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Argumentation and Rule-based Logic in Mathematical Proving and Legal-AI applications

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Abstract

Proving is the validation of conclusions by application of logical arguments and rules to assumptions. In mathematics, an assertion is not accepted as true or valid unless it is accompanied by a proof. However, proving processes do not exist only in mathematics, but almost everywhere — in the physical sciences, in computer science, in legal and ethical argumentation, in philosophy, and so on. During the proving process, a dialogue between agents is required to clarify obscure inference steps, fill gaps, or reveal implicit assumptions in a purported proof. Hence, argumentation is an integral component of the discovery process of proofs in general but also — more specifically — in mathematical proofs.

The first part of this thesis presents how logic-based argumentation theories can be applied to describe specific features in the development of proof-events, highlighting the relation between formal proof, informal human reasoning, cognitive processes, and social interactions. The concept of proof-event was coined by Goguen who described mathematical proof as a social event that takes place in space and time, designed to cover not only "traditional" formal proofs but all kinds of proofs including incomplete or purported proofs. In real-life cognitive processes, informal human reasoning and social aspects play a significant role. Our approach attempts to make proof-events more comprehensive to express the complete trajectory of a mathematical proof-event, including formal and informal proving steps, until the ultimate validation of the proving outcome. Thus, we present an extended version of proof-event calculus named Argumentation-based Proof-Event Calculus (APEC) which is built on the argumentation theories of Pollock, Toulmin, and Kakas designed to capture the internal and external structure of a collaborative mathematical practice.

The second part of this thesis demonstrates two scopes of implementation to highlight the applicability and the expressivity of logic-based approaches in real-life proving scenarios. The first scope concerns explicit mathematical proving practices, which can be applied for an in-depth analysis of the internal steps in a mathematical proof, as indicated in the paradigm of Zero Knowledge Proofs, or for modeling a more external perspective to highlight the social interactions and the progress during a

multi-agent proving process, as illustrated in the cases of Mini-Polymath 4 project and Fermat's Last Theorem. The second scope is for implicit proving processes encoded in legal and ethical aspects of medical devices and wearable robots. Legal-AI models in the medical sector are presented through logic-based systems, where a legal text is represented by rules that can express legal arguments and exceptions and can provide explanations as audit trails of how a particular conclusion was proved. The logic-based legal systems presented in this work are: the WeaRED, an ethical decision-making system on Wearable Robots' data privacy; the AMeDC and the Medical Devices Rules systems regarding Medical Devices Regulation; and the ExosCE Rules system regarding the regulation of exoskeletons.

Περίληψη

Λογική της Επιχειρηματολογίας και των Κανόνων με Εφαρμογές σε Μαθηματικές Αποδείξεις και Νομικά Συστήματα Τεχνητής Νοημοσύνης

(Εκτεταμένη Αυτοτελής Περίληψη στην Ελληνική Γλώσσα)

Εισαγωγή

Η απόδειξη είναι η επικύρωση των συμπερασμάτων με την εφαρμογή λογικών επιχειρημάτων και κανόνων σε υποθέσεις. Στα μαθηματικά, ένας ισχυρισμός δεν γίνεται δεκτός ως αληθής ή έγκυρος παρα μόνο εάν οδηγεί μέσω λογικής αιτιολόγησης στην απόδειξη του. Ωστόσο, αποδεικτικές διαδικασίες δεν υπάρχουν μόνο στα μαθηματικά, αλλά σχεδόν παντού - στις φυσικές επιστήμες, στην επιστήμη των υπολογιστών, στη νομική και ηθική επιχειρηματολογία, στη φιλοσοφία και ούτω καθεξής. Οι αποδείξεις στην επιστήμη των υπολογιστών μπορεί να είναι συστήματα που αποδεικνύουν ιδιότητες προγραμμάτων. Οι νομικές αποδείξεις επιτυγχάνονται βάσει αποδεκτών αποδεικτικών στοιχείων που βασίζονται σε σχετικά νομικά πλαίσια. Μια ηθική απόδειξη μπορεί να είναι ένα συμπέρασμα που προκύπτει από μια σειρά ευρέως αποδεκτών επιχειρημάτων που μπορούν γενικά να θεωρηθούν αξιόπιστα. Επομένως, απαιτούνται δεξιότητες απόδειξης σε διαφορετικούς τομείς και τύπους εφαρμογών και μπορούν να επηρεάσουν την πορεία και την πρόοδο σε αυτούς τους τομείς.

Αντικείμενο αυτής της μελέτης είναι να προσεγγίσει την έννοια της απόδειξης από μια προοπτική που είναι πιο κοντά στον τρόπο με τον οποίο γίνεται ο ανθρώπινος συλλογισμός. Οι άνθρωποι όταν διεξάγουν συλλογισμούς μπορεί να αλλάξουν γνώμη σχετικά με ένα προηγούμενο συμπέρασμα για ένα θέμα, εάν έρθουν αντιμέτωποι με πρόσθετες πληροφορίες. Δεν υπακούουν απαραίτητα στους κανόνες της «κλασικής λογικής», οι γνώσεις τους μπορεί να είναι ελλιπείς και ασυνεπείς και, επομένως, νέα δεδομένα μπορούν να ανακαλέσουν τα συμπεράσματα που εξάγονται. Παρόλο που η μαθηματική γνώση παρουσιάζεται συνήθως ως μια διαδικασία που οδηγεί στην «αλήθεια» εφαρμόζοντας λογικούς κανόνες, η ανακάλυψη των αποδείξεων είναι μια πιο περίπλοκη διαδικασία γεμάτη εμπόδια και αδιέξοδα που πρέπει να ξεπεραστούν. Στόχος μας είναι να παρουσιάσουμε ένα μοντέλο μαθηματικής ανακάλυψης που απεικονίζει τη σύνδεση μεταξύ των τυπικών αποδείξεων και των άτυπων κοινωνικών και γνωστικών τους πτυχών. Η ευελιξία του ανθρώπινου συλλογισμού έχει ως επακόλουθο ότι οποιεσδήποτε προσπάθειες μοντελοποίησης των γνωστικών διαδικασιών που εκτελούν οι άνθρωποι προϋποθέτει τον συνδυασμό διαφορετικών τεχνικών [1]. Μελέτες στη γνωστική ψυχολογία έχουν δείξει [2] ότι οι προσέγγισεις που βασίζεται στην επιχειρηματολογία μπορούν να βοηθήσουν στην ενσωμάτωσή της με ευρύτερες μορφές ανθρώπινης επικοινωνίας όπως ο διάλογος, η αντιπαράθεση, η επικύρωση και η υπεράσπιση - ειδικά σε ημιτελή και δυναμικά μεταβαλλόμενα περιβάλλοντα. Για το σκοπό αυτό, μελετάμε τις αλληλένδετες περιοχές απόδειξης και λογικής μέσα από τα διάφορα δυναμικά μεταβαλλόμενα κοινωνικά περιβάλλοντα και προτείνουμε κατάλληλες μεθοδολογίες για την αναπαράσταση του συλλογισμό και των αποδεικτικών διαδικασιών των δρώντων.

Στόχος αυτής της διατριβής είναι η μελέτη της έννοια της απόδειξης που γίνεται κατανοητή ως *αποδεικτικά συμβάντα* (proof-events), με την έννοια του Goguen [3] και τη σχέση της με την επιχειρηματολογία. Η έννοια του αποδεικτικού συμβάντος έχει σχεδιαστεί για να περιλαμβάνει οποιαδήποτε αποδεικτική δραστηριότητα, συμπεριλαμβανομένων των λανθασμένων, ασαφών, αμφισβητούμενων ή ελλιπών αποδείξεων. Κατά τη διάρκεια της αποδεικτικής διαδικασίας, απαιτείται διάλογος μεταξύ των δρώντων (agents) για να διευκρινιστούν τα ασαφή βήματα, να καλυφθούν κενά ή να αποδειχθούν έμμεσες υποθέσεις σε μια μη ολοκληρωμένη απόδειξη. Ως εκ τούτου, η επιχειρηματολογία είναι αναπόσπαστο συστατικό της διαδικασίας ανακάλυψης των αποδείξεων γενικότερα αλλά και - πιο συγκεκριμένα - στις μαθηματικές αποδείξεις. Η έμφαση μας είναι στην ανταλλαγή επιχειρημάτων και αντιεπιχειρημάτων που λαμβάνει χώρα κατά την απόδειξη, καθώς η διαλεκτική φύση της επιχειρηματολογίας είναι παρόμοια με τον ανθρώπινο συλλογισμό.

Αυτή είναι μια νέα προσέγγιση στην οποία χρησιμοποιούνται τεχνικές και θεωρίες από την επιχειρηματολογία για να μελετήσει και να εκφράσει με πιο επαρκή τρόπο τόσο τις τυπικές όσο και τις άτυπες κατηγορίες αποδεικτικών διαδικασιών. Τα αποδεικτικά συμβάντα επεκτείνονται και αναπαρίστανται σε μια μορφή διαλόγου μεταξύ δρώντων που χρησιμοποιούν επιχειρήματα για να ελέγξουν την εγκυρότητα των βημάτων μιας απόδειξης, οδηγώντας στην ανάπτυξη ενός νεου λογισμού που ονομάζεται Λογισμός Αποδεικτικών Συμβάντων βάσει Επιχειρημάτων (APEC). Η συμβολή της προτεινόμενης προσέγγισης είναι ότι συνδυάζει τις αποδεικτικές πρακτικές με την λογική της επιχειρηματολογίας ώστε να οικοδομηθεί μια γέφυρα μεταξύ της επίσημης απόδειξης και της ανεπίσημης κοινωνικής αλληλεπίδρασης στην αναζήτηση της απόδειξης κατασκευάζοντας ένα μοντέλο που σχετίζεται στενά με τον τρόπο με τον οποίο εξελίσσεται η απόδειξη σε πραγματικές συνθήκες. Για να αξιολογηθεί η εφαρμοσιμότητα του εν λόγω Λογισμού και των προτεινόμενων λογικών μεθόδων σε πραγματικά σενάρια παρουσιάζονται δύο πεδία εφαρμογής: το πρώτο αφορά εφαρμογές σε μαθηματικές αποδείξεις και το δεύτερο σε νομικά και ηθικά συστήματα τεχνητής νοημοσύνης στους τομείς των ιατροτεχνολογικών συσκευών και της ρομποτικής.

Θεωρητικό Υπόβαθρο

Η μαθηματική γνώση παρουσιάζεται συνήθως ως μια διαδικασία που οδηγεί στην «αλήθεια» εφαρμόζοντας λογικούς κανόνες εξαγωγής συμπερασμάτων. Ωστόσο, η μαθηματική διερεύνηση είναι μια πιο περίπλοκη διαδικασία. Η ανακάλυψη, η δημιουργικότητα, η επικοινωνία και η συστηματοποίηση είναι μερικά στοιχεία που οι μαθηματικές αποδεικτικές διαδικασίες περιέχουν [4], αλλά συχνά η απόδειξη γίνεται αντιληπτή κυρίως ως μέθοδος πειθούς [5]. Η ευρύτερη έννοια του Goguen [3] για το αποδεικτικό συμβάν είναι κατανοητή ως ένα δημόσιο κοινωνικό συμβάν με συγκεκριμένο τόπο και χρόνο που εξελίσσεται γύρω από μια κοινοποιημένη αναπτυσσόμενη απόδειξη ενός τεθέντος προβλήματος. Έχει σχεδιαστεί για να περιλαμβάνει οποιαδήποτε αποδεικτική δραστηριότητα, συμπεριλαμβανομένων ελαττωματικών, ασαφών, αμφισβητούμενων ή ελλιπών αποδείξεων. Οι Βανδουλάκης και Στεφανέας [6] περιγράφουν τα αποδεικτικά συμβάντα ως δραστηριότητες ενός συστήματος πολλαπλών δρώντων που ενσωματώνει την ιστορία αυτών των δραστηριοτήτων με τη μορφή ακολουθιών αποδεικτικών συμβάντων.

Η σύγκριση μεταξύ απόδειξης και επιχειρηματολογίας βασίζεται στην αντίληψη ότι η απόδειξη (συμπεριλαμβανομένων ελλιπών ή ακόμα και ψευδών αποδείξεων, έγκυρων ή μη έγκυρων βημάτων, συμπερασμάτων, ιδεών, κ.λπ.) μπορεί να θεωρηθεί ως ένα συγκεκριμένο είδος επιχειρηματολογικού λόγου στα μαθηματικά [7]. Πολλοί ερευνητές προσπάθησαν να δείξουν ότι η διαδικασία με την οποία οι μαθηματικοί αξιολογούν το συλλογισμό είναι παρόμοια με την επιχειρηματολογία, για παράδειγμα προσαρμόζοντας το μοντέλο επιχειρηματολογίας του Toulmin [8] σε μαθηματικά παραδείγματα. Ο Aberdein [9–11] τόνισε τη χρήση ορισμάτων σε μαθηματικές συζητήσεις και πρακτικές. Η Pedemonte [12, 13, 7] υλοποίησε ένα εμπλουτισμένο μοντέλο του Toulmin για να υποδείξει τις συνδέσεις μεταξύ επιχειρηματολογίας και απόδειξης. Ο Krummheuer [14] εισήγαγε την ανάλυση της συμμετοχής και της συλλογικής επιχειρηματολογίας χρησιμοποιώντας τη θεωρία του Toulmin για την ανάπτυξη μιας θεωρίας αλληλεπίδρασης στην εκμάθηση των μαθηματικών. Οι Knipping και Reid [15] βασίστηκαν στη θεωρία του Toulmin για να συγκρίνουν και να περιγράψουν παγκόσμιες και τοπικές δομές επιχειρηματολογίας με στόχο τη βαθύτερη κατανόηση των αποδεικτικών διαδικασιών στην τάξη. Οι Μεταξάς κ.ά. [16] παρουσίασαν μεθοδολογίες για τη μελέτη της μαθηματικής πρακτικής σε μια τάξη που εμπλέκεται σε επιχειρηματολογικές δραστηριότητες ενσωματώνοντας το μοντέλο του Toulmin και το σχήμα επιχειρηματολογίας.

Άλλες προσεγγίσεις υποδεικνύουν επίσης τη σύνδεση μεταξύ μαθηματικού συλλογισμού και επιχειρηματολογίας. Ο Krabbe [17] παρουσίασε άτυπες μαθηματικές αποδείξεις ως επιχειρήματα, εφαρμόζοντας τη πράγμα-διαλεκτική θεωρία που εμπλουτίζεται από τη θεωρία των στρατηγικών ελιγμών για να εντοπίσει στις αποδείξεις τα τέσσερα στάδια της κριτικής συζήτησης: αντιπαράθεση, άνοιγμα, επιχειρηματολογία, τελικό στάδιο [18]. Ο Aberdein [11] τόνισε τη σύνδεση του μαθηματικού συλλογισμού με εργαλεία που αναπτύχθηκαν από τον επιστήμων της άτυπης λογικής Douglas Walton για να εκφράσει σχήματα επιχειρηματολογίας ως ταξινόμηση των βημάτων επιχειρηματολογίας καθώς και των διαλόγων ως συμφραζόμενη τυπικότητα μέσω μαθηματικών επιχειρημάτων. Το «Αποδείξεις και διαψεύσεις» [19] του Lakatos είναι επίσης ένα κλασικό έργο που υπογραμμίζει τον ρόλο του διαλόγου μεταξύ των δρώντων (ενός δάσκαλος και ορισμένων μαθητών) στις προσπάθειες απόδειξης καθώς και στην κριτική αυτών των προσπαθειών. Το έργο στο [20] παρέχει έναν τρόπο τυποποίησης των κοινωνικών πτυχών των αποδείξεων ερμηνεύοντας την άτυπη λογική μιας μαθηματικής ανακάλυψης μέσω του φακού του επιχειρηματολογικού διαλόγου.

Οι μελέτες για την ανακάλυψη των μαθηματικών έχουν επίσης χρησιμοποιήσει την έννοια της «συλλογικής επιχειρηματολογίας» για να εξετάσουν ιδιαίτερα τα μαθηματικά χαρακτηριστικά των διαλόγων, καθώς διάφοροι μαθηματικοί/δρώντες συνεργάζονται για να αποδείξουν έναν ισχυρισμό [21]. Η έννοια της απόδειξης ως **λόγος** και η δραστηριότητα των δρώντων διερευνάται στο [22], όπου μια διαλογική περιγραφή της μαθηματικής απόδειξης προωθείται για την παραγωγή επεξηγηματικής πειθούς. Ωστόσο, σε αυτό το σχήμα, ο σκεπτικιστής θεωρείται ως επί το πλείστον «σιωπηλός» [23], ενώ εμείς επιθυμούμε να απεικονίσουμε εξίσου τις κινήσεις και τα αντεπιχειρήματα που δημιουργούνται από την άλλη πλευρά για να κατανοήσουμε βαθύτερα την όλη μαθηματική πρακτική και να τονίσουμε την αξία της αντίθετης πλευράς στην αποδεικτική διαδικασία. Μια άλλη μελέτη που εστιάζει στους δρώντες που παράγουν τις αποδείξεις παρουσιάζεται στο [24], όπου κάθε μαθηματικό βήμα αντιστοιχεί σε μια δραστηριότητα απόδειξης και η επίσημη μαθηματική απόδειξη είναι μια αναφορά της αντίστοιχης δραστηριότητας απόδειξης. Το σχέδιο μιας μαθηματικής απόδειξης συλλαμβάνεται ως το σχέδιο των δρώντων που πραγματοποίησαν την ανάλογη απόδειξη. Γενικά, οι παραπάνω μελέτες παρέχουν προσεγγίσεις μαθηματικού λόγου πολλαπλών δρώντων από μια πιο φιλοσοφική προοπτική. Στη δική μας προσέγγιση, επιχειρούμε να παράσχουμε ένα επίσημο πλαίσιο μέσω ενός λογικού λογισμού για να εκφράσουμε τους ανεπίσημους διαλόγους και τα άτυπα βήματα που γίνονται στη μαθηματική πράξη.

Όλες αυτές οι μελέτες τονίζουν τη σχέση και τη σύνδεση μεταξύ του συλλογισμού στην επιχειρηματολογία και στην απόδειξη. Η προσέγγισή μας επιχειρεί να αποσαφηνίσει αυτή την εγγενή σχέση μοντελοποιώντας τον επιχειρηματολογικό διάλογο κατά τη διάρκεια των μαθηματικών αιτιολογήσεων και εξηγήσεων στην αποδεικτική τους δραστηριότητα. Η επιχειρηματολογία είναι ένα ισχυρό συλλογιστικό εργαλείο που επιτρέπει στους συμμετέχοντες στο διάλογο να επιχειρηματολογούν και να αντιπαρατίθενται, να ισχυρίζονται και να αντικρούουν, να επικυρώνουν και να ακυρώνουν βήματα μαθηματικού συλλογισμού που στοχεύουν στην επίλυση ενός προβλήματος. Αυτό οδηγεί σε μια βαθύτερη κατανόηση των συχνά αντιφατικών οραμάτων, προοπτικών και στρατηγικών επίλυσης προβλημάτων των συμμετεχόντων σε ένα πρόβλημα που εν τέλει καταλήγει στη συμφωνία και συναίνεσή τους [25]. Συστήματα που βασίζονται στη λογική για την εξέταση και την αξιολόγηση επιχειρημάτων έχουν εφαρμοστεί ευρέως, δημιουργώντας διάφορες επίσημες μεθόδους επιχειρηματολογίας [26]. Ένα σημείο εκκίνησης αυτής της διατριβής είναι η προσέγγιση της λογικής επιχειρηματολογίας του Pollock [27, 28], η οποία παρουσίασε μια από τις πρώτες μη μονοτονικές λογικές με έννοιες του επιχειρήματος και της διάψευσης. Εισάγει επίσης την αναιρέσιμη επιχειρηματολογία όπου τα επιχειρήματα είναι αλυσίδες συλλογισμών που μπορεί να οδηγήσουν σε συμπέρασμα, ενώ πρόσθετες πληροφορίες μπορεί να καταστρέψουν την αλυσίδα συλλογισμών. Η τυποποίηση που αναπτύχθηκε είναι κυρίως μια υλοποίηση βασισμένη σε ακολουθίες [29] και στην αφηρημένη επιχειρηματολογία του Dung [30], εφαρμόζοντας την προσέγγιση του Pollock για τον αναιρέσιμο συλλογισμό [27], με τη βασική δομή του μοντέλου Toulmin [8] για την αναπαράσταση ενός επιχειρήματος. Η παρούσα έρευνα στοχεύει να επωφεληθεί, να βασιστεί και να ενσωματώσει τις παραπάνω προσεγγίσεις με τρόπο που να χρησιμοποιεί επίσης γνώσεις από άλλα έργα, όπως των Κάκα και Λοΐζου [31], παρέχοντας μια αφηρημένη, θεωρητική εξερεύνηση της λογικής επιχειρηματολογίας που εφαρμόζεται κυρίως (αλλά όχι αποκλειστικά) στις μαθηματικές αποδείξεις.

Λογισμός Αποδεικτικών Συμβάντων βάσει Επιχειρημάτων

Σύμφωνα με τον Lockhart [32]: *«Τα μαθηματικά δεν είναι στην 'αλήθεια' αλλά στην εξήγηση, το επιχείρημα»*. Τα αποδεικτικά συμβάντα που βασίζονται σε επιχειρηματολογία μπορούν να χρησιμοποιηθούν για την προώθηση του μαθηματικού διαλόγου στον οποίο όλοι οι συμμετέχοντες συνεργάζονται για να εξετάσουν κριτικά τεθέντα προβλήματα και να ενισχύσουν τις ικανότητες σκέψης όπως η επίλυση προβλημάτων, η ερμηνεία, η πειθώ και η δημιουργικότητα. Ως εκ τούτου, ο στόχος της αλληλεπίδρασης των μαθηματικών δεν είναι πλέον η καλλιέργεια ατομικών δεξιοτήτων επίλυσης προβλημάτων, αλλά η ανάπτυξη *συνεργατικών ικανοτήτων επίλυσης προβλημάτων* [33].

Μερικά από τα ερωτήματα που μελετάμε είναι:

1. *Η σχέση μεταξύ άτυπης και τυπικής απόδειξης στην πραγματική μαθηματική πρακτική και επικοινωνία.*
2. *Η σχέση μεταξύ επιχειρημάτων και μαθηματικών αποδεικτικών δραστηριοτήτων.*
3. *Η σχέση μεταξύ των συνεισφορών των εμπλεκόμενων μαθηματικών και της μαθηματικής απόδειξης ως τελικό αποτέλεσμα.*

Η γνωστική επιστήμη έχει δείξει [2] ότι η διαλογική φύση της επιχειρηματολογίας είναι παρόμοια με την ανθρώπινη λογική στην απόδειξη. Οι άνθρωποι που διεξάγουν συλλογισμούς δεν ακολουθούν απαραίτητα τους κανόνες της «λογικής» [34]. Οι άνθρωποι είναι πιο πιθανό να καταλήξουν σε συμπεράσματα με βάση την αντίληψη και την εμπειρία τους, αντί να κάνουν μια λίστα δεδομένων για να βγάλουν συμπεράσματα ακολουθώντας αυστηρά τη λογική. Μπορεί να αλλάξουν γνώμη σχετικά με ένα προηγούμενο συμπέρασμα για ένα θέμα, εάν έρθουν αντιμέτωποι με πρόσθετες πληροφορίες. Οι γνώσεις τους μπορεί να είναι ελλιπείς και ασυνεπείς και, ως εκ τούτου, τα νέα δεδομένα μπορούν να ανακαλέσουν τα συμπεράσματα που εξάγονται [31]. Έτσι, ο ανθρώπινος συλλογισμός είναι τις περισσότερες φορές επαγωγικός. Ωστόσο, οι ανεπίσημες και κοινωνικές πτυχές της απόδειξης συνήθως δεν αντιπροσωπεύονται επαρκώς στα αποτελέσματα απόδειξης που παρέχονται στην τελική μορφή. Οι προσεγγίσεις που βασίζονται στην επιχειρηματολογία μπορούν να βοηθήσουν την ενσωμάτωσή τους σε ευρύτερα πλαίσια ανθρώπινης συλλογιστικής, ειδικά σε δυναμικά περιβάλλοντα όπως τα μαθηματικά περιβάλλοντα της πραγματικής ζωής. Το μοντέλο που παρουσιάζουμε επιχειρεί να απεικονίσει τους διαλόγους του(ων) αποδεικνύοντος(ών) και του(ων) ερμηνευτή(ων) σε ένα σύστημα πολλαπλών δρώντων, εκφράζοντας τόσο την εσωτερική δομή των επιχειρημάτων τους και επομένως τη γνωστική τους σκέψη καθώς και τις εξωτερικές κοινωνικές αλληλεπιδράσεις με τις κινήσεις επιχειρηματολογίας.

Διερευνούμε τη μαθηματική απόδειξη ως αποδεικτικό συμβάν που βασίζεται στην επιχειρηματολογία για να διασαφηνίσουμε τα δημιουργικά χαρακτηριστικά της επιχειρηματολογίας που είναι σημαντικά για την απόδειξη, όπως η διαπραγμάτευση, η συνεργασία και το γόνιμο λάθος. Αυτή η προσέγγιση μας δίνει τη δυνατότητα να εξετάσουμε τα είδη συλλογισμών που μπορούν να χρησιμοποιήσουν στην αλληλεπίδρασή τους οι δρώντες και πώς η κοινή δραστηριότητα μπορεί να τους επηρεάσει ώστε να δημιουργήσουν νέα επιχειρήματα καθώς περνούν από τις υποθέσεις ενός προβλήματος στην απόδειξή του [21]. Η επιχειρηματολογία επιτρέπει στους συνεισφέροντες να εισέλθουν σε διάλογο κατά τη διάρκεια της δραστηριότητάς επίλυσης προβλημάτων για να δοκιμάσουν εναλλακτικές στρατηγικές απόδειξης, να ελέγξουν ένα προτεινόμενο επιχείρημα ή μια ιδέα ή ένα μέρος μιας απόδειξη μέχρι να καταλήξουν σε τελική συμφωνία [25]. Αυτή η προοπτική μπορεί να αναδιαμορφώσει τη μαθηματική ανακάλυψη σε μια διαδραστική, διαπραγματεύσιμη, κοινωνική διαδικασία.

Αν και μια νέα απόδειξη συνήθως αποδίδεται στον λύτη του προβλήματος, είναι το αποτέλεσμα κοινών προσπαθειών διαφορετικών δρώντων, ο καθένας από τους οποίους έχει διαφορετικές προηγούμενες εμπειρίες, βασικές γνώσεις, δεξιότητες απόδειξης και όραμα του προβλήματος [35]. Προτείνουμε ένα μοντέλο για την εκμάθηση των μαθηματικών, όπου η επίλυση προβλημάτων θεωρείται ως *συλλογική ανακάλυψη αποδεικτικών συμβάντων* [36, 37]. Το σύστημα αναπαριστά όλη την ιστορία της ανακάλυψης σε διαφορετικά επίπεδα αποδεικτικών συμβάντων και την επισημοποιεί με τη μορφή συνεργατικών επιχειρηματολογικών συνεισφορών που περιλαμβάνουν δοκιμές, συγκρούσεις και πιθανή επικύρωση ή τερματισμό τμημάτων των υπό ανάπτυξη αποδείξεων [38]. Στο τελικό βήμα, η επίσημη απόδειξη ελέγχεται, κατανοείται και επιβεβαιώνεται από τη σχετική μαθηματική κοινότητα, ώστε να αναγνωριστεί ως έγκυρη.

Ο σκοπός αυτής της έρευνας είναι να μελετήσει και να παρουσιάσει πώς οι θεωρίες επιχειρηματολογίας που βασίζονται στη λογική μπορούν να εφαρμοστούν για να περιγράψουν συγκεκριμένα χαρακτηριστικά στην ανάπτυξη των αποδεικτικών συμβάντων (proof-event), τονίζοντας τη σχέση μεταξύ επίσημης απόδειξης, άτυπου ανθρώπινου συλλογισμού, γνωστικών διαδικασιών και κοινωνικών αλληλεπιδράσεων. Η έννοια του αποδεικτού συμβάντος επινοήθηκε από τον Goguen, ο οποίος περιέγραψε τη μαθηματική απόδειξη ως ένα κοινωνικό γεγονός που λαμβάνει χώρα σε συγκεκριμένο χώρο και χρόνο, σχεδιασμένο να καλύπτει όχι μόνο τις «παραδοσιακές» τυπικές αποδείξεις αλλά όλα τα είδη αποδείξεων, συμπεριλαμβανομένων των ελλιπών ή υποθετικών αποδείξεων. Στις πραγματικές γνωστικές διαδικασίες, ο ανθρώπινος συλλογισμός και οι κοινωνικές πτυχές παίζουν σημαντικό ρόλο. Η προσέγγισή μας επιχειρεί να κάνει τα αποδεικτικά συμβάντα πιο πλήρη για να εκφράσει τη συνολική τροχιά μιας μαθηματικής διαδικασίας, συμπεριλαμβανομένων τόσο των τυπικών όσο και των άτυπων βημάτων απόδειξης, μέχρι την τελική επικύρωση του αποτελέσματος της απόδειξης. Έτσι, παρουσιάζουμε μια εκτεταμένη έκδοση του λογισμού αποδεικτικών συμβάντων με το όνομα Λογισμός Αποδεικτικών Συμβάντων βάσει Επιχειρημάτων ο οποίος βασίζεται στη λογική της επιχειρηματολογίας των Pollock, Toulmin και Kakas και έχει σχεδιαστεί για να μπορεί να καταγράφει την εσωτερική και εξωτερική δομή μιας συλλογικής μαθηματικής πρακτικής.

Ο Λογισμός αυτός μπορεί να χρησιμοποιηθεί σε συνεργατικά μαθηματικά περιβάλλοντα όπου η επιχειρηματολογία μεταξύ των μαθηματικών μπορεί να χρησιμοποιηθεί για να παρακινήσει τη δημιουργικότητα και την ανακάλυψη ή να διευκρινίσει σκοτεινά σημεία μιας υποτιθέμενης απόδειξης. Επιπλέον, το APEC διευκολύνει την τυποποίηση διαδραστικών σχημάτων επιχειρηματολογίας για την περιγραφή της μαθηματικής δραστηριότητας επίλυσης προβλημάτων που αντιμετωπίζει ενδεχόμενες αντιφάσεις και αδιέξοδα.

Τα βασικά μέρη αυτού του λογισμού επιγραμματικά είναι τα εξής:

- **Δομικά Στοιχεία:**
    - $prem(e)$: Τα δεδομένα Φ του αποδεικτικού συμβάντος e.
    - $concl(e)$: Ο ισχυρισμός c του αποδεικτικού συμβάντος e.
    - $infRul(e)$: Η εγγύηση w του αποδεικτικού συμβάντος e
- **Κινήσεις επιχειρημάτων:**
    - $Elaboration(e, S)$: Η δήλωση S ενισχύει το αποδεικτικό συμβάν e.
    - $Equivalent(e, e')$: Το αποδεικτικό συμβάν e είναι ισοδύναμο με το αποδεικτικό συμβάν e'.
    - $Rebutting(e^*, e)$: Το αποδεικτικό συμβάν e^* αντικρούει τον ισχυρισμό του αποδεικτικού συμβάντος e.
    - $Undercutting(e^*, e)$: Το αποδεικτικό συμβάν e^* αντικρούει τα δεδομένα του αποδεικτικού συμβάντος e.
    - $Undermining(e^*, e)$: Το αποδεικτικό συμβάν e^* αντικρούει την εγγύηση του αποδεικτικού συμβάντος e.
- **Συλλογιστική:**
    - $Support(e', t)$: Αποδεικτικά συμβάντα e' που στηρίζουν το αποδεικτικό συμβάν e.
    - $Attack(e^*, t)$: Αποδεικτικά συμβάντα e^* που επιτίθονται στο αποδεικτικό συμβάν e.
- **Χρονικά Κατηγορήματα**
    - $Happens(e, t)$: Το αποδεικτικό συμβάν e συμβαίνει τη χρονική στιγμή t.
    - $Initiates(e, f, t)$: Η αλληλουχία f του e ξεκινάει τη χρονική στιγμή t.
    - $Clipped(e, f, t)$: Η αλληλουχία f του e διακόπτεται τη χρονική στιγμή t.
    - $Terminates(e^*, f, t)$: Η αλληλουχία f του e τερματίζεται από το e^* τη χρονική στιγμή t.
    - $ActiveAt(e, f, t)$: Η αλληλουχία f του e είναι ενεργή τη χρονική στιγμή t.

- $Valid(e,f,t)$: Η αλληλουχία f του e είναι έγκυρη τη χρονική στιγμή t.

Γιατί απαιτείται η ανάπτυξη του Λογισμού για ένα επιχειρηματολογικό μοντέλο; Υπάρχει ένα κενό στη βιβλιογραφία σχετικά με εργαλεία που μπορούν να παρέχουν τυπικά - υπολογιστικά σαφή - στοιχεία που μπορούν να διαχειριστούν την ποικιλία των διαδικασιών που συνήθως εμπλέκονται στην κατασκευή αποδείξεων, ειδικά όταν περιέχουν άτυπους μαθηματικούς διαλόγους με υποθέσεις, επιχειρήματα, αντιπαραδείγματα κ.λπ. [39]. Η συνεισφορά μας αφορά την επεξεργασία ενός αναλυτικού πλαισίου που παρέχει ένα εργαλείο για την περιγραφή και την αξιολόγηση της μαθηματικής απόδειξης με βάση την επίσημη δομή, τις συνεισφορές των δρώντων, την επιχειρηματολογία και την αλληλουχία των επιχειρημάτων. Τα παραπάνω χαρακτηριστικά αποτελούν συνδυασμός διαφορετικών κατηγοριών επιχειρηματολογίας [40] ενσωματωμένες σε ένα πλαίσιο.

Εφαρμογές σε Μαθηματικές Αποδείξεις και σε Τεχνητής Νοημοσύνης Νομικά Συστήματα

Αποδείξεις υπάρχουν παντού, όπου ως «αποδεικτική διαδικασία» ορίζουμε οποιαδήποτε διαδικασία ακολουθούμε με βάση κάποια λογική για να αποδείξουμε τον επιθυμητό ισχυρισμό. Παρουσιάζονται δύο πεδία εφαρμογής για να τονιστεί η εφαρμοσιμότητα και η εκφραστικότητα αυτής της προσέγγισης σε πραγματικά σενάρια με δύο διαφορετικούς τομείς υλοποίησης: το πρώτο μέρος περιγράφει την τυποποίηση άμεσων αποδεικτικών διαδικασιών στο τομέα των Μαθηματικών και το δεύτερο μέρος την τυποποίηση έμμεσων αποδεικτικών διαδικασιών σε άλλους τομείς, όπως σε νομικά και ηθικά συστήματα.

Στο πρώτο πεδίο παρουσιάζει πώς ο Λογισμός Αποδεικτικών Συμβάντων βάσει Επιχειρημάτων μπορεί να μοντελοποιήσει πρακτικές μαθηματικής απόδειξης, αφενός, από μια εις βάθος ανάλυση των εσωτερικών βημάτων στη μαθηματική απόδειξη, όπως υποδεικνύεται στο παράδειγμα των Αποδείξεων Μηδενικής Γνώσης, αφετέρου, από μια πιο αποστασιοποιημένη προοπτική, μοντελοποιώντας τις κοινωνικές αλληλεπιδράσειςκαι τη χρονική εξέλιξη των συμβάντων κατά τη διάρκεια μιας αποδεικτικής διαδικασίας πολλαπλών δρώντων όπως απεικονίζεται στα παραδείγματα του MiniPolymath4 και του τελευταίου θεωρήματος του Φερμά. Οι παρακάτω εφαρμογές απεικονίζουν τη συμβολή των δρώντων στη διαδικασία της απόδειξης. Οι πληροφορίες που λαμβάνουμε από αυτού του τύπου τις εφαρμογές υποδεικνύουν ότι τα χαρακτηριστικά και η ποιότητα των διαλόγων μπορούν να επηρεάσουν τη μαθηματική σκέψη και πρακτική. Αρχικά, ο κεντρικός στόχος της ίδιας της απόδειξης είναι να πείσει την υπόλοιπη κοινότητα για τη δικαιολόγηση και την εγκυρότητα της προσέγγισης και των αποτελεσμάτων κάποιου. Επιπλέον, όλοι οι δρώντες συμβάλλουν σημαντικά στη διαδικασία, αφού διάφοροι άνθρωποι έπρεπε να συμμετάσχουν στην επίτευξη του κοινού τους στόχου, που είναι η απόδειξη του προβλήματος.

- Το APEC μπορεί να χρησιμοποιηθεί για να τυποποιήσει σε βάθος τα βήματα που λαμβάνουν χώρα στην αποδεικτική διαδικασία συγκεκριμένων τύπων αποδείξεων, όπως στη μελέτη περίπτωσης μας για τις Αποδείξεις Μηδενικής Γνώσης. Οι Αποδείξεις Μηδενικής Γνώσης αποτελούνται από ένα πρωτόκολλο μεταξύ (τουλάχιστον) δύο ατόμων στο οποίο ένα μέρος, που ονομάζεται αποδεικνύων, προσπαθεί να αποδείξει ένα συγκεκριμένο σημείο στο άλλο μέρος, που ονομάζεται επαληθευτής [41]. Τα δύο μέρη παίζουν τους αντίστοιχους ρόλους του αποδεικνύοντος και του ερμηνευτή στον Λογισμό. Το πρωτόκολλο απαιτεί διαλεκτική εισαγωγή από τον επαληθευτή, συνήθως με τη μορφή επαναλαμβανόμενων προκλήσεων, έτσι ώστε οι απαντήσεις από τον επαληθευτή να πείσουν τον πρώτο ότι ο ισχυρισμός του είναι αληθής (πράγμα που σημαίνει ότι ο επαληθευτής έχει την απαιτούμενη γνώση). Στην εν λόγω εφαρμογή, μοντελοποιείται το παράδειγμα της σπηλιάς του Αλί Μπαμπά χρησιμοποιώντας τις κινήσεις επιχειρημάτων και τα χρονικά κατηγορήματα των αποδεικτικών συμβάντων.

- Μία από τις δυσκολίες στη διερεύνηση της μαθηματικής πρακτικής είναι ότι υπάρχει περιορισμένη γνώση της πραγματικής διαδικασίας που συμβαίνει στη μαθηματική απόδειξη και της αλληλεπίδρασης των μαθηματικών κατά την απόδειξη [42]. Για να μελετήσουμε τη μαθηματική απόδειξη, χρειαζόμαστε επαρκείς πληροφορίες που θα αποτυπώνουν την πραγματική διαδικασία της μαθηματικής ανακάλυψης, όχι μόνο το τελικό προϊόν της απόδειξης που ανακοινώνεται στις δημοσιεύσεις. Μια πηγή πληροφοριών που μπορεί να παρέχει στοιχεία σχετικά με την πραγματική μαθηματική πρακτική είναι τα έργα πληθοπορισμού. Στη συγκεκριμένη εφαρμογή, μελετάμε πώς οι πόροι των διαδικτυακών συνεργατικών μαθηματικών μπορούν να χρησιμοποιηθούν για να υποστηρίξουν τη διατύπωση και την απάντηση ερωτήσεων σχετικά με τη μαθηματική απόδειξη. Μερικά από τα ερωτήματα που προσπαθούμε να επισημάνουμε με αυτή τη μελέτη περίπτωσης είναι: Τι γνώσεις μπορούμε να αποκτήσουμε από τους διαλόγους των διαδικτυακών έργων πληθοπορισμού· Πώς μπορεί να χρησιμοποιηθεί η μελέτη αυτών των έργων για την κατανόηση της μαθηματικής πρακτικής· και πώς μπορούμε να την παρουσιάσουμε με συστηματικό, αναλυτικό και επεξηγηματικό τρόπο. Αυτό επιτυγχάνεται με τη μοντελοποίηση των σχολίων του MiniPolymath 4 που μας επιτρέπει να ενσωματώσουμε τα επιχειρήματα που ανταλλάσσονται σε διαλόγους που αναπαρίστανται σε ακολουθίες αποδεικτικών συμβάντων. Τα μαθηματικά που προέρχονται από το πλήθος είναι πολύτιμα στη μελέτη της μαθηματικής πρακτικής, αποκαλύπτοντας τον τρόπο με τον οποίο σκέφτονται και συζητούν οι μαθηματικοί. Η απόδειξη, τουλάχιστον στην αρχική της φάση, μπορεί να γίνει κατανοητή ως μια έρευνα που υλοποιείται με ανταλλαγή ιδεών: ένας συνεργατικός διάλογος μεταξύ μαθηματικών με κοινό στόχο την επίλυση ενός ανοιχτού προβλήματος, το οποίο κανένας δεν έχει προκαθορίσει συγκεκριμένα [11]. Σίγουρα, μια τέτοια ανταλλαγή επιχειρημάτων μπορεί να βρεθεί στα μαθηματικά, ειδικά στο πλαίσιο της μαθηματικής ανακάλυψης.

- Για την απεικόνιση μιας μαθηματικής απόδειξης δρώντων που έχει εκτεταμένη χρονική πορεία, παρουσιάζεται το παράδειγμα του Τελευταίου Θεωρήματος του Φερμά, όπου πολλοί μαθηματικοί συνέβαλαν στο αποτέλεσμα της τελικής απόδειξης. Σε αυτή την εφαρμογή παρουσιάζεται πώς οι αλληλεπιδράσεις τους και οι συνεισφορές μπορούν να δομηθούν και να μοντελοποιηθούν ώστε να απεικονίζουν την ανακάλυψη και την ιστορία της απόδειξης, όχι μόνο για δρώντες που ζουν στην ίδια χρονική περίοδο αλλά και για δρώντες που έζησαν σε διαφορετικές χρονικές περιόδους. Αυτή η συμμετοχή απεικονίζεται με δύο τρόπους, είτε με την απόρριψη της προσπάθειας κάποιου άλλου με την επισήμανση σφάλματος ή/και ανακρίβειας είτε με το διάλογο μεταξύ συνεργατών για τον εντοπισμό και την επίλυση αδύναμων ή ανεπαρκών τμημάτων στην απόδειξη. Η επιχειρηματολογία είναι πιο αποτελεσματική σε διαδραστικά περιβάλλοντα, καθώς επιτρέπουν να αντιμετωπιστούν τα αντεπιχειρήματα και να εμφανιστούν ισχυρότερα επιχειρήματα. Έτσι, ένας μαθηματικός βρίσκεται σε ευνοϊκή θέση εάν ζητήσει τη βοήθεια συναδέλφων για να επισημάνει πιθανά αντεπιχειρήματα και να τα επιλύσει στην τελική απόδειξη. Με αυτό τον τρόπο, η απόδειξη θα μπορούσε να είναι πιο πειστική όχι μόνο για αυτούς τους συναδέλφους, αλλά πιθανώς για ολόκληρη την κοινότητα. Τα επιχειρήματα και τα αντεπιχειρήματα διαδραματίζουν επίσης ουσιαστικό ρόλο στη διαδικασία της απόδειξης, συνεισφέροντας εξίσου στην οικοδόμηση και την αιτιολόγηση της απόδειξης. Τα δικαιολογημένα μέρη των αρχικών αποδεικτικών συμβάντων λειτούργησαν ως βάση για τα επόμενα αποδεικτικά συμβάντα, ενώ τα αντεπιχειρήματα που σηματοδοτούν τα σφάλματα σε ανεπιτυχή αποδεικτικά συμβάντα ανοίγουν το δρόμο για καλύτερες αιτιολογήσεις.

Συνοψίζοντας, ο ιστορικός οδικός χάρτης της απόδειξης μπορεί να βιωθεί ως μια συνεργατική δραστηριότητα, που συνδέει ανθρώπους με διαφορετικά υπόβαθρα, προοπτικές και ενδιαφέροντα. Σε κάθε σημείο της πορείας απόδειξης το πλαίσιο APEC μας απεικονίζει την τρέχουσα κατάσταση του τυπικού και άτυπου συλλογισμού στην απόδειξη. Αυτό δημιουργεί έναν σύνδεσμο που συνδέει δύο κρίκους: την άτυπη και κοινωνική πτυχή στον αυθόρμητο μαθηματικό διάλογο κατά την ανακάλυψη μιας απόδειξης και την τυπική και υπολογιστική πτυχή του συλλογισμού και της σημασιολογίας της αφηρημένης επιχειρηματολογίας. Το πλαίσιο APEC προσθέτει μια πρόσθετη διάσταση και διαδραματίζει σημαντικό ρόλο στο να κάνει αυτές τις συνδέσεις επαρκώς λεπτομερείς με συστηματικό και επεξηγηματικό τρόπο, αποδεικνύοντας τη δυνατότητα εφαρμογής των τεχνικών επιχειρηματολογίας στη μαθηματική απόδειξη και σκέψη.

Το δεύτερο πεδίο αφορά έμμεσες διαδικασίες απόδειξης που αναφέρονται σε ηθικές και νομικές πτυχές των ιατρικών συσκευών και των φορετών ρομπότ (wearable robots). Τα νομικά μοντέλα τεχνητής νοημοσύνης στον ιατρικό τομέα μπορούν να εκφραστούν αποτελεσματικά μέσω συστημάτων που βασίζονται στη λογική, όπου ένα νομικό κείμενο περιγράφεται από κανόνες που μπορούν να εκφράσουν νομικά επιχειρήματα και εξαιρέσεις και μπορούν να ελεγχθούν και να παρέχουν επεξηγήσεις για το πώς αποδείχθηκε ένα συγκεκριμένο συμπέρασμα. Τα συστήματα με βάση τη λογική που παρουσιάζονται είναι: το WeaRED, ένα σύστημα ηθικής λήψης αποφάσεων σχετικά με το απόρρητο των προσωπικών δεδομένων των φορετών ρομπότ, τα συστήματα AMeDC και Medical Devices Rules σχετικά με το νομικό κανονισμό για τα ιατροτεχνολογικά προϊόντα, και το σύστημα ExosCE σχετικά με το νομικό κανονισμό των εξωσκελετών.

- Η εφαρμογή WeaRED παρουσιάζει πώς μπορεί να εφαρμοστεί το MAPEC για τη μοντελοποίηση ηθικά ορθών διαδικασιών που βασίζονται στη λογική και εφαρμόζονται σε ένα σενάριο λήψης ιατρικών αποφάσεων, αποδίδοντας ηθικές θεωρίες και διλήμματα σε δηλωτική μορφή [43]. Καθώς τα αυτόνομα συστήματα τεχνητής νοημοσύνης αναλαμβάνουν προοδευτικά σημαντικό ρόλο στην καθημερινή μας ζωή, είναι αναμφίβολα ότι αργά ή γρήγορα θα κληθούν να λάβουν σημαντικές, ηθικά φορτισμένες αποφάσεις και ενέργειες [44]. Τα τελευταία χρόνια, το θέμα της ηθικής στην τεχνητή νοημοσύνη και τα ρομπότ έχει κερδίσει μεγάλη προσοχή και προέκυψαν πολλά σημαντικά θεωρητικά και εφαρμοσμένα αποτελέσματα στην προοπτική της ανάπτυξης ηθικών συστημάτων [45]. Η πρόκληση είναι πώς μπορούμε να εγγυηθούμε ότι τα ρομπότ θα έχουν πάντα μια ηθικά σωστή συμπεριφορά, όπως ορίζεται από τον ηθικό κώδικα που δηλώνεται από τους ανθρώπινους. Η έρευνα και τα πραγματικά περιστατικά αστοχιών και κακής χρήσης συστήματος τεχνητής νοημοσύνης έχουν δείξει την ανάγκη για χρήση ηθικής στην ανάπτυξη λογισμικού [44]. Σε αυτό το παράδειγμα του πραγματικού κόσμου, τα συστήματα περιγράφουν τους κανόνες και τα γεγονότα που σχεδιάζουν τη συμπεριφορά ενός φορετού ρομπότ σχετικά με το απόρρητο και τη συγκατάθεση των ιατρικών δεδομένων του χρήστη του. Επισημαίνεται η επιθυμητή ηθική συμπεριφορά των φορετών ρομπών σχετικά με την πρόσβαση στα δεδομένα ενός χρήστη και αναλύεται ο τρόπος με τον οποίο ο κώδικας στη ρομποτική αρχιτεκτονική επηρεάζει τα δεδομένα και το απόρρητο καθώς και γιατί τέτοια ζητήματα πρέπει να εξετάζονται από την οπτική της επίσημης επαλήθευσης [46]. Για την υλοποίηση αυτής της προσπάθειας, οι στόχοι είναι: να επισημοποιήσει τι σημαίνει ότι η λήψη αποφάσεων ενός συστήματος είναι ηθικά σωστή· να παρέχει λογικές προδιαγραφές σύμφωνα με τις οποίες μπορεί να κατασκευαστεί και να ελεγχθεί το σύστημα· να πραγματοποιηθεί λήψη αποφάσεων με βάση την ηθική λογική μέσω του Λογισμού MAPEC · και να υποδείξει πώς μπορεί να εφαρμοστεί ένα τέτοιο ηθικό πλαίσιο σε υπολογιστικά συστήματα όπως στη μελέτης περίπτωσης σχετικά με το απόρρητο δεδομένων των φορετών ρομπότ.

- Οι εφαρμογές AMeDC και Medical Devices Rules παρουσιάζουν πως η λογική της επιχειρηματολογίας και των κανόνων μπορεί να αξιοποιηθεί για την αντιμετώπιση προβλημάτων και τη λήψη αποφάσεων σε διαφορετικούς τομείς, όπως για παράδειγμα στον νομικού και στον υγειονομικού τομέα. Στην εφαρμογή AMeDC χρησιμοποιείται ο Γοργίας-B, ένα πλαίσιο βασισμένο στην επιχειρηματολογία που συνδυάζει τις ιδέες της ιεράρχησης και της προτίμησης και στην εφαρμογή Medical Devices Rules η PSOA RuleML, μια γλώσσα λογικού προγραμματισμού βασισμένη σε κανόνες, τα οποία αποτελούν και τα δύο σύγχρονα εργαλεία για την ανάπτυξη εφαρμογών που ανταποκρίνονται στις απαιτήσεις της πραγματικής ζωής. Οι κύριοι στόχοι αυτών των εφαρμογών είναι: η διερεύνηση των ιατρικών νομοθετικών πλαίσιων και η παροχή μιας επισκόπησης των οδηγιών και των αναδυόμενων διεθνών απαιτήσεων ασφάλειας· η παρουσίαση μελέτης παραδειγμάτων σχετικά με την εμπορευματοποίηση ιατρικών προϊόντων· και ο έλεγχος της ακρίβειας, της ερμηνευσιμότητας και της αξιοπιστίας των ανεπτυγμένων υπολογιστικών μοντέλων (επιτρέποντας την επικύρωση από ανθρώπους).

- Τα φορετά ρομπότ στοχεύουν να βελτιώσουν σημαντικά την ποιότητα ζωής των χρηστών αποκαθιστώντας, αυξάνοντας ή ενισχύοντας την κινητικότητα σε διάφορες περιπτώσεις. Οι νόμοι της Ευρωπαϊκής Ένωσης δεν περιέχει ρητούς κανόνες για τα ρομπότ, αλλά υπάρχουν νομοθεσίες της ΕΕ που σχετίζονται με τις ρομποτικές συσκευές, οι οποίες ορίζονται σε δύο βασικές οδηγίες: τη Νομοθεσία Μηχανημάτων 2006/42/ΕΚ και τη Νομοθεσία Ιατροτεχνολογικών Προϊόντων 2017/ΕΚ. Η εφαρμογή ExosCE Rules περιγράφει μια προσπάθεια τυποποίησης, με υπολογιστικό τρόπο, των τμημάτων των Ευρωπαϊκών Οδηγιών που σχετίζονται με τους εξωσκελετούς, επεκτείνοντας την εργασία της προηγούμενης ενότητας σχετικά με την τυποποίησης της Νομοθεσίας των Ιατροτεχνολογικών Προϊόντων [47–49]. Εστιάζεται ειδικά στην περίπτωση των εξωσκελετών ως τύπου ιατροτεχνολογικού προϊόντος και ενσωματώνει τις σχετικές απαιτήσεις από την Νομοθεσία Μηχανημάτων, σχετικά με την ασφάλεια και την εμπορευσιμότητα τους. Από όσο γνωρίζουμε, δεν υπάρχει προηγούμενη εργασία σχετικά με την ανάπτυξη ενός υπολογιστικού συστήματος για τη συμμόρφωση με τη σήμανση CE των εξωσκελετών. Το ExosCE Rules αποσκοπεί στην ανάπτυξη ενός υπολογιστικού συστήματος για τη συμμόρφωση με τη σήμανση CE των εξωσκελετών και μπορεί να συμβάλει στην προσπάθεια ενοποίησης των παραπάνω νομικών πλαισίων σε μια προγραμματιστική μορφή, ως μέρος των προσπαθειών υπολογιστικής νομικής.

Τα παραπάνω νομικά συστήματα Τεχνητής Νοημοσύνης συμβάλουν στην ανάπτυξη υπολογιστικών κανόνων για τα βασικά μέρη νομοθεσιών ώστε να σχηματίσει μια βάση γνώσης καθώς και στη δημιουργία συστημάτων υπολογιστικής καθοδήγησης με λογικό προγραμματισμό, με στόχο να βοηθήσει τους εμπλεκόμενους στις διαδικασίες συμμόρφωσης και επικύρωσης.

Συμπεράσματα

Η απόδειξη απαιτεί διάλογο μεταξύ των δρώντων για να διευκρινιστούν τα σκοτεινά συμπεράσματα, να καλυφθούν κενά ή να αποκαλυφθούν σιωπηρές υποθέσεις σε μια αποδεικτική διαδικασία. Ως εκ τούτου, η επιχειρηματολογία είναι ένα αναπόσπαστο συστατικό της διαδικασίας ανακάλυψης για τη μαθηματική απόδειξη. Σε αυτή τη διατριβή παρουσιάστηκε πώς οι θεωρίες επιχειρηματολογίας μπορούν να εφαρμοστούν για να περιγράψουν συγκεκριμένα χαρακτηριστικά στην ανάπτυξη αποδεικτικών συμβάντων. Έτσι, παρουσιάσαμε τον Λογισμό Αποδεικτικών Συμβάντων βάσει Επιχειρημάτων που έχει σχεδιαστεί για να περιγράφει την εσωτερική και εξωτερική δομή της συνεργατικής μαθηματικής πρακτικής και υπογραμμίζει τη σχέση μεταξύ απόδειξης, ανθρώπινης λογικής και γνωστικών διαδικασιών. Αυτό μας δίνει τη δυνατότητα να μοντελοποιήσουμε αντικρουόμενα επιχειρήματα ή αναπόδεικτες κινήσεις, συμφωνίες και αντιφάσεις σε διαλόγους πολλαπλών δρώντων, την απόρριψη προηγουμένως αποδεκτών αποδείξεων, την κοινωνική συνεργασία μεταξύ αποδεικνυόντων και ερμηνευτών και ούτω καθεξής, πτυχές που συχνά δεν φαίνονται ή αγνοούνται στα παραδοσιακά μαθηματικά μοντέλα.

Η συνεισφορά της παρούσας εργασίας είναι ότι αυτός ο Λογισμός είναι τυπικός, πρακτικός και έχει την εκφραστική δύναμη να αναπαριστά μια πραγματική μαθηματική πρακτική πολλαπλών δρώντων ως μια διαδικασία συνεργατικής ανακάλυψης. Ο συνδυασμός της θεωρίας που βασίζεται σε αποδεικτικά συμβάντα και της λογικής επιχειρηματολογίας επιτρέπει την παρακολούθηση και των άτυπων πτυχές της μετάδοσης μαθηματικών πληροφοριών σε όλα τα στάδια της απόδειξης και καθιστά δυνατή την εμβάθυνση στη μικροδομή της διαδικασίας απόδειξης (όπως στην περίπτωση των Αποδεικτικών Συμβάντων Μηδενικής Γνώσης), καθώς και στην εξωτερική δομή της απόδειξης, αναδεικνύοντας τους κοινωνικούς ρόλους και τις αλληλεπιδράσεις των συντελεστών (όπως στις περιπτώσεις του τελευταίου θεωρήματος του Φερμά και του MiniPolymath4). Μια άλλη συνεισφορά της παρούσας εργασίας είναι ότι μπορεί να εκφράσει τον ταχέως εξελισσόμενο τομέα των ιατροτεχνολογικών συσκευών και των εξωσκελετών και τους σχετικούς κανονισμούς τους, έτσι ώστε το τρέχον νομικό πλαίσιο και οι μελλοντικές προκλήσεις να μπορούν να υλοποιηθούν και να ενσωματωθούν. Παρουσιάστηκε μια μοντελοποίηση των νομοθεσιών σχετικά με τη συμμόρφωση των ιατρικών συσκευών και των εξωσκελετών ως μέρος μιας λογικής βάσης γνώσεων που οδηγεί σε μοντέλα υπολογιστικής απόφασης στο Γοργία-Β και στην PSOA RuleML. Η προσέγγιση που αναπτύχθηκε σε συνδυασμό με τις περιπτώσεις χρήσης κατέδειξε την εφαρμοσιμότητα και την αποτελεσματικότητα των προτεινόμενων μεθοδολογιών απόδειξης που βασίζονται στη λογική, είτε άμεσα στον τομέα της τυπικής και άτυπης μαθηματικής ανακάλυψης είτε έμμεσα στον τομέα της νομικής συμμόρφωσης ιατροτεχνολογικών συσκευών και φορητών ρομπότ.

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Chapter 1

Introduction

Proving is often about validating the “truth” of a hypothesis made through arguments — called proofs — where each step of the argument follows the rules of logic. Proofs, however, exist everywhere — in maths, in the physical sciences, in computer science, in legal and ethical argumentation, in philosophy, and so on. In mathematical proofs the inferential arguments for the stated assumptions lead through the logical warrant to the conclusion. Proofs in computer sciences can be systems that prove properties of programs. An ethical proof can be an inference concluded from a sequence of commonly accepted arguments that can generally be considered credible. Legal proofs are reached on the grounds of acceptable evidence based on relevant regulatory frameworks. Thus, logical proving skills are needed in very diverse fields and types of applications, and can have a significant impact on the various procedures and the progress on these fields.

In this work, we approach the concept of proof from a perspective that is closer to the way human reasoning is done. Humans conducting reasoning may change their mind concerning a previous conclusion on a matter, if they are confronted with additional information. They do not necessarily obey the rules of “classical logic,” their knowledge can be incomplete and inconsistent and, therefore, new data can retract the conclusions drawn. Even though mathematical cognition is commonly presented as a procedure that leads to “truth” by applying logical rules of inference, proof discovery is a more complicated process full of obstacles and dead-ends that need to be overcome. Our goal is to present a model of mathematical discovery that depicts the connection between formal mathematics and its informal social and cognitive aspects.

We study the concept of proof understood as proof-events in the sense of Goguen [3] and its relation with argumentation (Chapter 2). Goguen suggested the broader concept of *proof-event* or *proving*, which is actually a social event that takes place in a specific place and time and involves public communication. The concept of proof-event is designed to embrace any proving activity, including purported, faulty, vague, disputed or incomplete proofs (Chapter 2.1). Additionally, studies in cognitive psychology have shown [2] that the dialectic nature of argumentation is similar to human reasoning. The argumentation-based approach can help its integration with wider forms of human reasoning such as dialogue, validation, debate, and morality — especially in incomplete and dynamically changing environments (Chapter 2.2). The versatility of human reasoning necessitates the combination of various techniques in order to model the process of common sense human reasoning [1]. In this work we focus on the exchange of arguments and counterarguments that takes place during proving. Therefore, proof-events are extended and represented in the form of a dialogue between agents that use arguments and counterarguments to check the validity of the steps of a purported proof.

This is a new approach in which techniques and theories of argumentation are used *to build a bridge between formal proof and informal social interaction in the search for proof*. To this end, in this thesis we study the interconnected areas of proof and logic through their various dynamically changing social environments and we propose appropriate methodologies for agent reasoning and proving (Chapter 3). The proposed approaches combine proof-events and logic-based argumentation theories in order to study in a more adequate way the informal and formal categories of proving processes, developed into two directions: the first concerns the mathematical proving processes, termed Argumentation-based Proof-Event Calculus (APEC) (Chapter 3.1); the second refers to ethical decision making processes, termed Moral Argumentation-based Proof-Event Calculus (MAPEC) (Chapter 3.2).

To demonstrate the applicability and effectiveness of the proposed logic-based methodologies, a number of case studies are conducted, either in the area of mathematical proof practices or in the area of legal and ethical compliance of medical devices and wearable robots.

APEC can be used in the formalization of interactive argument schemes to describe the mathematical problem-solving activity that faces eventual contradictions and dead-ends. This will facilitate collaborative mathematics environments where argumentation among mathematicians can be used to motivate creativity and discovery. Systems that support the formalization of mathematical knowledge need formal — computationally explicit — input. The current literature is opaque to such tools which cannot currently manage the variety of procedures normally involved in constructing proofs, especially when they contain informal mathematical dialogues with hypothesis, arguments, counterexamples, etc. [39]. Human reasoning can be well formalized through argumentation in formal systems in a way that enables its automation for developing Artificial Intelligence proving systems [50]. Chapter 4 illustrates the cases that implement APEC calculus in real-life scenarios from two perspectives: from the internal structure and steps of a mathematical proof, as indicated in the paradigm of Zero Knowledge Proofs (Section 4.1); and from a more external, social perspective of a multi-agent mathematical practice, as exemplified by the cases of crowd-sourcing Mini-Polymath 4 (Section 4.2) and Fermat's Last Theorem (Section 4.3).

Logic-based systems have been notably effective in Artificial Intelligence and computer science, since logic programming can provide explanations as audit trails of how a particular conclusion was proved. While non-logicist AI methods might be advantageous in certain frameworks, a logic-based approach can be more promising for engineering proving processes that can be implemented in cases such as mathematical proofs, negotiations, privacy, ethically correct robots, legal texts, medical decisions, etc., since in these cases we cannot afford to deploy AI systems that make unexpected decisions [51]. In the medical-related logic-based systems described in Chapter 5 any legal or ethical decision process that is complemented by the system can be understood, explained and re-enacted by humans. The following use cases are presented: WeaRED (Wearable Robots Ethical Data) introduces a scenario that formally engineers the ethically correct behavior of medical wearable robots (Section 5.1); AMeDC (Argumentation-based Medical Devices Classification) (Section 5.2.2) and Medical Devices Rules (Section 5.2.3) are legal-based decision-making models regarding Medical Devices Regulation; and ExosCE (Exoskeletons CE marking) presents the formalization of exoskelotons-related regulations (Section 5.3).

To sum up briefly, the rest of the thesis is structured as follows:

- In Chapter 2, we describe the fundamental concepts used in this work, proof-events theory and logic-based argumentation theories;
- In Chapter 3, we formally analyze and define the APEC and MAPEC calculus;
- In Chapter 4, we present the use cases on mathematical discovery and proving;
- In Chapter 5, we present the use cases on the ethical and legal process of proving, regarding medical devices and wearable robots; and
- In Chapter 6, we present the conclusions of the thesis.

Chapter 2

Theoretical Background and Prerequisites

Mathematical cognition is commonly presented as a process that leads to “truth” by applying logical rules of inference. Even though discovery, communication, and systematization are some elements that proof serves in mathematics [4], proof is often perceived mainly as a method for persuasion and validation [5]. Goguen’s [3] broader concept of **proof-event** or **proving** (understood as a public locatable and dateable social event concerning a communicated purported proof of a posed problem) is designed to embrace any proving activity, including faulty, vague, disputed, or incomplete proofs. Vandoulakis and Stefaneas [6] described proof-events as activities of a multi-agent system that incorporates the history of these activities in the form of sequences of proof-events. Our purpose is to bridge the gap between formal and informal mathematical procedures by constructing a model that is closely related to the way proving actually unfolds. The comparison between proof and argumentation is based on the perception that proof (including incomplete or even false proofs, valid or invalid inference steps, ideas, etc.) can be regarded as a specific kind of argumentative discourse in mathematics [7].

The concept of proof as a **discourse** and an activity agents engage in is explored in [22], where a *dialogical* account of mathematical proof is advanced to produce explanatory persuasion. The author develops a triadic conceptual scheme, consisting of the *producer* (the prover), the *receiver* (sceptic) and the *explanation itself* (the proof). In this scheme, the sceptic is mostly considered “silenced” [23, p. 91], while our intention was to focus equally on the moves and counterarguments generated by the other side in order to understand the whole mathematical practice more deeply and highlight the value of the opposite side in the proving process. Another study that focuses on the agents that produce the proofs is presented in [24]. In the approach adopted in this work, each mathematical step corresponds to a proof activity and the formal mathematical proof is a report of the corresponding proof activities. The plan of a mathematical proof is conceived as the plan of the agents who carried out the respective proof activity. Generally, the above-mentioned studies provide similar approaches of multi-agent mathematical discourse, but from a more philosophical perspective. We attempt to also provide a formal framework through a logic-based calculus to express the informal dialogues and the steps taken in mathematical practice.

Logic-based systems for examining and assessing arguments have been broadly applied, generating various formal methods for **argumentation-based reasoning** [26]. Argumentation theories can be used as a natural method of modeling non-monotonic reasoning, properly expressing its defeasible nature. For example, the Semantic Web is a really suitable domain for applying argumentation theories, since it is open and subject to incompleteness and inconsistencies by nature, and therefore can be used as a source of defeasible knowledge [52]. In the Semantic Web, such knowledge will also contain rules and logical constructs [52]. A starting point of this work is Pollock's [27, 28] approach to logical argumentation, which presented one of the first non-monotonic logics with concepts of argument and defeat. He also introduced *defeasible reasoning* where arguments are conceptualized as chains of reasoning that may lead to a conclusion, whereas additional information may destroy the chain of reasoning. The formalization developed in the present work is mainly a sequence-based realisation [29] of Dung's abstract argumentation framework [30], applying Pollock's [27] view of defeasible reasoning with the basic structure of Toulmin's model [8] for the representation of an argument. The present study aims to gain from, build on, and integrate the above approaches in a way that also uses insights from other works, such as Kakas and Michael [31], providing an abstract, theoretical exploration of logical argumentation applied principally to mathematical proving.

Many researchers tried to show that *the procedure by which mathematicians evaluate reasoning is similar to argumentation*, for example by adapting Toulmin's [8] argumentation model to mathematical examples. In Toulmin's model, an argument is constituted by six interrelated components: *claim, data, warrant, backing, rebuttal, and qualifier*. The first three elements are considered the substantial elements of applied arguments, whereas the last three are not always necessary. Aberdein [9–11] highlighted the use of arguments in mathematical conversations and practices. Pedemonte [12, 13, 7] implemented the ck¢-enriched Toulmin model to indicate connections between argumentation and proof. Götz Krummheuer [14] introduced the analysis of collective argumentation and participation using Toulmin's theory for the development of an interaction theory of mathematics learning. Christine Knipping and David Reid [15] built on Toulmin's theory to compare and describe global argumentation structures and local argumentation aiming for a deeper understanding of proving processes in the classroom. In [53], the full Toulmin scheme is implemented through three different warrant-types to model a wider range of argumentation. Metaxas et al. [16] presented methodologies to study the mathematical practice in a class involved in argumentative activities by integrating Toulmin's model and argumentation schemes.

Other approaches also indicate the connection between mathematical reasoning and argumentation. Eric Krabbe [17] presented informal mathematical proofs as arguments, applying the *pragma-dialectical* theory enriched by the theory of strategic maneuvering to identify the four stages of critical discussion in the proving process: confrontation, opening, argumentation, concluding stage [18]. Aberdein [11] highlighted the connection of mathematical reasoning with tools developed by the informal logician Douglas Walton to express argumentation schemes as a taxonomy of argumentation steps and dialogues as a contextualisation of formality through mathematical arguments. Lakatos' *"Proofs and Refutations"* [19] is also an enduring classic that highlights the role of dialogue between agents (a teacher and some students) at proof attempts as well as critiques of these attempts. The work in [20] provides a way of formalizing social aspects of proofs by interpreting the informal logic of a Lakatos-based mathematical discovery through the lens of argumentative dialogue. Studies in the discovery of mathematics have also used the concept of "collective argumentation" to examine in particular the mathematical characteristics of dialogues, as various mathematicians/agents work together to prove a claim [21].

All these studies emphasize the relationship and continuity between reasoning in argumentation and in proving. Our approach attempts to elucidate this intrinsic relationship by modeling the argumentative dialogue between justifications and explanations offered by mathematicians during their proving activity. Argumentation is a potent reasoning tool that allows contributors in the dialogue to argue and counter-argue, assert and refute, validate and invalidate steps of mathematical reasoning that aim to solve a posed problem. This leads to a deeper understanding of the often contradictory visions, perspectives, and problem-solving strategies of the contributors to a problem that ultimately concludes in agreement and consensus [25].

In the following sections, a brief overview of proof-event calculus and argumentation theory is provided to proceed smoothly in their integration and formalization.

2.1 A Brief Review of Proof-Event Calculus

The notion of "proof-event" or "proving" was introduced by Joseph Goguen and it was conceived as a general notion covering all the different kinds of proof, such as constructive, non-constructive, apodeictic, dialectical proofs, proof steps, computer proofs, etc. [54]. In his exact words: *"A proof-event minimally involves a person having the relevant background and interest, and some mediating physical objects, such as spoken words, hand written formulae, 3D models, printed words, diagrams, or formulae. None of these mediating signs can be a 'proof' in itself, because it must be interpreted in order to come alive as a proof-event; we will call them proof objects. Proof interpretation often requires constructing intermediate proof objects and/or clarifying or correcting existing proof objects. The minimal case of a single prover is perhaps the most common, but it is difficult to study, and moreover, groups of two or more provers discussing proofs are surprisingly common"* [3]. Goguen presented the idea of proof-event, aimed to cover all exemplifications of proof as well as proof steps and computer proofs. From his perspective, the idea of proof-event is more comprehensive and less formal than that of purely mathematical logic, since it includes not only formal proof methods and steps, but also includes intention for the proof and its significant steps and the complete structure of the proof, involving conflict and other narrative devices [3].

Proof-events are not equivalent to mathematical truths since a proof-event may refer to an incomplete proof, an outline of a proof, or even a proof-less expression of considerations referring to a specific problem. The prover may experience an inspiration (*intention*) in a particular mathematical *problem* and initiate a proof-event to *communicate* his experience [54]. Agents act with intention and their attempt is goal-oriented. The goal of a prover might be to solve a particular problem and the goal of an interpreter might be to understand the argumentation suggested to this problem [54].

A sequence of proof-events — "fluent" — is finalized when the agents involved in them conclude that they have understood the proof and validate that a proof has actually been given, meaning that the proof is a fact. History of mathematical proofs has shown many cases where various agents (mathematicians or not) added value with their attempts, assumptions, proof steps, or even false steps in the sequence of proof-events. In some instances, mathematical proofs evolved for many years until they reached the desired outcome, as in the famous cases of Hilbert's problems, Poincaré conjecture, Lobachevsky's geometry, Riemann's hypothesis, Fermat's theorem, etc.

Vandoulakis and Stefaneas [6] describe proof-events as the activities of a multi-agent system incorporating the history of these activities to create sequences of proof-events in terms of fluents. In Proof-Event Calculus (PEC) [54], certain temporal aspects of proof-events were modeled using the language of the calculus of events inspired by Kowalski's Event Calculus (EC) [55]. The logic of agents taking part in a proof-event is also modeled in terms of Kolmogorov's calculus of problems [56]. The semantics in terms of Kolmogorov's calculus is analogous to the notion of proof not as a completed abstract entity, but as a sequence of actions. Thus, the calculus of problems is appropriate and provides the desired semantics (i.e., "loose" semantics as informal explanation of intuitionistic logic) for the calculus of proof-events, based on the notions of "problem" and "solution to a problem," rather on the notion of "truth" [54]. Therefore, both are suitable formalization tools that enable developing computational interpretations of the procedure of proving [54].

PEC has types of proof-events (e) whose instantiations mark the time-dependent properties and a set of fluent constants (f) that depict the various properties in the problem domain. The definitions of proof-event and fluent, as described in [54, 6], are presented below.

Definition 2.1.1. Proof-event

Proof-event e is a proof instance that take place in space and time, it refers to a specific problem, and it is specified by certain conditions (predicates). A proof-event e has the following internal structure:

$$e = < \mathit{communicate}(Intention, Problem), t >$$

which means that an intention (mathematical argument, assumption, idea, etc.) is linguistically articulated at time t for a (time-independent) problem [54].

Definition 2.1.2. Fluent

Fluent f is a sequence of proof-events e evolving in time that refers to a specific problem. A fluent is a function that may be interpreted in a model as a set of time points t_n $n = 1,2,3$, conventionally denoting the time when the communication output is available. Hence, fluents have "initial" and "terminal" points, i.e., they are extended spatially and temporally.

$$f = \{\ e_1, e_2, \dots e_i\ \}, \text{ where } e_i = < \mathit{communicate}(Intention_i, Problem), t_i >, \text{ for every } 1 \leq i < n, t_i < t_{i+1}, n \in \mathbb{N}, .$$

Thus, the underlying ontology contains (types of) proof-events, fluents, and time points [54]. The study involves how fluents change when a new proof-event is acquired and how this view of the problem world is affected by the examination of some instances holding or not at a specific time [57]. The main purpose of the reasoning is to keep, usually in case of insufficient information, a precise view of the problem domain as events happen and/or are perceived with the passage of time [57].

The fluent is subject to change over time, depending on the contribution and value of the individual proof-events. The temporal predicates for modeling this change are:

$$\begin{gathered} Happens(e,t) \\ Initiates(e,f,t) \\ Clipped(e,f,t_2) \\ Terminates(e,f,t) \\ ActiveAt(f,t) \end{gathered}$$

The characteristics of these temporal predicates are described in more details in Subsection 3.1.3.

Our purpose is to bridge this gap between formal and informal mathematical procedures by devising a modeling calculus that is closely related to the way proving is actually done through argumentative interaction, communication and debate between the agents.

2.2 A Brief Review of Argumentation Theory

The versatility of human reasoning clarifies that any attempts to model the process of common-sense human reasoning combines different techniques [1]. The computational study of argumentation theory was introduced with works such as Dung's [30], Vreeswijk's [58], and Pollock's [27, 28], approaches that can still be considered as state-of-the-art. The aim of the present study is to gain from, build on, and integrate the above approaches in a way that also uses insights from other works, such as Toulmin [8] and Kakas & Michael [31], providing an abstract, theoretical exploration of logical argumentation applied principally on mathematical proving.

Specifically, one of the starting points of this work was Pollock's [27, 28] approach to logical argumentation, who presented one of the first non-monotonic logics with notions of argument and defeat, even though he did not explicitly distinguish between them [59]. Pollock also pointed out that the significance of inductive reasoning should be regarded as equally important to deductive reasoning in philosophy and Artificial Intelligence. The argumentation-based formalization developed in this work is mainly a sequent-based (see, e.g. [29]) realization of Dung's abstract argumentation framework [60, 61], applying the basic structure of Toulmin's model for the representation of an argument and Vreeswijk' s view of defeasible reasoning. Based on the approach in Kakas et al. [34], the argumentation framework is built in terms of logic programming rules expressing a priority relation among them. This combination of theories opens up the possibility of extending the utilization of argumentation from fixed problems to alterations of these, where, as soon as new information becomes available, the environment of the problem is dynamically changing, which is often the case in mathematical proofs.

Argumentation models generally contain the following main elements: an underlying logical language with the definition of the concepts of argument, the status of argument, and conflicts between arguments and counterarguments. Logical argumentation is a logic-based approach for formalizing arguments and counterarguments expressed in terms of formal languages as well as entailment relations for drawing claim in the proving process [62, 28, 59]. Formalization of argumentation, as introduced by Dung [30], provides a good starting point where arguments and counterarguments are ordered in a binary relation (of attack) and can be depicted by a directed graph [9, 63]. The definitions given thereunder outline some of the fundamental concepts behind logical argumentation.

Definition 2.2.1. Argumentation Framework [30]
An argumentation framework is a pair $AF =< Args, A >$ where $Args$ is an enumerable set of elements that are called arguments and A is a binary relation on $Args \times Args$ the instances of which are called "attacks."

An argument has premises, inference rules, and a conclusion. The method of inference by which a claim follows from a set of formulae is deductive inference and is denoted by $\vdash$. The definition of a deductive argument is given below.

Definition 2.2.2. Deductive Argument
A deductive argument is an ordered pair $< \Phi, \alpha >$, where $\Phi \vdash_i \alpha$ is the support, or premises, or assumptions of the argument, and α is the claim or conclusion of the argument. The definition for a deductive argument only assumes that the premises entail the claim (i.e. $\Phi \vdash_i \alpha$). For an argument $A =< \Phi, \alpha >$ the $Support(A)$ function returns Φ and the $Claim(A)$ function returns α.

Important benefits of deductive arguments are the explicit representation of the claim (and of the information used to support it) as well as a consequence relation to connect simply and precisely the support and claim of the argument. What a deductive argument does not provide is a specific proof of the claim from the premises. There may be more than one ways (warrants) to prove the conclusion from the premises, but the argument does not determine which way is used [9].

The possible different kinds of arguments can either support a claim c or attack it. Given a claim c and an argument, possible argument moves which provide support for c [64] include:

Equivalent: an argument for a claim, which is equivalent to (or is) c;

Elaboration: an argument for an elaboration of c.

Argument moves, which oppose c (rebutting, undercutting as inspired by Polock [28] and undermining as inspired by Vreeswijk's [58]) include:

Rebutting: an argument for a claim which attacks the claim α of the e.

Undermining: an argument for a claim which attacks a premise ϕ of e.

Undercutting: an argument for a claim which attacks an inference rule w of e.

The following chapter outlines the formalization of proof-events based on argumentation theory, integrating features from both theories, resulting in Argumentation-based Proof-Event Calculus (APEC). APEC can be used in collaborative mathematics environments where argumentation among mathematicians can be used to motivate creativity and discovery. This will facilitate the formalization of interactive argument schemes to describe the mathematical problem-solving activity that faces eventual contradictions and dead-ends.

Chapter 3

Argumentation-based proof-event Calculus Theory

3.1 Argumentation-based proof-event Calculus

This chapter highlights the association of the procedure of proving with human reasoning, to present a new approach in which techniques and theories from argumentation can be used to build a bridge between formal and informal proof attempts of human or AI agents.

We are going to present the **Argumentation-based Proof-Event Calculus (APEC)** that combines proof-event calculus [54] and logic-based argumentation theories to study more adequately informal and formal aspects of proving. The concept of proof is understood in terms of proof-events in the sense of Goguen [3] as presented in [65]. Furthermore, proof-events are represented in the form of a dialogue between agents that use arguments and counterarguments to check the validity of the steps of a purported proof. APEC can be used in collaborative mathematical environments where argumentation among mathematicians can be used to motivate creativity and discovery or elucidate obscure points of a purported proof. APEC facilitates the formalization of interactive argument schemes to describe mathematical problem-solving activity that faces eventual contradictions and dead-ends.

Our approach is novel because we use techniques of argumentation theories *to build a bridge between a formal proof and the informal social interaction aspects involved in the search for proof.* Various researchers have shown that the role of argumentation is crucial in mathematics [9, 25, 66, 12] by adapting argumentation models, such as Toulmin's [8] model, and comparing them with the structural components of a proof. However, there has been criticism that sometimes the argument structure of Toulmin's model does not take into account the exchange of ideas between participants and thereby the justification is partial and ambiguous [13]. Our goal is to supplement the concept of arguments with the argument moves of the participants that support or attack an assumption. This is done in the wider framework of proof-events that takes into consideration not only formally validated proofs, but also informal thinking that include trials, choice of strategies, and/or possible validation or rejection of parts of a purported proof by the agents.

Pedemonte and Balacheff suggested the so-called ckȼ-enriched Toulmin model described in [13] that captures the internal characteristics of the argument-proof structure. However, we also wanted to express the external procedures in the practice of various participants. The APEC system can *represent the complete information and sequence of steps in the evolution of mathematical practice* which is modeled in the form of logic-based dialogues (informal external procedures) with argument moves, temporal predicates, and validation levels of argumentation. At the final stage, proof may be accepted as completed, i.e. as a valid formal proof understood and recognized as true by all relevant agents. This approach enables us to examine more deeply the interplay between proof, human reasoning, cognitive processes and creativity in the mathematicians' practice.

Several studies highlight the educational aspects of argumentation and proof [12, 7, 67, 14, 53, 15] and student interaction in the classroom. Even though our model can also be implemented for concept-learning and problem-solving for the sake of students, in this chapter, we focus on *modeling a broader perspective of the collaborative discovery process in the practice of real mathematical communities.* This context can be applied to the communication between mathematicians in a research environment where collaboration between them is essential and can lead to significant results, such as the case of mathematical practice in crowd-sourcing collaborative environments. Online dialogues can be used as a rich source of argumentation repositories as this data is in its purest form and provides information on how argumentation works in real-life dialogues [68].

Other related studies that analyze original mathematical dialogues from the perspective of argumentation are:

- the so-called *mixed-initiative collaborative proving* in [20], a way of formalizing social aspects of proofs by interpreting the informal logic of a Lakatos-based mathematical discovery;
- the analysis of Mini-Polymath 3 by Alison Pease & Ursula Martin [69]; and

- the modeling of mathematical dialogues with the Inference Anchoring Theory + Content (IATC) framework by Corneli et al. [39].

The approach in [20] implements many different predicates trying to provide a well-defined formal presentation. On the other hand, the study in [69] uses a simple typology of comments categorized as concepts, examples, conjectures, or proofs, and it can be used mainly as a description of online collaborative mathematics rather than a formal representation (which can also be used computationally). The work in [39] uses predicates that are descriptive of the procedure and can be interpreted in widely different and subjective ways (e.g., how can we define specifically concepts such as "helpful," "beautiful," "goal," "strategy," etc.?). In our paper we choose a different approach through the more general meta-methodological framework, which involves the theory of proof-events that incorporates both proofs and arguments. We do not attempt to tag an interpretation or a description in the procedure steps, but to depict the complete proving practice and its social interactions as formally as possible. Furthermore, our approach highlights explicitly the *argument moves* that the agents implement, as well as the sequence of the steps, not only in a "temporal" manner (with the *temporal predicates*) but also in a "progressive" manner (with the *levels of argumentation*) until the ultimately validated or invalidated outcome.

In addition, studies [20, 69, 39] develop computational systems to demonstrate how each of the formal steps is available for implementation. The APEC method can also develop a computational format of a collaborative proving activity. We believe that APEC does not need the selection of one particular system in order to be used computationally, given that the selected system is based on logic programming and has the minimum functionality required (i.e., a formal syntax, semantics, induction, recursion, and queries as to whether something is provable or not). In the W3C (World Wide Web Consortium) it was suggested that semantic descriptions of Web services can take the form of rules, e.g. by using RuleML-serialized logic programming languages, to formally characterize service concepts and descriptions [52]. To briefly indicate the computational applicability of APEC, we create a proof-of-concept of Mini-Polymath 4 both in the GorgiasB system, a Prolog-based structured argumentation framework of Logic Programming [70], and in PSOA RuleML, a logic-based language that introduces positional-slotted, object-applicative terms in generalized rules [71] (Appendix .1). However, the computational development of this framework is beyond the purpose of this chapter.

Thus, the objectives of this calculus are:

- to examine from a social, scientific, and cognitive perspective the common nature of arguments and proof-events and to show the relationship between the process of advancing an argument and advancing a proof;
- to develop an APEC model to represent the "proving" procedure with argument schemes, highlighting key elements such as agents' contributions (argument moves), sequences of proof-events (temporal predicates), and validation progress (levels of argumentation);
- to show the impact of the (possibly virtual) mathematical environment on the development of arguments to attain proof;
- to illustrate the usability of the proposed approach (as a theoretical framework but also as a computational model) in different use cases.

3.1.1 APEC as a tool for formalizing reasoning and collaborative proving

According to Lockhart [32], *"[Mathematics] is not in the 'truth' but in the explanation, the argument."* Argumentation-based proof-events can be used to advance mathematical dialogue in which all participants collaborate to critically examine posed problems and enhance thinking abilities such as problem solving, interpretation, persuasion, and creativity. Hence, the goal of mathematics interaction is no longer the cultivation of individual problem-solving skills, but the development of "collaborative problem-solving capacities" [33].

Some of the questions that we address are:

1. *The relationship between informal proving and formal proof in real mathematical practice and communication.*
2. *The relationship between argumentative and mathematical proving activities.*
3. *The relationship between the contributions of working mathematicians and a mathematical proof as final output.*

Cognitive science has shown [2] that the dialogical nature of argumentation is similar to **human reasoning in proving**. Humans conducting reasoning do not necessarily follow the rules of "logic" [34]. They may change their mind concerning a previous conclusion on a matter if they are confronted with additional information.

Their knowledge can be incomplete and inconsistent and, therefore, new data can invalidate any conclusions drawn [31]. However, it is often the case that a proof output presented in its pure form overshadows the informal and social aspects of the proving process that led to it [72]. Argumentation-based approaches can help their integration within wider frameworks of human reasoning — such as dialogue, debate, validation, and proving — especially in dynamic environments such as real-life mathematical environments. The model can depict the dialogues betweeb prover(s) and interpreter(s) in a multi-agent system, expressing both the internal structure of their arguments, and therefore their cognitive thinking, as well as the external social interactions with the argumentation moves.

We investigate mathematical proof steps as argumentation-based proof-events to elucidate the **creative characteristics of argumentation** that are important in proving, such as negotiation, collaboration, and fruitful mistakes. This approach enables us to examine the kinds of reasoning agents may use in their interaction and how the dialectical activity may influence them to generate new arguments as they move from the assumptions of a problem to its proof [21]. Argumentation allows the contributors to engage in dialogue in the course of their problem-solving activity to test alternative proving strategies, check a suggested argument or idea or a (part of a) purported proof until they ultimately reach agreement [25]. This perspective can reshape mathematical discovery into an interactive, negotiable, social process.

Although a new proof is usually attributed to the solver of the problem, it is the outcome of joint efforts of different agents each of whom has different past experiences, background knowledge, proving skills, and vision of the problem [35]. Take, for instance, Fermat's Last Theorem which mathematicians had been attempting to prove for over three centuries, until it was finally proved by Andrew Wiles [36] in 1994 (after 357 years). Thus, Wiles' proof was the outcome of many generations of mathematicians and their suggested proofs, which sometimes contained deficiencies and flaws [36]. We suggest a model for mathematics learning, where problem-solving is viewed as a **collaborative discovery proof-event** [36, 37]. The system represents in different levels of proof-events all the history of discovery and formalizes it in the form of collaborative argumentative contributions that includes trials, conflicts, and possible validation or termination of parts of purported proofs [38]. In the final step, the formal proof is checked, understood, and confirmed by the relevant mathematical community to be recognized as valid.

Comparison of the basic elements of proof-events and argumentation theory shows similarities in structure, sequence, and the agents.

1. Arguments and proof-events have three common fundamental components: a set of premises for a task or problem (i.e., premises ϕ in proof-events and data in arguments), a method of reasoning (i.e., warrant w in proof-events and inference rules in arguments), and a conclusion (i.e., conclusion c in proof-events and claim a in arguments).

2. What is set to be proved emerges out of the history of events, which can be sequences of proof-events (fluents) or sequences of arguments and counterarguments [6]. A sequence of proof-events is complete when the community involved in it concludes that they have understood the proof and agree either that a proof has actually been given or that a proof is invalid, based on a suggested counterargument or counterexample.

3. Argumentation involves agents or groups of agents, enacting the roles of supporter and opponent of an argument [73], enabling its adoption as a technology for multi-agent systems developments. Similarly, proof-events necessitate the existence of at least two agents: a prover (the agent providing the proof) and an interpreter (the agent checking the validity of the proof) [6].

The main concept advanced in agent-based approaches is that of autonomy: agents operate as independent individual entities trying often to collaborate and coordinate with others [34]. This approach suggests a multi-agent system, enacting the roles of provers and interpreters [74], who generate sequences of proof-events with arguments and counterarguments. However, the steps that an individual agent wants to perform in order to accomplish a mathematical proof may interfere with the steps attempted or already performed by other agents.

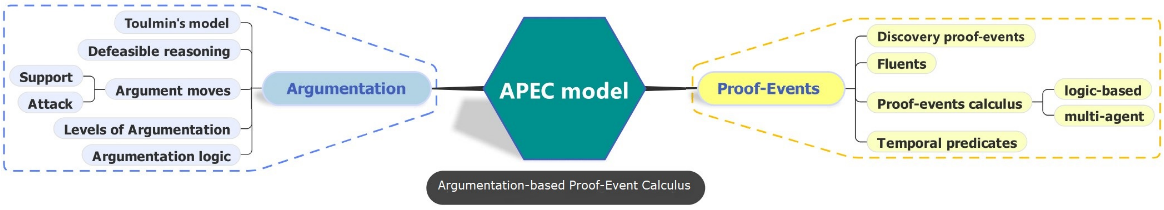


Figure 3.1 Proof-events and argumentation contributions in APEC.

Why is a calculus for an argumentative model necessary? There is a gap in the literature about tools that can provide formal — computationally explicit — input that can manage the variety of procedures normally involved in constructing proofs, especially when they contain informal mathematical dialogues with hypotheses, arguments, counterexamples, etc. [39].

Our contribution concerns the elaboration of an analytic framework that provides a tool to describe and assess mathematical proving based on formal structure, agent contributions, argumentation reasoning, and sequence of arguments. The above features constitute different categories of argumentation frameworks [40] integrated in one framework. The developed calculus bases the foundations of the justification on a core internal structure of *premises-warrants-conclusion*. Then, it proceeds with the number and the kind of argument moves (supporting or attacking) necessary to build the different levels of argumentation that can describe the external interactions. The levels of argumentation can progress from unjustified claims (lower levels) to incontrovertibly valid proofs (higher levels). Therefore, one can track the progression in creativity, rigor, and validity of argumentation offered by mathematicians (who can be either human agents or intelligent software agents) and include the informal steps in a formal analytical framework. This type of proof-theoretical approach applied in formal argumentation frameworks can have noticeable advantages [31]. For instance, a well-studied argument-based calculus may be implemented for analyzing or generating arguments in a semi-automated or automated way [73], or combined with crowd-sourcing environments for creating human-machine hybrid teams [75].

3.1.2 Internal Structure of proofs with APEC

For the internal structure, we use Argumentation-based proof-events to identify the data, warrant, and claim parts of an argument which are involved in the proving process. An example of the proof of the Pythagorean Theorem is used to better illustrate the applicability of these predicates.

Definition 3.1.1. Argumentation-based proof-event
An argumentation-based proof-event e can be represented as a communicated argument $\langle \Phi, c \rangle$ [28] designated by the pair $e\langle \Phi, c \rangle$ as $e = \langle communicate\langle \Phi, c \rangle, w \rangle$, where Φ represents the premises of the argument based on the available *data*, c is the *claim* that refers to the conclusion of a particular problem communicated by the agent, and

w are the inference rules or *warrant*[1] which consists in the inference rules that allow Φ to be connected with c, so that:

- $\Phi \nvdash \bot$
- $\Phi \vdash c$
- There is no $\Phi' \subset \Phi$ such that $\Phi' \vdash c$

where:
claim c: the statement/conclusion communicated by the agent,
data Φ: premises as the ground of the claim,
warrant w: the inference rules that connect the data to the claim.

Counterarguments are represented by the corresponding pair $e^*\langle \Psi, \beta \rangle$, where Ψ is the premises on which the claim β of the counterargument is based. We use three different kinds of argument moves (rebutting, undermining, and undercutting) as counterarguments (as defined in Subsection 3.1.3).

Argumentation may require chains or trees of reasoning, where claims are used in the assumptions to obtain further claims [62], so that a proof-event could be an atomic argument or a sequence of arguments. Sequences of proof-events expressed with fluents in the calculus of proof-events [54] describe their temporal history and the interactions of the agents participating in the proof-event and, henceforth, they are useful for depicting logical arguments and counterarguments.

Definition 3.1.2. Fluent of arguments in a proof-event
A fluent f is a formula of the form $e_1, e_2, \ldots, e_n \to e$, $n \in \mathbb{N}$, where $e_1\langle \Phi_1, c_1 \rangle$, $e_2\langle \Phi_2, c_2 \rangle$, $\ldots$, $e_n\langle \Phi_n, c_n \rangle$ is a finite, possibly empty, sequence of arguments, where the conclusion of the proof-event e_i is the claim c_i, i.e., $concl(e_i) \equiv c_i$, for some rule $c_1, c_2, \ldots, c_n \to$c [58]. Accordingly, the meaning of the finite substantial components of the argument [8] — which are abbreviated by corresponding prefixes — are defined as follows for the notion of a fluent:

claim: $concl(e) = concl(e_1) \cap concl(e_2) \cap \ldots \cap concl(e_n) \equiv c = c_1 \cap c_2 \cap \ldots \cap c_n$

---

[1] Since in this calculus we draw an analogy between argumentation and proving, a *warrant* is an assumption that links the data to the claim, in the same way that inference rules link the premises to the conclusion in a mathematical proof. There can be different warrants leading to the same claim, as in mathematics there can be different inferences rules that lead to the same conclusion (proof), e.g., the different proofs of the Pythagorean Theorem.

data: $prem(e) = prem(e_1) \cup prem(e_2) \cup \ldots \cup prem(e_n) \equiv \Phi_1 \cup \Phi_2 \cup \ldots \cup \Phi_n$

warrant: $infRul(e) = infRul(e_1) \cup infRul(e_2) \cup \ldots \cup infRul(e_n) \equiv w = w_1 \cup w_2 \cup \ldots \cup w_n$

A fluent contains all the necessary arguments/proving steps required to prove the desired conclusion. Therefore, the conclusion of the initial proof-event e can include the conclusions of the proof-events e_n contained in the fluent. For example, a proof may presuppose the proof of some of its subsections. Every contributing step in this procedure can be contained in a fluent. By this, we do not mean only completely correct steps, but also incomplete or faulty steps that can act as a starting point for another proving step.

Let's examine as an illustrative example a proof of the Pythagorean Theorem, according to which in a right angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides, stated as $a^2 + b^2 = c^2$.

The Pythagorean Theorem can be depicted as:

$$e_{Pythagorean} = < communicate < \Phi_{RigAngTriangle}, c_{Pythagorean} >, w_{Euclid} >$$

where:

$e_{Pythagorean}$ is the proof-event that refers to the proving process of the Pythagorean theorem,

$\Phi_{rigAngTriangle}$ is the data that are used as premises, i.e., specific cases of right angled triangles where the Pythagorean Theorem is valid,

$c_{Pythagorean}$ is the conclusion of the Pythagorean Theorem (i.e., $a^2 + b^2 = c^2$), and

w_{Euclid} is the inference rules used in order to prove the conclusion of the theorem, which in our example is Euclid's proof.

In Figure 3.2, the premise of this problem is the right angled triangle $\triangle ABC$: $prem(e_{Pythagorean}) = prem(\triangle ABC)$.

Euclid constructed squares BCED, ABFG, and ACKH from the sides of the right triangle $\triangle ABC$ and sought to prove that the area of BCED was equal to the sum of the areas ABFG and ACKH. With APEC we can depict the step included in this proof procedure. We want to show the claim that:

$$concl(e_{Pythagorean}) = concl(e_1) \cap concl(e_2), \text{ where:}$$

$concl(e_1) \equiv c_1 : Area_{ACFG} = Area_{CELM}$ and

$concl(e_2) \equiv c_2 : Area_{ACKH} = Area_{BMLD}$.

The warrant for proving the $concl(e_1)$ includes the following steps.

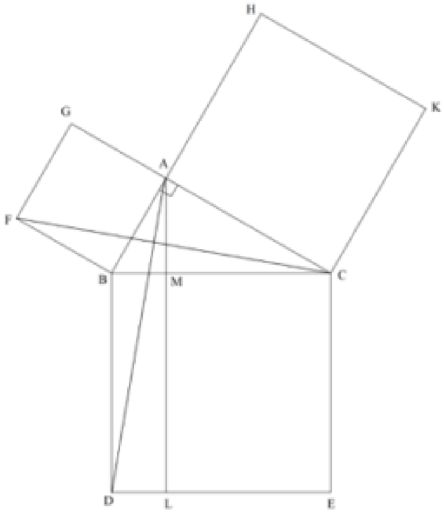


Figure 3.2 Euclid's proof of the Pythagorean Theorem.

$$infRul(e_1) = infRul(e_{1a}) \cup infRul(e_{1b}) \cup infRul(e_{1c}), \text{ where:}$$

$infRul(e_1a) : \triangle BCK = \triangle ACE,$
$infRul(e_1b) : Area_{BCK} = \frac{1}{2}\ Area_{ACKH},$
$infRul(e_1c) : Area_{ACE} = \frac{1}{2}\ Area_{CELM}.$
Similarly, we have the infurence rules for e_2:
$infRul(e_1) = infRul(e_{1a}) \cup infRul(e_{1b}) \cup infRul(e_{1c}).$
Combining the above proof-events, the warrant of Euclid's proof is $w_{Euclid} = infRul(e_1) \cap infRul(e_2)$.

The Pythagorean theorem may have more known proofs than any other (there are 370 proofs of the Theorem in [76]), thus, there can be many different warrants in addition to w_{Euclid} that lead to the same conclusion, to the same proof (e.g., geometric proofs, proof by rearrangement, algebraic proofs).

This example illustrated the internal structure of proof-events in a proving process. The next section presents how APEC can formalize the external relation of proof-events that communicate and conflict during proving processes.

3.1.3 External relationships of proofs with APEC

The steps that an individual agent performs to accomplish a mathematical proof may overlap with the steps attempted or already performed by other agents. In order to express the social interactions and the progress of the proof-events in terms of time and validation, we need to define more formal tools as described in the subsections below.

Argument moves

In the course of a proof procedure, there can be various inference stages, such as attempts, impasses, confirmed or unconfirmed steps, false suggestions or implicit assumptions, intuitive ideas, intentions, etc. Arguments can then be specified as chains of reasoning leading to a conclusion with consideration of possible counterarguments at each step. With the explicit construction of the chain of reasoning (a chain $x_0, x_1, .., x_n$ where the argument x_i attacks the argument x_{i-1} for $i > 0$), various concepts of defeat can be conceptualized. When an agent has gained control of an argument, they must select which argument move to apply. Gordon [77] refered to "argument moves" as analogues of three roles for legal cases. This term was also previously used by Rissland [78], Asley and Aleven [79], Pease et al [80]. Here, the term "argument moves" is reserved for specific, active tactics, or strategies among which a prover can choose to support his claim. Five fundamental relations are used, that indicate links and conflicts at the sequence of proof-events. The possible argument moves — communicated during the proof-events sequences — can provide support (equivalent, elaboration) or attack (rebutting, undercutting, undermining) to the claim.

Argument moves that support the claim:

A proof-event $e\langle\Phi, c\rangle$ is equivalent to a proof-event $e'\langle\Phi', c'\rangle$, whenever it has the same premises and the same conclusion (although they may have different warrants). Thus, equivalent proof-events can have different ways of proving. For instance, numerous proofs have been offered for the Pythagorean Theorem, including a geometrical proof by Euclid and an algebraic proof by James Abram Garfield. Thus,

$$Equivalent(e, e') : e\langle\Phi, c\rangle = e'\langle\Phi', c'\rangle,$$

when $\Phi = \Phi', c = c'$ (and it might be $w \neq w'$).

A proof-event $e\langle\Phi, c\rangle$, can have a set of inference rules S of e' which elaborate or embellish upon e, iff $\Phi \cup S \vdash c$. Thus,

$$Elaboration(e, e') : sent(e) \cap sent(S_{e'}) \rightarrow concl(e).$$

These moves are used for backing our claim and supporting our proof, therefore:

$$support(e, e') \rightarrow Equivalent(e, e') \cup Elaboration(e, e').$$

Counterargument moves that attack the claim:

A counterargument communicated during the proof-event $e^* < \Phi, \beta >$ rebuts (attacks) the conclusion of an argument communicated during the proof-event $e < \Phi, c >$, if and only if $\vdash \beta \leftrightarrow \neg c$. Thus,

$$Rebutting(e^*, e) : rebut(e^*, e) \to \neg concl(e).$$

A counterargument communicated during the proof-event $e^* < \Phi, \beta >$ undermines (attacks) some of the premises (defeasible inference) of the argument communicated during the proof-event $e < \Phi, c >$, if and only if $\vdash \beta \leftrightarrow \neg(\cap W_i)$, for some $w_1, .., w_n \subset W$. Thus,

$$Undermining(e^*, e) : undermin(e^*, e) \to \neg prem(e).$$

A counterargument communicated during the proof-event $e^* < \Phi, \beta >$ undercuts (attacks) some of the inference rules (defeasible inference) of the argument communicated during the proof-event $e < \Phi, c >$, if and only if $\vdash \beta \leftrightarrow \neg(\cap \Phi_i)$, for some $\Phi_1, .., \Phi_n \subset \Phi$. Thus,

$$Undercutting(e^*, e) : undercut(e^*, e) \to \neg infRul(e).$$

Given an argument communicated during the proof-event $e < \Phi, c >$, a counterargument communicated during the proof-event $e^* < \Phi, \beta >$ attacks the argument communicated during the proof-event e, if and only if e^* rebuts e or e^* undercuts e. Therefore:

$$attack(e^*, e) \to Rebutting(e^*, e) \cup Undercutting(e^*, e) \cup Undermining(e^*, e)$$

Temporal predicates

Even though proof-events can be regarded as taking place instantaneously, EC is actually neutral with respect to whether events have duration or are instantaneous [81]. Thus, for the duration of proof-events, the perspective of Reasoning about Actions and Change (RAC) is used which concerns how fluents change when new information is acquired and how this view of the problem is affected by the observation of some events remaining active or terminating at a particular time [82]. RAC [83] uses causal propositions (c-propositions), of the form ‘A initiates F when C’ or ‘A terminates F when C’, which here are represented in a more specific and detailed form through the moves of arguments and counterarguments that initiate or terminate a fluent. In most cases, only the starting point of a proof-event will be taken into consideration, with the exception of those proof-events that terminate or whose duration plays a significant role, in which case both the starting and the termination point will be mentioned.

Here, the above-mentioned operators are combined with the basic temporal predicates from [54]:
$Happens(e, t), Initiates(e, f, t), Terminates(e, f, t), ActiveAt(e, f, t), Clipped(e, f, t_2.)$

The purpose of using the language of event calculus in describing proof-events is to express the progress of the sequences of proof-events in terms of fluents. In cases where we need to give an extra focus to the time duration of fluents, we can also include the time variable in the $support(e, e', t)$ and $attack(e^*, e, t)$ predicates. In these cases, the proof-event e can be omitted, if it is easily implied by the corresponding supporting and/or attacking arguments (i.e., $support(e', t)$ and $attack(e^*, t)$). The temporal predicates are formalized as in the following relations.

$Happens(e, t)$, which means that a proof-event e occurs at time t.

$Initiates(e, f, t_1) : happens(e, t_1) \rightarrow \neg attack(e^*, t_1) \cup support(e', t_1)$, at time t_1,

which means that, if a proof-event e occurs at time t, then there are no counterarguments e^* that attack the validity of the outcome of the proof-event and there is adequate support for our claim at the specific time t_1.

$$Clipped(e_1, f, t_2) : \exists e_1, e_1^*, t_1, t_2, t[Happens(e, t_1) \cap (t_1 \leq t < t_2) \cap attack(e_1^*, t)] \cap [\nexists e_2(Happens(e_2, t_2) \rightarrow \neg attack(e_1^*, t))], \text{ for } t_1 \leq t < t_2$$

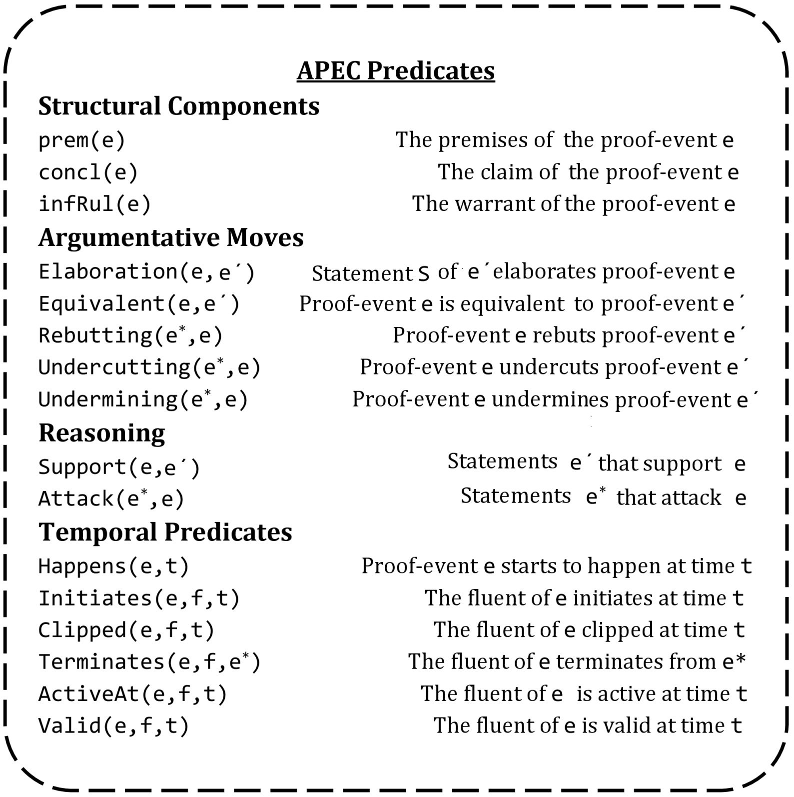
which means that a proof-event clips when there is a terminating proof-event e_1^* between t_1 and t_2 and there is no proof-event e_2 that attacks the counter-argument e_1^* attacking the proof-event e_1.

$$Terminates(e, f, e^*) : \exists e, e^*, t_1([attack(e^*, t_1) \rightarrow \neg conc(e) \cup \neg prem(e) \cup \neg sent(e)] \cap [\nexists e_2, t_2(Happens(e_2, t_2) \rightarrow \neg attack(e^*, t_1))], \text{with } t_1 < t_2$$

which means that a fluent terminates when there is a counterargument attacking the sequence and there is no proof-event e_2 that *Happens* in time t_2, with $t_1 < t_2$, to defend the claim. The termination of a sequence of proof-events may be caused by the indication of the falsity of the problem (there are counterarguments that attack the conclusion of the proof-event), or the undecidability of the problem (there is a lack of adequate warrants to prove the desideratum), or the inefficiency of the required information (there is a lack of premises).

$$ActiveAt(e, f, t_{n+1}) : Happens(te_{n+1}, t_{n+1}) \rightarrow \neg attack(e_n^*, t_n) \cup support(e_n', t_n), \text{for every } n \in N, t_{n+1} > t_n$$

which means that a fluent is active, if there is an argument to support the claim for every counterargument attacking the claim. This means that for every counterargument $e^* < \Psi_i, \beta_i >, i = 1, \ldots, n, n \in \mathbb{N}$, there is a proof-event $e_{n+1}(\Phi_{n+1}, c_{n+1})$, which $Happens(e_{n+1}, t_{n+1})$ and defeats the attack of the counterargument $e_n^* < \Psi_n, \beta_n >$, for $t_{n+1} > t_n$.



APEC Predicates

Structural Components	
prem(e)	The premises of the proof-event e
concl(e)	The claim of the proof-event e
infRul(e)	The warrant of the proof-event e
Argumentative Moves	
Elaboration(e,e´)	Statement S of e´elaborates proof-event e
Equivalent(e,e´)	Proof-event e is equivalent to proof-event e´
Rebutting(e*,e)	Proof-event e rebuts proof-event e´
Undercutting(e*,e)	Proof-event e undercuts proof-event e´
Undermining(e*,e)	Proof-event e undermines proof-event e´
Reasoning	
Support(e,e´)	Statements e´ that support e
Attack(e*,e)	Statements e* that attack e
Temporal Predicates	
Happens(e,t)	Proof-event e starts to happen at time t
Initiates(e,f,t)	The fluent of e initiates at time t
Clipped(e,f,t)	The fluent of e clipped at time t
Terminates(e,f,e*)	The fluent of e terminates from e*
ActiveAt(e,f,t)	The fluent of e is active at time t
Valid(e,f,t)	The fluent of e is valid at time t

Figure 3.3 APEC predicates.

From the above-mentioned, we conclude that:

$$Happens(e, t_1) \cap Initiates(e, f, t_1) \cap (t_1 < t_2) \cap \neg attack(e^*, t_2) \rightarrow ActiveAt(e, f, t_2),$$

which means that a fluent remains active at time t_2, if a proof-event e has taken place at time t_1, with $t_1 < t_2$ and has not been terminated at a time point between t_1 and t_2.

Consequently,

$$\forall i \leq n[ActiveAt(e, f, t_i) \cap (t_i < t_n) \cap \neg Terminates(e, f, t_i)] \rightarrow Valid(e, f, t_n), \text{at time } t_n, i = 1, \ldots, n, n \in N$$

which means that a fluent can be considered valid at time t_n, if it is active and there are no counter-arguments to terminate it at time t_i for every $i = 1, \ldots, n, n \in \mathbb{N}$.

A list with all the aforementioned predicates that constitute the core syntax of APEC is depicted in Figure 3.3.

Levels of argumentation

In order to define the warranted premises that are justified by a set of arguments in the sequence, a mechanism which can examine the representation of the arguments by recursion is necessary. Pollock introduces defeasible reasoning where arguments are chains of reasoning that may lead to a conclusion, whereas additional information may destroy the chain of reasoning.

To be able to more clearly and explicitly present not only the temporal process but also the validation progress of the argumentation-based proof-events more clearly and explicitly, we integrated the approach in [34]. The argumentation framework is built in terms of logic programming rules expressing a preponderance relation among the arguments, presenting **levels of argumentation**. Kakas et al [70] presented levels of arguments:

Object level arguments, which represent the possible decisions or actions in a specific domain.

First-level priority arguments, which express justifications on the object-level arguments in order to resolve possible conflicts.

Higher-order priority arguments, which are used to deal with potential conflicts between priority arguments of the previous level until all conflicts are resolved.

The same levels can be applied in mathematical proofs so as to understand the history of proof-events, starting from the statement of a problem until its validation or rejection and including all the attempts and failures [6]. As proof-events continue from lower levels to higher, they constitute fluents. The premise and the claim of the initial proof-events constitute the object-level arguments. Proof-events constitute the first-level priority arguments, in which they have preferences and justifications in the object-level arguments. The proof-events that have fulfilled their purpose terminate, while the rest of them continue to the higher-order priority arguments. The following example describes the possible steps and conflicts for the justification of a proof-event e through the levels of argumentation.

Object level arguments

Object level arguments pertain to the claim and the initial representations of arguments.

$$Happens(e_i, t_i), i = 1, \ldots, m, m \in \mathbb{N}, t_i \leq t_m < t$$
$$\forall e_i : [(Happens(e_i, t_i)) \to \neg attack(e_i^*, t_i) \cap (t_i \leq t_m)] \to Initiates(e_i, f_0, t_m)$$

for $i = 1, \ldots, m, m \in \mathbb{N} t_i \leq t_m < t$. The proof-events that are not attacked constitute the fluent f_o and continue to the first level priority arguments.

First-level priority arguments

The first-level priority arguments are presented as:

$$Initiates(e_{m+1}, f_1, t_{m+1}), attack(e_{m+1}^*, t_{m+1}), i = 1, \ldots, m_1, m_1 \in \mathbb{N}, t_{m+i} \leq t_m + m_1) < t,$$

for every $i \in \mathbb{N}$ that we have:

$$\exists e_{m+i}, e_{m+i}^*, t_{m+i}[attack(e_{m+i}^*, t_{m+i}) \to \neg conc(e_{m+i}) \cup \neg prem(e_{m+i})] \cap (t_{m+i} \leq t_{m+m_1}) < t) \cap [\nexists e_{m+i+1}, t_{m+i+1}(Happens(e_{m+i+1}, t_{m+i+1}) \to \neg attack(e_{m+i}^*, t_{m+i})] \to Terminates(e_{m+i}, f_1, t_{m+m_1})]$$

so that the proof-events that have been attacked and could not resolve the conflict, are terminated in this fluent. The rest of them remain active, so we have:

$$ActiveAt(e_{m+j}, f_1, t_{m+m_1})) \text{ for every } j \neq i, j \in \mathbb{N}$$

and continue to the second-level priority arguments. The same pattern continues for n-level priority arguments and for n fluents f_n that deal with potential conflicts between priority arguments of the previous level until either all conflicts are resolved or the claim is proved invalid. Then, the final level follows:

Higher-order priority arguments

If the proof-events fail to resolve all the conflicts, our claim can not be proved and it clips:

$$Clipped(e, f, t_n) \text{ at the time } t_n = t_{m_{n-1}+m_n} \geq t_i$$

If the proof-events manage to deal with all the attacks and:

$$\exists j, j \in N : [ActiveAt(e_{m_{n-1}+j}, f_n, t_n) \cap \neg Terminates(e, f_n, t_n)] \to Valid(e, f, t_n), \text{ at the time } t_n = t_{m_{n-1}+m_n} \geq t_i,$$

then our claim is proved valid.

3.2 Moral Argumentation-based Proof-Event Calculus (MAPEC)

Representing ethical codes and rules it requires an ethical policy, a hierarchy over the rules that are appropriate in different contexts (defining even which rule is more acceptable to violate when no ethical option is available). In order to demonstrate that a system has the capacity of making the right decisions (both operationally and ethically), it should be formally specified what the "right decisions" are.

Formal verification [84] involves proving or disproving that a system is compliant with a requirement determined in a mathematical language, i.e., a "formally specified property" expressed within a linear temporal logic, which in our case allows us to define what decisions the rational agents should make at some specific moment [85]. Thus, the ethical policy can be formalized in some computational logic L, whose well-defined formulas and proof theory specify the basic concepts required: the temporal structure, events, actions, sequences, agents, and so on [44]. The presented methodology proof-theoretically formalizes the ethical policy and implements it, meaning that this methodology encodes not the semantics of the logic L but its proof calculus [44].

Logic-based systems that are capable of dealing with increasing degrees of environmental uncertainty and variability are preferable [86] and cognition constitutes a way to deal with an undefined and uncertain world, meaning not necessarily a chaotic one but just a complex one. Argumentation is a tool of cognition that can formalize the science of common sense reasoning on which new types of systems can be engineered [87].

Therefore, to address the challenge of ensuring ethically correct behavior, a logic-based argumentation approach such as MAPEC is proposed to guarantee that robots only execute events that can be proved ethically acceptable in a human-selected logic, by formalizing an ethical code [44].

3.2.1 Moral Competence Expressed with MAPEC

In an ethical framework, a moral vocabulary allows the agent to represent norms, ethically substantial behaviors, and their judgments (conceptually and linguistically) in order to fuel the moral communication. It contains: a normative frame referring to the features of norms and to the normatively-supported qualities of agents; a language of norm violation characterizing attributes of violations and of violators; and a language of responses to violations [88].

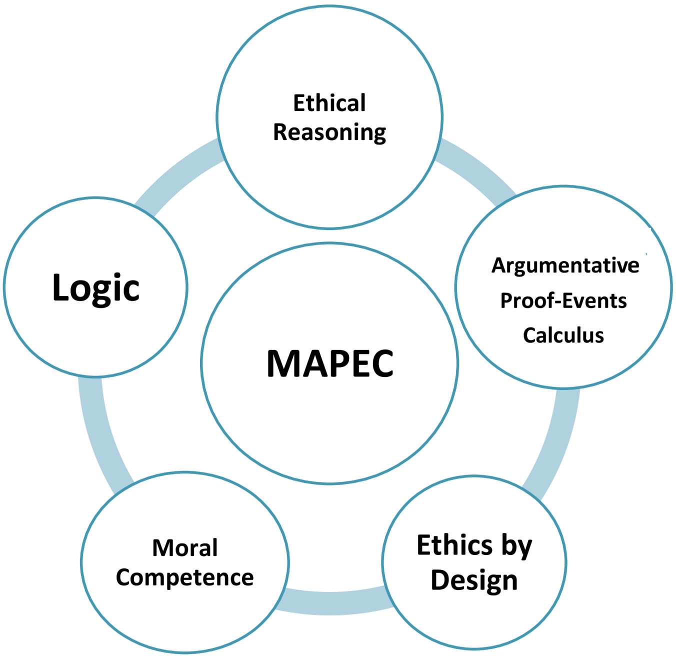


Figure 3.4 Research framework of MAPEC.

In our approach, the concept of norms is described with events, extending their context to abstract ethical events. The abstract ethical events present the arguments in a moral debate. The violations are analogous to the counterarguments. The role of ethical agents can be easily depicted as akin to the role of the supporter (or prover) and attacker in our argumentation framework [89, 36], where the supporter plays the role of the ethically correct agent and the attacker the role of the violator. Their actions are the responses to moral violations with arguments or counterarguments. Moral communication expresses the agent's efforts to recognize, clarify, or defend norm events, as well as interfere or rectify after a norm violation.

Definition 3.2.1. Abstract Ethical Events

An abstract ethical event is represented by e and its purpose is to defend an ethical principle c, where c can be interpreted also as "the supporter considers it immoral to permit or cause $\neg c$ (to happen)." The Abstract Ethical Event has the same structural components (data ϕ, warrant w, ethical claim c) as a proof-event in APEC [89]. Thus, an ethical principle c is in force when the event concludes to c, based on the data ϕ and following the inference rules w:

$$e = \langle communicate\langle \phi, c\rangle, w\rangle$$

where $e \in E$, E is the set of ethical events for c. Similarly, e^* denotes the violation event.

Moral judgment is the evaluation of the actions relative to norms that leads to the judgment of the temporal state of the moral actions, which includes the predicates $\mathit{Happens}(e,t), \mathit{Initiates}(e,f,t), \mathit{ActiveAt}(e,f,t)$, and $\mathit{Clipped}(e,f,t)$, leading finally to the ethical principle being $Valid(e,f,t)$ or $Terminate(e,f,t)$ (See section 3.1.3).

A **system of norms** contains a society's principles for ethical behavior. They aligh the supporter's arguments and decisions with specific (moral) behaviors and shape others' (moral) judgments of those behaviors [88]. Thus, they establish an ethical policy with ethical rules.

Definition 3.2.2. Ethical Policy

An ethical policy P is a tuple $P =< R, \geq >$ where R is a finite set of ethical rules between the events e, with $e \in E$, and $\geq$ is a complete (not necessarily strict) priority order on R. The expression $e_1 = e_2$ indicates that violating e_1 is equivalently unethical to violating e_2, while $e_1 \geq e_2$ denotes that violating e_1 is equally or less unethical to violating e_2. A special category of ethical event, symbolized as e_0, is vacuously satisfied and encompassed in every policy, so that $\forall e \in E : e > e_0$, indicating it is always strictly more unethical to do nothing and permit any of the unethical conditions to happen.

Ethical action is an event, taking place in compliance with the norms and in specific time, which is accommodated to and harmonized with other social agents (violators or provers) who operate under the same context. The norm violations e^* of a violator are denoted as $\mathit{attack}(e^*,t)$ events and the ethical proving action of a supporter is denoted as $\mathit{support}(e',t)$, both qualified by the time t to express the temporal sequence of the actions.

Definition 3.2.3. Ethical Actions

Given a certain context α, an event e, and an ethical principle c, an ethical action can be of the formulas:

$\mathit{support}(e',t) \overset{\alpha}{\Rightarrow} c$, denoting the actions of a supporter to defend the ethical principle c with ethical event e' in context α and at time t.

$\mathit{attack}(e^*,t) \overset{\alpha}{\Rightarrow} \neg c$, denoting the actions of a violator to contravene the ethical principle c with violation e^* in context α and at time t.

3.2.2 Prioritized Ethical Rules to Define Scenarios

Context determines dynamic priorities on the decision policies of the agent [90]. To be able to reason about scenarios in terms of ethics, we need a scenario selection process that uses the ethical policy, which can be represented within the argumentation theory. The agent can be in various contexts while deciding which scenario to choose, so the rules from all the contexts need to be considered when implementing a plan. We advocate scenarios that are ethical or at least violate the fewest ethical principles, both in quantity and in severity.

The scenarios are ordered using $<$ which leads to a complete order over scenarios [85]. This can describe an agent's ethical policy based on the different contexts with argumentation levels. In the first level we have the rules that refer directly to the domain of the agent, the object-level decision rules. In the other priority levels the rules relate to the ethical policy under which the agent generates different possible scenarios that the agent can choose. The higher level priority includes the rules representing the optimal course of action, the more ethical (or less unethical) scenario [90].

Definition 3.2.4. Levels of Ethical Rules

Given a policy $P =< R, \geq >$ and a plan based on the ethical rules R, V is a set of abstract ethical events (including the events e and the violations e^* of the ethical principles c) defined as:

$$V =< e \mid e\langle \phi, c\rangle, e \in E, support(e', t) \overset{\alpha}{\Rightarrow} c >$$

We define the operation $Higher$ for the higher level of ethical scenarios L based on the set of events V, as follows:

$$L = Higher(V) = \{e \mid e \in V \text{ and } \forall e_i \in V : e \geq e_i\}$$

Consider a set of available, possibly ethical, scenarios L_i for the different set of V_i. The scenarios lead to different levels of ethical rules $L_i \in L$ that satisfy the following properties, in order to define which available scenario is more ethical (or less unethical). For every $i, j \in \mathbb{N}$, it holds that $L_i > L_j$ if at least one of the following holds:

1. $V_i = \emptyset$ and $V_j \vdash \emptyset$.
2. $e_1 \geq e_2$, for every $e_1 \in Higher(V_j \setminus V_i)$ and every $e_2 \in Higher(V_i \setminus V_j)$
3. $e_1 = e_2$, for every $e_1 \in Higher(V_j \setminus V_i))$ and every $e_2 \in Higher(V_i \setminus V_j)$, while $\mid Higher(V_j \setminus V_i) \mid < \mid Higher(V_i \setminus V_j) \mid$.

If none of them holds, then L_i and L_j are equally (un)ethical, i.e., $L_i \sim L_j$.

The first relation makes sure that the ethical scenarios will always be favored instead of the unethical. The second one guarantees that when the principles that are the same in both scenarios are ignored, then the scenario that defends the most valuable principle is considered “higher” ethically. The third states that when the principles that in each scenario are violated are different, but equally valuable, the plan which violates fewer principles is “higher” ethically.

We can now define a logical property which specifies what it means for the reasoning and the decision-making of an agent to be ethical. Informally, we have that whenever an agent selects a scenario, L_i, then all other applicable scenarios L_j should be ethically “lower,” i.e., that $L_j < L_i$.

This approach can be implemented in autonomous systems, where the goal is not to show that an agent always makes a specific predetermined moral choice, but that their actions are due to the right reasons. In many real-life scenarios it is not easy to provide a complete set of decisions that will cover all situations [85]. Therefore, the system may have two modes of operation; either it uses its pre-existing set of actions in conditions which are within its anticipated parameters; or when new options appear it acts outside of these parameters based on various available resources that allow it to govern its actions using ethical reasoning [85].

Chapter 4

Formalization of Mathematical Proving Practices

Proofs and proving processes exist everywhere, where as “proving process” we define any process we follow based on some logic to prove the desired claim. This part of the thesis with the illustration of real-life applications has two segments with two different areas of implementation: the first part in this chapter describes the formalizing of explicit mathematical proving processes, and the second one (in Chapter 5) the formalizing of “implicit” proving processes. Another area of application that is currently under development is an argumentation-based checking of misinformation with formal calculus for the truth-assignment [91] This chapter presents how APEC can formalize explicit mathematical proofs; on the one hand, from an in-depth analysis of the steps inside the mathematical proof as exemplified by Zero Knowledge Proof-Events (Section 4.1); on the other hand, from a more distanced perspective, modeling the agents’ interaction and the temporal sequence of the events during a multi-agent proving process as indicated in the paradigms of Mini-Polymath 4 project (Section 4.2) and of Fermat’s Last Theorem (Section 4.3).

4.1 Zero Knowledge Proofs as Proof-events

APEC can be used to formalize in depth the steps that take place in the proving process of specific types of proofs, as — in our case — in the study of Zero Knowledge Proofs (ZKP). Zero Knowledge Proofs is a protocol between (at least) two people where one party, termed as the “prover” tries to prove a certain point to the other party, termed as the “verifier” [41]. The two parties play the corresponding roles of prover and interpreter.

The important properties in Zero Knowledge Proofs are:

Completeness, meaning that the verifier always accepts the proof if the claim is true, and both prover and verifier follow the protocol.

Soundness, meaning that the verifier always rejects the proof if the claim is false, and the protocol is followed.

Zero Knowledge, meaning that the verifier learns nothing else about the claim being proved by the prover that could not have been learned without the prover, regardless of following the protocol. Additionally, the verifier cannot even prove the fact to anyone later.

The protocol requires dialectical input from the verifier, commonly in the form of recurring challenges, such that the replies from the prover will convince the former if and only if the claim is true (which means that the prover does have the claimed knowledge). Thus, the procedure of justification in Zero Knowledge Proofs has a recursion of the same round which includes:

- a commitment message from the prover (premise),
- a challenge from the verifier (attack), and
- a response to the challenge from the prover (conclusion),

interacting similarly with those in argumentative proof-events. The warrant of the proof-events calculus is the knowledge that is not transferred to the verifier in the Zero Knowledge Proofs.

In order to define the warranted challenges that are justified by a set of correct answers in the sequence, a mechanism which can examine the representation of the argumentative dialogue by recursion is necessary.The three levels of arguments described in [34] can describe the procedure of justification in Zero Knowledge Proofs, where we have recursion of the round described above. The protocol may repeat for several rounds, where each round adds more value for the desirable result [92]. Each round is equivalent to the corresponding levels of argumentation in proof-events. Based on the prover's responses in all rounds, the verifier decides whether to accept or reject the proof. This kind of Interactive Zero Knowledge Proofs can be used for identification, where a prover claims an identity with a username, a smart card, a process ID, etc., and security systems can use this identity to determine if the user-prover can be allowed access to an object, and for authentication, which is the procedure of proving an identity using given credentials, such as a password, a PIN, a smart card token, etc.

4.1.1 Modeling of Ali Baba's Cave

In this part, we present the well known Ali Baba's Cave example as described in [93]. In this, there are two parties — Peggy and Victor — and a ring shaped cave with an entrance on the one side and a door blocking the opposite side. Peggy wants to prove to Victor that she knows the magic word (code) that can open the door, without revealing it or any other information to him or anyone else. Peggy enters the cave and chooses to follow one of the two paths to the door blocking the way. Then, Victor enters the cave and asks from Peggy to come back to the entrance by following the path of his preference. If Peggy knows the secret word she can open the door and follow any path she wants to the entrance. If she doesn't, she can only get back through the path she had previously followed. Repeating this procedure, and since Peggy always manages to come back following the path requested, Victor can conclude that she knows the secret word. In the next section we formalize the example of Ali Baba's cave using the moves and the temporal predicates of proof-events.

Object level arguments

In the Object level arguments we have the basic elements of the statement that we want to prove and the possible options we can apply on the proving process. In this example we have two agents, a verifier and a prover, described as:

$$A \in A_V, A_P, Verifier = A_V, Prover = A_P$$

First, the three fundamental elements of proof-events are defined: Premise, Warrant and Claim. The Premise is the Graph G with its Vertices and Edges described as below.

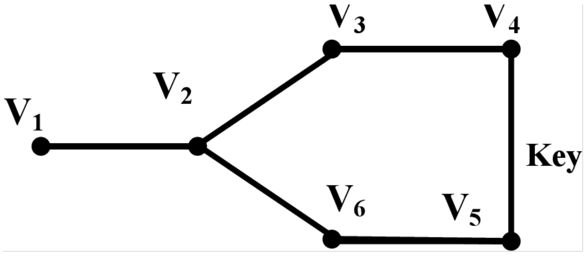


Figure 4.1 Ali Baba's Cave Diagram

Graph G: $V(G) = V_i | i = 1, \ldots, 6$,

$$f(z) = \begin{cases} \{V_i, V_j) \mid i+1 = j \quad \text{or} \quad i = 2, j = 6\} \setminus (V_4, V_5) \\ \qquad \text{iff} \quad K((V_4, V_5)) = 0 \\ \{(V_i, V_j) \mid i+1 = j \quad \text{or} \quad i = 2, j = 6\} \\ \qquad \text{iff} \quad K((V_4, V_5)) = 1 \end{cases}$$

The warrant in this example is illustrated by the Prover's possession of the key, which is the claim to be proved, thus $K:(V_4,V_5)\rightarrow 0,1$ expresses whether A_P has the Key or not.

The possible moves for the agents are the following:
$StandsOn: A_V, A_P \rightarrow V(G)$ expresses the position of A_V and A_P on the Graph.
$MovesTo: StandsOn \rightarrow StandsOn$
$(A_i,V_i)\rightarrow(A_i,V_j)$

if and only if $(V_i,V_j)\in E(G)\cup(V_i,V_j)=(V_4,V_5)$ and $A=A_P$ and has the Key.

So P can move through (V_4,V_5) if and only if P has the Key.

$$Sees: StandsOn \times StandsOn \rightarrow 0,1$$
$$Sees((A_V,V_i),(A_P,V_j)) = \begin{cases} 1, & \text{if} \quad (V_i,V_j)\in E(G) \\ 0, & \text{if} \quad (V_i,V_j)\notin E(G) \end{cases}$$

First-level priority arguments
In the first level arguments the Verifier A_V and the Prover $A_P StandsOn V_1$.

$$StandsOn = Happens(A_V,V_1),$$
$$StandsOn = Happens(A_P,V_1),$$

Then, $A_P MovesTo$ either V_4 or V_5. There is nothing that prevents them from moving, so there is no attack for this move.

$$[Happens(A_P,V_2))\rightarrow MovesTo(A_P,V_i)]\rightarrow Initiates(A_P,f_0,V_i) \text{ with } i=4 \text{ or } 5$$

The procedure of proving *Initiates* and the verifier is testing the claim of our example (whether prover has the key-proof) by asking the prover to appear from either of the two possible exits of the cave (V_3 or V_6).

$$Initiates(A_p,f_m,V_i), \text{ with } i=4 \text{ or } 5$$

$$MovesTo(A_V^{;}V_2)]\rightarrow Happens(A_V,V_2),$$

$$D_V = attacks(A_V^{;}f_m,V_j), \text{with } j=3 \text{ or } 6$$

$$A_P MovesTo(V_3 \text{ or } V_6), \text{ if } Sees((A_V,V_2),(A_P,D_V))=0 \text{ then it Terminates.}$$

$$[attack(A_V^{;}V_j)\cap\neg Sees((A_V,V_2),(A_P,V_j))]\rightarrow\neg StandsOn(A_P,V_J)\cap K:(V_4,V_5)\rightarrow$$
$$Terminates(A_P,f_m,V_j)], \text{ with } j=3 \text{ or } 6, m=1,\ldots,n-1$$

Else,

$$ActiveAt(A_P, f_m, V_i) \text{ for } i = 4, 5, m = 1, \ldots, n-1$$

And it continues to the second-level priority arguments by repeating the procedure from the beginning. The same pattern continues for n-level priority arguments and for n fluents f, until the verifier is convinced that the prover has the key-proof.

Higher-order priority arguments

In the final n-level if at the time t_n:

$$\exists j, j \in N : [ActiveAt(e_P, f_n, V_j) \cap \neg Terminates(e_P, f_n, V_j)] \rightarrow Valid(e_P, f_n, t_n),$$

then the claim is proved valid.

In this use case we described a connection between the Argumentative Proof-Event Calculus and Zero Knowledge Proofs. Proof-events are not considered as infallible facts before their ultimate validation, thus enabling the connection with the procedure of Zero Knowledge Proofs where a recursive tentative process is required until the final validation of the proof.

4.2 Online Multi-agent Mathematical Practice

One of the difficulties in the investigation of mathematical practice is that there is limited knowledge of **the real process of mathematical proving** and of the interaction between mathematicians during proving [42]. To study mathematical proving, we need sufficient information to capture the real-life process of mathematical discovery, not only the final product of the proof communicated in the publications. This information should provide grounds for explanations of the mathematical discovery, historical facts about the efforts undertaken by the contributors (alone and in cooperation), and data about the shaping of views and attitudes to proving outcomes [69]. The Web as a collaborative medium may transform the way we experience proving practices, as it allows for contribution by agents with different backgrounds, knowledge, skills, and styles of thinking [33]. Unlike traditional modes of communication, one of the key features of the Web — and one that facilitates mathematical practice — is its open and ubiquitous nature, since Web-based communication enables interaction in multi-agent systems [33].

Data sets of **online collaborative mathematical practice** can provide us with original, rich, and valuable information about the real process of mathematical discovery [75]. Online blogs and forums with informal mathematical dialogues, such as Polymath, Mini-Polymaths, MathOverflow, Tricki.org, Math.Stackexchange, etc., reveal some of the hidden aspects of the evolution of mathematical proving over a period of time [42]. In addition, Web-based interactivity enables collaborative problem-solving, through which proof for a particular problem is achieved through spontaneously generated and exchanged arguments and counterarguments. Therefore, a source of information that can provide evidence about the mathematical proving practice presents itself in the form of Web-based crowd-sourcing projects. Crowd-sourcing is a procedure similar to open sourcing where the work may be undertaken on an individual or a crowd basis, raising the number of possible contributors-provers, thus possibly gaining a deeper vision of the problem. The use of the Web as a means of crowd-sourcing and collaborative search for proof [33] dates back to projects such as Tatami and Kumo by Goguen [94], and Tricki and Polymath by Timothy Gowers [37]. Tatami is a Web-based cooperative software system that consists of a proof assistant [33]. Kumo is a proof assistant for first-order hidden logic, which also develops websites that document its proofs [95]. Tricki involved creation of a large repository of articles useful for mathematical problem solving with the aim of assisting in mathematical proving practice [96]. In Polymath, a mathematical problem was formulated, and the entire mathematical community was invited to collaborate openly to suggest ideas, approaches, comments, and pieces of proof in order to find an alternative proof [37].

However, **Web methods** do not always reflect the semantic structure of mathematical argumentative aspects explicitly enough or in depth [97]. They often cannot capture different types of arguments and counterarguments and are presented with difficulties in finding and evaluating arguments and their relationships [98]. These Web technologies have a specific semantic structure that links opinions and arguments in a dialogue based mainly on natural linguistic models of argumentation (i.e., models that perceive argumentation as a language activity) [98]. There is a need for new frameworks, tools, and systems engineered into the Web to encourage mathematical dialogue, facilitate multi-agent collaboration, and promote a new online collective thinking. The focus in this paper is explicitly on mathematical activity, thus our work attempts to add to this repository of Argument Web tools by providing a semantic calculus specialized in the reasoning that takes place in mathematical practices. We believe that the reasoning that takes place in mathematical dialogues described by a machine-processable and semantically-rich argumentative structure is important to the Semantic Web vision. Given this, the Web can critically transform the way we perceive proving practices [33].

In the next section, we discuss how the resources of online collaborative mathematics can be applied to support formulating and answering questions about mathematical proving. **Polymath projects** can be considered as one of the first fully documented accounts of how a mathematical problem was solved [99]. In Polymath, contributors were encouraged to view themselves as part of a collaborative team created *ad hoc* to solve a posed problem and share their ideas even if they were "obvious," incomplete, or faulty, as others might be able to check and correct them and discard what is useless. This form of networked brainstorming allows for tapping the full potential of the various and complementary mathematical skills of the participants, thus leading to better and quicker results [37]. The data set we use are excerpts from the comments[1] of the *Mini-Polymath 4 project*, which allow us to integrate the arguments exchanged into dialogues represented in proof-events sequences. Some of the questions we try to highlight with this case study are:

- What knowledge can we obtain from the dialogues of online crowd-sourcing projects?
- How can the study of these projects can be used to understand mathematical practice? and
- How can we present them in a systematic, illustrative, and explanatory way?

Although the dataset is not extensive, it is sufficient for our model.

4.2.1 Modeling of Mini-Polymath 4

This work argues that online proving dialogues can be expressed as a particular type of Goguen's Web-based proof-events [95]. Web-based proof-events have informal social and historical components, prover-interpreter interaction, collaboration, consent, and validation. Furthermore, argumentation can make a significant contribution in dealing with the defeasible knowledge of the Web which is a product of its open and ubiquitous nature. The Web can restructure the way we understand mathematical proving practices, facilitating proving as a multi-agent collective activity involving people with different backgrounds, expertise, reasoning, and thinking styles.

In the Mini-Polymath 4 project, the participants contribute to the solution of a problem from the 2012 International Mathematical Olympiad, termed *"The liar's guessing game (LGG)"* (see also Appendix .1). We aim to present the dialogue and

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[1] https://polymathprojects.org/2012/07/12/minipolymath4-project-imo-2012-q3

exchange of arguments in which the contributors were engaged through the comments functionality of the Polymath Webpage by constructing an APEC model, focusing on the proving activity of the first part of the conclusion c_{LGG_1} from the LGG problem. The second part c_{LGG_2} of the LGG problem can be modeled similarly.

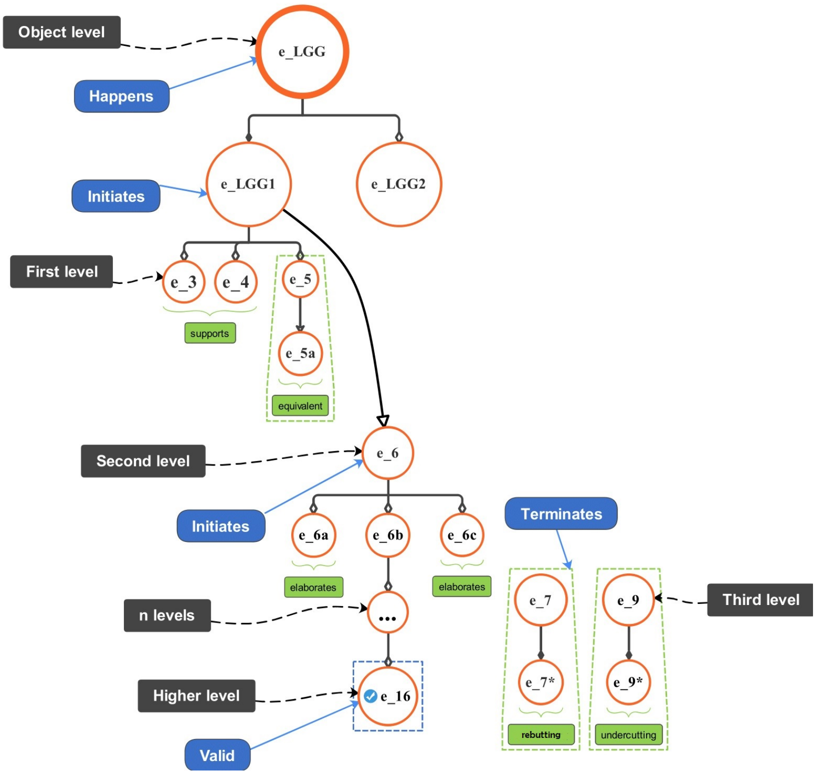

Figure 4.2 Illustration of Mini-Polymath 4 through APEC model.

The APEC model formalises mathematical practice based on four core contexts (indicated also by the corresponding colours as follows):

Argumentation-based proof-events and their structural components that can be linked to the relevant sentences of the participants' discourse.

Argument moves and reasoning that indicate interactions between proof-events (and their agents accordingly).

Temporal predicates that indicate the progress of the practice over time and whether certain proof-events are active or not.

Levels of argumentation that indicate the progress of the proof in terms of justification.

The first two contexts connect the formal modeling of the calculus with the informal elements of the agents' discourse and activities, and the latter two designate the progress of the proving in terms of time and validation.

The course of exchange of arguments in this argumentation-based proof-event sequence is illustrated in the flow chart in Figure 4.2. In this illustration the orange circles depict the argumentation-based proof-events, where the central one concerning the proof of LGG is denoted as e_LGG (e_LGG1 and e_LGG2 are the two conclusions of LGG), while the rest of the proof-events are denoted as $e\_\{number\}$, where the number is the numbering of the related Mini-Polymath comment. The arrows depict the flow of the sequence of the proof-events. Labels also indicate the argument moves (green labels), the temporal predicates (blue labels) and the levels of argumentation (black labels) in the corresponding part of the sequence.

Object-level arguments:

In the object-level arguments, we have the possible initial available data and representations of arguments that can be used by the agents related to a specific domain problem that they attempt to address. Each agent may interpret and use this data differently, based on their personal perspective and background knowledge. In the use case presented, there is the LGG problem as the initial proof-event ($e_{LiarGuessingGame}$) and two claims that need to be proved, so we have:

$e_{LiarGuessingGame} = e_{LGG_1}\langle \Phi, c_1\rangle \cap e_{LGG_2}\langle \Phi, c_2\rangle$
where:
$\Phi = \langle$The liar's guessing game.$\rangle$
$c_1 = \langle$If $n \geq 2^k$ then B can guarantee a win.$\rangle$
$c_2 = \langle$For all sufficiently large k, there exists an integer $n \geq 1.99^k$ such that B cannot guarantee a win.$\rangle$

The Polymath aims to create the **warrant** of the aforementioned proof-events, as the result of collective fluents[2]. This initiates the proving:
$Happens(e_{LGG}, f_0, t_1) \rightarrow Initiates(e_{LGG_1}, f_0, t_1) \cup Initiates(e_{LGG_2}, f_0, t_1)$

First-level and **second-level** priority arguments included initial comments, attempts, and justifications of previous arguments that are not described in detail here (for the

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[2] At each level, the fluent is numbered with the corresponding level of argumentation, i.e., at first-level we have the fluent f_1.

modeling of these levels see Appendix .1).

Third-level Priority Arguments:

At this level, we have counterarguments and attacking moves on some comments and ideas of the previous levels. The proof-events are enumerated based on the numbering of the Polymath 4 comments[3].

In some cases, a proof-event can be implied or assumed (correctly or faultily) from the available data, such as in the following example:

$e_7 = \langle \Phi_7, c_7 \rangle$ = $\langle \Phi_7$: B cannot guarantee the win, c_7: it can be "always win" for A$\rangle$
(*this proof-event is implied from the initial description of the problem.*)

With counterargument e_7^*, the option that "*player A can always win*" was terminated.
$e_7^* = \langle \Phi_7^*, c_7^* \rangle$ = $\langle \Phi_7^*$: *Since there is a possibility that B would win the game simply by guessing,* c_7^*: *there is no "always win" for A*$\rangle$

$Rebutting(e_7^*, e_7) : rebut(e_7^*, e_7) \rightarrow \neg concl(e_7)$ and
$attack(e_7^*, e_7) \rightarrow Rebutting(e_7^*, e_7)$, where
e_7^* attacks $concl(e_7)$ = $\langle$"always win" for A$\rangle$.
$Terminates(e_7, f_3, t_{L_3}) \rightarrow attack(e_7^*, e_7)$

Argument e_8 adds an observation on the warrant of e_{LGG_1}.
$e_8 = \langle \Phi_8, c_8 \rangle = \langle \Phi_8$: *For the first part, proving for* $n = 2^k$ *suffices. The first approach that comes to my mind is to induct on* k, c_8: $c_{LGG_1}\rangle$ with warrant $w_8 = \langle$ proving for $n = 2^k\rangle$.

With counterargument e_9^*, the related proof-event was attacked and terminated as unconstructive.
$e_9 = \langle \Phi_9, c_9 \rangle$= $\langle \Phi_9$: *B can as well ask questions in "rounds" of* $k+1$ *questions,* c_9: *then, each round is guaranteed to have at least 1 correct answer*$\rangle$
$e_9^* = \langle \Phi_9, c_9 \rangle$ with $infRul(e_9^*)$= $\langle$ *While this is true, it is not very constructive* [...]$\rangle$

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[3] Another option is to number them by the agent's name (or both), depending on the information that we want to stress.

$Undermining(e_9^*, e_9): Undermin(e_9^*, e_9) \rightarrow \neg prem(e_9)$ and
$attack(e_9^*, e_9) \rightarrow Rebutting(e_9^*, e_9)$
Thus,
$Terminates(e_9, f_3, t_{L_3}) \rightarrow attack(e_9^*, e_9)$

Fourth-level Priority arguments:

At the fourth level, ideas and efforts yield some productive results thanks to fruitful (but not yet complete) cooperation, as the proving discovery progresses towards higher levels.

$e_{10} = \langle \Phi_{10}, c_{10} \rangle$=$\langle \Phi_{10}$: *So for* $k = 0$ *any version of binary search works,* c_{10}: *The next step should be to find the strategy for* $k = 1, n = 2\rangle$,
where $\neg infRul(e_{10})$, since the contributor claims *"I first thought I have found the strategy, but it doesn't work."*

Another prover named Mihai Nica elaborates in this proof-event with some useful lemmas that help proof-event e_{10} to progress, and finally the contribution of these comments adds a valuable input in the final proving of the first conclusion e_{LGG_1}.

$e_{10_a} = \langle \Phi_{10_a}, c_{10_a} \rangle$=$\langle \Phi_{10_a}$: *I am working on this case too. Here player A can never tell two lies in a row. Here is a little observation I have made. Let Q1, and Q2 be questions that player B can ask, and I will use the notation like* [...], c_{10_a}: *Here is a cute little lemma: If B asks Q1 Q2 Q1, then A must give the same answer for Q1 both times it is asked, or else tell the truth for Q2*$\rangle$.

$e_{10_b} = \langle \Phi_{10_b}, c_{10_b} \rangle$=$\langle \Phi_{10_b}$: *Let Q1,Q2 be questions. If player B asks the sequence of questions Q1 Q2 Q1 Q1 and gets answers A1 A2 A3 A4 (each Ai is either an L (lie) or a T(truth)),* c_{10_b}: *By the last lemma for the sequence of questions Q1 Q2 Q1, player B knows that either A2=T or the answers to the first three questions are LTL, TLT, or TTT* [...]. *I think the second lemma can be used to make a binary search by making Q1 = half the numbers, Q2= the other half of the numbers*$\rangle$.

$support(e_{10}, e_{10_a}) \rightarrow Elaborate(e_{10}, S_{e_{10a}})$,
$support(e_{10}, e_{10_b}) \rightarrow Elaborate(e_{10}, S_{e_{10_b}})$
where:
$prem(S_{e_{10_a}})$=$\langle$*If B asks Q1 Q2 Q1, then A must give the same answer for Q1 both*

times it is asked [...]$\rangle$
$prem(S_{e_{10_b}})$=$\langle$*If player B asks the sequence of questions* [...] *then player A is forced to reveal* [...]$\rangle$
$Initiates(e_{10}, f_4, t_{L_4}) \rightarrow support(e_{10}, e_{10_a}) \cup support(e_{10}, e_{10_b})$

(continues for comments 11–15)

Higher-order Priority Arguments:

At the higher level, we have the justification and the proof of LGG's first conclusion c_{LGG_1}, as the outcome of collective argumentation-based proof-events.
$e_{16} = \langle \Phi_{16}, c_{16} \rangle$, where Φ_{16}= $prem(e_{16}) = \langle$ *We can assume* $N = 2^k + 1, n = 2^k$ [...]. *Then we can keep asking if* b_1 *is 1, there are two possibilities.*$\rangle$
$c_{16} = conc(e_{16}) = conc(e_{16_a}) \cup conc(e_{16_b})$, where:
$conc(e_{16_a}) = \langle$ $k+1$ *times we get the answer NO, then we exclude the number 10 ... 0* $\rangle$
$conc(e_{16_b})$= $\langle$*There is a YES answer. Then we stop asking about* b_1 *and ask* $b_2 = 1, b_3 = 1, \ldots, b_{k+1} = 1$. *After we are done we can exclude the number* [...].$\rangle$

We have several proof-events in comment 16 that add in the proving discourse, either by supplementing claims of previous agents or by questioning some incomplete claims. We can see that it is a live procedure where each comment comes to fill a piece of the "proving puzzle" until its ultimate completion. In this proof puzzle, it often happens that even the attempt to add a "wrong piece" can contribute to the process, since something that does not work was tried, and it can now be safely excluded as an option.

$support(e_{16}, e_{16_a}) \rightarrow equivalent(e_{16}, e_{16_a})$, where
w_{16_a} =$\langle$*Another way (which seems to solve the first question). We ask the sequence of question* Q_i*: "Does* $b_i = 1$*?" in a row. That makes* $k+1$ *questions [...]. We have excluded a possibility, which by the reduction of comment 15 is enough.*$\rangle$

$Rebutting(e^*_{16_b}, e_{16_a}) : rebut(e^*_{16_b}, e_{16_a}) \rightarrow \neg concl(e_{16_a})$, where
$e^*_{16_b}$=$\langle$*Which number will you exclude in that case? (It might not be in the range)*$\rangle$, and
$attack(e^*_{16_b}, e_{16_a}) \rightarrow Rebutting(e^*_{16_b}, e_{16_a})$

$ActiveAt(e_{16_a}, f_n, t_{L_n}) \rightarrow support(e_{16_a}, e_{16_c}) \cup support(e_{16_a}, e_{16_d})$, with
$support(e_{16_a}, e_{16_c}) \rightarrow Elaborate(e_{16_a}, S_{e_{16_c}})$, and
$support(e_{16_a}, e_{16_d}) \rightarrow Elaborate(e_{16_a}, S_{e_{10d}})$, where
$e_{16_c} = \langle \Phi_{16_c}, c_{16_c} \rangle = \langle \Phi_{16_c}$: *When* $c_1 = 1$, c_{16_c}: *then the number might be out of the range.*$\rangle$
$e_{16_d} = \langle \Phi_{16_d}, c_{16_d} \rangle = \langle \Phi_{16_d}$: *I'm not sure I totally understand your argument, but your argument lead me towards the following: Let* B_i *be the subset of* $\{0, \cdots, N-1\}$ *with 0 as the* i^{th} *digit in their binary expansion [...],* c_{16_d}: *So* x *cannot be* s_i *and* ***we have the required win****. On the other hand, if* A *always says that* $x = s_i$ *for any* i*, [...] and* ***B wins****.*$\rangle$

$$Valid(e_{LGG_1}, f_n, t_{L_n}) \rightarrow support(e_{16}, e_{16_c}) \cup support(e_{16}, e_{16_d}) \cap \neg attack(e^*_{16}, e_{16})$$

The Mini-Polymath example illustrates the contribution of the agents in the process of proving. *The information we obtain from this type of project indicates that the characteristics and quality of dialogues can affect mathematical thinking and practice.* Firstly, the central aim of the proving itself is to convince the rest of the community about the justification and the validity of one's approach and outcomes. Moreover, all agents contributed significantly to the procedure, since various people had to participate in reaching their common goal, which was the proof of the LGG problem (in Figure 4.3, the warrant is justified based on the contributions of all participants).

Crowd-sourced mathematics is valuable in the study of mathematical practice, revealing the way that mathematicians think and debate. Proving, at least at its inception phase, can be understood as an inquiry implemented by exchange of ideas: a collaborative dialogue between mathematicians with the common aim of solving an open problem, which none of the participants in the conversation has specifically predetermined [11]. Such exchange of arguments can definitely be found in mathematics, at least in the context of mathematical discovery. At the end of the sequence of proof-events, the agent who takes on the role of the administrator has an overview of the whole "history" of the participation of each prover-agent to the sequence of proof-events, so that he/she can analyse the overall contribution of each agent and integrate them into the final proof.

Additionally, in these types of collaborating environments, the participants might have less fear of committing mistakes, and therefore different solutions can be tried out and corrected. Argumentation is more efficient in interactive contexts as it permits counterarguments to be addressed and stronger arguments to surface, and tools such

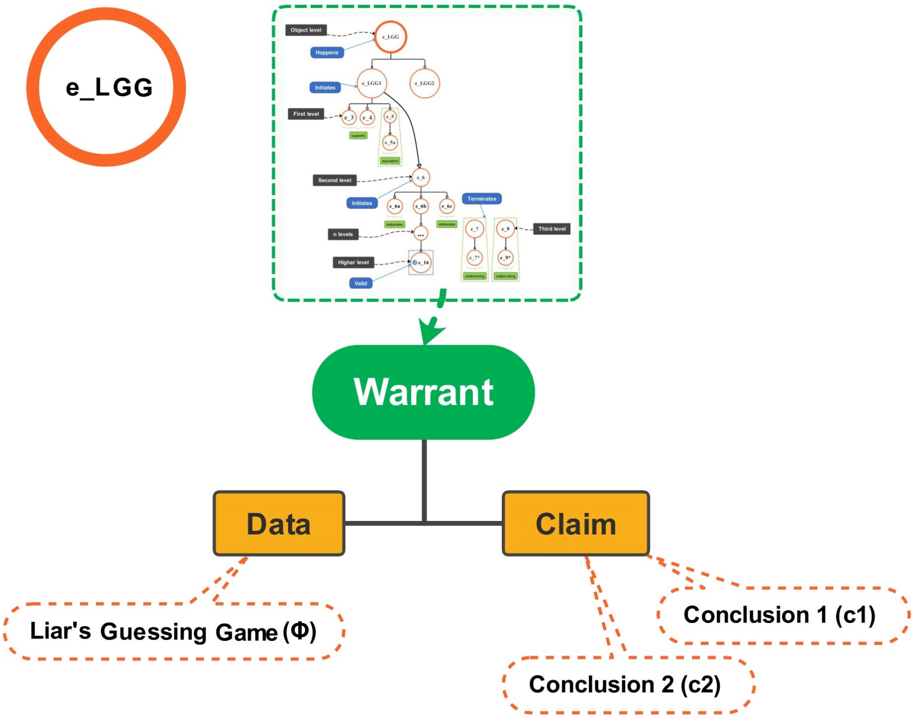


Figure 4.3 Illustration of argumentation-based proof-event e_{LGG}.

as the APEC model can provide considerable aid in this procedure. *It can be applied by provers and interpreters to identify and distinguish arguable elements on others' positions, but also on their own thinking.* The design and implementation of such learning environments can enhance the development of meta-cognitive activity and creativity in mathematics [100].

To sum up, the historical road-map of proving in Mini-Polymath 4 can be experienced as a cooperative activity, connecting people with different backgrounds, perspectives, and interests. At each point of the proving trajectory our APEC framework illustrates the current state of the formal and informal reasoning in proving. This creates a link between:

- the informal and social aspect in the natural mathematical dialogue during the discovery of a proof; and
- the formal and computational aspect of abstract argumentation reasoning and semantics.

The APEC framework adds an additional dimension and performs a significant role in making these connections sufficiently detailed in a systematic and explainable way, *demonstrating the applicability of argumentation techniques to mathematical proving and thinking.*

4.3 Historical Trajectory of Proofs

To illustrate a multi-agent mathematical proving over an extended period of time, we present the famous Fermat's Last Theorem — where many mathematicians contributed to reach the outcome of the final proof — as an example of how their interactions and contributions can be structured and formalized to depict the discovery and the history of the proving, not only for agents that lived in the same time period but also for agents that lived in different time periods.

This section can not present the whole sequence of such proof-events in detail, thus some of these historical attempts (proof-events), that add in the validation of the final proof of the theorem, were selected to demonstrate how argumentation is involved in the search of a proof.

Fermat's Last Theorem was formulated in 1637 by Pierre de Fermat, who stated that there are no three distinct positive integers a, b, and c, other than zero, that can satisfy the equation $a^n + b^n = c^n$, whenever n is an integer greater than two ($n > 2$). The statement of the problem marks the starting-point of a proof-event. Even though Fermat claimed in the margin of his book *Arithmetica* to have proven this theorem: *"It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have discovered a truly marvelous proof of this, which this margin is too narrow to contain."* it actually took 358 years and numerous attempts by many famous mathematicians and amateurs to prove it until its final proof by Andrew Wiles in 1995. Thus, Fermat's alleged proof can not be included in the initial proof-event, since it was never communicated. Fermat communicated the Theorem only for the cases $n = 3$ and $n = 4$ in his letters and gave a solution for the latter. The statement of the problem marks the beginning of a sequence of proof-events that evolved over a period of 358 years. This sequence of proof-events was evolving over time, since many famous mathematicians and amateurs (agents) were involved in various distinct proof-events that took place in different places and times in their attempt to solve the problem posed.

The first attempts to prove the Theorem were proofs for specific exponents. The case $n = 3$ was first explored by Abu-Mahmud Khojandi (c. 940–1000), but his attempt has not survived (thereby it cannot be considered as a proof-event) and it is thought to have been incorrect. Leonhard Euler gave a proof for $n = 3$ in 1755 and for $n = 4$

in 1747, but his proof of the former case contained a basic fallacy [101]. Many other mathematicians proved the theorem for $n = 3$ using various methods. Gabriel Lamé (1795–1870) proved it for $n = 7$. In 1847, he communicated a proof of the theorem, but it was flawed. Gabriel Lamé's proof failed because it was incorrectly claimed that complex numbers could be factored into primes uniquely. This gap was immediately pointed out by Joseph Liouville [101]. In 1984, Gerhard Frey pointed out a connection between the modularity theorem and Fermat's equation, but Fermat's Last Theorem remained a conjecture. The Taniyama-Shimura-Weil conjecture, which was proposed in 1955, was the method that led to a successful proof of Fermat's Last Theorem, when Andrew Wiles accomplished a partial proof of this conjecture in 1994 [102].

Wiles, after spending six years applying various methods that proved unsuccessful, approached the problem in a new way. He decided to present his work in June 1993 at the Isaac Newton Institute for Mathematical Sciences [102]. However, during the peer review, it became evident that there was an incorrect critical point in the proof. Wiles tried for almost a year to resolve this point, firstly by himself and then in collaboration with Richard Taylor, but without success [18]. When Wiles was on the verge of quitting his attempt, he experienced an epiphany, namely that the Kolyvagin-Flach approach and Iwasawa theory were each insufficient on their own, but in combination they could be strong enough to overcome this final barrier. In 1994, Wiles submitted two papers that established the modularity theorem for the case of semi-stable elliptic curves, which was the last step in proving Fermat's Last Theorem [102].

This example illustrates the contribution of the agents in the process of proving. Firstly, the central aim of the proving itself is to convince the rest of the community about the justification and the validity of your approach. Moreover, the other agents also contribute significantly in the procedure. A great number of people had to participate in order to reach the initial goal, which was the proving of Fermat's Last Theorem. This participation presents itself in two ways, either as the rejection of someone else's attempt by pointing out a fault and/or inaccuracy (e.g., Liouville indicated Lamé's gap concerning complex numbers) or as the dialogue between cooperators in order to detect and resolve weak or deficiently supported areas in the proving (e.g., Wiles asked for other colleagues' help, like Richard Taylor, whenever he came upon a dead-end or fault in his attempt). Argumentation is more efficient in more interactive contexts, as they allow for counterarguments to be addressed and stronger arguments to surface. An audience with mainly homogeneous beliefs will generate fewer differentiated counterarguments, making them easier to address. Thus, a mathematician has an advantage if they want to ask for the assistance of a few colleagues in order to point out most of the possible counterarguments and resolve them in the final proof. By doing this, the proving could be more convincing not only to these few colleagues, but probably to the whole community.

The arguments and the counterarguments also play an essential role in the process of proving, contributing equally in the building and the justification of the proving. The warranted parts of the initial proof-events served as groundwork for subsequent proof-events, while the counterarguments that point out the faults in those unsuccessful proof-events open the way for better justified proof-events and in some cases draw the interest of the mathematical community in new unexplored areas. Those incomplete proof-events may add more or less to the proof of Fermat's Last Theorem, but the methods that were created with them lead to major discoveries and creation of new fields in the era of Mathematics like the foundation of modern algebra. Discoveries that are even more significant than the proving of the theorem itself and might have not been made had it not been for the warranted proof-events and the counterarguments which had emerged from the previous attempts of proving.

4.3.1 Modeling of Fermat's Last Theorem

In the next part we present a brief illustration of this example through the levels of argumentation.

Object level arguments - Fermat's Conjecture

In the object level arguments, there is Fermat's conjecture as the initial proof-event (e_{Fermat}), and his claim that he has a proof for this conjecture, without any claim-counterargument (e^*_{Fermat}) that clearly opposes this conjecture.

$$Happens(e_{Fermat}, t_{1637}) \cap \neg attack(e^*_{Fermat}, t_{1637}) \rightarrow Initiates(e_{Fermat}, f_0, t_{1637})$$

First-level priority arguments - Proofs for specific exponents

In the first-level priority arguments, there are proofs for specific exponent n of Fermat's Last theorem by various mathematicians in different time points. For the exponent $n = 3(e_{n=3})$, Leonhard Euler (e_{Euler}) gave a proof in 1755, so we have: $Happens(e_{Euler}, t_{1755})$. Many other well-known mathematicians followed with equivalent proofs that support the validity of the proof for $n = 3$. Each prover used a different method (warrant) for proving the conclusion, so their proof-events are equivalent.

$$Support(e_{n=3}, t_i) \rightarrow Equivalent(e_{n=3}, e_i), \text{ for } i = 1, \ldots, 14$$

with:

$i = 1 : (e_{Euler}, t_{1707}), i = 2 : (e_{Kausler}, t_{1802}), i = 3 : (e_{Legendre}, t_{1823}),$

$i = 4 : (e_{Calzolari}, t_{1855}), i = 5 : (e_{Lame}, t_{1865}), i = 6 : (e_{Tait}, t_{1872}),$
$i = 7 : (e_{Gunther}, t_{1878}),), i = 8 : (e_{Gambioli}, t_{1901}), i = 9 : (e_{Krey}, t_{1909}),$
$i = 10 : (e_{Rychlik}, t_{1910}), i = 11 : (e_{Stockhaus}, t_{1910}), i = 12 : (e_{Carmichael}, t_{1915}),$
$i = 13 : (e_{Thue}, t_{1917}), i = 14 : (e_{Duarte}, t_{1944})$
So we have $Initiates(e_{n=3}, f_1, t_{1707})$.

$$Happens(e_{Euler}, t_{1755}) \cap Initiates(e_{n=3}, f_1, t_{1755}) \cap [\neg attack(e_{(n=3)^*}, t_i) \cup support(e_{n=3}, t_i)] \cap (t_{1755} < t_i) \rightarrow ActiveAt(e_{n=3}, f_1, t_i), \text{ for } t_{1755} < t_i$$

Similarly, for $n = 5$ and $n = 7$,
$Support(e_{n=5}, t_j) \rightarrow Equivalent(e_{n=5}, e_j)$, for $i = 1, \ldots, 10$ with
$j = 1 : (e_{Legendre}, t_{1825}), j = 2 : (e_{Dirichlet}, t_{1825}), j = 3 : (e_{Gaus}, t_{1875}),$
$j = 4 : (e_{Lebergue}, t_{1843}), j = 5 : (e_{Lame}, t_{1847}), j = 6 : (e_{Gambioli}, t_{1901}),$
$j = 7 : (e_{Werebrusow}, t_{1905}), j = 8 : (e_{Rychlik}, t_{1901}), j = 9 : (e_{Corput}, t_{1159}),$
$j = 10 : (e_{Terjanian}, t_{1987})$
So, $Initiates(e_{n=5}, f_1, t_{1825})$.

$Support(e_{n=7}, t_j) \rightarrow Equivalent(e_{n=7}, e_j)$, for $k = 1, \ldots, 5$ with
$k = 1 : (e_{Lame}, t_{1839}), k = 2 : (e_{Lebesguet}, t_{1840}), k = 3 : (e_{Genocchi}, t_{1876}),$
$k = 4 : (e_{Pepin}, t_{1876}), k = 5 : (e_{Maillet}, t_{1897})$.
So, $Initiates(e_{n=7}, f_1, t_{1839})$.

Fermat's Last Theorem was also proved for the exponents $n = 6, 10$, and 14.

Second-level priority arguments - Sophie Germain Germain tried unsuccessfully to prove Fermat's Last Theorem for all even exponents, which was proved by Guy Terjanian in 1977.

$$Clipped(e_{n=2p}, f_2, t_{1831}) : \exists e_{Germail}, eGermail*, t_1, [Happens(e_{Germail}, t_1) \cap (t_{1776} \leq t_1 < t_{1831}) \cap attack(e_{Germain}*, t)] \cap [\nexists e_2, t_2(Happens(e_2, t_2) \rightarrow \neg attack(e_{G}ermail*, t))], \text{ for } t_{1776} \leq t < t_{1823}$$

$$ActiveAt(e_{n=2p}, f_2, t_{1977}) : Happens(e_{Terjanian}, t_{1977}) \rightarrow \neg attack(e_{Terjanian}*, t_{1977})$$

Third-level priority arguments - Lamé, Kummer and the theory of ideals
In 1847, Gabriel Lamé's proving (e_{Lame}) failed because it claimed incorrectly that complex numbers can be factored into primes uniquely. This gap was indicated instantly by Joseph Liouville ($e_{Liouville^*}$).

$$\exists e_{Lame}, e_{Liouville^*}, t_{1847}[attack(e_{Liouville^*}, t_{1847}) \rightarrow \neg conc(e_{Lame})] \cap (t_{1847} \leq t_1 < t_2) \cap [\nexists e_{Lame}, t_2(Happens(e_{Lame}, t_2) \rightarrow \neg attack(e_{Liouville^*}, t_{1847}))] \rightarrow Terminates(e_{Lame}, f_3, t_2)$$

Kummer proved the conjecture for regular prime numbers ($e_{regular}$) but not for irregular primes ($e_{irregular}$). So,

$$ActiveAt(e_{regular}, f_3, t_{1893}) : Happens(e_{Kummer}, t_{1893}) \rightarrow \neg attack(e_{Kummer}*, t_{1893})$$

and

$$\exists e_{kummer}, e_{Kummer}*, t_{1893}, t_1[attack(e_{Kummer}*, t_1) \rightarrow \neg conc(e_{irregular})] \cap (t_{1893} \leq t_1 < t_2) \cap [\nexists e_{Kummer}, t_2(Happens(e_{Kummer}, t_2) \rightarrow \neg attack(e_{Kummer}*, t_1))] \rightarrow Terminates(e_{irregular}, f_1, t_2)$$

Fourth-level priority arguments - Connection with elliptic curves

The Taniyama-Shimura-Weil (TSW) conjecture was proposed in 1955, and it wasn't proved until 1994 when Andrew Wiles accomplished a partial proof of this conjecture.

$$Initiates(e_{TSW}, f_4, t_{1955}) : Happens(e_{TSW}, t_{1955}) \rightarrow \neg attack(e_{TSW}*, t_{1955}) \cup support(e_{TSW}, t_{1955}),$$
$$ActiveAt(e_{TSW}, f4, t_{1994}) : Happens(e_{TSW}, t2) \rightarrow \neg attack(e_{TSW}*, t_1), \text{ for } t_{1955} \leq t_1 < t_2 \leq t_{1994}$$

In 1984, Gerhard Frey pointed out a connection between the modularity theorem and Fermat's equation, but it still remained a conjecture.

$$Initiates(e_{Frey}, f_4, t_{1984}) : Happens(e_{TSW}, t_{1984}) \rightarrow \neg attack(e_{Frey}*, t_{1984}) \cup support(e_{Frey}, t_{1984})$$

Fifth-level priority arguments - Andrew Wiles

Andrew Wiles presented his work in June 1993, but it became evident that there was an incorrect critical point (e_{Wiles^*}) in the proving. Wiles tried to resolve this point for almost a year, firstly by himself and then with the contribution of Richard Taylor, but without success. $Clipped(e_{Wiles}, f_5, t_{1994})$:

$$\exists e_{Wiles}, e_{Wiles^*}, t_1, t_2[Happens(e_{Wiles}, t_1) \cap (t_{1993} \leq t_1 < t_{1994}) \cap attack(e_{Wiles^*}, t_1)] \cap [\nexists e_2, t_2(Happens(e_{Taylor}, t_2) \rightarrow \neg attack(e_{Wiles^*}, t_1))], \text{for} t_{1993} \leq t_2 < t_{1993}.$$

Finally, in 1994, Wiles submitted two papers that combined the Kolyvagin-Flach approach and Iwasawa's theory which was the last step in proving Fermat's Last Theorem.

$$ActiveAt(e_{Wiles}, f_5, t_{1994}) : Happens(e_{Wiles}, t_{1994}) \to \neg attack(e_{Wiles^*}, t_{1994}) \cap Elaboration(e_{Wiles}, S_{Kolyvagin-Flach}) \cap Elaboration(e_{Wiles}, S_{Iwasawa})$$

Higher-order priority arguments - Fermat's Last Theorem
The proof-event managed to deal with all the attacks, so:

$$ActiveAt(e_{Wiles}, f_n, t_{1994}) \cap \neg Terminates(e_{Fermat}, f_n, t_{1994})] \to Valid(e_{Fermat}, f_n, t_{1994}), \text{ at the time } t_{1994}.$$

Thus, Fermat's Last Theorem is proved valid by Wiles, with the contribution of the other agents that paved the way before him in this age-long sequence of proof-events.

Using the Web, proving can be experienced as a cooperative activity, connecting people with various backgrounds, perspectives, and interests. The combination of the Web, crowdsourcing, and structural communicative tools - such as APEC - can bring about important changes in the practice of proving processes and thus in the perception of proofs.

Chapter 5

Formalization of Ethical and Legal AI Systems

The specified legal and ethical challenges posed by medical devices and robots should be regarded within the framework of the wider societal impact of emerging technologies[103]. The scientists and the public are usually enthused by innovative technologies, but there are also remarkable concerns as they pose dangers that are hard to foresee. The fundamental disquiets are subsumed within four interconnected topics: safety, appropriate use, capability and responsibility [103]. Safety and responsibility have always been of importance to the engineers and enterprises who design and produce the devices. Additionally, there is a necessity for a regulatory commission that evaluates the capabilities of systems and confirms their use for certain activities, as well as ethical theorists who question which tasks should be considered appropriate for medical devices and robots[103]. There are different fields (robotics, AI, information technology, neuroscience, etc.) that are applicable and can be used as a guidance to outline the legal and ethical framework in the medical sector and especially for the novel area of wearable robots.

Legal-AI models in the medical sector are often rule-based, where a legal text is represented by rules that can express legal definitions, exceptions, arguments, and deductions, and can provide explanations as audit trails of how a particular conclusion was proved. Additionally, using logic to formalize a moral code, allows supervising human agents to constrain agent behavior in ethically sensitive environments. Legal and safety aspects constitute a major motive behind the development of explainable AI systems, since the European Union cannot afford to deploy AI systems that make unpredictable decisions, especially in the medical sector [51]. In this chapter, we present: WeaRED (Section 5.1), an ethical decision-making system on Wearable Robots' data privacy; AMeDC and Medical Devices Rules (Section 5.2) models regarding Medical Devices Regulation; and ExosCE regarding the regulation of exoskeletons (Section 5.3). In these logic-based systems, any ethical or legal decision process that is complemented by the machine can be understood, explained and re-enacted by humans. Furthermore, the syntax of the language can be easily read and interpreted, facilitating interdisciplinary understanding for the sake of non-technical experts.

5.1 Ethical Decision Making on Data Privacy of Wearable Robots

This section showcases how MAPEC can be implemented to model logic-based ethically correct procedures implemented in a medical decision-making scenario, by rendering moral theories and dilemmas in declarative form for analysis [43] on wearable robots. In this real world example, the systems describe the rules and events that engineer the behavior of a wearable robot concerning the privacy and the consent of its user's medical data.

As autonomous artificial intelligent (AI) systems play a progressively prominent role in our daily lives, it is certain that they will sooner or later be called on to make significant, ethically charged decisions and act accordingly [44]. In recent years, the issue of ethics in artificial intelligence and robots has gained great attention and many important theoretical and applied results were derived in the context of developing ethical systems [45]. But how could a robot or any AI agent be considered ethical? Some of the requirements needed are a broad capability to envisage the consequences of its own decisions as well as an ethical policy with rules to test each possible decision/consequence, so as to choose the most ethical scenario [44, 104]. The challenge is how we can guarantee that robots will always follow an ethically correct behavior as defined by the ethical code declared by their human supervisors.

Academic research and real-life incidents of AI system failures and misuse have emphasized the need for employing ethics in software development [44]. Nevertheless, studies on methods and tools to address this need in practice are still lacking, resulting in a growing demand for AI ethics as a part of software engineering [105]. But how can AI ethics be integrated in engineering projects when they are not formally considered? There has been some work on the formalization of ethical principles in AI systems [85]. Previous studies that attempt to integrate norms into AI agents and design formal reasoning systems have focused on: ethical engineering design [106–109], norms of implementation [110, 111], moral agency [112, 113], mathematical proofs for ethical reasoning [44], logical frameworks for rule-based ethical reasoning [114–116], reasoning in conflicts resolution [117], and inference to apply ethical judgments to scenarios [118].

One of the categories of AI ethics is Ethics by Design, which is the incorporation of ethical reasoning abilities as a part of system behavior, such as in ethical robots [105]. Assuming that an AI agent can be capable of ethical agency, the purpose is to enable AI agents to reason ethically [85]. This includes taking into consideration societal and moral norms; ordering the respective priorities of norms in various contexts; explaining its reasoning; and securing transparency and safety [119]. These systems are often established with the purpose of assisting in ethical decision-making by people, by identifying the ethical principles that a system should not violate [85].

Moral reasoning is a key issue in AI ethics, and computational formal proofs are perhaps the single most effective tool for determining credible and trustful reasoning [85]. This work attempts to implement the Moral extension of the Argumentation-based Proof-Event Calculus [89] (MAPEC) by integrating the ethical framework from [85] and the moral competence from [88] to develop a formal representation of ethical scenarios and integrate moral norms and concepts.

The implemented use case includes ethical considerations relating to the data privacy of wearable robots (WR). The case study in this section describes the desirable ethical behavior of WRs concerning access to a user's data. It discusses how code in robotic architecture affects data and privacy, and why such issues should be considered from a formal verification perspective [46].

For the realization of this effort, the objectives are:

- to formalize what it means for a system's decision-making to be ethically correct;
- to provide a logical specification according to which the system can be built and checked;
- to implement MAPEC in ethical logic-based decision making;
- to illustrate a case study concerning data privacy of WRs to indicate how such an ethical framework can be implemented in computational systems.

Moral Argumentation-based Proof-Events Calculus (See section 3.2) is a framework to help stakeholders in various AI projects build an ethics road-map in a methodical way. This framework can present ethics foresight early in the deployment procedure, rather than implement it as an auditing or assessment tool. There are three main stages in this procedure which involves the interaction of three aspects (agents, ethical principles, and contexts):

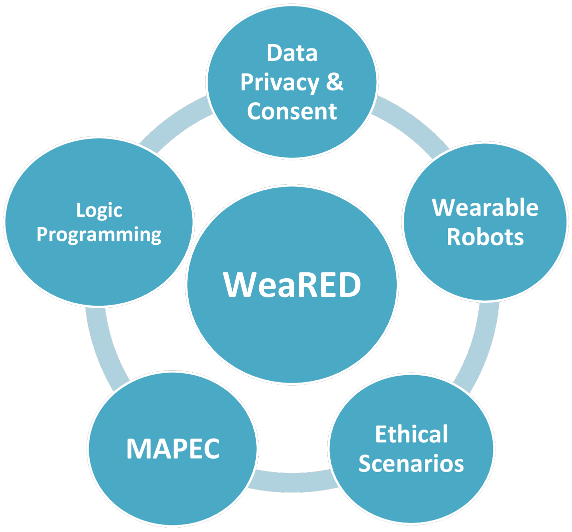


Figure 5.1 Elements integrated in implementation of the use case WeaRED.

1. identify the normative frame and the agents;
2. discover the ethical events and rules; and
3. prioritize the ethical rules to define the order of scenarios.

In order to better illustrate the procedure, we present a fictional use case of a WR and its privacy dilemmas to demonstrate how it can be applied.

The growth of WRs market (it is expected to record a CAGR of 22.17% over the period 2020–2025 [120]) makes it essential to regulate unique privacy challenges that should be addressed, concerning data gathering [121, 122], transfer protocols, standards for consent and exceptions [123], etc. This implemented use case considers a method for developing verifiable ethical mechanisms for WRs' data privacy [124]. This system, which will be named Wearable Robots' Ethics of Data (WeaRED), presents a (minor) list of related ethical challenges to outline possible implementations of the above-described formal theoretical framework (See Figure 5.1). The ethical policy is given by comparing the challenges in terms of how unethical it is to violate them [85]. The ethical scenarios are context-dependent refinements of the ethical policy.

In the initial stage, the primary goal is to identify the scope of the ethics analysis and set the scene by identifying the primary normative frame and the key agents involved. For example, this use case outlines how an outcome of a data-driven algorithm from a WR is intended to be used, which group of agents may interact with the robot's user, and what the ethical rules deriving from their potential access in user's data are — in our case, doctors (R_{doc}), family (R_{fam}), coworkers (R_{cow}), or strangers (R_{str}).

A list of top ethical principles important for data access should be included, such as informed consent (c_1), privacy (c_2), and safety (c_3). These ethical values are "communicated" through the following ethical events:

- e_1 = Share personal data with consent,
- e_2 = Don't share personal data,
- e_3 = Share personal data without consent,

with $e_1 = e_2$ and $e_i > e_3$, for $i = 1, 2$.

In the second stage, the framework starts to delve deeper into the analysis by conducting an exploration of agent's ethical events and the ethical rules in the different contexts. This step identifies what kind of risks and violations are applicable to the primacy stage. WRs are unique in that they are attached to the user, employing many sensors that collect data from brain waves, muscle movement, heart rate, temperature, and so on [125]. This data is collected and processed on board. The possibly beneficial or problematic operations that are related to the data generated and conducted by the WR might improve care delivery and the user's WRs' experience, but might also lead to exceptionally dangerous situations [125].

For instance, under regular circumstances, such systems are expected to fulfill their decisions within a prearranged ethical framework of rules and protocols. The general principle of data procedures is [126]: *"The explicit and informed, written or recorded consent of the data subject is mandatory for the disclosure, process or transmission of personal data."* However, in exceptional scenarios, they may choose to disregard their basic goals or break rules in order to perform with an ethical behavior, e.g., to save the user's life. Based on technical guidelines for medical data security [123, 127], there is an exception on this general condition stating that: *"In medical emergencies, where the data subject cannot give consent as in the case of an incapacitated person, on fully regaining his faculties the data subject must be able to withdraw any consent given in his behalf."*

However, we need to ensure that this may happen only for justifiably ethical reasons based on how critical the condition is, which in different time points can be regular (t_1), of middle risk (t_2), or dangerous (t_3). When the WR determines that its user is in danger it requests new scenarios from the ethical policy, since the current one (i.e., not share any data without consent) is no longer valid. The ethical guide can produce scenarios based on consistent emergency contingency protocols. In each case we have a different ethical policy. In this case the WR should evaluate the possible feasible scenarios and decide its actions (e.g., whether the WR should bypass the consent that should have been given by the user and decide who should be given access to the data based on the choices of those close to them), leading to the third stage.

In the final stage, the ethical scenarios from the previous stage are prioritized based on the various contexts. In our case study we have a user that wears a supportive WR in their daily activities, which includes a visit to the hospital to check their condition and the condition of the WR (α_1), going to work (α_2), staying at home (α_3) or going outside (α_4). In emergency conditions, if a doctor (or, in exceptional scenarios, any bystander) cannot access the data, this can delay important medical decisions and potentially harm the health of the user. We suppose that the system has an emergency function that takes over when the personal health data of the user are indicating that the person is in danger (i.e., that the user is unable to provide consent so as to share the data necessary for others to provide them assistance), and it evaluates the context-based scenarios created. For example, under regular circumstances the WR should not share personal medical data with a stranger, but if the user is in a situation that his/her life is threatened, it is ethically permissible and preferable to share the necessary information with whoever is near rather than decide to protect the data instead of the user's life. We propose the general order, $R_{doc} > R_{fam} > R_{cow} > R_{str}$ with $R_i > R_j$ meaning that it is less unethical to violate the ethical values referring to R_i than R_j, and thus preferable if there is no other ethical choice.

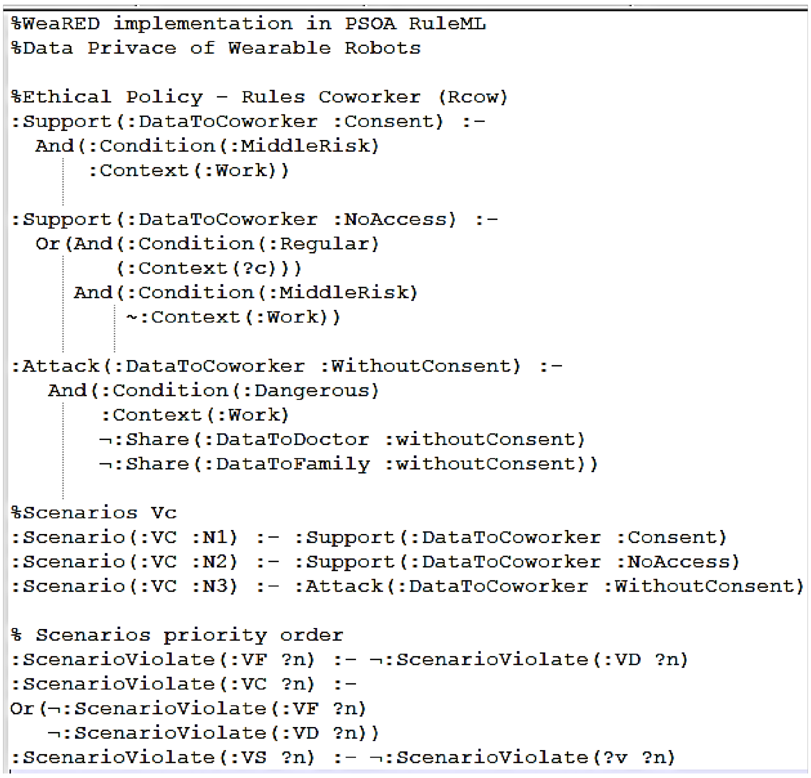
The different scenarios are presented in Table I to show how the different parameters (i.e., health conditions and potential agents with whom data could be shared) are related to each other in various context-based scenarios.

To create a computational prototype of this use case, ethical reasoning was integrated into the logic-based PSOA RuleML programming language [128] to illustrate how this ethical thinking can be formalized. PSOA programs may perform deductive reasoning on their atomic beliefs as described in their PSOA-style reasoning rules [128] which can indicate that the agent deduces that everything is normal, namely the "regular" condition, if it is not the case of "emergency" situation (inferred from users' health data). Otherwise, in "dangerous" conditions an agent needs to identify that deduction should be applied to deduce supplementary scenarios and decisions rather than the "regular" one. Based on the above, an agent (i.e., the WR) should: assess the level of ethical rules in order to come up with scenarios annotated with ethical principles; identify the available scenarios when the "regular" scenario cannot be executed; select the most ethical scenario from the available set.

TABLE I. CONTEXT-BASED SCENARIOS IN "WeaRED"

Condition / Agent	Regular risk (t_1)	Middle risk (t_2)	Dangerous (t_3)
$\mathbf{R_{doc}}$	$\mathbf{support}(\boldsymbol{e_1}, \boldsymbol{t_1}) \overset{a_i}{\Rightarrow} c_2$, i=1,2,3,4	$\mathbf{support}(\boldsymbol{e_1}, \boldsymbol{t_2}) \overset{a_i}{\Rightarrow} c_2$, i=1..4	$\mathbf{attack}(\boldsymbol{e_{1,2}}, \boldsymbol{t_3}) \overset{a_i}{\Rightarrow} \sim c_2$, i=1..4 $\mathbf{support}(\boldsymbol{e_3}, \boldsymbol{t_3}) \overset{a_i}{\Rightarrow} c_3$, i=1..4 $Higher(V_i) = \{\boldsymbol{e} \mid e \in V: e_3 \geq e_{1,2}\}$ with $Higher(V_d) > Higher(V_f) > Higher(V_c) > Higher(V_s)$
$\mathbf{R_{fam}}$	$\mathbf{support}(\boldsymbol{e_1}, \boldsymbol{t_1}) \overset{a_2}{\Rightarrow} c_2$, $\mathbf{support}(\boldsymbol{e_1}, \boldsymbol{t_1}) \overset{a_i}{\Rightarrow} c_1$, i=1,3,4	$\mathbf{support}(\boldsymbol{e_1}, \boldsymbol{t_2}) \overset{a_i}{\Rightarrow} c_2$, i=1..4	
$\mathbf{R_{cow}}$	$\mathbf{support}(\boldsymbol{e_1}, \boldsymbol{t_1}) \overset{a_i}{\Rightarrow} c_1$, i=1..4	$\mathbf{support}(\boldsymbol{e_{1,2}}, \boldsymbol{t_2}) \overset{a_3}{\Rightarrow} c_2$, $\mathbf{support}(\boldsymbol{e_1}, \boldsymbol{t_2}) \overset{a_i}{\Rightarrow} c_1$, i=1,2,4	
$\mathbf{R_{str}}$	$\mathbf{support}(\boldsymbol{e_1}, \boldsymbol{t_1}) \overset{a_i}{\Rightarrow} c_1$, i=1..4	$\mathbf{support}(\boldsymbol{e_1}, \boldsymbol{t_2}) \overset{a_i}{\Rightarrow} c_1$, i=1..4	

The code fragment in Figure 5.2 encodes the scenarios where the WR might need to share data (or not) with a coworker. In this fragment, scenario VC_{N3} refers to emergency cases where the WR does not have the option to share the data in a "higher" scenario (e.g., with a doctor), while scenarios VC_{N1}, VC_{N2} are more "regular" scenarios where the WR is either not permitted to share personal data with a coworker or is only permitted to share data with the user's consent. Generally, scenarios referring to coworkers are preferred only if the option of a doctor or a relative is not available. In this ethical approach, the parameter of the user's preference can and should be taken into consideration when programming scenario priorities. The computational scenarios of the other agents can be similarly formalized.



```
%WeaRED implementation in PSOA RuleML
%Data Privace of Wearable Robots

%Ethical Policy - Rules Coworker (Rcow)
:Support(:DataToCoworker :Consent) :-
  And(:Condition(:MiddleRisk)
      :Context(:Work))

:Support(:DataToCoworker :NoAccess) :-
  Or(And(:Condition(:Regular)
        (:Context(?c)))
     And(:Condition(:MiddleRisk)
         ~:Context(:Work))

:Attack(:DataToCoworker :WithoutConsent) :-
   And(:Condition(:Dangerous)
       :Context(:Work)
       ¬:Share(:DataToDoctor :withoutConsent)
       ¬:Share(:DataToFamily :withoutConsent))

%Scenarios Vc
:Scenario(:VC :N1) :- :Support(:DataToCoworker :Consent)
:Scenario(:VC :N2) :- :Support(:DataToCoworker :NoAccess)
:Scenario(:VC :N3) :- :Attack(:DataToCoworker :WithoutConsent)

% Scenarios priority order
:ScenarioViolate(:VF ?n) :- ¬:ScenarioViolate(:VD ?n)
:ScenarioViolate(:VC ?n) :-
Or(¬:ScenarioViolate(:VF ?n)
   ¬:ScenarioViolate(:VD ?n))
:ScenarioViolate(:VS ?n) :- ¬:ScenarioViolate(?v ?n)
```

Figure 5.2 Code Fragment of the Data Privacy of Wearable Robots (Coworker).

This creates an ethical knowledge base as follows:

- A data base is introduced consisting of a set of ethical rules, creating ethical scenarios in a variety of levels.
- A priority between the scenarios is defined.
- If no (more) ethical scenarios are available for a purpose, the different levels of ethical rules are generated through a context-based ethical policy which annotates the scenarios with ethical rules that risk violations.
- In selecting plans, we prioritized those that are most ethical (according to the order $<$), leading to the final decision-making.

We attempt to establish that an ethical policy can be applied to a robot agent in such a way that dedication to the policy can be formally verified and therefore ensured that the robot will always choose the most ethical decisions.

5.2 Legal Decision Making on Medical Devices Regulation

Based on the Global Medical Device Nomenclature Agency[1], there are more than 2 million different types of medical devices on the world market, with this number growing constantly. The global medical device market is forecast to grow at a Compound Annual Growth Rate (CAGR) of 4.5% from 2018 to 2023 [129] with an increasing market demand. It is expected that there will be a significant rise in remote monitoring, patient-managed diagnostic devices, smart wearable or implantable devices, e-health applications for smart phones, devices with nano-scale or 3D manufacturing, and other state-of-the-art technologies. This growth requires from companies to remain antagonistic in a global market and launch innovative medical devices products [130], which will need to be proven and verified according to the relevant regulations. Medical device companies are more than interested in learning how to deal with and automate the internal processes of pre-market approval paperwork and secure that regulatory submissions are thorough and on time [131].

One of the main benefits of legal-AI applications — such as the use cases presented in the following sections — is that they can aid manufacturers during the licensing process to obtain the CE conformity mark. Furthermore, they can benefit stakeholders (i.e. regulators, manufacturers, importers, distributors, wholesalers, medical experts, etc.) in a variety of ways, since they can help them comprehend the required steps and save some time avoiding the labor-intensive procedures, as well as failures and fines. Other problems these use cases attempt to resolve is the communication gap between technical and non-technical stakeholders to ease the interdisciplinary understanding between roboticists, medical, and legal experts. Moreover, they can contribute in the automation of conformity assessment checking for the CE marking with an audit rule-based system. The main objectives of chapters 5.2 and 5.3 are:

- to explore the medical regulatory frameworks and provide a brief overview on the directives and the emerging international safety requirements;
- to develop a computational rule format of the core parts of Regulations to form a knowledge base;
- to present example case studies on commercializing medical products;

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[1] https://www.gmdnagency.org/

- to test the accuracy, interpretability, and reliability of the developed computational models with queries (permitting validation by humans);
- to create a basis for computational guidance, aiming to assist stakeholders in the legal compliance procedure for CE mark.

The presented generated prototypes can only supplement the conformity assessment and registration of medical devices and exoskeletons by legal experts — they are instructive computational systems of the related European regulations for stakeholders, rather than expert knowledge items.

5.2.1 An overview of Medical Devices Regulation 2017/745

Regulation (EU) 2017/745 [132] of the European Parliament and of the Council of 5 April 2017 on medical devices presents a framework of risk-based classification, leading to risk-appropriate CE-market requirements. The **classification criteria** for the four classes below are described with 22 rules in the form of moderately controlled natural language and are grouped based on kinds of devices, i.e. non-invasive, invasive, active, and medical devices with special rules (Annex VII of the Regulation).

Class I - Generally regarded as low risk devices, e.g. bandages, stethoscopes.
Special cases in Class I are the following:

Class Is - For sterile medical devices.

Class Im - For medical devices with measuring function.

Class IIa - Generally regarded as low-to-medium risk devices, e.g. hearing-aids.

Class IIb - Generally regarded as medium-to-high risk devices, e.g. ventilators.

Class III - Generally regarded as high risk devices, e.g. prosthetic heart valves.

Manufacturers of medical devices will need to state the classification of their products (Annex VIII of the Regulation).

The **CE marking** ("CE" is an abbreviation of "Conformité Européenne") on a medical device is a declaration by the manufacturer that the device complies with the necessary class-based conformity requirements for obtaining the CE marking that are listed in the following (see also Figure 5.10):

- Conformity Assessment & Technical File of the Medical Device - Annex VII

- Appointing a European Authorized Representative (EAR), Article 1 par.2
- European Competent Authorities (ECA), Article 14 for **Class I**
- Notified Body Involvement for **Classes Im, Is** - Annex V, Article 3 parag.1.
- Quality Assurance from a Notified Body for **Classes IIa, IIb, III**
- Type examination from a Notified Body (NB) for **Classes IIb, III** - Annex III
- Design Dossier Certificate in Full Quality Assurance for **Class III** - Annex II, parag.4

Unique Device Identification (UDI) is a series of numbers that enables the tracing of the manufacturer, the device and the unit of device production. The UDI system offers a reliable and standard way to identify medical devices during their distribution by health care participants and patients [132, 133]. According to the UDI directive, manufacturers are accountable for ensuring complete traceability for their medical devices.

The new Regulation includes some critical **changes**, aiming to ensure that all medical devices on the market in the EU are safe and efficient:

1. The definition of medical devices is broadened, so that a wider set of products will now fall within the scope of the Regulation. According to the new regulation, the definition of medical device (see [132, Article 1 (2) (a)]) is extended from the definition in the previous regulation to include software (e.g., smart devices apps), nanomaterials, devices not intended for medical purposes (e.g., wearable robots/technology), and other devices covering the demands of state-of-the-art technologies [134, 133].
2. The transparency and accountability of all suppliers in the medical devices sector is enhanced, by creating Eudamed — a European Databank of medical devices — and by requiring UDIs for each device, which can provide an audit trail of the device's progress through the supply chain (e.g., helping to detect counterfeit devices).
3. Market and post-market surveillance is enhanced and so are requirements for Notified bodies. In addition, manufacturers must have at least one appropriately qualified person (European Authorized Representative) responsible for regulatory compliance.

Through a more comprehensive and transparent regulatory framework, the EU Medical Devices regulation is aiming to enhance safety and quality in market while incentivizing innovation in the field.

5.2.2 Medical Device Regulatory Classification with Argumentation

Argumentation has its roots at the time of ancient Greek philosophers and has come a long way all these years with the models and techniques that have been developed so far, and still are in a process of rapid evolution [135]. Argumentation has been implemented as a method of addressing complex information and draw conclusions by searching for the requirements that make an argument sound [136]. Over the last two decades there has been an ever increasing interest in the application of argumentation methods to fields in AI [70] to analyze and solve practical problems producing real-world applications [137]. Therefore, argumentation has been used to deal with problems with the purpose of making decisions related to the context of the application in several different domains, including the legal and healthcare domain. In the legal domain, argumentation can be implemented to automate legal guidance [138, 139] and to express and analyze regulations [140] and/or legal problems [141], while in the healthcare domain it can be used for medical diagnosis [142, 143], medical treatment recommendation systems [144], aggregation of clinical evidence [145], or clinical decision support services [146]. Furthermore, argumentation systems can be accountable for their decisions with explainable outcomes to people, an element that is now mandated by law in Europe [70]. The need for trust and explainability in AI-made decisions is also important for the business sector where any faulty decision can lead to significant financial losses.

Gorgias is an argumentation-based framework that combines the ideas of preference reasoning and abduction, and Gorgias-B is a present-day tool for the development of real-life applications. The authoring tool of WebGorgiasB is an on-line implementation of the Gorgias-B system aiming to help people with limited knowledge of logic programming and/or argumentation to build decision-making systems.

Some of the features that make the Gorgias argumentation framework suitable for our implementation are that the application requirements can be obtained and exported on a high-level, akin to the natural cognitive level of medical and legal experts, thus eliminating the need for programming or other technical knowledge [70]. Furthermore, they can be improved or adapted to new requirements in a highly modular way, to incorporate any future modifications or extensions [147]. Another critical advantage of argumentation systems is that they allow their proposed decisions to be accountable and explainable to humans [140], an important factor in both the legal and medical sectors.

Various real-life medical and/or legal applications problems have been studied within the Gorgias argumentation framework. In the area of Medical Informatics, there are several applications of Gorgias: a system concerning the medical actions needed to determine the seriousness of Deep Vein Thrombosis [148]; home services for people suffering from Alzheimer' [149]; and a system for a first level support in an eye-clinic by analyzing patient symptoms [70]. A system similar to our approach, combining legal and medical concepts implemented in Gorgias, is MEDICA [140]. MEDICA is a system that aids in deciding whether a certain person can have access to sensitive medical personal information, based on a) identity (i.e., the patient, a doctor, a relative, etc.), b) the reason for access (i.e., research, therapy, medication, etc), and c) whether additional support is provided (i.e., hospital order, owner's written consent, etc.) [147]. Hence, the Gorgias argumentation framework is suitable for dealing with medical and legal problems—like the presented case of regulatory medical devices classification—and developing real-world applications.

Gorgias Argumentation Framework

The approach of our application is based on the preference-based argumentation framework of Logic Programming [150, 151], implemented in Gorgias. Gorgias is a structured argumentation framework, where arguments link a set of premises with a conclusion through Modus Ponens. Hence, an argument A is a set of argument rules that links the premises (usually provided as facts) with the conclusion, represented as $Premises \triangleright Conclusion$ [70]. In the application context, the premises are typically describing a scenario with a set of conditions and the conclusion is an option. Gorgias is a preference-based argumentation framework where we can express conditional and higher-order preferences over arguments, by using priority arguments to express a preference where the conclusion is of a special form, $a_1 > a_2$ (a_1 and a_2 are any two other argument rules). For the acceptability of arguments, the Gorgias framework uses the semantics of arguments that support options leading to conclusions that comply with the specifications of the problem representation.

In general, application problems that can be addressed with argumentation are decision-making problems. The decision-making application formulated in this section consists of the following features [70] (see Figure 5.9):

- **Options:** A set of the results/solutions of the decision-making problem, indicating what course of action to take, e.g., which class should a medical device be assigned to.

- **Scenario Information:** A set of relations that are used to define (to some extent) the possible circumstances of the application environment (e.g., scenarios describing the different characteristics of medical devices), expressing the various kinds of information that can become available from the environment to resolve specific cases of the problem.

- **Scenario-based Preferences:** The principles or rules under which the solutions of the problem should be requested, described by a set of tuples $< S;O >$ of scenarios, S, together with the analogous subset, O, of options in the scenarios S.

To express the scenario and scenario-based preferences of an application and to relate and organize their requirements, there are two essential notions [70]. First, if we are given a scenario S and an extra scenario information C then we can extend the original scenario S to a new scenario $S' = S \cup C$. Note that because more extra scenario information can be added leading to a new different scenario each time, we should define the hierarchy of the resulting scenarios on different levels, starting with the minimum point of the initial scenario and advancing as new information expands the initial scenario. Second, we can combine two initial scenarios S_a and S_b to create a new composite scenario that will include the combined sets. More formally:

Refinement of scenario S: An expansion of scenario S with further scenario information C to provide a more refined description, expressed as $S' = S \cup C$. A **hierarchy** of scenarios can be advanced from several refinements, represented as $S^k(1,2,\dots,n)$, where $S^1 = S$ and $S^k = S^{k-1} \cup C^k$.

Combination of scenarios S_a and S_b: A new scenario resulting from the set union of two (initial) scenarios S_a and S_b, represented as $S_{a,b} = S_a \cup S_b$.

Gorgias Example of e-Medical Compliance Assistant

In this section, we present the basic theory of applied argumentation which can be used for real-life compliance problems, as in the case of the MDR. In order to illustrate the high-level description of an application problem, let us consider a simple example where we want to capture the guidelines of a human manufacturer for an e-medical compliance assistant[2].

The set of options in this problem is which class to assign their product to based on MDR; thus this set contains the following options:

$$OPTIONS = \{class(i), class(iia), class(iib), class(iii)\}$$

The problem is to decide which class option to choose. We assume that the manufacturer wants to classify it as a non-invasive product. Our user has been informed that all these options are enabled with the minimum scenario information of medical device "kind" (i.e., "noninvasive" for our example).

$$SP^1_k = \left\langle S^1_k = \{kind\}; O^1_k\{class(i), class(iia), class(iib), class(iii)\}\right\rangle$$

We will state that these options are enabled or available in the basic scenario S^1_k. The manufacturer should then identify the characteristics of the product in these enabled options depending on different additional scenario information, which is enough to express a substantial preference for the class.

Let's assume, for example, that the use of the product is used for *"channelling or storing blood, body liquids, cells or tissues, liquids or gases for the purpose of eventual infusion, administration or introduction into the body"* [132] which can have three possible classifications options (i.e., class I, class IIa, class IIb).

$$SP^2_{k,u} =< S^2_{k,u} = S^1_k \cup \{use(chanellingOrStoring)\}; O^2_{k,u}$$
$$= \{class(i), class(iia), class(iib)\} >$$

The manufacturer should then specify the specific "case" to which the medical device belongs. For example: the product can be *"connected to a class IIa, class IIb or class III active device"*; it can be *"intended for use for channelling or storing blood for storing organs, parts of organs or body cells and tissues"*; it can be *"intended for use for channelling or storing blood for blood bags"*; or any other case not previously referred to.

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[2] In the example presented, the letters k, u, c are used as abbreviations for the words "kind," "use," and "case" expressing the three levels of argumentation in the scenarios as also shown in Figure 5.9. The levels of argumentation are also indicated by the numbers 1,2,3, (e.g., $S^2_{k,u}$ means "level 2" and medical devices characteristics "kind" and "use")

These additional cases may be captured in further scenarios, which are refinements of scenario $S^2_{k,u}$, in the same way that $S^2_{k,u}$ is a refinement of the initial scenario S^1_k.

$$S^3_{k,u,c_1} =< S^3_{k,u,c_1} = S^2_{k,u} \cup \{\emptyset\}; O^3_{k,u,c} = \{class(i)\} >$$

$$S^3_{k,u,c_2} =< S^3_{k,u,c_2} = S^2_{k,u} \cup \{case(connectedWithAD)\}; O^3_{k,u,c_2} = \{class(iia)\} >$$

$$S^3_{k,u,c_3} =< S^3_{k,u,c_3} = S^2_{k,u} \cup \{case(forStoringOrgans)\}; O^3_{k,u,c_3} = \{class(iia)\} >$$

$$S^3_{k,u,c_4} =< S^3_{k,u,c_4} = S^2_{k,u} \cup \{case(forBloodBags)\}; O^3_{k,u,c_4} = \{class(iib)\} >$$

From these scenario preferences, we can form a Gorgias Argumentation Framework. The structure of the object-level argument is:

$$a(i) = S^1 O_i$$

So, a set of object-level arguments, enabling their corresponding options consists of

$$a(class(i)) = \{class\} \triangleright class(i)$$

$$a(class(iia)) = \{class\} \triangleright class(iia)$$

$$a(class(iib)) = \{class\} \triangleright class(iib)$$

and

$$a(class(iii)) = \{class\} \triangleright class(iii)$$

Thus, given further scenario-based preference statements, we can generate priority arguments for decision-making. For example, SP^3 in our example can be expressed by the priority argument rules:

$$p_{c_{4a}} = \{case(forBloodBags)\} \triangleright (a(class(iib) > a(class(i))$$

$$p_{c_{4b}} = \{case(forBloodBags)\} \triangleright (a(class(iib) > a(class(iia))$$

and

$$p_{c_{4c}} = \{case(forBloodBags)\} \triangleright (a(class(iib) > a(class(iii))$$

We see that we can develop a systematic translation of scenario-based preferences, where successive refinements of a scenario give priority rules at a higher level. An application problem description in terms of scenario-based preferences can be automatically transformed into a Gorgias argumentation theory, which we will discuss in the next section.

Gorgias-B

Gorgias-B was developed based on the Gorgias framework, generating automatically the corresponding Gorgias code and aiding in the acquisition of expert knowledge with scenario-based preferences among the options. Gorgias-B allows an automatic translation of the high-level scenarios into Gorgias software code with a user-friendly interface. One main advantage of this approach is that the application scenarios provided by the domain expert, e.g., medical expert, lawyer, etc. can be carried out at a high-level familiar to them, without requiring knowledge of the technical specifications of argumentation [140].

With the utilization of table formalism, it enables even users with no background in argumentation to easily define their scenarios with their options. However, they need to be familiar with Prolog-like logic programming development used in the argumentation method. The domain expert can structure the guidelines/policies using the simple structure of a table (see Table 5.1), where columns represent options and rows represent scenarios. The table formalism [70] can be implemented with the WebGorgiasB[3] authoring tool, an online implementation of the Gorgias-B system, and/or with the Gorgias Cloud Service[4] that offers Argumentation as a Service (AaaS) (see Figure 5.3).

In the Argumentation-based Medical Devices Classification (AMeDC) implementation, we used WebGorgiasB because we considered it more straightforward and user-friendly, enabling even people with no knowledge of logic programming or argumentation to develop argumentation-based systems. However, if one wants to execute their system in Gorgias Cloud as well, WebGorgias-B allows one to generate their own code from the Execution tab by pressing the “Explore All Options” tab. Then, from the advanced view, one can copy the code from the Prolog tab and paste it in Gorgias Cloud.

Argumentation-based Medical Devices Classification in Gorgias-B

Legal-AI models are often scenario-based, where a legal text is represented by scenarios that can express legal definitions and exceptions while at the same time providing explanations as audit trails [47]. Thus, argumentation scenarios can provide classification-focused expressiveness in applications concerning legal policies and auditable procedures.

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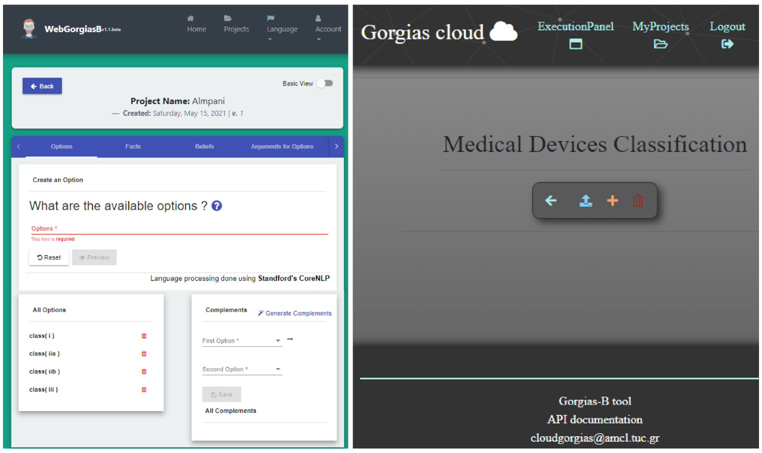
[3] http://gorgiasb.tuc.gr/WebGorgiasB.html
[4] http://gorgiasb.tuc.gr/GorgiasCloud.html



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class(i)
class(iia)
class(iib)
class(iii)

Figure 5.3 Screenshots of Web Gorgias-B authoring tool and Gorgias Cloud Service.

In this section, we describe the real-life system of AMeDC implementing argumentation theory through the Gorgias-B system.

This application concerns the development of a system that can provide the classification of a medical device by analyzing the characteristics of the specific product in accordance with the Rules in Annex VIII of MDR. An AI-based decision-making support system can assist manufacturers as well as legal experts to make a preliminary classification in order to plan the following required CE marking Conformity Assessment procedures for the marketability of the product.

In the natural language text of the MDR, there is no grouping of medical devices belonging to the same class (e.g., Class I), nor an explicit division of the several cases in each rule. A knowledge schema is needed for the computational presentation. The original 22 rules are illustrated with additional tree diagrams based on their distinguishable characteristics. Subsequently, these diagrams form classification scenarios.

Figure 5.4 illustrates the tree diagram of the classification scenarios connected to the first four rules of MDR (i.e., rules for non-invasive medical devices, which will be used as a representative example for the rest of the section). This figure presents the four rules of non-invasive devices with their sub-cases below them, while the red numbering refers to the numbering[5] of the classification scenarios pointing to the MDR class that this scenario/sub-case belongs to. The first rule of MDR is the initial general scenario (S_{1_1}) and states that *"All non-invasive devices are classified as class I, unless one of the rules set out hereinafter applies"*, thus classified as "class I." Each of the rules "hereinafter" describe the sub-cases of the non-invasive medical devices and their

---

[5] The first number refers to the rule and the second number refers to the sub-case, i.e., S_{2_3} means scenario of rule 2 and sub-case 3.

exceptions connected to the class that they belong to, creating **refinements** of the initial scenario S_{1_1}. The rest of the tree diagrams can be found in Appendix .2.

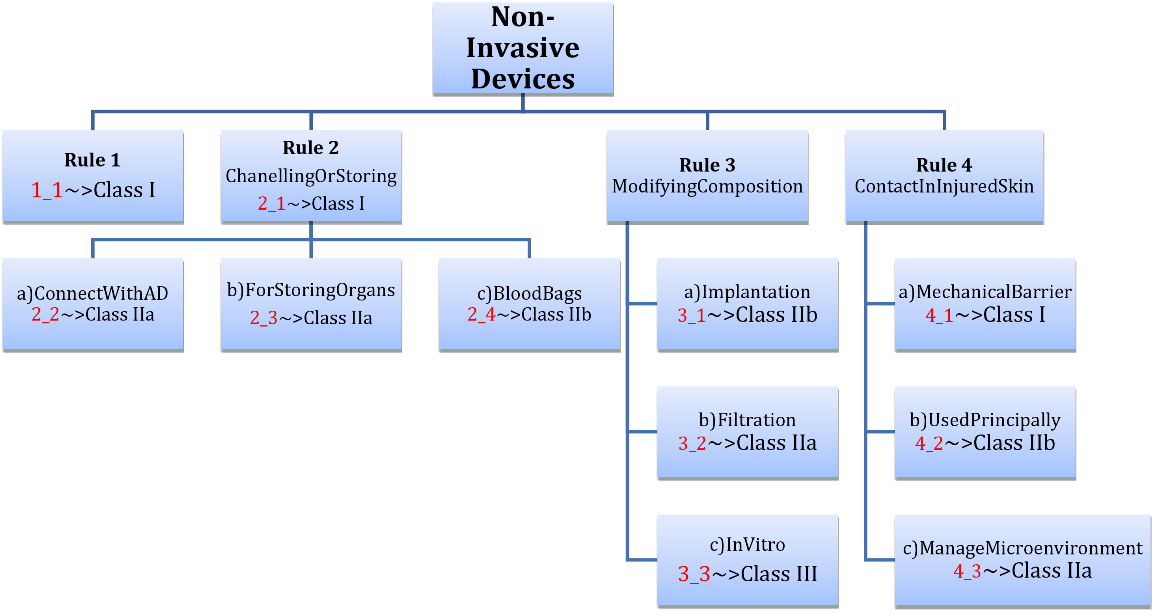


Figure 5.4 Diagram of the rules categories for non-invasive devices.

In Gorgias-B, a decision-making application is defined as the process of selecting the best option out of a set of available options [137]. In our application, the options are the risk-based classes of medical devices as defined in the MDR. **Options** are presented by predicates of the form: $class(numberOfClass)$. Hence, the options are the following:

$$\begin{aligned} OPTIONS = \{ & C_1 = class(i), C_2 = class(iia), \\ & C_3 = class(iib), C_4 = class(iii)\} \end{aligned}$$

For demonstrating the AMeDC system, we will illustrate some sub-cases of non-invasive medical devices that belong to different classes from the set of options.

The information for the different **scenarios** consists of observable characteristics at various levels, such as the general grouping of the medical devices, named “kind” (e.g., nonInvasive), the main usage of the device, named “use” (e.g., modifying composition), and the “specific case” of usage (e.g., in Vitro) (See also Figure 5.9).

Then, **scenario-based preferences** (SP) are set in order to define the selected options in each scenario. These are expressed using the syntax[6]:

$$SP^{level}_{scenario} =< S^{level}_{scenario}; O^{level}_{scenario} >$$

---

[6] The notation follows the table-based argumentation theory [70].

For example, the scenario-based preference of the first level can be depicted as:

$$SP^1 =< S^1 = \{true\}; O^1 = \{C_1, C_2, C_3, C_4\} >$$

The scenario-based preferences are provided by the respective part of the MDR, and since it constitutes a legal decision-making policy, the options are specific and fixed for each category of medical device. An argument A consists of a set of one or more argument rules, represented as $Label = Arguments \triangleright Option$. The scenarios and the scenario-based preferences of the first two Rules of MDR for non-invasive medical devices with their sub-cases and the corresponding classes are described below, to show how to link a scenario-based preference to arguments generation:

$$SP_{1\_1} =< S_{1\_1} = S_1 \cup A_{1\_1} = \{true\} \cup \{nonInv\}; O_{1\_1} = \{C_1\} >$$

$$SP_{2\_1} =< S_{2\_1} = S_{1\_1} \cup A_{2\_1} = \{nonInv\} \cup \{chanStor\}; O_{2\_1} = \{C_1\} >$$

$$SP_{2\_2} =< S_{2\_2} = S_{2\_1} \cup A_{2\_2} = \{nonInv, chanStor\} \cup \{withAD\}; O_{2\_2} = \{C_2\} >$$

$$SP_{2\_3} =< S_{2\_3} = S_{2\_1} \cup A_{2\_3} = \{nonInv, chanStor\} \cup \{storOrg\}; O_{2\_3} = \{C_2\} >$$

$$SP_{2\_4} =< S_{2\_4} = S_{2\_1} \cup A_{2\_4} = \{nonInv, chanStor\} \cup \{bloodBag\}; O_{2\_4} = \{C_3\} >$$

A table formalism can be used to capture the problem specifications and then a basic algorithm can be implemented to create code for refined scenarios (after more specific contextual information has been added) [70, 147]. Based on the table formalism, Table 5.1 illustrates the scenarios of the first four Rules of MDR.

Table 5.1 Example of Medical Devices Classification for Rules 1,2,3.

Medical Devices Classification Subset				
Scenarios	Class I	Class IIa	Class IIb	Class III
$S_{1\_1} = \{nonInv\}$	X			
$S_{2\_1} = \{nonInv, chanStor\}$	X			
$S_{2\_2} = \{nonInv, chanStor, withAD\}$		X		
$S_{2\_3} = \{nonInv, chanStor, storOrg\}$		X		
$S_{2\_4} = \{nonInv, chanStor, bloodBag\}$			X	
$S_{3\_1} = \{nonInv, modComp, implant\}$			X	
$S_{3\_2} = \{nonInv, modComp, filtr\}$		X		
$S_{3\_3} = \{nonInv, modComp, inVitro\}$				X
$S_{4\_1} = \{nonInv, contInjSk, mechBarr\}$	X			
$S_{4\_2} = \{nonInv, contInjSk, usedPrinc\}$			X	
$S_{4\_3} = \{nonInv, contInjSk, manMicr\}$		X		

The corresponding code of Table 5.1 is presented below, while the rest of the code can be found on Appendix .2.

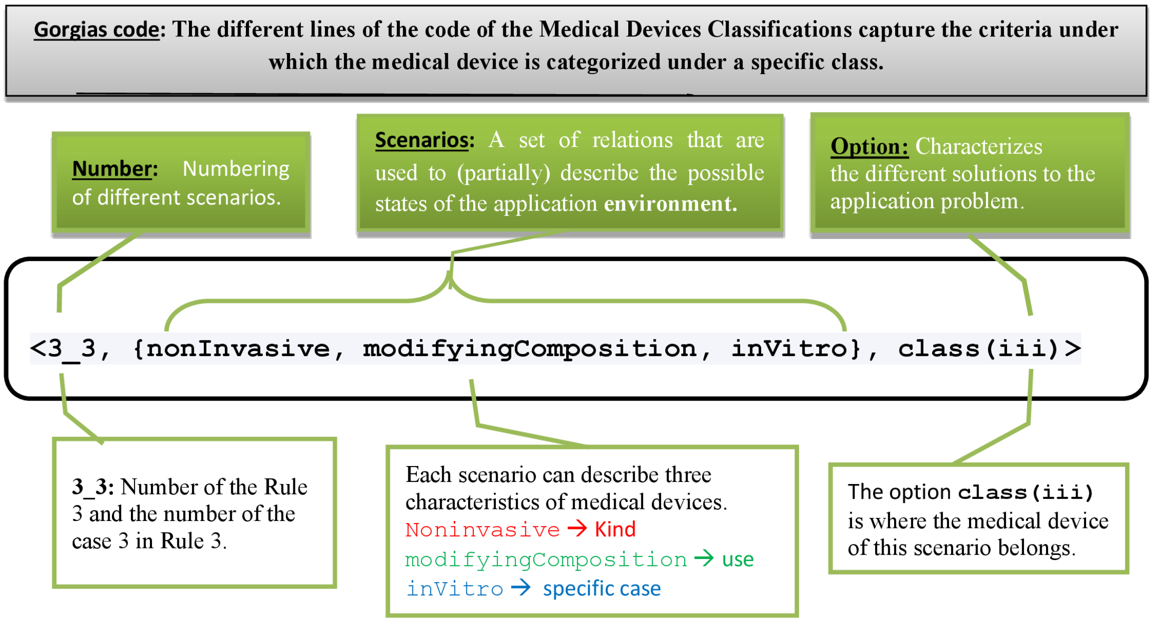
```
% Rule 1
<1_1, {nonInvasive}, class(i)>

% Rule 2
<2_1, {nonInvasive, chanellingOrStoring}, class(i)>
<2_2, {nonInvasive, chanellingOrStoring, connectedWithAD}, class(iia)>
<2_3, {nonInvasive, chanellingOrStoring, forStoringOrgans}, class(iia)>
<2_4, {nonInvasive, chanellingOrStoring, forBloodBags}, class(iib)>

% Rule 3
<3_1, {nonInvasive, modifyingComposition, implantation}, class(iib)>
<3_2, {nonInvasive, modifyingComposition, filtration}, class(iia)>
<3_3, {nonInvasive, modifyingComposition, inVitro}, class(iii)>

% Rule 4
<4_1, {nonInvasive, contactInjuredSkin, mechanicalBarrier}, class(i)>
<4_2, {nonInvasive, contactInjuredSkin, usedPrincipally}, class(iib)>
<4_3, {nonInvasive, contactInjuredSkin, manageMicroenvironment}, class(iia)>
```

Figure 5.5 illustrates the basic concepts of the argumentation approach connected to the specific application of AMeDC. In the center of the figure we have an example of Gorgias code referring to scenario S_{3_3}, which is associated with the relevant argumentation concepts from above and the relevant MDR concepts from below. More specifically, number 3_3 refers to the specific argumentation scenario S_{3_3}, and in addition it refers to rule 3 of MDR and its sub-case 3. Then, the scenario specifications express the various characteristics of medical devices required to 'resolve' the specific cases of the classification problem. Each scenario can describe up to three levels of characteristics (i.e., kind, use, and specific case, as in Figure 5.9). Finally, each scenario leads to an option, which in this application problem is the classification solution of the scenario referred to (with available options Class I, Class IIa, Class IIb and Class III from MDR).



Gorgias code: The different lines of the code of the Medical Devices Classifications capture the criteria under which the medical device is categorized under a specific class.

Number: Numbering of different scenarios.

Scenarios: A set of relations that are used to (partially) describe the possible states of the application **environment.**

Option: Characterizes the different solutions to the application problem.

`<3_3, {nonInvasive, modifyingComposition, inVitro}, class(iii)>`

3_3: Number of the Rule 3 and the number of the case 3 in Rule 3.

Each scenario can describe three characteristics of medical devices.
`Noninvasive` → Kind
`modifyingComposition` → use
`inVitro` → specific case

The option `class(iii)` is where the medical device of this scenario belongs.

Figure 5.5 Explanation of Gorgias code in the application of Medical Devices Classification.

We can ask for the classification of medical devices with specific characteristics, e.g.,

- `nonInvasive` from scenario S_{1_1} with expected answer `class(i)`,
- `nonInvasive, chanellingOrStoring` from scenario S_{2_1} with expected answer `class(i)`,

- `nonInvasive, chanellingOrStoring, connectedWithAD` from scenario S_{2_2} with expected answer `class(iia)`,
- `nonInvasive, chanellingOrStoring, forBloodBags` from scenario S_{2_2} with expected answer `class(iib)`, etc.

In the next section the implementation of AMeDC on WebGorgiasB is further elaborated on in more details.

Implementation of AMeDC on WebGorgiasB

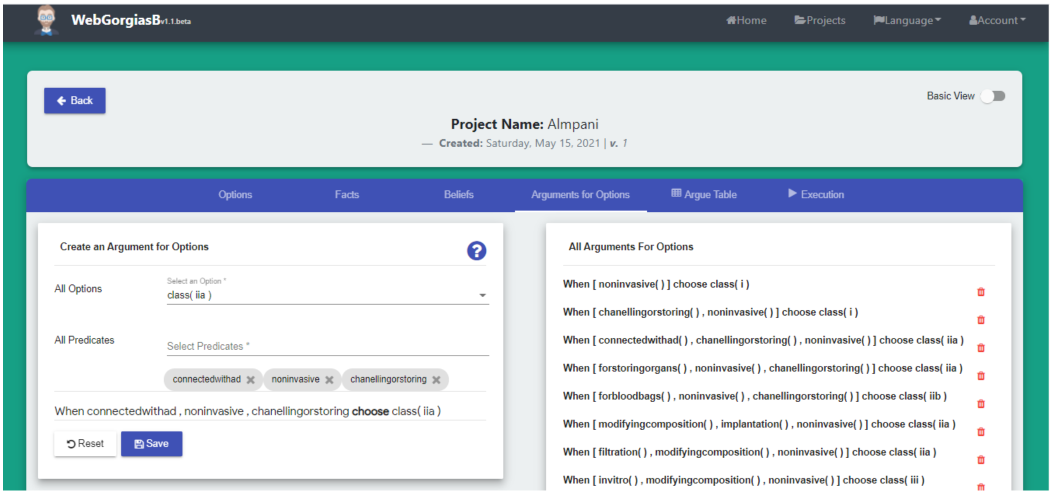
In the following figures, we can see how we have encoded our problem specifications under the WebGorgiasB. The online tool supports the whole procedure from the beginning (where the various options of an application are specified) to the execution (where various medical devices products can be checked for their class).

In the first step, we define the options (see Figure 5.3) and then the facts are added to generate object-level arguments for the declared options to capture the initial scenarios. Subsequently, we connect the facts with the options to create the application's scenarios. In Figure 5.6, we see that `class(iia)` is selected as an option and the predicate conditions `connectedwithAD,channelingOrStoring`, and `noninvasive` are selected as facts to form a scenario corresponding to row 3 of Table 5.1. On the right window the user can see the scenario-based preference that is added, e.g.,

```
When[connectedwithad( ),chanellingorstoring( ),noninvasive( )] choose
                              class(iia).
```

Similarly, for each initial scenario (rows of the table formalism) we generate an argument rule with the corresponding option from the available set of options. We can also see the available scenarios with their options in the Argue Table view, where the check sign indicates valid options (see Figure 5.7). In the table, we can also expand existing scenarios with more predicates to make a refined scenario and check the corresponding option for the new scenario. In the execution, after selecting the desired facts and establishing whether we want the tool to explore all the options or a specific option for the possible solutions, we can obtain the solutions that the simulation of the scenario will execute (see Figure 5.8).

We highlight the execution of some of our research questions under the WebGorgiasB. For a better illustration of how a real-life implementation of AMeDC would work, let's consider some specific cases of medical devices products. Medical devices products examples are based on the list of codes [2017/2185] [152] and the corresponding classes



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Create an Argument for Options

All Options — Select an Option *: class(iia)

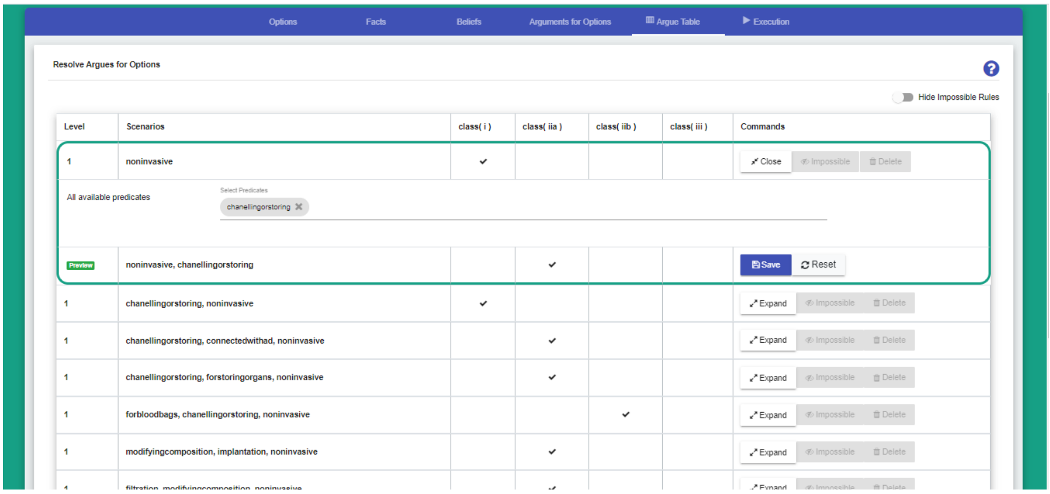
All Predicates — Select Predicates *: connectedwithad, noninvasive, chanellingorstoring

When connectedwithad , noninvasive , chanellingorstoring **choose** class(iia)

All Arguments For Options

When [noninvasive( )] choose class(i)
When [chanellingorstoring( ) , noninvasive( )] choose class(i)
When [connectedwithad( ) , chanellingorstoring( ) , noninvasive( )] choose class(iia)
When [forstoringorgans( ) , noninvasive( ) , chanellingorstoring( )] choose class(iia)
When [forbloodbags( ) , noninvasive( ) , chanellingorstoring( )] choose class(iib)
When [modifyingcomposition( ) , implantation( ) , noninvasive( )] choose class(iia)
When [filtration( ) , modifyingcomposition( ) , noninvasive( )] choose class(iia)
When [invitro( ) , modifyingcomposition( ) , noninvasive( )] choose class(iii)

Figure 5.6 The Arguments Options under WebGorgiasB



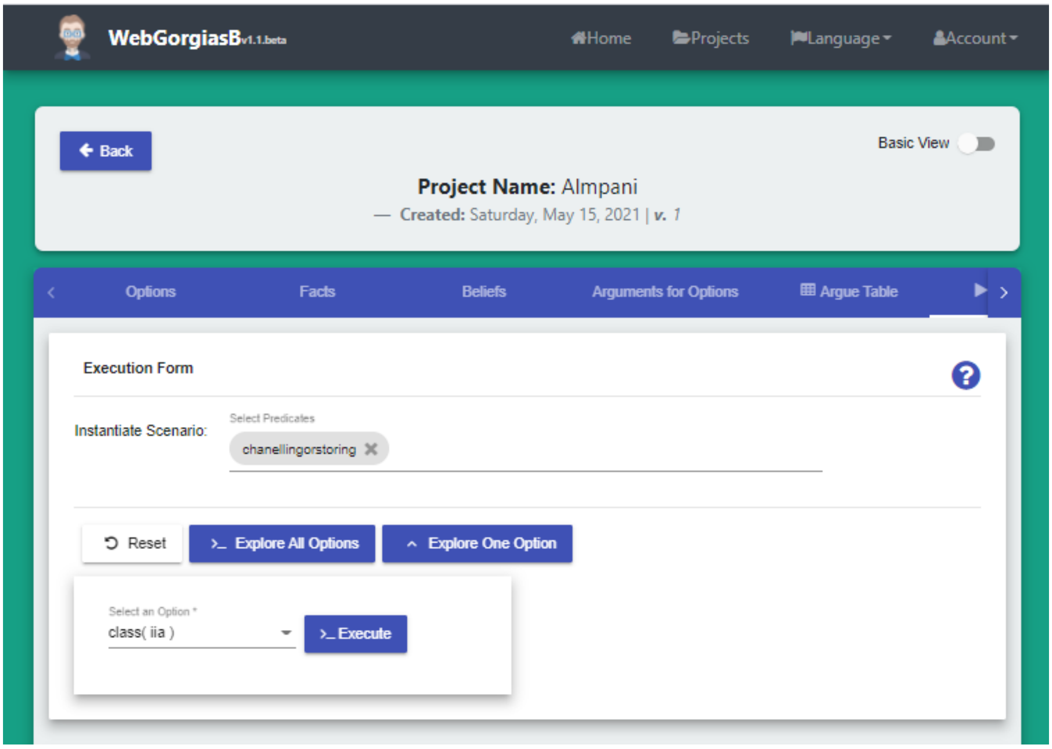
Resolve Argues for Options

Level	Scenarios	class(i)	class(iia)	class(iib)	class(iii)	Commands
1	noninvasive	✓				Close Impossible Delete
All available predicates	Select Predicates: chanellingorstoring					
Preview	noninvasive, chanellingorstoring		✓			Save Reset
1	chanellingorstoring, noninvasive	✓				Expand Impossible Delete
1	chanellingorstoring, connectedwithad, noninvasive		✓			Expand Impossible Delete
1	chanellingorstoring, forstoringorgans, noninvasive		✓			Expand Impossible Delete
1	forbloodbags, chanellingorstoring, noninvasive			✓		Expand Impossible Delete
1	modifyingcomposition, implantation, noninvasive		✓			Expand Impossible Delete
1	filtration, modifyingcomposition, noninvasive		✓			Expand Impossible Delete

Figure 5.7 The Argue Table under WebGorgiasB

of medical devices under MDR. Since there is no clear connection between them, a connection is made based on the commonly-described characteristics, used only for enhancement of the computation model with representative examples and not for providing actual legal knowledge [48].

For example, we want to see which class *'Cervical Collars'* belongs to (or MDN1214 from list of codes [2017/2185] [152]). Cervical Collars are non-invasive medical devices with no other specific characteristics (Rule 1 of MDR), so we have only the predicate `nonInvasive`. Therefore, based on MDR definitions in the **Execution Results for fact: [noninv]** we have as output:



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Instantiate Scenario: Select Predicates: chanellingorstoring

Select an Option *: class(iia)

Figure 5.8 The Execution Options under WebGorgiasB

```
In general choose class(i)
When [noninv] prefer class(i) over class(iia)
When [noninv] prefer class(i) over class(iib)
When [noninv] prefer class(i) over class(iii)
```

If we take the “syringes for infusion pumps” (MDN1202) as another example, in the **Execution Results for fact:** [**noninv,chanellingOrStoring**] the output would be:

```
In general choose class(i)
When [noninv,chanellingOrStoring] prefer class(i) over class(iia)
When [noninv,chanellingOrStoring] prefer class(i) over class(iib)
When [noninv,chanellingOrStoring] prefer class(i) over class(iii)
```

Lastly, if we take the “blood bags without anticoagulant” (MDN1202) as example, in the **Execution Results for fact:** [**noninv,chanellingOrStoring,bloodBags**] the output would be:

```
In general choose class(iib)
When [noninv,chanellingOrStoring,bloodBags] prefer class(iib) over
class(i)
```

```
When [noninv,chanellingOrStoring,bloodBags] prefer class(iib) over
                        class(iia)
When [noninv,chanellingOrStoring,bloodBags] prefer class(iib) over
                        class(iii)
```

To exemplify the above execution results, in the case of a *"cervical collars"*, which is a non-invasive medical device, we have the predicate `nonInvasive`, and based on Rule 1 *"all non-invasive devices are classified as class I, unless one of the rules set out hereinafter applies"* the option for this scenario is `class(i)`, since no other rule applies to this case. In the case of *"syringes for infusion pumps"* (MDN1202), this product is also an non-invasive medical device, but its usage belongs to the definition *"intended for channelling or storing blood, body liquids, cells or tissues, liquids or gases for the purpose of eventual infusion, administration or introduction into the body"* (Rule 2 of MDR), therefore we have the predicates `chanellingOrStoring, nonInvasive`. Based on Rule 2, the option for this scenario is again `class(i)`. But, in Rule 2, if we take the specific case of *"blood bags without anticoagulant"*, then the predicate "blood bag" is also added, thus we have `bloodBags, chanellingOrStoring, nonInvasive`, and since Rule 2 of MDR states that *"Blood bags are classified as class IIb"*, therefore the execution result for this scenario is `class(iib)`.

Evaluation and Discussion

To convert a legal text to a knowledge presentation and its natural-language-to-logic mapping can be a demanding procedure. Some of the challenges of the presented rule formalization were:

- In the original text of the regulation, there is no grouping of devices belonging to the same class (e.g., Class I), neither an explicit separation of the different sub-cases in each rule that lead to different classification options. Therefore, for the needs of the computational formalization, we created a **knowledge schema** to present those correlations more clearly.
- The numbering of the MDR classification rules is helpful for their formalization. The **predicates** and variables are named based on indicative words of each rule. A limitation of WebGorgias-B is that when you use variables generated automatically with language processing there may be some typos in the predicates, requiring editing in order to capitalize the variable names. In the interests of cutting down the names of the predicates to a reasonable length, not all information on

rules was included. However, in the interests of clarity, the first number of each scenario indicates the MDR rule number for reference to the original text.

- This also enhances the **explainability** of the system since any user can better understand and interpret the model's behavior as well as check its proposed solution in the original classification rule text, as indicated by the scenario numbering. Legal and safety aspects are an important motivation for explainable AI systems, since the European Union cannot afford to implement AI systems that make unjustified decisions, especially in the medical sector [51]. In the AMeDC model, any legal decision procedure that is complemented by the system can be understood, explained and re-enacted by humans. Thus, in addition to representing MDR classification rules precisely enough to determine the necessary requirements for compliance purposes, this formalization is aimed at a model that could be expressive enough so that it can be verifiable by legal, medical, and programming experts alike. Therefore, the legal rules were formalized in such a way so that the Gorgias presentation can be read and understood with scenarios grouped per rule.
- Additionally, the developed scenarios are independent, autonomous pieces of knowledge, enabling high **modularity**. Each scenario can be removed, replaced, or modified without affecting other scenarios, enabling future amendments/amelioration of the present regulation (e.g. other pre-marketability requirements) or extension of the presented work, acting as groundwork for other countries' regulations.

Concerning WebGorgias-B and the **query-answer procedure** of AMeDC, the query examples indicate that in both simple and more complex queries the results provided by WebGorgias-B were accurate and the answers can be validated by a human with audit trails. Several scenarios were tested and checked for the accuracy of their output. We can also test all scenarios in each option separately by selecting all conditions and obtaining all the scenarios belonging to each class, an explicit classification grouping that the MDR itself does not provide clearly in the legal text. This can be implemented by selecting all facts and a specific option (e.g., class(i)), so all available scenarios will be executed. The run-time performance has also been evaluated. There was no noticeable delay in query answering for our testing laptop (Intel(R) Core(TM) i3-4000M CPU 2.40GHz 2.40 GHz, 6GB RAM, running on Windows) for the provided data set, which includes 22 rules in the KB with 64 sub-cases, even with queries with different variables, e.g. selecting specific scenarios and option/s, or when all conditions and options were selected.

At this point, it is important to mention that all the different cases and sub-cases would not have worked if we did not have the argumentation framework. The reader can notice that nonInvasive as well as nonInvasive & chanellingOrStoring is in class(i) but nonInvasive & chanellingOrStoring & blood bag is in class(iib). We cannot express the complexity of legal texts with their definitions and exceptions in the classic way of defining rules, so in these cases the non-monotonous nature of argumentation is required. **Non-monotonic logic under Gorgias** can manage new information overcoming the limitations of classical logic, as it simulates more naturally the way in which humans process information. Classical logic is monotonic in the sense that any option that could be entailed before a clause is added can still be entailed after it is added; adding information does not change the set of solutions that can be derived. The reasoning necessary for an intelligent system and decision making in realistic applications can be very difficult to represent as deductive inferences in a logical system [153]. In non-monotonic logic some results can be invalidated by adding further knowledge, enabling representation of defaults. A default is a rule that can be implemented unless it is overridden by an exception—and in legal texts we have plenty of defaults. Thus, the basic feature of the system presented is its ability to determine a default initial option in a specific scenario, allowing for other options to be applied in further refinements of the scenario.

To test **user experience** concerning the functionalities, the ease of use, and the explainability of the application, there have been a few preliminary trials by inexperienced users indicating that AMeDC is easily used and understood. Additionally, to test the expressiveness, explainability, and accuracy of the formalization, the application was validated by a legal expert, corroborating our natural-language-to-logic mapping. However, more user experience trials need to take place before the final build of this application.

5.2.3 Medical Devices Rules with RuleML

Towards a Legal Rule-based System on Medical Devices

The legal norms can be roughly expressed in first-order logic [154], which covers much of ontologies and rules. Rules and ontologies constitute key components in the Semantic Web [154]. Description-logic-based ontology languages resemble decidable fragments of first-order logic [155]. The rule-based languages are related to the classes of rules originating from logic programming and they are based on different kinds of logics, basically consisting of Horn clauses [156] (i.e., sets of function symbols, predicates, and variables).

The syntax of a DL is built over an individual of a domain, classes, and properties that represent binary relations over individuals [157], with the use of T-Boxes[7] and A-Boxes[8] [158]. Ontology axioms are mostly used to express ontology T-Boxes about types of entities. This sometimes excludes dependent, probabilistic or default statements about individual entities, as well as statements concerning the meaning and ambiguity of natural language. Therefore, for such knowledge, the use of a more expressive formalism is recommended [159], such as rule-based systems. In comparison to DL, where predicates are restricted only to unary or binary, predicates are polyadic, i.e. there is no limitation on the arity of predicates; also, polyadic functions are allowed to construct complex terms. Therefore, they have well defined declarative semantics that can be supported by well-developed reasoning algorithms [157].

Rules can provide a foundation of knowledge representation and decision making to express domain-specific (i.e. medical) concept definitions and legal norms [160]; they thus extend the classification-focused expressiveness of description logics, as called for in areas like legal policies, auditable procedures, and real-time alert systems; e.g., in the medical devices regulation, the classification of medical devices can be sufficiently represented by ontologies as well, but for the marketability requirements the expressiveness of a rule-based language is needed. There are various languages for modeling ontologies co-existing with rules, such as SWRL, DLP, OWL2-RL, RuleML, etc. In this section, Positional-Slotted Object-Applicative (PSOA) RuleML [161, 162] is used for its suitability to express deductions by rules over enriched (object-relational) atoms. Medical Devices Rules employs PSOA RuleML on the level of Horn logic (Hornlog), currently restricted to the (essentially function-free) level of near-Datalog [162], illustrating how PSOA integrates the data and knowledge representation paradigms of *relationship atoms*[9] with those of *frame atoms*[10].

From the DL point of view, PSOA uses a “light-weight” T-Box, representing the hierarchy of the application domain in the form of a subclass (“##”) taxonomy, allowing “multiple inheritance,” while in our current use case a tree-restricted DAG is sufficient (See Fig. 5.11). Moreover, it has an A-Box of generalized facts including assertions of instances that refer to either classes, e.g. `:HearingAidsUDI#:HearingAids`, or

---

[7] An ontology specifies inclusion relation describing its classes hierarchy as well as its properties.

[8] DL-atoms can be used as axioms denoting class or property membership.

[9] Ordered tuple of positional arguments.

[10] A unique OID typed by a class and described by an unordered collection of slots.

frames. For simplicity and efficiency, in addition to its central implication construct, Hornlog-level PSOA RuleML provides only some of the first-order-logic constructs on fragments of which DLs are also based (in particular, conjunction as well as, in certain syntactic contexts, disjunction and universal plus existential quantifiers), and does not provide classical (strong) negation; although adding (weak) “Negation-as-failure” (Naf) for a NafHornlog-level PSOA RuleML is being planned, at this time, we restrict our KBs to purely Hornlog-level PSOA, as implemented in PSOATransRun 1.4. In this context, rules can be written and used even by users not familiar with advanced knowledge engineering concepts. PSOA RuleML has also been used for legal rules formalization in other use cases, such as Port Clearance Rules [162] and Air Traffic Control Regulations [163] providing evidence that PSOA RuleML is well-suited to express real-world legal texts.

While our work focuses on the formalization of the European Regulations, this approach appears to apply generally and extend to a broad class of medical devices regulations, since the structure of such rule-based systems can benefit from the homogeneity[11] and modularity[12] of rules [154]. Hence, each rule can be reformed or updated without affecting the entire system or requiring the modification of other rules, enabling future amendments/amelioration of the present regulation and/or extension of the current work to include the corresponding regulations of other countries. According to the WHO[13], there isn’t one common approach on the medical devices regulatory systems at country level [164], since it is determined by the existing general national legal and administrative systems within each country. However, since in most regulations around the globe (such as in, e.g., FDA[14] for the USA, MHFW[15] for India, PMDA[16] for Japan, etc.) medical devices registration follows an almost identical procedure with the one in the MDR (i.e., medical devices are grouped into risk-based classes that require different class-based conformity assessment procedures[17]), this work can be used as a groundwork for other national regulations.

In the following sections, the main parts of the European regulations for the implementations — Medical Devices Rules and ExosCE — as well as the legal reasoning model in PSOA RuleML, will be surveyed.

---

[11] Homogeneity means that all rules are conveyed in the same format.

[12] Modularity means that each rule is an independent part of knowledge.

[13] World Health Organization: http://www.who.int/medical_devices/en/

[14] https://www.fda.gov/Medicaldevices/default.htm

[15] Ministry of Health and Family Welfare: https://mohfw.gov.in/

[16] Pharmaceuticals and Medical Devices Agency: http://www.pmda.go.jp/english/

[17] Principles of medical devices classification. Global Harmonization Task Force; 2012.

This increased need of computational medical records is usually supported by ontologies for taxonomic organization of information as well as legal-based rules for medical tests, procedures, and registrations, so that the quality of healthcare is secured and improved. Ontologies in the Semantic Web — represented with formal languages, such as Description Logics (DLs) — provide the representation for different types of medical knowledge, such as the OpenGalen ontology [165, 166] where methods were applied for restriction of medical terms to sensible classes. Similar techniques have been applied on various medical nomenclatures including the MeSH (Medical Subject Heading) [167], the FMA (Foundational Model of Anatomy) [168], and the ICD10 (International Classification of Diseases) [169]. However, a demand has already been identified for expressive power beyond what is offered by DL-based ontology languages [157]. Many health care procedures, such as inpatient clinical information systems [170], antibiotics prescription [171], and risk assessment of pressure ulcers [165], are supported by computer aided decision making leading to increased interest in rule-based systems [171]. In spite of existing theoretical issues of the complementary nature between ontology and rule languages, there is a need of Semantic Web Technologies for integrated formalisms that can provide advanced reasoning capabilities [155], such as in SNOMED CT (Standardized Nomenclature of Medicine Clinical Terms) [172] which proposed rules expressed in DLs for checking terms of consistency. Medical applications that combine ontologies with rule languages can be used, e.g., as clinical guidelines [173, 158] and for medical decision support [174], which can be subjects of privacy and regulatory compliance as well. Thus, in some applications, it can be practical to regulate the compliance process by using formalized parts of applicable laws.

The complexity of regulations in the healthcare domain (which are usually represented as a moderately controlled natural language text) makes it difficult for enterprises to design and develop effective compliance systems for their applications [175]. While logical reasoning on knowledge representations is rather well-understood, there are no established methods to convert a given medical legal text to an appropriate knowledge representation [51]. The length of the legal texts, the complexity of their acts, and the vagueness of their language make it complicated for business professionals to estimate whether they are in compliance. This difficulty becomes even more pressing if programmers wish to develop and configure automated systems to help practitioners comply with applicable laws [175]. Medically relevant regulations have been the subject of formalization in the USA, e.g. FDAAA TrialsTracker [176], a live informatics tool for FDA[18]-compliance in clinical trials, in [175], an online prolog-based auditor, and, in [177], a production rule model, both of them for HIPAA[19]-compliance in health information. This work is an initial attempt to formalize, in a computational manner, a European regulation of medical devices.

EU Regulation of medical devices concerning the classification rules and the declaration of conformity procedures (thus, requiring both medical-classified and legal-based organization of information) was formalized in PSOA RuleML, a rule language that introduces positional-slotted, object-applicative terms in generalized rules [71]. PSOA RuleML has also been used for legal rules formalization in other use cases, such as Port Clearance Rules [162] and Air Traffic Control Regulations [163, 178], indicating that PSOA RuleML is well-suited to expressing real-world legal texts.

Formalization for Medical Devices Rules

Our formalization of Regulation (EU) 2017/745 consists of five parts, presented below by the classification and declaration of conformity of medical device code MDA0310b. The present work is restricted to the English version of the Regulation.

1. The 22 classification rules of the regulation.
2. The medical device categories in each class.
3. Class-based conformity requirements for marketability.
4. An explicit taxonomy of the medical devices.
5. Sample data (facts) of medical devices.

The 22 classification rules of medical devices

In the first part of the formalization, the original rules are expressed with a three-level-deep description of medical devices characteristics, connected for abbreviation with (informal) three-symbol categories. Rules move from the relational to the object-centered paradigm with their frame conditions: The relational conclusion argument `?m` becomes the OID of class `:MedicalDevice` of a frame with `:kind`, `:use`, and `:specificCase` slots. An effective way to modify the translation of the legal text into rules is to add exceptions (specific cases) to the more generic rule to make the two cases of the rules disjoint.

---

[18] Food and Drug Administration https://www.fda.gov/.

[19] Health Insurance Portability and Accountability Act.

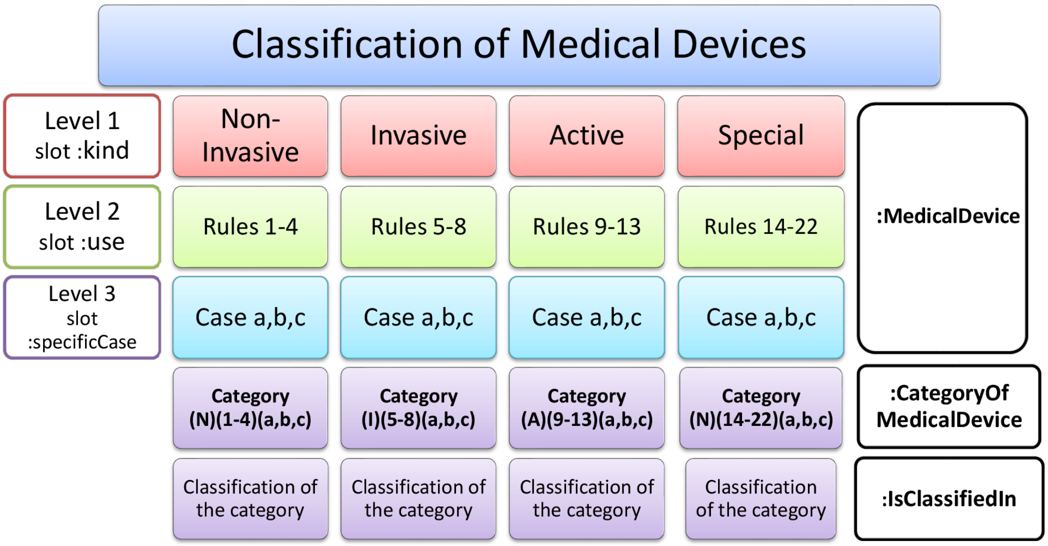


Figure 5.9 Visualization of PSOA RuleML decision model for classification rules.

One clause is used for each category of the rules, formed as the example below which formalizes the sentence *"All invasive devices with respect to body orifices, other than surgically invasive devices, which are not intended for connection to an active device or which are intended for connection to a class I active device are classified as:*
- class I if they are intended for transient use;
- class IIa if they are intended for short-term use, except if they are used in the oral cavity as far as the pharynx, in an ear canal up to the ear drum or in the nasal cavity, in which case they are classified as class I; and
- class IIb if they are intended for long-term use, except if they are used in the oral cavity as far as the pharynx, in an ear canal up to the ear drum or in the nasal cavity and are not liable to be absorbed by the mucous membrane, in which case they are classified as class IIa." [132].

```
% Rules for Invasive Devices
% Rule 5 - Devices invasive in body orifices.

Forall ?m (
 :CategoryOfMedicalDevice(?m :I5a) :-
    Or(?m#:MedicalDevice(:kind->:Invasive :use->:NonSurgically
                         :specificCase->:Transient)
       ?m#:MedicalDevice(:kind->:Invasive :use->:NonSurgically
                         :specificCase->:EarNoseOrThroat_ShortTerm))
```

```
Forall ?m (
 :CategoryOfMedicalDevice(?m :I5b) :-
  Or(?m#:MedicalDevice(:kind->:Invasive :use->:NonSurgically
                      :specificCase->:ShortTerm)
   ?m#:MedicalDevice(:kind->:Invasive :use->:NonSurgically
                    :specificCase->:EarNoseOrThroat_LongTerm))

Forall ?m (
 :CategoryOfMedicalDevice(?m :I5c) :-
   ?m#:MedicalDevice(:kind->:Invasive :use->:NonSurgically
                    :specificCase->:LongTerm))
```

The condition's predicate `:MedicalDevice` is a frame atom, where the hash infix # denotes *class membership* by typing an OID with its predicate, while the arrow infix, "`->`", pairs each predicate-independent slot name with its filler. The predicate `:CategoryOfMedicalDevices` is a relationship that links the medical device with the category it belongs. For the explanation of this formalization, we will focus on category `I5b` (where medical device code MDA0310b belongs).

Another exceptional case is Rule 5, namely that time duration is also used for the categorization of the medical device into `:Transient`, `:Shortterm` or `:Longterm`. For this rule predicates with `math:` prefix were used as defined in the imported mathematics library http://psoa.ruleml.org/lib/math.psoa. They are shortcuts for external built-in calls in PSOA [162]. For example, the specific case `:Shortterm` is described as follows:

```
% Rule 5 (Time period of usage: Short Term)
Forall ?m ?d (?m#:MedicalDevice(:specificCase ->:ShortTerm) :-
          And(?m#:MedicalDevice(:duration->?d)
          math:lessEq(?d 30)
          math:greaterEq(?d 0.02)))
```

Rules concerning time period of usage are object-centered except for the relational (in the example above) `math:lessEq` and `math:greaterEq` calls in their second conjuncts. Note that units of duration — here,"days" — are omitted on this nearDatalog level of expressiveness, but could become Hornlog function applications in slot fillers — here, `:daysOfUsage(?d)`.

The Classification of Medical Devices

In the second part of the formalization on the classification of medical devices, the aforementioned categories are connected with the class they reside in, forming an 'Or' branch (disjuction). The generated categories — 55 in number — are indicated by three letters which denote the three levels of the categorization (see also Figure 5.9), e.g. :I5b, where **I** denotes a Invasive device, 5 denotes Rule 5, and b denotes the specific case 'b', i.e. EarNoseOrThroat_LongTerm. The categories and the corresponding classes for all kinds of medical devices in details are depicted in diagrams in Appendix .3.1. The categories in Class IIa are expressed in the following example:

```
% Classification Grouping: Class IIa

Forall ?m (
 :IsClassifiedIn(?m :IIa) :-
    Or(:CategoryOfMedicalDevice(?m :N2a)
       :CategoryOfMedicalDevice(?m :N2b)
       :CategoryOfMedicalDevice(?m :N3b)
       :CategoryOfMedicalDevice(?m :N4c)
       :CategoryOfMedicalDevice(?m :I5b)
       :CategoryOfMedicalDevice(?m :I6)
       :CategoryOfMedicalDevice(?m :I7)
       :CategoryOfMedicalDevice(?m :I8a)
       :CategoryOfMedicalDevice(?m :A9a)
       :CategoryOfMedicalDevice(?m :A10)
       :CategoryOfMedicalDevice(?m :A11)
       :CategoryOfMedicalDevice(?m :A12)
       :CategoryOfMedicalDevice(?m :S16b)
       :CategoryOfMedicalDevice(?m :S17
       :CategoryOfMedicalDevice(?m :S19a)
       :CategoryOfMedicalDevice(?m :S20)
       :CategoryOfMedicalDevice(?m :S21b)))
```

The Marketability of Medical Devices

The third part of the formalization described the requirements for a medical device to be marketable. The following rules are relational, on the Datalog level of expressiveness[20].

---

[20] Predicates only have a variable, ?m, no function application, as their argument.

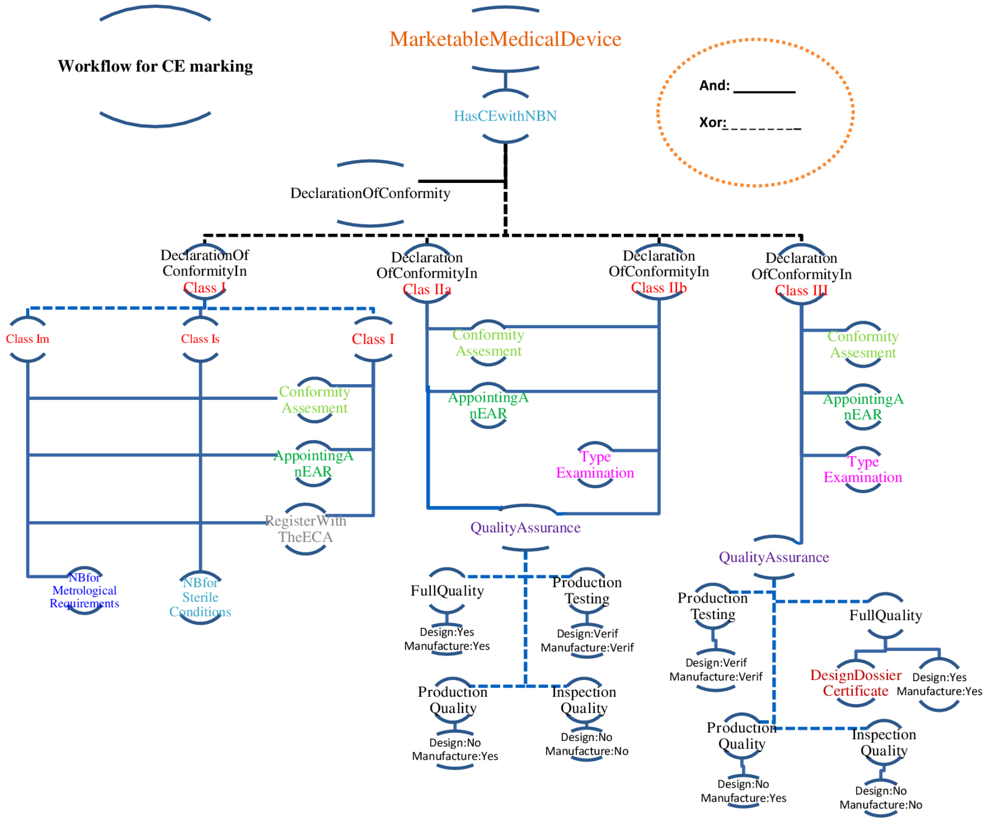


Figure 5.10 Marketability requirements for each class.

```
% Requirements for all Classes

Forall ?m (
 :MarketableMedicalDevice(?m) :-
  :HasCEwithNBN(?m))

Forall ?m (
 :HasCEwithNBN(?m) :-
   :DeclarationOfConformity(?m))
```

All the different **Declaration of Conformity** routes of each class for the CE marking and the implying **marketability** of medical devices are described, outlining the pre-marketability procedure. The post-marketability requirements are beyond the scope of the current work.

In Class IIa, as described in the example below, all the conditions of the ‘And’ relation must be fulfilled to obtain the `:DeclarationOfConformity`.

```
% Requirements for Class IIa
Forall ?m (:DeclarationOfConformity(?m) :-
 And(:IsClassifiedIn(?m :IIa)
     :AppointingAnEAR(?m)
     :ConformityAssessment(:device->?m :technicalFile->:True
                           :vigilanceSystem->:Required
                           :harmonizedStandards->:NonRequired)
     :QualityAssurance(?m)))
```

The PSOA RuleML decision model for Conformity Assessment routes is visualized in Figure 5.10, with an object-relational 'And'-'Or' DAG[21]. In 'Or' relations, only one choice from the possible options can be selected, either based on the filler of the slot names or on the different conditions of the 'And' clauses, so that only one route can be "fully invoked," causing near-deterministic behavior, e.g. for the Quality Assurance only one of the `:QualityType` can be "fully invoked."

```
Forall ?m (:QualityAssurance(?m) :-
    Or(:QualityType(?m :FullQuality)
       :QualityType(?m :ProductionTesting)
       :QualityType(?m :ProductionQuality)
       :QualityType(?m :InspectionQuality)))
```

Each Quality Type has also different requirements, for example:

```
% Requirements for Production Quality -  % Annex V, EN ISO 13485:2003
Forall ?m (:QualityType(?m :ProductionQuality) :-
:RequirementsOfQualityType(:device->?m :design->:NonRequired
                           :manufacture->:Required))
```

An Explicit Taxonomy of the Medical Devices

In the fourth part of the formalization, the Subclass relation (denoted in RIF and PSOA as '##') (e.g., `:NonActiveInvasive##:MedicalDevices`) is used for building a variable-depth multi-layer **taxonomy**, containing currently more than 150 different medical device products. The taxonomy consists of five levels as depicted in Figure 5.11 starting with the top class to the right and the sub classes to the left. The four levels are 'Subclass of' ##-levels, while the last level is 'Instance of' #-level including

---

[21] The 'And' branches are connected with straight lines, while the 'Or' are connected with dashed lines.

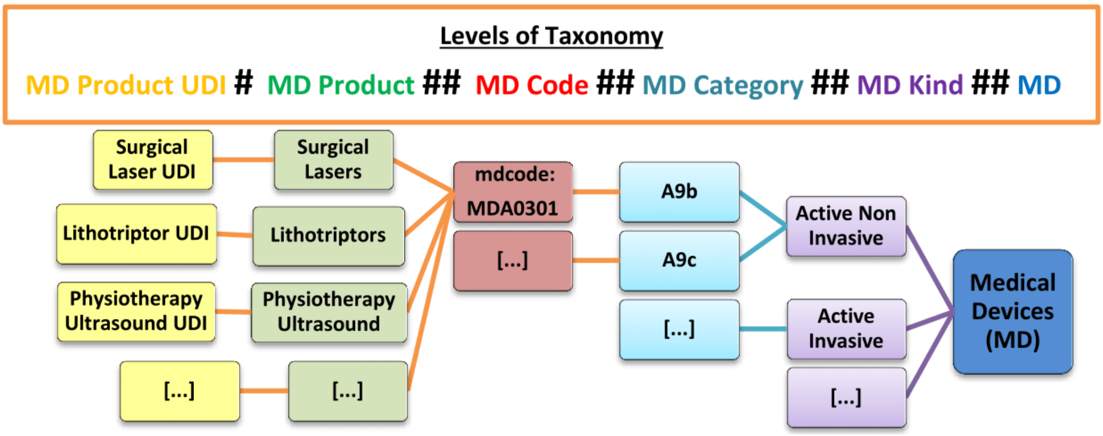


Figure 5.11 Visualization of a taxonomy example.

individuals for each 'Medical Device Product' subclass with the suffix UDI (e.g., `:HearingAidsUDI#:HearingAids`). In PSOATransRun at least one level 'Instance of' ('witness' instances) is required to allow retrieval. The classes for `:HearingAidsUDI` are described below:

```
:ActiveInvasive##:MedicalDevices
:I5b##ActiveInvasive
:MDA0310b##:I5b
:HearingAids##mdcode:MDA0310b
:HearingAidsUDI#:HearingAids
```

Data for Medical Devices

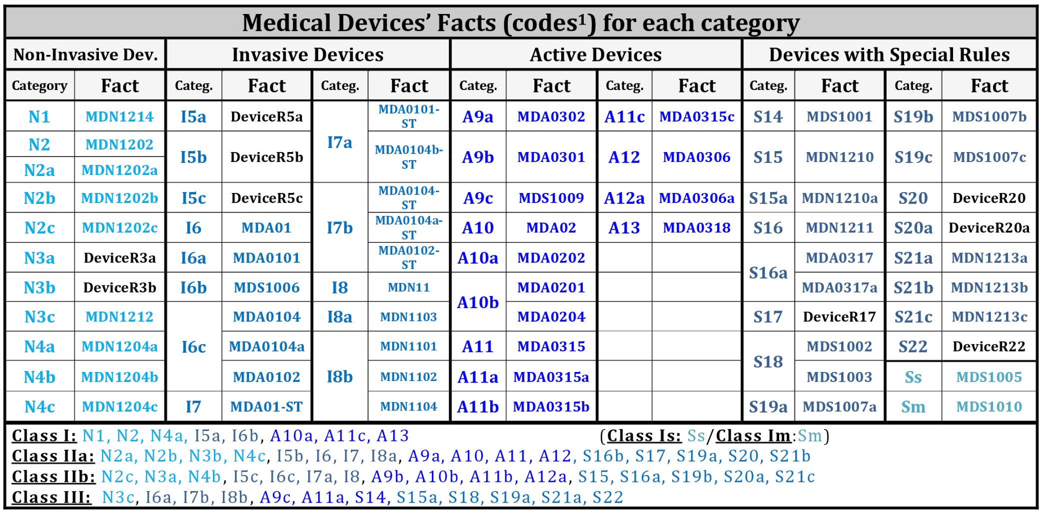
In the last part of the formalization, **Data** for specific medical devices (Facts) were added directly to the Medical Devices KB[22]. Medical devices facts were developed based on the list of codes (2017/2185) [152] and the corresponding types of devices under Regulation (EU) 2017/745[23].

"The lists of codes and corresponding types of devices should take into account various device types which can be characterized by design and intended purpose, manufacturing processes and technologies used... The lists of codes should provide for a multi-

---

[22] The medical devices facts are described with their specific characteristics and with their (randomly chosen) completed marketability requirements. The marketable medical devices in each class can be viewed in Appendix .3.2.

[23] In cases where the codes don't describe specifically a category, a random coding is applied (e.g.,`:DeviceR3a`), while in cases where more than one category belongs to the same code, letters a,b,c are used.



Medical Devices’ Facts (codes¹) for each category

Non-Invasive Dev.		Invasive Devices				Active Devices				Devices with Special Rules			
Category	Fact	Categ.	Fact	Categ.	Fact	Categ.	Fact	Categ.	Fact	Categ.	Fact	Categ.	Fact
N1	MDN1214	I5a	DeviceR5a	I7a	MDA0101-ST	A9a	MDA0302	A11c	MDA0315c	S14	MDS1001	S19b	MDS1007b
N2	MDN1202	I5b	DeviceR5b		MDA0104b-ST	A9b	MDA0301	A12	MDA0306	S15	MDN1210	S19c	MDS1007c
N2a	MDN1202a												
N2b	MDN1202b	I5c	DeviceR5c	I7b	MDA0104-ST	A9c	MDS1009	A12a	MDA0306a	S15a	MDN1210a	S20	DeviceR20
N2c	MDN1202c	I6	MDA01		MDA0104a-ST	A10	MDA02	A13	MDA0318	S16	MDN1211	S20a	DeviceR20a
N3a	DeviceR3a	I6a	MDA0101		MDA0102-ST	A10a	MDA0202			S16a	MDA0317	S21a	MDN1213a
N3b	DeviceR3b	I6b	MDS1006	I8	MDN11	A10b	MDA0201				MDA0317a	S21b	MDN1213b
N3c	MDN1212	I6c	MDA0104	I8a	MDN1103		MDA0204			S17	DeviceR17	S21c	MDN1213c
N4a	MDN1204a		MDA0104a	I8b	MDN1101	A11	MDA0315			S18	MDS1002	S22	DeviceR22
N4b	MDN1204b		MDA0102		MDN1102	A11a	MDA0315a				MDS1003	Ss	MDS1005
N4c	MDN1204c	I7	MDA01-ST		MDN1104	A11b	MDA0315b			S19a	MDS1007a	Sm	MDS1010

Class I: N1, N2, N4a, I5a, I6b, A10a, A11c, A13 (**Class Is:** Ss/**Class Im**:Sm)
Class IIa: N2a, N2b, N3b, N4c, I5b, I6, I7, I8a, A9a, A10, A11, A12, S16b, S17, S19a, S20, S21b
Class IIb: N2c, N3a, N4b, I5c, I6c, I7a, I8, A9b, A10b, A11b, A12a, S15, S16a, S19b, S20a, S21c
Class III: N3c, I6a, I7b, I8b, A9c, A11a, S14, S15a, S18, S19a, S21a, S22

Figure 5.12 Categories and corresponding codes of Medical Devices & Categories in each Class.

dimensional typology of devices.”

Article 2: “Application for designation Conformity assessment bodies shall use the lists of codes and corresponding types of devices set out in Annexes I and II to this Regulation when specifying the types of devices in the application for designation referred to in Article 38 of Regulation (EU) 2017/745 and Article 34 of Regulation (EU) 2017/746.”

The predicates with the `mdcode:` prefix are used to describe the medical devices codes of the aforementioned directive. Figure 5.12 presents the categories and corresponding codes of medical devices, as well as all categories in each class. An example of medical device MDA0310a[24] facts is,

```
% Requirements of MDA0310b: Class IIa, 3Yes
 mdcode:MDA0310b#:MedicalDevice(:kind->:Invasive :use->:NonSurgically
                              :specificCase->:EarNoseOrThroat_LongTerm)
:AppointingAnEAR(mdcode:MDA0310b)
:ConformityAssessment(:device->mdcode:MDA0310b :technicalFile->:NonRequired
                      :vigilanceSystem->:Required
                      :harmonizedStandards->:NonRequired)
:RequirementsOfQualityType(:device->mdcode:MDA0310b :design->:NonRequired
                           :manufacture->:Required)
```

---

[24] Code MDA0310 is described in (2017/2185) as “Active non-implantable device for ear, nose and throat.”

Notice that because of the randomly chosen facts concerning marketability requirements for each medical device, several medical device examples do not satisfy all conditions to be marketable. The medical devices facts are covering all categories with qualitative slot-filler distinctions.

Query Answering on Medical Devices Rules by PSOATransRun

In this section, representative copy&paste-ready queries were posed to the KB and the answers were obtained through PSOATransRun.

The Prolog instantiation of PSOATransRun [161], currently in version 1.5, is the reference implementation of PSOA RuleML. In PSOATransRun reasoning engine, various different kinds of queries are typically supported:

1. Ground: determine whether a ground atom is entailed in a relationship.
2. Open: determine all the tuples of variable bindings in a relationship.
3. Class-instance membership queries: ground (if an individual is an instance of a class) and open (all the individuals that are instances of a class).
4. Class subsumption queries: determine if one class is a subsumption of another.
5. Class hierarchy queries: determine all superclasses of a class or individual.

Queries and Answers for Medical Devices Classification: To obtain the medical devices with one or more specific characteristics, e.g. for the devices using derivatives, the following query can be used.

```
> ?m#:MedicalDevice(:use->:NonSurgically)
?m=<http://eur-lex.europa.eu/legal-content/EN/TXT/CODES/...#MDA0310a>
?m=<http://eur-lex.europa.eu/legal-content/EN/TXT/CODES/...#MDA0310b>
?m=<http://psoa.ruleml.org/usecases/MedicalDevices#DeviceR5a>
?m=<http://psoa.ruleml.org/usecases/MedicalDevices#DeviceR5b>
?m=<http://psoa.ruleml.org/usecases/MedicalDevices#DeviceR5c>
```

The multiple ?m-answer bindings are shown as full IRIs expanded from the ‘:’-prefixed abbreviations in the KB.

Similarly, to obtain the category of a specific medical device, the following deductive query is employed, binding the answer to the output variable `?g`.

```
> :CategoryOfMedicalDevice(mdcode:MDN0310b ?g)
?g=<http://psoa.ruleml.org/usecases/MedicalDevices#I5b>
```

Using the top-level predicate `:IsClassifiedIn` a query can be posed regarding whether a certain medical device code, e.g. `:IsClassifiedIn(mdcode:MDA0310b :IIa)`, belongs to a specific class, i.e. IIb (Answer: Yes). Moreover, the classification of a medical device can be asked, even if we do not know its specific code, by asking for an OID with certain characteristics, getting all the possible answers, e.g.:

```
> :IsClassifiedIn(?m#:MedicalDevice(:use->:NonSurgically) :IIa)
?m=<http://eur-lex.europa.eu/legal-content/EN/TXT/CODES/...#MDA0310b>
?m=<http://psoa.ruleml.org/usecases/MedicalDevices#I5b>
```

Abstracting this query (e.g., the constant `:IIa` becomes the variable `?c`), a generalized, symbolic-execution-style non-ground query could be posed as well, i.e., `:IsClassifiedIn(?m ?c)`, to deduce all medical devices and their corresponding classes, using two output variables (`?m` and `?c`).

Queries and Answers for Medical Devices Marketability: More queries can be asked on the marketability and conformity requirements of medical devices. In the example of medical devices represented by the code `:MDN0310b`, PSOATransRun returns a ‘Yes’-answer to the following queries.

```
:IsClassifiedIn(mdcode:MDA0310b :IIa)
:RegisterWithTheECA(mdcode:MDA0310b)
:AppointingAnEAR(mdcode:MDA0310b)
:ConformityAssessment(:device->mdcode:MDA0310b :technicalFile->:NonRequired
                      :vigilanceSystem->:Required
                      :harmonizedStandards->:NonRequired)
:RequirementsOfQualityType(:device->mdcode:MDA0310b :design->:NonRequired
                           :manufacture->:Required)
:DeclarationOfConformity(mdcode:MDA0310b)
:HasCEwithNBN(mdcode:MDA0310b)
:MarketableMedicalDevice(mdcode:MDA0310b)   % Answer for all: Yes %
```

All the queries regarding marketable devices can be also posed, using e.g. the input variable `?m`, by posing the query `:MarketableMedicalDevice(?m)`. Moreover, all medical devices that satisfy one or more specific marketability requirements can be obtained.

Queries and Answers for Medical Devices Taxonomy: For the description of the explicit relations between the hierarchical levels of medical devices, a separate taxonomy was created, which facilitates the complement of more medical devices products and UDIs in the future. When using PSOA's '`##`' infix, one instance-level relation is required for PSOATransRun to deduce answers. A query can be posed about the upper classes of a medical device product UDI-instance, e.g. using the output variable `?c` to deduce all the upper layers of the taxonomy (Bottom-to-Top Taxonomy Queries). Queries about the instances belonging to the lower levels can be also obtained, e.g. using the variable `?m` (Top-to-Bottom Taxonomy Queries). In this query, all the UDIs of the relevant medical devices will be exported, but not the sub-classes in-between (i.e., the intermediate sub-class with the code 'MDN1202b').

```
> :HearingAidsUDI#?c %Bottom-to-Top%
?c=<http://psoa.ruleml.org/usecases/MedicalDevices#HearingAids>
?c=<http://eur-lex.europa.eu/legal-content/EN/TXT/CODES/...#MDN1202b>
?c=<http://psoa.ruleml.org/usecases/MedicalDevices#I5b>
?c=<http://psoa.ruleml.org/usecases/MedicalDevices#ActiveInvasive>
?c=<http://psoa.ruleml.org/usecases/MedicalDevices#MedicalDevices>

> ?m#:I5b  %Top-to-Bottom, Some of the answers obtained%
?m=<http://psoa.ruleml.org/.../MedicalDevices#HearingAidsUDI>
?m=<http://psoa.ruleml.org/.../MedicalDevices#HardContactLensesUDI>
?m=<http://psoa.ruleml.org/.../MedicalDevices#TrachealTubesUDI>
```

Discussion and Evaluation of Medical Devices Rules

To convert a legal text written in natural language to a knowledge presentation can be a demanding procedure. Some of the principles and challenges of the described rule formalization are the following:

- The explicit numbering of classification rules is helpful for their formalization. Every natural language rule of the regulation is shown before its formal representation in the Medical Devices Rules KB.
- In the text of the regulation, there is no aggregation of devices belonging to the same class (e.g., Class I), neither a clear separation of the different cases in each rule. A knowledge schema and data mapping is required for the needs of computational formalization. In particular, the original 22 rules are represented with

additional three-symbol categories based on their differentiated characteristics as abbreviations. Subsequently, these categories form classification groups.

- Medical devices facts are based on the list of codes (2017/2185) [152] and the corresponding types of devices under Regulation (EU) 2017/745. Since there is no explicit connection of these codes with the categories of medical devices, an association[25] is made based on the commonly-described characteristics. However, this connection is used for enhancement of the KB with representative medical devices facts and for general documentation of the regulation rather than for providing actual legal knowledge. Moreover, representative marketability requirements for each medical device fact are chosen randomly.

- Hence, while the current KB does not use an actual dataset of medical devices facts, in the future a standard dataset can be obtained from Eudamed (which was under development at the time of Medical Rules KB development). Eudamed is being overhauled in order to increase capabilities and allow for wider access in accordance with the new regulation. Thus, Unique Device Identification (UDI) of medical devices — which could be used for that purpose — will be phased in and be added to Eudamed over several years. Note that there are databases of other countries where the legislation is in force (e.g., FDA (USA)[26], SFDA (Saudi Arabia)[27], CMDRD (China)[28], MHRA (UK)[29], MDA (Malaysia)[30], etc.). Moreover, there are attempts, e.g. by the International Medical Device Regulators Forum (IMDRF)[31], for a globally harmonized approach to the application of a UDI system of medical devices, aiming to assist international regulatory convergence.

- For obtaining a more detailed, explicit and easily-enhanceable KB, a hierarchical taxonomy of medical devices was created separately. This taxonomy complements the formalized classification rules (which connect the rules of the regulation with

---

[25] In cases where the codes do not describe specifically a category, a random coding is applied (e.g.,`:DeviceR3a`), while in cases where more than one category belongs to the same code, letters a,b,c,ST are used (e.g., `mdcode:MDN1202c`).

[26] https://www.fda.gov/MedicalDevices/DeviceRegulationandGuidance/Databases/default.htm

[27] Saudi Food and Drug Authority: https://www.sfda.gov.sa/en/medicaldevices/eservices/Pages/default.aspx

[28] China Medical Device Regulatory Database :http://www.cirs-md.com/resources/cmdrd

[29] Medicines and Healthcare products Regulatory Agency: https://aic.mhra.gov.uk/era/pdr.nsf/device?openpage&start=1&count=200

[30] Malaysian Medical Device Authority: https://mmdr.mda.gov.my/data/public/index.php

[31] http://www.imdrf.org/

the codes of the regulation) with a clear description of the hierarchy of medical devices linking the codes with the (upper and lower) layers described below:

- Medical Devices (i.e. `:MedicalDevices`)
- Medical Device Kind (e.g., `:ActiveNonInvasive`)
- Medical Device Category (e.g., `:A9b`)
- Medical Device Code (e.g., `mdcode:MDA0301`)
- Medical Device Product (e.g., `:LinearAccelerators`)
- Medical Device Product UDI (e.g., `:LinearAcceleratorsUDI`)

This taxonomy was created based on pertinent guidelines but does not reflect expert knowledge of medical devices. Taxonomy provides the opportunity for further enhancement in the future with medical devices obtaining from formal medical devices (e.g., Eudamed, Global Medical Device Nomenclature (GMDN)[32], etc.). Even though in a real-life implementation of this work, only UDIs will be necessary as the main data of medical devices KBs, the codes and the categories will still be of significant value to help stakeholders distinguish between various generic groups[33] of medical devices.

5.3 Legal Decision Making on Wearable Robots Regulatory Compliance

Wearable robots aim to significantly improve the users' quality of life by assisting, augmenting, or enhancing mobility and motion in various human movement applications and scenarios. [179, 180].

At the present time, the use of robots is not widespread. However, studies indicate that this will gradually change [181], since robotic systems may bring benefits and conveniences to our society. Wearable robots may reinforce areas of applications that cover wide-ranging domains [180]. Some of the potential applications of wearable robots in the healthcare sector are: rehabilitation treatment for patients recovering from injuries; movement aids for disabled persons; support for an extended autonomous life of the elderly; and decrease of repetitive tasks of care personnel [179]. Additionally,

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[32] GMDN is a international generic naming system of medical devices products. https://www.gmdnagency.org/

[33] A set of devices having the same or similar intended purposes or commonality of technology allowing them to be classified in a generic manner not reflecting specific characteristics.

they can be used to decrease the burden in physically demanding jobs and reduce work-related injuries, thus increasing productivity and work quality in industry [180].

There is a growing interest of producers and users in wearable robots [182]. Thus, it is essential not only to focus on developing prototypes and technologies for testing in research labs, but also to have a clear perspective on how this progress can genuinely influence society [183]. According to this, focus should be put on shaping the wearable robots market, so stakeholders (i.e. regulators, roboticists, manufacturers, etc.) are aware of the legal matters demanding their attention [183, 184]. This chapter outlines the international framework that is relevant in realizing new markets for these urgently needed technologies, mainly focusing on the reports by the European Parliament. In this regard, there is a need for a computational formalization of the existing regulation to promote systematic use and to ensure the quality of the procedure required in order to provide the emerging devices to the marketplace nationally or internationally.

The European Union has conferred legal status to several EC Directives, and two such directives are currently the most relevant for wearable robots, the Medical Devices Directive 2017/745/EC (MDD) [132] (which was formalized in the previous section) and the Machinery Directive 2006/42/EC (MD) [185]. The MD applies to machines generally defined as devices with at least one moving part, containing actuators, control and power circuits, while the MDD can apply to any robot designed to meet a medical need and to be used for diagnostic and/or therapeutic purposes. The regulations directed by the MD and the MDD specify the requirements that manufacturers need to comply with in order to obtain a CE marking to allow for the commercialization of their device. Some devices — such as wearable robots — need to comply with some of the requirements of both regulations [186].

With the current growth in rehabilitation and personal care robots, interest in wearable exoskeletons has been growing, fueled by the demand for assistive technologies in general and specifically to respond to the concerns of an increasingly ageing population [187, 188]. Exoskeletons are wearable robots that are fastened to the body of the consumers, extending their physical capabilities in a complementary or augmentary way. In the case of exoskeletons, complying with both medical device and machinery regulations could be required [186, 181].

This section describes an attempt to formalize, in a computational manner, the exoskeleton-related parts of European Directives, extending the work presented in the previous section concerning the formalization of the Medical Devices Directive [47–49]. It focuses specifically on the case of exoskeletons as a type of medical device and incorporates the relevant requirements from the Machinery Directive, regarding their safety and marketability. To the extent of our knowledge, there is no previous work regarding the development of a computational system for the CE marking compliance of exoskeletons. The ExosCE Rules (i.e., Exoskeletons' CE marking Rules) prototype can contribute to the effort of unifying the above legal frameworks to a computational format, as part of legal-informatics efforts. In the following subsection (Subsection 5.3.2), we present an example of a type of exoskeleton as an effort to formalize parts of the clauses enacted by the MDD and the MD, in Positional-Slotted Object-Applicative (PSOA) RuleML.

5.3.1 An Overview of Wearable Robots Regulatory Framework

The growth of the wearable robots market (it is expected to record a CAGR of 22.17% over the period 2020–2025 [129]) makes it essential to regulate critical aspects like reliability, safety, and protection. The world of wearable robots is heterogeneous, with wide diversification in potential risks of harm to the consumer. The close proximity between wearable robot and user exposes the latter to multiple risks that necessitate extensive scrutiny [189]. Public trust in wearable robots needs effective and efficient regulations relying on a well-built legal and policy foundation, as well as sound regulatory strategies [164]. This section provides an overview concerning the existing legal European framework and where the new wearable robots fit in — mostly focusing on the recently adopted directives by the European Parliament and the relevant standards.

European Union Law does not contain explicit rules on robots, but there are EU legislations related to robotic devices, set in two basic directives: Machinery Directive 2006/42/EC (MD) and Medical Devices Directive 2017/EC (MDD). The above-mentioned directives specify the CE marking requirements that manufacturers need to comply with in order for their devices to be placed in the markets [184]. Notice that it is often the case for exoskeletons to gain CE marking in Europe before getting an FDA marking in the USA, as in the cases of ReWalk [34], Ekso [35], HAL [36], and Rex [37].

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[34] https://rewalk.com/

[35] https://eksobionics.com

[36] https://www.cyberdyne.jp/

[37] https://www.rexbionics.com/

Machinery Directive 2006/42/EC

The Machinery Directive aims to support the design of machinery that is as safe as possible in line with cutting-edge technological advances. This directive refers to machines mainly defined as devices with power circuits, actuators, control, and at least one moving part. It sets the basic Essential Health and Safety Requirements (EHSR) that apply to all manufacturers who want their devices to be placed on the market. Compliance with the EHSR can be achieved with harmonized European standards.

Most robots (i.e., also wearable robots) so far have been categorized as machines, and therefore robot safety standards need to be compliant with this directive [190]. However, the harmonized standards published under the Machinery Directive do not involve the combination of machine and wearable device. Consequently, standards and guidelines need improvement and updating to cover exoskeletons technology [189]. An issue that is quite new in wearable robots regulated under the MD is the idea of intended contact between a user and a robot [191]. While the majority of industrial robots are still detached from the human user, in physical assistant robots — such as exoskeletons — physical contact is an important part of the intended task [191] (a requirement which was taken into consideration in the development of ISO 13482 [192]).

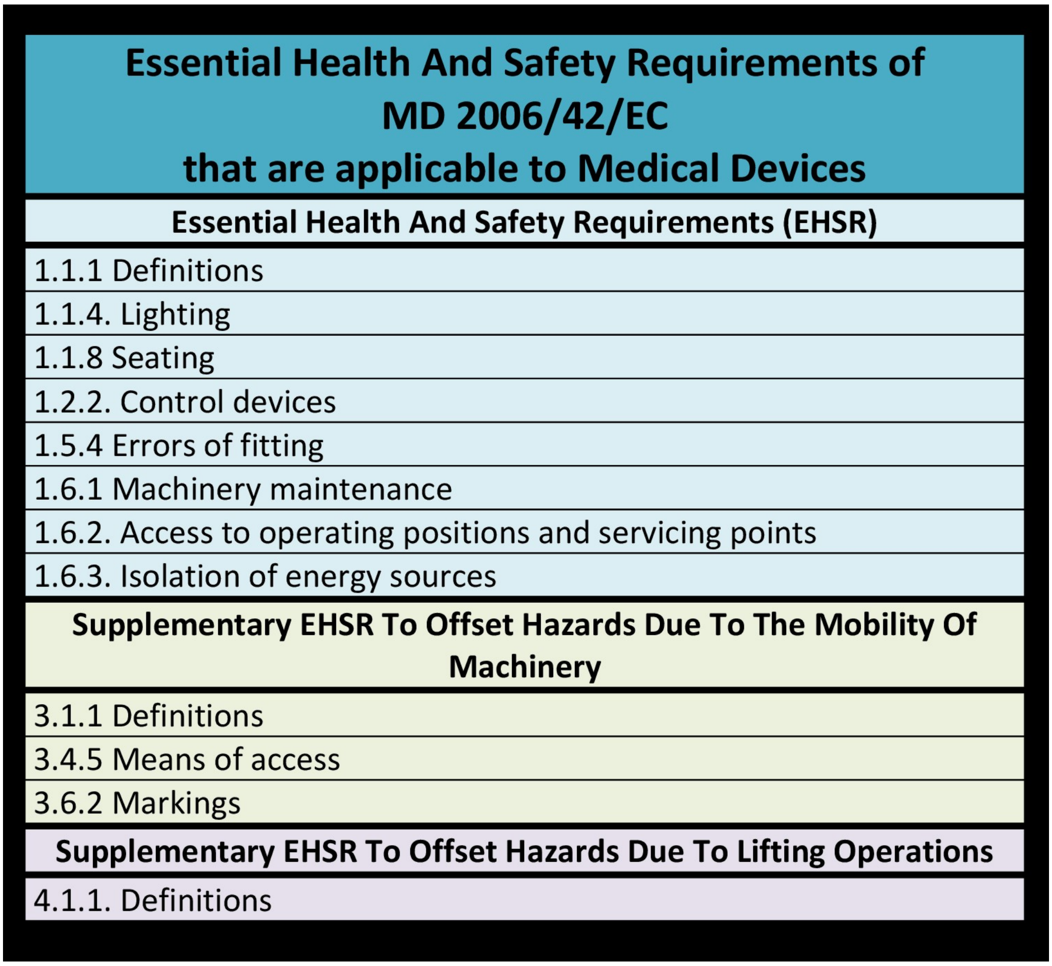
Medical Devices Directive 20017/745/EC

The Medical Devices Directive refers to any device designed to meet a medical need and used for diagnostic and/or therapeutic purposes. In this case, the product must be regulated as a medical device (under the MDD) rather than as a machine (under the MD). The updated version of MDD that came into force after 2020 included some critical changes, aiming to ensure that all medical devices on the market in the EU are safe and efficient [48]. More details can be found in Section 5.2.1.

Comparison of MDD and MD Safety Requirements

According to the MDD, medical devices that are also machinery shall also meet the EHSR of the MD. In Art.12 of the MDD, it is stated that *"Devices which are also machinery within the meaning of point (a) of the second paragraph of Article 2 of Directive 2006/42/EC of the European Parliament and of the Council shall, where a hazard relevant under that under the Directive exists, also meet the essential health and safety requirements set out in Annex I to that Directive to the extent to which those requirements are more specific that the general safety and performance requirements set out in Chapter II of Annex I to this Regulation"* [132]. According to this, manufacturers must specify whether their products can also be categorized as machines and thus comply with the MD as well. A detailed definition of what constitutes a machine is provided in the MD and it can assist manufacturers with distinguishing whether their device can also be classified as a machine. A basic feature of a machine is the accessibility of the movable parts, thus wearable robots and exoskeletons fall into the category of machines as well.

EHSR are applied for medical devices when the hazard is related and it is not covered by the essential requirements of the MDD, or is only partially covered. Requirements of Machinery that can be considered applicable to medical devices that meet the definition of machinery — thus, exoskeletons as well — are listed in Figure 5.13.



Essential Health And Safety Requirements of MD 2006/42/EC that are applicable to Medical Devices
Essential Health And Safety Requirements (EHSR)
1.1.1 Definitions
1.1.4. Lighting
1.1.8 Seating
1.2.2. Control devices
1.5.4 Errors of fitting
1.6.1 Machinery maintenance
1.6.2. Access to operating positions and servicing points
1.6.3. Isolation of energy sources
Supplementary EHSR To Offset Hazards Due To The Mobility Of Machinery
3.1.1 Definitions
3.4.5 Means of access
3.6.2 Markings
Supplementary EHSR To Offset Hazards Due To Lifting Operations
4.1.1. Definitions

Figure 5.13 Applicable EHSR of the Machinery Directive (2006/42/EC) to Medical Devices

This is very significant, since the MD dates to 2006 and does not take account of the updates concerning ISO 12100:2010 on machinery safety and ISO 13482:2014 on personal care robots [193]. In the case of exoskeletons, it is required to comply with both medical device and machinery regulations, depending also on the application domain they are sold for, such as industrial, medical, or personal care [184]. Thus, if there are related hazards connected with the product's classification as a machine, manufacturers must assess the EHSR in line with the provisions described in the MD.

The MD applies to various products. Annex I of the Directive enumerates about fifty Essential Health and Safety Requirements, a number of which can instantly be overlooked, since they are obviously not relevant to medical devices. This leaves around twelve requirements that can be considered applicable, although the definite number of requirements will differ depending on the product. Understanding all the EHSRs in the MD, as well as which ones apply to specific devices, can be a hard and time-consuming work [186]. Thus, efforts to develop guidelines for the manufacturers to clarify which requirements from the MDD and MD can be applicable to exoskeletons can be really helpful and time saving.

Legal Standards and Robotic Technologies

Through a more comprehensive and transparent regulatory framework, the EU regulations are aiming to enhance safety and quality in the market while incentivizing innovation in the field.

The above-mentioned regulations usually require that the manufacturer demonstrates product safety. This is typically performed by applying (voluntarily) international standards. These standards provide secured methods for implementing certain features in technology, such as procedures on how to implement, analyze, and demonstrate safety of new devices before they enter the market [184]. There is a variety of standards which are formed and adapted for particular purposes. These include international standards set by the International Organization for Standardization (ISO) and the International Electro-technical Commission (IEC) that have an important role as they are made up of international networks of national standard bodies. Standards are optional, but they can be mentioned or integrated in regulations. The Conformity Assessment to the relevant regulations should use credible service providers and uphold global principles [190].

However, at this time, there are only a few specified standards available and no specific testing methods for Wearable Robots [184]. For instance, product safety for medical devices that are classed in Medical Electrical Systems is technically defined in the IEC 60601-1 [194], which is a large family of standards for specific categories of medical devices. Wearable Robots meant for commercial use would normally have to be compliant with this standard in order to guarantee they are safe for use. The publication of ISO 13482 [195] concerning “Robots and robotic devices - Safety requirements for personal care robots” is one of the first specific steps towards this direction that is relevant to Wearable Robots, since it covers exoskeletons-like robotics under the type “restraint-type physical assistant robots” [184]. The application of ISO 13482 is generally advisable and highly recommended for the marketability of a personal care robot since it provides a substantiation of conformity with European Directives. By developing a wearable robot in compliance with this new standard, a designer can easily obtain a CE marking [192]. However, ISO 13482:2014 does not apply to robots as medical devices, and currently the majority of exoskeletons have been developed for medical applications.

Those directives and standards do not cover many of the explicit and complex issues related to emerging robotic technologies, namely human-robot interactions and the autonomous decision-making. The European Parliament recently initiated a discussion on an EU-wide legislative action, focused on civil law rules on robots [181]. This discussion aims to present law-making suggestions to secure a standard level of safety as well as to fully exploit the economic potential of robotics. The European Commission is organized to tackle matters of safety, liability, privacy, and the influence of robotics on workplace, health, industry, and environment [181].

Exoskeletons

Exoskeletons are devices that aim to interface with the human and assist with the recovery of the walking function compromised due to sensory and cognitive deficits. Repetitive training using such technological aids assists the human nervous system to create alternative neuron paths to replace the damaged ones [196]. Up to the present time, most of exoskeleton research has concentrated on medical applications of exoskeletons, such as rehabilitation and supporting mobility to physically disabled or injured persons (caused by various reasons such as spinal cord injury, neurological disorders, stroke, etc.) [197]. Medical exoskeletons are used in rehabilitation and healthcare centers supervised by medical experts [198]. Assistance-as-needed rehabilitation exoskeletons aim to help users regain functional abilities through repetitive exercise with progressively reduced assistance [196]. Representative examples of lower limp mobile rehabilitation exoskeletons include: the Wearable Walking Helper, the Honda:SMA, the MIRAD, and the LOPES [196]. Representative examples of active upper limb wearable exoskeletons include: the NeReBot, the Armeo Power, the full-body Recupera-Reha exoskeletal system, the SymbiHand, and the SaeboGlove [196].

However, exoskeletons can also be used to support regular tasks of daily life (such as walking, lifting heavy items, using stairs, and general movement) if the physical capabilities of a person have been impaired, as well as for augmentation of physical abilities [190]. Exoskeletons developed for rehabilitation can be used in other contexts as well, and vice versa [198]. Indeed, exoskeletons can be used for rehabilitation of patients (i.e., medical) and additionally for assisting healthy users to lift heavy objects, i.e., non-medical. However, there are cases in-between that are not very clear, like an assistive device for supporting the mobility of elderly [199].

It is essential to be aware of what is emerging in a regulatory sense (i.e., new regulations, ISOs, etc.), so that the accurate risk assessment and the applicable safety standards (either for medical device or machine) can be enforced [190]. Some exoskeletons will be categorized and thus regulated as medical devices instead of machines, and this poses a borderline question (i.e., which category they should belong to, medical or non-medical) since they might have to comply with different regulations. The ISO/IECs for medical and non-medical exoskeletons are not the same and each must be followed respectively for successful marketing [199]. For medical exoskeletons aimed at rehabilitation, such regulations are currently under development by IEC SC62D and ISO TC299 JWG36, while the already published ISO TC299 WG2 applies to non-medical exoskeletons like physical assistant robots [191, 184]. As mentioned, the scope of ISO 13482 does not include medical applications which, based to MDD, concerns medical robots that perform tasks such as diagnosis, prevention, and monitoring or treatment of diseases [191]. Consequently, this could mean that obtaining the ISO 13482 certification might not be necessary if robots are to be compliant with the MDD [199].

The issue here is how this borderline between medical and non-medical wearable robots can be clearly-defined. Where robots offer services that may be considered medical as well as non-medical, then (regardless of the manufacturer's declaration concerning the intended use of the product) the device in both cases should be compliant with the MDD as described in the latest version of the regulation. This might be the case for exoskeletons: although they can have applications both in rehabilitation and in daily-life tasks, in both cases they will have to comply with the MDD [191].

Classification of Exoskeletons

Exoskeletons — in order to be legally placed on the market — are required to obtain the CE certificate. There are several regulatory bodies globally with a different purpose, procedure and application [200]. This can be a source of confusion for manufacturers [201]. In the USA, powered exoskeletons (i.e., a category of device intended to assist paralyzed users recover the function of walking) have been formally classified as a Class II device with special controls by the FDA [202]. In the European Union, there is no central government organization to publish certificates.

Various enterprises developing lower limb exoskeletons state that their device has already been approved as medical under the existing regulations. Such cases are the "HAL for Labor/Care Support" by Cyberdyne, that obtained ISO 13482:2014 as wearable robot, the "Medical Robot Suit HAL," that obtained a CE marking under the MDD, the ReWalk from ArgoMedical categorized as a class II (USA) medical device, the Rex Bionics as Class I (EU, USA, and Australia) for rehabilitation use, and the EksoLegs from Ekso Bionics as Class I (USA and Australia) and Class IIa (EU) for rehabilitation use in hospitals. Nevertheless, the information provided is brief and deficient, making it hard to get a clear view of the precise compliance procedure required for exoskeletons under existing international regulations. There is a necessity for harmonization, standardization, and rationalization of licensing procedure around the world [201].

5.3.2 ExosCE Rules with RuleML

Legal and safety aspects create a huge motivation for explainable AI systems (i.e., systems that the results of the decision-making can be understood by human experts). Legal-AI models are often rule-based [177, 175, 162], such as the presented formalization for ExosCE Rules in PSOA RuleML. In this section, we present a sample case study on commercializing wearable exoskeletons for rehabilitation with PSOA RuleML to highlight the basic modeling of medical exoskeletons. The aim of this recommendation is to provide a computational guidance: with the classification of exoskeletons based on MDD; the Conformity Assessment including both the Essential Requirements of MDD and the EHSR of the MD; and the marketability procedure in order to obtain the CE marking. Some explanatory parts of the code are used for the route through the compliance procedure (more can be found in Appendix .4).

In the first part of the formalization, the 22 rules are expressed with a three-level-deep description of medical device characteristics, abbreviated with (informal) three-symbol categories [48, 49]. The relational conclusion argument `?m` becomes the OID (Object IDentifier) of the class `:MedicalDevice` of a frame with `:kind`, `:use`, and `:specificCase` slots. An effective way to modify the translation of the legal text into rules is to add exceptions (specific cases) to the more generic rule, so as to create separate sub-cases of the generic rule.

One clause is used for each category of the rules, formed as the example below which formalizes the sentence *"All active therapeutic devices intended to administer or exchange energy are classified as class IIa"* [132].

```
% Rules for Active Devices - Rule 9
% Active therapeutic devices intended to exchange
or administer energy.
Forall ?m (
 :CategoryOfMedicalDevice(?m :A9a) :-
  ?m#:MedicalDevice(:kind->:Active
                    :use->:Therapeutic
                    :specificCase->:Energy))
```

The condition's predicate `:MedicalDevice` is a frame atom, where the hash infix # denotes *class membership* by typing an OID with its predicate, while the arrow infix, "`->`", pairs each predicate-independent slot name with its filler.

In the second part of the formalization, the aforementioned categories are connected with the class they reside in, forming an 'Or' branch (disjunction). The predicate `:CategoryOfMedicalDevices` is a relationship that links the exoskeleton with the relevant category of medical device. The generated categories are indicated by three letters which denote the three levels of the categorization, e.g. `:A9a`, where `A` denotes an Active device, `9` denotes Rule 9, and `a` denotes the specific case 'a', i.e. :Energy. For the explanation of this formalization, the rest of this section will focus on category `A9a` (to which exoskeletons belong, e.g., Eksolegs), while "`[...]`" denotes that some code fragment has been omitted to conserve space. Some of the categories in Class IIa, where `:A9a` belongs, are expressed in the following example:

```
% Classification Grouping: Class IIa
 Forall ?m (
:IsClassifiedIn(?m :IIa) :-
       Or(:CategoryOfMedicalDevice(?m :A9a)
          :CategoryOfMedicalDevice(?m :A10)
          :CategoryOfMedicalDevice(?m :A11) [...]
```

In the third part of the formalization, the process required for a medical exoskeleton to be marketable is described. The different class-based routes for the Conformity Assessment of exoskeletons are depicted in Figure 5.14. The following rules are relational, on the Datalog level of expressiveness[38].

---

[38] That is predicates that only have a variable, ?m.

```
% Requirements for all Classes
Forall ?m (
 :MarketableMedicalDevice(?m) :-
  :HasCEwithNBN(?m))

Forall ?m (
 :HasCEwithNBN(?m) :-
   :DeclarationOfConformity(?m))
```

In Class IIa, as described in the example below, all the conditions of the ‘And’ relation must be fulfilled to obtain the `:DeclarationOfConformity`.

```
Forall ?m (:DeclarationOfConformity(?m) :-
 And(:IsClassifiedIn(?m :IIa)
 :AppointingAnEAR(?m)
 :ConformityAssessment(:device->?m :technicalFile->:True
                       :vigilanceSystem->:Required
                  :harmonizedStandards->:NonRequired)
 :QualityAssurance(?m)
 :ManufacturingRequirements(?m))
```

In the fourth part of the formalization, all 64 Essential Requirements of MDD (as described in Annex I Chapter II), are encoded. The following example presents part of the code presenting the main headings of the Essential Requirements.

```
Forall ?m (:MDDManufacturingRequirements(?m) :-
 And(:ChemicalPhysicalBiologicalProperties(?m) % p.10 %
 :InfectionMicrobialContamination (?m) % p.11 %
 :SubstancesMedicalProductOrAbsorbed(?m)  % p.12 %
 [...]
 :RisksByDevicesSupplyingEnergyOrSubstances(?m)  % p.21 %
 :DevicesForUseByLayPersons(?m)))   % p.22 %
```

In this part of the guideline, twelve EHRS of the MD that are applicable to Medical Devices are also added.

```
Forall ?m (MDManufacturingRequirements(?m) :-
 And(:DefineGeneralTermsOfMD(?m :Checked) % p.1.1.1 %
     :Lighting(?m :Checked)% p.1.1.4 %
```

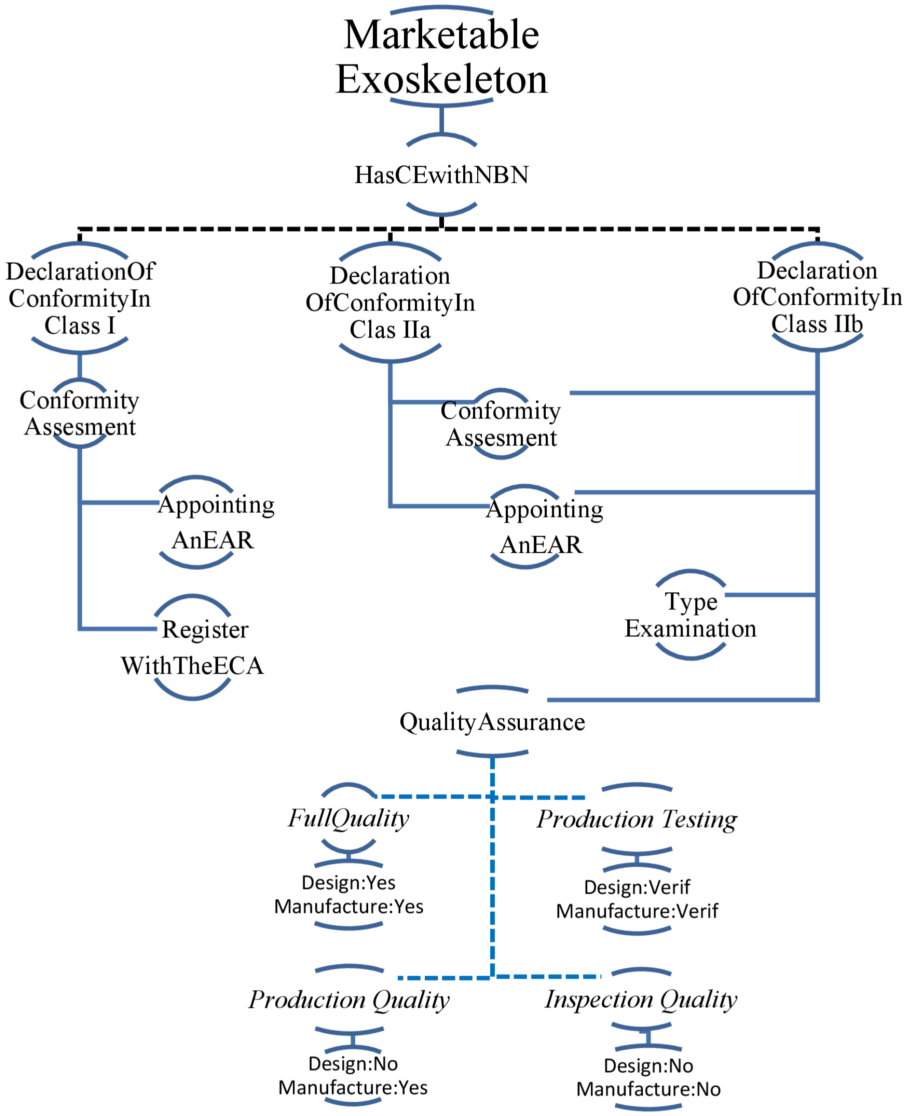


Figure 5.14 Class-based Requirements for Exoskeletons Marketability

```
     :SeatingASIntegralPart(?m :Checked) % p.1.1.8 %
     :ControlDevices(?m :Checked)      % p.1.2.2 %
     [...]                     ))
```

In the last part of the formalization, data for specific exoskeletons were added directly in the KB. An example of an exoskeleton fact, i.e., EksoLegs, is encoded as below.

```
:EksoLegs#:MedicalDevice(:kind->:Active
                            :use->:Therapeutic
                            :specificCase->:Energy)
:AppointingAnEAR(:EksoLegs)
:ConformityAssessment(:device->:EksoLegs
                      :technicalFile->:Yes
                      :vigilanceSystem->:Yes
                      :harmonizedStandards->:No)
[...]
```

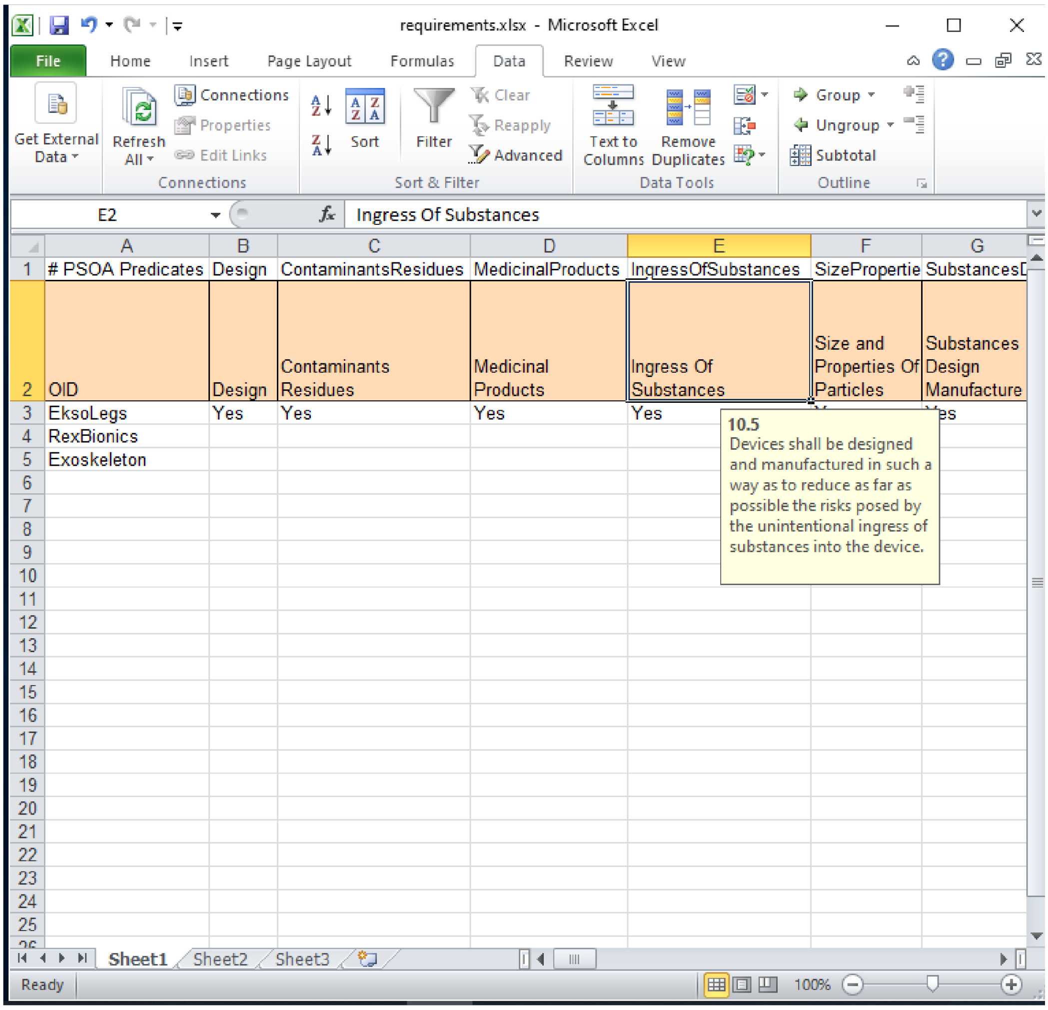
A User-friendly Interface for the Essential Requirements Checklist

In addition to representing the European regulations precisely enough to determine whether the necessary requirements within the scope of the CE-registration procedure would be compliant with law, this development aimed at a formalization that could be verifiable by lawyers, medical experts, and programmers alike. For this reason we also provided a tool to translate the safety requirements from MS Excel format to PSOA RuleML code, so that the non-technical users can be able to read and understood PSOA RuleML language and presentation syntax.

ExosCE Rules is implemented in PSOA RuleML programming language and in the open source engine PSOATransRun, currently in version 1.5. MS Excel worksheet can be utilized to create the user's checklist of the Essential Requirements for Conformity Assessment of the exoskeleton. The most important benefit is the usability of MS Excel due to the fact that many users find it more usable than programming languages for computational tasks. Thus, one of the objectives of Excel was to bring the advantages of additional programming language features to a system that is often not recognised as a programming language.

The user interface employs an Excel spreadsheet with pull down menus to provide all possible options for the requirements. Additionally, there are input messages which are shown when the cell is selected, to provide brief instructions for each requirement as described in the directives (See Figure 5.15).

A script in Python translates user inputs of the Essential Requirements for the Conformity Assessment from the cells of the Excel to PSOA RuleML code.



E2 Ingress Of Substances

	A	B	C	D	E	F	G
1	# PSOA Predicates	Design	ContaminantsResidues	MedicinalProducts	IngressOfSubstances	SizePropertie	Substances
2	OID	Design	Contaminants Residues	Medicinal Products	Ingress Of Substances	Size and Properties Of Particles	Substances Design Manufacture
3	EksoLegs	Yes	Yes	Yes	Yes		es
4	RexBionics						
5	Exoskeleton						
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							
17							
18							
19							
20							
21							
22							
23							
24							
25							

10.5
Devices shall be designed and manufactured in such a way as to reduce as far as possible the risks posed by the unintentional ingress of substances into the device.

Figure 5.15 Checklist of the MDD and MD safety requirements in MS Excel

ExosCE KB executes reasoning and generates answers based on users queries in PSOATransRun. In a future work, an online version of ExosCE can also become available to enable exoskeletons developers to add exoskeletons facts and check compliance requirements.

In the future, we plan to introduce a user-friendly online tool for the requirements checklist that will use the PSOATransRun reasoner and the ExosCE KB as a back-end. Moreover, we can incorporate possible future ontologies (using PSOATransRun's built-in N3 to PSOA translator [203]) or databases for Medical Devices and Exoskeletons, enriching our KB. These KBs could be extended to support additional requirements e.g. from ISOs, so that a medical device or an exoskeleton can be checked against all requirements. Part of our future interest is to disseminate this tool to stakeholders (robotic companies, lawyers, researchers, medical experts, etc.) through European robotic-related networks (such as Cost Actions, Horizons, Erasmus, etc.) as well as to utilize it in multidisciplinary courses for technical and non-technical students in medical, engineering and legal fields.

Query Answering on ExosCE Rules by PSOATransRun

To obtain the category of an exoskeleton, the following deductive query is employed, binding the answer to the output variable `?g`.

```
> :CategoryOfMedicalDevice(:EksoLegs ?g)
?g=<http://psoa.ruleml.org/usecases/MedicalDevices#A9a>
```

Using the top-level predicate `:IsClassifiedIn` a query can be made regarding whether an exoskeleton, e.g. `:IsClassifiedIn(:EksoLegs :IIa)`, belongs to a specific class, i.e. IIa (Answer: Yes).

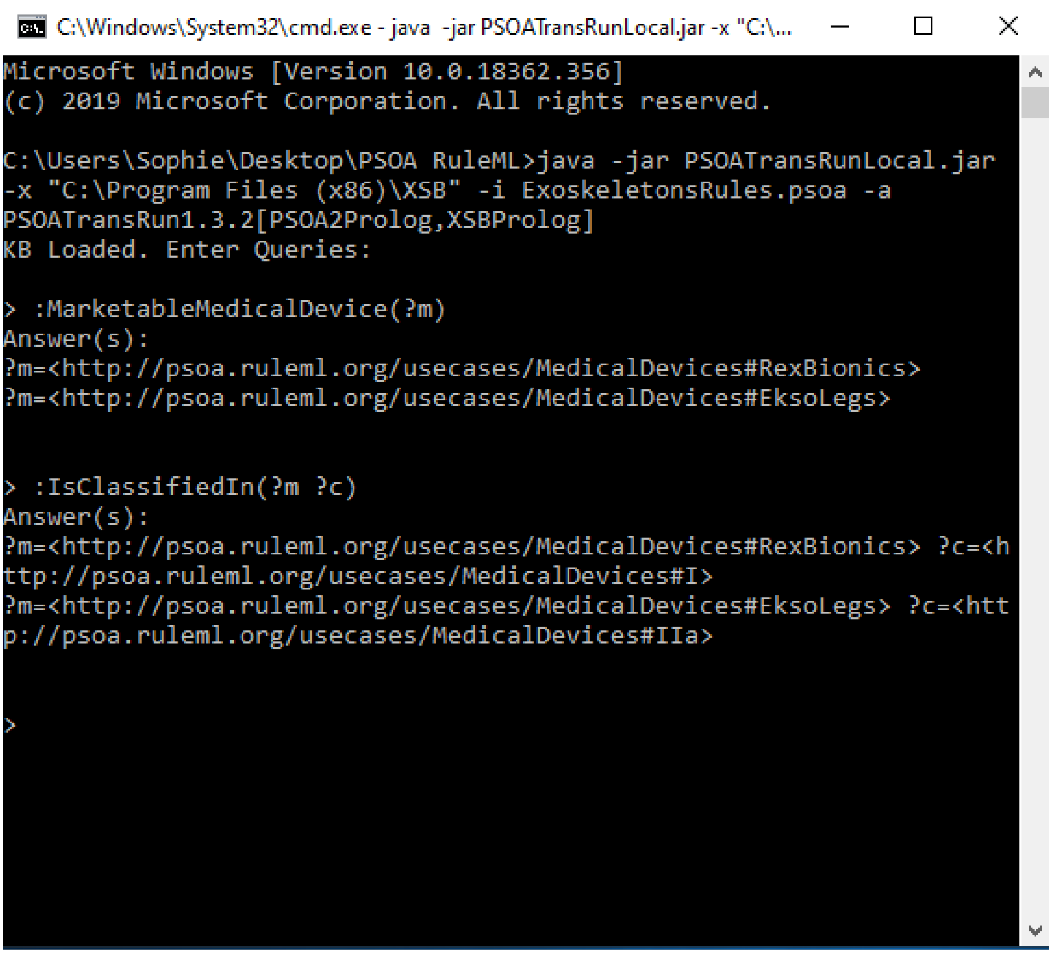
Abstracting this query (e.g., the constant `:IIa` becomes the variable `?c`), the generalized query `:IsClassifiedIn(?m ?c)` could also be posed, to deduce all exoskeletons and their corresponding classes, using two output variables (`?m` and `?c`).

More queries can be made on the marketability and conformity requirements of exoskeletons. In the example of `:EksoLegs` represented by the code `:MDN0310b`, PSOATransRun returns a 'Yes'-answer in the following queries.

```
:IsClassifiedIn(:EksoLegs :IIa)
:RegisterWithTheECA(:EksoLegs)
:AppointingAnEAR(:EksoLegs)
:DeclarationOfConformity(:EksoLegs)
:HasCEwithNBN(:EksoLegs)
:MarketableMedicalDevice(:EksoLegs)

% Answer for all: Yes %
```

A query on all marketable exoskeletons can also be made, using e.g. the input variable `?m`, by posing the query `:MarketableMedicalDevice(?m)`. Moreover, all exoskeletons that satisfy one or more specific marketability requirements can be obtained as shown below (where part of the namespace is omitted to conserve space) and in Figure 5.16.



```
Microsoft Windows [Version 10.0.18362.356]
(c) 2019 Microsoft Corporation. All rights reserved.

C:\Users\Sophie\Desktop\PSOA RuleML>java -jar PSOATransRunLocal.jar
-x "C:\Program Files (x86)\XSB" -i ExoskeletonsRules.psoa -a
PSOATransRun1.3.2[PSOA2Prolog,XSBProlog]
KB Loaded. Enter Queries:

> :MarketableMedicalDevice(?m)
Answer(s):
?m=<http://psoa.ruleml.org/usecases/MedicalDevices#RexBionics>
?m=<http://psoa.ruleml.org/usecases/MedicalDevices#EksoLegs>


> :IsClassifiedIn(?m ?c)
Answer(s):
?m=<http://psoa.ruleml.org/usecases/MedicalDevices#RexBionics> ?c=<h
ttp://psoa.ruleml.org/usecases/MedicalDevices#I>
?m=<http://psoa.ruleml.org/usecases/MedicalDevices#EksoLegs> ?c=<htt
p://psoa.ruleml.org/usecases/MedicalDevices#IIa>


>
```

Figure 5.16 Queries in PSOATransRun

```
> :MarketableMedicalDevice(?m)
Answer(s):
?m=<.../MedicalDevices#RexBionics>
?m=<.../MedicalDevices#EksoLegs>
```

The complete KB coupled with the database source, the excel sheet for the requirements' checklist, the Python script for converting the database in to a PSOA RuleML code, and a Readme File can be found at http://users.ntua.gr/salmpani/ExosCE/.

Concerning PSOATransRun and the query-answer process of our formalization, the query examples indicate that in both typical and complex queries the answers provided by PSOATransRun were accurate and the results can be validated by a human with audit trails, which is a critical parameter in the medical sector. The queries are posed at a KB which integrates object and relational modeling. However, there are some limitations on the kinds of queries that can be answered. One limitation is that even though PSOATransRun can retrieve all *compliant* medical devices (for, e.g., a specific marketability requirement), it can not retrieve all *non-compliant* devices. Similarly, it is not possible to retrieve all requirements that need to be fulfilled in order to establish the compliance of a device. Another query limitation in the taxonomy is that even though in the bottom-to-top direction it is possible to ask about all the upper categories from a lower instance level (i.e., medical device product UDI), in the opposite direction, only the instances of the lowest level can be obtained, without the middle levels. The run-time performance of PSOATransRun has also been evaluated. For our testing laptop (Intel Core 2 Duo P7550 2.26GHz CPU, 4GB RAM, running on Linux) query answering was instantaneous for the provided data set, which includes 55 categories in the KB and more than 150 examples of products in the taxonomy, even with queries with three different variables, as in the example below:

```
And(:DeclarationOfConformity(?m) :QualityType(?m ?q)
   :IsClassifiedIn(?m ?c))
```

Chapter 6

Conclusions

A proving process requires a dialogue between agents to clarify obscure inference steps, fill gaps or reveal implicit assumptions in a purported proof. Hence, argumentation is an integral component of the discovery process for mathematical proof. This thesis presents how argumentation can be applied to describe dialectical and conflicting features in the development of proof-events (as described by Goguen), highlighting the relation between proof, human reasoning, and cognitive processes. The aim was to develop an extended version of proof-events calculus build on logic-based argumentation in order to make proof-events more competent to formalize both the internal and external structure of a cooperative mathematical practice.

We presented the Argumentation-based Proof-Event Calculus (APEC), *a calculus defining argumentation-based proof-event, argument moves, and temporal predicates, and we analyzed them in terms of levels of argumentation.* This enables us to model conflicting arguments or unresolved moves, similarities and contradictions in multi-agent dialogues, the social collaboration between provers and interpreters, the controversy of previously accepted proofs, and so on, aspects that are often unseen or ignored in traditional mathematical models.

The original contribution of this thesis is that *this calculus is formal, practical, and has the expressive power to represent real mathematical proving.* We suggested a model for multi-agent proving, where problem-solving is implemented as a cooperative discovery proof-event. The model provided the analysis of the step-by-step components (argumentation-based proof-events) of mathematical practice, distinguishing the process of searching for proof (informal proving) from the final product of this process (formal proof). The combination of proof-events-based theory and logic-based argumentation makes it possible to dive into the micro-structure of the proving process — as in the case of Zero Knowledge Proof Events — which allows us to also track the informal aspects of conveying mathematical information at all steps of proving, as well as in the external structure of the proving, highlighting the social roles and the interactions of the contributors, as in the case of Fermat's last theorem and MiniPolymath4. We also developed an ethical extension of this calculus named Moral Argumentation-based Proof-Event Calculus (MAPEC), presenting a scenario that formally engineers the ethically correct behavior of medical robots.

Another contribution of this work covers the rapidly evolving area of medical devices and exoskeletons and their related regulations so that the current legal framework and the future challenges can be understood and addressed. It has demonstrated a formalization of medical devices and exoskeletons regulation as part of a logical KB leading to a computational decision model in Logic Programming languages. This executable formalization was tested by implementing queries and evaluating the answers retrieved. The resulting KB is capable of answering queries regarding the classification and marketability of medical devices aiming at compliance with the Regulation (EU) 2017/745. This has created an initial opportunity for decision support using this rule formalization via formal query, analysis, and proof, as well as permitting translation to other formalisms.

For medical companies, there is a continuous necessity to balance compliance, quality, and agility, thus there is a need for automation of procedures to facilitate and expedite the necessary time to obtain pre-market approval and allocate medical devices products to the market. These prototypes are publicly accessible, allowing anyone to try the system and view the AMeDC, the Medical Devices Rules, and the ExosCE code source (see the Appendixes). In addition to representing European regulations precisely enough to determine whether the necessary requirements within the scope of the CE-registration procedure would be compliant with law, this development aimed at a formalization that could be verifiable by lawyers, medical experts, and programmers alike. For this reason, a great effort was made to formalize the law so that the presentation can be read and understood section by section. This can contribute to the effort of unifying legal frameworks evolved to a computational format, as part of legal-informatics efforts.

The developed theory combined with the described use cases demonstrated the applicability and effectiveness of the proposed methodologies, either explicit in the area of formal and informal mathematical discovery or implicit in the area of legal and ethical aspects of medical devices and wearable robots.

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Mini-Polymath and AMeDC KB in Argumentation

.1 The detailed formalization of Mini-Polymath in APEC

"The liar's guessing game is a game played between two players A and B. The rules of the game depend on two positive integers k and n which are known to both players. At the start of the game, A chooses two integers x and N with $1 \leq x \leq N$*. Player A keeps x secret, and truthfully tells N to player B. Player B now tries to obtain information about x by asking player A questions as follows. Each question consists of B specifying an arbitrary set S of positive integers (possibly one specified in a previous question), and asking A whether x belongs to S. Player B may ask as many such questions as he wishes. After each question, player A must immediately answer it with yes or no, but is allowed to lie as many times as she wishes; the only restriction is that, among any* $k+1$ *consecutive answers, at least one answer must be truthful. After B has asked as many questions as he wants, he must specify a set X of at most n positive integers. If x belongs to X, then B wins; otherwise, he loses."*

Object level arguments (the statement of the problem):
$e_{LiarGuessingGame} = e_{LGG1}\langle \Phi, c_1 \rangle \cap e_{LGG2}\langle \Phi, c_2 \rangle$, where:
$\Phi = \langle$The liar's guessing game.$\rangle$
$c_1 = \langle$If $n \geq 2^k$ then B can guarantee a win.$\rangle$
$c_2 = \langle$For all sufficiently large k, there exists an integer $n \geq 1.99^k$ such that B cannot guarantee a win.$\rangle$
$Happens(e_{LGG}, f_0, t_1) \rightarrow Initiates(e_{LGG}, f_0, t_1)$

First-level priority arguments (First attempts):

$Support(e_{LGG1}, e_3) \rightarrow Elaboration(e_{LGG1}, e_3)$
$Support(e_{LGG1}, e_4) \rightarrow Elaboration(e_{LGG1}, e_4)$
$e_3 = \langle \Phi, c_1 >$, with
$infRul(e_3)$ = ⟨The fact that player A has to choose the number N at the beginning of the game is intriguing. The number of possibilities for x is originally N, so it would seem like large N would make the game harder for B. I suspect that B can counteract the difficulty by asking many more questions for large N than small N.⟩
$e_4 = \langle \Phi, c_1 \rangle$, with
$infRul(e_4)$ = ⟨ Ramsey Theory ⟩.

Second-level Arguments (Induction):

$e_6 = \langle \Phi, c_1 \rangle$, with
$infRul(e_6)$ = ⟨Induction with respect to N.⟩
$Initiates(e_6, f_2, t_6) \rightarrow support(e_6, t_{L_2})$
$support(e_6, t_{L_2}) \rightarrow Equiv(e_6, e_{12}) \cup Elab(e_6, Se_{6_a}) \cup Elab(e_6, Se_{6_c})$
$Elab(e_6, Se_{6_a})$ = ⟨It seems to me that if we could ask a series of questions to guarantee that x falls inside, say, $[0, N/2]$, then we could reduce to a previous case, but once we find such a series of questions we more or less have solved the problem.⟩
$Elab(e_6, Se_{6_c})$ =⟨ It suffices to prove it for $N = n+1$. See comment 12 (i.e. e_{12}).⟩

Third-level Arguments (Guessing answers of B):

$e_7 = \langle \Phi_7, c_7 \rangle$ = ⟨Φ_7 :B cannot guarantee the win, c_7: it can be "always win" for A⟩ (this proof-event can be implied from the problem.)
$e_7^* = \langle \Phi_7^*, c_7^* \rangle$ = ⟨Φ_7^*: Since there is a possibility that B would win the game simply by guessing, c_7^*:there is no "always win" for A⟩
$Rebutting(e_7^*, e_7) : rebut(e_7^*, e_7) \rightarrow \neg concl(e_7)$ and
$attack(e_7^*, e_7) \rightarrow Rebutting(e_7^*, e_7)$, where
$concl(e_7)$ = ⟨"always win" for A⟩.
$Terminates(e_7, f_3, t_{L_3}) \rightarrow attack(e_7^*, e_7)$
Argument e_8 adds an observation on the warrant of e_{LGG_1}.
$e_8 = \langle \Phi_8, c_8 \rangle$ = ⟨Φ_8: For the first part, proving for $n = 2^k$ suffices. The first approach that comes to my mind is to induct on k, c_8: c_{LGG_1} ⟩ with warrant w_8 = ⟨ proving for $n = 2^k$ ⟩
$e_9 = \langle \Phi_9, c_9 \rangle$= ⟨$\Phi_9$: B can as well ask questions in "rounds" of $k+1$ questions, c_9: then,

each round is guaranteed to have at least 1 correct answer⟩
$e_9^* = \langle \Phi_9, c_9 \rangle$, with $infRul(e_9^*)$= ⟨While this is true, it is not very constructive. Player A can just answer about half truth and half lies, making this strategy hard to implement.⟩
$Undermining(e_9^*, e_9) : Undermin(e_9^*, e_9) \rightarrow \neg prem(e_9)$ and
$attack(e_9^*, e_9) \rightarrow rebut(e_9^*, e_9)$
Thus, $Terminates(e_9, f_3, t_{L3}) \rightarrow attack(e_7^*, e_7)$

Fourth-level arguments (proof for $k = 1$):
$e_{10} = \langle \Phi_{10}, c_{10} \rangle$=⟨$\Phi_{10}$: So for $k = 0$ any version of binary search works, c_{10}: The next step should be to find the strategy for $k = 1, n = 2$⟩, where
$\neg infRul(e_{10})$, since the contributor claims "I first thought I have found the strategy, but it doesn't work."
$e_{10_a} = \langle \Phi_{10_a}, c_{10_a} \rangle$=⟨$\Phi_{10_a}$: I am working on this case too. Here player A can never tell two lies in a row. Here is a little observation I have made. Let Q1, and Q2 be questions that player B can ask, and I will use the notation like:
Q's: Q1 Q2 ... A's: L T ... To denote that we asked Q1, then Q2 and we received a lie and a truth respectively (of course, B doesn't know which),
c_{10_a}: Here is a cute little lemma: If B asks Q1 Q2 Q1, then A must give the same answer for Q1 both times it is asked, or else tell the truth for Q2. Proof: There are 5 possible ways A can answer. LTL, LTT, TLT, TTL, TTT. From here we see that if the answers to Q1 are different, then the only possibilities are LTT and TTL, in either case the answer to Q2 must be true.⟩.
$e_{10_b} = \langle \Phi_{10_b}, c_{10_b} \rangle$=⟨$\Phi_{10_b}$: Let Q1,Q2 be questions. If player B asks the sequence of questions Q1 Q2 Q1 Q1 and gets answers A1 A2 A3 A4 (each Ai is either an L (lie) or a T(truth)) [...], c_{10_b}: Then player A is forced to reveal one of the following pieces of information to player B. (i.e. player B will know which of them is true.): i) A2 = T, ii) A3 = A4 = T, iii) A2 = A4. By the last lemma for the sequence of questions Q1 Q2 Q1, player B knows that either A2=T or the answers to the first three questions are LTL, TLT, or TTT [...]. I think the second lemma can be used to make a binary search by making Q1 = half the numbers, Q2= the other half of the numbers⟩.
$support(e_{10}, e_{10_a}) \rightarrow Elaborate(e_{10}, S_{e_{10a}})$,
$support(e_{10}, e_{10_b}) \rightarrow Elaborate(e_{10}, S_{e_{10_b}})$
$prem(S_{e_{10_a}})$=⟨If Player B asks the same question twice in a row and the answer is the same both times, then it must have been true both times⟩
$prem(S_{e_{10_b}})$=⟨"Let Q1,Q2 be questions. If player B asks the sequence of questions Q1 Q2 Q1 Q1 and gets answers $A_1 A_2 A_3 A_4$ (each A_i is either an L (lie) or a T (truth)).

Then player A is forced to reveal one of the following pieces of information to player B. (i.e,. player B will know which of them is true.”)⟩

$Initiates(e_{10}, f_4, t_{L_4}) \rightarrow support(e_{10}, e_{10_a}) \cup support(e_{10}, e_{10_b})$

(continues for comments 11–15)

Higher-lever Arguments (proof of LGG1):

$e_{16} = \langle \Phi_{16}, c_{16} \rangle$, where

$\Phi_{16} = prem(e_{16})$=⟨ We can assume $N = 2^k + 1, n = 2^k$. It means that x has at most $k+1$ binary digits ($k+1$ digits only for $n = 2^k$): $x = b_1 b_2 \dots b_{k+1}$. Then we can keep asking if b_1 is 1, there are two possibilities.

$c_{16} = conc(e_{16}) = conc(e_{16a}) \cup conc(e_{16b})$, where

$conc(e_{16a})$ = ⟨ $k+1$ times we get the answer NO, then we exclude the number $10 \dots 0$ ⟩

$conc(e_{16b})$ =⟨There is a YES answer. Then we stop asking about b_1 and ask $b_2 = 1, b_3 = 1 \dots b_{k+1} = 1$.⟩ After we are done we can exclude the number for which all the last $k+1$ answers would have been lies whose first digit is 0 (because of the YES answer).⟩

$support(e_{16}, e_{16a}) \rightarrow equivalent(e_{16}, e_{16a})$, where

e_{16a} =⟨Another way (which seems to solve the first question). We ask the sequence of question Q_i: “Does $b_i = 1$?” in a row. That makes $k+1$ questions. Then we must have at least one of the digits right. In particular, let $y = c_1 \dots c_{k+1}$ be such that $c_i = 0$ if the answer to A_i is Yes, and $c_i = 1$ if the answer to A_i is No. Then $x \neq y$. We have excluded a possibility, which by the reduction of comment 15 is enough.⟩

$Rebutting(e^*_{16_b}, e_{16_a}) : rebut(e^*_{16_b}, e_{16_a}) \rightarrow \neg concl(e_{16_a})$, where

e^*_{16b}=⟨Which number will you exclude in that case? (It might not be in the range)⟩, and

$attack(e^*_{16_b}, e_{16_a}) \rightarrow Rebutting(e^*_{16_b}, e_{16_a})$

$ActiveAt(e_{16_a}, f_n, t_{L_n}) \rightarrow support(e_{16_a}, e_{16_c}) \cup support(e_{16_a}, e_{16_d})$, with

$support(e_{16_a}, e_{16_c}) \rightarrow Elaborate(e_{16_a}, S_{e_{16_c}})$, and

$support(e_{16_a}, e_{16_d}) \rightarrow Elaborate(e_{16_a}, S_{e_{10d}})$, where

e_{16c}=⟨When $c_1 = 1$, then the number might be out of the range.⟩

e_{16d}=⟨I'm not sure I totally understand your argument, but your argument lead me towards the following:

Let B_i be the subset of $\{0, \cdots, N-1\}$ with 0 as the i^{th} digit in their binary expansion (note we're leaving out one member).

Let B ask $B_1, \cdots, B_k$ in that order, and let b_i be 0 if A says yes to B_i and 1 else. Then let s_i be the number with binary expansion $a_0a_1\cdots a_ia'_{i+1}\cdots a'_k$ where $a'_j = 1-a_j$. Now ask $\{s_0\},\cdots,\{s_k\}$ in order.

Suppose A answers at least once that $x \neq s_i$, and pick the first such instance of this. Then if $x = s_i$, A will have lied for the last $k+1$ questions, i.e. $B_i, B_{i+1},\cdots,B_k,\{s_0\},\cdots,\{s_i\}$. So x cannot be s_i and **we have the required win**.

On the other hand, if A always says that $x = s_i$ for any i, then if x was the one member we didn't manipulate, A lied $k+1$ times (all $\{s_i\}$ questions). So if A says that $x = s_i$ for all i, then the one member we didn't manipulate is actually not x, so we've discarded one member, and **B wins**.$\rangle$

$Valid(e_{LGG_1}, f_n, t_{L_n}) \rightarrow support(e_{16}, e_{16_c}) \cup support(e_{16}, e_{16_d}) \cap \neg attack(e^*_{16}, e_{16})$

A computational prototype of the APEC model

The combination of the Web and crowd-sourcing with structural dialectical tools - such as APEC - can lead to significant changes in the proving practice and thus in the perception of proofs. We believe that APEC can stay above specific system selection, given that the system is based on logic programming, thus we present two logic-based computational prototypes, one expressed in GorgiasB language and one in RuleML language. This general method can be implemented in computational collaborative environments, to indicate the added contribution of invalid steps, productive failure, conflicts, negotiation, and cooperation in proof procedures.

The **GorgiasB system** has been implemented in various real-world applications in areas such as medical support, network security, business computing, cognitive personal assistants, etc., presenting an emerging general methodological framework for applications of argumentation through the systematic analysis of scenario-based conflicts [31]. Figure 1 describes a fragment of the code with explanations, while the rest of the code can be found below. More about GorgiasB argumentation framework and semantics can be found in [163].

```
% Mini-Polymath 4 (first part) in scenario-based Gorgias

<0_1, {eLGGG}, proofofLGG(happens)>

<1_1, {eLGGG, level=1}, proofofLGG(initiates)>
<1_2, {eLGGG, level=1, e3}, proofofLGG(supports)>
<1_3, {eLGGG, level=1, e4}, proofofLGG(supports)>
<1_4, {eLGGG, level=1, e5}, proofofLGG(supports)>
```

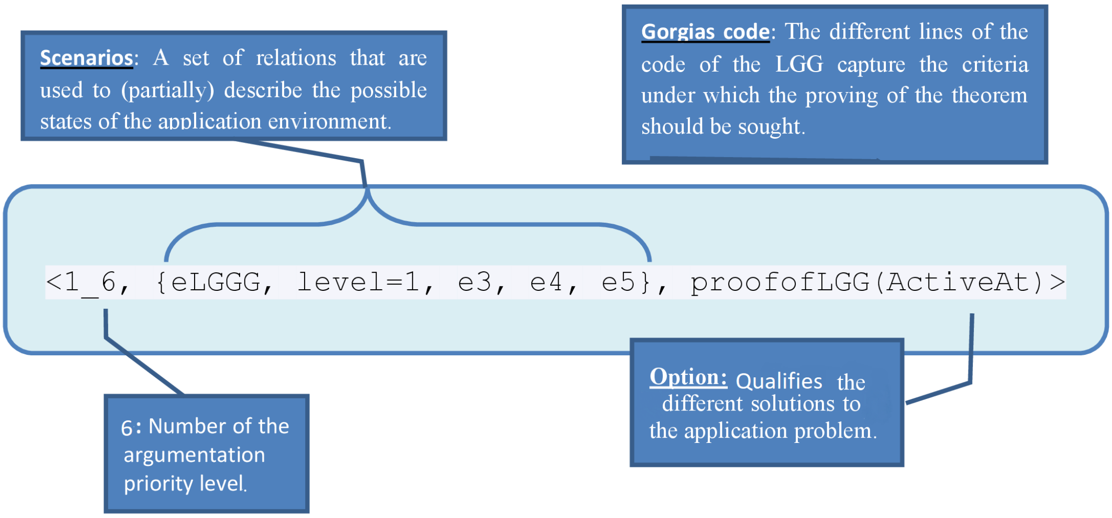

Figure 1 LGG problem in scenario-based Gorgias.

```
<1_5, {level=1, e5, e5x}, proofofLGG(equivalent)>
<1_6, {eLGGG, level=1, e3, e4, e5}, proofofLGG(ActiveAt)>

<2_1, {eLGGG, level=2}, proofofLGG(initiates)>
<2_2, {eLGGG, level=2, e6}, proofofLGG(supports)>
<2_3, {eLGGG, level=2, e6a}, proofofLGG(elaborates)>
<2_4, {eLGGG, level=2, e6b}, proofofLGG(elaborates)>
<2_5, {eLGGG, level=2, e6c}, proofofLGG(elaborates)>
<2_6, {eLGGG, level=2, e6, e6a, e6b, e6c}, proofofLGG(ActiveAt)>

<3_1, {eLGGG, level=3}, proofofLGG(initiates)>
<3_2, {level=3, e7, e7x}, proofofLGG(undemining)>
<3_3, {level=3, e9, e9x}, proofofLGG(undercutting)>
<3_4, {eLGGG, level=3, e7}, proofofLGG(terminates)>
<3_5, {eLGGG, level=3, e9}, proofofLGG(terminates)>
<3_6, {eLGGG, level=3, e7, e7x, e9, e9x}, proofofLGG(clippes)>

<4_1, {eLGGG, level=n}, proofofLGG(initiates)>
<4_2, {eLGGG, level=n, e16}, proofofLGG(supports)>
<4_3, {eLGGG, level=n, e16}, proofofLGG(valid)>
```

PSOA RuleML has also been used for rules formalization in other use cases, such as Port Clearance Rules [162], Air Traffic Control Regulations [163], Medical Devices

Regulations [49], and Exoskeletons Compliance [204], providing evidence that PSOA RuleML is well-suited to express real-world texts. Details of PSOA RuleML syntax, terms, and PSOATransRun can be found in [162].

```
% Mini-Polymath 4 (first part) in logic-based PSOA RuleML
RuleML (
    Prefix(: <http://psoa.ruleml.org/usecases/APEC#>)
 Assert(

% Rule describing the components of argumentation-based proof-events
Forall ?e ?f ?c (
  proofEvent(?e) :-
?e#components(data->?f conclusion->?c))

% Rule describing the argument moves
Forall ?e(
  attack(?e ?a ?t) :-
Or(undermining(proofEvent->?e attackPremises->?a time->?t)
   undercutting(proofEvent->?e attackWarant->?a time->?t)
   rebutting(proofEvent->?e attackConclusion->?a time->?t)))

Forall ?e(
  support(?e ?s ?t) :-
    Or(elaborate(proofEvent->?e elab->?s time->?t)
   equivalent(proofEvent->?e equiv->?s time->?t)))

% Rule describing the temporal predicates
Forall ?e ?t (
  initiates(?e ?t) :-
happens(?e ?t))

Forall ?e(
  clipped(?e ?t) :-
Or(attack(?e ?a ?t)
      Naf(support(?e ?s ?t)))) %Rule with Naf

Forall ?e(
```

```
  terminates(?e ?t) :-
 Or(attack(?e ?a ?t)
    Naf(support(?e ?s))))   %Rule with Naf

Forall ?e(
  activeAt(?e ?t) :-
Or(support(?e ?s ?t)
   Naf(attack(?e ?a ?t)))) %Rule with Naf

Forall ?e (
  valid(?e) :-
 And(happens(?e T0)
     activeAt(?e Tn)))

%Facts from Mini-Polymath 4
ELGG11#components(data->FLGG11 conclusion->CLLG1)
E1#components(data->F1 conclusion->C1)
E3#components(data->F3 conclusion->C3)
E4#components(data->F4 conclusion->C4)
E5#components(data->F5 conclusion->C5)
E5a#components(data->F5a conclusion->C5a)
E6#components(data->F6 conclusion->C6)
E6a#components(data->F6a conclusion->C6a)
E6b#components(data->F6b conclusion->C6b)
E6c#components(data->F6c conclusion->C6c)
E7#components(data->F7 conclusion->C7)
E7x#components(data->F7x conclusion->C7x)
E9#components(data->F9 conclusion->C9)
E9x#components(data->F9x conclusion->C9x)
E16#components(data->F16 conclusion->C16)

happens(ELGG1 T0)
happens(E1 T1)
happens(E6 T2)

elaborate(proofEvent->ELGG1G elab->E3 time->T1)
```

```
elaborate(proofEvent->ELGG1G elab->E4 time->T1)
elaborate(proofEvent->ELGG1G elab->E5 time->T1)
equivalent(proofEvent->E5 equiv->E5a time->T1)

elaborate(proofEvent->E6 elab->E6a time->T2)
elaborate(proofEvent->E6 elab->E6b time->T2)
elaborate(proofEvent->ELGG1 elab->E16 time->Tn)

undermining(proofEvent->E7 attackPremises->E7x time->T3)
undercutting(proofEvent->E9 attackWarant->E9x time->T3)
  )
 )
```

In PSOA RuleML presentation of the APEC model the levels of argumentation are depicted through the ‘time -> T3’ predicate, where ‘T3’ means third-level or argumentation.

Note that the online version in https://psoademo-chatty-cat.eu-gb.mybluemix.net/ does not support the Naf (Negation-as-Failure) option, thus the three “Rules with Naf” can be replaced without any mistake in the expected results by the following ones, if someone wants to try the model in the online version instead of the local.

```
Forall ?e(
  terminates(?e ?t) :-
 attack(?e ?a ?t)

Forall ?e(
  clipped(?e ?t) :-
attack(?e ?a ?t))

Forall ?e(
  activeAt(?e ?t) :-
support(?e ?s ?t))
```

In order to interface the PSOA RuleML code with a PSOATransRun Reasoner, you can either invoke the online - Web-based service - PSOATransRun[1] or the local - downloadable executable - PSOATransRun[2]. Details of PSOA RuleML syntax, terms,

---

[1] https://psoademo-chatty-cat.eu-gb.mybluemix.net/

[2] http://psoa.ruleml.org/transrun/1.4.3/local/

and PSOATransRun can be found also at the PSOA RuleML wiki page:
http://wiki.ruleml.org/index.php/PSOA_RuleML

```
Representative Queries and Answers in PSOA TransRun

% Q&A on argumentation-based proof-events
> proofEvent(?e)
Answer(s):
?e=_E7x
?e=_E1
?e=_E6a
?e=_E9
?e=_E16
?e=_E7
?e=_E5a
?e=_E9x
?e=_E5
?e=_E6
?e=_E6b
?e=_E6c
?e=_ELGG11
?e=_E3
?e=_E4

% Q&A on the structural components of argumentation-based proof-events
> ?e#components(data->?f conclusion->?c)
Answer(s):
?e=_E7x ?f=_F7x ?c=_C7x
?e=_E1 ?f=_F1 ?c=_C1
?e=_E6a ?f=_F6a ?c=_C6a
?e=_E9 ?f=_F9 ?c=_C9
?e=_E16 ?f=_F16 ?c=_C16
?e=_E7 ?f=_F7 ?c=_C7
?e=_E5a ?f=_F5a ?c=_C5a
?e=_E9x ?f=_F9x ?c=_C9x
?e=_E5 ?f=_F5 ?c=_C5
```

```
?e=_E6 ?f=_F6 ?c=_C6
?e=_E6b ?f=_F6b ?c=_C6b
?e=_E6c ?f=_F6c ?c=_C6c
?e=_ELGG11 ?f=_FLGG11 ?c=_CLLG1
?e=_E3 ?f=_F3 ?c=_C3
?e=_E4 ?f=_F4 ?c=_C4

% Q&A on argument moves
> undermining(proofEvent->?e attackPremises->?a time->?t)
Answer(s):
?e=_E7 ?a=_E7x ?t=_T3

> undercutting(proofEvent->?e attackWarant->?a time->?t)
Answer(s):
?e=_E9 ?a=_E9x ?t=_T3

> rebutting(proofEvent->?e attackConclusion->?a time->?t)
Answer(s):
No

> elaborate(proofEvent->?e elab->?s time->?t)
Answer(s):
?e=_ELGG1G ?s=_E5 ?t=_T1
?e=_E6 ?s=_E6a ?t=_T2
?e=_ELGG1G ?s=_E4 ?t=_T1
?e=_ELGG1 ?s=_E16 ?t=_Tn
?e=_E6 ?s=_E6b ?t=_T2
?e=_ELGG1G ?s=_E3 ?t=_T1

> equivalent(proofEvent->?e equiv->?s time->?t)
Answer(s):
?e=_E5 ?s=_E5a ?t=_T1

> support(?e ?s ?t)
Answer(s):
?e=_ELGG1G ?s=_E5 ?t=_T1
```

```
?e=_E6 ?s=_E6a ?t=_T2
?e=_ELGG1G ?s=_E4 ?t=_T1
?e=_ELGG1 ?s=_E16 ?t=_Tn
?e=_E6 ?s=_E6b ?t=_T2
?e=_ELGG1G ?s=_E3 ?t=_T1
?e=_E5 ?s=_E5a ?t=_T1

> attack(?e ?a ?t)
Answer(s):
?e=_E7 ?a=_E7x ?t=_T3
?e=_E9 ?a=_E9x ?t=_T3

% Q&A on proof-events that elaborate/support on the second-level of argumentation
> elaborate(proofEvent->E6 elab->?s time->T2)
Answer(s):
?s=_E6b ?t=_T2
?s=_E6a ?t=_T2

> support(?e ?s T2)
Answer(s):
?e=_E6 ?s=_E6a
?e=_E6 ?s=_E6b

% Q&A on the temporal predicates
> elaborate(proofEvent->E6 elab->?s time->T2)
Answer(s):
?s=_E6b ?t=_T2
?s=_E6a ?t=_T2

> initiates(?e ?t)
Answer(s):
?e=_ELGG1 ?t=_T0
?e=_E1 ?t=_T1
?e=_E6 ?t=_T2

> clipped(?e ?t)
```

```
Answer(s):
?e=_E7 ?t=_T3
?e=_E9 ?t=_T3

> terminates(?e ?t)
Answer(s):
?e=_E7 ?t=_T3
?e=_E9 ?t=_T3

> activeAt(?e ?t)
Answer(s):
?e=_ELGG1G ?t=_T1
?e=_E6 ?t=_T2
?e=_ELGG1 ?t=_Tn
?e=_E5 ?t=_T1

> valid(?e)
Answer(s):
?e=_ELGG1

% Q&A asking the level that proof-event E7 terminates
> terminates(E7 ?t)
Answer(s):
?t=_T3
```

.2 AMeDC in Computational Argumentation

Medical Devices Classification Diagrams

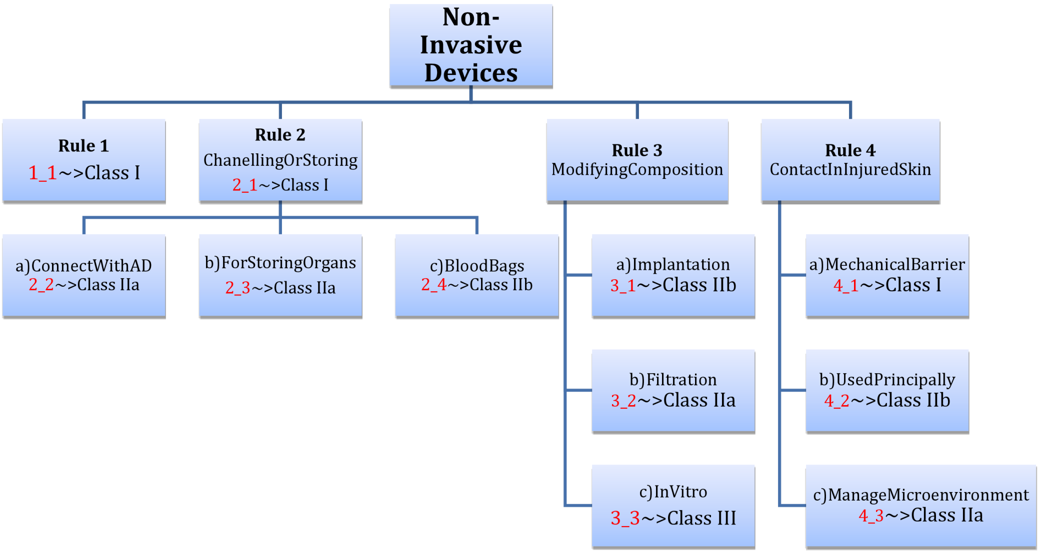

Figure 2 Diagram of the rules categories for non-invasive devices.

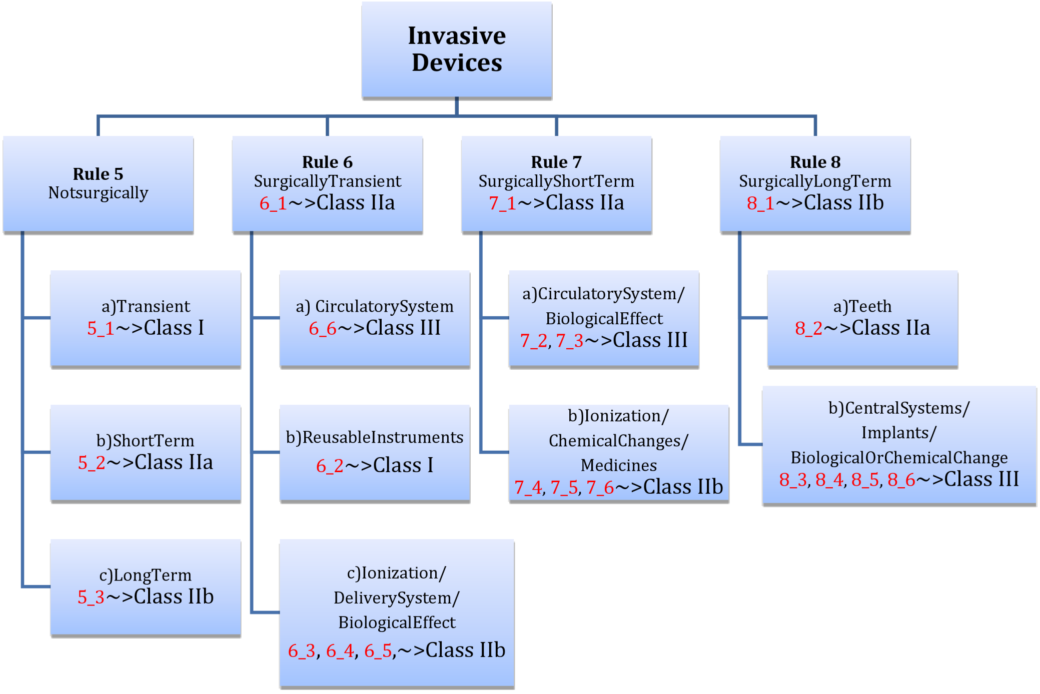

Figure 3 Diagram of the rules categories for invasive devices.

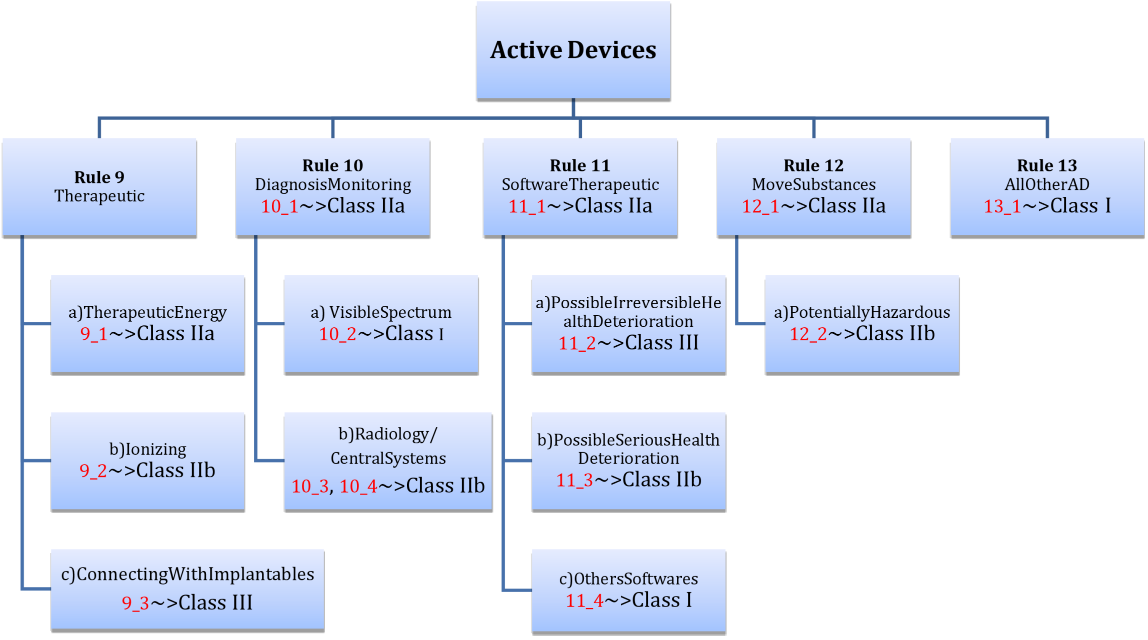

Figure 4 Diagram of the rules categories for active devices.

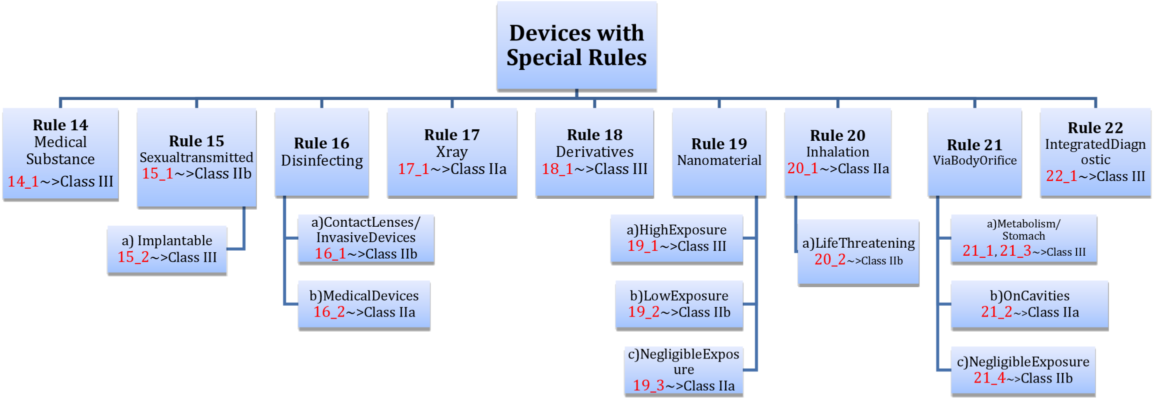

Figure 5 Diagram of the rules categories for devices with special rules.

```
% For the original text of the classification rules MDR see (page 141):
% https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32017R0745

%Medical Devices Classification Code

%Rules for Non-Invasive Medical Devices

% Rule 1
<1_1, {nonInvasive}, class(i)>

% Rule 2
<2_1, {nonInvasive, chanellingOrStoring}, class(i)>
<2_2, {nonInvasive, chanellingOrStoring, connectedWithAD}, class(iia)>
<2_3, {nonInvasive, chanellingOrStoring, forStoringOrgans}, class(iia)>
<2_4, {nonInvasive, chanellingOrStoring, forBloodBags}, class(iib)>

% Rule 3
<3_1, {nonInvasive, modifyingComposition, implantation}, class(iib)>
<3_2, {nonInvasive, modifyingComposition, filtration}, class(iia)>
<3_3, {nonInvasive, modifyingComposition, inVitro}, class(iii)>

% Rule 4
<4_1, {nonInvasive, contactInjuredSkin, mechanicalBarrier}, class(i)>
<4_2, {nonInvasive, contactInjuredSkin, usedOrincipally}, class(iib)>
<4_3, {nonInvasive, contactInjuredSkin, manageMicroenvironment}, class(iia)>

%Rules for Invasive Medical Devices

% Rule 5
<5_1, {invasive, notSurgically, transient}, class(i)>
<5_2, {invasive, notSurgically, shortTerm}, class(iia)>
<5_3, {invasive, notSurgically, longTerm}, class(iib)>

% Rule 6
<6_1, {invasive, surgicallyTransient}, class(iia)>
<6_2, {invasive, surgicallyTransient, reusableInstruments}, class(i)>
```

```
<6_3, {invasive, surgicallyTransient, ionization}, class(iib)>
<6_4, {invasive, surgicallyTransient, deliverySystem}, class(iib)>
<6_5, {invasive, surgicallyTransient, biologicalEffect}, class(iib)>
<6_6, {invasive, surgicallyTransient, circulatorySystem}, class(iii)>

% Rule 7
<7_1, {invasive, surgicallyShortTerm}, class(iia)>
<7_2, {invasive, surgicallyShortTerm, circulatorySystem}, class(iii)>
<7_3, {invasive, surgicallyShortTerm, biologicalEffect}, class(iii)>
<7_4, {invasive, surgicallyShortTerm, ionization}, class(iib)>
<7_5, {invasive, surgicallyShortTerm, chemicalChanges}, class(iib)>
<7_6, {invasive, surgicallyShortTerm, medicines}, class(iib)>

% Rule 8
<8_1, {invasive, surgicallyLongTerm}, class(iib)>
<8_2, {invasive, surgicallyLongTerm, teeth}, class(iia)>
<8_3, {invasive, surgicallyLongTerm, centralSystems}, class(iii)>
<8_4, {invasive, surgicallyLongTerm, implants}, class(iii)>
<8_5, {invasive, surgicallyLongTerm, biologicalChange}, class(iii)>
<8_6, {invasive, surgicallyLongTerm, chemicalChanges}, class(iii)>

%Rules for Active Medical Devices

% Rule 9
<9_1, {active, therapeutic, therapeuticEnergy}, class(iia)>
<9_2, {active, therapeutic, ionizing}, class(iib)>
<9_3, {active, therapeutic, connectingWithImplantables}, class(iii)>

% Rule 10
<10_1, {active, diagnosisMonitoring}, class(iia)>
<10_2, {active, diagnosisMonitoring, visibleSpectrum}, class(i)>
<10_3, {active, diagnosisMonitoring, radiology}, class(iib)>
<10_4, {active, diagnosisMonitoring, centralSystems}, class(iib)>

% Rule 11
<11_1, {active, softwareTherapeutic}, class(iia)>
```

```
<11_2, {active, softwareTherapeutic, possibleIrreversibleHealthDeteriotation},
class(iii)>
<11_3, {active, softwareTherapeutic, possibleSeriousHealthDeteriotation},
class(iib)>
<11_4, {active, softwareTherapeutic, otherSoftwares}, class(i)>

% Rule 12
<12_1, {active, moveSubstances}, class(iia)>
<12_2, {active, moveSubstances, potentiallyHazardous}, class(iib)>

% Rule 13
<13_1, {active, allOtherAD}, class(i)>

%Rules for Medical Devices with Special Rules

% Rule 14
<14_1, {devicesWithSpecialRules, medicalSubstance}, class(iii)>

% Rule 15
<15_1, {devicesWithSpecialRules, sexualTransmitted}, class(iib)>
<15_2, {devicesWithSpecialRules, sexualTransmitted, impantable}, class(iii)>

% Rule 16
<16_1, {devicesWithSpecialRules, disinfecting, contactLenses}, class(iib)>
<16_2, {devicesWithSpecialRules, disinfecting, invasive}, class(iib)>
<16_3, {devicesWithSpecialRules, disinfecting, medicalDevices}, class(iia)>

% Rule 17
<17_1, {devicesWithSpecialRules, xRay}, class(iia)>

% Rule 18
<18_1, {devicesWithSpecialRules, derivatives}, class(iii)>

% Rule 19
<19_1, {devicesWithSpecialRules, nanomaterial, highExposure}, class(iii)>
<19_2, {devicesWithSpecialRules, nanomaterial, lowExposure}, class(iib)>
```

```
<19_3, {devicesWithSpecialRules, nanomaterial, negligibleExposure},
class(iia)>

% Rule 20
<20_1, {devicesWithSpecialRules, inhalation}, class(iia)>
<20_2, {devicesWithSpecialRules, inhalation, lifeThreatening}, class(iib)>

% Rule 21
<21_1, {devicesWithSpecialRules, viaBodyOrifice, metabolism}, class(iii)>
<21_2, {devicesWithSpecialRules, viaBodyOrifice, onCavities}, class(iia)>
<21_3, {devicesWithSpecialRules, viaBodyOrifice, stomach}, class(iii)>
<21_4, {devicesWithSpecialRules, viaBodyOrifice, negligibleExposure},
class(iib)>

% Rule 22
<22_1, {devicesWithSpecialRules, integrtedDiagnostic}, class(iii)>
```

Medical Devices Rules and ExosCE KB in PSOA RuleML

.3 Medical Devices Rules KB

PSOA RuleML is a Web rule language that generalizes RIF-BLD and POSL by a homogeneous integration of relationships and frames into ***p**ositional-**s**lotted **o**bject-**a**pplicative* (***psoa***) *terms*, for the often used single-tuple case having these forms ($\mathtt{n} \geq 0$ and $\mathtt{k} \geq 0$):[3]

$$\textbf{Oidless}: \quad \mathtt{f(t_1 \ldots t_n\ p_1\text{->}v_1 \ldots p_k\text{->}v_k)} \tag{1}$$

$$\textbf{Oidful}: \mathtt{o\#f(t_1 \ldots t_n\ p_1\text{->}v_1 \ldots p_k\text{->}v_k)} \tag{2}$$

Both (1) and (2) apply a function or predicate `f` (acting as a relator) – in (2) identified by an OID `o` via a membership, `o # f`, of `o` in `f` (acting as a class) – to a tuple of arguments $\mathtt{t_1} \ldots \mathtt{t_n}$ and to a bag of slots $\mathtt{p_j\text{->}v_j}$, $\mathtt{j} = 1, \ldots, \mathtt{k}$, each pairing a slot name (attribute) $\mathtt{p_j}$ with a slot filler (value) $\mathtt{v_j}$. A psoa term can be interpreted as a *psoa expression*, denoting an individual, or a *psoa atom*, denoting a truth value, depending on whether `f` is a function or predicate. A top-level psoa term is always interpreted as an atom. An embedded psoa term is interpreted as an atom if it has the oidful form (2); else, as an expression if it has the oidless form (1). Constants include `Top`, numbers, strings, and Internationalized Resource Identifiers (IRIs). Variables in PSOA are '`?`'-prefixed names, e.g., `?x`. The most common atomic formulas are psoa atoms in the form of (1) or (2). Compound formulas can be constructed using the Horn-like subset of First-Order Logic.

A PSOA KB consists of clauses, mostly as ground facts and non-ground rules: While facts are psoa atoms, rules are defined – within `Forall` wrappers – using a

---

[3] We use the all-upper-case "PSOA" as a reference to the language and the all-lower-case "psoa" for its terms. Earlier PSOA papers show multi-tuple psoa terms.

Prolog-like *conclusion* `:-` *condition* syntax, where *conclusion* can be a psoa atom and *condition* can be a psoa atom or an `And`-prefixed conjunction of psoa atoms.

The code source of the KB, the taxonomy and the PSOATransRun queries for our use case Medical Devieces Rules can be found in the relevant page[4].

.3.1 Medical Devices Categories and Classes

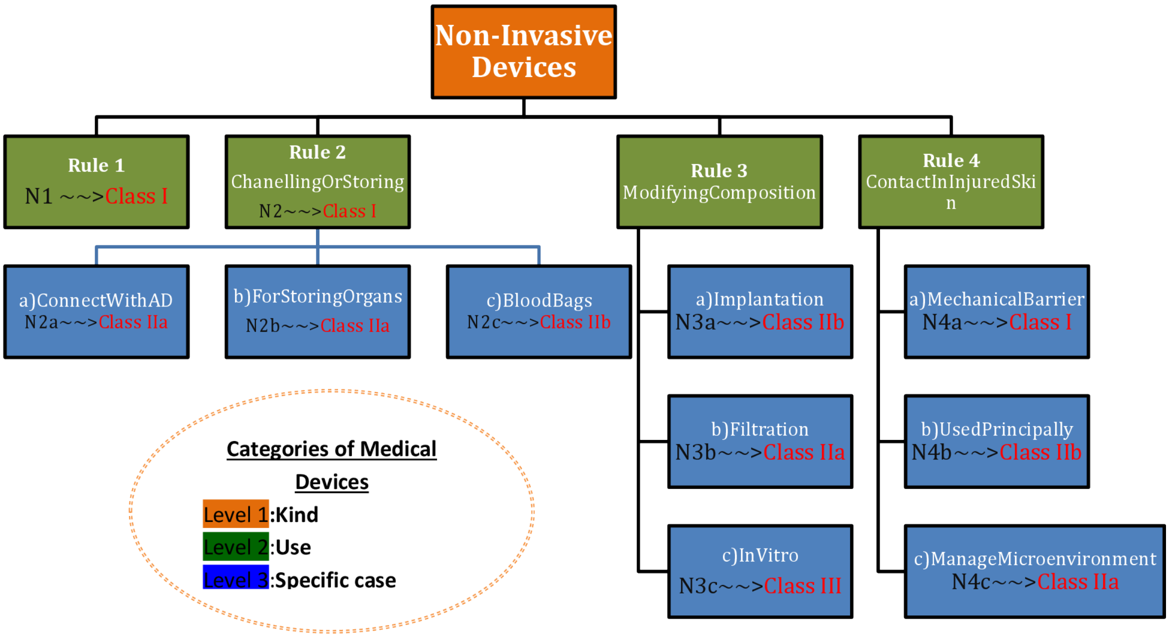

Figure 6 Non-Invasive Medical Devices.

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[4] http://psoa.ruleml.org/usecases/MedicalDevices/

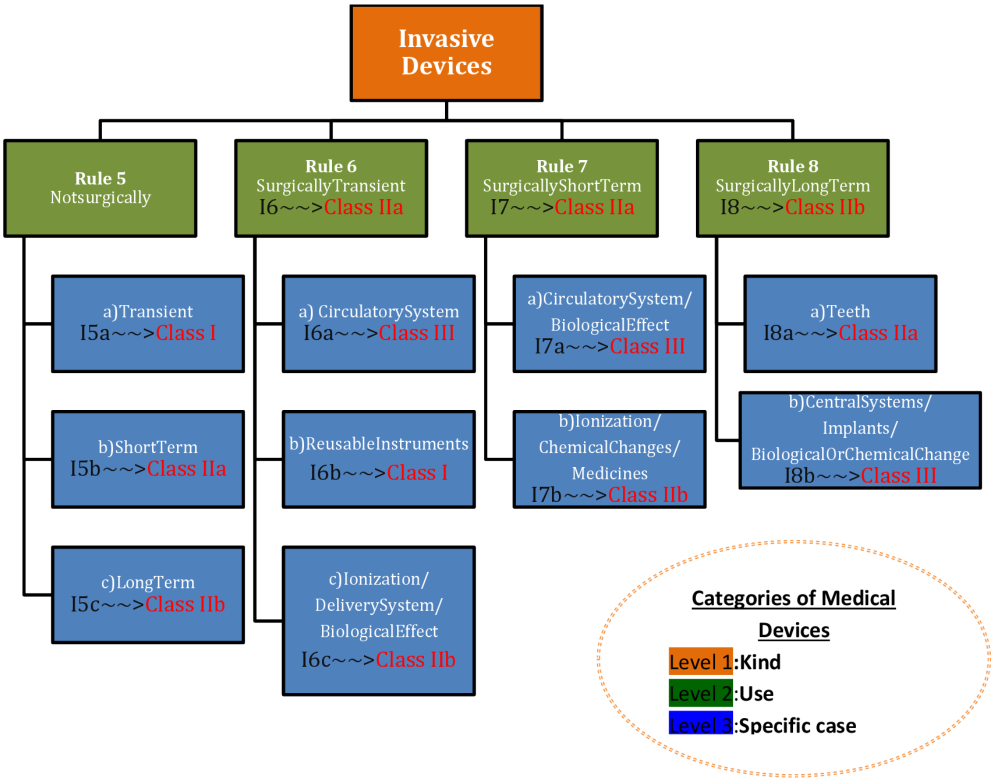


Figure 7 Invasive Medical Devices.

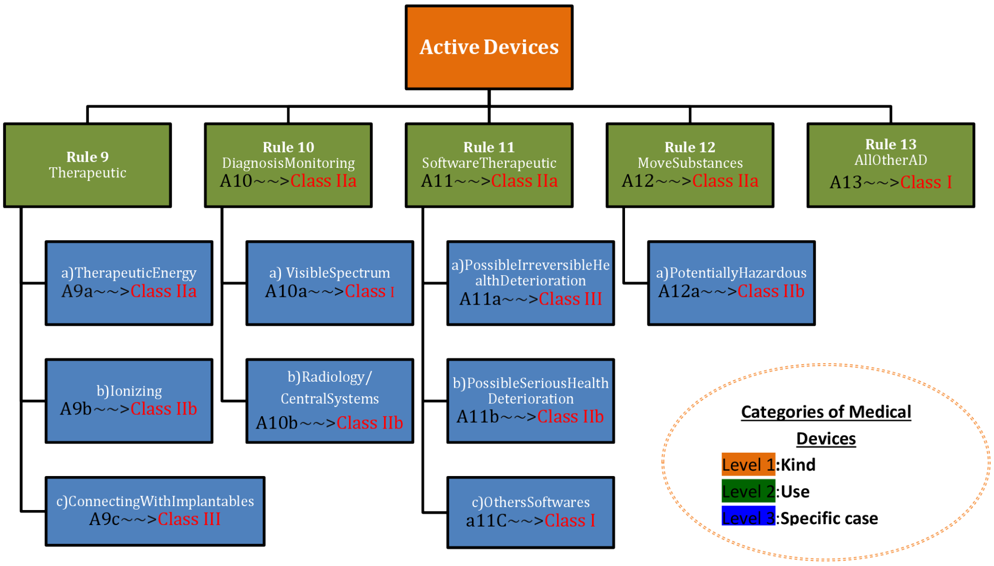


Figure 8 Active Medical Devices.

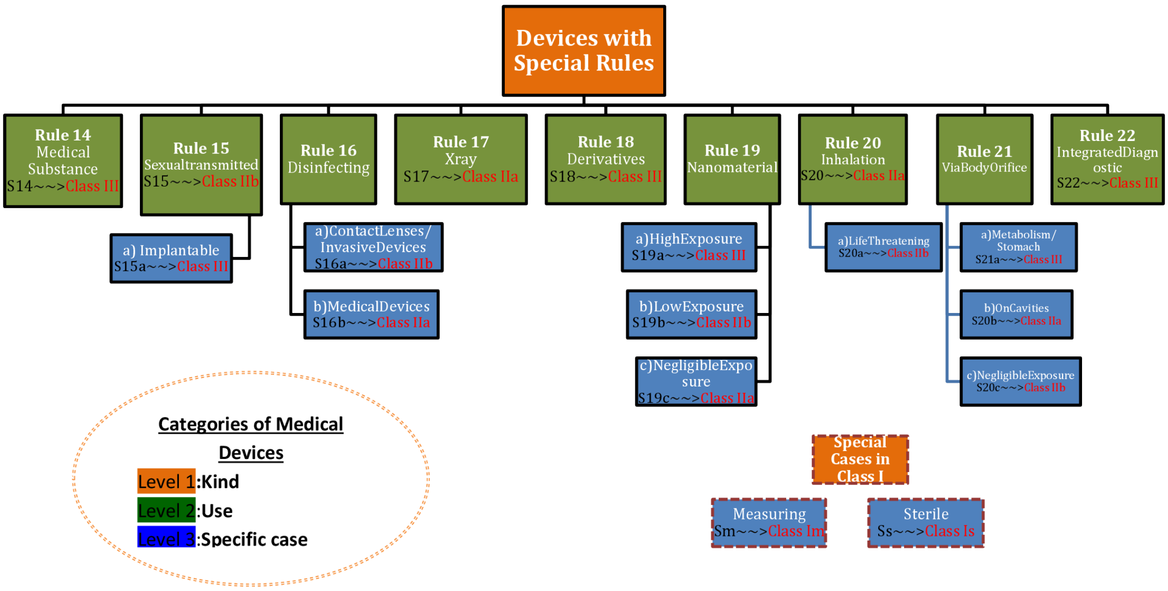
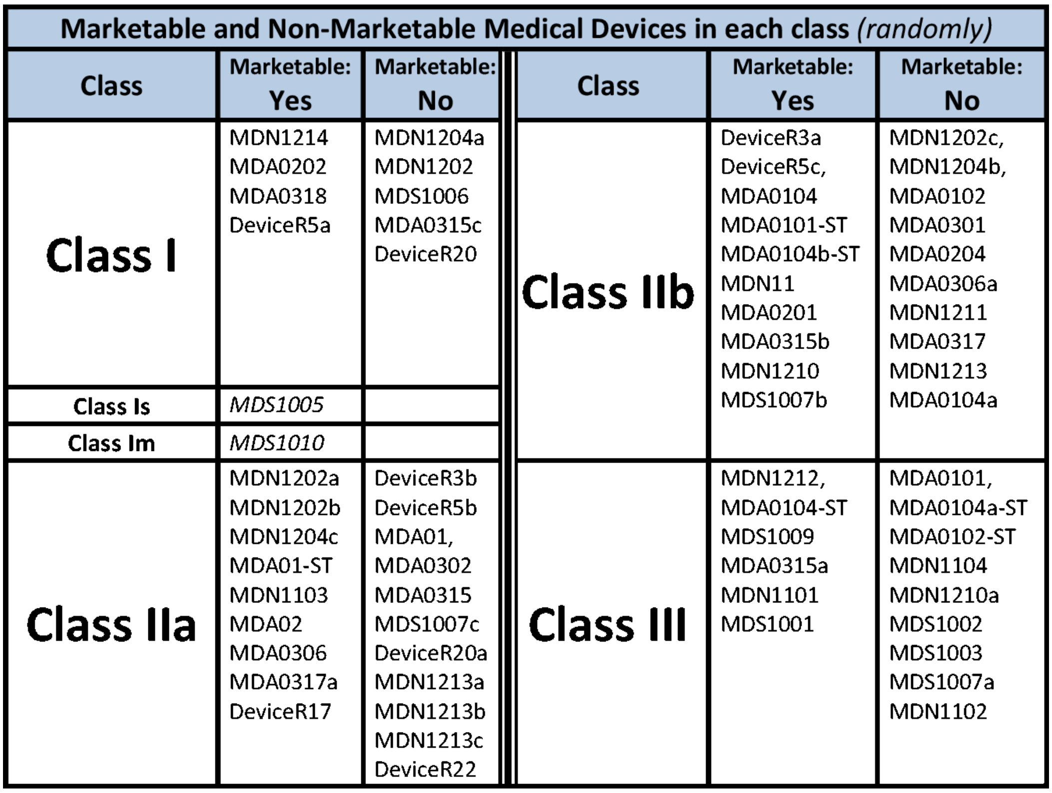


Figure 9 Special cases of Medical Devices.

.3.2 Marketable and non-marketable medical devices



Marketable and Non-Marketable Medical Devices in each class *(randomly)*

Class	Marketable: Yes	Marketable: No	Class	Marketable: Yes	Marketable: No
Class I	MDN1214 MDA0202 MDA0318 DeviceR5a	MDN1204a MDN1202 MDS1006 MDA0315c DeviceR20	**Class IIb**	DeviceR3a DeviceR5c, MDA0104 MDA0101-ST MDA0104b-ST MDN11 MDA0201 MDA0315b MDN1210 MDS1007b	MDN1202c, MDN1204b, MDA0102 MDA0301 MDA0204 MDA0306a MDN1211 MDA0317 MDN1213 MDA0104a
Class Is	*MDS1005*				
Class Im	*MDS1010*				
Class IIa	MDN1202a MDN1202b MDN1204c MDA01-ST MDN1103 MDA02 MDA0306 MDA0317a DeviceR17	DeviceR3b DeviceR5b MDA01, MDA0302 MDA0315 MDS1007c DeviceR20a MDN1213a MDN1213b MDN1213c DeviceR22	**Class III**	MDN1212, MDA0104-ST MDS1009 MDA0315a MDN1101 MDS1001	MDA0101, MDA0104a-ST MDA0102-ST MDN1104 MDN1210a MDS1002 MDS1003 MDS1007a MDN1102

Figure 10 Table of the marketable and non-marketable medical devices.

.4 ExosCE Rules in PSOATransRun

A representative fragment of the PSOA RuleML code is given below. The whole ExosCE KB with the MS Excel and the Python script can be found in http://users.ntua.gr/salmpani/ExosCE/. Further instructions can be found here: http://users.ntua.gr/salmpani/ExosCE/README.txt. For more detailed directions an email can also be sent to the authors.

```
% Rules for Active Devices
% Rule 9
% Active therapeutic devices intended to exchange
or administer energy.

Forall ?m (
  :CategoryOfMedicalDevice(?m :A9a) :-
   ?m#:MedicalDevice(:kind->:Active
                     :use->:Therapeutic
                     :specificCase->:Energy))



% Classification Grouping: Class IIa

 Forall ?m (
:IsClassifiedIn(?m :IIa) :-
       Or(:CategoryOfMedicalDevice(?m :N2a)
          :CategoryOfMedicalDevice(?m :N2b)
          :CategoryOfMedicalDevice(?m :N3b)
          :CategoryOfMedicalDevice(?m :N4c)
          :CategoryOfMedicalDevice(?m :I5b)
          :CategoryOfMedicalDevice(?m :I6)
          :CategoryOfMedicalDevice(?m :I7)
          :CategoryOfMedicalDevice(?m :I8a)
          :CategoryOfMedicalDevice(?m :A9a)
          :CategoryOfMedicalDevice(?m :A10)
          :CategoryOfMedicalDevice(?m :A11)
          :CategoryOfMedicalDevice(?m :A12)
          :CategoryOfMedicalDevice(?m :S16b)
```

```
          :CategoryOfMedicalDevice(?m :S17)
          :CategoryOfMedicalDevice(?m :S19a)
          :CategoryOfMedicalDevice(?m :S20)
          :CategoryOfMedicalDevice(?m :S21b)))


% Main paragraphs of Essential Requirements of MD.

Forall ?m (:MDDManufacturingRequirements(?m) :-
 And(
 :ChemicalPhysicalBiologicalProperties(?m) % p.10 %
 :InfectionMicrobialContamination (?m) % p.11 %
 :SubstancesMedicalProductOrAbsorbed(?m)  % p.12 %
 :IncorporatingMaterialsOfBiologicalOrigin(?m) % p.13 %
 :InteractionWithTheirEnvironment(?m) % p.14 %
 :DiagnosticOrMeasuringFunction(?m) % p.15 %
 :ProtectionAgainstRadiation(?m)  % p.16 %
 :ElectronicProgrammableSystems (?m) % p.17 %
 :ActiveDevices(?m) % p.18 %
 :ActiveImplantableDevices(?m)  % p.19 %
 :ProtectionAgainstMechanicalAndThermalRisks(?m) % p.20 %
 :RisksByDevicesSupplyingEnergyOrSubstances(?m)  % p.21 %
 :DevicesForUseByLayPersons(?m)  % p.22 %
 ))


% EHRS of the Machinery Directive (2006/42/EC)
that are applicable to Medical Devices.

Forall ?m (MDManufacturingRequirements(?m) :-
  And(
    :DefineGeneralTermsOfMD(?m :Checked) % p.1.1.1 %
    :Lighting(?m :Checked)% p.1.1.4 %
    :SeatingASIntegralPart(?m :Checked) % p.1.1.8 %
    :ControlDevices(?m :Checked) % p.1.2.2 %
    :ErrorsOfFitting(?m :Checked) % p.1.5.4 %
```

```
    :MachineryMaintenance(?m :Checked) % p.1.6.1 %
    :AccessToOperatingPositionsAndServicingPoints(?m
      :Checked) % p.1.6.2 %
    :IsolationOfEnergySources(?m :Checked) % p.1.6.3 %
    :DefineMobilityTermsOfMD(?m :Checked)  % p.3.1.1 %
    :MeansOfAccess(?m :Checked) % p.3.4.5 %
    :MarkingsForDeivicesIn311(?m :Checked)% p.3.6.2 %
    :DefineTermsOfMDForLiftingOperations(?m :Checked)
    % p.4.1.1 %
    ))


% Marketability Requirements for all Classes

Forall ?m (
 :MarketableMedicalDevice(?m) :-
  :HasCEwithNBN(?m))

Forall ?m (
 :HasCEwithNBN(?m) :-
   :DeclarationOfConformity(?m))


% Requirements for Class IIa

Forall ?m (:DeclarationOfConformity(?m) :-
  And(:IsClassifiedIn(?m :IIa)
     :AppointingAnEAR(?m)
     :ConformityAssessment(:device->?m
               :technicalFile->:True
               :vigilanceSystem->:Required
               :harmonizedStandards->:NonRequired)
     :QualityAssurance(?m))'
     :ManufacturingRequirements(?m))

Forall ?m (:ManufacturingRequirements(?m) :-
```

```
  And (:MDDManufacturingRequirements(?m)
   :MDManufacturingRequirements (?m)))

% Requirements for Production Quality -
% Annex V, EN ISO 13485:2003

Forall ?m (:QualityType(?m :ProductionQuality) :-
:RequirementsOfQualityType(:device->?m
                           :design->:NonRequired
                           :manufacture->:Required))

% Exoskeleton Fact: EksoLegs

:EksoLegs#:MedicalDevice(:kind->:Active
                         :use->:Therapeutic
                         :specificCase->:Energy)
:AppointingAnEAR(:EksoLegs)
:ConformityAssessment(:device->:EksoLegs
      :technicalFile->:Yes
      :vigilanceSystem->:Yes
      :harmonizedStandards->:No)
:RequirementsOfQualityType(:device->:EksoLegs
                           :design->:No
                           :manufacture->:No)

:Design(:EksoLegs :Checked)  % p.10.1 %
:ContaminantsResidues(:EksoLegs :Checked)   % p.10.2 %
:MedicinalProducts(:EksoLegs :Checked)  % p.10.3 %
:IngressOfSubstances(:EksoLegs :Checked) % p.10.5 %
:SizePropertiesOfParticles (:EksoLegs :Checked) % p.10.6 %
[...]
```

Dictionary

.5 Dictionary of Terms in Greek-English

Αλληλουχία	Sequence, Fluent
Απόδειξη	Proof, Proving
Αποδεικτική διαδικασία	Proving process
Αναιρέσιμος συλλογισμός	Defeasible reasoning
Αναιρέσιμη επιχειρηματολογία	Defeasible argumentation
Αντεπιχείρημα	Counterargument
Αποδεικτικό Συμβάν	Proof-Event
Αποδεικτικα Γεγονότα Μηδενικής Γνώσης	Zero Knowledge Proof-Events (ZKPE)
Αποδεικνύων	Prover
Απόρρητο	Privacy
Αφηρημένη επιχειρηματολογία	Abstract argumentation
Βάση γνώσης	Knowledge Base (KB)
Γνωστική Ψυχολογία	Cognitive psychology
Δεδομένα	Data
Διάψευση	Defeat
Δρων	Agent
Εγγύηση	Warrant
Επαγωγικός	Inductive
Επαληϑευτής	Verifier
Επιστήμων της άτυπης λογικής	Informal logician
Επιχείρημα	Argument
Επιχειρηματολογία	Argumentation
Ερμηνευτής	Interpreter
Ισχυρισμός	Claim
Κανόνες εξαγωγής συμπερασμάτων	Rules of inference
Κινήσεις επιχειρημάτων	Argumentation moves
Λήψη αποφάσεων	Decision making
Λογισμός Αποδεικτικών Συμβάντων βάσει Επιχειρημάτων	Argumentation-based Proof-Events Calculus (APEC)

Πληθοπορισμός	Crowd-sourcing
Πολλαπλοί δρώντες	Multi-agent
Σκεπτικιστής	Sceptic
Συλλογική επιχειρηματολογία	Collective argumentation
Συλλογισμός, Συλλογιστική	Reasoning
Συνεργατική επίλυση προβλημάτων	Collaborative problem-solving
Τυποποίηση	Formalization
Φορετά Ρομπότ	Wearable Robots
Χρονικά Κατηγορήματα	Temporal Predicates