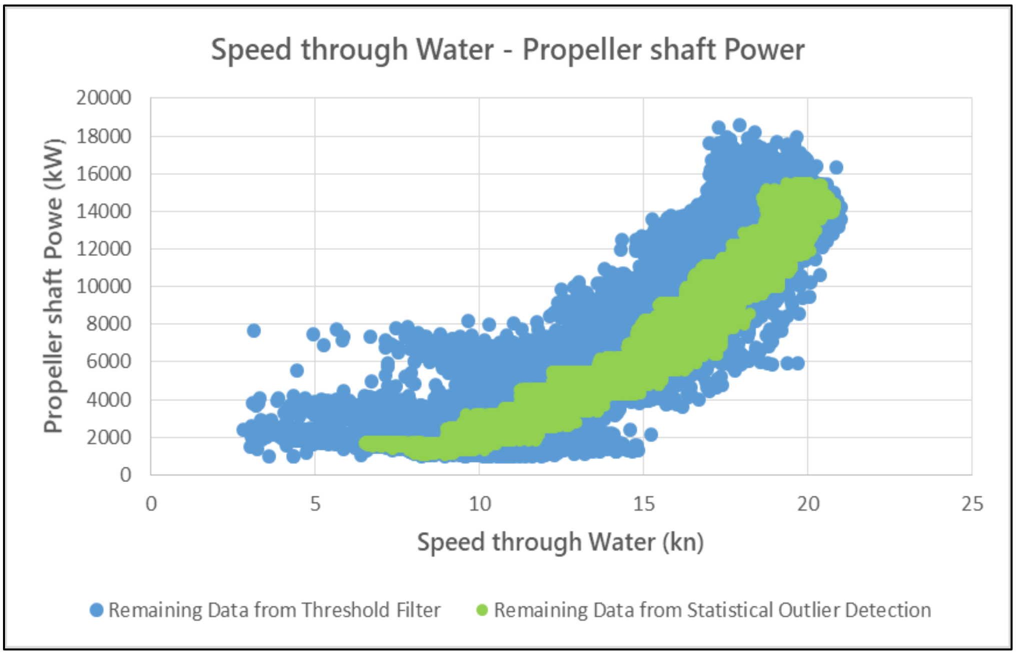


Figure 12. Comparison between the remaining data of the threshold filter (Color = blue) and the actual remaining data of the dataset (Color = green), mean draft = 9.3 m (2)

Comparison graphs of the reaming data, after the application of the statistical outlier detection, (green colour) and the dataset provided after the first filtering procedure of chapter 2.4 (blue colour) for mean Draft = 10.5 m are displayed in the following pages (Figure 13, Figure 14):

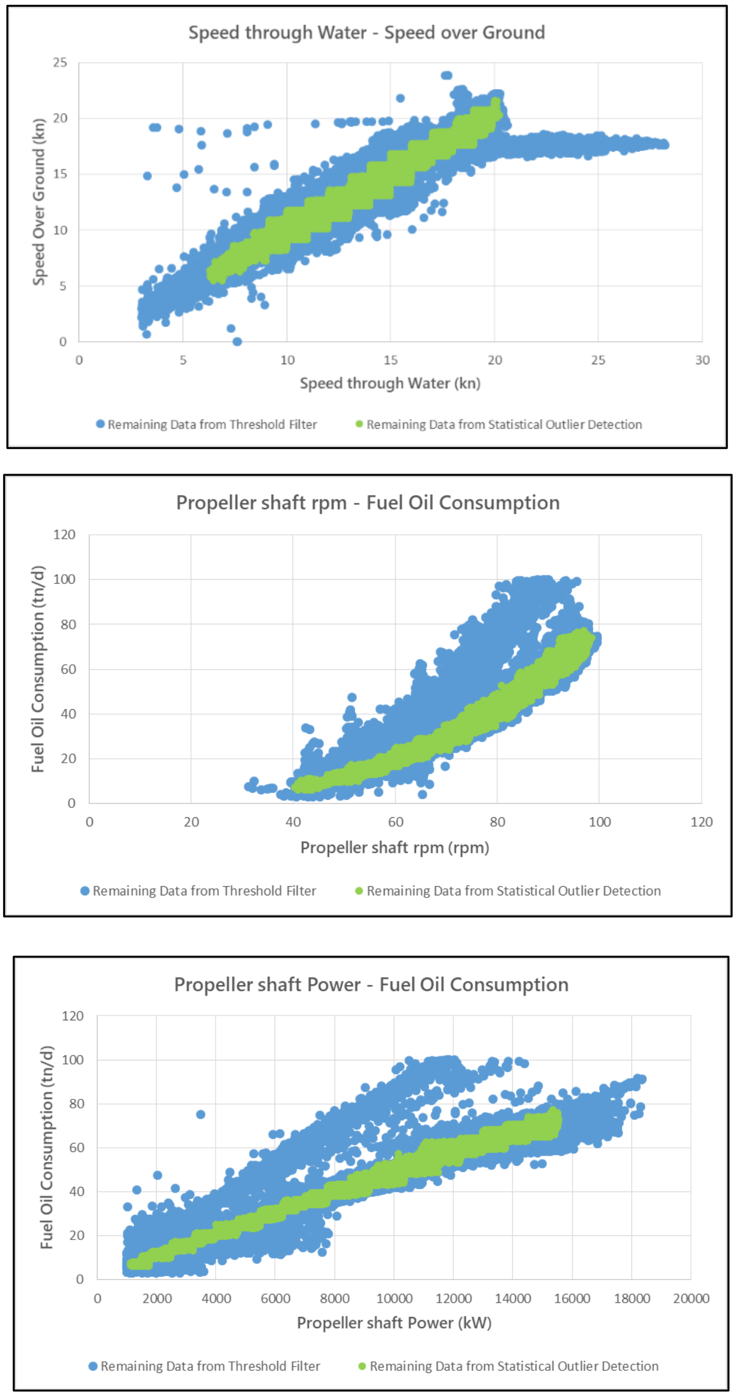
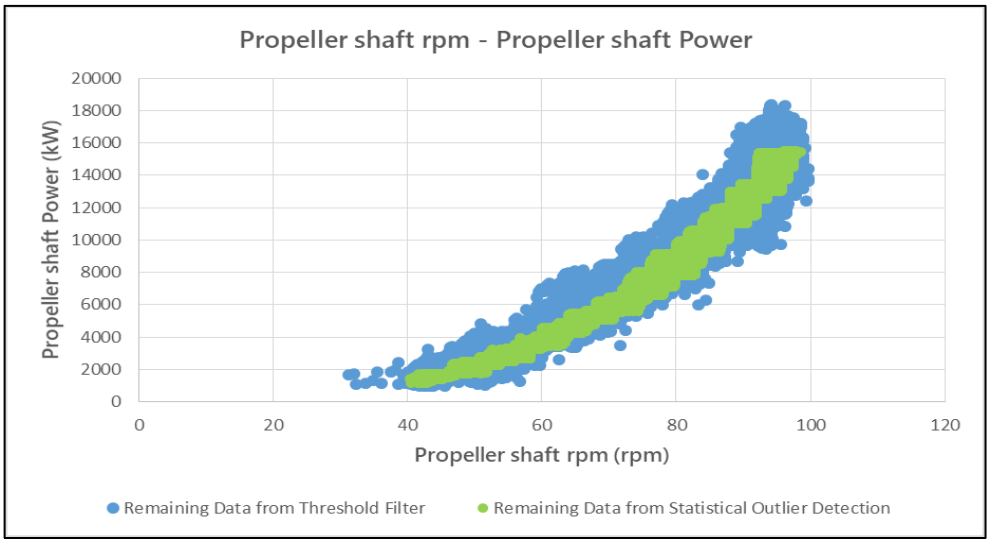


Figure 13. Comparison between the remaining data of the threshold filter (Color = blue) and the actual remaining data of the dataset (Color = green), mean draft = 10.5 m (1)



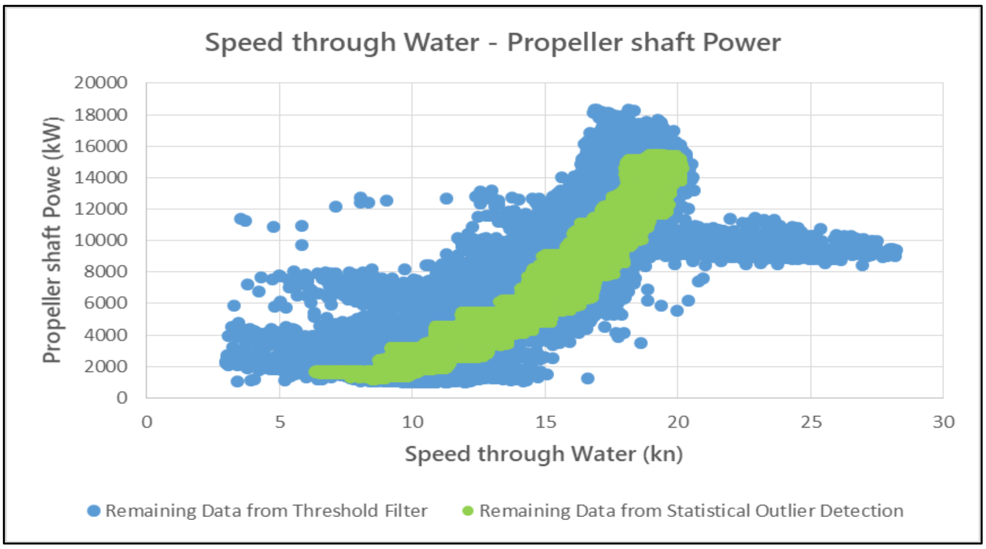


Figure 14. Comparison between the remaining data of the threshold filter (Color = blue) and the actual remaining data of the dataset (Color = green), mean draft = 10.5 m (2)

In the legends of the graphs of the above figures:

- ❖ Remaining data from Threshold Filter corresponds to the dataset provided after the application of the threshold values, such as Speed over Ground > 0 knots (sub-section 2.4)
- ❖ Remaining data from Statistical Outlier Detection corresponds to the final dataset proposed after the application of all the sub – filters and the threshold values filter (initial sub-section (2.5))

The graphs above are used, in order to understand the correction of the dataset via the outlier detection procedure and to show that the dataset, is now reliable and realistic, and ready to be used in the statistical model and in the theoretical analysis of the prediction of the main engine Fuel Oil Consumption.

Due to this particular omission of data and filtering procedures it's considered that all the parameters refer to different loading conditions, but not at anchorage, unloading conditions in ports, waiting for port births or bunkering in the port. The data used are for open sea conditions and fully developed seas. The graphs for the remaining data from the statistical outlier detection procedure is presented in Appendix A.

3. Theoretical Models

The goal of this chapter is to estimate the Fuel Oil Consumption of the Containership, using two different theoretical methods. To achieve that it's necessary to use the correct dataset (Chapter 2) and describe the framework for the theoretical models.

The first theoretical model is strongly connected to the modified Kwon Method provided by Kwon, 2008. The method analyses different operational conditions of the ship, in regards to the relative wind speed and relative wind direction, to eventually calculate the brake power of the ship (P_B) and as a result estimate the Fuel Oil Consumption of the ship, for different weather conditions (Wind Speed & Wind Direction).

The second theoretical model is implemented by the International Towing Tank Committee (ITTC) in the article ''Preparation, Conduct and Analysis of Speed/Power Trials'' (7.5–04–01–01.1) 2017. The ITTC method, as it's going to be called moving forward, is based on the correction of the dataset that is provided by the ship sensors. The correction method is based on the resistance increase due to wind and wave effects. To be more specific the added resistance that we use is separated in three parts the added resistance due to the wind effects, the added resistance due to the effect of waves and the added resistance due to water temperature and salt content. The basic principle of the ITTC method is the conversion of added resistance to ''added shaft Power'' and then the combination of the already existent Propeller shaft Power with the ''added shaft Power''. Furthermore, provided the corrected Propeller shaft Power it's possible to estimate the theoretical fuel oil consumption using the formula: $FOC = P \cdot SFOC$. It's mandatory to mention that the method used for the calculation of the added resistance due to the wind effects, as presented in the ITTC, is developed by Fujiwara, 2005 and that the method used for the calculation of the added resistance due to the waves is the Stawave – 1 method, which is described in the ITTC and the STA – JIP, is developed by Boom, 2013.

In order to be able to carry out the Kwon method and the ITTC correction it's necessary to transform the relative wind speed and relative wind direction, in a way so that it can be placed at a widely accepted reference height of 10 meters.

3.1 Kwon Method

3.1.1 Wind Speed & Wind Direction correction

The wind speed and wind direction correction are based on the suggested instructions provided in the ITTC (7.5–04–01–01.1, 2017) and in ISO 19030 (ISO, 2016). In order to begin with the suggested methodology it's necessary to understand that the wind speed and wind direction are given for the height of the anemometer sensor. Furthermore, it's mandatory to know the exact vertical position of the anemometer, for the design draft. Also, the reference height for the wind resistance, for the design draft is necessary. Moreover, the area of maximum transverse section exposed to the winds is necessary. The said parameters are provided with the following values:

- A_{ref} = 485 m^2
- $Z_{a,ref}$ = 10 m
- $Z_{ref,ref}$ = 25 m

Moreover, it's necessary to describe the sign convention for the wind directions developed by ISO, 2015. The following figure (Figure 15) is illustrated.

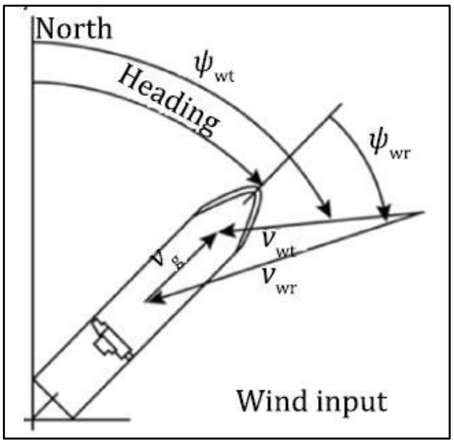


Figure 15. Sign conventions for wind directions (ISO, 2015)

The first step towards the correction of the wind speed and wind direction is to calculate the true wind velocity at the vertical position of the anemometer v_{wt} in m/s and the true wind direction at the vertical position of the anemometer ψ_{wt} in degrees.

$$v_{wt} = \sqrt{v_{wr}^2 + v_g^2 - 2 \cdot v_{wr} \cdot v_g \cdot \cos\psi_{wr}} \quad (3.1.1)$$

$$\psi_{wt} = \tan^{-1}\left\{\frac{v_{wr}\cdot\sin(\psi_{wr}+\psi_0) - v_g\cdot\sin(\psi_0)}{v_{wr}\cdot\cos(\psi_{wr}+\psi_0) - v_g\cdot\cos(\psi_0)}\right\}, \quad v_{wr}\cdot\cos(\psi_{wr}+\psi_0) - v_g\cdot\cos(\psi_0) \geq 0 \quad (3.1.2)$$

$$\psi_{wt} = \tan^{-1}\left\{\frac{v_{wr}\cdot\sin(\psi_{wr}+\psi_0) - v_g\cdot\sin(\psi_0)}{v_{wr}\cdot\cos(\psi_{wr}+\psi_0) - v_g\cdot\cos(\psi_0)}\right\} + 180, \quad v_{wr}\cdot\cos(\psi_{wr}+\psi_0) - v_g\cdot\cos(\psi_0) < 0 \quad (3.1.3)$$

Where,

- v_{wr}: Relative wind velocity at the vertical position of the anemometer (m/s)
- v_g: Ship's Speed over Ground (m/s)
- ψ_{wr}: Relative wind direction at the vertical position of the anemometer (degrees)
- ψ_0: Ship's heading (degrees)

The second and final step in the wind velocity and direction correction is the calculation of the relative wind speed at the reference height and relative wind direction at the reference height. However a few calculations are necessary in order to achieve the correct weather data.

$$\Delta T = T_{ref} - T \quad (3.1.4)$$
$$A = A_{ref} + \Delta T \cdot B \quad (3.1.5)$$
$$Z_\alpha = Z_{\alpha,ref} + \Delta T \quad (3.1.6)$$

$$Z_{ref} = \frac{A_{ref} \cdot (Z_{ref,ref} + \Delta T) + 0{,}5 \cdot B \cdot \Delta T^2}{A} \quad (3.1.7)$$

The described calculations are used to calculate the true wind velocity at the reference height (m/s).

$$v_{wt,ref} = v_{wt} \cdot \left(\frac{Z_{ref}}{Z_a}\right)^{1/7} \quad (3.1.8)$$

Where.

- ❖ ΔT (m): The difference between design draft $T_{ref} = T_d$ = 11,5 m and mean draft that is measured by the sensors.
- ❖ A (m^2): Corrected area of maximum transverse section exposed to the winds.
- ❖ A_{ref} (m^2): Area of maximum transverse section exposed to the winds.
- ❖ B (m): Breadth.
- ❖ Z_a (m): Vertical position of the anemometer.
- ❖ $Z_{a,ref}$ (m): Vertical position of the anemometer, for the design draft.
- ❖ Z_{ref} (m): Reference height for the wind resistance coefficient.
- ❖ $Z_{ref,ref}$ (m): Reference height for the wind resistance, for the design draft

The above calculations lead us to the relative wind speed at the reference height and relative wind direction at the reference height.

$$v_{wr,ref} = \sqrt{v_{wt,ref}^2 + v_g^2 + 2 \cdot v_{wt,ref} \cdot v_g \cdot \cos(\psi_{wt} - \psi_0)} \quad (3.1.9)$$

$$\psi_{wr,ref} = \tan^{-1}\left\{\frac{v_{wt,ref} \cdot \sin(\psi_{wt} - \psi_0)}{v_g + v_{wt,ref} \cdot \cos(\psi_{wt} - \psi_0)}\right\}, \quad v_g + v_{wt,ref} \cdot \cos(\psi_{wt} - \psi_0) \geq 0 \quad (3.1.10)$$

$$\psi_{wr,ref} = \tan^{-1}\left\{\frac{v_{wt,ref} \cdot \sin(\psi_{wt} - \psi_0)}{v_g + v_{wt,ref} \cdot \cos(\psi_{wt} - \psi_0)}\right\} + 180, \quad v_g + v_{wt,ref} \cdot \cos(\psi_{wt} - \psi_0) < 0$$

(3.1.11)

The aforementioned functions are described in the ITTC, 2017. The following table (Table 8) is used to sum up the basic information used to correct the relative wind speed and direction in the reference height (10 m).

Table 8. Standard Values for Wind Speed & Wind Direction correction

Parameter Name	*Values*	*Units*
Area of maximum transverse section exposed to the winds (A_{ref})	485	m^2
Breadth (B)	30.2	m
Vertical position of the anemometer, for the design draft ($Z_{a,ref}$)	25	m
Reference height for the wind resistance, for the design draft ($Z_{ref,ref}$)	10	m
Design Draft (T_d)	11.5	m

3.1.2 Beaufort Scale & Encounter Angle

The purpose of this sub – section is to create a Beaufort scale, given the new relative wind velocity for the reference height, and define the meaning of an encounter angle between the ship and the weather effects (Waves, Wind). The encounter angle of a ship is defined by the following formula:

$$\text{Encounter Angle} = \beta - \alpha \quad (3.1.12)$$

Where, β is the wave angle and α is the ship heading. This study considers that the wave angle is similar and equal to the wind direction.

However, the above formula provides values from 0 degrees to 360 degrees, in order to be in the designated direction limits [0 – 180] degrees, it’s necessary to correct the encounter angle using the following formula:

Corrected Encounter Angle = Encounter Angle – 2 · (Encounter Angle – π) (rad) (3.1.13). After this particular correction four encounter angle categories occur (Kwon, 2008):

- Head Sea (0 – 30) degrees
- Bow Sea (30 – 60) degrees
- Beam Sea (60 – 150) degrees
- Following Sea (1500 – 180) degrees

The aforementioned categories are used only for an orthogonal coordinates system. Hence the above Encounter Angle formulas are used. The figure provided by Soares & Varela (2011) (Figure 16) and Figure 17, provided by Kwon (2008), are used for the better understanding of the above formulas.

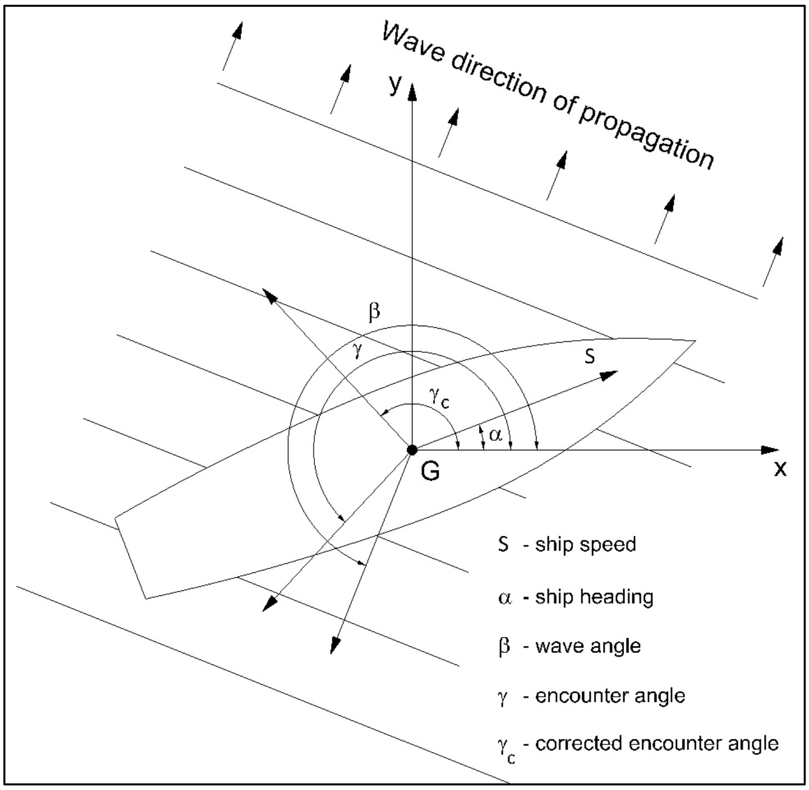


Figure 16. Definition of the Encounter Angle (Soares & Varela, 2011)

Furthermore, it shall be mentioned that the wave direction and the wind direction are thought to be the same.

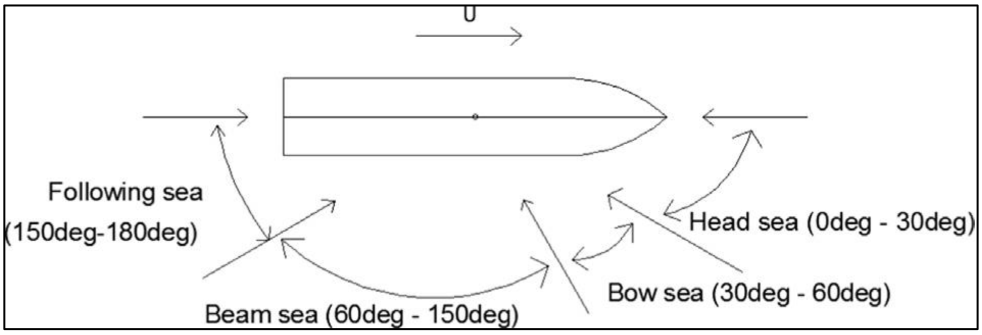


Figure 17. Encounter Angle Categories (Kwon, 2008)

The Beaufort scale is defined by the ITTC (7.5–04–01–01.1) and is described in the following table (Table 9), as well as shown in Figure 18.

Table 9. Beaufort scale

Beaufort Number	Descriptive term	m/s
0	Calm	0-0.2
1	Light Air	0.3-1.5
2	Light breeze	1.6-3.3
3	Gentle breeze	3.4-5.4
4	Moderate breeze	5.5-7.9
5	Fresh breeze	8-10.7
6	Strong breeze	10.8-13.8
7	Near gale	13.9-17.1
8	Gale	17.2-20.7
9	Strong gale	20.8-24.4
10	Storm	24.5-28.4
11	Violent Storm	28.5-32.6
12	Hurricane	32.7 and over

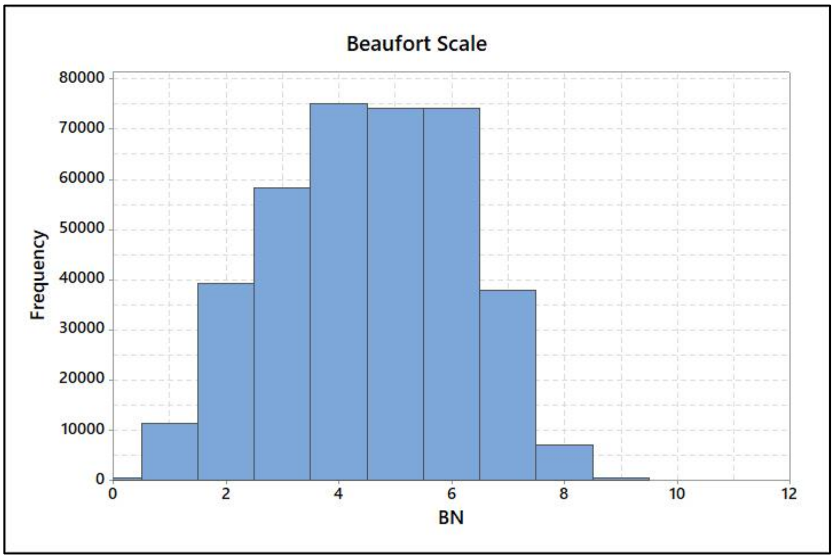


Figure 18. Histogram of Wind Velocity data (Beaufort Number)

3.1.3 Added Resistance Modelling

This sub – section describes the semi – empirical modified Kwon method (Kwon, 2008) that helps express the added resistance, as an absolute speed loss. The weather effect, presented as speed loss, compares the ship speed in different sea conditions to the ship's expected speed in calm water conditions.

The goal, of the added resistance modelling chapter, is to calculate the ship's speed in calm water condition and be able to use it to define the effective engine Power using the Holtrop & Mennen method (Holtrop & Mennen, 1982). Furthermore, after defying the effective engine Power the goal is to create the relationship between the actual ship's speed (irregular waves) and the required engine Power. If the aforementioned goals are achieved, the estimation of the Fuel Oil Consumption for a Specific Ship under Specific Speed, Encounter Angle, Draft, and Sea State can easily be calculated with the formula: FOC = P · SFOC.

The following Flowchart is used to describe the whole procedure mentioned above (Figure 19).

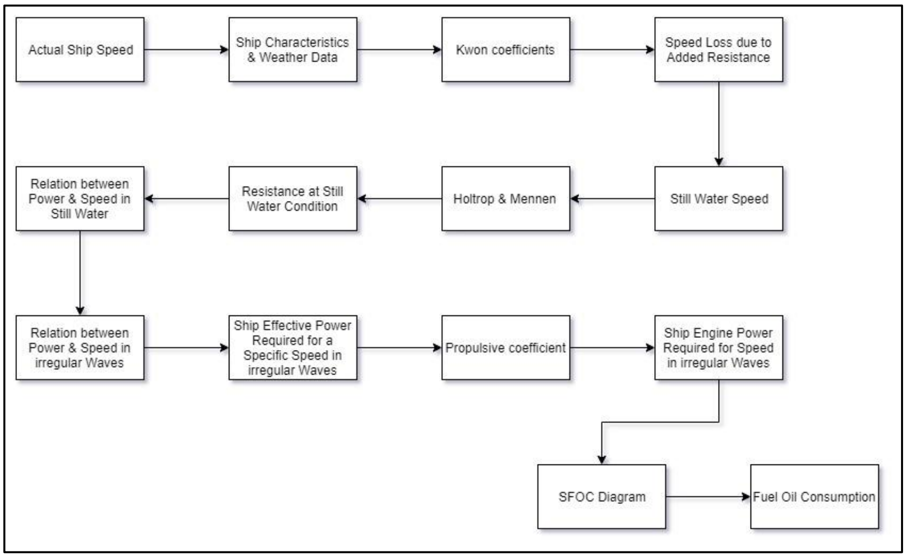


Figure 19. Flow Diagram (Kwon Method)

The formula used for the Kwon method (Kwon, 2008) is the following:

$$\frac{\Delta V}{V1} \cdot 100\% = C_u \cdot C_{form} \cdot C_\beta \quad (3.1.13)$$

$$V_2 = V_1 - (C_u \cdot C_{form} \cdot C_\beta) \cdot \frac{1}{100\%} \cdot V_1 \quad (3.1.14)$$

Where,

- ❖ V_1: Design (nominal) operating ship speed in still water conditions (no wind, no waves), given in m/s.
- ❖ V_2: Actual ship speed in selected weather (wind and irregular waves) conditions given in m/s.
- ❖ $\Delta V = V_1 - V_2$: Absolute speed loss, given in m/s
- ❖ C_β: Direction reduction coefficient, dependant on the weather direction angle (with respect to the ship's bow) and the Beaufort Number (BN), as shown in Table 12.
- ❖ C_{form}: Ship form coefficient, as shown in Table 10.

- ❖ C_u: Speed reduction coefficient, dependant on the ship's block coefficient C_b. The loading condition and the Froude Number (Fn), as shown in Table 11.

For the calculation of the Froude Number we use the following formula:

$$Fn = \frac{V}{\sqrt{g \cdot L}} \quad (3.1.15)$$

The calculation of the block coefficient is provided by the following empirical formula:

$$C_b = C_{b0} \cdot (T/T_0) \wedge ((C_{WL}/C_{b0})\text{-}1) \quad (3.1.16)$$

Where,

- ❖ T_0: Is considered to be the water ballast draft = 6.07 m
- ❖ C_{b0}: Is considered to be the water ballast block coefficient = 0.5573 (Sea Trials)

The calculation of the displacement is done using the following formula:

$$\Delta = C \cdot \gamma \cdot L \cdot B \cdot T \cdot C_b \quad (3.1.17)$$

The following Tables (Table 10, Table 11, and Table 12) are necessary to define the aforementioned coefficients.

Table 10. Ship form coefficient (C_{form})

Type of (displacement) ship	Ship form coefficient (C_{form})
All ships (except container ships) in loaded loading condition	$0.5BN + BN^{6.5}/(2.7 \cdot \Delta^{2/3})$
All ships (except container ships) in ballast loading condition	$0.7BN + BN^{6.5}/(2.7 \cdot \Delta^{2/3})$
Container ships in normal loading conditions	$0.7BN + BN^{6.5}/(22 \cdot \Delta^{2/3})$

Table 11. Speed reduction coefficient (C_u)

Block Coefficient	Ship Loading Condition	Speed reduction coefficient (C_u)
0.55	Normal	$1.7 – 1.4 \cdot Fn – 7.4 \cdot Fn^2$
0.6	Normal	$2.2 – 2.5 \cdot Fn – 9.7 \cdot Fn^2$
0.65	Normal	$2.6 – 3.7 \cdot Fn – 11.6 \cdot Fn^2$
0.7	Normal	$3.1 – 5.3 \cdot Fn – 12.4 \cdot Fn^2$
0.75	Loaded or Normal	$2.4 – 10.6 \cdot Fn – 9.5 \cdot Fn^2$
0.8	Loaded or Normal	$2.6 – 13.1 \cdot Fn – 15.1 \cdot Fn^2$
0.85	Loaded or Normal	$3.1 – 18.7 \cdot Fn + 28.0 \cdot Fn^2$
0.75	Ballast	$2.6 – 12.5 \cdot Fn – 13.5 \cdot Fn^2$
0.8	Ballast	$3.0 – 16.3 \cdot Fn – 21.6 \cdot Fn^2$
0.85	Ballast	$3.4 – 20.9 \cdot Fn + 31.8 \cdot Fn^2$

Table 12. Weather direction reduction coefficient (C_β)

Weather Direction	Encounter angle (degrees)	Direction reduction coefficient (C_β)
Head sea (irregular wave) and wind	0 – 30	$2 \cdot C_\beta = 2$
Bow sea (irregular wave) and wind	30 – 60	$2 \cdot C_\beta = 1.7 – 0.03 \cdot ((BN\text{-}4)^2)$
Beam sea (irregular wave) and wind	60 – 150	$2 \cdot C_\beta = 0.9 – 0.06 \cdot ((BN\text{-}6)^2)$
Following sea (irregular wave) and wind	150 – 180	$2 \cdot C_\beta = 0.4 – 0.03 \cdot ((BN\text{-}8)^2)$

In this study a linear interpolation is used in order to find the speed reduction coefficient. The area of the linear interpolation is:

$$C_b = [0.55 – 0.7].$$

The following figures (Figure 20, Figure 21) are provided to justify the need of a new filtering procedure, only for the Kwon Method.

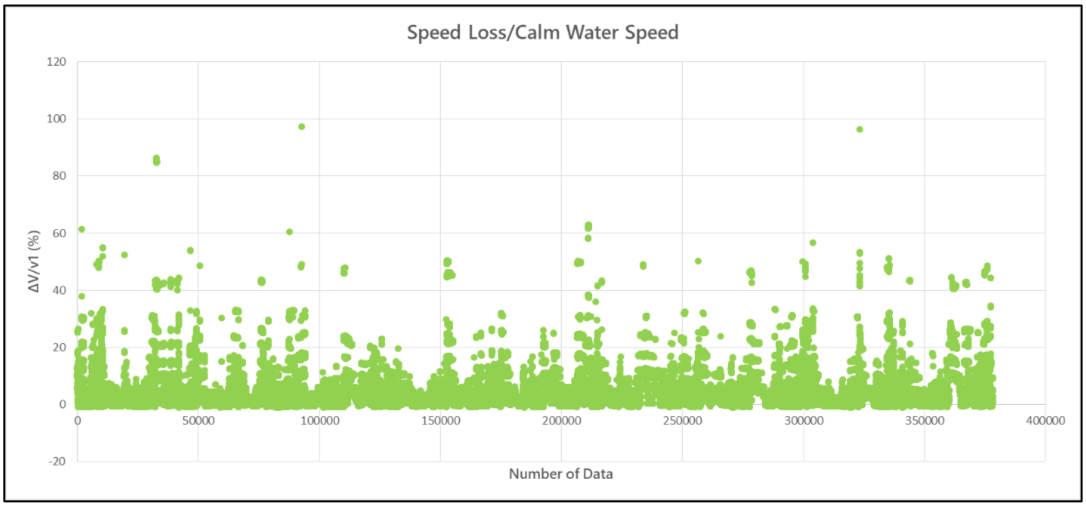


Figure 20. Speed Loss/Calm Water Speed (Unfiltered)

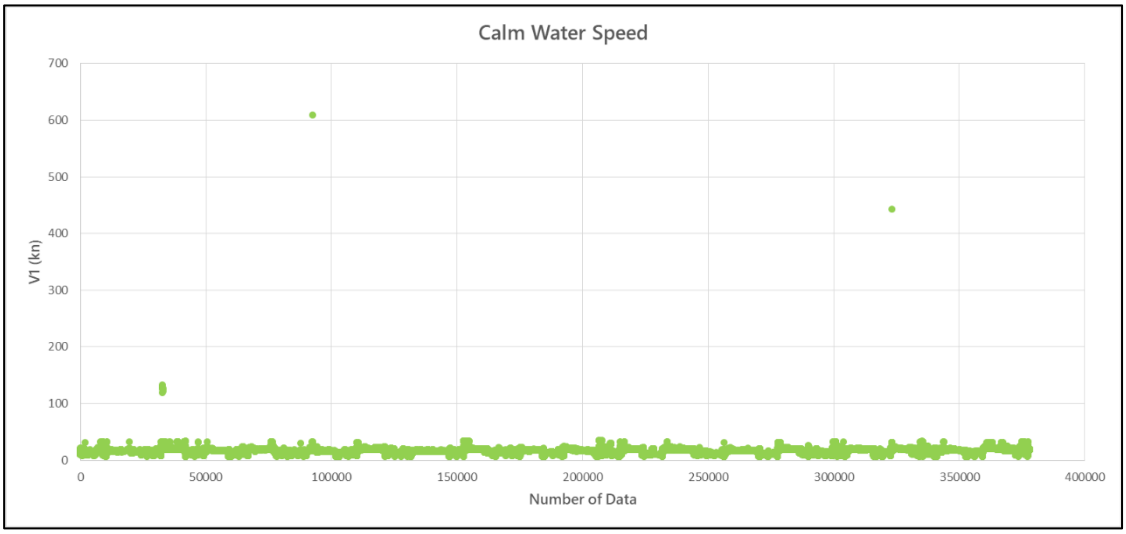


Figure 21. Calm Water speed (Unfiltered Data)

As it can be seen from the two graphs, a few data points appear to have very high calm water speed and the Speed Loss/Calm Water Speed is in need of a threshold value, in order to discard the data points that have higher values than the threshold parameter.

3.1.4 Filtering Procedure (Kwon Method)

In this sub-section, two new filters are applied to the dataset. The first one is a simple inequality. The inequality is the following: Calm Water speed > 24 kn. If this criterion is met, then the data points are considered unfit and will be excluded from the dataset. The threshold value of 24 knots is provided by the maximum amount of speed the ship can achieve in the sea trials.

The second filter is used, because Kwon's method is empirical. To achieve a fair estimation of fuel oil consumption, it's necessary to discard the data points that comply with the following formula:

$$\Delta V = (V_1 - V_2) / V_1 \% > 20\% \quad (3.1.18)$$

Where,

- V_1: Design (nominal) operating ship speed in still water conditions (no wind, no waves), given in m/s.
- V_2: Actual ship speed in selected weather (wind and irregular waves) conditions.

The absolute speed loss is heavily influenced by the weather conditions that the ship has to overcome (Beaufort scale, Encounter angle). The reasoning behind this filter is to eliminate the values of the Beaufort scale that cause a huge change between calm water speed and speed in irregular waves and wind.

For the following calculation a specific loading condition is used (Displacement – Draft – Block coefficient). The variable that is being changed through the whole filtering procedure is the calm water speed. The loading condition is the following:

- Δ = 35000 tons.
- C_b = 0.6250

The specific values illustrate the mean value of the displacements and block coefficient, the ship achieves during its 18 months of travel (data sampling period). The variation of the calm water speed is from 5 m/s to 9 m/s, the Beaufort scale is from 0 – 10 BN and there are four different categories of Encounter angles (Head, Bow, Beam, Following). Moreover, the direction reduction coefficient (C_β) is presented in the following tables (Table 13, Table 14, Table 15, Table 16) and figures (Figure 22, Figure 23) to illustrate the problematic nature of the semi – empirical Kwon method (Kwon, 2008).

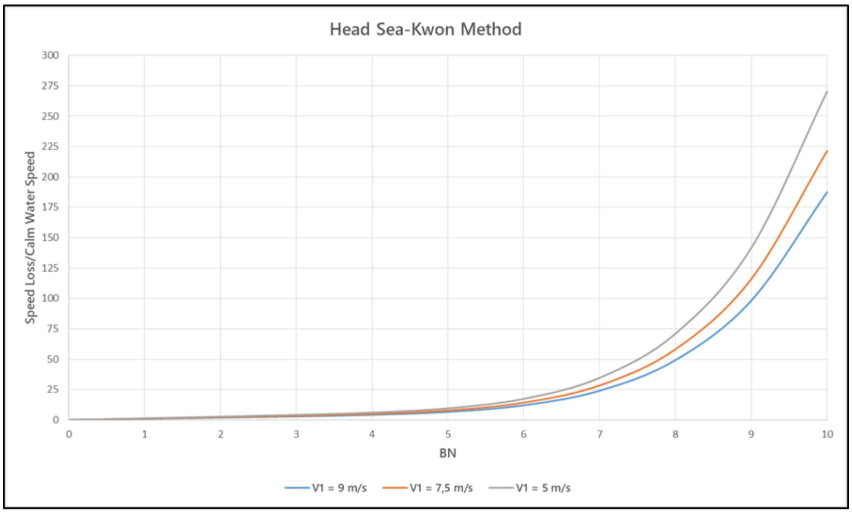
Table 13. Speed Loss/Calm Water speed for Head Sea.

Head Sea				
BN	ΔV/V1 for V1 = 9 m/s	ΔV/V1 for V1 = 7,5 m/s	ΔV/V1 for V1 = 5 m/s	10 · Cβ
0	0	0	0	10
1	0.9287	1.0969	1.3390	10
2	1.8624	2.1997	2.6852	10
3	2.8571	3.3745	4.1194	10
4	4.1763	4.9326	6.0215	10
5	6.6123	7.8099	9.5338	10
6	12.0127	14.1883	17.3203	10
7	24.0432	28.3975	34.6661	10
8	49.2163	58.12964	70.9613	10
9	98.2110	115.9975	141.6031	10
10	187.5067	221.4651	270.3520	10

Table 14. Speed Loss/Calm Water speed for Bow Sea.

Bow Sea				
BN	ΔV/V1 for V1 = 9 m/s	ΔV/V1 for V1 = 7.5 m/s	ΔV/V1 for V1 = 5 m/s	10 · Cβ
0	0	0	0	6.1
1	0.6640	0.7842	0.9574	7.15
2	1.4713	1.7377	2.1213	7.9
3	2.3857	2.8177	3.4397	8.35
4	3.5498	4.1927	5.1182	8.5
5	5.5213	6.5213	7.9607	8.35
6	9.4900	11.2088	13.6830	7.9
7	17.1908	20.3042	24.7862	7.15
8	30.0219	35.4590	43.2864	6.1
9	46.6502	55.0988	67.2615	4.75
10	58.1270	68.6542	83.8091	3.1

In the following page a series of graphs is going presented to show the Speed Loss/Calm Water Speed variations for the encounter angles (Head, Bow Sea) as a function of the Beaufort Number. The yellow part of each Table is the outlying regions for each encounter angle respectively. For example, for Bow Sea (30 – 60 degrees) any relative wind speed greater than 7 BN is not used for the estimation of the fuel oil consumption (Figure 22). The direction reduction coefficient is used to show the impact of the weather condition.



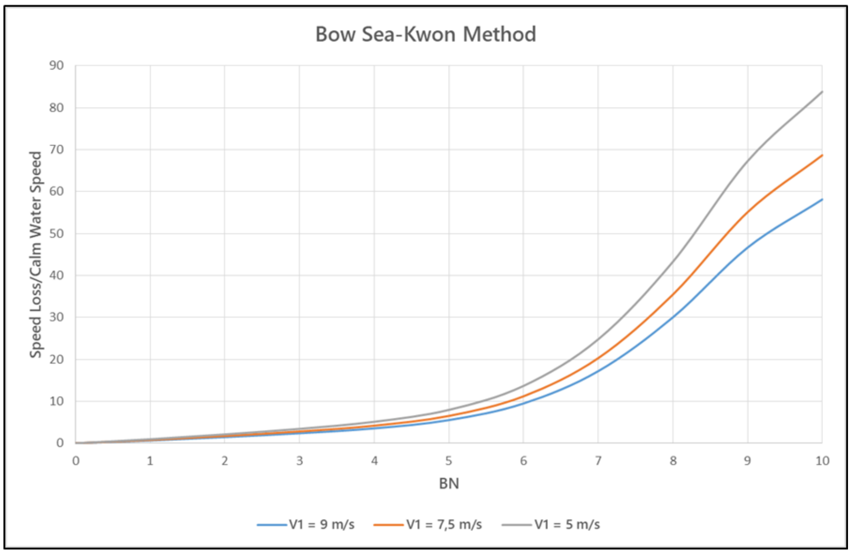


Figure 22. Speed Loss/Calm Water Speed (Head, Bow Sea)

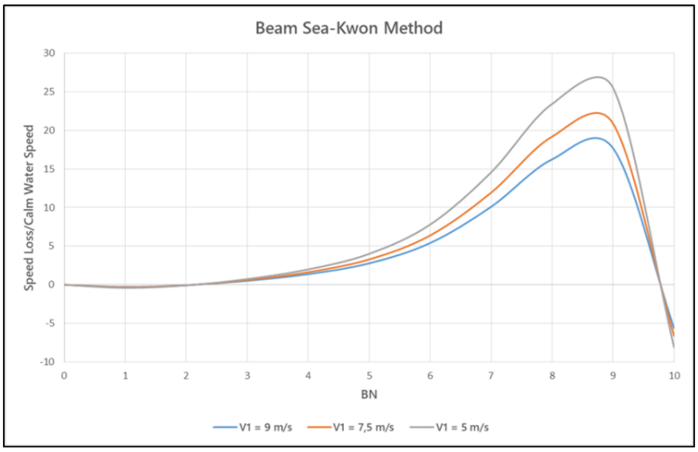
Table 15. Speed Loss/Calm Water speed for Beam Sea.

Beam Sea				
BN	ΔV/V1 for V1 = 9 m/s	ΔV/V1 for V1 = 7.5 m/s	ΔV/V1 for V1 = 5 m/s	10 · Cβ
0	0	0	0	-6.3
1	-0.2786	-0.3291	-0.4017	-3
2	-0.0558	-0.0660	-0.0805	-0.3
3	0.5142	0.6074	0.7415	1.8
4	1.3781	1.6277	1.9871	3.3
5	2.7772	3.2801	4.0042	4.2
6	5.4057	6.3847	7.7941	4.5
7	10.0981	11.9270	14.5598	4.2
8	16.2413	19.1827	23.4172	3.3
9	17.6779	20.8796	25.4885	1.8
10	-5.62520	-6.6439	-8.1105	-0.3

Table 16. Speed Loss/Calm Water speed for Following Sea.

Following Sea				
BN	ΔV/V1 for V1 = 9 m/s	ΔV/V1 for V1 = 7.5 m/s	ΔV/V1 for V1 = 5 m/s	$10 \cdot C\beta$
0	0	0	0	-7.6
1	-0.4968	-0.5868	-0.7163	-5.35
2	-0.6332	-0.7479	-0.9129	-3.4
3	-0.4999	-0.5905	-0.7209	-1.75
4	-0.1670	-0.1973	-0.2408	-0.4
5	0.4298	0.5076	0.6197	0.65
6	1.6817	1.9863	2.4248	1.4
7	4.4479	5.2535	6.4132	1.85
8	9.8432	11.6259	14.1922	2
9	18.1690	21.4595	26.1965	1.85
10	26.2509	31.0051	37.8492	1.4

In the following page two graphs are going to be presented to show the Speed Loss/Calm Water Speed variations for the Beam Sea and the Following Sea (Encounter Angle) as a function of the Beaufort Number (Figure 23). The yellow part of each Table is the outlying regions for each encounter angle respectively.



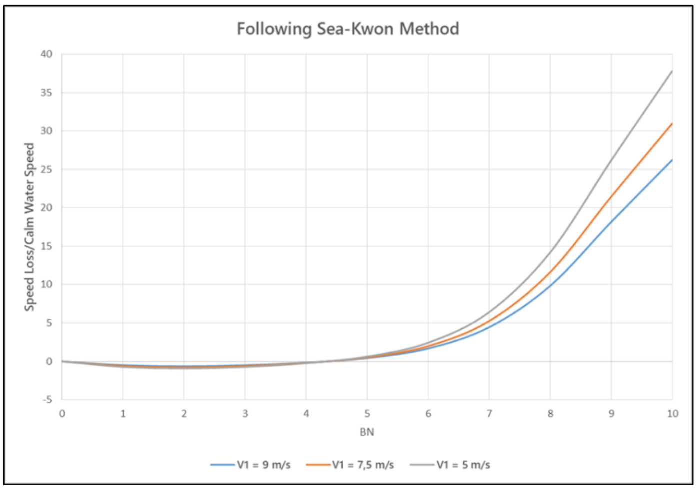


Figure 23. Speed Loss/Calm Water Speed (Beam, Following Sea)

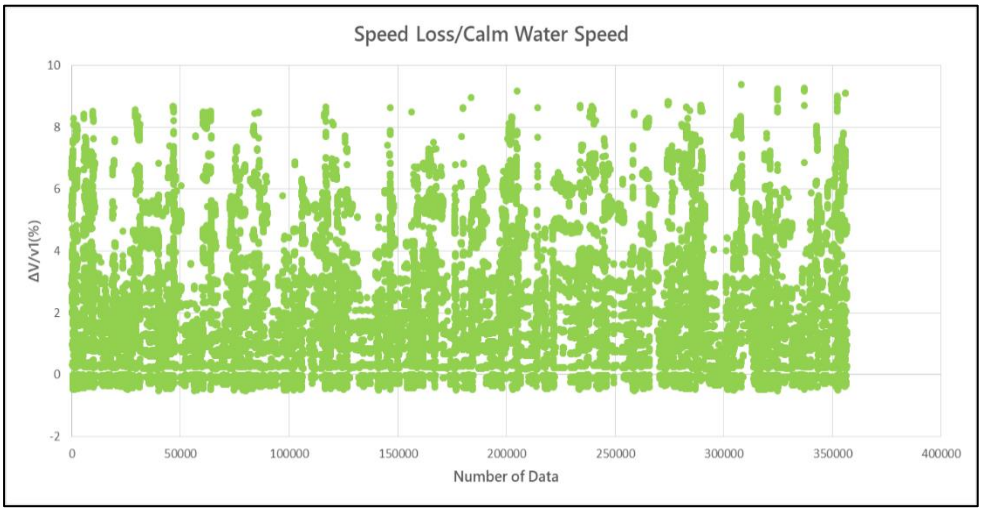
The following limitations arise:

- Head Sea (0-30 degrees): BN ≥ 7
- Bow Sea (30-60 degrees): BN ≥ 7
- Beam Sea (60-150 degrees): BN ≥ 8
- Following Sea (150-180 degrees): BN ≥ 9

The fundamental mistake of the method is that it doesn't provide the best results for Wind Speed greater than 6 BN. Mostly because the wind, when it is regarded as Following Sea, under normal circumstances provides a boost to the vessel but with the use of the Kwon method it appears that for wind speeds higher than 5 BN the direction reduction coefficient, causes a speed loss and not an increase in speed. Moreover, the graphs of the weather direction reduction coefficient (C_β), which explains the aforementioned fundamental mistake are illustrated in Appendix B.

As a result the end result graphs will illustrate the relationship of the actual ship speed and the fuel oil consumption in the area of 1 BN to 6 BN in the Beaufort scale, in which areas the error of the empirical formula (Kwon, 2008) is minimal.

The results of the two new filters are a 0.625 % elimination and a 5.12 % elimination of data points for each filter respectively and the results of the filters are shown in the following figures (Figure 24, Figure 25).



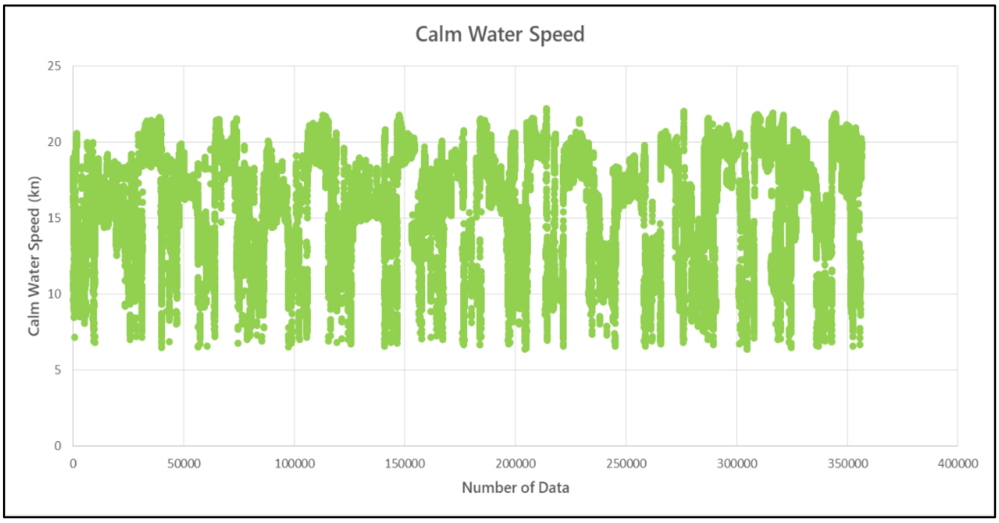


Figure 24. Results of the Filtering Procedure (Kwon Method)

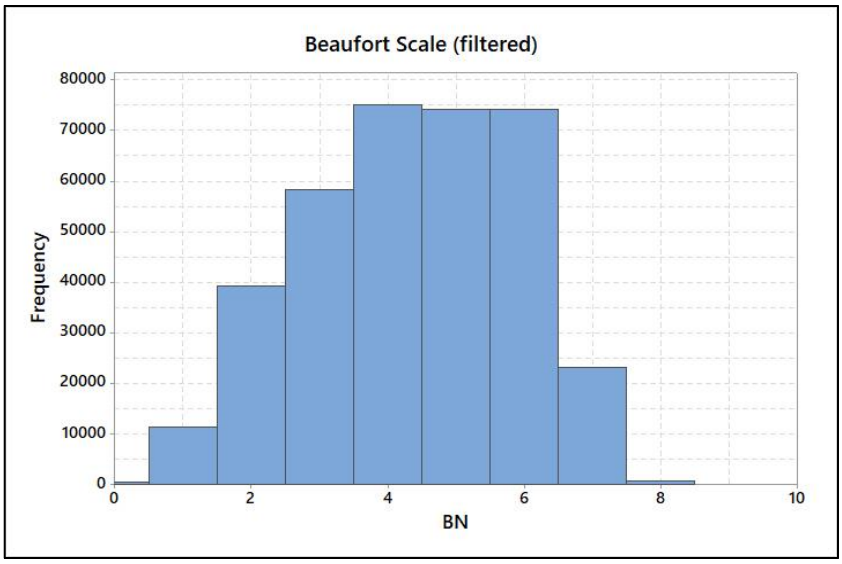


Figure 25. Beaufort scale for Kwon method (filtered data)

3.1.5 Ship Operational performance modelling

In this particular sub – section the resistance in calm water is going to be calculated using the Holtrop & Mennen method (Holtrop & Mennen, 1982). Moreover, the effective power of the ship is going to be calculated. All the formulas provided in this particular sub – section are developed by either Holtrop & Mennen (1982) or Krinstensen & Lützen (2013).

$$P_e = R_{total} \cdot V \quad (3.1.19)$$

The resistance in calm water is provided by the following formula:

$$R_{total} = R_F \cdot (1+k_1) + R_{APP} + R_w + R_B + R_{TR} + R_A \quad (3.1.20)$$

Where,

- ❖ R_{total}: Total resistance of the ship in calm water.
- ❖ R_F: Frictional resistance according to the ITTC – 1957 friction formula.

$$R_F = 0.5 \cdot V^2 \cdot S \cdot C_F \cdot \rho \quad (3.1.21)$$

$$C_F = 0.075/ (Log_{10}Re - 2)^2 \quad (3.1.22)$$

$$Re = \rho \cdot V \cdot L/\mu \quad (3.1.23)$$

$1 + \kappa_1$: Form Factor describing the viscous resistance of the hull form in relation to R_F

- ❖ R_{APP}: Resistance of appendages

$$R_{APP} = 0.5 \cdot \rho \cdot V^2 \cdot S_{APP} \cdot (1+k_2)_{eq} \cdot C_F \quad (3.1.24)$$

Where, ρ is the water density, V the speed of the vessel in calm water, S_{APP} the wetted area of the appendages, $1+\kappa_2$ the appendage resistance factor and C_F the coefficient of frictional resistance of the ship according to the ITTC – 1957 formula.

❖ R_w: Wave – making and wave – breaking resistance

$$R_w = c_1 \cdot c_2 \cdot c_5 \cdot \nabla \cdot \rho \cdot g \cdot \exp\{m_1 \cdot F_n^d + m_2 \cdot \cos(\lambda \cdot F_n^{-2})\} \quad (3.1.25)$$

❖ R_B: Additional pressure resistance of bulbous bow near the water surface

$$R_B = 0.11 \exp(-3 \cdot P_B^{-2}) \cdot F_{ni}^3 \cdot A_{BT}^{1.5} \cdot g \cdot \rho / (1+F_{ni}^2) \quad (3.1.26)$$

❖ R_{TR}: Additional pressure resistance of immersed transom stern

$$R_{TR} = 0.5 \cdot V^2 \cdot A_T \cdot C_6 \cdot \rho \quad (3.1.27)$$

❖ R_A: model – ship correlation resistance

$$R_A = 0.5 \cdot V^2 \cdot S \cdot C_A \cdot \rho \quad (3.1.28)$$

$$C_A = 0.006 \cdot L + 100^{-0.16} - 0.00205 + 0.003 \cdot \sqrt{\frac{L}{7.5}} \cdot C_B^4 \cdot C_2 \cdot (0.04 - C_4) \quad (3.1.29)$$

It’s also necessary to provide the basic parameters that were used in order to calculate the total resistance (Table 17).

Table 17. Holtrop & Mennen (Basic Parameters)

Name	*Symbol*	*Formula*	*Units*
Length of Waterline	L	$1.015 \cdot L_{PP}$	m
Length between perpendiculars	L_{BP}	199	m
Breadth	B	30.2	m
Draught on F.P	T_F	T + trim	m
Draught on A.P	T_A	T – trim	m
Displacement Volume	∇	$C_B \cdot L_{BP} \cdot B \cdot T$	m^3
Longitudinal center of buoyancy	lcb	$8.8 - 38.9 \cdot F_n$	m
Transverse bulb area	A_{BT}	$C_{BB} \cdot T \cdot C_M \cdot B$	m^2
Center of bulb area above keel line	h_b	$C_{ZB} \cdot T$	m
Midship section coefficient	C_M	$1.006 - 0.0056 \cdot C_B^{-3.56}$	
Waterplane area coefficient	C_{WP}	$0.95 \cdot C_P + 0.17 \cdot (1 - C_P)^{(1/3)}$	
Transom area	A_T	$0.07 \cdot A_M$	m^2
Wetted area appendages	S_{APP}	50	m^2
Propeller diameter	Dp	7200	mm
Clearance propeller with keel line		$0.035 \cdot D$	m
Number of propeller blades	Z	5	
Ship’s Speed	V	Calm Water Speed	m/s
Stern Shape Parameter	C_{stern}	10	
Block coefficient	C_B	$C_{b0} \cdot (T/T_0)$^$((C_{WL}/C_{b0})-1)$	
Prismatic coefficient	C_P	C_M/C_B	
Midship section area	A_M	$B \cdot T \cdot C_M$	m^2
Depth	D	16.7	m
Height coefficient	C_{ZB}	0.405	
Breadth coefficient (Kracht, 1970)	C_{BB}	0.17	
Froude Number	F_n	g/(L · V)^(1/2)	
Froude Number based on the immersion	Fn_i	$V/\sqrt{g \cdot (T_F - h_b - 0.25 \cdot \sqrt{A_{BT}}) + 0.15 \cdot V^2}$	
Coefficient of emergence of the bow	P_B	$0.56 \cdot \sqrt{A_{BT}}/(T_F - 1.5 \cdot h_b)$	

By using the aforementioned formulas, the total resistance, of the ship is calculated, for different loading conditions. The results of the total resistance calculation are summed in a graph using the total resistance divided by the displacement force as a function of the Froude number (Fn) (Figure 26).

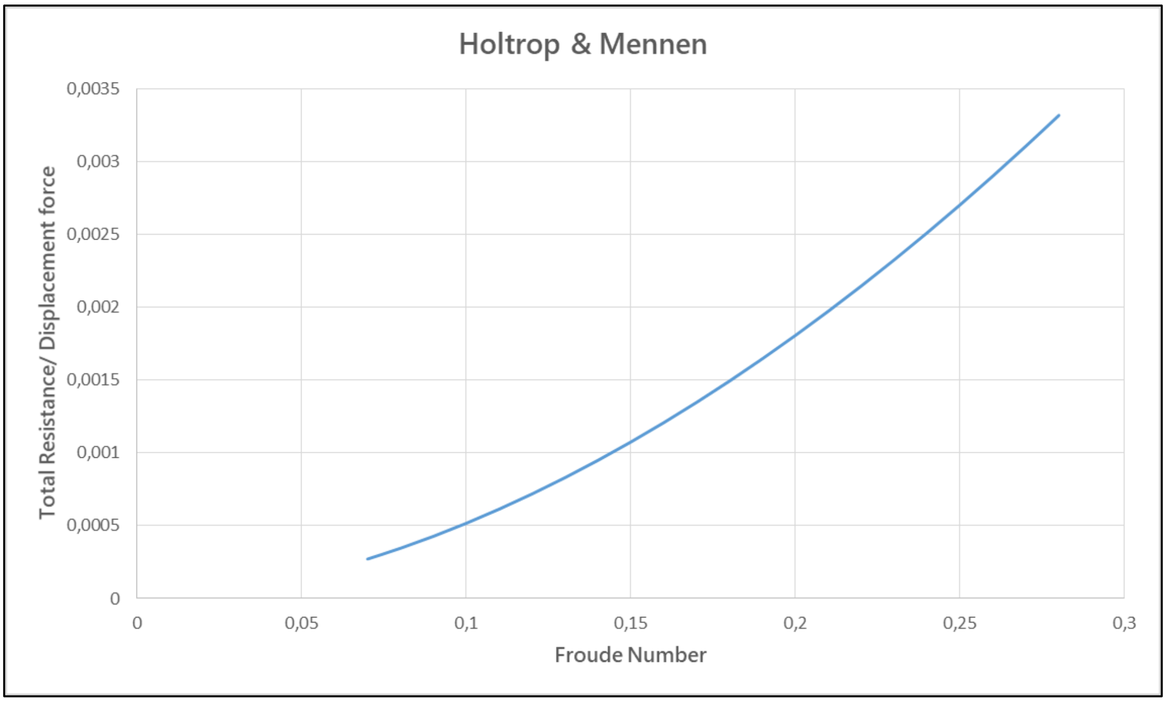


Figure 26. Relation between Total resistance (calm water) / Displacement force and Froude number

Since the total resistance in calm water conditions is found, the effective Power can be calculated using the formula:

$$P_e = R_{total} \cdot V \quad (3.1.30)$$

The next step, in order to calculate the brake Power, is to calculate the propulsive coefficient, which consists of four coefficients. The hull efficiency, the open water efficiency, the relative rotative efficiency and the shaft efficiency. The following formulas, for the calculation of the open water efficient are developed by Krinstensen & Lützen (2013), and are based on the Andersen & Berslin (Andersen, 1994) graphs. The arithmetical approximation, introduced by Krinstensen, applies only for propellers of the Wageningen B – series. Since, the propeller type is not provided in the Sea Trials, it's considered to be a propeller of the Wageningen B – series. The rest of the functions are proposed by Holtrop & Mennen (1982).

$$n_T = n_o \cdot n_H \cdot n_R \cdot n_S \quad (3.1.31)$$

Where,

- ❖ n_H: The hull efficiency

$$n_H = \frac{1-t}{1-w} \quad (3.1.32)$$

$$t = 0.25014 \cdot (B/L)^{0.28956} \cdot (\sqrt{BT}/D)^{0.2624} / (1 - C_P + 0.0225 \cdot lcb)^{0.01762} + 0.0015 \cdot C_{stern}$$

$$(3.1.33)$$

$w = C_9 \cdot C_V \cdot L/T_A \cdot (0.0661875 + 1.21756 \cdot C_{11} \cdot \frac{CV}{1-CP1}) + 0.24558 \cdot \sqrt{\frac{B}{L \cdot (1-CP1)}} - \frac{0.09726}{0.95-CP} + \frac{0.11434}{0.95-CP} + 0.75 \cdot C_{stern} \cdot C_V \cdot 0.002 \cdot C_{stern}$ (3.1.34)

Where,

- t: Thrust coefficient (Holtrop & Mennen, 1982)
- w: Wake friction (Holtrop & Mennen, 1982)
- C_P: The prismatic coefficient (mentioned in Table 10)
- n_R: The relative rotative efficiency (Holtrop & Mennen, 1982)

$$n_R = 0.9922 - 0.05908 \cdot A_E/A_O + 0.07424 (C_P - 0.0225 \cdot lcb) \quad (3.1.35)$$

- n_S: The shaft efficiency (shaft line, gearbox)

$$n_S = 0.99 \quad (3.1.36)$$

- n_o: open water efficiency

The calculation of the open water efficiency is based on the Breslin & Andersen curves. The open water efficiency is described with the help of the thrust loading coefficient C_{Th} (Krinstensen & Lützen, 2013).

$$C_{Th} = \frac{T}{0.5 \cdot Adisk \cdot \rho \cdot VA^2} \quad (3.1.37)$$

$$C_{Th} = 8/\pi \cdot \frac{R}{(1-t) \cdot \rho \cdot (VA \cdot Dp)^2} \quad (3.1.38)$$

$$C_{Th} = 8/\pi \cdot K_T/J^2 \quad (3.1.39)$$

$$J = V_A/n \cdot D \quad (3.1.40)$$

$$K_T = \frac{R}{(1-t) \cdot \rho \cdot n^2 \cdot Dp^4} \quad (3.1.41)$$

$$R = (1 - t) \cdot T \quad (3.1.42)$$

$$V_A = (1 - w) \cdot V \quad (3.1.43)$$

In the particular study it's considered that the open water efficiency is calculated by the following formula:

$$n_o = \frac{2}{1+\sqrt{CTh+1}} \cdot \text{Max}\ \{0.65; (0.81 - 0.014 \cdot C_{Th})\} \quad (3.1.44)$$

In this manner the propulsion coefficient is provided, according to equation (3.1.31).

The distribution of the open water efficiency is shown in the following histogram (Figure 27). The propulsive coefficient can be concluded based on the specific ship operational performance conditions and sea trial data.

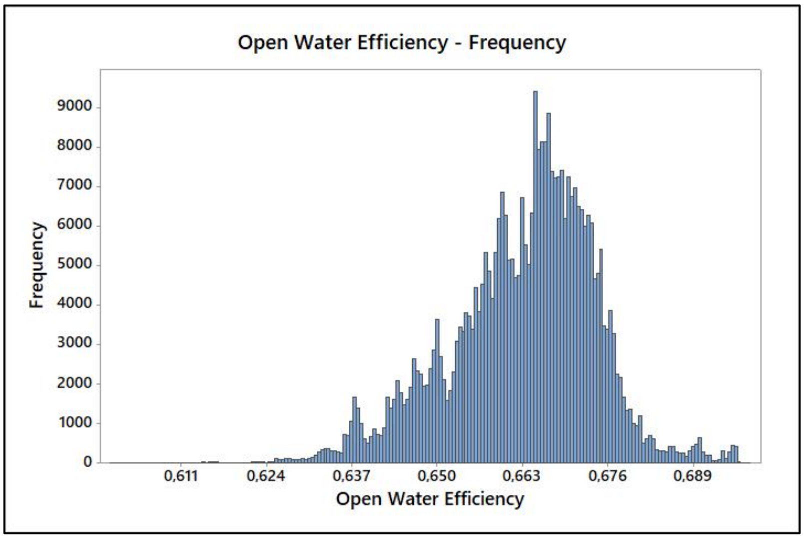


Figure 27. Distribution of the open water efficiency

After calculating the propulsive coefficient, the brake Power can be defined by using the formula:

$$P_B = P_e/n_T \quad (3.1.45)$$

It shall be noted at this point that the most appropriate solution for the calculation of the delivered power of the vessel is the use of open water diagrams, provided they are available for the specific propeller type. In that case, the open water efficiency can be calculated accurately, from the respective open water diagram is provided. For the sake of completion, in Appendix D this calculation procedure is presented.

3.1.6 Estimation of the fuel oil consumption

The calculation of the fuel oil consumption is completed by finding the Specific fuel oil consumption (SFOC) of the main engine. To calculate the SFOC for a significant amount of loads it's necessary to review the main engine project guide and find the SFOC for an acceptable amount of data, as well as the fitted data (polynomial interpolation) and the error margins of the polynomial interpolation (Table 18).

Table 18. SFOC from Project Guide, SFOC fitted and the polynomial

Engine type: Hyundai – Wärtsilä 7RTA72U-B			
Load (%)	SFOC (g/kWh)	SFOC fitted (g/kWh)	Error
25	181.6012	181.6079	0,004%
30		180.1859	
40		177.1323	
50	174.3329	174.2277	0,060%
55	172.9918	172.9668	0,014%
60	171.6508	171.8926	0,141%
75	170.3098	170.1147	0,115%
85	170.3098	170.3420	0,019%
95	171.6508	171.7690	0,069%
100	172.9918	172.9178	0,043%

Since the basic SFOC curve is provided, a 4^{th} degree polynomial interpolation is used to describe the SFOC curve for loads smaller than 25% of the MCR.

The used polynomial interpolation is the following:

$SFOC = -4.45385E\text{-}07 \cdot Load^4 + 0.000150266 \cdot Load^3 - 0.01296698 \cdot Load^2 + 0.12428457 \cdot Load + 184.4313098$ (3.1.54)

Where,

- ❖ Load = P_B/MCR (%)
- ❖ MCR = 21560 kW (Sea Trials)

The following figure (Figure 28) is used to show the curve of the specific fuel oil consumption.

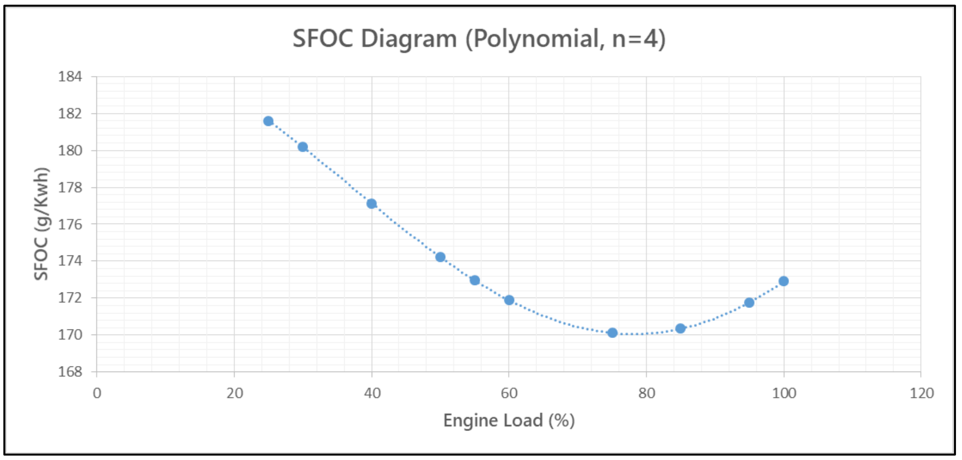


Figure 28. Specific fuel oil consumption (curve)

The maximum amount of error is 0.15%. In this case the polynomial interpolation is considered robust and acceptable.

The relationship between the speed through water, normalized for the two reference loading conditions (Mean Draft = 9.3m, 10.5 m), and the estimated fuel oil consumption provided by the theoretical method (Kwon) is in display in the following figures (Figure 29, Figure 30).

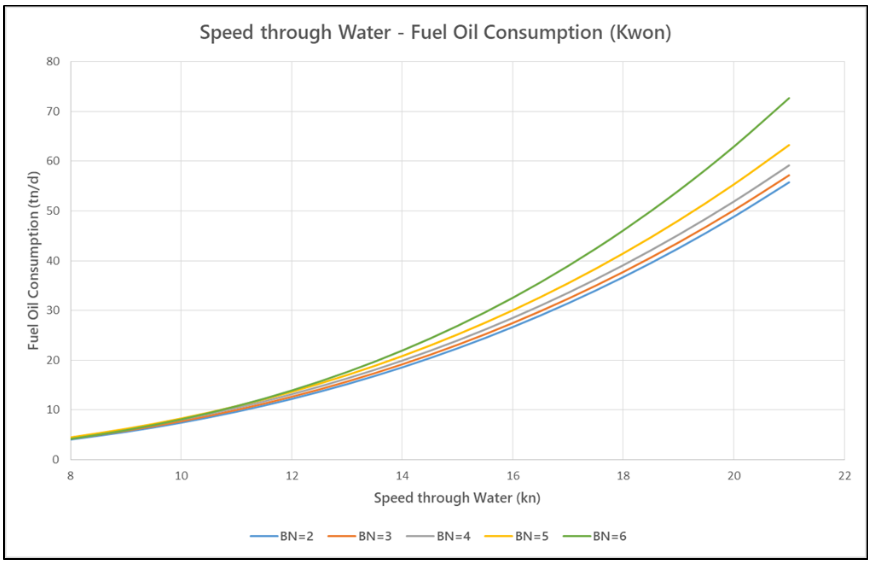


Figure 29. Estimation of the Fuel Oil Consumption for different Beaufort Number (Kwon method, draft = 9.3m)

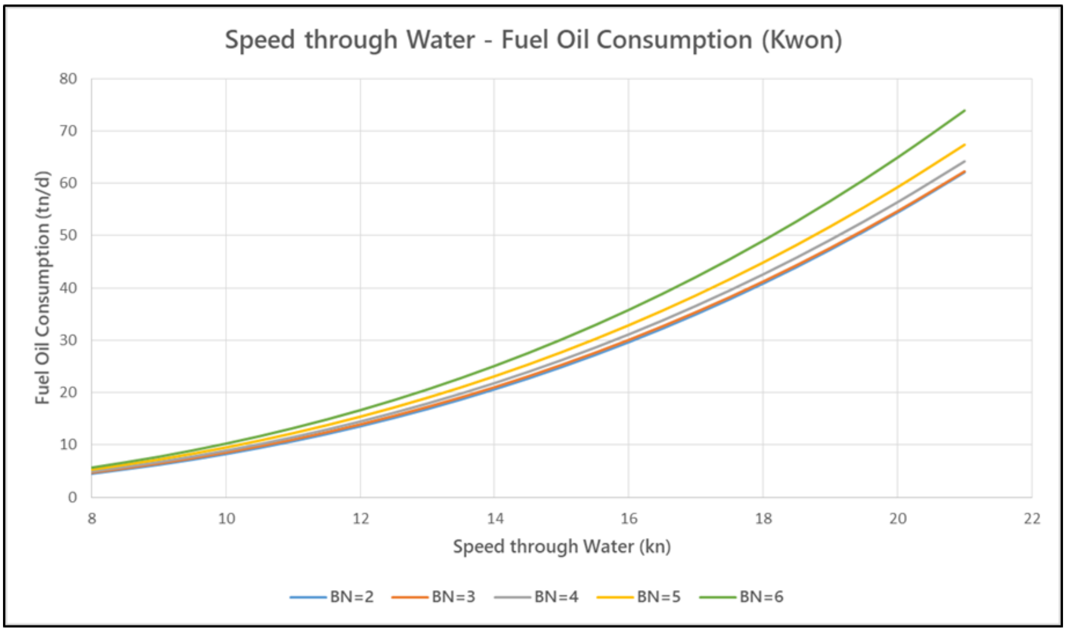


Figure 30. Estimation of the Fuel Oil Consumption for different Beaufort Number (Kwon method, draft = 10.5m)

3.2 ITTC Method

The second theoretical model is based on the methodology, provided by the ITTC (7.5–04–01–01.1). The ITTC method is based on the power correction due to the added resistance caused by wind and wave effects. The three basic formulas, used in the ITTC method, are the following:

$$\Delta R = R_{wind} + R_{wave} \quad (3.2.1)$$

$$\Delta P = \Delta R \cdot \frac{Vs}{n_{Do}} + P_D \cdot \left(1 - \frac{n_{DM}}{n_{Do}}\right) \quad (3.2.2)$$

$$P_{Dcorr} = P_D - \Delta P \quad (3.2.3)$$

Where,

- R_{wind}: The added resistance due to the wind effects (N).
- R_{wave}: The added resistance due to the wave effects (N).
- Vs: Ship's speed through water (m/s).
- n_{DM}: Propulsive efficiency coefficient during sea trial.
- n_{Do}: Propulsive efficiency coefficient in ideal condition obtained from standard towing tank test and interpolated to the speed Vs.
- P_D: Delivered engine power, in this case it's considered to be the propeller shaft Power, that is provided by the ship's sensors and measuring devices.

The following flow diagram is necessary for the explanation of the ITTC method (2017), (Figure 31).

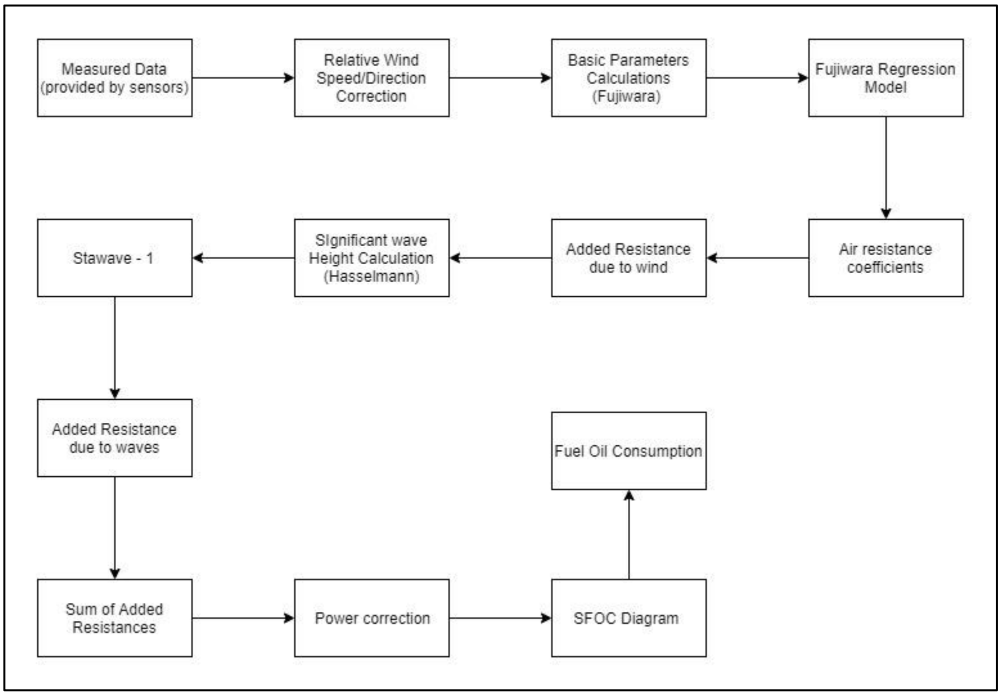


Figure 31. Flow Diagram for the ITTC method (2017)

3.2.1 Added resistance due to wind

In order to calculate the added resistance due to wind, it's necessary to use the relative wind speed and relative wind direction in the reference height (Chapter 3.1.1). The resistance due to relative wind is calculated by:

$$R_{AA} = R_{rw} – R_{ow} \quad (3.2.4)$$

Where,

$$R_{rw} = 0.5 \cdot \rho_\alpha \cdot V_{wr}^2 \cdot A \cdot C_{DA} (\psi_{wr,ref}) \quad (3.2.5)$$

$$R_{ow} = 0.5 \cdot \rho_\alpha \cdot V_g^2 \cdot A \cdot C_{DA} (0) \quad (3.2.6)$$

Where,

- ρ_α: Mass density of air (kg/m^3)
- V_{wr}: Relative wind speed at reference height (m/s)
- A_{XV}: Area of maximum transverse section exposed to the wind (m^2)
- $\psi_{wr,ref}$: Relative wind direction at reference height (degrees)
- C_{DA}: Wind resistance coefficient
- V_g: Measured ship's speed over ground (m/s)

The calculation of the wind resistance coefficient is provided by the regression formula Fujiwara, 2015 (ITTC (7.5-04-01-01.1)).

$$C_{DA} = C_{LF} \cdot cos\,(\Psi_{wr}) + C_{XLI} \cdot (sin\,(\Psi_{wr}) – 0.5 \cdot sin\,(\Psi_{wr}) \cdot cos\,(\Psi_{wr})^2\,) \cdot sin\,(\Psi_{wr}) \cdot cos\,(\Psi_{wr}) + C_{ALF} \cdot sin\,(\Psi_{wr}) \cdot cos\,(\Psi_{wr})^3 \quad (3.2.7)$$

Where,

Για 0 < Ψ_{wr} < 90 ° :

$$C_{LF} = \beta 10 + \beta 11 \cdot \frac{A_{YV}}{L_{OA} \cdot B} + \beta 12 \cdot \frac{C_{MC}}{L_{OA}} \text{ (3.2.8)}$$

$$C_{XLI} = \delta 10 + \delta 11 \cdot \frac{A_{YV}}{L_{OA} \cdot h_{BR}} + \delta 12 \cdot \frac{A_{YV}}{h_{BR} \cdot B} \text{ (3.2.9)}$$

$$C_{LF} = \varepsilon 10 + \varepsilon 11 \cdot \frac{A_{OD}}{A_{YV}} + \varepsilon 12 \cdot \frac{B}{L_{OA}} \text{ (3.2.10)}$$

Για 90 < Ψ_{wr} < 180 ° :

$$C_{LF} = \beta 20 + \beta 21 \cdot \frac{A_{YV}}{L_{OA} \cdot B} + \beta 22 \cdot \frac{C_{MC}}{L_{OA}} + \beta 23 \cdot \frac{A_{OD}}{L_{OA}^2} + \beta 24 \cdot \frac{A_{XV}}{B^2} \text{ (3.2.11)}$$

$$C_{XLI} = \delta 20 + \delta 21 \cdot \frac{A_{YV}}{L_{OA} \cdot h_{BR}} + \delta 22 \cdot \frac{A_{XV}}{A_{YV}} + \delta 23 \cdot \frac{B}{L_{OA}} + \delta 24 \cdot \frac{A_{XV}}{B \cdot h_{BR}} \text{ (3.2.12)}$$

$$C_{LF} = \varepsilon 20 + \varepsilon 21 \cdot \frac{A_{OD}}{A_{YV}} \text{ (3.2.13)}$$

Για Ψ_{wr} = 90 °

$$C_{DA_{\psi wr=90°}} = 0.5 \cdot (C_{DA_{\psi wr=90°-\mu}} + C_{DA_{\psi wr=90°+\mu}}) \text{ (3.2.14)}$$

Where,

- A_{OD}: Lateral projected area of superstructures etc. on deck
- A_{XV}: Area of maximum transverse section exposed to winds
- A_{YV}: Projected lateral area above the waterline
- B: Ship breadth
- C_{DA}: Wind resistance coefficient
- C_{MC}: Horizontal distance from Midship section to center of lateral projected area A_{YV}.
- h_{BR}: Height of top of superstructure (bridge etc.)
- h_C: Height from waterline to center of lateral projected area A_{YV}.
- L_{OA}: Length overall
- μ: smoothing range, normally 10 degrees
- Ψwr: relative wind direction, where 0 means heading winds

The non – dimensional parameters $\beta ij, \delta ij, \varepsilon ij$ used in the formulae are shown in Table 19.

Table 19. Non - dimensional parameters

	i	j				
		0	1	2	3	4
β_{ij}	1	0,922	-0,507	-1,162	-	-
	2	-0,018	5,091	-10,367	3,011	0,341
δ_{ij}	1	-0,458	-3,245	2,313	-	-
	2	1,901	-12,727	-24,407	40,310	5,481
ε_{ij}	1	0,585	0,906	-3,239		-
	2	0,314	1,117	-	-	-

For the better understanding of the parameters used in the regression formula the following figure (Figure 32) is provided.

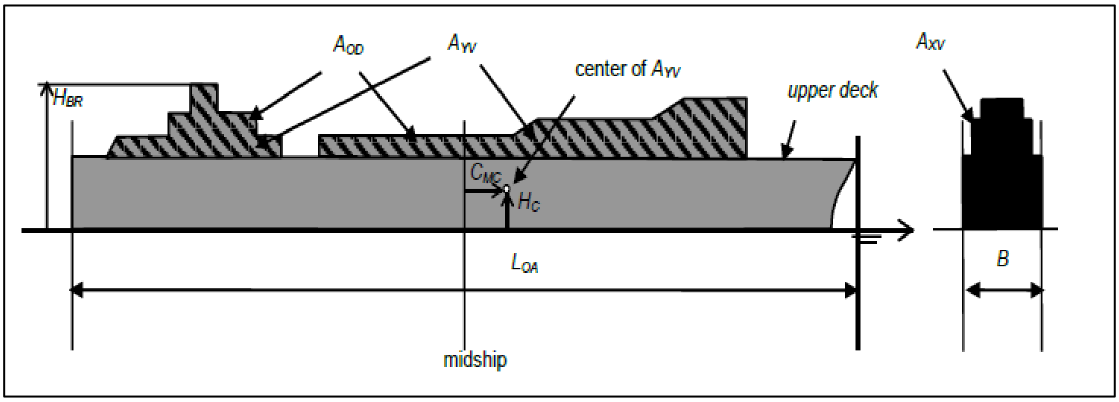


Figure 32. Parameters of the Fujiwara regression formula (Fujiwara, 2015)

For the calculation of the wind resistance coefficient the parameters are necessary and are provided in the following table (Table 20):

Table 20. Wind resistance coefficient parameters

	Ballast	Design Draft	Draft = 9.3 m	Draft = 10.5 m
g (m/s^2)	9.81	9.81	9.81	9.81
ρ_s (kg/m^3)	1025	1025	1025	1025
A_{OD} (m^2)	439	2311	1552.55	1966.25
A_{XV} (m^2)		485	551.44	515.2
C_{MC} (m)	-6.185	-12.193	-9.759	-11.087
H_c (m)	7.027	5.080	5.869	5.439
h_{BR} (m)	40	40	40	40
L_{WL} (m)	195.231	202.015	199.32	200.84
B (m)	30.2	30.2	30.2	30.2
T (m)	6.07	11.5	6.07	11.5
D (m)	16.7	16.7	16.7	16.7

Furthermore, the area of maximum transverse section exposed to winds is calculated by the following formula:

$$A_{XV} = 485 + B \cdot \Delta T \quad (3.2.15)$$

Moreover, the projected lateral area above the waterline is calculated by the following formula:

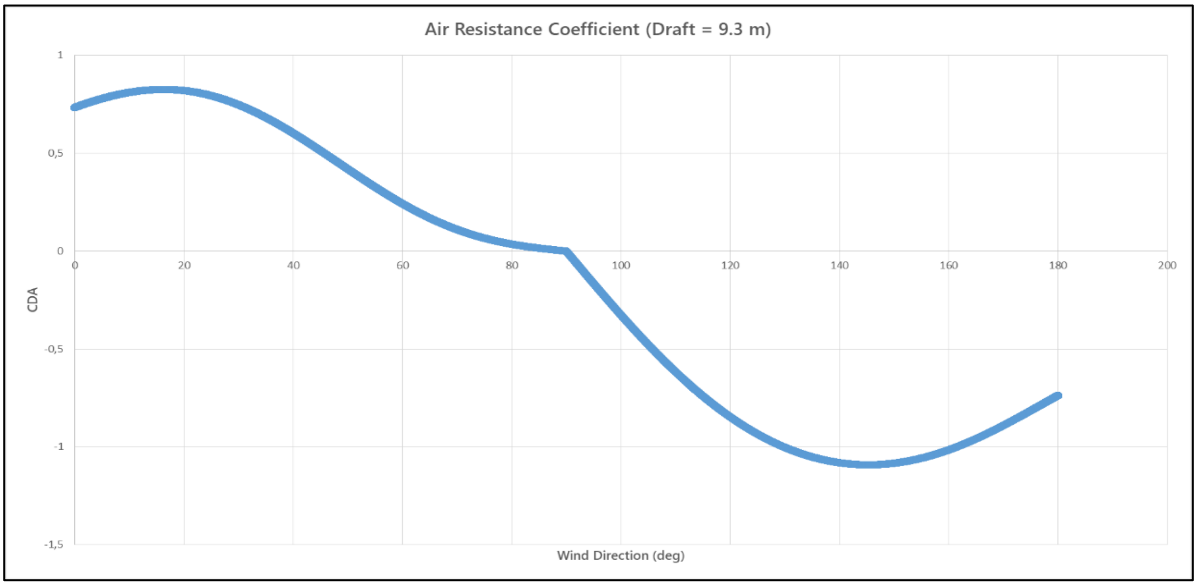
$$A_{YV} = A_{OD} + (D - T) \cdot L_{WL} \quad (3.2.16)$$

The values of C_{MC}, H_C, A_{OD}, and L_{WL} consistently remain the same, depending on the current mean draft of the ship. To be more specific it's considered that for mean drafts from 7 m to 9.875 m, the wind resistance coefficient is obliged to adopt the curve provided by the mean draft of 9.3 m. Respectively for mean drafts greater than 9.875 m the wind resistance coefficient follows the curve provided by the mean draft value of 10.5 m.

This particular procedure occurs because the used data refer only to laden condition and for that reason the curve created for the ballast condition cannot be used. Moreover, this particular separation of the dataset is similar to the separation of the dataset used in the Speed through Water – Propeller shaft Power, Speed through Water – Fuel Oil Consumption graphs, which are illustrated in the 2nd Chapter of the thesis. In addition the wind resistance coefficient cannot use the curve provided for the Design Draft = 11.5 m, because the dataset doesn't surpass this particular draft. In the following pages the wind resistance coefficients are illustrated in a different pair of graphs. All in all the dataset uses the wind resistance coefficient for the mean drafts values of 9.3 m and 10.5 m.

The following figure (Figure 33) illustrates the two wind resistance coefficients used to calculate the added resistance due to wind for every data point in the dataset. The data with mean draft lower than 9.875 m use the air resistance coefficient for Draft 9.3 m (*first graph of Figure 33*) and the data with mean draft value greater than 9.875 m use the air resistance coefficient for Draft 10.5 m (*second graph of Figure 33*).

To sum it all up a comparative set of graphs are displayed, in order to show the difference between the four available wind resistances coefficients (Figure 34).



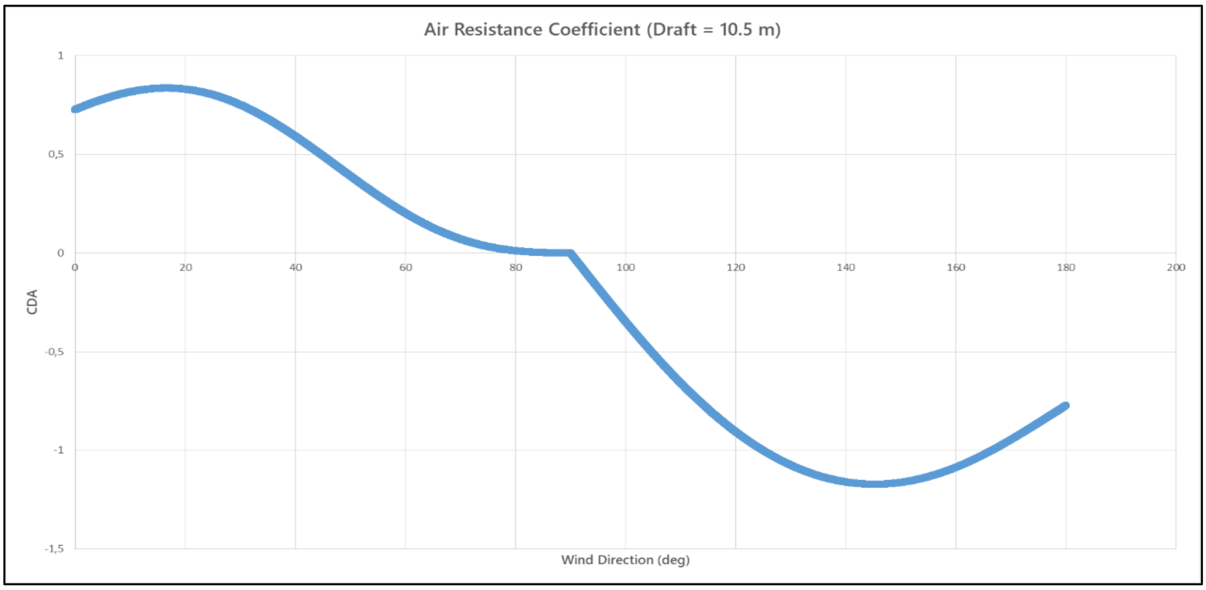


Figure 33. Air resistance coefficient draft = 9.3m, 10.5 m respectively

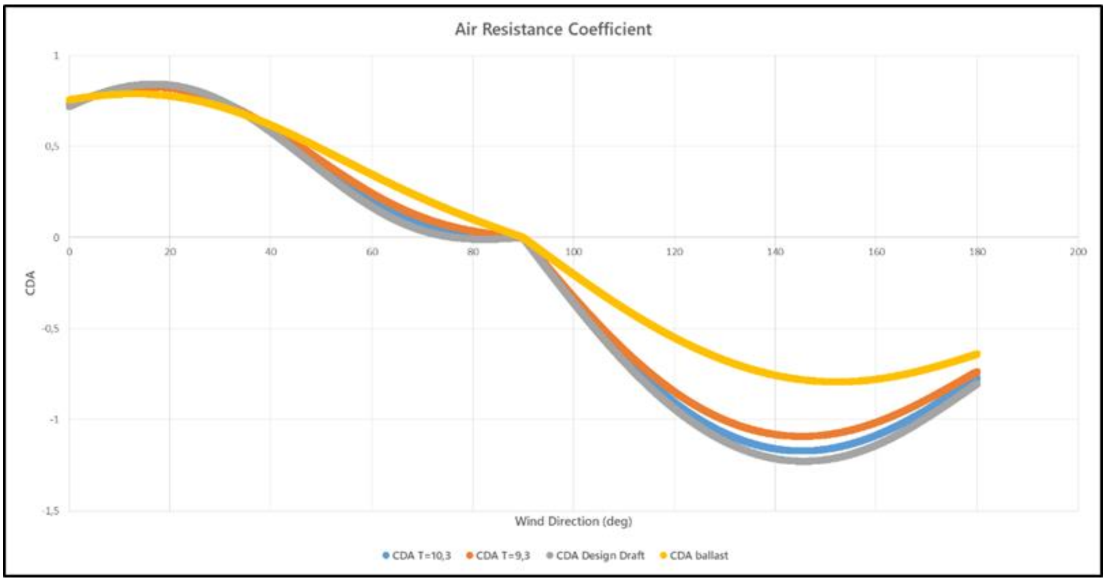


Figure 34. Comparative graph of the Fujiwara method for the calculation of the air resistance coefficient

3.2.2 Added resistance due to waves

The goal of this sub – section is to examine and calculate the added resistance due to waves. Given the lack of parameters describing the sea state the method that is going to be used is the Stawave – 1 (Boom, 2013). The Stawave – 1 method estimates the resistance increase in head waves provided that heave and pitch motions are smalls. The application is restricted for the area of the bow, to be more specific the encounter angle needs to be within the area of ± 45 degrees. For wave directions outside this area no wave correction is applied. In addition, in this condition, it's considered that the effect of wave inducted motions is considered negligible and the added resistance of the ship is dominated by the wave reflection of the hull on the waterline.

The added resistance is calculated by the following formula:

$$R_{AWL} = \frac{1}{16} \cdot \rho_s \cdot g \cdot H^2 \cdot B \sqrt{\frac{B}{L_{BWL}}} \quad (3.2.17)$$

Where,

- ρ_s: Water density for actual water temperature and salt content (kg/m^3)
- g: Acceleration of gravity (m/s^2)
- H: Significant wave height (m)
- B: Beam of the ship (m)
- L_{BWL}: Length of the bow on the waterline to 95% of maximum beam as shown in Figure 35 (m)

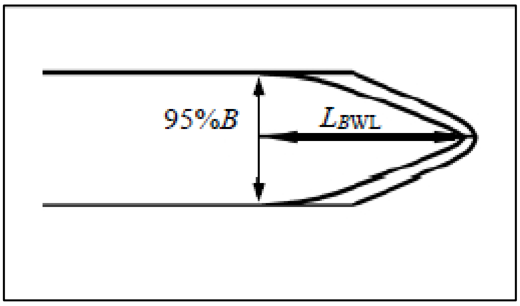


Figure 35. L_{BWL} for Stawave – 1 (Boom, 2013)

Stawave – 1 is validated for the following conditions:

I. Heave and pitch during speed/power trial are small (vertical acceleration at bow < 0.05 · g
II. Head Waves. Wave direction within 0 to ± 45 degrees from bow are corrected as head waves

For the calculation of the added resistance using the Stawave – 1 method it's necessary to calculate the significant wave height. The wave height is going to be calculated for every data point separately and the calculation are based on the Hasselmann formula, for fully developed sea (Chen et al 2002).

The formula that calculates the significant wave height as a function of the wind speed for a fully developed sea is the following:

$$H_S = 0.01614 \cdot U^2,\ 0 \le U \le 7.5 \text{ m/s} \quad (3.2.18)$$

$$H_S = 0.01 \cdot U^2 + 0.0008134 \cdot U^3,\ 7.5 \le U \le 50 \text{ m/s} \quad (3.2.19)$$

Where, U is the true wind speed at the reference height of 10 meters calculated in m/s. Subsequently a series of graphs are used to illustrate the relationship between the significant wave height and the absolute value of the added resistance due to waves (Figure 36).

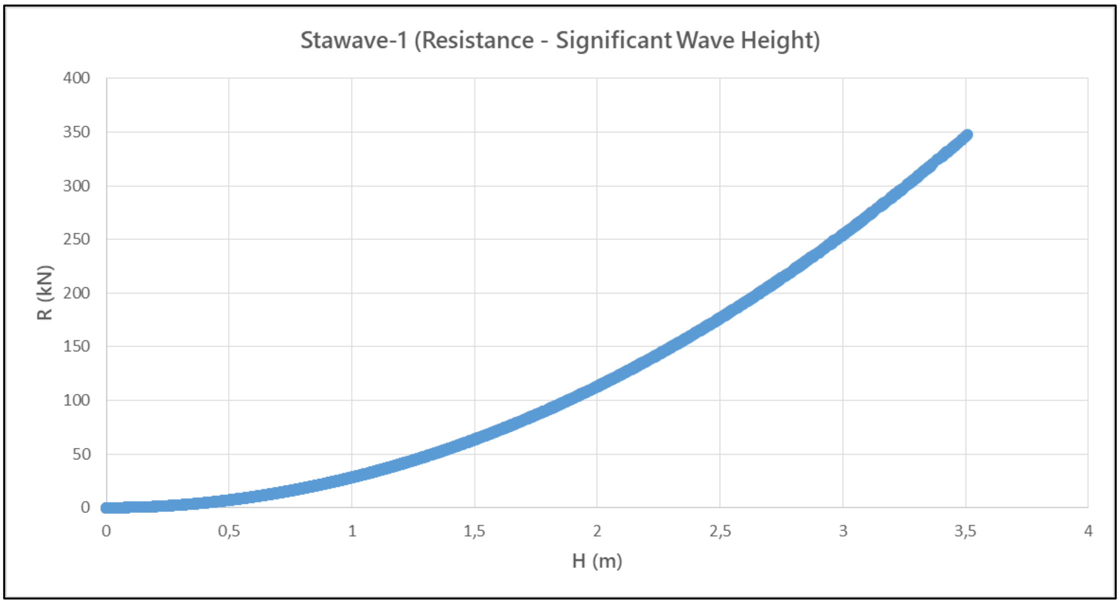


Figure 36. Added Resistance – Significant Wave height

After calculating the two added resistance and considering the added resistance due to water temperature and salt content negligible, the formula that calculates the corrected power is the following (ITTC, 2017):

$$P_{Dcorr} = P_D - \Delta P \quad (3.2.20)$$

$$\Delta P = \Delta R \cdot \frac{Vs}{n_{Do}} + P_D \cdot \left(1 - \frac{n_{DM}}{n_{Do}}\right) \quad (3.2.21)$$

Where, both the propulsive efficiency coefficient during sea trial (n_{DM}) and propulsive efficiency coefficient in ideal condition obtained from standard towing tank test and interpolated to the speed Vs (n_{Do}) are a standard value and to be more specific it's considered to be 0.7. The power deviation graphs provide additional details for the results of the ITTC method (Figure 37).

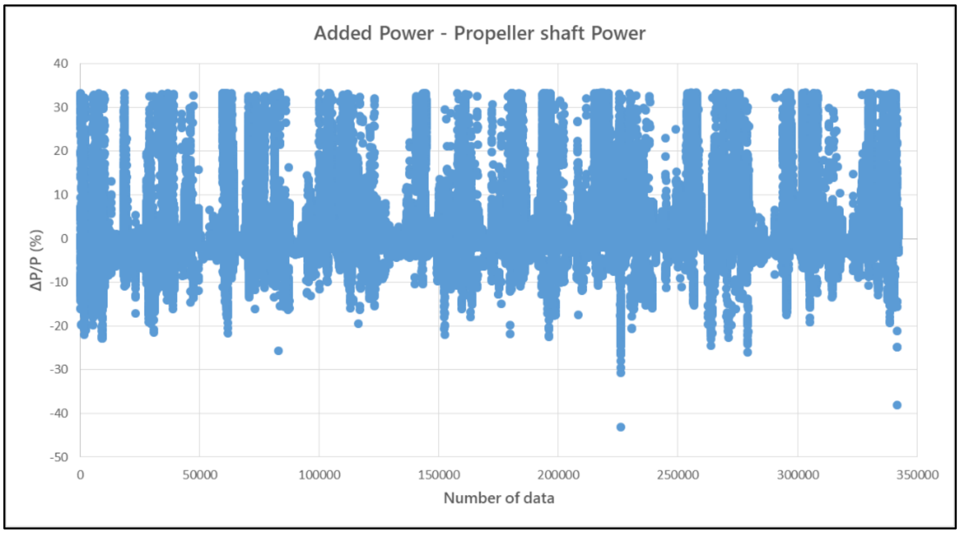


Figure 37. Power Variation for ITTC method (Fujiwara + Stawave-1)

Since the propeller shaft Power is corrected the calculation of the theoretical Fuel oil Consumption is finalized using the following formula:

$$FOC_{ITTC} = P_{Dcorr} \cdot SFOC \quad (3.2.22)$$

The relationship between the speed through water, normalized for the two reference sailing conditions (Mean Draft = 9.3m, 10.5 m), and the estimated fuel oil consumption provided by the ITTC method is illustrated in the following pages (Figure 38, Figure 39).

Normally the ITTC shaft Power correction is used for data that comply with the following limitations:

- Water temperature > 2 °C. The ship isn't travelling in a region that has ice
- The true wind velocity is between 0 – 7.9 m/s (0 – 4 BN)
- The depth of the sea is greater than:

$$H = 3 \cdot \sqrt{B \cdot T_M} \text{ or } H = 2.75 \cdot \frac{V_g}{g} \quad (3.2.23)$$

- The propeller shaft power is within the acceptable limits (Sea trials)
- The displacement is within ± 5 % of the displacement of the reference curve
- The rudder angle is lower than 5 degrees

In this thesis the ITTC method is used for all possible data points which are not filtered in Chapter 2 or are not excluded after the use of the filters applied to the Kwon method.

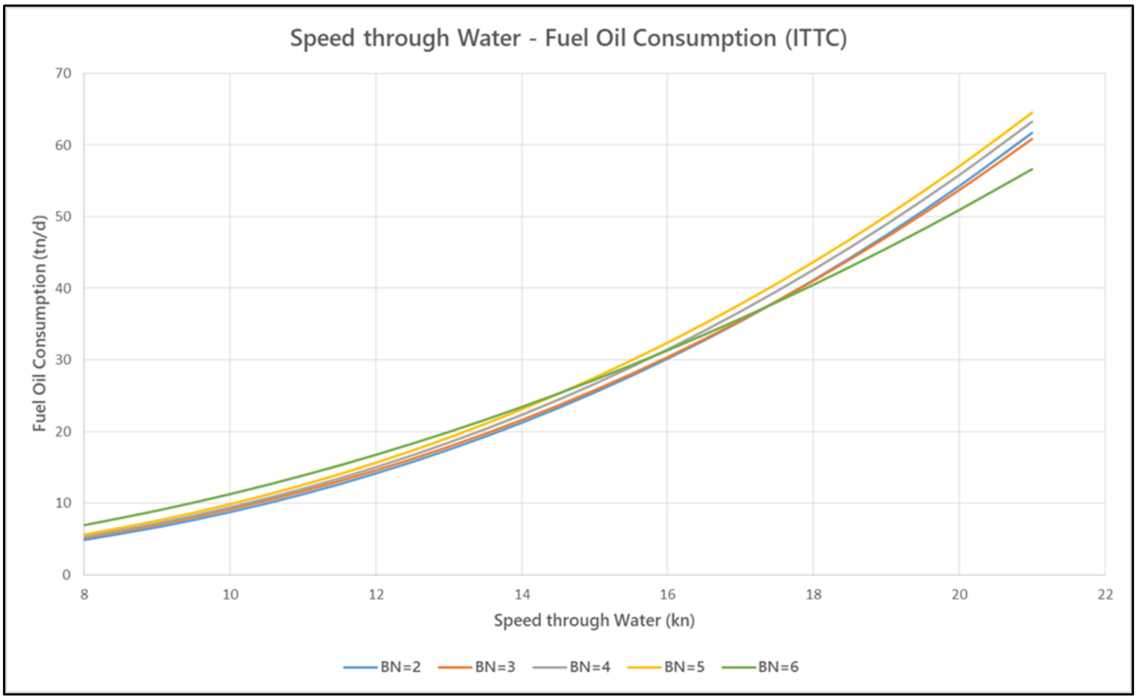


Figure 38. Estimation of the Fuel Oil Consumption for different Beaufort Number (ITTC method, draft = 9.3 m)

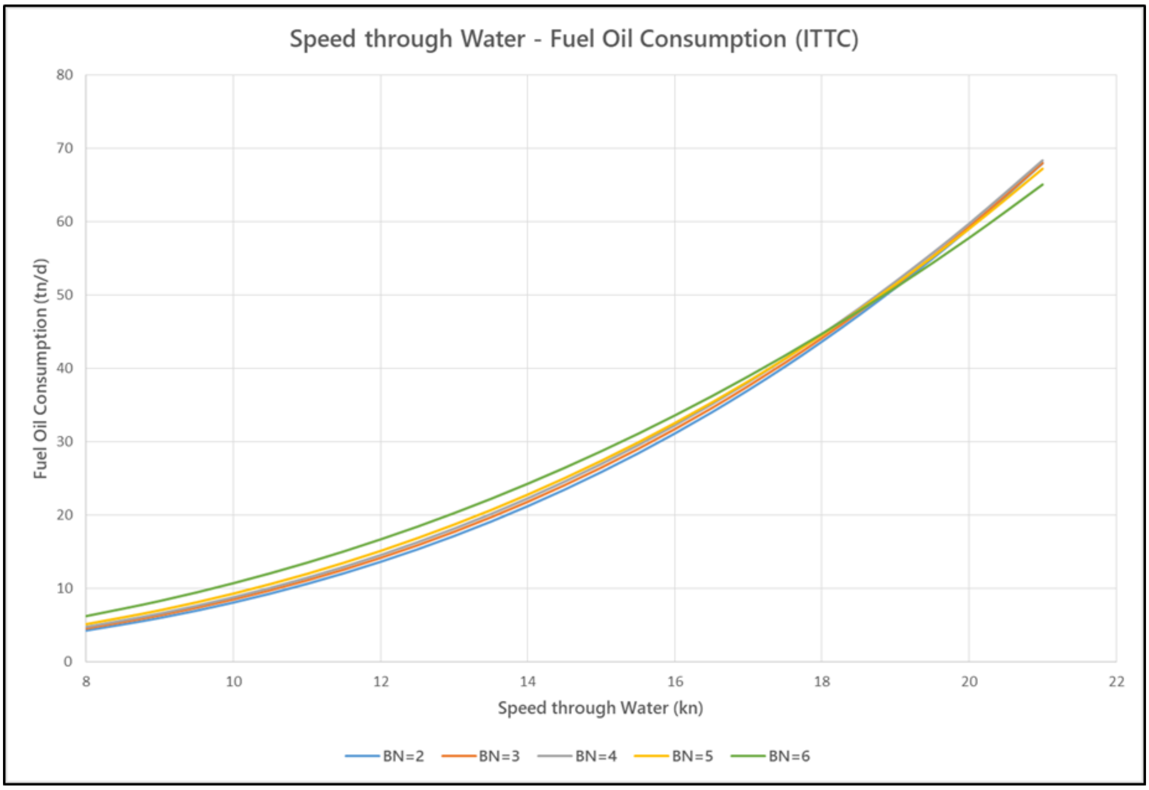


Figure 39. Estimation of the Fuel Oil Consumption for different Beaufort Number (ITTC method, draft = 10.5m)

3.3 Comparison of the results of the theoretical models

This sub – section provides a comparison analysis between the estimated FOC by the two theoretical models and the respective measured values. In addition, after the creation of the statistical model, a more thorough analysis of results and performance parameters will be presented.

The results of the estimation of the fuel oil consumption for Mean Draft = 9.3 m are displayed in the following pages:

Table 21. Mean Value of the Estimated Fuel Oil Consumption (Draft = 9.3 m)

Beaufort Scale	Measured Fuel Oil Consumption	Estimated Fuel Oil Consumption (Kwon Method)	Estimated Fuel Oil Consumption (ITTC Method)
	Average	Average	Average
1	25.99	19.77	22.73
2	23.21	17.76	20.31
3	26.63	20.77	23.19
4	31.64	24.64	27.22
5	30.97	23.48	25.52
6	33.39	26.46	26.01
7	42.11	34.7	34.75

The aforementioned fuel oil consumptions are the mean values provided by the whole dataset (Table 21). The mean values are provided using the average formula and calculating each fuel oil consumption for a specific weather condition.

For example, for the calculation of the Measured Fuel Oil Consumption the dataset that is used is the one corresponding to relative wind speed of 1 BN and in the loading condition of 9.3 m (Draft between 7 m to 9.875 m). After the separation of the dataset in 14 categories (Draft, Weather Condition) the average of each dataset for the 2 methods and the measured data, provided by the sensors, is calculated.

The FOC/Nautical miles is the division of the Average Fuel Oil Consumption calculated from Table 14 and the distance covered under a specific weather condition and is displayed in Figure 40 and Figure 41.

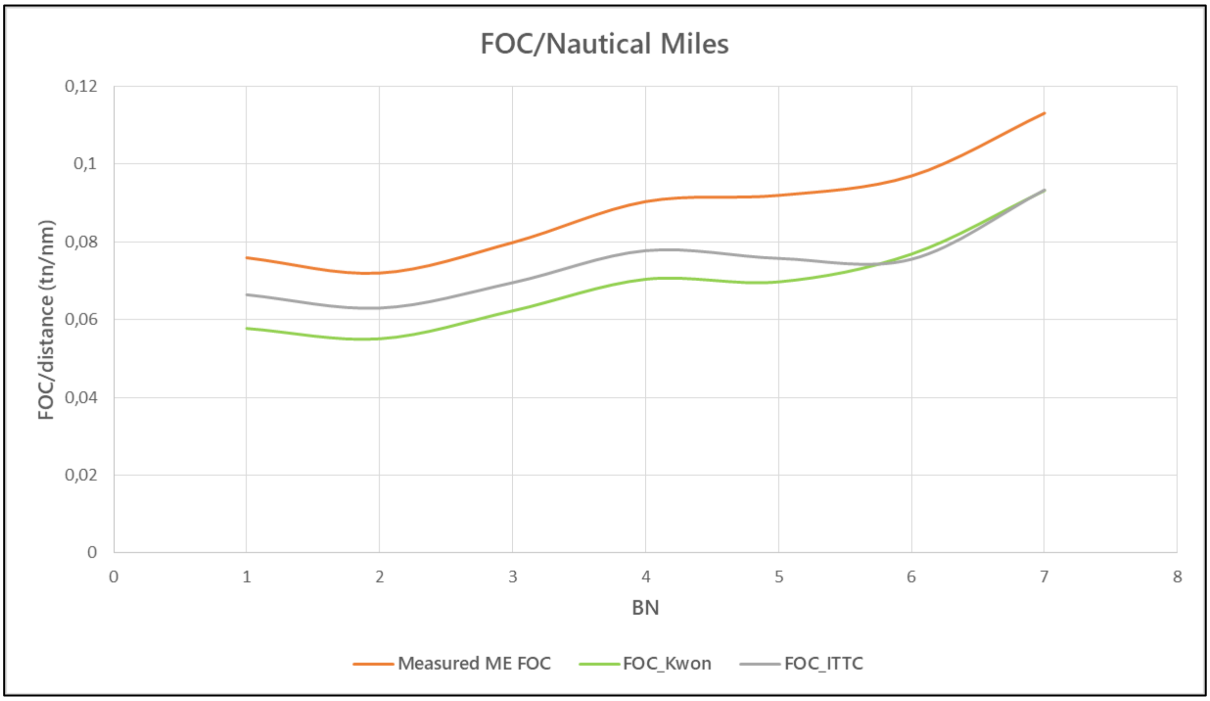


Figure 40. Fuel Oil Consumption to nautical miles (mean draft = 9.3 m)

The results of the estimation of the fuel oil consumption for Mean Draft = 10.5 m are displayed in the following pages:

Table 22. Mean Value of the Estimated Fuel Oil Consumption (Draft = 10.5 m)

Beaufort Scale	Measured Fuel Oil Consumption	Estimated Fuel Oil Consumption (Kwon Method)	Estimated Fuel Oil Consumption (ITTC Method)
	Average	Average	Average
1	32.04	26.28	28.1
2	36.95	30.49	32.5
3	37.66	30.9	33.08
4	38.89	32.24	33.77
5	41.61	35.14	35.03
6	46.58	40.81	37.74
7	47.5	41.82	39.63

The aforementioned fuel oil consumptions are the mean values provided by the whole dataset (Table 22). The mean values are provided using the average formula and calculating each fuel oil consumption for a specific weather condition.

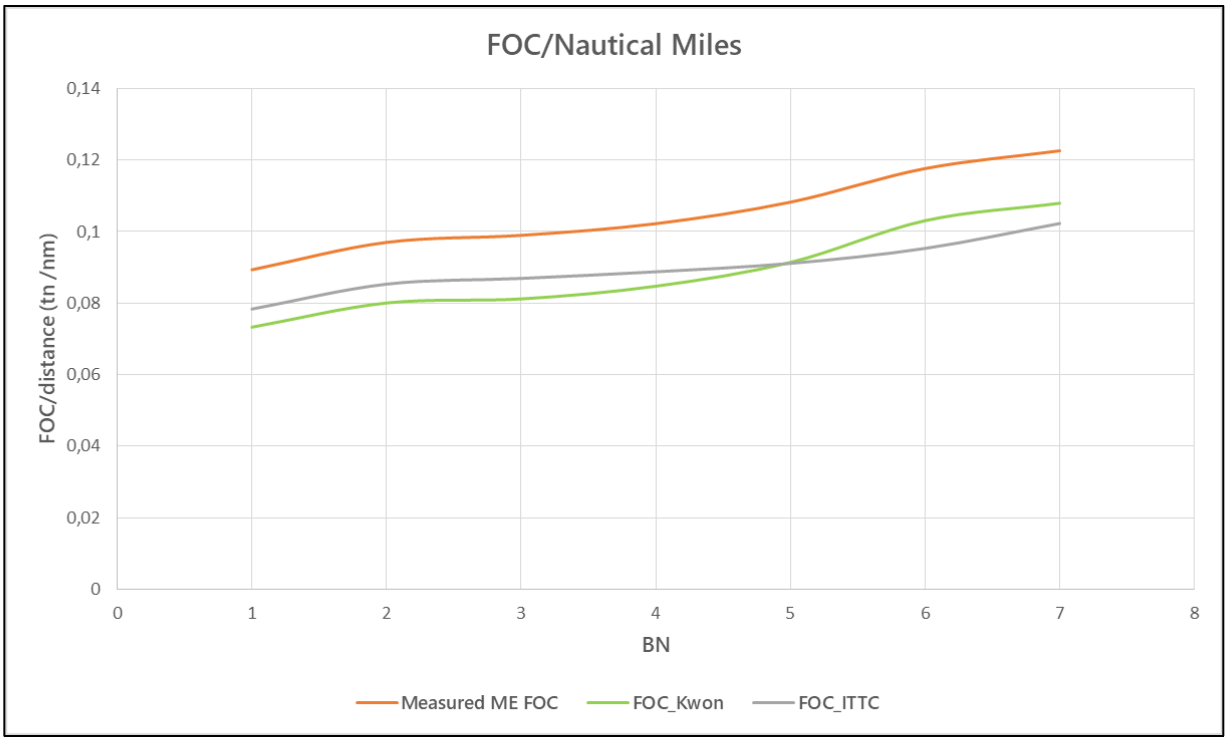


Figure 41. Fuel Oil Consumption to nautical miles (mean draft = 10.5 m)

The results graphs display the relationship between the speed through water and estimated fuel oil consumption for each theoretical method respectively.

The main takeaway of the results is that the theoretical fuel oil consumption is lesser than the actual fuel oil consumption measured by the ship's sensors.

The Kwon method (Kwon, 2008) doesn't take into consideration the hull condition of the ship, hull fouling, and the condition of the main engine and provides results for clean hulls and for a main engine that's perfectly maintained and has no problems for the whole data sampling period. In addition, the Kwon method doesn't take into consideration the special circumstances a ship has to encounter in open sea. It's worth mentioning that the Kwon method isn't conditioned or corrected to acknowledge the hull condition and the hull problems that can appear from 18 months of sailing. Moreover, for the purpose of the thesis it's considered that the ship isn't dry docked throughout the whole data sampling period.

The Kwon method provides logical results considering the fact that if the ship encounters worse weather condition, then the ships resistance is affected and is positively substantial, than the previous value. Hence the effective Power required to overcome the weather condition is bigger and in that way the fuel oil consumption is a greater value, when the ship sails in weather condition of 6 BN than in 2 BN.

The ITTC method (ITTC, 2017) is practically the correction of the required power in order to overcome specific weather condition (Head Sea, Specific relative wind Speed). The ITTC method provides better estimations for the fuel oil consumption for Beaufort Numbers of 1 – 5, comparing them to the results of the Kwon method, and for BN greater or equal to 6 the ITTC method provides drastically smaller Fuel oil consumption than the Kwon method. This particular deviation from the norm happens, because the ITTC method is strongly connected to the ship's operation profile, the condition of the ship's hull and the decisions being made by the crew members. The ITTC method is the correction of the already measured Propeller shaft Power, hence it’s highly correlated to the actual fuel oil consumption of the ship.

The ITTC method doesn’t deviate from the main curve, as shown in Figure 38, Figure 39, all of the Speed through Water – Fuel Oil Consumption curves seem to be close to each other, except the curve for 6 BN for mean draft = 9.3 m. The problematic region of results for this method is the lack of data for the sea state, so the added resistance for waves is calculated by the Stawave – 1 method (Boom, 2013) that is applied only in 20 % of the Propeller shaft Power data.

All in all, the ITTC method provides better results for relative wind direction of head waves and relative Wind velocity lesser or equal to 4 BN. The estimated values for relative wind speed equal to 5 BN is the same and for 6 BN the recommended estimation method is the Kwon method. Although the particular figures/results don’t take into consideration the encounter angle a better representation of the estimated fuel oil consumptions for the theoretical methods will be provided after the evaluation of the statistical model (Multiple Linear Regression).

4. Statistical Model

The aim of this chapter is the creation of the statistical model and the evaluation of the capabilities of the model. The model is based on a multiple linear regression method. The chapter of the statistical model will be separated into two sections the regression analysis and the evaluation of the statistical method and its results, for the estimation of the fuel oil consumption.

4.1 Regression analysis

Regression analysis is the procedure used in order to estimate the relationships between a dependant variable (response) and an independent variable (predictor). The goal of this section is to create a model that has the capability to estimate the fuel oil consumption of the ship for the whole dataset.

The relationship between the Propeller shaft Power and the Fuel Oil Consumption is linear, hence a robust model, would be a linear regression between the Shaft Power and the fuel oil consumption. Although the aforementioned linear regression, as well as a regression using the propeller shaft torque or the propeller shaft revolutions, seem like the best scenarios for a statistical model, this particular thesis utilizes the speed through water, the weather data (relative wind speed, relative wind direction), the ship's heading, the operational profile (Mean Draft, trim) and the rudder angle as the main variables to predict the fuel oil consumption of the vessel. In addition the 2nd and 3rd power of the Speed through water variable are used as parameters in the multiple linear regression, because of the relationship of the total resistance of the ship ($R_T = 0.5 \cdot V^2 \cdot S \cdot C_T \cdot \rho$) and the relationship of the effective power ($P = R_T \cdot V$). The following equations lead us to the assumption that the power is a function of the 3rd degree of power of the ship's speed, however in a more detailed study, even the value of the power in the speed could be a parameter to be calculated. The fuel oil consumption is calculated by a linear relation with the ship's power. Hence, the fuel oil consumption is assumed to be highly correlated with the 3rd degree of power of the ship's speed ($FOC \sim V^3$).

The dataset used for the regression analysis is the same one that is described in chapter 2. The corrections and the data exclusions for the Kwon method, ITTC method (Chapters 3.1 & 3.2) are reverted. Moreover an additional filter is applied, to be more specific any data point that produces speed through water less than 10 knots is discarded. The aforementioned filter affects the regression model positively, because it helps predict the fuel oil consumption accurately and as shown in Figure 11 and Figure 13 (Speed over Ground – Speed through Water, graphs) the dataset has a limited amount of speeds lesser than 10 knots.

The 4th Chapter is separated in four sub – sections. The first sub – section is the introduction of basic principles, statistical quantities and the actual meaning of each calculated statistical parameter. The second sub – section describes the correlation of the variables and finds the best subset. The third sub – section presents the main characteristics of the model and the formula used for the estimation of the fuel oil consumption. The fourth and last sub – section presents the behaviour of the statistical model, in random conditions.

4.1.1 Multiple linear regression

The most popular multiple linear regression model that relates the dependant (y) to a number of independent (x) is the following:

$$Y_i = \beta_0 + \beta_1 \cdot X_{i1} + \beta_2 \cdot X_{i2} + \ldots + \beta_p \cdot X_{ip} + \varepsilon_i \quad (4.1)$$

Where,

- y_i: The I – th observation of the dependant variable
- x_{ij}: The I – th observation of the independent variable
- β_o: The regression intercept term
- β_j: The slope coefficient of the j – th independent variable
- ε_i: The error term of the I – th observation, usually considered to be normal distribution

The model assumes that there is a linear relationship between the dependent variable and the predictors. Each slope coefficient (β) represents the change in the mean response per unit increase with the associated variable when all the other predictors remain constant. The intercept term represents the mean response when all the predictors are zero.

The MLR model that is able to make predictions for the y variable can be represented using the following formula:

$$\hat{y}_i = b_0 + b_1 \cdot X_{i1} + b_2 \cdot X_{i2} + \ldots + b_p \cdot X_{ip} \quad (4.1)$$

Where,

- $\hat{y}_i$: The predicted value of the I – th observation of the dependent variable y
- x_{ij}: The I – th observation of the j – th independent variable
- b_j: The sample estimates of the β_j coefficients.

The estimates of the slope coefficients are calculated using the following formula (matrix):

$$Y = X \cdot B + E \Rightarrow \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & X_{i\,1} & X_{i\,2} \\ 1 & X_{i+1\,1} & X_{i+1\,2} \\ \vdots & \vdots & \vdots \end{bmatrix} \cdot \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \end{bmatrix} \quad (4.2)$$

Where,

$$B = (X^T \cdot X)^{-1} \cdot X^T \cdot Y \quad (4.3)$$

Where,

- X: Is the design matrix
- Y: The observed dependent variable

The residual error term is calculated for each observation:

$$e_i = Y_i - \hat{y}_I \quad (4.4)$$

4.1.2 Coefficient of determination

The coefficient of determination, known as R^2 is a valuable characteristic of regression models. It explains how the variation in the response can be explained by the variation in the independent variables. To be more specific it's considered that y is the dependent variable and f the fitted value predicted by the regression model. Then the R^2 is calculated by the following formula:

$$R^2 = 1 - \frac{SS_{RES}}{SS_{TOT}} \quad (4.5)$$

Where,

- $SS_{TOT} = \sum(y_i - \bar{y})^2$: The total sum of squares of the dependent variable
- $\bar{y}$: The mean value of the y variable
- SS_{RES}: $\sum(y_i - f_i)^2$: The residual sum of squares

R^2: Receives values in the area of [0, 1], interval that expresses the fitting of the regression model:

- $R^2 = 0$: The model always predicts $\bar{y}$. The outcome cannot be predicted by any of the independent variables
- $R^2 = 1$: The model always predicts the observed y_i value and has no residuals. The outcome can be predicted without error from the independent variables

The coefficient of determination increases as more predictors are added to the model. However, it's worth mentioning that adding a predictor to a model can actually lead to worse estimations despite the increase in the coefficient of determination. The uncontrollable increase of predictors to a model makes it overly customized. This phenomenon is called overfitting.

An adjusted coefficient of determination is used in regression analysis to account for the increase of the coefficient of determination when new predictors are added to the model. For a model with z data points and p independent variables the adjusted coefficient of determination is calculated by the following formula:

$$R^2_{ADJ} = 1 - (1 - R^2) \cdot \left(\frac{z-1}{z-p-1}\right) \quad (4.6)$$

Finally the predicted R^2 is calculated by the following formula:

$$R^2_{PRED} = 1 - \frac{PRESS}{SS_{TOT}} \quad (4.7)$$

The predictive residual errors sum of squares, PRESS, is calculated as the sum of the squares of all the resulting prediction errors that occur by removing each observation in turn and refitting the model with the remaining observations.

4.1.3 Multicollinearity – Variance inflation factors

In a multiple linear regression model, it's valuable to know the relationship between the predictors with each other and not only the relationship of the predictors with the response. In the ideal regression model all the independent variables are correlated with the dependent parameter but not with each other. However that ideal model cannot be achieved and the predictors correlate with each other. Multicollinearity occurs when a predictor of the model can be predicted from the other independent variables with a substantial degree of accuracy. The existence of collinearity causes the coefficient of the model to change drastically in response to small changes in the dataset. The phenomenon doesn't reduce the reliability of the model and its predicting strength within the dataset, but it may produce a regression model that gives invalid results about individual predictors and cannot distinguish which variables are redundant with others.

The severity of the multicollinearity can be quantified by the Variance inflation factor (VIF). The numerical value of the VIF is the percentage to which the variance is inflated for each coefficient due to multicollinearity. The Variance inflation factor is calculated for each independent parameter of the model based on the following procedure:

- For the following regression model:

$$Y_i = \beta_0 + \beta_1 \cdot X_1 + \beta_2 \cdot X_2 + \ldots + \beta_p \cdot X_p + \varepsilon \quad (4.8)$$

- For each independent variable, a regression is calculated with x_j, as the response and the rest of the variables as the predictors. For example:

$$X_2 = a_0 + a_1 \cdot X_1 + a_3 \cdot X_3 + \ldots + a_p \cdot X_p \quad (4.9)$$

- The coefficient of determination is calculated for the aforementioned model. The variance inflation factor for the X variable is provided by the following formula:

$$VIF = \frac{1}{1-R^2} \quad (4.10)$$

Hence, high values of the Variation inflation factor are indicators of multicollinearity since they occur for high values of the coefficient of determination (R^2). This suggest that the respective predictor can be accurately predicted by the rest of the independent variables. In addition a VIF value of 1 means that the examined variable is not correlated with the other predictors. The VIF threshold value is considered to be 2.5 anything greater than that is considered to be unfit to participate in the regression model.

4.1.4 Standard deviation & Standard error of coefficient

The regression analysis includes the calculation of the standard deviation S of the distance between the data values (y) and the fitted values (f). The standard deviation is calculated in the units of the response.

The formula used to calculate Standard deviation is the following:

$$S = \sqrt{\frac{\sum_i^N (x_i - \bar{x})^2}{n-1}} \quad (4.11)$$

Where,

- ❖ X = y – f
- ❖ n: Is the number of observations in the dataset

In a regression model the standard error of the coefficient (SE) is calculated for each predictor variable x according to following formula:

$$SE = \frac{S}{\sqrt{\sum_i^N (x_i - \bar{x})^2}} \quad (4.12)$$

Where, S: Is the standard error of the model.

The standard error of the coefficient is always positive and it measures how precisely the models estimates the coefficient’s unknown value. The smaller the standard error the more precise the estimate.

4.1.5 T – Value

The T – statistic is the ratio of the departure of the estimated value of a parameter from its hypothesized value to its standard error. In regression models, the T – Value is used to measure, for each variable, the ratio between the coefficient b and its standard error (SE). The T – Value is calculated by the following formula:

$$T - Value = b/SE \quad (4.13)$$

4.1.6 Mallow’s C_p

The regression analysis often incudes a preliminary procedure called best subsets, which aims to identify the subset or subsets that best meet some fitting criteria, such as a large R^2 value or a small Mean Squared Error ($MSE = S^2$).

During this process, a statistical quantity named Mallow’s C_p is calculated in order to assess the size of the bias introduced into the responses by the presence of a model that lacks important predictors, an underspecified model.

Mallow’s C_p is calculated for each one of the examined regression models, all the possible combinations among the predictors, by using the following formula:

$$C_p = k + 1 + \frac{(MSE_j - MSE_{ALL})}{MSE_{ALL}} \cdot (n - k - 1) \quad (4.14)$$

Where,

- ❖ k: Is the number of variables of the examined model
- ❖ n: The number of data points.

- MSE_j: The mean squared error of the examined model
- MSE_{ALL}: The mean squared error of the unique model that combines all the predictors. The usage of MSE_{all} guarantees that the full model has a $C_p = k + 1$.

Models with a small value of Mallow's C_p have a small estimated total variation in predicted responses. If Mallow's C_p is near or below k + 1 the bias is low or none but when it is much greater than k + 1 the bias is significant. In general, when conducting a best subset analysis, the model or models with C_p values near k + 1 are more preferable for selection.

4.1.7 F – Value

The dependent variable is expressed by y and the fitted value, which is predicted by the regression model, with f. The F – Value of the model is calculated by the following method:

$$\text{F – Value} = \frac{\frac{SS_{REG}}{DF_{REG}}}{\frac{SS_{RES}}{DF_{RES}}} = \frac{R^2}{1-R^2} \cdot \frac{n-p-1}{p} \quad (4.15)$$

Where,

- $SS_{REG} = \sum(f_i - \bar{y})^2$: The regression sum of squares
- $DF_{REG} = p$: The degrees of freedom of the regression model and p is the number of the model's predictors
- $SS_{RES} = \sum(y_i - f_i)^2$: The residual sum of squares
- $DF_{RES} = n - 1 - p$: The degrees of freedom of the residual errors and n is the number of the observations

4.1.8 P – Value

The P-value is a probability that measures the evidence against the null hypothesis. Lower probabilities provide stronger evidence against the null hypothesis.

To determine whether each main effect and the interaction effect is statistically significant, the P-value of each term is compared to a significance level α that is usually set at 0.05. The alpha value indicates the percentage of the risk of concluding that an effect exists when it does not. If the P-value is greater than the selected significance level then the effect is not statistically significant, whereas if it's equal or less then the effect of the term is statistically significant. The P – Value is calculated with the T – Value or the F – Value, hence it's highly correlated to this particular statistical variables.

4.2 Best Subsets Procedure

The variables that are going to participate in the creation of the linear regression model for the estimation of the fuel oil consumption are the following:

- Speed through Water (knots)
- 2nd Power of the Speed through Water (knots)
- 3rd Power of the Speed through Water (knots)
- Mean Draft (m)
- Trim (m)
- Relative wind speed at the anemometer height (m/s)
- Relative wind direction at the anemometer height (degrees)
- Ship's heading (degrees)
- Wind Effect. It's essentially a calculation of the combination of the relative wind speed at the anemometer height and the relative wind direction at the anemometer height. The formula, in order to calculate the wind effect is:

$$\text{Wind Effect} = \text{Wind Speed} \cdot \text{Cos (wind direction)} \quad (4.16)$$

The correlations between the response and the predictors is provided by the calculation of the Pearson Correlation Coefficient.

$$PCC = \frac{\sum_{i=1}^{n}(x_i-\bar{x}) \cdot (y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2} \cdot \sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2}} \quad (4.17)$$

Where,

- x_i: The data points of the x parameter
- $\bar{x}$: The mean value of the x dataset
- y_i: The data points of the y parameter
- $\bar{y}$: The mean value of the y dataset
- n: The number of data in the dataset

The results of the Pearson correlation coefficient are described in the following tables (Table 23, Table 24).

Correlation using the STW^2:

Table 23. Correlation using STW^2

Correlations	STW	STW^2	Mean Draft	Trim	Wind Speed	Wind Direction	Wind Effect	FOC
STW	1							
STW^2	0.995	1						
Mean Draft	0.292	0.303	1					
Trim	0.462	0.47	0.585	1				
Wind Speed	0.086	0.087	0.186	0.072	1			
Wind Direction	-0.042	-0.05	-0.068	-0.129	-0.301	1		
Wind Effect	0.031	0.035	0.109	0.136	0.462	-0.88	1	
FOC	0.955	0.971	0.328	0.419	0.224	-0.091	0.097	1

Correlation using the STW^3:

Table 24. Correlation using STW^3

Correlations	STW	STW^3	Mean Draft	Trim	Wind Speed	Wind Direction	Wind Effect	FOC
STW	1							
STW^3	0.995	1						
Mean Draft	0.292	0.31	1					
Trim	0.462	0.475	0.585	1				
Wind Speed	0.086	0.087	0.186	0.072	1			
Wind Direction	-0.042	-0.057	-0.068	-0.129	-0.301	1		
Wind Effect	0.031	0.038	0.109	0.136	0.462	-0.88	1	
FOC	0.955	0.977	0.328	0.419	0.224	-0.091	0.097	1

The fuel oil consumption and the speed through water are highly correlated from the start of the analysis (PCC = 0.9, Chapter 2) and it's the cornerstone of the statistical model. Furthermore the value is close to 0.96, because the dataset that is used for the statistical model is filtered, hence it provides better results and higher Pearson correlation coefficients for particular variables.

A best subsets analysis is conducted in order to determine the most suitable regression model for the examined dataset and variables. The results of this analysis are presented in the tables below. More specifically, for each possible number of variables the best two models are picked and their respective R^2, Mallow's C_p, and S are shown. The aforementioned parameters and the whole best subset procedure is calculated with the use of the statistical program Minitab, developed by the University of Pennsylvania (B. Ryan, 1972).

The first set of subsets have the STW^2 variable as the most significant variable (Table 25).

Table 25. Best Subsets for STW ^{2}

Variables	R^2	C_p	S	STW	STW^2	T	trim	Wind Effect
1	94.3	222089.9	4.357					
1	91.3	514135.4	5.368					
2	95.5	99378.6	3.855					
2	94.7	183475	4.206					
3	95.8	66092.2	3.707					
3	95.8	74038.9	3.743					
4	96.2	31859.2	3.548					
4	96.1	39555.8	3.584					
5	96.5	6	3.394					

The second set of subsets have the STW^3 variable as the most polarizing variable (Table 26).

Table 26. Best Subsets for STW^3

Variables	R^2	C_p	S	STW	STW^3	T	trim	Wind Effect
1	95.4	109119.8	3.8827					
1	91.3	520506.5	5.3675					
2	95.8	73696.9	3.7273					
2	95.7	83871.4	3.7726					
3	96.1	39181.1	3.5693					
3	96.1	45833.2	3.6003					
4	96.5	4168.2	3.4016					
4	96.2	33264.8	3.5415					
5	96.5	6	3.3811					

The green blocks show the variables that are used for the model. On the other hand the red blocks show the variables that are not used for the regression model.

The best Subset is the one that has the following variables:

- 3rd Power of the Speed through Water (knots)
- Mean Draft (m)
- Trim (m)
- Wind Effect

This particular decision was made, after considering the values of the coefficient of determination, the Mallow's C_p and the standard deviation. The subsets containing the 2nd power of the Speed through water appear to have the phenomenon of multicollinearity, because the Speed through water is used in all of the Subsets and it's obvious that the STW and STW^2 are highly correlated with each other. Moreover the subsets using the STW^2 have quite large Mallow's C_p, greater than 40000, and the subsets using the STW3 variable tend to have at least 3 subsets with lower Mallow's C_p. The coefficient of determination doesn't provide enough clarity, because all of the subsets have at least 90 %.

On the other hand the 3rd Power of the Speed through Water appear to have a minimum amount of subsets that contain STW, hence there are no problems with multicollinearity and the VIF numbers are lower than the threshold of 2.5. Although the model that contains the ship's all of the possible predictors has a small Mallow's C_p, it's not used because it will cause overfitting and multicollinearity, because of the use of the STW as a predictor. Moreover the fact that the mean draft and trim are used doesn't cause the variance inflation factor to become greater than the pre – determined threshold of 2.5. The standard deviation in all of the proposed subsets varies from 4 to 3.5, for that reason the standard deviation isn't the determining factor for the selection of the regression model. The worst standard deviation is 5.1 and appears when the sole predictor is the Speed through Water, which appears to be the worst individual subset. The primary factor for the selection of, the best subset, is the Mallow's C_p. The bias of the regression model is the most significant checking variable.

4.3 Regression Model

The following model is the best option for the statistical method and it uses the following formula:

$$FOC = -11.6426 + 0.00886 \cdot STW^3 + 0.183332 \cdot WE + 1.9511 \cdot T - 6.5528 \cdot trim \quad (4.18)$$

Where,

- STW^3**:** 3rd Power of the Speed through Water (knots)
- T: Mean Draft (m)
- Trim: The trim of the ship (m)
- WE: The Wind Effect, proposed in chapter (4.2) as the combination of the relative wind velocity and the relative wind direction.

The fuel oil consumption is calculated in tons per day.

In the following tables a series of important statistical information is provided (Table 27, Table 28, and Table 29):

Table 27. Coefficient of the regression model

Coefficients					
Term	Coefficient	SE coefficient	T – Value	P – Value	VIF
Constant	-11.6426	0.0936	-124.33	0.000	-
STW^3	0.008860	0.000003	2758.86	0.000	1.29
WE	0.183332	0.000904	202.9	0.000	1.02
T	1.9511	0.0105	186	0.000	1.53
trim	-6.5528	0.0258	-254.27	0.000	1.8

Table 28. Regression model summary

Model Summary				
S	R^2	R^2 adjusted	R^2 predicted	Mallow's C_p
3.40162	96.5%	96.5%	96.5%	4168.2

Table 29. Variance analysis

Analysis of Variance					
Source	DF	Adj SS	Adj MS	F – Value	P – Value
Regression	4	109318009	27329502	2361890.09	0.000
STW^3	1	88070582	88070582	7611299.78	0.000
WE	1	476339	476339	41166.47	0.000
T	1	400292	400292	34594.33	0.000
trim	1	748092	748092	64652.12	0.000
Error	342401	3961932	12		
Total	342405	113279941			

The F – Test indicates whether the linear regression model provides a better fit to the data, than a model that contains no independent variables. The T – Value is used to measure, the ratio between the coefficient b and its standard error (SE) and the p – value is the probability of getting results, as extreme as, the observed values under the null hypothesis. The larger the absolute value of the T – value and the larger the F – value, provided by the F – Test, the smaller the P – value and the greater the evidence against the null hypothesis. A null hypothesis is a hypothesis that says there is no statistical significance between the variables. In this thesis, the absolute T – Values are quite high and also the F – values are greater than the designated threshold. Hence, the P – Value is equal to zero and the presence of the null hypothesis is negligible. Moreover the VIF values are smaller than the threshold of 2.5, hence the regression model is robust and the values well fitted.

The fitting of the model is illustrated in a figure that consists of the Predicted Fuel Oil Consumption (statistical model) and the observed Fuel Oil Consumption (measured data from the ship's sensors). The fitted model is compared with the y = x line (Figure 42).

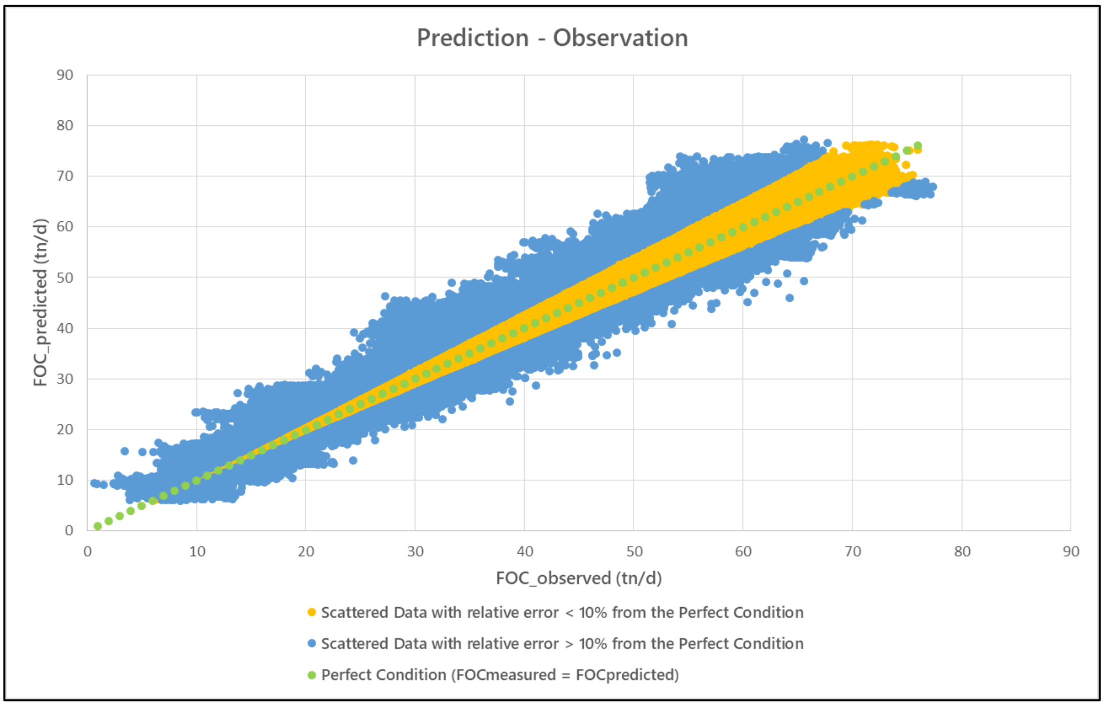


Figure 42. Observed Fuel Oil Consumption compared to Predicted Fuel Oil Consumption

The scattered data, which have a relative error of above 10% from the perfect condition (blue color), account only for 32.3% of the actual data. The rest of the data appear to have a relative error value smaller or equal than 10% (yellow color). Hence the model appears to have good results. Moreover, the formula used to calculate the relative error is the following:

$$relative\ error = \left| \frac{FOC_{measured} - FOC_{predicted}}{FOC_{measured}} \right| \quad (4.19)$$

$$percent\ error = 100 \cdot relative\ error \quad (4.20)$$

4.4 Behavior of the statistical model

In order to determine, if the statistical model, predicts the Fuel Oil Consumption with a fairly decent accuracy, it’s mandatory to test it for random weather data (relative wind velocity & relative wind direction), random Draft and random Trim and the whole speed spectrum of: 10 knots – 20 knots. The aforementioned variables of the tests are the following:

I. Condition number I:
- ❖ Draft = 9 m,
- ❖ Trim = 0.8 m
- ❖ Wind Speed = 4 BN
- ❖ Wind direction = 49 degrees

II. Condition number II:
- ❖ Draft = 10 m,
- ❖ Trim = 1.096 m
- ❖ Wind Speed = 4 BN
- ❖ Wind direction = 83.56 degrees

III. Condition number III:
- ❖ Draft = 10 m,
- ❖ Trim = 1.17 m
- ❖ Wind Speed = 6 BN
- ❖ Wind direction = 67.2 degrees

IV. Condition number IV:
- ❖ Draft = 10.5 m,
- ❖ Trim = 1.088 m
- ❖ Wind Speed = 5 BN
- ❖ Wind direction = 55.8 degrees

V. Condition number V:
- ❖ Draft = 9.55 m,
- ❖ Trim = 0.99 m
- ❖ Wind Speed = 3 BN
- ❖ Wind direction = 77.954 degrees

The aforementioned parameters are the mean values of a sample dataset around the mean Draft. In addition, the used data for each condition are summarized in a table and the tolerances of each variable are presented in the following tables (Table 30, Table 31).

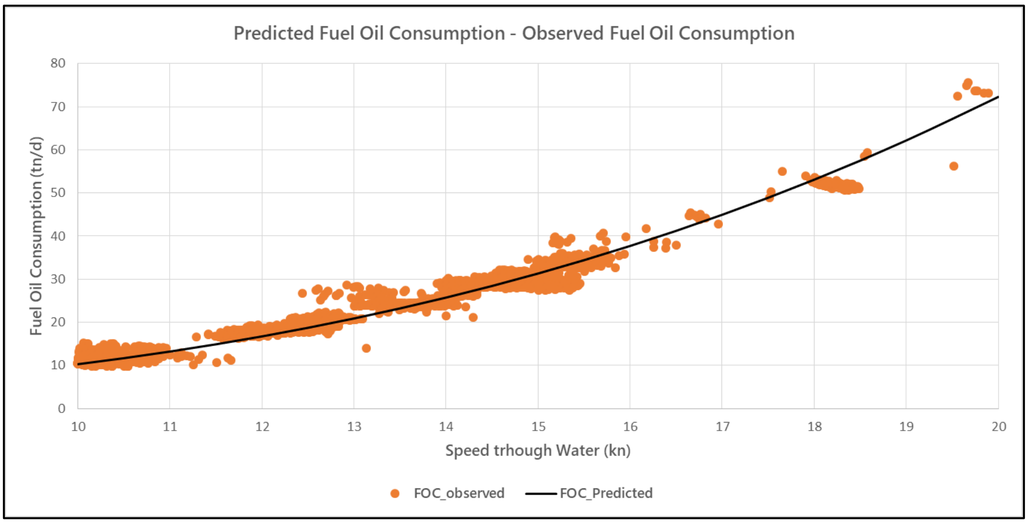
Table 30. Tolerances for testing conditions

	Tolerances (Orange Color Data, Scatter Data)			
Conditions	Draft (m)	Trim (m)	Wind Speed (BN)	Wind Effect (m/s)
I	8.85 – 9.15	0 – 1.206	5.5 – 8	-7.89 – 7.98
II	9.85 – 10.15	0 – 1.73	5.5 – 8	-8 – 8
III	9.85 – 10.15	0.37 – 1.652	10.8 – 13.9	-13.9 – 13.9
IV	10.35 – 10.65	-0.114 – 1.809	8 – 10.8	-10.8 – 10.8
V	9.35 – 9.65	-0.214 – 1.774	3.4 – 5.5	-5.5 – 5.5

Table 31. Used Data for testing conditions

	Used Data (Black Color Curve, Statistical Method)			
Conditions	Draft (m)	Trim (m)	Wind Speed (BN)	Wind Direction (deg)
I	9	0.8	4	49.00
II	10	1.096	4	83.56
III	10	1.17	6	67.20
IV	10.5	1.088	5	55.80
V	9.55	0.99	3	77.95

The following figures describe the ability of the regression model to estimate the fuel oil consumption for random conditions (Figure 43, Figure 44).



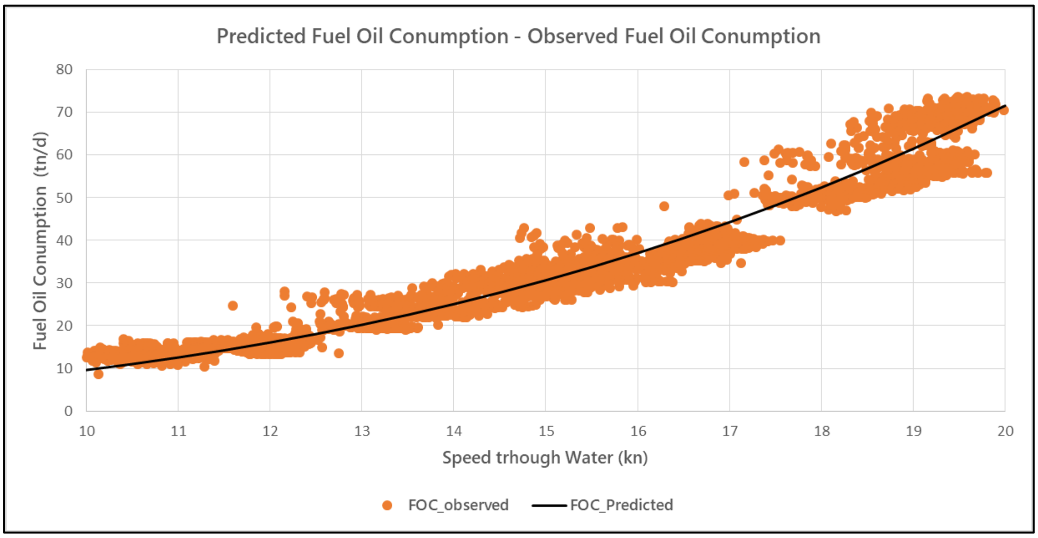


Figure 43. Comparison of measured and estimated FOC for the conditions No I – II, up and lower graph respectively.

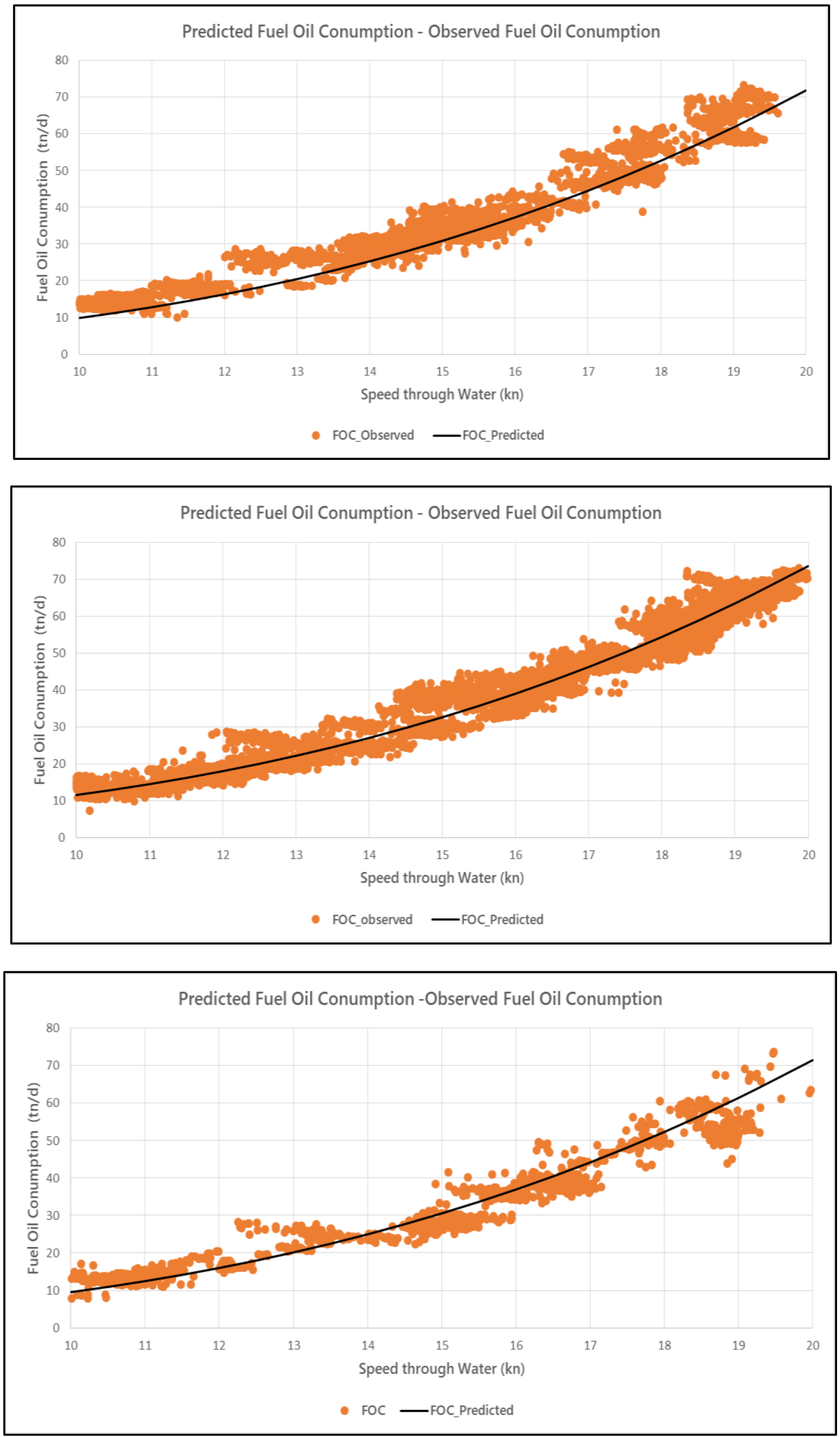


Figure 44. Comparison of measured and estimated FOC for the conditions No III – V, up, middle and lower graph respectively.

In addition, the behavior of the statistical model is tested for the relationship between its own variables. For example if the relative wind velocity is steadily increased what is the response of the regression model, considering that the other three variables remain the same (Wind direction, Mean Draft, and Trim). In this particular analysis the Speed through Water is considered to be integral part of the graphs and variate from 10 knots to 20 knots (Figure 3, Figure 5). The summary of the constant variables, used in Figure 45, can be summarized by the following table (Table 32).

Table 32. Constant Variables

	Constant Variables			
Graph	Draft (m)	Trim (m)	Wind Speed (BN)	Wind Direction (deg)
I	10.13	1.081	0 – 10	60
II	10.13	1.081	5	0 – 180
III	7 – 10.5	1.081	5	60
IV	10.13	0 – 2	5	60

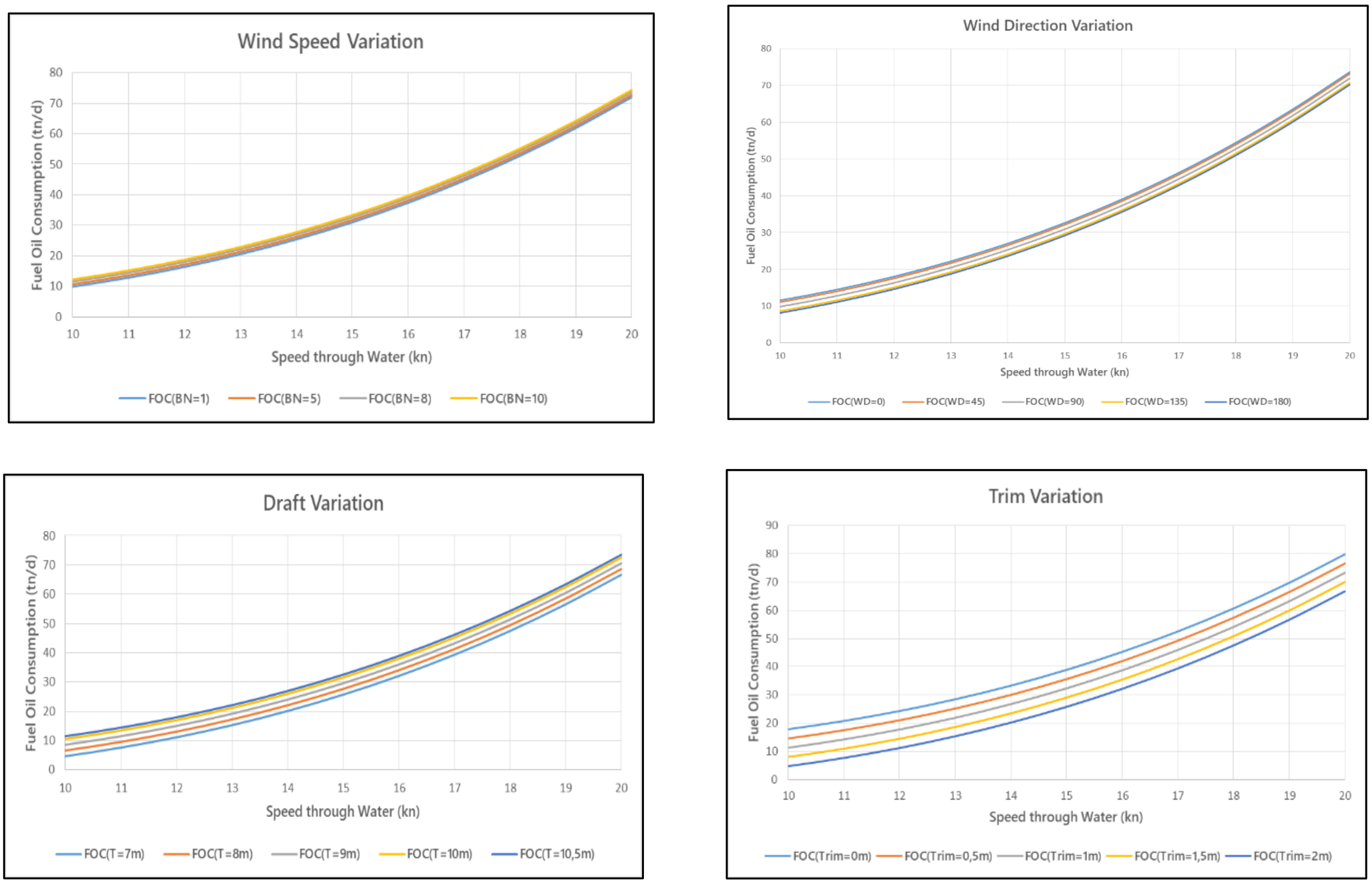


Figure 45. Relationship of the statistical model with each parameter

After analyzing the behavior of the model, an operational range for each predictor of the statistical model is defined, in order for the statistical model to be robust and not generate negative fuel oil consumption values.

- Speed through Water: 10 knots ≤ Speed through Water ≤ 20 knots
- Trim: - 1 m ≤ Trim ≤ 1.5 m
- Draft: 8.5 m ≤ Draft ≤ 11.5 m
- Wind Speed: 0 m/s ≤ Wind Speed ≤ 13.8 m/s
- Wind Direction: 0 degrees ≤ Wind Direction ≤ 180 degrees

5. Comparison between the Statistical model & Theoretical models

The aim of this chapter is to provide the results of the aforementioned methods (Kwon Method, ITTC Method, and Statistical Method) for different weather conditions (Encounter angle and Wind speed) in shared graphs. Furthermore the dataset is separated into loading conditions:

- ❖ Loading Condition with Mean Draft: 9.3 m (6.65 m – 9.875 m)
- ❖ Loading Condition with Mean Draft: 10.5 m (9.875 m – 11.65 m)

The figures that are going to be illustrated in the following pages are summed up in the following table (Table 33):

Table 33. Comparative Figures

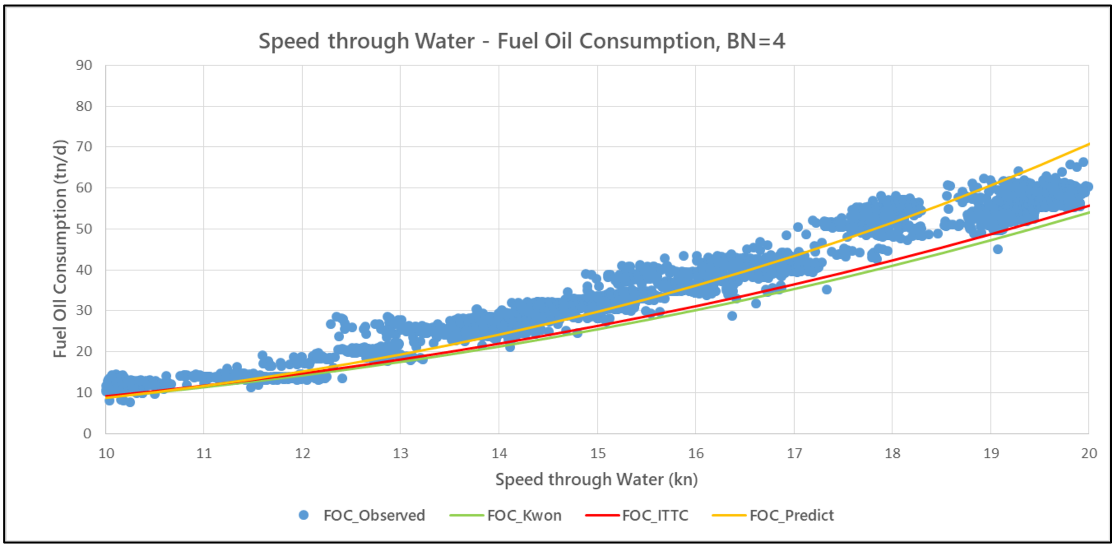
Set of Graphs	Encounter Angle	Wind Speed (BN)	Mean Draft (m)
I	Head Sea	4, 6	9.3
II	Beam Sea	6	9.3
III	Following Sea	4	9.3
IV	Head Sea	4	10.5
V	Bow Sea	6	10.5
VI	Following Sea	4	10.5
VII	All	4 – 6	9.3
VIII	All	4 – 6	10.5

Additionally a table form is introduced to describe the coefficient of determination each method achieves for the conditions, mentioned in Table 33 (Table 34).

Table 34. Coefficient of Determination for the theoretical and statistical models in different weather & loading Conditions

Encounter Angle	Wind Speed (BN)	Mean Draft (m)	R^2 Kwon Method	R^2 ITTC Method	R^2 Statistical Model
Head Sea	4	9.3	0.9651	0.9652	0.9807
Head Sea	6	9.3	0.8664	0.8222	0.9219
Beam Sea	6	9.3	0.971	0.9699	0.9699
Following Sea	4	9.3	0.9562	0.9527	0.9670
Head Sea	4	10.5	0.9679	0.978	0.9785
Bow Sea	6	10.5	0.9557	0.8202	0.9584
Following Sea	5	10.5	0.9601	0.9632	0.9705

It appears that the statistical model provides the best overall results. Moreover, the most suitable theoretical method for wind speed greater than 5 BN is the Kwon method (2008). The following figures are illustrated, in order to confirm the aforementioned table. (Figure 46, Figure 47, Figure 48, Figure 49).



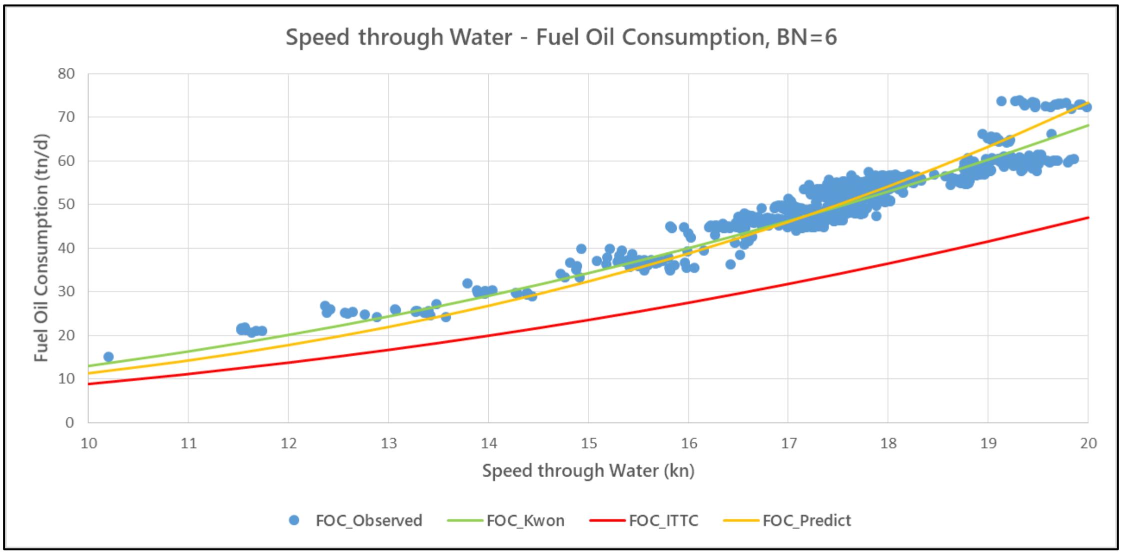
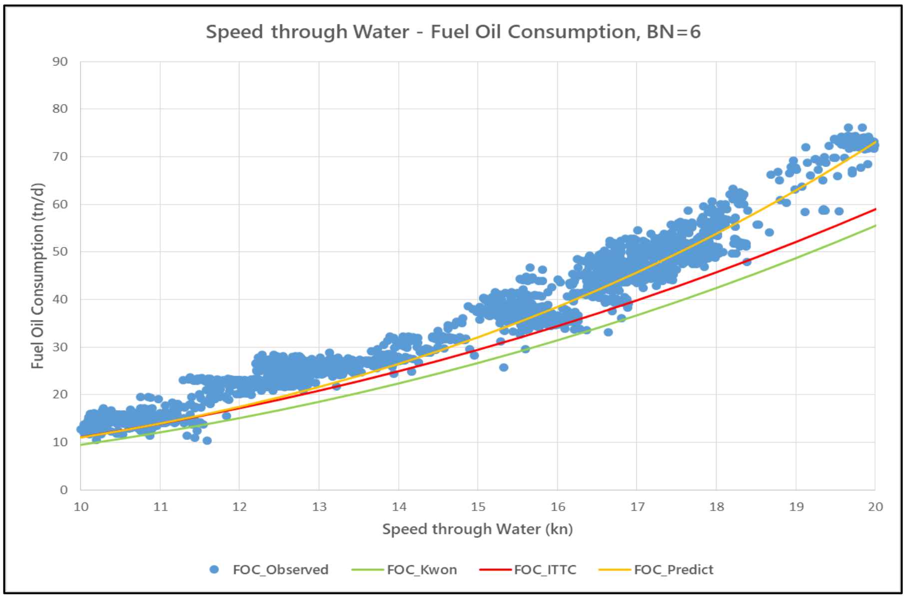


Figure 46. Head Sea for mean draft = 9.3 m



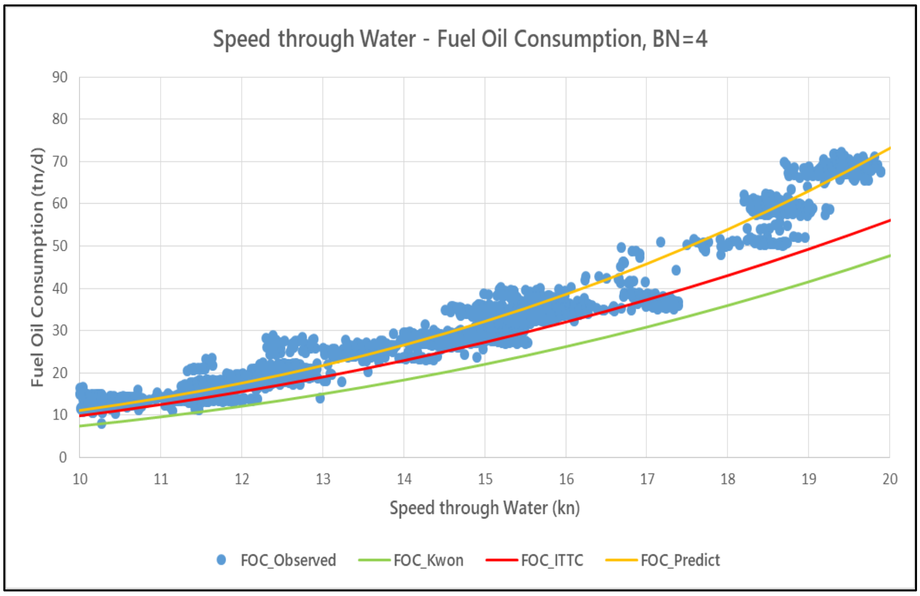


Figure 47. Beam Sea (up graph) & Following Sea (lower graph), mean draft = 9.3 m

It's obvious that for Head Sea and for Wind Speed > 3 BN the ITTC method estimates the fuel oil consumption to be lesser than the actual consumption. On the other hand the Kwon method for Wind Speed greater than 5 BN seems to be able to estimate the consumption with good accuracy. The statistical model provides lower fuel oil consumption estimations for speed through water in the area of 10 – 18 knots, than the actual fuel oil consumption.

For Bow Sea and for Wind Speed > 4 BN the ITTC method estimates the fuel oil consumption to be lesser than the actual consumption. On the other hand the Kwon method for Wind Speed greater or equal than 6 BN seems to be able to estimate the consumption with great accuracy. (Appendix C). The statistical model provides lower fuel oil consumption estimations for speed through water in the area of 10 – 16 knots, than the actual fuel oil consumption and for speed through water greater than 16 knots the statistical model estimates the fuel oil consumption to be bigger than the actual (ship's) consumption.

For Following Sea and for Beam Sea, both the Kwon Method and the ITTC method estimate the fuel consumption to be smaller than the actual fuel oil consumption. The statistical model is robust and provides great results on the estimation of the fuel oil consumption. However, it appears to not be able to estimate the fuel oil consumption, with the ideal accuracy, for speed through water greater than 17 knots.

Hence the statistical model for mean draft = 9.3 m can provide a good estimation for the fuel oil consumption for speeds that variate from 10 knots to 17 knots.

Generally speaking the statistical model provides the best estimation of the fuel oil consumption because it's been trained and fitted to have the same anomalies as the actual dataset of the ship.

The following figures (Figure 48, Figure 49) provide the necessary results for the second loading condition, mean Draft = 10.5 m.

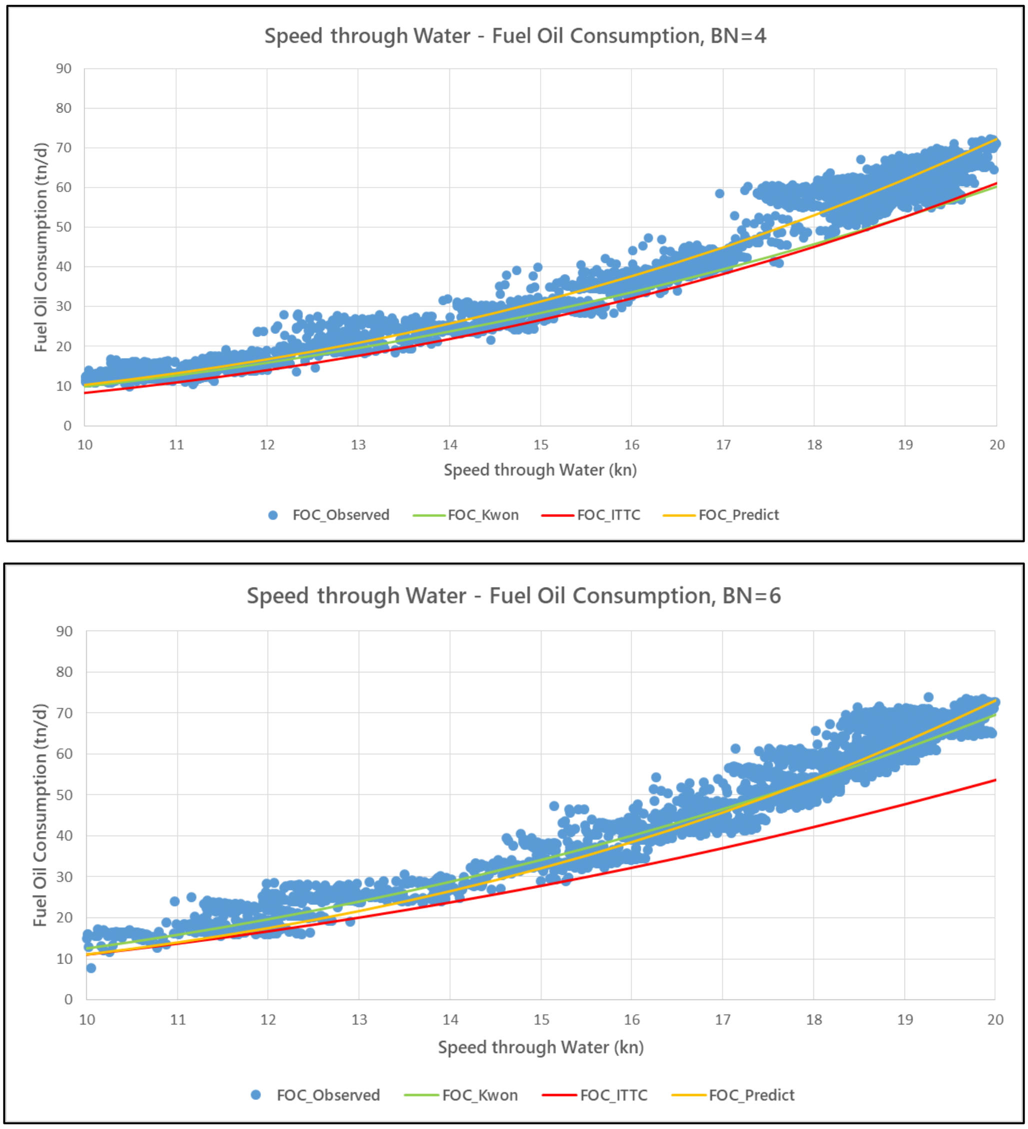


Figure 48. Head Sea (up graph) & Bow Sea (lower graph), mean draft =10.5 m

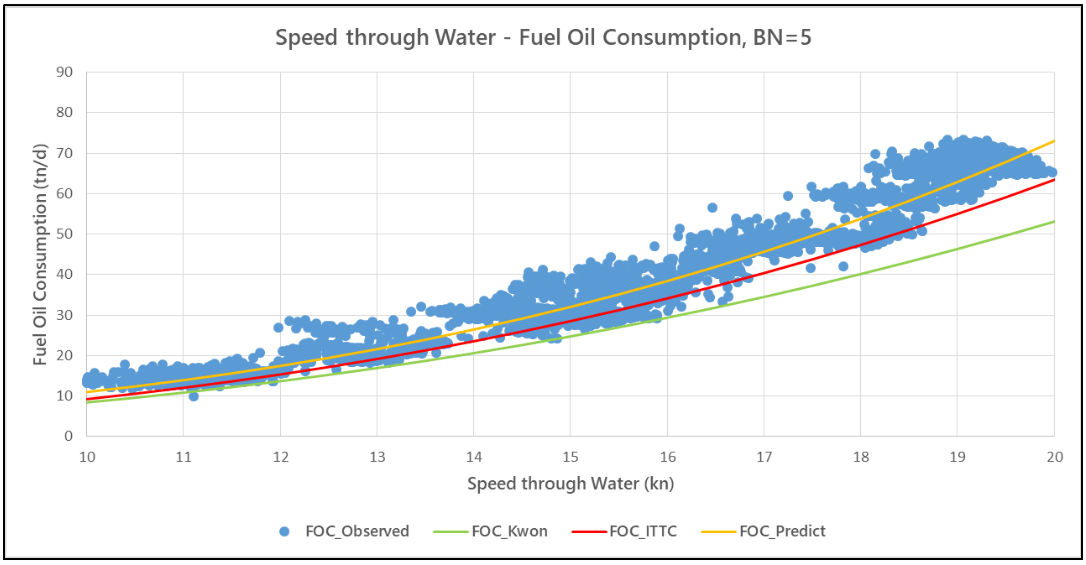


Figure 49. Following Sea, mean draft = 10.5 m

As seen in Figure 48, the Kwon method estimates the fuel oil consumption for wind speed = 4 BN perfectly and anything lower than 4 BN the model deviates from the actual dataset (Appendix C). The ITTC method estimates the fuel oil consumption to be much lower than the actual consumption. The reason for that is the correction of the Power (decrease of Power) due to the Stawave – 1. The statistical model is robust and estimates the fuel consumption accurately.

For Bow Sea the Kwon method estimates the fuel oil consumption for wind speed = 6 BN perfectly and anything lower than 6 BN the model deviates from the actual dataset (Appendix C). In addition, the ITTC method continues to estimate the fuel oil consumption 5 tons per day lower, than the actual fuel oil consumption of the ship.

For Following Sea and for Beam Sea (Appendix C), both the Kwon Method and the ITTC method estimate the fuel consumption to be smaller than the actual fuel oil consumption and the results are similar to Figure 47.

The statistical model is robust and provides great results on the estimation of the fuel oil consumption. However, it appears to not be able to estimate the fuel oil consumption, with the ideal accuracy, for speed through water greater than 19 knots. The results that are not presented for the above encounter angles and wind velocity are portrayed in Appendix C.

The following pages consist of the results for the wind velocity range of: [4 BN, 6 BN], while including the real data (FOC_observed), the two theoretical models and the statistical model are illustrated in the following figures (Figure 50, Figure 51). The dataset is separated only due to the wind speed criterion and the separation, due to the encounter angle isn't used for the particular graphs. In addition, a new graph is used: FOC Deviation – Speed through Water. This will be used to show the variations of the estimated fuel consumption between the two theoretical models and the statistical model. The FOC Deviation is calculated following the formula:

$$\text{FOC Deviation} = \frac{\Delta FOC}{FOC_S} = \frac{FOC_S - FOC_{th}}{FOC_S} \quad (5.1)$$

Where,

- FOC_S: Is the standard part of the equation and is calculated by the statistical model
- FOC_{th}: Is the variable of the formula and its calculated by the theoretical models

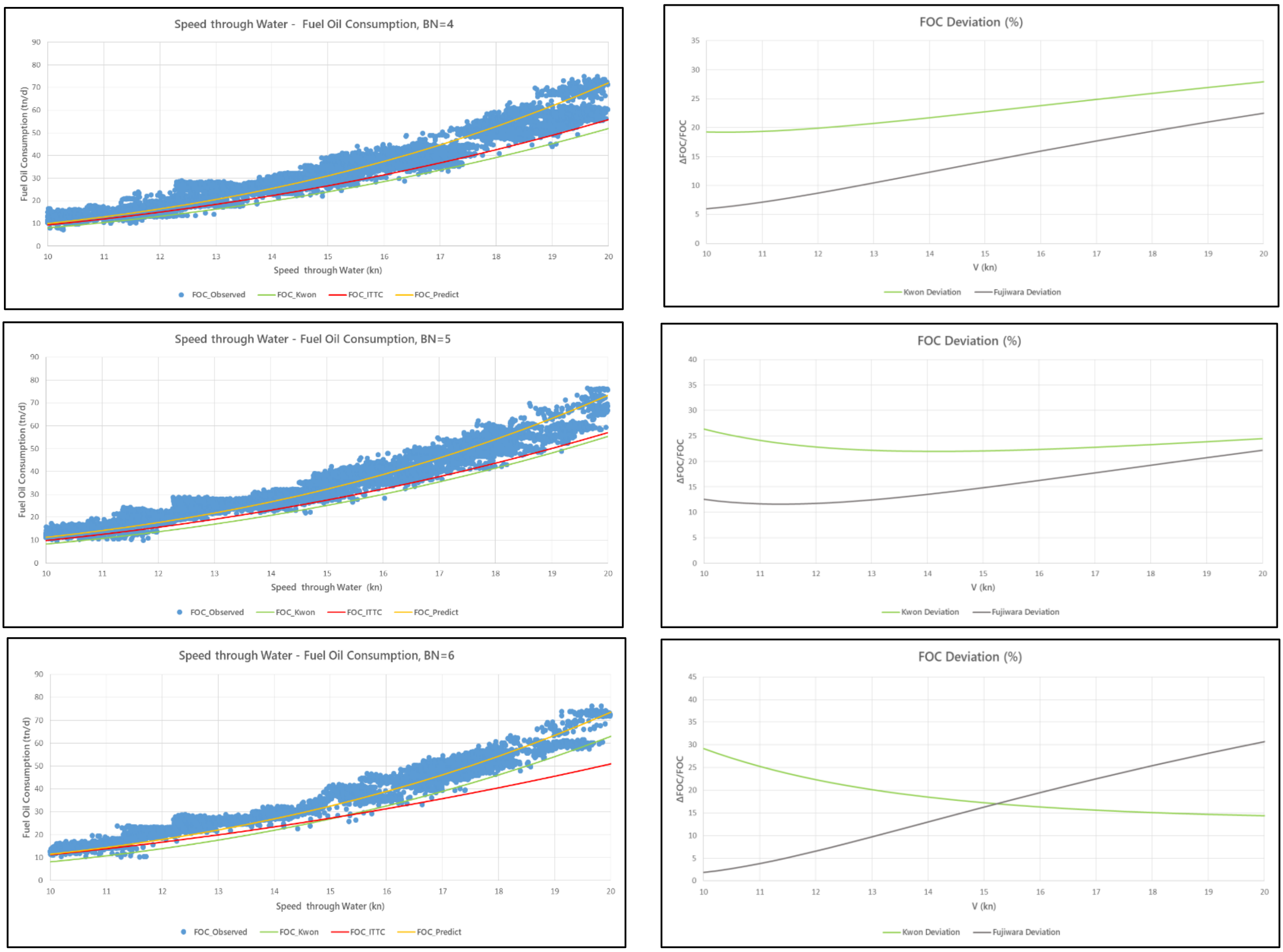


Figure 50. Speed through Water – Fuel Oil Consumption & FOC margins for mean draft = 9.3 m

It is observed that only in the case of wind speed = 6 BN Kwon method has a higher fuel consumption compared to the ITTC model. For the statistical model and wind speed of 4 BN the fuel consumption is identical with the Scatter, of the actual data, for speeds lower than 17 knots. For the case of the wind velocity of 5, 6 BN the statistical fuel consumption is lower for 11 – 17 knots and higher for the other speeds.

In addition the average difference in fuel consumption for the Kwon model compared to the consumption of the statistical model is: 23%. While in the case of the ITTC method the difference between the theoretical consumption and the consumption of the statistical model is in the area of 15.2%

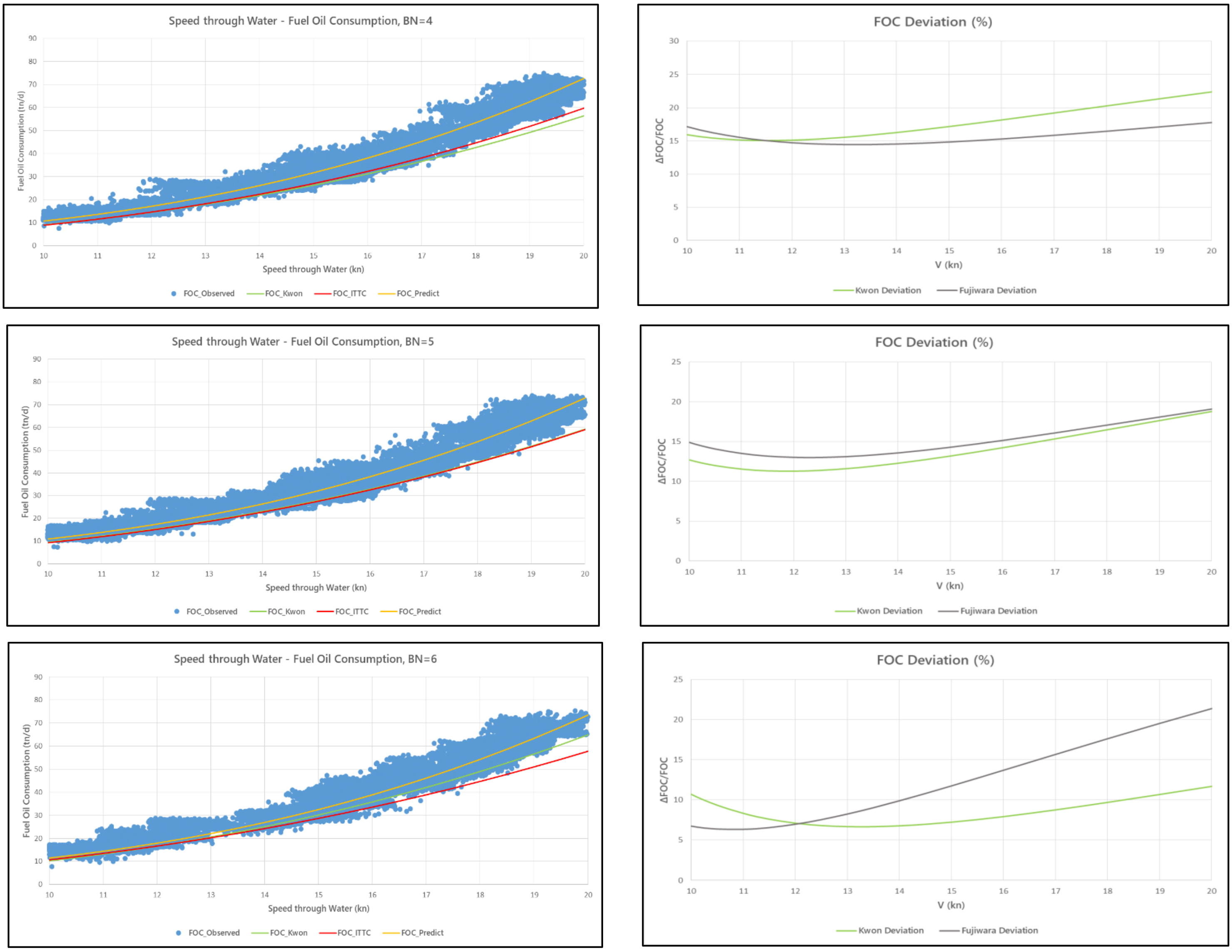


Figure 51. Speed through Water – Fuel Oil Consumption & FOC margins for mean draft = 10.5 m

It is observed, from the graphs of Figure 51, that for a speed through water from 10 knots to 12 knots the difference between the theoretical consumption of the ITTC method and the actual fuel consumption is greater than that of the theoretical method of Kwon. This phenomenon occurs in all weather conditions under consideration. In addition, it is observed that only in the case of wind velocity = 6 BN, a higher fuel consumption in the theoretical model of Kwon, rather than in the ITTC model, is estimated. For the statistical model and for wind speed equal to 4, 5, and 6 BN the fuel oil consumption is less than the actual consumption (10 - 16 knots). For the remaining speed range the fuel consumption is higher than the actual fuel consumption of the container ship.

Moreover, the average fuel oil consumption difference for the Kwon model compared to the statistical model is 14.6%. While in the case of ITTC the difference between the theoretical consumption and the consumption of the statistical model is in the area of 15.4%.

6. Conclusions

A summary of the proposed procedure for the estimation of the fuel oil consumption and the comparison of the results is necessary. Firstly, operational data are collected, by a variety of sensors and measuring devices. The data are implemented in a set of pre – processing procedures, which use a set of filters that are meaningful (Speed over ground < 0) for the function of the ship, in order to exclude unusual and random data. The most significant part of the data exclusion is the use of statistical outlying detection procedures. Once the operational data are filtered, two theoretical models are used in order to find the corrected propeller shaft power, which is necessary for the estimation of the actual fuel oil consumption of the ship. The Kwon method (2008) and the ITTC method (2017), are used for the calculation of the fuel oil consumption for different weather and loading conditions. In addition a regression model is used to fit the available operational data and to provide predictions about the fuel oil consumption (FOC) for different operational scenarios. The regression analysis is a different approach for evaluating the actual fuel oil consumption of the vessel.

The theoretical model proposed by the International Towing Tank Committee appears to estimate the fuel oil consumption with better accuracy for wind speed lesser or equal than 4 BN. For wind velocity = 5 BN the estimation of the theoretical models are pretty similar. For wind speed greater than 5 BN the best theoretical model is Kwon's method and it achieves similar values to the one of the actual fuel oil consumption, In addition for Head, Bow Sea (Encounter angle < 60 degrees) and wind speed greater than 6 BN the Kwon method estimates the fuel consumption to be higher than the actual fuel oil consumption. The best way to use the models is to combine them depending on the weather data, provided by the ship.

The graphs display the relationship between the speed through water and estimated fuel oil consumption for each method respectively. The main takeaway of the results is that the theoretical fuel oil consumption is lesser than the actual fuel oil consumption measured by the ship's sensors.

As mentioned in chapter 3, the Kwon method doesn't take into consideration the ship's hull condition, and the condition of the main engine and provides results for clean hulls and for a main engine that's perfectly maintained and also considers that the propeller is clean and has no corrosion. In addition, the Kwon method doesn't take into consideration the special circumstances a ship has to encounter in open sea. It's worth mentioning that the Kwon method isn't conditioned or corrected to acknowledge the hull condition and the hull problems that can appear from 18 months of sailing.

To be more specific a coefficient could be used, and the clean hull resistance could be multiplied with the aforementioned coefficient after each month. In this thesis the particular coefficient isn't used and the ship is considered to be in perfect condition for the whole data sampling time (18 months). Moreover, for the whole purpose of the thesis it's considered that the ship isn't cleaned for the whole sampling data, energy saving devices aren't used and aren't installed in the vessel throughout the whole data sampling period and its considered that the ship hasn't gone through any significant maintenance process (Dry Dock).

The Kwon method provides logical results considering the fact that if the ship encounters worse weather condition then the ships resistance is affected and substantial, than the previous value. Hence the effective Power required to overcome the weather condition is bigger and in that way the fuel oil consumption is a greater value, when the ships sails in Weather condition of 6 BN than in 4 BN.

The ITTC method is practically the correction of the required power in order to overcome specific weather condition (Head Sea, Specific relative wind Speed). The ITTC method provides bigger values in the fuel oil consumption for Beaufort Numbers of 1 – 5, compared them to the results of the Kwon method, and for BN greater or equal to 6 the ITTC method provides drastically smaller Fuel oil consumption than the Kwon method. This particular deviation from the norm happens, because the ITTC method is strongly connected to the ship's operation profile, the condition of the ship's hull and the decisions being made by the crew members. To be more specific the ITTC is the correction of the already measured Propeller shaft Power, hence it's highly correlated to the fuel oil consumption of the ship. In addition a strong variable is the existence of the head waves and the use of the Stawave – 1 formula. It's considered that the deviation of the water temperature doesn't affect the resistance in a substantial way, so it's considered to be negligible.

The ITTC method is reliable and it's the better method to use for wind speed lower than 7.9 m/s, which is considered to be the limit for a wind speed to be considered as 4 BN. The Kwon method provides better results for wind speeds greater than 5 BN, although it's not advised to use the Kwon method for the filtered regions of the data set. For example wind speed = 8 BN and encounter angle = Head sea (0 – 30 degrees).

In conclusion the ITTC method provides better results for relative wind direction of Head waves and relative Wind velocity lesser or equal to 4 BN. The estimated values for relative wind speed equal to 5 BN is the same and for 6 BN and greater the recommended estimation method is the Kwon method. Although the particular figures/results don't take into consideration the encounter angle.

The statistical model, multiple linear regression, is the best way to estimate the fuel oil consumption of a specific ship. It's the best way possible, because it's fitted to the actual dataset of the ship. For example if the hull condition is worsens and the ship consumes more fuel oil, the statistical model can understand this variation and adapt to the new dataset. Practically the statistical model is a regression formula of the Scatter dataset.

The fuel oil consumption margins between the statistical model and the theoretical model is approximately 15 – 20 %. This is a good margin considering both the theoretical models don't take into consideration the propeller condition, the main engine condition and the drift of the sensors. The best margin, for the Fuel oil consumption, is lower than 10 % and that happens for the ITTC method for speed through water in the region of 10 – 15 knots.

In conclusion the estimated fuel oil consumption for the theoretical methods appears to be more accurate for speed between 10 – 15 knots and wind speed between 5 to 6 BN. The statistical model is well fitted and can be used for any weather conditions and operational profile (Draft, trim).

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Appendix

Appendix A: Data Analysis

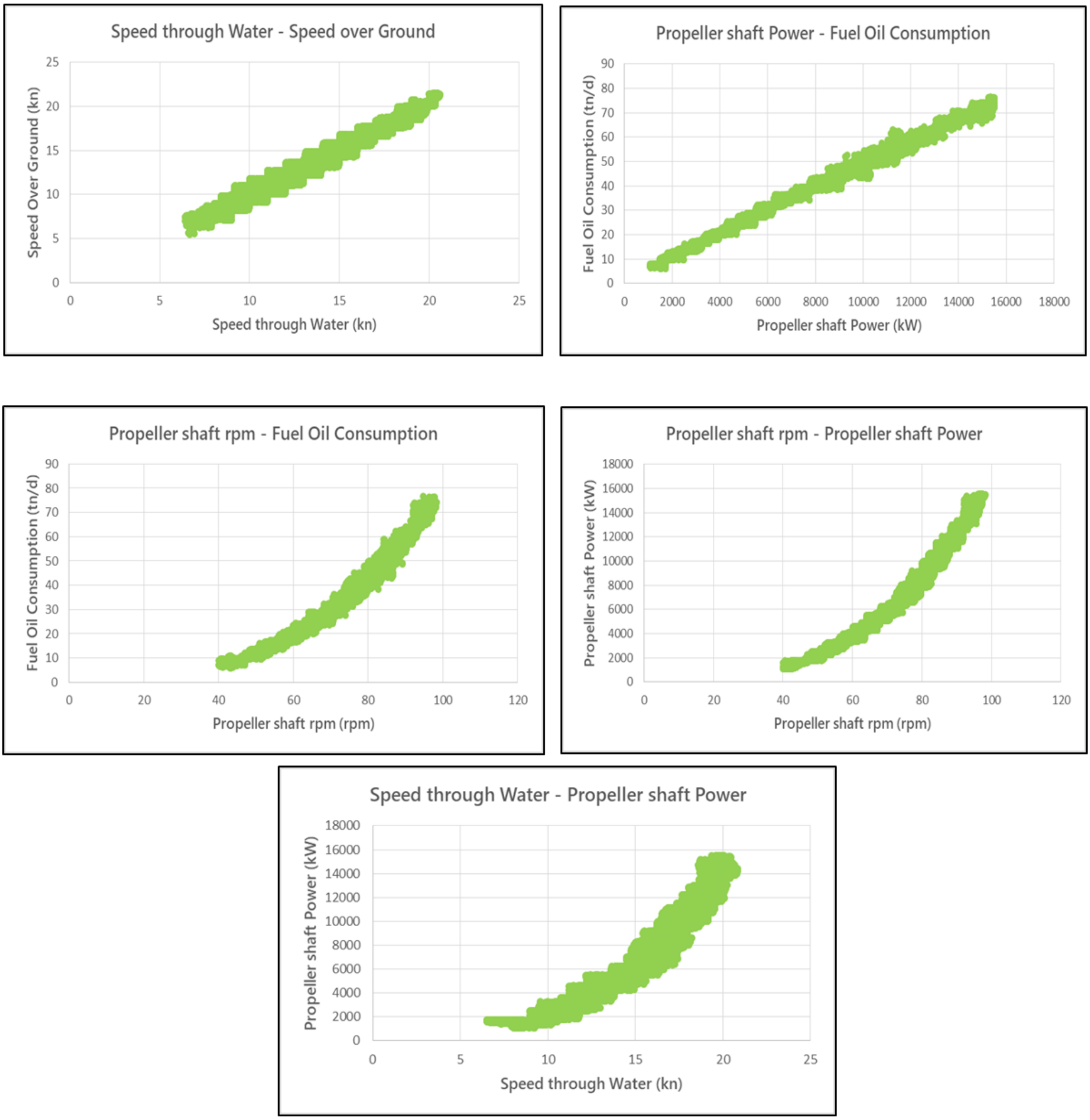


Figure 52. Scatter Plot for filtered data (Statistical Outlier detection), mean draft = 9.3 m

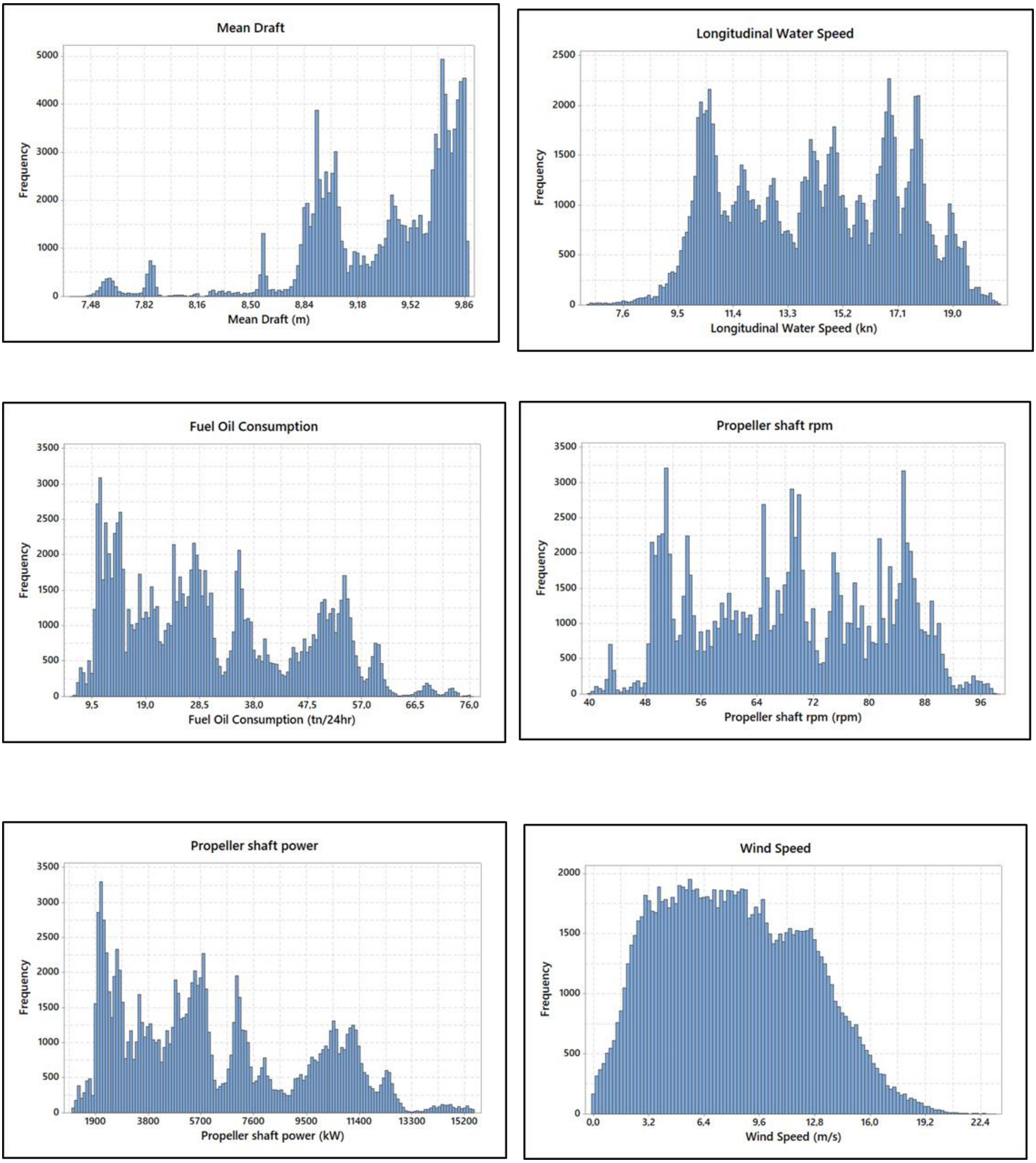


Figure 53. Histograms for the filtered data (Statistical Outlier detection), mean draft = 9.3 m

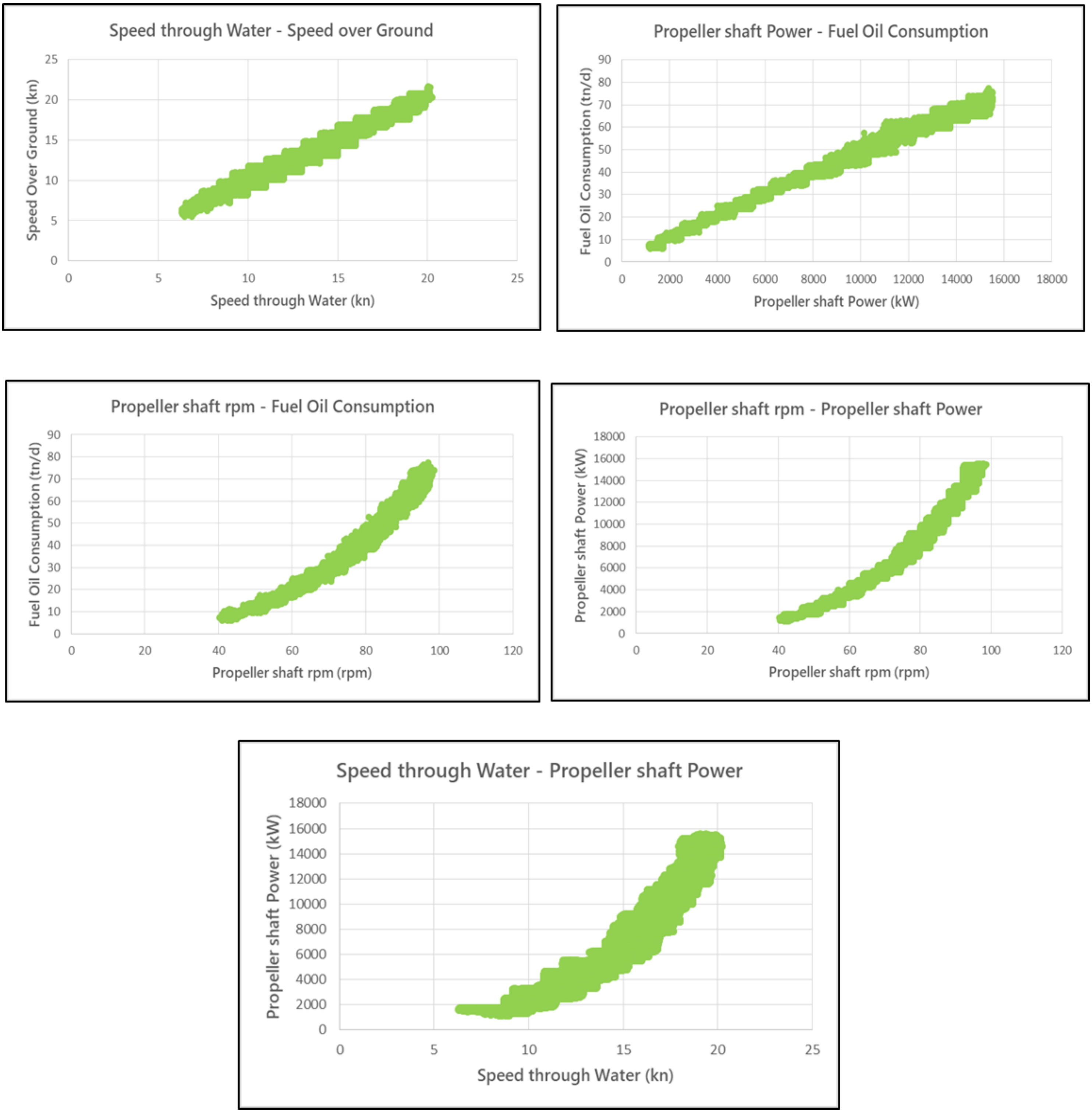


Figure 54. Scatter Plot for filtered data (Statistical Outlier detection), mean draft = 10.5 m

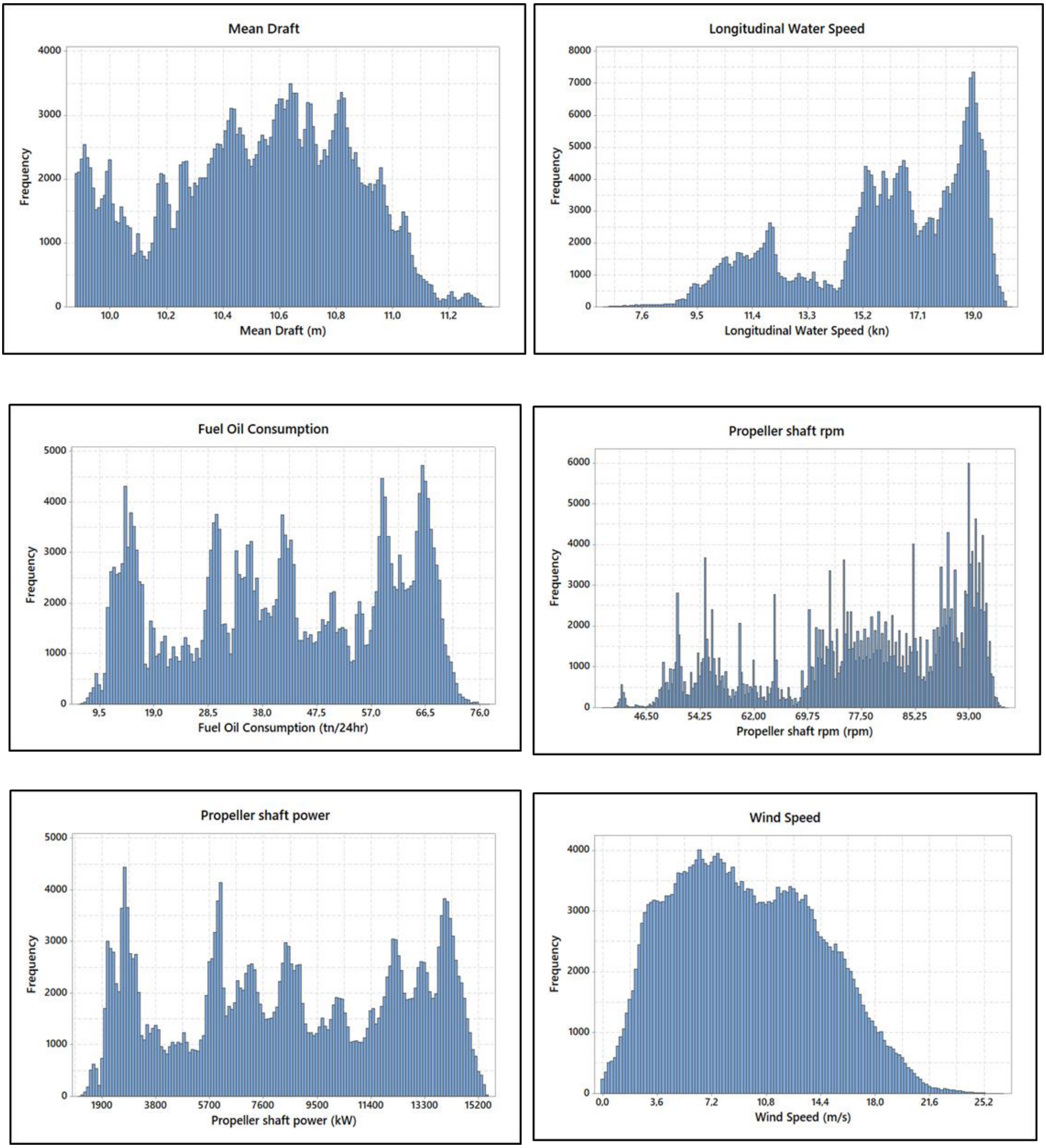


Figure 55. Histograms for the Filtered data (Statistical Outlier detection), mean draft = 10.5 m

Appendix B: Theoretical Models

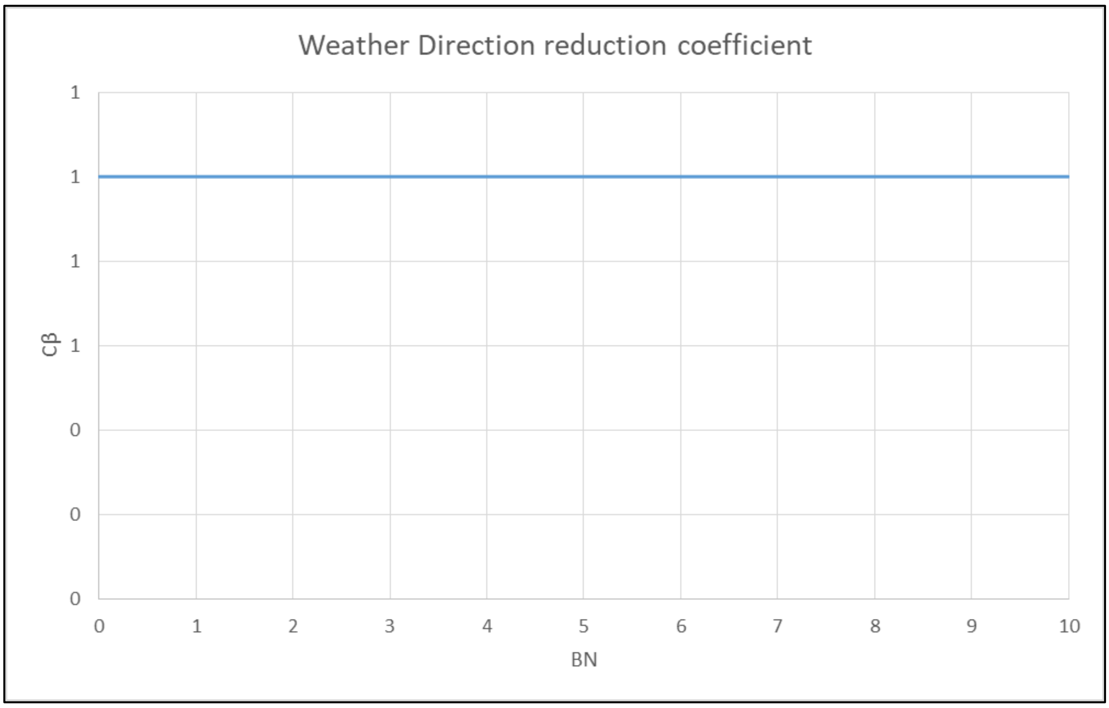


Figure 56. Weather Direction Reduction coefficient for Head Sea

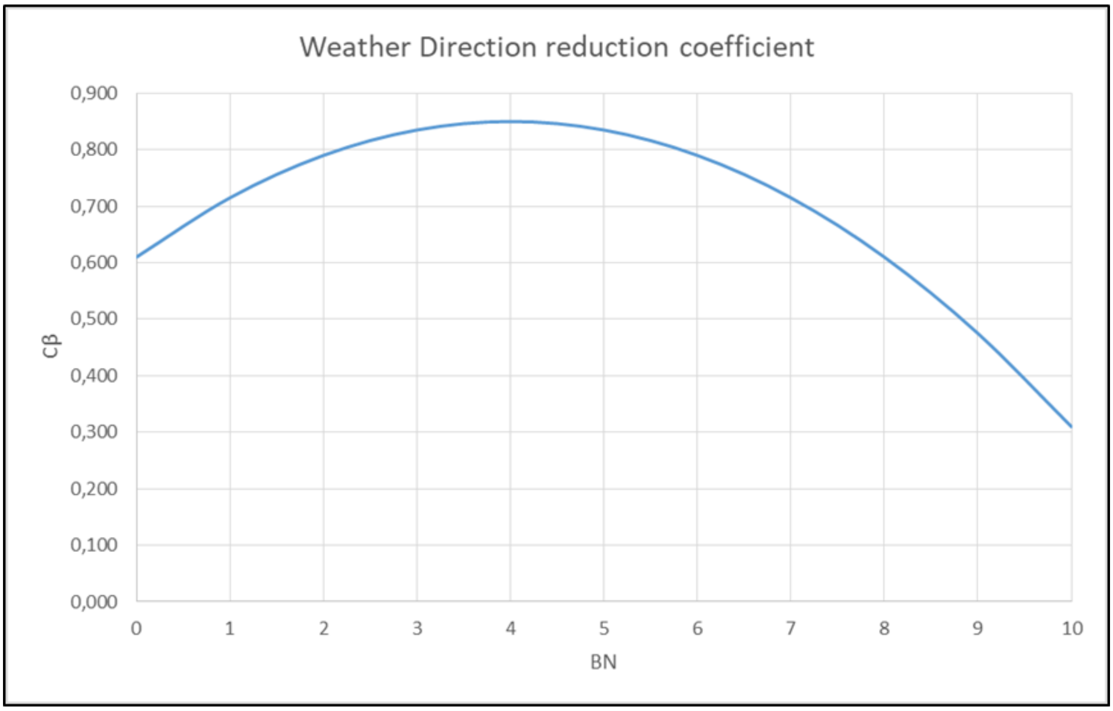


Figure 57. Weather Direction Reduction coefficient for Bow Sea

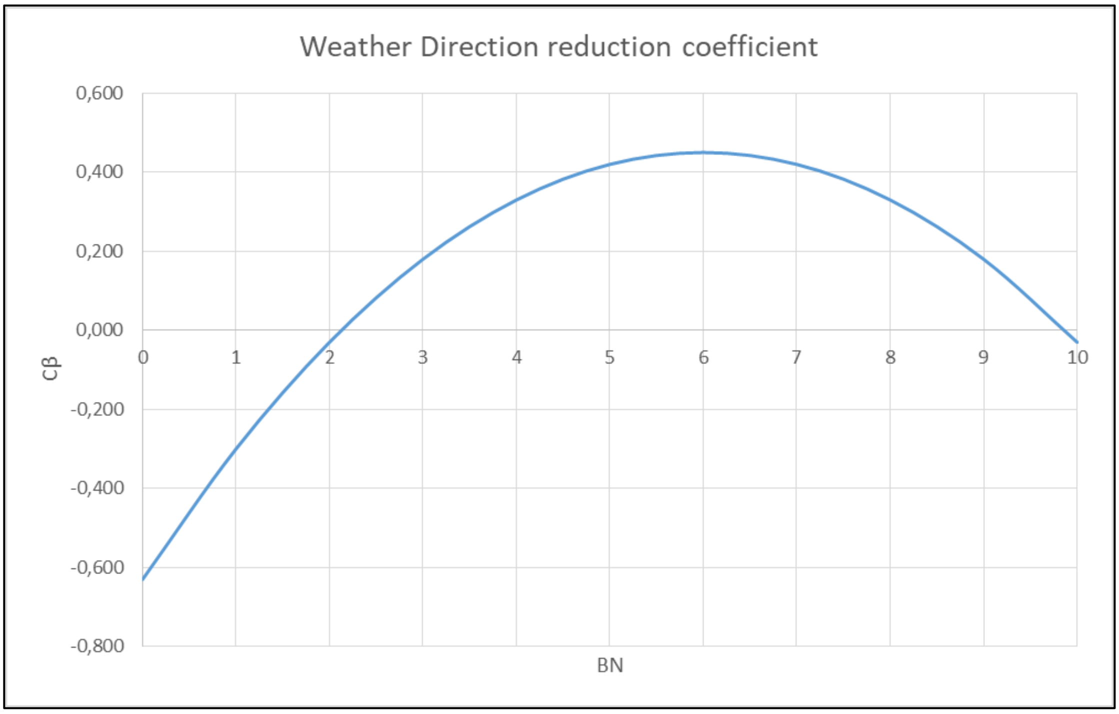


Figure 58. Weather Direction Reduction coefficient for Beam Sea

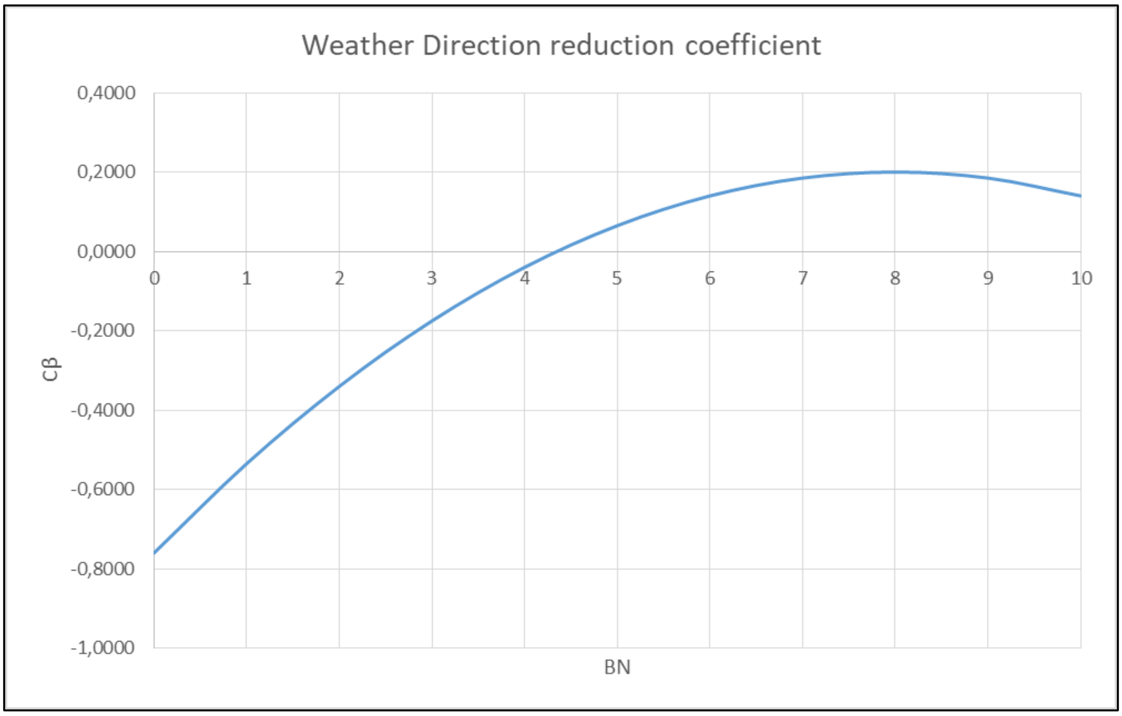
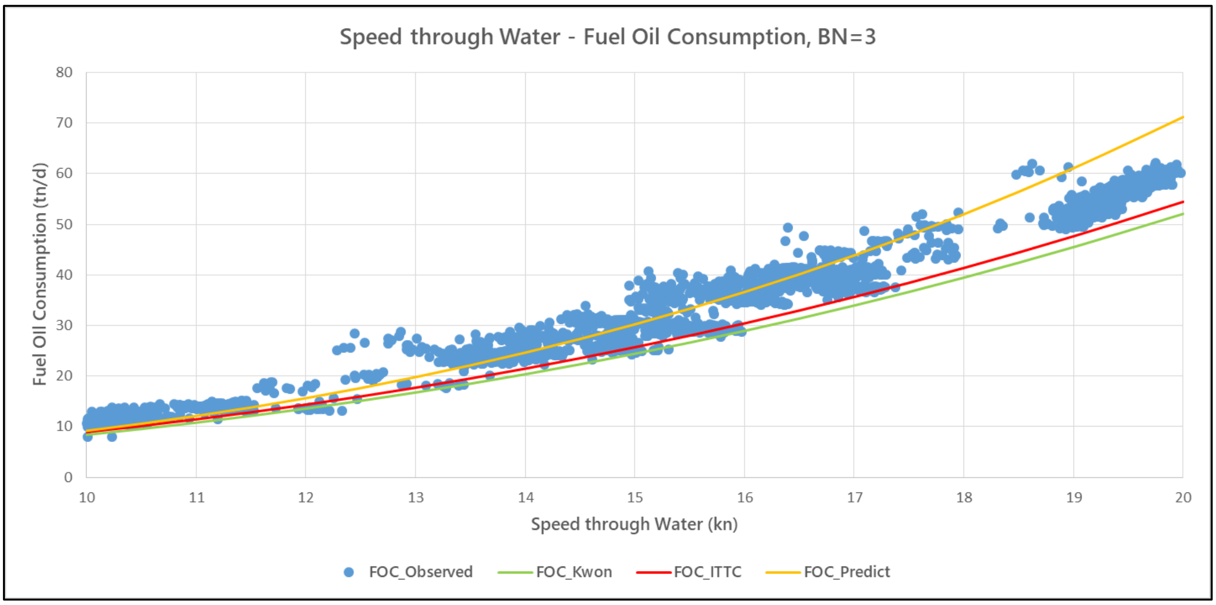


Figure 59. Weather Direction Reduction coefficient for Following Sea

Appendix C: Comparison between the Statistical model & Theoretical models



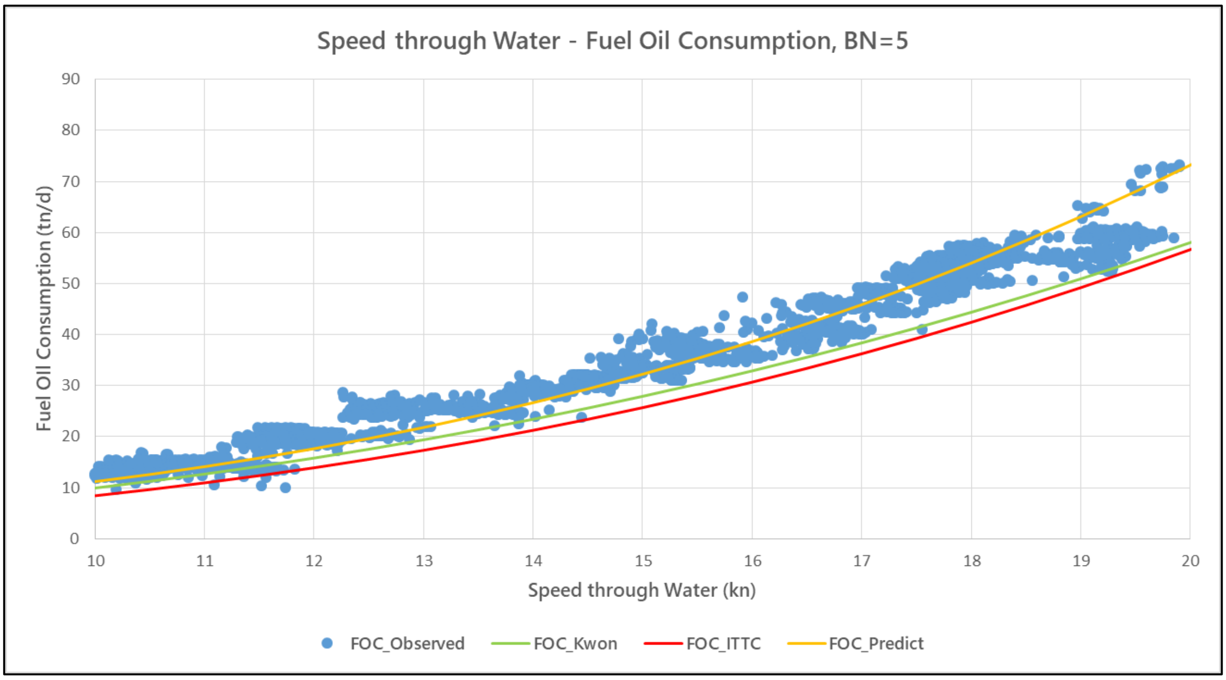


Figure 60. Head Sea, mean draft = 9.3 m

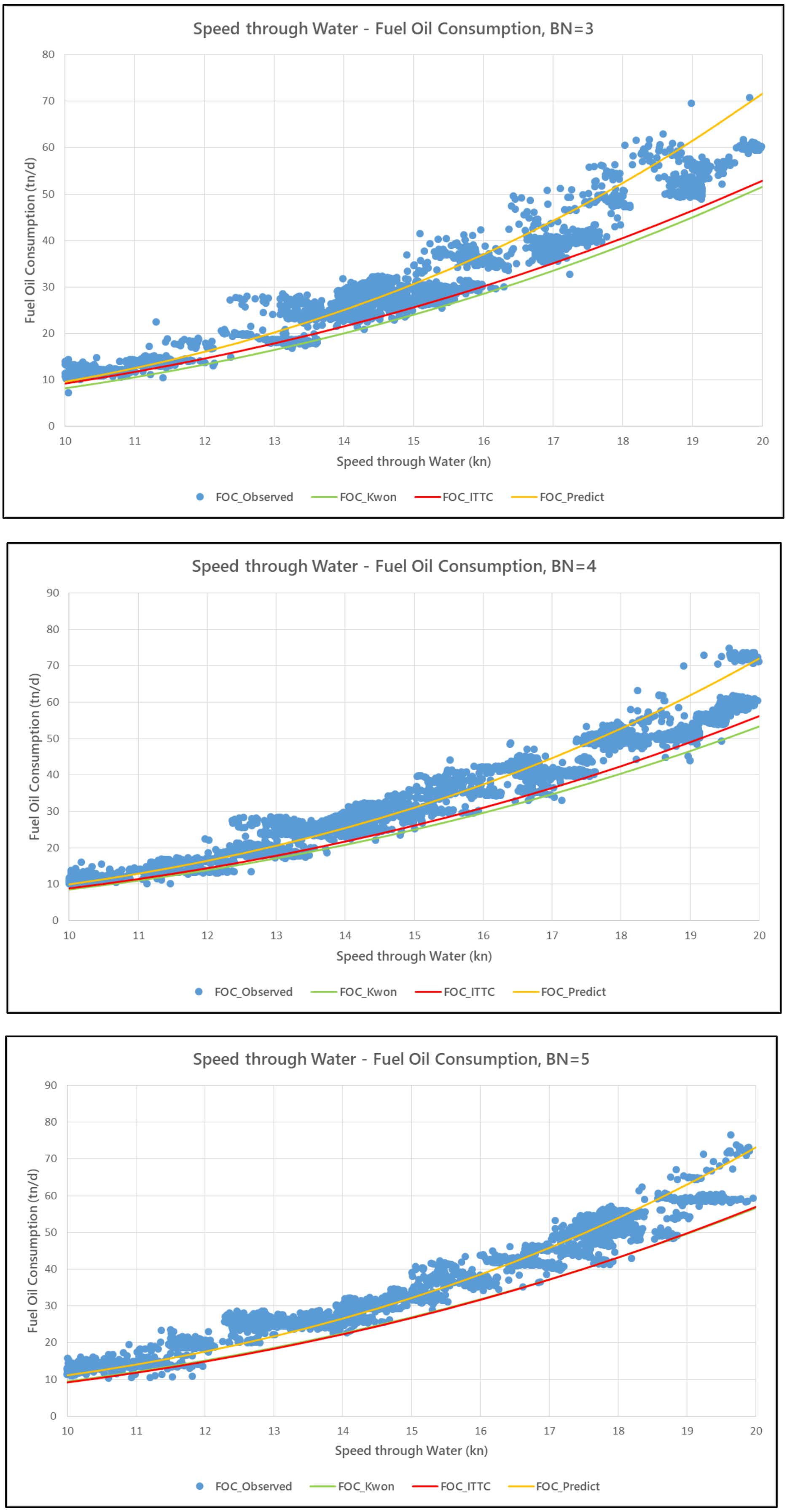


Figure 61. Bow Sea, mean draft = 9.3 m

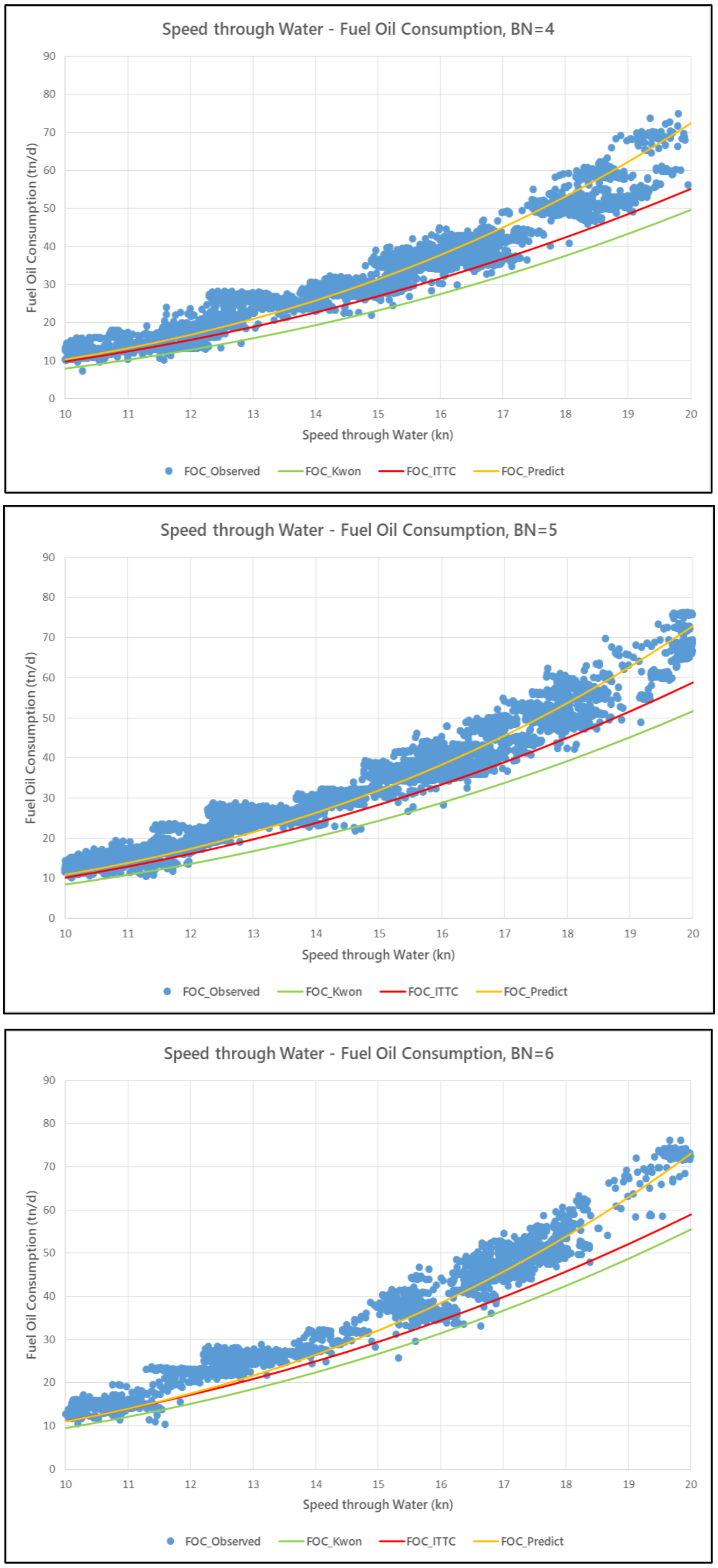


Figure 62. Beam Sea, mean draft = 9.3 m

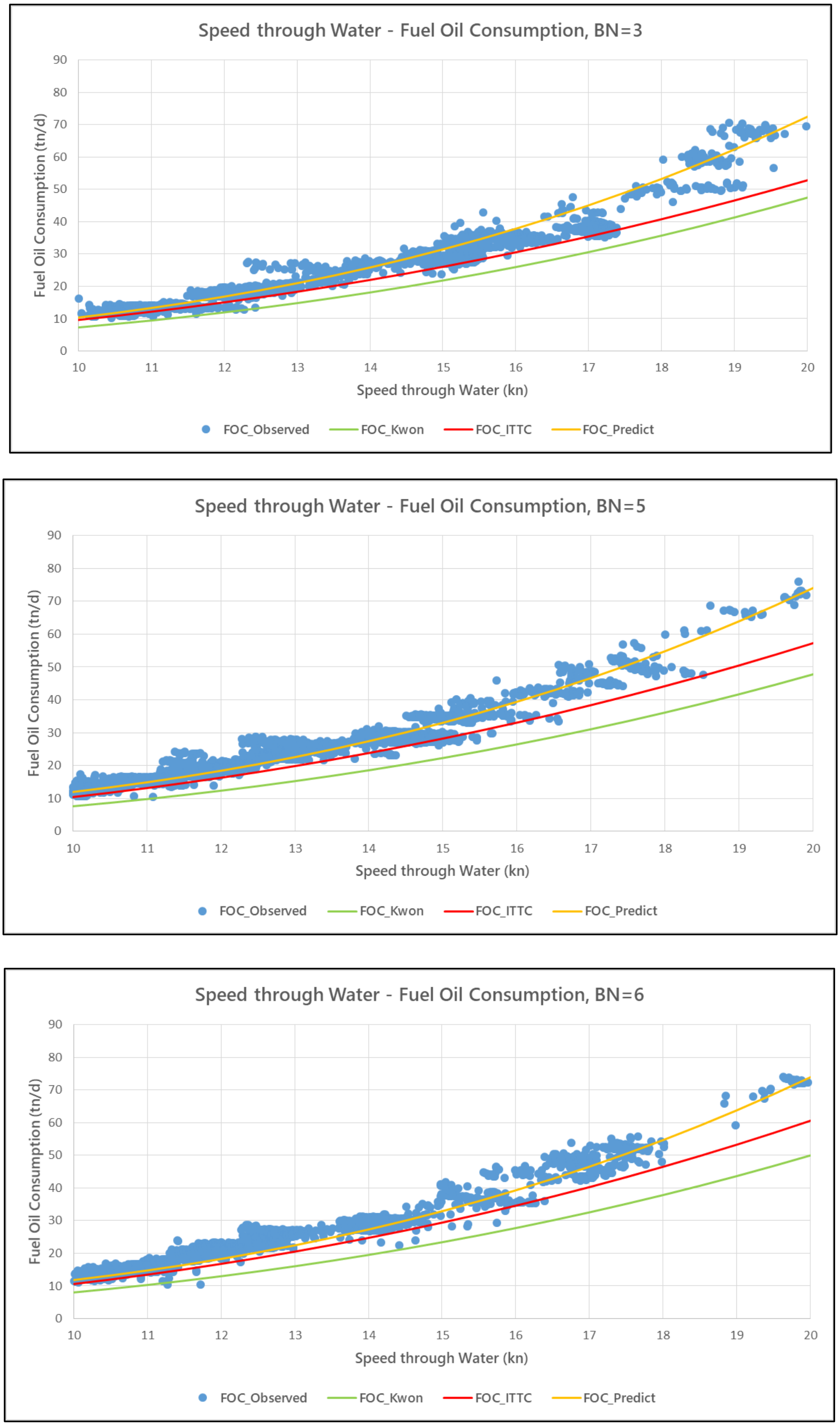
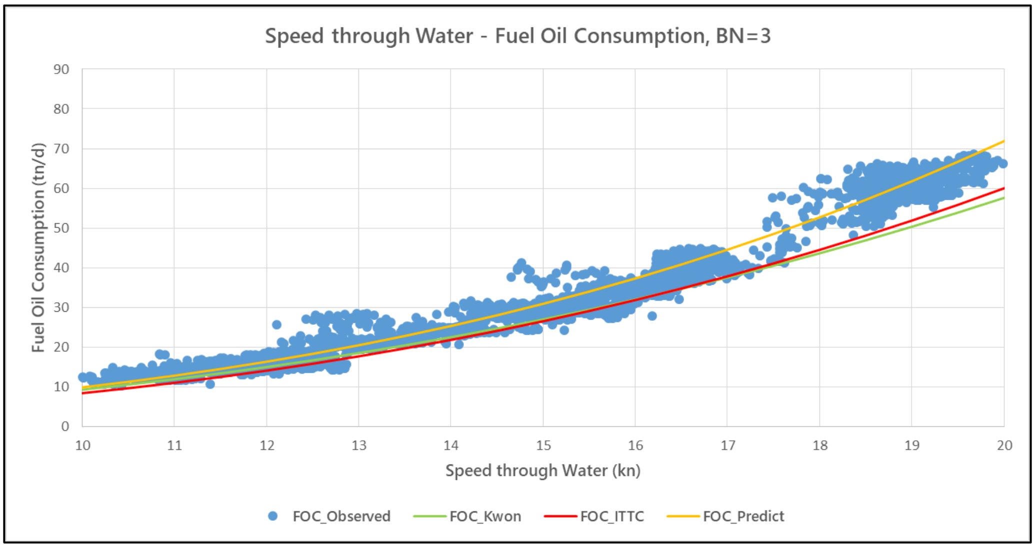
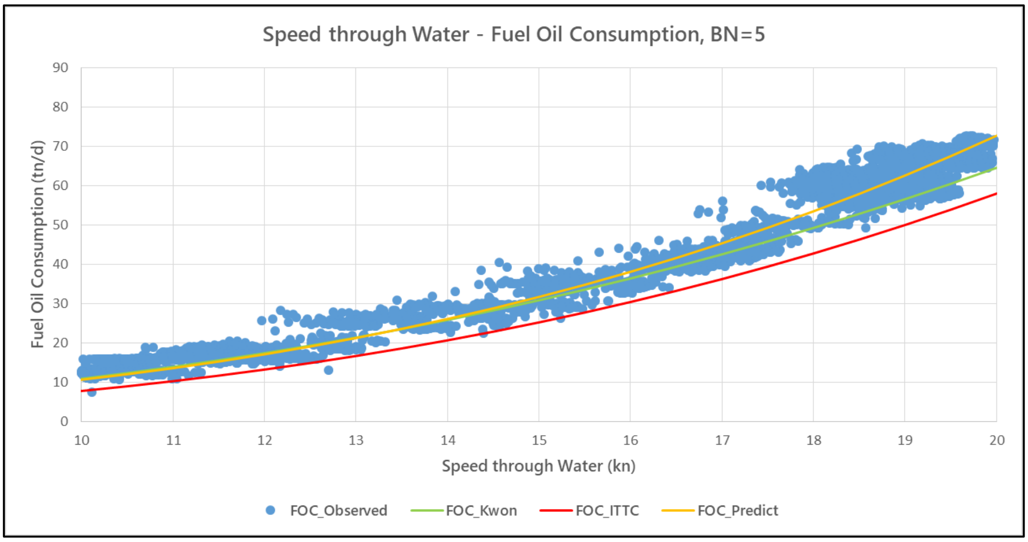


Figure 63. Following Sea, mean draft = 9.3 m





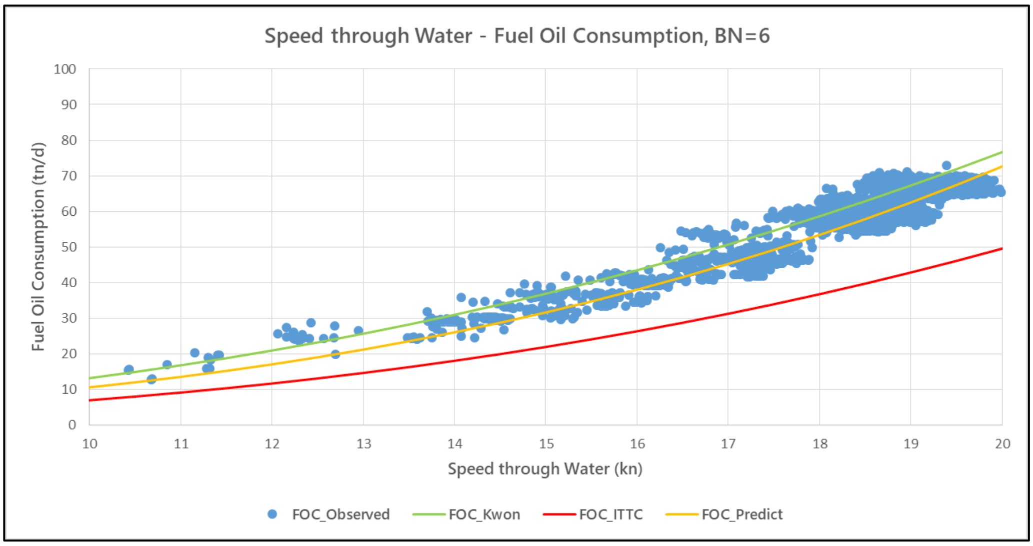
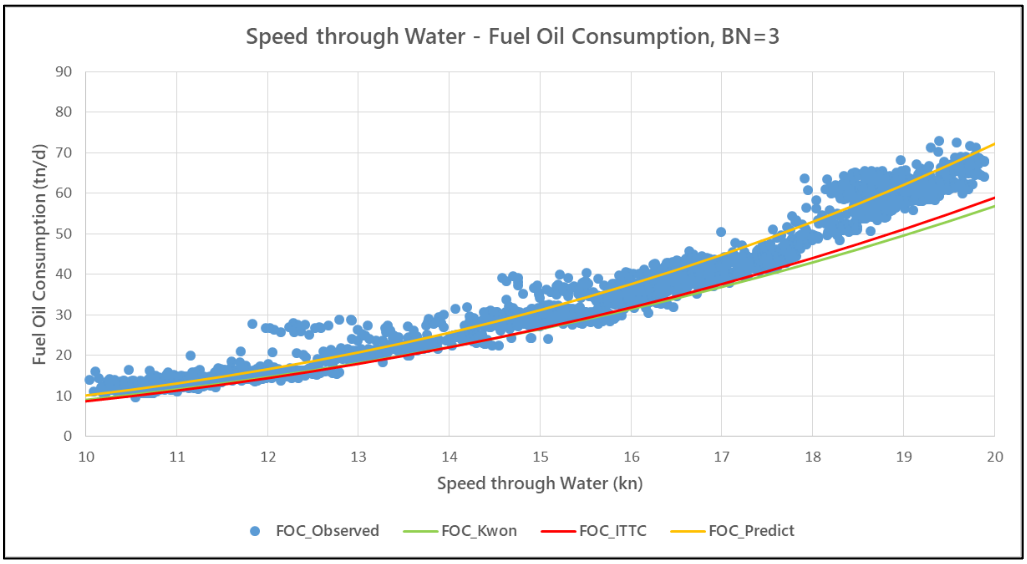
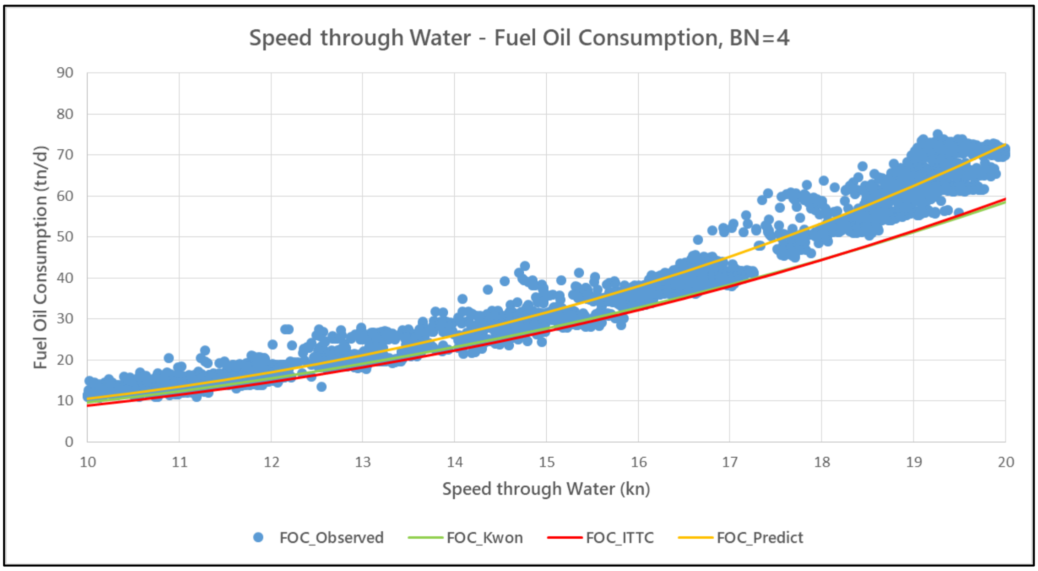


Figure 64. Head Sea, mean draft =10.5 m





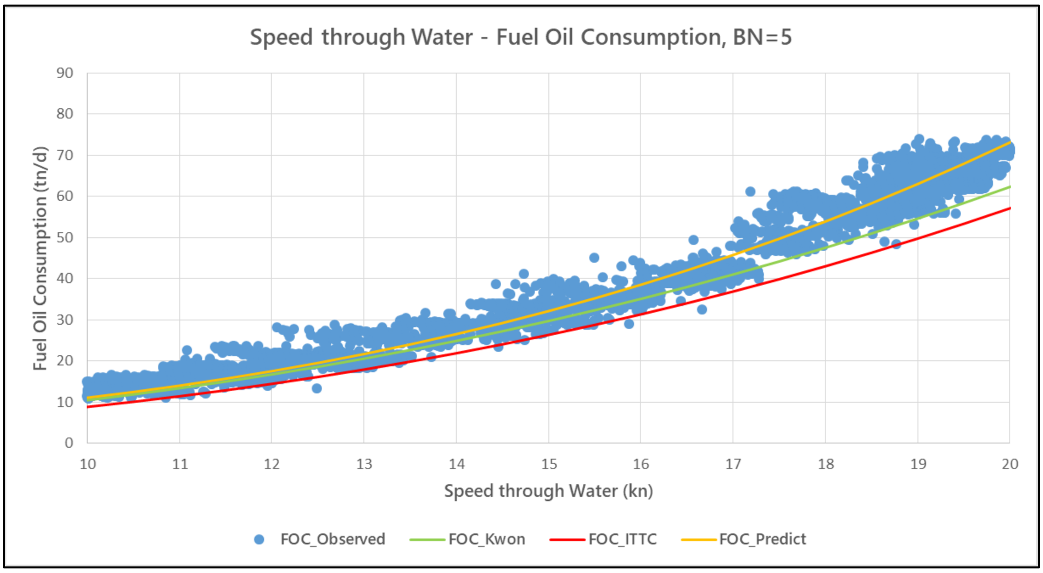
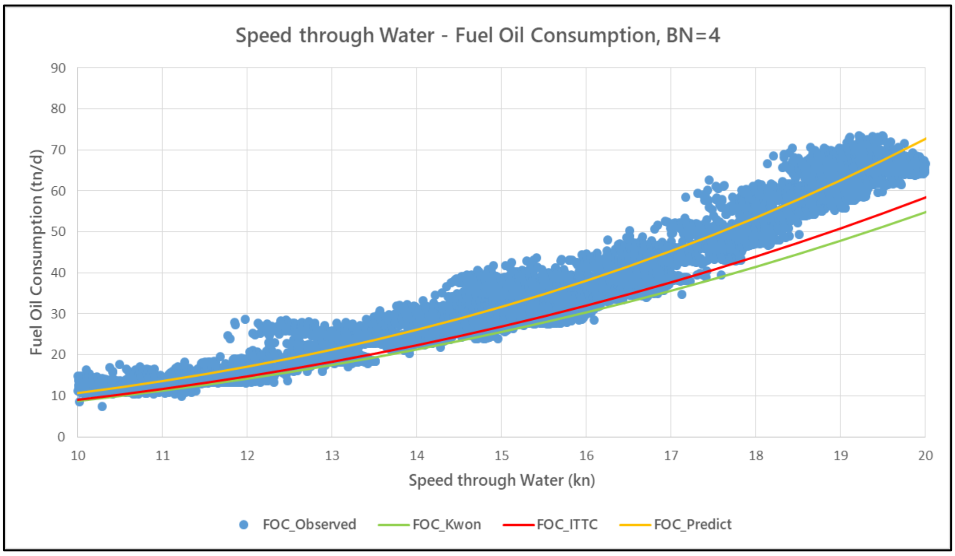
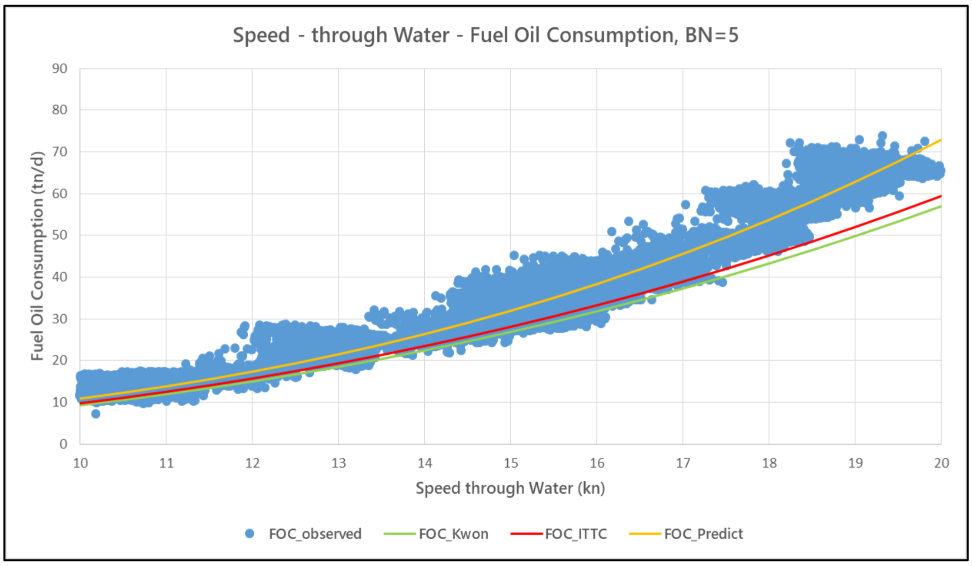


Figure 65. Bow Sea, mean draft =10.5 m





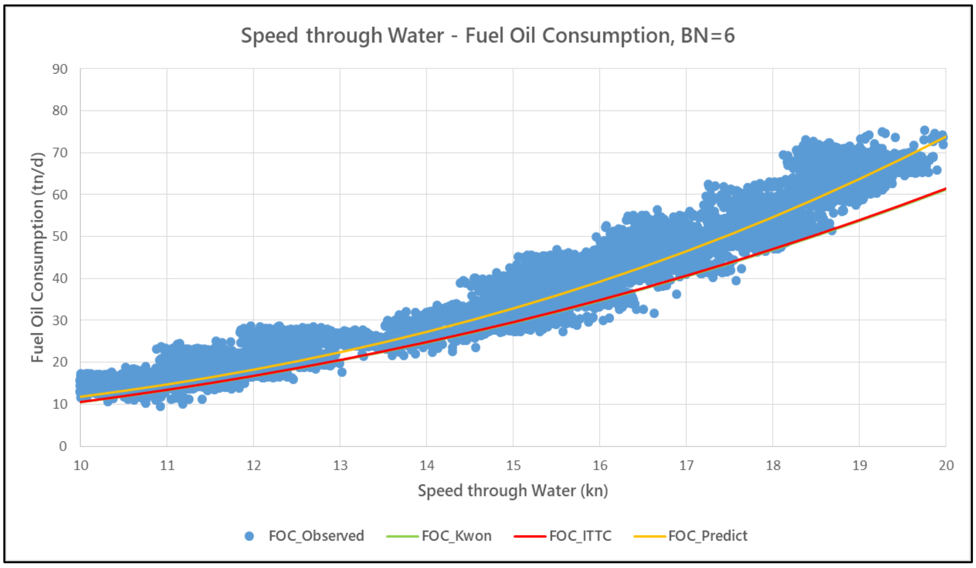


Figure 66. Beam Sea, mean draft =10.5 m

Figure 67. Following Sea, mean draft =10.5 m

Appendix D: Propeller & Main Engine's modelling

The most efficient solution in the calculation of the delivered power of the vessel is the use of open water diagrams, provided they are available for the specific propeller type. The proposed methodology is described in the following paragraphs.

For a specific vessel, the propeller's characteristics (diameter (D), pitch ratio (P/D), number of blades (z), expanded blade area ratio (A_e/A_o), thrust coefficient (K_T) and torque coefficient (K_Q) curves are provided.

Since the vessel's speed (V) is provided and the total resistance can be calculated, with the use of the aforementioned equations (sub – section 3.1.5), the thrust coefficient (K_T), torque coefficient (K_Q), advance coefficient (J), number of revolutions per second (n) and the propeller's efficiency (n_O) can be determined (Politis, 2019).

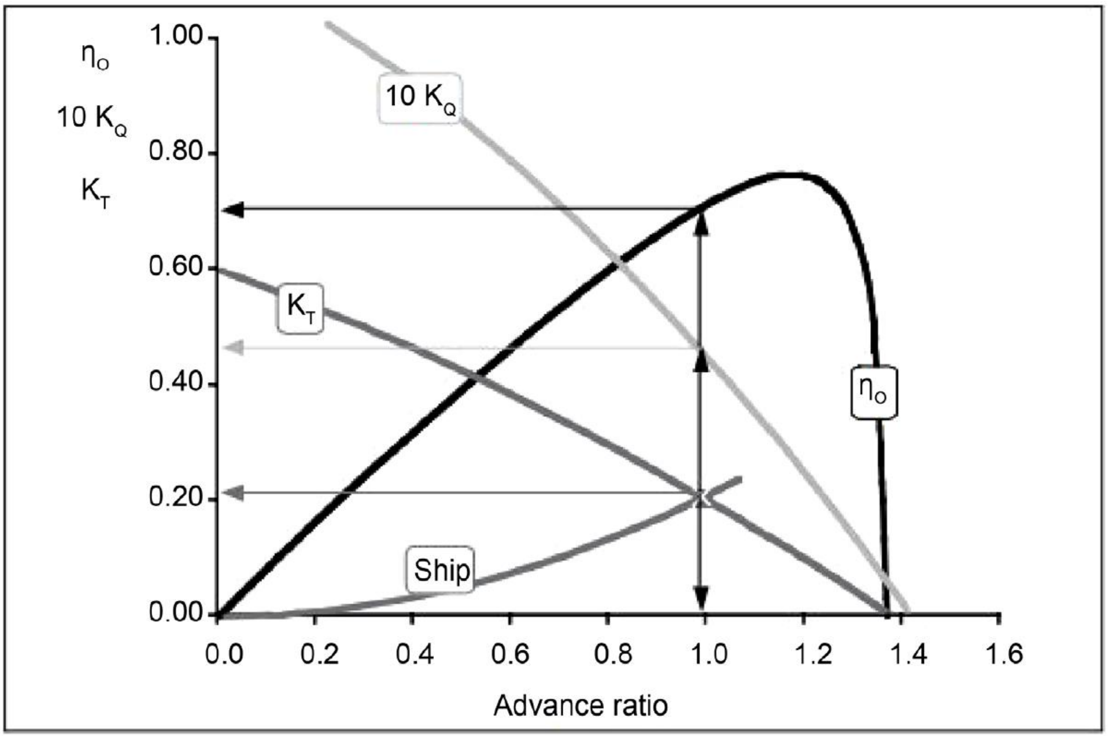


Figure 68. Open Water diagram

Firstly, the thrust deduction factor (t) and the wake friction (w) can be obtained either from models tests or empirical formula. The required thrust of the propeller can be calculated using the following equations:

$$T = \frac{R_O}{(1-t)} = R \quad \text{(D.1)}$$

$$K_T(J) = \frac{T}{\rho \cdot n^2 \cdot D_P^4} \quad \text{(D.2)}$$

$$J = \frac{V_a}{n \cdot D_P} \quad \text{(D.3)}$$

The following quantity is calculated from the aforementioned equations.

$$\frac{K_T(J)}{J^2} = \frac{\frac{T}{\rho \cdot n^2 \cdot D_P^4}}{\left(\frac{V_a}{n \cdot D_p}\right)^2} = \frac{T}{\rho \cdot V_a^2 \cdot D_P^2} \quad (D.4)$$

Where,

- D_p: Propeller diameter
- $K_T(J)$: Open water propeller thrust coefficient
- $V_a = V_s \cdot (1\text{-}w)$
- ρ = 1,025 kg/m^3

Then the curve $K_T = CJ^2$ is plotted on the open water diagram, as it is shown in Figure 68. The advance coefficient J is determined by the intersection point of that curve and the thrust coefficient curve (K_T – J).

After the advance coefficient is determined, the thrust coefficient (K_T), torque coefficient (K_Q) and the propeller's efficiency (n_O) are determined by the propeller's open water diagram, as it is presented in Figure 68.

The rotation rate of the propeller (n), in revolutions per second, is found from the relation:

$$n = \frac{V_a}{J \cdot D_P} \quad (D.5)$$

After calculating the propeller shaft revolutions, the torque (Q) of the propeller can be defined from the following equations:

$$K_Q(J) = \frac{Q_O}{\rho \cdot n^2 \cdot D_P^5} \quad (D.6)$$

$$Q_O = \rho \cdot n^2 \cdot D_P^5 \cdot K_Q(J) \quad (D.7)$$

The delivered power (DHP) is calculated using the following equation:

$$n_R = \frac{Q_O}{Q} \quad (D.8)$$

$$DHP = 2 \cdot \pi \cdot Q \cdot n \quad (D.9)$$

$$DHP = 2 \cdot \rho \cdot \pi \cdot n^3 \cdot D_P^5 \cdot K_Q(J) \quad (D.10)$$

The delivered power is calculated then the shaft power is derived using the following equation:

$$SHP = DHP \cdot n_S \quad (D.11)$$

Where, n_S is the shaft efficiency.

The propulsive coefficient is calculated using the following equation:

$$P.C. = n_T = \frac{EHP}{SHP} = \frac{EHP \cdot THP_O \cdot DHP_O \cdot DHP}{THP_O \cdot DHP_O \cdot DHP \cdot SHP} = \frac{R_O \cdot V}{T \cdot V_O} \cdot n_O \cdot \frac{Q_O}{Q} \cdot n_s = n_o \cdot \frac{1-t}{1-w} \cdot n_R \cdot n_S \quad (D.12)$$

Where, $THP_O = T \cdot V_O$ (D.13)

In the particular thesis the open water diagram of the propeller is not provided, hence the use of statistical and semi – empirical methods is proposed.