ΕΘΝΙΚΟ ΜΕΤΣΟΒΙΟ ΠΟΛΥΤΕΧΝΕΙΟ

ΣΧΟΛΗ ΕΦΑΡΜΟΣΜΕΝΩΝ ΜΑΘΗΜΑΤΙΚΩΝ ΚΑΙ ΦΥΣΙΚΩΝ ΕΠΙΣΤΗΜΩΝ

ΤΡΙΣΔΙΑΣΤΑΤΑ ΕΝΕΡΓΕΙΑΚΑ ΚΕΝΑ ΑΠΟ TYPE II ΘΕΩΡΙΕΣ ΥΠΕΡΒΑΡΥΤΗΤΑΣ

ΔΙΔΑΚΤΟΡΙΚΗ ΔΙΑΤΡΙΒΗ

ΓΕΩΡΓΙΟΥ ΤΡΙΓΓΑ

ΕΠΙΒΛΕΠΩΝ:

ΑΛΕΞΑΝΔΡΟΣ ΚΕΧΑΓΙΑΣ

Καθηγητής ΕΜΠ

Ιούλιος 2022

ΕΘΝΙΚΟ ΜΕΤΣΟΒΙΟ ΠΟΛΥΤΕΧΝΕΙΟ

ΣΧΟΛΗ ΕΦΑΡΜΟΣΜΕΝΩΝ ΜΑΘΗΜΑΤΙΚΩΝ ΚΑΙ ΦΥΣΙΚΩΝ ΕΠΙΣΤΗΜΩΝ

ΤΡΙΣΔΙΑΣΤΑΤΑ ΕΝΕΡΓΕΙΑΚΑ ΚΕΝΑ ΑΠΟ TYPE II ΘΕΩΡΙΕΣ ΥΠΕΡΒΑΡΥΤΗΤΑΣ

ΔΙΔΑΚΤΟΡΙΚΗ ΔΙΑΤΡΙΒΗ

ΓΕΩΡΓΙΟΥ ΤΡΙΓΓΑΣ

Διπλωματούχου Τμήματος Φυσικής Πανεπιστημίου Ιωαννίνων

ΤΡΙΜΕΛΗΣ ΣΥΜΒΟΥΛΕΥΤΙΚΗ ΕΠΙΤΡΟΠΗ:

1. Αλέξανδρος Κεχαγιάς, Καθ. ΕΜΠ (Επιβλέπων)
2. Νικόλαος Τετράδης, Καθ. ΕΚΠΑ
3. Antonio Riotto, Καθ. Παν. Γενεύης

ΕΠΤΑΜΕΛΗΣ ΣΥΜΒΟΥΛΕΥΤΙΚΗ ΕΠΙΤΡΟΠΗ:

1. Αλέξανδρος Κεχαγιάς, Καθ. ΕΜΠ (Επιβλέπων)
2. Νικόλαος Τετράδης, Καθ. ΕΚΠΑ
3. Antonio Riotto, Καθ. Παν. Γενεύης
4. Νικόλαος Ήργες, Αν. Καθ. ΕΜΠ
5. Παναγιώτα Καντή, Καθ. Παν. Ιωαννίνων
6. Νικόλαος Μαυρόματος, Καθ. ΕΜΠ
7. Δημήτριος Τσίμπης, Καθ. Παν. Λυών

Αθήνα, Ιούλιος 2022

Περίληψη

Η παρούσα διδακτορική διατριβή επικεντρώνεται στη μελέτη τρισδιάστατων ενεργών θεωριών πεδίου συζευγμένων με τη βαρύτητα Einstein προερχόμενες από το χαμηλοενεργειακό όριο της θεωρίας υπερχορδών. Πιο συγκεκριμένα μελετώνται οι ιδιότητες τρισδιάστατων ενεργειακών κενών που προέρχονται από τη δεκαδιάστατη υπερβαρύτητα Type II, η οποία περιγράφει το χαμηλοενεργειακό όριο της κλειστής υπερχορδής, ύστερα από συμπαγοποίηση των πλεοναζουσών διαστάσεων σε εφταδιάστατους χώρους με δομή G2. Ξεκινάμε με την παρουσίαση βασικών εννοιών της άμαζης θεωρίας υπερχορδών και του χαμηλοενεργειακού ορίου της παραθέτοντας έννοιες και ορισμούς που χρησιμοποιούνται στο κύριο μέρος της ανάλυσης. Στη συνέχεια επικεντρωνόμαστε στην κατασκευή τρισδιάστατων ενεργειακών κενών και ως πρώτο βήμα παρέχουμε την εσωτερική γεωμετρία όπου συμπαγοποιούνται οι πλεονάζουσες διαστάσεις όπως επίσης παραθέτουμε την ελάσσονα τρισδιάστατη υπερβαρύτητα που περιγράφει την ενεργή θεωρία πεδίου. Ξεκινώντας από τη θεωρία Type IIA συμπαγοποιούμε σε χώρους G2 μηδενικής καμπυλότητας και λαμβάνοντας υπόψιν την παρουσία O2-υπερεπιφανειών καταλήγουμε σε ενεργειακά κενά τύπου Minkowski. Προσθέτοντας στο φάσμα τη μάζα Romans και τη συνεισφορά O6-υπερεπιφανειών βρίσκουμε υπερσυμμετρικά και μη-υπερσυμμετρικά ενεργειακά κενά τύπου AdS τα οποία χαρακτηρίζονται από ιδιότητες όπως: η σταθεροποίηση όλων των πεδίων moduli, παραμετρικός διαχωρισμός της ενεργειακής κλίμακας των Kaluza-Klein καταστάσεων από την κοσμολογική σταθερά της τρισδιάστατης θεωρίας, όπως επίσης τη συνεπή κβάντωση των σταθερών της θεωρίας. Οι παραπάνω λύσεις παραμένουν στο όριο της κλασικής θεωρίας υπερχορδών καθιστώντας έτσι την περιγραφή μέσω του ορίου της υπερβαρύτητας αξιόπιστη. Ωστόσο η μη-εντοπισμένη περιγραφή των υπερεπιφανειών που αποτελεί συνήθη προσέγγιση στην κατασκευή ενεργών θεωριών από συμπαγοποίηση χρήζει περεταίρω ανάλυσης με σκοπό τη μελέτη της αξιοπιστίας των αποτελεσμάτων. Ξεκινώντας από μια γενική μορφή της μετρικής και αναλύοντας τη μορφή των πεδίων σε δυναμοσειρά βρίσκουμε τη μη-τετριμμένη μορφή τους (μέσω διορθώσεων επόμενης τάξης) κοντά στα σημεία όπου οι υπερεπιφάνειες είναι εντοπισμένες. Μετέπειτα εκτιμούμε την απόσταση από τα σημεία αυτά όπου οι προσεγγιστικοί υπολογισμοί μπορούν να θεωρηθούν αξιόπιστοι. Στη συνέχεια ερευνάται η δυνατότητα κατασκευής ενεργειακών κενών τύπου de-Sitter με την προσθήκη αντί-βρανών σπάζοντας την εναπομένουσα υπερσυμμετρία, συνεισφέροντας επίσης θετικά στο ενεργειακό κενό. Η μελέτη των κρίσιμων σημείων του δυναμικού δείχνει πως η σταθερότητα του κενού μπορεί να επιτευχθεί όταν η πυκνότητα ενέργειας παραμένει θετική αλλά σχετικά μικρή, ωστόσο η απαίτηση της κβάντωσης των σταθερών της θεωρίας και η συνέπεια των εξισώσεων κίνησης οδηγούν στην ανάγκη για γενίκευση του εσωτερικού χώρου. Ακόμα και σε αυτή την περίπτωση μπορεί να μελετηθεί ποιοτικά πως τα κλασικά τρισδιάστατα ενεργειακά κενά de-Sitter είτε θα οδηγηθούν σε διάσπαση είτε σε παρουσία ταχυονικών καταστάσεων. Ακολουθώντας την ίδια μεθοδολογία μελετάμε τη συμπαγοποίηση της Type IIB σε εσωτερικούς χώρους G2 με καμπυλότητα. Υπολογίζοντας τις εξισώσεις κίνησης που περιγράφουν τις υπερσυμμετρικές λύσεις της χαμηλοενεργειακής θεωρίας και χρησιμοποιώντας την S-δυαδικότητα επαληθεύουμε τις λύσεις της Type IIB, με την παρουσία O9-υπερεπιφανειών, μέσω αυτών της Ετεροτικής υπερβαρύτητας. Στη συνέχεια παρουσιάζουμε αναλυτικά παραδείγματα ενεργειακών κενών τύπου Minkowski και AdS δίνοντας έμφαση στο ρόλο των σταθερών δομής του εσωτερικού χώρου στη σταθεροποίηση των πεδίων moduli καθώς επίσης παρουσιάζουμε σταθερές λύσεις στο κλασικό όριο της θεωρίας. Ωστόσο ο διαχωρισμός της ενεργειακής κλίμακας στην περίπτωση των AdS ενεργειακών κενών από Type IIB έρχεται σε αντίθεση με την κβάντωση των σταθερών δομής. Τέλος λαμβάνοντας υπόψιν επιτρεπόμενες υπερεπιφάνειες που σπάνε την εναπομένουσα υπερσυμμετρία μελετάμε την κατασκευή ενεργειακών κενών de-Sitter. Η κατασκευή τέτοιων κενών, ικανοποιώντας τη κβάντωση σταθερών, την αποφυγή ταχυονίων και παραμένοντας ταυτόχρονα στο κλασικό όριο της θεωρίας χορδών είναι μη-τετριμμένη. Ωστόσο η ύπαρξη συνεπών de-Sitter ενεργειακών κενών στην περίπτωση αυτή μπορεί να αποφανθεί ύστερα από εκτενή μελέτη του παραμετρικού χώρου των σταθερών δομής.

Three-dimensional flux vacua from Type II supergravity theories

PhD thesis

of

Georgios Tringas

in

Physics Division

School of Applied Mathematical and Physical Sciences

National and Technical University of Athens

July 2022

Abstract

In this thesis we concentrate on the study of minimal three-dimensional effective field theories coupled to Einstein gravity originating from the low-energy limit of string theory. The main motivation behind this work is to use it as an accurate but simpler testing ground for the study of flux vacua with moduli stabilization and scale separation but also to study aspects of the swampland program. The existence of vacua with such properties is the foundation of string phenomenology. More specifically we work with Type II supergravities at classical string theory regime and after compactification on seven-dimensional G2 spaces we study the properties of three-dimensional flux vacua. We start with a review of basic concepts of the massless string theory and its supergravity limit in order to provide definitions and concepts which are used in the main part of the analysis. We focus on the construction of three-dimensional vacua and as a first step we provide the internal geometry where we compactify on as well as the minimal three-dimensional supergravity which apply for all the parts of the analysis. Starting from Type IIA we perform compactification on Ricci flat G2-manifolds and considering O2 orientifolds we find no-scale Minkowski vacua and the proper truncated three-dimensional minimal supergravity. Considering the Romans mass in the spectrum and net contribution of O6-planes we find supersymmetric and non-supersymmetric AdS vacua which posses elementary properties of consistent vacua such as full moduli stabilization, parametric separation of scales, quantization of fluxes while remaining at the classical string theory regime. Both the supersymmetric and non-supersymmetric vacua are stable in the smeared approximation of sources. As a consequence of the latter we extend our calculations and focus on the backreaction of the local sources whose exact formulation was approximated. Starting from a general metric ansatz and performing a proper expansion for the fields we find the next-to-leading order corrections which describe the non-trivial profile of the fields close to the source. We estimate the relevant distance to the source loci where the smeared approximation and subsequently our previous results can be trusted. Furthermore, we explore the possibility of de-Sitter vacua construction using the previous stable models including anti-branes which break completely the supersymmetry and contribute positively (uplift) to the energy vacua. A critical point analysis indicates that shallow de-Sitter vacua can be stable, however performing flux quantization and solving properly the equations of motion shows that consistent solutions can exist when going to more general spaces including warping. Nevertheless a qualitative calculation reveals that an interplay of fluxes and charges which might lead to either brane-flux decay or tachyonic vacuum setting once again the de-Sitter vacua from string theory difficult to control. Along the same line we perform compactification of Type IIB on co-calibrated G2 spaces with internal curvature. We study the supersymmetric equations of the effective field theory and use S-duality to verify that the supersymmetric conditions of Type IIB including O9-planes in the smeared approximation are in agreement with those of Heterotic supergravity. This verifies our results and possibly indicates that the non-closeness of the field-forms (non-trivial profile) capture the backreaction effects of the sources. We present two explicit examples of Minkowski and AdS vacua pointing out the dependence of our results on the choice of the structure group of the internal space and their role on moduli stabilization. It is shown that scale separation for AdS vacua is in contrast with the quantization of the group structure constants. Lastly we include supersymmetry breaking sources in order to perform de-Sitter uplift however it seems non-trivial to satisfy flux quantization, avoid tachyonic states and remain in the classical string theory regime at the same time. An exhaustive scan over various parameter values of structure groups should be performed in order to have a final verdict on the existance of consistent classical de-Sitter vacua from Type IIB.

Acknowledgements

I would like to take the opportunity to thank my supervisors, colleagues, friends and my family who have been supporting me during the last years. My postgraduate studies have been a complete journey with struggles and success but always full of passion.

I am very thankful to my supervisor Alex Kehagias for giving me the opportunity to conduct research. He has always been a great paradigm of a nice man and great scientist. His remarkable vision, ideas and knowledge in different topics of theoretical physics definitely inspired me, giving me momentum to be constantly trying my best. I really appreciate the opportunity you gave me to collaborate with several people and for letting me work on topics I was interested in. Most importantly thank you for being caring to me and the rest of your students.

I am thankful to my friend and collaborator Fotis Farakos for being supportive and patient with me all these years. His role as unofficial co-supervisor and collaborator was valuable and definitely helped me progress in physics. It has always been a great pleasure working together and discussing endless hours online during the last years. Also I am really indebted for inviting me to KU Leuven and U.Padova, giving me this way the opportunity to be part and interact with people from the local groups.

I am grateful to Thomas Van Riet, a great person and scientist, for giving me such an interesting topic to work on during my visit at KU Leuven and generally for introducing me to flux compactifications. Our interaction completely changed my field of research which was actually something I was looking for at that time.

I would like to thank my friends and collaborators Ioannis Dalianis and Niccolo Cribiori for our nice collaboration during the first years of this journey. You have both inspired me in many and different ways. Also I want also to thank Maxim Emelin for our fruitful collaboration and nice discussions. I am thankful also to my friends Fotis Koutroulis and John Taskas for the great company we've been together, our long conversations and nice co-existance in the office.

I thank my closest friends who have been patient with me during all these years. Paris for your ethos, our deep conversations and friendship since our undergraduate years. Christos, for reminding me to enjoy the beauty of life. I thank my partner Vanessa for her comprehension at difficult times, support and patience too.

Most importantly I thank my parents, siblings, aunt Dimitra and uncle Thanasis for being unconditionally supportive during these years.

Publications

This dissertation is based on the following four publications:

1. M. Emelin, F. Farakos and G. Tringas, "O6-plane backreaction on scale-separated Type IIA AdS_3 vacua," [arXiv:2202.13431 [hep-th]]. (Accepted in JHEP)

2. M. Emelin, F. Farakos and G. Tringas, "Three-dimensional flux vacua from IIB on co-calibrated G2 orientifolds," Eur. Phys. J. C **81**, no.5, 456 (2021) [arXiv:2103.03282 [hep-th]].

3. F. Farakos, G. Tringas and T. Van Riet, "Classical de Sitter solutions in three dimensions without tachyons?," Eur. Phys. J. C **80**, no.10, 947 (2020) [arXiv:2007.12084 [hep-th]].

4. F. Farakos, G. Tringas and T. Van Riet, "No-scale and scale-separated flux vacua from IIA on G2 orientifolds," Eur. Phys. J. C **80**, no.7, 659 (2020) [arXiv:2005.05246 [hep-th]].

The author also worked on the following publications during his postgraduate studies which are not included in this dissertation:

1. I. Dalianis, A. Kehagias, I. Taskas and G. Tringas, "On the Vacuum Structure of the N=4 Conformal Supergravity," Universe **7**, no.11, 409 (2021) [arXiv:2110.05463 [hep-th]].

2. N. Cribiori, F. Farakos and G. Tringas, "Three-forms and Fayet-Iliopoulos terms in Supergravity: Scanning Planck mass and BPS domain walls," JHEP **05**, 060 (2020) [arXiv:2001.05757 [hep-th]].

3. I. Dalianis and G. Tringas, "Primordial black hole remnants as dark matter produced in thermal, matter, and runaway-quintessence postinflationary scenarios," Phys. Rev. D **100**, no.8, 083512 (2019) [arXiv:1905.01741 [astro-ph.CO]].

4. I. Dalianis, A. Kehagias and G. Tringas, "Primordial black holes from α-attractors," JCAP **01**, 037 (2019) [arXiv:1805.09483 [astro-ph.CO]].

He was also invited to sign the following white paper

1. A. Kashlinsky, *et al.* "Electromagnetic probes of primordial black holes as dark matter," [arXiv:1903.04424 [astro-ph.CO]].

Αφιερώνεται στην οικογένειά μου...

Contents

Chapter 1

Introduction

The Standard Model of particle physics describes successfully physical phenomena at low energies well below the Planck scale, however it cannot provide satisfactory answers to several fundamental questions such as the hierarchy problem, the presence of dark matter and dark energy, the universe evolution in the very beginning etc. Therefore the Standard Model can be considered as an effective field theory that originates from a more fundamental theory that its description is not known yet. On the other hand General Relativity seems to pass all the tests for the description of large scale interactions however it is decoupled from the rest of the forces and not included in the Standard Model. Trying to fit General Relativity into the framework of quantum field theory leads to ultraviolet divergences spoiling the validity of the theory. However it is reasonable for someone to ask for a theory where all these fundamental forces are unified at higher energies, in other words to ask for a theory of quantum gravity. The root of this problem seems to be that particles are treated as point-like objects. The best proposed candidate for quantum gravity we have so far is string theory where strings can be described by one-dimensional objects embedded in higher dimensional backgrounds. In string theory the oscillations of the strings define the mass spectrum and predict the existance of graviton, a quantum mechanical particle that carries the gravitational force, thus string theory can be considered as a theory of quantum gravity. In addition to graviton, several fields emerge from the string oscillations together with special properties and symmetries such as supersymmetry which relates the bosonic and fermionic degrees of freedom. Different types of strings has lead to the discovery of five versions of superstring theory : type I, type IIA, type IIB and heterotic string theories $SO(32)$ and $E8 \times E8$, which are related via dualities [1–3]. Requiring consistency of the vacua one finds that superstring theory lives in ten dimensions. The low-energy limit of these theories can be described by the relevant ten-dimensional supergravities which exhibit different spectrum and symmetries and it is the starting point of our analysis. At this limit strings can be considered as point particles and not as strings. In order string theory to be the correct quantum gravity should be able to reproduce both the Standard Model and the spacetime properties of our universe however both higher dimensions and supersymmetry have not be observed in our experiments and for this reason we have to treat them in special manner in order to justify their existance only at high energies.

So the goal of research in string theory is to find a solution of the theory that reproduces the Standard Model and at the same time be compatible with the cosmological observations for an accelerated universe expansion. This seems to be non-trivial since we have not yet managed to obtain successfully any of these. The accelerated expansion can be achieved by the presence of either a constant vacuum energy (cosmological constant) or with slowly rolling scalar field(s). In the current analysis we study the possibility of getting general cosmological constants which are characterized by elementary properties of the cosmological constant of our Universe. This is a crucial starting point in order to understand if string theory can accommodate consistent vacua.

In order to describe our four-dimensional spacetime using string theory it is reasonable to ask what is the mechanism that keeps the rest of dimensions unobserved at low energies. Since we detect only four out of the ten dimensions that string theory predicts close to the Planck scale, a canonical mechanism to hide the remaining dimensions is to assume that going to low energies the extra-dimensions become small. This means that at low energies they cannot be observed and physics are well described by an effective field theory. To proceed we assume the ten-dimensional metric is a product

$$M_{10} = M_d \times X^{10-d}\,, \tag{1.0.1}$$

where M_d for $d = 4$ is the spacetime we experience while X^{10-d} is a compact manifold with small size directions. This mechanism is the so-called Kaluza-Klein (KK) compactification firstly introduced by Kaluza and Klein [4]. The mass states of the Kaluza-Klein modes after compactification are dictated by $m^2 \sim 1/R^2$ where R stands for the radius of the periodically compactified dimension $y^m \sim y^m + 2\pi R$. Thus at energies $E \ll 1/L_{KK}$ with L_{KK} the typical scale of the compactified dimension, the observed physics should be the Standard Model and the General Relativity while effects of the internal space should be suppressed. We will define the criteria for keeping the extra dimensions unobserved in the following section.

A crucial step to obtain a d-dimensional effective action out of the ten-dimensional one is to integrate over the compactified extra dimensions, this step is called *dimensional reduction*. The reduced effective action comes with a number of free massless scalar fields however their existance cannot be interpreted in our spacetime. For example, with the presence of free massless fields one does not reproduce 4d relativity since scalar fields mediate forces. These fields contain information about the shape of internal space and usually combine with gauge potentials from the 10d theory. Since their dynamics should not affect the effective theory we want them to obtain large masses. This method is called *moduli stabilization* for compactifications [3, 6] and is one of the most relevant activities in string phenomenology and central point of this analysis. A crucial ingredient for moduli stabilization in a computable way are fluxes together with local objects and we refer to [3, 6–9]. If such ingredients are not included the effective field theory will contain massless moduli. It is fair to say that by now, our best understanding of flux compactifications involve compactifications to anti-de-Sitter (AdS) space, whereas compactifications to de-Sitter (dS) space are notoriously difficult to control, if they exist at all [10, 11]. Compactifications down to Minkowski space with stabilized moduli are not known to us. Thus the presence of non-trivial fluxes turns on an effective potential for the moduli and minimizing it we attempt to stabilize them. For example a bosonic d-dimensional effective field theory, with Einstein gravity and scalar moduli, will have the following form

$$S = M_p^{d-2} \int \mathrm{d}^d x \sqrt{-g_d} \Big(\frac{1}{2} R_d + \frac{1}{2} G_{ij} \partial_\mu \phi^i \partial^\mu \phi^j - \tilde{V}(\phi) \Big) \tag{1.0.2}$$

where ϕ stands for scalar moduli fields[1]. The stabilization of moduli is not a trivial task and usually not all of them are stabilized even with the presence of fluxes. Apart from the difficulty to control all the moduli, their vacuum expectation value at the minimum of the potential can be interpreted as a cosmological constant Λ in the effective field theory. This is a mechanism to derive and interpret the cosmological constant from flux compactifications.

As discussed previously a very essential feature of string compactifications is that the compact dimensions should be "small" enough in order to find a genuine four-dimensional effective field theory. Since we described the method for deriving the cosmological constant it is important to estimate whether the extra-dimension effects are absent from the effective theory. Usually one defines "small" with respect to the four-dimensional Hubble scale. This feature is also referred to as *scale separation* since the KK scale is supposed to be parametrically separated/decoupled from the Hubble scale

$$\frac{L_{KK}}{L_\Lambda} \ll 1\,. \tag{1.0.3}$$

For an isotropic compactified internal space (e.g torus) the Kaluza-Klein scale can be considered to be the volume of the internal space while the cosmological constant totally depends on the effective potential and its ingredients after the dimensional reduction. Since the goal of this analysis is to find consistent effective field theories with maximally symmetric vacua coupled to Einstein gravity we attempt to construct consistent AdS vacua too (also Minskowski) and study possible features such as separation of scales and moduli stabilization.

Another feature of the effective field theory is the possibility of preserving some amount of the initial supersymmetry of the ten-dimensional supergravity. The presence of supersymmetry guaranties the absence of tachyon but also can provide a way to cross-check the validity of some of our results. A method to break the desirable amount of supersymmetry is the specific choice of internal space accompanied with discrete symmetries under which supersymmetric spinors at the vacuum are not invariant. Apart from this, extended local objects related to the sting endpoints like D-branes can break or preserve some amount of supersymmetry while orientifold planes can further reduce it. In this analysis we will study cases where the effective field theory preserves the minimal supersymmetry which is phenomenologically desirable but we also study the possibility of constructing non-supersymmetric anti-de Sitter vacua and de Sitter including anti-D-branes.

[1] $\tilde{V}(\phi) = \int \mathrm{d}^{10-d} y \sqrt{g_{10-d}}\, V(F,\phi)$

As we discussed, the low-energy dynamics of string theory compactifications can be described by lower-dimensional effective theories whose properties are determined by the specifics of the internal geometry, fluxes and other ingredients. The range of effective theories that can be obtained in this fashion is vast, but it appears that not all otherwise internally consistent lower-dimensional effective theories can appear as low-energy limits of string compactifications. The delineation of criteria that determine whether an effective theory can be realized in string theory (or be consistent with quantum gravity more generally) has come to be known as the *swampland program* [12, 13], with theories that fail to satisfy these criteria said to reside in the swampland.

One general expectation that has come out of the swampland program is that compactifications to non-supersymmetric anti-de Sitter space should be able to decay [14], and that supersymmetric compactifications cannot have an arbitrarily small internal mean radius compared to the external AdS radius [15]. In contrast to these conjectures, the effective theories describing specific compactifications to non-susy AdS constructed in the literature appear to be both fully stable and also enjoy a separation of scales when the supersymmetry breaking effects are switched off [16–20]. This discrepancy has motivated the further scrutiny of such constructions. For instance, the supersymmetric vacua appearing in [16] have been further analyzed and partially challenged in a series of publications [21–23], where the gaugino condensation backreaction is properly taken into account as proposed in [24]. With regards to the constructions in [17], which are classical, one could suspect that the inconsistent approximation is the use of "smeared" orientifold sources. In the meantime, various difficulties for achieving scale separation in Type II are discussed in [25–27], further recent developments can be found in [28–32], and implications on the holographic side are discussed in [33–35].

The generality of the swampland conjectures means they should also apply to flux compactifications with any number of external dimensions, unless of course there are quantum gravity reasons to expect a specific dimensional dependence. Conversely, if the qualitative properties of flux compactifications depend on the external dimensions, then from the perspective of the swampland this means that some aspects of quantum gravity are intrinsically different across dimensions. Therefore, string flux compactifications down to dimensions different than four are a valuable resource for our understanding of the swampland. In particular, three-dimensional compactifications are especially interesting for a number of reasons. Firstly they are dual to two-dimensional field theories living on the boundary and in the case of supersymmetric AdS vacua, this means two-dimensional supersymmetric CFTs (for a sample of recent work see e.g. [36–41]). As a result the properties of such vacua can be cross-checked with 2d CFT methods. Secondly, from a technical point of view, the field content of a 3d flux compactification is considerably simpler than the 4d counter-parts which allows to perform a more thorough study of such vacua in this analysis. For example, the minimal supersymmetric background in 3d allows half of the number of supersymmetries than the minimal 4d background. Thirdly, since gravitation in three dimensions is intrinsically different than four dimensions or beyond, the study of the 3d swampland offers a unique ground to test the dependence of the conjectures on the dimensions of the external space.

Outline

In chapter 2 we review basic concepts of superstring theory starting from the Polyakov action and discuss the massless spectrum and perturbative limits such as the weak coupling and large volume regime. Next we write down the low-energy actions of Type II and the local sources which are extensively used in the analysis. We end this chapter with few examples on Kaluza-Klein compactifications and the introduction of the concept of orientifold plane.

In chapter 3 we introduce the internal space where we compactify on. We start by reviewing some general properties of seven-dimensional manifolds with G2-structure and focus on the identities of the fundamental three-form and its relation to curvature. Next we choose the internal compact space to be a seven-torus accompanied with an orbifold group leading to singular space. We introduce properties of the final singular geometry that will be used extensively in the analysis.

In chapter 4 we demonstrate our first compactification example of Type IIA on G2 orientifolds. We consider net O2-plane contribution and find the effective field theory to be minimal three-dimensional supergravity with no-scale Minkowski vacua with a flat direction. Next we consider the Romans mass and net contribution of O6-planes leading to supersymmetric AdS vacua with full moduli stabilization, separation of scales, flux quantization, parametrically large volume and small weak coupling. Furthermore it is shown that for specific choice of fluxes stable non-supersymmetric exists. We mention that we use the smeared approximation for all our sources.

In chapter 5 we calculate the backreaction of the local sources on the fields of the theory. We start from a general warped metric and performing a proper expansion for the fields we find the next-to-leading order corrections due to the presence of local sources. We estimate the exact first order corrections to all the fields as well as the relevant distance from the source loci where the smeared approximation and subsequently our previous smeared results can be trusted.

In chapter 6 we construct de-Sitter vacua by adding anti-branes to the previous Type IIA setup. We check the consistency of the new vacua by performing critical point analysis of the potential considering the bounds arising from the quantization of fluxes and the equations of motion. We assume the existance of more general spaces with warp factor in order the solution to be consistent however the interplay of fluxes and charges signifies a possible decays of the vacuum.

Finally in chapter 7 perform compactification of Type IIB on co-calibrated G2 spaces with internal curvature. We solve the supersymmetric equations of the effective field theory and use S-duality to compare our results with supersymmetric vacua from Heterotic supergravity. We present two explicit examples of Minkowski and AdS vacua which significantly depend on the choice of the structure group after the twisting of internal manifold. Then we examine the possibility of constructing AdS vacua with scale separation and proper flux quantization. Lastly we attempt to perform de-Sitter uplift by adding several combinations of supersymmetry breaking.

Chapter 2

Low energy actions from String theory

In this chapter we give an introduction to the superstring theory, we review the basic steps to obtain the spectrum and derive low-energy effective actions of string theory. Since our main topic of the analysis involves low-energy effective actions we try to link and justify how the actions are derived from string theory but also introduce concepts and definitions that will appear repeatedly in the main passage.

2.1 Basics on String theory

We start from the two-dimensional Polyakov action which describes the bosonic string. We discuss how boundary conditions on the equations of motion are satisfied and how they define the string oscillations. Next we comment on the spectrum of the bosonic string and finally arrive at the dynamics which can describe the universal part of the low-energy target space action. Next we introduce worldsheet fermions which leads us to the relevant for our analysis RR and NSNS sectors (which contain the fluxes for the effective vacua emergence) in order to write down the complete bosonic target space action. For a detailed description of the quantization of the string, where we do not go through, we refer to the textbooks [1–3] where also this chapter is based on.

2.1.1 The bosonic string sector

The two-dimensional action of a moving string can be described by the Polyakov action

$$S_P = \frac{1}{4\pi\alpha'} \int \mathrm{d}^2\sigma \sqrt{\gamma}\gamma^{ab} G_{MN}(X) \partial_a X^M \partial_b X^N \,, \tag{2.1.1}$$

where γ is the flat worldsheet metric with coordinates $\sigma^1 : -\infty < \tau < \infty$ and $\sigma^2 : 0 < \sigma < l$. The metric G_{MN} is the D-dimensional target space metric of a general background and $X^M \equiv X^M(\sigma, \tau)$ are real scalar fields which can be thought to parametrize the manifold and define a map from the string worldsheet to the physical D-dimensional spacetime. The action describes the propagation of a string in the D-dimensional spacetime. The overall coefficient is the Regge slope, a dimensionful parameter which is related to the string scale l_s^2 via $\alpha' = \frac{l_s^2}{2}$ and the string tension

$$T = \frac{1}{2\pi\alpha'} = \frac{1}{2\pi l_s^2} \,. \tag{2.1.2}$$

It is important to mention that the Polyakov action is characterized by Poincare invariance, diffeomorphism invariance, worldsheet Weyl invariance at classical level, while the Weyl invariance is not a symmetry at curved backgrounds at quantum level and imposing this condition to hold will lead to significant outcome in the next subsection.

2.1.2 Mass spectrum of the strings

The variation principle of the Polyakov action with respect to the metric and the scalar fields gives the equations of motion together with boundary terms. The vanishing conditions on the boundary terms define the nature of the string; open strings that oscillate with endpoints moving independently or fixed on the boundaries and different kind of solutions with closed strings. The classical solutions of these fields describe the propagation of the string in the target space and the oscillation part is described by mode operators. These theories can be quantized either with canonical or gauge fixing light-cone quantization by constructing the Hamiltotian and making use of the Virasoro algebra. The quantization follows by defining the Hilbert space of string states with a ground state $|0, k^M\rangle$ annihilated by the positive modding $n > 0$ oscillators a_n. Higher energy states are described by momentum eigenstates $|N, k^M\rangle$ where N is a number operator and stands for counting the excited states. Excited states are constructed by acting on the ground state with creation operators a_n with $n < 0$. Acting with the negative mode oscillators increases the mass of the state and the masses of spacetime particles increase with the number of oscillators in the corresponding state. We neither go through the quantization nor the explicit wavefunction solutions of the string however we just present the spectrum which will help us to understand the origin of terms in the low-energy action.

Closed bosonic string
For the closed string the boundary conditions which make the boundary terms vanish are

$$X^M(\tau, 0) = X^M(\tau, l) \,. \tag{2.1.3}$$

For completeness we indicate that the scalar fields X^M can be expressed in the light-cone coordinates as linear combination of right and left movers $X^M(\tau,\sigma) = X_L^M(\tau+\sigma) + X_R^M(\tau-\sigma)$. The left and right-moving coordinates are frequently noted as $\sigma^{\pm} = \tau \pm \sigma$. The closed string has two oscillatory operators α_n^M and $\tilde{\alpha}_n^M$ which describe the amplitude of the momentum mode n of the left and right movers and make up the number operators

$$N = \frac{1}{2} \sum_M \sum_{n>0} \alpha_{-n}^M \alpha_n^M \,, \tag{2.1.4}$$

and the mass relation

$$M^2 = \frac{2}{\alpha'}(N + \tilde{N} - 2)\,, \tag{2.1.5}$$

which is the sum of left and right movers mass. For physical states the number operators of left and right movers should match $N = \tilde{N}$. Requiring Poincare invariance and absence of tachyons on finds the critical dimensions for the bosonic string to be $D = 26$ which has been considered in the mass relation. It is easy to see that the ground state $M^2|0, k^M\rangle$ of the closed string is a tachyon. The massless state created by $M^2(a^i_{-1}\tilde{a}^j_{-1})|0, k^M\rangle$ corresponding to the tensor product of two massless vectors, one left mover and one right mover. The part of the tensor product that is symmetric and traceless transforms under $SO(D-2)$ as a massless spin-2 particle, the *graviton*. The trace term is a massless scalar which is called the *dilaton* and the antisymmetric part is an antisymmetric second-rank tensor. Thus the reducible representation of the massless state decomposes into a symmetric traceless tensor, an antisymmetric tensor and a scalar consisting the spectrum of the state. As stated also in the introduction string theory predicts gravity which is not trivial. Acting further with creation operators one can construct infinite massive states with more complicated structure.

Open bosonic string
For the open string there are two conditions that can be imposed in order to make the boundary terms vanish. The first possibility comes with Poincare invariance of the ten-dimensional theory and the fields satisfy the following conditions

$$\partial^\sigma X^M(\tau, 0) = \partial^\sigma X^M(\tau, l) = 0\,, \tag{2.1.6}$$

which are known as *Neumann* boundary conditions applied on both open string endpoints at $\sigma = 0, l$. In this case the endpoints are free to move through spacetime. The other possibility of boundary conditions breaks the Poincare invariance

$$X^M(\tau, 0) = x_0^M\,, \quad X^M(\tau, l) = x_l^M\,, \tag{2.1.7}$$

and is known as *Dirichlet* boundary conditions while the endpoints of the string are fixed and not free to move. Considering Dirichlet conditions for $M = m = 1, \ldots, p+1$ and Neumann conditions for $M = i = p+2, \ldots, D$ dimensions the Lorentz invariance is broken in the following way $SO(1, D-1) \to SO(1, p) \times SO(D-p-1)$. These hypersurfaces are called D-branes and their low-energy effective action has an important role in our analysis.

The worldsheet field expansion for fields with Dirichlet boundary conditions is similar to what we have discussed before, it includes propagating and oscillating parts of the string. However the main difference is that the string center of mass is localized on the D-brane and thus the target-space particles propagate along the $(p+1)$-dimensional worldvolume. Performing the quantization of the open string one finds that the massless states transform as a vector, originating from the $(p+1)$ creation operator α^m, while scalars are coming from the transverse oscillator α^i. Following the quantization procedure one can find the mass state for the open string to be

$$M^2 = \frac{1}{\alpha'}(N-1)\,, \tag{2.1.8}$$

and it is obvious that the ground state $|0,k^M\rangle$ is a tachyon. The first excited state for oscillations longitudinal to the brane transforms as massless vector boson on the D-brane $\alpha^i_{-1}|0,k^M\rangle = |1,k^M\rangle$ in a vector representation of $SO(1,p)$. For oscillations transverse to the brane the states transform as scalars under $SO(1,p)$ which live on the D-brane and can be interpreted as fluctuations of the D-brane in the transverse directions while they transform as vectors under the $SO(D-p-1)$ transverse to the D-brane. We notice that the massive spectrum for both closed and open strings scales inversely proportional to the length of the string $M^2 \sim 1/\alpha' \sim 1/l_s^2$ which means that at energies lower than the string scale one can neglect these states and consider only the massless spectrum.

2.1.3 Perturbative limits and regimes

It useful to express the massless states found after the quantization the Polyakov action solutions into a new perturbative-limit action including so-called Neveu-Schwarz (NS) sector: the graviton, the antisymmetric rank two field and the dilaton

$$S = \frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\sqrt{\gamma}\Big[\gamma^{ab}G_{MN}(X)\partial_a X^M\partial_b X^N + \epsilon^{ab}B_{MN}(X)\partial_a X^M\partial_b X^N + \alpha'\phi R\Big] \tag{2.1.9}$$

and for our purposes we focus on the last term of the 2d action. For constant values of the dilaton it corresponds to the Einstein-Hilbert term for the metric. However the integrated curvature scalar over the 2d surface is a topological invariant quantity, the Euler characteristic, which is defined by the handles h, boundaries b and cross-caps c of the surface in the following way

$$\chi = \frac{1}{4\pi}\int \mathrm{d}^2\sigma\sqrt{\gamma}R = 2-2h-b-c\,. \tag{2.1.10}$$

The coupling of the string to a constant dilaton weights each worldsheet diagram in the path integral by a factor exp($-\phi\chi$) such that the dilaton vacuum expectation value is the string coupling constant g_s

$$g_s = e^{\phi}\,. \tag{2.1.11}$$

The path integral sums over topologies is a loop expansion in the worldsheet with the coupling g_s as the loop counting parameter. For example for one loop effect we add one handle to the tree-level diagram g_s, the coupling constant is g_s^2 and the amplitudes are weighted by exp(2ϕ). The 2d worldsheet theory is non-interacting however there are non-trivial interaction in the spacetime theory which arise from the non-trivial worldsheet controlled by the string coupling. In our analysis we want classical string coupling thus we work in the weakly coupled regime $g_s \ll 1$ where string loop effects are neglected. As we will see when allow the presence of fluxes the action obtains a potential and it's vacuum expectation value determines the value of the dilaton and thus the string coupling constant.

The Polyakov action describes a 2d field theory in non-trivial curved background which is known as non-linear sigma model and usually is not exactly solvable but can be done perturbatively. It can be seen in the Polyakov action (but also in (2.1.9)) that the Regge slope α' can be considered as expansion parameter since the action becomes large in the limit where α' goes to zero. One can expand in perturbation series $X^M(\sigma) = X_0^M + Y^M(\sigma)$ where Y^M are the variations around the point X_0^M. Then the integrand of the action is given by the expansion

$$G_{MN}(X)\partial_a X^M \partial_b X^N = \Big(G_{MN}(X_0) + \partial_S G_{MN}(X_0)Y^S + \dots\Big)\partial_a Y^M \partial_b Y^N\,. \tag{2.1.12}$$

In this expansion the first term simply gives the kinetic terms for the scalar field fluctuations while the second term is a cubic interaction term with coupling constant $\partial_S G_{MN}(X_0)$ which includes derivative of the target space metric at the point X_0. For L_R the length scale of the variation, the coupling constant $\partial_S G_{MN}(X_0)$ is of order $1/L_R$. Since the dimensionless coupling is α'/L_R^2, if L_R is much larger than the length scale of the string

$$\frac{\alpha'}{L_R^2} \ll 1\,, \tag{2.1.13}$$

then the coupling constants in the expansion are small and we can use perturbation theory in the two-dimensional theory. Also when this limit is satisfied no massive string states are created and the theory describes only massless backgrounds. At this limit we ignore the internal structure of the string and use point-particle low-energy effective field theory. We will consider this regime extensively in the compactification of internal dimensions.

As we saw there a double perturbative expansions that we will consider. We want small coupling constant to perform spacetime loop expansion α' expansion is worldsheet loop expansion for fixed topology and suppresses the stringy effects.

Weyl invariance and the universal sector

We have mentioned before that the bosonic string action is invariant under some symmetries at classical level, however it is not Weyl invariant at quantum level. This can be also verified from the trace of the stress-energy tensor which vanishes classically but not in the quantum theory. The requirement of the theory to be invariant at quantum level lead to the low-energy supergravity limit description. The stress-energy tensor contains the renormalization group beta functions for the NS fields and imposing Weyl invariance they should vanish. We write down the beta function expansion including all the fields which appear in the massless states

$$\beta^G_{MN} = \alpha'\Big(R_{MN} + 2\nabla_M\nabla_N\phi - \frac{1}{4}H^2_{MN}\Big) + \mathcal{O}(\alpha'^2)\,, \tag{2.1.14}$$

$$\beta^B_{MN} = \alpha'\Big(-\frac{1}{2}\nabla^S H_{SMN} + \nabla^S\phi H_{SMN}\Big) + \mathcal{O}(\alpha'^2)\,, \tag{2.1.15}$$

$$\beta^\phi = \frac{D-26}{6} + \alpha'\Big(-\frac{1}{2}\nabla^2\phi + \nabla_M\nabla^M\phi - \frac{1}{24}H^2_{MN}\Big) + \mathcal{O}(\alpha'^2)\,. \tag{2.1.16}$$

Now setting the beta functions to zero $\beta^G_{MN} = \beta^B_{MN} = \beta^\phi = 0$ gives us spacetime equations of motion at one loop for the massless closed fields. It is worth to mention that the dilaton beta function vanishes exactly for $D = 26$ which states the critical dimensions of the bosonic string. These equations can be also obtained from the following spacetime action

$$S_U = \frac{1}{2\kappa_0^2}\int \mathrm{d}^D x\sqrt{-G}e^{-2\phi}\Big(R - \frac{1}{12}H_{MNL}H^{MNL} + 4\partial_M\phi\partial^M\phi + \mathcal{O}(\alpha'^2)\Big)\,. \tag{2.1.17}$$

Computing the β-function to higher orders in α' one can obtain higher derivative corrections to the effective action.

As we will see this is the universal action for all the low-energy effective actions. The normalization constant κ_0 that appears in the target space action can be absorbed by ϕ, while in the Einstein frame with a canonical Einstein-Hilbert term, κ_0 should be observed as the gravitational coupling constant. As we saw the Polyakov action which describes a two-dimensional field theory can be replaced by a field theory living in a 26-dimensional spacetime when imposing Weyl invariance in curved background and considering the α' expansion limit.

2.1.4 The fermionic string sector

So far we have discussed the bosonic string and the universal low-energy sector that arises from its massless states. It is natural to include worldsheet fermions in the theory and also suggested in some cases to cure the tachyons. The inclusion of such worldsheet fermions will also give rise to target space fermions and extra bosons which are physical and desired in order to construct flux vacua which potentially describe the vacuum state of the universe.

In analogy to the bosonic sector we introduce two component spinor free fermionic action

$$S_F = \frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\,\bar{\Psi}^M\rho^a\partial_a\Psi_M\,, \tag{2.1.18}$$

where Ψ are Dirac spinors and ρ^a are 2×2 matrices. There are actually the superpartners of X^M and this action is related to the bosonic part via two-dimensional worldsheet supersymmetry

$$\partial X^M = \bar{\epsilon}\Psi^M\,, \quad \delta\Psi^M = \rho^a\partial_a X^M\epsilon\,, \tag{2.1.19}$$

where ϵ is the supersymmetric parameter, an anti-commuting Majorana spinor. Similarly to what was described for the bosonic string, from the equations of motion one can derive the mode expansion and use the quantization process to construct the spectrum of the theory. One can express the action above in the Majorana representation with two real Majorana spinors and in the light-cone coordinates the action can be written as

$$S_F = \frac{1}{4\pi\alpha'} \int \mathrm{d}^2\sigma (\psi_- \partial_+ \psi_- + \psi_+ \partial_- \psi_+)\,. \tag{2.1.20}$$

where ψ_+ is the left-mover and ψ_- the right-mover. The variation of this action gives Dirac type solutions in these coordinates but also the following boundary terms which should vanish independently

$$\delta S_F \sim \int \mathrm{d}\tau (\psi_- \delta\psi_- - \psi_+ \delta\psi_+)\Big|_{\sigma=l} - \int \mathrm{d}\tau (\psi_- \delta\psi_- - \psi_+ \delta\psi_+)\Big|_{\sigma=0}\,. \tag{2.1.21}$$

The combinations to make these boundary terms vanish leads again to open and closed strings with different sectors which we study separately.

Superstring states

Similar to the bosonic string, the fermionic modes arising from the closed-string boundary conditions can be expressed as left and right-movers. There are two possible periodicity conditions which make the boundary terms vanish

$$\psi^M_\pm(\tau,\sigma) = +\psi^M_\pm(\tau,\sigma+l) \quad \text{Ramond boundary condition} \tag{2.1.22}$$
$$\psi^M_\pm(\tau,\sigma) = -\psi^M_\pm(\tau,\sigma+l) \quad \text{Neveu-Schwarz boundary condition} \tag{2.1.23}$$

where the positive sign describes *periodic* boundary conditions, Ramond conditions (R), while the negative sign describes *anti-periodic* boundary conditions Neveu-Schwarz conditions (NS). The solution of the former have oscillators d^i_n and $\tilde{d}^i_n$ with integer modes $n \in \mathbb{Z}$ while the later b^i_r and $\tilde{b}^i_r$ half-integer modes $r \in \mathbb{Z} + 1/2$. Either R or NS boundary conditions can be imposed on the left and right-movers separately. This leads to the two choices for the mode expansion[1] for the left-movers and two for the right-movers.

Next we define ground states annihilated by positive modding oscillators $d^i_n|0,k^M\rangle_R$ and $b^i_r|0,k^M\rangle_{NS}$ (respectively for the left movers) while the excited states are constructed by acting on the ground state with the negative oscillators which are the creation operators. Quantizing properly the fermionic modes using anticommutation relations and considering the bosonic oscillators on the states $a^i_n|0,k^M\rangle_R$ and $a^i_n|0,k^M\rangle_{NS}$ one arrives at the critical dimension $D = 10$ for the superstring theory. We write down indicatively the mass of the

[1]Left movers R: $\psi^M_+(\tau,\sigma) = \sum_n \tilde{d}^M_n e^{-2in(\tau+\sigma)}$, NS: $\psi^M_+(\tau,\sigma) = \sum_r \tilde{b}^M_r e^{-2ir(\tau+\sigma)}$
Right movers R: $\psi^M_-(\tau,\sigma) = \sum_n d^M_n e^{-2in(\tau-\sigma)}$, NS: $\psi^M_-(\tau,\sigma) = \sum_r b^M_r e^{-2ir(\tau-\sigma)}$

right movers

$$R: \quad \frac{\alpha' M_r^2}{2} = \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i + \sum_{n=1}^{\infty} n d_{-n}^i d_n^i \,, \tag{2.1.24}$$

$$NS: \quad \frac{\alpha' M_r^2}{2} = \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i + \sum_{r=1/2}^{\infty} r b_{-r}^i b_r^i - \frac{1}{2} \,. \tag{2.1.25}$$

It is easy to see that the groundstate of the NS sector is a tachyon which can be removed from the physical spectrum by the GSO projection such that the massless states of the left and right movers match. Then the physical mass is given by the sum $M^2 = M_l^2 + M_r^2$. The left and right massless states form $SO(D-2)$ representations. Applying the critical dimension $D = 10$ the first excited state of the NS is the vector representation 8_V of $SO(8)$ while the R sector first excited state forms one spinor of positive and one of negative chirality representation, 8_S and 8_C respectively. The true state is given by tensoring together a left-mover with a right-mover

$$|+\rangle \otimes |-\rangle \,, \tag{2.1.26}$$

and since there are two choices for the left-movers and two choices for the right-movers, we get a total of four different sectors: the RR sector, the R-NS sector, the NS-R sector and finally the NSNS sector. Note that the states in RR and NSNS sectors are background spacetime bosons, while states in the NS-R and R-NS sectors are background spacetime fermions. The decomposition in the following table

	type IIA		type IIB	
Sector	$\vert+\rangle \otimes \vert-\rangle$	$SO(8)$	$\vert+\rangle \otimes \vert-\rangle$	$SO(8)$
NSNS	$8_V \otimes 8_V$	$1 \oplus 28_V \oplus 35_V$	$8_V \otimes 8_V$	$1 \oplus 28_V \oplus 35_V$
NS-R	$8_V \otimes 8_S$	$8_C \oplus 56_C$	$8_V \otimes 8_C$	$8_S \oplus 56_S$
R-NS	$8_C \otimes 8_V$	$8_S \oplus 56_S$	$8_C \otimes 8_V$	$8_S \oplus 56_S$
RR	$8_C \otimes 8_S$	$8_V \oplus 56_V$	$8_C \otimes 8_C$	$1 \oplus 28_C \oplus 35_C$

Table 2.1: Bosonic and fermionic spectrum of the closed string arising from the tensoring left and right movers constructing this was the type II theories.

2.2 Low energy actions

In the previous section we have naively discussed the worldsheet superstring action and the massless spectrum of the theory as well as the low-energy limit of the bosonic string. We saw that the presence of worldsheet fermions gives rise to states which transform as fermions, vectors and bosons and make up two types of theories based on their decomposition. In this section introduce properties of so called Type II supergravities focusing on the universal NSNS and the RR sector.

2.2.1 Type II supergravities

Two different theories, the Type IIA and Type IIB supergravities can be obtained from the closed string massless spectrum discussed before and differ on the RR sector due to their different chirality. The low-energy effective action is described by the analogous supergravities in ten dimensions and they admit $N = 2$ supersymmetry due to their 32 supercharges described by two ten-dimensional Majorana-Weyl spinors. The spectrum is demonstrated in Table 2.2 and corresponds to the states in Table 2.1. The RR sector of Type IIA contains odd p-forms C_p with their respective field strengths $F_{p+1} = \mathrm{d}C_p$[2] and the NS-R and R-NS sector contain two gravitinos and two dilatinos with opposite chirality. The Type IIB RR sector contains even p-forms C_p with self-dual F_5, while the NS-R and R-NS contain two left-handed gravitinos and two right-handed dilatinos.

	Type IIA	Type IIB
NSNS	g, B_2, ϕ	g, B_2, ϕ
NS-R	$\tilde{\lambda}_1$, $\tilde{\psi}_{1M}$	λ_1 , ψ_{1M}
R-NS	λ_2 , ψ_{2M}	λ_2 , ψ_{2M}
RR	C_1, C_3	C_0, C_2, C_4

Table 2.2: The massless spectrum of Type IIA and IIB supergravities.

Since we are looking for background solutions in this analysis the fermionic fields vanish at the vacuum in order to be compatible with Poincare symmetry after compactification. Thus we introduce only the bosonic part of the Type II supergravity action in *string* frame

$$S_{\mathrm{II}} = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}X\sqrt{-G}\left\{e^{-2\phi}\left(R_{10} + 4G^{MN}\partial_M\tau\partial_N\tau - \frac{1}{2}|H_3|^2\right) - \frac{1}{2}|\tilde{F}_{p+1}|^2\right\}, \tag{2.2.1}$$

where $2\kappa_{10}^2 = (2\pi)^7\alpha'^4$, the closed field strength of the Kalb-Ramond field $H_3 = \mathrm{d}B$ the ten-dimensional determinant of the metric $G \equiv \det(G_{MN})$. The above supergravity action is invariant under ten-dimensional diffeomorphisms and as referred before admits N=2 local supersymmetry. The presence of field strengths in the tree-level action signifies the gauge invariance under the transformation of the Kalb-Ramond field $B_2 \to B_2 + \mathrm{d}\tilde{\lambda}_1$ which is naturally involved in the worldsheet action and the gauge invariance under the transformation of the Ramond-Ramond potentials $C_p \to C_p + \mathrm{d}\tilde{\lambda}_{p+1}$.

Type IIA

These bosonic states that we are interested in are obtained by tensoring a pair of Majorana-Weyl spinors as shown in Table 2.2. In the IIA case, the two Majorana -Weyl spinors have opposite chirality and one obtains a one-form C_1 and a three-form C_3 which enter the

[2] $|F_p|^2 = \frac{1}{p!}F_{M_1\ldots M_p}F^{M_1\ldots M_p}$

action (2.2.1) in the following combinations

$$\tilde{F}_2 = F_2 + F_0 \wedge B_2 \,, \tag{2.2.2}$$

$$\tilde{F}_4 = F_4 + C_1 \wedge H_3 + \frac{1}{2} F_0 \wedge B_2 \wedge B_2 \,. \tag{2.2.3}$$

In the definitions above we mention that the Romans mass F_0 is included such that the Type IIA supergravity admits a deformation by a mass parameter, which may be considered as a background field strength. This field is auxiliary, it is not propagating any degrees of freedom $\mathrm{d}F_0 = 0 = \mathrm{d} \star F_{10}$, while we need to put a nine-form potential $F_{10} = \mathrm{d}C_9$ in the perspective. However the Romans mass has physical effects since it contributes to the energy density and contains additional contributions to Chern-Simons couplings too. Setting it to zero the Type IIA supergravity can be obtained by dimensional reduction of 11d supergravity on a circle of radius R. The presence of the Romans mass will for the construction of consistent vacua in this analysis.

Type IIB

In the IIB case the two Majorana-Weyl spinors have the same chirality, and one obtains a zero-form C_0, a two-form C_2 and a C_4 gauge field as shown in Table 2.2 with the field strength of the later to respect the self-dual relation $F_5 = \star F_5$ and the enter the supergravity action (2.2.1) in the following way

$$\tilde{F}_3 = F_3 - H_3 \wedge C_0 \,, \tag{2.2.4}$$

$$\tilde{F}_5 = F_5 - \frac{1}{2}(B_2 \wedge F_3 - C_2 \wedge H_3) \,. \tag{2.2.5}$$

2.2.2 D-branes

D-branes (Dp-branes) are lower-dimensional planes which span $(p+1)$-dimensional subspaces within the 10d spacetime with open strings ending one them as described in 2.1.2. They are dynamical and non-perturbative objects in the low-energy limit of string theory. The $(p+1)$-dimensional low-energy action which describes the gravitational part of the massless bosonic open string is the Dirac-Born-Infeld (DBI) action. In addition to the standard Nambu-Goto part, this action describes the coupling of the D-brane to the rest NSNS fields and includes the number of scalars interpreted as transverse fluctuations of the D-brane which parametrize its position in the transverse space

$$X^M(\xi) = 2\pi\alpha' \phi^M(\xi) \,, \quad M = p+2, \ldots, D \,, \tag{2.2.6}$$

where $\xi = 1, \cdots, p+1$ are the longitudinal coordinates (2.1.7). For our purposes we ignore the supersymmetric part and write down only the bosonic part of the action

$$S_{\mathrm{DBI}} = -T_{\mathrm{D}p} \int \mathrm{d}^{p+1}\xi e^{-\phi} \sqrt{-\det\big(g_{ab} + \mathcal{F}_{ab} + (2\pi\alpha')^2 \partial_a\phi\partial_b\phi\big)} \,, \tag{2.2.7}$$

where $\mathcal{F}_{ab} = B_{ab} + 2\pi\alpha' F_{ab}$ with B_{ab} the NSNS massless two-form and F_{ab} is the field strength of the U(1) gauge boson $F = \mathrm{d}A$ living on the worldvolume of the D-brane. The existence of the $\mathcal{F}_{ab}$ term in the action is reasonable because it is the only invariant combination under the gauge symmetries of the worldsheet action. The metric g_{ab} indicates the induced metric

$$g_{ab}(\xi) = \frac{\partial X^M}{\partial \xi^a}\frac{\partial X^N}{\partial \xi^b} G_{MN}(X(\xi)) \,. \tag{2.2.8}$$

We note also in (2.2.7) that in this frame the dilaton is an overall factor and can be considered as part of the tension $T_{\mathrm{D}p}$. Its dependence to the string coupling constant is inversely proportional which means that in the weak coupling limit $g_s \ll 1$ the D-branes are non-perturbative states and behave as rigid objects. The tension of the D-brane is related to the string length via

$$T_{\mathrm{D}p} = (2\pi)^{-p} \alpha'^{(-1-p)/2} \,. \tag{2.2.9}$$

Apart from the gravitational coupling described by the DBI action, D-branes are electrically charged under RR charges. They emit closed strings and this is expressed via the coupling with the Chern-Simons action which describes the C_p coupling of D-branes

$$S_{\mathrm{CS}} = Q_{\mathrm{D}p} \int \mathbf{C} \wedge e^{\mathcal{F}}|_{p+1} \,, \tag{2.2.10}$$

where $Q_{\mathrm{D}p}$ is the electric charge of the D-brane. $\mathbf{C}$ stands for the polyform $\mathbf{C} = \sum_n C_p$ for p the gauge potentials allowed in Type II theories. However the presence of background flux $\mathcal{F}$ can make the D-brane carry also lower RR charges. Performing Taylor expansion for the exponential term $e^{\mathcal{F}} = 1 + \mathcal{F} + \mathcal{F}^2/2! + \dots$ and considering the polyform $\mathbf{C}$ we keep only the $(p+1)$ piece of the integral which corresponds to the D-brane worldvolume. For example the first term of this expansion in the Chern-Simons action gives the D-brane sourcing C_p gauge fields. The next term describes the worldvolume gauge field inducing lower-dimensional D-brane charges $C_{p-1} \wedge \mathcal{F}$. Since we have already given the C_p potentials for Type II theories we can read off from the Dp-brane action that Type IIB theory is compatible with Dp-branes with odd p and even in the number of $p+1$ dimensions while Type IIA theory contains only Dp-branes with even p and odd number of $p+1$ dimensions

$$\begin{aligned} \text{Type IIA} &: \mathrm{D0}, \mathrm{D2}, \mathrm{D4}, \mathrm{D6}, \mathrm{D8} \,, \\ \text{Type IIB} &: \mathrm{D1}, \mathrm{D3}, \mathrm{D5}, \mathrm{D7}, \mathrm{D9} \,. \end{aligned}$$

Having introduced the gravitational and electric part, there is a condition for supersymmetric D-branes that relates the tension to the charge $T_{\mathrm{D}p} = Q_{\mathrm{D}p}$ and indicates the cancelling of the forces among parallel branes ensuring stability of a D-brane system. Furthermore, D-branes preserve on their worldvolume one half of the original supersymmetry making them stable objects. For this to happen their charge has to be equal to their tension.

Anti-D-branes

We end this subsection by considering the so called anti-D-branes or $\overline{\text{Dp}}$-branes which are similar to the common D-branes but they carry the opposite charge. Anti-D-brane have the same tension as D-branes which means that they have the same coupling to fields coming from the NSNS closed string. On the other hand, the coupling to the RR fields comes with the opposite sign. This means that for a system of parallel D-branes and anti-D-branes the total charge vanishes (brane-antibrane annihilation) while the tensions sum up. This also indicates that this state of the system is not BPS anymore since the total charge vanishes while the tension does not and BPS relation is not satisfied. Actually for parallel D-branes and anti-D-branes all the supersymmetries are broken since the anti-D-branes break exactly the preserved D-branes supersymmetries. This property will be used in the de-Sitter uplift where all the supersymmetries preserved in the Minkowski or AdS vacua should vanish while the tension will contribute positively to the energy density.

2.2.3 Toroidal compactification

Starting from the worldsheet perspective we have discussed the effective action of the superstrings which gives rise to bosons and fermions living in a 10d target space. However as described in the introduction in order to make contact with the real world we have to obtain an effective field theory by compactifying the extra dimensions. The first step for the *compactification* of the extra dimensions is to express the ten-dimensional metric as the product of two manifolds $M_d \times X^{10-d}$, where X^{10-d} stands for the compact manifold, so called internal space while M_d stands for the external space. In our examples the external space will be three-dimensional. The idea of compactification is based historically on the Kaluza-Klein mechanism while as we will see for a proper and realistic compactification not just compact dimensions needed to be small but also a hierarchy on scales will play crucial role.

Compactification in field theory

In this first example we consider a field theory in five-dimensional spacetime with the presence of a scalar field. We consider the fifth dimensions, internal manifold, to be a circle $x^5 = S^1$ and so it is compactified in following way $x^5 \cong x^5 + 2\pi R$. Since we want to investigate the effect of the extra dimension we consider the five-dimensional scalar field which can be expanded in Fourier modes in the following way

$$\phi(x^M) = \sum_{k\in\mathbb{Z}} e^{\frac{ikx^5}{R}} \phi_k(x^\mu) \,, \tag{2.2.11}$$

due to the periodicity of the extra dimension. The indices here $M = 1, \ldots, 5$ and $\mu = 1, \ldots, 4$. We assume that the scalar field is massless such that there are only kinetic terms in the five-dimensional action. It is straightforward to see that the momentum of the field in the extra dimension is quantized $p = k/R$. From the five dimensional equations of motion we find that

$$\partial_\mu \partial^\mu \phi_k - \frac{k^2}{R^2}\phi_k = 0\,, \tag{2.2.12}$$

and the mass of the four-dimensional fields is a tower of states $m_k^2 = (k/R)^2$ which depends one the radius of the fifth periodical dimension. The effect of the fifth dimensions appear in the four dimensional equation. However at energies lower than the compactification scale

$$L \ll M_{KK} \sim R^{-1} \tag{2.2.13}$$

the modes are heavy and only the four-dimensional part contributes. On the other hand when $L \gg R^{-1}$ we see the tower of Kaluza-Klein states entering the scale. If one considers initially a massive scalar field the effects of the extra dimension appear as a shift in the mass spectrum $m_k^2 = M^2 + (k/R)^2$.

In order to study further the effective field theory which arises after the compactification of extra dimensions and make a connection to string theory field content we consider the universal action (2.1.17) and $D = 5$. Assume the existance of a scale factor (a scalar component of the initial metric) for the fifth dimension $ds^2 = g_{\mu\nu}dx^\mu dx^\nu + e^{2b}dx^5 dx^5$ and we compactify over the periodic dimension to get the following action

$$S = \frac{\pi R}{\kappa_0^2}\int \mathrm{d}^4x\sqrt{-g_4}e^{-2\tilde{\phi}}\big(R_4 - \partial_\mu b\partial^\mu b + 4\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} - \frac{1}{12}H_{\mu\nu\sigma}H^{\mu\nu\sigma} - \frac{1}{4}e^{-2b}H_{5\mu\nu}H^{5\mu\nu}\big) \tag{2.2.14}$$

for $\tilde{\phi} = \phi - b/2$. Firstly we observe that the gravitational coupling of the effective theory has changed and depends on the radius of the internal space and the scale factor. Second the effective action contains massless scalar fields which do not have a potential thus they can mediate gravitational-like forces which are not observed in our experiments. These fields are so called *moduli* which we want to stabilize with the presence of fluxes and supersymmetry in order to obtain masses but also extra fields come due to the decomposition of the three-rank antisymmetric field after the reduction. For a more general analysis for compactifications with warped metric see [5].

Bosonic string compactification

The effect of compactification in string theory is global and arises in the boundary conditions for the worldsheet fields. More specifically consider a ten-dimensional bosonic field where one of the spatial directions satisfy the boundary condition

$$X^i(\tau, \sigma + l) = X^i(\tau, \sigma) + 2\pi R w\,, \tag{2.2.15}$$

while the rest satisfy the standard closed string boundary condition (2.1.3). The presence of winding w it's due to the 2d nature of the strings and the standard closed string boundary condition is restored for $w = 0$. Obsiously this periodicity condition changes the mode expansion for both bosons and fermions and affect the spectrum as we will see. In contrast to the example of the compactification on a circle in field theory, the momenta in the compactified direction of closed strings contains extra winding terms in addition to the quantized standard ones $p_{R/L} = \frac{k}{R} \pm \frac{wR}{\alpha'}$ while these terms squared enter to the mass for the closed string (2.1.5)

$$M^2 = \left(\frac{k^2}{R^2} + \frac{w^2 R^2}{\alpha'^2}\right) + \frac{2}{\alpha'}(N + \tilde{N} - 2)\,. \tag{2.2.16}$$

It is important to note that compactifying over more than one dimensions there are extra contributions to closed string spectrum due to the scalars arising from the two-rand antisymmetric tensor decomposition. Regarding the mass of the closed string in this example it is obvious that the standard massless string limit is derived for $k = w = 0$. For non-zero winding and $R/\alpha' \gg 1$ the stringy modes (α' effects) decouple from the standard KK modes. This is the large internal volume approximation where the radius is much larger than the string length. On the other hand for $R/\alpha' \ll 1$ the winding states become important and we cannot trust the field theory approximation. The effective field theory with energy scale L has to be $L \ll M_{KK} \ll M_{string}$. In addition to the compactification scale now there is an extra constrain in order the stringy scales not to be present in the low energy theory.

Toroidal orbifold, untwisted and twisted sector

So far we have seen the compactification of a string on a circle which can be generalized to a torus. However compactifying on such a smooth manifold the remaining effective theory preserves a large amount of supersymmetry resulting effective field theories which are not phenomenology friendly. Now we want to construct a d-dimensional internal space imposing the periodic condition (2.2.15) to fields X^i with the presence of discrete symmetries. In short one can define an orbifold as the quotient space of a smooth manifold T divided by the isometry group Γ leading to the singular space $X^d = T^d/\Gamma$. Actually considering our smooth manifold to be a torus divided by some discrete symmetry we construct the so called toroidal orbifolds.The group Γ acts with fixed points on the coordinates of the torus.

One might think that a singular space could be problematic due to the singularities on the worldsheet. However the nature of strings which are extended objects are compatible with propagating in such spaces. The spectrum of closed strings considering an orbifold group Γ falls in two cases. To discuss them we write down the periodic boundary condition of a compactified closed string up to the action of $g \in \Gamma$ on the field

$$X^M(\tau, \sigma + l) = gX^M(\tau, \sigma) + 2\pi Rw\,. \tag{2.2.17}$$

The first case is the so called *untwisted states* where $g = 1$ and corresponds to the string theory compactified on a manifold e.g a torus with boundary condition (2.2.15). The mode expansions and the mass spectrum is the one described in the compactification of bosonic string subsection thus the string states are invariant under the group action. The second case is the so called *twisted states* which correspond to the fixed points by the orbifold, for example for the group element $g = -1$. This implies that for the center of mass of the string is located at a fixed plane. The presence of such twisted states is also required by a strong symmetry of type IIB, the modular invariance. The complete massless spectrum is anomaly-free thus both the untwisted and twisted sectors should be taken into account.

2.2.4 Orientifolds

Unoriented strings

So far have discussed the spectrum of oriented strings which means that the worldsheet has an orientation. In order to introduce unoriented strings we consider the worldsheet parity operator Ω which acts on the worldsheet fields as a coordinate transformation in the following way

$$\Omega : \; X^M(\tau, \sigma) = X^M(\tau, l - \sigma) \,, \tag{2.2.18}$$

changing the orientation of the worldsheet. This condition holds for both open and closed strings and changes only the oscillatory part of the X^M field expansion acting as $a^M \leftrightarrow (-1)^N a^M$ for the open string and $a^M \leftrightarrow \tilde{a}^M$ for the closed string. This corresponds to the exchanges of right and left moving sectors. Applying the worldsheet operator on the ground states we get

$$\Omega|N, k\rangle = (-1)^N |N, k\rangle \,, \tag{2.2.19}$$

$$\Omega|N, \tilde{N}, k\rangle = |N, \tilde{N}, k\rangle \,, \tag{2.2.20}$$

while only the invariant states, which have eigenvalues $+1$, survive. As we notice for the open string there is change of the sign for odd states which signifies that the worldsheet parity truncates the spectrum of the oriented open string. In addition to this, the open string carries degrees of freedom at the endpoints, known as Chan-Paton indices, which can be interpreted as indices of a gauge group. The worldsheet parity acts non-trivially on them leading to the O^- projections giving $SO(N)$ gauge bosons, and the projection O^+ giving $USp(N)$ gauge bosons. The former are related to the usual orientifold planes which we describe next while the later will be used in the construction of de-Sitter vacua in our Type IIB example.

Orientifold involution and O-planes

Another important local object in string theory is the p-dimensional hyperplane with fixed loci, so called *orientifolds* or Op-planes. An orientifold plane is a result of the orientifold projection $\mathcal{O}$ that we will introduce in a short, a combination of the worldsheet parity Ω which produces unoriented strings and target space reflections which create orbifolds and does not affect the external spacetime. The action σ is a reflection in target space and the subspace fixed under σ correspond to a region where the orientation of a string can flip. In addition, the involutions leave invariant certain submanifolds and the product of these submanifolds to the external space are referred as Op-planes where p stands for the worldvolume similar to Dp-branes. Finally the orientifold projection is given to be

$$\mathcal{O} = \Omega\sigma(-1)^{F_L} \,, \tag{2.2.21}$$

where F_L is the spacetime fermion number in the left-moving sector and is needed to ensure $\mathcal{O}^2 = \mathbb{1}$ on all states.

In contrast to D-branes, these objects are non-dynamical and strings are not tied up to the orientifold plane and thus there are no gauge fields in their worldvolume. However they also couple to the RR sector via a Chern-Simons part and their effect on the target space can be described at tree-level by an action similar to the one we have introduced for Dp-branes

$$S_{\mathrm{O}p} = -T_{\mathrm{O}p} \int \mathrm{d}^{p+1}\xi e^{-\phi}\sqrt{-\det(g_{ab})} + Q_{\mathrm{O}p}\int_{p+1} \mathbf{C} \,, \tag{2.2.22}$$

which consists of two pieces and these objects have a tension $T_{\mathrm{O}p}$ and a charge $Q_{\mathrm{O}p}$. The most common type of O-plane has tension opposite to this D-brane and are related the following way

$$T_{\mathrm{O}p} = -2^{p-5}T_{\mathrm{D}p} \,. \tag{2.2.23}$$

In this analysis we take the charge of D-branes to be positive thus the orientifold planes have negative charge. This is the most common kind of orientifold plane but in a special case we will consider also orientifold planes with positive tension and charge which are usually denoted as O$^+$-planes. These objects should not be confused with anti-O-planes which have the same tension but opposite charge to this of the regular O-planes. Next we list the O-planes allowed in Type II depending on their worldvolume

$$\begin{aligned} \text{Type IIA} &: \ \mathrm{O2}, \mathrm{O4}, \mathrm{O6}, \mathrm{O8} \,, \\ \text{Type IIB} &: \ \mathrm{O3}, \mathrm{O5}, \mathrm{O7}, \mathrm{O9} \,. \end{aligned}$$

An extensive review of orientifold planes in string theory can be found in [42].

The presence of orientifolds determine the spectrum of the effective theory since they project out fluxes which are not invariant under the orientifold involution. In order the NSNS sector to be invariant under the local orientifold projection $\mathcal{O}$, the fields should transform in the following way under the target space involutions

$$\sigma\phi = \phi \,, \qquad \sigma g = g \,, \qquad \sigma H_3 = -H_3 \,. \tag{2.2.24}$$

For the RR fields we need

$$\sigma F_p = -\alpha\, F_p \,, \qquad (\mathrm{O4}, \mathrm{O5}, \mathrm{O8}, \mathrm{O9}) \tag{2.2.25}$$

$$\sigma F_p = +\alpha\, F_p \,, \qquad (\mathrm{O2}, \mathrm{O3}, \mathrm{O6}, \mathrm{O7}) \tag{2.2.26}$$

where α is the operator which reverses all the indices of a p-form see [43]. The orientifold is supersymmetric if the orientifold operator leaves the supersymmetry generator invariant.

2.2.5 Tadpole cancellation and flux quantization

Consider now a simple example where the background flux $\mathcal{F} = 0$. Integrating by part the kinetic terms of the RR fields and considering the simplified CS term we get

$$S_{C_p} \sim \int C_p \wedge \mathrm{d}F_{9-p} + \sum_i N \int C_p \wedge \delta_{i,9-p}\,, \tag{2.2.27}$$

where we have also included the number N of D-branes which in general can be arbitrary but integer. We have used the delta function with support on the transverse directions in order to express the Chern-Simons part in ten dimensions. Performing the variation for the C_p field we arrive at the Bianchi identity

$$\mathrm{d}F_{8-p} = Q_{\mathrm{D}p} \sum_i N\delta_{i,9-p}\,. \tag{2.2.28}$$

From this equation one can calculate the nonzero charge of the D-brane which acts as a source term in the equations of motion for the C_p field. However, considering the internal indices and performing the integral over the internal space there are inconsistencies arising. In complete analogy to Gauss law there should not be net charge on compact manifolds since the magnetic lines emanating from the charge either have to extend to infinity or end up on another source with opposite charge. Actually in Type II string theory there is a tadpole in the string perturbation series for RR-fields reflected in target space by non-trivial total RR-field flux on compact spaces. This inconsistency cancels if the charge can be neutralized by adding opposite charge source. So far we have seen that anti-D-branes are charged with the opposite charge of D-branes so one can include them to cancel the tadpole. This can be done however implies that the model will be necessarily non-supersymmetric. The introduction of orientifold planes with negative tension and opposite charge to those of D-branes can satisfy the Bianchi identity but also reduce the amount of supersymmetry leading to friendly phenomenological vacua. Thus we have to consider orientifold backgrounds.

So far we have seen in short that the presence of D-branes in string theory arises as open string endpoints that at the low-energy description of supergravity they appear as non-perturbative objects. However when solving the supergravity equations of motion such local objects are involved in the vacua solution and their charges are fixed together with the fluxes of RR and NSNS fields. One restriction of string theory on the supergravity solutions is the *quantization of the fluxes* which we consider it here as a consistency condition. The quantization rules in our conventions for the 10d theory are

$$\int F_p = (2\pi)^{p-1}(\alpha')^{(p-1)/2} f_p\,, \qquad f_p \in \mathbb{Z}\,, \tag{2.2.29}$$

thus in order the flux vacua to be of string origin the fluxes and the charges have to be integer numbers.

An other condition that has to be satisfied to avoid inconsistencies both in the world-sheet and in the target space is the tadpole cancellation which was discussed before for the simple case of RR charges sourced by a number of D-branes. The equation of motion for the RR field, Bianchi identity. We we consider the Bianchi identity including extra flux and sources

$$\mathrm{d}F_{8-q} = H_3 \wedge F_{6-q} + (2\pi)^7 \alpha'^4 (N_{\mathrm{O}p} Q_{\mathrm{O}p} + N_{\mathrm{D}p} Q_{\mathrm{D}p}) \delta_{9-p} \, . \tag{2.2.30}$$

where except the local sources the presence of fluxes appeared on the right hand side(+). However now except the sources with opposite charge that can neutralize the total charge the extra flux term has to vanish. In order the right hand side to vanish after integration over the internal space perform the *smearing* approximation

$$\delta_{9-n} \rightarrow j_{9-n} \, , \tag{2.2.31}$$

where we have replaced the delta function with a regular function. Then the integral of the Bianchi identity gives the tadpole cancellation condition

$$\int_X \mathrm{d}F_{8-p} = 0 \;\; \rightarrow \;\; \frac{1}{(2\pi)^7 \alpha'^4} \int_X H_3 \wedge F_{6-p} + Q_{source} = 0 \, . \tag{2.2.32}$$

where $Q_{source} = (N_{\mathrm{O}p} Q_{\mathrm{Dp}} + N_{\mathrm{D}p} Q_{\mathrm{Dp}})$ where as we referred before the number of O-planes is fixed by the geometry while of the D-branes is not. To satisfy this equation the number of D-branes $N_{\mathrm{D}p}$ should be chosen in such a way to cancel the flux, roughly speaking

$$Q_{source} \sim h_3 f_{6-p} \, , \tag{2.2.33}$$

the left hand side is the fluxes of the relevant NSNS and RR forms. This condition bounds significantly the values of the fluxes which are responsible for scale separation.

Chapter 3

Internal space: G2-structures and toroidal orbifold

In this section we start by introducing basic concepts of smooth manifolds with G2-structure. We review properties of G2-structures such as the curvature of the manifold, the structure equations which characterize the fundamental three-form of G2 as well as the torsion classes which correspond to the irreducible representation of G2-structures and will appear explicitly in our analysis. Next we specify the choice of our internal space to be seven-dimensional torus with G2-structure together with a specific orbifold group. As previously discussed the presence of orbifold will lead us to the construction of effective field theories with reduced amount of supersymmetry.

3.1 G2-structures

In this subsection we introduce the basic features of the seven-dimensional internal space X_7 to be used in our compactifications. Our scope is to define the general properties of a seven-dimensional manifold X_7 which admits a G2-structure and we establish we basic tools for our analysis. The G2-structure is characterized by the invariant three-form in an oriented seven dimensional manifold X_7

$$\Phi = e^{127} - e^{347} - e^{567} + e^{136} - e^{235} + e^{145} + e^{246} \,, \tag{3.1.1}$$

where $e^{127} = e^1 \wedge e^2 \wedge e^7$, etc and e^i are one-form basis coordinates of X_7. Given this three-form on X_7 it is possible to define a unique Riemannian metric $g(\Phi) = g$ associated to Φ and thus a Hodge duality operation. Then we can write down the co-associative invariant four-form

$$\star\Phi = \Psi = e^{3456} - e^{1256} - e^{1234} + e^{2457} - e^{1467} + e^{2367} + e^{1357} \,. \tag{3.1.2}$$

A convenient expression for the internal volume in terms of the fundamental forms is

$$\mathrm{vol}(X_7) = \frac{1}{7} \int \Phi \wedge \star\Phi \,. \tag{3.1.3}$$

The fundamental three-form and the relevant metric are uniquely defined in the following way: a stable three-form Φ on the tangent spaces of X_7 reduces the structure group $GL(7) \to G2$ so that X_7 is a G2-structure manifold [45]. Then the three-form (3.1.1) is uniquely defined together with the symmetric bilinear B_{ab}

$$g_{ab}\sqrt{\det g} = B_{ab} = \frac{1}{144}\Phi_{amn}\Phi_{bpq}\Phi_{rst}\epsilon^{mnpqrst}\,, \tag{3.1.4}$$

where $\epsilon^{mnpqrst}$ is the seven dimensional Levi-Civita symbol and the metric can be read off in terms of the three-form. See [46] for a detailed derivation.

Irreducible representations of G2

To study further manifolds with G2-structure it is important to understand the decomposition of representations of p-forms living in Λ^p spaces after the reduction $GL(7) \to G2$. We denote by $\Lambda^p_l(X_7) \equiv \Lambda^p_l$ the l-dimensional irreducible representation of G2 and define the Hodge duality operation $\star\Lambda^p_l \cong \Lambda^{7-p}_l$ that we have used already. The $\Lambda^1 = \Lambda^1_7$ is irreducible representation of G2 while the **21**-dimensional space of two-forms on X_7 follows the direct sum decomposition $\Lambda^2 = \Lambda^2_7 \oplus \Lambda^2_{14}$. The **35**-dimensional space of three-forms decomposes as $\Lambda^3 = \Lambda^3_1 \oplus \Lambda^3_7 \oplus \Lambda^3_{27}$. The higher form representations are induced by Hodge duality. We write down the properties and conditions of each irreducible representation in terms of the fundamental three-form

$$\Lambda^2_7 = \{\star(\alpha \wedge \star\Phi) \,|\, \alpha \in \Lambda^1\}\,, \tag{3.1.5}$$
$$\Lambda^2_{14} = \{\beta \in \Lambda^2 \,|\, \beta \wedge \star\Phi = 0\}\,, \tag{3.1.6}$$
$$\Lambda^3_1 = \{f\Phi \,|\, f \in \mathcal{C}^\infty\}\,, \tag{3.1.7}$$
$$\Lambda^3_7 = \{\star(\alpha \wedge \Phi) \,|\, \alpha \in \Lambda^1\}\,, \tag{3.1.8}$$
$$\Lambda^3_{27} = \{\gamma \in \Lambda^3 \,|\, \gamma \wedge \Phi = 0\,,\, \gamma \wedge \star\Phi = 0\}\,. \tag{3.1.9}$$

3.1.1 Intrinsic torsion, curvature and closeness of the three-form

In this subsection we introduce algebraic and analytic properties of the three-form and geometric quantities like torsion and curvature which will be crucial for analytical calculations and for understanding the physical notion of curvature on manifolds with group structure.

We have seen that the fundamental three-form on a smooth manifold X_7 defines a Riemannian metric g_{ij} and therefore a unique connection ∇ (Levi-Civita covariant derivative). The exterior derivative on the three-form does always vanish, it is called the intrinsic torsion of the G2-structure and is defined as

$$\nabla_l\Phi_{abc} = T_{lm}g^{mn}\Psi_{n\,abc}\,, \tag{3.1.10}$$

which lies in the product space $\nabla\Phi \in \Lambda^1_7 \otimes \Lambda^3_7$. The expression above is written in terms of the full torsion tensor T_{lm} which has **49**-components and fully defines the intrinsic torsion

$\nabla\Phi$. The fundamental three-form is not necessarily covariantly constant while whether the covariant derivative of the three-form vanishes depends on the fully antisymmetric torsion T_{lm} that we define next

$$T_{lm} = \frac{1}{4}W_1 g_{lm} + \frac{1}{3}(W_7)_{lm} - \frac{1}{2}(\star W_{14})_{lm} - (\star W_{27})_{lm}\,, \tag{3.1.11}$$

which is expressed in terms of the metric and the W_i p-differential forms (we follow the notation of [47]). The latter, which are the so-called *torsion classes*, correspond to the irreducible representations $W_1 \in \mathbf{1}$ is a function, $W_7 \in \mathbf{7}$, $W_{14} \in \mathbf{14}$, $W_{27} \in \mathbf{27}$ of the G2 structure after the decomposition $\mathbf{49} = \mathbf{1} \oplus \mathbf{7} \oplus \mathbf{14} \oplus \mathbf{27}$. The torsion classes can be calculated in terms of the torsion tensor as in [45] however in our examples we will only define them after twisting the torus with the presence of group structure constants. The presence of torsion forms was classified by Fernandez and Gray [48] in 16 different classes and they showed that torsion free G2-structures are equivalent to the fundamental three-form being closed and co-closed.

However one can get more analytic and handy expressions than the covariant derivative which relate derivatives on the fundamental forms to the torsion classes. It was shown in [49] that the exterior derivatives on the three-form and its dual give the structure equations and can be decomposed in terms of the torsion classes

$$\mathrm{d}\Phi = W_1 \star \Phi - \Phi \wedge W_7 + W_{27}\ , \tag{3.1.12}$$

$$\mathrm{d}\star\Phi = \frac{4}{3}\star\Phi \wedge W_7 + W_{14}\,, \tag{3.1.13}$$

together with the useful relations $\Phi \wedge W_{27} = 0$ and $\star W_{14} \wedge \Phi = W_{14}$. From the structure equations it is easy to read that W_1 is a zero-form, W_7 a two-form, W_{14} a five form and W_{27} a four-form. Having defined the metric and the covariant derivative one can find the non-vanishing Ricci tensor and scalar which are a key feature for studying more general internal spaces. For this reason we introduce the form of the Ricci scalar is

$$R^{(7)} = -4\star\mathrm{d}\star W_7 + \frac{21}{8}W_1^2 + \frac{30}{9}|W_7|^2 - \frac{1}{2}|W_{14}|^2 - \frac{1}{2}|W_{27}|^2\,, \tag{3.1.14}$$

for all non-zero torsions. This expression will be simplified for our specific examples.

A case of great interest for compactifications that we also study here arise when all torsions are simultaneously set to zero, the G2-structure is torsion free. This directly imposes that the full torsion is zero and thus $\nabla\Phi = 0$ while the structure equations simplify to

$$\mathrm{d}\Phi = 0\,, \tag{3.1.15}$$

$$\mathrm{d}\star\Phi = 0\,. \tag{3.1.16}$$

Since the fundamental three-form determines a Riemannian metric we can also find expressions for the Ricci curvature of the manifold and can be expressed in terms of the torsion. and then the G2-structure group is equivalent to the *G2-holonomy* of the manifold $\mathrm{Hol}(g) \subseteq \mathrm{G2}$, the internal space is *Ricci flat*. Non-vanishing covariant derivative $\nabla\Phi \neq 0$ signify deviation from G2 holonomy.

3.2 The T^7 orbifold with G2-structure

The first step to a specific model is the choice of G2 manifold and then study the possible O-plane involutions that can be defined over it. The easiest examples are G2 spaces that arise from *toroidal orbifolds* whose singularities are blown up or not. For singularities that can be resolved by a known geometric blow-up procedure the canonical set of examples were constructed in the original work by Joyce [50, 51]. But truly singular G2 spaces are physical as well in the context of string or M-theory and they are even required to get more interesting lower-dimensional phenomenology [52]. Both regular and singular G2 spaces constructed from toroidal orbifolds come with extra modes not visible at the level of the torus covering space. Either these modes are really the extra moduli of cycles introduced by the geometric blow up, or they come from the twisted sector of the string. In our examples in this thesis we will use the simplest singular toroidal orbifold and be careless about the unresolved orbifold singularities, which we assume can be resolved in string theory at the cost of extra twisted sectors. In any case the restricted set of seven *real* circle radii we consider are present in most models. So in that sense we capture the "universal" sector of many toroidal G2 compactifications, just like the STU truncation in four-dimensional N=1 flux reductions.

So far we have discussed the basic setup of a seven dimensional manifold with G2-structure. Now we want to specify the internal manifold to be toroidal orbifolds of the form

$$X_7 = \frac{T^7}{\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2} \,. \tag{3.2.1}$$

We introduce the seven vielbeins of the torus

$$e^m = r^m \mathrm{d}y^m \,, \tag{3.2.2}$$

and the seven internal coordinates are labeled as y^m

$$y^m \simeq y^m + 2\pi r^m \,. \tag{3.2.3}$$

The finite group of isometries Γ, forming the orbifold group, should preserve the three-form in Eq.(3.1.1) and the co-associative four-form in Eq.(3.1.2). For our orbifold group Γ we use the following $\mathbb{Z}_2$ involutions

$$\begin{aligned}
\Theta_\alpha &: (y^1, \dots, y^7) \to (-y^1, -y^2, -y^3, -y^4, y^5, y^6, y^7)\,,\\
\Theta_\beta &: (y^1, \dots, y^7) \to (-y^1, -y^2, y^3, y^4, -y^5, -y^6, y^7)\,,\\
\Theta_\gamma &: (y^1, \dots, y^7) \to (-y^1, y^2, -y^3, y^4, -y^5, y^6, -y^7)\,,
\end{aligned} \tag{3.2.4}$$

and then $\Gamma = \{\Theta_\alpha, \Theta_\beta, \Theta_\gamma\}$. Note that the Θ commute, they square to the identity, and they preserve the calibration Φ. All of the three Θ, the three Θ^2 and the single Θ^3 have each 16 copies of $\mathbb{T}^3$ as fixed points, but they do not act on each other freely, therefore we have a singular G2;[1] to find the singular space of the full Γ and to perform the blow-up is studied in this work.

Let us focus now on the untwisted sector of this orbifold. The untwisted Betti numbers (i.e. the Betti numbers before we resolve the singularities) can be simply found by counting the number of linearly independent p-forms $\mathrm{d}y^{i_1\cdots i_p}$ invariant under the orbifold action Γ. For example in the simplest case where the invariant forms are the fundamental three and four-form and the volume the Betti numbers will be $b_0 = 1$, $b_1 = 0$, $b_2 = 0$, $b_3 = 7$. The seven invariant three-forms build a basis

$$\Phi_i = \left(\mathrm{d}y^{127}, -\mathrm{d}y^{347}, -\mathrm{d}y^{567}, \mathrm{d}y^{136}, -\mathrm{d}y^{235}, \mathrm{d}y^{145}, \mathrm{d}y^{246}\right) \;, \quad i = 1, \dots, 7 \,, \tag{3.2.5}$$

on which we have already expanded the calibration as

$$\Phi = \sum_{i=1}^{7} s^i(x)\Phi_i \,, \quad \text{or} \quad \Phi = s^i \Phi_i \,, \tag{3.2.6}$$

where the $s^i(x) = s^i$ are the metric moduli and describe the internal metric deformations and relate to the vielbeins as $e^1 = (s^1 s^6 s^7)^{1/2} (\prod_i s^i)^{-1/6} dy^1$, etc. (we refer to them as shape moduli or deformations too). Indeed, the s^i can be related to the seven torus radii r^m as follows

$$s^1\Phi_1 = e^{127} \;\rightarrow\; s^1 = r^1 r^2 r^7 \;, \quad s^2\Phi_2 = -e^{347} \;\rightarrow\; s^2 = r^3 r^4 r^7 \;, \quad \text{etc.} \tag{3.2.7}$$

Now one can re-express the internal volume (3.1.3) in terms of the seven torus radii

$$\mathrm{vol}(X_7) = \prod_{m=1}^{7} r^m = \left(\prod_{i=1}^{7} s^i\right)^{1/3} = \frac{1}{7}\int \Phi \wedge \star\Phi \,, \tag{3.2.8}$$

where we use $\int_{\mathbb{T}^7} dy^1 \wedge \cdots \wedge dy^7 = 1$ in the covering space. For later convenience we also define here a basis of closed and co-closed four-forms that are left invariant under the orbifold involutions

$$\Psi_i = \left(\mathrm{d}y^{3456}, -\mathrm{d}y^{1256}, -\mathrm{d}y^{1234}, \mathrm{d}y^{2457}, -\mathrm{d}y^{1467}, \mathrm{d}y^{2367}, \mathrm{d}y^{1357}\right) \;, \quad i = 1, \dots, 7 \,. \tag{3.2.9}$$

which together with the basis of harmonic three-forms satisfy an orthogonality condition of the form We denote a generic G2 space as X_7 and introduce the basis for the dual four-form $\Psi = \star\Phi$ on X_7 as

$$\int \Phi_i \wedge \Psi_j = \delta_{ij} \tag{3.2.10}$$

Notice that the three-forms (3.2.5) and the four-forms (3.2.9) satisfy the relation (3.2.10) and that in the Ψ_i basis the co-associative calibration takes the form

$$\star\Phi = \sum_{i=1}^{7} \frac{\mathrm{vol}(X_7)}{s^i} \Psi_i \,, \qquad \star\Phi_i = \frac{\mathrm{vol}(X_7)}{(s^i)^2} \Psi_i \,. \tag{3.2.11}$$

[1] In [50] these singularities are referred to as ‘bad’ not because they cannot be resolved but rather because there is no straightforward prescription to do so.

Chapter 4

Three-dimensional vacua in Type IIA

4.1 Introduction

In this part of the thesis we focus mainly on the construction of AdS vacua from Type IIA orientifold compactifications. So far our best motivated scenarios for achieving moduli stabilization in scale-separated AdS vacua are [16,17,53], where only reference [17] succeeds in finding an arbitrary separation between the KK scale and the AdS scale by taking an unconstrained flux quantum to infinity. On the downside this AdS vacuum does have scalars of the order of the AdS mass scale and hence these vacua are not so useful for true phenomenological models.

Interestingly, the very basic assumption of being able to achieve scale separation is under debate [15, 25, 28, 54, 55] since the compactifications used in string phenomenology always feature ingredients which obscure fully explicit computations. One can hope that holography could settle this discussion once it can be shown there are huge families of CFTs that could reproduce qualitative features like full moduli stabilization and scale separation. We refer to [29, 56–60] for recent work in that direction.

These issues motivate us to investigate moduli stabilization and scale separation in 3d vacua. Such vacua or not obviously relevant for phenomenology but the ingredients used in the construction of 4d vacua exist there as well, whereas that seems not to be true in 5d or higher in our opinion. This already makes 4 dimensions special in string theory. The nice feature of 3 dimensions is that supersymmetric AdS vacua can be dual to SCFTs in 2d. Since 2d CFTs are somewhat more studied than 3d CFTs our hope is that we can settle the issue for 3d AdS vacua[1]. Although, in this thesis we focus entirely on the supergravity problem and do not venture into a holographic description. We hope to come back to this in the future.

So we are led to consider compactifications of string theory on seven-dimensional manifolds that preserve some amount of supersymmetry. Furthermore, we want to have some handle on the moduli problem for such manifolds. This restricts us to manifolds with G2 holonomy, since by now there is an extended literature on the moduli problem for such

[1] One can of course also ponder AdS vacua in 2d, see for instance [30].

spaces; see e.g. [52, 61–64] for a sample of papers. In contrast, manifolds with a G2 structure group but no G2 holonomy are much less understood when it comes to fluctuation theory.

Spaces with G2 holonomy, abbreviated G2 spaces from here onwards, are Ricci flat and compactifying over them without fluxes and sources will lead to a 3d supergravity theory with 4 supercharges and a moduli space. We are interested in lifting the moduli space using fluxes and therefore one should worry whether the flux backreaction will drive the system away from the Ricci flat internal G2 space. It is believed that orientifolds ameliorate this issue to some extend as we explain below. Orientifolds are anyhow needed to cancel the RR tadpoles induced by the fluxes and also to circumvent certain nogo arguments for having Minkowski solutions or scale-separated AdS solutions[2] [25].

The negative tension of orientifold planes can help in keeping the deformation away from the Ricci flat manifold (G2, Calabi–Yau) under control. This is best seen when the orientifolds are smeared over the internal dimensions as to reflect the course graining over distances smaller than the KK scale. Smeared orientifolds provide the necessary negative energy/momentum in order to cancel the positive energy/momentum of the fluxes, thus providing a well behaved "solution" with a flat internal space [65–67].

Of course orientifolds are localized objects in string theory and if one wants a more sensible solution that can be probed at distances smaller than the Kaluza-Klein scale one needs to find 10d supergravity solutions with orientifold singularities. This can be done explicitly for flux solutions that do not involve intersecting planes, but only parallel ones [65, 67, 68]. Such solutions have internal manifolds that differ from the smeared solution, and furthermore have a warp factor in front of the external space metric. Nonetheless the backreaction of the orientifolds is mild since the alteration of the internal metric can be described in terms of a conformal factor multiplying the space transversal to the wrapped planes[3]. From the 10d equations one can then verify that derivatives of the warp and conformal factor are crucial in canceling the flux energy/momentum. Nonetheless the smeared solution is well approximated away from the sources, especially in the limit of weak coupling and large volume [69]. This can be understood from the fact that those limits dilute the fluxes sufficiently such that the original flat space Ansatz was sensible. This connection between smeared and localized solutions is not proven for flux vacua with intersecting sources but there is some recent evidence that points towards it [70, 71].

Crucially, the low energy effective field theories that are commonly used for flux compactifications do not take into account warping and other backreaction effects, so they effectively probe the smeared orientifold solutions. This is not strange and just means that the EFT course grains over distances smaller than the KK scale. There are constructions however which do take it into account and go under the name of *warped effective field theory*; see [72–78] for a biased sample of papers on the topic.

Four-dimensional flux vacua of interest to phenomenology that are obtained within

[2] Scale separation here means a decoupling of the KK scale from the AdS scale [26].

[3] To be more precise: parallel branes and planes wrap a fibre of a bundle. The conformal factor of the base equals the warp factor and the conformal factor of the fibre equals the inverse in string frame.

the classical realm of 10d supergravity with Dp-brane and Op-plane sources come in two kinds: 1) there are the no-scale Minkowski vacua in Type IIB from three-form fluxes and their T-duals [65, 79, 80] or 2) the scale-separated AdS vacua in Type IIA [17, 81] and their Type IIB cousins [27, 82]. Whereas the no-scale Minkowski solutions tend to feature parallel sources and their 10d description in terms of localized orientifolds is more or less understood, the same does not hold for the scale-separated AdS vacua, which always feature intersecting sources. For the latter we are also not aware of an EFT improvement, like warped effective field theory, to describe the backreaction effects in the scale-separated AdS vacua. Although as we mentioned earlier there is recent progress towards justifying the smeared approach [70, 71].

In this chapter we follow the "standard" strategies used in compactifications to 4d: we are led to find vacua in 3d using fluxes and sources as described directly in the 3d supergravity. Similarly to 4d, we find the two classes: no-scale Minkowski vacua and scale-separated AdS vacua, and again it is only clear for the no-scale vacua how to "backreact" the orientifolds properly and find 10d solutions.

4.2 Fluxes, sources and G2 spaces

Before we dive into concrete flux compactifications we wish to understand what the possibilities are, insisting that the internal manifold is of *G2 holonomy*. Note that the expected 3d supergravity theories will only have 2 real supercharges. The reason is that the orbifold Γ group already restricts the 32 supercharges of Type IIA theories to 4 real supercharges and then the O2-planes which are allowed in our setup further cut them in half:

$$\text{Type IIA supercharges}: \quad 32 \quad \xrightarrow{\Gamma \text{ orbifold}} \quad 4 \quad \text{real} \quad \xrightarrow{\text{O2-plane}} \quad 2 \quad \text{real} \tag{4.2.1}$$

To list the possibilities we work in the democratic formalism for the fluxes and then take all fluxes to be internal (also called magnetic), without loss of generality. For instance an electric H_3 flux is then simply described by magnetic H_7 flux. All fluxes we can have are

$$\text{Type IIA}: \; H_3, H_7, F_0, F_2, F_4 \,,$$

where we used that a G2 space has no one- or six-cycles and we assume that fluxes are closed but non-exact.

For reasons explained in the first chapters, we make sure that the Op-plane charges in the compactification are always cancelled by fluxes and not only by Dp-branes. So we will ignore Dp-branes in what follows, although they will eventually be impossible to avoid in some of our concrete models. The relevant Bianchi identities and flux equations of motion are summarized by

$$\mathrm{d}F_{8-p} = H_3 \wedge F_{6-p} + \delta_{\mathrm{O}p} \,. \tag{4.2.2}$$

The possible planes that can be used, bearing in mind there are no one- or six-cycles, are

$$\begin{aligned} \text{Type IIA} &: \; \mathrm{O2}, \mathrm{O4}, \mathrm{O6}\,, \\ \text{Type IIB} &: \; \mathrm{O5}, \mathrm{O7}\,. \end{aligned}$$

The charges of O7 or O5-planes can never be cancelled by fluxes. For O7-planes this is obvious and for O5-planes we have to realize that the Bianchi identity requires non-zero F_1 flux, which cannot be present because of a lack of one-cycles. This rules out Type IIB and G2 holonomy altogether and we are now left with Type IIA.

We ignore O4-planes all together since they wrap two-cycles and G2 has calibrated two-cycles, but they are never supersymmetric [62]. We thus have O2 and O6-planes left. Consider an O2-plane filling the 3d space. The parities of the fluxes are

$$\begin{aligned} \text{even} &: \; F_0, F_4, H_7\,, \\ \text{odd} &: \; F_2, H_3\,. \end{aligned}$$

But there are no odd two-forms in the space transversal to the O2-plane, neither even seven-forms, so H_7 and F_2 are removed. Then, in absence of O6-planes we also have to remove F_0 because of the F_2 Bianchi identity (note that we do not want to eliminate H_3 since then we cannot cancel the O2-plane tadpole), which brings us to

$$\text{O2 allowed flux:} \quad F_4, H_3\,.$$

The solutions in this model have been considered earlier in [67] and are of the no-scale Minkowski type where $\star_7 H_3 \sim F_4$. If the internal space would be $S^1 \times CY_6$ this would be T-dual to the well-known 4d Minkowski solutions in Type IIB with three-form fluxes. The "solutions" described in [67] are completely general and only the conditions to solve the 10d equations were stated. Neither supersymmetry nor the moduli problem was treated. One of the aims of this thesis is to fill this gap and construct the 3d supergravity in case the internal manifold is G2.

Since space-filling O2-planes can intersect space-filling O6-planes in a supersymmetric manner, we can consider their combination. Then the above reasoning goes through but without removing the F_0 flux. So we have

$$\text{O2/O6 allowed flux:} \quad F_0, F_4, H_3\,. \tag{4.2.3}$$

Imagine we want to have an O6-plane and cancel its tadpoles with fluxes and that we do not want an O2-plane. O6-planes wrap four-cycles inside the G2 space that need to be calibrated in a supersymmetric manner if one wants to achieve a 3d supergravity description. Note that $H_3 \wedge F_2$ and $H_3 \wedge F_4$ then have to vanish from the Bianchi identities. From the tadpole cancellations we get $F_2 = 0$, whereas we can keep $F_4 \neq 0$ as long as it wedges to zero with the H_3, which brings us to

$$\text{O6 allowed flux:} \quad F_0, F_4, H_3\,, \quad F_4 \wedge H_3 \equiv 0\,. \tag{4.2.4}$$

To summarize, assuming G2 holonomy we could prove that the only models in the market are O2, O6-planes and possibly O2/O6-planes together.

4.2.1 Brief summary of minimal 3d supergravity

Here we list few basic aspects of three-dimensional N=1 supergravity and later we will present the specific theory of our interest. More details can be found in Appendix 8.1.

The bosonic sector of N=1 supergravity in 3d has a metric field, real scalar fields ϕ^I and (abelian) vectors $A^{(A)}$, which can be dualized to scalars. The bosonic part of the general action is

$$e^{-1}\mathcal{L} = \frac{1}{2}R - g^{\mu\nu}G_{IJ}(\phi)\partial_\mu\phi^I\partial_\nu\phi^J - \frac{1}{4}f(\phi)F^{(A)}_{\mu\nu}F^{\mu\nu(A)} - V(\phi)\,, \tag{4.2.5}$$

with

$$V(\phi) = G^{IJ}P_IP_J - 4P^2\,. \tag{4.2.6}$$

We name the function P the real superpotential and P_I is shorthand for $\partial_I P$. The gauge kinetic function $f(\phi)$ is real but otherwise unrestricted. From here onwards G^{IJ} is the inverse of the target space metric G_{IJ}.

On the fermion side we have the gravitino ψ_μ, the Majorana spinors χ^I which are the superpartners of the real scalars ϕ^I, and the superpartners of the vectors (gaugini) which are also Majorana and denoted $\lambda^{(A)}$. We need the supersymmetry variations in order to understand the supersymmetry invariance of flux solutions

$$\begin{aligned} \delta\psi_\mu\Big|_{\text{shift}} &= \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\epsilon - P\gamma_\mu\epsilon\,, \\ \delta\chi^I\Big|_{\text{shift}} &= G^{IJ}P_J\epsilon\,, \\ \delta\lambda^{(A)}\Big|_{\text{shift}} &= 0\,, \end{aligned} \tag{4.2.7}$$

where ϵ is the 2-component fermionic Majorana local supersymmetry parameter.

4.3 The no-scale model

In this section we discuss the "no-scale" type backgrounds obtained from space-filling O2-planes whose tadpole is cancelled by a combination of F_4 and H_3 fluxes that obey the proportionality rule $H_3 \sim \star F_4$. The 10d description of such backgrounds has appeared in [67], but without any details; rather, it was only explained what is needed to solve the 10d equations of motion. Here we discuss how to obtain the effective field theory in 3d when warping can be sufficiently ignored. These backgrounds are inspired from T-duality of the GKP backgrounds in Type IIB [79].

Because we compactify on G2 spaces we expect maximally four of the original 32 supercharges to be preserved. The O2-plane further reduces the number of supersymmetries in half (i.e. two supercharges remain) by essentially truncating the spectrum. As a result the low energy theory will be a 3d N=1 supergravity. Our interest here is in finding the exact low energy description of such theory by matching the 3d objects to the 10d data. We first start with a general discussion of 3d minimal supergravity.

4.3.1 A 10d view on the effective theory

We will work with Type IIA supergravity with space-filling O2-planes and we will have only non-vanishing F_4 and H_3 fluxes as dictated by

$$\mathrm{d}F_6 = H_3 \wedge F_4 + Q_{\mathrm{O2}}\delta_7\,, \tag{4.3.1}$$

where Q_{O2} is the negative O2-plane charge. The relevant bosonic part of the 10d action in 10d Einstein frame is

$$S = \int_{10} \left(\star_{10} R - \tfrac{1}{2} e^{-\phi} \star_{10} H_3 \wedge H_3 - \tfrac{1}{2} e^{\phi/2} \star_{10} F_4 \wedge F_4\right) + e^{-\phi/4} T_{\mathrm{O2}} \int_{\mathrm{O2}} \sqrt{|g_3|}\,. \tag{4.3.2}$$

The O2-plane fills the external space, has its world-volume perpendicular to the seven dimensional internal G2 space on which we compactify on. From the 10d action one can readily find the 10d form of the scalar potential in 3d as we now show. A direct dimensional reduction gives the following 10d expression for the 3d potential V_{3d} given by

$$V = \int_7 \left(\tfrac{1}{2} e^{-\phi} \star_7 H_3 \wedge H_3 + \tfrac{1}{2} e^{\phi/2} \star_7 F_4 \wedge F_4 - e^{-\phi/4} T_{\mathrm{O2}} \frac{\epsilon_7}{\mathrm{vol}_7}\right)\,, \tag{4.3.3}$$

where now the stars are 7d Hodge stars and ϵ_7 is the 7d volume form. Because the O2-planes are BPS objects their tension and charge are related as $T_{\mathrm{O2}} = \pm Q_{\mathrm{O2}}$ where the minus sign refers to anti-O2-planes. That charge, however, can be obtained from the Bianchi identity (4.3.1), or equivalently from the RR tadpole condition

$$-\int_7 H_3 \wedge F_4 = Q_{\mathrm{O2}} = Q_{\mathrm{O2}} \int_7 \frac{\epsilon_7}{\mathrm{vol}_7}\,. \tag{4.3.4}$$

As a result one can replace the charge of the source in the action and the effective potential becomes

$$V_{\mathrm{3d}} = \int_7 \left(\tfrac{1}{2} e^{-\phi} \star_7 H_3 \wedge H_3 + \tfrac{1}{2} e^{\phi/2} \star_7 F_4 \wedge F_4 \mp e^{-\phi/4} H_3 \wedge F_4\right)\,, \tag{4.3.5}$$

which can be recast in a form that is a manifest total square[4]

$$V_{\mathrm{3d}} = \frac{1}{2} \int_7 \sqrt{g_7} \left(e^{-\phi/2} H_3 \mp e^{\phi/4} \star_7 F_4\right)^2\,. \tag{4.3.6}$$

One of the $\pm$ signs corresponds to O2-planes while the other to anti-O2-planes. Since the scalar potential is a total square, the vacuum must live at configurations for which the square vanishes, that is

$$H_3 \mp e^{3\phi/4} \star_7 F_4 = 0\,, \tag{4.3.7}$$

and therefore the solutions are Minkowski. Whether supersymmetry is present or not is discussed below.

[4] In our notation the square of a form is defined as: $\sqrt{g} F^2 \mathrm{d}^7 x = \star F \wedge F = \sqrt{g} \frac{1}{p!} F_{\mu_1 \cdots \mu_p} F^{\mu_1 \cdots \mu_p} \mathrm{d}^7 x$

Let us now discuss the spectrum of bosonic fields in 3d. The graviton of course is given directly by the dimensional reduction of the 10d metric tensor, so let us now focus on what can be the scalars of the 3d theory. Recall that in going from 11d to 4d on a G2, see e.g. [61], the scalars come from the metric and the gauge three-form C of the 11d supergravity, which are decomposed as

$$C = c^i \Phi_i + \dots \ , \quad \Phi = s^i \Phi_i \ , \tag{4.3.8}$$

where the s^i and the c^i are real scalar moduli. The real scalars combine into complex holomorphic moduli as: $z^i = c^i + is^i$. The low-energy theory is standard 4d N=1 supergravity and the kinetic terms of this sector are

$$\mathcal{L}_{kin.} = -g_{i\bar{j}} \partial_\mu z^i \partial^\mu \bar{z}^{\bar{j}} \ , \tag{4.3.9}$$

where the Kähler metric is given by

$$g_{i\bar{j}} = \frac{1}{4} \mathrm{vol}(X_7)^{-1} \int_{X_7} \Phi_i \wedge \star \Phi_j \ . \tag{4.3.10}$$

Let us now turn to compactifications of Type IIA to three dimensions. At first sight we would expect to have moduli from C_3, C_1 and B_2. However, since there are no one-cycles, there are no scalars coming from reducing C_1. There is also no scalar coming from dualizing the vector C_1 to a scalar in 3d since the vector is projected out by the O2-plane, thus the dilaton also remains a genuine real scalar. Indeed, without the orientifold, that dualized vector would have paired up with the dilaton to a complex field. Moreover there are no scalars coming from C_3 either, again because of the O2-plane projection since all three-cycles are necessarily odd whereas C_3 is even under O2-plane Eq.(2.2.26). There are furthermore no scalars from reducing B_2 over two-cycles because there are no odd two-cycles for an O2-plane here. There are however even two-cycles and reducing C_3 over them gives vectors in 3d that dualize to scalars. These are actually axions with compact field ranges. Without the O2-plane projections they would have paired up with the scalars from B_2 to give complex fields. We therefore find that exactly those scalars that would have paired up with the metric scalars s^i to give complex scalars are absent consistent with the real formulation of minimal 3d supergravity.

To sum it up, all scalars come from the metric (ignoring for the moment the dilaton) which one obtains from expanding Φ over a basis of harmonic three-forms Eq.(3.2.5). So we have b_3 amount of metric scalars s^i and b_2 axions from C_3 expanded along harmonic two-forms. There can also be scalars corresponding to D2 or D6 positions if D2- or D6-branes are needed for tadpole cancellation. Also such scalars will have compact field ranges.

Our fluxes F_4 and H_3 can give masses to the metric scalars but not to the axions and brane positions, which instead should get a mass from quantum effects. However, we will find that a linear combination of the volume and the dilaton necessarily remains massless in the Minkowski vacua where only (4.3.7) holds.

Indeed, at this point we can readily study the generic formulas that arise from our discussion, and ask how many moduli can be stabilized from imposing (4.3.7). In the spirit of this section we use generic G2 ingredients instead of restricting to specific models. We define

$$G_{ij} = \frac{7}{4}\frac{\int \Phi_i \wedge \star\Phi_j}{\int \Phi \wedge \star\Phi} = \frac{1}{4}\mathrm{vol}(X_7)^{-1}\int \Phi_i \wedge \star\Phi_j \;, \tag{4.3.11}$$

where we used the volume relation in (3.1.3). The bilinear form G_{ij} becomes a metric on the moduli space. We will expand on that in the next subsection. Then the expansion of the $\star_7\Phi_i$ in the four-form basis (3.2.10) generically can be expressed as

$$\star\Phi_i = B_i{}^l(s)\,\Psi_l\,, \tag{4.3.12}$$

and by taking the wedge product with Φ_j and using the orthonormality in (3.2.10) we find

$$B_i{}^l(s) = 4\mathrm{vol}(X_7)G_{ij}\delta^{jl}\,, \tag{4.3.13}$$

which means that the moduli-dependent coefficients $B_i{}^l(s)$ are completely specified by the geometry. Note that $B_i{}^l(s)$ is a $b_3(X) \times b_3(X)$ invertible matrix.

We expand the fluxes as

$$F_4 = f^i\Psi_i \;, \quad H_3 = h^i\Phi_i\,, \tag{4.3.14}$$

with f^i and h^i flux quanta that should be properly quantized. If we insert them into (4.3.7) together with (4.3.12) and (4.3.13) we find

$$h^i B_i{}^k\Psi_k = \pm e^{3\phi/4} f^k\Psi_k \;\rightarrow\; h^i B_i{}^j(s) = \pm e^{3\phi/4} f^j\,. \tag{4.3.15}$$

We see that we have in principle b_3 conditions that can thus fix up to b_3 moduli, which are essentially all the s^i. We will show below that a linear combination of dilaton and volume remains free and has to be fixed by different mechanisms. The flux parameters f^i and h^i are a total of $2b_3 - 1$ independent parameters because of the tadpole condition (4.3.4) which requires

$$f^i\delta_{ij}h^j = -Q_{\mathrm{O2}}\,. \tag{4.3.16}$$

One can therefore expect that all of the s^i will be fixed in terms of the dilaton because there is a large freedom in choosing the fluxes. Of course for special values of the f^i and h^i not all s^i will be fixed.

4.3.2 The 3d supergravity effective theory

Let us now turn to the specific low-energy supergravity theory we want to study. For compactifications of Type IIA on Calabi–Yau orientifolds with fluxes the 4d N=1 supergravity theory was constructed in [83], here we perform a similar investigation for Type IIA on G2 orientifolds. Since we readily have the scalar potential given by (4.3.6), we need also the kinetic terms in order to match the 3d supergravity with the compactified effective theory. In 10d Einstein frame, the reduction Ansatz for the metric is

$$\mathrm{d}s_{10}^2 = e^{2\alpha v}\mathrm{d}s_3^2 + e^{2\beta v}\widetilde{\mathrm{d}s}_7^2\,, \tag{4.3.17}$$

where v is a 3d scalar that accounts for the compactification volume and hence $\widetilde{\mathrm{d}s}_7^2$ is the metric on a unit-volume G2 space. With the specific choice of numbers $\alpha^2 = 7/16$ and $-7\beta = \alpha$ we find canonical kinetic terms in 3d

$$e^{-1}\mathcal{L} = R_3 - \tfrac{1}{2}(\partial v)^2 - \tfrac{1}{2}(\partial\phi)^2 - V_{3\text{d}}\,. \tag{4.3.18}$$

Taking now into account the volume scalings, the expression for the potential (4.3.6) takes the form

$$V_{3\text{d}} = \frac{1}{2}\int_7 \sqrt{\tilde{g}_7}e^{-21\beta v}\,\left(e^{-\phi/2}e^{\beta v/2}H_3 \mp e^{-\beta v/2}e^{\phi/4}\tilde{\star}_7 F_4\right)^2\,, \tag{4.3.19}$$

where $\tilde{g}_7$ is the unit-volume metric $\mathrm{vol}(\tilde{X}_7) = 1$. Finally we consider the following *orthonormal* redefinition of scalars[5]

$$\frac{x}{\sqrt{7}} = -\frac{3\phi}{8} + \frac{\beta}{2}v\,, \qquad 2y = -21\beta v - \frac{1}{4}\phi\,, \tag{4.3.20}$$

and the action takes the form

$$e^{-1}\mathcal{L} = R_3 - \tfrac{1}{2}(\partial x)^2 - \tfrac{1}{2}(\partial y)^2 + \ldots - V_{3\text{d}}\,, \tag{4.3.21}$$

where the ... denote kinetic terms for all other geometric scalars and

$$V_{3\text{d}} = \frac{1}{2}\int_7 \sqrt{\tilde{g}_7}e^{2y}\,\left(e^{\frac{x}{\sqrt{7}}}H_3 \mp e^{-\frac{x}{\sqrt{7}}}\tilde{\star}_7 F_4\right)^2\,. \tag{4.3.22}$$

In the rest of this section we will derive the effective 3d supergravity theory that gives rise to this action, meaning we fill in the dots of equation (4.3.21). Note already that y is the massless "no-scale" direction we mentioned earlier and it indeed corresponds to a linear combination of dilaton ϕ and volume v.

Let us also establish the normalization of the kinetic terms for the volume-preserving fluctuations, as it will be helpful to cross-check our results later. For that we consider a single extra fluctuation, say z, of a unit-volume torus that serves as a proxy for our G2 internal space. We will also keep the volume modulus v in our discussion because we want to keep track of the relative normalizations. We have for the unit-volume metric

$$\widetilde{\mathrm{d}s}_7^2 = e^{2\xi z}[\mathrm{d}y_1^2 + \mathrm{d}y_2^2 + \mathrm{d}y_7^2] + e^{2\delta z}[\mathrm{d}y_4^2 + \mathrm{d}y_5^2 + \mathrm{d}y_6^2 + \mathrm{d}y_3^2]\,, \tag{4.3.23}$$

where unit volume implies $3\xi = -4\delta$. Direct dimensional reduction of the 10d Einstein–Hilbert term gives

$$\int_3 \sqrt{-g_3}\left(R_3 - \tfrac{1}{2}(\partial v)^2 - \tfrac{21}{4}\xi^2(\partial z)^2\right)\,, \tag{4.3.24}$$

[5] For later convenience we also note that $\phi = -\frac{3\sqrt{7}}{8}x - \frac{1}{8}y$ and $7\beta v = \frac{\sqrt{7}}{32}x - \frac{21}{32}y$.

therefore we set $\xi^2 = 2/21$ to have canonical kinetic terms. We will use this normalization as a way to double check later our kinetic terms for the v the s^i moduli. We can express the relevant part of the G2 form, keeping only the modulus z while freezing the rest, as

$$\tilde{\Phi} = e^{3\xi z} dy^{127} + \dots\,, \quad \tilde{\star}\tilde{\Phi} = e^{4\delta z} dy^{3456} + \dots\,, \tag{4.3.25}$$

where $\mathrm{d}y^{127} = \mathrm{d}y^1 \wedge \mathrm{d}y^2 \wedge \mathrm{d}y^7$ and $\mathrm{d}y^{3456} = \mathrm{d}y^3 \wedge \mathrm{d}y^4 \wedge \mathrm{d}y^5 \wedge \mathrm{d}y^6$. If we wish to generate this term from a general formula using the Φ-form we find that it comes from

$$e^{-1}\mathcal{L}_{\rm kin} = -\frac{1}{2}\mathrm{vol}(X_7)^{-1}\int_7 \Phi_i \wedge \star\Phi_j \partial s^i \partial s^j\,. \tag{4.3.26}$$

This is consistent with the kinetic term one derives in going from 11d to 3d [61][6]. But there is a small subtlety compared with the literature on G2 compactifications from 11d. For that, let us use that

$$s^i = e^{3\beta v}\tilde{s}^i \tag{4.3.27}$$

with $\tilde{s}^i$ the fluctuations for unit-volume spaces. Equation (4.3.26) contributes the following piece, $-\frac{9}{32}(\partial v)^2$, to the kinetic term for the volume. Hence we deduce that the full kinetic term should be

$$\begin{aligned} e^{-1}\mathcal{L}_{\rm kin} =& -\frac{1}{2}(\partial\phi)^2 - \frac{7}{32}(\partial v)^2 - \frac{1}{2}\mathrm{vol}(X_7)^{-1}\int_7 \Phi_i \wedge \star\Phi_j \partial s^i \partial s^j \\ =& -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial v)^2 - \frac{1}{2}\mathrm{vol}(\tilde{X}_7)^{-1}\int_7 \Phi_i \wedge \tilde{\star}\Phi_j \partial \tilde{s}^i \partial \tilde{s}^j\,. \end{aligned} \tag{4.3.28}$$

One can verify that in contrast to our situation, in going from 11d to 4d, the contribution of (4.3.26) to the volume kinetic term is complete. Notice also that in the second line of (4.3.28) we have used the tilde notation to refer to the G2 expressions that instead of the s^i make use of their unit-volume counter-parts, the $\tilde{s}^i$. For the later in fact it holds that $\mathrm{vol}(\tilde{X}_7) = 1$, but we have chosen to keep the expressions with the unit volume manifest to avoid any confusion.

From here onwards we change to different normalizations to make contact with the literature on 3d gravity. This we can simply do by rescaling the 3d metric as $g_{\mu\nu} \to \frac{1}{4}g_{\mu\nu}$, such that we end with

$$e^{-1}\mathcal{L}_{\rm kin} = \tfrac{1}{2}R_3 - \tfrac{1}{4}(\partial x)^2 - \tfrac{1}{4}(\partial y)^2 - \tfrac{1}{4}\mathrm{vol}(\tilde{X}_7)^{-1}\int_7 \Phi_i \wedge \tilde{\star}\Phi_j \partial\tilde{s}^i \partial\tilde{s}^j\,. \tag{4.3.29}$$

In the language of subsection 4.2.1 we have the real scalars $\phi^I = x, y, \tilde{s}^i$. The x and y are a combination of the volume modulus and of the dilaton, whereas the $\tilde{s}^i$ are moduli for the

[6]There is a 1/2 factor different from [61] but can be understood from the different normalization of the Einstein Hilbert term.

G2 metric deformations (but not volume) and the sigma model metric of (4.2.5) is given by

$$G_{IJ} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & G_{ij} \end{bmatrix} , \tag{4.3.30}$$

with G_{ij} defined earlier in (4.3.11) where now all s^i are replaced with $\tilde{s}^i$. But note that this is overcounting the degrees of freedom since the above notation seems to imply that the $\tilde{s}^i$ are independent whereas they should multiply to a fixed number as they describe the unit-volume G2 space - indeed we know that after all $\mathrm{vol}(\tilde{X}_7) = 1$. In what follows we will demonstrate that we can pretend the $\tilde{s}^i$ to be independent and at the very end use that their product equals one. This is fully consistent with all our calculations in the bosonic sector and also with supersymmetry as we shown in the appendix.

To fix the 3d supergravity theory we need to find the real superpotential function P, which will inform us about the supersymmetry shifts via equation (4.2.7). Instead of doing a full derivation using 10d supersymmetry rules, we will guess the answer for P and verify that indeed P leads to the scalar potential (4.3.22) we derived already from 10d, through equation (4.2.6). The answer for P can be guessed in analogy with existing superpotentials for flux compactifications to be

$$P = \frac{e^y}{8} \left[\gamma e^{x/\sqrt{7}} \int \star\Phi \wedge H_3 \, \mathrm{vol}(X_7)^{-\frac{4}{7}} + e^{-x/\sqrt{7}} \int \Phi \wedge F_4 \, \mathrm{vol}(X_7)^{-\frac{3}{7}} \right] . \tag{4.3.31}$$

The number γ will turn out to be ± 1, depending on whether we look at theories with O2 planes or anti-O2-planes. The reader may notice that we have used the s^i in (4.3.31) instead of the $\tilde{s}^i$, however, it is easy to check that $P(s^i) \equiv P(\tilde{s}^i)$. When we take derivatives of P with respect to $\tilde{s}^i$, i.e. P_i, we will of course use the $P(\tilde{s}^i)$ version.

Now we want to evaluate the scalar potential. As we explained the 3d supergravity built from G_{IJ} and P will contain one additional degree of freedom because of the double counting of the volume that will in any case appear in the kinetic terms and from the one additional fermionic field. For the moment we ask the reader to bear with the extra degree of freedom until we show that it can be fixed consistently. So our approach here is to simply verify that our choice of P leads to the correct scalar potential derived from 10d.

We first note that y is special since

$$(G_{yy})^{-1} P_y P_y - 4P^2 = 0 \, , \tag{4.3.32}$$

and as a result the scalar potential takes the from

$$V = G^{ij} P_i P_j + (G_{xx})^{-1} P_x P_x \; . \tag{4.3.33}$$

This is what we call no-scale in 3d. First we evaluate P_x which gives

$$P_x = \frac{\partial P}{\partial x} = -\frac{1}{\sqrt{7}} \frac{e^y}{8} \int_7 \mathrm{vol}(\tilde{X}_7)^{-\frac{3}{7}} \, \tilde{\Phi} \wedge \mathcal{F}_4 \; , \tag{4.3.34}$$

where we remind the reader that we use the notation $\tilde{\Phi} = \Phi_i \tilde{s}^i$, etc. and

$$\mathcal{F}_4 = e^{-x/\sqrt{7}} F_4 - \gamma e^{x/\sqrt{7}} \tilde{\star} H_3 \, \text{vol}(\tilde{X}_7)^{-\frac{1}{7}} \; . \tag{4.3.35}$$

Therefore we find

$$(G_{xx})^{-1} P_x P_x = \frac{4}{7} \left(\frac{e^y}{8} \right)^2 \left(\int_7 \text{vol}(\tilde{X}_7)^{-\frac{3}{7}} \; \tilde{\Phi} \wedge \mathcal{F}_4 \right)^2 \; . \tag{4.3.36}$$

Now we want to evaluate the P_i. To do this we need a series of properties that we now list. The derivatives with respect to $\tilde{s}^i$ are defined as

$$\frac{\partial}{\partial \tilde{s}^i} \Phi = \Phi_i \; , \quad \frac{\partial}{\partial \tilde{s}^i} (\tilde{\star} \tilde{\Phi}) = \frac{4}{3} \tilde{\star} \pi^1(\Phi_i) - \tilde{\star} \pi^{27}(\Phi_i) \; , \quad \partial_i \text{vol}(\tilde{X}_7) = \frac{1}{3} \int_7 \Phi_i \wedge \tilde{\star} \tilde{\Phi} \; , \tag{4.3.37}$$

where the π^1 and π^{27} are projections to irreducible G2 representations defined for instance in [61]. They obey the orthogonality properties

$$\tilde{\star} \pi^1(\Phi_i) \wedge \pi^{27}(\Phi_j) = 0 \; , \quad \tilde{\star} \pi^1(\Phi_i) \wedge \pi^1(\Phi_j) + \tilde{\star} \pi^{27}(\Phi_i) \wedge \pi^{27}(\Phi_j) = \Phi_i \wedge \tilde{\star} \Phi_j \, . \tag{4.3.38}$$

In practice $\pi^1(\Phi_i)$ is defined as

$$\pi^1(\Phi_i) = \left(\frac{\int \Phi_i \wedge \tilde{\star} \tilde{\Phi}}{\int \tilde{\Phi} \wedge \tilde{\star} \tilde{\Phi}} \right) \tilde{\Phi} \, , \tag{4.3.39}$$

therefore we can simplify our expressions such that only $\tilde{\Phi}$ or Φ_i appear instead of $\pi^1(\Phi_i)$. Moreover, the $\pi^{27}(\Phi_i)$ can be also traded for $\tilde{\Phi}$ and Φ_i by using the manipulations

$$\pi^{27}(\Phi_i) \wedge B = \Phi_i \wedge B - \pi^1(\Phi_i) \wedge B = \Phi_i \wedge B - \left(\frac{\int \Phi_i \wedge \tilde{\star} \tilde{\Phi}}{\int \tilde{\Phi} \wedge \tilde{\star} \tilde{\Phi}} \right) \tilde{\Phi} \wedge B \, . \tag{4.3.40}$$

To prove this relation one has to consider that the four-form B can be expanded as $B = \tilde{\star} \Phi_j B^j$. Within this setup one can prove also that for the four-forms A and B we have

$$G^{ij} \int \Phi_i \wedge A \int \Phi_j \wedge B = \frac{4}{7} \int \tilde{\Phi} \wedge \tilde{\star} \tilde{\Phi} \int \tilde{\star} A \wedge B \, , \tag{4.3.41}$$

which can be checked by expanding both $A = \tilde{\star} \Phi_i A^i$ and $B = \tilde{\star} \Phi_i B^i$. Notice that $G^{ij} \int \tilde{\star} \Phi_i \wedge \tilde{\Phi} \int \tilde{\star} \Phi_j \wedge \tilde{\Phi} = \frac{4}{7} \left(\int \tilde{\Phi} \wedge \tilde{\star} \tilde{\Phi} \right)^2$. We can then evaluate the derivative of P with respect to $\tilde{s}^i$ to be given by

$$P_i = \frac{e^y}{8} \left[\int \Phi_i \wedge \mathcal{F}_4 \, \text{vol}(\tilde{X}_7)^{-\frac{3}{7}} - \left(\frac{\int \tilde{\star} \Phi_i \wedge \tilde{\Phi}}{\int \tilde{\star} \tilde{\Phi} \wedge \tilde{\Phi}} \right) \int \tilde{\Phi} \wedge \mathcal{F}_4 \, \text{vol}(\tilde{X}_7)^{-\frac{3}{7}} \right] . \tag{4.3.42}$$

Then using the form of P_i and the aforementioned properties of the G2 three-form we find

$$G^{ij}P_iP_j = 4\left(\frac{e^y}{8}\right)^2 \int \mathcal{F}_4 \wedge \tilde{\star}\mathcal{F}_4 \,\mathrm{vol}(\tilde{X}_7)^{\frac{1}{7}} - \frac{4}{7}\left(\frac{e^y}{8}\right)^2 \left(\int \mathrm{vol}(\tilde{X}_7)^{-\frac{3}{7}}\,\tilde{\Phi}\wedge\mathcal{F}_4\right)^2 . \quad (4.3.43)$$

We notice that the second term of this equation exactly corresponds to minus the expression in equation (4.3.36). Hence, from the no-scale structure (4.3.33) we finally find

$$V = \frac{e^{2y}}{16}\int_7 \mathcal{F}_4 \wedge \tilde{\star}\mathcal{F}_4 \,\mathrm{vol}(\tilde{X}_7)^{\frac{1}{7}} \,, \quad (4.3.44)$$

which can then be rewritten as

$$V = \frac{e^{2y}}{16}\int \sqrt{\tilde{g}_7}\left(e^{-x/\sqrt{7}}F_4 - \gamma e^{x/\sqrt{7}}\tilde{\star}H_3\right)^2 . \quad (4.3.45)$$

This almost concludes our proof, since, up to a factor of $1/8$ we reproduce the 10d potential (4.3.22). The extra $1/8$ factor is due to the rescaling of the metric mentioned around equation (4.3.29).

One issue remains; the double-counting of the volume modulus, in the sense that we have been working with one scalar too much as we never enforced that the $\tilde{s}^i$ should describe fluctuations of the unit-volume G2 space.[7] The overall $\int\tilde{\Phi}\wedge\tilde{\star}\tilde{\Phi}$ fluctuation has to be eliminated and we would like to impose the constraint

$$\int \tilde{\Phi}\wedge\tilde{\star}\tilde{\Phi} = 7 \,, \quad (4.3.46)$$

which equivalently means, in case of a seven-torus, that the scalars $\tilde{s}^i$ would be restricted to satisfy

$$\prod_i^7 \tilde{s}^i = 1 \qquad \text{(for seven-torus)} . \quad (4.3.47)$$

Both equation (4.3.46) and (4.3.47) follow from each other in the case of a seven-torus and can be imposed on the final bosonic action to give us the correct scalar potential with the true degrees of freedom. However, at this point we have to be careful not to spoil supersymmetry, therefore we have to impose this constraint on the superfield level (or alternatively on the full multiplet level). In other words, the supersymmetry transformations have to respect the constraint (4.3.46). This in fact will reduce also the fermionic degrees of freedom by one. We leave the derivation of this technical point to Appendix 8.2.

[7] Let us point out that a similar situation does occur in compactifications of the Type IIB theory where one has $\int\Omega\wedge\overline{\Omega}\sim V||\Omega||$ as a result one may be double-counting the Calabi–Yau metric g_{mn} volume V. This however does not happen because the volume modulus is extracted from the metric and one has $\det[g_{mn}] = 1$.

Finally we note that, by introducing an arbitrary constant c, there is an infinite family of P-functions that gives rise to the same scalar potential V

$$P = \frac{e^y}{8}\left[\gamma e^{x/\sqrt{7}}\int \star\Phi\wedge H_3\,\mathrm{vol}(X_7)^{-\frac{4}{7}} + e^{-x/\sqrt{7}}\int \Phi\wedge F_4\,\mathrm{vol}(X_7)^{-\frac{3}{7}} + c\right]. \qquad (4.3.48)$$

It was noticed already in [84] that this is a generic way for finding "fake superpotential" in no-scale models. This way, any no-scale solution can be made supersymmetric in a different supergravity theory.

4.3.3 Open string moduli, axions, quantum corrections and uplifts

We have provided a first analysis of no-scale Minkowski vacua in 3d from flux compactifications. If our ultimate goal is full moduli stabilization then we need to take care of the axions, the D2/D6 moduli (if any) and the closed string y-modulus. Especially the y field is worrisome since it is the only one with a non-compact moduli space. In the next section we will stabilize the y field using further fluxes (Romans mass). But we could equally be tempted to parallel the history of moduli stabilization in 4D as pioneered in [16, 53, 85]. We furthermore could contemplate the further construction of de Sitter solutions from uplifting any AdS vacuum one obtains after fixing the y-modulus supersymmetrically (if at all possible). In what follows we merely mention the possible paths and difficulties for achieving this.

In 4D the massless no-scale moduli are potentially fixed in a controllable fashion through quantum effects, and most notably a leading non-perturbative correction to the superpotential that involves the no-scale direction [16][8]. But this approach does not seem feasible to us in our 3d models for a simple reason: we have minimal supergravity in 3d which does not come with holomorphic protection. Hence we expect no non-renormalization theorems to exist for perturbative corrections to the superpotential, neither holomorphic arguments to restrict the form of non-perturbative corrections. This seems to rime with the fact that there seem no possible supersymmetric wrappings of Euclidean D2- or D4-branes. Of course quantum effects induced by the strongly coupled gauge theories on multiple D6 branes wrapping calibrated four-cycles will be there. But they are not easily computable and we think it seems realistic that we are faced with a standard Dine–Seiberg problem [87].

Imagine there is nonetheless a computable AdS vacuum that is sufficiently weakly coupled and with high enough masses of the moduli such that some additional "mild" supersymmetry breaking does not immediately destabilize the vacuum. Then we could contemplate which supersymmetry breaking sources can provide an uplift to dS, if ever. In that respect it is interesting to realize that the analogue of Klebanov–Strassler throats does exist in such backgrounds and they were constructed in [88]. Anti-D2-branes are then the natural SUSY-breaking uplift ingredient for which a probe computation à la [89] would suggest the solutions can be metastable. However this probe computation has been refuted in [90], but that criticism in turn was argued to be essentially harmless because of

[8] Although see [86] for some interesting worries about the self consistency of this approach.

the arguments in [91–94], although other problems associated to anti-brane uplifting could persist as reviewed in [10].

4.3.4 Toroidal orientifolds

Let us now turn to the Op-planes allowed in out Type IIA setup. As we have seen we need to include O2-planes in our setup. To this end consider the target space part of the O2-plane action, denoted σ_{O2}, as the following $\mathbb{Z}_2$ involution

$$\sigma_{\text{O2}} : (y^1, \ldots, y^7) \to (-y^1, -y^2, -y^3, -y^4, -y^5, -y^6, -y^7)\,. \tag{4.3.49}$$

The σ_{O2} has 2^7 fixed points, or alternatively different O2-plane sources, in the torus covering space. For the periodic condition of the torus in Eq.(3.2.3), they sit at the points $y^i = 0, 1/2$. Notice that the calibration is odd under the O2-plane involution

$$\sigma_{\text{O2}} : \Phi \to -\Phi\,, \tag{4.3.50}$$

and that the Γ and the σ_{O2} commute. We want the orbifold image of an O2-plane to be again some physical object, and as we will see it is an O6-plane. For that it is sufficient to consider the combination of the Γ involutions with the σ_{O2}

$$\sigma_\alpha = \Theta_\alpha \sigma_{\text{O2}} : (y^1, \ldots, y^7) \to (y^1, y^2, y^3, y^4, -y^5, -y^6, -y^7)\,, \tag{4.3.51}$$
$$\sigma_\beta = \Theta_\beta \sigma_{\text{O2}} : (y^1, \ldots, y^7) \to (y^1, y^2, -y^3, -y^4, y^5, y^6, -y^7)\,, \tag{4.3.52}$$
$$\sigma_\gamma = \Theta_\gamma \sigma_{\text{O2}} : (y^1, \ldots, y^7) \to (y^1, -y^2, y^3, -y^4, y^5, -y^6, y^7)\,. \tag{4.3.53}$$

These three involutions can be interpreted as intersecting O6-planes on the positions displayed in the Table 4.1.

	y^1	y^2	y^3	y^4	y^5	y^6	y^7
O6_α	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$-$	$-$	$-$
O6_β	$\otimes$	$\otimes$	$-$	$-$	$\otimes$	$\otimes$	$-$
O6_γ	$\otimes$	$-$	$\otimes$	$-$	$\otimes$	$-$	$\otimes$

Table 4.1: Indicative positions of a O6-planes filling the external space three-dimensional space (not displayed here) and wrapping the four-cycles Ψ_2,Ψ_3 and Ψ_7 in the internal space.

Here "$\otimes$" means the O6-plane world-volume contains these direction on the internal manifold and "$-$" means the O6-plane positions are localized at $y^i = 0, 1/2$ in that direction. These intersections are nicely consistent with the rules for preserving supersymmetry and this is no coincidence because the orbifold actions were chosen such as to preserve the G2 three-form.

One can explicitly verify the supersymmetric calibration of the O6-planes. According to [95], the condition is that a four-cycle is calibrated if and only if Φ restricted to it vanishes identically. This is the case as can be checked for each of the O6-planes. Consider for instance the planes in the directions spanned by e^1, e^2, e^3, e^4. Not a single component of Φ carries indices only in that subspace. The same for the other O6-planes.

Equivalent conditions are that $\star\Phi$ restricted to the O6-plane four-cycle equals exactly the volume form on it [95]. Yet another condition is that the source three-forms j_3 appearing in the F_2 Bianchi identity wedge to zero against Φ. There is an unambiguous way to find these source forms from the involution as explained in Appendix C of [96]. When applied to our case one finds for the involutions in Eq.(4.3.51-4.3.53)

$$j_\alpha = -e^{567}\,, \qquad j_\beta = -e^{347}\,, \qquad j_\gamma = e^{246}\,, \tag{4.3.54}$$

which speak for themselves. We find each time that $\Phi \wedge j_3 = 0$.

We are still not finished with the images however. We also have

$$\sigma_{\alpha\beta} = \Theta_\alpha\Theta_\beta\sigma_{O2} : (y^1, \ldots, y^7) \to (-y^1, -y^2, y^3, y^4, y^5, y^6, -y^7)\,, \tag{4.3.55}$$
$$\sigma_{\beta\gamma} = \Theta_\beta\Theta_\gamma\sigma_{O2} : (y^1, \ldots, y^7) \to (-y^1, y^2, y^3, -y^4, -y^5, y^6, y^7)\,, \tag{4.3.56}$$
$$\sigma_{\gamma\alpha} = \Theta_\gamma\Theta_\alpha\sigma_{O2} : (y^1, \ldots, y^7) \to (-y^1, y^2, -y^3, y^4, y^5, -y^6, y^7)\,, \tag{4.3.57}$$
$$\sigma_{\alpha\beta\gamma} = \Theta_\alpha\Theta_\beta\Theta_\gamma\sigma_{O2} : (y^1, \ldots, y^7) \to (y^1, -y^2, -y^3, y^4, -y^5, y^6, y^7)\,. \tag{4.3.58}$$

In total we have 7 different directions for O6-planes

	y^1	y^2	y^3	y^4	y^5	y^6	y^7
$\mathrm{O6}_\alpha$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$-$	$-$	$-$
$\mathrm{O6}_\beta$	$\otimes$	$\otimes$	$-$	$-$	$\otimes$	$\otimes$	$-$
$\mathrm{O6}_\gamma$	$\otimes$	$-$	$\otimes$	$-$	$\otimes$	$-$	$\otimes$
$\mathrm{O6}_{\alpha\beta}$	$-$	$-$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$-$
$\mathrm{O6}_{\beta\gamma}$	$-$	$\otimes$	$\otimes$	$-$	$-$	$\otimes$	$\otimes$
$\mathrm{O6}_{\gamma\alpha}$	$-$	$\otimes$	$-$	$\otimes$	$\otimes$	$-$	$\otimes$
$\mathrm{O6}_{\alpha\beta\gamma}$	$\otimes$	$-$	$-$	$\otimes$	$-$	$\otimes$	$\otimes$

Table 4.2: We demonstrate the O6-planes localized positions with "-" and the warped directions $\otimes$ in the internal space.

These intersections are mutually supersymmetric as one can verify. This is because again all O6-planes are calibrated supersymmetrically. The total O6-plane source form that enters the Bianchi is but the sum of the forms appearing in Φ for unit value of the moduli s^i. In our current no-scale model we will cancel the O6-plane tadpole by simply introducing two D6-branes for each O6-plane. This adds new open string fields, i.e. gauge fields and scalar moduli. However, in the next section we will solve tadpoles differently and find AdS vacua instead.

In total the geometric sector contains seven moduli that come from the seven-torus and together with the dilaton we have overall *eight* real scalar moduli. From the form of the scalar potential (4.3.44) we found that the vacua satisfy

$$e^{-x/\sqrt{7}} F_4 = \gamma e^{x/\sqrt{7}} \tilde{\star} H_3 \,\mathrm{vol}(\tilde{X}_7)^{-\frac{1}{7}}\,. \tag{4.3.59}$$

As before we expand the fluxes as

$$H_3 = h^i \Phi_i \ , \quad F_4 = f^i \Psi_i \, , \tag{4.3.60}$$

where f^i and h^i are appropriately quantized real constants restricted by the O2-plane tadpole

$$\delta_{ij}\, h^i\, f^j = -Q_{\mathrm{O2}} \, . \tag{4.3.61}$$

We want to evaluate $\tilde{\star} H_3$ and insert it into (4.3.59). This means we have to evaluate $\tilde{\star}\Phi_i$ in terms of Ψ_i. We find[9]

$$\tilde{\star}\Phi_i = \frac{\mathrm{vol}(\tilde{X}_7)}{(\tilde{s}^i)^2} \Psi_i \, , \tag{4.3.62}$$

where there is no summation over i implied. Now we insert everything into (4.3.59) to find the seven conditions

$$(\tilde{s}^i)^2 = \gamma e^{2x/\sqrt{7}} \mathrm{vol}(\tilde{X}_7)^{6/7} \frac{h^i}{f^i} \ , \tag{4.3.63}$$

for every i. To handle (4.3.63) we can take the product of all these seven equations together, i.e. evaluate $\prod_i (\tilde{s}^i)^2 = 1$ and use this to get a condition that their product gives $\mathrm{vol}(\tilde{X}_7)$. Indeed we find, taking into account that $\gamma^7 = \gamma = \gamma^{-1}$, that

$$x = \frac{1}{2\sqrt{7}} \mathrm{Log}\Big(\gamma \prod_i \frac{f^i}{h^i}\Big) \, , \tag{4.3.64}$$

and x is fixed. Inserting the expression for x into (4.3.63) and using the definition of the $\tilde{s}^i$ gives

$$(\tilde{s}^i)^2 = \Big(\prod_j (f^j/h^j)\Big)^{1/7} \frac{h^i}{f^i} \, , \tag{4.3.65}$$

for every i. We see that 7 out of the 8 universal moduli are fixed, i.e. the y scalar still remains undetermined and is the no-scale modulus. Searching now for a supersymmetric vacuum will require that the derivatives of the superpotential with respect to the scalars $x, y, \tilde{s}^i$ to be zero. The derivatives (4.3.34) and (4.3.42) are proportional to $\mathcal{F}_4$ and they vanish due to the Minkowski condition $\mathcal{F}_4 = 0$, setting no extra conditions on the fluxes. However, the derivative of the superpotential with respect to the scalar y sets the following conditions

$$P_y = P = 0 \ \rightarrow \sum_i f_i^{1/2} \Big[(\gamma h^i)^{-1/2} + (\gamma h^i)^{1/2} \Big] = 0 \, . \tag{4.3.66}$$

[9]We use $\star dy^{ijk} = \frac{\varepsilon^{ijklmnp}}{4!\sqrt{g_7}}\, g_{lq} g_{mr} g_{ns} g_{pt}\, dy^{qrst}$, where $\epsilon^{ijklmnp}$ is a tensor density and takes values $\varepsilon^{1234567} = 1$.

4.4 AdS vacua in Type IIA with scale separation

4.4.1 Indication for scale separation

To find what the necessary conditions are for scale separation we apply a reasoning similar to the one in (section 4 of) [27] and simply consider the dependence of the potential on the dilaton and volume modulus. This gives us necessary but non-sufficient conditions for scale separation. We first start with a discussion that is valid for compactifications down to any d-dimensions. Consider 10d string frame and the following metric Ansatz

$$\mathrm{d}s_{10}^2 = \eta_0^2\eta^{-2}\mathrm{d}s_d^2 + \rho\mathrm{d}s_{10-d}^2\,, \tag{4.4.1}$$

with $\rho^{(10-d)/2}$ the volume in 10d string frame and where

$$\eta^{d-2} = e^{-2\phi}\rho^{\frac{10-d}{2}}\,, \tag{4.4.2}$$

in order to find d-dimensional Einstein frame. In our notation τ_0, ρ_0 describe the vacuum expectation values such that in a vacuum we have

$$\mathrm{d}s_{10}^2 = \mathrm{d}s_d^2 + \rho_0\mathrm{d}s_{10-d}^2\,. \tag{4.4.3}$$

We use the notation in which the $(10-d)$-dimensional internal metric at unit volume is denoted $\tilde{g}_{10-d}$. In this analysis so far we left out the compensating η_0^2-factor in the reduction Ansatze which effectively means we work in Planck units since

$$S_d \supset \int_d \sqrt{g_d}\left(R_d + \ldots - V\right)\,, \tag{4.4.4}$$

where the dots represent all omissions such as kinetic terms for the scalars. When we use instead the formula (4.4.1) we find

$$S_d \supset \int_d \sqrt{g_d}\left(\eta_0^{d-2}R_d + \ldots - \eta_0^d V\right)\,, \tag{4.4.5}$$

such that we conclude that the Planck scale is fixed by

$$M_p = l_p^{-1} = \eta_0\,, \tag{4.4.6}$$

in string units. An approximation for the KK scale in 10d string frame is $L_{KK}^2 = \rho$. In a vacuum the Einstein equations tell us that

$$R_d = \frac{d}{d-2}M_p^2 V\,. \tag{4.4.7}$$

So there are two length scales associated with this vacuum:

$$\text{curvature radius } (L_\Lambda): \qquad L_\Lambda^{-2} = M_p^2|V|\,, \tag{4.4.8}$$

$$\text{vacuum energy length scale } (L_\rho): \qquad L_\rho^{-2} = M_p^2|V|^{2/d}\,. \tag{4.4.9}$$

We thus have two notions of scale separation that are expressed as

$$\mathrm{I}: \ \frac{L_{KK}^2}{L_\Lambda^2} = \rho_0\tau_0^2 V \to 0\,, \qquad \mathrm{II}: \ \frac{L_{KK}^2}{L_\rho^2} = \rho_0\tau_0^2 V^{2/D} \to 0\,. \tag{4.4.10}$$

The combination $\rho_0\eta_0^2$ exactly equals the volume modulus ρ_0^E in 10d Einstein frame. So if we apply the definitions to compactifications down to three-dimensions we have simply

$$\mathrm{I}: \ e^{16\beta v} V \to 0\,, \qquad \mathrm{II}: \ e^{16\beta v} V^{2/3} \to 0\,. \tag{4.4.11}$$

We now verify some minimal conditions for compactifications of Type IIA to achieve scale separation. We will use criterion I from now on. If we assume the internal space is Ricci flat (G2) and we assume O6-planes, Romans mass F_0, and F_4, H_3 fluxes and no net O2/D2 tension, then the scalar potential in 3d goes like

$$V = \tfrac{1}{3!}|\tilde{H}|^2\rho^{-3}\eta^{-2} + \tfrac{1}{4!}|\tilde{F}_4|^2\rho^{-1/2}\eta^{-3} + |F_0|^2\rho^{7/2}\eta^{-3} + T_6\rho^{1/4}\eta^{-5/2}\,. \tag{4.4.12}$$

Here the tilde symbols denote contractions with the unit-volume metric. We hope to generate a separation of scales by cranking up the F_4 form since that flux could be unbounded by tadpoles in case it wedges to zero with H_3. We first verify that the above ingredients are the necessary minimal requirements to find AdS vacua. That they are sufficient will be demonstrated with an explicit example below.

The equations of motion for stabilizing ρ and η; $\eta\,\partial_\eta V = 0 = \rho\,\partial_\rho V$, can be regrouped to obtain

$$2T_6\rho^{1/4}\eta^{-5/2} = -\tfrac{4}{3!}|\tilde{H}_3|^2\rho^{-3}\eta^{-2} - \tfrac{3}{4!}|\tilde{F}_4|^2\rho^{-1/2}\eta^{-3} < 0\,, \tag{4.4.13}$$

$$4|F_0|^2\rho^{7/2}\eta^{-3} = \tfrac{4}{3!}|\tilde{H}_3|^2\rho^{-3}\eta^{-2} + \tfrac{1}{4!}|\tilde{F}_4|^2\rho^{-1/2}\eta^{-3} > 0\,. \tag{4.4.14}$$

We learn that we need net O6 tension and non-zero Romans mass to achieve moduli stabilization with non-zero F_4 flux, just like in the 4d models [17]. The on-shell potential then becomes

$$V = -\frac{1}{4}|\tilde{F}_4|^2\rho^{-1/2}\eta^{-3} < 0\,, \tag{4.4.15}$$

which is indeed AdS$_3$. To verify whether scale separation could be possible we perform a scaling analysis to the potential and we consistently realize the following scalings which are compatible with the tadpole conditions

$$F_4 \sim N\,, \qquad F_0 \sim H_3 \sim T_6 \sim N^0\,. \tag{4.4.16}$$

Remarkably there is a possible scaling for the dilaton and volume with N such that every term in the potential scales indeed in the same way:

$$\eta \sim N^{13/4}\,, \quad \rho \sim N^{1/2} \qquad \to \qquad V \sim N^{-8}\,,\ \exp(\phi) \sim N^{-3/4}\,. \tag{4.4.17}$$

Notice that for large N the modulus ρ grows while the dilaton is damped so we are guaranteed to be in the supergravity limit. This scaling indeed implies separation since

$$\rho\tau^2 V \sim N^{-1}\,. \tag{4.4.18}$$

4.4.2 10d view on the effective theory from toroidal orbifolds

We now hunt for a concrete example by changing the Minkowski no-scale solution of the previous section. We add Romans mass and realize tadpoles differently and this will turn out to be sufficient.

The following Bianchi identities lead to non-trivial RR tadpoles

$$\begin{aligned} \mathrm{d}F_2 =& H_3 \wedge F_0 + Q_{\mathrm{O6}}\delta_{\mathrm{O6}}\,, \\ \mathrm{d}F_4 =& H_3 \wedge F_2\,, \\ \mathrm{d}F_6 =& H_3 \wedge F_4 + Q_{\mathrm{O2}}\delta_{\mathrm{O2}} + Q_{\mathrm{D2}}\delta_{\mathrm{D2}}\,, \end{aligned} \tag{4.4.19}$$

where for completeness we have also indicated the presence of D2-branes. We now take

$$F_2 = 0\ , \quad F_4 = F_{4A} + F_{4B} \neq 0\ , \quad F_0 \neq 0\ , \quad H_3 \neq 0\,, \tag{4.4.20}$$

where the F_4 splitting refers to the way the flux wegdes with H_3, that is

$$H_3 \wedge F_{4A} \equiv 0\ , \quad H_3 \wedge F_{4B} = -Q_{\mathrm{O2}}\delta_{\mathrm{O2}} - Q_{\mathrm{D2}}\delta_{\mathrm{D2}}\,. \tag{4.4.21}$$

The F_2 tadpole cancellation works by cancelling the contributions from the O6-planes with $H_3 \wedge F_0$. We can allow for an O2-plane source but we assume it is canceled by a correct amount of D2-branes when $F_{4B} = 0$. Otherwise we will not have any D2-branes and it will be $H_3 \wedge F_{4B}$ that cancels the O2-plane tadpole.

The bosonic part of the 10d action that contributes to the potential is now

$$\begin{aligned} S =& \int_{10} \sqrt{-G_{10}}\, \left(-\tfrac{1}{2}e^{-\phi}|H_3|^2 - \tfrac{1}{2}e^{\phi/2}|F_4|^2 - \tfrac{1}{2}e^{5\phi/2}F_0^2\right) \\ &+ e^{-\phi/4}Q_{\mathrm{O2/D2}}\int_{\mathrm{O2/D2}}\sqrt{-g_3} + e^{3\phi/4}Q_{\mathrm{O6/D6}}\sum_{\{\alpha,\beta,\gamma\}}\int_{\mathrm{O6}}\sqrt{-g_7}\,, \end{aligned} \tag{4.4.22}$$

and now we use the notation $Q_{\mathrm{O2/D2}} = Q_{\mathrm{O2}} + Q_{\mathrm{D2}}$ and $Q_{\mathrm{O6/D6}} = Q_{\mathrm{O6}} + Q_{\mathrm{D6}}$. Our notation in the above formula for the integral over the O6 sources already anticipates our toroidal orientifold example. In particular we use the orbifold setup that we studied in the previous section. Since we have calibrated O6 sources $j_{\{\alpha,\beta,\gamma\}}$, the $\mathrm{d}F_2$ Bianchi (in the smeared approximation) then gives

$$0 = F_0\, H_3 + Q_{\mathrm{O6/D6}}\, J_3\ , \quad J_3 = \sum \frac{j_{\{\alpha,\beta,\gamma\}}}{\mathrm{vol}(\{\alpha,\beta,\gamma\})_3} = \sum_i \Phi_i\,. \tag{4.4.23}$$

The notation $j_{\{\alpha,\beta,\gamma\}}/\mathrm{vol}_3$ reflects one should normalize each volume three-form j_3 transverse to each orientifold with respect to its own three-cycle volume. For example we have

$$\frac{j_{\alpha\beta}}{\mathrm{vol}(\alpha\beta)_3} = \frac{e^{127}}{r^1 r^2 r^7} = \frac{s^1\Phi_1}{s^1} = \Phi_1\,. \tag{4.4.24}$$

We take the following fluxes consistent with the tadpoles

$$\begin{aligned} H_3 &= h_3 \sum_i \Phi_i && \Longrightarrow && h_3 m_0 = \pm Q_{\mathrm{O6/D6}}\,, \\ F_{4A} &= \sum_i f_4^i \Psi_i && \Longrightarrow && \sum_i f_4^i = 0\,, \\ F_{4B} &= \sum_i \hat{f}^i \Psi_i && \Longrightarrow && \sum_i \hat{f}_4^i = \pm Q_{\mathrm{O2/D2}}/h_3\,. \end{aligned} \tag{4.4.25}$$

We now compute the scalar potential for the 3d compactification and use (4.3.17) with the internal metric given by the seven-torus orbifold. We split the scalar potential V into two parts, one that relates to the fluxes together with the O2-plane that we call collectively V_{Flux} and one that relates to the O6-planes V_{O6}. For the fluxes/O2 we have

$$V_{\mathrm{Flux}} = \frac{1}{2}\int_7 \sqrt{\tilde{g}_7}\left[e^{-21\beta v}\left(e^{-\phi/2}e^{\beta v/2}H_3 \mp e^{-\beta v/2}e^{\phi/4}\tilde{\star}_7 F_4\right)^2 + e^{-14\beta v}e^{5\phi/2}F_0^2\right]\,, \tag{4.4.26}$$

where $\sqrt{\tilde{g}_7} = 1$. Note that all internal space contractions are with the unit-volume metric $\tilde{g}_{mn}$ as indicated by the tilde symbol. Let us now turn to the O6 contributions. For the $\mathrm{O6}_{\alpha\beta}$ orientifold for example we have

$$S_{\mathrm{O6}_{\alpha\beta}} = e^{3\phi/4}T_{\mathrm{O6}}\int_{\mathrm{O6}_{\alpha\beta}}\sqrt{-g_7} = e^{3\phi/4}T_{\mathrm{O6}}\int_{\mathrm{O6}_{\alpha\beta}}\sqrt{-g_7}\int\frac{e^{127}}{s^1} = e^{3\phi/4}T_{\mathrm{O6}}\int\sqrt{-g_{10}}\,\frac{1}{s^1}\,. \tag{4.4.27}$$

The contribution of all planes to the effective potential is then[10]

$$V_{\mathrm{O6}} = -e^{3\phi/4}T_{\mathrm{O6}}\,e^{-14\beta v}\sum_i \frac{1}{s^i} = -e^{3\phi/4}T_{\mathrm{O6}}\,e^{-17\beta v}\sum_i\frac{1}{\tilde{s}^i}\,, \tag{4.4.28}$$

where in the last step we have inserted the unit-volume fluctuations $\tilde{s}^i$ with the use of $s^i = e^{3\beta v}\tilde{s}^i$. Note that $-17\beta v + 3\phi/4 = (-10\beta v - \phi/2) + (-7\beta v + 5\phi/4)$ which are exactly the combinations of volume and dilaton that appear with H_3 and m terms respectively, as one can see in (4.4.26). In the end the most convenient form for the O6 contribution is the one that is written in terms of the unit-volume scalars and the x and y, and reads

$$V_{\mathrm{O6}} = -T_{\mathrm{O6}}\,e^{\frac{3}{2}y - \frac{5}{2\sqrt{7}}x}\sum_i\frac{\mathrm{vol}(X)^{3/7}}{s^i} = -T_{\mathrm{O6}}\,e^{\frac{3}{2}y-\frac{5}{2\sqrt{7}}x}\sum_i\frac{1}{\tilde{s}^i}\,, \quad T_{\mathrm{O6}} = \gamma\, h_3 m_0\,, \tag{4.4.29}$$

where $\gamma = \pm 1$. Now we are ready to derive the full scalar potential $V_{\mathrm{Flux}} + V_{\mathrm{O6}}$ from a 3d N=1 supergravity.

[10] We could calculate this result also by taking into account that the volume form on the associated $\alpha\beta$ four-cycle is $\star j_{\alpha\beta} = \frac{\mathrm{vol}(X)}{s^1}\Psi_1$, which then would allow the following manipulations $\int_{\mathrm{O6}}\sqrt{-g_7} = e^{3\alpha v}\int_3\sqrt{-\tilde{g}_3}\int_{\text{4-cycle}}\sqrt{g_4} = e^{3\alpha v}\int_3\sqrt{-\tilde{g}_3}\int_{\Psi^1\text{ 4-cycle}}\star j_{\alpha\beta} = e^{-14\beta v}\int_3\sqrt{-\tilde{g}_3}(s^1)^{-1}$.

4.4.3 The 3d supergravity

Our aim here is to find the superpotential P that defines our 3d supergravity theory.

Taking into account that for our 3d supergravity constructions we always use the normalization $R/2$ for the Hilbert–Einstein term we have to rescale the 3d metric of the previous subsection by $1/4$. This means that with our superspace formulation the total scalar potential we want to reproduce is

$$\begin{aligned} V^{\mathrm{Total}} =& \frac{e^{2y}}{16} \int \Big(e^{-2x/\sqrt{7}} F_4 \wedge \tilde{\star} F_4 \,\mathrm{vol}(\tilde{X}_7)^{\frac{1}{7}} + e^{2x/\sqrt{7}} H_3 \wedge \tilde{\star} H_3 \,\mathrm{vol}(\tilde{X}_7)^{-\frac{1}{7}} \pm 2F_4 \wedge H_3 \Big) \\ &+ \frac{F_0^2}{16} e^{y-\sqrt{7}x} - \frac{1}{8} Q_{\mathrm{O6/D6}} e^{\frac{3}{2}y - \frac{5}{2\sqrt{7}}x} \sum_i \frac{1}{\tilde{s}^i} \,, \end{aligned} \tag{4.4.30}$$

with the fluxes and $Q_{\mathrm{O6/D6}}$ given by (4.4.25). As we have explained one should in the end set $\prod_i \tilde{s}^i = 1$ to restrict to the true degrees of freedom of the toroidal orbifold. We will verify that our new superpotential still satisfies the conditions that allow us to impose the constraint on the $\tilde{s}^i$ without spoiling supersymmetry.

One can easily verify that in case there would only be Romans mass, the N=1 superpotential equals

$$P^R = \frac{m_0}{8} \exp\left[\frac{1}{2}y - \frac{\sqrt{7}}{2}x\right] , \tag{4.4.31}$$

where the R superscript is to stress this is the pure Romans mass contribution to the superpotential. Note that $P^R_i = 0$. To reproduce the total scalar potential (4.4.30) we work with the target space metric given by (4.3.30) and we suggest that the total superpotential is simply a sum

$$P^{\mathrm{Total}} = P + P^R \,, \tag{4.4.32}$$

where P is given is (4.3.31) and P^R is (4.4.31). Indeed, if one writes down all the contributions to the scalar potential we have

$$V^{\mathrm{Total}} = G^{IJ} P^{\mathrm{Total}}_I P^{\mathrm{Total}}_J - 4(P^{\mathrm{Total}})^2 = V + V^R + 8P^R_x P_x + 8P^R_y P_y - 8P^R P \,, \tag{4.4.33}$$

where $V = G^{IJ} P_I P_J - 4P^2$, was our no-scale potential. Once we evaluate the cross-terms we find

$$8P^R_x P_x + 8P^R_y P_y - 8P^R P \equiv V_{\mathrm{O6}} \,. \tag{4.4.34}$$

Finally note that

$$\int \Phi_i \wedge \tilde{\star}\tilde{\Phi} \; G^{ij} \, P^{\mathrm{Total}}_j = 0 \,, \tag{4.4.35}$$

which as we explain in the appendix is the condition that guarantees that we can set on the superspace level $\prod_i \tilde{S}^i = 1$ such that we reduce consistently to the true degrees of freedom.

4.4.4 Supersymmetric AdS vacua

Now that we have found the superpotential[11] for our toroidal orbifold to be

$$P = \frac{m_0}{8} e^{\frac{y}{2} - \frac{\sqrt{7}x}{2}} + \frac{\gamma h_3}{8} e^{y + \frac{x}{\sqrt{7}}} \sum_{i=1}^{7} \frac{1}{\tilde{s}^i} + \frac{1}{8} e^{y - \frac{x}{\sqrt{7}}} \sum_{i=1}^{7} (f^i + \hat{f}^i) \tilde{s}^i \ , \quad \tilde{s}^7 = \prod_{a=1}^{6} \frac{1}{\tilde{s}^a} \ , \tag{4.4.36}$$

we can look for supersymmetric vacua. We first consider what happens in the simplest case where $F_{4A} \neq 0$ and $F_{4B} = 0$ and we take the following simple concrete set of fluxes

$$\hat{f}_4^i = 0 \ , \quad f_4^i = (-f, -f, -f, -f, -f, -f, +6f) \ , \quad \gamma = 1 \ , \quad f \, , h_3 \, , m_0 > 0 \, . \tag{4.4.37}$$

Because of the O6 tadpole the values of h_3 and m_0 are in fact very limited - all our freedom is essentially in f. At a later stage, once we established our solutions, we will properly quantize all fluxes and charges. With this flux choice, the superpotential simplifies to

$$P = -\frac{f}{8} e^{y - \frac{x}{\sqrt{7}}} \left[\sum_{a=1}^{6} \tilde{s}^a - 6 \prod_{a=1}^{6} \frac{1}{\tilde{s}^a} \right] + \frac{h}{8} e^{y + \frac{x}{\sqrt{7}}} \left[\sum_{a=1}^{6} \frac{1}{\tilde{s}^a} + \prod_{a=1}^{6} \tilde{s}^a \right] + \frac{m}{8} e^{\frac{y}{2} - \frac{\sqrt{7}x}{2}} \, . \tag{4.4.38}$$

We search for solutions which are as isotropic as possible: meaning all $\tilde{s}^a$ $(a = 1 \ldots 6)$ have the same value, which we denote σ, namely

$$\langle \tilde{s}^a \rangle = \sigma \, . \tag{4.4.39}$$

The supersymmetric conditions ($\partial P = 0$) become:

$$\begin{aligned} 0 &= a\sigma^6 + 6\sigma + \frac{6a}{\sigma} - \frac{6}{\sigma^6} - \frac{7b}{2} \, , \\ 0 &= a\sigma^6 - 6\sigma + \frac{6a}{\sigma} + \frac{6}{\sigma^6} + \frac{b}{2} \, , \\ 0 &= a\sigma^5 - \frac{a}{\sigma^2} - \frac{6}{\sigma^7} - 1 \, , \end{aligned} \tag{4.4.40}$$

where

$$a = \frac{h}{f} e^{\frac{2x}{\sqrt{7}}} \ , \quad b = \frac{m}{f} e^{-\frac{y}{2} - \frac{5x}{2\sqrt{7}}} \, . \tag{4.4.41}$$

One can either do a numerical integration or solve analytically to find

$$a = 0.515696 \ldots \ , \quad b = 3.43111 \ldots \ , \quad \sigma = 1.32691 \ldots \, . \tag{4.4.42}$$

We thus have for the dilaton and the volume modulus scalings

$$g_s = e^{\phi} \sim f^{-3/4} \ , \quad \mathrm{vol}(X_7) = e^{7\beta v} \sim f^{49/16} \, , \tag{4.4.43}$$

[11] We now simply refer to P^{Total} as P.

and we can thus verify that our vacuum corresponds to weak string coupling and to large volume for large f. The AdS vacuum energy is given by

$$\langle V\rangle = -\frac{1}{64a^6b^4}\left(6\sigma^2+\frac{36}{\sigma^{12}}\right)\frac{m^4h^6}{f^8}\,. \tag{4.4.44}$$

Finally we can check the scale separation in our example

$$|\rho\,\tau^2\,V| \sim f^{-1}\,, \tag{4.4.45}$$

which matches exactly with (4.4.18). So we find small coupling, large volume and scale separation. We see that in our solution all of the six $\tilde{s}^a$ moduli take the same numerical value, σ, by construction and the seventh is slightly different. But the torus remains, as a whole, at large values and no separate directions get small.

Since the scalars $\tilde{s}^a$ are not canonically normalized the Hessian of the potential does not directly correspond to the mass matrix, but does inform us about the possible existence of tachyons (above the BF bound). We find

$$\frac{\langle V_{IJ}\rangle}{|\langle V\rangle|} = \begin{pmatrix} 4.282 & 3.321 & 3.321 & 3.321 & 3.321 & 3.321 & 6.823 & 2.132 \\ 3.321 & 4.282 & 3.321 & 3.321 & 3.321 & 3.321 & 6.823 & 2.132 \\ 3.321 & 3.321 & 4.282 & 3.321 & 3.321 & 3.321 & 6.823 & 2.132 \\ 3.321 & 3.321 & 3.321 & 4.282 & 3.321 & 3.321 & 6.823 & 2.132 \\ 3.321 & 3.321 & 3.321 & 3.321 & 4.282 & 3.321 & 6.823 & 2.132 \\ 3.321 & 3.321 & 3.321 & 3.321 & 3.321 & 4.282 & 6.823 & 2.132 \\ 6.823 & 6.823 & 6.823 & 6.823 & 6.823 & 6.823 & 21.286 & 4.913 \\ 2.132 & 2.132 & 2.132 & 2.132 & 2.132 & 2.132 & 4.913 & 5. \end{pmatrix}, \tag{4.4.46}$$

where the lines/columns are $\tilde{s}^a, x, y$. The important thing to see in this matrix is that the 8 eigenvalues read: 39.296, 4.441, 3.434, 0.961, 0.961, 0.961, 0.961, 0.961. This means that all the masses are positive.

Since we did not perform an exhaustive analysis of all possibilities, one can wonder if starting with different F_{4A} flux values instead of (4.4.37) leads to more solutions. One can try for example configurations like $f^i = f(1,1,1,-1,-1,-1,0)$ or $f^i = -f(2,1,1,1,1,1,-7)$. But we were not able to find supersymmetric AdS vacua in these cases. We therefore postpone performing a complete analysis of the vacuum structure for a future work.

In the appendix we explicitly show that there is an almost identical AdS$_3$ solution which can be found by taking $f \to -f$. This is sometimes called skew-whiffing and breaks supersymmetry [97]. For the rest the solution seems essentially the same.

4.4.5 More flux

We now briefly discuss what happens when we turn on also the F_{4B} component of the F_4 flux. There is a good reason for doing this. In the previous solution we had to cancel the O2 tadpole with D2-branes, leaving a compact moduli space of D2 positions on the G2 space. But by adding F_{4B} fluxes we can satisfy the O2 tadpole without any explicit D2 sources.

First one can think of turning on only the F_{4B} component, in which case we will have $\hat{f}^i \neq 0$ and $f^i = 0$. The $\hat{f}^i$ that enter the superpotential (4.4.36) however are restricted by flux quantization and by the tadpole cancellation and thus cannot be large enough to give a scale separation. As a result, after one solves the eight equations $P_x = P_y = P_a = 0$ the eight scalars will generically be stabilized in a supersymmetric AdS vacuum, which does not have a scale separation. In fact even if we turn on the F_{4A} together with the F_{4B} component, but we do not take the former to be large, again we obtain vacua without scale separation. We can ask how the large F_{4A} component will influence the vacuum in the presence of an F_{4B}. Let us assume we have some unspecified values for the $\hat{f}^i$ but we choose the values (4.4.37) for the other fluxes. In this case the supersymmetric conditions read

$$\begin{aligned}
0 &= \frac{7b}{2} - a\left(\sum_{a=1}^{6}\frac{1}{\tilde{s}^a} + \prod_{a=1}^{6}\tilde{s}^a\right) - \sum_{a=1}^{6}\tilde{s}^a + 6\prod_{a=1}^{6}\frac{1}{\tilde{s}^a} + \sum_{a=1}^{6}\frac{\hat{f}_a}{f}\tilde{s}^a + \frac{\hat{f}_7}{f}\prod_{a=1}^{6}\frac{1}{\tilde{s}^a}\,,\\
0 &= \frac{b}{2} + a\left(\sum_{a=1}^{6}\frac{1}{\tilde{s}^a} + \prod_{a=1}^{6}\tilde{s}^a\right) - \sum_{a=1}^{6}\tilde{s}^a + 6\prod_{a=1}^{6}\frac{1}{\tilde{s}^a} + \sum_{a=1}^{6}\frac{\hat{f}_a}{f}\tilde{s}^a + \frac{\hat{f}_7}{f}\prod_{a=1}^{6}\frac{1}{\tilde{s}^a}\,,\\
0 &= -\frac{a}{\tilde{s}^b} + a\prod_{a=1}^{6}\tilde{s}^a - \tilde{s}^b - 6\prod_{a=1}^{6}\frac{1}{\tilde{s}^a} + \frac{\hat{f}_b}{f}\tilde{s}^b - \frac{\hat{f}_7}{f}\prod_{a=1}^{6}\frac{1}{\tilde{s}^a}\,, \quad \text{no } b \text{ summation}\,.
\end{aligned} \tag{4.4.47}$$

The equations in the last line are a total of six equations, as they should be, because they will specify the six $\tilde{s}^a$. The a and b are given by (4.4.41). From (4.4.47) it is now clear what happens in the generic case with parametrically large F_{4A}. We see that the parametrically large f will damp the $\hat{f}^i$ contribution, therefore (4.4.47) will essentially converge to (4.4.40), and we will get again a vacuum with $\tilde{s}^a = \sigma$ with the a, b and σ values determined by our previous solution up to negligible parametrically small corrections.

4.4.6 Flux quantization

We consider the relation in Eq.(2.2.23), an O6-plane has the charge of 2 anti-D6-branes. If the charge quantum of the Romans mass F_0 is m and that of the H_3-flux is h_3, then the integrated form of the F_2 Bianchi identity is the RR tadpole condition for each cycle

$$h_3 m = 2N_{\rm O6} - N_{\rm D6}\,, \tag{4.4.48}$$

where $N_{\rm O6}$ and $N_{\rm D6}$ denote number of O6-planes and D6-branes respectively. There are two ways to approach this: either one works on the covering space in which a single O6 involution leaves multiple 8 O6 fixed points in a transversal $\mathbb{T}^3$ or one works in the orbifolded space in which a single O6 has 6 other orbifold images. We will do the first. Then we have for each O6 involution 8 fixed points. These make a total of 56 O6 planes with 8 corresponding to each one of the seven cycles. Hence for our supersymmetric AdS vacuum considering the quantization condition in Eq.(2.2.29) with $\alpha' = 1$ we find

$$H_3 = (2\pi)^2 h_3\,, \quad F_0 = (2\pi)^{-1} m\,, \quad h_3 m = 16 \qquad F_4 = (2\pi)^3 f_4\,, \tag{4.4.49}$$

where $h_3, m, f_4 \in \mathbb{Z}$ and we assumed no D6-branes.

4.5 Outlook

Let us summarize what we have done in this section. We have found the form of the 3d real superpotential P for G2 compactifications of Type IIA supergravity with O2, O6 sources and H_3, F_4, F_0 fluxes to be

$$P = \frac{e^y}{8}\left[e^{\frac{x}{\sqrt{7}}}\int \star\Phi \wedge H_3\,\mathrm{vol}(X_7)^{-\frac{4}{7}} + e^{-\frac{x}{\sqrt{7}}}\int \Phi \wedge F_4\,\mathrm{vol}(X_7)^{-\frac{3}{7}}\right] + \frac{F_0}{8}\,e^{\frac{1}{2}y - \frac{\sqrt{7}}{2}x}\,, \tag{4.5.1}$$

where x and y are specific linear combinations of the 10d dilaton and volume defined in Eq. (4.3.20) and Φ the G2 invariant three-form. The 3d theory is a minimal supergravity with two real supercharges and a scalar potential given by Eq. (4.2.6)

$$V(\phi) = G^{IJ} P_I P_J - 4P^2\,, \tag{4.5.2}$$

where G_{IJ} is the metric on the scalar manifold. To our knowledge this is the first investigation of compactifications of Type II supergravity on G2 orientifolds.[12]

We then found two classes of (supersymmetric) solutions from the critical points of P or V. The first class consists of no-scale Minkowski solutions and the second class of scale-separated AdS$_3$ vacua. Both were established on the same toroidal orbifold, although the RR tadpoles were solved differently each time. These types of solutions are expected to exist on general classes of G2 spaces that allow involutions for O2- and O6-planes.

The no-scale solution had one clear massless direction, the y-scalar, but more importantly we argued that there is no obvious obstacle in stabilizing all other moduli with non-compact moduli spaces. The scalars with compact moduli spaces are D-brane positions and Abelian vectors (axions). This is in contrast with the no-scale vacua in 4d from three-form fluxes in Type IIB [79] where all Kähler moduli are massless and to get a single massless direction one needs to restrict to special Calabi–Yau spaces with single Kähler moduli.

In the AdS$_3$ solutions on the other hand we also stabilized the remaining y-scalar. Note that all of our vacua are at tunably small coupling and large volume. This is not any special for no-scale vacua, because of the flat y-direction, but it is for the AdS vacua where all non-compact scalars can be stabilized. Especially the separation of scales is an extra cherry on top of these AdS solutions. In contrast to 4d compactifications the bridge from

[12] Although see [98] for G2 compactifications of Type II without orientifolds and fluxes. Furthermore, some of the AdS$_3$ solutions in [36, 99, 100] could be related to our findings.

no-scale vacua to moduli stabilized vacua can be done in one and the same model. We simply added Romans mass and solved the tadpoles differently.

Our main focus was on demonstrating with simple examples what one expects to find in three dimensions and so for none of our examples have we delved into a detailed discussion of the twisted sector neither the axions and D-brane moduli, and this should be understood better. Also increasing the number of examples would be a relevant task for the future. We have summarized the classes of vacua in a schematic fashion in Fig. 1.

$$\begin{tikzcd} \text{No-scale} \arrow[r, "F_0 \neq 0", "F_{4B} \neq 0"'] \arrow[d, "F_0 \neq 0"', "F_{4A} \neq 0"] & \text{AdS w/out scale sep.} \arrow[d, "F_{4A} \neq 0"] \\ \text{AdS w/ scale sep.} \arrow[r, "F_{4B} \neq 0"] & \text{AdS w/ scale sep.} \end{tikzcd}$$

Fig. 1: The backgrounds/vacua depending on the form of the F_4 flux and the value of the Romans mass. The parametrically large F_{4A} component generates the scale separation for the AdS vacua. The F_{4B} fluxes can be used to cancel O2-plane tadpoles.

We want to emphasize that the no-scale vacua can be understood in the form of 10-dimensional solutions with localized and backreacting orientifolds [67]. On the other hand, this is not understood for the AdS_3 vacua since they feature seven intersecting O6 planes and it is not known how to find backreacted solutions of this kind, although the recent results of [70, 71] can most likely be applied here as well and could be encouraging.

Our main motivation for constructing the AdS_3 vacua comes from holography. Already for a while there is an interest in settling the discussions about the consistency of flux vacua with scale separation. The existence of such vacua is the foundation of conventional string phenomenology. Constructing the would-be CFTs dual to scale-separated AdS vacua or show they do not exist ("bootstrap them away") would be the natural way forward [56–59]. This endeavor has not yet materialized in actual concrete statements and this is why we prefer to establish a landscape of scale-separated flux vacua in 3d using the "standard techniques" in string phenomenology whose consistency is being debated. The reason is that 2d CFTs have been studied in more detail and especially recently some novel results seem to point against the existence of certain AdS_3 vacua with very high moduli stabilization (such that one arrives at pure gravity in the IR) [101]. If the same can be argued for our scale-separated AdS_3 vacua it almost certainly implies that also the AdS_4 vacua in massive Type IIA [17] neither have a holographic dual CFT [102] because something is inconsistent about their construction [103, 104].

Chapter 5

AdS vacua and O6-plane backreaction

5.1 Introduction

The consistency of the smearing approximation is often challenged since it is not yet clear whether it leads to consistent truncation and misleading results. On one hand, the resulting internal manifolds and localized source configurations are some highly complicated solutions of the full higher-dimensional equations of motion, whose explicit construction is prohibitively difficult. On the other hand, below the compactification scale, one could expect the lower-dimensional effective theory to be somewhat insensitive to the local details of the internal manifold, at least to leading order in the compactification scale. For this reason, one expects to obtain the same lower-dimensional effective description from a "smeared" solution, in which the charge density from the "localized" sources is distributed in a continuous fashion over the internal manifold. These solutions are much easier to construct explicitly with many examples in the literature, and in some cases the approximation is controllable [67, 69, 105]. More importantly, this logic also suggests that properties of the true "unsmeared" solution are encoded in higher order corrections in some appropriate perturbative expansion. Explicit procedures for finding such an expansion and computing leading corrections to the internal geometry have been only recently proposed [71, 106], and applications of this procedure have already appeared in [107, 108].

In this chapter we go one step further and we apply the procedure proposed in [106] to the AdS_3 vacua of the previous chapter. We evaluate the backreaction of the localized sources and explicitly verify the parametric control over the corrections in the scale-separated limit. Our analysis indicates that, assuming such AdS_3 solution with localized sources exists, the smeared source approximation captures useful information about it - at least to leading order in the backreaction. Such assumption has of course the caveat that one has to assume that the O6-plane singularities we encounter here can be resolved within string theory.

5.2 Unsmearing the sources

5.2.1 The setup

We start from the bosonic part of the Type IIA supergravity action in Eq.(2.2.1) but in the *string* frame

$$S_{\mathrm{IIA}} = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}X\sqrt{-G}\Big(\tau^2(\mathcal{R}_{10} - \frac{1}{2}|H_3|^2) + 4G^{MN}\partial_M\tau\partial_N\tau - \frac{1}{2}|F_p|^2\Big)\,, \tag{5.2.1}$$

and have considered a redefinition for the dilaton field $\tau = e^{-\phi}$ in order to be in agreement with [106]. For the Ricci curvature we have choose the calligraphic notation since in this study contains the warp factor of the metric as well. For the local sources we write down only the DBI part in the effective action which is relevant for our analysis and we ignore the Chern–Simons terms and the fluctuations of world-volume fields of the Dp-branes. The Chern–Simons terms will of course be properly taken into account when we check the Bianchi identities/tadpole conditions. For the contribution of the localized sources to the effective action we thus have

$$S_{\mathrm{Op/Dp}} = -T_p\int \mathrm{d}^{10}X\,\sqrt{-G}\sum_i \tau\delta(\pi_i)\,, \tag{5.2.2}$$

where $\delta(\pi_i)$ is a unit-normalized delta-like distribution denoting the locus of the sources that wrap the cycle π_i. For example, in our three-dimensional compactification, for a space-filling O6-plane π_i refers to four-cycles and $\tilde{\pi}_i$ to 3-cycles. The coefficient T_p is given by

$$T_p = N_{\mathrm{Op}}\mu_{\mathrm{Op}} + N_{\mathrm{Dp}}\mu_{\mathrm{Dp}}\,, \tag{5.2.3}$$

and denotes total tension of all the sources wrapping a given cycle and the individual D-brane and O-plane tensions are given by

$$\mu_{\mathrm{Dp}} = (2\pi)^{-p}(\sqrt{\alpha'})^{-(p+1)}\,, \tag{5.2.4}$$
$$\mu_{\mathrm{Op}} = -2^{p-5}\times\mu_{\mathrm{Dp}}\,. \tag{5.2.5}$$

It is important to stress that the reason we have N_{Op} and N_{Dp} appearing is because the delta-distributions $\delta(\pi_i)$ integrate to unit.

We will be interested in a flux background where the external space is (warped) AdS_d and the internal space is compact. To this end we make an ansatz for the ten dimensional metric, always in the string frame, of the form

$$\mathrm{d}s_{10}^2 = w^2(y)g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + g_{mn}\mathrm{d}y^m\mathrm{d}y^n\,, \tag{5.2.6}$$

where $g_{\mu\nu}$ is the unwarped d-dimensional external metric and g_{mn} is the $(10-d)$ dimensional internal one. For convenience in computing the stress tensor, we write the local sources in terms of the ten dimensional metric. Note however that, for external spacetime filling sources, the action can be expressed in terms of the source worldvolume metric by using the relation

$$\delta(\pi_i) \equiv \frac{\sqrt{g_{\pi_i}}}{\sqrt{g_{(10-d)}}}\delta^{(9-p)}(y)\,, \tag{5.2.7}$$

where $g_{\pi_i} \equiv \det((g_{\pi_i})_{\alpha\beta})$ is the metric determinant of the wrapped cycle. The $\delta^{(9-p)}(y)$ function collectively denotes the localized positions of the sources in the internal space and integrates to one

$$\int_{\tilde{\pi}_i} \mathrm{d}^{9-p}y\delta^{(9-p)}(y) = 1\,, \tag{5.2.8}$$

over the dual cycle $\tilde{\pi}_i$.

5.2.2 Equations of motion

To compare the localized solutions to the smeared ones and find the next to leading order corrections we first need to calculate the equations of motions for the metric, dilaton and the fluxes. We start with a general discussion including all the possible sources in the equations, eventually restricting to the specific choice of sources and fluxes that we are interested in.

Equations with localized sources

In this section we set

$$2\pi\sqrt{\alpha'} = 1\,, \tag{5.2.9}$$

and write down the equations of motion for the fluxes together with their sources which are given by the Bianchi identities. For sources that wrap cycles of the internal space we have

$$\int_{\pi_i} \mathrm{vol}_{\pi_i} = \int \mathrm{vol}_{\pi_i} \wedge \delta_{i,9-p} = \int \mathrm{d}^{10-d}y\sqrt{g_{10-d}}\,\delta(\pi_i)\,, \tag{5.2.10}$$

where vol_{π_i} is the volume density of the wrapped π_i cycles, and $\delta_{i,9-p}$ is a *unit-normalized* $(9-p)$-form with legs transverse to the sources wrapping the i-th cycle and with support on the source locus

$$\delta_{i,9-p} = \delta(\pi_i)\,\mathrm{d}^{9-p}y_\perp\,, \tag{5.2.11}$$

where $y_\perp$ are the coordinates transverse to the sources and wedge products are implied. For the massive Type IIA supergravity considered in the previous chapter the relevant Bianchi identities, including the number of sources wrapping each cycle, are

$$\mathrm{d}F_2 = H_3 \wedge F_0 - 2N_{\mathrm{O6}}\sum_i^7 \delta_{i,3} + N_{\mathrm{D6}}\sum_i^7 \delta_{i,3}\,, \tag{5.2.12}$$

$$\mathrm{d}F_4 = H_3 \wedge F_2\,, \tag{5.2.13}$$

$$\mathrm{d}F_6 = H_3 \wedge F_4 - 2^{-3}N_{\mathrm{O2}}\delta_7 + N_{\mathrm{D2}}\delta_7\,. \tag{5.2.14}$$

Here $N_{\mathrm{O6/O2}} = 0$ if there are no O-planes, otherwise it is non-vanishing and depends on the number of fixed points the relevant orientifold involution has in the internal manifold. Our specific case will involve $N_{\mathrm{O2}} = 2^7$ for the total "number" of O2-planes. For O6-planes we will have $N_{\mathrm{O6}} = 2^3$ for each three-cycle, which are in fact all images of a single O-plane under the G2 orbifold.

In order to proceed further, we need the equations of motion for the dilaton and the metric; we have performed this analysis for d-external dimensions in the Appendix 8.4. Note that throughout this work we assume that the dilaton profile does not depend on the external space coordinates but only on the internal ones: $\tau(y)$. The equations of motion for the dilaton are found in Eq.(8.4.2), and for three external dimensions ($d = 3$) they become

$$\begin{aligned} 0 =& - 8\nabla^2\tau + 2\frac{\tau}{w^2}R_3 - \frac{24}{w}(\partial_m w)(\partial^m\tau) - 12\frac{\tau}{w}\nabla_m\nabla^m w - 12\frac{\tau}{w^2}\nabla_m w\nabla^m w \\ &+ 2\tau R_7 - \tau|H_3|^2 + 2\mu_6\sum_i \delta(\pi_i) + 2^{-3}\mu_2\delta(\pi)\,, \end{aligned} \tag{5.2.15}$$

where we use the notation

$$\mu_6 = N_{\mathrm{O6}} - 2^{-1}N_{\mathrm{D6}}\,, \quad \mu_2 = N_{\mathrm{O2}} - 2^3 N_{\mathrm{D2}}\,. \tag{5.2.16}$$

To find the variation with respect to the metric we need the stress-energy tensor of the localized sources in the internal space, given by the projector

$$\Pi_{i,mn} = -\frac{2}{\sqrt{g_{\pi_i}}}\frac{\delta\sqrt{g_{\pi_i}}}{\delta g_{mn}} = (g_{\pi_i})^{\alpha\beta}\frac{\partial y^l}{\partial\xi_i^\alpha}\frac{\partial y^p}{\partial\xi_i^\beta}g_{ml}g_{np}\,, \tag{5.2.17}$$

where ξ_i^α are worldvolume coordinates of the branes/planes wrapping the i-th cycle. The Einstein equation in Eq.(8.4.8) becomes

$$\begin{aligned} 0 =& -\frac{\tau^2}{w^2}R_3 + 3\tau^2\Big(w^{-1}\nabla^2 w + 2w^{-2}\nabla_m w\nabla^m w\Big) + \frac{9}{4}\frac{\tau}{w}\partial_m w\partial^m\tau + \frac{3}{4}\tau\nabla^2\tau + \frac{3}{4}(\partial\tau)^2 \\ &- \frac{3}{8}\tau^2|H_3|^2 - \frac{3}{2}\sum_{p=0}^{6}\frac{p-1}{8}|F_n|^2 + \frac{3}{8}\mu_6\tau\sum_i\delta(\pi_i) + \frac{15}{8}2^{-3}\mu_2\tau\delta(\pi)\,. \end{aligned} \tag{5.2.18}$$

The trace-reversed Einstein equations using Eq.(8.4.6) and Eq.(8.4.9) become

$$\begin{aligned} 0 =& -\tau^2 R_{mn} + 3\frac{\tau^2}{w}\nabla_m\partial_n w + \frac{3}{4}\frac{\tau}{w}g_{mn}(\partial w)(\partial\tau) + \frac{1}{4}g_{mn}\tau\nabla^2\tau \\ &+ \frac{1}{4}g_{mn}(\partial\tau)^2 + 2\tau\nabla_m\partial_n\tau - 2(\partial_m\tau)(\partial_n\tau) \\ &+ \frac{1}{2}\tau^2\Big(|H_3|^2_{mn} - \frac{1}{4}g_{mn}|H_3|^2\Big) + \frac{1}{2}\sum_{p=0}^{6}\Big(|F_p|^2_{mn} - \frac{p-1}{8}g_{mn}|F_p|^2\Big) \\ &+ \mu_6\sum_i\Big(\Pi_{i,mn} - \frac{7}{8}g_{mn}\Big)\tau\delta(\pi_i) - 2^{-4}\mu_2\frac{3}{8}g_{mn}\tau\delta(\pi)\,, \end{aligned} \tag{5.2.19}$$

for $|F_p|^2_{mn} = \frac{1}{(p-1)!} F_{m\mu_2\ldots\mu_p} F_n^{\mu_2\ldots\mu_p}$. Since in the case of our interest O2/D2 sources fill the external space, the projector Π_{mn} for them is zero.

Smearing the sources

Having found the localized equations of motion we can directly find the ones in the smeared approximation. In this approximation the sources take the form

$$\delta(\pi_i) \to j_{\pi_i} = \frac{\mathcal{V}_{\pi_i}}{\mathcal{V}_7} = \frac{\int_{\pi_i} \mathrm{d}^4 y \sqrt{g_{\pi_i}}}{\int \mathrm{d}^7 y \sqrt{g_7}} \,, \qquad \delta_{i,3} \to j_{i,3} = \frac{\mathrm{vol}_{\tilde{\pi}_i}}{\mathcal{V}_{\tilde{\pi}_i}} = \frac{\mathrm{vol}_{\tilde{\pi}_i}}{\int_{\tilde{\pi}_i} \mathrm{d}^3 y \sqrt{g_{\tilde{\pi}_i}}} \,, \tag{5.2.20}$$

with the three-form volume density given by $\mathrm{vol}_{\tilde{\pi}_i} = \sqrt{g_{\tilde{\pi}_i}} \mathrm{d}y^i \wedge \mathrm{d}y^j \wedge \mathrm{d}y^k = e^i \wedge e^j \wedge e^k$, where i, j, k are directions transverse to the O6-plane. For clarity we note that in our work π_i refer to four-cycles and $\tilde{\pi}_i$ to the corresponding/dual three-cycles; we will specify these once we turn to the G2 example. Thus each smeared source that enters the Bianchi is normalized with respect to its own three-cycle volume. The $\mathcal{V}_7$ is the internal space volume, which we will explicitly define for our example later. We accompany the smeared approximation by the following additional assumptions, which will be justified by the equations of motion:

- The warp factor $w(y)$ of the external space as well as the dilaton $\tau(y)$ are slowly varying with respect to the internal coordinates and can be considered to be constant $w(y) \equiv const.$ and $\tau(y) \equiv const.$
- The background field strengths satisfy $\mathrm{d}F_n = 0 = \mathrm{d}\star F_n$ and similarly for the H-flux, and are thus expanded on the harmonic forms of the 7d internal space, while the latter is chosen to be Ricci-flat, that is $R_{mn} = 0$.

The equations of motion of the dilaton in Eq.(5.2.15) in the smeared approximation simplify to

$$0 = 2\frac{\tau}{w^2} R_3 - \tau |H_3|^2 + 2\mu_6 \sum_i j_{\pi_i} \,. \tag{5.2.21}$$

The Einstein equation in Eq.(5.2.18) becomes

$$0 = -\frac{\tau^2}{w^2} R_3 - \frac{3}{8}\tau^2 |H_3|^2 - \frac{3}{2} \sum_{p=0}^{6} \frac{p-1}{8} |F_p|^2 + \frac{3}{8}\mu_6 \tau \sum_i j_{\pi_i} \,, \tag{5.2.22}$$

and the trace-reversed Einstein equations of Eq.(5.2.19) reduce to

$$\begin{aligned} 0 = {} & \frac{1}{2}\tau^2 \Big(|H_3|^2_{mn} - \frac{1}{4} g_{mn} |H_3|^2 \Big) + \frac{1}{2} \sum_{p=0}^{6} \Big(|F_p|^2_{mn} - \frac{p-1}{8} g_{mn} |F_p|^2 \Big) \\ & + \mu_6 \sum_i \Big(\Pi_{i,mn} - \frac{7}{8} g_{mn} \Big) \tau j_{\pi_i} \,. \end{aligned} \tag{5.2.23}$$

Considering our assumptions for the smeared localized objects and the harmonic expansion of the field strength forms, the smeared Bianchi identities of Eqs.(5.2.12)-(5.2.14) become

$$0 = H_3 \wedge F_0 - 2N_{\mathrm{O6}} \sum_i^7 j_{i,3} + N_{\mathrm{D6}} \sum_i^7 j_{i,3}\,, \tag{5.2.24}$$

$$0 = H_3 \wedge F_2\,, \tag{5.2.25}$$

$$0 = H_3 \wedge F_4 - 2^{-3} N_{\mathrm{O2}} j_7 + N_{\mathrm{D2}} j_7\,. \tag{5.2.26}$$

Here j_7 is the seven dimensional form of the internal space because the O2-planes fill the full three-dimensional external one. At this point, we also impose by fiat $F_6 = 0$. When we specialize to the case of G2 holonomy, this will be justified by the absence of six-cycles.

The F_4 background flux actually splits into two parts

$$F_4 = F_{4A} + F_{4B} = \sum_i \left(f^i + \hat{f}^i \right) \Psi_i\,, \tag{5.2.27}$$

where, postponing further details for later, we only note that

$$\Psi_i = \text{basis of harmonic four-forms of the internal space}\,. \tag{5.2.28}$$

The H-flux is also expanded on the harmonic three-forms of the internal space and takes the form

$$H_3 = \sum_i h^i \Phi_i\,, \quad \Phi_i = \text{basis of harmonic three-forms of the internal space}\,. \tag{5.2.29}$$

The F_4 splitting refers to the way the RR-flux wedges with the H-flux, that is

$$H_3 \wedge F_{4A} \equiv 0\,, \qquad H_3 \wedge F_{4B} = 2^{-3} N_{\mathrm{O2}} j_7 - N_{\mathrm{D2}} j_7\,. \tag{5.2.30}$$

The term $H_3 \wedge F_{4A}$ vanishes by construction, leaving the f^i unconstrained, except for quantization conditions. Meanwhile, the second equation can either be satisfied by balancing the fluxes terms against the smeared source terms, or by demanding that $H_3 \wedge F_{4B}$ vanish independently by setting $F_{4B} = 0$ (or equivalently $\hat{f}^i = 0$). In the latter case, we require a net charge cancellation between the D2-branes and O2-planes, i.e. $N_{\mathrm{O2}} = 8N_{\mathrm{D2}}$.

The integral of $\mathrm{d}F_6$ over the internal closed manifold is zero and the tadpole relation is satisfied for fixed "orientation" of the F_{4A} flux while at the same time its magnitude remains unbounded. In the case when the D2/O2 cancellation happens, we have

$$\int_7 \mathrm{d}F_6 = 0\,, \qquad \hat{f}^i = 0\,, \qquad \sum_i h^i f^i = 0\,, \qquad \sum_i f^i f^i = \text{free}\,, \qquad 0 = 16 - N_{\mathrm{D2}}\,, \tag{5.2.31}$$

always for properly quantized flux coefficients h^i and f^i. Scale separation can be achieved parametrically in the limit of *large* f^i, that is

$$\sum_i f^i f^i \gg 1 \quad \Rightarrow \quad \text{separation of KK and AdS scales}\,, \tag{5.2.32}$$

therefore it is not prohibited by flux quantization. The appropriate flux quantization can be found in 4.4.6. When there is no net D2/O2 cancellation one has to consider the appropriate amount of D2-branes because the last equation in (5.2.31) is altered to $\sum_i h^i \hat{f}^i = 2^{-3} N_{\rm O2} - N_{\rm D2}$. For the rest of the article we will have

$$F_4 \equiv F_{4A} \ , \quad \text{unless otherwise noted,} \tag{5.2.33}$$

so that we do not clutter the formulas.

5.2.3 Scaling of the fields

Now we use the *smeared* equations of motion with net D2/O2 cancellation that we found in the previous subsection and require each term in the equations to have the same scaling. As expansion parameter of the fluxes we use the parameter n, and we will see that the smeared equations of motion are invariant under its variation. The expansion parameter can have a physical interpretation as the vacuum expectation value of some field or flux, and it will later serve as our expansion parameter when we evaluate the backreaction. The fact that the smeared solution leaves the n undetermined means that we can make it parametrically large so that we can have a good control over the corrections.

To start we assume that the metric of the internal space has the following scaling at smeared level

$$g_{mn} \sim n^a \, . \tag{5.2.34}$$

Then we consider the smeared O6-plane sources in Eq.(5.2.20) which enter the Bianchi identity and the Einstein equations, and we find the following scaling

$$j_{\pi_i} \sim \frac{\sqrt{g_4}}{\sqrt{g_7}} \sim n^{-\frac{3}{2}a} \, , \qquad j_{i3} \sim \frac{\sqrt{g_3}}{\sqrt{g_3}} \sim n^0 \, . \tag{5.2.35}$$

We notice that the smeared O6-planes which enter the Einstein equations have the same scaling as the ones in the 4d compactification on a Calabi–Yau [106]. This happens because the difference of the dimensions between the wrapped volume and the internal space is the same in both cases $j_{\pi_i} \sim \sqrt{g_3}/\sqrt{g_6} \sim \sqrt{g_4}/\sqrt{g_7}$. The dual current of the wrapped cycles $j_{i,3}$ is a three-form and therefore it has no scaling because it does not depend on the metric. The next step is to consider the dilaton and the Einstein equations of motion as well as the Bianchi identities to find the scaling of the fluxes. We will work with the ansatz

$$F_0 \sim n^c \ , \ F_4 \sim n^f \ , \ H_3 \sim n^b \ , \ \tau \sim n^t \ , \ w \sim n^w \, . \tag{5.2.36}$$

Moreover, the square of a form of n-rank has the following scaling

$$|F_p|^2 = \frac{1}{p!} g^{a_1 a_1'} \ldots g^{a_p a_p'} F_{a_1 \ldots a_p} F_{a_1' \ldots a_p'} \sim n^{-p \times a} \times n^{2\tilde{k}} \, , \tag{5.2.37}$$

where the $\tilde{k}$ is the RR or NSNS flux, therefore $\tilde{k} = c, f, b$ for our case. Let us first check the Bianchi identities which will define the scaling of the RR and H fluxes. From the first Bianchi identity in Eq.(5.2.12) we get

$$b + c = 0\,, \tag{5.2.38}$$

since the smeared source in the Bianchi is not scaling. The second Bianchi in Eq.(5.2.13) does not give us any scaling information, and the same goes for the third equation in Eq.(5.2.14), because the specific combinations of fluxes vanish. From equation (5.2.22) we find the following scaling relation

$$2\tau - 2w = 2\tau - 3a + 2b = 2c = -4a + 2f = \tau - \frac{3}{2}a\,, \tag{5.2.39}$$

and from the traced Einstein equations (5.2.23) we find

$$2\tau - 2a + 2b = a + 2c = -3a + 2f = \tau - \frac{1}{2}a\,. \tag{5.2.40}$$

Then the dilaton equations of motion in Eq.(5.2.21) give the following scaling relation

$$t - 2w = t - 3a + 2b = -\frac{3}{2}a\,. \tag{5.2.41}$$

Solving (5.2.39)-(5.2.41) and (5.2.12) we get

$$a \to -\frac{2}{3}t + \frac{4}{3}w\ , \quad b \to -t + w\ , \quad c \to t - w\ , \quad f \to -\frac{1}{3}t + \frac{5}{3}w\,. \tag{5.2.42}$$

We need an extra condition to find the proper scaling and this comes from the Romans mass, F_0, which has no scaling because it is a quantized constant, thus $c = 0$. The parametric scaling of the fluxes at smeared/leading order then is

$$F_4 \sim n\,, \quad F_0 \sim n^0\,, \quad H_3 \sim n^0\,, \quad \tau \sim n^{\frac{3}{4}}\,, \quad w \sim n^{\frac{3}{4}}\,, \quad g_{mn} \sim n^{\frac{1}{2}}\,, \tag{5.2.43}$$

which is the same scaling as in [106]. Another way to see this scaling would be to impose the dilaton and the warp factor to have the same scaling $n^t = n^w$, which would then fix the Romans mass to $c = 0$. It is gratifying to see that the scaling of the fluxes we found here from analysing the full higher-dimensional equations does actually agree with the one found in subsection 4.4.1 where the low-energy effective theory was instead analyzed.

When the flux F_{4B} is not zero its wedge with H_3 has to be cancelled by a non-vanishing O2/D2 charge in the Bianchi identity. From the variation of the dilaton, including now the net O2/D2 contribution, we find

$$0 = 2\frac{\tau}{w^2}R_3 - \tau|H_3|^2 + 2\mu_6 \sum_i j_{\pi_i} + 2^{-3}\mu_2 j_\pi\,. \tag{5.2.44}$$

Performing the scaling analysis for the smeared sources we see that $j_{\pi_i} \sim n^{-3a/2}$ and $j_\pi \sim n^{-7a/2}$, and requiring the equation to be invariant under the $1/n$ scaling we see that the scaling of the metric as well as the rest of the fields have to be zero. From the Bianchi identity in Eq.(5.2.30) and considering the scaling of $H_3 \sim n^0$ and $j_7 \sim n^0$ we directly see that

$$F_{4B} \sim n^0 \,. \tag{5.2.45}$$

We will later discuss the contribution of the O2/D2 and F_{4B} in the potential and see how they affect the smeared potential.

5.2.4 Next to leading order equations of motion

In this subsection we expand the RR, NSNS fields and the warp factor in terms of a scaling parameter n, which can be interpreted as tracking the leading order scaling of the F_{4A} flux responsible for the scale separation. The fields in the smeared approximation are the leading order terms of a $1/n^p$ expansion. We then perform the $1/n^p$ expansion to find the first order equations of motion. The power p for each field, i.e. the scaling rate of the next to leading order terms, is not uniquely dictated by the system of equations we have at our disposal. However, with a proper ansatz we can calculate all the next to leading order RR flux corrections. Our ansatz is

$$F_6 = F_6^{(0)} n + F_6^{(1)} n^0 + \mathcal{O}(n^{-1}) \,, \tag{5.2.46}$$
$$F_4 = F_4^{(0)} n + F_4^{(1)} n^0 + \mathcal{O}(n^{-1}) \,, \tag{5.2.47}$$
$$F_2 = F_2^{(0)} n^{1/2} + F_2^{(1)} n^0 + \mathcal{O}(n^{-1/2}) \,, \tag{5.2.48}$$
$$H_3 = H_3^{(0)} n^0 + H_3^{(1)} n^{-1} + \mathcal{O}(n^{-2}) \,, \tag{5.2.49}$$
$$\tau = \tau^{(0)} n^{3/4} + \tau^{(1)} n^{-1/4} + \mathcal{O}(n^{-5/4}) \,, \tag{5.2.50}$$
$$w = w^{(0)} n^{3/4} + w^{(1)} n^{-1/4} + \mathcal{O}(n^{-5/4}) \,, \tag{5.2.51}$$
$$g_{mn} = g_{mn}^{(0)} n^{1/2} + g_{mn}^{(1)} n^{-1/2} + \mathcal{O}(n^{-3/2}) \,. \tag{5.2.52}$$

Starting with the Bianchi identities, we expand the fluxes in Eq.(5.2.12) at first order and we get

$$\mathrm{d}\Big(F_2^{(0)} n^{1/2} + F_2^{(1)} + \dots\Big) = \Big(H_3^{(0)} + H_3^{(1)} n^{-1} + \dots\Big) \wedge F_0^{(0)} - 2\mu_6 \sum_i \delta_{i,3} \,, \tag{5.2.53}$$

where at leading order we recover the smeared expression along with the first order correction of the Bianchi identity

$$\mathrm{d}F_2^{(0)} = 0 \,, \tag{5.2.54}$$
$$\mathrm{d}F_2^{(1)} = H_3^{(0)} \wedge F_0^{(0)} - 2\mu_6 \sum_i \delta_{i,3} \,. \tag{5.2.55}$$

For the Bianchi identity in Eq.(5.2.13) we get

$$\mathrm{d}\Big(F_4^{(0)} n^1 + F_4^{(1)} n^0 \dots\Big) = \Big(H_3^{(0)} n^0 + H_3^{(1)} n^{-1} + \dots\Big) \wedge \Big(F_2^{(0)} n^{1/2} + F_2^{(1)} n^0 + \dots\Big) \,, \tag{5.2.56}$$

from which we deduce

$$\mathrm{d}F_4^{(0)} = 0\,, \tag{5.2.57}$$

$$\mathrm{d}F_4^{(1)} = H_3^{(0)} \wedge F_2^{(1)} \neq 0\,. \tag{5.2.58}$$

We notice that the first order correction of this Bianchi contains the RR two-form correction $F_2^{(1)}$ whose exact form is calculated in the next section using the Einstein equations. For the Bianchi identity in Eq.(5.2.14) we have

$$\mathrm{d}\Big(F_6^{(0)}n + F_6^{(1)}n^0 ...\Big) = \Big(H_3^{(0)}n^0 + H_3^{(1)}n^{-1} + ...\Big) \wedge \Big(F_4^{(0)}n + F_4^{(1)}n^0 + ...\Big) - 2^{-3}\mu_2\delta_7\,. \tag{5.2.59}$$

In our case of interest, we will set $F_{4B} = 0$ and we also consider vanishing net O2/D2 charge. In addition $H_3^{(0)} \wedge F_{4A}^{(0)}$ and $F_6^{(0)}$ vanish in the smeared approximation. Then we have

$$\mathrm{d}F_6^{(0)} = H_3^{(0)} \wedge F_{4A}^{(0)} = 0\,, \tag{5.2.60}$$

$$\mathrm{d}F_6^{(1)} = H_3^{(0)} \wedge F_{4A}^{(1)} + H_3^{(1)} \wedge F_{4A}^{(0)} \neq 0\,. \tag{5.2.61}$$

At leading order the orientation of the fluxes leads to the desired cancellation, while at subleading order we can always set

$$\int_7 \mathrm{d}F_6^{(1)} = \int_7 \Big(H_3^{(0)} \wedge F_{4A}^{(1)} + H_3^{(1)} \wedge F_{4A}^{(0)}\Big) = 0\,, \tag{5.2.62}$$

by adjusting the harmonic parts of $F_{4A}^{(1)}$ and $H_3^{(1)}$ such that no new sources are required for the tadpole cancellation.

We now turn to the first order expression of Einstein and dilaton equations of motion Eq.(5.2.15)-(5.2.19). The dilaton equation is

$$\begin{aligned} 0 =& -8\nabla^2\tau^{(1)} + 2\frac{\tau^{(0)}}{(w^{(0)})^2}R_3 - 12\frac{\tau^{(0)}}{w^{(0)}}\nabla_m\nabla^m w^{(1)} + 2\tau^{(0)}R^{(1)}_{mn}g^{(0)mn} - \tau^{(0)}|H_3^{(0)}|^2 \\ &+ 2\mu_6\sum_i \delta(\pi_i)\,, \end{aligned} \tag{5.2.63}$$

and the next to leading order expansion of the Einstein equation in Eq.(5.2.18) is

$$\begin{aligned} 0 =& -\frac{(\tau^{(0)})^2}{(w^{(0)})^2}R_3 + 3\frac{(\tau^{(0)})^2}{w^{(0)}}\nabla^2 w^{(1)} + \frac{3}{4}\tau^{(0)}\nabla^2\tau^{(1)} \\ &- \frac{3}{8}(\tau^{(0)})^2|H_3^{(0)}|^2 - \frac{3}{2}\sum_{p=0}^{4}\frac{p-1}{8}|F_p^{(0)}|^2 + \frac{3}{8}\mu_6\tau^{(0)}\sum_i \delta(\pi_i)\,. \end{aligned} \tag{5.2.64}$$

Next, the first order correction to the trace reversed Einstein equation in Eq.(5.2.19) becomes

$$\begin{aligned} 0 =& - (\tau^{(0)})^2 R^{(1)}_{mn} + 3\frac{(\tau^{(0)})^2}{w^{(0)}}\nabla_m\partial_n w^{(1)} + \frac{1}{4}g^{(0)}_{mn}\tau^{(0)}\nabla^2\tau^{(1)} + 2\tau^{(0)}\nabla_m\partial_n\tau^{(1)} \\ &+ \frac{1}{2}(\tau^{(0)})^2\Big(|H^{(0)}_3|^2_{mn} - \frac{1}{4}g^{(0)}_{mn}|H^{(0)}_3|^2\Big) + \frac{1}{2}\sum_{p=0}^{4}\Big(|F^{(0)}_p|^2_{mn} - \frac{p-1}{8}g^{(0)}_{mn}|F^{(0)}_p|^2\Big) \\ &+ \mu_6\sum_i\Big(\Pi^{(0)}_{i,mn} - \frac{7}{8}g^{(0)}_{mn}\Big)\tau^{(0)}\delta(\pi_i)\ . \end{aligned} \tag{5.2.65}$$

We combine the smeared and the first order equations of motion to find the following relations for the RR, the dilaton and the warping

$$\mathrm{d}F^{(1)}_2 = 2\mu_6\sum_i(j_{i,3} - \delta_{i,3})\,, \tag{5.2.66}$$

$$\nabla^2\tau^{(1)} = -\frac{3}{2}\mu_6\sum_i(j_{\pi_i} - \delta(\pi_i))\,, \tag{5.2.67}$$

$$\nabla^2 w^{(1)} = \frac{1}{2}\frac{w^{(0)}}{\tau^{(0)}}\mu_6\sum_i(j_{\pi_i} - \delta(\pi_i))\,. \tag{5.2.68}$$

For the backreaction on the internal metric we have

$$\tau^{(0)}R^{(1)}_{mn} - 3\frac{\tau^{(0)}}{w^{(0)}}\nabla_m\partial_n w^{(1)} - 2\nabla_m\partial_n\tau^{(1)} = \mu_6\sum_i\Big(\frac{1}{2}g^{(0)}_{mn} - \Pi^{(0)}_{i,mn}\Big)(j_{\pi_i} - \delta(\pi_i))\ . \tag{5.2.69}$$

These equations determine the backreaction of the localized sources on the solution from the smeared approximation and can be used in different setups. To proceed further we need to work on a specific example therefore we focus on a G2 orientifold.

5.3 The G2 orbifold example

5.3.1 Calculation for a single O6-plane

The internal space

So far we have found the formal expressions for the first order corrections to some of the fields using just the presence of O6-planes and the dimensions of the internal space. To find the exact form of the corrections at first order we need to specify the internal geometry and solve Eqs. (5.2.66)-(5.2.69). We consider again the toroidal orbifold $T^7/(Z_2\times Z_2\times Z_2)$ and the structure properties of the G2.

The diagonal metric of the internal space and the metric elements can be written as

$$\mathrm{d}s^2_7 = \sum_m^7(r_m)^2\mathrm{d}y^m\mathrm{d}y^m\,, \qquad g^{(0)}_{ij} \equiv (r^{(0)}_m)^2 n^{1/2}\ , \qquad i=j=1,...,7\,. \tag{5.3.1}$$

For more details on this orbifold, and a series of different applications, see e.g. [47]. Considering the $1/n$ expansion form the metric in Eq.(5.2.52) the radii get corrections

$$r_m = r_m^{(0)} n^{1/4} + r_m^{(1)} n^{-3/4} + \mathcal{O}(n^{-3/2})\,. \tag{5.3.2}$$

Corrections to the RR flux

In order to proceed and calculate the first order corrections to the fluxes one should solve the equations in Eq.(5.2.66)-(5.2.69) for the seven intersected O6-planes in Eq.(4.3.51)-(4.3.58). As a first step we solve the equations with the presence of a single O6-plane, indicatively we choose the O6$_\alpha$-plane with involution given by Eq.(4.3.51) which wraps the four-cycle π_3. We start from the Bianchi in Eq.(5.2.66) in order to calculate the RR fluxes and we write it in terms of the internal geometry basis

$$\mathrm{d}F_2^{(1)} = 2\rho_3 \Big(\mathrm{d}y^5 \wedge \mathrm{d}y^6 \wedge \mathrm{d}y^7\Big) = -2\rho_3\, \Phi_3\,, \tag{5.3.3}$$

where the ρ_3 refers to the appropriate "backreaction density". For the specific O6-plane Eq.(4.3.51) which wraps the π_3, this backreaction density term is

$$\rho_3 = \mu_6 \Big(j_{\pi_3} - \delta(\pi_3)\Big) = \mu_6 \left\{ 1 - \frac{1}{N_{\mathrm{O6}}} \sum_{m\in\{0,1\}} \delta\Big(y^5 - \frac{m}{2}\Big)\delta\Big(y^6 - \frac{m}{2}\Big)\delta\Big(y^7 - \frac{m}{2}\Big) \right\}. \tag{5.3.4}$$

To avoid clutter we do not include the subscript "3" in ρ_3 in this part because it is always implied. As we will verify momentarily, an inspection of Eq.(5.3.3) leads us to guess that the F_2 is of the form

$$F_2^{(1)} = -2 \star_7 (\mathrm{d}\beta_3 \wedge \Psi_3)\,. \tag{5.3.5}$$

Here we have introduced the function $\beta_3 \equiv \beta_3(y)$ which as we will see satisfies a Poisson equation and it will be further specified in the next section. For the few next steps we suppress the subscript 3 to avoid clutter. Indeed the derivative on (5.3.5) gives

$$\mathrm{d}F_2^{(1)} = -2(\nabla^2\beta)\Phi_3\,, \tag{5.3.6}$$

which can be verified with the following series of steps

$$\mathrm{d}(\star_7(\mathrm{d}\beta\wedge\Psi_3)) = \mathrm{d}(\star_7\mathrm{d}(\beta\wedge\Psi_3)) = \star_7((\nabla^2\beta)\Psi_3) = \nabla^2\beta(\star_7\Psi_3) = (\nabla^2\beta)\Phi_3\,. \tag{5.3.7}$$

This is easily seen by the fact that $\star\mathrm{d}\star\mathrm{d}(\beta\Psi_3) = \nabla^2(\beta\Psi_3)$. Comparing this to Eq.(5.3.3) we get a Poisson equation for β that reads

$$\nabla^2\beta = \rho\,. \tag{5.3.8}$$

Similar to [106] the transverse space at each point on the O6-plane is a three-torus. Note that because $F_2^{(1)}$ is not closed we do not need to expand it on harmonic cycles. From (5.3.5) we see however that $\sigma : F_2^{(1)} \to -F_2^{(1)}$ so it is odd, as it should be, and that $\sigma_\alpha : F_2^{(1)} \to -F_2^{(1)}$ so it is again odd as it should be under the O6 involutions, and finally that $\Theta_\alpha : F_2^{(1)} \to F_2^{(1)}$ therefore it is invariant under the orbifold (as it should be). The parities under the other orbifold/orientifold involutions can also be checked to be consistent. It is also straightforward to check using (5.3.5) that $\mathrm{d} \star_7 F_2^{(1)} = 0$ which means that the equation of motion for F_2 (that is $\mathrm{d} \star_{10} F_2 + H_3 \wedge \star_{10} F_4 = 0$) is satisfied to leading order in the $1/n$ expansion.

Since we have found the explicit form of $F_2^{(1)}$ we are able to calculate the first order corrections to the rest of the RR forms. Using the Bianchi identity in Eq.(5.2.58) the later becomes

$$\mathrm{d}F_4^{(1)} = H_3^{(0)} \wedge F_2^{(1)} = \mathrm{d}\Big(-2 \sum_i h^i \Psi_i \wedge \beta(y) \Big) . \tag{5.3.9}$$

This can be seen from the following steps

$$\begin{aligned} H_3^{(0)} \wedge F_2^{(1)} &= -\sum_i h^i \Phi_i \wedge \star_7 \mathrm{d}\Big(\beta(y) \wedge \sum_j \Psi_j\Big) = \sum_i h^i \Phi_i \wedge \star_7 \mathrm{d}\Big(\Psi_i \wedge \beta(y)\Big) \\ &= -2 \sum_i h^i \Psi_i \wedge \mathrm{d}\beta(y) = \mathrm{d}\Big(-2 \sum_i h^i \Psi_i \wedge \beta(y) \Big) . \end{aligned} \tag{5.3.10}$$

Thus the co-closed part of F_4 which appears beyond the smeared approximation is

$$F_4^{(1)} = -2\beta(y) \sum_i h^i \Psi_i . \tag{5.3.11}$$

Once more, for distances far from the source the $F_4^{(1)}$ becomes negligible as expected in the smeared limit and this becomes clear when we calculate the explicit form of $\beta(y)$. Adding the harmonic part we have

$$F_4^{(1)} = G^i \Psi_i - 2\beta(y) \sum_i h^i \Psi_i , \tag{5.3.12}$$

where G^i is the corrected flux and can be chosen to be

$$G^i = 2h^i \int_{\Psi_i} \mathrm{d}^4 y \beta(y) . \tag{5.3.13}$$

Thus from Eq.(5.2.61) we have

$$\begin{aligned} \mathrm{d}F_6^{(1)} = H_3^{(0)} \wedge F_4^{(1)} + H_3^{(1)} \wedge F_4^{(0)} &= H_3^{(0)} \wedge \Big(G^i \Psi_i - 2\beta(y) \sum_i h^i \Psi_i \Big) + H_3^{(1)} \wedge F_4^{(0)} \\ &= h^i \Phi_i \wedge \Big(G^j \Psi_j - 2\beta(y) \sum_j h^j \Psi_j \Big) + H_3^{(1)} \wedge F_4^{(0)} . \end{aligned} \tag{5.3.14}$$

As we will show in the next subsection, $H_3^{(1)}$ is also fully specified by the function β, up to harmonic pieces, which can be tuned to ensure $\int \mathrm{d}F_6^{(1)} = 0$.

Corrections to the dilaton, warp factor, NS flux and the metric

So far we used the Bianchi identity of $F_2^{(1)}$ and the internal geometry in order to specify the explicit form of all the first order corrections of the RR fluxes. However we have not found yet the exact first order corrections to the dilaton, the warp factor and the internal metric.

In order to solve Eq.(5.2.69) and identify the first order corrections to the remaining fluxes we start from the following definition of the Ricci tensor of the internal space

$$R^{(1)}_{mn} = -\frac{1}{2}g^{(0)rs}\nabla_m\nabla_n g^{(1)}_{rs} + \frac{1}{2}g^{(0)rs}\Big(\nabla_s\nabla_m g^{(1)}_{rn} + \nabla_s\nabla_n g^{(1)}_{rm}\Big) - \frac{1}{2}\nabla^2 g^{(1)}_{mn}\,, \tag{5.3.15}$$

and the relation for the Ricci tensor R_{mn} from Eq.(5.2.69) we have

$$R^{(1)}_{mn} = \frac{3}{w^{(0)}}\nabla_m\partial_n w^{(1)} + \frac{2}{\tau^{(0)}}\nabla_m\partial_n\tau^{(1)} + \frac{1}{\tau^{(0)}}\sum_i\left(\frac{1}{2}g^{(0)}_{mn} - \Pi^{(0)}_{i,mn}\right)\frac{\sqrt{g_{\pi_i}}}{\sqrt{g_7}}\rho_i\,, \tag{5.3.16}$$

where ρ_i refers to the appropriate backreaction density for the i-th cycle. Combining Eq.(5.3.15) and Eq.(5.3.16) we get the equation

$$\begin{aligned}
&-\frac{1}{2}g^{(0)rs}\nabla_m\nabla_n g^{(1)}_{rs} + \frac{1}{2}g^{(0)rs}\Big(\nabla_s\nabla_m g^{(1)}_{rn} + \nabla_s\nabla_n g^{(1)}_{rm}\Big) - \frac{1}{2}\nabla^2 g^{(1)}_{mn} \\
&= \frac{3}{w^{(0)}}\nabla_m\partial_n w^{(1)} + \frac{2}{\tau^{(0)}}\nabla_m\partial_n\tau^{(1)} + \frac{1}{\tau^{(0)}}\sum_i\left(\frac{1}{2}g^{(0)}_{mn} - \Pi^{(0)}_{i,mn}\right)\frac{\sqrt{g_{\pi_i}}}{\sqrt{g_7}}\rho_i\,.
\end{aligned} \tag{5.3.17}$$

Focusing now on the 3rd cycle (and again suppressing the subscript on ρ_3 and β_3), we write the volume of the four-cycles wrapping the internal space and the current of the smeared source

$$\mathcal{V}_{\pi_3} = r^{(0)}_1 r^{(0)}_2 r^{(0)}_3 r^{(0)}_4\,, \qquad j_{\pi_3} = \frac{1}{r^{(0)}_5 r^{(0)}_6 r^{(0)}_7}\,. \tag{5.3.18}$$

Next we calculate the Ricci tensor for cases depending on parallel, transverse and mixed leg components. For the calculation we make the following assumption

$$g^{(0)11}g^{(1)}_{11} = g^{(0)22}g^{(1)}_{22} = g^{(0)33}g^{(1)}_{33} = g^{(0)44}g^{(1)}_{44}\,, \qquad g^{(0)55}g^{(1)}_{55} = g^{(0)66}g^{(1)}_{66} = g^{(0)77}g^{(1)}_{77}\,. \tag{5.3.19}$$

First, when both the legs of the Ricci tensor are along the wrapped cycle, the stress-energy tensor in Eq.(5.2.17) gets the simple form

$$\Pi_{3,mn} = (g_{\pi_3})_{mn}\,, \tag{5.3.20}$$

for m, n the directions of the wrapped four-cycle. The O6-plane wrapping the π_3 is parallel to the directions y^1, y^2, y^3, y^4 and the fields w, τ and g_{mn} are sourced by $\delta(y^5 - \hat{y}^5)\delta(y^6 - \hat{y}^6)\delta(y^7 - \hat{y}^7)$ which depend only on the transverse y^5, y^6, y^7 directions. We label the wrapped directions with with indices i, j and investigate first the case where the components are parallel and same, the relation Eq.(5.3.17) gives the following solution

$$\nabla^2 g_{ii}^{(1)} = \frac{(r_i^{(0)})^2}{r_5^{(0)} r_6^{(0)} r_7^{(0)} \tau^{(0)}} \rho \,, \qquad i = j = 1, 2, 3, 4 \,. \tag{5.3.21}$$

Now let us check the Ricci tensor for transverse and same directions, R_{kl} with $k = l = 5, 6, 7$

$$\begin{aligned} &- 2g^{(0)11} \partial_5 \partial_5 g_{11}^{(1)} - \frac{1}{2} g^{(0)55} \partial_5 \partial_5 g_{55}^{(1)} - \frac{1}{2} \nabla^2 g_{55}^{(1)} \\ &= \frac{3}{w^{(0)}} \partial_5 \partial_5 w^{(1)} + \frac{2}{\tau^{(0)}} \partial_5 \partial_5 \tau^{(1)} + \frac{1}{2} \frac{r_5^{(0)}}{r_6^{(0)} r_7^{(0)} \tau^{(0)}} \rho \,. \end{aligned} \tag{5.3.22}$$

For one parallel and one transverse direction, R_{jk}, the equation is trivially satisfied. For two different transverse directions, i.e. R_{kl} with $k \neq l$, we can work-out for example the case R_{56} which gives

$$0 = -\frac{3}{w^{(0)}} \partial_5 \partial_6 w^{(1)} - 2g^{(0)11} \partial_5 \partial_6 g_{11}^{(1)} - \frac{1}{2} g^{(0)55} \partial_5 \partial_6 g_{55}^{(1)} - \frac{2}{\tau^{(0)}} \partial_5 \partial_6 \tau^{(1)} \,. \tag{5.3.23}$$

From Eq.(5.3.22)-(5.3.23) we have

$$\nabla^2 g_{kk}^{(1)} = -\frac{(r_i^{(0)})^2}{r_5^{(0)} r_6^{(0)} r_7^{(0)} \tau^{(0)}} \rho \,, \qquad k = l = 5, 6, 7 \,, \tag{5.3.24}$$

and we write again the solution of the warp factor and the dilaton but expressed in terms of the ρ source

$$\nabla^2 \tau^{(1)} = -\frac{3}{2} \frac{1}{r_5^{(0)} r_6^{(0)} r_7^{(0)}} \rho \,, \tag{5.3.25}$$

$$\nabla^2 w^{(1)} = \frac{1}{2} \frac{w^{(0)}}{\tau^{(0)}} \frac{1}{r_5^{(0)} r_6^{(0)} r_7^{(0)}} \rho \,. \tag{5.3.26}$$

With the use of the same function $\beta(y)$ as in Eq.(5.3.8), and Eqs.(5.3.21), (5.3.22), (5.3.25) and (5.3.26), we get the relations

$$-\frac{g_{kk}^{(1)}}{r_i^{(0)2}} = \frac{g_{ii}^{(1)}}{r_i^{(0)2}} = -\frac{2\tau^{(1)}}{3\tau^{(0)}} = \frac{2w^{(1)}}{w^{(0)}} = \frac{1}{r_5^{(0)} r_6^{(0)} r_7^{(0)}} \frac{\beta(y^5, y^6, y^7)}{\tau^{(0)}} \,. \tag{5.3.27}$$

Note that until now $H_3^{(1)}$ was not required to solve for the leading correction to any other fields. On the other hand, the equation of motion for H_3 reads

$$\mathrm{d} \left(\tau^2 \star_{10} H_3 \right) = \star_{10} F_2 \wedge F_0 + \star_{10} F_4 \wedge F_2 \,, \tag{5.3.28}$$

which in our case becomes

$$(\tau^{(0)})^2 \mathrm{d}(\star_{10} H_3^{(1)}) = \mathrm{d}(\tau^{(0)})^2 \wedge \star_{10} H_3^{(0)} + \star_{10} F_0 \wedge F_2^{(1)} + \star_{10} F_4^{(0)} \wedge F_2^{(1)} , \tag{5.3.29}$$

and involves the leading corrections to several other fields including the warp factor (from the Hodge star inside the derivative on the left-hand side). All the corrections to the fields involved in this equation are related to β in such a way that the final equation for for H_3 takes the form

$$\nabla^\rho H^{(1)}_{\rho\mu\nu} = \nabla^\rho(\beta) X_{\rho\mu\nu} , \tag{5.3.30}$$

where X is a harmonic 3-form (i.e. $\mathrm{d}X = 0 = \mathrm{d} \star_7 X$), which can be expanded on the Φ_i basis with coefficients that are determined by the other leading order corrections. This is sufficient to determine the precise expression for $H_3^{(1)}$, which takes the form

$$H_3^{(1)} = \beta X + H^i \Phi_i , \tag{5.3.31}$$

where H^i are constants of integration, which give us the freedom to ensure tadpole cancellation without new sources as in (4.4.48). Thus determining the β is sufficient to determine all the leading order backreaction, including the form of $F_6^{(1)}$ from (5.3.14).

5.3.2 Solution of Poisson equation

To solve the Poisson equation in Eq.(5.3.8) we mostly follow the steps of [106]. We introduce a formal solution in terms of Fourier series and estimate the backreaction, without specifying the regularization, because it is in any case independent of the choice.

We start from the β_3 and we suppress the subscript 3 for now as usual and we also take into account that $\mu_6 = N_{\mathrm{O6}} = 8$. This means we have to solve the equation

$$\nabla^2 \beta = 8 - \sum_{m,n,p\in\{0,1\}} \delta\Big(y^5 - \frac{m}{2}\Big)\delta\Big(y^6 - \frac{n}{2}\Big)\delta\Big(y^7 - \frac{p}{2}\Big) . \tag{5.3.32}$$

To solve this we expand β as

$$\beta = \sum_{m,n,k\in\{0,1\}} \phi_{mnp} , \tag{5.3.33}$$

where

$$\nabla^2 \phi_{mnk} = 1 - \delta\Big(y^5 - \frac{m}{2}\Big)\delta\Big(y^6 - \frac{n}{2}\Big)\delta\Big(y^7 - \frac{p}{2}\Big) . \tag{5.3.34}$$

We first look at one of the fixed points and use the Fourier transform of the delta distribution to get

$$1 - \delta(y^5)\delta(y^6)\delta(y^7) = 1 - \sum_{\vec{k}\in\mathbb{Z}^3} e^{2\pi i \vec{k}\cdot\vec{y}} = - \sum_{\vec{k}\in\mathbb{Z}^3\setminus\{0\}} e^{2\pi i \vec{k}\cdot\vec{y}} , \tag{5.3.35}$$

where $\vec{y}_{\pi_3} = (y^5, y^6, y^7)$ and we use the discrete Fourier transforms of the delta functions to respect the toroidal periodicity.From this we deduce that

$$\phi_{000} = \sum_{\vec{k}\in\mathbb{Z}^3\backslash\{0\}} \frac{1}{4\pi^2 k^2} e^{2\pi i \vec{k}\cdot\vec{y}}\,, \tag{5.3.36}$$

and similarly for the other ϕ_{mnk}. So the Poisson equation in (5.3.32) is solved for

$$\begin{aligned} \beta(y) &= \sum_{m_a\in\{0,1\}} \sum_{\vec{k}\in\mathbb{Z}^3\backslash\{0\}} \frac{1}{4\pi^2 k^2} e^{2\pi i \vec{k}\cdot(\vec{y}-\frac{\vec{m}}{2})} + \text{const.} \\ &= \sum_{\vec{k}\in\mathbb{Z}^3\backslash\{0\}} \frac{1}{2\pi^2 k^2} e^{4\pi i \vec{k}\cdot\vec{y}} + \text{const.} \end{aligned} \tag{5.3.37}$$

The notation is $\vec{m} = (m_5, m_6, m_7)$, $\vec{k} = (k_5, k_6, k_7)$ and $k^2 = k_5^2/r_5^{(0)2} + k_6^2/r_6^{(0)2} + k_7^2/r_7^{(0)2}$.

Since (5.3.37) is not convergent one may wish to regularize it by following [106,109–111], or by simply introducing a hard cut-off on the magnitude of the momenta $\vec{k}$. However, to estimate the backreaction of the O-planes we just need the behavior near one of the loci. This means we want to evaluate (5.3.37) at, say, $\vec{y} \to 0$. Clearly, near such point the impact of the other sources can be ignored and the divergence will be dominated only by the source at $\vec{y} = 0$. Therefore, near the source at $\vec{y} \to 0$, the equation (5.3.32) can be approximated by $\nabla^2\beta \simeq -\delta(\vec{y})$ which has the text-book solution $\beta \simeq r^5 r^6 r^7/(4\pi\sqrt{y^2})$. This means that near the O-plane we simply have a $1/|y|$ singularity. For completeness we can verify this intuitive behavior in the following way. We first define, $\hat{y}_i = y_i/\epsilon$ and $\kappa_i = \epsilon k_i/r_i$, such that

$$k^2 = g^{ij}k_i k_j = \frac{1}{\epsilon^2}(\kappa_5^2 + \kappa_6^2 + \kappa_7^2) \equiv \frac{1}{\epsilon^2}\kappa^2\,, \tag{5.3.38}$$

$$r^2 \equiv g_{ij}\hat{y}^i\hat{y}^j = \epsilon^2(r_5 y_5^2 + r_6 y_6^2 + r_7 y_7^2)\,. \tag{5.3.39}$$

Dropping finite contributions we can write

$$\beta(\vec{y}) = \sum_{\kappa_i\in(\epsilon/r_i)\mathbb{Z}\backslash\{0\}} \frac{\epsilon^2}{2\pi^2\kappa^2} e^{4\pi i \vec{\kappa}\cdot\hat{y}} = \frac{1}{\epsilon} \sum_{\kappa_i\in(\epsilon/r_i)\mathbb{Z}\backslash\{0\}} \frac{r_5 r_6 r_7}{2\pi^2\kappa^2} e^{4\pi i \vec{\kappa}\cdot\vec{y}} \Delta\kappa_5\Delta\kappa_6\Delta\kappa_7, \tag{5.3.40}$$

where $\Delta\kappa_i = \epsilon/r_i$. The near-brane limit is captured by sending $\epsilon \to 0$, in which case the sum becomes an integral and we obtain

$$\beta(\vec{y}) \to \frac{r_5 r_6 r_7}{2\pi^2\epsilon} \int d^3\kappa \frac{e^{4\pi i \vec{\kappa}\cdot\hat{y}}}{\kappa^2} = \frac{1}{4\pi}\frac{r_5 r_6 r_7}{\epsilon|\hat{y}|} = \frac{1}{4\pi}\frac{r_5 r_6 r_7}{r}\,. \tag{5.3.41}$$

Where the first equality follows from recognizing the Fourier transform of the Coulomb potential.[1] This fixes the behavior of β near a single O6-plane.

[1] Note that carrying out this Fourier transform properly also requires the use of a regularization scheme.

From the β behavior derived in (5.3.41) and the relation in Eq.(5.3.27) we can see that the first order correction on the fields near the locus of a *single* O6-plane is

$$\tau = \tau^{(0)} n^{3/4} - \frac{3}{8\pi r} n^{-1/4} + \mathcal{O}(n^{-5/4}) \,, \tag{5.3.42}$$

$$w = w^{(0)} n^{3/4} + \frac{w^{(0)}}{\tau^{(0)}} \frac{1}{8\pi r} n^{-1/4} + \mathcal{O}(n^{-5/4}) \,, \tag{5.3.43}$$

$$g_{kk} = g^{(0)}_{kk} n^{1/2} - \frac{r_i^{(0)2}}{\tau^{(0)}} \frac{1}{4\pi r} n^{-1/2} + \mathcal{O}(n^{-3/2}) \,, \qquad k = 5,6,7 \tag{5.3.44}$$

$$g_{ii} = g^{(0)}_{ii} n^{1/2} + \frac{r_i^{(0)2}}{\tau^{(0)}} \frac{1}{4\pi r} n^{-1/2} + \mathcal{O}(n^{-3/2}) \,. \qquad i = 1,2,3,4 \tag{5.3.45}$$

Near the local sources the $1/|y|$ corrections play against the n suppression, but for large enough n they are always subdominant. Conversely, for any value of n there is always a region close enough to the O-plane where the leading order backreaction dominates.

For the rest of the O6-planes, we have that each one of them wraps one Ψ_i four-cycle, thus the Bianchi identity can be immediately generalized to

$$\mathrm{d}F_2^{(1)} = -2 \sum_i^7 \rho_i \, \Phi_i \,, \tag{5.3.46}$$

and the source term is

$$\rho_i = 1 - \frac{1}{8} \sum_{m \in \{1,2\}} \delta\Big(y^A - \frac{m}{2}\Big) \delta\Big(y^B - \frac{m}{2}\Big) \delta\Big(y^C - \frac{m}{2}\Big) \,, \tag{5.3.47}$$

where the combination of A, B and C is given by

$$(A,B,C)_i = \{(1,2,7),(3,4,7),(5,6,7),(1,3,6),(2,3,5),(1,4,5),(2,4,6)\} \,. \tag{5.3.48}$$

Then similarly to Eq.(5.3.46) we have

$$F_2^{(1)} = -2 \star_7 \sum_i^7 \Big(\mathrm{d}\beta_i(y^A, y^B, y^C) \wedge \Psi_i \Big) \,, \tag{5.3.49}$$

where each β_i satisfies a condition of the form (5.3.8). Then the backreaction near each O-plane has an equivalent form as (5.3.41) and therefore can be controlled for large enough n. The full form of $\tau^{(1)}, w^{(1)}$ and the metric follow similarly from the equivalent equations to (5.3.27) to find individual contributions of the form (5.3.42)-(5.3.45) for each three-cycle and adding them together.

5.4 Corrections to the effective scalar potential

5.4.1 Corrections in the absence of net D2/O2 charge

We want to investigate whether the backreaction corrections affect the leading order 3d scalar potential and as a consequence the scale-separation. Considering the metric decomposition in Eq.(5.2.6), the dimensional reduction of the ten-dimensional action (2.2.1) gives

$$S_{10} = 2\pi \int \mathrm{d}^3x\sqrt{g_3} \int \mathrm{d}^7y\sqrt{g_7}w^3\Big(\tau^2 R_{10} + \mathcal{L}_m\Big)\,. \tag{5.4.1}$$

Here R_{10} is the ten-dimensional Ricci scalar given in Eq.(8.4.1) and the $\mathcal{L}_m$ the rest of the kinetic and potential terms. In order to get the effective 3d action one should integrate over the internal coordinates. However, we just need to write down the action from a three-dimensional point of view and study the contribution of the corrections. The 3d effective action is

$$S_3 = \int \mathrm{d}^3x\sqrt{g_3}\Big(\tilde{\mathcal{V}}_7 R_3 - V_3\Big)\,, \qquad \tilde{\mathcal{V}}_7 = \int \mathrm{d}^7y\sqrt{g_7}w^3\tau^2\,, \tag{5.4.2}$$

where the scalar potential takes the form

$$\begin{aligned} V_3 = \int \mathrm{d}^7y\sqrt{g_7}w^3\bigg(&-\tau^2 R_7 + 6\frac{\tau^2}{w}\nabla_m\nabla^m w + 6\frac{\tau^2}{w^2}\nabla_m w\nabla^m w \\ &- 4g^{mn}\partial_m\tau\partial_n\tau + \frac{1}{2}\tau^2|H_3|^2 + \frac{1}{2}|F_p|^2 - 2\mu_6\sum_i \tau\delta(\pi_i)\bigg)\,. \end{aligned} \tag{5.4.3}$$

To see the effect of the first order correction we replace the delta function corresponding to the O6-plane by our next-to-leading order solution for the $\mathrm{d}F_2$ Bianchi identity. We start from the volume of the wrapped cycle in Eq.(5.2.10) which gives

$$\int \mathrm{d}^7y\sqrt{g_7}\delta(\pi_i) = \int \mathrm{vol}_{\pi_i}\wedge\delta_{i,3} = \frac{1}{2}\int \mathrm{vol}_{\pi_i}\wedge\Big(H_3\wedge F_0 - \mathrm{d}F_2\Big)\,. \tag{5.4.4}$$

Replacing this into the effective scalar potential gives, after some manipulations,

$$\begin{aligned} V_3 = \int \mathrm{d}^7y\sqrt{g_7}w^3\bigg(&-\tau^2 R_7 + 6\frac{\tau^2}{w}\nabla_m\nabla^m w + 6\frac{\tau^2}{w^2}\nabla_m w\nabla^m w - 4g^{mn}\partial_m\tau\partial_n\tau \\ &+ \frac{1}{2}\tau^2|H_3|^2 + \frac{1}{2}|F_p|^2\bigg) - \mu_6\sum_i\int \mathrm{vol}_{\pi_i}\wedge\Big(\tau w^3 H_3\wedge F_0 + \mathrm{d}(\tau w^3)\wedge F_2\Big)\,. \end{aligned} \tag{5.4.5}$$

We now want to bring the effective action Eq.(5.4.2) to the Einstein frame. To do this we perform the rescaling $g^S_{\mu\nu} = g^E_{\mu\nu}(2\pi\tilde{\mathcal{V}}_7)^{-1/2}$. The Einstein-frame scalar potential of the 3d effective theory is

$$V^E = \frac{V_3}{(2\pi)^2\tilde{\mathcal{V}}_7^{3/2}}\,, \tag{5.4.6}$$

and we will be ignoring from now on the superscript E. We can use this form of the effective potential to estimate the impact of the backreaction. We will do this by comparing the contributions from the unsmeared terms to the leading order smeared potential.

Let us find the scaling of the smeared potential first. At leading order the volume $\tilde{\mathcal{V}}_7$ scales like $\tilde{\mathcal{V}}_7^{(0)} \sim n^{11/2}$ and $\mathrm{vol}_{\pi_i}^{(0)} \sim n$. To find the scaling of R_7 we need the scaling of the Ricci tensor in (5.3.15). We see that $R_{mn} \sim g^{(0)rs}\nabla_m\nabla_n g_{rs}^{(1)} + ...$ where nabla contains products of the metric and its inverse so it doesn't scale. The scaling of the internal Ricci scalar at leading order is $R_7^{(0)} \sim n^{-3/2}$. The zeroth order potential (after few integrations by parts) takes the form

$$
\begin{aligned}
V_3^{smeared} = & \frac{1}{(2\pi)^2 \tilde{\mathcal{V}}_7^{(0)1/2} \tau^{(0)2}} \left(\frac{1}{2}\tau^{(0)2}|H_3^{(0)}|^2 + \frac{1}{2}\sum_{p=0,4} |F_p^{(0)}|^2 \right) n^{-17/4} \\
& - \frac{\mu_6}{(2\pi)^2 \tilde{\mathcal{V}}_7^{(0)3/2}} \sum_i \int \mathrm{vol}_{\pi_i}^{(0)} \wedge \left(\tau^{(0)} w^{(0)3} H_3^{(0)} F_0 \right) n^{-17/4} .
\end{aligned} \tag{5.4.7}
$$

We see that the leading order potential scales as $n^{-17/4}$.

We will now estimate the impact of the backreaction by evaluating the scaling of the terms that correspond to the unsmearing corrections by inserting the expansions (5.2.46)-(5.2.52) in the effective potential. First we can check the term that originates from the leading order correction to the last term in (5.4.5). The leading order in n is

$$
\delta V_3 \ni -\frac{\mu_6}{(2\pi)^2 \tilde{\mathcal{V}}_7^{(0)3/2}} \sum_i \int \mathrm{vol}_{\pi_i}^{(0)} \wedge \left(\mathrm{d}(3\tau^{(0)} w^{(0)2} w^{(1)} + \tau^{(1)} w^{(0)3}) \wedge F_2^{(1)} \right) n^{-21/4} . \tag{5.4.8}
$$

Note that there are derivatives of the dilaton and the warp factor. We see that this correction term is damped faster for large values of n compared to the smeared term, thus the potential matches to the smeared one at the leading order, *assuming* that the formal singularities of the near-brane regions are somehow resolved from string theory. Indeed, the formal expression (5.4.8) hides singularities related to the fact that the solution clearly breaks down in the regions of the internal space surrounding the O-plane loci because the $1/r$ terms dominate over the n suppression. A way to see this is by focusing on the π_3 four-cycle backreaction in (5.4.8) and estimating the term

$$
\delta V_3^{\pi_3} \sim \frac{n^{-21/4}}{\tilde{\mathcal{V}}_7^{(0)3/2}} \int \mathrm{vol}_{\pi_3}^{(0)} \wedge \left(\mathrm{d}(3\tau^{(0)} w^{(0)2} w^{(1)} + \tau^{(1)} w^{(0)3}) \wedge F_2^{(1)} \right) , \tag{5.4.9}
$$

in the near-brane region. At that limit from (5.3.6), (5.3.27), (5.3.32) and (5.3.41) we have,

$$
\text{near the O6}_\alpha \text{ central locus: } F_2^{(1)} \sim \frac{1}{|\vec{y}|^2} \ , \quad \mathrm{d}F_2^{(1)} \sim \delta(\vec{y})\, \Phi_3 \ , \quad \frac{\tau^{(1)}}{\tau^{(0)}} \sim \frac{w^{(1)}}{w^{(0)}} \sim \frac{1}{|\vec{y}|\, \tau^{(0)}} \, . \tag{5.4.10}
$$

We can regularize the divergence of the integral in (5.4.9) by excising regions around the O6 locus, which we take to be three-spheres of radius r_0 and denote $S_3(r_0)$. Integrating by parts now produces a non-vanishing boundary term. This leads to an estimation of the near-O6-plane backreaction of the form

$$
\begin{aligned}
\delta V_3^{\pi_3}(\mathrm{O6}_\alpha \text{ locus}) \sim \frac{n^{-21/4} w^{(0)3} \mathcal{V}_{\pi_3}^{(0)}}{\tilde{\mathcal{V}}_7^{(0)3/2}} \Big(& \int_{\partial S_3(r_0)} (3\tau^{(0)} \frac{w^{(1)}}{w^{(0)}} + \tau^{(1)}) F_2^{(1)} \\
& - \int_{\tilde{\pi}_3 \backslash S_3(r_0)} (3\tau^{(0)} \frac{w^{(1)}}{w^{(0)}} + \tau^{(1)}) \mathrm{d}F_2^{(1)} \Big) \\
\sim \frac{n^{-21/4} w^{(0)3} \mathcal{V}_{\pi_3}^{(0)}}{\tilde{\mathcal{V}}_7^{(0)3/2}} \frac{6\pi^2}{r_0} \, .
\end{aligned}
\tag{5.4.11}
$$

In the last line we used the relations in (5.4.10) and only the boundary contribution survives, because $\mathrm{d}F_2^{(1)}$ vanishes outside the excised regions. Clearly, the resulting expression depends on r_0, and diverges as we try to shrink the excised regions. This simply signals the breakdown of the leading order solution near the O6 planes, where stringy corrections to the 10d dynamics are expected to appear. These corrections, in principle determine a physical value of r_0 such that (5.4.11) would accurately capture the contribution to the potential from fields away from the O6 locus. Indeed, if we require the backreaction to be negligible we need

$$
V_3^{smeared} \gg \delta V_3 \quad \Rightarrow \quad n^{-17/4} \gg n^{-21/4} r_0^{-1} \quad \Rightarrow \quad n \gg r_0^{-1} \, . \tag{5.4.12}
$$

This suggests that we can have a good approximation of the true solution for distances from the loci much greater than $1/n$.

We can also estimate the backreaction from other terms to see if the $1/n$ estimate for the safety distance from the loci is good enough. We can check for example the dilaton term from the first line in (5.4.5) focusing on the higher order terms

$$
\delta V_3 \ni \frac{1}{(2\pi)^2 \tilde{\mathcal{V}}_7^{3/2}} \int \mathrm{d}^7 y \sqrt{g_7}\, w^3(y) \bigg(-4\, \partial_m \, \delta\tau \partial^m \delta\tau \bigg) \, . \tag{5.4.13}
$$

Following the same reasoning as before we find for the leading n unsmearing correction

$$
\delta V_3^{dilaton}(\mathrm{O6}_\alpha \text{ locus}) \sim n^{-21/4} r_0^{-1} \, , \tag{5.4.14}
$$

which agrees with (5.4.12).

It is however suggested in [106] that for a 10d "observer" the backreaction is stronger and would require $r_0 \gg n^{-1/4}$ to be able to safely ignore the unsmearing effect. The argument in [106] for this is to compare for example $\tau^2 |H|^3$ to $(\partial\delta\tau)^2$ and see that one needs $n \gg r^{-4}$. We note that this condition delineates the regions of the internal space where leading order corrections to the 10d solution already give approximately the correct field profiles.

The purpose of r_0, however, is to properly separate out the additional g_s corrections to the 10d solution, over and above the $1/n$ corrections. Thus, the choice of r_0 should be determined by the regions where the string coupling becomes large, i.e. $1/n$. It therefore appears that there is a region $1/n < r < 1/n^{1/4}$, where although the 10d equations of motion can be trusted, the resulting $1/n$ expansion of their *solution* can not. The contributions to the scalar potential coming from integrating over those regions likely have to be computed to all orders and appropriately resummed.

Meanwhile the degrees of freedom near the O6 locus, i.e. at $r < 1/n$, would have to be captured by a strong-coupling description of the O6 planes, as the string coupling truly becomes large in those regions even at leading order in $1/n$. Unfortunately, in the presence of a Romans mass, such a strong coupling description is currently unavailable.

5.4.2 Corrections including a net O2/D2 charge contribution

When there is no net O2/D2 cancellation, such contribution needs to cancel by fluxes in the tadpole/Bianchi. Then there is an extra contribution in the potential that comes from the RR field $|F_4|^2 = |F_{4A} + F_{4B}|^2$ and has the form

$$\begin{aligned} V_3^{extra} &= \frac{1}{(2\pi)^2\tilde{\mathcal{V}}_7^{3/2}} \int_7 w^3 \left(F_{4A} \wedge \star_7 F_{4B} + \frac{1}{2} F_{4B} \wedge \star_7 F_{4B} - 2^{-3}\mu_2 \tau \frac{j_7}{\mathcal{V}_7} \right) \\ &= \frac{1}{(2\pi)^2\tilde{\mathcal{V}}_7^{(0)3/2}} \int_7 w^{(0)3} \left(F_{4A}^{(0)} \wedge \star_7 F_{4B}^{(0)} n^{-21/4} + \frac{1}{2} F_{4B}^{(0)} \wedge \star_7 F_{4B}^{(0)} n^{-25/4} \right) \\ &\quad - \frac{1}{(2\pi)^2\tilde{\mathcal{V}}_7^{(0)3/2}} \int_7 w^{(0)3} \left(2^{-3}\mu_2 \tau^{(0)} \frac{j_7^{(0)}}{\mathcal{V}_7^{(0)}} n^{-7} \right), \end{aligned} \tag{5.4.15}$$

since the extra terms scale as

$$F_{4A} \wedge \star_7 F_{4B} \sim n^{3/4} , \quad F_{4B} \wedge \star_7 F_{4B} \sim n^{-1/4} , \quad j_7/\mathcal{V}_7 \sim n^{-7/4} , \tag{5.4.16}$$

and indicatively $F_{4A} \wedge \star_7 F_{4A} \sim n^{7/4}$. The scaling of F_{4B} is dictated by the Bianchi identity (5.2.59) with $H^{(0)}$ scaling as n^0. Considering the scaling of the extra contributions it seems that neither the O2/D2 contributions nor the terms which contain the F_{4B} scale the same way as the potential in Eq.(5.4.7) and are subleading at large values of the parameter n.

We stress that we do not unsmear the O2-plane here, this requires additional analysis which we leave for a future work. However, the analysis of [67], where *space-filling* localized and smeared O2 sources are compared, shows that at least for supersymmetric solutions the backreaction is not expected to lead to inconsistencies.

5.5 Outlook

In this work we have analyzed the backreaction of localized sources on the scale-separated AdS$_3$ N=1 vacua of massive Type IIA supergravity. We have found that when one applies the scale separation limit to the various ingredients then the corrections from the localized sources can be made arbitrarily small. Therefore away from the sources the solution seems to be under control and its uplift to an actual solution of string theory seems plausible. Of course, unless the O6-plane singularities that we encountered can be resolved within string theory the smeared approximation is bound to fail. Moreover, our analysis here was only the first step that accounts only for the leading order backreaction, and therefore we do not know at this point if some intricate inconsistency can show up at the next order, as the AdS conjecture would imply [15]. One could further check the consistency of the backreacted solutions by matching with the supersymmetric analysis of Type II AdS_3 vacua performed in [37]. These questions and checks are left for future work.

One equally interesting question that could be now addressed is the stability of non-SUSY AdS_3 flux vacua, which should be unstable according to the swampland conjectures [14]. In particular from the supersymmetric construction in the previous chapter one can also find the non-supersymmetric "skew-whiffed" AdS_3 vacua, where the F_4 flux has flipped sign. For the moment our leading order analysis has not indicated some pathology of such vacua but it may be that by going to next to leading order in the backreaction some pathology may show up thus verifying [14]. For example, four-dimensional "skew-whiffed" vacua were studied recently in [108, 112] and possible instabilities were detected. We also leave the analysis of the non-supersymmetric vacua for a future work.

Finally, on a more general note, the understanding of 3d non-supersymmetric vacua of string theory is interesting on its own right due to the applications in holography, but also as a way to scrutinize the 3d swampland. A clear classification of classical de Sitter vacua (as is done in 4d [113, 114]) would have its own merits and in addition would verify or challenge the conspiracy of string theory against de Sitter [10, 115–118]. It would be interesting to see how the unsmearing procedure we discussed here would change these results in the three-dimensional solutions.

Chapter 6

Type IIA : De–Sitter uplift

6.1 Introduction

Here we continue the scrutiny of the conservative setting; can we achieve small extra dimensions, with all moduli stabilized and SUSY broken in a dS vacuum? This question has been studied, for obvious reasons, mostly for flux vacua in 4d, see [6–9, 119] for general reviews and [10, 11, 120] for reviews with an emphasis on de Sitter vacua. It is however useful to think of other dimensions as well. From a landscape viewpoint this is anyways required if one wishes to understand what the total space of vacua is. Generic vacua can have any number of compact dimensions up to 9. It should also be obvious that there are more vacua in lower dimensions since the amount of compact manifolds, ways to wrap branes and fluxes increases dramatically with every extra compact dimensions. Very intriguingly there is not a single suggestion known to achieve moduli stabilization with small compact dimensions in $d > 4$. That makes $d = 4$ rather special in a flux context similarly to the string gas picture [121].

In this work in particular we will consider 3d vacua. In the previous sections we have argued that compactifications of massive IIA supergravity on G2 orientifolds with fluxes can lead to full moduli stabilization in 3d (SUSY) AdS vacua that allow a tuning to arbitrary weak string coupling, large radii and a parametric separation of scales between the AdS length and the KK scale. This is very analogous to flux compactifications of massive IIA on Calabi–Yau orientifolds to four dimensions [17, 81].

Such striking parametric separation of scales at weak coupling is in contradiction with some Swampland conjectures [15] (see also [55]). These conjectures are loosely derived from the distance conjecture [122] and inspired from no-go theorems with given assumptions [25]. Recently however a suggestion was made how the massive IIA vacua can nonetheless be consistent with the web of swampland conjectures in a very interesting way [29]. So purely based on the conceptual ideas surrounding the Swampland it could be that such scale separation is consistent with our understanding of string theory and the question is much open right now.

On the technical side however, there have always been reasons to doubt the consistency of the massive IIA vacua [103, 104]. These worries are related to the backreaction of the O6-planes which have only been taken into account in a *smeared* fashion [66]. Relatedly there is also no 11-dimensional uplift of the strongly coupled region near an O6 singularity. However, partial results about the backreaction of the localized sources are known and encouraging [70,71,105] and simple flux vacua exist for which it can be shown that smearing is harmless on the condition there is a large volume/weak coupling limit [69].

It is our hope that having an infinite family of 3d AdS vacua with scale separation at weak coupling allows an easier holographic CFT study than with 4d vacua which hopefully shines a complementary light on these issues.

In this chapter we continue our study of compactifications of massive IIA supergravity on G2 orientifolds with fluxes but turn to the question of the existence of meta-stable dS vacua which have been conjectured to be impossible completely [10, 115, 123] or impossible at sufficiently weak coupling [116–118, 124]. Our setup is entirely classical in the sense that we stick to 10d supergravity at the two-derivative level with orientifold and D-brane sources.

The quest for classical dS vacua has already a history and started with the suggestive works [125, 126] and later the more concrete proposals of [127–129]. A thorough but outdated scan and overview can be found in [130] whereas an update of the recent situation is described in [113, 131–134] and [135–138]. Most of these works focussed on 4d vacua, but some preliminary results about higher dimensions are known [123] and a complicated suggestion for a meta-stable solution in 3d was proposed in [139].

In here we follow a route towards classical dS solutions similar to to [139] (but considerably simpler) and consider general mixtures of orientifold and anti-brane sources such that the lower-dimensional would-be EFT has no (linearly realized) supersymmetry. Of course the dangers are around every corner in that case since one should worry about the anti-brane stability as well as their backreaction. In contrast, most of the works on classical dS solutions start with calibrated orientifold and D-brane sources and break SUSY spontaneously instead of explicitly which provides slightly more control at first sight. Nonetheless we will argue for a certain amount of control directly from ten dimensions, by verifying whether the approximations made are justified. This means that all curvature and inverse length scales should be small in string units and that the string coupling is small. In our concrete example this will be the case but not parametrically in contrast with the AdS vacua constructed in the previous chapter. The most surprising outcome of our analysis is that by adding *two* different species of anti-branes together, namely anti-D2 and anti-D6, we can achieve dS critical points with the following properties: 1) with some tuning of coefficients we can get rid of the typical tachyons present in classical dS vacua, 2) the internal manifold does not need to be negatively curved, 3) the resulting model is very simple. Unfortunately the tuning required is impossible for the simple torus examples in this analysis, but there is no reason to expect that a general G2 construction with warped throats would not allow it. Interestingly we will find evidence that exactly those ingredients could trigger perturbative brane-flux decay. This can be taken as non-trivial circumstantial evidence for the no-dS conjecture although more concrete models should be constructed to verify our general findings.

6.2 Mass producing 3d de Sitter?

We have argued in a previous chapter that G2 compactifications of massive IIA supergravity with O2/O6 sources allow the stabilization of all moduli if enough fluxes are turned on showed in the previous sections. The vacua are then (SUSY) AdS_3 at tunably weak coupling, large length scales and separation between KK and AdS scale. It is then tempting to somehow uplift these vacua to meta-stable dS by adding SUSY-breaking ingredients with positive energy. It is known that this is not a good strategy in 4d [140][1] and the same applies in 3d. Heuristically this works as follows: AdS vacua that are well suited for uplifting have the feature that the mass m of the lightest (non-axionic) scalar is large in AdS units, that is

$$m^2L^2 >> 1\,, \tag{6.2.1}$$

where L is the AdS length. Such vacua are at the bottom of a scalar potential that approaches zero from below while being very narrow as depicted in figure 6.1 below. Due

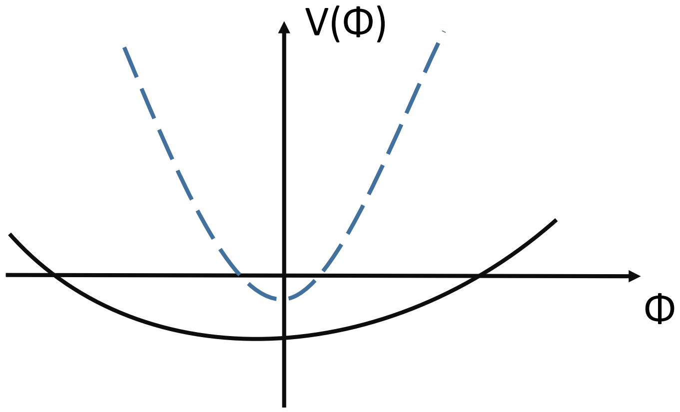

Figure 6.1: *The potential with the dashed line is better suited for uplifting than the potential with the solid line.*

to the large L a small SUSY-breaking source necessarily brings one to positive energy and due to the high m^2 it will not destabilize the system. So models that achieve (6.2.1) very well were therefore conjectured to be in the Swampland [54]. This Swampland conjecture is of course inspired by the no-dS conjectures but applies to AdS vacua and should therefore be easier to prove or disprove. Despite the difficulty in finding vacua obeying (6.2.1) this conjecture is furthermore inspired by the bizarre properties the dual CFT would have since m^2L^2 determines the dual conformal operator weight. The KKLT and LVS AdS vacua [16, 53] are our most concrete suggestions for vacua that get close to obey (6.2.1), but they cannot do so tunably. Racetrack models are built to achieve (6.2.1) if they would allow the "Kallosh–Linde" fine-tuning [142], but there is not a single string theory

[1] Although see [141].

example showing such behavior and it might also be in tension with Swampland bounds on axion decay constants [143], see however [144, 145]. Finally it has been demonstrated that non-geometric flux backgrounds can achieve (6.2.1) arbitrary well because they allow moduli-stabilized Minkowski solutions [146, 147], where effectively $m^2L^2 = \infty$. But such backgrounds are far from being shown to be trustworthy because the EFTs derived from non-geometry are difficult to control, see [148] for a review.

Although KKLT and LVS do not obey (6.2.1) tunably it is suggested they achieve it sufficiently well to allow anti-D3 uplifts to dS vacua. However that procedure would require warped throats in order to make the uplift energy tunably small, but recently it has been appreciated that demanding the throat volume to fit inside the total Calabi–Yau volume is so constraining that the tuning freedom might be lost [22]. We will come back to issues related to gluing throat regions into compact spaces later.

This analysis is about following a somewhat related strategy but in 3d and with only classical[2] ingredients (fluxes, branes and orientifolds). So we turn to massive IIA supergravity with O2/O6-planes, (anti) D2/D6-branes. We will not attempt to uplift the AdS vacua of the previous sections, but we will instead search directly for meta-stable de Sitter critical points arising from uplifting moduli-stabilized non-SUSY Minkowski critical points. Obviously any non-SUSY Minkowski minimum that is not of the no-scale type will arise from a fine-tuning that is almost certainly impossible after quantization of fluxes and charges, but we will use it nonetheless as a guiding principle and afterwards compute what the effects of the quantization are. For the sake of meta-stable dS_3 solutions quantization will turn out problematic for simple set-ups but we argue that we do not expect this to be a real issue for more involved set-ups.

We start our analysis by restricting to the universal bulk moduli, that is the dilaton and the volume. Generically these scalars are the typical place where the classical tachyonic instability would show up when we study Minkowski/de Sitter vacua with broken and full moduli stabilization.[3]As we will see this situation is seemingly not the case here because these tachyons don't show up in a generic setup. A reason for why we can avoid tachyons is related to our discussion around figure 6.1 as the "Minkowski limit" of our models is not of the no-scale type, see also [150].

The two universal bulk moduli we have here are the real scalars x and y and they are a linear combination of the dilaton and the volume modulus defined in the previous sections and equation (4.3.20) further below.The scalar potential then reads[4]

$$V = A\,e^{2y} + F\,e^{2y-\frac{2x}{\sqrt{7}}} + H\,e^{2y+\frac{2x}{\sqrt{7}}} + C\,e^{y-\sqrt{7}x} + T\,e^{\frac{3y}{2}-\frac{5x}{2\sqrt{7}}}\,, \tag{6.2.2}$$

where we have used the symbols A, F, H, C, T to indicate the various contributions from the fluxes and the sources. These are functions of all other scalar fields specific to a

[2]Here 'classical' refers to string theory ingredients whose leading order contributions to the energy can be captured using 10d supergravity at the two-derivative level, with inclusion of source terms.

[3]And if not in that 2-scalar subsector, the tachyon is in a 3-scalar sector with the third scalar representing the overall volume of the orientifold cycles [149].

[4]If we had included the curvature of the internal manifold its contribution would be $V_{R7} \sim -R_7\,e^{\frac{3y}{2}-\frac{x}{2\sqrt{7}}}$. However here we work directly with a Ricci flat internal space therefore $R_7 = 0$.

compactification. We will give the exact origin of these terms and their form for a toroidal orbifold in the next section. The coefficient A relates to the O2-plane tension and T to the O6-plane tension and both are negative. The coefficients F, H and C are related to $|F_4|^2$, $|H_3|^2$ and $|F_0|^2$ respectively, and they are positive definite. For technical simplicity we also absorb the vacuum values of x and y in these coefficients such that

$$\langle x \rangle = 0 = \langle y \rangle \,. \tag{6.2.3}$$

Similar to [125, 126] we then analyze the three conditions

$$V_x = 0 \,, \quad V_y = 0 \,, \quad V = \epsilon \,, \tag{6.2.4}$$

where we use $\epsilon > 0$ to parametrize the small vacuum energy. The "Minkowski limit" is therefore $\epsilon = 0$ and we will consider it essentially as a crude estimation to test stability, and building on that, only a small uplifting will then give meta-stable de Sitter. Once we apply these equations to the scalar potential we get after few manipulations

$$\begin{aligned} A &= -8\epsilon - 2F \,, \\ H &= F + C + 5\epsilon \,, \\ T &= 4\epsilon - 2C \,. \end{aligned} \tag{6.2.5}$$

Note that the consistency of these vacua simply requires $H > C$, $H > F$, $A < 0$ and $T < 0$ because we are assuming small ϵ. The mass matrix of the x and y scalars has eigenvalues given by

$$m_\pm = \frac{2}{7}\left(9C + 2H \pm \sqrt{88C^2 + 29CH + 4H^2}\right) + \mathcal{O}(\epsilon) \,, \tag{6.2.6}$$

where we have used the vacuum conditions. Since we can make ϵ arbitrarily small, to check the positivity of the masses we only need to look at the ϵ-independent parts (i.e. the Minkowski limit). The reader can verify that

$$m_\pm > 0 \quad \to \quad H > C \,, \tag{6.2.7}$$

which is in complete agreement with the consistency of the de Sitter solutions.

We thus conclude that we have at hand a classical framework for "mass production" of 3d de Sitter. The reason it works (at the level of the 2 universal scalars) is exactly because of our arguments surrounding figure 6.1: we have assumed small ϵ (effectively put it to zero) and found a positive mass matrix in the universal directions. So the presence of such regions in the scalar potential implies there are good reasons to expect meta-stable vacua.

We will see however for our specific example that careful consideration of quantization and tadpole conditions changes this naive estimate and makes us wonder if these vacua are in the swampland instead. Indeed notice first that if we did not take ϵ small then we could have instabilities because the leading order contribution of ϵ to the masses is

$$7\, m_+ \, m_- = 4C(H - C) - 4\epsilon\,(39C + 4H) + \mathcal{O}(\epsilon^2) \,. \tag{6.2.8}$$

From the second term we can see why an instability can arise if the de Sitter vacuum is not shallow enough, i.e. if ϵ is not small enough[5]. In fact due to flux quantization the vacuum conditions (6.2.5) put strong constraints on the possible values of ϵ, especially once we take the tadpole cancellation conditions into account which bound the fluxes. Therefore having arbitrarily small ϵ should not be taken for granted.

Note that our analysis has not used any curvature of the internal 7D manifold, which would contribute another piece to the 3d potential. The fact that the equations $\partial_x V = \partial_y V = 0$ are consistent with the absence of such a term is different from the usual classical dS vacua constructions. The reason this is possible now is because of a *mixture* of different brane types, i.e., O2/D2 and O6/D6, which is usually not considered. One could worry that a Ricci-flat Ansatz is not consistent with the 10d equations of motion due to backreaction of fluxes. However the self-consistency of the approach is guaranteed if a critical point is found and from a 10d viewpoint one finds that the negative tension of the smeared orientifolds exactly cancels the positive tension of the branes and fluxes inside the compact dimensions. Whether or not the smearing of the orientifolds is a problem depends probably on how small the coupling can be and how large the internal volume is [69].

6.3 A toroidal example

In the previous section from the generic form of the scalar potential we have argued that massive IIA compactified on a 7D Ricci flat space with O-plane and anti-D-brane sources could potentially lead to meta-stable de Sitter vacua. Now we attempt at finding a specific example: we specify the ingredients, we derive the potential (6.2.2), and we also discuss the tadpoles and flux quantization conditions. We set $\alpha' = 1$, we utilize the orientifold and orbifold of the previous sections, and we focus directly on solutions with an isotropic seven-torus for simplicity but also as we will show, it can guarantee stability of the non-universal scalars.

6.3.1 Tadpoles, flux quantization and potential

Let us first look at the Bianchi identities which we require to satisfy without using D2- or D6-branes. For the O2 Bianchi identity we have

$$0 = \int_7 H_3 \wedge F_4 - (2\pi)^5 \left(N_{\overline{\mathrm{D2}}} + 16\right) , \tag{6.3.1}$$

where we have used $N_{\mathrm{O2}} = 2^7$. For the O6-planes we have $N_{\mathrm{O6}} = 2^3$ per 3-cycle and we allow the same number ($N^{(i)}_{\overline{\mathrm{D6}}}$) of anti-D6-branes per i'th 3-cycle:

$$N^{(i)}_{\overline{\mathrm{D6}}} \equiv N_{\overline{\mathrm{D6}}} \ , \quad \text{total number of } N_{\overline{\mathrm{D6}}} = 7 \times N_{\overline{\mathrm{D6}}} \, . \tag{6.3.2}$$

[5]Notice that had we set $F = 0$ then $H - C$ would be of order ϵ and so the expression (6.2.8) would not be reliable. In this sense we are implicitly assuming that F has a considerable contribution. However in our examples latter we explicitly check that the masses are positive in any case thus verifying our generic analysis.

Therefore for the O6 Bianchi we have for each 3-cycle

$$0 = \int_{3-cycle} H_3 \wedge F_0 - 2\pi\,(N_{\overline{\mathrm{D6}}} + 16) \int_{3-cycle} J_3\,, \tag{6.3.3}$$

where J_3 is the unit-normalized 3-form source: $J_3 = \sum_i \Phi_i$. Fluxes, consistent with a isotropic seven-torus are the following:

$$H_3 = (2\pi)^2 K \sum_i \Phi_i\ , \quad F_0 = (2\pi)^{-1} M\ , \quad F_4 = (2\pi)^3 G \sum_i \Psi_i\,, \tag{6.3.4}$$

where $K, M, G \in \mathbb{Z}$. The F_2 tadpole condition gives the same result for each 3-cycle:

$$16 = KM - N_{\overline{\mathrm{D6}}}\,. \tag{6.3.5}$$

Whereas the F_6 tadpole implies

$$16 = 7KG - N_{\overline{\mathrm{D2}}}\,. \tag{6.3.6}$$

We now spell out the various contributions to the 3d scalar potential. The contribution from the fluxes to the 10d action in Einstein frame (with $F_0 = m$, the Romans mass) is

$$S_{\text{kin. flux}} = \int_{10} \sqrt{-G}\left(R_{10} - \frac{1}{2}e^{-\phi}|H_3|^2 - \frac{1}{2}e^{\phi/2}|F_4|^2 - \frac{1}{2}e^{5\phi/2}m^2\right)\,, \tag{6.3.7}$$

where we kept the ten-dimensional Ricci scalar to keep track of normalizations, the contributions from the sources are (ignoring open string moduli)[6]

$$S_{\text{p=2}} = -(2\pi)^7\,(\mu_{\mathrm{O2}} + \mu_{\overline{\mathrm{D2}}})\ e^{-\phi/4} \int_3 \sqrt{-g_3}\,, \tag{6.3.8}$$

$$S_{\text{p=6}} = -(2\pi)^7\,(\mu_{\mathrm{O6}} + \mu_{\overline{\mathrm{D6}}})\ e^{3\phi/4} \sum_{i=1}^{7} \int_{M_i} \sqrt{-g_7}\,, \tag{6.3.9}$$

where the M_i denote the 7D worldvolumes of the anti-D6 branes and where

$$\begin{aligned} &\mu_{\overline{\mathrm{D2}}} = N_{\overline{\mathrm{D2}}}\,(2\pi)^{-2}\ , \quad \mu_{\mathrm{O2}} = -(2)^{-3} N_{\mathrm{O2}}\,(2\pi)^{-2} = -16\,(2\pi)^{-2}\,, \\ &\mu_{\overline{\mathrm{D6}}} = N_{\overline{\mathrm{D6}}}\,(2\pi)^{-6}\ , \quad \mu_{\mathrm{O6}} = -2N_{\mathrm{O6}}\,(2\pi)^{-6} = -16\,(2\pi)^{-6}\,. \end{aligned} \tag{6.3.10}$$

When one considers spaces with warped regions then the anti-D2-brane tension can redshift and we postpone the discussion of this until later.

Starting now from 10d Einstein frame we finally perform a direct dimensional reduction:

$$ds^2_{10} = e^{2\alpha v} ds^2_3 + e^{2\beta v} \widetilde{ds}^2_7\,, \tag{6.3.11}$$

[6] Note that our p-brane actions are multiplied with an overall factor $(2\pi)^7$ because the total action we are using is $(2\pi)^7 \times (S_{IIA} + S_{sources})$.

where $\alpha^2 = 7/16$, $\alpha = -7\beta$ and $\widetilde{ds}_7^2$ is the metric on a unit-volume G2 space. Similar to the first part of the analysis we perform a rescaling of the 3d metric $g_{\mu\nu} \to \frac{1}{4} g_{\mu\nu}$ to match to more conventional units for 3d supergravity theories. We extract the volume from the metric deformation moduli by setting

$$s^i = \mathrm{vol}(X)^{3/7} \tilde{s}^i \ , \quad \mathrm{vol}(X) = e^{7\beta v} \ , \tag{6.3.12}$$

where the unit-volume deformations satisfy

$$\prod_{i=1}^{7} \tilde{s}^i = 1 \ \rightarrow \ \tilde{s}^7 = \prod_{a=1}^{6} (\tilde{s}^a)^{-1} \, . \tag{6.3.13}$$

Notice for example that because of these definitions (taking into account that $\tilde{\star}\Psi_i = (\tilde{s}^i)^2 \Phi_i$, where the i does not sum) we have

$$\begin{aligned} |F_4|^2 = F_4 \wedge \tilde{\star} F_4 &= (2\pi)^6 G^2 \sum_{i,j} \Psi_i \wedge \tilde{\star} \Psi_j = (2\pi)^6 G^2 \sum_{i=1}^{7} (\tilde{s}^i)^2 \\ &= (2\pi)^6 G^2 \left(\sum_{a=1}^{6} (\tilde{s}^a)^2 + \prod_{a=1}^{6} (\tilde{s}^a)^{-2} \right) \ , \end{aligned} \tag{6.3.14}$$

and similarly for $|H_3|^2$ and the other terms. We combine the volume and the dilaton again in the useful combinations (x and y universal moduli) in Eq.(4.3.20) and once we include all these ingredients, the 3d scalar potential becomes

$$V = Ae^{2y} + F(\tilde{s}^i) e^{2y - \frac{2x}{\sqrt{7}}} + H(\tilde{s}^i) e^{2y + \frac{2x}{\sqrt{7}}} + C e^{y - \sqrt{7} x} + T(\tilde{s}^i) e^{\frac{3y}{2} - \frac{5x}{2\sqrt{7}}} \ , \tag{6.3.15}$$

with the coefficients given by

$$\begin{aligned} A &= \frac{(2\pi)^5}{8} \left(2N_{\overline{\mathrm{D2}}} - 7KG \right) e^{2y_0} \, , \\ F &= \frac{(2\pi)^6 G^2}{16} \left(\sum_{a=1}^{6} (\tilde{s}^a)^2 + \prod_{a=1}^{6} (\tilde{s}^a)^{-2} \right) e^{2y_0 - \frac{2x_0}{\sqrt{7}}} \ , \\ H &= \frac{(2\pi)^4 K^2}{16} \left(\sum_{a=1}^{6} (\tilde{s}^a)^{-2} + \prod_{a=1}^{6} (\tilde{s}^a)^{2} \right) e^{2y_0 + \frac{2x_0}{\sqrt{7}}} \ , \\ C &= \frac{M^2}{16 (2\pi)^2} e^{y_0 - \sqrt{7} x_0} \ , \\ T &= \frac{2\pi}{8} \left(2N_{\overline{\mathrm{D6}}} - KM \right) \left(\sum_{a=1}^{6} \frac{1}{\tilde{s}^a} + \prod_{a=1}^{6} \tilde{s}^a \right) e^{\frac{3y_0}{2} - \frac{5x_0}{2\sqrt{7}}} \ , \end{aligned} \tag{6.3.16}$$

where y_0 and x_0 have been inserted in the same places where x and y appear such that in this way the vacuum is always at $x = 0 = y$, by appropriately defining x_0 and y_0. For completeness let us note again the kinetic terms

$$e^{-1}\mathcal{L}_{\text{kin}} = \tfrac{1}{2}R_3 - \tfrac{1}{4}(\partial x)^2 - \tfrac{1}{4}(\partial y)^2 - \tfrac{1}{4}\int_7 \Phi_i \wedge \tilde{\star}\Phi_j \partial\tilde{s}^i \partial\tilde{s}^j \,, \tag{6.3.17}$$

where $\tilde{\star}\Phi_i = (\tilde{s}^i)^{-2}\Psi_i$ (no summation over i implied). The conditions (6.2.4) and (6.2.5) that we studied in the previous section should now be enforced.

6.3.2 Moduli stabilization

In this part we turn to the stabilization of the 8 universal real scalar moduli $(x, y, \tilde{s}^a)$ of toroidal orbifold compactifications. The very first observation we can make right away from the form of the scalar potential (6.3.15) is that all the unit-volume toroidal moduli $\tilde{s}^a$ are always stabilized at

$$\langle\tilde{s}^a\rangle = 1 = \langle\tilde{s}^7\rangle \,, \tag{6.3.18}$$

giving a totally isotropic torus as we anticipated. In this way we always have

$$\frac{\partial V}{\partial\tilde{s}^a}\bigg|_{\tilde{s}^a=1} = 0 \,. \tag{6.3.19}$$

We assume these vevs for the $\tilde{s}^a$ in what follows. On this background we can then notice that

$$\frac{\partial^2 V}{\partial x \partial\tilde{s}^a} = 0 = \frac{\partial^2 V}{\partial y \partial\tilde{s}^a} \,, \tag{6.3.20}$$

which means there is no mass mixing between the x, y moduli and the $\tilde{s}^a$. As a result we can directly evaluate the eigenvalues for the mass matrix of the moduli $\tilde{s}^a$ independently. Using the vacuum conditions (6.2.4) one can express the second derivatives of the moduli on the scalar potential in terms of the functions H and C. First we find that the derivatives of the functions $F(\tilde{s}^a), H(\tilde{s}^a)$ and $T(\tilde{s}^a)$ at the vacuum are proportional to themselves, i.e.

$$F_{ab} = \frac{4}{7}F\,(1+\delta_{ab}) \quad , \quad H_{ab} = \frac{4}{7}H\,(1+\delta_{ab}) \quad , \quad T_{ab} = \frac{1}{7}T\,(1+\delta_{ab}) \,, \tag{6.3.21}$$

where δ_{ab} is the Kronecker delta. As a result we can evaluate V_{ab} (for $a, b = 1, \ldots, 6$) on the vacuum, and find

$$\text{Eigenvalues}[V_{ab}] = \frac{1}{7}\,(4F + 2H + 4\epsilon + 2(H-C)) \times \{7,1,1,1,1,1\} \,. \tag{6.3.22}$$

Since we already require $H > C$, the stability of the $\tilde{s}^a$ is granted on any background of the type we study here. In other words the tachyon that generically plagues de-Sitter string vacua is making its appearance here in the dilaton-volume sector (i.e. x and y) and not in the internal space deformations. A possible underlying reason for the positive masses of the $\tilde{s}^a$ scalars is that $\langle P_a\rangle = 0$ (where P is the superpotential) so they are in some sense stabilized in their supersymmetric positions[7].

[7] To construct the total superpotential including the SUSY-breaking sectors one would have to use 3d nilpotent superfields, see e.g. [151].

Now let us focus on the explicit x and y stabilization because we will see it gives few more consistency conditions than our generic discussion, especially due to flux quantization. For a de Sitter vacuum we ask as before that

$$V\Big|_{x=0=y} = \epsilon = \frac{1}{4}\left(2C+T\right) . \tag{6.3.23}$$

The condition (6.3.23) once combined with the scalar potential readily gives

$$e^{y_0/2} = \frac{e^{-\frac{9x_0}{2\sqrt{7}}} M(1-\tilde{\epsilon})}{7(2\pi)^3 K\left(1-2w\right)} , \tag{6.3.24}$$

where we have introduced $w, \tilde{\epsilon}$ defined through

$$N_{\overline{\mathrm{D6}}} = wMK \ , \quad \tilde{\epsilon} = 32(2\pi)^2 M^{-2} e^{\sqrt{7}x_0 - y_0}\, \epsilon \, . \tag{6.3.25}$$

Then for the consistency of the solution (6.3.24) we find that

$$w < 1/2 \, . \tag{6.3.26}$$

The condition $\partial V/\partial x = 0$ can be simplified to give

$$e^{4x_0/\sqrt{7}} = -\frac{2(2\pi)^2 G^2 (1-\tilde{\epsilon})^2}{K^2\left(-2\tilde{\epsilon}^2 + 4(3+14(w-1)w) + \tilde{\epsilon}(39+140(w-1)w)\right)} , \tag{6.3.27}$$

where again we have to check the self-consistency of this solution. Since $\tilde{\epsilon}$ is small, we only need to satisfy

$$3 + 14(w-1)w < 0 \ \rightarrow \ 0.311... < w < 0.688... , \tag{6.3.28}$$

with w a rational number. Here we can see the crucial contribution from the anti-D6-branes: if they are set to vanish then the solution becomes inconsistent. Since we already have an upper bound on w we find it more convenient to readily set

$$N_{\overline{\mathrm{D6}}} = MK/3 \ , \quad w = 1/3 , \tag{6.3.29}$$

which gives

$$e^{4x_0/\sqrt{7}} = \frac{2(2\pi)^2 G^2 (1-\tilde{\epsilon})^2}{K^2\left(\frac{4}{9} - \frac{71}{9}\tilde{\epsilon} + 2\tilde{\epsilon}^2\right)} . \tag{6.3.30}$$

From (6.3.30) we see that the solution is consistent indeed as long as $\tilde{\epsilon} \ll 1$. Note that by setting $w = 1/3$ we are assuming that MK is an integer multiple of 3. If we chose different values of w it would still have to be a rational number, and thus we would have to assume MK to be an integer times a bigger number than 3 which would most probably be in tension with the tadpole conditions if we stay on the toroidal setup.

Finally, inserting the above conditions into the equation for $\partial V/\partial y = 0$ gives an equation that we should solve for $N_{\overline{\mathrm{D2}}}$ because we have already fixed y_0, which gives

$$N_{\overline{\mathrm{D2}}} = 7KG\left(\frac{1}{2} - \frac{1}{6\sqrt{2}}\frac{4 - 43\tilde{\epsilon} + 18\tilde{\epsilon}^2}{\sqrt{4 - 71\tilde{\epsilon} + 18\tilde{\epsilon}^2}(1-\tilde{\epsilon})}\right) . \tag{6.3.31}$$

Equation (6.3.31) has a strong impact. Since $N_{\overline{\text{D2}}}$ is integer and the values of K and G are bounded to be rather small due to the tadpole conditions it is essentially impossible to satisfy (6.3.31) and have $\tilde{\epsilon} \ll 1$. To see this difficulty let us set $\tilde{\epsilon} = 0$ which is the Minkowski limit, and we get

$$N_{\overline{\text{D2}}}\Big|_{\text{Mink.}} = 7KG\left(\frac{1}{2} - \frac{1}{3\sqrt{2}}\right), \tag{6.3.32}$$

which can never be satisfied for any choice of integers due to the $\sqrt{2}$. Note that one could choose a different value for w instead of $1/3$, but still within the bounds (6.3.28), such that no $\sqrt{2}$ appears in (6.3.31). In such case one is faced with the problem that the flux quanta would acquire very large values which is in tension with our toroidal tadpole conditions. To summarize, here we see that if we had ignored tadpole cancellation or flux quantization we could not see the inconsistency of this solution.

Before turning to extensions let us see if the x and y masses and the $\tilde{s}^a$ masses are positive. From (6.2.7) and (6.3.22) we know that all these positivity condition boil down to

$$H > C\,. \tag{6.3.33}$$

For the positivity of the x-y masses we also need $\tilde{\epsilon} \ll 1$ as we have explained. One can directly verify that $H > C$ by using (6.3.24) as long as $1 > 2w > 0$ (which is satisfied for $w = 1/3$). Thus we see that our solution guarantees the positivity of the masses.

6.3.3 Beyond the toroidal orbifold

The toroidal orbifold example could only get us this far. However we have learned a few things from this simple example. For example we have seen that the tachyon lies in the dilaton-volume sector and not in the $\tilde{s}^a$ sector, we have seen that we need two sources of supersymmetry breaking in order to even have a chance of achieving moduli stabilization in de Sitter, and of course we have seen that a careful consideration of the tadpole conditions reveals possible inconsistencies.

Let us then discuss how we could go beyond a toroidal orbifold. To address the difficulty of consistently solving an equation like (6.3.31) one would have to study more general spaces. Either spaces that allow a much larger range for the flux quanta or spaces that induce warping to the anti-D2-brane. Indeed, if we had included anti-D2 warping, the scalar potential would only change in the A term which would become

$$A = \frac{(2\pi)^5}{8}\left(2\alpha N_{\overline{\text{D2}}} - 7KG\right)e^{2y_0}\,, \tag{6.3.34}$$

where α is the effect of the warping. Then our calculations would follow through exactly in the same way, but instead of (6.3.31) we would have

$$\alpha N_{\overline{\text{D2}}} = 7KG\left(\frac{1}{2} - \frac{1}{6\sqrt{2}}\frac{4 - 43\tilde{\epsilon} + 18\tilde{\epsilon}^2}{\sqrt{4 - 71\tilde{\epsilon} + 18\tilde{\epsilon}^2(1-\tilde{\epsilon})}}\right) \sim \frac{7}{4}KG\,, \tag{6.3.35}$$

which means we could easily solve (6.3.35) by assuming an appropriate value for α as long as $N_{\overline{\text{D2}}} > 7KG/4$ (always for small $\tilde{\epsilon}$). This small sample calculation we did here of course does not guarantee that such procedure will work but rather it points to the possible extensions that may lead to a classically stable 3d de Sitter.

Let us now elaborate on a specific setup for the fluxes. If we assume that somehow we introduce warping then one can have for example

$$G = 4\,,\;\; K = 1\,,\;\; M = 24\,,\;\; N_{\overline{\text{D6}}} = 8\,, \tag{6.3.36}$$

and

$$(24)^2\,\tilde{\epsilon} = 10^{-4}\;\;, \quad \alpha N_{\overline{\text{D2}}} = 7.40034\;\;, \quad N_{\overline{\text{D2}}} = 12\,. \tag{6.3.37}$$

The vacuum energy is now of order 10^{-24} in string units whereas the $x, y, \tilde{s}^a$ moduli masses are all positive and are of order 10^{-17}. In particular we have for the $x-y$ mass matrix

$$m^2_{x-y} = \begin{pmatrix} 1.468.. \times 10^{-16} & 4.085.. \times 10^{-17} \\ 4.085.. \times 10^{-17} & 1.2.. \times 10^{-17} \end{pmatrix}\,, \tag{6.3.38}$$

which gives eigenvalues 1.58×10^{-16} and 5.95×10^{-19}, whereas the matrix of the $\tilde{s}^a$ masses is totally independent (i.e. $\langle V_{,x,\tilde{s}^a}\rangle = 0 = \langle V_{,y,\tilde{s}^a}\rangle$) and has eigenvalues of order $\langle V_{,\tilde{s}^a,\tilde{s}^b}\rangle \sim 10^{-17} > 0$ in agreement with our general discussion. For this example the string coupling and volume reads

$$g_s = e^{\phi} = 0.112...\;\;, \quad \text{vol}(X_7) = 1.2669 \times 10^7\,, \tag{6.3.39}$$

which potentially makes the supergravity approximation reliable. In fact taking into account the very strict constraints we have here it makes it hard to find other combination of fluxes in the hope to reduce g_s even more to weak coupling. This is of course the Dine–Seiberg problem that says we cannot expect to be far from a strong coupling, which seems specific to dS, not AdS [118, 131, 133]. In any case, one should keep in mind that we are doing a sample computation and one should look into more suitable internal spaces to find realistic and trustable solutions.

Finally, we face a possible problem that we elaborate on in the next section: generically it seems that we have

$$\frac{N_{\text{anti-brane}}}{\text{flux}\;\;\text{quanta}} > 1\,, \tag{6.3.40}$$

except for the comparison of the anti-D6 with the Romans quanta which gives

$$\frac{N_{\overline{\text{D6}}\,\text{per 3-cycle}}}{M} = \frac{1}{3}\,. \tag{6.3.41}$$

In the next section we explain that this may be another source of instabilities implying the need to for internal spaces that allow larger numbers for the flux quanta.

6.4 Open string instabilities?

Anti-branes break supersymmetry because they are "anti" with respect to the background fluxes or orientifolds. With respect to the orientifolds they carry the same charge but opposite tension and preserve different supercharges. They are repelled from the orientifolds both gravitationally and electromagnetically and need to find a position in between the orientifolds where forces cancel out. We have not checked the details of that, but assume such positions exist due to the compactness of the internal manifold. Even if the anti-branes find such stable positions, there could still be perturbative instabilities lurking around the corner. For instance anti-branes can annihilate against surrounding fluxes [89]. This is possible when fluxes induce opposite brane charges via the transgression terms in type II supergravity theories

$$\mathrm{d}F_q = H_3 \wedge F_{q-2} + Q\delta\,, \tag{6.4.1}$$

where the δ denotes a $(q+1)$-form distribution describing the localized magnetic charge of (anti-) D$(8-q)$ branes. When that form does not have the same orientation as $H_3 \wedge F_{q-2}$ parts of that background flux can lower their flux quanta together with Q as to preserve the charge. Heuristically one assumes that some D$(8-q)$ branes materialized out of the flux cloud and annihilated with the actual anti-D$(8-q)$ branes. One could think that this process is always non-perturbative because it requires the nucleation of branes out of fluxes. But this picture is too heuristic and a more detailed approach, first pioneered by Kachru, Pearson and Verlinde (KPV) in [89], shows that this process can even be perturbative. The specific mechanism relies on brane polarization *aka* the Myers effect *aka* the dielectric effect [152]. We will not go in any details about this for the case at hand but instead draw some basic lessons from known backgrounds with brane-flux instabilities and then comment on the case at hand.

Let us start with the most well studied example of anti-D3 branes at the bottom of the Klebanov–Strassler (KS) throat. This was the situation described by KPV [89]. The bottom of this throat is an S^3 with radius squared given by $R^2 = b_0^2 g_s M$, where b_0 is a numerical factor close to 1 and M is the RR 3-form flux quantum piercing the S^3. KPV found that

$$\frac{N_{\overline{\mathrm{D3}}}}{M} < 0.08\ldots\,, \tag{6.4.2}$$

otherwise the anti-branes would decay perturbatively leaving $M - N_{\overline{\mathrm{D3}}}$ SUSY D3-branes behind and one less unit of NSNS flux. Other simple models of anti-branes down throats come with similar bounds. For instance anti-M2 branes down the "M-theory CGLP throat" have [153]

$$\frac{N_{\overline{\mathrm{M2}}}}{M} < 0.05\ldots\,, \tag{6.4.3}$$

where now M is an F_4 flux quantum. Or anti-D6 branes in environments with Romans mass require [94]

$$\frac{N_{\overline{\mathrm{D6}}}}{M} < 0.5\ldots\,, \tag{6.4.4}$$

where now M is the Romans mass quantum. Note that our torus model has $N_{\overline{\mathrm{D6}}}/M = 1/3$ and so satisfies the inequality but not parametrically.

Note that all these inequalities were derived using brane probe actions in regimes where it is unclear they should be trusted. Not only due to possible strong coupling effects but also the classical backreaction of the anti-branes (see [154] for some pioneering work) could be a worry and potentially enhance the instabilities [155]. However non-trivial evidence in favor of the probe action results came from the complementary blackfold treatments carried out in [93, 156, 157] as well as from arguments pointing to the absence of dangerous singularities due to backreaction [91, 92, 94, 158]. It is however quite likely that the actual bounds are a bit more strict than the probe results.

From the inequalities (6.4.2), (6.4.3) and (6.4.4) one could be tempted to think that in general perturbative brane flux annihilation for anti-Dk branes is prevented if

$$\frac{N_{\overline{\mathrm{Dk}}}}{N_{\mathrm{flux}}} < \lambda \,, \tag{6.4.5}$$

where N_{flux} is some NSNS or RR flux integer[8], $N_{\overline{\mathrm{Dk}}}$ is the number of anti-Dk branes and λ some specific number. We believe that (6.4.5) is morally correct but there is no general formula for λ. It will be highly dependent on the specific example, the details of the manifold, the fluxes, etc. For instance anti-branes as defined earlier can be completely stable against brane-flux decay in AdS backgrounds [159–161]. In general we expect λ to depend non-trivially on the values of the stabilized moduli and this is why a case by case analysis is necessary. So it is impossible for us to discuss the general constraints from brane-flux decay until we have a specific model in which the closed string tachyons are absent after quantization of fluxes is taken into account. As we argue in this analysis our torus example at least has shown there is no fundamental reason to expect tachyons to be present in more involved models, unlike the situation with classical dS vacua without anti-branes [149, 162]. In what follows we will stick to explaining why we expect a dependence of λ on stabilized moduli. The only exception being the case of anti-D6 branes where (6.4.4) seems independent of details. Our strategy will be to use the most well-studied example of anti-D3 branes and demonstrate how moduli dependences creep into λ.

We start with demonstrating g_s dependences. For instance in [163] anti-D3 decay in the "S-dual" KS throat[9] was studied. This decay is against RR fluxes instead of NSNS fluxes and find

$$\frac{g_s N_{\overline{\mathrm{D3}}}}{K} < 0.08\ldots \,, \tag{6.4.6}$$

with K the NSNS flux quantum. Furthermore there will be a dependence on the size of the cycle that harbors the anti-brane. In the examples of warped throats this is not obvious since the cycles at the tip are localized cycles and do not change their volume when the overall volume of the compactification manifold adjusts. So in models without anti-branes

[8] The flux quanta should either coming from fluxes piercing the cycle that harbors the anti-brane or is "Poincaré dual" to it.

[9] This is the weakly coupled supergravity solution obtained by S-dualizing the KS solution and then dialing to small coupling.

living near deformed conifolds there is a dependence. Using the KPV computation we can compute this by keeping the cycle size L arbitrary. From the brane-polarization potential in [89] one then finds that

$$\pi\left(\frac{N_{\overline{D3}}}{M}\right)_{\max} = \operatorname{arccot}\left(\frac{L^2}{g_s M}\right) - \frac{1}{2}\sin\left(2\operatorname{arccot}\left(\frac{L^2}{g_s M}\right)\right). \tag{6.4.7}$$

So when $L^2 = b_0^2 g_s M$ we will find (6.4.2), but if the cycle size is set by different effects one can easily infer that large volume and small coupling enhances the decay. This simple observation shows that using anti-branes that do not live at the bottom of warped throats generated by deformed conifolds, is risky. In fact the above formula even suggests the problems worsen at weak coupling and large volume. Hence, similar to our discussion about closed string stability we are led to inserting anti-branes in warped throats from desingularized conifolds since then these complicated dependences might be washed away. Most studies on brane-flux decay assume there are no "compactification effects" on the brane-flux decay when the throats are inserted in compact spaces. Although this is still somewhat unclear and preliminary results can be found in [164, 165]. Interestingly other compactification effects exist on stability when throats are inserted in compact spaces, see for instance [22, 166–169].

We have used anti-D2 branes and anti-D6 branes. Given the existence of smooth supersymmetric G2 holonomy throats with D2 charges that cap off in a finite S^4 [88], we expect (in analogy with the KS throat) that also compact G2 manifolds can have such throats. The probe computation for brane-flux decay of anti-D2 branes has not yet been carried out[10] and is somewhat obscured by having to use the self-dual 3 forms on the IIA NS5 brane. We expect the computation to go along the lines of [170] and give a maximal value of $\frac{N_{\overline{\mathrm{D2}}}}{N_{\mathrm{flux}}}$ of a few percent, just like for anti-D3 branes (6.4.2) and anti-M2 branes (6.4.3). This will then lead to a new problem for our 3d de Sitter vacua. Since equation (6.3.35) implies

$$\frac{N_{\overline{\mathrm{D2}}}}{G} = 7K\alpha^{-1}\left(\frac{1}{2} - \frac{1}{6\sqrt{2}}\frac{4 - 43\tilde{\epsilon} + 18\tilde{\epsilon}^2}{\sqrt{4 - 71\tilde{\epsilon} + 18\tilde{\epsilon}^2}(1-\tilde{\epsilon})}\right), \tag{6.4.8}$$

we will have to increase $\tilde{\epsilon}$ such that the value of $N_{\overline{\mathrm{D2}}}/G$ drops and the system is safe from such decays (in our discussion until now we always assumed $\tilde{\epsilon} \ll 1$). However, we know that increasing $\tilde{\epsilon}$ may lead to tachyons. Indeed, from (6.3.25) we have $\tilde{\epsilon} = 2\epsilon/C$ and combined with (6.2.8) we find that an absence of tachyons requires (up to order $\tilde{\epsilon}^2$)

$$\epsilon < C\frac{H-C}{39C+4H} < 1 \quad \rightarrow \quad \frac{\tilde{\epsilon}}{2} < \frac{H-C}{39C+4H}. \tag{6.4.9}$$

From now on we work only up to linear order in $\tilde{\epsilon}$. This will turn out consistent since we will verify we need $\tilde{\epsilon} < 0.012$. Indeed, using the properties of our solution and in particular (6.3.24) we have

$$H = \frac{9}{7}(1-2\tilde{\epsilon})C + \mathcal{O}(\tilde{\epsilon}^2). \tag{6.4.10}$$

[10] But results about the backreaction do exist at first order in the SUSY-breaking charge [90].

Then the no-tachyon condition (6.4.9) reads

$$\frac{4-36\tilde{\epsilon}}{309-72\tilde{\epsilon}}-\tilde{\epsilon}>0\,. \tag{6.4.11}$$

One can see that the condition (6.4.11) gives small values for $\tilde{\epsilon}$ which essentially lead to large values of $N_{\overline{\mathrm{D2}}}/G$ from (6.4.8). This tension is depicted in figure (6.2). To conclude, let us

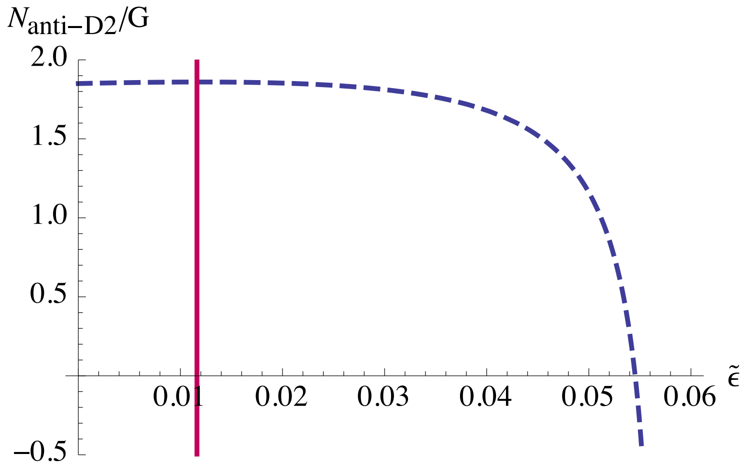

Figure 6.2: *The behavior of the ratio $N_{\overline{\mathrm{D2}}}/G$ in units of $K\alpha^{-1}$ that determines brane-flux decay. The smaller the ratio the more certain one can be there is no decay. The region to the right of the vertical red line suffers from closed string tachyons.*

translate our findings in terms of F_4 flux quanta G. We have here a rather striking interplay between the parameters and the instabilities they relate to: trying to avoid brane-flux decay we have to increase G which in turn works against a tachyon-free vacuum. Instead a small G leads to tachyon-free vacua but then such vacua are jeopardized by brane-flux decay. So it seems that in our tachyon-free setup the fraction $N_{\overline{\mathrm{D2}}}/G$ is order one, and far well above in case there is warping. This however could be due to our isotropic way of stabilizing fluxes. We have not considered an alternative stabilization, but one would have to study different internal spaces that would naturally point towards a non-isotropic setup. It would be interesting to push this further and try to check whether this tension can somehow be relaxed.

Note that anti-D6 branes, wrapping 4d cycles will probably extend in the whole bulk. But it seems that the arguments of [94] leading to (6.4.4) are model independent, although the actual bound might be tighter than (6.4.4) due to backreaction. We have shown that in our model we could obey the bound (6.4.4), but not parametrically.

6.5 Outlook

We have shown that 3d flux compactifications of massive IIA supergravity with fluxes, O2/O6 and anti-D2/anti-D6 sources are an interesting environment for classical dS model building for multiple reasons:

- The standard tachyons (or better: reasons for tachyon) in classical dS vacua [149, 150, 162] seem absent for generic models. However flux quantization in the simplest toroidal model does lead to tachyons. As we argued throat models in which the anti-D2 branes have warped down tensions should be safe from this in case the throat can be long enough. The existence of such throats is inferred from the actual construction for non-compact G2 spaces [88].

- The internal manifold can be Ricci flat and our understanding of the moduli problem is better for such manifolds.

- If warping can be present one can achieve "border-line" numbers for the string coupling (order 0.1) and for the volume (order 10^7 in string units)[11].

Note that anti-D6 uplifts have been found to be useful in 4d compactifications as well. For instance [171, 172] claims they lead to a dS landscape but the construction involves a mixture of racetrack potentials and classical fluxes and it is unclear whether it is truly top down. On the other hand, reference [141] added anti-D6 branes to the classical vacua of [130] and found the tachyons were absent. But the problem of large coupling and small volume persists. What we suggest in this analysis is that 3d compactifications allow classical solutions with better numbers and with flat internal spaces. Even more, if we use models with local throats the flux quantization problems we encountered seem alleviated.

If the no-dS conjecture is correct the problem with this scenario for constructing dS solutions can come from multiple directions, which could actually work against each other. Maybe the issues we noticed with flux quantization bringing one away from the meta-stable minimum persist. Although as we argued, at the same level of precision of the uplift procedure in KKLT one could argue that sufficiently large throats can do the job. We gave an explicit example around equation (6.3.35) with very mild warping where already a meta-stable minimum was reached. A different problem with these vacua that could enforce the no-dS Swampland conjecture is the open string stability. We argued that the conditions for open string stability would be of the form (6.3.40). We have explicitly verified that the stability of the anti-D6 was satisfied in case we can trust existing probe results [94] but that the stability of the anti-D2 is a worry due to equation (6.4.8), although no concrete brane-flux decay computation is carried out in lack of a concrete model without closed string tachyons after flux quantization. At this point it makes sense to assume that the no-dS conjecture would actually be enforced from the open-string sector once the closed-string sector seems stable. So dS model building continues to share many analogies with the "whack-a-mole" game.

[11] So the radii of the separate circles are order 10.

Chapter 7

Three-dimensional vacua in Type IIB

7.1 Introduction

As shown in the previous chapters, Type II on G2 holonomy manifolds seem to offer the possibility to scrutinize swampland conjectures. Here we pursue this direction further by working instead with manifolds with G2-structure thus non-zero internal curvature. There is an extended bibliography on flux compactifications with G2-structure, for example 4d vacua have been studied in [47, 61, 173–176] and 3d vacua of Heterotic strings have been studied for example in [177, 178]. A simple deviation from G2 holonomy is *co-calibrated* G2-structures, which will be our main focus. As we will see, since we want to reduce the amount of preserved supersymmetry to the minimum, i.e. N=1 in 3d, Type IIB offers a preferred framework, compared to Type IIA, due to the fact that O5/O9 planes are naturally compatible with the co-calibrated G2-structure.

In the rest of this work we first present the background geometry and then perform a direct dimensional reduction of Type IIB supergravity on toroidal orbifolds with co-calibrated G2-structure. We then work out the 3d superpotential and verify our findings via an appropriate S-duality. As an application, we study moduli stabilization within the 3d EFT framework which yields supersymmetric AdS_3 vacua. We also consider a related setup involving brane-supersymmetry-breaking (BSB) that has been developed and studied for example in [179–185], which allows for non-supersymmetric Anti-de Sitter as well as de Sitter vacua. For the de Sitter vacua we study first only the volume-dilaton sector and we see that stable critical points are allowed, however when we also switch-on the shape moduli we find that they pose a threat to stabilization in de Sitter. In addition, in all cases we find that scale-separation is in tension with certain quantization conditions.

7.2 Type IIB on toroidal orbifolds

7.2.1 Co-calibrated G2-structures and twisting the torus

We again restrict our attention to the simplest toroidal example where $X_7 = T^7/(Z_2 \times Z_2 \times Z_2)$ which is a seven torus orbifolded under the action of the Z_2 involutions in Eq.(3.2.4). The orbifold group is $\Gamma = \{\Theta_\alpha, \Theta_\beta, \Theta_\gamma\}$ and therefore one automatically has to take into account the combined involutions

$$\begin{aligned}
\Theta_\alpha\Theta_\beta : (y^1,\dots,y^7) &\to (+y^1,+y^2,-y^3,-y^4,-y^5,-y^6,+y^7)\,,\\
\Theta_\beta\Theta_\gamma : (y^1,\dots,y^7) &\to (-y^1,-y^2,+y^3,+y^4,-y^5,-y^6,+y^7)\,,\\
\Theta_\gamma\Theta_\alpha : (y^1,\dots,y^7) &\to (+y^1,-y^2,+y^3,-y^4,-y^5,+y^6,-y^7)\,,\\
\Theta_\alpha\Theta_\beta\Theta_\gamma : (y^1,\dots,y^7) &\to (-y^1,+y^2,+y^3,-y^4,+y^5,-y^6,-y^7)\,.
\end{aligned} \tag{7.2.1}$$

The above involutions allow us to twist the torus by introducing non-zero (geo)metric fluxes $\tau^i_{jk} \neq 0$ which introduce non-trivial components of the curvature and thus deviate from holonomy to G2-structure. We follow the steps of [47, 175] and twist the torus *à la* Scherk–Schwarz [186]. For the twisted torus one replaces the straight differential forms $\mathrm{d}y^i$ with twisted one-forms $\mathrm{d}y^i \to \eta^i$ which satisfy the Maurer–Cartan equation

$$\mathrm{d}\eta^i = \frac{1}{2}\tau^i_{jk}\eta^j \wedge \eta^k\,. \tag{7.2.2}$$

where τ^i_{jk}[1] describes a twisting of the direction i over the directions j and k, hence named twisted torus. This means we also have twisted vielbeins

$$e^i = r^i\eta^i\,, \tag{7.2.3}$$

and in particular in the previous expressions one does the replacements $\mathrm{d}y^{ijk} \to \eta^{ijk}$ and $\mathrm{d}y^{ijkl} \to \eta^{ijkl}$. From the Scherk–Schwarz reduction the geometric flux is constrained by

$$\tau^i_{ji} = 0\,, \quad \tau^l_{[ij}\tau^m_{k]l} = 0\,. \tag{7.2.4}$$

These conditions restrict the possible τ^l_{ij} values. In particular the specific orbifold group further projects out the torsion classes W_7 and W_{14}, therefore the structure equations in Eq.(3.1.12) become

$$\begin{aligned}
\mathrm{d}\Phi &= W_1 \star \Phi + W_{27}\,,\\
\mathrm{d}\star\Phi &= 0\,,
\end{aligned} \tag{7.2.5}$$

which is the case of *co-calibrated* G2-structures due to the closure of $\star\Phi$. This actually happens because the eliminated torsion classes, which were one- and two-forms, were not

[1] The geometric fluxes have an algebraic interpretation where for each η^i there exist a dual tangent vector z^i in the following way $\eta^i = B^i_j(y)\mathrm{d}y^j \to z_i = (B^{-1})^j_i\partial/\partial y^j$ where can be interpreted as the structure constants of a Lie algebra $[z_i, z_j] = \tau^l_{ij}z_l$.

invariant under the orbifold action. The Betti numbers, which depend on the presence of W_i, now coincide with those of the G2 holonomy case

$$b_0(X_7) = 1 \ , \quad b_1(X_7) = 0 \ , \quad b_2(X_7) = 0 \ , \quad b_3(X_7) = 7 \ , \tag{7.2.6}$$

and this means that the torsion class W_{27} can be expanded in the fundamental basis Ψ_i, which will be important for our calculations later.

In addition, following [47], we can also define the geometric flux matrix

$$\mathcal{M}_{ij} = \int_7 \Phi_i \wedge \mathrm{d}\Phi_j \ , \tag{7.2.7}$$

such that $\mathrm{d}\Phi_i = \sum_j \mathcal{M}_{ij}\Psi_j$. The values of $\mathcal{M}_{ij}$ depend on the coefficients τ^i_{jk} in the following way

$$\mathcal{M}_{ij} = \frac{1}{2}\begin{pmatrix} 0 & -\tau^7_{5,6} & -\tau^7_{3,4} & +\tau^1_{4,5} & +\tau^2_{4,6} & +\tau^1_{3,6} & -\tau^2_{3,5} \\ -\tau^7_{5,6} & 0 & +\tau^7_{1,2} & +\tau^3_{2,5} & -\tau^3_{1,6} & -\tau^4_{2,6} & -\tau^4_{1,5} \\ -\tau^7_{3,4} & +\tau^7_{1,2} & 0 & +\tau^6_{2,4} & +\tau^5_{1,4} & -\tau^5_{2,3} & +\tau^6_{1,3} \\ +\tau^1_{4,5} & +\tau^3_{2,5} & +\tau^6_{2,4} & 0 & -\tau^3_{4,7} & +\tau^1_{2,7} & +\tau^6_{5,7} \\ +\tau^2_{4,6} & -\tau^3_{1,6} & +\tau^5_{1,4} & -\tau^3_{4,7} & 0 & -\tau^5_{6,7} & -\tau^2_{1,7} \\ +\tau^1_{3,6} & -\tau^4_{2,6} & -\tau^5_{2,3} & +\tau^1_{2,7} & -\tau^5_{6,7} & 0 & +\tau^4_{3,7} \\ -\tau^2_{3,5} & -\tau^4_{1,5} & +\tau^6_{1,3} & +\tau^6_{5,7} & -\tau^2_{1,7} & +\tau^4_{3,7} & 0 \end{pmatrix} . \tag{7.2.8}$$

With the use of this matrix one can show that

$$W_1 = \frac{1}{7\mathrm{vol}(X_7)} \sum_{i,j} s^i \mathcal{M}_{ij} s^j = \frac{1}{7}\left(\prod_k s^k\right)^{-1/3} \sum_{i,j} s^i \mathcal{M}_{ij} s^j \ , \tag{7.2.9}$$

which gives an exact expression for the W_1 torsion class in terms of the moduli s^i and the geometric fluxes.

7.2.2 O5-planes and O9-planes

Now we turn the discussion to the relation of the orientifolds and the orbifold group Γ. In the previous sections we worked with Type IIA and space-filling O2-planes, however now the presence of both torsion and O2-planes is forbidden by the Maurer–Cartan equation, which automatically sets the structure constants to zero and brings us back to G2 holonomy. Thus one can only get N>1 Type IIA vacua with the presence of both O2-planes and internal curvature induced by twisting the torus.

Here instead we will focus on Type IIB where we can have O3, O5, O7 and O9-planes. Due to the lack of one-cycles and five-cycles in co-calibrated G2 the O3-planes and O7-planes are excluded in our setup. Therefore we focus on the O5-planes and O9-planes which as we will see fit nicely within the co-calibrated G2 setup. For the O5-planes, we choose the calibration for the source current to be proportional to the associated four-form

$$J_4(\mathrm{O5}) \sim \sum_i \Psi_i \ , \tag{7.2.10}$$

i.e. O5-planes wrap three-cycles inside the G2 space that need to be calibrated in a supersymmetric manner, if one wants to achieve a 3d supergravity. In this way the O5-plane involutions match with the orbifold group Γ. Then the O5-planes sit at the fixed points of the involutions (3.2.4) and their positions are shown in the following diagram

	y^1	y^2	y^3	y^4	y^5	y^6	y^7
$\mathrm{O5}_\alpha$	$-$	$-$	$-$	$-$	$\otimes$	$\otimes$	$\otimes$
$\mathrm{O5}_\beta$	$-$	$-$	$\otimes$	$\otimes$	$-$	$-$	$\otimes$
$\mathrm{O5}_\gamma$	$-$	$\otimes$	$-$	$\otimes$	$-$	$\otimes$	$-$
$\mathrm{O5}_{\alpha\beta}$	$\otimes$	$\otimes$	$-$	$-$	$-$	$-$	$\otimes$
$\mathrm{O5}_{\beta\gamma}$	$\otimes$	$-$	$-$	$\otimes$	$\otimes$	$-$	$-$
$\mathrm{O5}_{\gamma\alpha}$	$\otimes$	$-$	$\otimes$	$-$	$-$	$\otimes$	$-$
$\mathrm{O5}_{\alpha\beta\gamma}$	$-$	$\otimes$	$\otimes$	$-$	$\otimes$	$-$	$-$

Table 7.1: Localized positions and warped directions by the O5-planes. In contrast to the Type IIA setup, the O5-planes are interpreted simply by the Γ group involutions.

Similarly to the O6-plane analysis, the "$\otimes$" symbol denotes the directions on the internal X_7 manifold spanned by the O5-plane worldvolume, while the "$-$" denotes the "localized" (modulo smearing) positions (i.e. 0 and 1/2) of the O5-planes, related to the wrapped cycles by Hodge duality. This gives the following currents

$$j_\alpha = -e^{1234}\,, \quad j_\beta = -e^{1256}\,, \quad j_\gamma = e^{1357}\,, \tag{7.2.11}$$

and also $j_{\alpha\beta}$, etc. One can also deduce the smeared contribution of the O5-planes to the three-dimensional effective action. For example, for the $\alpha\beta$ three-cycle

$$\frac{j_{\alpha\beta}}{\mathrm{vol}(\alpha\beta)_4} = \frac{e^{3456}}{r^3r^4r^5r^6} = \Psi_1\,, \tag{7.2.12}$$

we would have

$$S_{\mathrm{O5}} \sim \int_{O5_{\alpha\beta}} \sqrt{-g_6} = \int_3 \sqrt{-g_3}\int_{3-\mathrm{cycle}} \sqrt{g_3} = \int_3 \sqrt{-g_3}\int_{\Phi^1} \star j_{\alpha\beta} = \int_3 \sqrt{-g_3}s^1\,. \tag{7.2.13}$$

We will give the exact and more compact form of (7.2.13) in the next section.

We see that the O5-planes are compatible with the G2 involutions. However, we should also ask that when we combine the O5-plane involutions $\sigma(\mathrm{O5})$ with the G2 then the generated involution is also due to a physical object. In other words we always ask the images of Op-planes to be Op-planes. For example if we take

$$\sigma(\mathrm{O5}_\alpha)\Theta_\beta : \eta^i \equiv \sigma(\mathrm{O5}_{\alpha\beta}) : \eta^i\,, \tag{7.2.14}$$

we verify that the web of O5-planes is generated. Now, there are six non-trivial combinations that generate all the O5-planes even if we assumed the existence of only one of them, but there is also a combination that leads to the identity. That is

$$\sigma(\mathrm{O5}_\alpha)\Theta_\alpha : \eta^i \to \eta^i\,, \tag{7.2.15}$$

which has to be identified as an involution arising from an Op-plane. Clearly the only candidate is the O9-plane, which is also 10d space-filling, and can be mutually supersymmetric with the O5-planes. To this end we have

$$\sigma(\mathrm{O5}_\alpha)\Theta_\alpha : \eta^i \equiv \sigma(\mathrm{O9})\eta^i \,. \tag{7.2.16}$$

This means our setup really resides in Type I string theory. Naturally, the configuration must also include a suitable number of D9-branes, resulting in an open string sector, which we will largely ignore in this work. Alternatively, we could consider a similar setup with O5- and O9$^+$-planes resulting in the brane-supersymmetry-breaking scenario [183]. We will return to this case in section 7.5.

7.2.3 The scalar potential from 10d

Since we plan to perform a dimensional reduction of Type IIB on a background that includes Op-planes we now discuss the possible background fluxes we can introduce and the field content of the 3d effective theory. We will need to discuss only the bosonic sector as the fermionic sector is fixed by supersymmetry. The latter, because of the O5-planes on top of the G2 and the O9-planes, is left with only two independent Killing spinors, that is we have 3d N=1 local supersymmetry

$$\text{Type IIB supercharges}: \quad 32 \xrightarrow{\text{O9-plane} \,-\, \text{Type I}} 16 \xrightarrow{\Gamma\ \text{orbifold}} 2 \ \text{ real}\,. \tag{7.2.17}$$

The gravity sector will essentially include the 3d external metric $g_{\mu\nu}$ and the seven s^i moduli that parametrize the *twisted* torus radii. We will further split them into the overall volume modulus v and the unit-volume deformations $\tilde{s}^i$ (which we will often refer to as shape moduli). These seven moduli, together with the dilaton ϕ, form the full set of eight real scalar moduli that will enter the 3d theory. Indeed, other scalar moduli would only arise from the reduction of the RR fields or the NS two-form and we will outline now why they are not a part of the 3d effective theory.

We will follow again [123] for the rules of the parities of the various fields

$$\begin{aligned} \text{even} &: \ F_1, F_5, H_3, H_7\,, \\ \text{odd} &: \ F_3, F_7\,, \end{aligned}$$

and we focus explicitly on the parities under the O5-planes. First we note that the H_3 has to be odd and so does the H_7 and since there is no odd three-form basis to expand H_3 on (or a seven-form to expand H_7), the H flux has to vanish. In addition the co-calibrated toroidal G2 has no one- or two-cycles (the Betti numbers are given by (7.2.6)) and so the 3d fluctuations of the B_2 NS gauge two-form are truncated. Now we turn to the RR sector. The C_0 RR field, which would be a scalar, is odd under the O5-plane and so its 3d fluctuations are truncated. The F_1 flux cannot be part of the background as there are no one-cycles. The C_2 RR field is even under the O5-plane, however, due to the lack of one- or two-cycles it does not give rise to vector or scalar fluctuations in 3d. In addition, two-forms in 3d are auxiliary fields and so they only contribute via their three-form background flux. Indeed, the three-form RR flux F_3 can have non-vanishing values. The F_3 is even under the O5 parity and therefore can be expanded on the basis of the even forms Φ_i, whereas the F_7, which is also even under O5, will just be proportional to the volume form of the internal space. Finally the C_4 is odd under O5 parity and since there are no odd three- or four-cycles it does not give rise to any 3d fluctuations. In addition, its F_5 flux would need to be expanded in a basis of odd five-forms which do not exist in the co-calibrated G2. As a result F_5 (and C_4) are completely truncated. This verifies that the 3d supergravity will only have the seven radii of the torus together with the dilaton ϕ as scalar moduli.

The (pseudo) action for the Type IIB supergravity in the Einstein frame is given by the sum of the NSNS and the RR parts bellow

$$\begin{aligned} S_{\text{NS}} &= \frac{1}{2\kappa_0^2}\int \mathrm{d}^{10}X\sqrt{-G}\Big(R - \frac{1}{2}\partial_M\phi\partial^M\phi - \frac{1}{2}e^{-\phi}|H_3|^2\Big)\,, \\ S_{\text{RR}} &= \frac{1}{2\kappa_0^2}\int \mathrm{d}^{10}X\sqrt{-G}\Big(-\frac{1}{2}\sum_n e^{\frac{5-n}{2}\phi}|F_n|^2\Big)\,, \end{aligned} \tag{7.2.18}$$

where n runs over 1, 3 and 5. The Born–Infeld part of either of the Dp-brane or Op-plane actions in the Einstein frame is

$$S_{\text{loc}} = \mu_p\int e^{\frac{p-3}{4}\phi}\sqrt{-g_{p+1}}\,, \tag{7.2.19}$$

where the tension of the local object is $\mu_p > 0$ for Op-planes and $\mu_p < 0$ for Dp-branes. We will give momentarily the details about the Bianchi identities that are related to the couplings of these objects to the RR fields.

We can now perform a direct dimensional reduction down to 3d. In 10d Einstein frame, our reduction Ansatz for the metric is written in Eq.(4.3.17)where v is a 3d scalar that accounts for the compactification volume and hence $\widetilde{ds}_7^2$ is the metric on a unit-volume G2 space. The world indices then break into external and internal respectively as $M = (\mu, m)$, where $\mu = 0, 1, 2$ and $m = 1, \ldots, 7$. The potential energy contributions to the three-dimensional action, that arise after the compactification from the ten-dimensional action considering the reduction Ansatz, are

$$\begin{aligned} V_R &= -\tilde{R}^{(7)}\, e^{-2\beta v}e^{2\alpha v}\,, \\ V_{\text{flux}} &= \frac{1}{2}|F_q|^2\, e^{\frac{5-q}{2}\phi}e^{2\beta v(\frac{7}{2}-q)}e^{3\alpha v}\,, \\ V_{\text{D}_\text{p}/\text{O}_\text{p}} &= -\mu_p\, e^{\frac{p-3}{4}\phi}\, e^{2\beta v(\frac{2p-11}{4})}e^{\frac{5}{2}\alpha v}\,, \end{aligned} \tag{7.2.20}$$

where the Ricci scalar of the co-calibrated G2 internal space is

$$\tilde{R}^{(7)} = \frac{21}{8}\tilde{W}_1^2 - \frac{1}{2}|\tilde{W}_{27}|^2\,, \tag{7.2.21}$$

Note however that $\sqrt{\tilde{g}} = 1$. Now with the specific choice of numbers

$$\alpha^2 = 7/16 \ , \quad \beta = -\frac{1}{4\sqrt{7}} \ , \quad -7\beta = \alpha \, , \tag{7.2.22}$$

we find canonical kinetic terms for the volume-dilaton in three dimensions Eq.(4.3.18). Let us now recall that the only background RR fluxes that we can switch on due to the O5-plane truncation (or the parity restrictions) are given by

$$F_7 = -\mathcal{G}\, \mathrm{d}y^{1234567} \ , \quad F_3 = \sum_i f^i \Phi_i \, , \tag{7.2.23}$$

which are consistent with tadpole cancellation, since $H_3 = 0$ and

$$\mathrm{d}F_7 = 0 \ , \quad \mathrm{d}F_3 \neq 0 \, . \tag{7.2.24}$$

The latter holds due to the co-calibrated G2-structure which gives rise to torsion. Because of that, the Bianchi identity for the F_3 is satisfied as

$$\mathrm{d}F_3 = -\mu_{\mathrm{O5}} J_4(\mathrm{O5}) \ , \quad \text{no D5-branes} \, . \tag{7.2.25}$$

As a result such background does not require D5-branes for the cancellation of the O5-plane source even though the NS H-flux is identically vanishing. In the presence of D5-planes the Bianchi identity (again for $H_3 = 0$) becomes

$$\mathrm{d}F_3 = -\mu_{\mathrm{O5}}\, J_4(\mathrm{O5}) - \mu_{\mathrm{D5}i}\, J_4(\mathrm{D5}i) \, , \tag{7.2.26}$$

where $\mu_{\mathrm{O5}} > 0$ and $\mu_{\mathrm{D5}i} < 0$, and we readily identify the O5-plane/D5-brane charges with their tension (up to the dilaton factors) because they are supersymmetric BPS objects. Here we indicate with $J_4(\mathrm{D5}i)$ the source current for the D5-branes wrapping the i-th 4-cycle. Even though the tadpole cancellation is seemingly possible without the use of D5-branes, namely as in (7.2.25), as we will see when we turn to explicit examples we will often need to use (7.2.26). For a review of the Type IIB ingredients we have used here see e.g. [67].

One may be worried about $\mathrm{d}F_3 \neq 0$ because it implies the existence of magnetic sources and that the F_3 is not closed any more, which means that there can be inconsistencies if in our theory a bare C_2 RR field also appears. However, it is important to appreciate that Type IIB supergravity does not have an honest Lorentz invariant Lagrangian, and as a result the full information of the consistent reduction is captured by the 10d equations of motion and the 10d tadpole conditions. In these equations the C_2 in fact does not appear, it is indeed introduced only after one solves the tadpoles with the condition $\mathrm{d}F_3 = 0$. Then the effective pseudo-action for Type IIB can be written down which will also include the C_2. However, a priori one only has a set of 10d equations of motion and Bianchi equations to solve. In our approach we first make sure we satisfy these conditions in the internal space and then we look directly at the resultant 3d effective theory.

7.3 The 3d N=1 superpotential

7.3.1 The scalar potential of 3d N=1 supergravity

Now we will construct the superpotential for the 3d N=1 supergravity by matching with the scalar potential that we derived from dimensional reduction in the previous parts. Since we want to have a 3d Einstein frame with the conventional 1/2 factor in front of the Hilbert–Einstein term, we perform a Weyl rescaling of the external 3d space metric of the form

$$g_{\mu\nu}^{\text{from dim. reduction}} = \frac{1}{4} \times g_{\mu\nu}^{\text{in 3d } N=1 \text{ supergravity}} . \tag{7.3.1}$$

This brings the kinetic terms for the scalar moduli and the scalar potential from the dimensional reduction to the form

$$e^{-1}\mathcal{L}_{\text{kin}} = \frac{1}{2}R_3 - \frac{1}{4}\partial v \partial v - \frac{1}{4}\partial\phi\partial\phi - \frac{1}{4}\text{vol}(\tilde{X}_7)^{-1}\int_7 \Phi_i \wedge \tilde{\star}\Phi_j \partial\tilde{s}^i\partial\tilde{s}^j - \frac{1}{8}V^{\text{dim. red.}} , \tag{7.3.2}$$

where we set the 3d Planck scale to unit and $V^{\text{dim. red.}}$ is the scalar potential from the direct dimensional reduction. We use the tilde "$\sim$" symbol to denote that the internal metric used is now the unit-volume one, and the internal metric shape moduli ($\tilde{s}^i$) are also the ones corresponding to the unit-volume. We will see momentarily exactly how this works.

In general, once we are given the kinetic terms of a 2-derivative 3d N=1 supergravity theory, the scalar potential is uniquely fixed by the superpotential, the latter being a real function of the scalar multiplets. In contrast to 4d N=1 here the superfields are real and so the superpotential is also real. In addition the scalar manifold is only required to be Riemannian and there is no pre-potential required to generate it. To be precise, the scalar sector of 3d N=1 supergravity has the form

$$e^{-1}\mathcal{L}_{\text{scalar}} = \frac{1}{2}R_3 - G_{IJ}\partial\varphi^I\partial\varphi^J - \left(G^{IJ}P_IP_J - 4P^2\right) , \tag{7.3.3}$$

where φ^I are the various real scalar moduli, the real function $P(\varphi^I)$ is the superpotential, and $P_I = \partial P/\partial\varphi^I$. For our setup, the moduli are $\varphi^I = (\tilde{s}^i, v, \phi)$, and therefore the scalar potential has the form

$$V = G^{IJ}P_IP_J - 4P^2 = G^{ij}P_iP_j + 4P_v^2 + 4P_\phi^2 - 4P^2 , \quad I = i, v, \phi , \tag{7.3.4}$$

where G^{ij} is the inverse of $\frac{1}{4}\text{vol}(\tilde{X}_7)^{-1}\int_7 \Phi_i \wedge \tilde{\star}\Phi_j$ and

$$P_i = \frac{\partial P}{\partial \tilde{s}^i} , \quad P_v = \frac{\partial P}{\partial v} , \quad P_\phi = \frac{\partial P}{\partial \phi} . \tag{7.3.5}$$

Note that the $\tilde{s}^i$ satisfy the condition

$$\text{vol}(\tilde{X}_7) = 1 = \left(\prod_i \tilde{s}^i\right)^{1/3} , \tag{7.3.6}$$

and therefore for our toroidal orbifold we find explicitly

$$G_{ij} = \frac{1}{4}\mathrm{vol}(\tilde{X}_7)^{-1}\int_7 \Phi_i \wedge \tilde{\star}\Phi_j = \frac{\delta_{ij}}{4(\tilde{s}^j)^2}\,. \tag{7.3.7}$$

We can solve the condition (7.3.6) by setting

$$\tilde{s}^7 = \prod_{a=1}^{6}\frac{1}{\tilde{s}^a}\,, \tag{7.3.8}$$

which we will often invoke throughout this work and in the examples later. Then (7.3.7) should not be used as the true scalar manifold metric for the $\tilde{s}^a$ scalars. Instead we have to take into account that $\partial_\mu\tilde{s}^7$ also contains derivatives with respect to the $\partial_\mu\tilde{s}^a$. Therefore from (7.3.3) once we take into account (7.3.7) and (7.3.8) we find

$$\tilde{G}_{ab} = \frac{1+\delta_{ab}}{4\,\tilde{s}^a\tilde{s}^b}\,,\quad a,b = 1,2,3,4,5,6\,, \tag{7.3.9}$$

such that $G_{ij}\partial\tilde{s}^i\partial\tilde{s}^j \equiv \tilde{G}_{ab}\partial\tilde{s}^a\partial\tilde{s}^b$. This matrix should be used when one wants to canonically normalize the scalars.

Let us now discuss an important technical point about the way that we evaluate the scalar potential from the superpotential. We first take the derivatives of the superpotential with respect to the *unrestricted* $\tilde{s}^i$, and then, after all derivatives have been evaluated, we impose the condition (7.3.6). This procedure is completely consistent because of the specific properties of our superpotential, otherwise such procedure would not preserve supersymmetry. In particular it was proven in previous section that a sufficient condition for doing this is

$$G^{ij}P_i\int \Phi_j \wedge \tilde{\star}\tilde{\Phi} = 0\,, \tag{7.3.10}$$

which we will see is always satisfied by our superpotential. In particular, when (7.3.10) holds then the condition (7.3.6) is fully supersymmetric in the sense that once we act on the latter with a supersymmetry transformation it also eliminates the fermion superpartner of the extra scalar and also the extra auxiliary field. In fact the condition (7.3.10) also guarantees that

$$P(s^i) \equiv P(\tilde{s}^i)\,, \tag{7.3.11}$$

where the s^i are the original seven moduli of the G2 that describe the internal metric deformations

$$s^i = \mathrm{vol}(X_7)^{3/7}\tilde{s}^i = e^{3\beta v}\tilde{s}^i\,. \tag{7.3.12}$$

As a result, due to (7.3.11), we can present our superpotentials in terms of s^i instead of $\tilde{s}^i$ to avoid cluttering, when possible, and without jeopardizing the result. However when we act with $\tilde{s}^i$ derivatives we have to recast them in terms of $\tilde{s}^i$ first and then take derivatives. We also stress once more that because we performed a 3d Weyl rescaling after the dimensional reduction we will have

$$V^{(7.3.4)\text{ from 3d N=1 superpotential}} = \frac{1}{8} \times V^{\text{from dim. reduction}} . \tag{7.3.13}$$

This means we multiply the scalar potential found from the dimensional reduction with 1/8 to match to the scalar potential we get from the superpotential calculation. In this way the supersymmetric theory (7.3.3) will agree with (7.3.2).

In the rest of this section we will present the total superpotential P in three steps: First we will present the superpotential that corresponds to the internal curvature contribution, then the one that corresponds to the F_3 flux, and then the one that corresponds to the F_7 flux. Since we essentially guess these contributions, we only need to check them by matching with the respective terms in the dimensional reduction scalar potential. Moreover, we will see that these three contributions to the superpotential can be combined without generating additional terms in the scalar potential, except one, which reproduces precisely the scalar potential term from the calibrated and smeared O5-planes (and possibly D5-branes). This cross-term is generated from the mixing of the internal curvature superpotential with the superpotential for F_3. Crucially it is the F_3 that is used in the tadpole cancellation conditions in the 10d supergravity and relates directly to the consistent incorporation of the O5-planes. This means that 3d N=1 supergravity is somehow aware of the 10d tadpole cancellation conditions and automatically takes them into account.

7.3.2 Superpotential from geometric flux

The superpotential for the internal curvature, i.e. the geometric flux, is

$$P^R = \frac{1}{16} e^{-8\beta v} \int \Phi \wedge \mathrm{d}\Phi \, \mathrm{vol}(X_7)^{-\frac{6}{7}} . \tag{7.3.14}$$

From (7.3.14) we directly see that

$$P^R_v = -\frac{\beta}{2} P^R , \quad P^R_\phi = 0 . \tag{7.3.15}$$

For the derivatives with respect to $\tilde{s}^i$ we have

$$\begin{aligned} P^R_i = & \frac{e^{-8\beta v}}{16} \Big(\int \Phi_i \wedge \mathrm{d}\tilde{\Phi} \, \mathrm{vol}(\tilde{X}_7)^{-\frac{6}{7}} + \int \tilde{\Phi} \wedge \mathrm{d}\Phi_i \, \mathrm{vol}(\tilde{X}_7)^{-\frac{6}{7}} \Big) \\ & - \frac{e^{-8\beta v}}{16} \Big(\frac{6}{7} \int \tilde{\Phi} \wedge \mathrm{d}\tilde{\Phi} \, \mathrm{vol}(\tilde{X}_7)^{-\frac{13}{7}} (\mathrm{vol}(\tilde{X}_7))_i \Big) , \end{aligned} \tag{7.3.16}$$

where $\tilde{\Phi} = \tilde{s}^i \Phi_i$ and we have $\mathrm{d}(A_3 \wedge B_3) = \mathrm{d}A_3 \wedge B_3 - A_3 \wedge \mathrm{d}B_3$. Then we use the third relation in Eq.(4.3.26) and the identity

$$\Phi_i \wedge \mathrm{d}\tilde{\Phi} = \tilde{\Phi} \wedge \mathrm{d}\Phi_i , \tag{7.3.17}$$

which bring the derivative of the superpotential with respect to $\tilde{s}^i$ to the form

$$P_i^R = \frac{1}{8}e^{-8\beta v}\Big(\int \Phi_i \wedge \mathrm{d}\tilde{\Phi} - \frac{\int \tilde{\star}\Phi_i \wedge \tilde{\Phi}}{\int \tilde{\star}\tilde{\Phi} \wedge \tilde{\Phi}} \int \tilde{\Phi} \wedge \mathrm{d}\tilde{\Phi}\Big)\, \mathrm{vol}(\tilde{X}_7)^{-\frac{6}{7}} \,. \tag{7.3.18}$$

We stress that the second formula in (7.3.17) is not an integration by parts, rather it is an exact algebraic identity. From this we can also deduce

$$P_i^R = \frac{1}{8}e^{-8\beta v} \int_7 \Phi_i \wedge \tilde{W}_{27}\, \mathrm{vol}(\tilde{X}_7)^{-\frac{6}{7}} \,, \tag{7.3.19}$$

where $\tilde{W}_{27} = W_{27}(\tilde{s}^i)$ (and we will similarly use $\tilde{W}_1$ shortly). This equation means that W_{27} sources the supersymmetry breaking due to torsion and that if it vanishes then the P_i^R vanish identically. A more extensive account of the properties we used here can be found in the previous sections, from which one can also prove that (7.3.18) satisfies (7.3.10). Now we insert the three pieces $P^R_{i,v,\phi}$ into the formula (7.3.4) and obtain

$$V^R\Big|_{\mathrm{vol}(\tilde{X}_7)=1} = \frac{1}{8}e^{-16\beta v}\Big(-\frac{21}{8}\tilde{W}_1^2 + \frac{1}{2}|\tilde{W}_{27}|^2\Big) = -\frac{\tilde{R}_7}{8}e^{-16\beta v} \,, \tag{7.3.20}$$

which is exactly the desired result. Note that this corresponds to the Ricci scalar found in [187], but here we write it in the notation of [47], and also it is automatically multiplied by the correct volume prefactor that appears from the dimensional reduction. As a technical remark, in deriving (7.3.20) we needed to contract (7.3.18) with G^{ij}, and to do this we have used in various instances the identity

$$G^{ij}\int \Phi_i \wedge A \int \Phi_j \wedge B = \frac{4}{7}\int \tilde{\Phi} \wedge \tilde{\star}\tilde{\Phi} \int \tilde{\star}A \wedge B \,, \tag{7.3.21}$$

which can be checked by expanding $A = \tilde{\star}\Phi_i A^i$ and $B = \tilde{\star}\Phi_i B^i$ (see e.g. [61]). For example, this identity was used to derive

$$\begin{aligned} G^{ij}\int \Phi_i \wedge \mathrm{d}\tilde{\Phi} \int \Phi_j \wedge \mathrm{d}\tilde{\Phi} &= \frac{4}{7}\int \tilde{\Phi} \wedge \tilde{\star}\tilde{\Phi} \int \tilde{\star}\mathrm{d}\tilde{\Phi} \wedge \mathrm{d}\tilde{\Phi} \\ &= 4\Big(\tilde{W}_1^2 \int \tilde{\Phi} \wedge \tilde{\star}\tilde{\Phi} + \int \tilde{W}_{27} \wedge \tilde{\star}\tilde{W}_{27}\Big) \\ &= 28\tilde{W}_1^2 + 4|\tilde{W}_{27}|^2 \,. \end{aligned} \tag{7.3.22}$$

Here of course we have considered that $\mathrm{d}\tilde{\Phi}$ is expanded in the basis Ψ_i, otherwise (7.3.21) cannot be used.

7.3.3 Superpotential from RR flux

The superpotential for F_3 takes the form[2]

$$P^{F3} = -\frac{q}{8}e^{-10\beta v + \frac{\phi}{2}} \int \star\Phi \wedge F_3\, \mathrm{vol}(X_7)^{-\frac{4}{7}} \,, \qquad q = \pm 1 \,. \tag{7.3.23}$$

[2]From the Type I perspective one could say that F_3 here is in fact $\tilde{F}_3 = F_3 - \frac{1}{4}(\omega_{YM} - \omega_L)$, but we largely ignore here the open string sector in any case.

The role of q and the ambiguity in choosing it is physical and reflects the ambiguity, from the 3d supergravity point of view, of introducing O5- or anti-O5-planes. In this section we will be working with O5-planes and we will see shortly how the sign of q can be fixed by matching with the potential from dimensional reduction.We can again directly evaluate

$$P_v^{F3} = -10\beta P^{F3}\,, \quad P_\phi^{F3} = \frac{1}{2}P^{F3}\,, \tag{7.3.24}$$

and

$$P_i^{F3} = \frac{q}{8}e^{-10\beta v+\frac{\phi}{2}}\Big(\int \Phi_i \wedge \tilde{\star}F_3 - \frac{\int \tilde{\star}\Phi_i \wedge \tilde{\Phi}}{\int \tilde{\star}\tilde{\Phi}\wedge\tilde{\Phi}}\int \tilde{\Phi}\wedge\tilde{\star}F_3\Big)\,\mathrm{vol}(\tilde{X}_7)^{-\frac{4}{7}}\,. \tag{7.3.25}$$

Note that (7.3.25) satisfies (7.3.10) as anticipated. We can also provide an alternative expression that has the form

$$P_i^{F3} = \frac{q}{8}e^{-10\beta v+\frac{\phi}{2}}\int_7 \Phi_i \wedge \tilde{\star}\pi^{27}(F_3)\,\mathrm{vol}(\tilde{X}_7)^{-\frac{4}{7}}\,, \tag{7.3.26}$$

where $\pi^{27}(F_3)$ denotes the projection of F_3 to the **27** representation of G2. Then we insert all these pieces into (7.3.4) and through a similar calculation as the one of the previous subsection we find

$$V^{F3}\Big|_{\mathrm{vol}(\tilde{X}_7)=1} = \frac{1}{16}e^{-20\beta v+\phi}\int_7 \tilde{\star}F_3\wedge F_3\,, \tag{7.3.27}$$

which is exactly the contribution to the scalar potential from the RR flux F_3. Note that we took into account that $q^2 = 1$ to get to this form. In addition, we are implicitly assuming $F_3 = f^i\Phi_i$ which means

$$\mathrm{d}F_3 = f^i\mathrm{d}\Phi_i \quad \text{with} \quad \mathrm{d}\Phi_i \neq 0\,, \tag{7.3.28}$$

due to torsion. However, $\mathrm{d}(\tilde{\star}F_3) = 0$ because our G2 is co-calibrated, i.e. $\mathrm{d}(\tilde{\star}\Phi_i) = 0$.

Since we have introduced and verified both P^R and P^{F3}, it is now a good time to combine them and uncover the O5-plane/D5-brane contribution to the scalar potential. To this end let us take

$$P^{R+F3} = P^R + P^{F3}\,. \tag{7.3.29}$$

Once we insert (7.3.29) into (7.3.4) we have

$$V^{R+F3} = V^{F3} + V^R + 2G^{IJ}P_I^R P_J^{F3} - 8P^R P^{F3}\,, \tag{7.3.30}$$

where the form of the cross-term is

$$\big(2G^{IJ}P_I^R P_J^{F3} - 8P^R P^{F3}\big)\Big|_{\mathrm{vol}(\tilde{X}_7)=1} = \frac{q}{8}e^{-18\beta v+\frac{\phi}{2}}\int_7\Big(\tilde{W}_1\,\tilde{\star}\tilde{\Phi}\wedge F_3 + \tilde{W}_{27}\wedge F_3\Big)\,. \tag{7.3.31}$$

This contribution has to be matched with the O5-plane/D5-brane contribution in the effective potential (7.2.20). We will now see how exactly this happens. First recall that each of the O5-planes wrap one internal three-cycle and therefore their currents wrap the dual four-cycles. Therefore for the total O5/D5 contribution we have

$$\begin{aligned} S_{\mathrm{O5}} + S_{\mathrm{D5}s} &= \frac{1}{8} e^{\phi/2} \sum_{3-\text{cycles}} (\mu_{\mathrm{O5}} + \mu_{\mathrm{D5}i}) \int_{\mathrm{3D} \times 3-\text{cycle}} \sqrt{-g_6} \\ &= \frac{1}{8} e^{\phi/2} \sum_{3-\text{cycles}} \left[\int_{\mathrm{3d} \times 3-\text{cycle}} \sqrt{-g_6} \int_{4-\text{cycle}} (\mu_{\mathrm{O5}} + \mu_{\mathrm{D5}i}) J_4(\mathrm{O5}) \right] , \end{aligned} \tag{7.3.32}$$

where the 1/8 factor comes from the 3d Weyl rescaling (7.3.1), and in going to the second line we have assumed a normalized integration over the four-cycles in the covering space such that

$$\int_{i\text{th}\, 4-\text{cycle}} J_4(\mathrm{O5}) = 1 = \int_{i\text{th}\, 4-\text{cycle}} J_4(\mathrm{D5}i) \,. \tag{7.3.33}$$

Let us stress that we are here explicitly ignoring open string moduli related to the D5s, which we assume to be fixed on their supersymmetric positions,otherwise we would have to include them in (7.3.32) - we leave this interesting development for a future work. Now we take into account that for each 6D integral that covers the 3d external space and one internal three-cycle we have

$$\int \sqrt{-g_6} = \int_{\mathrm{3d}} \sqrt{-g_3} \int_{\text{3-cyc.}} \sqrt{g_3} = \int_{\mathrm{3d}} \sqrt{-g_3} \int_{\text{3-cyc.}} \Phi = e^{3\beta v + 3\alpha v} \int_{\mathrm{3d}} \sqrt{-\tilde{g}_3} \int_{\text{3-cyc.}} \tilde{\Phi} \,, \tag{7.3.34}$$

which gives

$$S_{\mathrm{O5/D5}} = \frac{1}{8} e^{\frac{\phi}{2} + 3\beta v + 3\alpha v} \int_{\mathrm{3d}} \sqrt{-\tilde{g}_3} \sum_{\text{3-cycles}} \left[\int_{\text{3-cycle}} \tilde{\Phi} \int_{\text{4-cycle}} (\mu_{\mathrm{O5}} + \mu_{\mathrm{D5}i}) J_4(\mathrm{O5}) \right] . \tag{7.3.35}$$

To proceed it is instructive to work out the contribution for a specific three-cycle, and then recombine all the contributions including the other cycles. For example, for $i = 1$, we have

$$\begin{aligned} S_{\mathrm{O5/D5}(i=1)} &= \frac{1}{8} e^{-18\beta v + \frac{\phi}{2}} \int_{\mathrm{3d}} \sqrt{-\tilde{g}_3}\, \tilde{s}^1 \int_{\text{3-cyc.}} \phi_1 \int_{\text{4-cyc.}} (\mu_{\mathrm{O5}} + \mu_{\mathrm{D5}(i=1)}) J_4(\mathrm{O5}) \\ &= \frac{1}{8} e^{-18\beta v + \frac{\phi}{2}} \int_{\mathrm{3d}} \sqrt{-\tilde{g}_3} \int_7 (\tilde{s}^1\, \phi_1) \wedge J_4(\mathrm{O5}) (\mu_{\mathrm{O5}} + \mu_{\mathrm{D5}(i=1)}) \\ &= -\frac{1}{8} e^{-18\beta v + \frac{\phi}{2}} \int_{\mathrm{3d}} \sqrt{-\tilde{g}_3} \int_7 (\tilde{s}^1\, \phi_1) \wedge \mathrm{d}F_3 \,. \end{aligned} \tag{7.3.36}$$

The last step can be checked by acting with "$\tilde{s}^1 \phi_1 \wedge$" on (7.2.26). We then perform this procedure for the other six three-cycles and sum over the results to get the total contribution.

Taking into account that $\sum_i \tilde{s}^i \Phi_i = \tilde{\Phi}$, we conclude that

$$S_{\text{O5/D5}} = \sum_i S_{\text{O5/D5}i} = -\frac{1}{8} e^{-18\beta v + \frac{\phi}{2}} \int_{3\text{d}} \sqrt{-\tilde{g}_3} \int_7 \tilde{\Phi} \wedge dF_3 \,. \tag{7.3.37}$$

In addition we have that

$$\tilde{\Phi} \wedge \mathrm{d}F_3 = \mathrm{d}\tilde{\Phi} \wedge F_3 = \tilde{W}_1 \tilde{\star} \tilde{\Phi} \wedge F_3 + \tilde{W}_{27} \wedge F_3 \,, \tag{7.3.38}$$

where the first equality follows from $\tilde{\Phi} \wedge F_3 \equiv 0$. Then we conclude that the total contribution of the smeared O5-planes/D5-branes to the effective 3d potential is

$$V^{\text{O5/D5}} = \frac{1}{8} e^{-18\beta v + \frac{\phi}{2}} \int_7 \left(\tilde{W}_1 \tilde{\star} \tilde{\Phi} \wedge F_3 + \tilde{W}_{27} \wedge F_3 \right) \,, \tag{7.3.39}$$

which matches exactly with the extra term in (7.3.30) for

$$q = 1 \,. \tag{7.3.40}$$

Note that we could in principle split F_3 as $F_3 = F_{3A} + F_{3B}$ with $F_{3A} \neq 0$ such that $\mathrm{d}F_{3A} = 0$ but instead $\mathrm{d}F_{3B} = -\mu_{\text{O5}} J_4(\text{O5}) - \mu_{\text{D5}i} J_4(\text{D5}i)$, which would "free" one part of the F_3 flux from the tadpole condition.

Finally, for the F_7 flux (which is of Freund–Rubin type) the superpotential contribution is

$$P^{F7} = \frac{1}{8} \mathcal{G} \, e^{-14\beta v - \frac{\phi}{2}} \,, \tag{7.3.41}$$

where $\mathcal{G}$ is a real constant related to the F_7 flux (7.2.23). Then we evaluate the contribution to the scalar potential which gives

$$V^{F7} = \frac{1}{16} \mathcal{G}^2 e^{-28\beta v - \phi} \,. \tag{7.3.42}$$

The superpotential exponential $-14\beta v - \phi/2$ is compatible with the other exponentials and does *not* produce any new cross-terms. Note that we could have "$\pm\mathcal{G}$" in (7.3.41), but only one of the two would correspond to the 10d reduction with $F_7 = -\mathcal{G}\,\mathrm{d}y^{1234567}$, the other one would correspond to $F_7 = +\mathcal{G}\,\mathrm{d}y^{1234567}$. This ambiguity is fixed by S-duality which chooses the "+" sign as we will see momentarily. We conclude that the full superpotential that describes the dimensional reduction is given by $P^R + P^{F3} + P^{F7}$, and reproduces the 3d effective scalar potential (without the brane-supersymmetry-breaking term) which one can find by adding the contributions (7.3.20), (7.3.27), (7.3.39) and (7.3.42), and reads

$$\begin{aligned} V &= V^R + V^{F3} + V^{\text{O5/D5}} + V^{F7} \\ &= -\mathrm{R}_0(\tilde{s}^i) e^{-16\beta v} + \mathrm{F}_0(\tilde{s}^i) e^{-20\beta v + \phi} + \mathrm{T}_0(\tilde{s}^i) e^{-18\beta v + \frac{\phi}{2}} + \mathrm{G}_0 e^{-28\beta v - \phi} \,, \end{aligned} \tag{7.3.43}$$

with the coefficients given by

$$\mathrm{R}_0 = \frac{\tilde{R}_7}{8} = \frac{1}{64}\left(\sum_{i,j} \tilde{s}^i \mathcal{M}_{ij} \tilde{s}^j\right)^2 - \frac{1}{16}\sum_{i,j} \tilde{s}^i \mathcal{M}_{ij} \tilde{s}^j \sum_m \tilde{s}^m \mathcal{M}_{mj} \tilde{s}^j \,, \tag{7.3.44}$$

and

$$\mathrm{F}_0 = \frac{1}{16}\sum_i \left(\frac{f^i}{\tilde{s}^i}\right)^2 , \quad \mathrm{T}_0 = \frac{1}{8}\sum_{k,l} f^l \mathcal{M}_{lk} \tilde{s}^k \, , \quad \mathrm{G}_0 = \frac{\mathcal{G}^2}{16} \, . \tag{7.3.45}$$

7.4 Supersymmetric vacua

7.4.1 Supersymmetry cross-check

As a cross-check of the superpotential of the 3d theory, as well as the overall approach, we would like to verify that the 3d vacua that we will find truly describe supersymmetric configurations of the 10d theory. Because of the Op-plane truncations, the preservation of supersymmetry on our background boils down to the supersymmetry Killing equations that arise from Type I string theory with $F_{YM} = 0$, which can in turn be related to Heterotic string theory via S-duality. Earlier work on Heterotic string flux compactifications [178], has shown that backgrounds with

$$H_3^{(\mathrm{HET})} = \frac{1}{6} W_1 \Phi - \star W_{27} \, , \quad \mathrm{d}\,\phi^{(\mathrm{HET})} = 0 \, , \tag{7.4.1}$$

are supersymmetric. In other words $H_3^{(\mathrm{HET})} \equiv T(\Phi)$ is identified with the full antisymmetric G2 torsion (3.1.11). In [178] the W_7 is also present and relates to the dilaton via $\mathrm{d}\phi^{(\mathrm{HET})} = 2W_7$, however, the specific orbifolding we use for our twisted torus projects it out, such that we are strictly working with a co-calibrated G2. In addition an external component of the $H_3^{(\mathrm{HET})}$ flux is also allowed to be switched on, and is also related to the G2 torsion. The vacuum condition for the external $H_3^{(\mathrm{HET})}$ flux is

$$H_{\sigma\lambda\kappa}^{(\mathrm{HET\text{ - }ext})} = -\frac{7W_1}{6} e_\sigma^a e_\lambda^b e_\kappa^c \epsilon_{abc} \, , \tag{7.4.2}$$

where e_σ^a are the external drei-beins and ϵ_{abc} is the tangent space full antisymmetric symbol. This means $e_\sigma^a e_\lambda^b e_\kappa^c \epsilon_{abc}$ is indeed a tensor. Two comments are in order here. First, note that in [178] there is an overall factor e^n that relates the gravitino mass to the superpotential via $m_{3/2} = e^n P$. Here we have implicitly set it to unit, that is we have $n = 0$, because in our case the gravitino mass is given directly by $m_{3/2} = P$ on supersymmetric AdS as seen from (8.1.9). Secondly, in (7.4.2) we have not performed any additional Weyl rescalings, therefore it is still written in the original Heterotic string frame.

To match with Type I string backgrounds we perform an S-duality, which for the string frame fields is (see e.g. [2])

$$H_3^{(\mathrm{HET})} \to F_3 \, , \quad \phi^{(\mathrm{HET})} \to -\phi \, , \quad g_{MN}^{(\mathrm{HET})} \to e^{-\phi} g_{MN} \, , \quad \Phi \to e^{-3\phi/2}\Phi \, , \quad \star\Phi \to e^{-2\phi} \star \Phi \, , \tag{7.4.3}$$

and also affects the G2 torsion classes as

$$W_1 \to e^{\phi/2} W_1 \ , \quad W_{27} \to e^{-3\phi/2} W_{27} \, . \tag{7.4.4}$$

Therefore on a Type I supersymmetric background one would have the condition $e^{\phi} F_3 = \frac{1}{6} W_1 \Phi - \star W_{27}$ for the internal background flux and the condition $e^{\phi} F_3^{(\mathrm{ext})} = -\frac{7W_1}{6} \sqrt{g_3}\, \mathrm{d}t \wedge \mathrm{d}x \wedge \mathrm{d}z$ for the external background flux, with t, x and z being the external space coordinates.The background dilaton value would still satisfy $\mathrm{d}\phi = 0$. Then, going to Einstein frame $g_{MN} = e^{\phi/2} g^{(\mathrm{E})}_{MN}$, we find

$$e^{\frac{\phi}{2}} F_3 = \frac{1}{6} W_1 \Phi - \star W_{27} \ , \quad e^{\frac{\phi}{2}} F_3^{(\mathrm{ext})} = -\frac{7W_1}{6} \sqrt{g_3}\, \mathrm{d}t \wedge \mathrm{d}x \wedge \mathrm{d}z \ , \quad d\,\phi = 0 \, . \tag{7.4.5}$$

From the condition on the external $F_3^{(\mathrm{ext})}$ we find the required vacuum condition on F_7 to be

$$F_7 = -e^{\phi} \star_{10} F_3^{(\mathrm{ext})} = \frac{7W_1}{6} \, e^{\frac{\phi}{2}} \sqrt{g_7}\, \mathrm{d}y^{1234567} \, , \tag{7.4.6}$$

taking into account that $e^{(5-n)\phi/2} F_n = (-1)^{(n-1)(n-2)/2} \star F_{10-n}$ for $n > 5$. Finally these conditions become

$$e^{\frac{\phi}{2}} F_3 = \frac{1}{6} W_1 \Phi - \star W_{27} \ , \quad e^{-\frac{\phi}{2}} \mathcal{G} = -\frac{7W_1}{6} \, \mathrm{vol}(X_7) \, . \tag{7.4.7}$$

We conclude that the conditions (7.4.7) should hold for a supersymmetric vacuum of the (Einstein frame) Type I theory and we will re-derive them from our superpotential by requiring $P_I = 0$. This is a non-trivial cross-check.

We recall that the total superpotential in our setup (we choose $q = 1$) reads

$$P = \frac{\mathcal{G}}{8} e^{-14\beta v - \frac{\phi}{2}} - \frac{1}{8} e^{-10\beta v + \frac{\phi}{2}} \int \star\Phi \wedge F_3 \, \mathrm{vol}(X_7)^{-\frac{4}{7}} + \frac{1}{16} e^{-8\beta v} \int \Phi \wedge \mathrm{d}\Phi \, \mathrm{vol}(X_7)^{-\frac{6}{7}} \, . \tag{7.4.8}$$

We vary P with respect to the volume modulus v and the dilaton ϕ and we require $P_v = 0 = P_\phi$, which, after some manipulation, give

$$\mathcal{G}\, e^{-\phi/2} + \frac{13}{7} e^{\phi/2} \int \star\Phi \wedge F_3 = W_1 \mathrm{vol}(X_7) \, , \qquad e^{\phi/2} \int \star\Phi \wedge F_3 = \frac{11}{7} \mathcal{G}\, e^{-\phi/2} + 3\, W_1 \mathrm{vol}(X_7) \, . \tag{7.4.9}$$

Combining these two equations yields two conditions. First we find

$$\frac{6}{7} e^{-\phi/2} \mathcal{G} = -W_1 \mathrm{vol}(X_7) \, , \tag{7.4.10}$$

which matches exactly with the second condition in (7.4.7), and we also find

$$\frac{7}{6} W_1 \mathrm{vol}(X_7) = e^{\phi/2} \int \star\Phi \wedge F_3 \, , \tag{7.4.11}$$

which matches exactly with the first condition in (7.4.7) once we act on it with $\int \star\Phi \wedge (\cdot)$, taking into account that $\int \star\Phi \wedge \Phi = 7\text{vol}(X_7)$. Now we take the condition

$$\frac{\partial P}{\partial \tilde{s}^i} = 0\,, \tag{7.4.12}$$

where $i = 1, \ldots, 7$. This condition is sufficient to guarantee that $\partial P/\partial \tilde{s}^a = 0$, where $a = 1, \ldots 6$ are the true independent $\tilde{s}^a$ moduli. This happens because the unit-volume restriction can be solved as $\tilde{s}^7 = \prod_{a=1}^6 (\tilde{s}^a)^{-1}$ as we discussed earlier. The supersymmetry condition (7.4.12) gives

$$\int \Phi_i \wedge W_{27} = -e^{\phi/2} \int \star\Phi_i \wedge \pi^{27}(F_3)\,. \tag{7.4.13}$$

Note that $\partial P/\partial \tilde{s}^a = 0$ would at first sight correspond to only six equations, so one can wonder why in (7.4.13) we have seven equations. In fact the six $\partial P/\partial \tilde{s}^a = 0$ equations can only be solved once they are expanded in the Φ_i basis. The latter contains seven linearly independent elements and as a result the six $\partial P/\partial \tilde{s}^a = 0$ equations will eventually yield 7 equations (which are exactly (7.4.13)). Therefore, since the Φ_i are a complete basis, we can deduce

$$\star W_{27} = -e^{\phi/2} \pi^{27}(F_3)\,. \tag{7.4.14}$$

This equation matches exactly with the π^{27} part of the first equation in (7.4.7), taking into account that $\pi^{27}(\star W_{27}) \equiv \star W_{27}$. Interestingly we see that W_{27} can exist on a supersymmetric vacuum as long as it is cancelled by $\pi^{27}(F_3)$. This is in contrast to reductions of M-theory on co-calibrated G2 structures, where supersymmetry requires weak G2 holonomy, i.e. $W_{27} = 0$ [47].

Let us note at this point that the Type IIB background we have been considering contains smeared O5-planes, but interestingly, we see an exact match with the Heterotic supersymmetric background. This is a non-trivial check for the validity of the effective theory derived from the smeared solution and implies that there should be an underlying full solution in Type IIB where the orientifold sources are localized. Indeed, there are instances where the smearing can be "OK" [67, 69, 70]. We leave the interesting exercise of finding the underlying un-smeared solutions for the future.

7.4.2 Conditions for Minkowski and AdS

We can now examine the possibility of achieving full moduli stabilization and determine the required conditions thereof. From the conditions on the vacuum, that is equations (7.4.7), we find that the vacuum energy of a supersymmetric background is given by

$$V\Big|_{\text{SUSY}} = -\frac{\mathcal{G}^2}{16} e^{-\phi} \text{vol}(X_7)^{-4}\,. \tag{7.4.15}$$

From (7.4.15) we see that a Minkowski background would require $\mathcal{G} = 0$, which from (7.4.7) implies also that $\langle W_1 \rangle = \pi^1(F_3) = 0$. In other words, for a Minkowski vacuum we find the conditions

$$\text{SUSY Minkowski}: \quad \mathcal{G} \equiv 0\,, \quad \langle W_1 \rangle = 0 = \left\langle \int \Phi \wedge \star F_3 \right\rangle . \tag{7.4.16}$$

However we can still have non-trivial background flux and W_{27} torsion, as long as (7.4.14) is satisfied, which in fact also tells us that unless $W_{27} \neq 0$ the dilaton is not stabilized. Let us now go through the moduli stabilization on Minkowski in more detail. Because of the properties of the vacuum conditions it is more convenient to work directly with the s^i (and the dilaton of course), instead of treating the $\tilde{s}^a$ and the volume independently. Taking into account that $F_3 = f^i \Phi_i$, the last equation in (7.4.16) gives

$$\sum_i \frac{f^i}{s^i} = 0\,, \tag{7.4.17}$$

which for the moment fixes one of the 8 moduli (ϕ and s^i). Additionally, the condition $W_1 = 0$ gives an equation of the form

$$W_1 = 0 \quad \to \quad \sum_{i,j} s^i \mathcal{M}_{ij} s^j = 0\,, \tag{7.4.18}$$

which fixes one more of the seven s^i moduli. Equation (7.4.14) now, due to (7.4.16), reduces to

$$\mathrm{d}\Phi = -e^{\phi/2} \star F_3\,, \tag{7.4.19}$$

where we omit the VEV symbols, since they are implied. Then (7.4.19) gives

$$\sum_i s^i \mathcal{M}_{ij} = -e^{\phi/2} \left(\prod_k s^k \right)^{1/3} \frac{f^j}{(s^j)^2}\,, \tag{7.4.20}$$

which seemingly amounts to 7 vacuum conditions. Note however that (7.4.20) combined with (7.4.17) gives (7.4.18), which means one of the seven equations of (7.4.20) is already trivially satisfied. We therefore conclude that (7.4.20) provides only six additional equations, which are however enough to fix the positions of the dilaton and the five remaining s^i moduli. Clearly since this is a supersymmetric Minkowski vacuum, the absence of tachyonic instabilities is granted from supersymmetry.

Simple Minkowski vacua can be provided by the 2-step nilpotent examples of [47] which in our case read

$$\mathcal{M} = \begin{pmatrix} 0 & \sigma & \sigma & \sigma & -\sigma & -\sigma & -\sigma \\ \sigma & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma & 0 & 0 & 0 & 0 & 0 & 0 \\ -\sigma & 0 & 0 & 0 & 0 & 0 & 0 \\ -\sigma & 0 & 0 & 0 & 0 & 0 & 0 \\ -\sigma & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{7.4.21}$$

and we choose the values of F_3 and $\mathcal{G}$ to be

$$f^i = (0, f, f, f, -f, -f, -f) \ , \quad \mathcal{G} \equiv 0 \, . \tag{7.4.22}$$

The Minkowski vacuum can be found for the values

$$\tilde{s}^i = 1 \ , \quad f = -e^{-\frac{1}{14}(\sqrt{7}v_0 + 7\phi_0)}\sigma \, . \tag{7.4.23}$$

We also notice that when the $\tilde{s}^i$ are fixed on their vacuum values (7.4.23) but ϕ and v are left free the scalar potential (7.3.43) takes the form

$$V|_{\tilde{s}^i=1} = \frac{3}{8} e^{\frac{4v}{\sqrt{7}}} \left(e^{\frac{1}{14}(\sqrt{7}v + 7\phi)} f + \sigma \right)^2 \ , \tag{7.4.24}$$

which is consistent with (7.4.23) and also indicates the existence of at least one flat direction. This should not be confused with no-scale vacua because here $\langle P \rangle = 0$. Naturally, evaluating the determinant of the mass matrix, we find it to be vanishing.

Let us now turn to AdS supersymmetric vacua. Here we allow $P \neq 0$ and therefore we do not have to set $\mathcal{G}$ to vanish. As a result, the conditions (7.4.10) and (7.4.11) directly fix the dilaton and the volume fixed in terms of the six remaining independent s^i moduli. Indeed we find

$$e^{-\phi} = -\mathcal{G}^{-1} \int \star\Phi \wedge F_3 \, , \tag{7.4.25}$$

and

$$\mathrm{vol}(X_7)^2 = -\left(\frac{6}{7W_1} \right)^2 \mathcal{G} \int \star\Phi \wedge F_3 \, . \tag{7.4.26}$$

Then the seven conditions (7.4.14) fix the six remaining s^i. However, since they are readily contained in the first equation in (7.4.7), we can work directly with the latter. We can recast in fact the first equation in (7.4.7) to take the form

$$\frac{6\mathcal{G} \star F_3}{7W_1 \mathrm{vol}(X_7)} + \frac{7}{6} W_1 \star \Phi = \mathrm{d}\Phi \, , \tag{7.4.27}$$

which then explicitly gives seven equations once we expand on the Ψ_i basis. After some manipulations these equations read

$$\frac{6\mathcal{G}}{s^i} \left(\frac{f^i}{s^i} - \sum_{k=1}^{7} \frac{f^k}{s^k} \right) = \frac{\left(\sum_{m,n} s^m \mathcal{M}_{mn} s^n \right)}{\left(\prod_l s^l \right)^{1/3}} \sum_j s^j \mathcal{M}_{ji} \, , \tag{7.4.28}$$

and should be solved in terms of the s^i. Clearly (7.4.28) (or equivalently (7.4.27)) describes only six independent equations due to the condition (7.4.26)that is satisfied by the volume. We conclude that the RR and geometric fluxes give the possibility to stabilize all 8 moduli on a supersymmetric AdS_3 vacuum.

An example for the matrix $\mathcal{M}_{ij}$ (this is a specific instance of the $SO(p,q) \times U(1)$ example of [47]) that leads to full moduli stabilization is

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & 0 & h & 0 & 0 & m \\ 0 & 0 & 0 & h & 0 & 0 & m \\ 0 & 0 & 0 & -h & 0 & 0 & m \\ h & h & -h & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ m & m & m & 0 & 0 & 0 & 0 \end{pmatrix} , \tag{7.4.29}$$

and we further assume that there exists a supersymmetric AdS vacuum at the positions

$$\langle \tilde{s}^i \rangle = 1 \ , \quad \langle \phi \rangle = \phi_0 \ , \quad \langle v \rangle = v_0 \, . \tag{7.4.30}$$

To find this solution we start by evaluating $P_\phi = 0$ on the Ansatz (7.4.30) and find that it is solved by

$$\mathcal{G} = -e^{\phi_0 - \frac{v_0}{\sqrt{7}}} \left(\sum_i f^i \right) \, . \tag{7.4.31}$$

Similarly, we can evaluate $P_v = 0$ and using (7.4.31) and the Ansatz (7.4.30) to find

$$3 \left(\sum_i f^i \right) e^{\frac{\phi_0}{2} + \frac{v_0}{2\sqrt{7}}} = h + 3m \, . \tag{7.4.32}$$

This determines the explicit values for ϕ_0 and v_0.Then from the expressions $\partial P / \partial \tilde{s}^i = 0$, for $i = 1, \ldots, 7$, we get a series of equations which we simply satisfy by assigning the appropriate values to the f^i. Once we make use of the Ansatz and the conditions (7.4.31) and (7.4.32) the solutions read

$$\begin{aligned} f^1 &= f^2 = -\frac{2}{3} e^{-\frac{\phi_0}{2} - \frac{v_0}{2\sqrt{7}}} \, h \, , \\ f^3 &= \frac{4}{3} e^{-\frac{\phi_0}{2} - \frac{v_0}{2\sqrt{7}}} \, h \, , \\ f^4 &= -\frac{1}{3} e^{-\frac{\phi_0}{2} - \frac{v_0}{2\sqrt{7}}} \, (2h - 3m) \, , \\ f^5 &= f^6 = \frac{1}{3} e^{-\frac{\phi_0}{2} - \frac{v_0}{2\sqrt{7}}} \, (h + 3m) \, , \\ f^7 &= \frac{1}{3} e^{-\frac{\phi_0}{2} - \frac{v_0}{2\sqrt{7}}} \, (h - 6m) \, . \end{aligned} \tag{7.4.33}$$

One can also evaluate the vacuum energy which is given by

$$\langle V \rangle = -\frac{1}{144} e^{\frac{4v_0}{\sqrt{7}}} (h + 3m)^2 \, . \tag{7.4.34}$$

In general the above Ansatz/solution has full moduli stabilization on supersymmetric AdS because

$$\det[m^2] \neq 0 \,. \tag{7.4.35}$$

For example, for a specific setup we can have

$$\phi_0 = -3 \,, \; v_0 = -33\sqrt{7} \,, \; h = 1 \,, \; m = -1 \,, \tag{7.4.36}$$

which gives large volume and weak string coupling, and we can easily verify numerically that

$$\langle V \rangle < 0 \;, \quad \text{Eigenvalues}[m^2] > 0 \,, \tag{7.4.37}$$

which guarantees a full moduli stabilization. Note however that if we take $h = 3$ instead of $h = 1$ then we obtain a Minkowski solution with $\det[m^2] = 0$.

7.4.3 Indication for scale separation

Let us now discuss the possibility of having scale separation in the supersymmetric AdS vacua. The scalar potential in our setup has the form

$$V = \mathrm{F}_0 \, e^{-20\beta v + \phi} - \mathrm{R}_0 \, e^{-16\beta v} + \mathrm{T}_0 \, e^{-18\beta v + \frac{\phi}{2}} + \mathrm{G}_0 \, e^{-28\beta v - \phi} \,, \tag{7.4.38}$$

where $\mathrm{F}_0 = |F_3|^2/16$, $\mathrm{G}_0 = \mathcal{G}^2/16$, $\mathrm{T}_0 = -\mu_{O5}/8$ and $\mathrm{R}_0 = R_7/8$. Once we minimize (7.4.38) we find the vacuum values v_0 and ϕ_0, and the vacuum energy is given by (7.4.15). To study the scale separation we follow closely the steps of the previous sections, which means we ask that we can have flux values such that there is a limit where

$$\frac{L_{\text{KK}}^2}{L_\Lambda^2} = e^{16\beta v} \, V_{\text{vac}} \to 0 \,. \tag{7.4.39}$$

Here L_{KK} is the Kaluza–Klein scale that characterizes the internal space, and L_Λ is the scale that characterizes the external 3d Anti-de Sitter space.

To this end we consider a scaling limit where $\mathrm{G}_0 \sim N^a$ and $\mathrm{F}_0 \sim N^{a+2b}$ as $N \to \infty$ and we demand that each term in the potential has the same scaling behavior. Equating the scaling for the internal and external flux terms implies

$$N^{a+2b} e^{\phi - 20\beta v} \sim N^a e^{-\phi - 28\beta v} \implies N^{-2b} e^{-8\beta v} \sim e^{2\phi} \equiv N^{2p} \,, \tag{7.4.40}$$

which leads to

$$V_{\text{vac}} \sim N^{a+6p+7b} \;, \quad T_0 \sim N^{a+p+\frac{5}{2}b} \;, \quad R_0 \sim N^{a+2p+3b} \;, \quad e^{16\beta v} V_{\text{vac}} \sim N^{a+2p+3b} \,. \tag{7.4.41}$$

The fact that R_0 has the same scaling as $e^{16\beta v} V_{\text{vac}}$, means that in order to achieve scale separation, we need to be able to take R_0 small. We can also see that ${\mathrm{T}_0}^2 \sim \mathrm{R}_0 \mathrm{F}_0$, consistent with the supersymmetric origin of the O5-plane term.

In principle we expect our scaling limit to correspond to some large value for the fluxes. However,the tadpole condition $\mathrm{d}F_3 = -\mu_{\mathrm{O5}} J_4$ means thatthe only possible consistent scaling we can have is

$$\mathrm{T}_0 \sim \mathrm{F}_0 \sim \mathrm{R}_0 \sim N^0 \,, \tag{7.4.42}$$

making scale separation impossible due to flux quantization.

Thus, to achieve scale separation we have to first cancel the tadpole in such a way that the fluxes are not restricted neither from the Bianchi nor from the torsions. This will allow them to take parametrically large or small values independently. As a result we include D5s such that for the tadpole of the form (7.2.26) we get

$$\mathrm{d}F_3 = 0 = -\mu_{\mathrm{O5}}\, J_4(\mathrm{O5}) - \mu_{\mathrm{D5}}\, J_4(\mathrm{D5}) \,, \tag{7.4.43}$$

where we recall that $\mu_{\mathrm{O5}} > 0$ and $\mu_{\mathrm{D5}} < 0$, but we keep the F_3 flux to non-vanishing values, that is $F_3 \neq 0$ and $F_7 \neq 0$. This can be achieved by taking

$$\sum_i f^i \mathrm{d}\Phi_i = 0 \,, \tag{7.4.44}$$

which can have non-trivial solutions due to the freedom in choosing the torsion. Returning to the scalar potential, which is now missing the contribution from the O5-plane as it is cancelled by the D5-branes, we have

$$V = \mathrm{F}_0\, e^{-20\beta v+\phi} - \mathrm{R}_0\, e^{-16\beta v} + \mathrm{G}_0\, e^{-28\beta v-\phi} \,. \tag{7.4.45}$$

Note also that the supersymmetric minimization with respect to ϕ and v is bound to give (7.4.15), that is

$$\langle V\rangle = -\frac{\mathcal{G}^2}{16} e^{-\phi_0} \mathrm{vol}(X_7)^{-4} = -\mathrm{G}_0 e^{-28\beta v_0-\phi_0} \,. \tag{7.4.46}$$

This also guarantees that our moduli stabilization is consistent and non-tachyonic. Equivalently one can vary the scalar potential with respect to the volume and the dilaton to get

$$3\,\mathrm{G}_0\, e^{-28\beta v-\phi} = \mathrm{R}_0\, e^{-16\beta v} \;, \quad \mathrm{F}_0\, e^{-20\beta v+\phi} = \mathrm{G}_0\, e^{-28\beta v-\phi} \,, \tag{7.4.47}$$

which are consistent for this setup and give again the supersymmetric vacuum energy (7.4.46). To obtain scale separation, we take the scaling[3]

$$R \sim N^{-2} \;, \quad F_3 \sim N^0 \;, \quad F_7 \sim N^0 \,. \tag{7.4.48}$$

Asking that all the terms in the scalar potential scale in the same manner, we get

$$V \sim N^{-6} \;, \quad g_s = e^{\phi} \sim N^{-1} \;, \quad \mathrm{vol}(X_7) = e^{7\beta v} \sim N^{\frac{7}{4}} \,. \tag{7.4.49}$$

[3]Small values for the torsion can be also used, as in [188] and [132], to get de Sitter vacua. For constraints on geometric fluxes see e.g. [189].

Then we finally get

$$\frac{L_{\mathrm{KK}}^2}{L_\Lambda^2} = e^{16\beta v} V \sim N^{-2} \,. \tag{7.4.50}$$

We conclude that achieving parametric scale separation requires taking the internal curvature R_7 to extremely small but positive values. Note that in this limit we remain at weak string coupling and large volume, and are therefore well within the regime of validity of the supergravity approximation. Since the $\tilde{s}^i$ should be fixed to finite values (otherwise the volume becomes singular), requiring small internal curvature means we have to tune the structure constants in the twisted torus. However, this requirement can run into tension with quantization conditions on the structure constants [189], potentially making parametric scale separation impossible for the types of compactifications considered here. For further discussion on the intricacies of achieving scale separation in string theory see e.g. [17, 25–31, 56, 59, 71, 113]. Note in particular that in [27] the difficulty to get scale separation in Type IIB vacua has been anticipated.

7.5 Brane supersymmetry breaking

7.5.1 Introducing anti-D9-branes

Until now we have worked with O5-planes which due to the orbifold involutions gave rise to an image O9-plane, that is an object with negative tension and with charge with opposite sign than that of a D9-brane. However, instead of a conventional O9-plane one can consider a so-called "O9$^+$"-plane which has *positive* tension, and charge with the same sign as that of a D9-brane. Then the RR tadpole for the O9$^+$-plane is now to be cancelled by 16 anti-D9-branes. This combination of O9$^+/\overline{\mathrm{D9}}s$ is the so-called brane-supersymmetry-breaking (BSB) setup (a very recent review can be found in [183]). The gauge theory on such setup is USp(32), and because both the anti-D9-brane and the O9$^+$ have positive tensions, these add up and give a non-vanishing dilaton-dependent vacuum energy. In addition supersymmetry on the world-volume of this system is spontaneously broken and non-linearly realized. In particular the vacuum energy in the 10d Einstein frame has the form

$$V_{\text{10d-BSB}} = \mathrm{B}_0 \, e^{\frac{3}{2}\phi} \,. \tag{7.5.1}$$

In [181, 182] for example the coefficient B_0 is specified to be $\mathrm{B}_0 = 64 T_9$, where T_9 is the D9-brane tension up to the $e^{\frac{3}{2}\phi}$ dilaton factor, as in (7.2.20). Once we perform a direct dimensional reduction by inserting our metric Ansatz (4.3.17) it becomes in 3d

$$V_{\text{3d-BSB}} = \mathrm{B}_0 \, e^{-14\beta v + \frac{3}{2}\phi} \,. \tag{7.5.2}$$

To embed this new term in the 3d superpotetnial we have to include a real scalar nilpotent superfield [151], let us call it X, which satisfies

$$X^2 = 0 \,. \tag{7.5.3}$$

As in 4D (see e.g. [190]) such nilpotent superfields tend to rise the vacuum energy and capture the effects of anti-branes. The modification to the G_{IJ} metric to account for the coupling of X to 3d supergravity will be $G_{XX} = 1$ and $G_{Xi} = G_{Xv} = G_{X\phi} = 0$, whereas the superpotential contribution is

$$P^{\mathrm{BSB}} = \sqrt{\mathrm{B}_0}\, X\, e^{-7\beta v+\frac{3}{4}\phi}\,. \tag{7.5.4}$$

Let us stress that this non-linearity is intrinsic and it is inherited directly by the non-linear supersymmetry of the 10d BSB theory [179–182].

With the inclusion of the BSB term the total scalar potential for the volume-dilaton sector, i.e. ignoring the $\tilde{s}^a$ or assuming they are stabilized, reads

$$\begin{aligned} V &= \mathrm{F}_0 e^{-20\beta v+\phi} + \mathrm{G}_0 e^{-28\beta v-\phi} + \mathrm{T}_0 e^{-18\beta v+\frac{1}{2}\phi} - \mathrm{R}_0 e^{-16\beta v} + \mathrm{B}_0 e^{-14\beta v+\frac{3}{2}\phi} \\ &\equiv F + G + T - R + B\,. \end{aligned} \tag{7.5.5}$$

With $\mathrm{F}_0, \mathrm{G}_0, \mathrm{B}_0 \geq 0$ and $\mathrm{T}_0 \leq 0$ and we temporarily change notation such that $F = V^{F3}, G = V^{F7}$ etc. for visual convenience in the equations below. A critical point of this potential satisfies

$$\begin{aligned} 4F + 14G + 5T - 6R &= 0\,, \\ 2F - 2G + T + 3B &= 0\,. \end{aligned} \tag{7.5.6}$$

Which allows us to express the vacuum energy as

$$V_{\mathrm{vac}} = \frac{B}{2} - G\,. \tag{7.5.7}$$

The dependence of the vacuum energy only on two terms instead of three is remarkable. Solving (7.5.6) for different pairs of terms and substituting back into the potential results in an apparent dependence on all three remaining terms. However, note that the potential with only F, T, R terms would result in a no-scale or runaway potential, and thus vanishing vacuum energy, while the other terms give additive corrections to the scalar potential, without generating cross-terms. In other words, the cosmological constant is ultimately determined solely by the interplay of Freund–Rubin-type fluxes (F_7) and supersymmetry breaking terms. The reason internal fluxes do not contribute to the cosmological constant appears to be that satisfying the tadpole condition by O5 planes generates precisely the right tension to cancel their contribution. This is in line with the observation that although O-planes appear to evade the usual supergravity de Sitter no-go theorems [191, 192], once the flux they source is taken into account, the total stress-tensor does not produce a positive contribution to the vacuum energy [193].[4]

[4] From (7.5.7) we also see why one cannot get de Sitter vacua from Type I by simply adding anti-D5-branes, and instead we have to switch to the BSB setup to be able to even discuss such possibility. The fact that one may need two types of supersymmetry breaking sources to get classically stable de Sitter vacua was already alluded to in our previous dS analysis and as we will see we will need here both anti-D5s and anti-D9s as well, once we discuss the shape moduli stabilization.

While in the case of the Freund–Rubin term (7.3.41) the lack of additional cross-terms is justified by supersymmetry, with the BSB term, this amounts to ignoring backreaction from the anti-branes and therefore constitutes an important caveat to the analysis. It is possible that additional backreaction terms in the spirit of [194] are present. Nonetheless, let us press forward and explore the possibility of de Sitter minima of this potential. The mass matrix eigenvalues are

$$m_{\pm}^2 = \frac{1}{7}\left(20G - T - 2B \pm \sqrt{88B^2 - 3B(8G+T) + (8G+T)^2}\right), \tag{7.5.8}$$

which are positive when

$$B < 8G\,, \quad T < \frac{12B^2 + 8BG - 48G^2}{B - 8G}\,. \tag{7.5.9}$$

Note that this in principle allows for a positive vacuum energy when

$$2G < B < 8G \implies 2 < \frac{\mathrm{B}_0}{\mathrm{G}_0} e^{14\beta v + \frac{5}{2}\phi} < 8\,, \tag{7.5.10}$$

with the lower inequality giving positive energy, while the upper inequality guaranteeing (meta-) stability. Note however that this imposes a relation between the stabilized values of the dilaton and the volume. Finally, our findings are consistent with [185] because we have O5-plane sources. However the final verdict on the existence of such de Sitter vacuum can only be made after we stabilize the $\tilde{s}^a$ moduli and we take into account flux quantization.

As in the supersymmetric case, we can consider a scaling limit where $\mathrm{G}_0 \sim N^a$ and $\mathrm{F}_0 \sim N^{a+2b}$ as $N \to \infty$ and we demand that each term in the potential has the same scaling behavior. This once again determines the scalings

$$V_{\rm vac} \sim N^{a+6p+7b}\,, \qquad B_0 \sim N^{a+p+\frac{7}{2}b}\,, \qquad T_0 \sim N^{a+p+\frac{5}{2}b}\,, \qquad R_0 \sim N^{a+2p+3b}\,, \tag{7.5.11}$$

which lead to

$$\frac{B_0}{G_0} e^{14\beta v + \frac{5}{2}\phi} \sim N^{-b}\,, \qquad e^{16\beta v} V_{\rm vac} \sim N^{a+2p+3b}\,, \tag{7.5.12}$$

where we note that $b \neq 0$ means that we inevitably violate (7.5.10) as $N \to \infty$. This means that we need $\mathrm{F}_0 \sim \mathrm{G}_0$ to preserve the stable de Sitter vacua. As before, R_0 has the same scaling as the scale-separation parameter, $e^{16\beta v} V_{\rm vac}$, so we need to be able to take it small to achieve parametric scale separation, conflicting with the quantization of geometric flux. Furthermore, $b = 0$ also ensures that B_0 and T_0 have the same scaling, which we expect due to both terms arising from branes.

In fact we may further demand $\mathrm{T}_0 \sim \mathrm{B}_0 \sim N^0$, which requires $p = -a$ and $\mathrm{R}_0 \sim N^p$. This does indeed become small at large internal volume and weak coupling, yielding scale separation, but being in tension with quantization of the structure constants of the internal manifold.

On the other hand, if we don't demand parametric scale-separation, i.e. $\mathrm{R}_0 \sim N^0$ then we have $\mathrm{B}_0 \sim \mathrm{T}_0 \to \infty$ in our scaling limit. This, however is also unacceptable since the magnitude of B_0 is fixed.[5] Thus despite the scalar potential appearing to have de Sitter critical points, string theory does not seem to allow for parameter values such that these critical points appear at large internal volume and weak coupling, where this scalar potential is trustworthy.[6]

7.5.2 Explicit examples of 3d de Sitter solutions?

Actually, achieving full moduli stabilization including the $\tilde{s}^a$ is challenging, and we do not have a systematic way of tackling this question. However it is instructive to see first if we can generate the de Sitter vacua with the co-calibrated G2 geometry we have at hand following the methodology we also followed in our previous example with de Sitter in Type IIA. This does not give the most general de Sitter solution but it offers a simple way to obtain it. First we want to stabilize the $\tilde{s}^a$ in their "autonomous" supersymmetric positions, which means supersymmetric position of $\tilde{s}^a$ which do not require to fix the other moduli. From (7.4.14) we see that we would need

$$W_{27} = 0 \ , \quad \pi^{27}(F_3) = 0 \, , \tag{7.5.13}$$

such that the dilaton VEV is kept free and is to be determined independently. In addition the equation $W_{27} = 0$ can be also solved independent of the volume modulus. Indeed taking into account that $s^i = e^{3\beta v}\tilde{s}^i$, equation $W_{27} = 0$ takes the volume-independent form

$$\sum_i \tilde{s}^i M_{ij} - \frac{1}{7}\Big(\sum_{m,n} \hat{s}^m M_{mn} \tilde{s}^n\Big)\frac{1}{\tilde{s}^j} = 0 \ , \quad \prod_i \tilde{s}^i = 1 \, . \tag{7.5.14}$$

Then we notice that due to the structure of the scalar potential, even when it includes the BSB term, we have

$$\frac{\partial P}{\partial \tilde{s}^a}\Big|_{(7.5.14)} = 0 \quad \to \quad \frac{\partial V}{\partial \tilde{s}^a}\Big|_{(7.5.14)} = 0 \, . \tag{7.5.15}$$

This is because of the properties (7.3.15) and (7.3.24), but also $(G^{\phi\phi})_a = (G^{vv})_a = 0$, and of course from (7.3.41) we automatically have $P_a^{F7} \equiv 0$. Therefore we can find vacua where the shape moduli are stabilized at their autonomous supersymmetric positions, and then we need only to stabilize the volume and the dilaton. This is exactly how the stabilization happens in Chapter 6. Now let us see if under the assumptions (7.5.13) we can get de Sitter. We do not have to go into details, only check if the conditions we derived for de Sitter solution still hold. From (7.5.6) and (7.5.7) we see that

$$R = -F - G - 5V_{\text{vac}} \, , \tag{7.5.16}$$

[5] This situation is similar to [133], where weakly coupled, large volume 4d dS compactifications of massive Type IIA appear to require large numbers of O6 planes.

[6] A similar effect can be observed directly in gauged 4D $\mathcal{N}$=2 supergravity [195].

which means de Sitter critical points exist only for $\tilde{R}^{(7)} < 0$. In contrast to the latter, we see that (7.5.13) (or (7.5.14)) dictates $\tilde{R}^{(7)} \geq 0$. We conclude that there do not exist any de Sitter critical points that can be found with the method we followed in Chapter 6. As we said this does not exclude the possible existence of de Sitter, however it does leave much less room for it.

The fact that the shape moduli interfere with the construction of de Sitter solutions has been also discussed for example in [149, 168]. Indeed we believe that our example shows exactly how fixing the shape moduli into their "autonomous" supersymmetric positions creates problems to finding de Sitter. In other words, if we had the shape moduli fixed in such autonomous supersymmetric positions and then we tried to uplift the vacuum to de Sitter we would force them to move out of these supersymmetric positions, and so the stabilization procedure would have to be worked out from scratch. We conclude that one should not ignore the stabilization of shape moduli during the uplift, nor take it for granted when searching for realistic examples.

One could try to construct de Sitter vacua with the shape moduli in their supersymmetric positions by including also anti-D5-branes. Let us see what would happen if we included such objects - assuming momentarily they can be included consistently in our setup. Their contribution to the Bianchi identity would be

$$\mathrm{d}F_3 = -\mu_{\mathrm{O5}}\, J_4(\mathrm{O5}) - \mu_{\mathrm{D5}i}\, J_4(\mathrm{D5}i) + \mu_{\overline{\mathrm{D5}}i}\, J_4(\overline{\mathrm{D5}}i)\,, \tag{7.5.17}$$

where $\mu_{\overline{\mathrm{D5}}i} < 0$, and the brane action (ignoring open string moduli) is

$$S_{\overline{\mathrm{D5}}s} = \frac{1}{8} e^{-18\beta v + \frac{\phi}{2}} \int_{3\mathrm{d}} \sqrt{-\tilde{g}_3} \sum_{\text{3-cycles}} \left[\int_{\text{3-cycle}} \tilde{\Phi} \int_{\text{4-cycle}} \mu_{\overline{\mathrm{D5}}i}\, J_4(\overline{\mathrm{D5}}i) \right] . \tag{7.5.18}$$

Once we also take into account the O5-planes contributions to the 3d action, the net effect leads to the typical "doubling" of the anti-D5 terms due to (7.5.17). As a result, on top of all the previous contributions we had until now, we also have the additional term

$$2 \times V^{\overline{D5}} = 2 \times \frac{1}{8} e^{-18\beta v + \frac{\phi}{2}} \sum_i \mu_i \tilde{s}^i\,, \quad \mu_i = -\mu_{\overline{\mathrm{D5}}i} > 0\,. \tag{7.5.19}$$

Then the total scalar potential is

$$V = V^{\mathrm{BSB}} + V^R + V^{F3} + V^{F7} + V^{\mathrm{O5/D5}} + 2V^{\overline{\mathrm{D5}}}\,. \tag{7.5.20}$$

Note that here $V^{O5/D5}$ refers to the same contribution we had in (7.3.43). If one wanted to assign to the smeared O5-plane its honest correct contribution it would be $V^{O5} - V^{D5} + V^{\overline{D5}}$, and this the reason for the "doubling" of $V^{\overline{D5}}$ in (7.5.20) as well as the cancellation of the D5-brane contribution. Assuming now that $\tilde{s}^7 = 1/\prod_a \tilde{s}^a$, then we can have compatibility with an isotropic critical point of the shape moduli by requiring

$$\tilde{s}^a = 1\,, \quad \mu_i = \mu > 0\,. \tag{7.5.21}$$

However here we would directly run into two problems if we wanted to get a de Sitter solution with our prescription from the previous section. First of all the term (7.5.19) evaluated on the (7.5.21) critical point clearly affects only the T_0 term in the volume-dilaton scalar potential. Therefore it cannot change the fact that critical points still require $\tilde{R}^{(7)} < 0$ which as we said is not possible to achieve with the autonomous shape moduli stabilization. The second problem we would run into is that the tadpole (7.5.17) in the presence of a background with $\mathrm{d}F_3 = 0$ (which will probably be forced on us by the requirements (7.5.13)) will require various D5-branes for the cancellation of the O5-plane charge. Then clearly we cannot easily add anti-D5-branes as such system will be typically be inherently unstable.

As a means to escape the aforementioned issues we could still include anti-D5-branes but instead this time *not* ask that the shape moduli to be stabilized in their autonomous supersymmetric positions. This gives some more freedom in the construction and allows to find de Sitter critical points, albeit possibly inconsistent once flux quantization is taken carefully into account. However, here we want to give a general overview/exposition of the possibilities rather than proving the existence of a *bona fide* stable de Sitter solution. We will therefore be more liberal with the flux quantization and brane/plane tension constraints, but will still require basic self-consistency. In particular we do not include D5-branes, such that there is no obvious instability, and we also want to satisfy the tadpole condition (7.5.17), without D5s. Taking into account that $\mathrm{d}F_3 = \sum_{ij} f^j \mathcal{M}_{ij} \Psi_i$, the tadpole takes the form

$$\sum_j f^j \mathcal{M}_{ij} + \mu_i = -\mu_{\mathrm{O5}} < 0 \,, \quad \forall\, i \,. \tag{7.5.22}$$

This is forced on us by the fact that the Op-planes have the same contribution to each cycle tadpole and therefore, since we do not have D5-branes, we need all tadpole contributions related to dF_3 and μ_i to take the same value - otherwise the existence of D5-planes is implied. We shall work with the geometric fluxes that give rise to a matrix of the form (this is a specific choice of 2-step nilpotent example of [47])

$$\mathcal{M} = \begin{pmatrix} 0 & m & m & m & m & m & m \\ m & 0 & 0 & 0 & 0 & 0 & 0 \\ m & 0 & 0 & 0 & 0 & 0 & 0 \\ m & 0 & 0 & 0 & 0 & 0 & 0 \\ m & 0 & 0 & 0 & 0 & 0 & 0 \\ m & 0 & 0 & 0 & 0 & 0 & 0 \\ m & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} , \tag{7.5.23}$$

and with F_3 flux of the form

$$f^i = a\,(1, \omega, \omega, \omega, \omega, \omega, \omega) \,. \tag{7.5.24}$$

The shape moduli are stabilized at the positions

$$\tilde{s}^i = \left(\omega^{-\frac{6}{7}}, \omega^{\frac{1}{7}}, \omega^{\frac{1}{7}}, \omega^{\frac{1}{7}}, \omega^{\frac{1}{7}}, \omega^{\frac{1}{7}}, \omega^{\frac{1}{7}}\right) , \tag{7.5.25}$$

whereas the volume and dilaton are stabilized at the positions ϕ_0 and v_0. To get a de Sitter critical point we need to tune the geometric flux such that

$$m = -\frac{e^{\frac{\phi_0}{2}+\frac{v_0}{2\sqrt{7}}}\omega^{\frac{11}{7}}\left(a(5-30\omega)+\sqrt{a^2(33+4\omega(177\omega-29))-4\omega^{-\frac{12}{7}}\mathcal{G}^2(\omega-1)(24\omega+1)e^{\frac{2v_0}{\sqrt{7}}-2\phi_0}}\right)}{2+48\omega}. \tag{7.5.26}$$

Note that our solutions will have five parameters that we can in principle choose independently, which are

$$a\,,\ \omega\,,\ \mathcal{G}\,,\ \phi_0\,,\ v_0 \quad \text{(free parameters of the solution)}\,. \tag{7.5.27}$$

For the anti-D5-brane tensions we now have

$$\mu_1 = -\frac{1}{10}\left(30am\omega + 2a^2 e^{\frac{\phi_0}{2}+\frac{v_0}{2\sqrt{7}}}\omega^{\frac{18}{7}} + \mathcal{G}^2\omega^{\frac{6}{7}}e^{\frac{5v_0}{2\sqrt{7}}-\frac{3\phi_0}{2}} + 24m^2 e^{-\frac{\phi_0}{2}-\frac{v_0}{2\sqrt{7}}}\omega^{-\frac{4}{7}}\right)\,, \tag{7.5.28}$$

and

$$\mu_{2,3,4,5,6,7} = -\frac{1}{10}\left(5am + 2a^2 e^{\frac{\phi_0}{2}+\frac{v_0}{2\sqrt{7}}}\omega^{\frac{11}{7}} - e^{-\frac{3\phi_0}{2}-\frac{v_0}{2\sqrt{7}}}\omega^{-\frac{11}{7}}\left(e^{\phi_0}m^2 - \mathcal{G}^2 e^{\frac{3v_0}{\sqrt{7}}}\omega^{\frac{10}{7}}\right)\right)\,. \tag{7.5.29}$$

Note that μ_1 is different that the rest, this is because they need to cancel the different contribution of the dF_3 flux in each tadpole, even though the O5 contribution is the same. Finally, we also tune the BSB contribution to take the form

$$\mathrm{B}_0 = \frac{1}{40}e^{\frac{v_0}{2\sqrt{7}}-\frac{5\phi_0}{2}}\omega^{-\frac{10}{7}}\left(6e^{\phi_0}m^2 + 4\mathcal{G}^2\omega^{\frac{10}{7}}e^{\frac{3v_0}{\sqrt{7}}} - 7a^2\omega^{\frac{22}{7}}e^{\frac{v_0}{\sqrt{7}}+2\phi_0}\right)\,. \tag{7.5.30}$$

Then using our Ansatz and the specific aforementioned values for the various coefficients one can check that

$$\frac{\partial V}{\partial \phi} = 0\,, \quad \frac{\partial V}{\partial v} = 0\,, \quad \frac{\partial V}{\partial \tilde{s}^a} = 0\,, \tag{7.5.31}$$

with the vacuum energy given by

$$V_{\text{vac}} = \frac{1}{80}e^{\frac{4v_0}{\sqrt{7}}}\left(6m^2\omega^{-\frac{10}{7}} - \mathcal{G}^2 e^{\frac{3v_0}{\sqrt{7}}-\phi_0} - 7a^2\omega^{\frac{12}{7}}e^{\frac{v_0}{\sqrt{7}}+\phi_0}\right)\,. \tag{7.5.32}$$

Clearly the existence of de Sitter depends on the specific values one chooses. In addition one can check that if we ask that $\mu_i = 0$ we are driven to an AdS vacuum, therefore the inclusion of the anti-D5-branes is crucial.

Let us now give a few *numerical* examples. We can have

$$\text{Example 1}: \ \phi_0 = -3\,,\ v_0 = -3\sqrt{7}\,,\ \mathcal{G} = 0.01\,,\ a = 2\,,\ \omega = 0.09985\,, \tag{7.5.33}$$

which give self-consistent values

$$\mu_i > 0\,, \quad \mathrm{B}_0 > 0\,, \quad \mu_{O5} > 0\,, \tag{7.5.34}$$

and also satisfy (7.5.22). Note that in this example $m \simeq -0.0029 < 0$. For this numerical example we also find

$$V_{\text{vac}} \simeq 1.274 \times 10^{-13}\,, \quad \text{Eigenvalues}[V_{IJ}] > 0\,, \quad I = \phi, v, \tilde{s}^b\,, \tag{7.5.35}$$

implying a stable de Sitter critical point. Clearly from the values of the various coefficients we see that this example is in sharp contradiction with all sorts of flux quantization conditions but also clearly the values of μ_{O5}, $\mu_{\overline{\text{D5}}}$ and B_0 are unrealistic. In addition from the values of ϕ_0 and v_0 we see that we are definitely not safely within the large volume regime, however the string coupling is indeed small.

Another numerical example is to have

$$\text{Example 2 :} \;\; \phi_0 = -3\,, \;\; v_0 = -33\sqrt{7}\,, \;\; \mathcal{G} = 0.01\,, \;\; a = 10\,, \;\; \omega = 0.0998\,, \tag{7.5.36}$$

which still gives self-consistent values for the various coefficients and allows for slightly more realistic values for μ_{O5}, $\mu_{\overline{\text{D5}}}$ (but still overall unrealistic). We see that we are now safely within a weak coupling and large volume regime, but flux quantization is clearly not taken into account. For this example we find

$$V_{\text{vac}} > 0\,, \quad [V_{IJ}] < 0\,, \quad I = \phi, v, \tilde{s}^b\,, \tag{7.5.37}$$

therefore there are tachyons in the scalar sector.

We conclude that it seems that one can achieve (stable) de Sitter critical points from an effective theory model building perspective, but the required coefficients seem to be totally unrealistic from the string theory perspective. However, we believe that one needs to do an exhaustive scan over the various parameter values that are allowed by string theory in order to give a final verdict on the existence of classical 3d de Sitter vacua in string theory and on their stability. Our aim here was instead to highlight these open possibilities and we leave an exhaustive investigation for de Sitter solutions to future work. We expect that the study of 3d de Sitter vacua can further contribute to our understanding of such vacua from the perspective of the swampland program [10, 115–118, 134].

We finally stress that even if perturbative stability is achieved for the closed string moduli, including (anti) D5-branes can open up new decay channels, both perturbative and non-perturbative, in the open string sector, even if the various parameters are within a controlled string theory regime. Such instabilities may lead to very short lived vacua or completely destabilize them.

7.6 Type IIB – Outlook

In this work we have studied flux compactifications of string theory down to three external dimensions and have highlighted properties that make them an interesting playground to test various swampland conjectures. Our primary motivation was to provide the tools for the construction of the 3d N=1 supergravity, focusing in particular on the superpotential. Then we studied some simple examples that give us intuition for the vacuum structure. We focused in particular on discussing the possibility of having de Sitter and Anti-de Sitter vacua with scale separation and have seen how these vacua are allowed by the effective theory, but are hindered once we take into account proper quantization conditions as required in string theory.

As an outlook for future work we would like to discuss various possible extensions. One direction to expand on would involve a careful treatment of the open string sector, which we have mostly ignored here. This can be done in various ways. Firstly, as we have seen, it is un-avoidable to include O9-planes in this setup and so D9-branes also have to be included. This means that one must study carefully the D9-brane sector which leads to a non-abelian gauge theory in 3 dimensions. In addition since in principle we would also need to include D5-branes these would further contribute to the non-abelian gauge sector on the 3d external space as well as give rise to extra scalar moduli. Overall one would need to include new contributions also to the superpotential to correctly describe these sectors. Note that this setup could offer the basis for constructing a 3d toy-model version of the 4D KKLT construction. Indeed, the non-abelian gauge theory may give rise to gaugino condensation in the 3d EFT and including anti-D5-branes can give rise to a putative ulplift mechanism similar to the KKLT model. This may be a worthwhile endeavor as it may help to further understand the properties of de Sitter vacua in string theory, if such vacua truly exist, or simply a way to get more intuition about KKLT-type constructions. Along these lines one could also investigate the impact of Euclidean D-branes that wrap internal cycles, which we have ignored in the present work. These should give rise to non-perturbative contributions similar to the 4D case, however the absence of suitable non-renormalization theorems in 3d N=1 means their form is less constrained. On general grounds we can expect these contributions to take the form of non-perturbative exponentials dressed by a perturbative series in the moduli describing the volume of the wrapped cycle. It is also interesting to note that due to the dimensionality of the branes involved in our setup, it seems that the effects of gaugino condensation may differ in form from those of Euclidean D-branes, unlike the 4D scenario.

Another direction worth pursuing is to go beyond the co-calibrated toroidal G2 and include also the W_7 torsion. This is a very interesting development as it would allow to have more cycles in the theory and so more interesting backgrounds may be found. We have worked here only with toroidal oriefolds, however, one does not essentially need to restrict oneself to this set of compactifications. For example it would be important to study manifolds where the internal space allows warping, and this would also be important if one tries to build a 3d KKLT type of model, as we discussed earlier. Yet another direction to pursue would be finding the underlying un-smeared solutions of the orbifolds we discussed here. In a similar vein, one could also try to realize scenarios where the D5/O5 charge remains delocalized along the internal manifold but comes from topological flux and curvature terms in the D9/O9 worldvolume theory [196]. These scenarios should be related to resolutions of the orientifold singularities and therefore have a richer topology, with the un-smeared orientifold solutions as a limit. Such a study would undoubtedly shed more light on the properties of Op-planes and the consistency of working with the smeared solutions presented here.

Finally, one could try to classify all the 3d N=1 vacua that arise from flux compactifications on G2 with torsion and get important insight about the properties of the 3d swampland, especially by comparing to the dual 2d CFTs. Indeed, as we have seen (from the few sample examples we presented) the 3d N=1 low energy supergravity has a very rich vacuum structure, which however remains tractable due to its relatively simple ingredients. This means that a full classification of the *classical* 3d vacua (de Sitter and Anti-de Sitter alike) can be done and a thorough investigation of their properties is possible, especially using more advanced methods as for example proposed in [197].

Chapter 8

Appendix

8.1 3d minimal supergravity

To describe the matter-coupled 3d N=1 supergravity we will follow closely notation and conventions from [198]. The three-dimensional Clifford algebra has the 2×2 γ-matrices γ^a, where $a = 0, 1, 2$ are tangent space indices, and the matrices satisfy $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$, whereas the properties of Majorana spinors in 3d can be found in [199]. We will refer with Greek letters $\mu, \nu = 0, 1, 2$ to world indices. To make our presentation easier we will emulate a superspace description in terms of two real Grassmann variables θ_1 and θ_2, even though we will not enter into a specific superspace construction[1], instead we will follow closely the multiplet setup of [198]. The 3d N=1 supermultiplets we will utilize are given bellow:

- Supergravity sector: e_μ^a is the dreibein, and ψ_μ is the gravitino which is a spin-3/2 Majorana spinor. This multiplet has a real scalar auxiliary field S. These component fields appear in the Ricci scalar superfield

  $$\mathrm{S} = \mathrm{S} + i\theta^2 \left(R + 6\mathrm{S}^2\right) + \text{fermions}\,, \tag{8.1.1}$$

  where we abuse notation and use the same letter for the real superfield and for its lowest component, and R is the 3d Ricci scalar. Note that because of the Grassmann nature of the θ we have $(i\theta^2)^* = i\theta^2$. The supergravity sector appears also in the real super-density

  $$\mathcal{E} = e - 8i\theta^2 e\mathrm{S} + \text{fermions}\,, \tag{8.1.2}$$

  where $e = \sqrt{-g_3}$.

- Matter sector: ϕ^I are real scalars, χ^I are spin-1/2 Majorana spinors, and F^I are real scalar auxiliary fields. The indices $I = 1, \dots n$ take values on the target space scalar

[1] See for example [151, 200, 201] for a recent and full superspace presentation.

manifold with real coordinates ϕ^I and with Riemannian target space metric: $G_{IJ}(\phi)$. Their superspace expansion is

$$\phi^I = \phi^I + i\theta^2 F^I + \text{fermions}\,. \tag{8.1.3}$$

- Gauge sector: The gauge fields are denoted as $A_\mu^{(A)}$ where the indices (A) indicate that the field transforms in the adjoint, and the gaugini are denoted by $\lambda^{(A)}$ and are Majorana spin-1/2 fermions. These multiplets do not have independent auxiliary fields because the off-shell degrees of freedom of A_μ and λ match.

From the above ingredients we can built locally supersymmetric actions, by using a single superspace integral $i\int d^2\theta$ and taking into account that $i\int d^2\theta(i\theta^2) = 1$. For the supergravity sector we have

$$\frac{i}{2}\int d^2\theta \mathcal{E}\,\mathrm{S} = \frac{1}{2}eR - e\mathrm{S}^2 + \text{fermions}\,. \tag{8.1.4}$$

For the kinetic terms of the matter superfields we have

$$-\frac{i}{64}\int d^2\theta \mathcal{E}\, G_{IJ}(\phi)\overline{\chi}^I\chi^J = -eG_{IJ}\partial_\mu\phi^I\partial^\mu\phi^J + \frac{1}{16}G_{IJ}(\phi)F^IF^J + \text{fermions}\,, \tag{8.1.5}$$

where $G_{IJ}(\phi)$ is the real Riemannian target space metric and for the superpotential $P(\phi)$ we will always use

$$\frac{i}{2}\int d^2\theta \mathcal{E}\, P(\phi) = \frac{1}{2}eP_IF^I - 4eP\mathrm{S} + \text{fermions}\,. \tag{8.1.6}$$

Here $P(\phi)$ is a real function of the ϕ^I and $P_I = \partial P/\partial\phi^I$. Adding up these ingredients (and including the gauge sector which has no auxiliary fields) we can then built the most general Lagrangian for our purposes, which has bosonic sector

$$e^{-1}\mathcal{L} = \frac{1}{2}R - g^{\mu\nu}G_{IJ}(\phi)\partial_\mu\phi^I\partial_\nu\phi^J - \frac{1}{4}f(\phi)F_{\mu\nu}^{(A)}F^{\mu\nu(A)} - V(\phi)\,, \tag{8.1.7}$$

with

$$V(\phi) = G^{IJ}P_IP_J - 4P^2\,, \tag{8.1.8}$$

and the gauge kinetic function $f(\phi)$ real but otherwise unrestricted. The auxiliary fields F^I and S have been already integrated out in (8.1.7). The G^{IJ} is the inverse of the target space metric G_{IJ}. For completeness let us only point out that the quadratic gravitino sector has the form

$$e^{-1}\mathcal{L}_{3/2} = -\frac{1}{2}\overline{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\psi_\rho - \frac{1}{2}P\,\overline{\psi}_\mu\gamma^{\mu\nu}\psi_\nu\,. \tag{8.1.9}$$

Then we verify that for supersymmetric AdS$_3$ we have [199]

$$\langle P_i \rangle = 0 \ , \quad m_{3/2} = P = \frac{1}{2L_{\text{AdS}}} \ , \quad \langle V \rangle = -4P^2 = -\frac{1}{L^2_{\text{AdS}}} \ . \tag{8.1.10}$$

The fermionic shifts on a generic maximally symmetric background are given by

$$\begin{aligned} \delta\psi_\mu\Big|_{\text{shift}} &= \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\epsilon - P\gamma_\mu\epsilon \, , \\ \delta\chi^I\Big|_{\text{shift}} &= G^{IJ}P_J\epsilon \, , \\ \delta\lambda^{(A)}\Big|_{\text{shift}} &= 0 \, , \end{aligned} \tag{8.1.11}$$

where ϵ is the 2-component fermionic Majorana local supersymmetry parameter.

8.2 The unit-volume constraint

The way we will reduce the independent degrees of freedom in a consistent supersymmetric way is by using a superspace Lagrange multiplier. To this end we define the real three-form superfield (not a super three-form however)

$$\Phi = \Phi_i \tilde{S}^i \, , \tag{8.2.1}$$

where the $\tilde{S}^i$ are here the real superfields with lowest components $\tilde{s}^i$. We now postulate that the effective theory with the correct degrees of freedom is given by the Lagrangian

$$\mathcal{L} + i \int d^2\theta \left[\mathcal{E}\Lambda \left(\int_X \Phi \wedge \tilde{\star}\Phi - 7 \right) \right] \, , \tag{8.2.2}$$

where we have explicitly kept the X in $\int_X \Phi \wedge \tilde{\star}\Phi$ to indicate that that integration is over the internal space, and Λ is a real Lagrange multiplier superfield

$$\Lambda = L + \theta\chi^L + i\theta^2 F^L \, . \tag{8.2.3}$$

Once we vary the Lagrange multiplier superfield we have the superspace expression

$$\delta\Lambda \ : \ \int \Phi \wedge \tilde{\star}\Phi = 7 \, , \tag{8.2.4}$$

which guarantees that the condition (4.3.46) is imposed consistently on the full superspace level. Notice that this condition indeed eliminates one scalar degree of freedom due to $\int \Phi \wedge \tilde{\star}\Phi|_{\theta=0} = \int \tilde{\Phi} \wedge \tilde{\star}\tilde{\Phi}$, but it will also eliminate a fermion degree of freedom because the θ order term of (8.2.4) gives

$$\sum_j \int \Phi_j \wedge \tilde{\star}\tilde{\Phi}\, \chi^j = 0 \, . \tag{8.2.5}$$

Here we have referred to the superpartners of $\tilde{s}^i$ as χ^i.

We have until now seen that by introducing the Lagrange multiplier Λ we can restrict to the correct number of degrees of freedom and keep supersymmetry. However, we want to verify that the extra term we put in the action will not alter the form of our scalar potential. To see this we go to the bosonic sector of (8.2.2) and we focus on the extra term

$$
\begin{aligned}
& i \int d^2\theta \left[\mathcal{E}\Lambda \left(\Phi \wedge \tilde{\star}\Phi - 7\right)\right] \\
& = eF^{\Lambda}\left(\int \tilde{\Phi} \wedge \tilde{\star}\tilde{\Phi} - 7\right) - 8e\,\mathrm{S}L\left(\int \tilde{\Phi} \wedge \tilde{\star}\tilde{\Phi} - 7\right) + eL\int \Phi_j \wedge \tilde{\star}\tilde{\Phi}\, F^j \,.
\end{aligned}
\tag{8.2.6}
$$

Clearly when we vary F^L we get (4.3.46) and (4.3.47) as we want. Then notice that the second term that contains the supergravity auxiliary field S automatically drops out, and we are left only with the third term which crucially contains the matter auxiliary fields F^j.

Let us now describe how the variation of the third term in (8.2.6) works. First notice that this term essentially acts as a constraint once we vary L that reads

$$
\int \Phi_i \wedge \tilde{\star}\tilde{\Phi}\, F^i = 0 \,. \tag{8.2.7}
$$

This relation restricts the auxiliary fields of the $\tilde{s}^i$ multiplets and reduces them by one, consistently with the fact that we wanted to eliminate one complete multiplet. Since the auxiliary fields F^i are now subject to a restriction, in order to vary them and integrate them out to derive the scalar potential we have to include this new constraint in the variation. The simplest way to do this is to study the yet un-restricted auxiliary fields sector

$$
\mathcal{L}_{F^i} = \frac{1}{16} e G_{ij} F^i F^j + \frac{1}{2} e P_i F^i + eL\int \Phi_j \wedge \tilde{\star}\tilde{\Phi}\, F^j \,, \tag{8.2.8}
$$

where we have kept the Lagrange multiplier L and have not integrated it yet, such that the F^i are un-restricted and we can vary them normally. Now we vary both the F^i and L to find

$$
F^i = -4\,G^{ij} P_i - 8L\,G^{ij}\int \Phi_j \wedge \tilde{\star}\tilde{\Phi} \,, \tag{8.2.9}
$$

and of course also (8.2.7). Now we multiply (8.2.9) with $\int \Phi_i \wedge \tilde{\star}\tilde{\Phi}$ to find

$$
\int \Phi_i \wedge \tilde{\star}\tilde{\Phi}\, F^i = -4G^{ij}\int \Phi_i \wedge \tilde{\star}\tilde{\Phi}\, P_i - 224\, L \,, \tag{8.2.10}
$$

where we are now using the fact that $\mathrm{vol}(\tilde{X}_7) = 1$ because the $\tilde{s}^i$ have been already restricted from the variation of F^L. Now we observe that the left hand side term in (8.2.10) is vanishing because of the constraint (8.2.7), whereas the first term on the right hand side is also vanishing because an explicit calculation for our superpotentials shows that

$$G^{ij}P_i \int \Phi_j \wedge \tilde{\star}\tilde{\Phi} = 0\,. \tag{8.2.11}$$

Then from (8.2.10) we see that

$$L = 0\,. \tag{8.2.12}$$

Therefore going back to (8.2.9) we see that on-shell

$$F^i = -4\,G^{ij}P_i \;\rightarrow\; \mathcal{L}_{F^i} = -eG^{ij}P_iP_j\,, \tag{8.2.13}$$

and we get the standard contribution to the scalar potential from the $\tilde{S}^i$ multiplets even though they are restricted. We conclude that our scalar potential (4.3.45) is consistent and respects N=1 supersymmetry even when the $\tilde{s}^i$ are restricted to unit-volume. The same holds for (4.4.33) because of (4.4.35).

Let us give a different and more intuitive perspective now on why we were able to reduce to unit-volume and keep the scalar potential intact. We will discuss explicitly the toroidal case, but our discussion works for the other cases as well. As we said we wanted to have $\prod_i \tilde{S}^i = 1$. This condition could be imposed by setting $\tilde{S}^i = R^{3/7}T^i$ where $\prod_i T^i = 1$, with T^i and R real scalar superfields. Then we could re-derive the full theory using $P = P(x, y, r, t^i)$ with this new set of superfields, where $R|_{\theta=0} = r$ and $T^i|_{\theta=0} = t^i$. The most important property of the superpotential is that we would have $\partial P/\partial r = 0$, which would also give $F^R = 0$. Then to reduce to unit-volume, such that we do not double count the volume, we would have to set on the superfield level $R = 1$ which would give $r = 1$ and $F^R = 0$. The latter condition is completely compatible with the fact that $\partial P/\partial r = 0$ and this is exactly why our scalar potential does not change form.

8.3 Non-supersymmetric AdS$_3$

Here we verify that there is a non-supersymmetric AdS$_3$ when we flip the sign of the F_4 flux for the supersymmetric solution. We first write the scalar potential in terms of the $\tilde{s}^a$

$$V^{\text{Total}} = F(\tilde{s}^a)e^{2y-\frac{2x}{\sqrt{7}}} + H(\tilde{s}^a)e^{2y+\frac{2x}{\sqrt{7}}} + C\,e^{y-\sqrt{7}x} - T(\tilde{s}^a)e^{\frac{3y}{2}-\frac{5x}{2\sqrt{7}}}\,, \tag{8.3.1}$$

where $C = \frac{m^2}{16}$ and

$$\begin{aligned} F(\tilde{s}^a) &= \frac{f^2}{16}\left[\sum_a (\tilde{s}^a)^2 + 36\prod_a \frac{1}{(\tilde{s}^a)^2}\right]\,, \\ H(\tilde{s}^a) &= \frac{h^2}{16}\left[\sum_a \frac{1}{(\tilde{s}^a)^2} + \prod_b (\tilde{s}^b)^2\right]\,, \\ T(\tilde{s}^a) &= \frac{hm}{8}\left[\sum_a \frac{1}{\tilde{s}^a} + \prod_b \tilde{s}^b\right]\,. \end{aligned} \tag{8.3.2}$$

Here $F(\tilde{s}^a)$ is related to F_4 flux, $H(\tilde{s}^a)$ is related to the H_3, the constant C is related to the Romans mass which does not depend on the $\tilde{s}^a$, and the function $T(\tilde{s}^a)$ is related to the O-plane contribution. Note that all these functions are positive definite.

To minimize the potential we again search for vacua in which $\tilde{s}^a = \sigma$. We therefore calculate

$$\frac{\partial V}{\partial x}\Big|_{\tilde{s}^a=\sigma} = 0\ , \quad \frac{\partial V}{\partial y}\Big|_{\tilde{s}^a=\sigma} = 0\ , \quad \frac{\partial V}{\partial \tilde{s}^a}\Big|_{\tilde{s}^a=\sigma} = 0\,, \tag{8.3.3}$$

and we find after few manipulations respectively

$$\begin{aligned}
0 &= 12\left[\sigma^2 + \frac{6}{\sigma^{12}}\right] - 2a^2\left[\frac{6}{\sigma^2} + \sigma^{12}\right] + 7b^2 - 5ab\left[\frac{6}{\sigma} + \sigma^6\right]\,, \\
0 &= 12\left[\sigma^2 + \frac{6}{\sigma^{12}}\right] + 2a^2\left[\frac{6}{\sigma^2} + \sigma^{12}\right] + b^2 - 3ab\left[\frac{6}{\sigma} + \sigma^6\right]\,, \\
0 &= \left[\sigma^2 - \frac{36}{\sigma^{12}}\right] - a^2\left[\frac{1}{\sigma^2} - \sigma^{12}\right] + ab\left[\frac{1}{\sigma} - \sigma^6\right]\,,
\end{aligned} \tag{8.3.4}$$

where we see that the solution is given by the identical same values as the supersymmetric solution in Eq.(4.4.42). One can also evaluate the normalized V_{IJ} on this background and see that all the eigenvalues of this matrix are positive, and take the values of (4.4.46), which means that all 8 scalars are stable. Now let us check supersymmetry. First we see that for our background

$$P^T = \frac{3f}{4}e^{y-\frac{x}{\sqrt{7}}}\left[\sigma - \frac{1}{\sigma^6}\right] + \frac{h}{8}e^{y+\frac{x}{\sqrt{7}}}\left[\frac{6}{\sigma} + \sigma^6\right] + \frac{m}{8}e^{\frac{y}{2}-\frac{\sqrt{7}x}{2}}\,, \tag{8.3.5}$$

which once we evaluate for the values (4.4.42) we notice that

$$\langle V^{\text{Total}}\rangle \neq -4(P^T)^2\,, \tag{8.3.6}$$

which means we have a non supersymmetric vacuum.

8.4 Einstein equations

In this appendix we list some useful formulas and equations of motion of the Type II action in Eq.(5.2.1) with the presence of O6-planes in Eq.(5.2.2). For simplicity, and direct comparison to [106], we absorb N_{O6} in the $\sum_i \delta(\pi_i)$ part and then performing a dimensional reduction down to d-dimensions.

For the metric in Eq.(5.2.6) we find the Ricci scalar in terms of the warp factor

$$\mathcal{R}_{10} = w^{-2}R_d + R_{(10-d)} - 2dw^{-1}\nabla_m\nabla^m w - d(d-1)w^{-2}\nabla_m w\nabla^m w\,. \tag{8.4.1}$$

Here we define $\mathcal{R}_{MN}$ to be the Ricci tensor for the 10d string frame metric G_{MN}, and we use the same notation for the 7d and the 3d counterparts, i.e. $\mathcal{R}_{\mu\nu}$ and $\mathcal{R}_{mn}$, whereas when we work with the unwarped external or internal space metrics ($g_{\mu\nu}$ and g_{mn} respectively) we use the notation $R_{\mu\nu}$ and R_{mn}. The *dilaton* equations of motion for external d and internal $(10-d)$ metric are

$$\begin{aligned} 0 =& -8\nabla^2\tau + 2\frac{\tau}{w^2}R_d - \frac{8d}{w}(\partial_m w)(\partial^m \tau) - 2d(d-1)\frac{\tau}{w^2}\nabla_m w\nabla^m w - 4d\frac{\tau}{w}\nabla_m\nabla^m w \\ &+ 2\tau R_{(10-d)} - \tau|H_3|^2 + 2\sum_i \delta(\pi_i)\,. \end{aligned} \tag{8.4.2}$$

The variation of the action with respect to the ten dimensional metric G_{MN} in the *string* frame gives the following equations of motion

$$\begin{aligned} &\tau^2\Big(\mathcal{R}_{MN} - \frac{1}{2}G_{MN}\mathcal{R}_{10}\Big) + 2\tau G_{MN}\Big(\frac{d}{w}(\partial^\mu w)(\partial_\mu\tau) + \nabla^2\tau\Big) \\ &+ 2(\partial_M\tau)(\partial_N\tau) - 2\tau\nabla_M\nabla_N\tau \\ &- \frac{1}{2}\tau^2\Big(|H_3|^2_{MN} - \frac{1}{2}G_{MN}|H_3|^2\Big) - \frac{1}{2}\sum_{p=0}^{6}\Big(|F_p|^2_{MN} - \frac{1}{2}G_{MN}|F_p|^2\Big) - \frac{1}{2}T^{loc}_{MN} = 0\,, \end{aligned} \tag{8.4.3}$$

where we have used that

$$\nabla_M\nabla^M\tau = \frac{d}{w}\partial_\mu w\partial^\mu\tau + \nabla_m\nabla^m\tau\ , \quad \nabla_m\nabla^m\tau = \nabla^2\tau\,, \tag{8.4.4}$$

and

$$T^{loc}_{MN} = 2\tau G_{MN}|_{Op}\delta(\Sigma_{p+1}) = 2\tau\Pi_{i,MN}\delta(\Sigma_{p+1})\,. \tag{8.4.5}$$

We contract Eq.(8.4.3) with the 10d metric to find $\mathcal{R}_{10}$ and plugging this back in Eq.(8.4.3) gives

$$\begin{aligned} &-\tau^2\mathcal{R}_{MN} + \frac{d}{4}\frac{\tau}{w}G_{MN}(\partial w)(\partial\tau) + \frac{1}{4}G_{MN}(\tau\nabla^2\tau) + \frac{1}{4}G_{MN}(\partial_L\tau)(\partial^L\tau) + 2\tau\nabla_M\nabla_N\tau \\ &-2(\partial_M\tau)(\partial_N\tau) + \frac{1}{2}\tau^2\Big(|H_3|^2_{MN} - \frac{1}{4}G_{MN}|H_3|^2\Big) + \frac{1}{2}\sum_{p=0}^{6}\Big(|F_p|^2_{MN} - \frac{p-1}{8}G_{MN}|F_p|^2\Big) \\ &+\frac{1}{2}\Big(T^{loc}_{MN} - \frac{1}{8}G_{MN}T^{loc}\Big) = 0\,, \end{aligned} \tag{8.4.6}$$

where for T^{loc} we mean the contraction of 8.4.5 with the metric of the source worldvolume

$$T^{loc} = G^{MN}T^{loc}_{MN}\,. \tag{8.4.7}$$

Now we contract (8.4.6) with $g^{\mu\nu}$ and we get

$$\begin{aligned}
&-\tau^2\left(w^{-2}R_d-\frac{d}{w}\nabla^2 w-d(d-1)w^{-2}\nabla w\nabla w\right)\\
&+\frac{d^2}{4}\frac{\tau}{w}(\partial w)(\partial\tau)+\frac{1}{4}d(\tau\nabla^2\tau)+\frac{1}{4}d(\partial_L\tau)(\partial^L\tau)\\
&+\frac{1}{2}\tau^2\Big(-\frac{1}{4}d|H_3|^2\Big)+\frac{1}{2}\sum_{p=0}^{6}\Big(-\frac{p-1}{8}d|F_p|^2\Big)\\
&+\frac{1}{2}\Big(g^{\mu\nu}T^{loc}_{\mu\nu}-\frac{d}{8}T^{loc}\Big)=0\,.
\end{aligned} \tag{8.4.8}$$

Note that we have the relation

$$\mathcal{R}_{MN}\Big|_{M=m,N=n}=R_{mn}-\frac{d}{w}\Big(\partial_n\partial_m w-\partial_s w\Gamma^s_{mn}\Big)=R_{mn}-\frac{d}{w}\nabla_m\partial_n w\,. \tag{8.4.9}$$

Bibliography

[1] J. Polchinski, “String theory. Vol. 1: An introduction to the bosonic string,” , J. Polchinski, ‘String theory. Vol. 2: Superstring theory and beyond,”

[2] K. Becker, M. Becker and J. H. Schwarz, “String theory and M-theory: A modern introduction,” Cambridge University Press (2007).

[3] L. E. Ibanez and A. M. Uranga, “String theory and particle physics: An introduction to string phenomenology,”

[4] T. Kaluza, “Zum Unitätsproblem der Physik,” Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.) **1921**, 966-972 (1921) [arXiv:1803.08616 [physics.hist-ph]].
O. Klein, “Quantum Theory and Five-Dimensional Theory of Relativity.” Z. Phys. **37**, 895-906 (1926)

[5] S. M. Carroll, “Spacetime and Geometry,”

[6] E. Silverstein, “TASI / PiTP / ISS lectures on moduli and microphysics,” [arXiv:hep-th/0405068 [hep-th]].

[7] M. Grana, “Flux compactifications in string theory: A Comprehensive review,” Phys. Rept. **423**, 91-158 (2006) [arXiv:hep-th/0509003 [hep-th]].

[8] M. R. Douglas and S. Kachru, “Flux compactification,” Rev. Mod. Phys. **79**, 733-796 (2007) [arXiv:hep-th/0610102 [hep-th]].

[9] F. Denef, M. R. Douglas and S. Kachru, “Physics of String Flux Compactifications,” Ann. Rev. Nucl. Part. Sci. **57**, 119-144 (2007) [arXiv:hep-th/0701050 [hep-th]].

[10] U. H. Danielsson and T. Van Riet, “What if string theory has no de Sitter vacua?,” Int. J. Mod. Phys. D **27**, no.12, 1830007 (2018) [arXiv:1804.01120 [hep-th]].

[11] M. Cicoli, S. De Alwis, A. Maharana, F. Muia and F. Quevedo, “De Sitter vs Quintessence in String Theory,” Fortsch. Phys. **67**, no.1-2, 1800079 (2019) [arXiv:1808.08967 [hep-th]].

[12] E. Palti, “The Swampland: Introduction and Review,” Fortsch. Phys. **67**, no.6, 1900037 (2019) [arXiv:1903.06239 [hep-th]].

[13] M. van Beest, J. Calderón-Infante, D. Mirfendereski and I. Valenzuela, "Lectures on the Swampland Program in String Compactifications," [arXiv:2102.01111 [hep-th]].

[14] H. Ooguri and C. Vafa, "Non-supersymmetric AdS and the Swampland," Adv. Theor. Math. Phys. **21** (2017), 1787-1801 [arXiv:1610.01533 [hep-th]].

[15] D. Lüst, E. Palti and C. Vafa, "AdS and the Swampland," Phys. Lett. B **797**, 134867 (2019) [arXiv:1906.05225 [hep-th]].

[16] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, "De Sitter vacua in string theory," Phys. Rev. D **68**, 046005 (2003) [arXiv:hep-th/0301240 [hep-th]].

[17] O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, JHEP **07**, 066 (2005) [arXiv:hep-th/0505160 [hep-th]].

[18] V. Balasubramanian, P. Berglund, J. P. Conlon and F. Quevedo, "Systematics of moduli stabilisation in Calabi-Yau flux compactifications," JHEP **03** (2005), 007 [arXiv:hep-th/0502058 [hep-th]].

[19] P. G. Camara, A. Font and L. E. Ibanez, "Fluxes, moduli fixing and MSSM-like vacua in a simple IIA orientifold," JHEP **09** (2005), 013 [arXiv:hep-th/0506066 [hep-th]].

[20] P. Narayan and S. P. Trivedi, "On The Stability Of Non-Supersymmetric AdS Vacua," JHEP **07** (2010), 089 [arXiv:1002.4498 [hep-th]].

[21] Y. Hamada, A. Hebecker, G. Shiu and P. Soler, "Understanding KKLT from a 10d perspective," JHEP **06** (2019), 019 [arXiv:1902.01410 [hep-th]].

[22] F. Carta, J. Moritz and A. Westphal, "Gaugino condensation and small uplifts in KKLT," JHEP **08** (2019), 141 [arXiv:1902.01412 [hep-th]].

[23] F. F. Gautason, V. Van Hemelryck, T. Van Riet and G. Venken, "A 10d view on the KKLT AdS vacuum and uplifting," JHEP **06** (2020), 074 [arXiv:1902.01415 [hep-th]].

[24] Y. Hamada, A. Hebecker, G. Shiu and P. Soler, "On brane gaugino condensates in 10d," JHEP **04** (2019), 008 [arXiv:1812.06097 [hep-th]].

[25] F. F. Gautason, M. Schillo, T. Van Riet and M. Williams, "Remarks on scale separation in flux vacua," JHEP **03** (2016), 061 [arXiv:1512.00457 [hep-th]].

[26] D. Tsimpis, "Supersymmetric AdS vacua and separation of scales," JHEP **08**, 142 (2012) [arXiv:1206.5900 [hep-th]].

[27] M. Petrini, G. Solard and T. Van Riet, "AdS vacua with scale separation from IIB supergravity," JHEP **11**, 010 (2013) [arXiv:1308.1265 [hep-th]].

[28] A. Font, A. Herráez and L. E. Ibáñez, "On scale separation in type II AdS flux vacua," JHEP **03**, 013 (2020) [arXiv:1912.03317 [hep-th]].

[29] G. Buratti, J. Calderon, A. Mininno and A. M. Uranga, "Discrete Symmetries, Weak Coupling Conjecture and Scale Separation in AdS Vacua," JHEP **06**, 083 (2020) [arXiv:2003.09740 [hep-th]].

[30] D. Lüst and D. Tsimpis, "AdS$_2$ type-IIA solutions and scale separation," JHEP **07**, 060 (2020) [arXiv:2004.07582 [hep-th]].

[31] M. Emelin, "Effective Theories as Truncated Trans-Series and Scale Separated Compactifications," JHEP **11** (2020), 144 [arXiv:2005.11421 [hep-th]].

[32] D. Tsimpis, "Relative scale separation in orbifolds of S^2 and S^5," [arXiv:2201.10916 [hep-th]].

[33] J. P. Conlon, S. Ning and F. Revello, "Exploring The Holographic Swampland," [arXiv:2110.06245 [hep-th]].

[34] T. C. Collins, D. Jafferis, C. Vafa, K. Xu and S. T. Yau, "On Upper Bounds in Dimension Gaps of CFT's," [arXiv:2201.03660 [hep-th]].

[35] F. Apers, J. P. Conlon, S. Ning and F. Revello, "Integer Conformal Dimensions for Type IIA Flux Vacua," [arXiv:2202.09330 [hep-th]].

[36] G. Dibitetto, G. Lo Monaco, A. Passias, N. Petri and A. Tomasiello, "AdS$_3$ Solutions with Exceptional Supersymmetry," Fortsch. Phys. **66**, no.10, 1800060 (2018) [arXiv:1807.06602 [hep-th]].

[37] A. Passias and D. Prins, "On supersymmetric AdS$_3$ solutions of Type II," JHEP **08**, 168 (2021) [arXiv:2011.00008 [hep-th]].

[38] L. Eberhardt, "Supersymmetric AdS$_3$ supergravity backgrounds and holography," JHEP **02** (2018), 087 [arXiv:1710.09826 [hep-th]].

[39] F. Faedo, Y. Lozano and N. Petri, "New $\mathcal{N} = (0, 4)$ AdS$_3$ near-horizons in Type IIB," [arXiv:2012.07148 [hep-th]].

[40] A. Legramandi, N. T. Macpherson and G. L. Monaco, "All $\mathcal{N} = (8, 0)$ AdS$_3$ solutions in 10 and 11 dimensions," [arXiv:2012.10507 [hep-th]].

[41] S. Zacarias, "Marginal deformations of a class of AdS$_3$ $\mathcal{N} = (0, 4)$ holographic backgrounds," [arXiv:2102.05681 [hep-th]].

[42] A. Dabholkar, "Lectures on orientifolds and duality," [arXiv:hep-th/9804208 [hep-th]].

[43] P. Koerber and D. Tsimpis, "Supersymmetric sources, integrability and generalized-structure compactifications," JHEP **08**, 082 (2007) [arXiv:0706.1244 [hep-th]].

[44] P. G. O. Freund and M. A. Rubin, "Dynamics of Dimensional Reduction," Phys. Lett. B **97**, 233-235 (1980)

[45] K. Becker, D. Butter, W. D. Linch and A. Sengupta, "Components of eleven-dimensional supergravity with four off-shell supersymmetries," JHEP **07**, 032 (2021) [arXiv:2101.11671 [hep-th]].

[46] S. Grigorian and S. T. Yau, "Local geometry of the G(2) moduli space," Commun. Math. Phys. **287**, 459-488 (2009) [arXiv:0802.0723 [hep-th]].

[47] G. Dall'Agata and N. Prezas, "Scherk-Schwarz reduction of M-theory on G2-manifolds with fluxes," JHEP **10** (2005), 103 [arXiv:hep-th/0509052 [hep-th]].

[48] M. Fernández , A. Gray, "Riemannian manifolds with structure group G2." Annali di Matematica pura ed applicata 132, 19–45 (1982).

[49] S. Karigiannis, "Geometric Flows on Manifolds with G_2 Structure, I," math/0702077

[50] D. Joyce, "Compact Riemannian 7-manifolds with holonomy G2. I, II," *Journal of Differential Geometry,* **43** (1996) 291-328, 329-375.

[51] D. Joyce, "*Compact manifolds with spacial holonomy,*" Oxford Mathematical Monographs. Oxford University Press, 2000.

[52] B. S. Acharya, "M theory, G(2)-manifolds and four-dimensional physics," Class. Quant. Grav. **19**, 5619-5653 (2002)

[53] V. Balasubramanian, P. Berglund, J. P. Conlon, and F. Quevedo, "Systematics of moduli stabilisation in Calabi-Yau flux compactifications," *JHEP* **03** (2005) 007, arXiv:hep-th/0502058.

[54] F. F. Gautason, V. Van Hemelryck and T. Van Riet, "The Tension between 10D Supergravity and dS Uplifts," Fortsch. Phys. **67**, no.1-2, 1800091 (2019) [arXiv:1810.08518 [hep-th]].

[55] R. Blumenhagen, M. Brinkmann and A. Makridou, "Quantum Log-Corrections to Swampland Conjectures," JHEP **02**, 064 (2020) [arXiv:1910.10185 [hep-th]].

[56] J. Polchinski and E. Silverstein, "Dual Purpose Landscaping Tools: Small Extra Dimensions in AdS/CFT," [arXiv:0908.0756 [hep-th]].

[57] S. de Alwis, R. K. Gupta, F. Quevedo and R. Valandro, "On KKLT/CFT and LVS/CFT Dualities," JHEP **07**, 036 (2015) [arXiv:1412.6999 [hep-th]].

[58] J. P. Conlon and F. Quevedo, "Putting the Boot into the Swampland," JHEP **03**, 005 (2019) [arXiv:1811.06276 [hep-th]].

[59] L. F. Alday and E. Perlmutter, "Growing Extra Dimensions in AdS/CFT," JHEP **08**, 084 (2019) [arXiv:1906.01477 [hep-th]].

[60] E. Perlmutter, "Holography and the Swampland, talk at KITP "

[61] C. Beasley and E. Witten, "A Note on fluxes and superpotentials in M theory compactifications on manifolds of G(2) holonomy," JHEP **07**, 046 (2002) [arXiv:hep-th/0203061 [hep-th]].

[62] J. Gutowski and G. Papadopoulos, "Moduli spaces and brane solitons for M theory compactifications on holonomy G(2) manifolds," Nucl. Phys. B **615**, 237-265 (2001) [arXiv:hep-th/0104105 [hep-th]].

[63] S. Karigiannis, "Deformations of G2 and Spin(7) Structures," *Canadian Journal of Mathematics* **57** no. 05 (Oct,2005) 1012-1055.

[64] S. Grigorian, "Moduli Spaces of G2 Manifolds," *Reviews in Mathematical Physics* **22** no. 09 (Oct,2010) 1061-1097.

[65] M. Grana, R. Minasian, M. Petrini and A. Tomasiello, "A Scan for new N=1 vacua on twisted tori," JHEP **05**, 031 (2007) [arXiv:hep-th/0609124 [hep-th]].

[66] B. S. Acharya, F. Benini and R. Valandro, "Fixing moduli in exact type IIA flux vacua," JHEP **02**, 018 (2007) [arXiv:hep-th/0607223 [hep-th]].

[67] J. Blaback, U. H. Danielsson, D. Junghans, T. Van Riet, T. Wrase and M. Zagermann, "Smeared versus localised sources in flux compactifications," JHEP **12** (2010), 043 [arXiv:1009.1877 [hep-th]].

[68] D. Andriot, J. Blåbäck and T. Van Riet, "Minkowski flux vacua of type II supergravities," Phys. Rev. Lett. **118**, no.1, 011603 (2017) [erratum: Phys. Rev. Lett. **120**, no.16, 169901 (2018)] [arXiv:1609.00729 [hep-th]].

[69] S. Baines and T. Van Riet, "Smearing orientifolds in flux compactifications can be OK," Class. Quant. Grav. **37** (2020) no.19, 195015 [arXiv:2005.09501 [hep-th]].

[70] D. Junghans, "O-Plane Backreaction and Scale Separation in Type IIA Flux Vacua," Fortsch. Phys. **68**, no.6, 2000040 (2020) [arXiv:2003.06274 [hep-th]].

[71] F. Marchesano, E. Palti, J. Quirant and A. Tomasiello, "On supersymmetric AdS_4 orientifold vacua," JHEP **08**, 087 (2020) [arXiv:2003.13578 [hep-th]].

[72] S. B. Giddings and A. Maharana, "Dynamics of warped compactifications and the shape of the warped landscape," Phys. Rev. D **73**, 126003 (2006) [arXiv:hep-th/0507158 [hep-th]].

[73] M. R. Douglas and G. Torroba, "Kinetic terms in warped compactifications," JHEP **05**, 013 (2009) [arXiv:0805.3700 [hep-th]].

[74] G. Shiu, G. Torroba, B. Underwood and M. R. Douglas, "Dynamics of Warped Flux Compactifications," JHEP **06**, 024 (2008) [arXiv:0803.3068 [hep-th]].

[75] L. Martucci, "Warping the Kähler potential of F-theory/IIB flux compactifications," JHEP **03**, 067 (2015) [arXiv:1411.2623 [hep-th]].

[76] P. Koerber and L. Martucci, "From ten to four and back again: How to generalize the geometry," JHEP **08**, 059 (2007) [arXiv:0707.1038 [hep-th]].

[77] A. R. Frey and J. Roberts, "The Dimensional Reduction and Kähler Metric of Forms In Flux and Warping," JHEP **10**, 021 (2013) [arXiv:1308.0323 [hep-th]].

[78] A. R. Frey, G. Torroba, B. Underwood and M. R. Douglas, "The Universal Kahler Modulus in Warped Compactifications," JHEP **01**, 036 (2009) [arXiv:0810.5768 [hep-th]].

[79] S. B. Giddings, S. Kachru and J. Polchinski, "Hierarchies from fluxes in string compactifications," Phys. Rev. D **66**, 106006 (2002) [arXiv:hep-th/0105097 [hep-th]].

[80] K. Dasgupta, G. Rajesh and S. Sethi, "M theory, orientifolds and G - flux," JHEP **08**, 023 (1999) [arXiv:hep-th/9908088 [hep-th]].

[81] J. P. Derendinger, C. Kounnas, P. M. Petropoulos and F. Zwirner, "Superpotentials in IIA compactifications with general fluxes," Nucl. Phys. B **715**, 211-233 (2005) [arXiv:hep-th/0411276 [hep-th]].

[82] C. Caviezel, T. Wrase and M. Zagermann, "Moduli Stabilization and Cosmology of Type IIB on SU(2)-Structure Orientifolds," JHEP **04**, 011 (2010) [arXiv:0912.3287 [hep-th]].

[83] T. W. Grimm and J. Louis, "The Effective action of type IIA Calabi-Yau orientifolds," Nucl. Phys. B **718**, 153-202 (2005) [arXiv:hep-th/0412277 [hep-th]].

[84] J. Blåbäck, B. Janssen, T. Van Riet and B. Vercnocke, "BPS domain walls from backreacted orientifolds," JHEP **05**, 040 (2014) [arXiv:1312.6125 [hep-th]].

[85] A. Westphal, "de Sitter string vacua from Kahler uplifting," JHEP **03**, 102 (2007) [arXiv:hep-th/0611332 [hep-th]].

[86] S. Sethi, "Supersymmetry Breaking by Fluxes," JHEP **10**, 022 (2018) [arXiv:1709.03554 [hep-th]].

[87] M. Dine and N. Seiberg, "Is the Superstring Weakly Coupled?," Phys. Lett. B **162**, 299-302 (1985)

[88] M. Cvetic, G. W. Gibbons, H. Lu and C. N. Pope, "Supersymmetric nonsingular fractional D-2 branes and NS NS 2 branes," Nucl. Phys. B **606**, 18-44 (2001) [arXiv:hep-th/0101096 [hep-th]].

[89] S. Kachru, J. Pearson and H. L. Verlinde, “Brane / flux annihilation and the string dual of a nonsupersymmetric field theory,” JHEP **06**, 021 (2002) [arXiv:hep-th/0112197 [hep-th]].

[90] G. Giecold, E. Goi and F. Orsi, “Assessing a candidate IIA dual to metastable supersymmetry-breaking,” JHEP **02**, 019 (2012) [arXiv:1108.1789 [hep-th]].

[91] B. Michel, E. Mintun, J. Polchinski, A. Puhm and P. Saad, JHEP **09**, 021 (2015) doi:10.1007/JHEP09(2015)021 [arXiv:1412.5702 [hep-th]].

[92] D. Cohen-Maldonado, J. Diaz, T. van Riet and B. Vercnocke, JHEP **01**, 126 (2016) doi:10.1007/JHEP01(2016)126 [arXiv:1507.01022 [hep-th]].

[93] J. Armas, N. Nguyen, V. Niarchos, N. A. Obers and T. Van Riet, “Meta-stable non-extremal anti-branes,” Phys. Rev. Lett. **122**, no.18, 181601 (2019) [arXiv:1812.01067 [hep-th]].

[94] J. Blåbäck, F. F. Gautason, A. Ruipérez and T. Van Riet, “Anti-brane singularities as red herrings,” JHEP **12**, 125 (2019) [arXiv:1907.05295 [hep-th]].

[95] R. C. Mclean, “ Deformations of Calibrated Submanifolds,” *Commun. Analy. Geom* **6** (1996) 705-747.

[96] C. Caviezel, P. Koerber, S. Kors, D. Lust, D. Tsimpis and M. Zagermann, “The Effective theory of type IIA AdS(4) compactifications on nilmanifolds and cosets,” Class. Quant. Grav. **26**, 025014 (2009) [arXiv:0806.3458 [hep-th]].

[97] M. J. Duff, B. E. W. Nilsson and C. N. Pope, Phys. Lett. B **139**, 154-158 (1984) doi:10.1016/0370-2693(84)91234-6

[98] A. P. Braun and M. Del Zotto, “Mirror Symmetry for G_2-Manifolds: Twisted Connected Sums and Dual Tops,” JHEP **05**, 080 (2017) [arXiv:1701.05202 [hep-th]].

[99] G. Dibitetto and N. Petri, “AdS_3 vacua and surface defects in massive IIA,” PoS **CORFU2018**, 142 (2019) [arXiv:1904.02455 [hep-th]].

[100] A. Legramandi and N. T. Macpherson, “AdS_3 solutions with from $\mathcal{N} = (3, 0)$ from $\mathrm{S}^3 \times \mathrm{S}^3$ fibrations,” Fortsch. Phys. **68**, no.3-4, 2000014 (2020) [arXiv:1912.10509 [hep-th]].

[101] N. Benjamin, H. Ooguri, S. H. Shao and Y. Wang, “Light-cone modular bootstrap and pure gravity,” Phys. Rev. D **100**, no.6, 066029 (2019) [arXiv:1906.04184 [hep-th]].

[102] O. Aharony, Y. E. Antebi and M. Berkooz, “On the Conformal Field Theory Duals of type IIA AdS(4) Flux Compactifications,” JHEP **02**, 093 (2008) [arXiv:0801.3326 [hep-th]].

[103] T. Banks and K. van den Broek, "Massive IIA flux compactifications and U-dualities," JHEP **03**, 068 (2007) [arXiv:hep-th/0611185 [hep-th]].

[104] J. McOrist and S. Sethi, "M-theory and Type IIA Flux Compactifications," JHEP **12**, 122 (2012) [arXiv:1208.0261 [hep-th]].

[105] F. Saracco and A. Tomasiello, "Localized O6-plane solutions with Romans mass," JHEP **07** (2012), 077 [arXiv:1201.5378 [hep-th]].

[106] D. Junghans, "O-Plane Backreaction and Scale Separation in Type IIA Flux Vacua," Fortsch. Phys. **68** (2020) no.6, 2000040 [arXiv:2003.06274 [hep-th]].

[107] N. Cribiori, D. Junghans, V. Van Hemelryck, T. Van Riet and T. Wrase, "Scale-separated AdS_4 vacua of IIA orientifolds and M-theory," [arXiv:2107.00019 [hep-th]].

[108] F. Marchesano, D. Prieto and J. Quirant, "BIonic membranes and AdS instabilities," [arXiv:2110.11370 [hep-th]].

[109] S. Shandera, B. Shlaer, H. Stoica and S. H. H. Tye, "Interbrane interactions in compact spaces and brane inflation," JCAP **02** (2004), 013 [arXiv:hep-th/0311207 [hep-th]].

[110] D. Andriot and D. Tsimpis, "Gravitational waves in warped compactifications," JHEP **06**, 100 (2020) [arXiv:1911.01444 [hep-th]].

[111] D. Andriot, P. Marconnet and D. Tsimpis, "Warp factor and the gravitational wave spectrum," JCAP **07**, 040 (2021) [arXiv:2103.09240 [hep-th]].

[112] S. Giri, L. Martucci and A. Tomasiello, "On the Stability of String Theory Vacua," [arXiv:2112.10795 [hep-th]].

[113] D. Andriot, P. Marconnet and T. Wrase, "Intricacies of classical de Sitter string backgrounds," Phys. Lett. B **812** (2021), 136015 [arXiv:2006.01848 [hep-th]].

[114] D. Andriot, L. Horer and P. Marconnet, "Charting the landscape of (anti-) de Sitter and Minkowski solutions of 10d supergravities," [arXiv:2201.04152 [hep-th]].

[115] G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, "De Sitter Space and the Swampland," [arXiv:1806.08362 [hep-th]].

[116] D. Andriot, "On the de Sitter swampland criterion," Phys. Lett. B **785** (2018), 570-573 [arXiv:1806.10999 [hep-th]].

[117] S. K. Garg and C. Krishnan, "Bounds on Slow Roll and the de Sitter Swampland," JHEP **11** (2019), 075 [arXiv:1807.05193 [hep-th]].

[118] H. Ooguri, E. Palti, G. Shiu and C. Vafa, "Distance and de Sitter Conjectures on the Swampland," Phys. Lett. B **788** (2019), 180-184 [arXiv:1810.05506 [hep-th]].

[119] F. Denef, "Les Houches Lectures on Constructing String Vacua," *Les Houches* **87** (2008) 483–610, arXiv:0803.1194 [hep-th].

[120] D. Baumann and L. McAllister, *Inflation and String Theory*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 5, 2015. arXiv:1404.2601 [hep-th].

[121] R. H. Brandenberger and C. Vafa, "Superstrings in the Early Universe," *Nucl. Phys. B* **316** (1989) 391–410.

[122] H. Ooguri and C. Vafa, "On the Geometry of the String Landscape and the Swampland," Nucl. Phys. B **766**, 21-33 (2007) [arXiv:hep-th/0605264 [hep-th]].

[123] T. Van Riet, "On classical de Sitter solutions in higher dimensions," Class. Quant. Grav. **29**, 055001 (2012) [arXiv:1111.3154 [hep-th]].

[124] A. Bedroya and C. Vafa, "Trans-Planckian Censorship and the Swampland," arXiv:1909.11063 [hep-th].

[125] E. Silverstein, "Simple de Sitter Solutions," *Phys. Rev. D* **77** (2008) 106006, arXiv:0712.1196 [hep-th].

[126] M. P. Hertzberg, S. Kachru, W. Taylor, and M. Tegmark, "Inflationary Constraints on Type IIA String Theory," *JHEP* **12** (2007) 095, arXiv:0711.2512 [hep-th].

[127] C. Caviezel, P. Koerber, S. Kors, D. Lust, T. Wrase, and M. Zagermann, "On the Cosmology of Type IIA Compactifications on SU(3)-structure Manifolds," *JHEP* **04** (2009) 010, arXiv:0812.3551 [hep-th].

[128] R. Flauger, S. Paban, D. Robbins, and T. Wrase, "Searching for slow-roll moduli inflation in massive type IIA supergravity with metric fluxes," *Phys. Rev. D* **79** (2009) 086011, arXiv:0812.3886 [hep-th].

[129] U. H. Danielsson, S. S. Haque, G. Shiu, and T. Van Riet, "Towards Classical de Sitter Solutions in String Theory," *JHEP* **09** (2009) 114, arXiv:0907.2041 [hep-th].

[130] U. H. Danielsson, S. S. Haque, P. Koerber, G. Shiu, T. Van Riet, and T. Wrase, "De Sitter hunting in a classical landscape," *Fortsch. Phys.* **59** (2011) 897–933, arXiv:1103.4858 [hep-th].

[131] D. Junghans, "Weakly Coupled de Sitter Vacua with Fluxes and the Swampland," *JHEP* **03** (2019) 150, arXiv:1811.06990 [hep-th].

[132] D. Andriot, P. Marconnet and T. Wrase, "New de Sitter solutions of 10d type IIB supergravity," JHEP **08**, 076 (2020) [arXiv:2005.12930 [hep-th]].

[133] A. Banlaki, A. Chowdhury, C. Roupec and T. Wrase, "Scaling limits of dS vacua and the swampland," JHEP **03**, 065 (2019) [arXiv:1811.07880 [hep-th]].

[134] D. Andriot, "Open problems on classical de Sitter solutions," Fortsch. Phys. **67** (2019) no.7, 1900026 [arXiv:1902.10093 [hep-th]].

[135] C. Córdova, G. B. De Luca, and A. Tomasiello, "Classical de Sitter Solutions of 10-Dimensional Supergravity," *Phys. Rev. Lett.* **122** no. 9, (2019) 091601, arXiv:1812.04147 [hep-th].

[136] C. Córdova, G. B. De Luca, and A. Tomasiello, "New de Sitter Solutions in Ten Dimensions and Orientifold Singularities," arXiv:1911.04498 [hep-th].

[137] N. Cribiori and D. Junghans, "No classical (anti-)de Sitter solutions with O8-planes," *Phys. Lett. B* **793** (2019) 54–58, arXiv:1902.08209 [hep-th].

[138] N. Kim, "Towards an explicit construction of de Sitter solutions in classical supergravity," arXiv:2004.05885 [hep-th].

[139] X. Dong, B. Horn, E. Silverstein, and G. Torroba, "Micromanaging de Sitter holography," *Class. Quant. Grav.* **27** (2010) 245020, arXiv:1005.5403 [hep-th].

[140] R. Kallosh and M. Soroush, "Issues in type IIA uplifting," *JHEP* **06** (2007) 041, arXiv:hep-th/0612057.

[141] R. Kallosh and T. Wrase, "dS Supergravity from 10d," *Fortsch. Phys.* **67** no. 1-2, (2019) 1800071, arXiv:1808.09427 [hep-th].

[142] R. Kallosh and A. D. Linde, "Landscape, the scale of SUSY breaking, and inflation," *JHEP* **12** (2004) 004, arXiv:hep-th/0411011.

[143] J. Moritz and T. Van Riet, "Racing through the swampland: de Sitter uplift vs weak gravity," *JHEP* **09** (2018) 099, arXiv:1805.00944 [hep-th].

[144] R. Kallosh, A. Linde, E. McDonough, and M. Scalisi, "dS Vacua and the Swampland," *JHEP* **03** (2019) 134, arXiv:1901.02022 [hep-th].

[145] J. J. Blanco-Pillado, M. A. Urkiola, and J. M. Wachter, "Racetrack Potentials and the de Sitter Swampland Conjectures," *JHEP* **01** (2019) 187, arXiv:1811.05463 [hep-th].

[146] A. Micu, E. Palti, and G. Tasinato, "Towards Minkowski Vacua in Type II String Compactifications," *JHEP* **03** (2007) 104, arXiv:hep-th/0701173.

[147] B. de Carlos, A. Guarino, and J. M. Moreno, "Complete classification of Minkowski vacua in generalised flux models," *JHEP* **02** (2010) 076, arXiv:0911.2876 [hep-th].

[148] E. Plauschinn, "Non-geometric backgrounds in string theory," *Phys. Rept.* **798** (2019) 1–122, arXiv:1811.11203 [hep-th].

[149] U. H. Danielsson, G. Shiu, T. Van Riet and T. Wrase, "A note on obstinate tachyons in classical dS solutions," JHEP **03**, 138 (2013) [arXiv:1212.5178 [hep-th]].

[150] D. Junghans and M. Zagermann, "A Universal Tachyon in Nearly No-scale de Sitter Compactifications," *JHEP* **07** (2018) 078, arXiv:1612.06847 [hep-th].

[151] E. I. Buchbinder, J. Hutomo, S. M. Kuzenko and G. Tartaglino-Mazzucchelli, "Two-form supergravity, superstring couplings, and Goldstino superfields in three dimensions," Phys. Rev. D **96**, no.12, 126015 (2017) [arXiv:1710.00554 [hep-th]].

[152] R. C. Myers, "Dielectric branes," *JHEP* **12** (1999) 022, arXiv:hep-th/9910053.

[153] I. R. Klebanov and S. S. Pufu, "M-Branes and Metastable States," *JHEP* **08** (2011) 035, arXiv:1006.3587 [hep-th].

[154] I. Bena, M. Graña, and N. Halmagyi, "On the Existence of Meta-stable Vacua in Klebanov-Strassler," *JHEP* **09** (2010) 087, arXiv:0912.3519 [hep-th].

[155] U. H. Danielsson and T. Van Riet, "Fatal attraction: more on decaying anti-branes," *JHEP* **03** (2015) 087, arXiv:1410.8476 [hep-th].

[156] J. Armas, N. Nguyen, V. Niarchos, and N. A. Obers, "Thermal transitions of metastable M-branes," *JHEP* **08** (2019) 128, arXiv:1904.13283 [hep-th].

[157] N. Nguyen, "Comments on Stability of KPV Metastable State," arXiv:1912.04646 [hep-th].

[158] D. Cohen-Maldonado, J. Diaz, and F. F. Gautason, "Polarised antibranes from Smarr relations," *JHEP* **05** (2016) 175, arXiv:1603.05678 [hep-th].

[159] F. Apruzzi, M. Fazzi, D. Rosa, and A. Tomasiello, "All AdS_7 solutions of type II supergravity," *JHEP* **04** (2014) 064, arXiv:1309.2949 [hep-th].

[160] D. Junghans, D. Schmidt, and M. Zagermann, "Curvature-induced Resolution of Anti-brane Singularities," *JHEP* **10** (2014) 034, arXiv:1402.6040 [hep-th].

[161] F. F. Gautason, B. Truijen, and T. Van Riet, "Smeared antibranes polarise in AdS," *JHEP* **07** (2015) 165, arXiv:1502.00927 [hep-th].

[162] D. Junghans, "Tachyons in Classical de Sitter Vacua," *JHEP* **06** (2016) 132, arXiv:1603.08939 [hep-th].

[163] F. F. Gautason, M. Schillo, and T. Van Riet, "Is inflation from unwinding fluxes IIB?," *JHEP* **03** (2017) 037, arXiv:1611.07037 [hep-th].

[164] A. R. Frey, M. Lippert, and B. Williams, "The Fall of stringy de Sitter," *Phys. Rev. D* **68** (2003) 046008, arXiv:hep-th/0305018.

[165] C. M. Brown and O. DeWolfe, "Brane/flux annihilation transitions and nonperturbative moduli stabilization," *JHEP* **05** (2009) 018, arXiv:0901.4401 [hep-th].

[166] R. Blumenhagen, D. Kläwer, and L. Schlechter, “Swampland Variations on a Theme by KKLT,” *JHEP* **05** (2019) 152, arXiv:1902.07724 [hep-th].

[167] E. Dudas and S. Lüst, “An update on moduli stabilization with antibrane uplift,” arXiv:1912.09948 [hep-th].

[168] I. Bena, E. Dudas, M. Graña and S. Lüst, “Uplifting Runaways,” Fortsch. Phys. **67**, no.1-2, 1800100 (2019) [arXiv:1809.06861 [hep-th]].

[169] L. Randall, “The Boundaries of KKLT,” arXiv:1912.06693 [hep-th].

[170] I. Bena and A. Nudelman, “Warping and vacua of (S)YM(2+1),” *Phys. Rev. D* **62** (2000) 086008, arXiv:hep-th/0005163.

[171] N. Cribiori, R. Kallosh, C. Roupec, and T. Wrase, “Uplifting Anti-D6-brane,” *JHEP* **12** (2019) 171, arXiv:1909.08629 [hep-th].

[172] N. Cribiori, R. Kallosh, A. Linde, and C. Roupec, “Mass Production of IIA and IIB dS Vacua,” *JHEP* **02** (2020) 063, arXiv:1912.00027 [hep-th].

[173] U. Danielsson, G. Dibitetto and A. Guarino, “KK-monopoles and G-structures in M-theory/type IIA reductions,” JHEP **02**, 096 (2015) [arXiv:1411.0575 [hep-th]].

[174] A. Bilal, J. P. Derendinger and K. Sfetsos, “(Weak) g(2) holonomy from selfduality, flux and supersymmetry,” Nucl. Phys. B **628** (2002), 112-132 [arXiv:hep-th/0111274 [hep-th]].

[175] J. P. Derendinger and A. Guarino, “A second look at gauged supergravities from fluxes in M-theory,” JHEP **1409**, 162 (2014) [arXiv:1406.6930 [hep-th]].

[176] S. Andriolo, G. Shiu, H. Triendl, T. Van Riet, G. Venken and G. Zoccarato, “Compact G2 holonomy spaces from SU(3) structures,” JHEP **03** (2019), 059 [arXiv:1811.00063 [hep-th]].

[177] X. de la Ossa, M. Larfors and E. E. Svanes, “Restrictions of Heterotic G_2 Structures and Instanton Connections,” [arXiv:1709.06974 [math.DG]].

[178] X. de Ia Ossa, M. Larfors, M. Magill and E. E. Svanes, “Superpotential of three dimensional $\mathcal{N} = 1$ heterotic supergravity,” JHEP **2001**, 195 (2020) [arXiv:1904.01027 [hep-th]].

[179] I. Antoniadis, E. Dudas and A. Sagnotti, “Brane supersymmetry breaking,” Phys. Lett. B **464** (1999), 38-45 [arXiv:hep-th/9908023 [hep-th]].

[180] C. Angelantonj, I. Antoniadis, G. D’Appollonio, E. Dudas and A. Sagnotti, “Type I vacua with brane supersymmetry breaking,” Nucl. Phys. B **572** (2000), 36-70 [arXiv:hep-th/9911081 [hep-th]].

[181] E. Dudas and J. Mourad, "Consistent gravitino couplings in nonsupersymmetric strings," Phys. Lett. B **514**, 173-182 (2001) [arXiv:hep-th/0012071 [hep-th]].

[182] G. Pradisi and F. Riccioni, "Geometric couplings and brane supersymmetry breaking," Nucl. Phys. B **615**, 33-60 (2001) [arXiv:hep-th/0107090 [hep-th]].

[183] J. Mourad and A. Sagnotti, "An Update on Brane Supersymmetry Breaking," [arXiv:1711.11494 [hep-th]].

[184] I. Basile, J. Mourad and A. Sagnotti, "On Classical Stability with Broken Supersymmetry," JHEP **01** (2019), 174 [arXiv:1811.11448 [hep-th]].

[185] I. Basile and S. Lanza, "de Sitter in non-supersymmetric string theories: no-go theorems and brane-worlds," JHEP **10**, 108 (2020) [arXiv:2007.13757 [hep-th]].

[186] J. Scherk and J. H. Schwarz, "How to Get Masses from Extra Dimensions," Nucl. Phys. B **153**, 61 (1979).

[187] R. L. Bryant, "Some remarks on G(2)-structures," math/0305124 [math-dg].

[188] N. Cribiori, R. Kallosh, A. Linde and C. Roupec, "de Sitter Minima from M theory and String theory," Phys. Rev. D **101**, no.4, 046018 (2020) [arXiv:1912.02791 [hep-th]].

[189] F. Marchesano, "D6-branes and torsion," JHEP **05**, 019 (2006) [arXiv:hep-th/0603210 [hep-th]].

[190] E. A. Bergshoeff, K. Dasgupta, R. Kallosh, A. Van Proeyen and T. Wrase, "$\overline{\text{D3}}$ and dS," JHEP **05** (2015), 058 [arXiv:1502.07627 [hep-th]].

[191] G. W. Gibbons, "ASPECTS OF SUPERGRAVITY THEORIES," Print-85-0061 (CAMBRIDGE).

[192] J. M. Maldacena and C. Nunez, "Supergravity description of field theories on curved manifolds and a no go theorem," Int. J. Mod. Phys. A **16**, 822-855 (2001) [arXiv:hep-th/0007018 [hep-th]].

[193] K. Dasgupta, R. Gwyn, E. McDonough, M. Mia and R. Tatar, "de Sitter Vacua in Type IIB String Theory: Classical Solutions and Quantum Corrections," JHEP **07**, 054 (2014) [arXiv:1402.5112 [hep-th]].

[194] J. Moritz, A. Retolaza and A. Westphal, "Toward de Sitter space from ten dimensions," Phys. Rev. D **97**, no.4, 046010 (2018) [arXiv:1707.08678 [hep-th]].

[195] N. Cribiori, G. Dall'Agata and F. Farakos, "Weak gravity versus de Sitter," [arXiv:2011.06597 [hep-th]].

[196] K. Dasgupta, D. P. Jatkar and S. Mukhi, "Gravitational couplings and Z(2) orientifolds," Nucl. Phys. B **523**, 465-484 (1998) [arXiv:hep-th/9707224 [hep-th]].

[197] I. M. Comsa, M. Firsching and T. Fischbacher, "SO(8) Supergravity and the Magic of Machine Learning," JHEP **08** (2019), 057 [arXiv:1906.00207 [hep-th]].

[198] R. Andringa, E. A. Bergshoeff, M. de Roo, O. Hohm, E. Sezgin and P. K. Townsend, "Massive 3d Supergravity," Class. Quant. Grav. **27**, 025010 (2010) [arXiv:0907.4658 [hep-th]].

[199] D. Z. Freedman and A. Van Proeyen, "Supergravity,"

[200] M. Becker, D. Constantin, S. J. Gates, Jr., W. D. Linch, III, W. Merrell and J. Phillips, "M theory on spin(7) manifolds, fluxes and 3-D, N=1 supergravity," Nucl. Phys. B **683**, 67-104 (2004) [arXiv:hep-th/0312040 [hep-th]].

[201] S. M. Kuzenko, U. Lindstrom and G. Tartaglino-Mazzucchelli, "Off-shell supergravity-matter couplings in three dimensions," JHEP **03**, 120 (2011) [arXiv:1101.4013 [hep-th]].