



**NATIONAL TECHNICAL UNIVERSITY OF ATHENS  
SCHOOL OF NAVAL ARCHITECTURE AND MARINE ENGINEERING  
DIVISION OF MARINE ENGINEERING**

**Diploma Thesis**

**Analysis of the Shafting System Alignment  
for a Bulk Carrier Fleet**

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**Athens, July 2022**

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## **Acknowledgements**

As my undergraduate studies come to an end with this work, I want to take a moment to express my gratitude to all those people who helped me. First, I would like to thank my family, which supported me financially and mentally all those years. Second, I would like to thank my supervisor Dr. Christos Papadopoulos, who gave me the opportunity to learn many things on the domain of shaft alignment and he was always accessible when I wanted some advice. Furthermore, I want to thank George Rossopoulos, who guided me for the most part of this work and his help in programming was significant. Last but not least, I want to thank my friends, who encouraged me through my studying years. They will always be in my heart and memory.

## Abstract

The propulsion system may suffer various failures through a ship's life span. The failures may occur on the shaft lines, the crankshaft or the bearings, mainly due to excessive load. Bearings may also suffer from vibrations when unloaded or lightly loaded. A proper shaft alignment will lead to smooth distribution of bearing loads and smooth elastic deformation of the shafting system, thus reducing the probability of failures and increasing the reliability of the system. Although shaft alignment is a process studied for decades, classification societies released updated enhanced shaft alignment regulations a few years ago, as ships became larger and more flexible causing shorter and more stiff shaft lines that may also be significantly affected by hull deflections, which impact negatively along with vibrations the propulsion system.

In this study, a dimension analysis of the propulsion system of a bulk carrier fleet is carried out. Specifically, the fleet consists of 17 bulk carriers of different size categories. The analysis contains lengths and diameters of the three (3) shafts that consist of the shafting system (propeller shaft, intermediate shaft and main engine shaft) and their respective flanges for each vessel of the fleet.

The detailed shafting system of one of the vessels of the fleet is studied more extensively. The vessel is a typical 82k DWT bulk carrier powered by a 2-stroke Diesel engine. The shaft alignment process for this vessel was carried out by ABS for a specific propeller load and set of vertical bearing offsets (initial offsets) and the results were compared utilizing an in-house software for the same conditions. The shafting system is modelled as a beam divided into smaller segments and the bearings are modelled as supporting points. Radial shaft loads, thermal expansion of the engine and thrust eccentricity caused by the propeller are taken into account. The results (reaction forces, bending moments, etc.) are calculated using matrix analysis. Simulations are executed for 4 propeller immersion conditions (propeller in air, half immersed, 75% immersed, fully immersed) and for various new offsets combinations, which deviate with a specific manner from the initial offsets. The scenarios are classified as; (i) acceptable, (ii) marginal and (iii) not acceptable, in accordance with ABS regulations for the reaction forces and three (3) offset configurations (basis offsets) lead to results that meet the regulations. Simulations are executed for smaller deviations from the basis offsets of those three (3) different offset combinations and for the all propeller loading conditions.

Conclusions are drawn based on the above. Concerning the dimension analysis, comparisons between the ratios of the dimensions of the shaft and the flanges are made. Concerning the ship specific study, propeller immersion and vertical offsets of the bearings lead to the classification of various scenarios according to IACS regulations for acceptable bearing reaction forces. Depending on the occasion, conclusions can be made about the behavior of the elastic line of the shaft and the behavior of bearing reactions.

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## 1. Introduction

The propulsion system of conventional cargo ships typically consists of a two-stroke, or a four-stroke engine and a shafting system with the option of a reduction gearbox, which transmits the engine power to the propeller (Figure 1.1). In ships propelled by four-stroke Diesel engines, a reduction gearbox is required. Due to demand for high power output, the shafting system is usually subjected to a large amount of torque, whereas significant bending moments are also present, as a result of propellers getting larger in order to efficiently convert engine power into thrust. Proper design, therefore, demands that a shaft with large diameter is installed, which in turn increases the overall weight of the system. Radial shaft loads (propeller/shaft/engine weights) need to be supported by journal bearings (stern tube bearings, line bearings, crankshaft bearings), while axial loads are transmitted to the ship's structure by the main engine thrust bearing [7]. Nowadays the maritime industry demands larger and lighter ships to maximize the carrying capacity, while keeping the building and operational costs relatively low. Larger ships translate to larger power plants and consequently greater torsional loads applied to the shafting system. To overcome that threat, a shaft with increased diameter is installed, which in return increases the overall weight and stiffness of the system. On the other hand, hull structure is more flexible due to the reduced thickness of high tensile strength steel. We can observe that the synergy of the flexible hull structure and the stiff shafting system is an important issue for reliable operation of the vessel, which needs to be addressed in the design stage by means of detailed shaft alignment analysis.

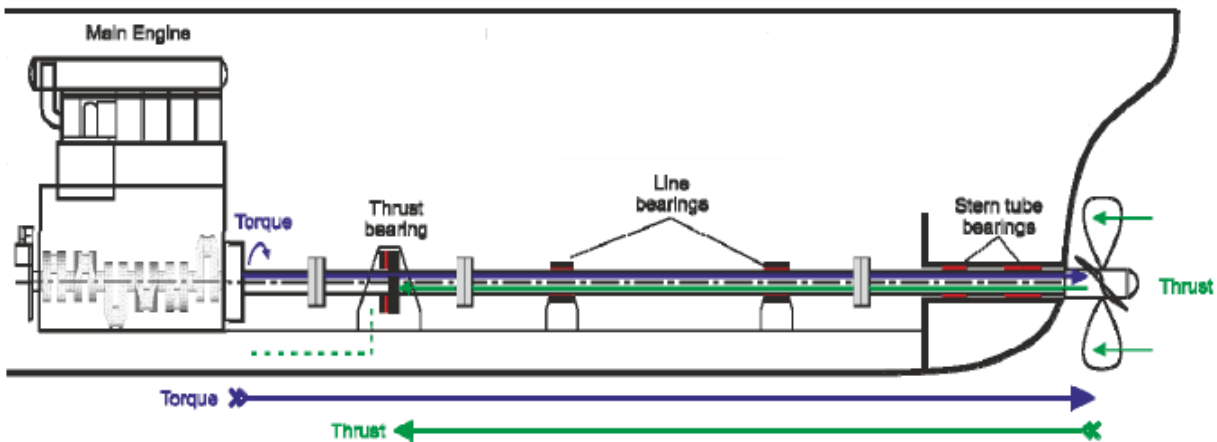


Figure 1.1: TYPICAL ARRANGEMENT OF THE PROPULSION SYSTEM OF A CARGO VESSEL

Shaft alignment is concerned with the determination of the number of bearings, their longitudinal position and their vertical offset, with the ultimate goal of an optimized shafting system complying to bearing loading criteria. The successful application of a shaft alignment plan is essential for stable and efficient operation of the propulsion system with decreased bearing wear, increased lifetime and decreased maintenance costs.

A key influence factor of the shaft alignment is the hull deformation of the vessel. Until recently, shaft alignment calculations were performed for the condition in which the alignment would be conducted. However, the ship operates in a wide range of different loading conditions throughout its lifetime. As

mentioned above, ships are getting larger, thus more flexible. It is imperative that the designer takes into account the hull deflections for all the anticipated loading conditions to ensure a fully optimized shaft alignment plan.

Another factor playing an important role in the shaft alignment process is the stiffness of the bearing foundation. The shafting system is modelled as an assembly of beam elements supported at the bearing locations. Contrary to that modelling assumption, all supporting points are very stiff, not completely rigid though. The bearings' foundation is expected to deform elastically under the weight of the shaft and all the other components of the system. Elastic deformation of the bearings' foundation should be taken into consideration in the design stage of the shaft alignment plan to approximate more accurately the performance of the shafting system.

During operation of the main engine, the engine components near the combustion chambers generally have a higher temperature (approximately 80-90 °C) in comparison to the components near the foundation of the engine (approximately 30-40 °C). Thermal expansion of the engine components is generally three-dimensional, but only the vertical component affects the alignment of the shafting system. The overall offsets of the main engine bearings, due to thermal expansion, can be assumed to be the composition of two components; a parallel vertical offset and a non-uniform parabolic offset. The parallel vertical offset is attributed mainly to the increased mean temperature of the main engine's foundation, whereas the parabolic is attributed to the larger expansion of the components near the combustion chambers, in comparison to the smaller expansion of the components near the engine foundation. Shaft alignment calculations need to incorporate vertical offsets of the M/E bearings due to thermal expansion to achieve a shaft alignment plan optimized for operational characteristics of the vessel as well.

Last but not least, the Marine Engineer must keep in mind the operating principle of the journal bearings, namely hydrodynamic lubrication. The existence of a thick lubricating oil film on which the shaft floats is another key parameter that contributes to the final vertical offsets of the bearings. Oil film thickness is dependent on a set of operational parameters such as oil temperature, oil viscosity and rotational speed of the shaft. It is obvious that the wide range of loading conditions and the uncertainty of shaft loads due to engine and propeller operation affect oil film formation and may alter all relevant calculations of shaft equilibrium.

## 1.1 Literature Review

Since the beginning of this century, ships tend to become larger and more flexible, which results in an increase in hull deformations. These deformations affect the propulsion system, causing bearing damages due to the change in bearing heights. Given this, ship – owners, classification societies and shipyards are striving to find solutions for a proper shaft alignment by strengthening the analysis and verification processes.

The reliability of propulsion systems has been studied for over 7 decades. The first study was carried out by the U.S Navy in the late 50s and it was a design analysis on propulsion system based on fatigue, torsional vibrations and corrosion on the shaft [18]. In the late 60s, fair curve alignment theory was introduced. According to this theory the shaft is proposed as a static beam supported on many bearings, which can be adjusted horizontally or vertically to change bearing load. Specifically, Mann in [15] used this theory in addition to methods of designing shafting systems based on strength and vibration considerations. Prior to 50s, the straight alignment method was used [8] in which the centers of all support bearings were aligned on the same straight line when aligning shafts. This method was abandoned due to irregular load distribution to each bearing. In the 70s and 80s the development of computer programs played an important role in establishing shaft alignment exercises [10].

In the early 2000s ships became longer with thinner hull in order to gain more efficiency. As a result, the alignment of propulsion system started to become more sensitive to hull deflections. Sverko [20] highlighted his concerns on shaft alignment sensitivity and concluded that not only hull deflections should be taken under consideration, but also bearing clearance, gear meshes, crankshaft modeling and alignment acceptability. Otherwise, serious consequences on the final alignment condition may occur. Indeed, 200 vessels suffered from bearing failures between 2013 and 2017 related to high flexibility of the hull and stern tube bearing misalignment [13]. In addition to the previous concept, Murawski [16] suggested that oil film damping characteristics of the journal bearings are as much important as hull deflections during shaft alignment. He also suggested, stern tube bearings should be modelled as continuous supports and intermediate and main engine bearings as points. Lee & Kim in [11] estimated hull deflections by reverse calculations using measurement data. Data was obtained by measuring bearing reaction forces from jack up method and bending moments from strain gauges and concluded in much higher bearing offset due to hull deflections caused by various loading conditions. Loading conditions, wave loads and environment temperature are the most common factors that affect hull deformations according to [12]. They used finite element analysis for the construction of the hull of a 76,000 DWT product oil tanker under gravity, buoyancy, temperature loads and elastic constraints to obtain the deformations of the double bottom. The light ship condition in calm water was taken as reference for the observation of relative bearing offsets. They concluded that shaft alignment is affected mostly by loading condition of the vessel. The shaft alignment behaves worse as the load of the ship is increasing. On the other hand, wave loads impact shaft alignment in some manner. Last but not least, environment temperature influences shaft alignment to a huge extent as there is a big difference in bearing reactions in summer and winter due to the difference in temperatures.

## 1.2 Goals - Outline

The goals of this present work are:

- Analysis of the dimensions of main parts of shafting systems of bulk carriers fleet
- Shaft alignment analysis for a specific vessel of the above fleet which includes:
  1. Comparison of influence factors and bearing reaction forces from ABS process of shaft alignment for a specific propeller immersion cases and offset configurations with the respective results utilizing the in-house software for shaft alignment.
  2. Simulation and classification of shaft alignment cases for various scenarios of propeller immersion and offset combinations following the IACS regulations for acceptable bearing reaction forces.

## 2. Shaft Alignment

### 2.1 Definition

The ship propulsion system typically comprises a two-stroke diesel engine and a shafting system transmitting power to the propeller. In ships with a four-stroke diesel engine installed, a reduction gearbox is required to transmit the generated power to the propeller in the most efficient rotational speed range. The shafting system comprises three individual parts: (a) the propeller shaft, (b) the intermediate shaft and (c) the crankshaft. Each shaft is supported by different (amount and type) journal bearings according to the weights to be supported. (pistons, connecting rod, crosshead, flywheel, chain tension, propeller, etc.).

The propeller shaft is usually supported in the stern tube by two stern tube bearings. The intermediate shaft is supported by at least one intermediate bearing. The crankshaft is supported by the crankshaft bearings, whose number depends on the number of cylinders of the main engine. In cases where the crankshaft comes as a single piece, the number of crankshaft bearings is the number of cylinders increased by one. On the contrary, in cases where the crankshaft is divided into two parts (i.e. large marine diesel engines) the number of crankshaft bearings is the number of cylinders increased by two.

The propulsion shafting alignment is a process involving the calculation and selection of proper support points to achieve the optimal operation of the shafting system for the service conditions of the vessel. Specifically, shaft alignment consists of two parts:

- The analysis of the shafting system and the determination of the shaft

alignment plan, and

- The application of the alignment procedure and the subsequent validation

through measurements.

**Shaft alignment is the process through which the following can be determined:**

- The number of support points along the shaft,
- The longitudinal position of every support point along the shaft,
- The vertical offset of each support point in relation to a pre-established reference line,
- The angle at which support bearings will be positioned in relation to a pre-established reference line, in order to minimize shaft/bearing misalignment
- The proper bearing dimensions that would ensure the shaft weight is supported adequately,
- The reaction forces at the support point of each bearing in cold and hot condition respectively

The reliability of a shafting system depends on the quality of the shaft alignment process. Stresses on the shaft and power losses of the system are reduced, when a proper shaft alignment is carried out. Working within acceptable boundaries is fundamental for both shaft and bearings in order to achieve longer durability, less fatigue and avoid putting safety and operation of the ship at risk. In case of shaft connection with a gearbox, a proper shaft alignment reduces loads applied on the gear teeth as a result

the gearbox is protected from devastating failures. Repair costs and maintenance are significantly decreased because of an accurate shaft alignment study.

### 2.1.1.1 Static and Running Condition

There are two main types of conditions that are presented during a ship's lifecycle. The first one is when the ship is anchored or dry docked (static condition) and the second one is when the ship is operating on the sea (running condition). The fundamental differences between these conditions are presented below:

In static condition:

- The main engine is not operating
- There is no thrust produced from the propeller, as a result additional bending moment due to eccentricity is not created.
- At the support point vertical motion is acceptable within the limits of diametrical clearance.
- Hydrodynamic lubrication is not enabled, because the shaft is stationary.
- Torsional vibrations cannot be applied, because the shaft is not rotating.

In running condition:

- The main engine is operating
- The main engine is in hot condition as a result thermal expansions affect the vertical position of all crankshaft bearings.
- The propeller is producing thrust, which is eccentric to the shaft line, causing additional bending moment to the shaft.
- The shaft is rotating, which leads to the development of the fluid between the shaft and each bearing, lifting the shaft above the lower half of the bushing.
- Any misalignment between the bearing and the shaft will result in a slight shift of the conceived single-point support position of the shaft along the bearings' length.

In the present work, the running condition is studied, where a fluid film is developed and the shaft is bent. All the above related to the running condition are taken into consideration for this study. Calculations will not be done for static condition, although they can easily be added given more information about the overall shaft alignment plan.

### 2.1.2 Influence Factors

In a shaft alignment procedure, support points are placed longitudinally and at specific vertical offset from the reference line. It is visible that changing the vertical offset of a bearing will lead to an altered distribution of reaction forces amongst the other bearings. In order to evaluate the correlation between support points, influence factors are used. They are a measure of the deviation of the reaction force of a bearing, while the number and longitudinal position of all bearings is constant and the vertical offset of one of the bearings is modified.

The influence factor of bearing  $i$  on bearing  $j$  is a measure of the change in reaction force of bearing  $j$ , caused by a unit vertical offset of bearing  $i$ .

As such, it can be calculated as:

$$\sigma_{ij} = \frac{W_{ij} - W_j^0}{y_i}$$

Where:

- $\sigma_{ij}$  is the influence factor of bearing i on bearing j,
- $W_{ij}$  is the reaction force of bearing j when bearing i has moved vertically by an amount of  $y_i$
- $W_j^0$  is the reaction force of bearing j while bearings have zero vertical offsets ( i.e straight line)

Using the formula above, it is very simple to predict the reaction force of each bearing, for a set of vertical offsets, when all influence factors have been calculated.

Influence factors are a good measure of sensitivity in the shafting plan to external disturbances. Ship's motion and loading can result to different vertical offsets which should not cause devastating conditions at the loading of the bearings. Small values of the influence factors signify a less sensitive system, where on the contrary large influence factors identify a system with great risk of bad alignment caused by a small change in vertical bearing offsets.

## 2.2 Regulations for Design and Analysis

Regulations regarding shaft alignment of rotating machinery, such as the prime mover of a ship, are more or less common between IACS classes. Most characteristic general requirements are the following:

- A detailed shaft alignment plan must be submitted for approval with all the assumptions made and consideration taken.
- The results of the analysis must be shared to the class before the plan implementation.
- Analyses must be carried out for various ship operations (cold/hot) and loading conditions (Ballast Arrival, Full load Departure etc.) taking into consideration hull deflections and thermal expansions of the crankshaft.
- Stern tube slope must be investigated under any plan and single, or even multi-slope, boring must be applied whenever, to prevent shaft misalignment.
- Reaction Forces from the bearings must be compatible within allowable limits.
- The details and procedures followed during the implementation of the plan must be available to the class reviewer.
- All the above must be checked and verified by the class.

### 2.2.1 Reaction Forces

Allowable bearing reaction forces are defined by a series of requirements. First of all, bearings must be in contact with the shaft at the bottom half of their geometry. Reaction forces are defined as “positive” if the statement above is true. Secondly, reaction forces must be within a range of acceptable limits. Typical upper limit for the mean pressure in white metal bearings is 0.8 MPa and 0.6 MPa for



oil-lubricated synthetic materials [1]. The limit definition for maximum pressure varies in different classes, NK sets the limit at 40MPa [17] and Bureau Veritas links reaction forces to lubricant film thickness and sets a limit at 30  $\mu\text{m}$  of minimum film thickness (correlated to the roughness of the material) [4]. ABS states that reaction loads are not the only criteria that are important for alignment acceptance, but relative misalignment between the shaft and the bearings has at least the same importance.

Concerning the verification of the reaction forces through testing, large deviations are allowed ( $\pm 20\%$  deviation) between prescribed calculations and measured reactions, due to significant amount of uncertainty [2]. In all cases, the measured reactions take precedence over calculations.

### 2.2.2 Deflection Curve

The most modern class regulations contain information concerning this curve. According to ABS in [2], relative misalignment between the shaft and the bearing may be evaluated by the information defined by the deflection curve. Deflection curvature defines the angle of the shaft inclination at each node of the system. This angle is measured from the theoretical zero alignment angle.

There are guides in which, hull deflections are taken into consideration and state that if the misalignment angle is immoderate, slope boring or inclination of the bearing may be necessary. At this point, it is important for “bearing misalignment” to be explained.

Bearing misalignment between the shaft and the bearing is a very important parameter to check as it was mentioned above. It is the inclination of the shaft, within the bearing length. The stern tube bearing requires special treatment due to its long length. This does not lead to the conclusion that the other support points must not be studied as much as the stern tube bearing. The limitations set for the maximum angle of misalignment between the shaft and the bearing bushing are relative to its dimensions. According to [4], the angle must not exceed the ratio of the radial clearance over the bearing length, which practically prevents the shaft from contacting the bearing bottom. If calculations prove otherwise, slope boring or bearing inclination must be applied to the bearing bush.

An example of a two-slope boring arrangement is shown below. Two-slope boring is applied when there is no forward stern tube bearing in the shafting system.

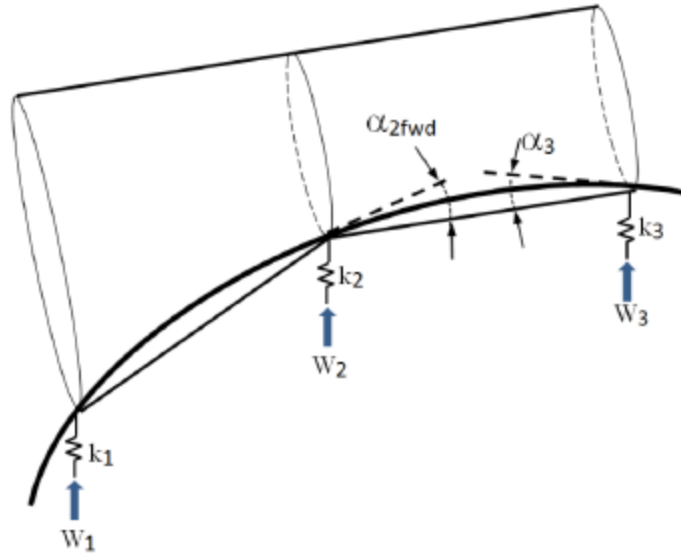


Figure 2.1: SHAFT BEARING WITH TWO SLOPE BORING

### 2.2.3 Slope Boring/ Bearing Inclination

Conditions inside the stern tube bearing change when the tail shaft starts rotating. The transition from a static shaft condition at zero RPM to a dynamic shaft condition and the development of the oil film need to be as swift as possible. The slope boring or the bearing inclination is applied to maximize contact area and facilitate a fast transition from a static to a dynamic condition. It assists the hydrodynamic lift of the shaft on the bearing lubricant during starting of the engine and minimizes metal-to-metal contact and oil film breakage when the engine comes to a stop. Classification societies established a misalignment angle limit of 0.3 mrad. The bearing misalignment angle is measured from the centerline through both the aft and forward stern tube bearings. A slope boring tolerance or deviation of  $\pm 0.1$  mrad, or approximately 1 millimeter per meter  $\pm 0.1$  mm, is acceptable [2].

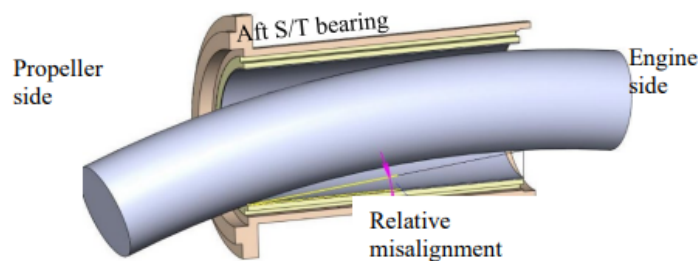


Figure 2.2: BEARING MISALIGNMENT [2]

Slope boring is the preferred method for reducing relative misalignment and is mostly applied to press-fitted stern bearings. During slope boring, the bearing's inner bore is machined in a lathe to the desired angle. The inner diameter centerline is offset at an angle with the outer diameter centerline.

The alternative of the slope boring is the bearing inclination, where the bearing is choked under the required angle with epoxy resin.

## Slope Boring

Slope boring is a process where the bearing shell is machined so as to ensure that the center line of the bearing's inner bore is misaligned to the desired angle. To allow provision for slope boring, the inner bearing diameter is initially pre-machined to the smaller diameter. The special boring machine is then attached to the stern block and aligned so as to match the required misalignment angle. Machining is then conducted by boring through the bearing in several passes, if required. Multiple passes may be necessary when larger amounts of bearing material are to be taken away because of a danger of bearing material overheating, as well as to ensure required machining tolerances.

## Bearing Inclination

An alternative approach to the pressure fitting is the choking of the bearings with epoxy resin. When bearings are choked the application of a single slope can be achieved by bearing inclination before epoxy casting. The bearings are inclined and fixed under the required angle during the final sighting process. This allows precise bearing setting and elimination of relative misalignment between the forward and aft stern tube bearings, which is otherwise difficult to completely eliminate with pressure fitted method. The preferred bearing inclination and choking procedure consists of individual choking of the aft and the forward stern tube bearing.

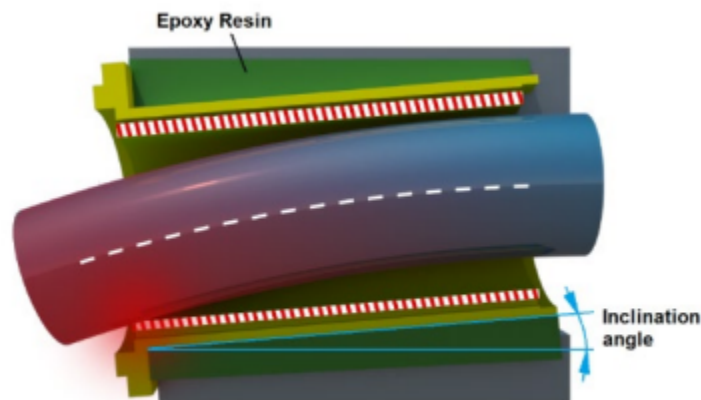


Figure 2.3: BEARING INCLINATION AND CHOKE IN STERN TUBE [2]

Bearing inclination is a less demanding procedure and has some benefits over slope boring. Stern tube bore does not require detailed machining, pre sighting is not required, machining of the outer diameter of the stern tube bearing is not needed and the installation of bearings based on bore sight only is very precise.

Although, the main disadvantage over slope boring procedure is the reduced heat conductivity due to the epoxy resin layer between the bearing and the stern structure.

#### 2.2.4 Single/Multi Point Contact

In conventional shaft alignment procedures, the contact point between the bearing and the shaft is modeled with a single point. The contact point represents the position of the bearing reaction. This approach is efficient when the requirement for slope boring does not have to be applied.

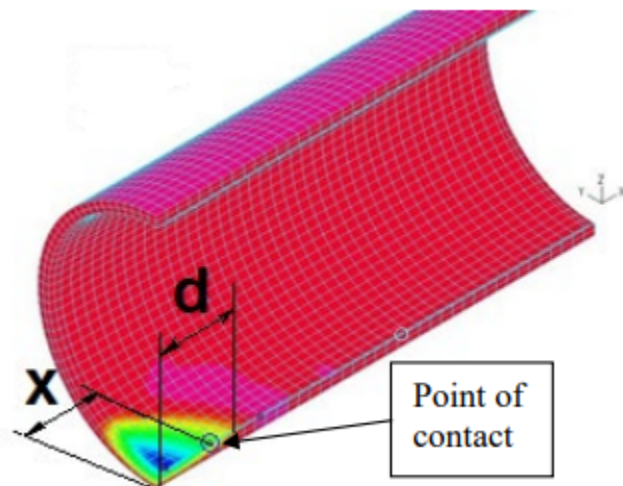


Figure 2.4: SINGLE POINT CONTACT MODEL [1]

According to [1], when a two-point or multiple-points contact approach is applied, the procedure is to first conduct the analysis with a single point contact to obtain the deflection curve. Based on the deflection curve zero misalignment bearing slope can be settled. The same bearing slope is then utilized to define the offsets on selected points along the bearing length. The selected points are the aft and the forward bearing edge, and the assumed point of maximum pressure is selected between  $D/3$  and  $L_b/4$  from the aft bearing edge, where  $D$  is shaft diameter and  $L_b$  is bearing length.

When the shaft alignment is applied to single slope bearing designs, the following approach is suggested for the aft stern tube bearing contact evaluation:

- 1) Two contact points are to be created in the shaft alignment model, at a minimum - one each at the aft and forward edge of the bearing
- 2) Define bearing offsets and calculate the aft stern tube-bearing slope in accordance with ABS Rules alignment requirements
- 3) Repeat the analysis with a single point contact by removing the forward contact point on the aft stern tube bearing

- 4) Apply the same bearing offsets used earlier for the two-point analysis
- 5) Evaluate bearing contact
- 6) Adjust the longitudinal distance of the aft point and recalculate.
- 7) Repeat 5) and 6) until satisfactory results are obtained.

The slope boring angle is now defined for applied contact point corrections. The results of this procedure for defining a more accurate static point of contact between the shaft and the bearing may be considered satisfactory when the longitudinal coordinate  $x$  of the selected single point of contact is equal or slightly smaller than calculated distance  $d$ , as shown on figure 2.4.

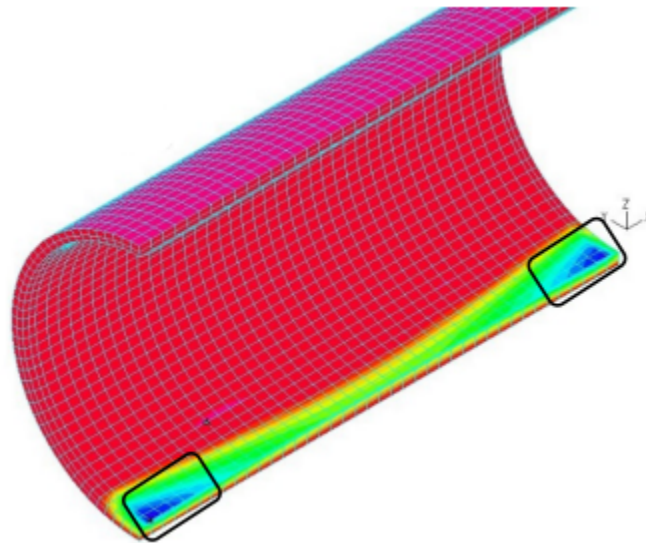


Figure 2.5: TWO POINT CONTACT MODEL [1]

Considering the number of contact points, neither the single nor the two-point contact approach is correct, because the bearing-to-shaft contact is actually established over an area of a bearing that the shaft penetrates into. ABS strategy is to initially assume either the combined approach, both the single and two-point contact, or the single point contact only.

Correction of the initial prediction of the bearing contact is recommended when the initially applied contact point falls outside the calculated contact area. If the analysis indicates contact at both edges of the bearing, the initially applied point of contact should be moved in accordance with the findings. Several iterations may be needed in order to stabilize the results.

### 2.2.5 Crankshaft Modeling

The crankshaft is very sensitive and the most complicated part of the shafting system. Bearings of large two stroke engines are sensitive to alignment condition and main engine damages and failure related to alignment are common. Thus, a proper shaft alignment is associated with a proper crankshaft modeling. Unfortunately, the crankshaft modeling is a controversial process.

The main problem in modeling the crankshaft for shaft alignment purpose is the definition of an equivalent system for crankshaft cranks. The cranks are made of complex shape and in big engines the cranks have flexibility, which cannot be ignored.

Most mercantile shaft alignment software is based on beam theory and does not support complex structures such as crankshafts. Instead, an equivalent beam model is utilized for shaft alignment calculations. The engine maker often provides this model.

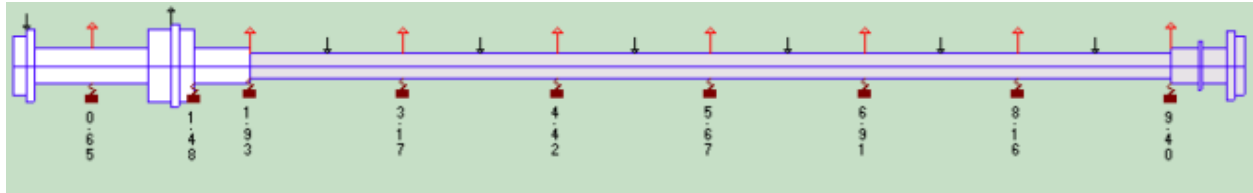


Figure 2.6: EQUIVALENT CRANKSHAFT MODEL [2]

Although there is not a regulation for the diameter of the equivalent crankshaft modeling, when a cylinder beam with a diameter of 60% of the crank journal is adopted the bearing reactions are very close, if not the same, to the bearing reactions in case of crankshaft [17]. Nevertheless, where recommended equivalent diameters by the engine manufacturer are available, the recommended values are to be used.

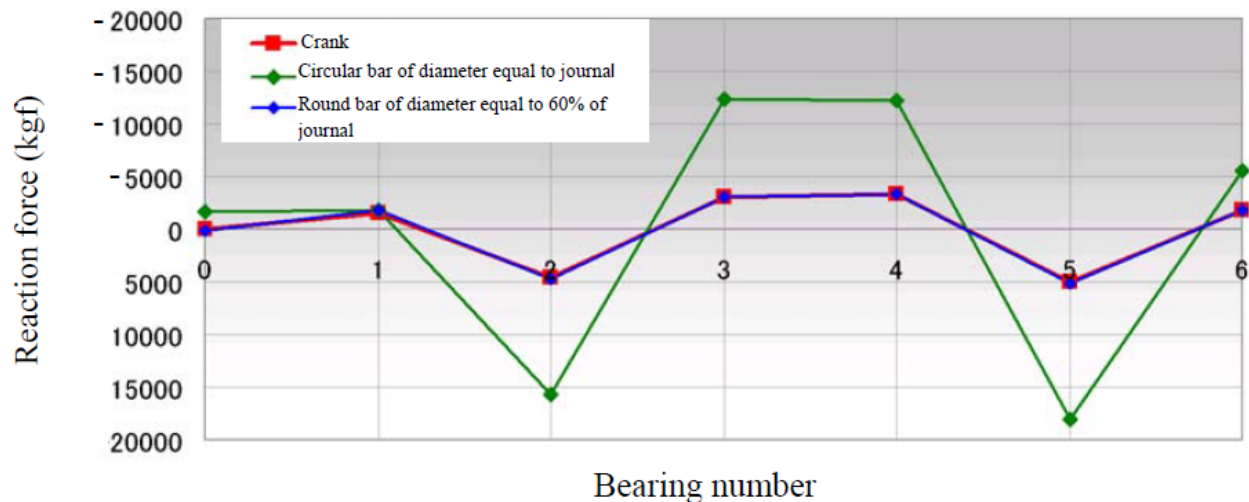


Figure 2.7: COMPARISON OF BEARING REACTION BETWEEN CRANK AND CYLINDER BEAM OF DIFFERENT DIAMETER [17]

## 2.3 Shaft Alignment Procedure

### 2.3.1 Preliminary Calculations

A couple of actions must be taken for the achievement of a proper alignment. The first step is to clarify the number and the longitudinal position of the bearings. Once it is settled, and all the bearings are in the same vertical position (zero vertical offset), a basic "straight line" is taking place in order to make an estimation for system's influence factors and reaction forces. The next step is to modify the vertical offsets in order to achieve an acceptable condition for the distribution of reaction forces. It is important

that reaction forces do not exceed the acceptable limits. It is recommended to aim for minimum misalignment between shaft and stern bushing. The last one is indeed difficult and in order to achieve, processes like slope boring or bearing inclination must be carried out. The last step is the application and evaluation of the shaft alignment.

### 2.3.2 Application

The shaft alignment procedure is not expected to start before the vessel stern blocks are fully welded and all of the heavy stern structure is in place. After that, the reference line for positioning the shafts, bearings, main engine and gearbox can be established. The next phase is the sighting through the shaft. Once this process is finished, the established reference line is further rectified (if it is necessary) by a slope boring or inclination of the stern tube bearing. The vessel is now ready for the shafts to be put in place, propeller installation and system assembly [2]. It is important to state that, temporary bearings are assisting the assembly at the same time. Propeller connection takes place, and if required, force is applied on the forward end of the tail shaft to hold it in contact with the forward stern tube bearing before assembling. At this stage, pre-assembly alignment condition of the shafting should be verified by the yard applying the “Sag & Gap” procedure. After that, bearing-shaft misalignment is evaluated and correction actions are carried out if necessary. Last but not least, when the shafts are coupled reaction measurements of the system parameters should be verified when the vessel is in dry dock condition in order to match the actual reaction forces of the pre-alignment phase. By doing this, the yard can control the alignment procedure against the analysis. Further verification of the alignment condition should take place when the vessel is afloat. It is more difficult to match with the calculated alignment in this case, because hull deflections are tough to predict precisely.

The “Sag & Gap” procedure, which is introduced above, is presented on the following figure

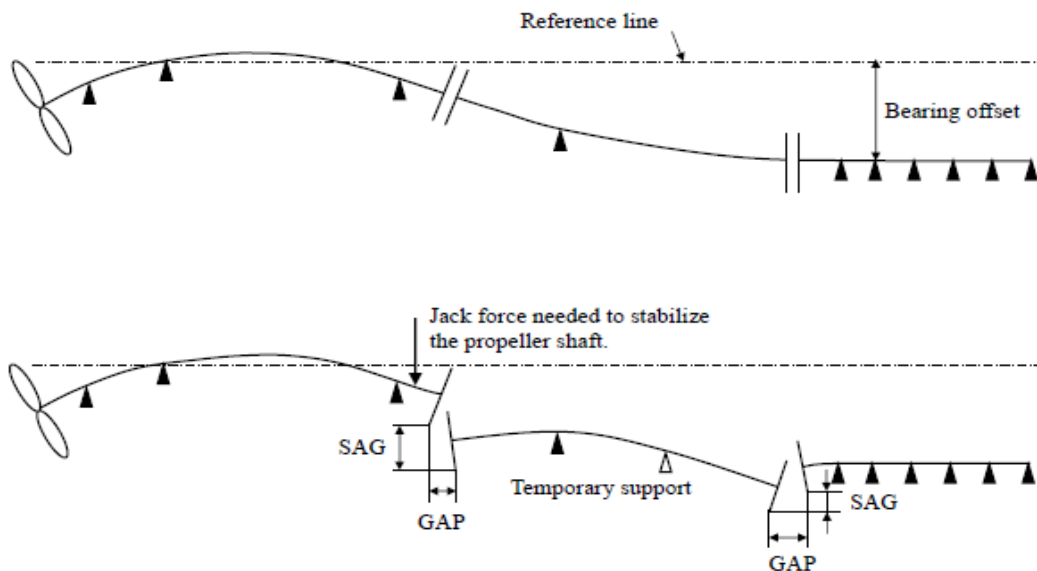


Figure 2.8: SAG AND GAP PROCEDURE [17]

## 2.4 Measurements

### 2.4.1 Sighting Through (Boring Sighting)

The process of reference line establishment is called sighting through or boring sighting. The procedure is conducted by optical instruments such as laser and piano wire.

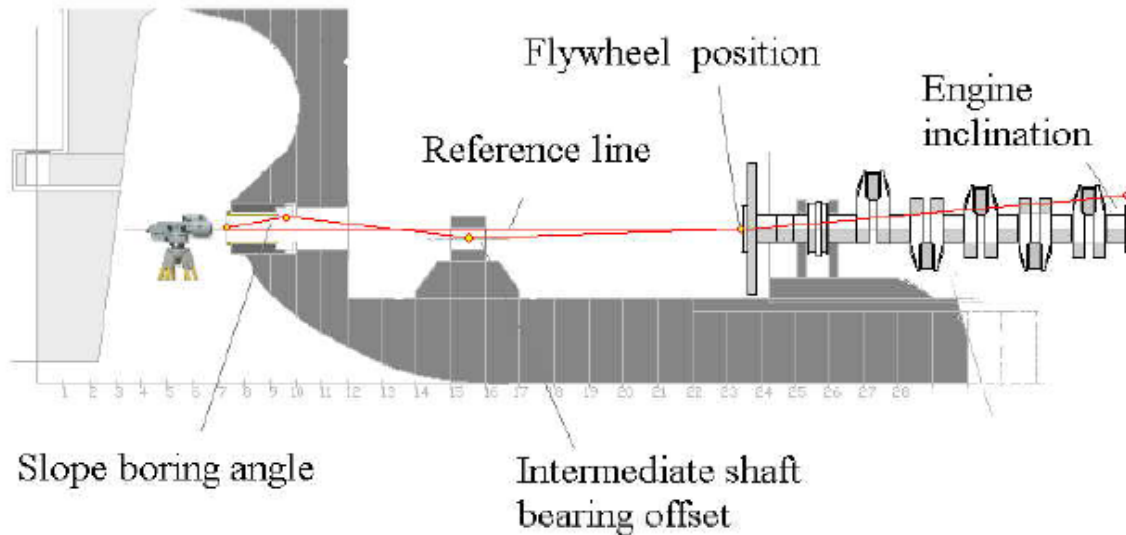


Figure 2.9: OPTICAL/LASER SIGHTING THROUGH [1]

Telescope, laser or piano wire is positioned in front of the aft stern tube bearing (American Bureau of Shipping, Guidance Notes on Propulsion Shafting System, 2014). The reference line is defined to match the center line of the aft stern tube bearing. Target points are defined at the location of the intermediate shaft bearings, gearbox flange or main engine flange. These points are offset for values corresponding to the prescribed bearing offsets for the dry dock condition. Afterwards, shaft line bearings and gearbox or main engine bearings are positioned into place. Slope boring angles are marked. In case of bearing inclination, the inclination angle is applied to the stern tube bearing and bearing is fixed in place inclined, ready for epoxy resin casting.

In order to prevent or minimize random disturbances of the established bearing location, engine position and slope boring the following are necessary:

- The temperature of the ship's structure must be as stable as possible. For this reason, sighting through must take place in early morning hours before sunrise.
- Sighting Through must be carried out, when the major welding work of stern block is complete. This is to prevent additional hull deformations, which are caused due to welding processing.
- Heavy structural parts of the structure such as superstructure, main engine shall be installed on the vessel prior to boring sighting.

### Piano Wire Application

In this method, a thin metal wire/string is bounced at the aft position of the engine, above the shaft and is connected with weight on the other side of the wire in order to keep as tense and straight as possible.



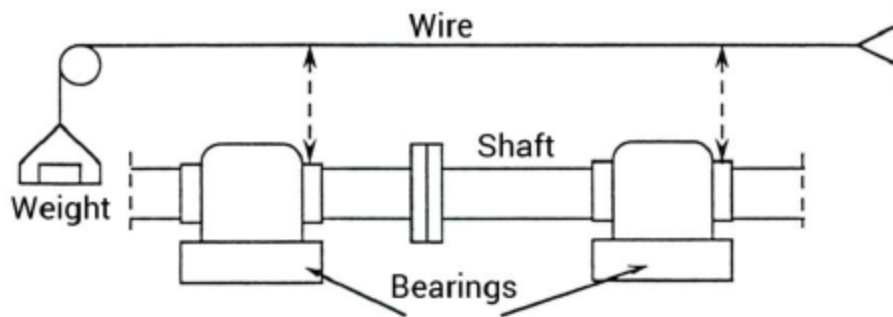


Figure 2.10: PIANO WIRE METHOD [6]

The prescribed bearing offset is now measured as a distance between the wire and the location of each support point. Positions of the bearings and a slope boring are defined using piano wire as a reference, but must be corrected in case of piano wire sagging.

Although it is not a complicated method, it has a great disadvantage. The measurement has to be made manually. Therefore, it is difficult for the personnel conducting the measurement to repeat measurements under a variety of conditions and measure the displacement at multiple points simultaneously. Another fact is concerns about the accuracy of the measurement arise when the displacement is small.

## Optical Methods

The most accurate methods for alignment implementation are the optical methods: optical telescope and laser. The optical means equipped are very accurate and therefore the outcome is high quality. In the first step of the application, the telescope/ laser is positioned on the aft end of the shafting system. A reference target is positioned on the other end and several targets are positioned at the exact vertical offsets and longitudinal positions. These targets have a narrow hole in the middle in order to allow laser beam to pass through. Once the reference line is established between telescope/laser and the fore target, the offsets of support points can be measured by simply adjusting the telescopes dials. The targets are adjusted starting from the fore one and moving aft end of the shafting system.

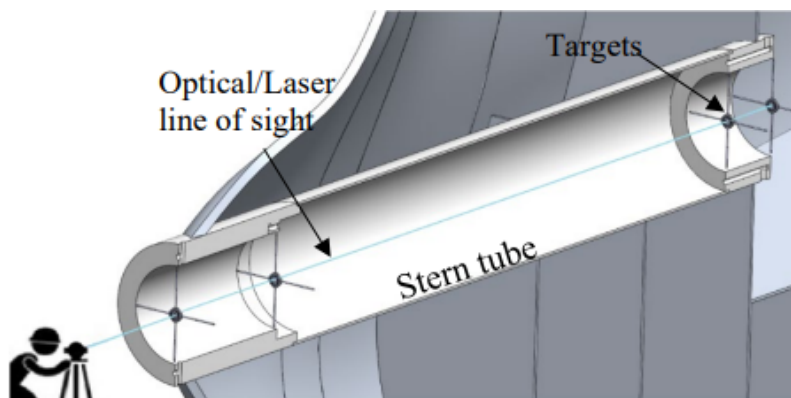


Figure 2.11: OPTICAL METHOD OF ALIGNMENT [2]

The targets are usually inscribed with concentric circles of various diameters in order to provide improved resolution, additionally to the scale on the graded dials. This method has the advantage of improved accuracy and additionally provides the benefit of being able to place objects/ targets at any pre- defined position simply by adjusting the telescope dials at a specific focus, distance and offset and matching the object's position with the selected target. Another advantage that is important for sloped stern tube bearings, is that the laser can set a reference line at any desired angle. In the case of single or double slope boring/ inclination of the stern tube bearing, the reference line is defined by the main slope of the stern tube bearing, therefore the optic methods are the only eligible method for accurate shaft alignment.

### 2.4.2 Sag and Gap Measurements

The “Sag and Gap” procedure is applied as an alignment verification prior to shafting assembly. Although it is a simple and fast method, it is questioned about its accuracy. During installation, the shafts (propeller shaft, intermediate shaft, crankshaft) are laid down on the supports uncoupled. In this state, the flanges are hanging freely and a significant amount of force needs to be applied in order to position them at the desired vertical and horizontal offset for coupling [2]. After decoupling a shafting system, flanges should be positioned at specific vertical and horizontal distances. The parameters of such measurements are “Sag and Gap”. Sag is defined by the vertical distance of edges (top or bottom) between two mating flanges and Gap is the horizontal distance between top or bottom edges of uncoupled flange pair. The following figure describes possible arrangements and how Sag and Gap values are measured for each case.

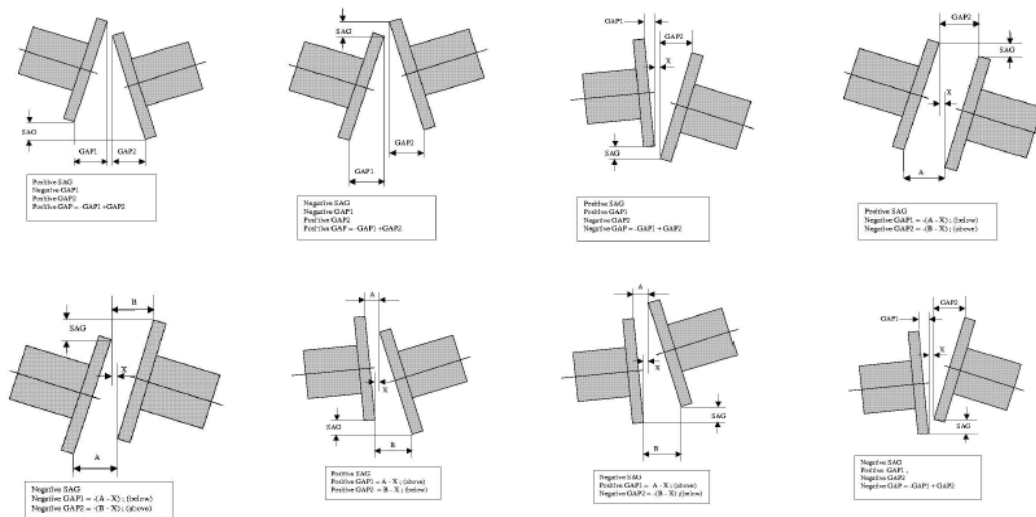


Figure 2.12: FLANGE ARRANGEMENT IN SAG AND GAP ANALYSIS [1]

It is recommended not to start the Sag and Gap procedure before the following are completed:

- Engine and reduction gear are installed
- Temporary supports are installed
- Shafts are placed inside the vessel and propeller is positioned
- The propeller shaft is in contact with the bottom shell at the foremost stern tube.

### 2.4.3 Bearing Reaction Forces Measurements

Once the shaft alignment process is completed, the shafts are coupled and the temporary supports have been removed, it is essential to measure the actual bearing reaction forces of the support points. The most common methods to measure the reaction forces are the following:

- Jack-up test
- Strain gauge measurement

#### Jack-up test

According to [17], jack up method is a direct way to check reaction forces. It is the most widely applied method in the industry, because it is very simple. A hydraulic jack, which is placed close to the bearing under examination, lifts the shaft and measures the load. Hydraulic pressure applied to lift and lower the shaft indicates the jacking load. A dial gauge is fixed on a magnetic stand, which is placed on the bearing housing, with the gauge spindle touching the shaft. The measurement points out the displacement of the shaft. Shaft displacement and jacking force are recorded and plotted on a graph from which the bearing reaction force is calculated.

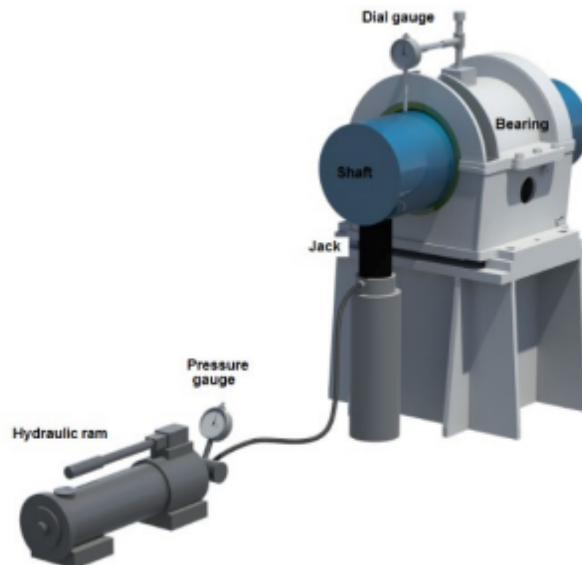


Figure 2.13: JACK UP REACTION MEASUREMENT [2]

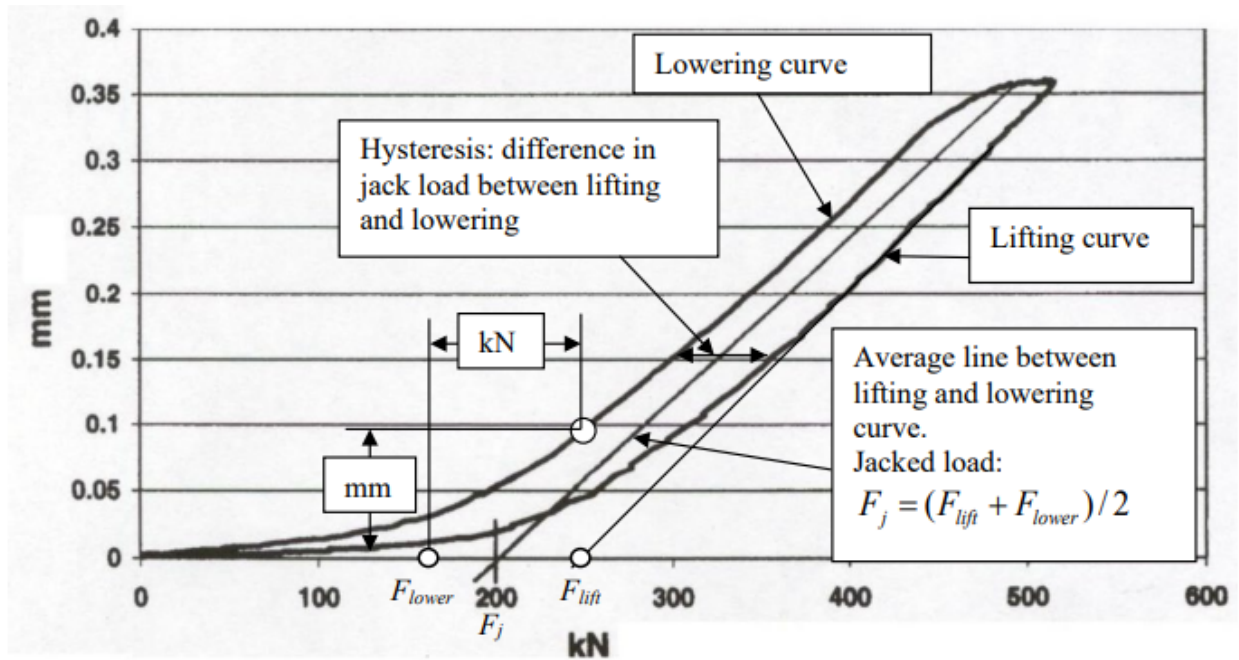


Figure 2.14: MEASURED JACK UP CURVE [2]

Initially, as the shaft lifts, the bearing is still carrying some of the load while the hydraulic jack takes over. This stage is represented by a relatively flat curve. The lifting gradient of the curve is increased slowly until the load is fully transferred from the bearing onto the hydraulic jack. As the lifting continues, the gradient reaches a constant value. The breakaway point is when the jack has completely lifted the shaft off the bearing and the load is completely transferred onto the jack. Jacking stops when an acceptable number of points has been recorded to define the slope of the curve. The process then reverses, and the shaft is lowered down as pressure is released from the hydraulic jack.

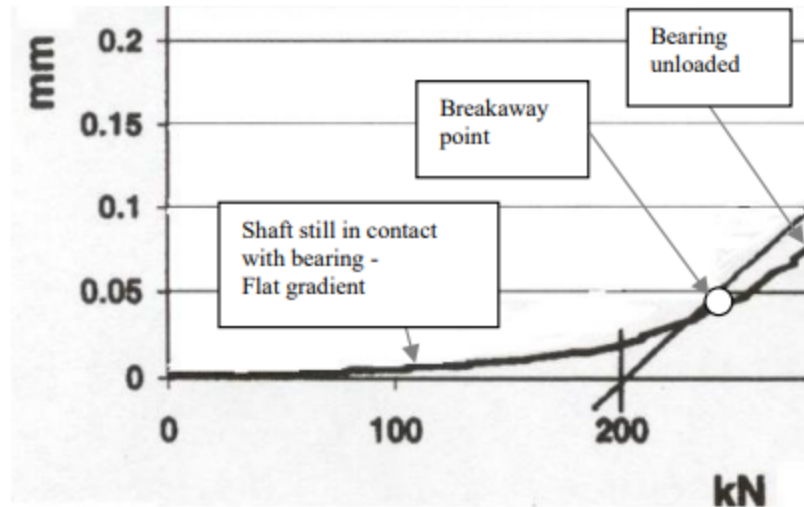


Figure 2.15: JACK UP LIFTING CURVE [2]

The lowering part of the process is not the same as the lifting part. The reason is due to the friction of the hydraulic jack and internal resistance of the shaft. The lowering shaft curve has the same shape as the lifting curve and the same number of points are recorded. The load reduces linear as the jack is lowered and the gradient of the curve remains constant until the breakaway point. From this point and onwards, the gradient of the curve changes continuously.

The jack measured load will not be exactly the same as the actual bearing load, because the jack is placed close to the bearing and not above or inside of it. The actual bearing force ( $F_b$ ) is correlated to jacked load by applying a correction factor ( $C_f$ ).

$$F_b = C_f \cdot F_j$$

The gradient of the jack up curve indicates the change of lifting or lowering force over a change of vertical offset. This is called also influence coefficient and is very similar to bearing influence factors since the jack is placed close to the bearing. The difference is very small since the longitudinal deviation is very small compared to the length of the shaft.

## Strain Gauge Measurement

In this method, strain gauges are installed on the surface of the shaft at multiple locations and can measure vertical and transverse bearing loads [2]. The measured strain is converted into bending moments and stress and thus bearing loads and shaft deflections can be estimated by reverse engineering. To increase the precision of the measurements more than one strain gauge is installed to measure the strain at the same location. Usually, two pairs of strain gauges are mounted on the shaft at the same longitudinal position 180 degrees apart and connected in a Wheatstone bridge. The strain gauges are attached to an input/output interface unit, which is connected to a computer. A main

advantage compared to jack up method is that it can provide simultaneous information on more than one bearing load. Furthermore, it can provide accurate information on the loading condition of bearings, which hydraulic jack setup is inaccessible, such as stern tube bearings. On the contrary, equipment installation demands a lot of time (one hour per measurement point). The equipment for the measurements is sophisticated and relatively expensive.

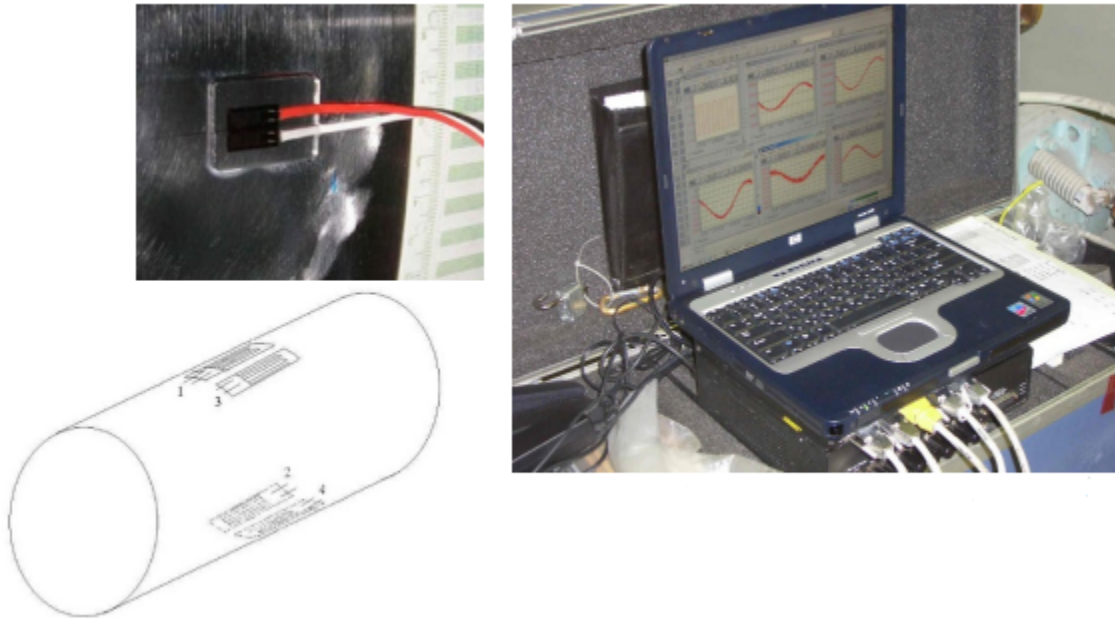


Figure 2.16: STRAIN GAUGE MEASUREMENT [1]

### 3. Journal Bearings

#### 3.1 Introduction

A journal bearing is the most common hydrodynamic bearing in use. It consists of a shaft or journal, which rotates freely on a supporting metal shell or sleeve with a complete  $360^\circ$  arc, also called bushing. It is used either to receive radical loads or to smoothly transmit the torque on the shaft. The inner part of the bushing is lined with a soft bearing material such as lead or tin babbitt, bronze or in rare cases plastic. Lubricant is supplied to the system from various arrangements such as inlet holes or more complex arrangements such as axial, circumferential and helical grooves [5].

As the shaft begins to rotate, it first rolls up the wall of the sleeve in the opposite direction of the rotation due to metal to metal friction between the shaft and the bearing bore. This motion, accompanied with an adequate lubricant supply, forms a supporting wedge-shaped film of lubricant, which lifts the shaft into its steady state position. Journal bearings are sensitive to a phenomenon called self-excited oil whirl. A lot of bearing designs have been established to deal with such vibration issues during the start-up. [9]

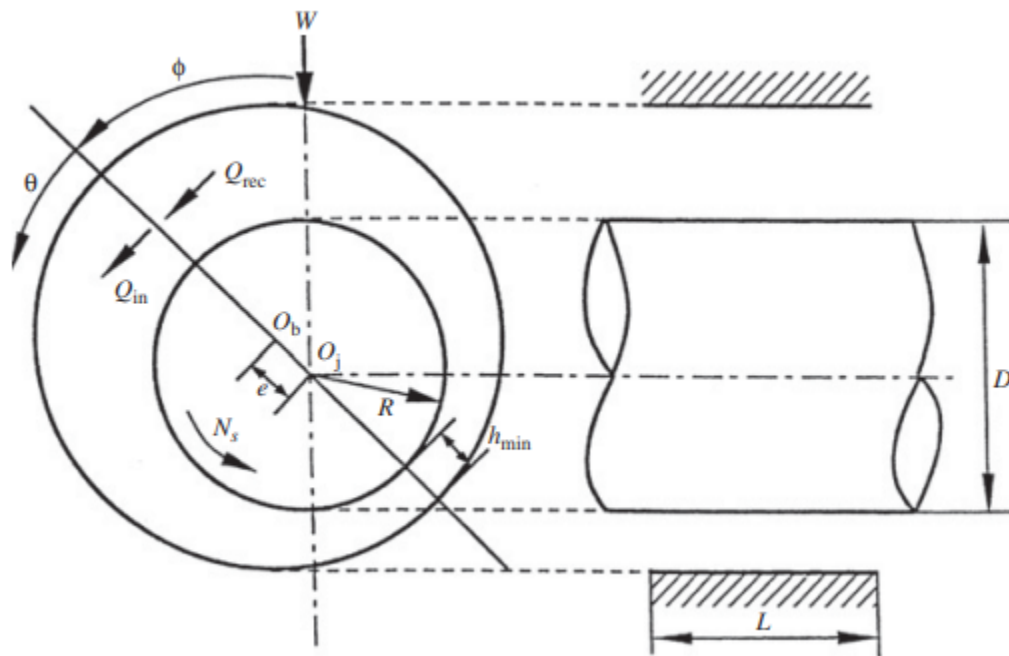


Figure 3.1: CROSS SECTION OF A JOURNAL BEARING [9]

Once the shaft is in a steady state, it assumes a position in the bearing clearance circle. This position can be defined by the eccentricity  $e$  and attitude angle  $\phi$ . Eccentricity is the displacement between the shaft center  $O_j$  and the bushing center  $O_b$ . Attitude angle is the angle between the load line and the line of the two referred centers. More vigorous oil film pumping at higher speed and increased viscosity will lead to decrease of eccentricity and increase of the attitude angle.

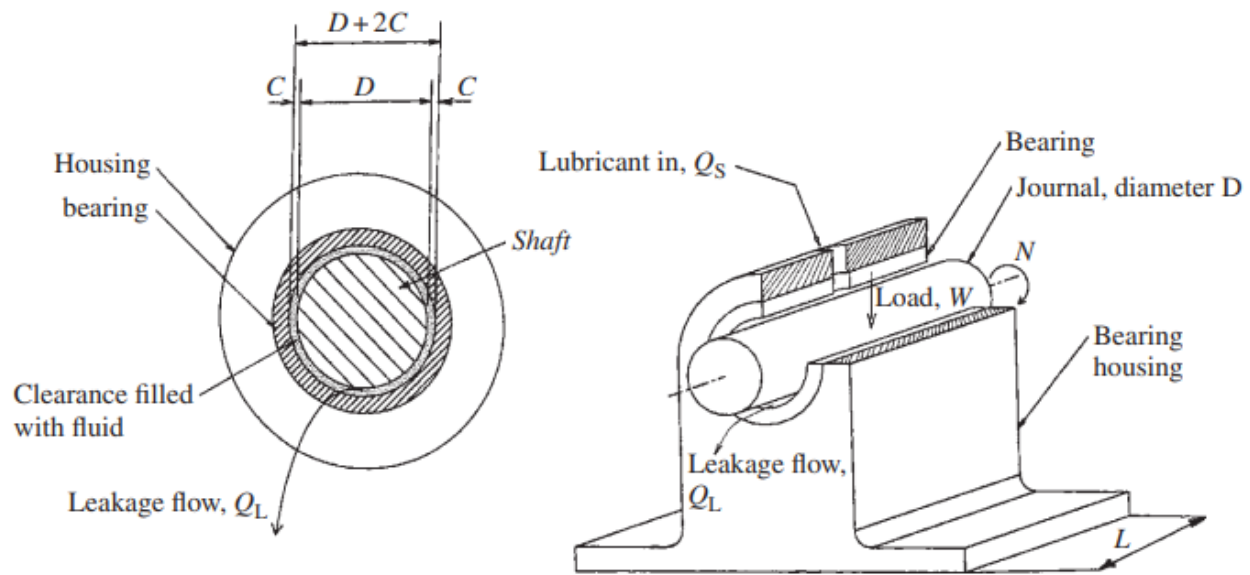


Figure 3.2: JOURNAL BEARING GEOMETRY AND NOMENCLATURE [9]

### 3.2 Hydrodynamic Lubrication Principles

According to [19], hydrodynamic lubrication occurs when two non-parallel surfaces move relative to each other separated by a pressurized thin lubricant film. The bottom surface, called “runner” is covered with lubricant and moves with a certain velocity. The top surface is inclined under a certain angle to the bottom surface. As the bottom surface moves and drags the lubricant along it into a converging wedge and a pressure field is generated. The pressure gradient causes the fluid velocity profile to bend inwards at the entrance of the wedge and outwards at the exit. The generated pressure field, which separates the two surfaces, has the ability to support a certain load. Hydrodynamic lubrication can be explained mathematically in a form of an equation, which was originally developed by Reynolds and is commonly known as “Reynolds Equation”.

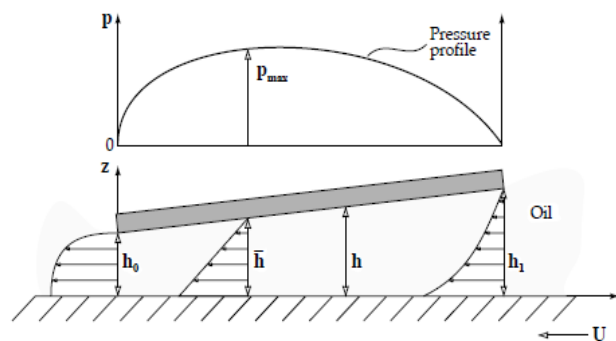


Figure 3.3: PRINCIPLE OF HYDRODYNAMIC PRESSURE GENERATION BETWEEN NON-PARALLEL SURFACES [19]



### 3.2.1 Mathematical Approach – Assumptions for the Construction of Reynolds Equation

The assumptions need to be made in order to derive the Reynolds equation are summarized below.

- Body forces are neglected

This assumption is always valid, since there are no outer fields of force on the fluids with an exception of magnetohydrodynamic fluids and their applications.

- Pressure is constant through the film

Always valid, since the thickness of hydrodynamic films is in the range of several micrometers. There might be some exceptions, however, with elastic films

- No slip at the boundaries

Always valid, since the velocity of the oil layer adjacent to the boundary is the same as that of the boundary.

- Lubricant behaves as Newtonian fluid

Usually valid with certain exceptions, e.g. polymeric oils.

- Flow is laminar

Usually valid, except large bearings, e.g. turbines.

- Fluid inertia is neglected

Valid for low bearing speeds or high loads. Inertia effects are included in more exact analyses.

- Fluid density is constant

Usually valid for fluids when there is not much thermal expansion. Definitely not valid for gases.

- Viscosity is constant through the generated fluid film

Crude assumption but necessary to simplify the calculations, although it is not true. Viscosity is not constant throughout the generated film.

There are some additional simplifications that can be implemented to the Reynolds equation in order to use it to study journal bearings. These are the following:

- Unidirectional Velocity Approximation

It is always possible to choose axis in such a way that one of the velocities is equal to zero. This is applied in many engineering systems such as a journal bearing slides along a rotating shaft.

- Steady Film Thickness Approximation

It is also possible to assume that there is no vertical flow across the film. This assumption requires that the distance between the two surfaces remains constant during the operation. Some inaccuracy may result from this analytical simplification since most bearings usually vibrate and consequently the distance between the operating surfaces cyclically varies. Movement of surfaces normal to the sliding

velocity is known as a squeeze film effect. Furthermore, in the case of porous bearings there is always some vertical flow of oil.

- Isoviscous Approximation

For many practical engineering applications it is assumed that the lubricant viscosity is constant over the film, i.e.  $\mu = \text{constant}$ . This approach is known in the literature as the ‘isoviscous’ model where the thermal effects in hydrodynamic films are neglected. Thermal modification of lubricant viscosity does, however, occur in hydrodynamic films and must be considered in a more elaborate and accurate analysis which will be discussed later. Assuming that  $\mu = \text{constant}$  the Reynold’s equation can be written:

$$\frac{\partial}{\partial x} \left( h^3 \cdot \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( h^3 \cdot \frac{\partial p}{\partial z} \right) = 6U\mu \cdot \frac{dh}{dx} \tag{1}$$

The pressure profile of the lubricant film is visualized on the following figure.

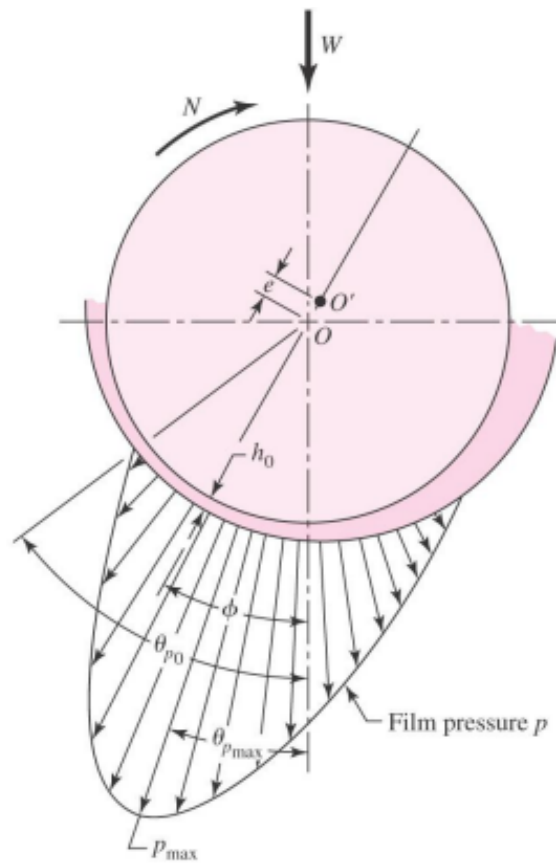


Figure 3.4: FILM PRESSURE DISTRIBUTION [3]

### 3.3 Operational and Performance Parameters

#### 3.3.1 Load Capacity

In journal bearings, the total load that can be supported with hydrodynamic lubrication can be derived from integration of the pressure at the circumference of the shaft. This load will be resolved into two

perpendicular loads. The first one is acting along the line of the shaft and bush centers and the second is a perpendicular load vector [19]. The sum of these vectors yields the total load and the angle between the total load and the first component is the attitude angle, which defines the line at which minimum and maximum film thickness occur at the side of converging and diverging section of the shaft respectively.

The expressions of the load components “ $W_x$ ”, “ $W_z$ ”, as demonstrated in the following figure, derive from assumption of a lubricant film area as:  $R \cdot dy \cdot d\theta$  where the hydrodynamic load is  $p \cdot R \cdot dy \cdot d\theta$ . The two force components are:

$$dW_x = p \cdot \cos(\varphi + \theta - \frac{\pi}{2}) \cdot R \cdot dy \cdot d\theta$$

$$\text{yielding a load of } W_x = \int_0^{2\pi L} \int_0 p \cdot \cos(\varphi + \theta - \frac{\pi}{2}) \cdot R \cdot dy \cdot d\theta \quad (2)$$

$$dW_z = p \cdot \sin(\varphi + \theta - \frac{\pi}{2}) \cdot R \cdot dy \cdot d\theta$$

$$\text{yielding a load of } W_z = \int_0^{2\pi L} \int_0 p \cdot \sin(\varphi + \theta - \frac{\pi}{2}) \cdot R \cdot dy \cdot d\theta \quad (3)$$

$$\text{The total load capacity is: } W = \sqrt{W_x^2 + W_z^2} \quad (4)$$

### 3.3.2 Sommerfeld Number

To determine the load-carrying capacity and other bearing performance parameters, the eccentricity ratio must be determined. This is defined by a dimensionless parameter called “Sommerfeld Number”. The Sommerfeld Number is calculated by the following formula:

$$S = \frac{\mu \cdot N_s \cdot D \cdot L_b}{W} \left( \frac{R}{C} \right)^2 \quad (5)$$

Where:

- $\mu$  is lubricant viscosity (Pa·s)
- $N_s$  is the rotor angular velocity (rps)
- $R$  is bearing radius (m)
- $C$  is bearing clearance (m)

- D is bearing diameter
- $L_b$  is bearing length
- W is the applied load (N)

### 3.3.3 Friction Force and Friction Coefficient

Integration of the x-component of shear stress, ' $\tau_x$ ', over the bearing area yield the total amount of Friction force:

$$F = \int_0^L \int_0^D \tau_x dx dy = \int_0^L \int_0^D \mu \cdot \frac{du}{dz} dx dy \quad (6)$$

The friction coefficient  $f$  is defined by the ratio of friction force F to the applied load W.

$$f = \frac{F}{W} \quad (7)$$

### 3.3.4 Power Loss

The power loss comes from the friction losses and is estimated as follows:

$$P_{loss} = F \cdot U = W \cdot f \cdot N_s \cdot R \quad (8)$$

Where:

- f is friction coefficient
- W is the applied load
- $N_s$  is the rotor angular velocity
- R is the bearing radius






## 4. Software

### 4.1 Shaft Alignment Tool

To validate bearing reaction forces and calculate influence factors, “Shaft Alignment Tool” is used. The software is created and developed in the department of Marine Engineering of School of Naval Architecture and Marine Engineering of National Technical University of Athens (NTUA). It is a simple program, which executes calculations relatively fast and precise. Besides reaction forces and influence factors, “Shaft Alignment Tool” and especially *“shaftAlignment\_bending\_momentst.exe”* is able to calculate bending moments, vertical displacements and rotation of the axis in specified positions, as it is shown on the following sections.

#### 4.1.1 Description

The workspace of the program is shown on the following figure. The shaft is divided into smaller beam elements, which are called segments. Between two segments or at the fore and after end of the shaft, there is a small circle, which is called node. A node can be free or constrained (support placement). A curvature is passing through constrained nodes, which is the elastic line of the shaft. On the top right of the figure, we can adjust (increase or reduce) the number of segments. Next to the adjustment of the number of segments, there are 5 icons, which their operations are described below:

-  enables/disables real time execution (elastic line, reaction forces, external loads)
-  shows/hides length value of each segment
-  shows/hides weight distribution of each beam element (kg/m)
-  displays the result table (see figure 4.2)
-  calculates and displays the model influence factors (see figure 4.3)

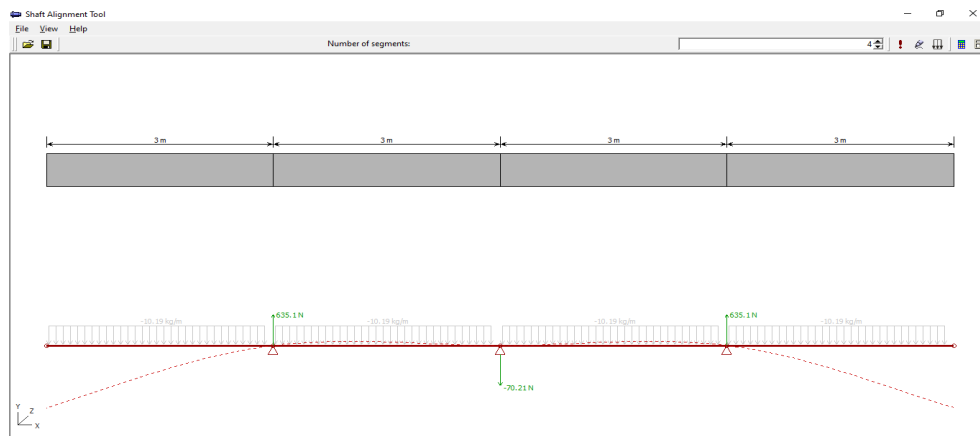


Figure 4.1: SHAFT ALIGNMENT TOOL WORKSPACE

The coordinates system is defined:

- Axis x: longitudinal axis
- Axis y: vertical axis
- Axis z: transverse axis

Shaft Model - Analysis Results

Model Data						
Nodes						
Node No	Coord. X (m)	Coord. Y (m)	Coord. Z (m)			
1	0	0	0			
2	3	0	0			
3	6	0	0			
4	9	0	0			
5	12	0	0			
Beam Members						
Member No	Length (m)	Distr. Weight (kg/m)	Total Weight (kg)	Moment of Inertia (m <sup>4</sup> )	Young's Modulus (N/m <sup>2</sup> )	
1	3	-100	-300	0.003	2.0601E+11	
2	3	-100	-300	0.003	2.0601E+11	
3	3	-100	-300	0.003	2.0601E+11	
4	3	-100	-300	0.003	2.0601E+11	
Forces						
Node No	Fx (N)	Fy (N)	Fz (N)	Mx (Nm)	My (Nm)	Mz (Nm)
1	0	-1.16529E-12	0	0	0	9.37916E-13
2	0	635.105	0	0	0	3.41061E-13
3	0	-70.2098	0	0	0	1.7053E-13
4	0	635.105	0	0	0	-3.41061E-13
5	0	-5.68434E-14	0	0	0	-2.27374E-13
Displacements						
Node No	Ux (m)	Uy (m)	Uz (m)	Rx (rad)	Ry (rad)	Rz (rad)
1	0	-3.08408E-06	0	0	0	1.20063E-06

Figure 4.2: RESULTS TABLE

Influence Factors,  $s(i,j)$

$s(1,1)=-33896.4$	$s(1,2)=67792.8$	$s(1,3)=-33896.4$
$s(2,1)=67792.8$	$s(2,2)=-135586$	$s(2,3)=67792.8$
$s(3,1)=-33896.4$	$s(3,2)=67792.8$	$s(3,3)=-33896.4$

Figure 4.3: CALCULATION OF INFLUENCE FACTORS

By right clicking on a beam element, the beam properties (length, distributed load, moment of inertia and Young's modulus) are shown, which can be adjusted as we wish. Last but not least, by right clicking on a node we may apply the following commands.

- ✓ If the node is constrained (support placement):
  - Apply displacement. We can adjust the vertical position (in mm) of the support (either above or below the reference condition, which is zero displacement)
  - Apply rotation. We can define the pitch (in mrad) of the shaft at a specific constrained node. It is positive if the shaft is heading upwards and negative if the shaft is heading downwards.
  - Unconstrain. Make a node free of support.
  
- ✓ If the node is unconstrained (free):
  - Apply vertical force (y axis).
  - Apply transverse force (z axis).
  - Apply vertical bending moment (y axis).
  - Apply transverse bending moment (z axis).
  - Constrain.

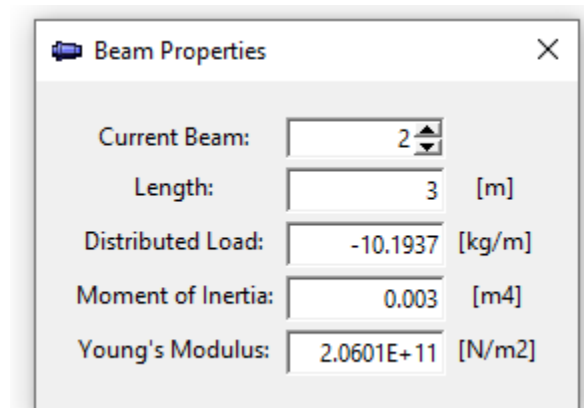


Figure 4.4: BEAM PROPERTIES

## 4.2 Notepad++

In most cases the shaft modeling is not simple as the example above. The diameter is not the same in all segments and external loads are not applied on equal spaced nodes. On the other hand, the beam elements are too many to adjust their beam properties one by one with ease. Therefore, these changes

cannot be done with “Shaft Alignment Tool” in the first place. Instead, Notepad++ is necessary and on the following section it is described how to handle it.

#### 4.2.1 Description

The first row of a notepad++ file should be filled with “Number\_of\_segments” and the second row with the actual number (e.g 53). The third row should be filled with “Number\_of\_constrained\_nodes” and the fourth row with the actual number (e.g 9). The fifth row should be filled with “Angle “ and the sixth row with the actual number, which in all cases is 0. The seventh row should be filled with “Segment\_Lengths”. The next rows should be filled with the actual value of the segment length (in meters) of each beam element one by one from first element to last. When all segment lengths are listed, the next blank row should be filled with “Segment\_Loads” and the next rows should be filled with the actual value of the distributed load (in N/m) of each beam element as described. When distributed loads of all beam elements are listed, the next blank row should be filled with “Segment\_Inertias” and the next rows should be filled with the actual value of the area moment of inertia of the beam section (in  $m^4$ ) of each segment one by one. When area inertias of all beam elements are listed, the next blank row should be filled with “Segment\_Youngs\_Moduli” and the next rows should be filled with Young’s Modulus actual value (in  $N/m^2$ ) of each segment. When this is done, the next blank row is filled with “Nodal\_Shifts” and the next rows are filled with the actual value of vertical offset (in meters) of each node one by one. When all nodal displacements are listed, the next blank row is filled with “Nodal\_Forces\_Y” and the next rows are filled with the actual values of vertical forces (in Newton) in every node one by one. After that, the same process goes for “Nodal\_Forces\_Z” (their actual values in Newton), “Nodal\_Bending\_Moments\_Y” (their actual values in Newton\*meters) and “Nodal\_Bending\_Moments\_Z” (their actual values in Newton\*meters). The next step is to fill the next blank row with “Nodal\_Slope” and the next rows should be filled with the actual values of the rotation of the shaft in every node in relevance to axis z (in rad). After that, the last step is the next blank row to be filled with “Constrained\_Nodes” and the next rows should be filled with the constrained nodes one by one represented by their number. For example, if the 13<sup>th</sup> node is constrained the row should be filled with 13, if the 25<sup>th</sup> node is constrained the row should be filled with 25, etc.). The notepad file should be saved as “.sft”. To understand better the whole process, an example is shown on the following figures.



Segment_Lengths	Segment_Loads	Segment_Inertias	Segment_Youngs_Moduli
0.02	-16354.42	0.003589081	2.10E+11
0.365	-20346.26	0.005554972	2.10E+11
0.03	-18295.91	0.004491803	2.10E+11
0.62	-23099.66	0.007160178	2.10E+11
0.92	-26067.42	0.009118194	2.10E+11
0.36	-29214.48	0.011452732	2.10E+11
0.15	-29214.48	0.011452732	2.10E+11
0.3875	-29214.48	0.011452732	2.10E+11
0.3875	-29214.48	0.011452732	2.10E+11
0.3875	-29214.48	0.011452732	2.10E+11
0.3875	-29214.48	0.011452732	2.10E+11
0.14	-29214.48	0.011452732	2.10E+11
2.035	-27557.26	0.010190254	2.10E+11
0.18	-29467.24	0.011651762	2.10E+11
0.27	-29467.24	0.011651762	2.10E+11
0.27	-29467.24	0.011651762	2.10E+11
0.15	-29467.24	0.011651762	2.10E+11
0.318	-29467.24	0.011651762	2.10E+11
0.052	-29467.24	0.011651762	2.10E+11
0.425	-29467.24	0.011651762	2.10E+11
0.5	-23249.41	0.007253317	2.10E+11
0.13	-70546.59	0.066782846	2.10E+11
0.13	-70546.59	0.066782846	2.10E+11
0.6625	-18967.26	0.004827497	2.10E+11
0.6625	-18967.26	0.004827497	2.10E+11
0.6625	-18967.26	0.004827497	2.10E+11
0.6625	-18967.26	0.004827497	2.10E+11
0.5	-19307.47	0.00500223	2.10E+11
0.5	-19307.47	0.00500223	2.10E+11
0.938	-18967.26	0.004827497	2.10E+11
0.938	-18967.26	0.004827497	2.10E+11
0.938	-18967.26	0.004827497	2.10E+11
0.938	-18967.26	0.004827497	2.10E+11
0.938	-18967.26	0.004827497	2.10E+11
0.13	-118545.38	0.188574099	2.10E+11
0.01	-118545.38	0.188574099	2.10E+11
0.2	-118545.38	0.188574099	2.10E+11
0.487	-46837.52	0.029437477	2.10E+11
0.504	-46837.52	0.029437477	2.10E+11
0.386	-46837.52	0.029437477	2.10E+11
0.646	-13360.55	0.002395308	2.10E+11
0.646	-13360.55	0.002395308	2.10E+11
0.646	-13360.55	0.002395308	2.10E+11
0.646	-13360.55	0.002395308	2.10E+11
0.646	-13360.55	0.002395308	2.10E+11
0.646	-13360.55	0.002395308	2.10E+11
0.001	-13360.55	0.002395308	2.10E+11

Figure 4.5: INPUT DATA EXAMPLE(PT.1)


Nodal_Shifts	Nodal_Forces_Y	Nodal_Forces_Z	Nodal_Bending_Moments_Y	Nodal_Bending_Moments_Z	Nodal_Slope
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	-385427	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0.0005	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0.0005	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
-0.00325	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	-58545.7005	0	0	0	0
0	0	0	0	0	0
-0.00438	0	0	0	0	0
0	47834	0	0	0	0
-0.00422	0	0	0	0	0
0	-280036	0	0	0	0
-0.00399	0	0	0	0	0
0	-280036	0	0	0	0
-0.00376	0	0	0	0	0
0	-280036	0	0	0	0
-0.00352	0	0	0	0	0
0	0	0	0	0	0

Constrained\_Nodes

9  
16  
29  
39  
41  
43  
45  
47

Figure 4.6: INPUT DATA EXAMPLE(PT.2)

### 4.3 Application

When the process described on 4.2.1 is finished, the notepad file should be saved as “.sft” in the same file directory as “*shaftAlignment\_bending\_momenst.exe*”. Then, the “.exe” software should be launched. On the top left of **figure 4.1** the  should be clicked. When this icon is pressed the following list of “.sft” files is shown.

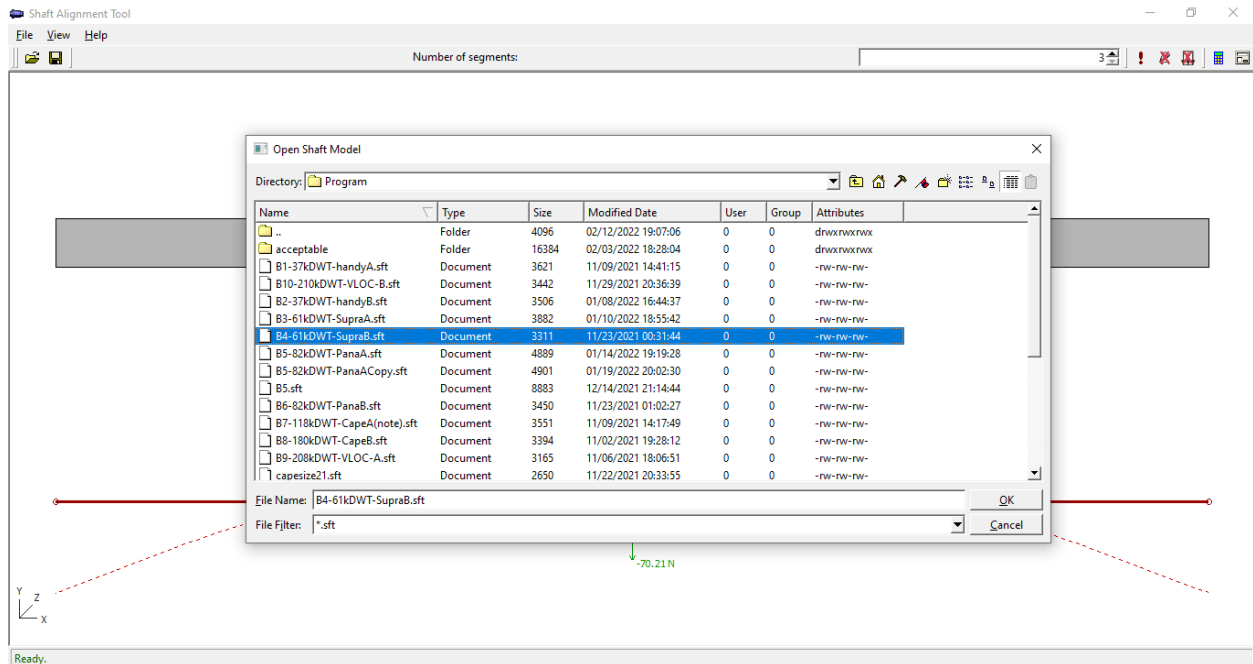


Figure 4.7: SFT FILES LIST

If any of these files is selected, then the shaft model with the elastic line and bearing reaction is appearing as on the following figure. To see the results analytically, we shall select the icons described on **4.1.1**.

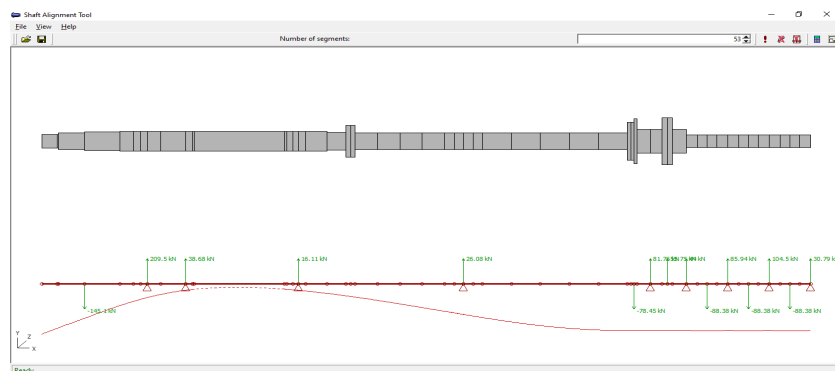


Figure 4.8: SHAFT MODELING EXAMPLE

## 5. Fleet Wise Study

The fleet consists of 17 bulk carriers of different DWT capacities. In this section, a dimension analysis of the propulsion system for each vessel is carried out. The analysis contains characteristics (length and diameter) of the 3 shafts (propeller shaft, intermediate shaft and main engine shaft) of the shafting system of every vessel. The fleet is presented below.

Vessel	DWT
ship 1	37000
ship 2	37000
ship 3	61000
ship 4	61000
ship 5	61462
ship 6	81714
ship 7	82000
ship 8	82000
ship 9	87144
ship 10	98681
ship 11	118000
ship 12	180000
ship 13	180274
ship 14	182511
ship 15	207939
ship 16	208000
ship 17	210000

Table 5.1: BULK CARRIERS FLEET

### 5.1 Dimension Analysis of Propulsion System

The lengths of the 3 shafts of the shafting system of each vessel are presented below.

Vessel	$L_{prop}$ (m)	$L_{int}$ (m)	$L_{M/E}$ (m)	$L_{tot}$ (m)	$\frac{L_{prop}}{L_{tot}}$	$\frac{L_{int}}{L_{tot}}$	$\frac{L_{mes}}{L_{tot}}$
ship 1	5.435	6.450	3.405	15.290	0.355	0.422	0.223
ship 2	6.140	6.450	3.405	15.995	0.384	0.403	0.213
ship 3	7.610	6.527	4.675	18.812	0.405	0.347	0.249
ship 4	6.550	5.950	3.800	16.300	0.402	0.365	0.233
ship 5	6.500	6.855	4.675	18.030	0.361	0.380	0.259
ship 6	6.557	6.290	5.411	18.258	0.359	0.345	0.296
ship 7	6.258	6.740	7.574	20.572	0.304	0.328	0.368
ship 8	6.175	5.754	5.406	17.335	0.356	0.332	0.312
ship 9	6.820	7.250	4.110	18.180	0.375	0.399	0.226
ship 10	6.437	7.077	5.411	18.925	0.340	0.374	0.286
ship 11	7.840	7.919	4.770	20.529	0.382	0.386	0.232
ship 12	8.842	8.456	6.309	23.607	0.375	0.358	0.267
ship 13	8.625	8.250	5.464	22.339	0.386	0.369	0.245
ship 14	8.415	9.135	4.770	22.320	0.377	0.409	0.214

<b>ship 15</b>	8.510	10.192	5.464	24.166	0.352	0.422	0.226
<b>ship 16</b>	8.485	8.600	5.464	22.549	0.376	0.381	0.242
<b>ship 17</b>	8.312	8.800	5.464	22.576	0.368	0.390	0.242
<b>AVERAGE</b>					0.368	0.377	0.255
<b>STANDARD DEVIATION</b>					0.023	0.028	0.040

Table 5.2: SHAFTS LENGTHS

- $L_{prop}$  is the length of the propeller shaft
- $L_{int}$  is the length of the intermediate shaft
- $L_{M/E}$  is the length of main engine shaft

The diameters of the 3 shafts of the shafting system of each vessel are presented below.

<b>Vessel</b>	<b><math>D_{prop}</math> (mm)</b>	<b><math>D_{int}</math> (mm)</b>	<b><math>D_{M/E}</math> (mm)</b>
<b>ship 1</b>	470	385	302
<b>ship 2</b>	450	365	302
<b>ship 3</b>	490	410	315
<b>ship 4</b>	500	415	315
<b>ship 5</b>	500	395	315
<b>ship 6</b>	550	450	389
<b>ship 7</b>	505	415	324
<b>ship 8</b>	540	445	389
<b>ship 9</b>	530	402	340
<b>ship 10</b>	570	470	389
<b>ship 11</b>	590	480	525
<b>ship 12</b>	660	570	454
<b>ship 13</b>	770	600	470
<b>ship 14</b>	670	570	447
<b>ship 15</b>	680	590	470
<b>ship 16</b>	695	560	470
<b>ship 17</b>	675	570	470

Table 5.3: DIAMETERS OF SHAFTS

- $D_{prop}$  is the diameter of propeller shaft
- $D_{int}$  is the diameter of intermediate shaft
- $D_{M/E}$  is the equivalent diameter of crankshaft

The lengths of the flanges of the shafting system of each vessel are presented below

<b>Vessel</b>	<b><math>L_{fl-prop}</math> (m)</b>	<b><math>L_{fl-int-aft}</math> (m)</b>	<b><math>L_{fl-int-fwd}</math> (m)</b>	<b><math>L_{fl-M/E}</math> (m)</b>
<b>ship 1</b>	0.12	0.1	0.103	0.065
<b>ship 2</b>	0.10	0.09	0.09	0.065
<b>ship 3</b>	0.08	0.08	0.08	0.065
<b>ship 4</b>	0.10	0.09	0.08	0.065
<b>ship 5</b>	0.10	0.09	0.08	0.065
<b>ship 6</b>	0.10	0.1	0.1	0.09
<b>ship 7</b>	0.09	0.085	0.085	0.065
<b>ship 8</b>	0.09	0.09	0.09	0.09
<b>ship 9</b>	0.12	0.12	0.09	0.15
<b>ship 10</b>	0.12	0.1	0.1	0.09
<b>ship 11</b>	0.12	0.1	0.1	0.095

ship 12	0.12	0.12	0.12	0.095
ship 13	0.15	0.12	0.12	0.01
ship 14	0.12	0.1	0.09	0.06
ship 15	0.12	0.12	0.12	0.01
ship 16	0.13	0.13	0.13	0.01
ship 17	0.06	0.06	0.12	0.12

Table 5.4: LENGTHS OF FLANGES

- $L_{fl-prop}$  is the length of flange of propeller shaft
- $L_{fl-int-aft}$  is the length of the rear flange of intermediate shaft
- $L_{fl-int-fwd}$  is the length of the forward flange of intermediate shaft
- $L_{fl-M/E}$  is the length of flange of main engine shaft

The diameters of the flanges of the shafting system of each vessel are presented below.

Vessel	$D_{fl-prop}$ (mm)	$D_{fl-int-aft}$ (mm)	$D_{fl-int-fwd}$ (mm)	$D_{fl-M/E}$ (mm)
ship 1	900	900	900	900
ship 2	800	800	885	885
ship 3	900	900	960	960
ship 4	820	820	960	960
ship 5	820	820	900	900
ship 6	950	950	1128	1128
ship 7	740	740	900	900
ship 8	870	870	1128	1128
ship 9	900	900	1100	1100
ship 10	990	990	1128	1128
ship 11	970	970	1200	1200
ship 12	985	985	1320	1320
ship 13	1020	1020	1320	1320
ship 14	880	880	900	900
ship 15	1120	1120	1400	1400
ship 16	1080	1080	1400	1400
ship 17	1240	1240	1395	1395

Table 5.5: DIAMETERS OF FLANGES

- $D_{fl-prop}$  is the diameter of flange of propeller shaft
- $D_{fl-int-aft}$  is the diameter of the rear flange of intermediate shaft
- $D_{fl-int-fwd}$  is the diameter of the forward flange of intermediate shaft
- $D_{fl-M/E}$  is the diameter of flange of main engine shaft

The ratios between flange lengths and shaft lengths of the propulsion system for each vessel are presented below.

Vessel	$L_{fl-prop} / L_{prop}$	$L_{fl-int-aft} / L_{int}$	$L_{fl-int-fwd} / L_{int}$	$L_{fl-M/E} / L_{M/E}$
ship 1	0.022	0.016	0.016	0.019
ship 2	0.016	0.014	0.014	0.019
ship 3	0.011	0.012	0.012	0.014
ship 4	0.015	0.015	0.013	0.017
ship 5	0.015	0.013	0.012	0.014
ship 6	0.015	0.016	0.016	0.017

ship 7	0.014	0.013	0.013	0.009
ship 8	0.015	0.016	0.016	0.017
ship 9	0.018	0.017	0.012	0.036
ship 10	0.019	0.014	0.014	0.017
ship 11	0.015	0.013	0.013	0.020
ship 12	0.014	0.014	0.014	0.015
ship 13	0.017	0.015	0.015	0.002
ship 14	0.014	0.011	0.010	0.013
ship 15	0.014	0.012	0.012	0.002
ship 16	0.015	0.015	0.015	0.002
ship 17	0.007	0.007	0.014	0.022
AVERAGE	0.015	0.014	0.014	0.015
STANDARD DEVIATION	0.003	0.002	0.002	0.008

Table 5.6: SHAFT-FLANGE LENGTH RATION

The ratios between flange diameters and shaft diameters of the propulsion system for each vessel are presented below.

Vessel	$D_{prop} / D_{fl-prop}$	$D_{int} / D_{fl-int-aft}$	$D_{int} / D_{fl-int-fwd}$	$D_{M/E} / D_{fl-M/E}$
ship 1	0.522	0.428	0.428	0.336
ship 2	0.563	0.456	0.412	0.341
ship 3	0.544	0.456	0.427	0.328
ship 4	0.610	0.506	0.432	0.328
ship 5	0.610	0.482	0.439	0.350
ship 6	0.579	0.474	0.399	0.345
ship 7	0.682	0.561	0.461	0.360
ship 8	0.621	0.511	0.395	0.345
ship 9	0.589	0.447	0.365	0.309
ship 10	0.576	0.475	0.417	0.345
ship 11	0.608	0.495	0.400	0.438
ship 12	0.670	0.579	0.432	0.344
ship 13	0.755	0.588	0.455	0.356
ship 14	0.761	0.648	0.633	0.497
ship 15	0.607	0.527	0.421	0.336
ship 16	0.644	0.519	0.400	0.336
ship 17	0.544	0.460	0.409	0.337
AVERAGE	0.617	0.506	0.431	0.355
STANDARD DEVIATION	0.066	0.057	0.055	0.044

Table 5.7: SHAFT-FLANGE DIAMETER RATIO

The ratios between shaft diameters and shaft lengths of the propulsion system for each vessel are presented below.

Vessel	$D_{prop} / L_{prop}$	$D_{int} / L_{int}$	$D_{M/E} / L_{M/E}$
ship 1	0.086	0.060	0.089
ship 2	0.073	0.057	0.089
ship 3	0.064	0.063	0.067

ship 4	0.076	0.070	0.083
ship 5	0.077	0.058	0.067
ship 6	0.084	0.072	0.072
ship 7	0.081	0.062	0.043
ship 8	0.087	0.077	0.072
ship 9	0.078	0.055	0.083
ship 10	0.089	0.066	0.072
ship 11	0.075	0.061	0.110
ship 12	0.075	0.067	0.072
ship 13	0.089	0.073	0.086
ship 14	0.080	0.062	0.094
ship 15	0.080	0.058	0.086
ship 16	0.082	0.065	0.086
ship 17	0.081	0.065	0.086
AVERAGE	0.080	0.064	0.080
STANDARD DEVIATION	0.006	0.006	0.014

Table 5.8: SHAFT DIAMETER-LENGTH RATIO

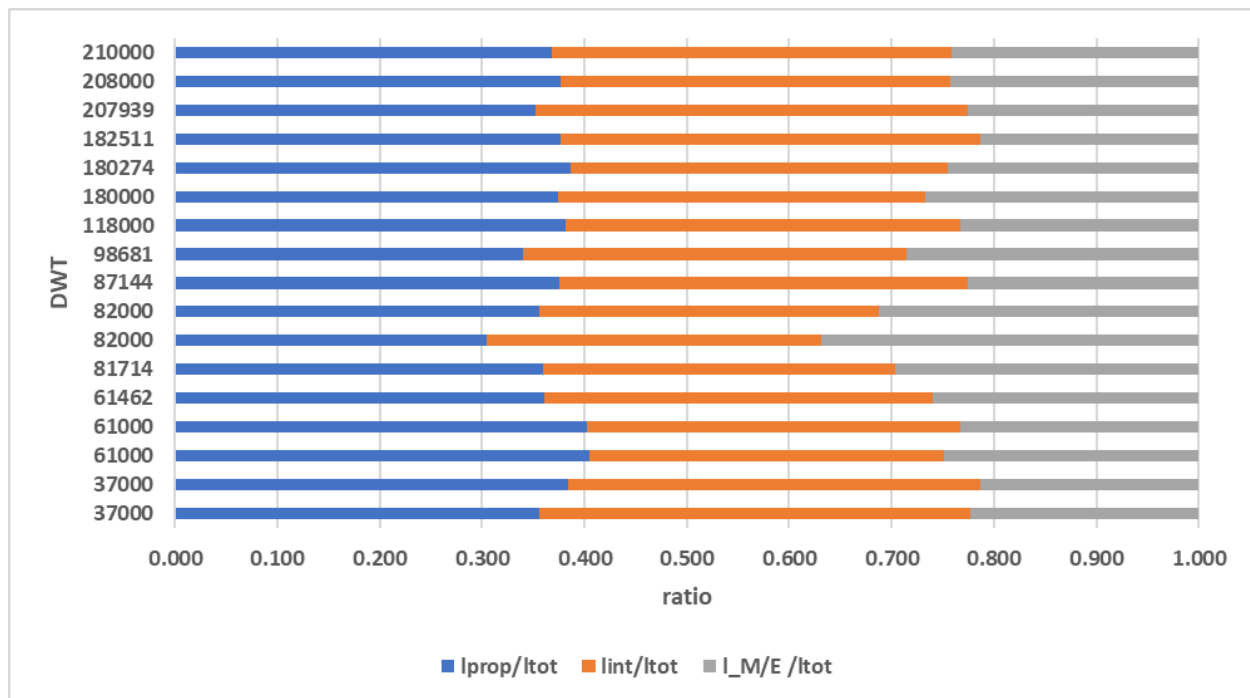


Figure 5.1: SHAFTS LENGTHS RATIOS



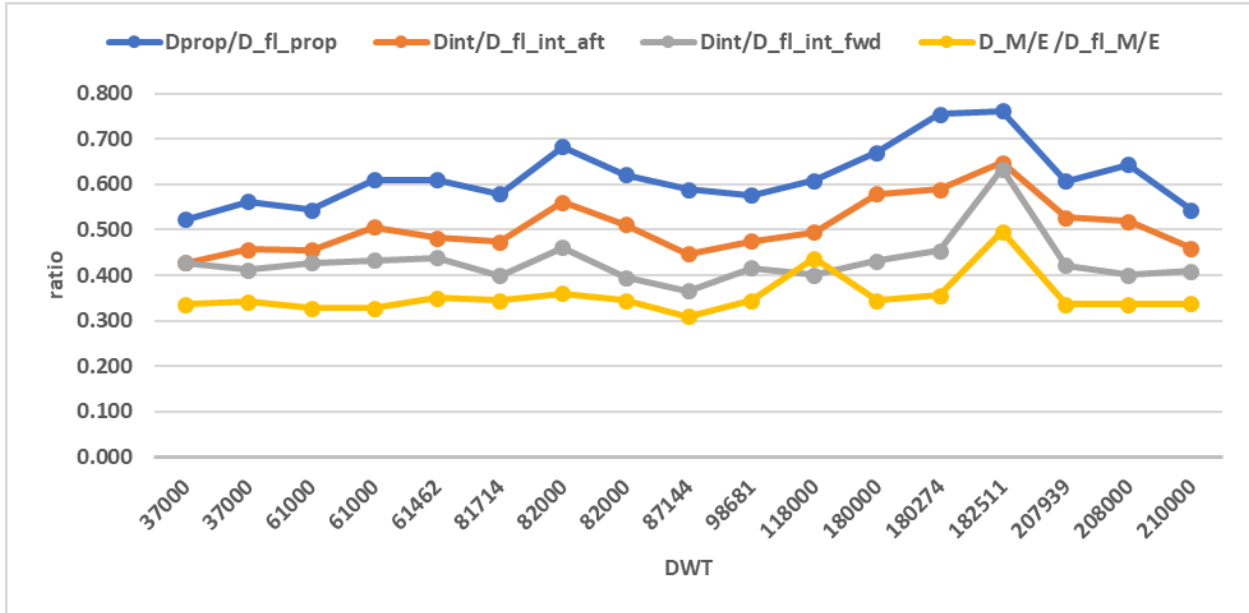


Figure 5.2: SHAFT-FLANGE DIAMETER RATIOS

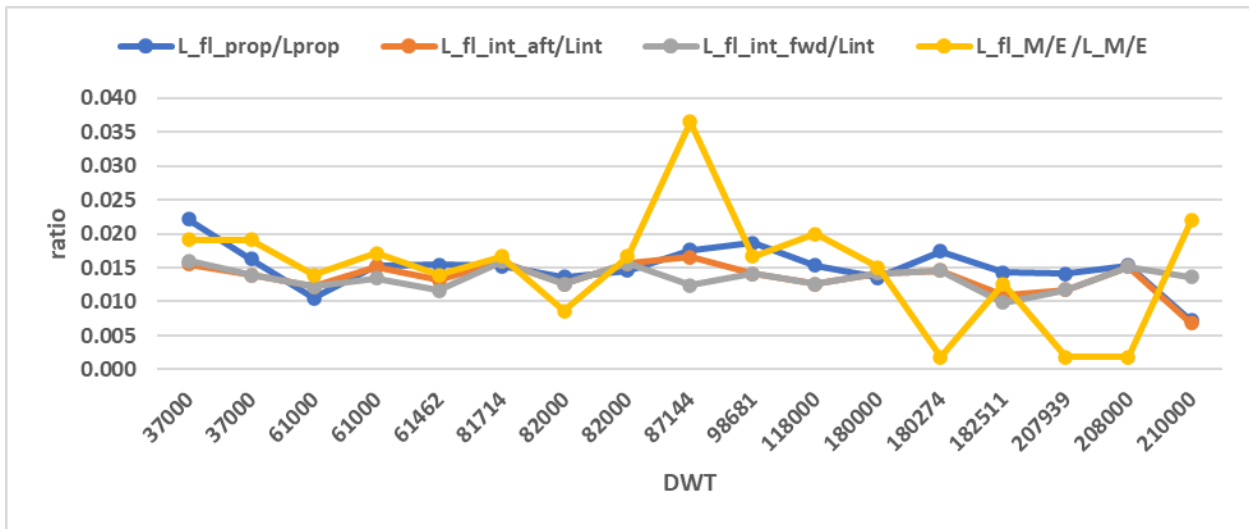


Figure 5.3: SHAFT-FLANGE LENGTH RATIOS

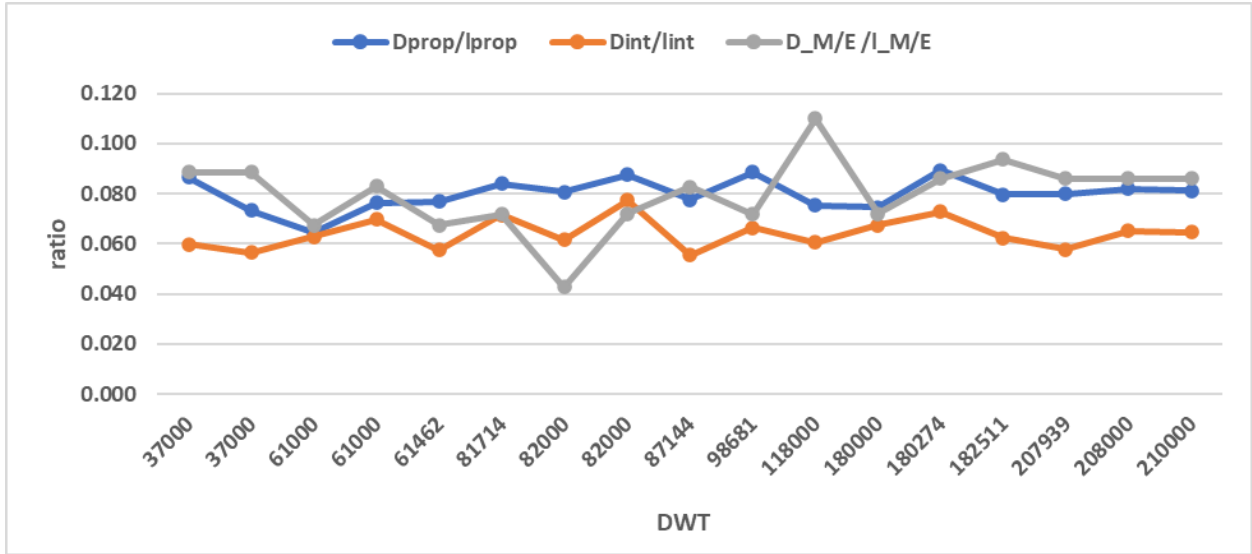


Figure 5.4: SHAFT DIAMETER-LENGTH RATIOS

## 6. Case Study

### 6.1 Principal Particulars of the Studied Vessel

In this current work, a typical 82000 DWT bulk carrier is studied. The principal particulars of the vessel under examination are shown on the following table. The studied vessel is under **American Bureau of Shipping** (ABS) classification.

Type	82000 DWT class bulk carrier
Length OA	229.02 m
Length BP	223.00 m
Breadth	32.25 m
Depth	20.10 m
Design Draft	12.20 m
Scantling Draft	14.50 m
Main Engine	Type: HYUNDAI B&W 7S50MC-C8
	MCR: 11620 kW at 127 rpm
	NCR: 10458 kW at 122.6 rpm
Propeller	Number of Blades: 4
	Material: Ni-Al-Bronze
	Diameter: 6200 mm
	Pitch (mean): 4101.8 mm
	Pitch ratio (mean): 0.6615
	Expanded area ratio: 0.5063
Keel Laid	2011

Table 6.1: PRINCIPAL PARTICULARS OF THE VESSEL

### 6.2 Characteristics of the Shafting System

The shafting system of the vessel is described on the following table in its three parts according to the shafting arrangement from ABS booklet (propeller shaft, intermediate shaft and main engine shaft).

Propeller Shaft	
Length (mm)	6258
Diameter (mm)	505
Number of Bearings	2
1) Aft Stern Tube Bearing (ASTB)	
Length (mm)	960
Diameter(mm)	505
Max permissible mean pressure	0.79 Mpa
2) Forward Stern Tube Bearing (FSTB)	
Length (mm)	290
Diameter(mm)	507
Max permissible mean pressure	0.79 Mpa
Intermediate Shaft	
Length (mm)	6740
Diameter (mm)	415
Number of Bearings	1
1) Intermediate Shaft Bearing (ISB)	
Length (mm)	280

Diameter(mm)	420
Max permissible mean pressure	1.47 Mpa
<b>Main Engine Shaft</b>	
Length (mm)	7574
Diameter until crankshaft (mm)	600
Diameter of equivalent crankshaft modeling (mm)	324
a) Number of bearings before crankshaft	2
1) Aftmost Bearing (#9)	
Length (mm)	194
Diameter(mm)	600
Max permissible mean pressure	2.91 Mpa
2) Aftmost M/E Bearing (#8)	
Length (mm)	97
Diameter(mm)	600
Max permissible mean pressure	2.91 Mpa
b) Number of Crankshaft bearings	7
M/E Bearings (#7 - #1)	
Length (mm)	194
Equivalent Diameter(mm)	324
Max permissible mean pressure	2.91 Mpa

Table 6.2: SHAFTING SYSTEM PROPERTIES

## 6.3 Input Data

### 6.3.1 Segments Properties

As it is discussed on section 4, in order to carry out the calculations on the “Shaft Alignment Tool”, the shaft should be divided into segments. This process is found on the respective booklet of ABS for the shafting system of the studied vessel. The geometrical and material characteristics of every segment is shown on the following table.

Segment Number	Distance (m)	Length (m)	Diameter (mm)	Inertia (m <sup>4</sup> )	Mass (kg)	Load Distribution (N/m)	Young Elasticity (N/m <sup>2</sup> )
1	0.05	0.05	340	0.000656	31	-6082.2	2.10E+11
2	0.079	0.029	360	0.000824	20	-6765.52	2.10E+11
3	0.16	0.081	360	0.000824	57	-6903.33	2.10E+11
4	0.28	0.12	360	0.000824	85	-6948.75	2.10E+11
5	0.3	0.02	345	0.000695	13	-6376.5	2.10E+11
6	0.91	0.61	449	0.001995	707	-11370	2.10E+11
7	1.42	0.51	479	0.002584	664	-12772.2	2.10E+11
8	1.707	0.287	505	0.003193	393	-13433.2	2.10E+11
9	1.807	0.1	505	0.003193	140	-13734	2.10E+11
10	1.837	0.03	505	0.003193	42	-13734	2.10E+11
11	2.157	0.32	505	0.003193	448	-13734	2.10E+11
12	2.797	0.64	505	0.003193	897	-13749.3	2.10E+11

13	2.827	0.03	505	0.003193	42	-13734	2.10E+11
14	4.257	1.43	505	0.003193	2004	-13747.7	2.10E+11
15	4.457	0.2	505	0.003193	281	-13783.1	2.10E+11
16	4.607	0.15	507	0.003243	211	-13799.4	2.10E+11
17	4.637	0.03	507	0.003243	42	-13734	2.10E+11
18	4.782	0.145	507	0.003243	204	-13801.7	2.10E+11
19	4.927	0.145	507	0.003243	204	-13801.7	2.10E+11
20	4.957	0.03	507	0.003243	42	-13734	2.10E+11
21	5.107	0.15	507	0.003243	211	-13799.4	2.10E+11
22	5.433	0.326	507	0.003243	516	-15527.5	2.10E+11
23	6.173	0.74	507	0.003243	972	-12885.6	2.10E+11
24	6.258	0.085	740	0.01472	286	-33007.8	2.10E+11
25	6.343	0.085	740	0.01472	286	-33007.8	2.10E+11
26	6.909	0.566	415	0.001456	600.75	-10412.3	2.10E+11
27	7.475	0.566	415	0.001456	600.75	-10412.3	2.10E+11
28	8.041	0.566	415	0.001456	600.75	-10412.3	2.10E+11
29	8.607	0.566	415	0.001456	600.75	-10412.3	2.10E+11
30	8.767	0.16	420	0.001527	174	-10668.4	2.10E+11
31	8.907	0.14	420	0.001527	152	-10650.9	2.10E+11
32	9.047	0.14	420	0.001527	152	-10650.9	2.10E+11
33	9.307	0.26	420	0.001527	282	-10640.1	2.10E+11
34	9.908	0.601	415	0.001456	638	-10413.9	2.10E+11
35	10.509	0.601	415	0.001456	638	-10413.9	2.10E+11
36	11.11	0.601	415	0.001456	638	-10413.9	2.10E+11
37	11.711	0.601	415	0.001456	638	-10413.9	2.10E+11
38	12.312	0.601	415	0.001456	638	-10413.9	2.10E+11
39	12.913	0.601	415	0.001456	638	-10413.9	2.10E+11
40	12.998	0.085	900	0.032206	424	-48934.6	2.10E+11
41	13.063	0.065	900	0.032206	321	-48446.3	2.10E+11
42	13.105	0.042	1060	0.061972	289	-67502.1	2.10E+11
43	13.295	0.19	600	0.006362	413	-21323.8	2.10E+11
44	13.392	0.097	600	0.006362	210	-21238.1	2.10E+11
45	13.489	0.097	600	0.006362	210	-21238.1	2.10E+11
46	13.646	0.157	600	0.006362	341	-21307.1	2.10E+11
47	13.752	0.106	1100	0.071869	786	-72742.1	2.10E+11
48	13.798	0.046	1100	0.071869	341	-72722	2.10E+11
49	13.836	0.038	1100	0.071869	316	-81577.9	2.10E+11
50	13.858	0.022	1230	0.112354	204	-90965.5	2.10E+11
51	14.05	0.192	600	0.006362	417	-21306.1	2.10E+11
52	14.147	0.097	600	0.006362	210	-21238.1	2.10E+11
53	14.244	0.097	324	0.000541	0	0	2.10E+11
54	14.572	0.328	324	0.000541	0	0	2.10E+11

55	14.9	0.328	324	0.000541	0	0	2.10E+11
56	14.997	0.097	324	0.000541	0	0	2.10E+11
57	15.094	0.097	324	0.000541	0	0	2.10E+11
58	15.422	0.328	324	0.000541	0	0	2.10E+11
59	15.75	0.328	324	0.000541	0	0	2.10E+11
60	15.847	0.097	324	0.000541	0	0	2.10E+11
61	15.944	0.097	324	0.000541	0	0	2.10E+11
62	16.272	0.328	324	0.000541	0	0	2.10E+11
63	16.6	0.328	324	0.000541	0	0	2.10E+11
64	16.697	0.097	324	0.000541	0	0	2.10E+11
65	16.794	0.097	324	0.000541	0	0	2.10E+11
66	17.122	0.328	324	0.000541	0	0	2.10E+11
67	17.45	0.328	324	0.000541	0	0	2.10E+11
68	17.547	0.097	324	0.000541	0	0	2.10E+11
69	17.644	0.097	324	0.000541	0	0	2.10E+11
70	17.972	0.328	324	0.000541	0	0	2.10E+11
71	18.3	0.328	324	0.000541	0	0	2.10E+11
72	18.397	0.097	324	0.000541	0	0	2.10E+11
73	18.494	0.097	324	0.000541	0	0	2.10E+11
74	18.822	0.328	324	0.000541	0	0	2.10E+11
75	19.15	0.328	324	0.000541	0	0	2.10E+11
76	19.247	0.097	324	0.000541	0	0	2.10E+11
77	19.344	0.097	324	0.000541	0	0	2.10E+11
78	19.672	0.328	324	0.000541	0	0	2.10E+11
79	20	0.328	324	0.000541	0	0	2.10E+11
80	20.097	0.097	324	0.000541	0	0	2.10E+11
81	20.194	0.097	324	0.000541	0	0	2.10E+11
82	20.572	0.378	324	0.000541	0	0	2.10E+11

Table 6.3: SEGMENT CHARACTERISTICS

Inertia is evaluated by the following formula:

$$I = \pi \cdot \frac{D^4}{64}$$

- D is the diameter of the segment

Distribution Load of each segment is estimated:

$$q = \frac{Mass}{Length}$$

According to the table above there are 82 segments. Due to the fact that the number of nodes is greater than the number of segments by one, the nodes are 83 in this case.

The crankshaft consists of segments 53 through 82. According to the engine's manufacturer the crankshaft is weightless for the calculations. Thus, the mass of those segments is zero.

### 6.3.2 Supporting Points

ABS suggests the longitudinal positions of the bearing supports for the shaft modeling. The longitudinal position of the support of ASTB is at the one third of its effective length. The longitudinal position of the support of FSTB is at the half of its effective length. The longitudinal position of the support of ISB is at the half of its effective length.

The engine manufacturer suggests the longitudinal positions of the main engine bearing supporting points. The longitudinal position of the supporting points of main engine bearings are at the half of their respective lengths.

The following table summarizes the longitudinal position of each bearing support.

Description	Node number	Distance to the rear end of the shaft (m)
ASTB	12	2.157
FSTB	19	4.782
ISB	32	8.907
Aftmost Bearing (#9)	45	13.392
Aftmost M/E Bearing (#8)	53	14.147
M/E Bearing. (#7)	57	14.997
M/E Bearing. (#6)	61	15.847
M/E Bearing. (#5)	65	16.697
M/E Bearing. (#4)	69	17.547
M/E Bearing. (#3)	73	18.397
M/E Bearing. (#2)	77	19.247
M/E Bearing. (#1)	81	20.097

Table 6.4: LONGITUDINAL POSITION OF SUPPORTS

### 6.3.3 External Loads

The external loads applied on the shaft are radial loads that are delivered by journal bearings described on “section 5.2”. Specifically, the external loads are: propeller load, nut & bonnet weight, mass of flywheel, chain tension minus mass of chain wheel and moving masses (including crank throw) through crankshaft length. These loads are steady and independent of the fact the vessel is seagoing or anchored or dry docked. Due to the fact that the studied vessel is operating, which means the main engine is on running condition, the propeller produces thrust, which is eccentric to the shaft line, causing additional bending moment to the shaft. The loads and bending moment are on the ABS booklet of the studied vessel and are presented below.

#### Propeller load

In this work, four different scenarios of propeller immersion are studied (propeller in air, half immersed, 75% immersed and fully immersed propeller). The vertical load is applied in the center of gravity of the propeller and is different for each scenario. The vertical load is estimated:

$$\text{Vertical load} = \text{Propeller weight} - \text{propeller buoyancy}$$

Propeller buoyancy depends on the percentage the propeller is immersed. Propeller buoyancy is estimated:

$$\text{Propeller buoyancy} = \text{density}_{\text{water}} \cdot \text{immersed volume of propeller}$$

Similarly, the vertical load of bonnet is calculated.

The vertical loads and bending moment due to thrust eccentricity, which are estimated in shaft alignment calculations booklet for the propeller and bonnet are shown on the following tables

Propeller immersion	Fy (kg)	Fy (N)	Mz (Nm)	Node	Distance to the rear end of the shaft (m)
0%	-16838	-165124	117700	7	0.91
50%	-15703	-153994	117700	7	0.91
75%	-15135	-148424	117700	7	0.91
100%	-14567	-142853	117700	7	0.91

Table 6.5: PROPELLER VERTICAL LOAD

Bonnet immersion	Fy (kg)	Fy (N)	Node	Distance to the rear end of the shaft (m)
0%	-309	-3030	3	0.079
50%	-213	-2089	3	0.079
75%	-165.5	-1623	3	0.079
100%	-118	-1157	3	0.079

Table 6.6: BONNET VERTICAL LOAD

## Other external loads

The rest of the external loads applied on the shaft (nut, flywheel mass, chain tension and moving masses of the crankshaft) are independent of the immersion condition. According to the shaft alignment calculations booklet of the studied vessel these loads are shown on the table below.

Description	Fy (N)	Node	Distance to the rear end of the shaft (m)
Nut	-4521	4	0.16
Flywheel	-27674	42	13.063
Chain Tension minus mass of Chain Wheel	67300	48	13.752
Moving Mass (#7)	-91400	55	14.572
Moving Mass (#6)	-91400	59	15.422
Moving Mass (#5)	-91400	63	16.272
Moving Mass (#4)	-91400	67	17.122
Moving Mass (#3)	-91400	71	17.972
Moving Mass (#2)	-91400	75	18.822
Moving Mass (#1)	-91400	79	19.672



Table 6.7: OTHER LOADS APPLIED ON THE SHAFT

The input data are visualized on the detailed model of the shafting system below. The model consists of 82 elements and 83 nodes.

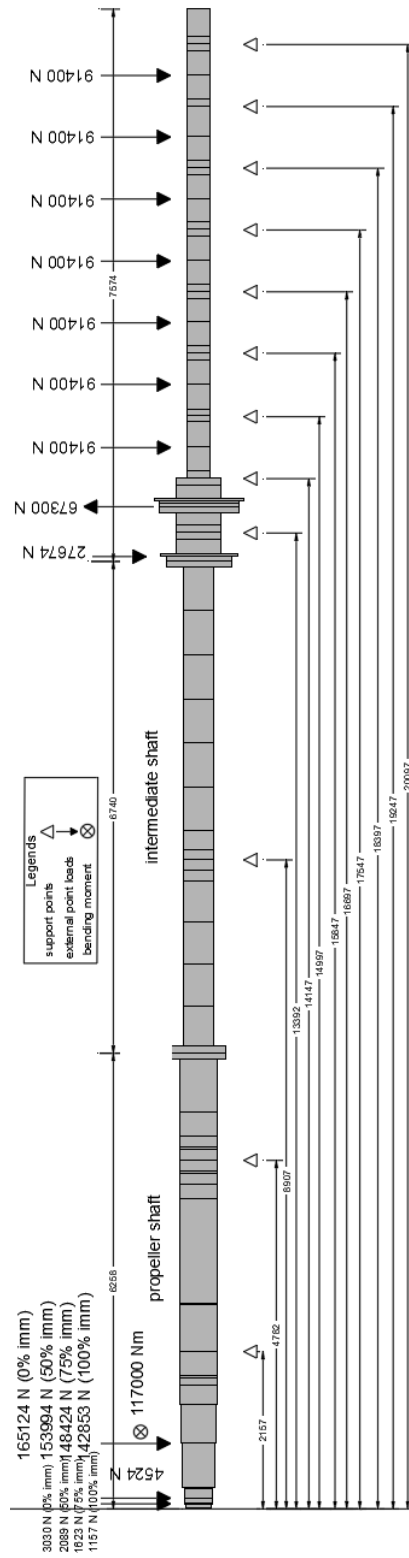


Figure 6.1: DETAILED SHAFTING SYSTEM MODEL

## 6.4 ABS Processing vs Shaft Alignment Tool Software

The first target is the comparison of the results produced by ABS with the results produced by “shaftAlignment\_bending\_momenst.exe” software and figure out if the “.exe” is reliable or not for the continuation of this work. The results from ABS are presented on the shaft alignment calculations booklet of the studied vessel. It is noted that the calculations are carried out for the case of fully immersed propeller.

In order the shaft alignment calculations take place, bearing offsets should be defined. In this study, the vessel is operating, which means the main engine is on running condition. Due to its operation, thermal expansions are caused, which affect the vertical offsets of main engine bearings. Specifically, the manufacturer of the main engine of the studied vessel states: “the thermal expansion of the engine structure, i.e from the engine seating to the center of the main bearing is 0.24 mm when the engine temperature is raised from cold (20<sup>0</sup> C) to normal running temperature (55<sup>0</sup>).”

Taking the above under consideration ABS suggests the vertical positions of the bearings. These are the initial offsets, which are used for the later part of this work.

Description	Node number	Offset (mm)
ASTB	12	0.00
FSTB	19	0.00
ISB	32	-3.88
Aftmost Bearing (#9)	45	-10.52
Aftmost M/E Bearing (#8)	53	-11.61
M/E Bearing. (#7)	57	-12.85
M/E Bearing. (#6)	61	-14.08
M/E Bearing. (#5)	65	-15.31
M/E Bearing. (#4)	69	-16.54
M/E Bearing. (#3)	73	-17.77
M/E Bearing. (#2)	77	-19.00
M/E Bearing. (#1)	81	-20.23

Table 6.8: BEARING OFFSETS (ABS PROCESSING)

### 6.4.1 Influence Factors

The number of supporting points is 12. Thus, the influence factors consist of a 12x12 matrix. The comparison takes place only for one quarter of the matrix (top left 6x6 matrix), because there is not information about the other 3 quarters of the respective matrix of ABS booklet. The following tables summarize the comparison of influence factors.

kN/mm	ASTB			FSTB		
	NTUA	ABS	DIFFERENCE	NTUA	ABS	DIFFERENCE
ASTB	-41.9	-39.6	-2.3	74.7	71.0	3.7
FSTB	74.7	71.0	3.7	-140.8	-134.7	-6.1
ISB	-41.9	-40.5	-1.4	93.4	91.3	2.1
Aftmost Bearing (#9)	29.3	29.7	-0.4	-88.5	-89.9	1.4

<b>Aftmost M/E Bearing (#8)</b>	-20.4	-20.9	<b>0.5</b>	61.5	63.2	<b>-1.6</b>
<b>M/E Bearing. (#7)</b>	0.1	0.3	<b>-0.2</b>	0.4	-0.9	<b>1.3</b>

Table 6.9: COMPARISON OF INFLUENCE FACTORS (PT.1)

kN/mm	ISB			Aftmost Bearing (#9)		
	NTUA	ABS	DIFFERENCE	NTUA	ABS	DIFFERENCE
<b>ASTB</b>	-41.9	-40.5	<b>-1.4</b>	29.3	29.7	<b>-0.4</b>
<b>FSTB</b>	93.4	91.3	<b>2.1</b>	-88.5	-89.9	<b>1.4</b>
<b>ISB</b>	-100.9	-100.7	<b>-0.2</b>	207.2	210.8	<b>-3.6</b>
<b>Aftmost Bearing (#9)</b>	207.2	210.8	<b>-3.6</b>	-1473.3	-1512.6	<b>39.3</b>
<b>Aftmost M/E Bearing (#8)</b>	-158.7	-162.9	<b>4.2</b>	2042.1	2113.3	<b>-71.2</b>
<b>M/E Bearing. (#7)</b>	1.0	2.4	<b>-1.4</b>	-867.4	-918.9	<b>51.5</b>

Table 6.10: COMPARISON OF INFLUENCE FACTORS (PT.2)

kN/mm	Aftmost M/E Bearing (#8)			M/E Bearing. (#7)		
	NTUA	ABS	DIFFERENCE	NTUA	ABS	DIFFERENCE
<b>ASTB</b>	-20.4	-20.9	<b>0.5</b>	0.1	0.3	<b>-0.2</b>
<b>FSTB</b>	61.5	63.2	<b>-1.6</b>	-0.4	-0.9	<b>0.5</b>
<b>ISB</b>	-158.7	-162.9	<b>4.2</b>	1.0	2.4	<b>-1.3</b>
<b>Aftmost Bearing (#9)</b>	2042.1	2113.3	<b>-71.2</b>	-867.4	-918.9	<b>51.5</b>
<b>Aftmost M/E Bearing (#8)</b>	-3678.2	-3826.9	<b>148.7</b>	2392.7	2519.4	<b>-126.7</b>
<b>M/E Bearing. (#7)</b>	2392.7	2519.4	<b>-126.7</b>	-2715.7	-2850.7	<b>135.0</b>

Table 6.11: COMPARISON OF INFLUENCE FACTORS (PT.3)

#### 6.4.2 Bearing Reactions

The detailed comparison between results of NTUA and ABS takes place not only for influence factors, but for reaction forces as well. This comparison refers to the condition, which the propeller is fully immersed and the vessel operates.

Bearing	Bearing Reaction (kN)		
	NTUA	ABS	DIFFERENCE
<b>ASTB</b>	245.4	250	<b>-4.6 kN</b>
<b>FSTB</b>	27.2	21	<b>6.2 kN</b>
<b>ISB</b>	25.28	28	<b>-2.72 kN</b>
<b>Aftmost Bearing (#9)</b>	22.45	24	<b>-1.55 kN</b>
<b>Aftmost M/E Bearing (#8)</b>	33.76	31	<b>2.76 kN</b>
<b>M/E Bearing. (#7)</b>	95.67	94	<b>1.67 kN</b>
<b>M/E Bearing. (#6)</b>	97.44	93	<b>4.44 kN</b>
<b>M/E Bearing. (#5)</b>	89.98	91	<b>-1.02 kN</b>
<b>M/E Bearing. (#4)</b>	92.4	93	<b>-0.6 kN</b>
<b>M/E Bearing. (#3)</b>	87.83	89	<b>-1.17 kN</b>
<b>M/E Bearing. (#2)</b>	108.1	108	<b>0.1 kN</b>
<b>M/E Bearing. (#1)</b>	31.87	33	<b>-1.13 kN</b>

Table 6.12: COMPARISON OF REACTION FORCES (NTUA VS ABS)

## 7. Simulations

The main part of the current work is to run simulations through “*Shaft\_Align\_Bending\_Moments.exe*” for various scenarios of vertical offsets and propeller immersions for the shafting system, which its detailed modeling is prescribed on sections “6.2 Characteristics of the Shafting System” and “6.3 Input Data”. These scenarios are evaluated and classified relatively to bearing reaction loads and their respective allowable limits. Specifically, ABS [2] states: “An alignment condition is acceptable as long as the bearing reactions remain positive under all service drafts, and no bearing is unloaded. In general, any positive static load is therefore acceptable. For practical reasons and to prevent unloading or overloading of the bearings due to unaccounted for disturbances, it is preferred at least 10% of the allowable bearing load is desired on each bearing and measured bearing reactions may not exceed 80% of the manufacturer’s maximum allowable load limit”. Due to the huge number of simulations, a script in MATLAB© is developed for the creation of input files, execution of them via “*Shaft\_Align\_Bending\_Moments.exe*” and save the results (vertical positions, reaction forces, bending moments, etc.) on output files. Specifically, the script adjusts the propeller load and the offsets of the input file, then calls the software for run and at last saves the results on a “.txt” file (output file). The process finishes, when all the possible scenarios are executed. Then, another script is created for the classification of the scenarios according to ABS regulations.

Taking ABS rule for bearing loads under account, the scenarios are classified into 3 categories. Scenarios which belong to “*acceptable*” category, scenarios which belong to “*marginal*” category and scenarios which belong to “*not acceptable*” category. The definitions of “acceptable”, “marginal” and “not acceptable” are:

- If there is a scenario, in which all bearing reactions are greater or equal to 10% of their respective maximum allowable load limits and less than 80% of their maximum allowable load limits, then the scenario belongs to “*acceptable*” category.
- If there is a scenario, in which all bearing reactions are positive and not exceed their respective maximum allowable load limits and at least one bearing reaction is less than 10% of its respective maximum allowable load limit or greater than 80% of its respective maximum allowable load limit, then the scenario belongs to “*marginal*” category.
- If there is a scenario, in which at least one bearing reaction is negative or exceed its respective maximum allowable load limit, then the scenario belongs to “*not acceptable*” category.

### 7.1 Allowable Bearing Load Limits

Generally, the vertical load carried by a journal bearing is estimated:

$$W = p_m \cdot A = p_m \cdot D_s \cdot L_b$$

Which,

- $P_m$  is the mean pressure of the pressure distribution of the lubricant film between the journal and the bushing, which is caused by the rotation of the shaft
- $A = D_s \cdot L_b$  is the projected area of the bearing
- $D_s$  is the diameter of the shaft
- $L_b$  is the effective length of the bearing

Maximum allowable mean pressure, diameter of the shaft and effective length for each bearing are shown on the following table

Description	Max $p_m$ (Pa)	$D_s$ (mm)	$L_b$ (mm)
ASTB	0.79	505	960
FSTB	0.79	507	290
ISB	1.47	420	280
Aftmost Bearing (#9)	2.91	600	194
Aftmost M/E Bearing (#8)	2.91	600	97
M/E Bearings (#7 - #1)	2.91	324	194

Table 7.1: MAXIMUM ALLOWABLE MEAN PRESSURE, SHAFT DIAMETER AND EFFECTIVE LENGTH OF BEARINGS

Knowing maximum  $p_m$ ,  $D_s$ ,  $L_b$ , the allowable load limits for each bearing can be calculated. The results are summarized below.

Description	A (mm <sup>2</sup> )	$W_{max}$ (kN)	10% $W_{max}$ (kN)	80% $W_{max}$ (kN)
ASTB	484800	383.0	38.3	306.4
FSTB	147030	116.2	11.6	92.9
ISB	117600	172.9	17.3	138.3
Aftmost Bearing (#9)	116400	338.7	33.9	271.0
Aftmost M/E Bearing (#8)	58200	169.4	16.9	135.5
M/E Bearings (#7 - #1)	62856	182.9	18.3	146.3

Table 7.2: ALLOWABLE BEARING LOAD LIMITS

## 7.2 Preliminary Approach

For the first set of simulations the following assumption was adopted. The offset of ASTB is fixed at 0mm, which is its respective “initial” offset. This assumption was made, because when the reference line is established (sighting through), the laser is passing through the center line of ASTB. The offsets of the rest bearings before crankshaft (FSTB, ISB, Aftmost Bearing (#9) and Aftmost M/E Bearing (#8)) can deviate 0,  $\pm 1$ mm,  $\pm 2$ mm,  $\pm 3$ mm,  $\pm 5$ mm,  $\pm 7$ mm or  $\pm 8$ mm from their respective initial offsets (see “Table 6.8: BEARING OFFSETS (ABS PROCESSING)”). As for the rest of the bearings, which are the bearings of the crankshaft, it is assumed that their offsets are linear related to offsets of Aftmost Bearing (#9) and Aftmost M/E Bearing (#8). The later assumption was made, because the crankshaft of the main engine is a stiff region due to its complicated geometry. Thus, the deflection curve at this part is linear similar to the deflection curve of the initial condition (see **APPENDIX A**).

The possible new offsets for the first five bearings are presented below.

- ✓ ASTB
  - Initial offset: 0mm
  - Deviation: -
  - New offset: 0mm

- ✓ FSTB

- Initial offset: 0mm
  - Deviation: -8mm, -7mm-5mm, -3mm, -2mm, -1mm, 0mm, 1mm, 2mm, 3mm, 5mm, 7mm, 8mm
  - New offset: -8mm, -7mm, -5mm, -3mm, -2mm, -1mm, 0mm, 1mm, 2mm, 3mm, 5mm, 7mm, 8mm
- ✓ ISB
- Initial offset: -3.88mm
  - Deviation: -8mm, -7mm-5mm, -3mm, -2mm, -1mm, 0mm, 1mm, 2mm, 3mm, 5mm, 7mm, 8mm
  - New offset: -11.88mm, 10.88, -8.88mm, -6.88mm, -5.88mm, -4.88mm, -3.88mm, -2.88mm, -1.88mm, -0.88mm, 2.88mm, 3.12mm, 4.12mm
- ✓ Aftmost Bearing (#9)
- Initial offset: -10.52mm
  - Deviation: -8mm, -7mm-5mm, -3mm, -2mm, -1mm, 0mm, 1mm, 2mm, 3mm, 5mm, 7mm, 8mm
  - New offset: -18.52mm, -17.52mm, -15.52mm, -13.52mm, -12.52mm, -11.52mm, -10.52mm, -9.52mm, -8.52mm, -7.52mm, -5.52mm, -3.52mm, -2.52mm
- ✓ Aftmost M/E Bearing (#8)
- Initial offset: -11.61mm
  - Deviation: -8mm, -7mm, -5mm, -3mm, -2mm, -1mm, 0mm, 1mm, 2mm, 3mm, 5mm, 7mm, 8mm
  - New offset: -19.61mm, -18.61mm -16.61mm, -14.61mm, -13.61mm, -12.61mm, -11.61mm, -10.61mm, -9.61mm, -8.61mm, -6.61mm, -4.61mm, -3.61mm

The combinations of possible vertical offsets take place for all cases of propeller immersion (0, 50%, 75%, 100%). Considering that there are 4 bearings with non-fixed offset, 13 possible offsets for each of them and 4 cases of propeller immersion, the scenarios, which should be executed are:

$$s = k \cdot p^q = 4 \cdot 13^4 = 114244$$

Which,

- ✓ k is the number of cases of propeller immersion
- ✓ q is the number of non-fixed offset bearings
- ✓ p is the number of possible offset values for each of the q bearings

Out of these cases, 0 were classified as “acceptable”, 11 were classified as “marginal” and the rest were classified as “not acceptable”. Only 3 unique offsets combinations are detected on these marginal occasions. More details are in the following section.

The unique offsets combinations presented with the format:

“ASTB offset deviation, FSTB offset deviation, ISB offset deviation, Aftmost Bearing (#9) offset deviation, Aftmost M/E Bearing (#8) offset deviation & propeller immersion condition”

### 7.2.1 Offset Deviations Combinations

The reaction forces of the bearings of each marginal case are presented on the following tables. If a reaction force is less than 10% of its maximum load, the value is highlighted.

	<b>0 0 0 0</b> (0% prop immersion)	<b>0 0 0 0</b> (50% prop immersion)	<b>0 0 0 0</b> (75% prop immersion)	<b>0 0 0 0</b> (100% prop immersion)
ASTB	283.7 kN	264.5 kN	254.9 kN	245.3 kN
FSTB	<b>11.2 kN</b>	19.2 kN	23.2 kN	27.2 kN
ISB	27.6 kN	26.6 kN	26.0 kN	25.5 kN
Aftmost Bearing (#9)	<b>16.3 kN</b>	<b>17.1 kN</b>	<b>17.5 kN</b>	<b>17.9 kN</b>
Aftmost M/E Bearing (#8)	43.9 kN	43.3 kN	43.1 kN	42.8 kN
M/E Bearings (#7)	79.0 kN	79.0 kN	79.0 kN	79.0 kN
M/E Bearings (#6)	102.9 kN	102.9 kN	102.9 kN	102.9 kN
M/E Bearings (#5)	85.1 kN	85.1 kN	85.1 kN	85.1 kN
M/E Bearings (#4)	98.2 kN	98.2 kN	98.2 kN	98.2 kN
M/E Bearings (#3)	76.9 kN	76.9 kN	76.9 kN	76.9 kN
M/E Bearings (#2)	118.1 kN	118.1 kN	118.1 kN	118.1 kN
M/E Bearings (#1)	28.6 kN	28.6 kN	28.6 kN	28.6 kN
STATUS	<b>MARGINAL</b>	<b>MARGINAL</b>	<b>MARGINAL</b>	<b>MARGINAL</b>

Table 7.3: “0 0 0 0 0”

	<b>0 -1 -2 -2 -2</b> (0% prop immersion)	<b>0 -1 -2 -2 -2</b> (50% prop immersion)	<b>0 -1 -2 -2 -2</b> (75% prop immersion)	<b>0 -1 -2 -2 -2</b> (100% prop immersion)
ASTB	292.8 kN	273.6 kN	264.0 kN	254.4 kN
FSTB	<b>2.6 kN</b>	<b>10.6 kN</b>	14.6 kN	18.6 kN
ISB	17.9 kN	<b>16.8 kN</b>	<b>16.3 kN</b>	<b>15.7 kN</b>
Aftmost Bearing (#9)	46.2 kN	47.0 kN	47.4 kN	47.8 kN
Aftmost M/E Bearing (#8)	23.1 kN	22.5 kN	22.3 kN	22.0 kN
M/E Bearings (#7)	79.1 kN	79.1 kN	79.1 kN	79.1 kN
M/E Bearings (#6)	102.9 kN	102.9 kN	102.9 kN	102.9 kN
M/E Bearings (#5)	85.1 kN	85.1 kN	85.1 kN	85.1 kN
M/E Bearings (#4)	98.2 kN	98.2 kN	98.2 kN	98.2 kN



M/E Bearings (#3)	76.9 kN	76.9 kN	76.9 kN	76.9 kN
M/E Bearings (#2)	118.1 kN	118.1 kN	118.1 kN	118.1 kN
M/E Bearings (#1)	28.6 kN	28.6 kN	28.6 kN	28.6 kN
STATUS	<b>MARGINAL</b>	<b>MARGINAL</b>	<b>MARGINAL</b>	<b>MARGINAL</b>

Table 7.4: "0 -1 -2 -2 -2"

	<b>0 3 7 8 8</b> (50% prop immersion)	<b>0 3 7 8 8</b> (75% prop immersion)	<b>0 3 7 8 8</b> (100% prop immersion)
ASTB	261.0 kN	251.4 kN	241.8 kN
FSTB	6.2 kN	10.2 kN	14.2 kN
ISB	57.9 kN	57.3 kN	56.8 kN
Aftmost Bearing (#9)	16.5 kN	16.9 kN	17.2 kN
Aftmost M/E Bearing (#8)	29.0 kN	28.8 kN	28.5 kN
M/E Bearings (#7)	79.1 kN	79.1 kN	79.1 kN
M/E Bearings (#6)	102.9 kN	102.9 kN	102.9 kN
M/E Bearings (#5)	85.1 kN	85.1 kN	85.1 kN
M/E Bearings (#4)	98.2 kN	98.2 kN	98.2 kN
M/E Bearings (#3)	76.9 kN	76.9 kN	76.9 kN
M/E Bearings (#2)	118.1 kN	118.1 kN	118.1 kN
M/E Bearings (#1)	28.6 kN	28.6 kN	28.6 kN
STATUS	<b>MARGINAL</b>	<b>MARGINAL</b>	<b>MARGINAL</b>

Table 7.5: "0 3 7 8 8"

Specifically, the actual offsets of the bearings from the 3 different combinations of offset deviations are presented below:

- "0mm, 0mm, 0mm, 0mm, 0mm"

- Offsets (mm)

ASTB	0.00
FSTB	0.00
ISB	-3.88
Aftmost Bearing (#9)	-10.52
Aftmost M/E Bearing (#8)	-11.61
M/E Bearings (#7)	-12.84
M/E Bearings (#6)	-14.06
M/E Bearings (#5)	-15.29
M/E Bearings (#4)	-16.52
M/E Bearings (#3)	-17.75
M/E Bearings (#2)	-18.97
M/E Bearings (#1)	-20.20

- **“0mm, -1mm, -2mm, -2mm, -2mm”**

- Offsets (mm)

ASTB	0.00
FSTB	-1.00
ISB	-5.88
Aftmost Bearing (#9)	-12.52
Aftmost M/E Bearing (#8)	-13.61
M/E Bearings (#7)	-14.84
M/E Bearings (#6)	-16.06
M/E Bearings (#5)	-17.29
M/E Bearings (#4)	-18.52
M/E Bearings (#3)	-19.75
M/E Bearings (#2)	-20.97
M/E Bearings (#1)	-22.20

- **“0mm,3mm,7mm,8mm,8mm”**

- Offsets (mm)

ASTB	0.00
FSTB	3.00
ISB	3.12
Aftmost Bearing (#9)	-2.52
Aftmost M/E Bearing (#8)	-3.61
M/E Bearings (#7)	-4.84
M/E Bearings (#6)	-6.06
M/E Bearings (#5)	-7.29
M/E Bearings (#4)	-8.52
M/E Bearings (#3)	-9.75
M/E Bearings (#2)	-10.97
M/E Bearings (#1)	-12.20

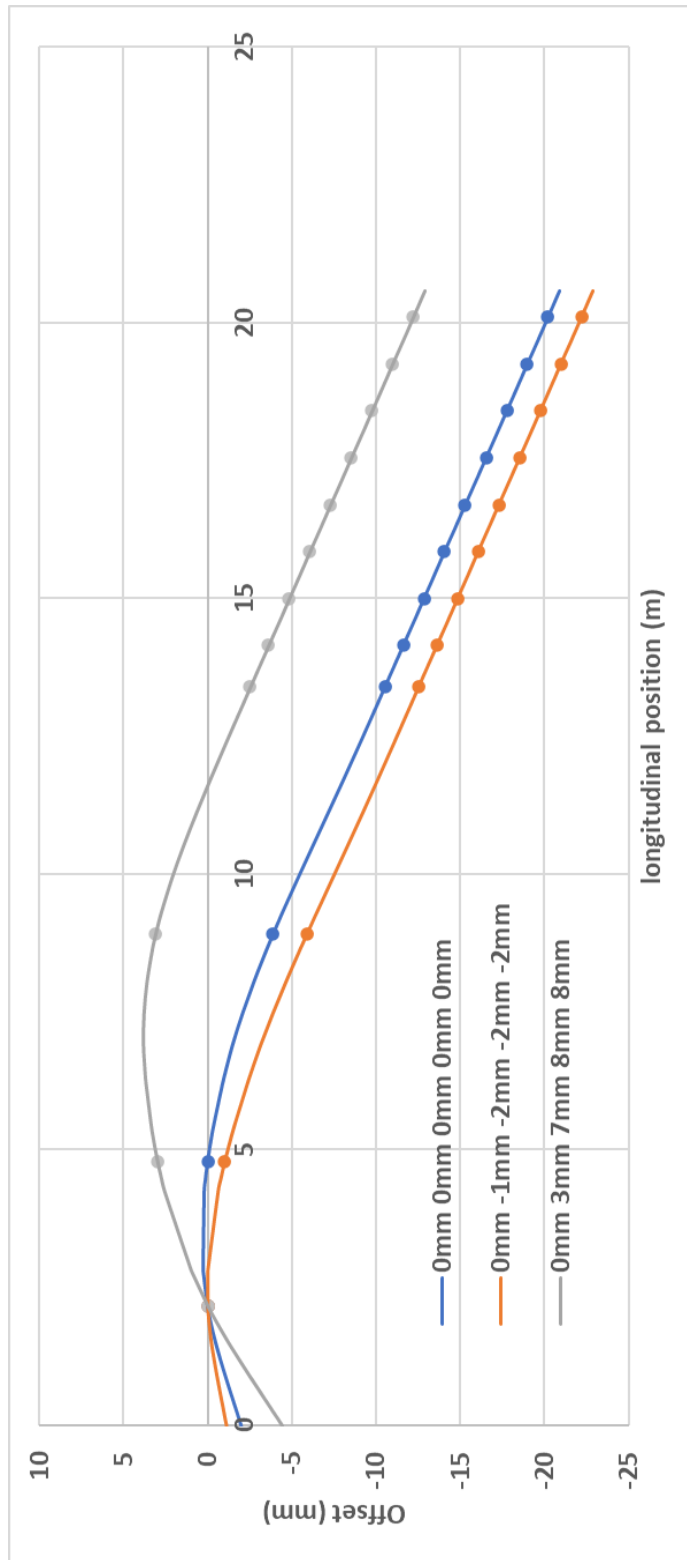


Figure 7.6: DEFLECTION CURVES

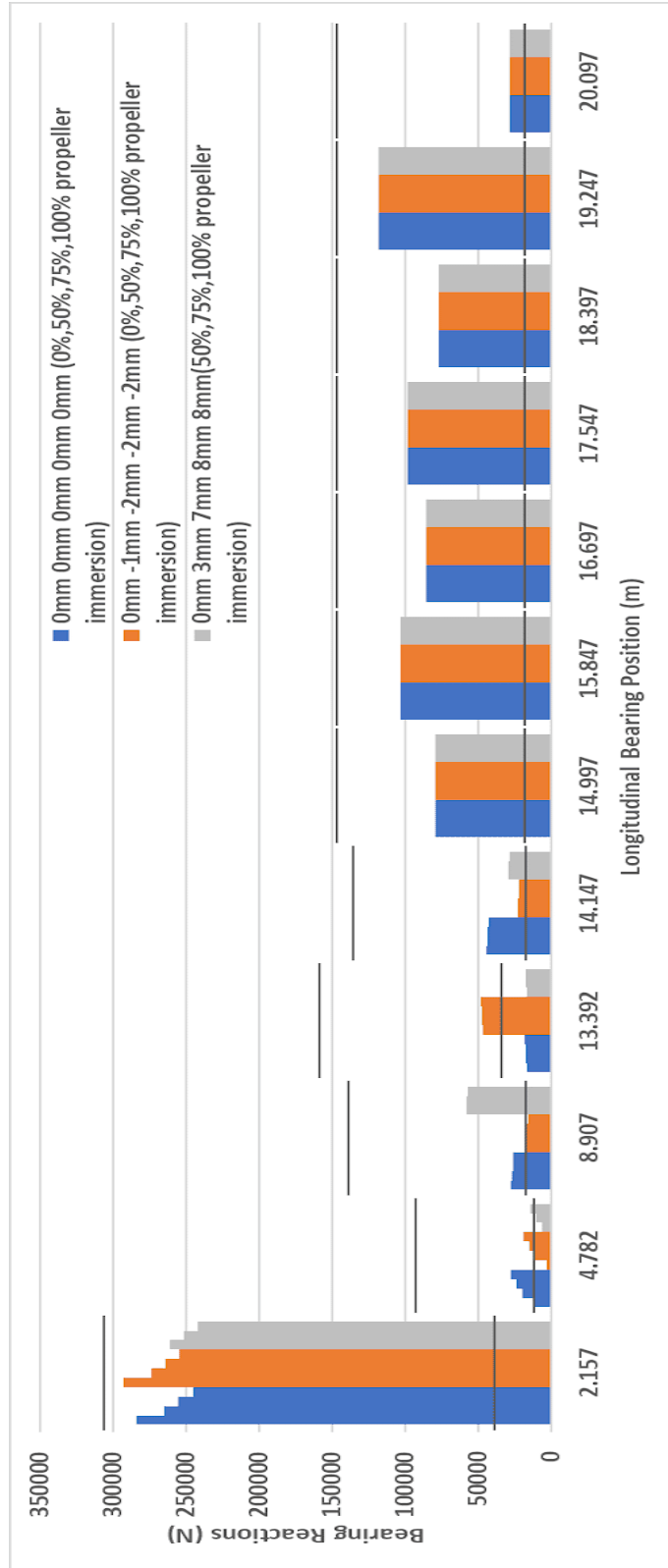


Figure 7.7: REACTION FORCES AND BOUNDARIES

Due to the limited number of unique offsets combinations of “marginal” occasions, a second approach is needed to be made based on the above offsets in order to find “acceptable” and more “marginal” occasions. The above offsets are defined as “basis” offsets for the second approach. More details are developed in the following section.

### 7.3 In Depth Approach

For the second set of simulations the following assumptions were made. The offset of ASTB is fixed at 0mm, which is its respective “basis” offset (see section “7.2 Initial Approach”). The offsets of the rest bearings before crankshaft (FSTB, ISB, Aftmost Bearing (#9) and Aftmost M/E Bearing (#8)) can deviate  $\pm 0.1\text{mm}$ ,  $\pm 0.2\text{mm}$ ,  $\pm 0.3\text{mm}$ ,  $\pm 0.5\text{mm}$  or  $\pm 0.8\text{mm}$  from their respective basis offsets. As for bearings of the crankshaft, it is assumed that their offsets are linear related to offsets of Aftmost Bearing (#9) and Aftmost M/E Bearing (#8) just as the first approach. These assumptions were developed for the 3 unique offsets combinations of the first set of simulations.

According to these assumptions, the input files are created, executed through “Shaft Align Tool” and the results of each scenario are saved on “output” files. The scenarios are classified with the same method the scenarios of the first set of simulations were classified (“acceptable”, “marginal”, “not acceptable”). For every possible combination of offsets, the supporting points of the first five bearings (ASTB, FSTB, ISB, Aftmost Bearing (#9) and Aftmost M/E Bearing (#8)) are projected alongside their coordinates. For each supporting point, x value is the longitudinal position of each supporting point relatively to total length of the shaft, while y value is the offset. A line is passing through these points, which its color is defined as follows:

- When for specific offsets combination and for the 4 conditions of propeller immersion, the respective scenarios are “acceptable”, or 3 of them are “acceptable” and 1 is “marginal”, the combination is called “**exceptional**” and is marked with **green** (see APPENDIX B).
- When for specific offsets combination and for 4 conditions of propeller immersion, the respective scenarios are “marginal”, or 3 of them are “marginal” and 1 is “acceptable”, or 2 of them are “marginal” and the other 2 are “acceptable”, and the combination is called “**moderate**” and is marked with **blue** (see APPENDIX B).
- The rest of the offsets combinations are called “**invalid**”.

It is feasible to find the distance of each point from the straight that is passing through the first two supporting points (ASTB, FSTB) by rotating the points around the supporting point of the first bearing according to the angle of the straight that is passing through the first two supporting points. The absolute value of y coordinate of the rotated points is the discussed distance of each supporting point.

The longitudinal position (relatively to shaft length) of each supporting point for the first five bearings is presented on the following table.

Description	Distance to the rear end of the shaft (m)	$\frac{\text{longitudinal position}}{\text{total shaft length}}$
ASTB	2.157	0.10
FSTB	4.782	0.23

ISB	8.907	0.43	
Aftmost Bearing (#9)	13.392	0.65	
Aftmost M/E Bearing (#8)	14.147	0.69	<b>Total shaft length = 20.572 m</b>

Table 7.6: DIMENSIONLESS LONGITUDINAL POSITIONS

The above dimensionless values are the x coordinates of the supporting points as well. These are defined as the initial coordinates.

The coordinates ( $x_R, y_R$ ) of the rotated points are calculated. The new coordinates of the first supporting point are the same as their respective initial coordinates, because it is the reference point of the rotation. The coordinates of the other four points are calculated:

$$x_R = x_I \cdot \cos\alpha + y_I \cdot \sin\alpha$$

$$y_R = -x_I \cdot \sin\alpha + y_I \cdot \cos\alpha$$

Which,

- $\alpha$  is the slope of the straight that is passing through the first two points,  $\alpha \in [-\pi/2, \pi/2]$
- $x_I, y_I$  are the initial coordinates of the point
- $x_R, y_R$  are the coordinates of the rotated point

The above equations are in charge, when the reference point of the rotation is (0,0). In this case, the reference point is (0.1,0). The modified equations of coordinates of rotated points are:

$$x_R = x_I' \cdot \cos\alpha + y_I \cdot \sin\alpha + x_1$$

$$y_R = -x_I' \cdot \sin\alpha + y_I \cdot \cos\alpha$$

Which,

- $x_1$  is the x coordinate of the first point
- $x_I' = x_I - x_1$

The rotated points are connected with a line, which its color is the same as the color of the line that is passing through the initial points.

### 7.3.1 "0mm, 0mm, 0mm, 0mm, 0mm"

The possible new offsets for the first five bearings are presented below.

- ✓ ASTB
  - Basis offset: 0mm
  - Deviation: -
  - New offset: 0mm
- ✓ FSTB

- Basis offset: 0mm
  - Deviation: -0.8mm, -0.5mm, -0.3mm, -0.2mm, -0.1mm, 0.1mm, 0.2mm, 0.3mm, 0.5mm,0.8mm
  - New offset: -0.8mm, -0.5mm, -0.3mm, -0.2mm, -0.1mm, 0.1mm, 0.2mm, 0.3mm, 0.5mm,0.8mm
- ✓ ISB
- Basis offset: -3.88mm
  - Deviation: -0.8mm, -0.5mm, -0.3mm, -0.2mm, -0.1mm, 0.1mm, 0.2mm, 0.3mm, 0.5mm,0.8mm
  - New offset: -4.68mm, -4.38mm, -4.18mm, -4.08mm, -3.98mm, -3.78mm, -3.68mm, -3.58mm, -3.38mm, -3.08mm
- ✓ Aftmost Bearing (#9)
- Basis offset: -10.52mm
  - Deviation: -0.8mm, -0.5mm, -0.3mm, -0.2mm, -0.1mm, 0.1mm, 0.2mm, 0.3mm, 0.5mm,0.8mm
  - New offset: -11.32mm, -11.02mm, -10.82mm, -10.72mm, -10.62mm, -10.42mm, -10.32mm, -10.22mm, -10.02mm, -9.72mm
- ✓ Aftmost M/E Bearing (#8)
- Basis offset: -11.61mm
  - Deviation: -0.8mm, -0.5mm, -0.3mm, -0.2mm, -0.1mm, 0.1mm, 0.2mm, 0.3mm, 0.5mm,0.8mm
  - New offset: -12.41mm, -12.11mm, -11.91mm, -11.81mm, -11.71mm, -11.51mm, -11.41mm, -11.31mm, -11.11mm, -10.81mm

The combinations of possible vertical offsets take place for all cases of propeller immersion (0, 50%, 75%, 100%). Considering that there are 4 bearings with non-fixed offset, 10 possible offsets for each of them and 4 cases of propeller immersion, the scenarios, which should be executed are:

$$s = k \cdot p^q = 4 \cdot 10^4 = 40000$$

Which,

- ✓ k is the number of cases of propeller immersion
- ✓ q is the number of non-fixed offset bearings
- ✓ p is the number of possible offset values for each of the q bearings

✓ Exceptional Cases

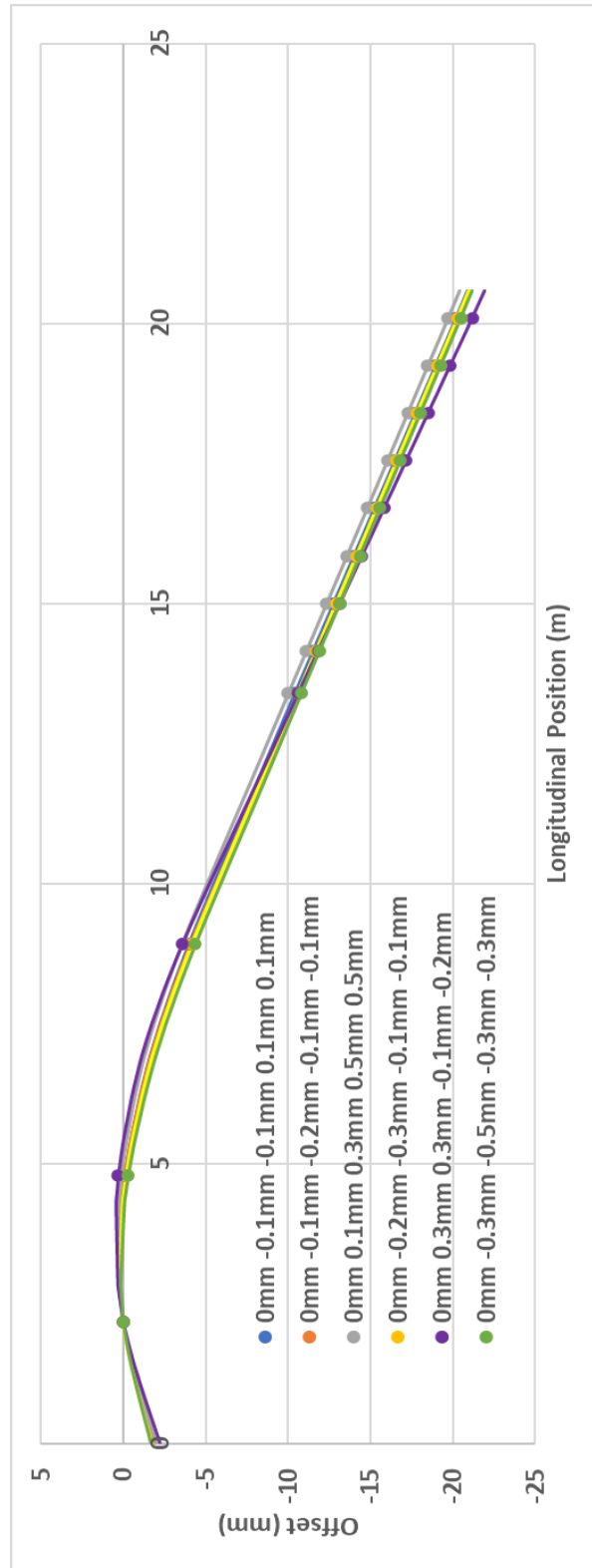


Figure 7.3: DEFLECTION CURVES OF EXCEPTIONAL CASES



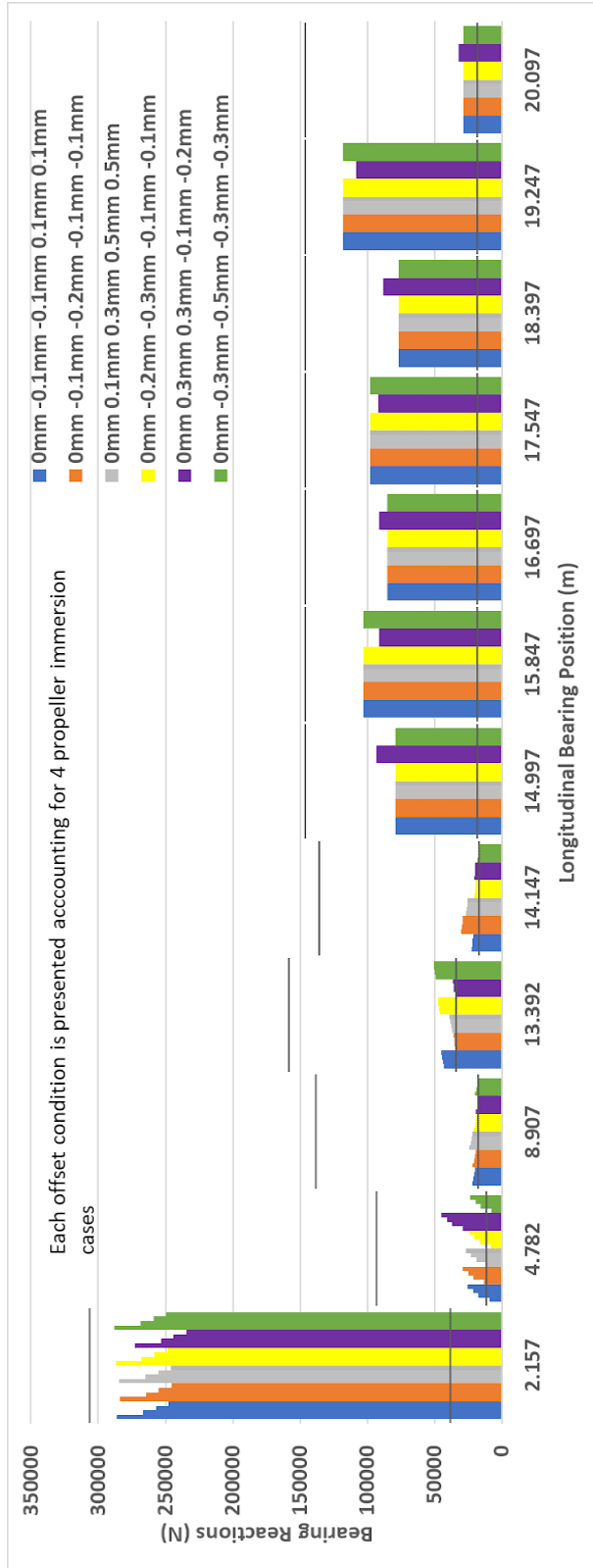


Figure 7.4: REACTION FORCES AND BOUNDARIES OF EXCEPTIONAL CASES

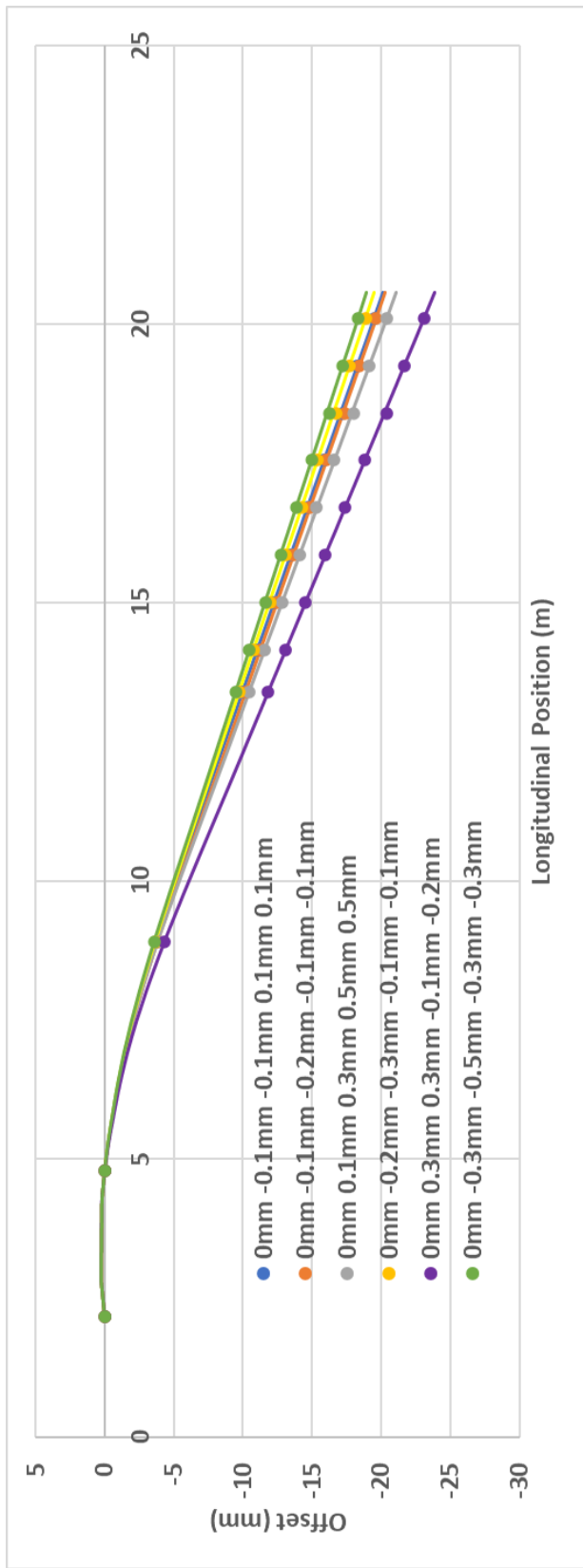


Figure 7.5: ROTATED DEFLECTION CURVES OF EXCEPTIONAL CASES

✓ Selective Moderate Cases

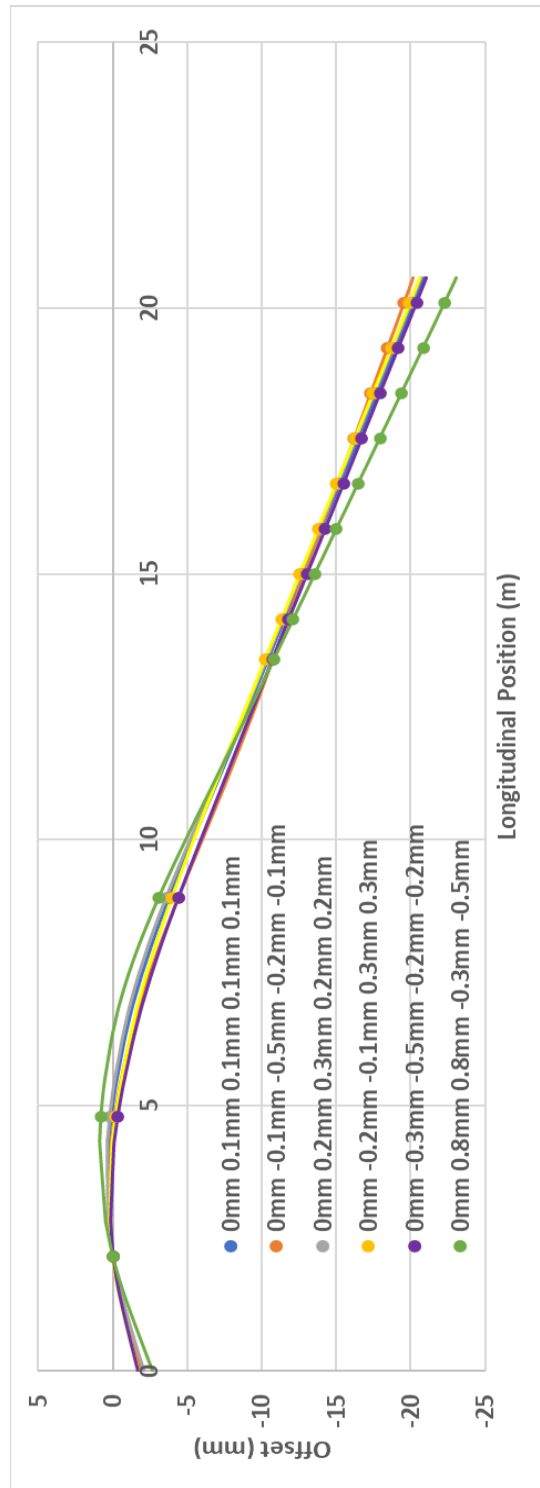


Figure 7.6: SELECTIVE DEFLECTION CURVES OF MODERATE CASES

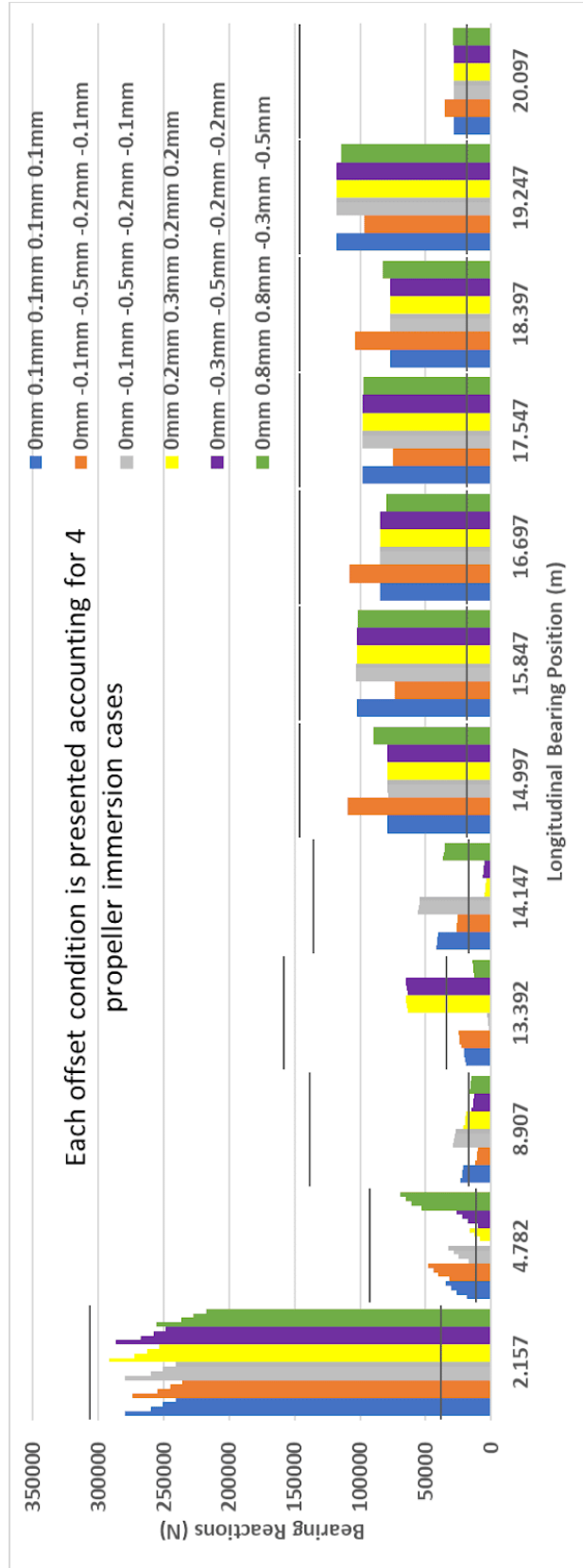


Figure 7.7: REACTION FORCES AND BOUNDARIES OF SELECTIVE MODERATE CASES

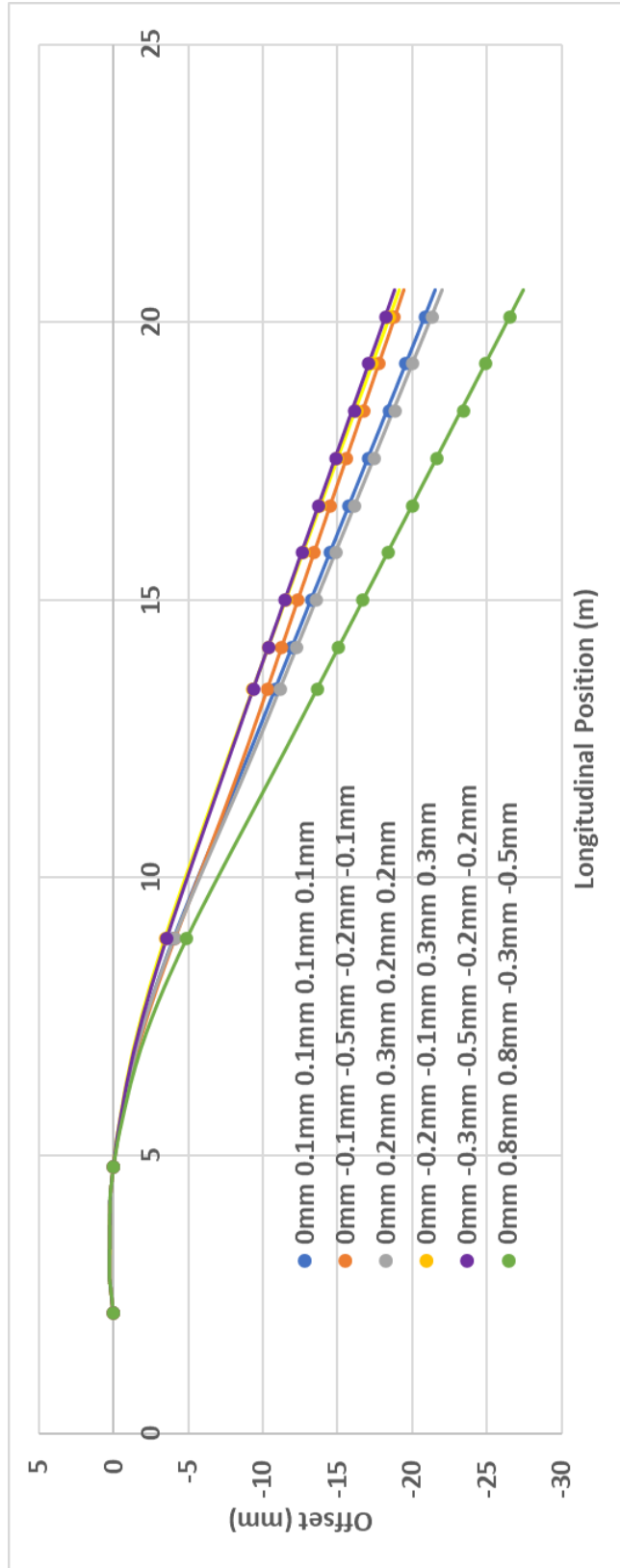


Figure 7.8: SELECTIVE ROTATED DEFLECTION CURVES OF MODERATE CASES

✓ Selective Invalid Cases

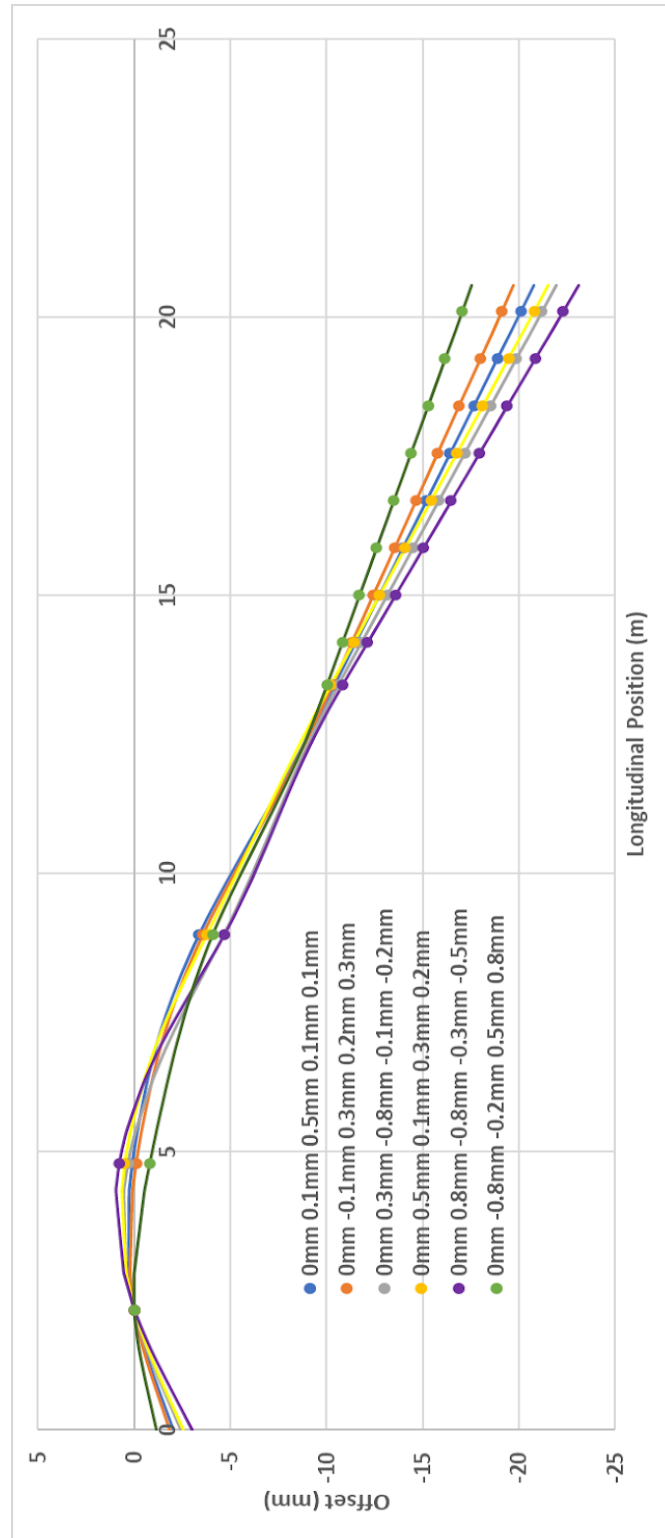


Figure 7.9: SELECTIVE DEFLECTION CURVES OF INVALID CASES

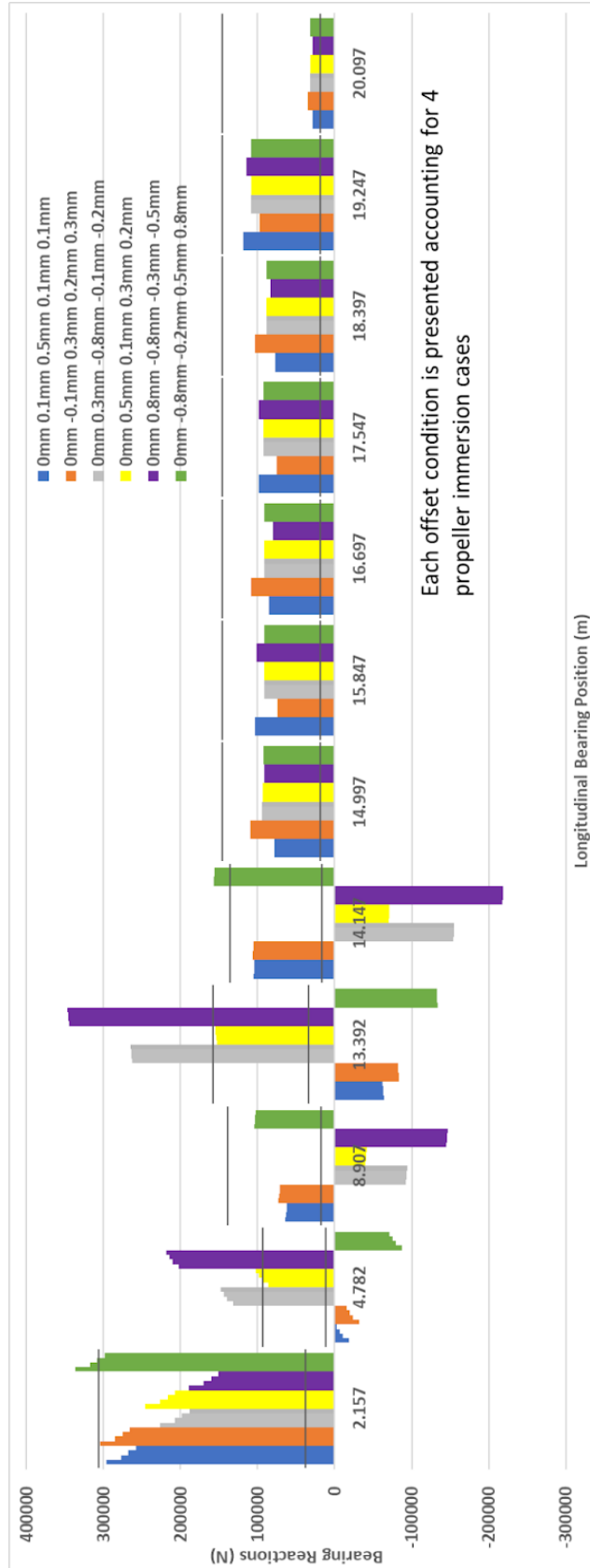


Figure 7.10: REACTION FORCES AND BOUNDARIES OF SELECTIVE INVALID CASES

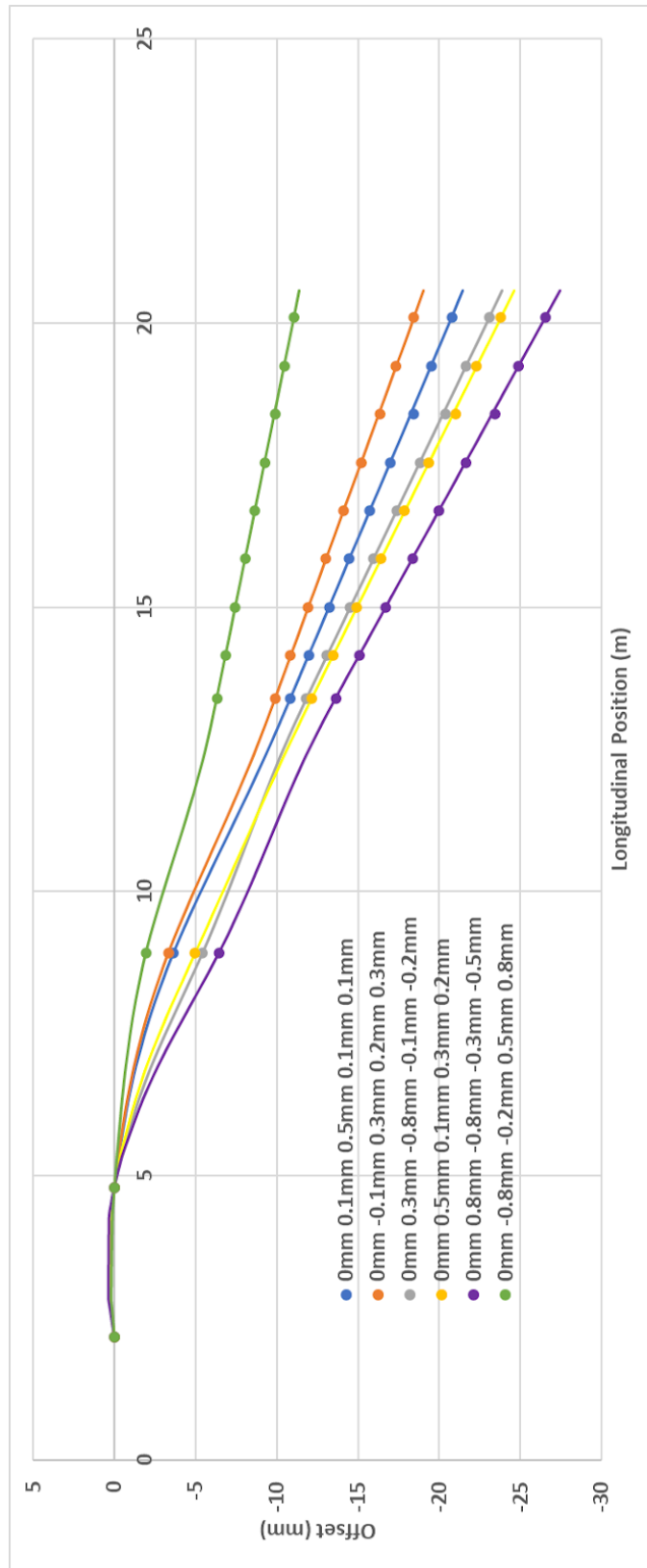


Figure 7.11: SELECTIVE ROTATED DEFLECTION CURVES OF INVALID CASES



### 7.3.2 “0mm, -1mm, -2mm, -2mm, -2mm”

The possible new offsets for the first five bearings are presented below.

- ✓ ASTB
  - Basis offset: 0mm
  - Deviation: -
  - New offset: 0mm
  
- ✓ FSTB
  - Basis offset: -1mm
  - Deviation: -0.8mm, -0.5mm, -0.3mm, -0.2mm, -0.1mm, 0.1mm, 0.2mm, 0.3mm, 0.5mm, 0.8mm
  - New offset: -1.8mm, -1.5mm, -1.3mm, -1.2mm, -1.1mm, -0.9mm, -0.8mm, -0.7mm, -0.5mm, -0.2mm
  
- ✓ ISB
  - Basis offset: -5.88mm
  - Deviation: -0.8mm, -0.5mm, -0.3mm, -0.2mm, -0.1mm, 0.1mm, 0.2mm, 0.3mm, 0.5mm, 0.8mm
  - New offset: -6.68mm, -6.38mm, -6.18mm, -6.08mm, -5.98mm, -5.78mm, -5.68mm, -5.58mm, -5.38mm, -5.08mm
  
- ✓ Aftmost Bearing (#9)
  - Basis offset: -12.52mm
  - Deviation: -0.8mm, -0.5mm, -0.3mm, -0.2mm, -0.1mm, 0.1mm, 0.2mm, 0.3mm, 0.5mm, 0.8mm
  - New offset: -13.32mm, -13.02mm, -12.82mm, -12.72mm, -12.62mm, -12.42mm, -12.32mm, -12.22mm, -12.02mm, -11.72mm
  
- ✓ Aftmost M/E Bearing (#8)
  - Basis offset: -13.61mm
  - Deviation: -0.8mm, -0.5mm, -0.3mm, -0.2mm, -0.1mm, 0.1mm, 0.2mm, 0.3mm, 0.5mm, 0.8mm
  - New offset: -14.41mm, -14.11mm, -13.91mm, -13.81mm, -13.71mm, -13.51mm, -13.41mm, -13.31mm, -13.11mm, -12.81mm

The combinations of possible vertical offsets take place for all cases of propeller immersion (0, 50%, 75%, 100%). Considering that there are 4 bearings with non-fixed offset, 10 possible offsets for each of them and 4 cases of propeller immersion, the scenarios, which should be executed are:

$$s = k \cdot p^q = 4 \cdot 10^4 = 40000$$

Which,

- ✓ k is the number of cases of propeller immersion
- ✓ q is the number of non-fixed offset bearings
- ✓ p is the number of possible offset values for each of the q bearings

✓ Exceptional Cases

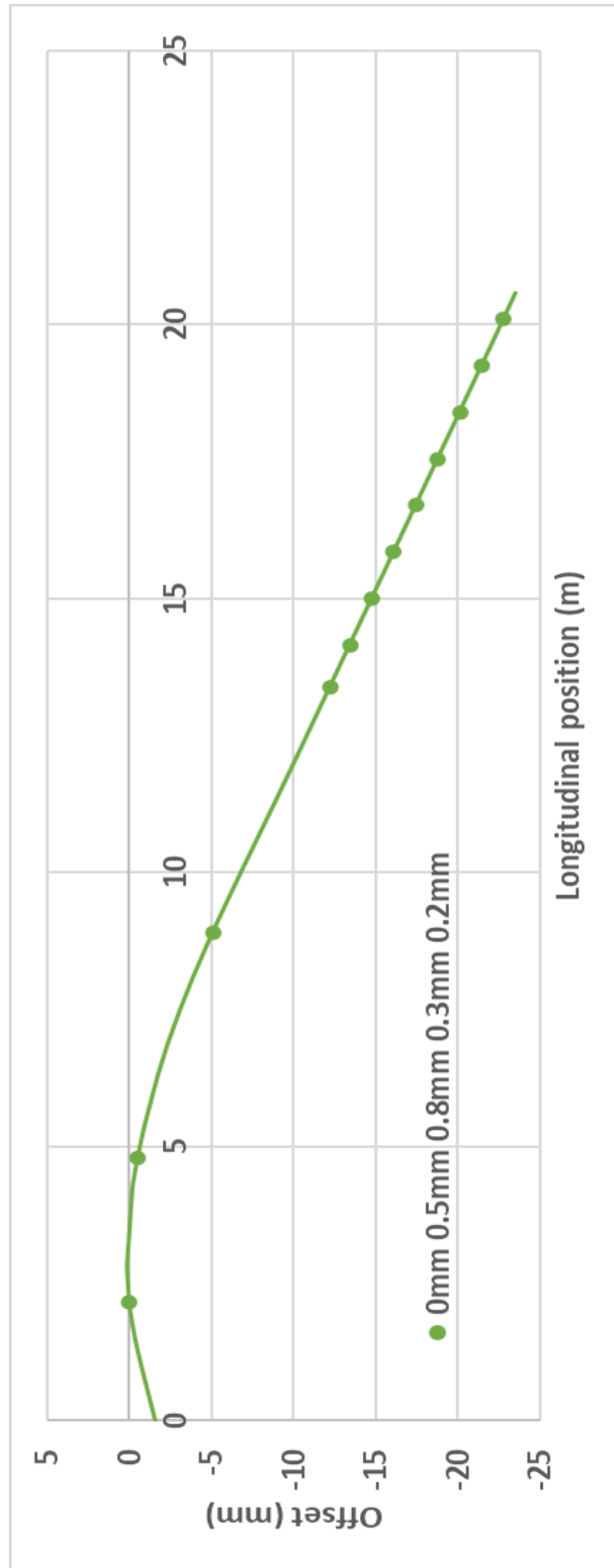


Figure 7.12: DEFLECTION CURVES OF EXCEPTIONAL CASES

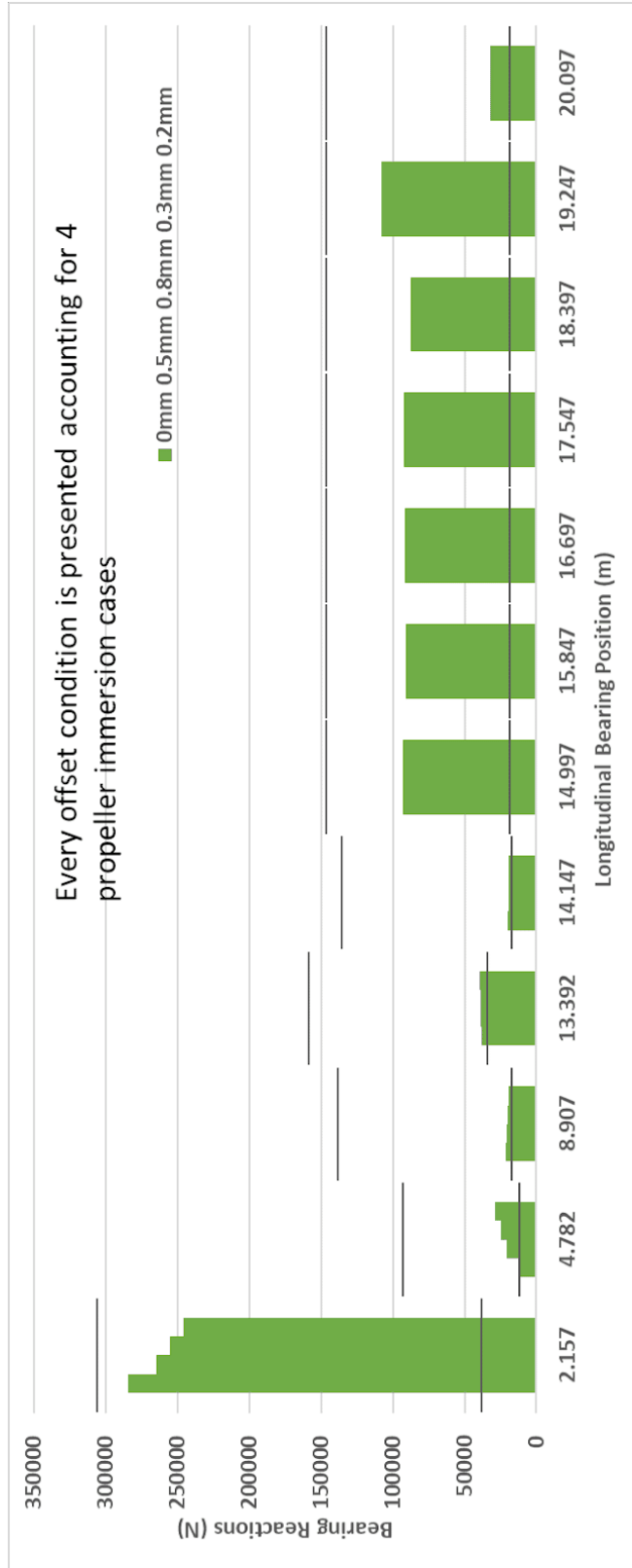


Figure 7.13: REACTION FORCES AND BOUNDARIES OF EXCEPTIONAL CASES

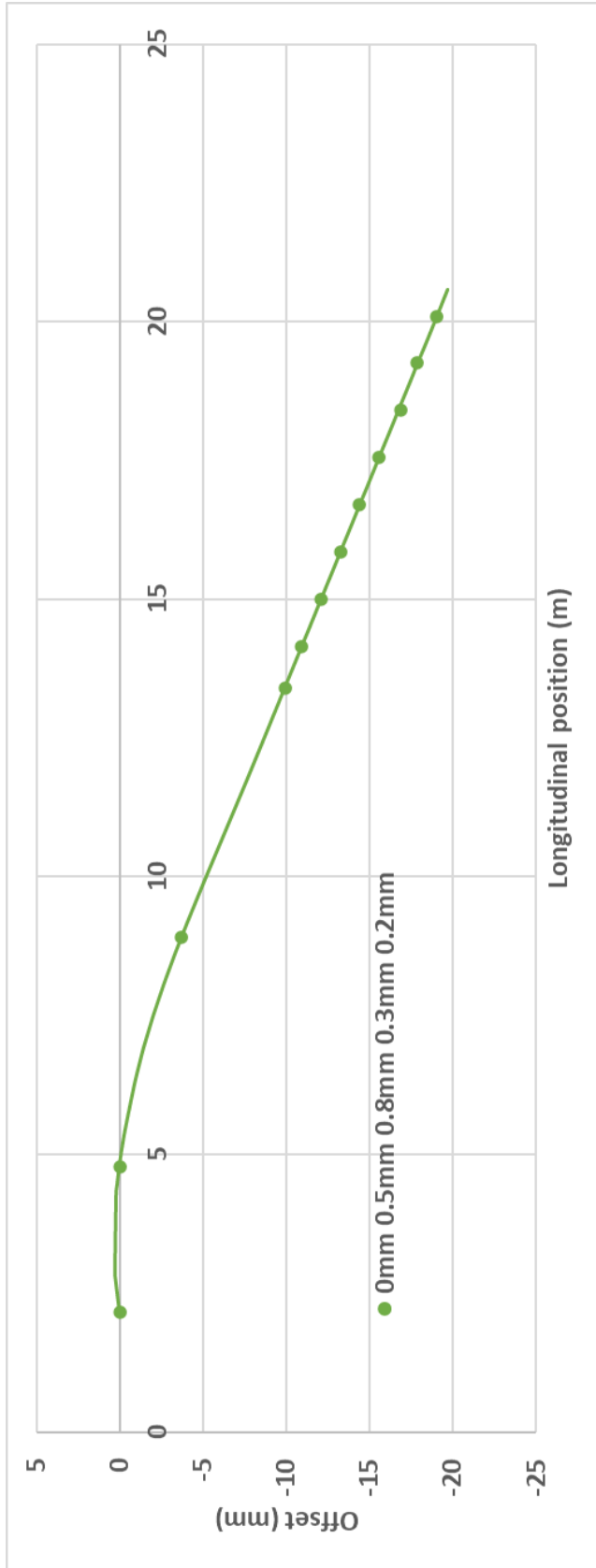


Figure 7.14: ROTATED DEFLECTION CURVES OF EXCEPTIONAL CASES

✓ Selective Moderate Cases

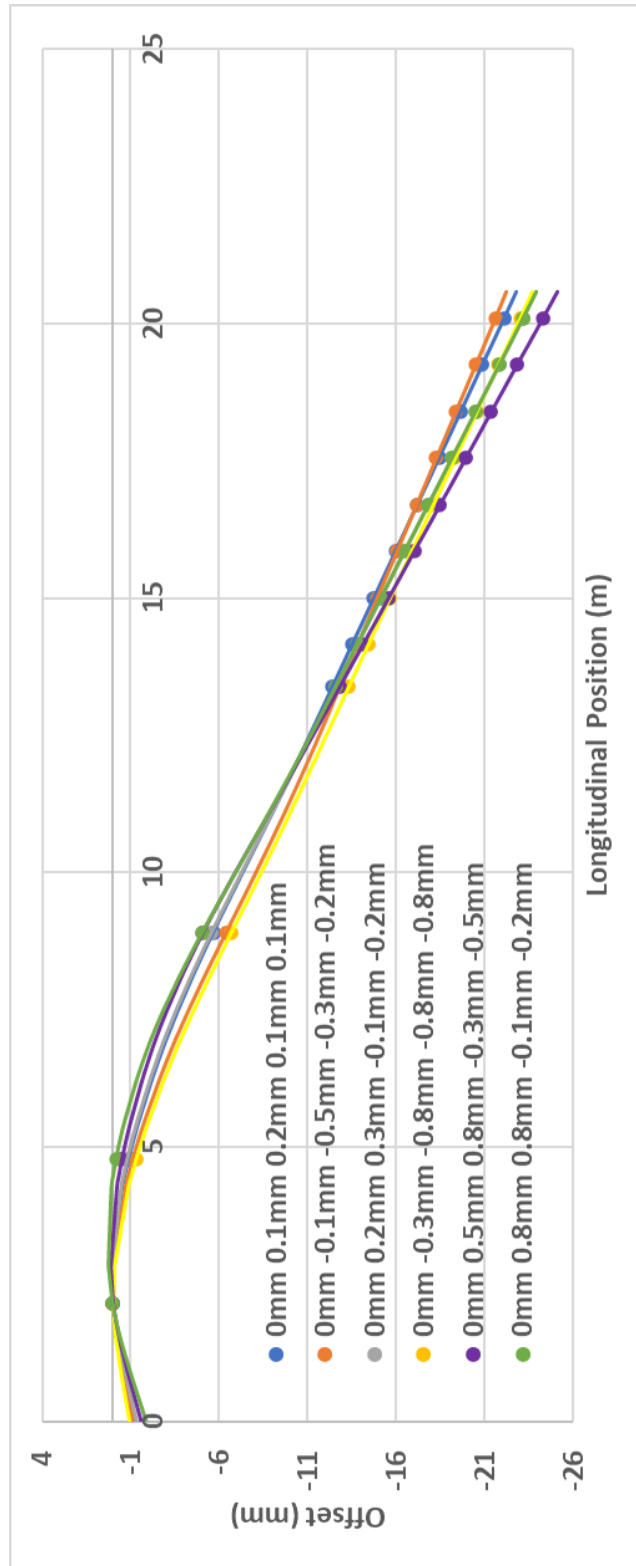


Figure 7.15: SELECTIVE DEFLECTION CURVES OF MODERATE CASES

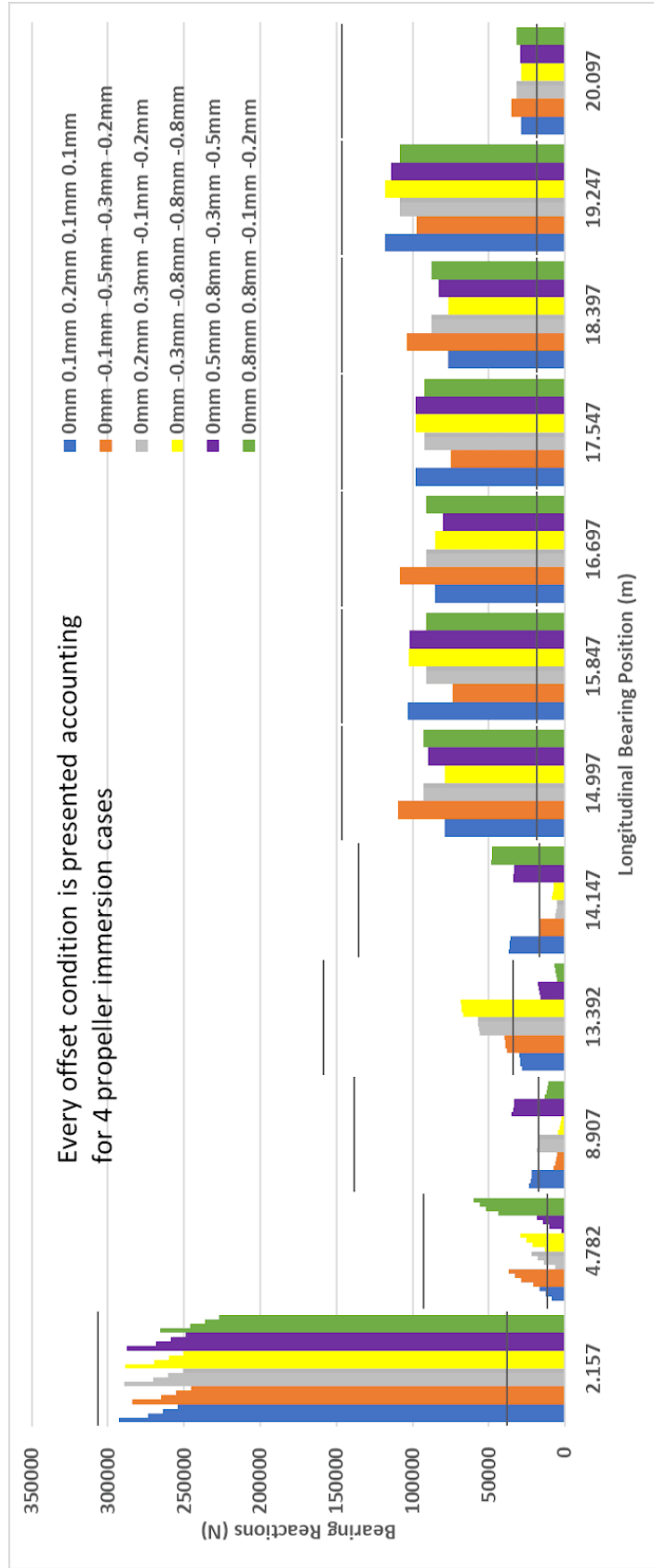


Figure 7.16: REACTION FORCES AND BOUNDARIES OF SELECTIVE MODERATE CASES

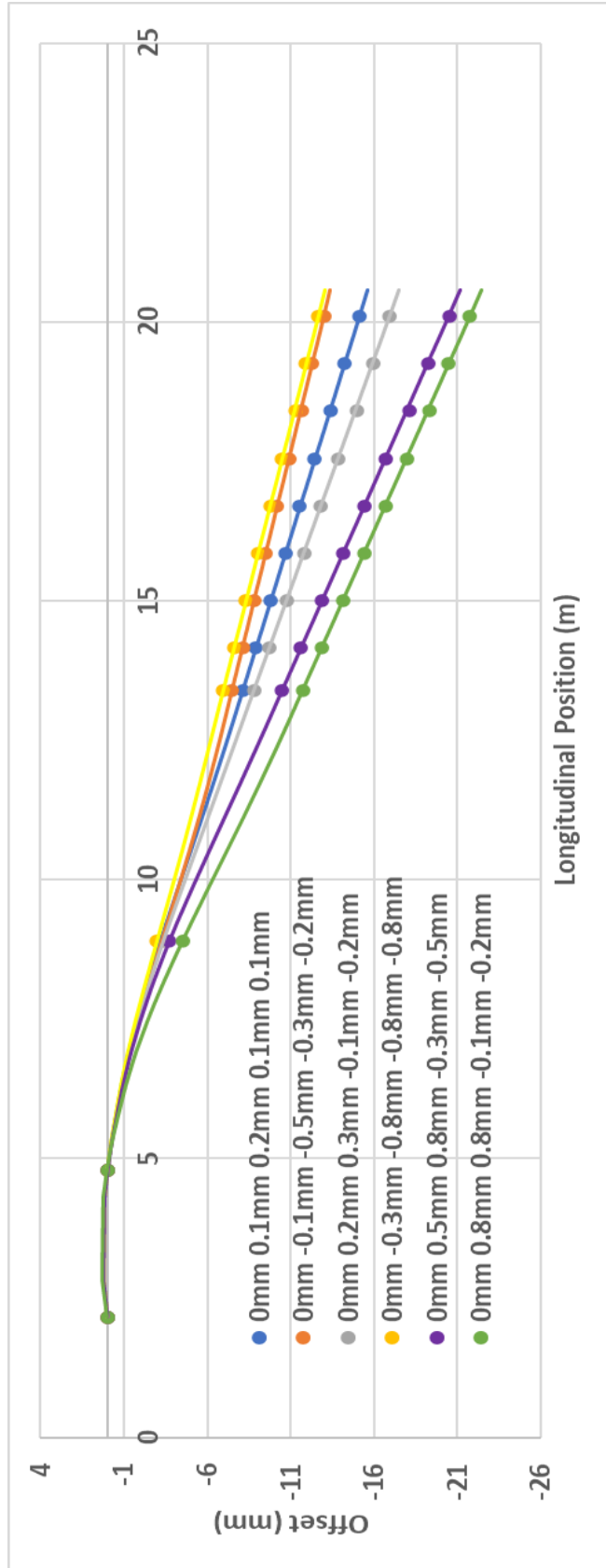


Figure 7.17: SELECTIVE ROTATED DEFLECTION CURVES OF MODERATE CASES



✓ Selective Invalid Cases

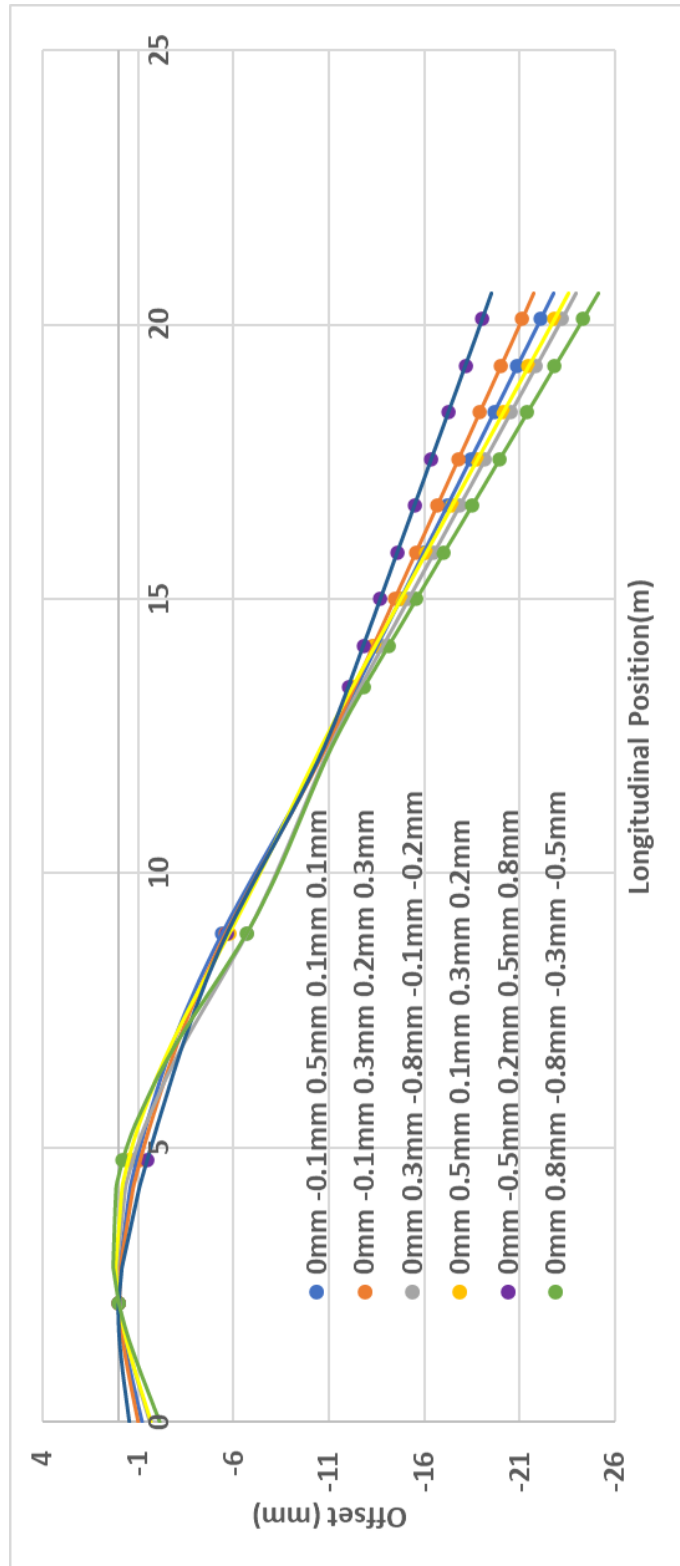


Figure 7.18: SELECTIVE DEFLECTION CURVES OF INVALID CASES

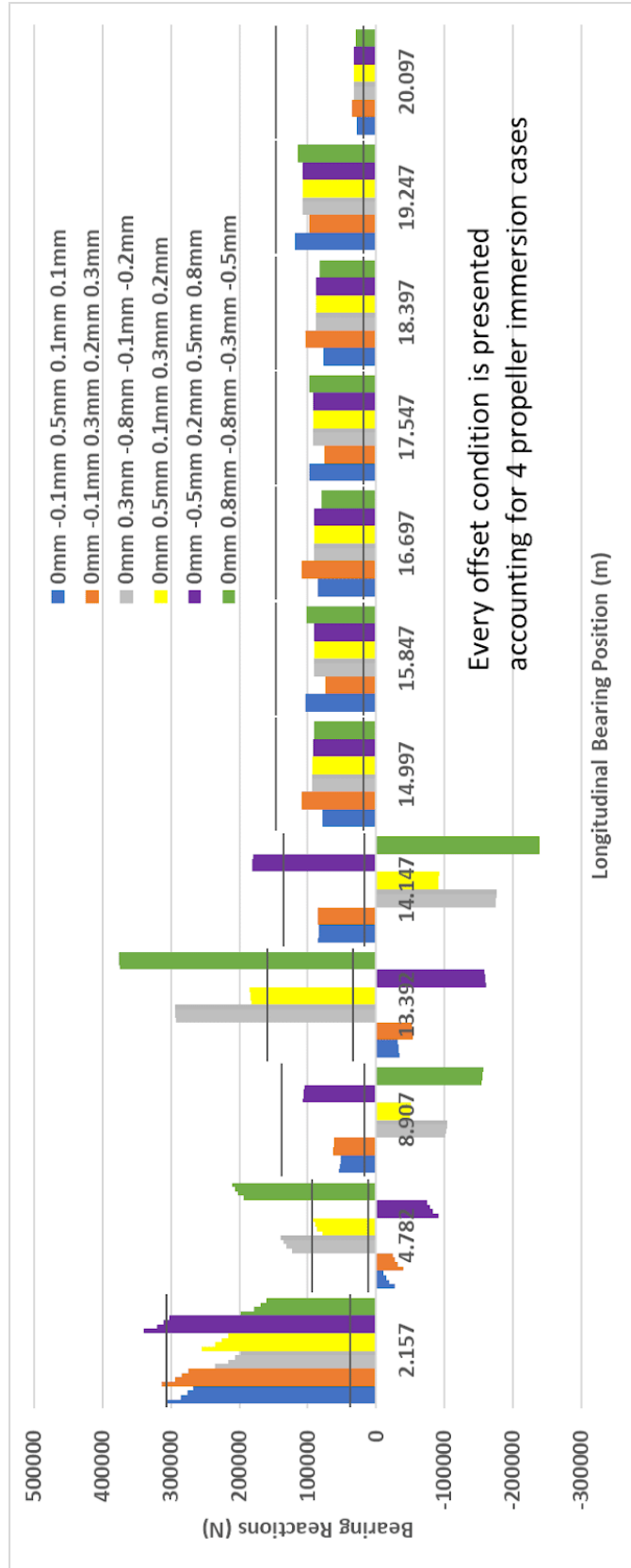


Figure 7.19: REACTION FORCES AND BOUNDARIES OF SELECTIVE INVALID CASES

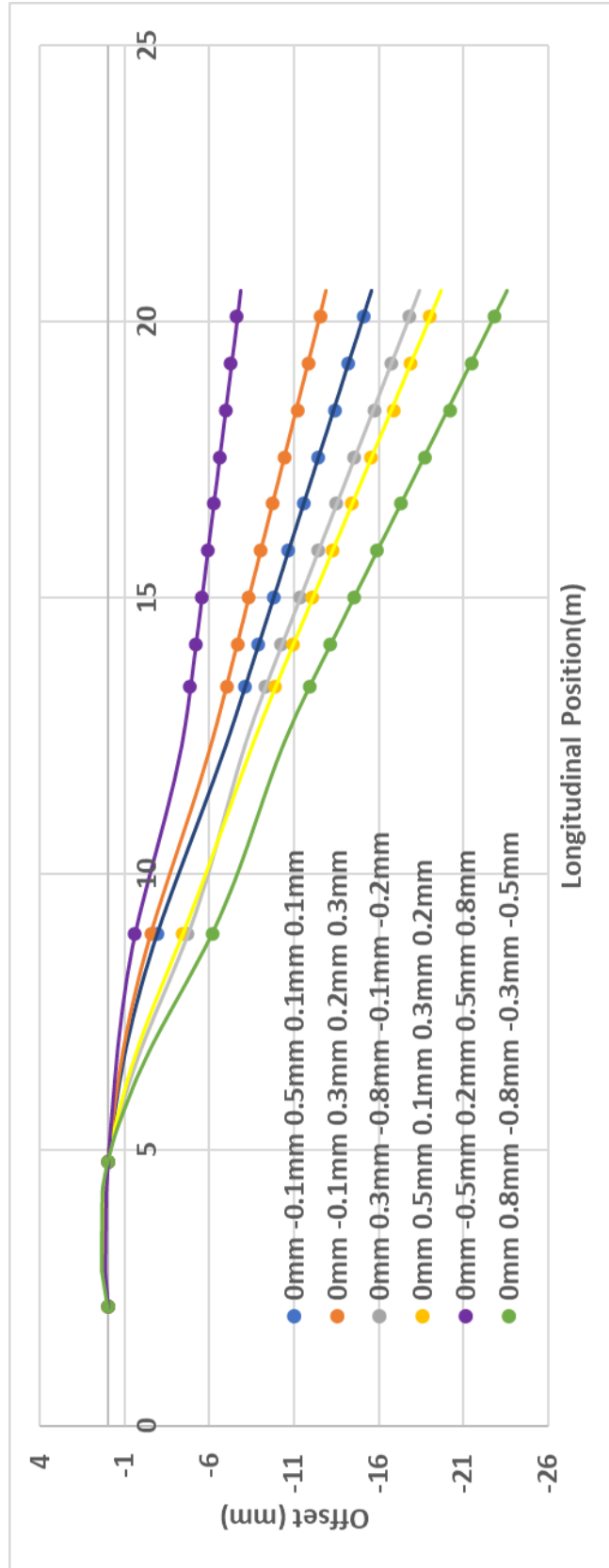


Figure 7.20: SELECTIVE ROTATED DEFLECTION CURVES OF INVALID CASES

### 7.3.3 “0mm, 3mm, 7mm, 8mm, 8mm”

The possible new offsets for the first five bearings are presented below.

- ✓ ASTB
  - Basis offset: 0mm
  - Deviation: -
  - New offset: 0mm
  
- ✓ FSTB
  - Basis offset: 3mm
  - Deviation: -0.8mm, -0.5mm, -0.3mm, -0.2mm, -0.1mm, 0.1mm, 0.2mm, 0.3mm, 0.5mm, 0.8mm
  - New offset: 2.2mm, 2.5mm, 2.7mm, 2.8mm, 2.9mm, 3.1mm, 3.2mm, 3.3mm, 3.5mm, 3.8mm
  
- ✓ ISB
  - Basis offset: 3.12mm
  - Deviation: -0.8mm, -0.5mm, -0.3mm, -0.2mm, -0.1mm, 0.1mm, 0.2mm, 0.3mm, 0.5mm, 0.8mm
  - New offset: 2.32mm, 2.62mm, 2.82mm, 2.92mm, 3.02mm, 3.22mm, 3.32mm, 3.42mm, 3.62mm, 3.92mm
  
- ✓ Aftmost Bearing (#9)
  - Basis offset: -2.52mm
  - Deviation: -0.8mm, -0.5mm, -0.3mm, -0.2mm, -0.1mm, 0.1mm, 0.2mm, 0.3mm, 0.5mm, 0.8mm
  - New offset: -3.32mm, -3.02mm, -2.82mm, -2.72mm, -2.62mm, -2.42mm, -2.32mm, -2.22mm, -2.02mm, -1.72mm
  
- ✓ Aftmost M/E Bearing (#8)
  - Basis offset: -3.61mm
  - Deviation: -0.8mm, -0.5mm, -0.3mm, -0.2mm, -0.1mm, 0.1mm, 0.2mm, 0.3mm, 0.5mm, 0.8mm
  - New offset: -4.41mm, -4.11mm, -3.91mm, -3.81mm, -3.71mm, -3.51mm, -3.41mm, -3.31mm, -3.11mm, -2.81mm

The combinations of possible vertical offsets take place for all cases of propeller immersion (0, 50%, 75%, 100%). Considering that there are 4 bearings with non-fixed offset, 10 possible offsets for each of them and 4 cases of propeller immersion, the scenarios, which should be executed are:

$$s = k \cdot p^q = 4 \cdot 10^4 = 40000$$

Which,

- ✓ k is the number of cases of propeller immersion
- ✓ q is the number of non-fixed offset bearings
- ✓ p is the number of possible offset values for each of the q bearings

✓ Exceptional Cases

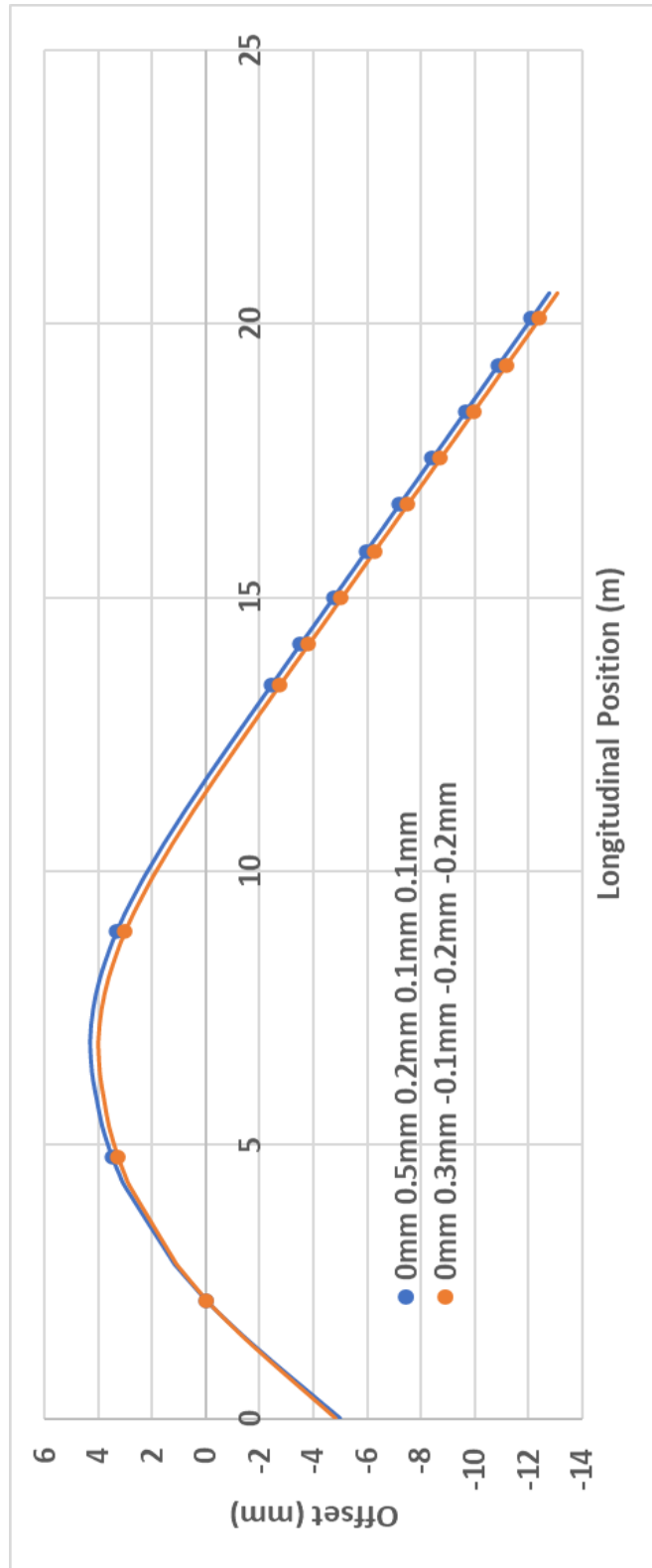


Figure 7.21: DEFLECTION CURVES OF EXCEPTIONAL CASES

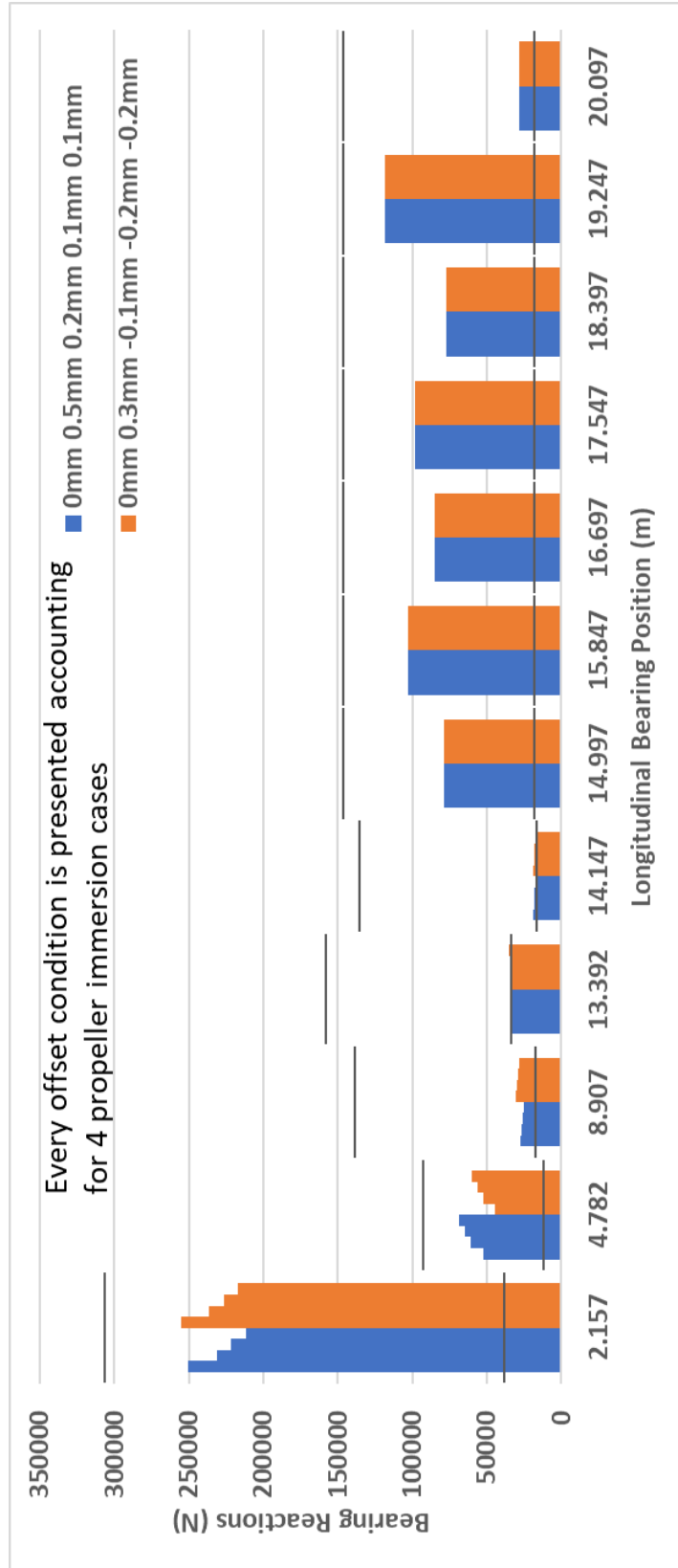


Figure 7.22: REACTION FORCES AND BOUNDARIES OF EXCEPTIONAL CASES

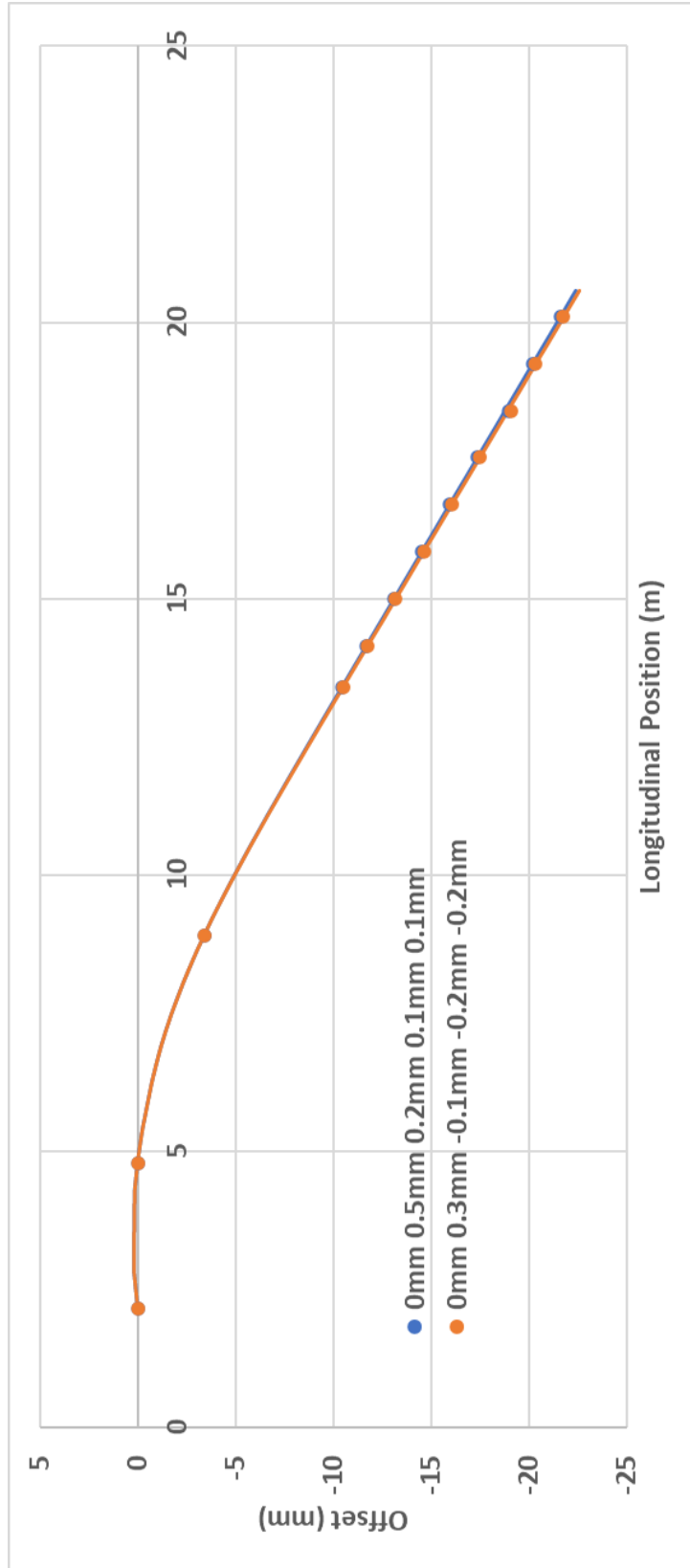


Figure 7.23: ROTATED DEFLECTION CURVES OF EXCEPTIONAL CASES



✓ Selective Moderate Cases

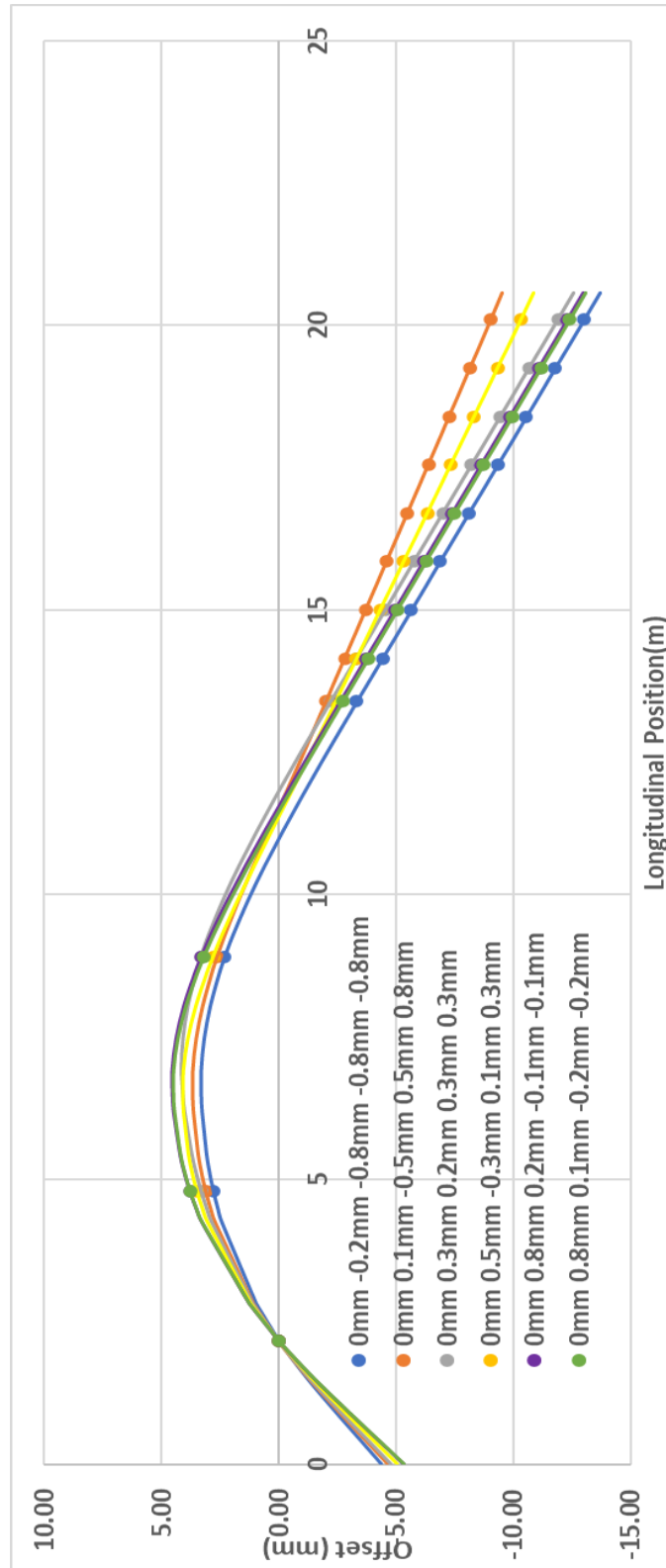


Figure 7.24: SELECTIVE DEFLECTION CURVES OF MODERATE CASES

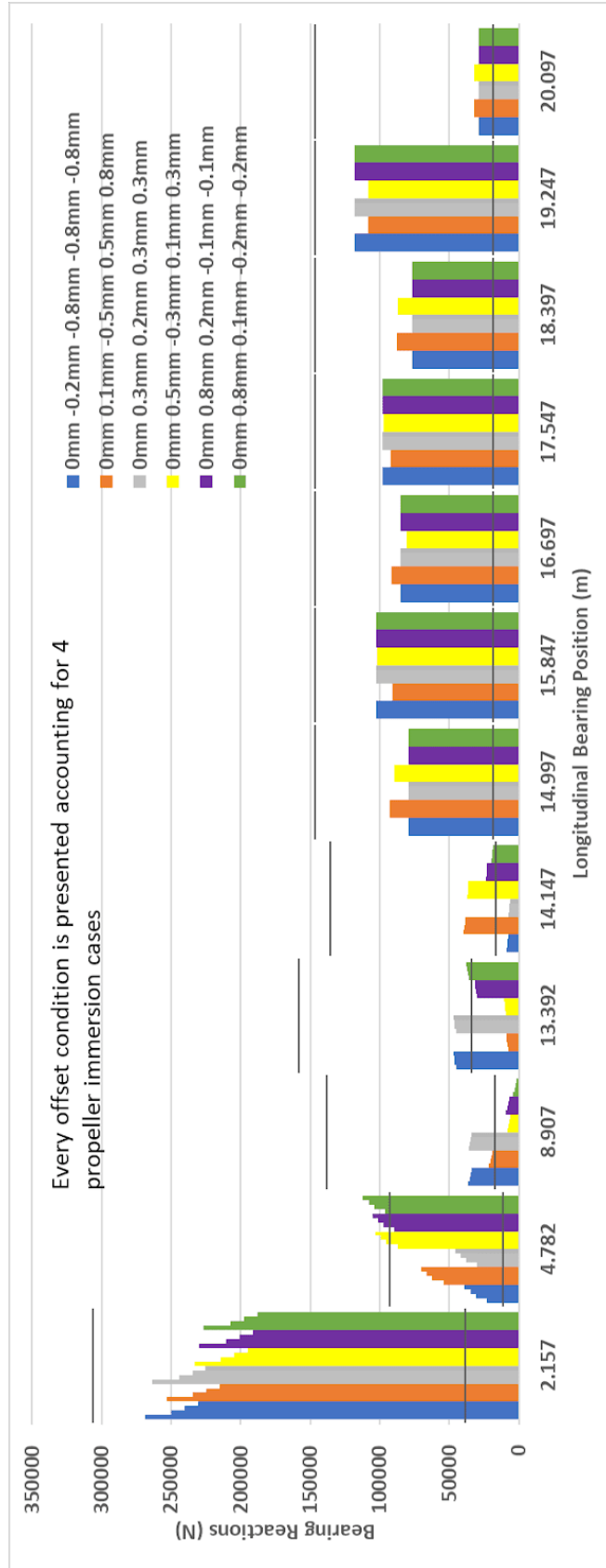


Figure 7.25: REACTION FORCES AND BOUNDARIES OF SELECTIVE MODERATE CASES

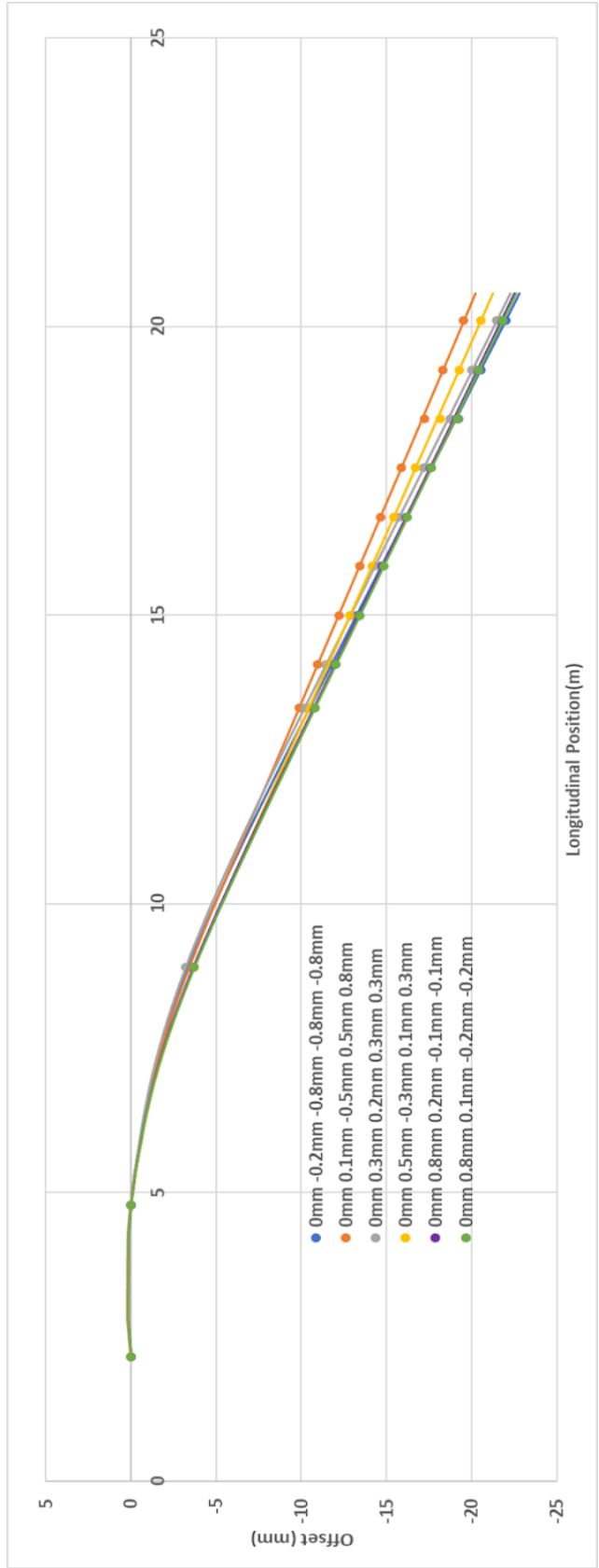


Figure 7.26: SELECTIVE ROTATED DEFLECTION CURVES OF MODERATE CASES

✓ Selective Invalid Cases

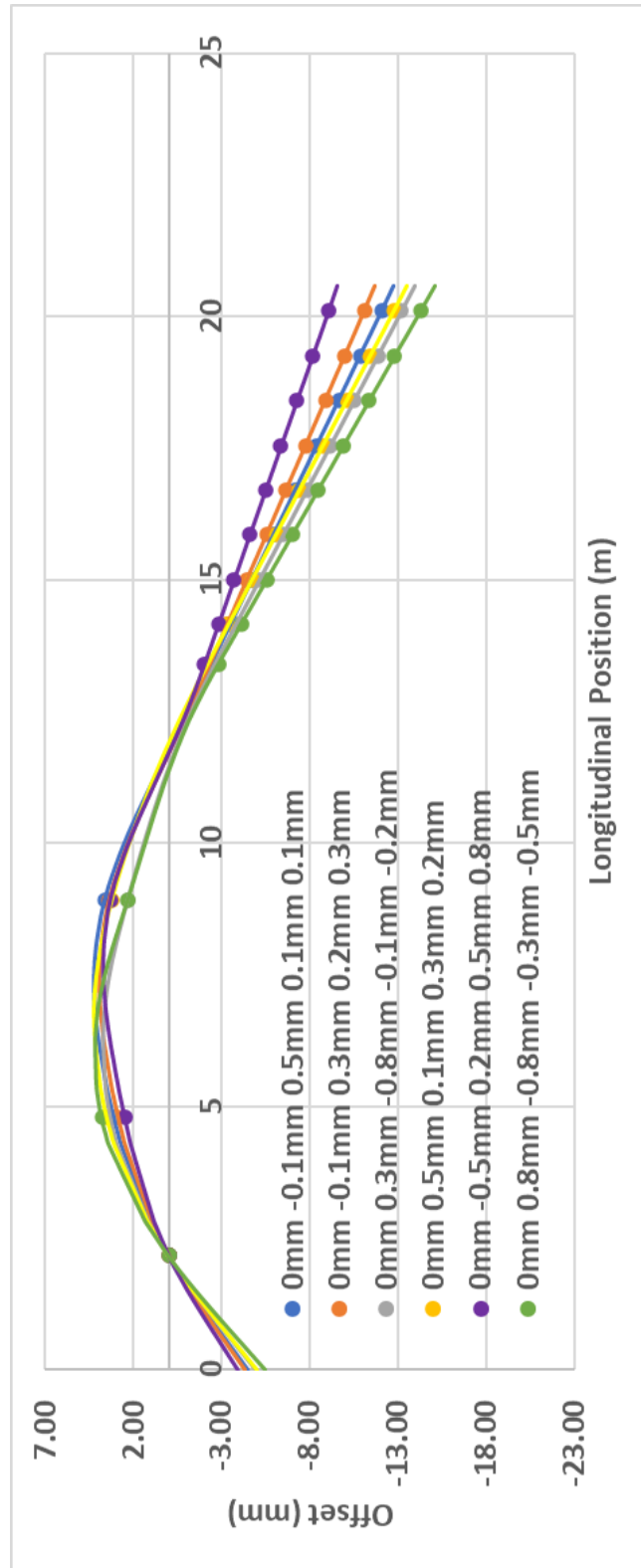


Figure 7.27: SELECTIVE DEFLECTION CURVES OF INVALID CASES

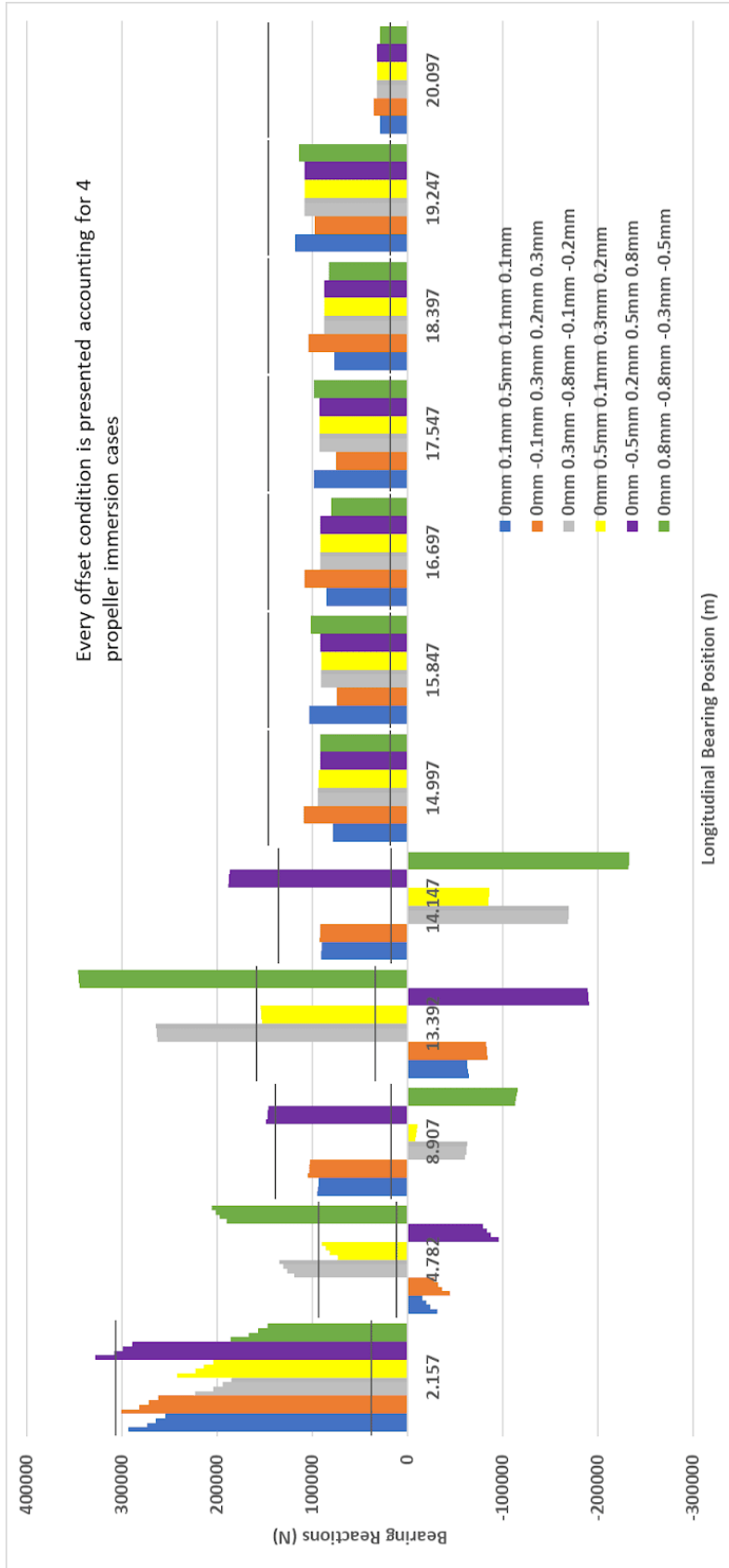


Figure 7.28: REACTION FORCES AND BOUNDARIES OF SELECTIVE INVALID CASES

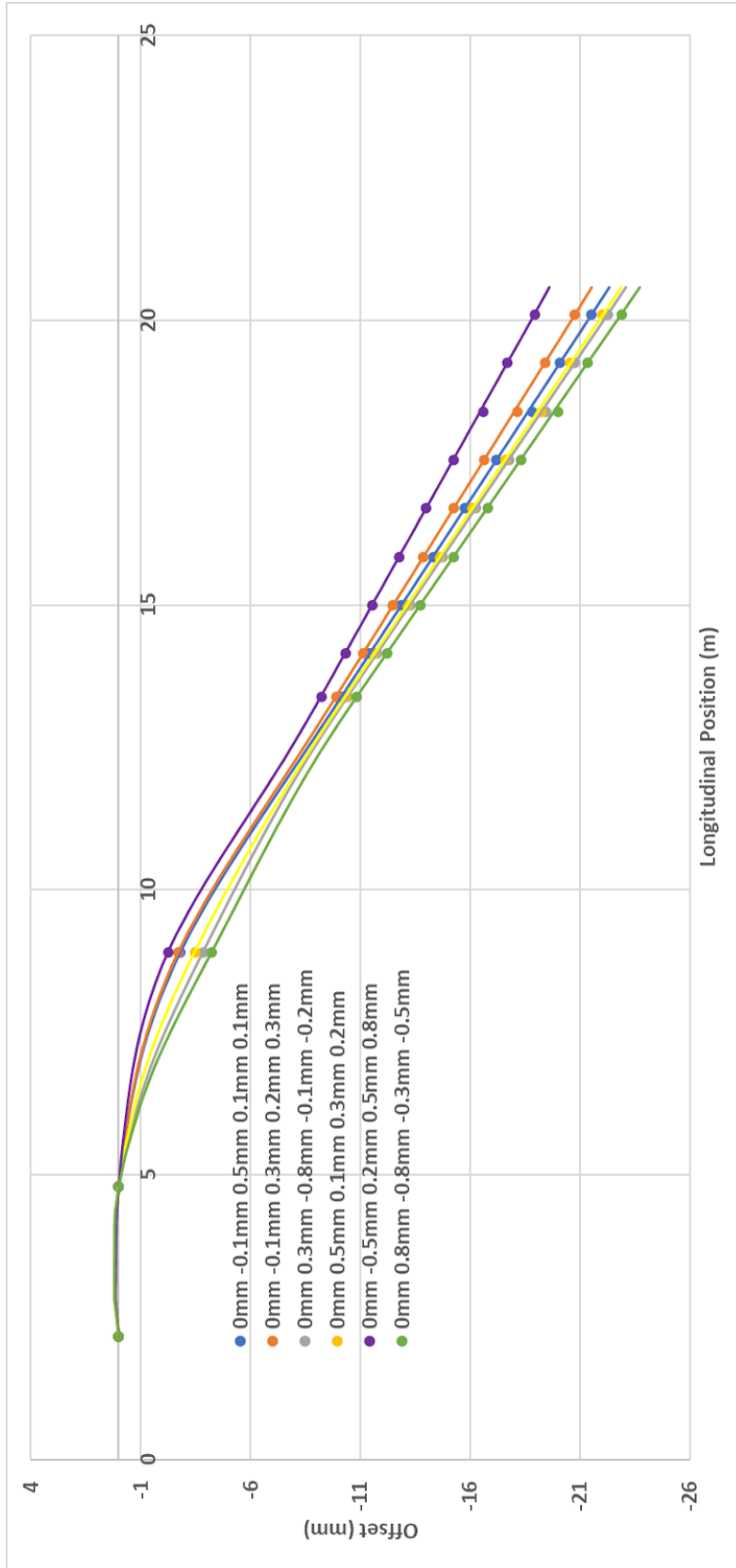


Figure 7.29: SELECTIVE ROTATED DEFLECTION CURVES OF INVALID CASES

## 7.4 Deflection Curve Comparison

In this section a comparison takes place among a representative deflection curve of an exceptional case, a representative deflection curve of a moderate case and a representative deflection curve of an invalid case

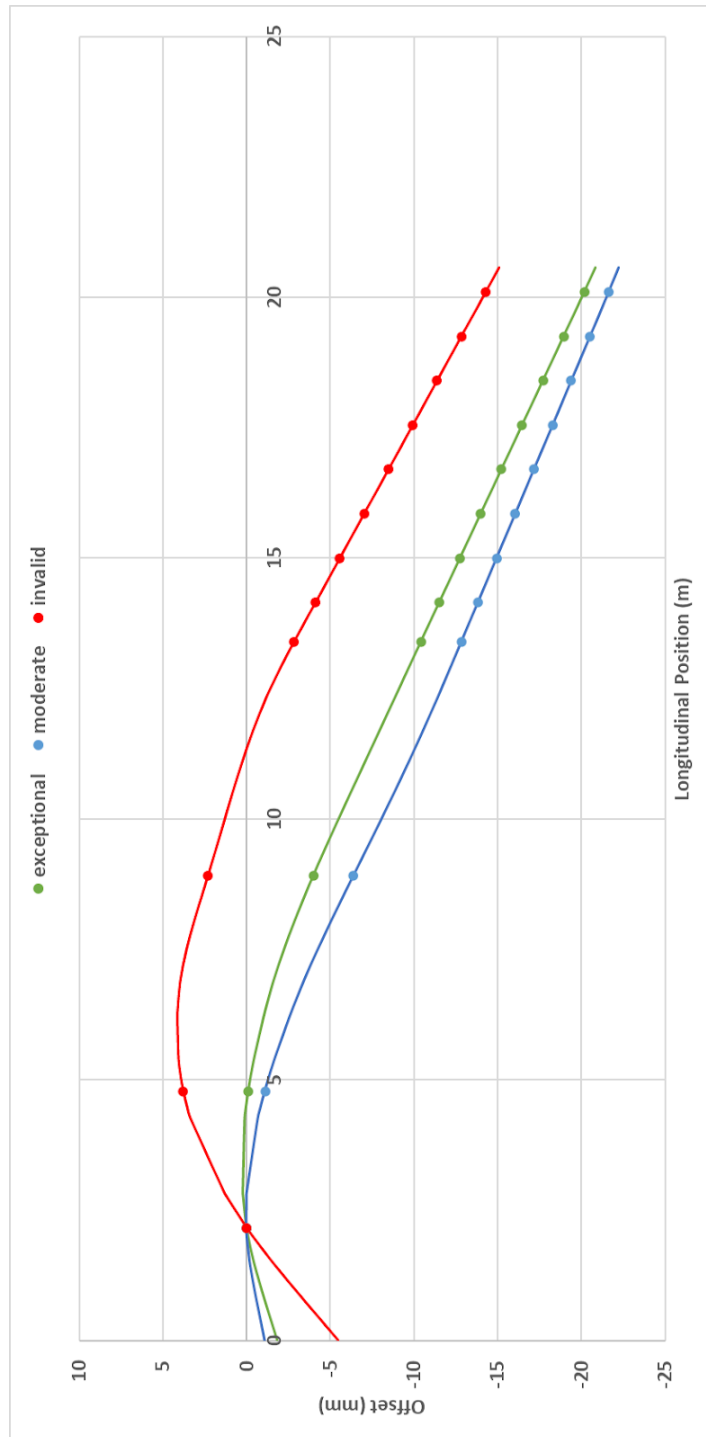


Figure 7.30: DEFLECTION CURVE COMPARISON

## 8. Conclusions – Future Work

### 8.1 Conclusions

Conclusions are drawn for the two parts of this work. Concerning the first part of the work, which is the dimension analysis of propulsion systems of a bulk carrier fleet of different size categories vessels, many interesting observations are made. First, it is observed that  $l_{prop}/l_{tot}$  values are significantly close regardless of the deadweight, with an average of 0.368 and standard deviation of 0.023. The same is observed for  $l_{int}/l_{tot}$  values, exhibiting an average value of 0.377 and a standard deviation of 0.028. Regarding the length ratios of main engine shaft ( $l_{M/E} / l_{tot}$ ) a bigger standard deviation is observed (0.04). Specifically, this ratio is increasing for the first seven vessels and then is decreasing with small fluctuations. The diameter shaft-flange ratios of the 3 shafts of the propulsion system (propeller shaft, intermediate shaft and main engine shaft) have a common characteristic. The ratios are increasing with increasing deadweight until ship 14, and, then, the ratios exhibit a decreasing trend. The highest ratios are detected on the propeller shaft ( $D_{prop}/D_{fl-prop}$ ) with an average value of 0.617 and standard deviation equal to 0.066, second highest are detected on the aft part of the intermediate shaft ( $D_{int}/D_{fl-int-aft}$ ) with an average value of 0.506 and standard deviation equal to 0.057, succeeded by ( $D_{int}/D_{fl-int-fwd}$ ) ratios with an average value of 0.431 and standard deviation equal to 0.055, while ( $D_{M/E} / D_{fl-M/E}$ ) ratios are the lowest with an average value equal to 0.355 and standard deviation equal to 0.044. As for shaft-flange length ratios, shaft-flange ratios of propeller shaft are identical, shaft-flange ratios of aft and forward part of the intermediate shaft are almost identical for each vessel with the same standard deviation values equal to 0.002. As for shaft-flange length ratios of the main engine shaft, they are not changing smoothly, as bigger fluctuations are observed. Shaft diameter-length ratios of propeller shaft are greater than the respective ratios for the intermediate shaft, but both stable, while shaft diameter-length ratios of the main engine shaft are unstable with sensible fluctuations.

Concerning the second part of this work, which is concerned with the detailed shaft alignment analysis of a specific vessel of the fleet, the following conclusions were drawn. The results (influence factors and bearing reaction forces) from “Shaft\_Align\_Tool” were very close to the respective results from the shaft alignment process carried out by ABS for specific propeller load and offsets. As for the preliminary approach, most of the cases were not acceptable, zero cases were acceptable and 11 were marginal. Only 3 different offset combinations (basis offsets) lead to these marginal cases and two of them are new combinations, as the other one contains the offsets suggested by ABS for the shaft alignment process of the shafting system. As for the “in depth approach”, the relative offset deviations are under 1mm (absolute value) from the basis offsets. Acceptable cases are detected as well as more marginal cases, but as the initial approach, the majority of the cases are “not acceptable”. A general observation is the deflection curves of exceptional cases are smooth, deflection curves of moderate cases have a slight change of camber upwards right before M/E shaft and deflection curves of invalid cases have many small changes of camber throughout the shaft line.

Concerning the reaction forces for the first two (2) regions (“0mm, 0mm, 0mm, 0mm, 0mm”, “0mm, -1mm, -2mm, -2mm, -2mm”), the behavior is similar. Specifically, in exceptional cases ASTB bearing reaction is high and within the 10%-80% of its permissible load, bearing reactions of FSTB, ISB, Aftmost Bearing (#9), Aftmost M/E Bearing (#8) are under or equal of 10% of their respective permissible loads and reaction forces of M/E bearings #7-1# are within the 10%-80% of their respective permissible load. In moderate cases ASTB & FSTB bearing reactions are within the 10%-80% of their permissible load, bearing reactions of ISB, Aftmost Bearing (#9), Aftmost M/E Bearing (#8) are under or equal of 10% of



their respective permissible loads and reaction forces of M/E bearings #7-1# are within the 10%-80% of their respective permissible load. In invalid cases, ASTB bearing reaction is above 80% of its permissible load reaction forces of M/E bearings #7-1# are within the 10%-80% of their respective permissible load and reaction forces of FSTB, ISB, Aftmost Bearing (#9) and Aftmost M/E Bearing (#8) are positive – negative alternately and vice versa. In these two (2) regions, offsets of FSTB are close to the reference line, while the offsets of the other bearings are far from the reference line.

Concerning the reaction forces for the last region (“0mm, 3mm, 7mm, 8mm”) the offsets of FSTB are more remote from the reference line compared to the FSTB offsets of the first two regions, while the offsets of the other bearings are closer to the reference line compared to the respective offsets of the first two regions. As for the reaction forces, the only difference detected compared to the previous regions is that reaction force of FSTB in exceptional and moderate case is much higher and within 10%-80% of its respective permissible load or even above 80% of its respective permissible load.

## 8.2 Future Work

Based on the current work the following topics can be studied and analyzed.

- Dimension Analysis of propulsion systems of other ship types

In this work, the analysis was carried out for a fleet of bulk carriers. The same process can be followed for other ship types (containerships, tankers, LNG carriers), for statistical comparisons.

- Geometry Optimization for FSTB, ISB, Aftmost Bearing (#9), Aftmost M/E Bearing (#8)

It was observed that reaction forces on these bearings were under 10% of their permissible load in most cases, even in exceptional cases. Based on current pressure limits for the bearings, a process can be carried out in order to establish the suitable bearing dimensions (length & diameter) for a better load distribution through the shaft line.

- AI Model Deflection Curve Optimization

An AI model can be developed using the results from this work (deflection curves and bearing reactions) in order to find the optimal deflection curve based on the load distribution on bearings.

- Shaft Alignment Study for bulk carriers of various sizes

Based on the methodology for the simulations of this work and the classification according to load limits of the bearings the same process can be carried out for the other bulk carriers of the fleet of this work. Comparisons can be made for the behavior of the deflection curves among them.

## Bibliography

- [1] American Bureau of Shipping. (2014). *Guidance Notes on Propulsion Shafting System*.
- [2] American Bureau of Shipping. (2019). *Guidance Notes on Propulsion Shafting Alignment*.
- [3] Budynas, R. G., & Keith Nisbett, J. (2011). Lubrication & Journal Bearings. In *Shigley's Mechanical Engineering Design, Ninth Edition* (pp. 627-636). McGraw-Hill.
- [4] Bureau Veritas. (2015). *Elastic Shaft Alignment*.
- [5] Cameron, A. (1976). Chapter 7. In *Basic Lubrication Theory* (pp. 71-100). Ellis Horwood Limited.
- [6] Frangopoulos, C. A. (2005). Volume C: Propulsion Shaft Alignment. In *Analysis and Optimization of Ship Energy Systems*.
- [7] Harrington, R. (1992). Propellers, Shafting and Shafting System Vibration Analysis. In *Marine Engineering* (pp. 353-356).
- [8] Jeon, H. (1986). *Ship Propulsion Transmission Equipment*. Busan, Korea. Taewha Publishing Company.
- [9] Khonsari, M. M., & Booser, R. E. (2017). Journal Bearings. In *Applied Tribology, Bearing Design and Lubrication, Third Edition* (pp. 256-260). John Wiley & Sons Ltd.
- [10] Larsen, O. C. (1981). Some Considerations on Marine Shafting Design. *Industrial Lubrication and Tribology*, 204-237.
- [11] Lee, Y. J., & Kim, U. K. (2005). A Study on Hull Deflection and Shaft Alignment Interaction in VLCC. *Korean Society of Marine Engineering*, 785-794.
- [12] Lei, S., Dongxin, X., & Xigeng, S. (2010). Research on Shafting Alignment Considering Ship Hull Deformations. *Marine Structures*, 103-114.
- [13] Lipsith, G. (2019, February 13). *Investigating an uptick in shaft bearing failures*. Retrieved from <https://www.rivieramm.com/opinion/opinion/investigating-an-uptick-in-shaft-bearing-failures-21806>
- [14] Low, K. H., & Lim, S. H. (2003). Propulsion shaft alignment method and analysis for surface crafts. *Elsevier*.
- [15] Mann, G. (1965). Analysis of Shafting Problems Using Fair Curve Alignment Theory. *American Soc. Nav. Eng.*, 117-133.
- [16] Murawski, L. (2005). Shaft Line Alignment Analysis Taking Ship Construction Flexibility And Deformations Into Consideration. *Marine Structures*, 62-84.
- [17] Nippon Kaiji Kyokai. (2006). *Guidelines on Shafting Alignment*.
- [18] Rudolph, M. (1958). A Quarter Century of Propulsion Shafting Design Practice and Operating Experience in the U.S Navy. *American Soc. Nav. Eng.*, 153-164.
- [19] Stachowiak, G. W., & Batchelor, A. W. (2005). Hydrodynamic Lubrication. In *Engineering Tribology* (pp. 101-115). Perth: Butterworth Heinemann.
- [20] Sverko, D. (2003). Design Concerns in Propulsion Shafting Alignment. *ABS Technical Papers*.
- [21] Szeri, A. Z. (2011). *Fluid Film Lubrication*. New York: Cambridge University Press.

# APPENDIX A

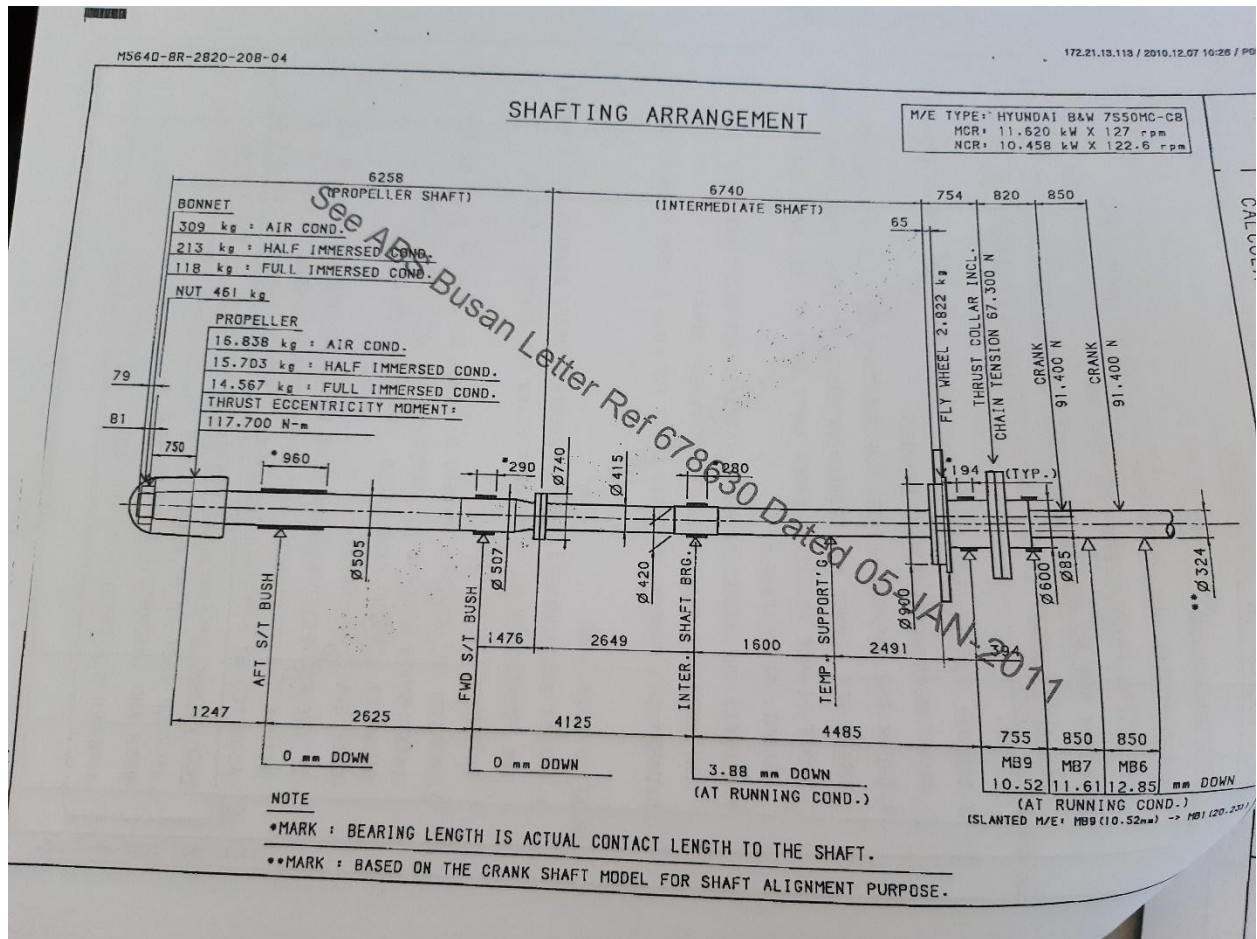


Figure A.1: SHAFT ARRANGEMENT

	Distance ***** m	Length ***** m	Diamet ***** m	diamet ***** m	Mass **** kg	
Propeller shaft e	0.050	0.050	0.340	0.000	31	
Propeller shaft n	0.079	0.029	0.360	0.000	20	
Propeller shaft n	0.160	0.081	0.360	0.000	57	
Propeller shaft n	0.280	0.120	0.360	0.000	85	
Propeller shaft	0.300	0.020	0.345	0.000	13	
Propeller shaft	0.910	0.610	0.449	0.000	707	
			0.479	0.000		
Propeller mass	0.910				14567	
Propeller shaft	1.420	0.510	0.479	0.000	664	
			0.505	0.000		
Propeller shaft	1.707	0.287	0.505	0.000	393	
Propeller shaft	1.807	0.100	0.505	0.000	140	
Propeller shaft	1.837	0.030	0.505	0.000	42	
Bearing element	2.157	0.320	0.505	0.000	448	
Aft stern tube be	2.157					Stiff
Bearing element	2.797	0.640	0.505	0.000	897	
propeller shaft	2.827	0.030	0.505	0.000	42	
propeller shaft	4.257	1.430	0.505	0.000	2004	
propeller shaft	4.457	0.200	0.505	0.000	281	
			0.507	0.000		
propeller shaft	4.607	0.150	0.507	0.000	211	
propeller shaft	4.637	0.030	0.507	0.000	42	
Bearing element	4.782	0.145	0.507	0.000	204	
Fwd stern tube be	4.782					Stiff
Bearing element	4.927	0.145	0.507	0.000	204	
Propeller shaft	4.957	0.030	0.507	0.000	42	
Propeller shaft	5.107	0.150	0.507	0.000	211	
Propeller shaft	5.493	0.326	0.507	0.000	516	
Propeller shaft	6.173	0.740	0.507	0.000	972	
			0.415	0.000		
Propeller shaft f	6.258	0.085	0.740	0.000	286	
Inter-shaft flang	6.343	0.085	0.740	0.000	286	
Intermediate shaf	8.607	2.264	0.415	0.000	2403	
Intermediate shaf	8.767	0.160	0.420	0.000	174	
Bearing element	8.907	0.140	0.420	0.000	152	
Inter-shaft beari	8.907					Stiff
Bearing element	9.047	0.140	0.420	0.000	152	
Intermediate shaf	9.307	0.260	0.420	0.000	282	
Intermediate shaf	12.913	3.606	0.415	0.000	3828	
Fwd inter-shaft f	12.998	0.085	0.900	0.000	424	
Thrust shaft flan	13.063	0.065	0.900	0.085	321	
Flywheel	13.105	0.042	1.060	0.085	289	
Thrust shaft	13.295	0.190	0.600	0.085	413	
Bearing element	13.392	0.097	0.600	0.085	210	
Aftmost bearing o	13.392					Stiff

Figure A.2: SHAFT SEGMENTS PART 1

Bearing element	13.489	0.097	0.600	0.085	210	
Thrust shaft	13.646	0.157	0.600	0.085	341	
Thrust shaft	13.752	0.106	1.100	0.085	786	
Thrust shaft	13.798	0.046	1.100	0.085	341	
Thrust shaft	13.836	0.038	1.100	0.085	316	
			1.230	0.085		
Thrust shaft	13.858	0.022	1.230	0.085	204	
Thrust shaft	14.050	0.192	0.600	0.085	417	
Bearing element	14.147	0.097	0.600	0.085	210	
Aftmost main bear	14.147					Stiff
Bearing element	14.244	0.097	0.324	0.000	0	
Thrust shaft	14.572	0.328	0.324	0.000	0	
Thrust shaft	14.900	0.328	0.324	0.000	0	
Bearing element	14.997	0.097	0.324	0.000	0	
Engine bearing 1	14.997					Stiff
Bearing element	15.094	0.097	0.324	0.000	0	
Thrust shaft	15.422	0.328	0.324	0.000	0	
Thrust shaft	15.750	0.328	0.324	0.000	0	
Bearing element	15.847	0.097	0.324	0.000	0	
Engine bearing 2	15.847					Stiff
Bearing element	15.944	0.097	0.324	0.000	0	
Thrust shaft	16.272	0.328	0.324	0.000	0	
Thrust shaft	16.600	0.328	0.324	0.000	0	
Bearing element	16.697	0.097	0.324	0.000	0	
Engine bearing 3	16.697					Stiff
Bearing element	16.794	0.097	0.324	0.000	0	
Thrust shaft	17.122	0.328	0.324	0.000	0	
Thrust shaft	17.450	0.328	0.324	0.000	0	
Bearing element	17.547	0.097	0.324	0.000	0	
Engine bearing 4	17.547					Stiff
Bearing element	17.644	0.097	0.324	0.000	0	
Thrust shaft	17.972	0.328	0.324	0.000	0	
Thrust shaft	18.300	0.328	0.324	0.000	0	
Bearing element	18.397	0.097	0.324	0.000	0	
Engine bearing 5	18.397					Stiff
Bearing element	18.494	0.097	0.324	0.000	0	
Thrust shaft	18.822	0.328	0.324	0.000	0	
Thrust shaft	19.150	0.328	0.324	0.000	0	
Bearing element	19.247	0.097	0.324	0.000	0	
Engine bearing 6	19.247					Stiff
Bearing element	19.344	0.097	0.324	0.000	0	
Thrust shaft	19.672	0.328	0.324	0.000	0	
Thrust shaft	20.000	0.328	0.324	0.000	0	
Bearing element	20.097	0.097	0.324	0.000	0	
Engine bearing 7	20.097					Stiff
Bearing element	20.194	0.097	0.324	0.000	0	
Shaft	20.572	0.378	0.324	0.000	0	
Total	20.572 m					37135 kg
Centre of gravity	4.671 m					

Figure A.3: SHAFT SEGMENTS PART 2

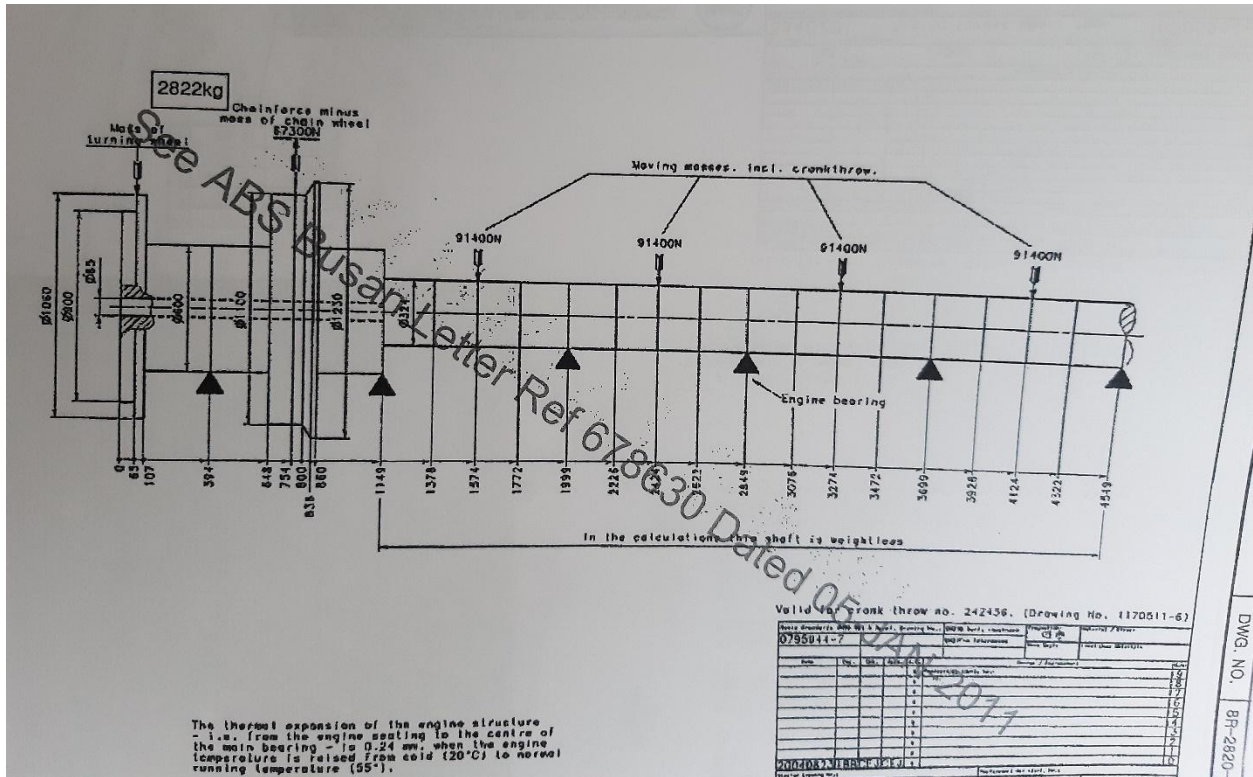


Figure A.4: M/E SHAFT MODEL

PROJECTED AREA (mm <sup>2</sup> )	484800	147030	117600
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INFLUENCE NUMBER

BEARINGS	BEARING LOAD VARIATION (kg/0.1m DOWN)					
	AFT STERN TUBE BUSH	FWD STERN TUBE BUSH	INTERMEDIATE SHAFT BRG.	AFTMOST BRG. OF ENGINE (#9)	AFTMOST MAIN BRG (#8)	#7 MAIN BRG
AFT STERN TUBE BUSH	-404	724	-413	303	-213	3
FWD STERN TUBE BUSH	724	-1374	931	-917	644	-9
INTERMEDIATE SHAFT BRG.	-413	931	-1027	2150	-1661	24
AFTMOST BRG. OF ENGINE (#9)	303	-917	2150	-15424	21550	-9370
AFTMOST MAIN BRG (#8)	-213	644	1661	21550	-39024	25691
#7 MAIN BRG	3	-9	24	-9370	25691	-29069

BEARING REACTIONS, VERTICAL OFF-SET & PRESSURE AT RUNNING CONDITION (DYNAMIC)

BEARINGS	AFT STERN TUBE BUSH	FWD STERN TUBE BUSH	INTERMEDIATE SHAFT BRG.	AFTMOST BRG. OF ENGINE (#9)	AFTMOST MAIN BRG (#8)	#7 MAIN BRG
VERTICAL OFF-SET (mm)	0 mm DOWNWARD	0 mm DOWNWARD	3.88 mm DOWNWARD	10.52 mm DOWNWARD	11.61 mm DOWNWARD	12.85 mm DOWNWARD
REACTION (kg)	23447	2064	2832	2410	3098	9498
BRG PRESSURE (kg/cm <sup>2</sup> )	5.25	1.41	2.41	PERMISSIBLE BEARING LOAD (kg)		
PERMISS. BRG. PRESSURE (kg/cm <sup>2</sup> )	8.1	8.1	15.0	0 ~ 29,690	1.531 ~ 29,690	

\*NOTE: EFFECT OF PROPELLER THRUST OF PROPELLER WAS REFLECTED ON THE INPUT DATA. THE APPLIED MOMENT IS 117700 N-m

BEARING REACTIONS & PRESSURE AT STATIC/HOT CONDITION. FOR REFERENCE

REACTION (kg)	19037	9110	2051	2984	2694	9504
BRG PRESSURE (kg/cm <sup>2</sup> )	3.93	6.20	1.74	PERMISSIBLE BEARING LOAD (kg)		
				0 ~ 29,690	1.531 ~ 29,690	

M5370-8R-2820-208-07

A4 (210x297)

Figure A.5: BEARING REACTIONS AND INFLUENCE FACTORS

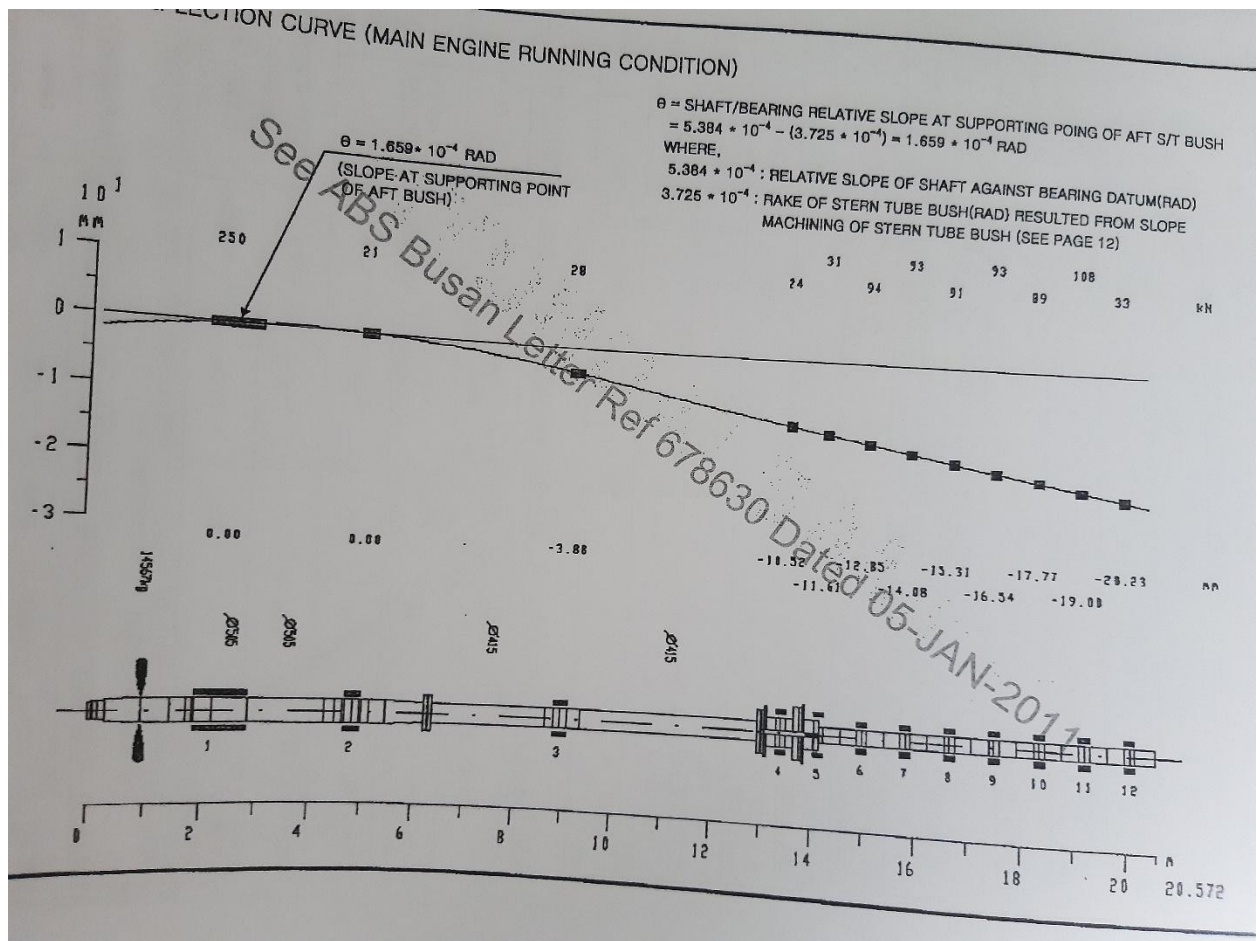


Figure A.6: DEFLECTION CURVE



# APPENDIX B

## “Omm Omm Omm Omm Omm”

Relative deviations (mm)	$y_l(mm)$	Propeller Immersion Status			
		0%	50%	75%	100%
0 0.1 0.1 0.1 0.1	0 0.1 -3.78 -10.41 -11.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 0.1 0.1	0 0.1 -3.98 -10.41 -11.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 -0.1 -0.1	0 0.1 -3.98 -10.61 -11.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.2 -0.1 -0.1	0 0.1 -4.08 -10.61 -11.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.1 0.1 0.1	0 -0.1 -3.98 -10.41 -11.51	MARGINAL	ACCEPTABLE	ACCEPTABLE	ACCEPTABLE
0 -0.1 -0.2 0.1 0.1	0 -0.1 -4.08 -10.41 -11.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.1 -0.1 -0.1	0 -0.1 -3.98 -10.62 -11.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.2 -0.1 -0.1	0 -0.1 -4.08 -10.62 -11.71	ACCEPTABLE	ACCEPTABLE	ACCEPTABLE	ACCEPTABLE
0 -0.1 -0.3 -0.1 -0.1	0 -0.1 -4.18 -10.62 -11.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 0.1 0.2	0 0.1 -3.98 -10.42 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.2 0.1 0.2	0 0.1 -4.08 -10.42 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.3 -0.1 -0.2	0 0.1 -3.58 -10.62 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.2 -0.1 -0.2	0 0.1 -3.68 -10.62 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.1 -0.1 -0.2	0 0.1 -3.78 -10.62 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.2 0.1 0.2	0 -0.1 -4.08 -10.42 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.3 0.1 0.2	0 -0.1 -4.18 -10.42 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.5 -0.2 -0.1	0 -0.1 -4.38 -10.72 -11.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.2 0.2 0.2	0 0.1 -3.68 -10.32 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.1 0.2 0.2	0 0.1 -3.78 -10.32 -11.41	ACCEPTABLE	ACCEPTABLE	MARGINAL	MARGINAL
0 0.1 -0.1 -0.2 -0.2	0 0.1 -3.98 -10.72 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.2 -0.2 -0.2	0 0.1 -4.08 -10.72 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.1 0.2 0.2	0 -0.1 -3.98 -10.32 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.2 -0.2 -0.2	0 -0.1 -4.08 -10.72 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.3 -0.2 -0.2	0 -0.1 -4.18 -10.72 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 0.2 0.3	0 0.1 -3.98 -10.32 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.2 0.2 0.3	0 0.1 -4.08 -10.32 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.2 -0.2 -0.3	0 0.1 -3.68 -10.72 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.1 -0.2 -0.3	0 0.1 -3.78 -10.72 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.2 0.2 0.3	0 -0.1 -4.08 -10.32 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.3 0.2 0.3	0 -0.1 -4.18 -10.32 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.8 -0.3 -0.1	0 -0.1 -4.68 -10.82 -11.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.5 -0.3 -0.2	0 -0.1 -4.38 -10.82 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.3 0.3 0.3	0 0.1 -3.58 -10.22 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.2 0.3 0.3	0 0.1 -3.68 -10.22 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.1 0.3 0.3	0 0.1 -3.78 -10.22 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 -0.3 -0.3	0 0.1 -3.98 -10.82 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.2 -0.3 -0.3	0 0.1 -4.08 -10.82 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.3 -0.3 -0.3	0 0.1 -4.18 -10.82 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.2 -0.3 -0.3	0 -0.1 -4.08 -10.82 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.3 -0.3 -0.3	0 -0.1 -4.18 -10.82 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.3 0.3 0.5	0 0.1 -4.18 -10.22 -11.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.5 0.3 0.5	0 -0.1 -4.38 -10.22 -11.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.3 0.5 0.5	0 0.1 -3.58 -10.02 -11.11	MARGINAL	ACCEPTABLE	ACCEPTABLE	ACCEPTABLE
0 0.1 0.2 0.5 0.5	0 0.1 -3.68 -10.02 -11.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.3 -0.5 -0.5	0 0.1 -4.18 -11.02 -12.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 0.1 0.5 0.5	0 -0.1 -3.78 -10.02 -11.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.5 -0.5 -0.5	0 -0.1 -4.38 -11.02 -12.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.5 0.8 0.8	0 0.1 -3.38 -9.72 -10.81	MARGINAL	MARGINAL	ACCEPTABLE	ACCEPTABLE
0 0.1 -0.5 -0.8 -0.8	0 0.1 -4.38 -11.32 -12.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.2 0.1 0.1	0 0.2 -3.68 -10.42 -11.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL

0 0.2 0.1 0.1 0.1	0 0.2 -3.78 -10.42 -11.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.1 -0.1 -0.1	0 0.2 -3.98 -10.62 -11.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.2 0.1 0.1	0 -0.2 -4.08 -10.42 -11.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.3 -0.1 -0.1	0 -0.2 -4.18 -10.62 -11.71	MARGINAL	ACCEPTABLE	ACCEPTABLE	ACCEPTABLE
0 0.2 -0.1 0.1 0.2	0 0.2 -3.98 -10.42 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.3 -0.1 -0.2	0 0.2 -3.58 -10.62 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.2 -0.1 -0.2	0 0.2 -3.68 -10.62 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.3 0.1 0.2	0 -0.2 -4.18 -10.42 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.5 -0.1 -0.3	0 0.2 -3.38 -10.62 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.5 0.2 0.1	0 0.2 -3.38 -10.32 -11.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.3 -0.2 -0.1	0 0.2 -4.18 -10.72 -11.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.5 -0.2 -0.1	0 -0.2 -4.38 -10.72 -11.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.3 0.2 0.2	0 0.2 -3.58 -10.32 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.2 0.2 0.2	0 0.2 -3.68 -10.32 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.1 0.2 0.2	0 0.2 -3.78 -10.32 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.1 -0.2 -0.2	0 0.2 -3.98 -10.72 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.3 -0.2 -0.2	0 -0.2 -4.18 -10.72 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.1 0.2 0.3	0 0.2 -3.98 -10.32 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.3 -0.2 -0.3	0 0.2 -3.58 -10.72 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.2 -0.2 -0.3	0 0.2 -3.68 -10.72 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.1 -0.2 -0.3	0 0.2 -3.78 -10.72 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.2 0.2 0.3	0 -0.2 -4.08 -10.32 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.3 0.2 0.3	0 -0.2 -4.18 -10.32 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.5 0.3 0.2	0 0.2 -3.38 -10.22 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.3 0.3 0.3	0 0.2 -3.58 -10.22 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.2 0.3 0.3	0 0.2 -3.68 -10.22 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.1 0.3 0.3	0 0.2 -3.78 -10.22 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.1 -0.3 -0.3	0 0.2 -3.98 -10.82 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.2 -0.3 -0.3	0 0.2 -4.08 -10.82 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.1 0.3 0.3	0 -0.2 -3.98 -10.22 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.3 -0.3 -0.3	0 -0.2 -4.18 -10.82 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.5 -0.3 -0.3	0 -0.2 -4.38 -10.82 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.5 0.3 0.5	0 -0.2 -4.38 -10.22 -11.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.5 0.5 0.5	0 0.2 -3.38 -10.02 -11.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.3 0.5 0.5	0 0.2 -3.58 -10.02 -11.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.2 0.5 0.5	0 0.2 -3.68 -10.02 -11.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.2 -0.5 -0.5	0 0.2 -4.08 -11.02 -12.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.3 -0.5 -0.5	0 0.2 -4.18 -11.02 -12.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.5 -0.5 -0.5	0 -0.2 -4.38 -11.02 -12.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.5 0.8 0.8	0 0.2 -3.38 -9.72 -10.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.8 -0.8 -0.8	0 -0.2 -4.68 -11.32 -12.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.2 0.1 0.1	0 0.3 -3.68 -10.42 -11.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.1 0.1 0.1	0 0.3 -3.78 -10.42 -11.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.1 -0.1 -0.1	0 0.3 -3.78 -10.62 -11.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.8 -0.1 0.1	0 -0.3 -4.68 -10.62 -11.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.3 -0.1 -0.2	0 0.3 -3.58 -10.62 -11.81	ACCEPTABLE	ACCEPTABLE	ACCEPTABLE	MARGINAL
0 0.3 0.2 -0.1 -0.2	0 0.3 -3.68 -10.62 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.5 0.1 0.2	0 -0.3 -4.38 -10.42 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.5 0.2 0.1	0 0.3 -3.38 -10.32 -11.51	ACCEPTABLE	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.5 -0.2 -0.1	0 -0.3 -4.38 -10.72 -11.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.3 0.2 0.2	0 0.3 -3.58 -10.32 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.2 0.2 0.2	0 0.3 -3.68 -10.32 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.5 -0.2 -0.2	0 -0.3 -4.38 -10.72 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.3 -0.2 -0.3	0 0.3 -3.58 -10.72 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.2 -0.2 -0.3	0 0.3 -3.68 -10.72 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.5 0.3 0.2	0 0.3 -3.38 -10.22 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL

0 -0.3 -0.8 -0.3 -0.2	0 -0.3 -4.68 -10.82 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.3 0.3 0.3	0 0.3 -3.58 -10.22 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.2 0.3 0.3	0 0.3 -3.68 -10.22 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.1 -0.3 -0.3	0 0.3 -3.98 -10.82 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.5 -0.3 -0.3	0 -0.3 -4.38 -10.82 -11.91	MARGINAL	ACCEPTABLE	ACCEPTABLE	ACCEPTABLE
0 0.3 0.5 -0.3 -0.5	0 0.3 -3.38 -10.82 -12.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.5 0.3 0.5	0 -0.3 -4.38 -10.22 -11.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.5 0.5 0.5	0 0.3 -3.38 -10.02 -11.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.3 0.5 0.5	0 0.3 -3.58 -10.02 -11.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.2 -0.5 -0.5	0 0.3 -4.08 -11.02 -12.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.5 -0.5 -0.5	0 -0.3 -4.38 -11.02 -12.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.5 0.8 0.8	0 0.3 -3.38 -9.72 -10.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.8 -0.8 -0.8	0 -0.3 -4.68 -11.32 -12.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.3 0.1 0.1	0 0.5 -3.58 -10.42 -11.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.8 0.1 -0.1	0 0.5 -3.08 -10.42 -11.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.2 -0.1 -0.1	0 0.5 -3.68 -10.62 -11.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.5 -0.1 -0.2	0 0.5 -3.38 -10.62 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.8 -0.1 -0.3	0 0.5 -3.08 -10.62 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.5 -0.8 0.1 0.3	0 -0.5 -4.68 -10.42 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.5 0.2 0.1	0 0.5 -3.38 -10.32 -11.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.5 -0.8 -0.2 -0.1	0 -0.5 -4.68 -10.72 -11.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.3 -0.2 -0.3	0 0.5 -3.58 -10.72 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.8 0.3 0.2	0 0.5 -3.08 -10.22 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.5 -0.8 -0.3 -0.2	0 -0.5 -4.68 -10.82 -11.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.5 0.3 0.3	0 0.5 -3.38 -10.22 -11.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.5 -0.3 -0.5	0 0.5 -3.38 -10.82 -12.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.5 -0.8 0.3 0.5	0 -0.5 -4.68 -10.22 -11.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.5 0.5 0.5	0 0.5 -3.38 -10.02 -11.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.5 -0.8 -0.5 -0.5	0 -0.5 -4.68 -11.02 -12.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.8 -0.5 -0.8	0 0.5 -3.08 -11.02 -12.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.5 -0.8 0.5 0.8	0 -0.5 -4.68 -10.02 -10.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.8 0.8 0.8	0 0.5 -3.08 -9.72 -10.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.8 -0.1 -0.3	0 0.8 -3.08 -10.62 -11.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.8 0.2 0.1	0 0.8 -3.08 -10.32 -11.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.8 0.3 0.2	0 0.8 -3.08 -10.22 -11.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.8 -0.3 -0.5	0 0.8 -3.08 -10.82 -12.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL

Table B.1: INITIAL POINTS AND LINE COLOR DEFINITION

$y_l(mm)$	$absolute y_R(mm)$
0 0.1 -3.78 -10.41 -11.51	0 0 4.04 10.87 11.97
0 0.1 -3.98 -10.41 -11.51	0 0 4.24 10.87 11.97
0 0.1 -3.98 -10.61 -11.71	0 0 4.24 11.07 12.17
0 0.1 -4.08 -10.61 -11.71	0 0 4.34 11.07 12.17
0 -0.1 -3.98 -10.41 -11.51	0 0 3.72 9.97 11.05
0 -0.1 -4.08 -10.41 -11.51	0 0 3.82 9.97 11.05
0 -0.1 -3.98 -10.62 -11.71	0 0 3.72 10.17 11.25
0 -0.1 -4.08 -10.62 -11.71	0 0 3.82 10.17 11.25
0 -0.1 -4.18 -10.62 -11.71	0 0 3.92 10.17 11.25
0 0.1 -3.98 -10.42 -11.41	0 0 4.24 10.87 11.87
0 0.1 -4.08 -10.42 -11.41	0 0 4.34 10.87 11.87
0 0.1 -3.58 -10.62 -11.81	0 0 3.84 11.07 12.27
0 0.1 -3.68 -10.62 -11.81	0 0 3.94 11.07 12.27
0 0.1 -3.78 -10.62 -11.81	0 0 4.04 11.07 12.27
0 -0.1 -4.08 -10.42 -11.41	0 0 3.82 9.97 10.95

0 -0.1 -4.18 -10.42 -11.41	00 3.92 9.97 10.95
0 -0.1 -4.38 -10.72 -11.71	00 4.12 10.27 11.25
0 0.1 -3.68 -10.32 -11.41	00 3.94 10.77 11.87
0 0.1 -3.78 -10.32 -11.41	00 4.04 10.77 11.87
0 0.1 -3.98 -10.72 -11.81	00 4.24 11.17 12.27
0 0.1 -4.08 -10.72 -11.81	00 4.34 11.17 12.27
0 -0.1 -3.98 -10.32 -11.41	00 3.72 9.87 10.95
0 -0.1 -4.08 -10.72 -11.81	00 3.82 10.27 11.35
0 -0.1 -4.18 -10.72 -11.81	00 3.92 10.27 11.35
0 0.1 -3.98 -10.32 -11.31	00 4.24 10.77 11.77
0 0.1 -4.08 -10.32 -11.31	00 4.34 10.77 11.77
0 0.1 -3.68 -10.72 -11.91	00 3.94 11.17 12.37
0 0.1 -3.78 -10.72 -11.91	00 4.04 11.17 12.37
0 -0.1 -4.08 -10.32 -11.31	00 3.82 9.87 10.85
0 -0.1 -4.18 -10.32 -11.31	00 3.92 9.87 10.85
0 -0.1 -4.68 -10.82 -11.71	00 4.42 10.37 11.25
0 -0.1 -4.38 -10.82 -11.81	00 4.12 10.37 11.35
0 0.1 -3.58 -10.22 -11.31	00 3.84 10.67 11.77
0 0.1 -3.68 -10.22 -11.31	00 3.94 10.67 11.77
0 0.1 -3.78 -10.22 -11.31	00 4.04 10.67 11.77
0 0.1 -3.98 -10.82 -11.91	00 4.24 11.27 12.37
0 0.1 -4.08 -10.82 -11.91	00 4.34 11.27 12.37
0 0.1 -4.18 -10.82 -11.91	00 4.44 11.27 12.37
0 -0.1 -4.08 -10.82 -11.91	00 3.82 10.37 11.45
0 -0.1 -4.18 -10.82 -11.91	00 3.92 10.37 11.45
0 0.1 -4.18 -10.22 -11.11	00 4.44 10.67 11.57
0 -0.1 -4.38 -10.22 -11.11	00 4.12 9.77 10.65
0 0.1 -3.58 -10.02 -11.11	00 3.84 10.47 11.57
0 0.1 -3.68 -10.02 -11.11	00 3.94 10.47 11.57
0 0.1 -4.18 -11.02 -12.11	00 4.44 11.47 12.57
0 -0.1 -3.78 -10.02 -11.11	00 3.52 9.57 10.65
0 -0.1 -4.38 -11.02 -12.11	00 4.12 10.57 11.65
0 0.1 -3.38 -9.72 -10.81	00 3.64 10.17 11.27
0 0.1 -4.38 -11.32 -12.41	00 4.64 11.77 12.87
0 0.2 -3.68 -10.42 -11.51	00 4.19 11.32 12.42
0 0.2 -3.78 -10.42 -11.51	00 4.29 11.32 12.42
0 0.2 -3.98 -10.62 -11.71	00 4.49 11.52 12.62
0 -0.2 -4.08 -10.42 -11.51	00 3.57 9.52 10.6
0 -0.2 -4.18 -10.62 -11.71	00 3.67 9.72 10.8
0 0.2 -3.98 -10.42 -11.41	00 4.49 11.32 12.32
0 0.2 -3.58 -10.62 -11.81	00 4.09 11.52 12.72
0 0.2 -3.68 -10.62 -11.81	00 4.19 11.52 12.72
0 -0.2 -4.18 -10.42 -11.41	00 3.67 9.52 10.5
0 0.2 -3.38 -10.62 -11.91	00 3.89 11.52 12.82
0 0.2 -3.38 -10.32 -11.51	00 3.89 11.22 12.42
0 0.2 -4.18 -10.72 -11.71	00 4.69 11.62 12.62
0 -0.2 -4.38 -10.72 -11.71	00 3.87 9.82 10.8
0 0.2 -3.58 -10.32 -11.41	00 4.09 11.22 12.32
0 0.2 -3.68 -10.32 -11.41	00 4.19 11.22 12.32
0 0.2 -3.78 -10.32 -11.41	00 4.29 11.22 12.32
0 0.2 -3.98 -10.72 -11.81	00 4.49 11.62 12.72
0 -0.2 -4.18 -10.72 -11.81	00 3.67 9.82 10.9
0 0.2 -3.98 -10.32 -11.31	00 4.49 11.22 12.22
0 0.2 -3.58 -10.72 -11.91	00 4.09 11.62 12.82
0 0.2 -3.68 -10.72 -11.91	00 4.19 11.62 12.82
0 0.2 -3.78 -10.72 -11.91	00 4.29 11.62 12.82
0 -0.2 -4.08 -10.32 -11.31	00 3.57 9.42 10.4
0 -0.2 -4.18 -10.32 -11.31	00 3.67 9.42 10.4
0 0.2 -3.38 -10.22 -11.41	00 3.89 11.12 12.32
0 0.2 -3.58 -10.22 -11.31	00 4.09 11.12 12.22
0 0.2 -3.68 -10.22 -11.31	00 4.19 11.12 12.22
0 0.2 -3.78 -10.22 -11.31	00 4.29 11.12 12.22
0 0.2 -3.98 -10.82 -11.91	00 4.49 11.72 12.82
0 0.2 -4.08 -10.82 -11.91	00 4.59 11.72 12.82
0 -0.2 -3.98 -10.22 -11.31	00 3.47 9.32 10.4
0 -0.2 -4.18 -10.82 -11.91	00 3.67 9.92 11

0 -0.2 -4.38 -10.82 -11.91	00 3.87 9.92 11
0 -0.2 -4.38 -10.22 -11.11	00 3.87 9.32 10.2
0 0.2 -3.38 -10.02 -11.11	00 3.89 10.92 12.02
0 0.2 -3.58 -10.02 -11.11	00 4.09 10.92 12.02
0 0.2 -3.68 -10.02 -11.11	00 4.19 10.92 12.02
0 0.2 -4.08 -11.02 -12.11	00 4.59 11.92 13.02
0 0.2 -4.18 -11.02 -12.11	00 4.69 11.92 13.02
0 -0.2 -4.38 -11.02 -12.11	00 3.87 10.12 11.2
0 0.2 -3.38 -9.72 -10.81	00 3.89 10.62 11.72
0 -0.2 -4.68 -11.32 -12.41	00 4.17 10.42 11.5
0 0.3 -3.68 -10.42 -11.51	00 4.45 11.77 12.88
0 0.3 -3.78 -10.42 -11.51	00 4.55 11.77 12.88
0 0.3 -3.78 -10.62 -11.71	00 4.55 11.97 13.08
0 -0.3 -4.68 -10.62 -11.51	00 3.91 9.27 10.14
0 0.3 -3.58 -10.62 -11.81	00 4.35 11.97 13.18
0 0.3 -3.68 -10.62 -11.81	00 4.45 11.97 13.18
0 -0.3 -4.38 -10.42 -11.41	00 3.61 9.07 10.04
0 0.3 -3.38 -10.32 -11.51	00 4.15 11.67 12.88
0 -0.3 -4.38 -10.72 -11.71	00 3.61 9.37 10.34
0 0.3 -3.58 -10.32 -11.41	00 4.35 11.67 12.78
0 0.3 -3.68 -10.32 -11.41	00 4.45 11.67 12.78
0 -0.3 -4.38 -10.72 -11.81	00 3.61 9.37 10.44
0 0.3 -3.58 -10.72 -11.91	00 4.35 12.07 13.28
0 0.3 -3.68 -10.72 -11.91	00 4.45 12.07 13.28
0 0.3 -3.38 -10.22 -11.41	00 4.15 11.57 12.78
0 -0.3 -4.68 -10.82 -11.81	00 3.91 9.47 10.44
0 0.3 -3.58 -10.22 -11.31	00 4.35 11.57 12.68
0 0.3 -3.68 -10.22 -11.31	00 4.45 11.57 12.68
0 0.3 -3.98 -10.82 -11.91	00 4.75 12.17 13.28
0 -0.3 -4.38 -10.82 -11.91	00 3.61 9.47 10.54
0 0.3 -3.38 -10.82 -12.11	00 4.15 12.17 13.48
0 -0.3 -4.38 -10.22 -11.11	00 3.61 8.87 9.74
0 0.3 -3.38 -10.02 -11.11	00 4.15 11.37 12.48
0 0.3 -3.58 -10.02 -11.11	00 4.35 11.37 12.48
0 0.3 -4.08 -11.02 -12.11	00 4.85 12.37 13.48
0 -0.3 -4.38 -11.02 -12.11	00 3.61 9.67 10.74
0 0.3 -3.38 -9.72 -10.81	00 4.15 11.07 12.18
0 -0.3 -4.68 -11.32 -12.41	00 3.91 9.97 11.04
0 0.5 -3.58 -10.42 -11.51	00 4.87 12.66 13.79
0 0.5 -3.08 -10.42 -11.71	00 4.37 12.66 13.99
0 0.5 -3.68 -10.62 -11.71	00 4.97 12.86 13.99
0 0.5 -3.38 -10.62 -11.81	00 4.67 12.86 14.09
0 0.5 -3.08 -10.62 -11.91	00 4.37 12.86 14.19
0 -0.5 -4.68 -10.42 -11.31	00 3.39 8.18 9.03
0 0.5 -3.38 -10.32 -11.51	00 4.67 12.56 13.79
0 -0.5 -4.68 -10.72 -11.71	00 3.39 8.48 9.43
0 0.5 -3.58 -10.72 -11.91	00 4.87 12.96 14.19
0 0.5 -3.08 -10.22 -11.41	00 4.37 12.46 13.69
0 -0.5 -4.68 -10.82 -11.81	00 1.79 8.58 9.53
0 0.5 -3.38 -10.22 -11.31	00 4.67 12.46 13.59
0 0.5 -3.38 -10.82 -12.11	00 4.67 13.06 14.39
0 -0.5 -4.68 -10.22 -11.11	00 3.39 7.98 8.83
0 0.5 -3.38 -10.02 -11.11	00 4.67 12.26 13.39
0 -0.5 -4.68 -11.02 -12.11	00 3.39 8.78 9.83
0 0.5 -3.08 -11.02 -12.41	00 4.37 13.26 14.69
0 -0.5 -4.68 -10.02 -10.81	00 3.39 7.78 8.53
0 0.5 -3.08 -9.72 -10.81	00 4.37 11.96 13.09
0 0.8 -3.08 -10.62 -11.91	00 5.14 14.21 15.56
0 0.8 -3.08 -10.32 -11.51	00 5.14 13.91 15.16
0 0.8 -3.08 -10.22 -11.41	00 5.14 13.81 15.06
0 0.8 -3.08 -10.82 -12.11	00 5.14 14.41 15.76

Table B.2: ROTATED POINTS AND LINE COLOR DEFINITION

**“0mm -1mm -2mm -2mm -2mm”**

Relative deviations (mm)	$y_i$ (mm)	Propeller Immersion Status			
		0%	50%	75%	100%
0 0.1 0.2 0.1 0.1	0 -0.9 -5.68 -12.42 -13.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.1 0.1 0.1	0 -0.9 -5.78 -12.42 -13.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.1 -0.1 -0.1	0 -0.9 -5.78 -12.62 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 -0.1 -0.1	0 -0.9 -5.98 -12.62 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.1 0.1 0.1	0 -1.1 -5.98 -12.42 -13.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.2 -0.1 -0.1	0 -1.1 -6.08 -12.62 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 0.1 0.2	0 -0.9 -5.98 -12.42 -13.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.2 0.1 0.2	0 -1.1 -6.08 -12.42 -13.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.3 0.1 0.2	0 -1.1 -6.18 -12.42 -13.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.3 0.1 0.3	0 -0.9 -6.18 -12.42 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.5 0.1 0.3	0 -1.1 -6.38 -12.42 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.2 -0.2 -0.1	0 -0.9 -6.08 -12.72 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.3 -0.2 -0.1	0 -0.9 -6.18 -12.72 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.3 -0.2 -0.1	0 -1.1 -6.18 -12.72 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.5 -0.2 -0.1	0 -1.1 -6.38 -12.72 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.2 0.2 0.2	0 -0.9 -5.68 -12.32 -13.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.1 0.2 0.2	0 -0.9 -5.78 -12.32 -13.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.1 -0.2 -0.2	0 -0.9 -5.78 -12.72 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 -0.2 -0.2	0 -0.9 -5.98 -12.72 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.2 -0.2 -0.2	0 -1.1 -6.08 -12.72 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.3 -0.2 -0.2	0 -1.1 -6.18 -12.72 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.1 0.2 0.3	0 -0.9 -5.78 -12.32 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 0.2 0.3	0 -0.9 -5.98 -12.32 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.2 0.2 0.3	0 -1.1 -6.08 -12.32 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.3 -0.3 -0.2	0 -0.9 -6.18 -12.82 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.5 -0.3 -0.2	0 -1.1 -6.38 -12.82 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.2 0.3 0.3	0 -0.9 -5.68 -12.22 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 -0.3 -0.3	0 -0.9 -5.98 -12.82 -13.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.2 -0.3 -0.3	0 -0.9 -6.08 -12.82 -13.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.3 -0.3 -0.3	0 -1.1 -6.18 -12.82 -13.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.2 0.3 0.5	0 -0.9 -6.08 -12.22 -13.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.3 0.3 0.5	0 -1.1 -6.18 -12.22 -13.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.8 -0.5 -0.3	0 -1.1 -6.68 -13.02 -13.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.3 0.5 0.5	0 -0.9 -5.58 -12.02 -13.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 -0.5 -0.5	0 -0.9 -5.98 -13.02 -14.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.2 -0.5 -0.5	0 -0.9 -6.08 -13.02 -14.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.3 -0.5 -0.5	0 -0.9 -6.18 -13.02 -14.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.3 -0.5 -0.5	0 -1.1 -6.18 -13.02 -14.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.5 -0.8 -0.8	0 -1.1 -6.38 -13.32 -14.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.3 0.1 0.1	0 -0.8 -5.58 -12.42 -13.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.2 0.1 0.1	0 -0.8 -5.68 -12.42 -13.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.1 0.1 0.1	0 -0.8 -5.78 -12.42 -13.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.2 -0.1 -0.1	0 -0.8 -5.68 -12.62 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.1 -0.1 -0.1	0 -0.8 -5.78 -12.62 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.3 -0.1 -0.2	0 -0.8 -5.58 -12.62 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.5 0.1 0.3	0 -1.2 -6.38 -12.42 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.2 -0.2 -0.1	0 -0.8 -6.08 -12.72 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.5 -0.2 -0.1	0 -1.2 -6.38 -12.72 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.3 0.2 0.2	0 -0.8 -5.58 -12.32 -13.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.2 0.2 0.2	0 -0.8 -5.68 -12.32 -13.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.1 -0.2 -0.2	0 -0.8 -5.78 -12.72 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.1 0.2 0.3	0 -0.8 -5.78 -12.32 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL

0 0.2 0.3 -0.2 -0.3	0 -0.8 -5.58 -12.72 -13.91	MARGINAL	MARGINAL	ACCEPTABLE	MARGINAL
0 0.2 0.2 -0.2 -0.3	0 -0.8 -5.68 -12.72 -13.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.2 -0.3 -0.2	0 -0.8 -6.08 -12.82 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.5 -0.3 -0.2	0 -1.2 -6.38 -12.82 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.3 0.3 0.3	0 -0.8 -5.58 -12.22 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.2 0.3 0.3	0 -0.8 -5.68 -12.22 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.1 -0.3 -0.3	0 -0.8 -5.98 -12.82 -13.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.1 0.3 0.5	0 -0.8 -5.98 -12.22 -13.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.5 0.3 0.5	0 -1.2 -6.38 -12.22 -13.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.1 -0.5 -0.5	0 -0.8 -5.98 -13.02 -14.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.2 -0.5 -0.5	0 -0.8 -6.08 -13.02 -14.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.5 -0.5 -0.5	0 -1.2 -6.38 -13.02 -14.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.5 0.5 0.8	0 -1.2 -6.38 -12.02 -12.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.3 -0.8 -0.8	0 -0.8 -6.18 -13.32 -14.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.3 0.1 0.1	0 -0.7 -5.58 -12.42 -13.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.2 0.1 0.1	0 -0.7 -5.68 -12.42 -13.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.2 -0.1 -0.1	0 -0.7 -5.68 -12.62 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.1 -0.1 -0.1	0 -0.7 -5.78 -12.62 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.1 0.1 0.2	0 -0.7 -5.78 -12.42 -13.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.5 -0.1 -0.2	0 -0.7 -5.38 -12.62 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.1 -0.2 -0.1	0 -0.7 -5.98 -12.72 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.3 0.2 0.2	0 -0.7 -5.58 -12.32 -13.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.2 -0.2 -0.2	0 -0.7 -5.68 -12.72 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.1 -0.2 -0.2	0 -0.7 -5.78 -12.72 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.2 0.2 0.3	0 -0.7 -5.68 -12.32 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.1 0.2 0.3	0 -0.7 -5.78 -12.32 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.3 -0.2 -0.3	0 -0.7 -5.58 -12.72 -13.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.8 0.2 0.5	0 -1.3 -6.68 -12.32 -13.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.8 -0.3 -0.1	0 -1.3 -6.68 -12.82 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.5 0.3 0.3	0 -0.7 -5.38 -12.22 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.3 0.3 0.3	0 -0.7 -5.58 -12.22 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.1 -0.3 -0.3	0 -0.7 -5.78 -12.82 -13.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.5 0.3 0.5	0 -1.3 -6.38 -12.22 -13.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.5 0.5 0.5	0 -0.7 -5.38 -12.02 -13.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.1 -0.5 -0.5	0 -0.7 -5.98 -13.02 -14.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.5 0.5 0.8	0 -1.3 -6.38 -12.02 -12.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.8 -0.8 -0.8	0 -1.3 -6.68 -13.32 -14.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.5 0.1 0.1	0 -0.5 -5.38 -12.42 -13.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.3 -0.1 -0.1	0 -0.5 -5.58 -12.62 -13.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.2 0.1 0.2	0 -0.5 -5.68 -12.42 -13.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.5 -0.1 -0.2	0 -0.5 -5.38 -12.62 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.8 -0.1 -0.3	0 -0.5 -5.08 -12.62 -13.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.8 0.2 0.1	0 -0.5 -5.08 -12.32 -13.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.5 0.2 0.2	0 -0.5 -5.38 -12.32 -13.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.5 -0.2 -0.3	0 -0.5 -5.38 -12.72 -13.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.8 0.3 0.2	0 -0.5 -5.08 -12.22 -13.41	ACCEPTABLE	ACCEPTABLE	ACCEPTABLE	ACCEPTABLE
0 0.5 0.5 0.3 0.3	0 -0.5 -5.38 -12.22 -13.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.2 -0.3 -0.3	0 -0.5 -5.68 -12.82 -13.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.8 -0.3 -0.5	0 -0.5 -5.08 -12.82 -14.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.5 -0.8 0.5 0.8	0 -1.5 -6.68 -12.02 -12.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.8 0.8 0.8	0 -0.5 -5.08 -11.72 -12.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.8 -0.1 -0.2	0 -0.2 -5.08 -12.62 -13.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.8 -0.3 -0.5	0 -0.2 -5.08 -12.82 -14.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL

Table B.3: INITIAL POINTS AND LINE COLOR DEFINITION

$y_l(mm)$	$absolute y_R(mm)$
0 -0.9 -5.68 -12.42 -13.51	0 0 3.37 8.38 9.4
0 -0.9 -5.78 -12.42 -13.51	0 0 3.47 8.38 9.4
0 -0.9 -5.78 -12.62 -13.71	0 0 3.47 8.58 9.6
0 -0.9 -5.98 -12.62 -13.71	0 0 3.67 8.58 9.6
0 -1.1 -5.98 -12.42 -13.51	0 0 3.15 7.49 8.79
0 -1.1 -6.08 -12.62 -13.71	0 0 3.25 7.69 8.69
0 -0.9 -5.98 -12.42 -13.41	0 0 3.67 8.38 9.3
0 -1.1 -6.08 -12.42 -13.41	0 0 3.25 7.49 8.39
0 -1.1 -6.18 -12.42 -13.41	0 0 3.35 7.49 8.39
0 -0.9 -6.18 -12.42 -13.31	0 0 3.87 8.38 9.2
0 -1.1 -6.38 -12.42 -13.31	0 0 3.55 7.49 8.29
0 -0.9 -6.08 -12.72 -13.71	0 0 3.77 8.68 9.6
0 -0.9 -6.18 -12.72 -13.71	0 0 3.87 8.68 9.6
0 -1.1 -6.18 -12.72 -13.71	0 0 3.35 7.79 8.69
0 -1.1 -6.38 -12.72 -13.71	0 0 3.55 7.79 8.69
0 -0.9 -5.68 -12.32 -13.41	0 0 3.37 8.28 9.3
0 -0.9 -5.78 -12.32 -13.41	0 0 3.47 8.28 9.3
0 -0.9 -5.78 -12.72 -13.81	0 0 3.47 8.68 9.7
0 -0.9 -5.98 -12.72 -13.81	0 0 3.67 8.68 9.7
0 -1.1 -6.08 -12.72 -13.81	0 0 3.25 7.79 8.79
0 -1.1 -6.18 -12.72 -13.81	0 0 3.35 7.79 8.79
0 -0.9 -5.78 -12.32 -13.31	0 0 3.47 8.28 9.2
0 -0.9 -5.98 -12.32 -13.31	0 0 3.67 8.28 9.2
0 -1.1 -6.08 -12.32 -13.31	0 0 3.25 7.39 8.29
0 -0.9 -6.18 -12.82 -13.81	0 0 3.87 8.79 9.7
0 -1.1 -6.38 -12.82 -13.81	0 0 3.55 7.89 8.79
0 -0.9 -5.68 -12.22 -13.31	0 0 3.37 8.18 9.2
0 -0.9 -5.98 -12.82 -13.91	0 0 3.67 8.78 9.8
0 -0.9 -6.08 -12.82 -13.91	0 0 3.77 8.78 9.8
0 -1.1 -6.18 -12.82 -13.91	0 0 3.35 7.89 8.89
0 -0.9 -6.08 -12.22 -13.11	0 0 3.77 8.18 9
0 -1.1 -6.18 -12.22 -13.11	0 0 3.35 7.29 8.09
0 -1.1 -6.68 -13.02 -13.91	0 0 3.85 8.09 8.89
0 -0.9 -5.58 -12.02 -13.11	0 0 3.27 7.98 9
0 -0.9 -5.98 -13.02 -14.11	0 0 3.67 8.98 10
0 -0.9 -6.08 -13.02 -14.11	0 0 3.77 8.98 10
0 -0.9 -6.18 -13.02 -14.11	0 0 3.87 8.98 10
0 -1.1 -6.18 -13.02 -14.11	0 0 3.35 8.09 9.09
0 -1.1 -6.38 -13.32 -14.41	0 0 3.55 8.39 9.39
0 -0.8 -5.58 -12.42 -13.51	0 0 3.52 8.83 9.86
0 -0.8 -5.68 -12.42 -13.51	0 0 3.62 8.83 9.86
0 -0.8 -5.78 -12.42 -13.51	0 0 3.72 8.83 9.86
0 -0.8 -5.68 -12.62 -13.71	0 0 3.62 9.03 10.06
0 -0.8 -5.78 -12.62 -13.71	0 0 3.72 9.03 10.06
0 -0.8 -5.58 -12.62 -13.81	0 0 3.52 9.03 10.16
0 -1.2 -6.38 -12.42 -13.31	0 0 3.28 7.04 7.83
0 -0.8 -6.08 -12.72 -13.71	0 0 4.02 9.13 10.06
0 -1.2 -6.38 -12.72 -13.71	0 0 3.29 7.34 8.23
0 -0.8 -5.58 -12.32 -13.41	0 0 3.52 8.73 9.76
0 -0.8 -5.68 -12.32 -13.41	0 0 3.62 8.73 9.76
0 -0.8 -5.78 -12.72 -13.81	0 0 3.72 9.13 10.16
0 -0.8 -5.78 -12.32 -13.31	0 0 3.72 8.73 9.66
0 -0.8 -5.58 -12.72 -13.91	0 0 3.52 9.13 10.26
0 -0.8 -5.68 -12.72 -13.91	0 0 3.62 9.13 10.26
0 -0.8 -6.08 -12.82 -13.81	0 0 4.02 9.23 10.16
0 -1.2 -6.38 -12.82 -13.81	0 0 3.29 7.44 8.33
0 -0.8 -5.58 -12.22 -13.31	0 0 3.52 8.63 9.66
0 -0.8 -5.68 -12.22 -13.31	0 0 3.62 8.63 9.66
0 -0.8 -5.98 -12.82 -13.91	0 0 3.92 9.23 10.26
0 -0.8 -5.98 -12.22 -13.11	0 0 3.92 8.63 9.46



0 -1.2 -6.38 -12.22 -13.11	00 3.29 6.84 7.63
0 -0.8 -5.98 -13.02 -14.11	00 3.92 9.43 10.46
0 -0.8 -6.08 -13.02 -14.11	00 4.02 9.43 10.46
0 -1.2 -6.38 -13.02 -14.11	00 3.29 7.64 8.63
0 -1.2 -6.38 -12.02 -12.81	00 3.29 6.64 7.33
0 -0.8 -6.18 -13.32 -14.41	00 4.12 9.73 10.76
0 -0.7 -5.58 -12.42 -13.51	00 3.78 9.28 10.31
0 -0.7 -5.68 -12.42 -13.51	00 3.88 9.28 10.31
0 -0.7 -5.68 -12.62 -13.71	00 3.88 9.48 10.51
0 -0.7 -5.78 -12.62 -13.71	00 3.98 9.48 10.51
0 -0.7 -5.78 -12.42 -13.41	00 3.98 9.28 10.21
0 -0.7 -5.38 -12.62 -13.81	00 3.58 9.48 10.61
0 -0.7 -5.98 -12.72 -13.71	00 4.18 9.58 10.51
0 -0.7 -5.58 -12.32 -13.41	00 3.78 9.18 10.21
0 -0.7 -5.68 -12.72 -13.81	00 3.88 9.58 10.61
0 -0.7 -5.78 -12.72 -13.81	00 3.98 9.58 10.61
0 -0.7 -5.68 -12.32 -13.31	00 3.88 9.18 10.11
0 -0.7 -5.78 -12.32 -13.31	00 3.98 9.18 10.11
0 -0.7 -5.58 -12.72 -13.91	00 3.78 9.58 10.71
0 -1.3 -6.68 -12.32 -13.11	00 3.34 6.19 7.17
0 -1.3 -6.68 -12.82 -13.71	00 3.34 6.99 7.77
0 -0.7 -5.38 -12.22 -13.31	00 3.58 9.08 10.11
0 -0.7 -5.58 -12.22 -13.31	00 3.78 9.08 10.11
0 -0.7 -5.78 -12.82 -13.91	00 3.98 9.68 10.71
0 -1.3 -6.38 -12.22 -13.11	00 3.04 6.39 7.17
0 -0.7 -5.38 -12.02 -13.11	00 3.58 8.88 9.91
0 -0.7 -5.98 -13.02 -14.11	00 4.18 9.88 10.91
0 -1.3 -6.38 -12.02 -12.81	00 3.04 6.19 6.87
0 -1.3 -6.68 -13.32 -14.41	00 3.34 7.49 8.47
0 -0.5 -5.38 -12.42 -13.51	00 4.09 10.18 11.23
0 -0.5 -5.58 -12.62 -13.71	00 4.29 10.38 11.43
0 -0.5 -5.68 -12.42 -13.41	00 4.39 10.18 11.13
0 -0.5 -5.38 -12.62 -13.81	00 4.09 10.38 11.53
0 -0.5 -5.08 -12.62 -13.91	00 3.79 10.38 11.63
0 -0.5 -5.08 -12.32 -13.51	00 3.79 10.08 11.23
0 -0.5 -5.38 -12.32 -13.41	00 4.09 10.08 11.13
0 -0.5 -5.38 -12.72 -13.91	00 4.09 10.48 11.63
0 -0.5 -5.08 -12.22 -13.41	00 3.79 9.98 11.13
0 -0.5 -5.38 -12.22 -13.31	00 4.09 9.98 11.03
0 -0.5 -5.68 -12.82 -13.91	00 4.39 10.58 11.63
0 -0.5 -5.08 -12.82 -14.11	00 3.79 10.58 11.83
0 -1.5 -6.68 -12.02 -12.81	00 2.82 5.29 5.96
0 -0.5 -5.08 -11.72 -12.81	00 3.79 10.48 10.53
0 -0.2 -5.08 -12.62 -13.81	00 4.57 11.72 12.9
0 -0.2 -5.08 -12.82 -14.11	00 4.57 11.92 13.2

Table B.4: ROTATED POINTS AND LINE COLOR DEFINITION

**"0mm 3mm 7mm 8mm 8mm"**

Relative deviations (mm)	$y_l(mm)$	Propeller Immersion Status			
		0%	50%	75%	100%
0 -0.2 -0.8 -0.8 -0.8	0 2.8 2.32 -3.32 -4.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.5 0.8 0.8	0 3.2 3.62 -1.72 -2.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.8 0.8 0.8	0 3.5 3.92 -1.72 -2.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.8 0.8 0.8	0 3.8 3.92 -1.72 -2.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.3 0.5 0.5	0 3.2 3.42 -2.02 -3.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.5 0.5 0.5	0 3.3 3.62 -2.02 -3.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.5 0.5 0.5	0 3.5 3.62 -2.02 -3.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.8 0.5 0.8	0 2.7 2.32 -2.02 -2.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.8 0.5 0.8	0 2.8 2.32 -2.02 -2.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.8 0.5 0.8	0 2.9 2.32 -2.02 -2.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.5 0.5 0.8	0 3.1 2.62 -2.02 -2.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.5 0.5 0.8	0 3.2 2.62 -2.02 -2.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.5 0.5 0.8	0 3.3 2.62 -2.02 -2.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 -0.3 0.5 0.8	0 3.5 2.82 -2.02 -2.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.8 0.3 0.2	0 3.5 3.92 -2.22 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.8 0.3 0.2	0 3.8 3.92 -2.22 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.2 0.3 0.3	0 3.1 3.32 -2.22 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.1 0.3 0.3	0 3.1 3.22 -2.22 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.3 0.3 0.3	0 3.2 3.42 -2.22 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.2 0.3 0.3	0 3.2 3.32 -2.22 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.3 0.3 0.3	0 3.3 3.42 -2.22 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.2 0.3 0.3	0 3.3 3.32 -2.22 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.5 0.3 0.3	0 3.5 3.62 -2.22 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.3 0.3 0.3	0 3.5 3.42 -2.22 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.5 0.3 0.3	0 3.8 3.62 -2.22 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.5 0.3 0.5	0 2.8 2.62 -2.22 -3.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.5 0.3 0.5	0 2.9 2.62 -2.22 -3.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.3 0.3 0.5	0 3.1 2.82 -2.22 -3.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.5 0.3 0.5	0 3.1 2.62 -2.22 -3.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.3 0.3 0.5	0 3.2 2.82 -2.22 -3.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.2 0.3 0.5	0 3.3 2.92 -2.22 -3.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.3 0.3 0.5	0 3.3 2.82 -2.22 -3.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 -0.2 0.3 0.5	0 3.5 2.92 -2.22 -3.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.5 0.2 0.1	0 3.3 3.62 -2.32 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.8 0.2 0.1	0 3.8 3.92 -2.32 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.1 0.2 0.2	0 3.1 3.22 -2.32 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.3 0.2 0.2	0 3.2 3.42 -2.32 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.2 0.2 0.2	0 3.2 3.32 -2.32 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.1 0.2 0.2	0 3.2 3.22 -2.32 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.3 0.2 0.2	0 3.3 3.42 -2.32 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.2 0.2 0.2	0 3.3 3.32 -2.32 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.1 0.2 0.2	0 3.3 3.22 -2.32 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.3 0.2 0.2	0 3.5 3.42 -2.32 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.2 0.2 0.2	0 3.5 3.32 -2.32 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.5 0.2 0.2	0 3.8 3.62 -2.32 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.3 0.2 0.2	0 3.8 3.42 -2.32 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.2 0.2 0.3	0 2.9 2.92 -2.32 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 0.2 0.3	0 3.1 3.02 -2.32 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.1 0.2 0.3	0 3.2 3.02 -2.32 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.1 0.2 0.3	0 3.3 3.02 -2.32 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.1 0.2 0.3	0 3.5 3.22 -2.32 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL

0 0.8 0.2 0.2 0.3	0 3.8 3.32 -2.32 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.8 0.2 0.5	0 2.9 2.32 -2.32 -3.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.8 0.2 0.5	0 3.1 2.32 -2.32 -3.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 0.1 0.1 0.1	0 3.1 3.22 -2.42 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.2 0.1 0.1	0 3.2 3.32 -2.42 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.1 0.1 0.1	0 3.2 3.22 -2.42 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.2 0.1 0.1	0 3.3 3.32 -2.42 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.1 0.1 0.1	0 3.3 3.22 -2.42 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.3 0.1 0.1	0 3.5 3.42 -2.42 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.2 0.1 0.1	0 3.5 3.32 -2.42 -3.51	MARGINAL	ACCEPTABLE	ACCEPTABLE	ACCEPTABLE
0 0.5 0.1 0.1 0.1	0 3.5 3.22 -2.42 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.3 0.1 0.1	0 3.8 3.42 -2.42 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.3 0.1 0.2	0 2.9 2.82 -2.42 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 0.1 0.2	0 3.1 3.02 -2.42 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.2 0.1 0.2	0 3.1 2.92 -2.42 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.1 0.1 0.2	0 3.2 3.02 -2.42 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.2 0.1 0.2	0 3.2 2.92 -2.42 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.1 0.1 0.2	0 3.3 3.02 -2.42 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 -0.1 0.1 0.2	0 3.5 3.02 -2.42 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.2 0.1 0.2	0 3.8 3.32 -2.42 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.1 0.1 0.2	0 3.8 3.22 -2.42 -3.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.8 0.1 0.3	0 2.7 2.32 -2.42 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.8 0.1 0.3	0 2.8 2.32 -2.42 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.5 0.1 0.3	0 3.1 2.62 -2.42 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.5 0.1 0.3	0 3.2 2.62 -2.42 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.5 0.1 0.3	0 3.3 2.62 -2.42 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 -0.3 0.1 0.3	0 3.5 2.82 -2.42 -3.31	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.8 -0.1 -0.3	0 3.5 3.92 -2.62 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.3 -0.1 -0.2	0 3.2 3.42 -2.62 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.3 -0.1 -0.2	0 3.3 3.42 -2.62 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.5 -0.1 -0.2	0 3.5 3.62 -2.62 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.5 -0.1 -0.2	0 3.8 3.62 -2.62 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.2 -0.1 -0.1	0 2.9 2.92 -2.62 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 -0.1 -0.1	0 3.1 3.02 -2.62 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.2 -0.1 -0.1	0 3.1 2.92 -2.62 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.1 -0.1 -0.1	0 3.2 3.02 -2.62 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.1 -0.1 -0.1	0 3.3 3.22 -2.62 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.1 -0.1 -0.1	0 3.3 3.02 -2.62 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.2 -0.1 -0.1	0 3.5 3.32 -2.62 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.1 -0.1 -0.1	0 3.5 3.22 -2.62 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.3 -0.1 -0.1	0 3.8 3.42 -2.62 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.2 -0.1 -0.1	0 3.8 3.32 -2.62 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.8 -0.1 0.1	0 2.7 2.32 -2.62 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.8 -0.1 0.1	0 2.8 2.32 -2.62 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.8 -0.1 0.1	0 2.9 2.32 -2.62 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.8 -0.1 0.1	0 3.1 2.32 -2.62 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.5 -0.1 0.1	0 3.3 2.62 -2.62 -3.51	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 0.2 -0.2 -0.3	0 3.2 3.32 -2.72 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.3 -0.2 -0.3	0 3.3 3.42 -2.72 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 0.2 -0.2 -0.3	0 3.3 3.32 -2.72 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.3 -0.2 -0.3	0 3.5 3.42 -2.72 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.5 -0.2 -0.3	0 3.8 3.62 -2.72 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.3 -0.2 -0.2	0 2.9 2.82 -2.72 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 -0.2 -0.2	0 3.1 3.02 -2.72 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.2 -0.2 -0.2	0 3.1 2.92 -2.72 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.1 -0.2 -0.2	0 3.2 3.02 -2.72 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL

0 0.2 -0.2 -0.2 -0.2	0 3.2 2.92 -2.72 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.1 -0.2 -0.2	0 3.3 3.02 -2.72 -3.81	MARGINAL	ACCEPTABLE	ACCEPTABLE	ACCEPTABLE
0 0.3 -0.2 -0.2 -0.2	0 3.3 2.92 -2.72 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 0.1 -0.2 -0.2	0 3.5 3.22 -2.72 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 -0.1 -0.2 -0.2	0 3.5 3.02 -2.72 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.2 -0.2 -0.2	0 3.8 3.32 -2.72 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.1 -0.2 -0.2	0 3.8 3.22 -2.72 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.5 -0.2 -0.1	0 2.8 2.62 -2.72 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.5 -0.2 -0.1	0 2.9 2.62 -2.72 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.3 -0.2 -0.1	0 3.2 2.82 -2.72 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.3 -0.2 -0.1	0 3.3 2.82 -2.72 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 -0.2 -0.2 -0.1	0 3.5 2.92 -2.72 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 -0.3 -0.2 -0.1	0 3.5 2.82 -2.72 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.8 -0.3 -0.5	0 3.8 3.92 -2.82 -4.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.3 -0.3 -0.3	0 2.9 2.82 -2.82 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.1 -0.3 -0.3	0 3.1 3.02 -2.82 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.2 -0.3 -0.3	0 3.1 2.92 -2.82 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.3 -0.3 -0.3	0 3.1 2.82 -2.82 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.1 -0.3 -0.3	0 3.2 3.02 -2.82 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.2 -0.3 -0.3	0 3.2 2.92 -2.82 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.3 -0.3 -0.3	0 3.2 2.82 -2.82 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.1 -0.3 -0.3	0 3.3 3.02 -2.82 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.2 -0.3 -0.3	0 3.3 2.92 -2.82 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 -0.1 -0.3 -0.3	0 3.5 3.02 -2.82 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 -0.2 -0.3 -0.3	0 3.5 2.92 -2.82 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.2 -0.3 -0.3	0 3.8 3.32 -2.82 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.8 0.1 -0.3 -0.3	0 3.8 3.22 -2.82 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.5 -0.3 -0.2	0 2.9 2.62 -2.82 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.5 -0.3 -0.2	0 3.1 2.62 -2.82 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.5 -0.3 -0.2	0 3.2 2.62 -2.82 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.3 -0.3 -0.2	0 3.3 2.82 -2.82 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 -0.3 -0.3 -0.2	0 3.5 2.82 -2.82 -3.81	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.8 -0.3 -0.1	0 2.9 2.32 -2.82 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.8 -0.3 -0.1	0 3.1 2.32 -2.82 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.8 -0.3 -0.1	0 3.2 2.32 -2.82 -3.71	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.2 -0.5 -0.5 -0.5	0 2.8 2.62 -3.02 -4.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.5 -0.5 -0.5	0 2.9 2.62 -3.02 -4.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.3 -0.5 -0.5	0 3.1 2.82 -3.02 -4.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.5 -0.5 -0.5	0 3.1 2.62 -3.02 -4.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.2 -0.5 -0.5	0 3.2 2.92 -3.02 -4.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.3 -0.5 -0.5	0 3.2 2.82 -3.02 -4.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.2 -0.5 -0.5	0 3.3 2.92 -3.02 -4.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.3 -0.5 -0.5	0 3.3 2.82 -3.02 -4.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 -0.1 -0.5 -0.5	0 3.5 3.02 -3.02 -4.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 -0.2 -0.5 -0.5	0 3.5 2.92 -3.02 -4.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 -0.3 -0.5 -0.5	0 3.5 2.82 -3.02 -4.11	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.8 -0.5 -0.3	0 3.3 2.32 -3.02 -3.91	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.3 -0.8 -0.8 -0.8	0 2.7 2.32 -3.32 -4.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 -0.1 -0.8 -0.8 -0.8	0 2.9 2.32 -3.32 -4.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.1 -0.5 -0.8 -0.8	0 3.1 2.62 -3.32 -4.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.2 -0.5 -0.8 -0.8	0 3.2 2.62 -3.32 -4.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.3 -0.5 -0.8 -0.8	0 3.3 2.62 -3.32 -4.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL
0 0.5 -0.3 -0.8 -0.8	0 3.5 2.82 -3.32 -4.41	MARGINAL	MARGINAL	MARGINAL	MARGINAL

Table B.5: INITIAL POINTS AND LINE COLOR DEFINITION

$y_l(mm)$	$absolute y_R(mm)$
0 2.8 2.32 -3.32 -4.41	0 0 4.88 15.88 17.2
0 3.2 3.62 -1.72 -2.81	0 0 4.61 16.07 17.42
0 3.5 3.92 -1.72 -2.81	0 0 5.08 17.41 18.79
0 3.8 3.92 -1.72 -2.81	0 0 5.85 18.76 20.16
0 3.2 3.42 -2.02 -3.11	0 0 4.81 16.37 17.72
0 3.3 3.62 -2.02 -3.11	0 0 4.86 16.82 18.18
0 3.5 3.62 -2.02 -3.11	0 0 5.38 17.71 19.09
0 2.7 2.32 -2.02 -2.81	0 0 4.62 14.13 15.14
0 2.8 2.32 -2.02 -2.81	0 0 4.88 14.58 15.6
0 2.9 2.32 -2.02 -2.81	0 0 5.14 15.02 16.05
0 3.1 2.62 -2.02 -2.81	0 0 5.35 15.92 16.96
0 3.2 2.62 -2.02 -2.81	0 0 5.61 16.37 17.42
0 3.3 2.62 -2.02 -2.81	0 0 5.86 16.82 17.88
0 3.5 2.82 -2.02 -2.81	0 0 6.18 17.71 18.79
0 3.5 3.92 -2.22 -3.41	0 0 5.08 17.91 19.39
0 3.8 3.92 -2.22 -3.41	0 0 5.85 19.26 20.76
0 3.1 3.32 -2.22 -3.31	0 0 4.65 16.12 17.46
0 3.1 3.22 -2.22 -3.31	0 0 4.75 16.12 17.46
0 3.2 3.42 -2.22 -3.31	0 0 4.81 16.57 17.92
0 3.2 3.32 -2.22 -3.31	0 0 4.91 16.57 17.92
0 3.3 3.42 -2.22 -3.31	0 0 5.06 17.02 18.38
0 3.3 3.32 -2.22 -3.31	0 0 5.16 17.02 18.38
0 3.5 3.62 -2.22 -3.31	0 0 5.38 17.91 19.29
0 3.5 3.42 -2.22 -3.31	0 0 5.58 17.91 19.29
0 3.8 3.62 -2.22 -3.31	0 0 6.15 19.26 20.66
0 2.8 2.62 -2.22 -3.11	0 0 4.58 14.78 15.9
0 2.9 2.62 -2.22 -3.11	0 0 4.84 15.22 16.35
0 3.1 2.82 -2.22 -3.11	0 0 5.15 16.12 17.26
0 3.1 2.62 -2.22 -3.11	0 0 5.35 16.12 17.26
0 3.2 2.82 -2.22 -3.11	0 0 5.41 16.57 17.72
0 3.3 2.92 -2.22 -3.11	0 0 5.56 17.02 18.18
0 3.3 2.82 -2.22 -3.11	0 0 5.66 17.02 18.18
0 3.5 2.92 -2.22 -3.11	0 0 6.08 17.91 19.09
0 3.3 3.62 -2.32 -3.51	0 0 4.86 17.12 18.58
0 3.8 3.92 -2.32 -3.51	0 0 5.85 19.36 20.86
0 3.1 3.22 -2.32 -3.41	0 0 4.75 16.22 17.56
0 3.2 3.42 -2.32 -3.41	0 0 4.81 16.67 18.02
0 3.2 3.32 -2.32 -3.41	0 0 4.91 16.67 18.02
0 3.2 3.22 -2.32 -3.41	0 0 5.01 16.67 18.02
0 3.3 3.42 -2.32 -3.41	0 0 5.06 17.12 18.48
0 3.3 3.32 -2.32 -3.41	0 0 5.16 17.12 18.48
0 3.3 3.22 -2.32 -3.41	0 0 5.26 17.12 18.48
0 3.5 3.42 -2.32 -3.41	0 0 5.58 18.01 19.39
0 3.5 3.32 -2.32 -3.41	0 0 5.68 18.01 19.39
0 3.8 3.62 -2.32 -3.41	0 0 6.15 19.36 20.76
0 3.8 3.42 -2.32 -3.41	0 0 6.35 19.36 20.76
0 2.9 2.92 -2.32 -3.31	0 0 4.54 15.32 16.55
0 3.1 3.02 -2.32 -3.31	0 0 4.95 16.22 17.46
0 3.2 3.02 -2.32 -3.31	0 0 5.21 16.67 17.92
0 3.3 3.02 -2.32 -3.31	0 0 5.46 17.12 18.38
0 3.5 3.22 -2.32 -3.31	0 0 5.78 18.01 19.29
0 3.8 3.32 -2.32 -3.31	0 0 6.45 19.36 20.66
0 2.9 2.32 -2.32 -3.11	0 0 5.14 15.32 16.35
0 3.1 2.32 -2.32 -3.11	0 0 5.65 16.22 17.26
0 3.1 3.22 -2.42 -3.51	0 0 4.75 16.32 17.66
0 3.2 3.32 -2.42 -3.51	0 0 4.91 16.77 18.12
0 3.2 3.22 -2.42 -3.51	0 0 5.01 16.77 18.12
0 3.3 3.32 -2.42 -3.51	0 0 5.16 17.22 18.58
0 3.3 3.22 -2.42 -3.51	0 0 5.26 17.22 18.58
0 3.5 3.42 -2.42 -3.51	0 0 5.58 18.11 19.49
0 3.5 3.32 -2.42 -3.51	0 0 5.68 18.11 19.49
0 3.5 3.22 -2.42 -3.51	0 0 5.78 18.11 19.49
0 3.8 3.42 -2.42 -3.51	0 0 6.35 19.46 20.86

0 2.9 2.82 -2.42 -3.41	0 0 4.64 15.42 16.65
0 3.1 3.02 -2.42 -3.41	0 0 4.95 16.32 17.56
0 3.1 2.92 -2.42 -3.41	0 0 5.05 16.32 17.56
0 3.2 3.02 -2.42 -3.41	0 0 5.21 16.77 18.02
0 3.2 2.92 -2.42 -3.41	0 0 5.31 16.77 18.02
0 3.3 3.02 -2.42 -3.41	0 0 5.46 17.22 18.48
0 3.5 3.02 -2.42 -3.41	0 0 5.98 18.11 19.39
0 3.8 3.32 -2.42 -3.41	0 0 6.45 19.46 20.76
0 3.8 3.22 -2.42 -3.41	0 0 6.55 19.46 20.76
0 2.7 2.32 -2.42 -3.31	0 0 4.62 14.53 16.64
0 2.8 2.32 -2.42 -3.31	0 0 4.88 14.98 16.1
0 3.1 2.62 -2.42 -3.31	0 0 5.35 16.32 17.46
0 3.2 2.62 -2.42 -3.31	0 0 5.61 16.77 17.92
0 3.3 2.62 -2.42 -3.31	0 0 5.86 17.22 18.38
0 3.5 2.82 -2.42 -3.31	0 0 6.18 18.11 19.29
0 3.5 3.92 -2.62 -3.91	0 0 5.08 18.31 19.89
0 3.2 3.42 -2.62 -3.81	0 0 4.81 16.97 18.42
0 3.3 3.42 -2.62 -3.81	0 0 5.06 17.42 18.88
0 3.5 3.62 -2.62 -3.81	0 0 5.38 18.31 19.79
0 3.8 3.62 -2.62 -3.81	0 0 6.15 19.66 21.16
0 2.9 2.92 -2.62 -3.71	0 0 4.54 15.62 16.95
0 3.1 3.02 -2.62 -3.71	0 0 4.95 16.52 17.86
0 3.1 2.92 -2.62 -3.71	0 0 5.05 16.52 17.86
0 3.2 3.02 -2.62 -3.71	0 0 5.21 16.97 18.32
0 3.3 3.22 -2.62 -3.71	0 0 5.26 17.42 18.78
0 3.3 3.02 -2.62 -3.71	0 0 5.46 17.42 18.78
0 3.5 3.32 -2.62 -3.71	0 0 5.68 18.31 19.69
0 3.5 3.22 -2.62 -3.71	0 0 5.78 18.31 19.69
0 3.8 3.42 -2.62 -3.71	0 0 6.35 19.66 21.06
0 3.8 3.32 -2.62 -3.71	0 0 6.45 19.66 21.06
0 2.7 2.32 -2.62 -3.51	0 0 4.62 14.73 15.84
0 2.8 2.32 -2.62 -3.51	0 0 4.88 15.18 16.3
0 2.9 2.32 -2.62 -3.51	0 0 5.14 15.62 16.75
0 3.1 2.32 -2.62 -3.51	0 0 5.65 16.52 17.66
0 3.3 2.62 -2.62 -3.51	0 0 5.86 17.42 18.58
0 3.2 3.32 -2.72 -3.91	0 0 4.91 17.07 18.52
0 3.3 3.42 -2.72 -3.91	0 0 5.06 17.52 18.98
0 3.3 3.32 -2.72 -3.91	0 0 5.16 17.52 18.98
0 3.5 3.42 -2.72 -3.91	0 0 5.58 18.41 19.89
0 3.8 3.62 -2.72 -3.91	0 0 6.15 19.76 21.26
0 2.9 2.82 -2.72 -3.81	0 0 4.64 15.72 17.05
0 3.1 3.02 -2.72 -3.81	0 0 4.95 16.62 17.96
0 3.1 2.92 -2.72 -3.81	0 0 5.05 16.62 17.96
0 3.2 3.02 -2.72 -3.81	0 0 5.21 17.07 18.42
0 3.2 2.92 -2.72 -3.81	0 0 5.31 17.07 18.42
0 3.3 3.02 -2.72 -3.81	0 0 5.46 17.52 18.88
0 3.3 2.92 -2.72 -3.81	0 0 5.56 17.52 18.88
0 3.5 3.22 -2.72 -3.81	0 0 5.68 18.41 19.79
0 3.5 3.02 -2.72 -3.81	0 0 5.78 18.41 19.79
0 3.8 3.32 -2.72 -3.81	0 0 6.45 19.76 21.16
0 3.8 3.22 -2.72 -3.81	0 0 6.55 19.76 21.16
0 2.8 2.62 -2.72 -3.71	0 0 4.58 15.28 16.5
0 2.9 2.62 -2.72 -3.71	0 0 4.84 15.72 16.95
0 3.2 2.82 -2.72 -3.71	0 0 5.41 17.07 18.32
0 3.3 2.82 -2.72 -3.71	0 0 5.66 17.52 18.78
0 3.5 2.92 -2.72 -3.71	0 0 6.08 18.41 19.69
0 3.5 2.82 -2.72 -3.71	0 0 6.18 18.41 19.69
0 3.8 3.92 -2.82 -4.11	0 0 5.85 19.86 21.46
0 2.9 2.82 -2.82 -3.91	0 0 4.64 15.82 17.15
0 3.1 3.02 -2.82 -3.91	0 0 4.95 16.72 18.06
0 3.1 2.92 -2.82 -3.91	0 0 5.05 16.72 18.06
0 3.1 2.82 -2.82 -3.91	0 0 5.15 16.72 18.06
0 3.2 3.02 -2.82 -3.91	0 0 5.21 17.17 18.52
0 3.2 2.92 -2.82 -3.91	0 0 5.31 17.17 18.52
0 3.2 2.82 -2.82 -3.91	0 0 5.41 17.17 18.52
0 3.3 3.02 -2.82 -3.91	0 0 5.46 17.62 18.98

0 3.3 2.92 -2.82 -3.91	0 0 5.56 17.62 18.98
0 3.5 3.02 -2.82 -3.91	0 0 5.98 18.51 19.89
0 3.5 2.92 -2.82 -3.91	0 0 6.08 18.51 19.89
0 3.8 3.32 -2.82 -3.91	0 0 6.45 19.86 21.26
0 3.8 3.22 -2.82 -3.91	0 0 6.55 19.86 21.26
0 2.9 2.62 -2.82 -3.81	0 0 4.84 15.82 17.05
0 3.1 2.62 -2.82 -3.81	0 0 5.35 16.72 17.96
0 3.2 2.62 -2.82 -3.81	0 0 5.61 17.17 18.42
0 3.3 2.82 -2.82 -3.81	0 0 5.66 17.62 18.88
0 3.5 2.82 -2.82 -3.81	0 0 6.18 18.51 19.79
0 2.9 2.32 -2.82 -3.71	0 0 5.14 15.82 16.95
0 3.1 2.32 -2.82 -3.71	0 0 5.65 16.72 17.86
0 3.2 2.32 -2.82 -3.71	0 0 5.91 17.17 18.32
0 2.8 2.62 -3.02 -4.11	0 0 4.58 15.58 16.9
0 2.9 2.62 -3.02 -4.11	0 0 4.84 16.02 17.35
0 3.1 2.82 -3.02 -4.11	0 0 5.15 16.92 18.26
0 3.1 2.62 -3.02 -4.11	0 0 5.35 16.92 18.26
0 3.2 2.92 -3.02 -4.11	0 0 5.31 17.37 18.72
0 3.2 2.82 -3.02 -4.11	0 0 5.41 17.37 18.72
0 3.3 2.92 -3.02 -4.11	0 0 5.56 17.82 19.18
0 3.3 2.82 -3.02 -4.11	0 0 5.66 17.82 19.18
0 3.5 3.02 -3.02 -4.11	0 0 5.98 18.71 20.09
0 3.5 2.92 -3.02 -4.11	0 0 6.08 18.71 20.09
0 3.5 2.82 -3.02 -4.11	0 0 6.18 18.71 20.09
0 3.3 2.32 -3.02 -3.91	0 0 6.16 17.82 18.98
0 2.7 2.32 -3.32 -4.41	0 0 4.62 15.43 16.74
0 2.9 2.32 -3.32 -4.41	0 0 5.14 16.32 17.65
0 3.1 2.62 -3.32 -4.41	0 0 5.35 17.22 18.56
0 3.2 2.62 -3.32 -4.41	0 0 5.61 17.67 19.02
0 3.3 2.62 -3.32 -4.41	0 0 5.86 18.12 19.48
0 3.5 2.82 -3.32 -4.41	0 0 6.18 19.01 20.39

Table B.6: ROTATED POINTS AND LINE COLOR DEFINITION