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Running Vacuum in String Inspired Cosmologies and Matter-Antimatter Asymmetry in the Universe -Baryogenesis through Leptogenesis

Master Thesis

of

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In accordance with the requirements of the degree of Interdepartmental Program of Postgraduate studies in Physics and Technological Applications. in the School of Applied Mathematical and Physical Sciences, I present the following thesis entitled,

Running Vacuum in String Inspired Cosmologies and Matter-Antimatter Asymmetry in the Universe Baryogenesis through Leptogenesis

This work was performed under the supervision of Professor Nikolaos Mavromatos. I declare that the work submitted in this thesis is my own, except as acknowledged in the text and footnotes, and has not been previously submitted for a degree at National Technical University of Athens or any other institution.

Despoina Farakou

To Nala.

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Abstract

The purpose of this Master thesis is to study the matter-antimatter asymmetry in the universe. In string inspired cosmologies, Baryogenesis through Leptogenesis can occur through the CP asymmetric decay of Right Handed Neutrinos (RHN). The first chapter is a general introduction to the topic, while the second chapter examines the physics of Leptogenesis occuring from the majorana RHN asymmetric decay, including the contribution of the one loop diagrams. Chapter 3 is a brief introduction to String theory, while also discusses the coupling between the Kalb-Ramond field and the RHN. Chapter 4 focuses on the asymmetric decay of the RHN, at tree level, and studies the Boltzmann equations that lead to Leptogenesis. Chapter 5 is aimed to study the Baryogenesis through Leptogenesis, due to sphalerons. Finally, conclusions are drawn in Chapter 6.

Περίληψη

Ο στόχος αυτής της μεταπτυχιακής εργασίας είναι να εξετάσει την ασυμμετρία ύλης-αντιύλης σε επεκτάσεις του Καθιερωμένου Προτύπου. Το πρώτο καφάλαιο αποτελεί μία εισαγωγή στην Βαρυογένεση και εξετάζει τις συνθήκες για να δημιουργηθεί η ασυμμετρία. Το δεύτερο κεφάλαιο επεκτείνει το Καθιερωμένο Πρότυπο κατά την ύπαρξη δύο ειδών Δεξιόστροφων Νετρίνων και επικεντρώνεται στον μηχανισμό Λεπτογένεσης λόγω της ασυμμετρικής διάσπασης του νετρίνων σε λεπτόνια και αντι-λεπτόνια. Το τρίτο κεφάλαιο εισάγει τον αναγνώστη στην Θεωρία Χορδών, και μελετάει την σύζευξη των φερμιονίων με ένα σταθερό πεδίο υποβάθρου. Το τέταρτο κεφάλαιο εξετάζει την Λεπτογένεση μέσω της ασυμετρικής διάσπασης των Δεξιόστροφων Νετρίνων, εξαιτίας της ύπαρξης του πεδίου υποβάθρου. Το πέμπτο κεφάλαιο συζητάει τον μηχανισμό Βαρυογένεσης μέσω Λεπτογενεσης, λόγω διαδικασιών παραβίασης της Λεπτονκής και Βαυονικής συμμετρίας στο Πρώιμο Σύμπαν. Τέλος, το τελευταίο κεφάλαιο περιλαμβάνει τον επίλογο.

Contents

Chapter 1

Introduction

It seems that most of our visible universe consists of matter, rather than anti-matter. The origin of such an asymmetry is yet unknown, although there are a few theories that offer a promising explanation.

Tracing back the matter-antimatter asymmetry, at the time of the last scattering, the imprints of the acoustic oscillations of the baryon-photon fluid can be seen in the measurements of the Cosmic Microwave Background anisotropies. Consequently, we can determine the baryon asymmetry, when the universe was 300.000 yrs old.

Another indication of the asymmetry is the Big Bang Nucleosynthesis (BBN), where the first nuclei were created. The Nucleosynthesis is affected by the baryon asymmetry density and the radiation density, e.g. the more the radiation density, the later would the Nucleosynthesis occur.

According to experimental results the baryon-to-photon ratio is, to a very good approximation, equal to

$$\Delta n(T \geq 1GeV) = \frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}} = \frac{n_B - n_{\bar{B}}}{s} = (8.4 - 8.9) \times 10^{-11} \tag{1.1}$$

where s, n_B are the entropy density and baryon density respectively, [4].

Although, in the beginning of time we assume that the universe consisted equally of matter and anti-matter, but this is not the case in the late universe ,or at least for $(T \leq T_{BBN})$.

Now that we have established that there really is a matter-antimatter asymmetry, we only have to find out the reason for such an asymmetry to exist.

- Sakharov's Conditions for Baryogenesis
  In 1967 Sakharov was the first that talked about the idea of Baryogenesis, which is the dynamical generation of the baryon asymmetry. Although, it took almost a decade for the idea to be taken seriously by the rest of the scientific community. In order for Baryogenesis to occur there are three conditions, the so-called Sakharov conditions,[26] [24]

- Baryon number B violation
  In order to initially have $B = n_b - n_{\bar{b}} = 0$ and later on $B \neq 0$, there have to take place some processes that violate the Baryonic number conservation.

  In the Standard Model physics, there are interactions that the Baryonic number is not conserved, [10].

$$\partial_\mu J^\mu_{B+L} = \frac{3g^2}{32\pi^2}\epsilon_{\alpha\beta\gamma\delta}W^{\alpha\beta}_\alpha W^{\gamma\delta}_\alpha \tag{1.2}$$

  where $W^{\gamma\delta}_\alpha$ is the SU(2) field strength, [33, 28, 6].

- C and CP violation
  Consider the following hypothetical interaction, $X^+ \to A^+ + B^0$. In order for the non-preservation of the baryon number, the production rate of the particles must be unequal to those of anti-particles,

$$\Gamma(X^+ \to A^+ + B^0) \neq \Gamma(X^- \to A^- + B^0) \tag{1.3}$$

  More precisely,

$$\begin{gathered}\Gamma(X^+_L \to A^+_R + B^0_L) + \Gamma(X^+_R \to A^+_L + B^0_R) \\ \neq \\ \Gamma(X^-_L \to A^-_R + B^0_L) + \Gamma(X^-_R \to A^-_L + B^0_R)\end{gathered}$$

  Considering C violation, we have that

$$\Gamma(X^+_L \to A^+_R + B^0_L) \neq \Gamma(X^-_L \to A^-_R + B^0_L) \tag{1.4}$$

  and

$$\Gamma(X^+_R \to A^+_L + B^0_R) \neq \Gamma(X^-_R \to A^-_L + B^0_R). \tag{1.5}$$

  While in the case of the CP violation,

$$\Gamma(X^+_L \to A^+_R + B^0_L) \neq \Gamma(X^-_R \to A^-_L + B^0_R) \tag{1.6}$$

  and

$$\Gamma(X^+_R \to A^+_L + B^0_R) \neq \Gamma(X^-_L \to A^-_R + B^0_L). \tag{1.7}$$

  Combining the above equations, we conclude that in order to have

$$\Gamma(i \to f) \neq \Gamma(\bar{i} \to \bar{f}), \tag{1.8}$$

  both the C and CP symmetry should be violated.

- Out-of-equilibrium
  The third condition for Baryogenesis, arises from the need for the interaction to be out of equilibrium.

In case of equilibrium, the reaction rate that creates the asymmetry between baryon-antibaryon, will be the same as the reaction rate that takes it back to the non asymmetrical phase,

$$\Gamma(i \to f) = \Gamma(f \to i). \tag{1.9}$$

Another way of explaining this, [24], is by considering that a certain species X is thermal equilibrium, and so is $\bar{X}$, with number densities

$$n_X \approx g_X (m_X T)^{3/2} e^{-\frac{m_X}{T} + \frac{\mu_X}{T}}, \qquad n_X \approx g_X (m_X T)^{3/2} e^{-\frac{m_X}{T} - \frac{\mu_X}{T}} \tag{1.10}$$

where $m_X = m_{\bar{X}} >> T$. Then the baryon asymmetry is equal to

$$B = n_X - n_{\bar{X}} = g_X (m_X T)^{3/2} e^{-\frac{m_X}{T}} sinh\left(\frac{\mu_X}{T}\right). \tag{1.11}$$

In addition, from condition 1, there can be interactions that violate the Baryon number, e.g. $XX \to \bar{X}\bar{X}$, which leads to $\mu_X = 0$.

Consequently, eq.1.11 for $\mu_X = 0$ gives $B = 0$, and no asymmetry would be generated.

- Baryogenesis via Leptogenesis

Another way for Baryogenesis to occur, is through Leptogenesis due to the sphaleron processes in the very early Universe.

This Thesis discuss two Leptogenesis processes in Standard Model extensions.

Chapter 2 discusses the case of the existence of majorana Right Handed Neutrinos (RHN). We consider the decay of the RHN into lepton and the Higgs field, and into anti-leptons and anti-charged Higgs field. One can show that at tree level interaction the decay rate of the production of leptons and anti-leptons, is exactly the same. While, if one considers the contributions of the one loop diagrams of the decay, when interfering a second species of RHN, one can prove that the RHN decays CP asymmetric. Afterwards, one may construct the Boltzmann equations for the two interactions, and hence combine them in order to study the evolution of the Lepton asymmetry in our Universe.

Chapter 3, is a brief introduction to String Theory. We consider the physics of a quantum string, and consider the first excited state of a closed string. The first excited state can be represented by three components, the graviton, the dilaton and the Kalb-Ramond (KR) field. One may write the effective action of String theory, including the first excited state, as also other excited states, e.g. the fermions. In this case, we see the non-trivial term of the coupling of the fermions with the Kalb-Ramond field.

Chapter 4, suggests the process of Leptogenesis in String inspired Cosmologies. Due to the coupling of the KR field and the fermions, the fermion spinors changes. That way, when calculating the decay rate of the RHN into leptons and anti-leptons, one will find out that they are not equal, even at tree level. Consequently, due to the CP asymmetric decay of the RHN, there is a Lepton asymmetry generation, that can be studied through the Boltzamann equations of the interactions.

Chapter 5, consists of two parts. The first part, studies the Baryon (B) and Lepton (L) number violating processes in the very early Universe. While also discusses briefly the sphaleron processes, that suggest the rate of the B and L violations in high temperature, and the suppression in low temperatures. The second part, discusses the procedure of the Baryon asymmetry generation, due to the non-conservation of the B+L number in the early Universe.

Finally, conclusions are drawn in Chapter 6.

Chapter 2

Leptogenesis

As mentioned in the previous Chapter there are certain conditions in order for Baryogenesis to occur. Although, there can be some alternatives, for example we can have Baryogenesis through Leptogenesis. This is exactly the case examined in this thesis.

Lepton number L may be violated by some non-SM physics, leading to a lepton asymmetry, that then gets converted into the observed baryon asymmetry, due to sphalerons. If one extend the Standard Model, by including Right Handed Neutrinos (RHN), one can see that that the lepton number L may be violated. Leptogenesis may occur that way, by the CP-asymetric decay of the RHN.

Although, as we will further discuss, one can not obtain CP-violating decay of the RHN at tree level. In order to have CP-asymmetric decay, one should include one-loop diagrams to the computation, with more than one RHN flavors included in the process. In that order of magnitude, $O(\lambda^2)$, we also take into account 2-2 scatterings.

2.1 Right Handed Neutrinos and the See-Saw Model

Neutrinos are electrically neutral, that means that their masses could be introduced in a different way than the rest of the fermions (quarks, electrons etc.).

- <u>Dirac mass term</u>

$$\nu = \nu_L + N_R \tag{2.1}$$

$$L = y\bar{L}HN_R + h.c. \tag{2.2}$$

that after the symmetry breaking would suggest a mass term like

$$L = m_D\bar{\nu}_L N_R + h.c. \tag{2.3}$$

where y is the Yukawa coupling, N_R is the right handed neutrino, H is the higgs field and L is the left handed lepton doublet. This the so-called Higgs portal .

- <u>Majorana mass term</u>

  Considering the neutrino is a majorana fermion means that, the neutrino is identical to its antiparticle. [3], [18]

$$\nu = N_R + N_R^c \tag{2.4}$$

$$L = m_R(\bar{N_R^c} N_R + h.c.) \tag{2.5}$$

  Here the subscript "c" indicates the charge conjugation, meaning $N_R^c = C\bar{N}_R^T$, while $C = i\gamma^2\gamma^0$.
  Note also, that the mass term violates the lepton number, [21]. Consider a global transformation, with $N_R' = e^{i\Lambda} N_R$ and $(N_R')^c = e^{-i\Lambda} N_R^c$, the above Lagrangian is not invariant under this transformation.

- <u>Combined Dirac Majorana mass term</u>

  One may combine the Dirac and Majorana mass term into one.

$$L = m_D \bar{\nu}_L N_R + \frac{1}{2} m_R \bar{N_R^c} N_R + h.c. \tag{2.6}$$

  That way through the see-saw mechanism, we can write the total mass term in the following way [15]

$$L = \begin{pmatrix} \bar{\nu_L} & \bar{N_R^c} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

  Meaning that, if we include in the theory both left handed and right handed neutrinos, and since neutrinos are neutral (and so they are their own antiparticles), we can have a Dirac mass term, while also a Majorana mass term. Both of the terms can combined and represented through the see-saw mechanism.

2.2 Standard Model Extension

In this section we review a mechanism that eventually produce a lepton asymmetry. Firstly, we extend the standard model, considering the existence of right handed neutrinos N_1, N_2, suggested by the seesaw model discussed in the previous section. The Lagrangian can be written as,

$$\begin{aligned} L = L_{SM} + N_1 i\partial\!\!\!/ N_1 + \lambda_1 \bar{L} H N_1 + \frac{M_1}{2} N_1^2 \\ + N_2 i\partial\!\!\!/ N_2 + \lambda_{2,3} \bar{L} H N_2 + \frac{M_2}{2} N_2^2 + h.c. \end{aligned} \tag{2.7}$$

where the notation is the same as in the previous subsection, [29].

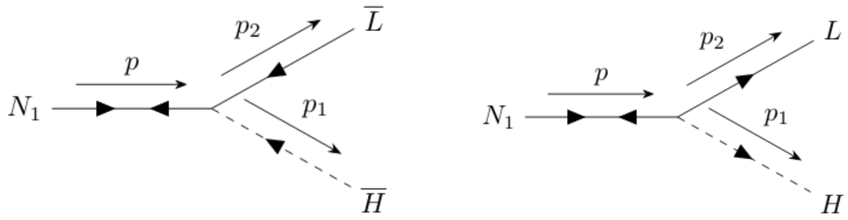

Figure 2.1: $N_1 \to LH$ and $N_1 \to \bar{L}\bar{H}$

2.2.1 Right Handed Neutrino Decay

In this subsection we study the decay of the Majorana RHN into leptons and anti-leptons, shown in fig.2.1. More precisely, we calculate in some detail the decay rate of N_1, at tree-level, following the steps shows in the nest pages .

- First of all we study the Lagrangian term of the interaction of the RHN with the Higgs field and the leptons, see eq. 4.1,

$$\begin{aligned} L \ni \lambda_1 \overline{e_L^-} H^+ N_{1R} + h.c &= \lambda_1 \overline{e^- \frac{1-\gamma^5}{2}} H^+ N_1 \frac{1+\gamma^5}{2} + h.c. \\ &= \lambda_1 \overline{e^-} \frac{1+\gamma^5}{2} H^+ N_1 \frac{1+\gamma^5}{2} + h.c. \\ &= \lambda_1 \overline{e^-} H^+ N_1 \left(\frac{1+\gamma^5}{2}\right)^2 + h.c. \\ &= \lambda_1 \overline{e^-} H^+ N_1 \frac{1+\gamma^5}{2} + \lambda_1^* \overline{N_1} \frac{1-\gamma^5}{2} H^{+\dagger} e^- \\ &= \lambda_1 \overline{e^-} H^+ N_1 \frac{1+\gamma^5}{2} + \lambda_1^* \overline{e^+} H^- \frac{1-\gamma^5}{2} N_1 \end{aligned} \tag{2.8}$$

  where we made use of the charge conjugation and the majorana conditions. In eq.2.8, we can see that the vertex rule corresponds to a factor of $-i\lambda_1(1+\gamma^5)/2$.

- Moving on to the computation of the decay amplitude $\mathcal{M}_1$ at the tree-level $N_1 \to LH$, shown in the left diagram of fig. 2.1

$$-i\mathcal{M}_1 = \bar{u}^s(p_2)(-i)\lambda_1 \frac{1+\gamma^5}{2} u^r(p) \tag{2.9}$$

$$
\begin{aligned}
\overline{|\mathcal{M}_1|^2} &= \frac{1}{2s+1}\sum_s\sum_r \left(\bar{u}^s(p_2)(-i)\lambda_1\frac{1+\gamma^5}{2}u^r(p)\right)^\dagger \times \left(\bar{u}^s(p_2)(-i)\lambda_1\frac{1+\gamma^5}{2}u^r(p)\right) \\
&= \frac{\lambda_1^\dagger\lambda_1}{2}Tr\left[\overline{\frac{1+\gamma^5}{2}}(\not{p}_2+m_e)\frac{1+\gamma^5}{2}(\not{p}+M_1)\right] \\
&= \frac{\lambda_1^2}{2}Tr\left[\frac{1-\gamma^5}{2}(\not{p}_2+m_e)\frac{1+\gamma^5}{2}(\not{p}+M_1)\right] \\
&= \frac{\lambda_1^2}{2}Tr\left[\not{p}_2\left(\frac{1+\gamma^5}{2}\right)^2(\not{p}+M_1)\right] = \frac{\lambda_1^2}{2}Tr\left[\not{p}_2\frac{1+\gamma^5}{2}(\not{p}+M_1)\right] \\
&= \frac{\lambda_1^2}{2}Tr\left[\frac{1}{2}\gamma^\mu(\gamma^\nu p_\nu+M_1)\right]p_{2\mu} + \frac{\lambda_1^2}{2}Tr\left[\frac{1}{2}\gamma^\mu\gamma^5(\gamma^\nu p_\nu+M_1)\right]p_{2\mu} \\
&= \frac{\lambda_1^2}{2}\frac{1}{2}p_{2\mu}p_\nu 4\eta^{\mu\nu} + \frac{\lambda_1^2}{2}\frac{1}{2}Tr\left[\gamma^\mu p_{2\mu}p_\nu M_1\right] \\
&+ \frac{\lambda_1^2}{2}\frac{1}{2}p_{2\mu}p_\nu Tr\left[\gamma^\mu\gamma^5\gamma^\nu\right] + \frac{\lambda_1^2}{2}\frac{1}{2}p_{2\mu}p_\nu Tr\left[\gamma^\mu\gamma^5 M_1\right] \\
&= \lambda_1^2(p\cdot p_2)
\end{aligned}
\tag{2.10}
$$

one can derive the result of eq. 2.10, using the trace identities of the gamma matrices [18].

- Consequently, one may now compute the decay width, in natural units

$$
\Gamma_1 = \frac{1}{2E_{N_1}}\int\frac{d^3p_1}{2E_1(2\pi)^3}\frac{d^3p_2}{2E_2(2\pi)^3}\overline{|\mathcal{M}_1|^2}(2\pi)^4\delta^4(p-p_1-p_2) \tag{2.11}
$$

replacing eq. 2.10 in eq. 2.11, we get

$$
\Gamma_1 = \frac{1}{2E_{N_1}}\int\frac{d^3p_1}{2E_1(2\pi)^3}\frac{d^3p_2}{2E_2(2\pi)^3}(p\cdot p_2)(2\pi)^4\delta^4(p-p_1-p_2) \tag{2.12}
$$

Taking into account the four momentum conservation, $p^\mu = p_1^\mu + p_2^\mu$, and the fact that $M_1 >> m_e, m_H$, one can derive the following equation

$$
\begin{aligned}
&(p-p_2)^2 = p_1^2 \\
&p^2 + p_2^2 - 2p\cdot p_2 = p_1^2 \\
&M_1^2 + m_e^2 - m_H^2 = 2p\cdot p_2 \\
&M_1^2 \approx 2p\cdot p_2
\end{aligned}
\tag{2.13}
$$

And then combine eq. 2.10 and2.13, in order to further simplify the decay width of eq. 2.12.

$$\begin{aligned}
\Gamma_1 &= \frac{1}{2E_{N_1}}\int \frac{d^3p_1}{2E_1(2\pi)^3}\frac{d^3p_2}{2E_2(2\pi)^3}(\frac{1}{2}\lambda_1^2M_1^2)(2\pi)^4\delta^4(p-p_1-p_2) \\
&= \frac{\lambda_1^2 m M_1^2}{4M_1}\int \frac{d^3p_1}{2E_1(2\pi)^3}\frac{d^3p_2}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p-p_1-p_2) \\
&= \frac{\lambda_1 M_1}{4}\int \frac{d^3p_1 d^3p_2}{2E_1(2\pi)^2}\delta(M_1-E_1-E_2)\delta^3(p-p_1-p_2) \\
&= \frac{\lambda_1^2 M_1}{4}\int \frac{4\pi\cdot d|p_2||p_2|^2}{4|p_2|^2(2\pi)^2}\delta(M_1-2E_2) \\
&= \frac{\lambda_1^2 M_1}{32\pi}
\end{aligned} \tag{2.14}$$

At this point one should have in mind that the N_1-decay could also produce a ν_e and a ϕ^0,

$$L \ni \lambda_1 \overline{\nu}_{eL} H^0 N_{1R} \tag{2.15}$$

that, after some calculations, would lead to decay width equal to Γ_1. Therefore, we get a total decay width

$$\boxed{\Gamma_{N_1\to LH} = \frac{\lambda_1^2 M_1}{16\pi}} \tag{2.16}$$

- Respectively, one can compute the decay width of the right diagram of fig. 2.1,Γ_2. Starting with the amplitude of the decay $\mathcal{M}_2$

$$-i\mathcal{M}_2 = \bar{v}^s(p)(-i)\lambda_1^*\frac{1-\gamma^5}{2}v^r(p_2) \tag{2.17}$$

$$\begin{aligned}
\overline{|\mathcal{M}_2|^2} &= \frac{1}{2s+1}\sum_s\sum_r\left(\bar{v}^s(p)(-i)\lambda_1\frac{1-\gamma^5}{2}v^r(p_2)\right)^\dagger \times \left(\bar{v}^s(p)(-i)\lambda_1\frac{1-\gamma^5}{2}v^r(p_2)\right) \\
&= \frac{\lambda_1^\dagger\lambda_1}{2}Tr\left[\overline{\frac{1-\gamma^5}{2}}(\not{p}-M_1)\frac{1-\gamma^5}{2}(\not{p}_2-m_e)\right] \\
&= \frac{\lambda_1^2}{2}Tr\left[\not{p}\frac{1-\gamma^5}{2}(\not{p}_2+m_e)\right] \\
&= \frac{\lambda_1^2}{2}Tr\left[\frac{1}{2}\gamma^\mu(\gamma^\nu p_{2\nu}+m_e)\right]p_\mu - \frac{\lambda_1^2}{2}Tr\left[\frac{1}{2}\gamma^\mu\gamma^5(\gamma^\nu p_{2\nu}+m_e)\right]p_\mu \\
&= \frac{\lambda_1^2}{2}\frac{1}{2}p_\mu p_{2\nu}4\eta^{\mu\nu} + \frac{\lambda_1^2}{2}\frac{1}{2}Tr\left[\gamma^\mu p_\mu m_e\right] \\
&- \frac{\lambda_1^2}{2}\frac{1}{2}p_\mu p_{2\nu}Tr\left[\gamma^\mu\gamma^5\gamma^\nu\right] - \frac{\lambda_1^2}{2}\frac{1}{2}p_\mu p_{2\nu}Tr\left[\gamma^\mu\gamma^5 m_e\right] \\
&= \lambda_1^2(p\cdot p_2)
\end{aligned} \tag{2.18}$$

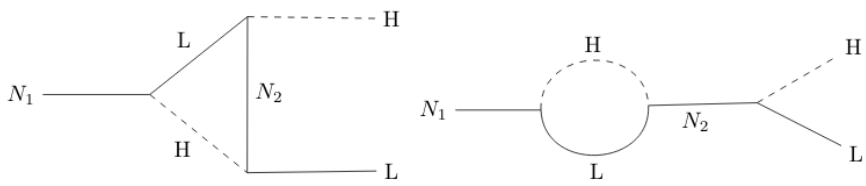

Figure 2.2: N_1 decay, vertex graph and self energy .

Comparing eq.2.10 and eq.2.18, one can see that they are identical. Consequently, the decay width Γ_2 of $N_1 \to \bar{L}\bar{H}$ at tree-level, is equal to Γ_1.

$$\boxed{\Gamma_{N_1 \to LH} = \Gamma_{N_1 \to \bar{L}\bar{H}} = \frac{\lambda_1^2 M_1}{16\pi}} \tag{2.19}$$

2.2.2 CP-asymmetry

We define the CP-asymmetry parameter ϵ_1, as

$$\epsilon_1 \equiv \frac{\Gamma(N_1 \to L + H) - \Gamma(N_1 \to \bar{L} + \bar{H})}{\Gamma(N_1 \to L + H) + \Gamma(N_1 \to \bar{L} + \bar{H})} \tag{2.20}$$

and calculate the parameter, in order to discuss the lepton asymmetry.

In the previous subsection, we proved that the decay width at tree-level of N_1 is $\Gamma(N_1 \to L + H) = \Gamma(N_1 \to \bar{L} + \bar{H}) = \lambda_1^2 M_1/8\pi$, hence there is no CP-asymmetry. Although, considering the interference of loop diagrams, shown in fig 2.2, one can show that N_1 decays CP-asymmetric [14], [11].

- **Vertex graph**

Starting with the vertex graph, one can see from the figures 2.3 and 2.4, that the decay amplitude for the N_1 decay into fermions, is equal to

$$\begin{aligned} \mathcal{M}_1 &= c_0 A_0 + c_1 A_1' + Z_\lambda c_1 \bar{u}^s(p_2) u^r(p) \\ &= c_0 A_0 + c_1 A_1 \end{aligned} \tag{2.21}$$

while for the N_1 decay into anti-fermions is,

$$\begin{aligned} \mathcal{M}_2 &= c_0^* \overline{A}_0 + c_1^* \overline{A'}_1 + Z_\lambda c_1^* \bar{v}^s(p) v^r(p_2) \\ &= c_0^* \overline{A}_0 + c_1^* \overline{A}_1 \end{aligned} \tag{2.22}$$

where c_0 and c_1 are the coupling constants for the tree level decay and the loop interference respectively, while the A_0 and A_1 are the rest of the amplitude. Furthermore, one should include the counter term Z_λ, in order to absorb the logarithmic divergence $1/\epsilon$ and the

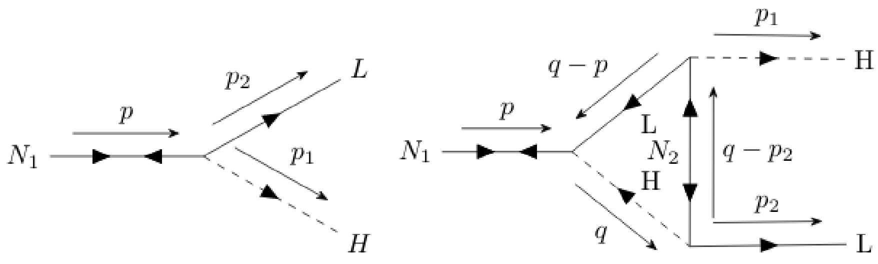

Figure 2.3: N_1 decay into fermions

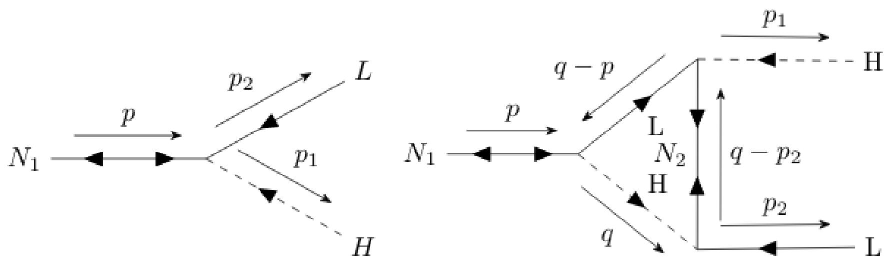

Figure 2.4: N_1 decay into anti-fermions

dependence on the unphysical transmutation mass μ, arising from the loop integral .More specifically,

$$-iA_0 = \bar{u}^s(p_2)(-i)\frac{1+\gamma^5}{2}u^r(p) \tag{2.23}$$

while,

$$\begin{aligned}-iA_1' &= \bar{u}^s(p_2)\int\frac{d^4q}{(2\pi)^4}\frac{i}{q^2}(-i)\frac{1+\gamma^5}{2}\frac{i}{\not{q}-\not{p}_2-m_{N_2}}\\ &(-i)\frac{1+\gamma^5}{2}\frac{i}{\not{q}-\not{p}}(-i)\frac{1-\gamma^5}{2}u^r(p)\\ &=\bar{u}^s(p_2)\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\cdot\frac{1+\gamma^5}{2}\cdot\frac{\not{q}-\not{p}_2+m_{N_2}}{(q-p_2)^2-m_{N_2}^2}\\ &\frac{1+\gamma^5}{2}\cdot\frac{\not{q}-\not{p}}{(q-p)^2}\cdot\frac{1-\gamma^5}{2}u^r(p).\end{aligned} \tag{2.24}$$

Subsequently, one may calculate the CP asymmetry parameter2.20 in the following way, [12],

$$\epsilon_1^{vertex} = \frac{1}{2\int d\Pi\cdot\tilde{\delta}\cdot|c_0A_0|^2}\int d\Pi\cdot\tilde{\delta}\cdot(|\mathcal{M}_1|^2-|\mathcal{M}_2|^2) \tag{2.25}$$

where $\tilde{\delta} = (2\pi)^4\delta(p_i - p_f)$ and $d\Pi = \frac{d^3p_1}{2E_1(2\pi)^3}\frac{d^3p_2}{2E_2(2\pi)^3}$. Computing separately the factor that contains the decay amplitudes, we obtain that,

$$\begin{aligned}
|\mathcal{M}_1|^2 - |\mathcal{M}_2|^2 = &\sum_{spin}(c_0^* A_0^\dagger c_0 A_0 + c_0^* A_0^\dagger c_1 A_1 + c_1^* A_1^\dagger c_0 A_0 \\
&+ c_1^* A_1^\dagger c_1 A_1 - c_0 \bar{A}_0^\dagger c_0^* \bar{A}_0 - c_0 \bar{A}_0^\dagger c_1^* \bar{A}_1 \\
&- c_1 \bar{A}_1^\dagger c_0^* \bar{A}_0 - c_1 \bar{A}_1^\dagger c_1^* \bar{A}_1) \\
= &\sum_{spin}(c_0^* A_0^\dagger c_1 A_1 + c_1^* A_1^\dagger c_0 A_0 \\
&- c_0 \bar{A}_0^\dagger c_1^* \bar{A}_1 - c_1 \bar{A}_1^\dagger c_0^* \bar{A}_0) \\
= &\sum_{spin}(c_0^* c_1 - c_0 c_1^*)(A_0^\dagger A_1 - A_1^\dagger A_0) \\
= &\,4\sum_{spin} Im[c_0^* c_1] \cdot Im[A_0^\dagger A_1] \\
= &\,4\sum_{spin} Im[(\lambda_1^* \lambda_2)^2] \cdot Im[A_0^\dagger A_1]
\end{aligned} \tag{2.26}$$

where we made use of the fact that $|A_i|^2 = |\bar{A}_i|^2$ and that $\bar{A}_i{}^\dagger \bar{A}_j = A_i^\dagger A_j$. Substituting eq. 2.26 into eq. 2.25, we can re-write the CP asymmetry parameter as,

$$\epsilon_1^{vertex} = \frac{\int d\Pi \cdot \tilde{\delta} \cdot 2\sum_{spin} Im[(\lambda_1^*\lambda_2)^2] \cdot Im[A_0^\dagger A_1]}{\int d\Pi \cdot \tilde{\delta} \cdot |c_0 A_0|^2}, \tag{2.27}$$

where in the denominator the only non-vanishing term is the one analogous to tree level amplitude.
Therefore, inserting the values of the A_0, A_1 from eq. 2.23 and eq. 2.24,

$$\epsilon_1^{vertex} = \mathcal{I}_{k1} \cdot 2 \cdot \frac{\frac{16\sqrt{x}\pi(M_{N_1}^2 + (M_{N_1}^2 + M_{N_2}^2)ln[x] - (M_{N_1}^2 + M_{N_2}^2)ln[1+x])}{16 M_{N_1}\pi \cdot 16\pi^2 \cdot 16}}{\frac{M_{N_1}}{16\pi}} \tag{2.28}$$

$$\epsilon_1^{vertex} = \mathcal{I}_{k1}\frac{\sqrt{x}(1 + (1+x)ln[(1+x)/x])}{8\pi} \tag{2.29}$$

where $\mathcal{I}_{k1} \equiv Im[(\lambda_1^*\lambda_k)^2]/\lambda_1^2 \quad ,k = 1, 2$, and $x = M_2^2/M_1^2$.

- **Self energy graph**
  Turning now to the self energy graph, of the fig. 2.2, one can prove that the corresponding CP-asymmetry parameter is equal to

$$\boxed{\epsilon_1^{self} = -\frac{1}{8\pi}\sum_{k=\neq 1}\frac{M_1 M_k}{M_k^2 - M_1^2} I_{k1}.} \tag{2.30}$$

The reason we need more than one right handed neutrinos to be included into our theory, is pointed out in eq. 2.29 and eq.2.30. Considering $k = 1$ in eq.2.29, then $Im[\lambda_1^\dagger \lambda_1] = 0$, hence no asymmetry will be generated. Furthermore, eq. 2.30, is not defined for $k = 1$.

2.3 Boltzmann Equations

To examine the lepton asymmetry that can occur through these decaying processes, one should proceed to the computations of the Boltzmann equations. Firstly, we study the Boltzmann equation considering no CP-violation, $\epsilon_1 = 0$. In more detail,

$$\Gamma(N_1 \to L + H) = \Gamma(\bar{L} + \bar{H} \to N_1) \tag{2.31}$$

$$\Gamma(N_1 \to \bar{L} + \bar{H}) = \Gamma(L + H \to N_1) \tag{2.32}$$

In case $\epsilon_1 = 0$, we have that

$$\Gamma(N_1 \to L + H) = \Gamma(L + H \to N_1) \tag{2.33}$$

Generally, the Boltzmann equations the evolution of the particle's phase space distribution function,$f(p^\mu, x^\mu)$, considering interactions [17].

$$\hat{L}[f] = C[F] \tag{2.34}$$

where $\hat{L}$ is the Liouville operator, and C is the collision operator. The Liouville operator for the FRW metric is

$$\hat{L} = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha} \tag{2.35}$$

$$\hat{L}[f] = E\frac{\partial f}{\partial t} - \frac{\dot{a}}{a}|p|^2 \frac{\partial f}{\partial E} \tag{2.36}$$

and having in mind the definition of the number density

$$n(t) = \frac{g}{2\pi^3} \int d^3p f(E, t) \tag{2.37}$$

eq. 2.34 can be re-written in the follwoing form

$$\frac{dn}{dt} + 3\frac{\dot{a}}{a}n = \frac{g}{2\pi^3} \int C[f] \frac{d^3p}{E} \tag{2.38}$$

Regarding processes of the form $1 \leftrightarrow 2 + 3$, eq. 2.38 becomes

$$\frac{dn_1}{dt} + 3Hn_1 = \int d\vec{p_1} \int d\vec{p_2} \int d\vec{p_3} (2\pi)^4 \delta(p_1 - p_2 - p_3) \times \\ \times [-|A_{1\to 23}|^2 f_1(1 \pm f_2)(1 \pm f_3) + |A_{23\to 1}|^2 (1 \pm f_1) f_2 f_3] \tag{2.39}$$

where H is the Hubble parameter, and $d\vec{p_i} = g d^3 p_i/(2E_i(2\pi)^3)$.

Since the reaction rate of the decay and the inverse reaction rate are equivalent, eq. 2.33, so are the squared amplitudes $|A_{1\to 23}|^2, |A_{23\to 1}|^2$.In addition, we consider that the interaction is fast enough that maintains kinetic equilibrium and so $f(p) = f_{eq}/n_{eq} * n$.

Here $f_{eq} = (e^{E/T} \pm 1)^{-1}$, the Bose-Einstein and Fermi-Dirac distributions. In case of no Bose condensation and Fermi degeneracy, $1 \pm f \approx 1$. Furthermore, the number density of relativistic and non-relativistic particles in equilibrium, evolves as

$$n^{eq}_{rel} = gT^3/\pi^2, \quad T >> m \tag{2.40}$$

$$n^{eq}_{non-rel} = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}, \quad T << m \tag{2.41}$$

Taking all the above suggestions into account, one can derive the following result

$$\frac{dn_1}{dt} + 3Hn_1 =< \Gamma_1 > n_1^{eq}\left(\frac{n_1}{n_1^{eq}} - \frac{n_2}{n_2^{eq}}\frac{n_3}{n_3^{eq}}\right) \tag{2.42}$$

where for convenience we replaced

$$< \Gamma_1 >= \frac{1}{2E_1}\int d\vec{p_2}d\vec{p_3}(2\pi)^4\delta^4(p_1 - p_2 - p_3)|A|^2 \tag{2.43}$$

Additionally, n_2, n_3 are in thermal equilibrium through the gauge interactions, and so we know that$n_2 = n_2^{eq}$ and $n_3 = n_3^{eq}$, that way eq.2.42 takes a simpler form

$$\frac{dn_1}{dt} + 3Hn_1 =< \Gamma_1 > (n_1 - n_1^{eq}) \tag{2.44}$$

The 3H term, comes from the dilution due to the expansion of the universe, and it can be shown that $3H = \dot{s}/s$. For convenience, we write eq. 2.3 in terms of the abundance of the species, $Y \equiv n/s$. Mainly because both number density and entropy density evolve as $\sim a^{-3}$ and so the fraction of those two is "constant".

Consequently, eq. is equivallent to

$$sHz\frac{dY_1}{dz} = \sum \Delta_1\gamma^{eq}(12... \leftrightarrow 34...)\left(\frac{Y_1}{Y_1^{eq}}\frac{Y_2}{Y_2^{eq}}... - \frac{Y_3}{Y_3^{eq}}\frac{Y_4}{Y_4^{eq}...}\right) \tag{2.45}$$

where we defined $z = m/T$, γ^{eq} the rate density. Also, $\Delta_1 = -1$ in case of $1 \leftrightarrow 23$, or $\Delta_1 = -2$ in case of $11 \leftrightarrow 23$, and so on.

2.3.1 Leptogenesis

In this subsection we are interested in the Boltzmann equations that would eventually lead to the Leptogenesis, consequently we have to take into account the CP-asymmetry [29],[12].

Considering the interactions $N_1 \leftrightarrow HL, \bar{H}\bar{L}$ where $\Delta L = 1$, eq. 2.45 becomes

$$sHz\frac{dYN_1}{dz} = -\gamma_D\left(\frac{Y_{N_1}}{Y^{eq}_{N_1}} - 1\right), \quad z = m_{N_1}/T \tag{2.46}$$

where γ_D is the decay rate density. Now, having in mind the CP-violating processes, and the eq. **??**, we write

$$\gamma^{eq}(N_1 \to HL) = \gamma^{eq}(\bar{H}\bar{L} \to N_1) = (1+\epsilon_1)\gamma_D/2 \tag{2.47}$$

$$\gamma^{eq}(N_1 \to \bar{H}\bar{L}) = \gamma^{eq}(HL \to N_1) = (1-\epsilon_1)\gamma_D/2 \tag{2.48}$$

Followingly, one can derive the Boltzmann equation, this time with respect to the L, since we eventually want to compute the lepton abundance. Therefore, considering the $HL \leftrightarrow N_1$ interaction , the eq. 2.39 becomes

$$sHz\frac{dY_L}{dz} = \frac{\gamma_D}{2}\left(-(1-\epsilon_1)\frac{Y_L}{Y^{eq}_L} + (1+\epsilon_1)\frac{Y_{N_1}}{Y^{eq}_{N_1}}\right) \tag{2.49}$$

where we used the eq. 2.45 and the two above equations 2.47,2.48. In the same way one can derive the Boltzmann equation for the evolution of the anti-lepton abundance, through the $\bar{H}\bar{L} \leftrightarrow N_1$

$$sHz\frac{dY_{\bar{L}}}{dz} = \frac{\gamma_D}{2}\left(-(1+\epsilon_1)\frac{Y_{\bar{L}}}{Y^{eq}_{\bar{L}}} + (1-\epsilon_1)\frac{Y_{N_1}}{Y^{eq}_{N_1}}\right) \tag{2.50}$$

Combing eq. 2.49 and 2.50, one can compute the abundance of the lepton excess in our universe, $\mathcal{L} - Y_L - Y_{\bar{L}}$.

$$\boxed{sHz\frac{d\mathcal{L}}{dz} = \epsilon_1\gamma_D\left(\frac{Y_{N_1}}{Y^{eq}_{N_1}} + 1\right) - \frac{\mathcal{L}}{2Y^{eq}_L}\gamma_D} \tag{2.51}$$

Avoiding Overcounting In order for the Boltzmann equation to be accurate, we should include all the related processes in the order of magnitude of λ that we examine [29], [12]. The CP-asymmetry is generated at $\gamma = \mathcal{O}(\lambda^4)$, which means that we have to take into account the interactions shown in fig. 2.5 and 2.6, with reaction rate densities

$$\gamma_t \equiv \gamma^{eq}(LH \leftrightarrow \bar{L}\bar{H}) \tag{2.52}$$

$$\gamma_s \equiv \gamma^{eq}(LL \leftrightarrow HH) \tag{2.53}$$

Figure 2.5: s-channel, $LH \leftrightarrow \bar{L}\bar{H}$

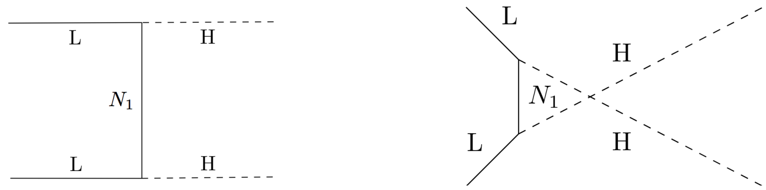

Figure 2.6: t-channel, $LL \leftrightarrow \bar{H}\bar{H}$

and $\gamma_{\Delta L=2} = 2(\gamma_t + \gamma_s)$.

Since the scattering $LH \leftrightarrow \bar{L}\bar{H}$ is mediated by the right handed neutrino $LH \leftrightarrow N_1 \leftrightarrow \bar{L}\bar{H}$, we have to consider that the CP asymmetric processes of $LH \leftrightarrow N_1$ and $N_1 \leftrightarrow \bar{L}\bar{H}$, are already included in the Boltzmann equation 2.58.

We compute the $\gamma_s^{on-shell}$ for the case explained above, and we remove it from our calculation.

$$\gamma_s^{on-shell}(LH \rightarrow \bar{L}\bar{H}) = \gamma^{eq}(LH \rightarrow N_1)BR(N_1 \rightarrow \bar{L}\bar{H}) \tag{2.54}$$

$$\gamma_s^{on-shell}(\bar{L}\bar{H} \rightarrow LH) = \gamma^{eq}(\bar{L}\bar{H} \rightarrow N_1)BR(N_1 \rightarrow LH) \tag{2.55}$$

and so,

$$\gamma_{eq}^{sub}(LH \rightarrow \bar{L}\bar{H}) = \gamma_s - (1 - \epsilon_1)^2\gamma_D/4 \tag{2.56}$$

$$\gamma_{eq}^{sub}(\bar{L}\bar{H} \rightarrow LH) = \gamma_s - (1 + \epsilon_1)^2\gamma_D/4 \tag{2.57}$$

Therefore, the final Boltzmann equation for the $\mathcal{L}$ is

$$\boxed{sHz\frac{d\mathcal{L}}{dz} = \gamma_D\left[\epsilon_1\left(\frac{Y_{N_1}}{Y_{N_1}^{eq}} - 1\right) - \frac{\mathcal{L}}{2Y_L^{eq}}\right] - 2\gamma_{\Delta L=2}^{sub}\frac{\mathcal{L}}{Y_L^{eq}}} \tag{2.58}$$

where we have included bothN_1 decay $\Delta L = 1$, and $\Delta L = 2$ scatterings.

Chapter 3

Kalb-Ramond Torsion

The previous chapter discussed the process of Leptogenesis, when the Standard Model is extended with the addition of majorana Right Handed Neutrinos in the theory. The CP asymmetric decay of such species would eventually lead to the domination of matter, rather than anti-matter.

In the following chapters we shall examine the process of Baryogenesis through Leptogenesis, in String inspired Cosmologies, including also Right Handed Neutrinos.

This chapter is a brief overview of the bosonic String Theory and introduces the gravitational multiplet. The gravitational multiplet of string theory consists of the graviton, the dilaton, and the Kalb-Ramond field. The coupling of the Kalb-Ramond field with the Right Handed Neutrinos, will be proved to be a potential origin of the matter-antimatter asymmetry in the Universe.

3.1 String Theory

At first, one should define what string theory is. Luckily, the name describes exactly the approach of this theory on physics. Basically, the point particles are replaced by strings, interactions by surfaces, and worldlines by worldsheets. This section is written based it the three following citations, [30][34][5].

3.1.1 Bosonic String Action

The action of relativistic point particles (0-brane), may be written the following way[34],

$$S_0 = \frac{1}{2}\int d\tau \left(g_{\mu\nu}(X)\dot{X}^\mu\dot{X}^\nu\right). \tag{3.1}$$

While from the variation of the action 3.1, one can obtain the geodesic equation

$$\ddot{X}^\mu + \Gamma^\mu_{kl}\dot{X}^k\dot{X}^l = 0, \tag{3.2}$$

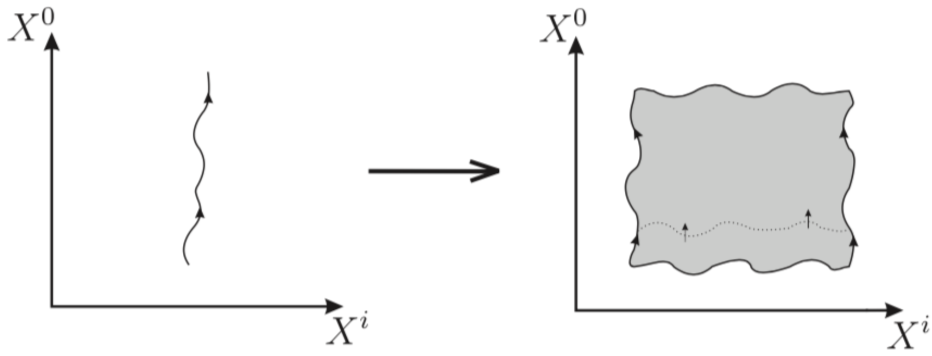

Figure 3.1: The worldsheet is the two-dimensional extension of the world-line, [34].

where Γ^{μ}_{kl} are the Christoffel symbols [8].

In a similar way, one can define the string action (1-brane). Consider the dimensional volume element , $d\mu$, equal to

$$d\mu = \sqrt{-det(G_{\alpha\beta}(X))}d^2\sigma \tag{3.3}$$

where

$$G_{\alpha\beta} = \frac{\partial X^{\mu}}{\partial \sigma^{\alpha}}\frac{\partial X^{\nu}}{\partial \sigma^{\beta}}g_{\mu\nu} \quad , \qquad \alpha, \beta = 0, 1 \tag{3.4}$$

the induced metric on the worldsheet, and $\mu, \nu = 0, ..., D-1$ [34]. We assume a string propagating in D-dimensional spacetime, and parametrize the worldsheet with $\sigma^0 \equiv \tau$ and $\sigma^1 \equiv \sigma$, (this is the two dimensional extension of the worldline, see fig. 3.1). The $g_{\mu\nu}$ is the background space time metric, while $G_{\alpha\beta}$ represent the metric on the string. Assuming the Minkowski background metric, the determinant of the string metric reads

$$det(G_{\alpha\beta}(X)) = (\dot{X}^2)(X'^2) - (\dot{X} \cdot X')^2, \tag{3.5}$$

where $\dot{X} = \partial X/\partial\tau$ and $X' = \partial X/\partial\sigma$. Consequently, the string action is written as

$$S_{NG} = -T \int d\tau d\sigma \sqrt{(\dot{X} \cdot X')^2 - (\dot{X}^2)(X'^2)} \tag{3.6}$$

and it is known as Nambu-Goto action. For convenience, one may re-write the action in a simpler form, by introducing an auxiliary field $h_{\alpha\beta}$.

$$S_{\sigma} = \frac{-T}{2} \int d\tau d\sigma \sqrt{-h} h^{\alpha\beta} \partial_{\alpha} X^{\mu} \partial_{\beta} X^{\nu} g_{\mu\nu} \tag{3.7}$$

3.1.2 Local Symmetries of the Bosonic String Theory Worldsheet

The action of eq. 3.7 is invariant under the re-parameterization and the Weyl transformation.

- Diffeomorphisms
  Is the invariance of the action under the change of a parameter, for example, $\sigma \longrightarrow \sigma' = f(\sigma)$.

- Weyl Symmetry
  Under this transformation the metric change as

$$h_{\alpha\beta}(\tau, \sigma) \longrightarrow h'_{\alpha\beta}(\tau, \sigma) = e^{2\phi(\sigma)} h_{\alpha\beta}(\tau, \sigma), \tag{3.8}$$

  while $\delta X^{\mu}(\tau, \sigma) = 0$.

Using the symmetries of the action, one can reduce the $h_{\alpha\beta}(\tau, \sigma)$ to a flat background metric [34]. That way, the action of eq. 3.7, in now equivalent to

$$S_{\sigma} = \frac{-T}{2} \int d\tau d\sigma (\dot{X}^2 - X'^2), \tag{3.9}$$

also known as Polyakov action.

3.1.3 Field equations and solutions

The next step would be to variate the action of eq. 3.9 with respect to the string field, and demand $\delta S_{\sigma} = 0$. Consequently,

$$\delta S_{\sigma} = \frac{-T}{2} \int d\tau d\sigma (2\dot{X}\delta\dot{X} - 2X'\delta X') = 0 \tag{3.10}$$

and integrating the above equation by parts, we get

$$(-\partial_{\tau}^2 + \partial_{\sigma}^2) X^{\mu} - T \int d\tau [X'\delta X^{\mu}|_{\sigma=\pi} + X'\delta X^{\mu}|_{\sigma=0}] = 0. \tag{3.11}$$

The field equation 3.11, can be simplified further to

$$(-\partial_{\tau}^2 + \partial_{\sigma}^2) X^{\mu} = 0, \tag{3.12}$$

when studying the following cases

- Closed String
  Boundary condition: $X^{\mu}(\tau, \sigma) = X^{\mu}(\tau, \sigma + n)$

- Open String
  Boundary condition: $\partial_{\sigma} X^{\mu}(\tau, \sigma) = \partial_{\sigma} X^{\mu}(\tau, \sigma + n) = 0$

- Open String
  Boundary condition: $X^{\mu}(\tau, \sigma) = X_0^{\mu}, \quad X^{\mu}(\tau, \sigma + n) = X_n^{\mu}$

From here and on, we will continue the calculations working on the light-cone coordinates,

$$\sigma^{\pm} = \tau \pm \sigma. \tag{3.13}$$

Therefore, the field equation in the light-cone coordinate system reads as

$$\partial_+\partial_- X^\mu(\sigma^+,\sigma^-) = 0, \tag{3.14}$$

where $\partial_\pm = \partial/\partial\sigma^\pm$. Regarding the solution of eq. 3.14, one may consider a linear combination of this form

$$X^\mu(\sigma^+,\sigma^-) = X^\mu_R(\sigma^+) + X^\mu_L(\sigma^-), \tag{3.15}$$

where $X^\mu_R(\sigma^+), X^\mu_L(\sigma^-)$ are some arbitrary functions with the indices R and L referring to the right and left propagation respectively.
In case of a Closed String, taking into account the aforementioned boundary condition, a general solution to eq. 3.14 is

$$\begin{aligned} X^\mu_R(\tau,\sigma) &= \frac{1}{2}x^\mu + \frac{1}{2}(\tau-\sigma)l_s^2 p^\mu + \frac{i}{2}l_s\sum_{n\neq 0}\frac{1}{n}a^\mu_n e^{-2in(\tau-\sigma)} \\ X^\mu_L(\tau,\sigma &= \frac{1}{2}x^\mu + \frac{1}{2}(\tau+\sigma)l_s^2 p^\mu + \frac{i}{2}l_s\sum_{n\neq 0}\frac{1}{n}\tilde{a}^\mu_n e^{-2in(\tau+\sigma)} \end{aligned} \tag{3.16}$$

suggesting a total field solution of the form

$$X^\mu(\tau,\sigma) = x^\mu + \frac{1}{2}\tau l_s^2 p^\mu + \frac{i}{2}l_s\sum_{n\neq 0}\frac{1}{n}(a^\mu_n e^{2in\sigma} + \tilde{a}^\mu_n e^{-2in\sigma})e^{2in\tau}, \tag{3.17}$$

where l_s is the string length, $T = 1/(2na')$ and $a' = l_s^2/2$.

The variation of the action of eq. 3.9, with respect to the metric $h_{\alpha\beta}$ gives the following field equations, in the light-cone coordinates,

$$\partial_+ X^\mu \partial_+ X_\mu = 0, \quad \partial_- X^\mu \partial_- X_\mu = 0. \tag{3.18}$$

Considering the second of the two above equations,

$$(\partial_- X^\mu_R)^2 \equiv a' \sum_n L_n e^{-in\sigma^-} = 0, \tag{3.19}$$

where we defined

$$L_n = \frac{1}{2}\sum_m a_{n-m}\cdot a_m. \tag{3.20}$$

Respectively, it can be shown that

$$\tilde{L}_n = \frac{1}{2}\sum_m \tilde{a}_{n-m}\cdot\tilde{a}_m. \tag{3.21}$$

and taking into consideration the eq. 3.19, we obtain that $L_n = \tilde{L}_m = 0$.

3.1.4 Quantum String

Since we are interested in the quantum string theory, like in Quantum Field Theory, we should quantize the fields X^μ. Firstly, we write the canonical momentum of the field $P^\mu(\tau,\sigma) = \delta L/\delta \dot{X}^\mu = \dot{X}^\mu/(2\pi a')$. Then, one should define the canonical equal-time commutation relations,

$$\begin{aligned}[X^\mu(\tau,\sigma), P_\nu(\tau,\sigma')] &= i\delta(\sigma-\sigma')\delta^\mu_\nu \\ [X^\mu(\tau,\sigma), X^\nu(\tau,\sigma')] &= [P_\mu(\tau,\sigma), P_\nu(\tau,\sigma')] = 0\end{aligned} \tag{3.22}$$

that will lead to commutation relations of the $x^\mu, p^\mu, a^\mu_n, \tilde{a}^\mu_n$,

$$[x^\mu, p_n] = i\delta^\mu_n, \quad [a^\mu_n, a^\mu_m] = [\tilde{a}^\mu_n, \tilde{a}^\mu_m] = n\eta^{\mu\nu}\delta_{m+n,0}, \tag{3.23}$$

and the rest commutations are equal to zero.

While, also we define

$$L_n = \frac{1}{2}\sum_m : a_{n-m}\cdot a_m :, \tag{3.24}$$

where the ":" symbol shows the normal order of the operators, (the same applies for $\tilde{L}$). Therefore, we also get that,

$$(L_0 - a)|\phi>= (\tilde{L}_0 - a)|\phi>= 0 \tag{3.25}$$

the parameter a, is a constant and it comes from the normal order of L_n

- At this point one can define the vacuum state of the string,

  $$a^\mu_n|0>= \tilde{a}^\mu_n|0>= 0 \tag{3.26}$$

  and now build up the Fock space, acting with the creation operators $a^\mu_n, \tilde{a}^\mu_n$ on the ground state,

  $$|\phi>= a^{\mu_1\dagger}_{n_1}\cdots\tilde{a}^{\mu_1\dagger}_{n_1}\cdots|0,p> . \tag{3.27}$$

  Note that, the ground state is $\psi(x)\cdot|0>$, hence in the momentum space these states are also eigenstates of the momentum operator.

- Ghosts
  Consider a physical state $|\psi>= a^{0\dagger}_m|0,p>$, this would mean that

  $$||\psi||^2 =< 0,p|a^0_m a^{0\dagger}_m|0,p>=< 0,p|[a^0_m a^{0\dagger}_m]|0,p>= -< 0||0> \tag{3.28}$$

  This result is unphysical and therefore we have to modify our model, in order to exclude the negative norm states. It can be proven that in the case of 26 dimensions, the unphysical states can be avoided, for further discussion see [34].

- Light-cone Gauge
In the same way that we considered the light-cone coordinates for representing the worldsheet, one can introduce the light-cone coordinates for gauge fixing the background spacetime. Hence we define,

$$X^{\pm} = \sqrt{\frac{1}{2}}(X^0 \pm X^{D-1}) \tag{3.29}$$

and so the Mikowski metric now reads,

$$ds^2 = -2dX^+dX^- + \sum_{i-1}^{D-2} dX^i dX^i \tag{3.30}$$

where

$$X^+ = X_L^+(\sigma^+) + X_R^+(\sigma^-) \equiv x^+ + \frac{1}{2}a'p^+(\sigma^+ + \sigma^-) \tag{3.31}$$

From the field equations that we derived, one may write the X^- as,

$$\begin{aligned} X_R^-(\sigma^-) &= \frac{1}{2}x^- + \frac{1}{2}(\sigma^-)l_s^2 p^- + \frac{i}{2}l_s \sum_{n\neq 0} \frac{1}{n} a_n^- e^{-2in(\sigma^-)} \\ X_L^-(\sigma^+) &= \frac{1}{2}x^- + \frac{1}{2}(\sigma^+)l_s^2 p^- + \frac{i}{2}l_s \sum_{n\neq 0} \frac{1}{n} \tilde{a}_n^- e^{-2in(\sigma^+)} \end{aligned} \tag{3.32}$$

where

$$a_n^- \equiv \sqrt{\frac{1}{2a'p^+}} \sum_{m=-\infty}^{+\infty} \sum_{i=1}^{D-2} a_{n-m}^i a_m^i. \tag{3.33}$$

The important result obtained from all the above considerations, is the effective mass of a string, which is now proved to be equal to

$$M^2 = \frac{4}{a'} \sum_{i=1}^{D-2} \sum_{m>0} a_{-m}^i a_m^i = \frac{4}{a'} \left(\sum_{i=1}^{D-2} \sum_{m>0} \tilde{a}_{-m}^i \tilde{a}_m^i - a \right) \tag{3.34}$$

and after the renormalization, the final result is

$$M^2 = \frac{4}{a'} \left(\sum_{i=1}^{D-2} \sum_{m>0} \tilde{a}_{-m}^i \tilde{a}_m^i - \frac{D-2}{24} \right) \tag{3.35}$$

- Ground State
Starting with the calculation of the ground state mass, we find that,

$$M^2 = -\frac{D-2}{6a'} \tag{3.36}$$

which is a negative result. These particles, are referred as tachyons.

- <u>First Excited States</u>
  Regarding the first excited states, one can act with the creation operators on the vacuum state as,

$$|\Omega^{ij} >= \tilde{a}^{i}_{-1} a^{j}_{-1} |0, p >, \tag{3.37}$$

and this would produce $(D-2) \cdot (D-2)$ particle states.This means that for$D = 26$, we have 576 particle states.

Massless particles have fewer degrees of freedom than the massive. Consider a massive particle in D dimensions, if we make a Lorentz transformation ton the rest frame, we obtain that (E,0....,0), and so it forms a representation of SO(D-1). Now, think about a massless particle, there is no rest frame and so a Lorentz transformation would give (E,E,0,...,0), and a final representation of SO(D-2). Hence, the first excited states of a close string can form a massless representation of the D-dimensional Poincare group.

Decomposing the $|\Omega^{ij} >$, one can obtain a symmetric and traceless spin-2 particle (called graviton), a scalar (called dilaton), and an antisymmetric second-rank tensor (called Kalb-Ramond field).

3.2 Vielbein Field and Spin Connection

In GR, conventionally, the "natural" differential basis is used as the coordinates basis, and they are given by

$$\hat{e}_{(\mu)} = \partial_{(\mu)} \quad , \qquad \hat{e}^{(\mu)} = dx_{(\mu)} \tag{3.38}$$

for the tangent T_p and contangent T^*_p space at a point p, respectively [35]. Also, the Greek indices are used to label the coordinates of the curved space-time. One may choose the local set of basis vectors $\hat{e}_{(a)}$, tetrad basis, to satisfy the

$$(\hat{e}_{(a)}, \hat{e}_{(b)}) = \eta_{a,b} \tag{3.39}$$

where $\eta_{a,b}$ is the Minkowski metric, and the small Latin indices will be used for reference in the non-coordinate frame.

The main goal is to find a coordinate chart that would cover the entire curved manifold. In order to achieve that, one can write any vector as a linear combination of the tetrad basis vectors. [35]

$$\hat{e}_{(\mu)}(x) = e^{a}_{\mu}(x)\hat{e}_{(a)} \quad , \qquad \hat{e}_{(a)} = e^{\mu}_{a}(x)\hat{e}_{(\mu)}(x) \tag{3.40}$$

where $e^{a}_{\mu}(x)$ is the so called vierbein field, which is like a transformation between curved space and flat space. While the metric can be written the following way,

$$g_{\mu\nu}(x) = e^{a}_{\mu}(x)e^{b}_{\nu}(x)\eta_{ab}. \tag{3.41}$$

The curvature of a manifold can be taken into account through the affine connection $\Gamma^{\alpha}_{\beta\gamma}$. While, in the tetrad basis, the affine connection is replaced by the spin connection, $\omega^{a}_{\mu b}$. It can be shown that it is connected to the affine connection through the following relation,[35]

$$\omega^{a}_{\mu b} = e^{a}_{\nu} e^{\lambda}_{b} \Gamma^{\nu}_{\mu\lambda} - e^{\lambda}_{b} \partial_{\mu} e^{a}_{\lambda}. \tag{3.42}$$

Consequently, one can show that the curvature and the torsion may be written the following way,

$$R^{a}_{b} = d\omega^{a}_{b} + \omega^{a}_{c} \wedge \omega^{c}_{b} \quad , \qquad T^{a} = de^{a} + \omega^{a}_{b} e^{b} \tag{3.43}$$

as a function of the spin connection and the frame field.

3.3 String theory induced geometry with torsion

In the section 3.1, we examined the bosonic string theory, and we established that the first excited state of a closed string consists of

- traceless, symmetric, dimensionless, spin-2 tensor field $g_{\mu\nu}$, the graviton.
- dimensionless spin-0 scalar field, the dilaton Φ, with string coupling $g^{s} = e^{\Phi}$
- dimensionless spin-1 antisymmetric tensor, Kalb-Ramond field $B_{\mu\nu}$

The field strength of the $B_{\mu\nu}$ is a three-form, $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$, where the [..] symbolize the antisymmetrization of the respective indices. The field strength, also, satisfies the Bianchi identity, $\partial_{[\mu} H_{\nu\rho\sigma]} = 0$. Hence, the effective action of the first excited state may be written as follows

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R - 2\partial^{\mu}\phi\partial_{\mu}\phi - e^{-4\phi} H_{\lambda\mu\nu} H^{\lambda\mu\nu} - \frac{2}{3}\delta c \cdot e^{2\phi} \right) \tag{3.44}$$

where the δc is the central charge deficit (in the previous sections we considered that c=D=26), and we define $\frac{1}{\kappa^2} \equiv \frac{M_s^2 V^c}{8\pi}$. Here the M_s is the string mass scale, while the V^c is the the compactification volume in terms of the Regge slope α' of the string. Note also, that in the action of eq. 3.44, there are only kinetic terms, since as we mentioned earlier the state of the string is massless.

In equation 3.44, the term containing the strength of the KR field can be absorbed in to the generalized curvature term $\bar{R}(\bar{\Gamma})$, defining a new "torsional connection" $\bar{\Gamma}$.

$$\bar{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} + e^{-2\phi} H^{\lambda}_{\mu\nu} \neq \bar{\Gamma}^{\lambda}_{\nu\mu} \tag{3.45}$$

Introducing the covariant derivative $\bar{\nabla}_a$ in the tetrad basis,

$$\bar{\nabla}_a \psi = e^{\mu}_{a}\left(\partial_\mu + \frac{i}{2}\bar{\omega}_{b\mu c}\Sigma^{bc}\right)\psi \tag{3.46}$$

where $\Sigma^{bc} = \frac{i}{4}[\gamma^b, \gamma^c]$, the generator of the Lorentz group. The spin connection is defined as in the previous section, with the small difference that the affine connection is "torsional" in the case examined

$$\omega^{a}_{\mu b} = e^{a}_{\nu} e^{\lambda}_{b} \bar{\Gamma}^{\nu}_{\mu\lambda} - e^{\lambda}_{b}\partial_\mu e^{a}_{\lambda}. \tag{3.47}$$

String anomaly cancellation requires to redefine the KR field strength, by including some extra terms.

$$\begin{aligned} &H = dB + \frac{\alpha'}{4}(\omega_{3L} - \omega_{3Y}) \\ &\omega_{3L} = Tr[\omega \wedge (d\omega + \frac{2}{3}\omega \wedge \omega)], \quad \omega_{3Y} = Tr[A \wedge (dA + \frac{2}{3}A \wedge A)] \end{aligned} \tag{3.48}$$

where ω_{3L} is the Lorentz Chern-Simons term which is a function of the spin connection ω, and A is the Yang-Mills gauge field. Consequently, the modified Bianchi identity reads as,

$$d\mathbf{H} = \frac{\alpha'}{4}\left(Tr[\mathbf{R} \wedge \mathbf{R}] - Tr[\mathbf{F} \wedge \mathbf{F}]\right). \tag{3.49}$$

where $\mathbf{F} = d\mathbf{A} + \mathbf{A} \wedge \mathbf{A}$, the Yang-Mills field strength.
In this case, one can set the constraint of eq. **??** in the path integral with respect to the action 3.44 and integrate over the field strength of the KR field. That way, one gets the following result,

$$\begin{aligned} \mathcal{Z} &= \int \Delta H e^{-iS}\delta(\epsilon^{\mu\nu\rho\sigma}\nabla_\mu H_{\nu\rho\sigma} - c_1 R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} + c_2 F_{\alpha\beta}\tilde{F}^{\alpha\beta}) \\ &= \int \Delta H e^{-iS}\int db e^{-i\int d^4x\sqrt{-g}(\epsilon^{\mu\nu\rho\sigma}\nabla_\mu H_{\nu\rho\sigma} - c_1 R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} + c_2 F_{\alpha\beta}\tilde{F}^{\alpha\beta})} \end{aligned} \tag{3.50}$$

where b is a pseudoscalar lagrange multiplier field, massless KR axion field. Consequently the effective action can be written as,

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu b\partial^\mu b - c_1 b R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} + ...\right). \tag{3.51}$$

In the early Universe, and under some circumstances [4], one may have a condensate of $< R\tilde{R} >$, that suggests an inflationary universe of the type encountered as "running vacuum". The vacuum energy density, which is written as a function of the Hubble parameter, is characterized by an equation of motion of the form,

$$p(H(t)) = -\rho(H(t)) \tag{3.52}$$

for further analysis of the model, see the Master Thesis of Vyron Chiotelis, [9].

Taking into account that

$$R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} = \nabla_\mu K^\mu \tag{3.53}$$

which is a mathematical property, we have that the equation of motion of the b-axion field is written as,

$$\frac{1}{\sqrt{-g}}\partial\left(\sqrt{-g}(\partial^\mu b - c_1 K^\mu)\right) = 0 \tag{3.54}$$

One solution of the above equation is

$$\dot{b} = c_1 K^0, \quad \partial_i b, R_i << \dot{b}, K^0 \tag{3.55}$$

where in this case the $b = b(t)$ violates the Lorentz symmetry. Considering $< R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} >= const.$, and combine it with eq. 3.53, we get that for an isotropic and homogeneous universe,

$$\frac{d}{dt}K^0 + 3HK^0 = const. \tag{3.56}$$

which accepts as a solution the $K^0 \simeq const.$ for an inflationary space-time where $H \simeq const.$. For further details regarding the running vacuum, see [9, 4]. Generally, the form of the running vacuum energy has the following structure,

$$\rho_{vac} \simeq \beta_1 H^2 + \beta_2 H^4 \tag{3.57}$$

where β_1 corresponds to the gravitational Chern-Simons term, and $\beta_2 > 0$. In the very early universe the second term is dominant, and leads to a running vacuum inflation. Due to the eq. 3.55, in the inflationary period,

$$\dot{b} = const. \tag{3.58}$$

and therefore the axion field remains undiluted at the end of inflation. While also,

$$H_{ijk} \sim \epsilon_{ijk0}\partial^0 b \tag{3.59}$$

which is spatially flat.

After this period, the decay of the inflation can generate the fermion fields. The generation of fermions will introduce the anomalies that can then cancel the primordial gravitational anomalies.

We now study the effective action of a higher excited state of the string, in order to include the fermions. Therefore, the action now reads,

$$\begin{aligned} S_{Dirac} &= \frac{1}{2}\int d^4x\sqrt{-g}i(\bar{\psi}\gamma^a\bar{\nabla}_a\psi - \bar{\nabla}_a\bar{\psi}\gamma^a\psi + 2im\bar{\psi}\psi) \\ &= \frac{1}{2}\int d^4x\sqrt{-g}i[\bar{\psi}\gamma^a\partial_a\psi - \partial_a\bar{\psi}\gamma^a\psi + \\ &+ \frac{i}{2}\bar{\psi}(\frac{i}{4}\gamma^a\gamma^b\gamma^c - \frac{i}{4}\gamma^a\gamma^c\gamma^b + \frac{i}{4}\gamma^b\gamma^c\gamma^a - \frac{i}{4}\gamma^c\gamma^b\gamma^a)e^\mu_a\bar{\omega}_{b\mu c}\psi \\ &+ m\bar{\psi}\psi]. \end{aligned} \tag{3.60}$$

Using the gamma matrices identities, and in particular

$$\gamma^\mu\gamma^\nu\gamma^\rho = \eta^{\mu\nu}\gamma^\rho + \eta^{\nu\rho}\gamma^\mu - \eta^{\mu\rho}\gamma^\nu - i\epsilon^{\sigma\mu\nu\rho}\gamma_\sigma\gamma^5, \tag{3.61}$$

and considering that $\bar{\nabla}_a\bar{\psi} = e^\mu_a\left(\partial_\mu - \frac{i}{2}\bar{\omega}_{b\mu c}\Sigma^{bc}\right)\bar{\psi}$, one can prove that the action is equal to,

$$\begin{aligned} S_{Dirac} &= \int d^4x\sqrt{-g}\bar{\psi}(i\gamma^a\partial_a + B_d\hat{\gamma}^5\gamma^d - m)\psi \\ &\equiv S^{free}_{Dirac} + \int d^4x\sqrt{-g}\hat{B}_\mu\bar{\psi}\gamma^\mu\gamma^5\psi \end{aligned} \tag{3.62}$$

where we have define the field $\hat{B}^d$ as

$$\hat{B}^d \equiv \frac{1}{4}\epsilon^{dabc}e^\mu_a\bar{\omega}_{b\mu c}. \tag{3.63}$$

Due to eq. 3.59,in four space time, one can see that

$$\hat{B}^\mu \approx \partial^\mu b \longrightarrow \boxed{\hat{B}^0 \approx \dot{b} \sim const.} \tag{3.64}$$

The second term of equation 3.62, expresses the coupling of the Kalb-Ramond axion field with the fermions. This term will play an important role in the next chapters, as it is the origin of the antisymmetric decay of the Right Handed Neutrinos, and eventually lead to Leptogenesis and Baryogenesis.

Chapter 4

Leptogenesis from Kalb-Ramond Torsion Background

In Chapter 2 we considered the existence of Right Handed Majorana Neutrinos, and stated the need of more than one RHN in order to obtain CP-asymmetric decay, and therefore Leptogenesis to occur.

In this Chapter we further extend the Standard Model, considering the interaction of the axion background field with the fermions.. As we will examine later on, one can obtain CP-violating decay of N_1 at tree-level, in contrast to Chapter 2, where we had to consider at least two Right Handed Neutrino flavors and study the loop diagrams.

Since there is an extra term in the Lagrangian density, one has to redefine the Dirac spinors. After that, we follow the same procedure as in Chapter 2, one should calculate the decay amplitude at tree-level, the decay rate and then continue with the Boltzmann equations, in order to derive the Lepton asymmetry abundance.

4.1 Spinors coupled to an Axial Background Field

As we showed in the previous chapter, one can write the Lagrangian including a non vanishing axion background field, as following

$$\begin{aligned}\mathcal{L}e^{-1} = &\frac{i}{2}e^{\mu}_{a}(\bar{\psi}_j\gamma^a\partial_\mu\psi_j - \partial_\mu\bar{\psi}_j\gamma^a\psi_j) + \bar{\psi}_j(\gamma^5\tilde{\not{B}} - m^{(j)})\psi_j \\ &+ \frac{3k^2}{16}(\bar{\psi}_j\gamma_\mu\gamma^5\psi_j)(\bar{\psi}_l\gamma_\mu\gamma^5\psi_l) + ...\end{aligned} \tag{4.1}$$

Now one can write the Euler-Lagange equation for the Lagrangian density of eq.4.1, and obtain the two following equations for the Dirac spinor and the charge conjugate spinor [13].

$$ie^{\mu}_{a}(\gamma^{a}\partial_{\mu}\psi_{j}) + (\gamma^{5}\tilde{\not{B}} - m^{(j)})\psi_{j} + \frac{3k^{2}}{16}(\gamma_{\mu}\gamma^{5}\psi_{j})(\bar{\psi}_{l}\gamma_{\mu}\gamma^{5}\psi_{l}) = 0 \tag{4.2}$$

$$ie^{\mu}_{a}(\gamma^{a}\partial_{\mu}\psi^{C}_{j}) + (\gamma^{5}\tilde{\not{B}} - m^{(j)})\psi^{C}_{j} - \frac{3k^{2}}{16}(\gamma_{\mu}\gamma^{5}\psi^{C}_{j})(\bar{\psi^{C}}_{l}\gamma_{\mu}\gamma^{5}\psi^{C}_{l}) = 0 \tag{4.3}$$

where $\psi^{C} = C\bar{\psi}^{T}$ and $C = i\gamma^{2}\gamma^{0}$. Since we know that $C\gamma_{\mu}C^{-1} = -\gamma_{\mu}^{T}$, and $C\gamma_{5}C^{-1} = \gamma_{5}^{T}$,

$$\bar{\psi}_{l}\gamma_{\mu}\gamma^{5}\psi_{l} = -\bar{\psi^{C}}_{l}\gamma_{\mu}\gamma^{5}\psi^{C}_{l} \tag{4.4}$$

We also define $F_{0} \equiv < \bar{\psi}_{l}\gamma_{\mu}\gamma^{5}\psi_{l} >$, and since the only non-vanishing component is the temporal component, we re-write the equations 4.2, 4.3

$$ie^{\mu}_{a}(\gamma^{a}\partial_{\mu}\psi_{j}) - m^{(j)}\psi_{j} - (\tilde{B}_{0} + \frac{3k^{2}}{16}F_{0})\gamma^{0}\gamma^{5}\psi_{j} = 0 \tag{4.5}$$

$$ie^{\mu}_{a}(\gamma^{a}\partial_{\mu}\psi^{C}_{j}) - m^{(j)}\psi^{C}_{j} - (\tilde{B}_{0} + \frac{3k^{2}}{16}F_{0})\gamma^{0}\gamma^{5}\psi^{C}_{j} = 0. \tag{4.6}$$

At this point, one may compute the Dirac spinors, by rewriting equation 4.6 in the following way,

$$(i\gamma^{\mu}\partial_{\mu} - m - B_{0}\gamma^{0}\gamma^{5})\psi = 0 \quad , \qquad B_{0} \equiv \tilde{B}_{0} + \frac{3k^{2}}{16}F_{0} \tag{4.7}$$

and inserting the plane wave solution, $\psi(x) = u_{r}(p)e^{-ipx}$. Resulting to

$$\begin{bmatrix} -mI & E_{r} - B_{0} - \vec{p}\cdot\vec{\sigma} \\ E_{r} - B_{0} + \vec{p}\cdot\vec{\sigma} & -m \end{bmatrix} u_{r} = 0, \tag{4.8}$$

here we used the chiral representation of the Dirac matrices ,

$$\gamma^{\mu} = \begin{bmatrix} 0 & \sigma^{\mu} \\ \overline{\sigma^{\mu}} & 0 \end{bmatrix} \quad , \qquad \gamma^{5} = \begin{bmatrix} -\mathbb{I} & 0 \\ 0 & \mathbb{I} \end{bmatrix} \tag{4.9}$$

where

$$\sigma^{\mu} \equiv \begin{bmatrix} \mathbb{I} \\ \sigma^{i} \end{bmatrix} \quad , \qquad \overline{\sigma^{\mu}} \equiv \begin{bmatrix} \mathbb{I} \\ -\sigma^{i} \end{bmatrix}. \tag{4.10}$$

Inserting $u_{r} = \begin{bmatrix} \xi_{1} \\ \xi_{2} \end{bmatrix}$ in eq. 4.8, we get that

$$\begin{aligned} -m\xi_{1} + (E_{r} - B_{0} - \vec{p}\cdot\vec{\sigma})\xi_{2} = 0 \\ (E_{r} - B_{0} + \vec{p}\cdot\vec{\sigma})\xi_{1} - m\xi_{2} = 0 \end{aligned} \tag{4.11}$$

and so,

$$\xi_{2} = \frac{-m}{E - B_{0} - \vec{p}\cdot\vec{\sigma}}\xi_{1} \tag{4.12}$$

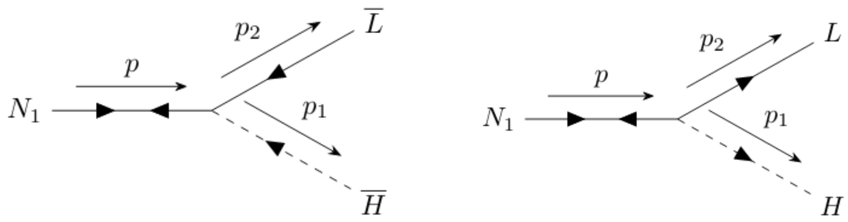

Figure 4.1: $N_1 \to LH$ and $N_1 \to \bar{L}\bar{H}$, in the presence of an axial background field

Normalising the spinors according to orthogonality, $\xi^{\dagger r}\xi^s = \delta^{rs}$, and

$$u_r^\dagger u_s = 2E_r\delta_{rs} \quad , \qquad v_r^\dagger v_s = 2E_r\delta_{rs} \tag{4.13}$$

while also consider the energy equation,

$$E_r^2 = m^2 + (B_0 + \lambda_r|\vec{p}|)^2 \tag{4.14}$$

,and we obtain that

$$u_r(p) = \begin{bmatrix} \sqrt{E_r - B_0 - \lambda_r|\vec{p}|}\xi^r \\ \sqrt{E_r + B_0 + \lambda_r|\vec{p}|}\xi^r \end{bmatrix} \tag{4.15}$$

where $\lambda_r \xi^r = \frac{\vec{p}\vec{\sigma}}{|\vec{p}|}\xi^r$, and $\lambda_r \equiv (-1)^{r-1}$, [7]. While the negative frequency spinors are

$$v_s(p) = \begin{bmatrix} \sqrt{E_s + B_0 - \lambda_s|\vec{p}|}\xi^s \\ -\sqrt{E_s - B_0 + \lambda_s|\vec{p}|}\xi^s \end{bmatrix} \tag{4.16}$$

Consequently, the Dirac field operator is defined in the same way that was derived in the absence of the axion field,

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3} \sum_{r=1,2} \frac{1}{\sqrt{2E_r}} (a^r u^r(p) e^{-ip\cdot x} + b^{\dagger r} v^r(p) e^{ip\cdot x}) \tag{4.17}$$

4.2 Right Handed Neutrino decay

In this section, we examine the RHN decay at tree-level in the presence of an axial background field, see fig. 4.1. Firstly we compute the decay amplitude, and then we proceed to the decay width in order to establish the CP-asymmetry.

4.2.1 Decay Amplitude

Starting with the Lagrangian density, the corresponding term to the interaction of fig. 4.1 is

$$L \ni -\lambda_1 \overline{L} H N_1 + h.c. \quad , \tag{4.18}$$

where N_1 is the majorana RHN, L is the lepton doublet $L = \begin{bmatrix} e^-_L \\ \nu_{e,L} \end{bmatrix}$, and H is the Higgs field $H = \begin{bmatrix} H^+ \\ H^0 \end{bmatrix}$. Considering the N_1 decay into electron and electron neutrino,

$$L \ni -\lambda_1 \overline{e^-}_L H^+ N_{1R} + h.c. = \lambda_1 \overline{e^-} H^+ \frac{1+\gamma^5}{2} N_1 + h.c. \tag{4.19}$$

therefore the decay amplitude is equal to

$$\begin{aligned} i\mathcal{M}_1 &= -i\overline{u_r}(p_2)\lambda_1 \frac{1+\gamma^5}{2} u_s(p) \\ &= -i\lambda_1 \left[\sqrt{E_r - B_0 - \vec{p_2}\cdot\vec{\sigma}}\xi^{r\dagger} \quad \sqrt{E_r + B_0 + \vec{p_2}\cdot\vec{\sigma}}\xi^{r\dagger} \right] \gamma^0 \\ &\times \frac{1+\gamma^5}{2} \begin{bmatrix} \sqrt{E_s - B_0 - \vec{p}\cdot\vec{\sigma}}\xi^s \\ \sqrt{E_s + B_0 + \vec{p}\cdot\vec{\sigma}}\xi^s \end{bmatrix} \\ &= -i\lambda_1 \sqrt{E_r - B_0 - \vec{p_2}\cdot\vec{\sigma}}\xi^{r\dagger} \sqrt{E_s + B_0 + \vec{p}\cdot\vec{\sigma}}\xi^s \end{aligned} \tag{4.20}$$

and so,

$$\begin{aligned} \overline{|\mathcal{M}_1|^2} &= \frac{\lambda_1^\dagger \lambda_1}{2 * \frac{1}{2} + 1} \sum_r \sum_s (E_r - B_0 - \vec{p_2}\cdot\vec{\sigma})(E_s + B_0 + \vec{p}\cdot\vec{\sigma})(\xi^{r\dagger}\cdot\xi^s)^\dagger(\xi^{r\dagger}\cdot\xi^s) \\ &= \frac{|\lambda_1|^2}{2} \sum_r \sum_s (E_s - B_0 - \lambda_s|\vec{p_2}|)(E_s + B_0 + \lambda_r|\vec{p}|)(\xi^{r\dagger}\cdot\xi^s)^\dagger(\xi^{r\dagger}\cdot\xi^s) \\ &= \frac{|\lambda_1|^2}{2}(E_s - B_0 - \lambda|\vec{p_2}|)(E_s + B_0 + \lambda|\vec{p}|) \end{aligned} \tag{4.21}$$

where $\lambda_r = \lambda_s = \lambda$ due to the helicity conservation, that enters the equation through the δ_{rs}. We also make the following considerations, in order to further simplify eq. 4.21. The electron and the higgs mass are negligible, compared to the RHN mass, and we set $|p| = 0$. Therefore, eq. 4.14 becomes respectively

$$E_{lepton} = |B_0 + \lambda|p_2|| \tag{4.22}$$

$$E_{N_1} = \sqrt{m_{N_1}^2 + B_0^2} \tag{4.23}$$

At this point, one should consider different cases for each possible value of the helicity λ.

- $\boxed{\lambda = 1}$
  In this case, eq. 4.21 becomes

$$\begin{aligned} \overline{|\mathcal{M}_1|^2} &= \frac{|\lambda_1|^2}{2} (|B_0 + |p_2|| - B_0 - |\vec{p_2}|) \left(\sqrt{m_{N_1}^2 + B_0^2} + B_0 \right) \\ &= \frac{|\lambda_1|^2}{2} (B_0 + |p_2| - B_0 - |\vec{p_2}|) \left(\sqrt{m_{N_1}^2 + B_0^2} + B_0 \right) \\ &= 0 \end{aligned} \tag{4.24}$$

  which means that no RHN can decay into leptons.

- $\boxed{\lambda = -1}$ On the other hand, the eq. 4.21 now becomes

$$\overline{|\mathcal{M}_1|^2} = \frac{|\lambda_1|^2}{2}\left(|B_0 - |p_2|| - B_0 + |\vec{p_2}|\right)\left(\sqrt{m_{N_1}^2 + B_0^2} + B_0\right) \quad (4.25)$$

  - If $\boxed{B_0 - |p_2| > 0}$, then eq. 4.25 becomes

$$\begin{aligned}\overline{|\mathcal{M}_1|^2} &= \frac{|\lambda_1|^2}{2}\left(B_0 - |p_2| - B_0 + |\vec{p_2}|\right)\left(\sqrt{m_{N_1}^2 + B_0^2} + B_0\right)\\ &= 0\end{aligned} \quad (4.26)$$

  - If $\boxed{B_0 - |p_2| < 0}$, then eq. 4.25 becomes

$$\begin{aligned}\overline{|\mathcal{M}_1|^2} &= \frac{|\lambda_1|^2}{2}\left(-B_0 + |\vec{p_2}| - B_0 + |\vec{p_2}|\right)\left(\sqrt{m_{N_1}^2 + B_0^2} + B_0\right)\\ &= \frac{|\lambda_1|^2}{2}(2)(|\vec{p_2}| - B_0)\left(\sqrt{m_{N_1}^2 + B_0^2} + B_0\right)\end{aligned} \quad (4.27)$$

Consequently the only non-vanishing decay amplitude, arises from the case of $\lambda = -1$, and $B_0 - |\vec{p_2}| < 0$.

$$\boxed{\overline{|\mathcal{M}_1|^2} = \frac{|\lambda_1|^2}{2}2(|\vec{p_2}| - B_0)\left(\sqrt{m_{N_1}^2 + B_0^2} + B_0\right)} \quad (4.28)$$

Moving on, one should also consider the RHN decay into anti-particles, see fig. 4.1. The decay amplitude of this process is equal to

$$\begin{aligned}i\mathcal{M}_2 &= -i\overline{v_r}(p)\lambda_1^*\frac{1-\gamma^5}{2}v_s(p_2)\\ &= -i\lambda_1^*\left[\sqrt{E_r + B_0 - \vec{p}\cdot\vec{\sigma}}\xi^{r\dagger} \quad -\sqrt{E_r - B_0 + \vec{p}\cdot\vec{\sigma}}\xi^{r\dagger}\right]\gamma^0\\ &\times \frac{1+\gamma^5}{2}\begin{bmatrix}\sqrt{E_s + B_0 - \vec{p_2}\cdot\vec{\sigma}}\xi^s\\ -\sqrt{E_s - B_0 + \vec{p_2}\cdot\vec{\sigma}}\xi^s\end{bmatrix}\\ &= i\lambda_1^*(-1\sqrt{E_r - B_0 + \vec{p}\cdot\vec{\sigma}}\xi^{r\dagger})\sqrt{E_s + B_0 - \vec{p_2}\cdot\vec{\sigma}}\xi^s\end{aligned} \quad (4.29)$$

and so,

$$\overline{|\mathcal{M}_2|^2} = \frac{|\lambda_1|^2}{2}(E_s - B_0 + \lambda|\vec{p}|)(E_s + B_0 - \lambda|\vec{p_2}|). \quad (4.30)$$

The energy equation for the anti-fermions takes the following form

$$E_r^2 = m^2 + (B_0 - \lambda_r|\vec{p}|)^2 \quad (4.31)$$

One can obtain non-vanishing decay amplitude, in the case of $\lambda = -1$. The non-zero case of $\lambda = +1$, was rejected due to the fact that we considered that $B_0 < |p_2|$ in eq. 4.28. Therefore, the decay amplitude of the RHN into anti-fermions is,

$$\boxed{\overline{|\mathcal{M}_2|^2} = |\lambda_1|^2(\sqrt{m_{N_1}^2 + B_0^2} - B_0)(B_0 + |\vec{p_2}|).} \tag{4.32}$$

The helicity of the antiparticle is $\frac{p\cdot\sigma}{|p|}\xi^r = -\lambda^r_{particle}\xi^r = \lambda^r_{antiparticle}\xi^r$, and so the helicity of the anti-neutrino and anti-lepton is $\lambda = +1$

In equations 4.25 and4.32, we consider that the $p = 0$. Nevertheless, this is an approximation, it will be more precise if we consider that the momentum is indeed very small, and write the Taylor expansion.

$$\boxed{\begin{aligned} \overline{|\mathcal{M}_1|^2} &= \frac{|\lambda_1|^2}{2}(|\vec{p_2}| - B_0)\frac{m_{N_1}^2}{p}\left(1 + \frac{B_0}{p} - \frac{m_{N_1}^2}{4p^2}\right) \\ \overline{|\mathcal{M}_2|^2} &= \frac{|\lambda_1|^2}{2})(B_0 + |\vec{p_2}|)\frac{m_{N_1}^2}{p}\left(1 - \frac{B_0}{p} - \frac{m_{N_1}^2}{4p^2}\right) \end{aligned}} \tag{4.33}$$

4.2.2 CP-asymmetry

In this subsection we compute the decay rate of the RHN into particles and antiparticles, in order to demonstrate the CP-assymetric decay.

It is convenient to define the variable Ω,

$$\Omega \equiv \sqrt{m_{N_1}^2 + B_0^2}. \tag{4.34}$$

- $\boxed{N_1 \longrightarrow LH}$

$$\Gamma_1 = \frac{1}{2E_{N_1}}\int \frac{d^3p_1}{2E_1(2\pi)^3}\frac{d^3p_2}{2E_2(2\pi)^3}\overline{|\mathcal{M}_1|^2}(2\pi)^4\delta^4(p - p_1 - p_2) \tag{4.35}$$

Considering that the total energy of the system is preserved, while $|p_1| = |p_2|$, and having in mind that $B_0 < p_2$, and $\lambda = -1$, we obtain the following equation,

$$|\vec{p_1}| - B_0 + |\vec{p_2}| = \Omega \quad \Leftrightarrow \quad |p_2| = \frac{\Omega + B_0}{2} \tag{4.36}$$

One can re-write eq. 4.35, and inserting eq. 4.36,

$$\Gamma_1 = \frac{|\lambda_1|^2}{2\Omega}\int \frac{d\omega}{16\pi^2}\frac{p_2}{\Omega - B_0}(p_2 - B_0)\left(\sqrt{m_{N_1}^2 + B_0^2} + B_0\right) \tag{4.37}$$

where we defined the following quantities, $p_2 \equiv |\vec{p_2}|$ and $d\omega$ is the solid angle element. Eventually after doing some calculations, one can obtain the following result,

$$\boxed{\Gamma_1 = \frac{|\lambda_1|^2}{16\pi}\frac{\Omega + B_0}{\Omega - B_0}\frac{m_{N_1}^2}{2\Omega}} \tag{4.38}$$

- $\boxed{N_1 \longrightarrow \bar{L}\bar{H}}$

  The decay rate of the RHN into antiparticles is equal to,

$$\boxed{\Gamma_2 = \frac{|\lambda_1|^2}{16\pi}\frac{\Omega - B_0}{\Omega + B_0}\frac{m_{N_1}^2}{2\Omega}} \tag{4.39}$$

Comparing the two decay rets of eq. 4.38 and eq. 4.39, one can see that $\Gamma_1 \neq \Gamma_2$. In particular, it seems that $\Gamma_1 > \Gamma_2$, and this is an indication of the asymmetry between particles and antiparticles, that favors the particles.

Furthermore, one may calculate the total RHN decay rate,

$$\Gamma = \Gamma_1 + \Gamma_2 = 2 \cdot \left(\frac{|\lambda_1|^2}{16\pi^2}\frac{\Omega^2 + B)^2}{\Omega} \right) \tag{4.40}$$

where we have doubled the result, because one has to consider also the RHN decay into the neutral Higgs and the electron neutrino, $N_1 \to \nu_e + \phi_0$.

4.3 CPT Violating term

The contribution of the of the Kalb-Ramond Torsion Background, in entering the Lagrangian through the term

$$L \in -\bar{N}\not{B}\gamma^5 N \tag{4.41}$$

if we apply a CPT transformation on this term,[20], we obtain the obtain the following result,

$$\begin{aligned}
L \in -\bar{N}_{CPT}\not{B}\gamma^5 N_{CPT} &= -\overline{ie^{i\delta}\gamma^5 N(-t,-x)}\gamma^0\gamma^5 B_0 ie^{i\delta}\gamma^5 N(-t,-x) = \\
&= -\overline{ie^{2i\delta}\gamma^5 i\gamma^1\gamma^3 N^*(t,x)\gamma^0}\gamma^0\gamma^5 B_0 ie^{2i\delta}\gamma^5 i\gamma^1\gamma^3 N*(t,x)\gamma^0 = \\
&= B_0\overline{N^*(t,x)}\gamma^5\gamma^1\gamma^3\gamma^0\gamma^0\gamma^5\gamma^5\gamma^1\gamma^3\gamma^0 N^*(t,x) = \\
&= \overline{N^*(t,x)}\gamma^0 B_0\gamma^5 N^*(t,x) \\
&= \bar{N}\not{B}\gamma^5 N
\end{aligned} \tag{4.42}$$

where $N^*(t,x) = N(t,x)$, due to the fact that the neutrino is majorana. The above equation shows that this term violates the CPT symmetry.

As we established in the previous subsections, this term is the origin of the asymmetric decay of the RHN, that eventually leads to Leptogenesis.

4.4 Leptogenesis from a CPTV Background field

The aim of this section is to examine the lepton asymmetry that would be produced, due to the CP-asymmetric decay of the right handed neutrino.

Basically, one has to solve the Boltzmann equations that describe the evolution of the RHN abundance, and the leptons and anti-leptons abundance. That way, we can examine the evolution of the lepton asymmetry through time, and eventually name it "Leptogenesis".

In order to form the Boltzmann equations for the interactions examined, there are a few variables that need to be computed first, e.g. the thermal equilibrium abundances and the thermal averaged decay rate.

4.4.1 Equilibrium Abundances

As we have defined in Chapter 2, the abundance of a particle species is defined as $Y_x = n_x/s$. Where n_x is the number density, while s is the entropy density.

Consequently, in order to compute the neutrino or lepton equilibrium abundance, one has to calculate the number density first

$$n_x^{eq} = g_x \int \frac{d^3p}{(2\pi)^3} f_x^{eq} \tag{4.43}$$

where, f^{eq} is the distribution function,

$$f_x^{eq} = \frac{1}{e^{E_x/T}+1} \quad . \tag{4.44}$$

Considering the number density of the RHN, one gets that

$$\begin{aligned} f_{N_1}^{eq} &= \frac{1}{e^{E_{N_1}/T}+1} = \frac{1}{e^{E_{N_1}/T}(1+e^{-E_{N_1}/T})} \\ &= e^{-E_{N_1}/T} \sum_{n=0}^{\infty} (-1)^n e^{-n \cdot E_{N_1}/T} \\ &\approx e^{-E_{N_1}/T} - e^{-2E_{N_1}/T} + e^{-3E_{N_1}/T} \end{aligned} \tag{4.45}$$

where we used the Taylor expansion of $1/(1+x) = \sum_{n=0}^{\infty}(-1)^n x^n$. Now, concerning the RHN total energy, one can expand the equation and re-write it as follows,

$$\begin{aligned} E_{N_1} &= \sqrt{m_{N_1}^2 + (B_0 + \lambda p_{N_1})^2} \\ &\approx p_{N_1} + \frac{m_{N_1}^2}{2p_{N_1}} - \frac{m_{N_1}^4}{8p_{N_1}^2} - \frac{m_{N_1}^2 \lambda B_0}{2p_{N_1}^2} + \lambda B_0 \end{aligned} \tag{4.46}$$

where we made use of the fact that $p_{N_1} >> m_{N_1}, B_0$.

Considering eq. 4.45, we obtain the following result for the equilibrium density of the RHN,

$$n_{N_1}^{eq} = g_{N_1} \int \frac{d^3 p_{N_1}}{(2\pi)^3} (e^{-E_{N_1}/T} - e^{-2E_{N_1}/T} + e^{-3E_{N_1}/T}) \tag{4.47}$$

In the above equation, one can see that the sub-integrals have the same form,

$$\begin{aligned} I_n &= \int \frac{d^3 p_{N_1}}{(2\pi)^3} e^{-nE_{N_1}/T} \\ &= \int \frac{d\omega \cdot dp_{N_1} p_{N_1}^2}{(2\pi)^3} e^{-nE_{N_1}/T} \end{aligned} \tag{4.48}$$

which, according to [7], is equal to

$$\begin{aligned} I_n &= T^3 e^{-n} e^{-n\frac{B_0}{T}} P_n \quad , \\ P_n &= \frac{n^2+2n+2}{n^3} - \frac{n+1}{2n}\frac{m_{N_1}^2}{T^2} + \frac{n}{8}\frac{m_{N_1}^4}{T^4}[1+e^n\Gamma(0,n)] + \frac{m_{N_1}^2 \lambda B_0}{2T^3} . \end{aligned} \tag{4.49}$$

where $\Gamma(0,n)$ is the Gamma function, [1].

Consequently, the RHN number density is

$$\boxed{\begin{aligned} n_{N_1}^{eq} &= \frac{5g_{N_1}T^3}{2\pi^2 e}(0.9251 - 0.1628\frac{m_{N_1}^2}{T^2} + 0.0278\frac{m_{N_1}^4}{T^4} \\ &- 0.8672\frac{B_0}{T} + 0.2203\lambda\frac{m_{N_1}^2 B_0}{T^3}) \end{aligned}} \tag{4.50}$$

Continuing with the lepton and ant-lepton number density, one obtains the following result

$$\begin{aligned} n_l^{eq} &= \frac{g_l}{2\pi^2}(J_1 - J_2 + J_3) \\ J_n &= \int_T^\infty dp_l p_l^2 e^{E_l/T} \end{aligned} \tag{4.51}$$

where we made expanded the exponential term in the distribution function, and having in mind that $p_l >> B_0$ we get that the energy is equal to

$$E_l = |B_0 + \lambda p_l| = B_0 + \lambda p_l \tag{4.52}$$

for either positive or negative λ. Therefore, replacing the energy E_l in eq. 4.51 and calculating the integral,

$$\boxed{n_l^{eq} = \frac{5g_l T^3}{2\pi^2 e}\left(0.9251 - 0.8672\lambda\frac{B_0}{T}\right)} \tag{4.53}$$

The last part of this subsection, is to calculate the abundance of the RHN, leptons, and anti-leptons. Since the entropy density is approximately$s \sim 14T^3$, the abundances have the following form

$$\boxed{\begin{aligned} &Y^{eq}_{N_1} \simeq 0.1652\frac{g_{N_1}}{\pi^2 e}\left(1 - 0.176z^2 + 0.0301z^4 0.9374\lambda\frac{B_0 z}{m_{N_1}} + 0.2381\lambda\frac{B_0 z^3}{m_{N_1}}\right), \\ &Y^{eq}_{l} \simeq 0.1652\frac{g_l}{\pi^2 e}\left(1 - 0.9374\lambda\frac{B_0 z}{m_{N_1}}\right), \\ &\mathcal{L}^{eq} \simeq 0.3097\frac{g_l}{\pi^2 e}\ \lambda\frac{B_0 z}{m_{N_1}}\Bigg) \end{aligned}} \tag{4.54}$$

where $z = m_{N_1}/T$.

4.4.2 Thermally Averaged Decay Rates

In order to derive the Boltzmann equations, one should also calculate the thermally averaged decay rates, $\gamma^{eq}(N_1 \longrightarrow LH)$ and $\gamma^{eq}(N_1 \longrightarrow \bar{L}\bar{H})$.

To begin with, we write the RHN decay into leptons and by similar calculations, one can also obtain the decay rate of the RHN into anti-leptons.

$$\begin{aligned} \gamma^{eq}(N_1 \longrightarrow LH) = &\int \frac{d^3p}{2E(2\pi)^3}\int \frac{d^3p_1}{2E_1(2\pi)^3}\int \frac{d^3p_2}{2E_2(2\pi)^3} f^{eq}_{N_1} \\ &\times \overline{|\mathcal{M}_{N_1 \longrightarrow LH}|^2}(2\pi)^4\delta^4(p - p_1 - p_2) \end{aligned} \tag{4.55}$$

where E, p is the RHN total energy and momentum, and the index 1 and 2 corresponds to the lepton and higgs field respectively, fig. 4.1. While, the decay amplitude has already been calculated in equation 4.33. One may integrated over the momentum delta function, which leads to the momentum conservation, $p_1 = p_{-}p_2$. That way, we re-write the above equation as

$$\begin{aligned} \gamma^{eq}(N_1 \longrightarrow LH) = &\frac{1}{8(2\pi^5)}\int d^3p \int dp_2 \int d\omega_2 f^{eq}_{N_1}\frac{p_2^2}{E_2 E_1 E} \\ &\times |\mathcal{M}_1|^2\delta(E - E_2 - E_1). \end{aligned} \tag{4.56}$$

Concerning the delta function, we know that

$$\begin{aligned} E - E_2 - E_1 = &E - |B_0 - p_2| - \sqrt{p_1^2} = \\ = &E + B_0 - p_2 - \sqrt{p^2 + p_2^2 - p_2 p \cdot cos(\theta)} \equiv f(p_2) \end{aligned} \tag{4.57}$$

where θ is the angle between the lepton and the neutrino momentum. Using the delta function property,

$$\int dp_2 \delta(f(p_2)) = \int dp_2 \frac{\delta(p_2 - p_{2,0})}{|f'(p_{2,0})|} \tag{4.58}$$

where $p_{2,0}$ is the solution of $f(p_2) = 0$, and so the eq. 4.56, becomes

$$\begin{aligned}\gamma^{eq}(N_1 \longrightarrow LH) =& \frac{\lambda_1^2 m_{N_1}^2}{16(2\pi^5)} \int d^3p \int dp_2 \int d\omega_2 \frac{f_{N_1}^{eq}}{E \cdot p} \left(1 + \frac{B_0}{p} - \frac{m_{N_1}}{4p}\right) \\ & \times \frac{p_{2,0}^2}{\sqrt{p^2 + p_{2,0}^2 - p_{2,0} p \cdot cos(\theta)}} \frac{1}{f'(p_{2,0})}\end{aligned} \tag{4.59}$$

After doing some further calculations, one can proof that, see [7],

$$\boxed{\begin{aligned}\gamma^{eq}(N_1 \longrightarrow LH) \simeq & \frac{3\lambda_1^2 m_{N_1}^4}{16(2\pi^3)} z^{-2/3} [0.2553 - 0.1447z^2 + \\ & + 0.0957z^4 + z\frac{B_0}{m_{N_1}}(0.6062 - 0.3063z^2)]\end{aligned}} \tag{4.60}$$

$$\boxed{\begin{aligned}\gamma^{eq}(N_1 \longrightarrow \bar{L}\bar{H}) \simeq & \frac{3\lambda_1^2 m_{N_1}^4}{16(2\pi^3)} z^{-2/3} [0.2553 - 0.1447z^2 + \\ & + 0.0957z^4 - z\frac{B_0}{m_{N_1}}(0.6062 - 0.3063z^2)]\end{aligned}} \tag{4.61}$$

4.4.3 Boltzmann Equations

In this subsection we will insert the results of the two previous subsection into the Boltzmann equations, in order to study the evolution of the RHN and the lepton-antilepton abundance.

First of all, we will write the Boltzmann equation, that shows how does the distribution function in time, due to collisions. The equations will be similar to those of Chapter 2, see eq. 2.39, with one extra term, [13]. The additional term appears due to the presence of the CPTV background field, as we will show.

The Liouvile operator, obtained in eq. 2.35, can be written as,

$$g_x \int \frac{d^3p}{(2\pi)^3 E} \mathcal{L}[f_x] = \frac{dn_x}{dt} - \frac{\dot{a}}{a} \frac{g_x}{(2\pi)^3} \int d\omega \cdot dp_x \frac{p_x^2 p_x^2}{E_x} \frac{\partial f_x}{\partial p_x} \frac{\partial p_x}{\partial E_x} \tag{4.62}$$

where we have already defined, for convenience, that $|p_x| \equiv p_x$, and $d\omega$ is the solid angle. One can also show that,

$$E_x^2 = m_x^2 + (p_x + \lambda \cdot B_0)^2 \Leftrightarrow \frac{\partial p_x}{\partial E_x} = \frac{E_x}{\lambda B_0 + p_x}, \tag{4.63}$$

and so eq. 4.62, is equivalent to

$$
\begin{aligned}
g_x \int \frac{d^3p}{(2\pi)^3 E}\mathcal{L}[f_x] &= \frac{dn_x}{dt} - \frac{\dot{a}}{a}\frac{g_x}{(2\pi)^3}\int d\omega \cdot dp_x \frac{p_x^4}{(\lambda B_0 + p_x)}\frac{\partial f_x}{\partial p_x} \\
&= \frac{dn_x}{dt} - \frac{\dot{a}}{a}\frac{g_x}{(2\pi)^3}\int d\omega \cdot dp_x \frac{\partial}{\partial p_x}\left(\frac{p_x^4 f_x}{(\lambda B_0 + p_x)}\right) \\
&+ \frac{\dot{a}}{a}\frac{g_x}{(2\pi)^3}\int d\omega \cdot dp_x \frac{\partial}{\partial p_x}\left(\frac{p_x^4}{(\lambda B_0 + p_x)}\right) f_x \\
&= \frac{dn_x}{dt} + \frac{\dot{a}}{a}\frac{g_x}{(2\pi)^3}\int d\omega \cdot dp_x \frac{\partial}{\partial p_x}\left(\frac{p_x^4}{(\lambda B_0 + p_x)}\right) f_x \\
&\simeq \frac{dn_x}{dt} + 3Hn_x - \frac{g_x}{2\pi^2}2\lambda B_0 \int dp_x p_x f_x + \mathcal{O}(B_0^2).
\end{aligned}
\tag{4.64}
$$

One can now obtain the Boltzmann equation for the RHN abundance,[7]. Starting with the RHN decay into leptons,which we will symbolize as $Y_{N_1}^-$, due to the fact that $\lambda = -1$

$$
zHs\frac{dY_{N_1}^-}{dz} + I \simeq -\left(\gamma^{eq}(N_1 \to LH)\frac{Y_{N_1}^-}{Y_{N_1}^{-,eq}} - \gamma^{eq}(LH \to N_1)\right) \tag{4.65}
$$

and for the RHN decay into antilepton, $Y_{N_1}^+$, with $\lambda = +1$

$$
zHs\frac{dY_{N_1}^+}{dz} - I \simeq -\left(\gamma^{eq}(N_1 \to \bar{L}\bar{H})\frac{Y_{N_1}^+}{Y_{N_1}^{+,eq}} - \gamma^{eq}(\bar{L}\bar{H} \to N_1)\right) \tag{4.66}
$$

here, the I is an integral that we will calculate later, and arose from the term analogue to λ on the right-hand-side of eq. 4.64.

Following the same procedure, one may obtain the Boltzmann equations for the lepton and anti-lepton abundance,

$$
zHs\frac{dY_L^-}{dz} + I \simeq -\left(\gamma^{eq}(LH \to N_1)\frac{Y_L^-}{Y_L^{-,eq}} - \gamma^{eq}(N_1 \to LH)\frac{Y_{N_1}^-}{Y_{N_1}^{-,eq}}\right) \tag{4.67}
$$

$$
zHs\frac{dY_{\bar{L}}^+}{dz} - I \simeq -\left(\gamma^{eq}(\bar{L}\bar{H} \to N_1)\frac{Y_{\bar{L}}^+}{Y_{\bar{L}}^{+,eq}} - \gamma^{eq}(N_1 \to \bar{L}\bar{H})\frac{Y_{N_1}^+}{Y_{N_1}^{+,eq}}\right) \tag{4.68}
$$

where we kept the same notation.

There are two useful quantities that is interesting to define, and then reconstruct the Boltzmann equations with dependence on them.

- The first one, is the average abundance for both the leptons and the RHN
$$\bar{Y_{N_1}} \equiv \frac{Y_{N_1}^{-} + Y_{N_1}^{+}}{2} = \frac{Y_{L}^{-} + Y_{\bar{L}}^{+}}{2} \equiv \bar{Y_L} \tag{4.69}$$
which are equal,since we consider that the leptons and anti-leptons are produced through the RHN decay.

- The second variable, and of most interest, is the lepton asymmetry
$$\mathcal{L} \equiv Y_{L}^{-} - Y_{\bar{L}}^{+}. \tag{4.70}$$

It can be shown that the Boltzmann equations for the RHN average abundance is

$$\boxed{\begin{aligned} 2zHs\frac{d\bar{Y}_{N_1}}{dz} &= \left(\gamma^{eq}(N_1 \to LH)\frac{Y_{N_1}^{-}}{Y_{N_1}^{-,eq}} - \gamma^{eq}(LH \to N_1)\right) \\ &+ \left(\gamma^{eq}(N_1 \to \bar{L}\bar{H})\frac{Y_{N_1}^{+}}{Y_{N_1}^{+,eq}} - \gamma^{eq}(\bar{L}\bar{H} \to N_1)\right) \end{aligned}} \tag{4.71}$$

while for the lepton asymmetry is

$$\boxed{\begin{aligned} zHs\frac{d\mathcal{L}}{dz} + 2I_l &= \left(\gamma^{eq}(LH \to N_1)\frac{Y_{L}^{-}}{Y_{L}^{-,eq}} - \gamma^{eq}(N_1 \to LH)\frac{Y_{N_1}^{-}}{Y_{N_1}^{-,eq}}\right) \\ &- \left(\gamma^{eq}(\bar{L}\bar{H} \to N_1)\frac{Y_{\bar{L}}^{+}}{Y_{\bar{L}}^{+,eq}} - \gamma^{eq}(N_1 \to \bar{L}\bar{H})\frac{Y_{N_1}^{+}}{Y_{N_1}^{+,eq}}\right) \end{aligned}} \tag{4.72}$$

where $I_l = 10.7052 g_l m_{N_1}^4 B_0/(\pi^2 e M_{pl} z^4)$, [7]. One can also show that, since we make the calculations in the very early universe, where the temperature was very high, the entropy is approximately $s \approx 14T^3$ and the Hubble parameter is $H \approx 6T^2/M_{pl}$. !!!!edw na dw giati isxuei ayto!!!!!!

We can now, insert the values of the variables calculated in the previous subsections, e.g. $Y_{N_1}^{-,eq}, \gamma^{eq}(N_1 \to LH, Y_{N_1}^{+,eq}), \gamma^{eq}(N_1 \to \bar{L}\bar{H}), Y_{\bar{L}}^{+}, Y_{L}^{-,eq}$. Consequently, the two equations 4.71 and 4.72, can be written in the following form, [7],

$$\boxed{\begin{aligned} &\frac{d\bar{Y}_{N_1}}{dz} + P(z)\bar{Y_{N_1}} = Q(z), z < 1 \\ &P(z) = a^2 z^{10/3}(1 - 0.3909z^2 + 0.2758z^4), \qquad a^2 \equiv \frac{0.0724|\lambda_1|^2 e M_{pl}}{168 g_{N_1}\pi m_{N_1}} \approx 0.167 \\ &Q(z) = b^2 z^{10/3}(1 - 0.5668z^2 + 0.3749z^4), \qquad b^2 \equiv \frac{0.0957|\lambda_1|^2 M_{pl}}{168(2\pi)^3 m_{N_1}} \approx 0.0056 \end{aligned}} \tag{4.73}$$

while for the lepton asymmetry abundance the Boltzmann equations, can be further simplified to

$$\boxed{\begin{aligned}&\frac{dL}{dz}+J(z)L=K(z), z<1\\&J(z)=\mu^2 z^{10/3}(1-0.5668z^2+0.3749z^4)\\&K(z)=\nu^2 z^{13/3}(1-0.2385z^2-0.3538z^4)\bar{Y}_{N_1}-\sigma^2 z^{13/3}(1-0.1277z^2-1.4067z^4)-\delta^2\\&\mu^2\equiv\frac{0.0362|\lambda_1|^2 eM_{pl}}{84g_l\pi m_{N_1}}\approx 0.227,\qquad \nu^2\equiv\frac{0.1041|\lambda_1|^2 eM_{pl}B_0}{84g_l\pi m_{N_1}^2}\approx 1.3055\frac{B_0}{m_{N_1}}\\&\sigma^2\equiv\frac{0.0479|\lambda_1|^2 M_{pl}B_0}{84(2\pi)^3 m_{N_1}^2}\approx 0.0056\frac{B_0}{m_{N_1}},\qquad \delta^2\equiv\frac{21.4104g_lB_0}{84\pi^2 em_{N_1}}\approx 0.038\frac{B_0}{m_{N_1}}\end{aligned}} \tag{4.74}$$

One may solve eq.4.74, [7], and obtain the the following result for the lepton asymmetry,

$$\boxed{\frac{\Delta L^{TOT}}{s}\equiv\frac{Y_L^- - Y_{\bar{L}}^+}{Y_L^- + Y_{\bar{L}}^+}=\frac{L}{2\bar{Y}_L}\simeq(0.008-0.014)\frac{B_0}{m_{N_1}}\quad,\qquad m_{N_1}/T_D=(1.44-1.62)} \tag{4.75}$$

while the phenomenological constraints suggest that,

$$\boxed{\frac{B_0}{m_{N_1}}\sim 10^{-9}-10^{-8}\quad,\qquad m_{N_1}/T_D=(1.44-1.62).} \tag{4.76}$$

Chapter 5

Baryogenesis through Leptogenesis

In the previous chapters we studied the process of Leptogenesis, in the Standard Model extensions. More precisely, we considered the case of the existence of Majorana Right Handed Neutrinos in a String Inspired Universe. While, as we have already examined, the decay of such particles can lead to Leptogenesis, it may also be the origin of Baryogenesis.

In this chapter we will discuss the baryon (B) and lepton (L) number violations, in the SM. One may examine the so called triangle anomaly, and find out that the B and L number are not conserved, in contrary to the B-L number which is exactly conserved.

In the following pages we will study the origin of this anomaly, as also different configurations of this anomaly, like the sphalerons. The sphalerons are saddle solutions of the field equation, making possible the passing over of the barrier connecting to vacuum states, when the energy is high enough, (like in the early Universe.)

Finally, we will introduce the Baryon asymmetry, generated by the B-L conservation in the early Universe.

5.1 B and L Number Violations

In this section we will study the processes that violate the Baryon and Lepton number.

5.1.1 Quantum Anomalies

During this subsection we will calculate the value expectation of the axial vector current, as also the divergence of the current.

Consider a simplified model, where there are only massless fermions that couple with an external U(1) gauge field [33], the action in this model reads as,

$$S = \int d^4x (i\bar{\psi}\partial\!\!\!/\psi - eJ_V^\mu \mathcal{A}_\mu) \tag{5.1}$$

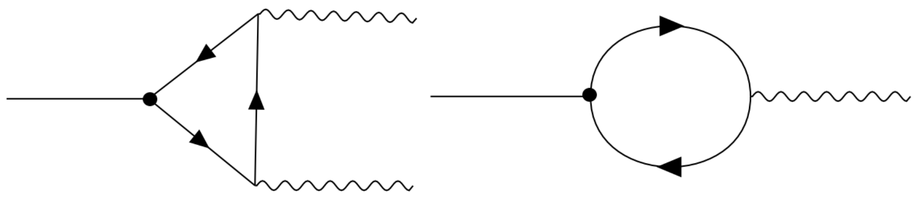

Figure 5.1: Diagrammatic representation of equation 5.4 .

where $J_V^\mu = \bar{\psi}\gamma^\mu\psi$. Now, the expectation value of the axial vector current, may be written as follows [20],

$$< J_A^\mu(x) >= \frac{\int D\psi D\bar{\psi} J_A^\mu(x) e^{i\int d^4x(i\bar{\psi}\partial\!\!\!/\psi - eJ_V^\mu \mathcal{A}_\mu)}}{\int D\psi D\bar{\psi} e^{i\int d^4x(i\bar{\psi}\partial\!\!\!/\psi - eJ_V^\mu \mathcal{A}_\mu)}}. \tag{5.2}$$

One may approach the expectation value with the perturbation theory, hence

$$\begin{aligned} < J_A^\mu(x) >= &- ie \int d^4x_1 < T[J_A^\mu(x) J_V^\nu(x_1]) > \mathcal{A}_\nu \\ &- \frac{e^2}{2} \int d^4x_1 \int d^4x_2 < T[J_A^\mu(x) J_V^\nu(x_1) J_V^\rho(x_2)] > \mathcal{A}_\nu \mathcal{A}_\rho \\ &+ ... \end{aligned} \tag{5.3}$$

and therefore,

$$\begin{aligned} \partial_\mu < J_A^\mu(x) >= &- ie \int d^4x_1 \partial_\mu < T[J_A^\mu(x) J_V^\nu(x_1)] > \mathcal{A}_\nu \\ &- \frac{e^2}{2} \int d^4x_1 \int d^4x_2 \partial_\mu < T[J_A^\mu(x) J_V^\nu(x_1) J_V^\rho(x_2)] > \mathcal{A}_\nu \mathcal{A}_\rho \\ &+ ... \end{aligned} \tag{5.4}$$

- The first term corresponds to the Feynman diagram shown in the right diagram of fig.5.1. It can be shown that the first term of eq.5.4, vanishes, and therefore do not contribute to the value of the current divergence, [33].

- The second term of eq.5.4, is of more interest. This term corresponds to the triangle diagram, shown in the left diagram of fig.5.1. Starting with the calculation of the loop, and considering the partial derivative in the momentum space [23][33],

$$\begin{aligned} e^2 \partial_\mu < T[J_A^\mu(0) J_V^\nu(x_1) J_V^\rho(x_2)] >= &\int \frac{d^4p}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} i(p+k)_\mu \Gamma^{\mu\nu\rho}(p,k) \\ &\cdot e^{i(px_1 + kx_2)} \end{aligned} \tag{5.5}$$

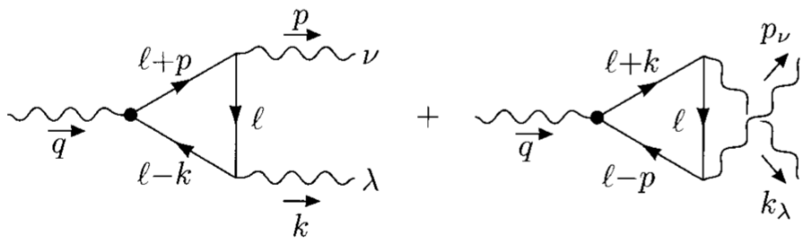

Figure 5.2: The triangle anomaly. [23]

where $\Gamma^{\mu\nu\rho}(p,k)$ is the loop contribution. There are two loop diagrams contributing to this calculation, shown in fig. 5.2, hence the loop contribution is equal to,

$$\begin{aligned} C^{\nu\rho} &= (p+k)_\mu \Gamma^{\mu\nu\rho}(p,k) \\ &= 2 \cdot e^2 \int \frac{d^4 l}{(2\pi)^4} Tr \left[(\not{p} + \not{k}) \gamma^5 \frac{i(\not{l} - \not{k})}{(l-k)^2} \gamma^\nu \frac{i \not{l}}{l^2} \gamma^\rho \frac{i(\not{l} + \not{p})}{(l+p)^2} \right] . \end{aligned} \tag{5.6}$$

Under calculations in Mathematica, one obtain the following result,

$$C^{\nu\rho}(p,k) = i \frac{e^2}{2\pi^2} \epsilon^{\nu\rho\alpha\beta} k_\alpha p_\beta . \tag{5.7}$$

After the above considerations, one may insert eq.5.7 to eq.5.4, and so, now the eq.5.4 reads

$$\begin{aligned} \partial_\mu < J_A^\mu(0) >= & \frac{e^2}{4\pi^2} \int d^4 x_1 \int d^4 x_2 \mathcal{A}_\nu \mathcal{A}_\rho \epsilon^{\nu\rho\alpha\beta} k_\alpha p_\beta \\ & \cdot e^{i(px_1 + kx_2)} \end{aligned} \tag{5.8}$$

The above equation can be further simplify, considering the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, hence we can write

$$\partial_\mu < J_A^\mu(0) >= \frac{e^2}{16\pi^2} \epsilon^{\nu\rho\alpha\beta} F_{\alpha\rho} F_{\nu\beta} \tag{5.9}$$

The above result, shows that vacuum expectation value of the axial current is not conserved!

Under similar computations [33, 28, 6], one can prove the following equations for the $J^\mu = \sum_{species} \bar{\psi}_L \gamma^\mu \psi_L = J^V + J^A$

$$\partial_\mu J^{B\mu} = \frac{N_f^2}{16\pi^2} F^\alpha_{\mu\nu} F^{\tilde{a}\mu\nu} + U_Y(1) contributions \tag{5.10}$$

$$\partial_\mu J^{L_f\mu} = \frac{g^2}{16\pi^2} F^\alpha_{\mu\nu} F^{\tilde{a}\mu\nu} + U_Y(1) contributions \tag{5.11}$$

where $F^{\alpha}_{\mu\nu}$ is the field strength of the SU(2) gauge bosons, while $F^{\tilde{a}\mu\nu} = \epsilon^{\mu\nu\rho\sigma}F^{\alpha}_{\rho\sigma}/2$, and N_f is the number of fermion families, [22].

Integrating equation 5.10 and 5.11, one may define the corresponding charge as

$$\Delta B(\Delta L)(t) = \int d^3x j^{B(L)0}(t) \tag{5.12}$$

where $\Delta B = B - \bar{B}$ and $\Delta L = L - \bar{L}$.

We define the winding number $\mathcal{N}$, that

$$\mathcal{N} = \frac{1}{16\pi^2}\int d^4x F^{\alpha}_{\mu\nu}F^{\tilde{a}\mu\nu}. \tag{5.13}$$

In perturbative gauge field configurations. the winding number $\mathcal{N} = 0$. While, in non-perturbative configurations the winding number is non-zero, such configurations are called sphalerons.

5.1.2 Sphalerons

In the previous subsection we studied the nature of the non-conservative currents in Electroweak theory, such currents lead to the violation of the Baryon number or the Lepton number. Another way of studying these violations, is through sphalerons [19, 16].

As we already discussed, the winding of the weak gauge fields is related to the Baryon (Lepton) number violations. Although, creating these gauge fields cost energy, and the barrier for such winding is of order $E_0 \approx M_W/a_W$,[2, 27]. In the case of low temperature, these processes occur by quantum tunnelling effects, described by instantons, [25, 32].

For the purpose of this Thesis, we are interested in the case of high temperature processes, due to the fact that Baryogenesis should take place in the very early Universe. In this case, it seems that one can pass over the barrier, and such a solution of the Electroweak theory is identified as the sphaleron processes,[2]. The jumping off the barrier between different vacua, corresponds to a different Baryon or Lepton number.

Since the energy barrier is the same from one transition to the next, one may consider a periodic potential, see fig.5.3, [25, 31]. The calculation of the transitions rate between different vacuums, and hence baryon (lepton) number violation, will play an important role for studying the Baryogenesis.

In Electroweak theory, there is an epoch that the Higgs vacuum expectation value is zero, and after a phase transition(at T=T_c) the Higgs develops a non-vanishing value, [25]. It can be shown that for $T < T_c$ the sphaleron decay rate has an exponential suppression by the Boltzmann factor,

$$\Gamma \sim e^{-E_{sph}/T} \tag{5.14}$$

while for $T > T_c$,

$$\Gamma \sim (a_w T)^4. \tag{5.15}$$

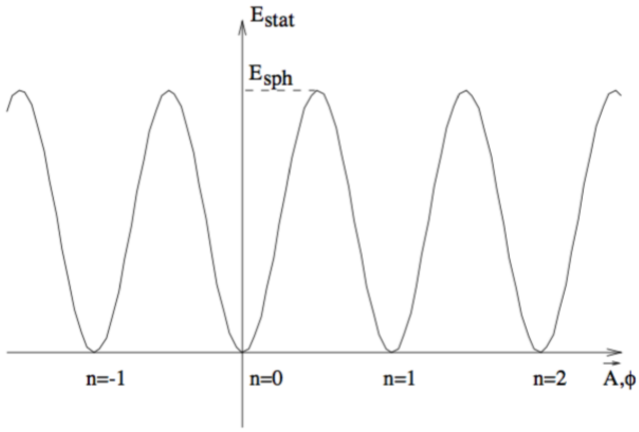

Figure 5.3: Schematic representation of the spaleron processes, [25]

where a_w is Consequently in the unbroken phase, the decay rate was much larger than the Hubble parameter, and expansion of the Universe, hence the B non-conserving reactions were fast. While after the phase transition, the B non-conserving reactions are not suppressed until the temperature gets lower than the sphaleron energy.

5.2 Baryogenesis from Kalb-Ramond Torsion Background

During this section, we consider in the Standard Model model both the Right Handed neutrinos, as also the presence of the Kalb-Ramond torsion background. As already discussed in the previous chapter, the presence of the KR field suggests the generation of the Lepton asymmetry in our Universe, due to the RHN CP asymmetric decay.

While, due to the sphaleron processes, one can study also the generation of the Baryon asymmetry. In the very early universe, when the temperature was very high, the Baryon and Lepton number was not conserved. That way, one may obtain the following equations,

$$\frac{d}{dt}\Delta B(t) = 3\frac{d}{dt}\Delta L_f(t) \quad , \qquad f = 3, \mu, \tau \tag{5.16}$$

while also the conservation equation,

$$\frac{d}{dt}(\Delta B(t) - \Delta L(t)) = 0. \tag{5.17}$$

Since we are interested in the evolution of the Baryon asymmetry, we re-write eq.5.16 as,

$$\frac{d}{dt}\Delta B(t) = \frac{1}{2}\frac{d}{dt}\Delta(B(t) + L(t)) \tag{5.18}$$

because the baryon and lepton number violation is equally violated. One may also consider the spaleron decay to be equal to the B+L violation, hence we can write,

$$\frac{d}{dt}\Delta(B(t)+L(t)) = \tau^{-1}\Delta(B(t)+L(t)) \tag{5.19}$$

where τ^{-1} is the sphaleron decay rate, [22]. Combining and integrating eq.5.21 and eq. 5.19, one may obtain the Baryon asymmetry at some time t to be

$$\Delta B(t) = \Delta B(t_{ini}) - \frac{1}{2}\Delta(B+L)(t_{ini}) + \Delta(B+L)(t_{ini})e^{-\tau^{-1}t}. \tag{5.20}$$

Due to the fact that $\Delta B(t_{ini}) = 0$, and the contribution of the exponential term is negligible, the Baryon asymmetry reads as,

$$\Delta B(t) \approx -\frac{1}{2}\Delta L(t_{ini}), \tag{5.21}$$

Finally, using the result of eq.6.1,

$$\boxed{\Delta B(t) \approx -\frac{q}{2}\frac{B_0(t_{ini})}{m_{N_1}}} \tag{5.22}$$

where the factor q is some numerical factor, and the minus sign does not have a physical significance, since it can can be absorbed in the definition of the currents.

Chapter 6

Conclusion

From observations of the Universe around us, as also from the Cosmic Microwave Background, and from studies regarding Nucleosynthesis, there are strong constraints on the matter-antimatter asymmetry. Although the origin of this asymmetry, is yet unrevealed.

During this Thesis, we discussed the process of Leptogenesis, when including Right Handed Neutrinos (RHN) in the Standard Model. The CP asymmetic decay of the RHN into lepton and antileptons, at one loop level, and through the interference of a second species of RHN, can produce a Lepton asymmetry in our Universe.

Furthermore, we discussed the decay of the RHN in String Inspired Cosmological Models. Where, due to the presence of the Kalb-Ramond Field, there is no need to add in the model a second species of RHN, in order to obtain Leptogenesis. In this model, The RHN decay CP asymmetric at tree level, hence one may construct the Boltzmann equations and study the evolution of the Lepton asymmetry. One may solve the equations with numerical methods, [7], and obtain the the following result for the lepton asymmetry,

$$\frac{\Delta L^{TOT}}{s} \simeq (0.008 - 0.014)\frac{B_0}{m_{N_1}} \quad , \qquad m_{N_1}/T_D = (1.44 - 1.62) \tag{6.1}$$

while the phenomenogical constraints suggest that,

$$\frac{B_0}{m_{N_1}} \sim 10^{-9} - 10^{-8} \quad , \qquad m_{N_1}/T_D = (1.44 - 1.62). \tag{6.2}$$

Finally through the sphaleron processes in the early Universe, one can study the Baryon and Lepton number non-conservation rate. Therefore, we suggested that the Baryon asymmetry is approximately equal to,

$$\Delta B(t) \approx -\frac{q}{2}\frac{B_0(t_{ini})}{m_{N_1}}. \tag{6.3}$$

In addition to all the aforementioned, one may also study the direct, due to the presence of the Kalb-Ramond field. In accordance to the previous calculations, it is possible to show that the quark spinors, will also be affected from the coupling with the KR field. Hence they can also contribute to Baryogenesis, although they will only cover just a small fraction of the Baryon asymmetry.

Concluding, this thesis suggests the Bayrogenesis through Leptogenesis in String inspired Cosmologies, due to the RHN decays. While also discuses the physics of Leptogenesis, when extended the Standard Model per two extra species of Right Handed Neutrinos.

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