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ΥΔΑΤΙΚΩΝ ΠΟΡΩΝ»

Εύα - Στυλιανή Στείρου

**Διερεύνηση των σφαλμάτων από
ομογενοποίηση σε υδροκλιματικές χρονοσειρές
με μακροπρόθεσμη εμμονή**

ΜΕΤΑΠΤΥΧΙΑΚΗ ΕΡΓΑΣΙΑ

Επιβλέπων Καθηγητής: Δημήτρης Κουτσογιάννης

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EXTENDED ABSTRACT

Introduction

Hydroclimatic time series contain inhomogeneities, which are errors introduced by replacements and calibration of instruments, station relocations, changes in the environment of the stations, etc. The most common expression of inhomogeneities is shifts between two parts of time series. The identification and correction of inhomogeneities is called homogenization and is usually done with statistical methods which compare a candidate station with one or more neighbouring reference stations, assuming that they belong to the same climatological region and they reflect the same weather and climate variations.

The homogenization of hydroclimatic data, mainly of temperature and precipitation time series, is a procedure of great importance and also a controversial subject because of its implications in the estimations of climate change. This study focuses on a generally ignored by the homogenization community, though important characteristic of hydroclimatic time series and of its effects on homogenization, the long-term persistence of hydroclimatic data, and has two components: (a) a literature review and (b) a computational approach.

1. Literature review

A systematic study of the scientific literature was made in order to examine types and causes of inhomogeneities, to identify and classify the existing homogenization methods, understand their stochastic background and to evaluate their output. This literature review focused mainly on previous evaluation studies of homogenization methods with synthetic data.

A main result of this study is that existing homogenization methods generally ignore the long-term persistence of hydroclimatic data expressed by the Hurst coefficient and examine only first-order autoregressive series (AR(1)) or series of identically and independently distributed Gaussian errors. No systematic studies of the relationship between the Hurst coefficient and the homogenization results have been identified.

It was also found that homogenization methods assume that the series of temperature differences or precipitation ratios between reference and candidate stations constitute series of random numbers (e.g. white noise). However a basic stochastic analysis indicates that the difference and ratio series reproduce the autocorrelation function of the reference and candidate series assuming that both have the same autocorrelation structure.

2. Computational approach

A computational approach based on Monte Carlo simulations permitted to understand and evaluate the behaviour of selected classical homogenization methods as a function of various parameters:

a) the Hurst coefficient of hydroclimatic time series (tested values = {0.50, 0.55, ..., 0.90}),

- b) the cross-correlation coefficient between candidate and reference time series (tested values: $\rho_{XY}=\{0.5, 0.6, 0.7, 0.8, 0.9, 0.95\}$),
- c) the number of reference time series and the way they were used to locate shifts (reference systems - shown in Table 1),
- d) the length of the time series (50 and 100 years), and
- e) the minimum distance between possible inhomogeneities or between inhomogeneities and the edge of series (tested values 5 and 10 years).

The percentage of time series with false alarms was regarded as the critical factor for this evaluation.

Table 1. Characteristics of the homogenization tests used.

Number of common shifts: minimum number of times a shift must be identified by different reference series so that it can be considered significant, L: length of simulated time series, MD: minimum distance between possible inhomogeneities or between inhomogeneities and the edge of series

Symbol	Number of reference series	Number of common shifts	Comments
1/1	1	1	L:100, MD:5
3/7	7	3	L:100, MD:5
3/1	3	1	An artificial time series is constructed as the mean of the simulated reference series L:100, MD:5
2/2	2	2	L:100, MD:5
3/3	3	3	L:100, MD:5
3/10	10	3	L:100, MD:5
3/20	20	3	L:100, MD:5
3/7-50y	7	3	L:50, MD:5
3/7-j10	7	3	L:100, MD:10

For temperatures, the homogenization method selected was SNHT for shifts (Alexandersson and Moberg, 1997) in combination with all systems of reference series summarized in Table 1 except 3/1 (see Figure 1). For multiple reference series a pairwise comparison of the candidate series with all reference series was applied. The SNHT was applied using a cutting algorithm described by Domonkos (2011a) and a 95% confidence level. For the precipitation data the Double Mass Curve (Kohler, 1949, Searcy and Hardison, 1960) was selected. The original method is subjective and involves the identification of the main inhomogeneity of the time series on a graph. An objective (automated) version was developed using a piecewise

linear algorithm based on the least squares approach. The reference series systems used were the most commonly applied 1/1 and 3/1.

The synthetic series with long-term persistence were simulated using a multiple time-scale fluctuation approach proposed by Koutsoyiannis (2002) and following the normal distribution. Temperature data were generated with zero mean and unit standard deviation and precipitation data with a mean of 1000mm and a standard deviation of 300mm. For every candidate series one or multiple correlated reference series with the same characteristics were generated (see Table 1). All simulations and computations were based on original Matlab codes.

3. Results and conclusions

Some main conclusions of this study are summarized in Figures 1, 2, 3 and 4.

- a) For time series with $H=0.5$ (i.e. characterized by white noise), the false alarm rate is not significant (below 5%), which is expected because of the design of the homogenization methods, but the percentage of series with false alarms increases with H .
- b) The number of reference series and of the minimum number required to locate shifts in the time series greatly affects the percentage of series with false alarms.

Furthermore, some more conclusions can be extracted concerning the application of the SNHT to temperature data and the Double mass curve to precipitation data.

For the temperature data (see Figures 1 and 2) it can be assumed that:

- a) For a Hurst coefficient $H = 0.85$, common shifts located by all (minimum number 3) reference time series correspond to percentage of series with false alarms higher than 5% and tend to indicate a real inhomogeneity (e.g. systems 2/2 and 3/3 in Figure 1).
- b) In the case of a common shift identified by some of the reference series only, this may only correspond to a false alarm (e.g. Figure 1). In such cases a possible inhomogeneity must be confirmed by analysis of the reference time series.
- c) The cross-correlation coefficient between reference and candidate series does not seem to influence the percentage of time series with false alarms.
- d) The percentage of series with false alarms for time series with length 50 years was lower than the percentage for 100 years (Figure 1).
- e) The minimum distance between possible inhomogeneities or between inhomogeneities and the edge of series influences but not greatly the percentage of series with false alarms. A minimum distance of 10 years leads to a lower percentage than a minimum distance of 5 years (Figure 1).
- f) For the case of a single reference series, corrections of the located shifts were applied. These corrections led to a similar percentage of series with increased and decreased trend after the homogenization. Therefore it seems that SNHT does not introduce significant changes in the temperature trends.
- g) For $H < 0.65$ the percentage of series with an increased Hurst coefficient after homogenization is similar to that with a lower Hurst coefficient. For $H > 0.65$ there is a different case. The percentage of series with an increased Hurst coefficient exceeds that of

series with a lower Hurst coefficient. This difference increases with the increase of the initial Hurst coefficient of the time series.

For the precipitation data (see Figures 3 and 4) it can be assumed that:

- For all values of the Hurst coefficient examined, the percentage of series with false alarms decreases with the increase of the ratio of the slopes of the two lines of the Double Mass Curve.
- Application of the Double Mass Curve with a reference time series produced by three time series (3/1) tends to decrease the percentage of false alarms in comparison to the application of the method with a single reference series (1/1).
- For the system 1/1 and all the parameters examined a slope ratio 1.5 corresponds to a percentage of series with false alarms lower than 5%. For the system 3/1 the same ratio is 1.3. These values seem to be indicative of a real inhomogeneity.

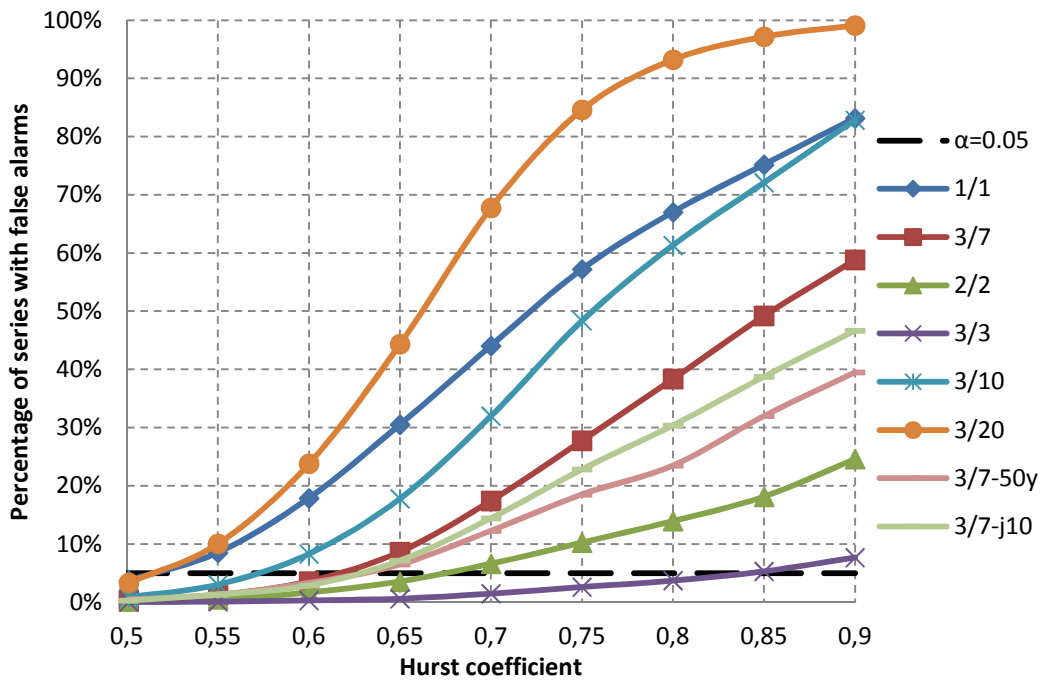


Figure 1. Percentage of series with false alarms as a function of the Hurst coefficient for various combinations of reference series analysed in Table 1. Results are shown for a typical cross-correlation coefficient between reference and candidate series, $\chi_{XY}=0.8$.

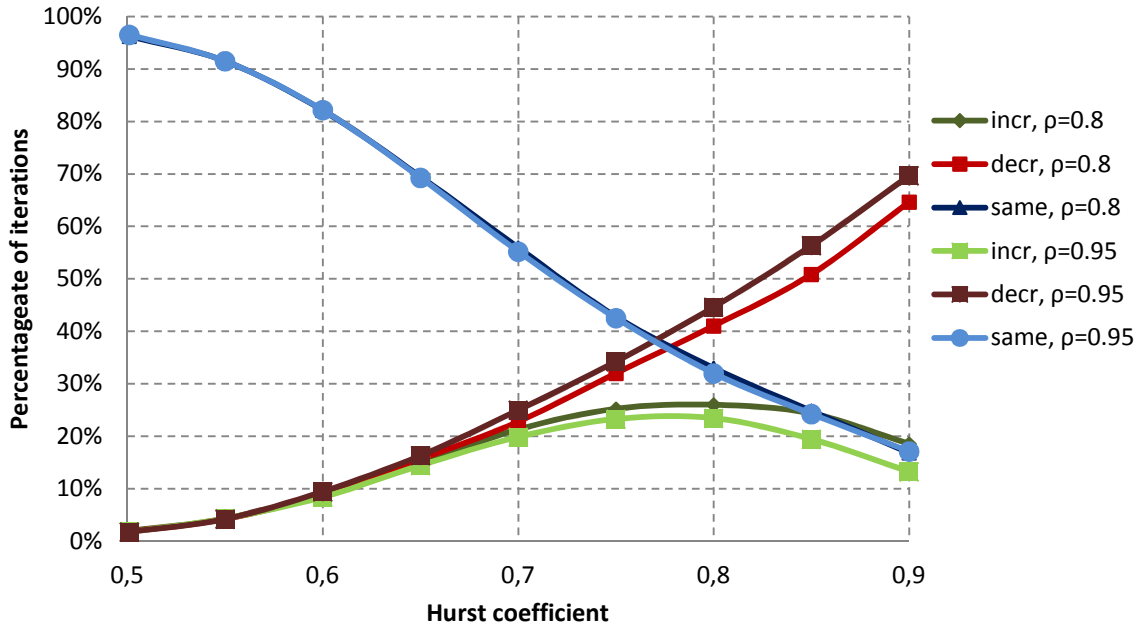


Figure 2. Influence of the SNHT homogenization in the Hurst coefficient of the homogenized times series. Results are shown for a typical cross-correlation coefficient between reference and candidate series, $\rho_{XY}=0.8$ and for $\rho_{XY}=0.95$ that led to slightly different results.

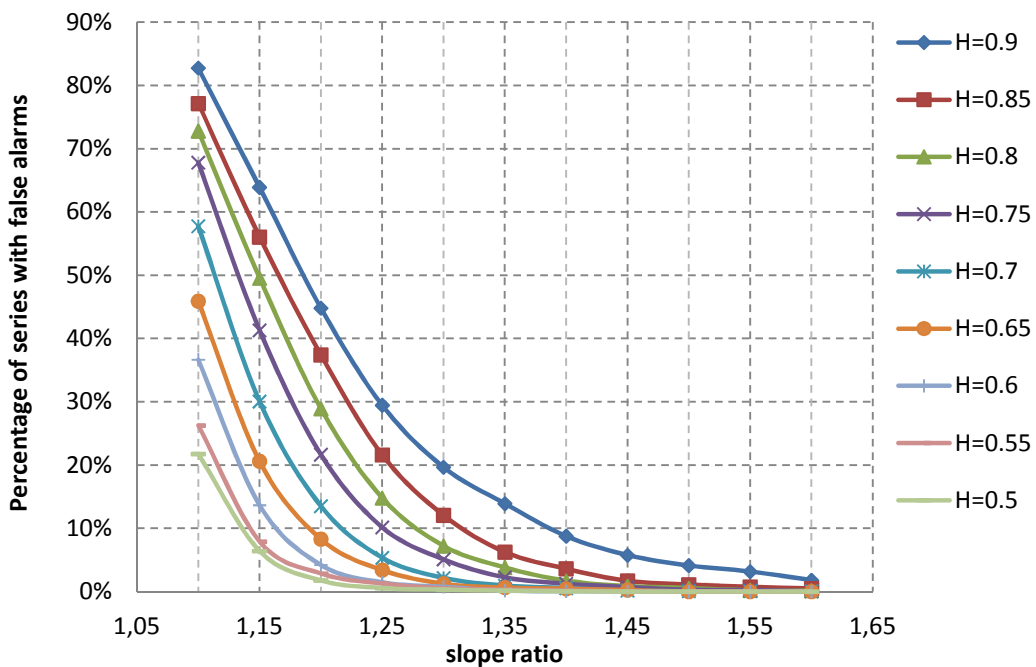


Figure 3. Percentage of series with false alarms as a function of the Hurst coefficient and the slope ratio of the lines of the Double Mass Curve for the reference system 1/1 analysed in Table 1. Results are shown for a typical cross-correlation coefficient between reference and candidate series, $\rho_{XY}=0.8$.

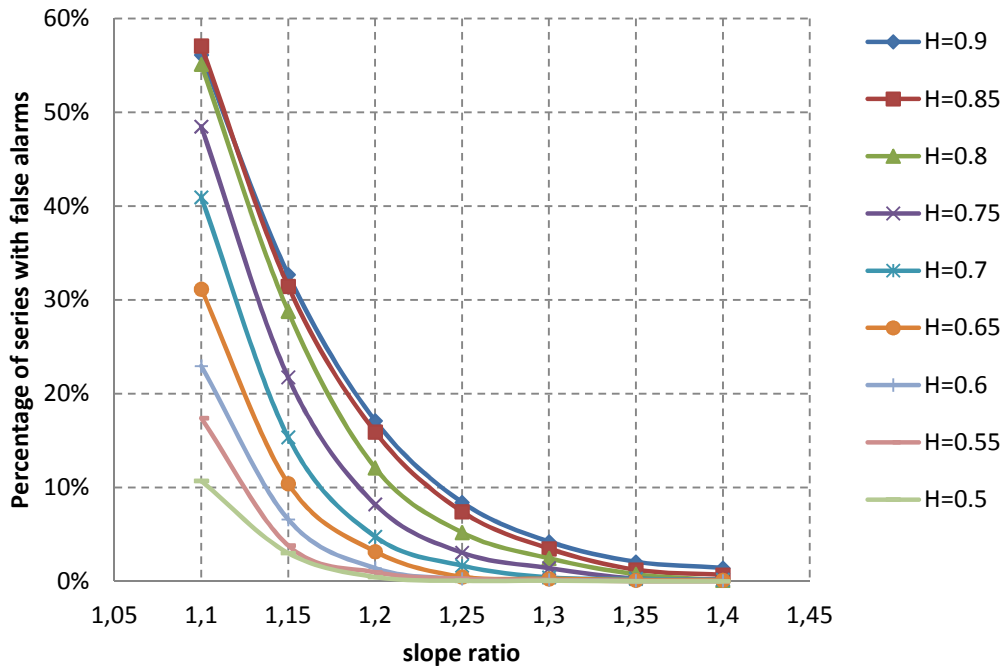


Figure 4. Percentage of series with false alarms as a function of the Hurst coefficient and the slope ratio of the lines of the Double Mass Curve for the reference system 3/1 analysed in Table 1. Results are shown for a typical cross-correlation coefficient between reference and candidate series, $\rho_{XY}=0.8$.

1.

1.1. μ

μ μ μ , μ μ μ
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 μ , (. . Auer *et al.*, 2007) . .
 μ μ μ μ (μ) ,
 μ μ , μ μ (Alexandersson and
Moberg, 1997). μ μ μ
 μ . , μ μ
 μ μ μ μ μ μ μ -
 μ (Venema *et al.*, 2012).
 μ μ μ μ μ μ
 μ μ (Aguilar *et al.*, 2003). μ μ μ μ μ μ
Koutsoyiannis, 2012), μ μ (, 2011, Steirou and
 μ , μ μ μ μ μ
 μ .
 μ μ , μ μ μ μ μ μ ,
 μ μ μ μ 100 ,
 μ μ μ μ .
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 μ μ (μ 1.1). μ μ μ μ μ
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 μ μ μ μ μ μ μ μ μ
(Koutsoyiannis, 2002). μ μ μ μ μ μ μ μ μ
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 μ μ .

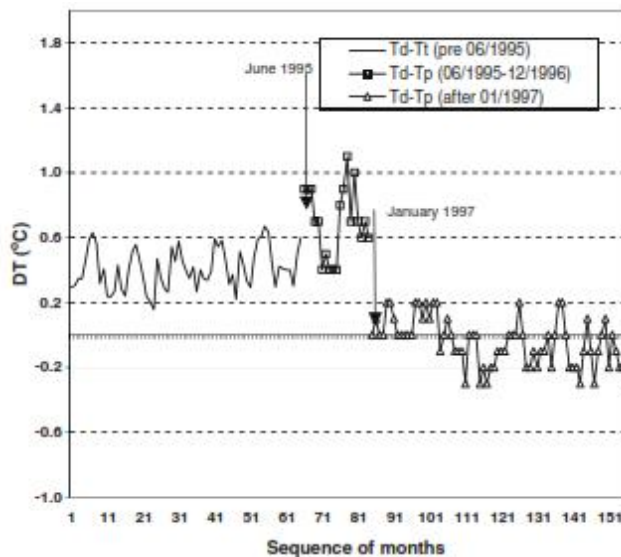
2.

(M, IPCC, 2007).

(Aguilar *et al.*, 2003).

2.1.

(Founda *et al.*, 2009).

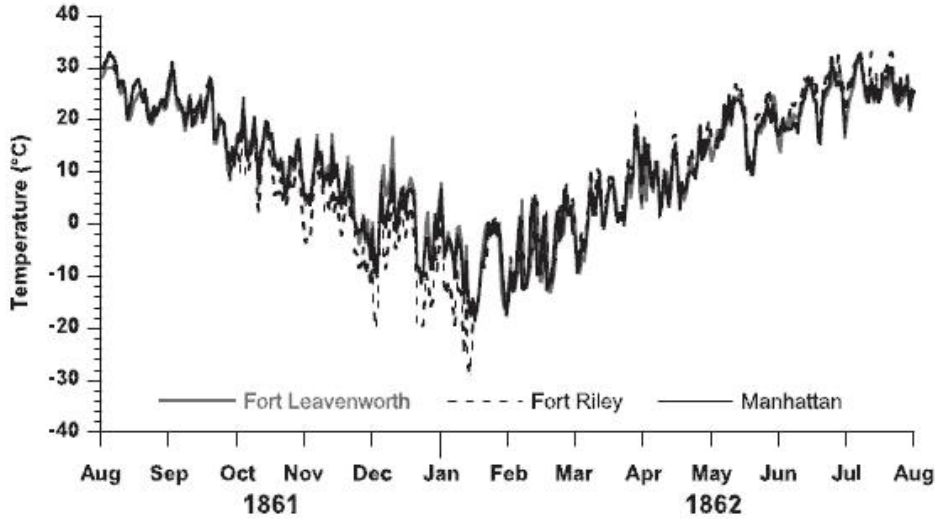


2.1. (DT)

: Founda *et al.* (2009).

2.1.1 μ / (measurement bias)

1. μ μ - (calibration) μ .
 μ μ (μ 2.2).



μ 2.2. μ μ μ μ Kansas
 : Fort Leavenworth, Fort Riley Manhattan Burnette et al. (2010).
 μ μ 1861 1862 μ μ
 μ Fort Riley Manhattan, μ 6.5 km.
 13.28 C Burnette et al.
 μ μ μ μ .

2. μ , μ μ , μ . μ , μ
 μ , . . . 9 ,
 , μ μ μ
 (Peterson et al., 1998). μ μ
 (Menne et al., 2009).

2.1.2 μ
 :

1) μ , μ (calibration) μ
 , μ .
 μ
 μ . μ μ
 μ μ (Easterling and Peterson, 1995).

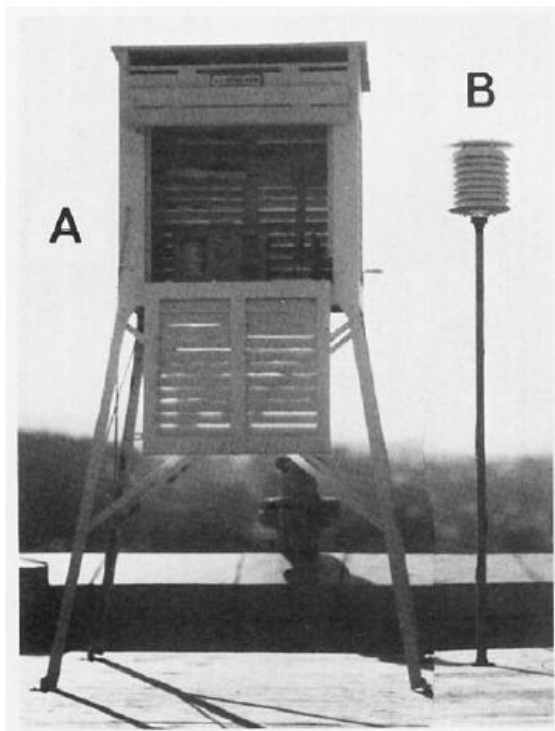
Quayle *et al.* (1991), 1980, (liquid-in-glass- LIG thermometers) (thermistor Maximum- Minimum Temperature System- MMTS).

2) Böhm *et al.* (2010). 1850-70 Böhm *et al.* (2010), « » (2.5 C) “Early Instrumental Bias”.

1980. (Cotton Region Shelter- CRS) (Quayle *et al.*, 1991).

- 3)
- (urbanization) .
 - (Alexandersson and Moberg, 1997) . .
 -

μ , μ μ (. . μ) . μ
 , (μ μ
 μ) μ μ μ
 μ μ μ .
 μ μ μ μ .
 μ μ μ μ μ
 (Beaulieu *et al.*, 2007).
 μ μ μ μ (. .
) μμ μ .
 μ μ μ μ .
 μ μ , μ . .
 μ (. . Fall *et al.*, 2011).



μ 2.3. () (CRS) μ
 , μ 1980 () : Quayle *et al.* (1991).

2.1.3. M

t_{max} , t_{min} / t_{07} , t_{14} , t_{21}
 t_{06} , t_{13} , t_{20} t_{08} , t_{14} , t_{21}
 t_{01} , t_{02}, \dots, t_{23} , t_{24}

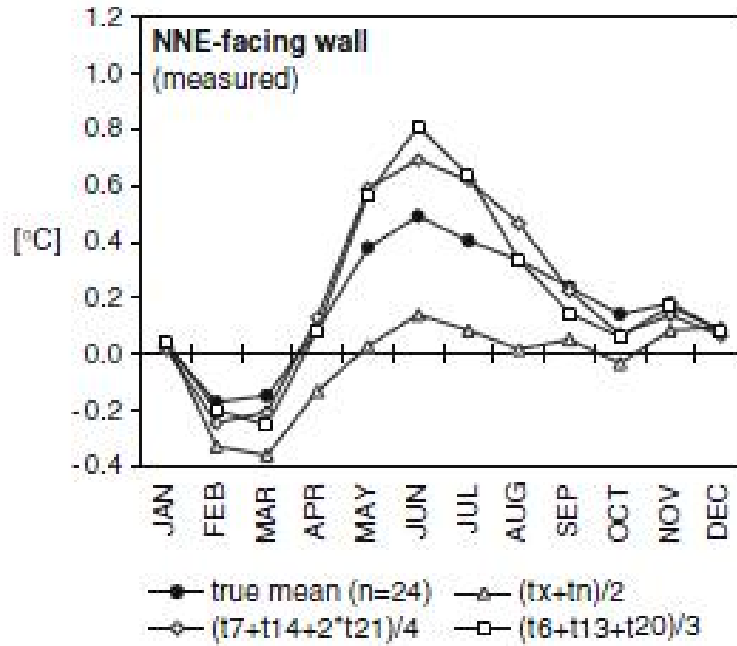
(Moberg and Bergström, 1997).

2.1 t_{07} , t_{14} , t_{21} t_{06} , t_{13} , t_{20} t_{08} , t_{14} , t_{21}
 t_{01} , t_{02}, \dots, t_{23} , t_{24} 2.4 2.5.

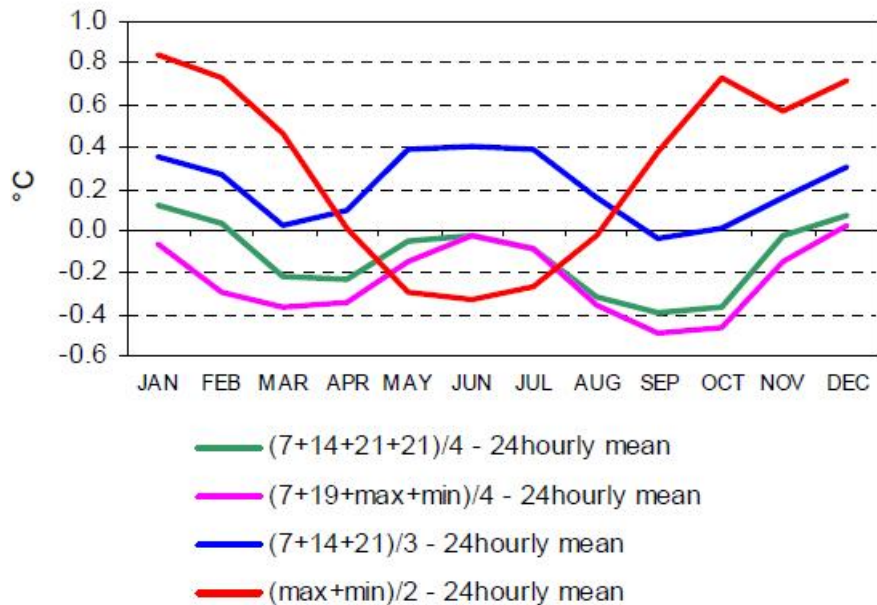
2.1. t_{max} , t_{min} t_{07} , t_{14} , t_{21} t_{06} , t_{13} , t_{20} t_{08} , t_{14} , t_{21}
 (1): Karl et al., 1986, (2): Böhm et al., 2010, (3): Moberg and Bergström, 1997,
 (4): Founda et al., 2009

t_{max} , t_{min}	$t = (t_{max} + t_{min}) / 2$	(1)
t_{07} , t_{14} , t_{21}	$t = (t_{07} + t_{14} + 2 \times t_{21}) / 4$	(2), (3)
t_{06} , t_{13} , t_{20}	$t = (t_{06} + t_{13} + t_{20}) / 3$	(3)
t_{08} , t_{14} , t_{21}	$t = (t_{08} + t_{14} + 5 \times t_{21}) / 7$	(3)
t_{01} , t_{02}, \dots, t_{23} , t_{24}	$t = (t_{01} + t_{02} + \dots + t_{24}) / 24$	(3), (4)

t_{07} , t_{14} , t_{21} t_{06} , t_{13} , t_{20} t_{08} , t_{14} , t_{21}
 t_{01} , t_{02}, \dots, t_{23} , t_{24} 24
 (Karl et al., 1986).



μ 2.4. μ μ μ μ Kremsmünster
 μ . μ μ
 μ : Böhm et al. (2010).



μ 2.5. μ μ μ μ Puchberg μ μ (1987-1996).
 μ : Aguilar et al. (2003).

3.

19 (early instrumental period) (Conrad, 2002).
Heidke, Helmert, and Abbe (Venema *et al.*, 2012).

(Kohler, 1949, Venema *et al.*, 2012).

20 (Domonkos 2011a).

2,

- (shifts),
- (trends).

(. . . USHCN V1, <http://www.ncdc.noaa.gov/oa/climate/research/ushcn/ushcn.html>),

1. (outliers),
2. ()
3. .

(. . . Menne and Williams, 2009)
2 3 (scalpel method)
(Rohde *et al.*, 2012).

(Beaulieu *et al.*, 2007):

- (subjective methods):

- **Absolute methods (objective methods):** These methods compare a candidate series with a reference series. They are used when the reference series is known and the candidate series is unknown. (Menne and Williams, 2009), (Sahin and Cigizoglu, 2010).

- **Relative methods (absolute methods):** These methods compare a candidate series with a reference series. They are used when the reference series is known and the candidate series is unknown. (Sahin and Cigizoglu, 2010).
- **Relative methods (relative methods):** These methods compare a candidate series with a reference series. They are used when the reference series is known and the candidate series is unknown. (reference series). (candidate series) μ

μ / μ (Conrad and Pollak, 1962).

(pairwise fashion, . . . Menne and Williams, 2009).

(Sahin and Cigizoglu, 2010).

μ Y_i μ X_i

3.1. Subjective methods

1.
 2.
 3.
 4.
- Beaulieu *et al.* (2007)

3.1.1. (Auer *et al.*, 2005) (Craddock, 1979).

3.1.1. (Kohler, 1949) (Searcy and Hardison, 1960).

3.2:

$$\sum y_i = y_1 + y_2 + \dots + y_i \quad (3.1)$$

$$\sum x_i = x_1 + x_2 + \dots + x_i \quad (3.2)$$

$i=1,2,\dots,$ (Dingman, 1994, Koutsoyiannis and Langousis, 2011).

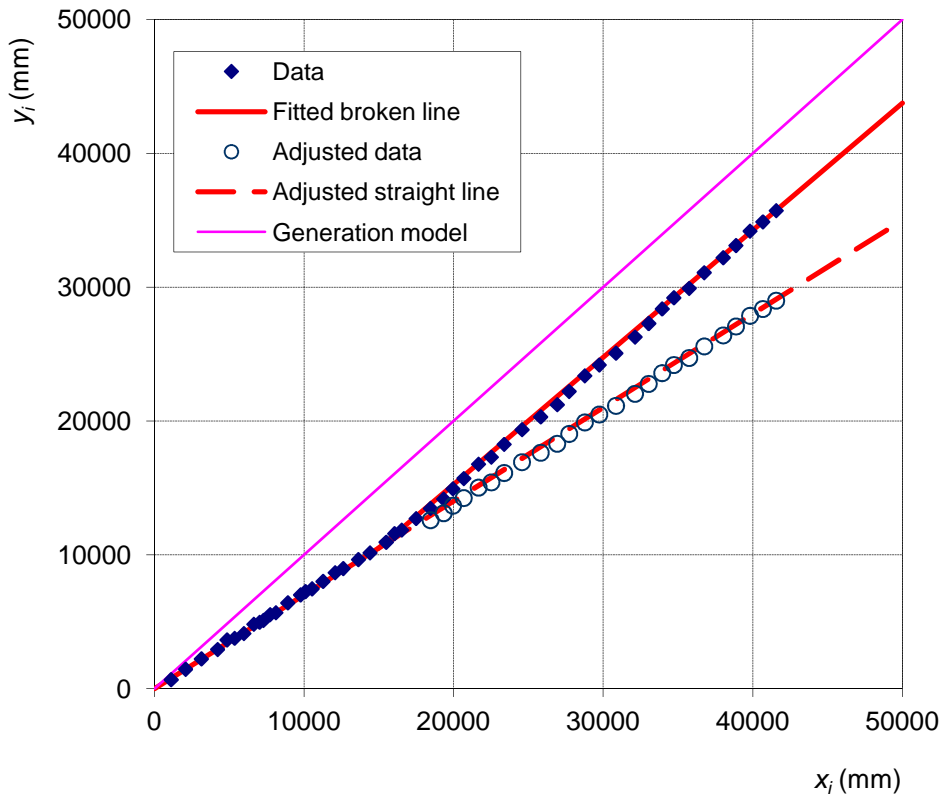
(Searcy and Hardison, 1960, Dingman, 1994) (Chang and Lee, 1974, Koutsoyiannis and Langousis, 2011).

1. 2. (Searcy and Hardison 1960). Dingman (1994),

(Dingman, 1994, Koutsoyiannis and Langousis, 2011).

$m = m/m$, m : m :

3.1 Koutsoyiannis and Langousis (2011)



3.1. $m = 0.737$, $m = 0.95$. : Koutsoyiannis and Langousis (2011).

$X(\mu)$ $Y(\mu)$, $X(50)$ $Y(250)$, $X(1000)$ $Y(25)$, $X(0.82)$ $Y(\text{false alarm})$.

Chang and Lee (1974)

3.2. (objective)

Beaulieu *et al.* (2007) Venema *et al.* (2012)

- (. . Buishand 1982, 1984, Alexandersson, 1986, Karl and Williams, 1987, Jaruskova, 1996, Alexandersson and Moberg, 1997, Wang *et al.*, 2007, Wang, 2008 . .),
- μ (. . Easterling and Peterson, 1995, Vincent, 1998),
- M (. . Lee and Heghinian, 1977, Ouarda *et al.*, 1999, Perreault *et al.*, 1999, 2000).

Student-t F-test. (Student-t test)

Venema *et al.* (2012), (. . 5), (. . Szentimrey, 1999, Mestre, 1999, Caussin and Mestre, 2004, Menne and Williams, 2009).

« »

(Moberg and Alexandersson, 1997, Beaulieu *et al.*, 2007, Domonkos, 2011a).

(Domonkos, 2011a).
 (. . Alexandersson and Moberg, 1997, Menne and Williams, 2009).

3.2.1-3.2.4. -

- (1995) (. . Easterling and Peterson),

- (. . Wijngaard *et al.*, 2003, Sahin and Cigizoglu, 2010, Venema *et al.*, 2012).

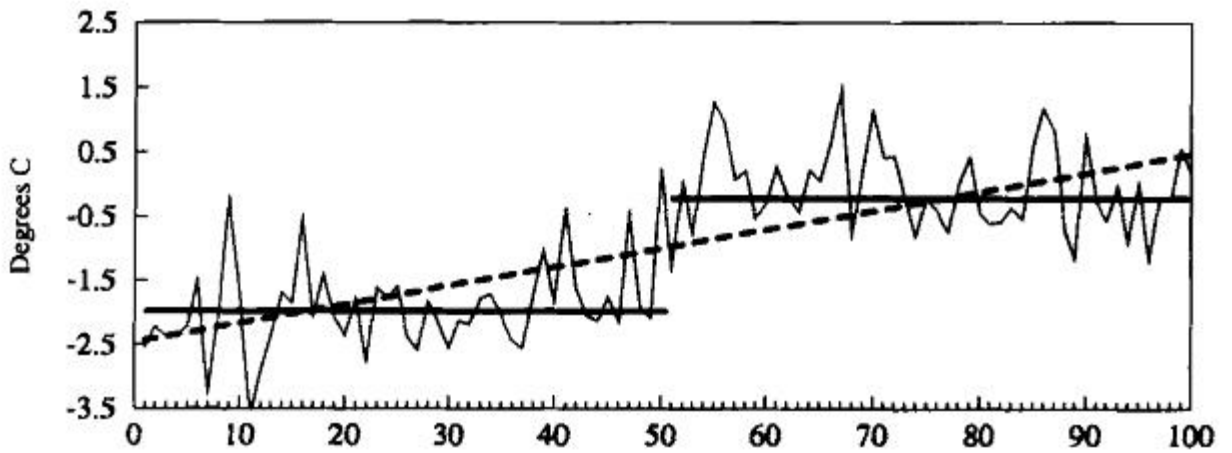
1. Easterling and Peterson test (Easterling and Peterson, 1995)
2. Standard Normal Homogeneity Test (SNHT) for single shifts (Alexandersson, 1986, Alexandersson and Moberg, 1997)
3. Standard Normal Homogeneity Test (SNHT) for trends (Alexandersson and Moberg, 1997)
4. Pairwise Homogenization Algorithm (PHA, Menne and Williams, 2009)

3.2.1. Easterling and Peterson (1995)

Easterling and Peterson (1995)
 (. .)

- 1) (reference stations)
 (candidate series).
 (dT/dt)
 (reference series)
- 2) .

- 3) $\sum_{i=1}^n (y_i - \hat{y}_i)^2$ (residual sum of squares- RSS_1)
- 4) $\sum_{i=1}^n (y_i - \mu)^2$ (RSS).
- 5) $RSS_2 = \sum_{i=1}^n (y_i - \hat{y}_i)^2$
- μ 3.2
Easterling and Peterson (1995).



μ 3.2. μ Easterling and Peterson (1995) μ

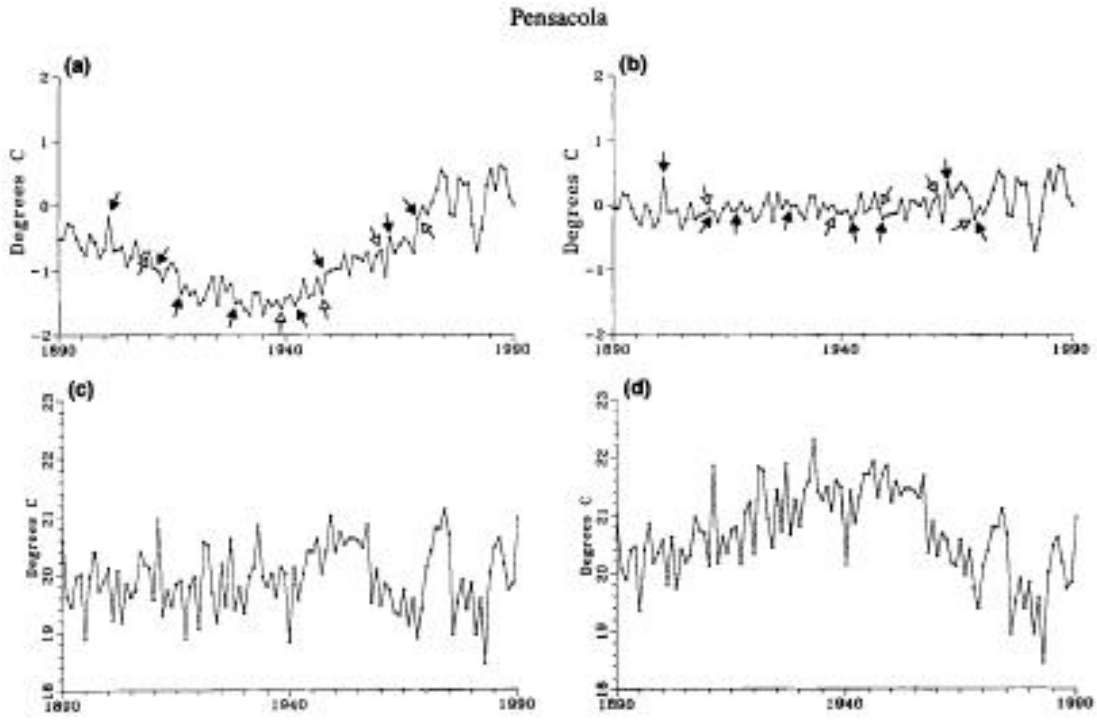
μ 2 C.

- 6) (two-phase fit, μ (likelihood ratio statistic) Solow (1987):

$$U = \left[\frac{RSS_1 - RSS_2}{3} \right] / \left[\frac{RSS_2}{(n-4)} \right] \quad (3.3)$$

μ U μ F μ 3 n-4 μ

- 7) Student *t*-test.
- 8) (6 7)
- 9) - multiresponse permutation procedures (MRPP, Mielke *et al.*, 1981).
- MRPP**
- Easterling-Peterson, (D_b) (D_a)
- 12
- 95%
- Easterling and Peterson (1995) Mielke *et al.* (1981). **MRPP**
- 10) 12 31 61 100 49-60 61-72. 1 60. 31
- 11) 10. Easterling and Peterson (1995) 3.3.



3.3. *Easterling-Peterson*
Pensacola, Florida
 (a) μ , μ
 (b) μ , μ
 (c) μ , μ
 (d) μ , μ
Peterson μ , μ *Easterling-*
 μ μ : *Easterling and Peterson (1995).*

3.2.2. Standard Normal Homogeneity Test for single shifts

μ μ μ μ ,
 μ μ μ .
 μ (. . Sahin and Cigizoglu, 2010).
 μ (candidate) Y : μ $X \mu$
 Q μ μ μ .
 μ Y_i μ μ μ μ
 (μ μ μ) i.
 X_j μ reference series (j k)
 $\mu X_{ji} \mu$ μ μ μ . j μ
 μ μ μ

$$Q_i = Y_i / \left\{ \left[\sum_{j=1}^k X_{ji} \bar{Y}_j / \bar{X}_j \right] / \sum_{j=1}^k \right\} \quad (3.4)$$

$$Q_i = Y_i - \left\{ \sum_{j=1}^k [X_{ji} - \bar{X}_j + \bar{Y}] / \sum_{j=1}^k \right\} \quad (3.5)$$

$j=1, \dots, k.$

SNHT

$$Z_i = (Q_i - \bar{Q}) / s_Q \quad (3.6)$$

Z $Y.$

SNHT for single shifts:

(H_0) $(H_1):$

$$H_0 : Z_i \in N(0,1) \quad i \in \{1, \dots, n\} \quad (3.7)$$

$$H_1 : \begin{cases} Z_i \in N(\sim_1, 1) & i \in \{1, \dots, a\} \\ Z_i \in N(\sim_2, 1) & i \in \{a+1, \dots, n\} \end{cases} \quad (3.8)$$

(test value) SNHT :

$$T_{\max}^s = \max_{1 \leq a \leq n-1} \{T_a^s\} = \max_{1 \leq a \leq n-1} [a\bar{z}_1^2 + (n-a)\bar{z}_2^2] \quad (3.9)$$

Alexandersson and Moberg (1997)
3.2.

t-test $Q_i,$

$$\bar{q}_1 = \dagger_{\bar{Q}} \bar{z}_1 + \bar{Q} \quad \bar{q}_2 = \dagger_{\bar{Q}} \bar{z}_2 + \bar{Q} \quad (3.10)$$

$$\mu_i = \bar{q}_2 / \bar{q}_1, \quad i = \{1, 2, \dots\} \quad (3.11)$$

$$u = \bar{q}_2 - \bar{q}_1 \quad (3.12)$$

SNHT for single shifts

3.2.4 Standard Normal Homogeneity Test for trends.

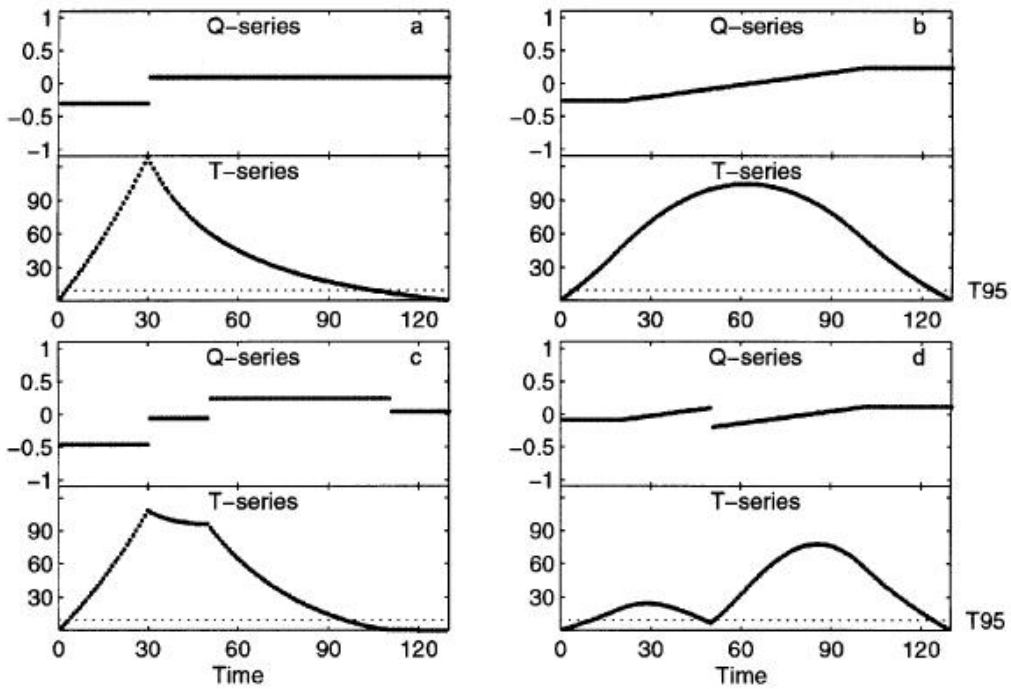
3.2.3. Standard Normal Homogeneity Test for trends

trends. SNHT for trends (H₀) : SNHT for single shifts (H₁) : SNHT for trends (H₁)

$$H_1 : \begin{cases} Z_i \in N(\tau_1, 1) & i \in \{1, \dots, a\} \\ Z_i \in N(\tau_1 + (1-a)(\tau_2 - \tau_1)(b-a), 1) & i \in \{a+1, \dots, b\} \\ Z_i \in N(\tau_2, 1) & i \in \{b+1, \dots, n\} \end{cases} \quad (3.14)$$

SNHT for trends SNHT for single shifts, μ max

Alexandersson and Moberg (1997).



3.4. : Alexandersson and Moberg (1997).
 (a) μ , (b) μ , (c) μ , (d) μ

3.1 μ μ μ SNHT
 Alexandersson and Moberg (1997)
 20000 μ μ μ .
 T_{crit} Khaliq and Ouarda (2007), μ
 ($\mu\mu$) μ Monte Carlo
 Alexandersson and Moberg (1997).

3.1. μ μ μ SNHT n
 : Alexandersson and Moberg (1997)

n	10	20	30	40	50	70	80	90	100	150	250
90	5.05	6.10	6.65	7.00	7.25	7.40	7.55	7.80	7.85	8.05	8.35
95	5.70	6.95	7.65	8.10	8.45	8.65	8.80	9.05	9.15	9.35	9.70
97.5	6.25	7.80	8.65	9.25	9.65	9.85	10.2	10.3	10.4	10.8	11.2

μ SNHT μ .
 , ' μ μ .
 μ μ Standard Normal Homogeneity Test
 for single shifts Standard Normal Homogeneity Test for trends Moberg
 and Alexandersson (1997) Domonkos (2011a). Domonkos (2011a)
 μ μ μ :

1) μ_1, \dots, μ_n , (cutting algorithm).
 μ_1, \dots, μ_n
 2) μ_1, \dots, μ_n .
 μ_1, \dots, μ_n .
 Domonkos (2011a)

5.4.1.

Moberg and Alexandersson (1997) μ_1, \dots, μ_n (μ_1, \dots, μ_n)
 μ_1, \dots, μ_n Standard Normal
 Homogeneity Test for single shifts. μ_1, \dots, μ_n
 μ_1, \dots, μ_n (μ_1, \dots, μ_n). μ_1, \dots, μ_n
 (Moberg and Alexandersson (1997))

3.2.4. Pairwise Homogenization Algorithm

μ_1, \dots, μ_n Menne and Williams (2009) μ_1, \dots, μ_n
 μ_1, \dots, μ_n Vincent (1998) SNHT.
 1) μ_1, \dots, μ_n (0.5). μ_1, \dots, μ_n
 μ_1, \dots, μ_n
 Corr($X_t - X_{t-1}, Y_t - Y_{t-1}$) μ_1, \dots, μ_n
 μ_1, \dots, μ_n .
 2) μ_1, \dots, μ_n 100 μ_1, \dots, μ_n 40 μ_1, \dots, μ_n 40 μ_1, \dots, μ_n
 μ_1, \dots, μ_n 40 μ_1, \dots, μ_n
 (μ_1, \dots, μ_n 7 μ_1, \dots, μ_n).
 3) μ_1, \dots, μ_n 60 μ_1, \dots, μ_n
 μ_1, \dots, μ_n 7 μ_1, \dots, μ_n .
 μ_1, \dots, μ_n

4) $D = X - Y$ (3.15)

$$X = \mu^X + S^X(mT + v) + u_X^{mT+v} + v_X^{mT+v}, Y = \mu^Y + S^Y(mT + v) + u_Y^{mT+v} + v_Y^{mT+v} \quad (3.16)$$

5) SNHT (Alexandersson and Moberg, 1997). (3.17)

$$H_0 : D_t \in N(\mu_0, \sigma^2), c_{k-1} \leq t \leq c_k \quad (3.17)$$

$$H_1 : \begin{cases} D_t \in N(\mu_1, \sigma^2), & c_{k-1} + 1 \leq t \leq c \\ D_t \in N(\mu_2, \sigma^2), & c + 1 \leq t \leq c_k \end{cases} \quad (3.18)$$

6) (BIC, Schwarz 1978).

7)

4.

3, μ , μ 20

μ , μ , μ (Domonkos 2011a).

μ , μ :

1. μ (hit rate),
2. μ (false alarms),
3. μ .

μ , μ , μ , μ , μ , μ , μ , μ .

μ (overdetection at the edges)

SNHT (Wang, 2008).

μ (. . . Venema *et al.*, 2012) 4.1

4.2:

1. μ - μ , μ , μ ,
2. μ , μ , μ .

4.1. μ , μ , μ , μ , μ , μ

μ , μ , μ , μ , μ , μ

- μ , μ / μ , μ , μ , μ , μ

μ (. . . Moberg and Alexandersson, 1997, Wijngaard *et al.*, 2003, Venema *et al.*, 2012).

μ , μ , μ , μ , μ , μ

μ , μ , μ , μ , μ , μ

μ , μ , μ , μ , μ , μ

(6), 4.2.1 μ

4.2.1.

μ (5) μ 1 (AR(1)). (Koutsoyiannis, 2002)

μ X_i, μ $i=1,2, \dots,$ μ $(\dots \mu, \dots)$.

)

$$\mu \mu, \mu X_i, \mu$$

$$\sim = E[X_i] = 0 \tag{4.1}$$

$$E[X_i^2] = \sigma^2, \tag{4.2}$$

$$E[X_i X_{i+j}] = 0 \tag{4.3}$$

:

$$\dots_j = \begin{cases} 1, & j = 0 \\ 0, & j \neq 0 \end{cases} \tag{4.4}$$

μ ($\mu \mu \dots \mu \mu$)

) μ 1 μ 1, μ AR(1), μ μ Markov .

Markov μ $X(t)$, μ Markov μ μ j. μ AR(1) :

$$X_i = \dots X_i + V_i \tag{4.5}$$

$1 \ (-1 < \dots < 1),$

$V_i: \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu$

$\mu \quad \mu \quad (1 - \dots)\mu \quad \mu \quad (1 - \dots)^2 \dots_0.$

$X_i \text{ (Markov)} :$

$\dots_j := \text{corr}[X_i, X_{i+j}] = \dots^{|j|} \tag{4.6}$

$\mu \quad \mu \quad \mu \quad \mu \quad \text{Markov} \quad \mu$

$1 \ (\dots).$

) M $\mu \quad \mu \quad \mu \quad \mu \mu$

$\mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu$

$\mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu$

$\mu \mu$, μ Hurst, μ Hurst-Kolmogorov $\mu \quad \mu \quad \mu \quad \mu$ (Koutsoyiannis, 2002)

$\mu \quad \text{AR}(1). \quad \mu \quad \mu \quad \mu \mu$

$\mu \quad \text{Hurst-Kolmogorov.}$

$\mu \quad \mu \quad \mu \mu \quad \text{Mandelbrot}$

(1965), $\mu \quad \mu \quad \mu \quad \mu$

$\mu \quad \dots \quad \mu \quad \mu$

:

$$Z_i^{(k)} - k \sim_d \left(\frac{k}{l}\right)^H (Z_j^{(l)} - l \sim) \tag{4.7}$$

$\sim_d \mu \quad \mu \quad \mu$

i: () ,

j: ($\mu \quad \mu$),

k, l: $\mu \quad (\mu)$,

$Z_i^{(k)} : \quad \mu \quad \mu \quad k$,

$\mu : \quad \mu \quad \mu \quad i^k$,

H: ($0 < H < 1$) () Hurst.

$\mu \quad :$

$$Z_i^{(k)} = \sum_{l=(i-1)k+1}^{ik} X_l \tag{4.8}$$

$$\chi_0^{(k)} = k^{2H} \chi_0 \quad (4.9)$$

$$\dots_j^k = \dots_j = (1/2)[(j+1)^{2H} + (j-1)^{2H}] - j^{2H}, \quad j > 0 \quad (4.10)$$

(4.7-4.10). Koutsoyiannis (2002) :

- -
 -
- (4.7). Hurst μ (0, 1), μ 0.5. Hurst $H < 0.5$ (4.10) μ, μ (Koutsoyiannis, 2002).

- 9. Beaulieu *et al.* (2008)
- 10. Domonkos (2008)
- 11. Domonkos (2011a)
- 12. Venema *et al.* (2012)

μ , μ
 μ . μ Buishand (1982), Lubes-Niel *et al.*
 (1998) Beaulieu *et al.* (2008) μ
 μ Venema *et al.* (2012) μ .

) μ
 / μ μ μ
 μ :

- 1. Wang (2008)
- 2. Rust *et al.* (2008)

μ μ Easterling and Peterson (1995) μ
 μ μ μ μ . μ μ
 μ μ μ μ
 μ (Venema *et al.*, 2012), μ
 . μ μ μ μ Reeves *et al.*

(2007) μ μ μ μ , μ
 μ .
 μ μ Monte
 Carlo, μ μ μ .

5.2.

- μ μ μ μ μ μ ,
- μ : μ ,
- μ μ μ μ ,
- μ μ (/ μ ,
- μ / μ),
- μ μ μ μ μ ,
- μ μ μ (. . μ ,
- μ μ μ),
- μ μ (, μ) μ μ
- μ ,
- μ (, μ ,) μ μ ,
- μ μ μ μ μ ,

- $\mu^2 = \mu$

5.3. μ

μ is a μ ring, where μ is a μ ideal, 5.2,

μ :

- μ is a μ ideal (μ / μ)
- μ is a μ ideal (μ / μ)
- μ is a μ ideal
- μ is a μ ideal
- μ is a μ ideal

μ is a μ ideal, μ is a μ ideal, μ is a μ ideal, μ is a μ ideal.

Menne and Williams (2005).

μ is a μ ideal, μ is a μ ideal, μ is a μ ideal, μ is a μ ideal.

5.4. μ μ μ

5.1, $12 \mu^2$

μ is a μ ideal.

μ is a μ ideal. 4.2.1.

μ is a μ ideal, μ is a μ ideal, μ is a μ ideal.

μ μ . μ μ
 μ ,
 μ μ μ , μ μ ,
 μ μ μ μ .

5.4.1. μ

1. Buishand (1982)

μ μ μ μ μ μ
 μ . μ μ μ μ μ
 μ μ μ μ :

- Von Neumann ratio test
- Cumulative deviations (, Craddock, 1979)
- Worsley's likelihood ratio test (Worsley, 1979)

μ μ 1999 30
 μ , μ .
 μ μ 0 m, m=15
m=5 μ μ .
Worsley's likelihood ratio test
Cumulative deviations μ μ μ Von
Neumann ratio test μ μ μ μ μ .

2. Easterling and Peterson (1992)

μ 8 μ μ μ μ μ
 μ μ 100 . μ :

- SNHT (Alexandersson, 1986)
- Potter's technique (Potter, 1981)
- t test
- Reg Y-hat (regression approach, Easterling and Peterson, 1992)
- Reg avg Y-hat (regression approach, Easterling and Peterson, 1992)
- Double mass analysis (Kohler, 1949)
- CUSUM (Cumulative sum technique, van Dobben de Bruyn, 1968)
- Two-phase regression (Solow, 1987)

1000 μ - μ Monte Carlo.
 μ 1 (AR(1)). μ , μ μ
 μ μ () μ

() 0.45-0.9
 0.5 2 ()
 0.5-2
 23.0
 25.0
 65
 0.5-2.0
 SNHT
 Potter, Two-phase regression
 SNHT Potter
 SNHT
 SNHT

3. Easterling and Peterson (1995)

Easterling and Peterson (1995)
 (Venema *et al.*, 2012).
 2
 100
 SNHT
 Easterling and Peterson (1992).
 :

- SNHT (Alexandersson, 1986),
- Easterling and Peterson test (Easterling and Peterson, 1995).

Easterling and Peterson (1995),
 0.75 ()
 5
 1000
 Monte Carlo, 1000
 :

-

- Alexandersson (1997) used a 5-year moving average to filter out short-term variability. The results show that the number of days with a minimum temperature below the freezing point of water (0°C) has increased by 26 days (51%) since 1950, and the number of days with a maximum temperature above 5°C has increased by 23 days (63%) since 1950 (Easterling and Peterson (1995)).

4. Lubes-Niel *et al.* (1998):

- rank correlation test (Kendall and Stuart, 1943, WMO, 1966, Olaniran, 1991)
 - Pettitt's test (Pettitt, 1979),
 - Buishand's test (Buishand, 1982,1984),
 - Lee and Heghinian's bayesian procedure (Lee and Heghinian, 1977),
 - Hubert and Carbonnel's segmentation procedure for hydrometeorological series (Hubert and Carbonnel, 1987, 1993, Hubert *et al.*, 1989),
- The results show that the number of days with a minimum temperature below the freezing point of water (0°C) has increased by 26 days (51%) since 1950, and the number of days with a maximum temperature above 5°C has increased by 23 days (63%) since 1950 (Easterling and Peterson (1995)).

5. Ducré-Robitaille *et al.* (2003):

- SNHT without trend (Alexandersson, 1986),
- SNHT with trend (Alexandersson and Moberg, 1997),
- MLR (multiple linear regression, Vincent, 1998),
- TPR (two-phase regression, Easterling and Peterson, 1995),
- WRS (Wilcoxon rank-sum, Karl and Williams, 1987),
- ST (sequential testing for equality of means, Gullett *et al.*, 1990),
- Bayesian approach without reference series (Ouarda *et al.*, 1999; Perreault *et al.*, 1999, 2000),
- Bayesian approach with reference series (Ouarda *et al.*, 1999; Perreault *et al.*, 1999, 2000).

$$X_t = 0.1X_{t-1} + e_t \quad e_t \sim N(0, 1) \quad (5.1)$$

$$Y_t = 1.5X_t + 0.1Y_{t-1} + e_t \quad e_t \sim N(0, 1), \quad (5.2)$$

$$Y_t = 1.5X_t + 0.1Y_{t-1} + e_t \quad e_t \sim N(0, 1), \quad (5.2)$$

MLR Bayesian with reference SNHT. (1.2%, 3.6%, 6.8%, 5%, 56.3%, SNHT with trend, TPR, WRS (13.3%, 41.3%,).

6. Menne and Williams (2005):

Ducre-Robitaille et al. (2003) :

- SNHT (standard normal homogeneity test without trend, Alexandersson, 1986),
- MLR (multiple linear regression, Vincent, 1998),
- Bayesian approach with reference series (Ouarda *et al.*, 1999; Perreault *et al.*, 1999, 2000).

m=5

7 1000 , μ 100
 1 (AR(1)), μ

$$x_{t+1} - \mu = (x_t - \mu) + e_{t+1} \quad e \sim N(0, 1) \quad (5.3)$$

(0), μ .
 ()
 United State Historical Climatology Network (USHCN).

7. DeGaetano (2006):

- SNHT without trend (Alexandersson, 1986),
- POTT (Potter’s method, Potter, 1981),
- MLR (multiple linear regression, Vincent, 1998),
- TPR (two-phase regression, Easterling and Peterson, 1995),
- BAYE (Bayesian test, Perreault *et al.*, 1999, 2000),
- PMETA (Parametric metadata-based test, Karl and Williams, 1987),
- NMETA (Nonparametric metadata-based test, Allen and DeGaetano, 2000).

– National Climatic Data Center (NCDC),
 0.70.
 0.8.
 5.6:

$$Z_t = [A]Z_{t-1} + [B]e_t \quad (5.6)$$

Z_t , $[A]$, $[B]$,
 e_t ,
 $[A]$ (detrended) DeGaetano (2006)
 4.4.

$$Z_t = Z_t + tb \quad (5.7)$$

$[A]$, $[B]$, e_t , $[A]$

μ 70% μ 90% μ AIC μ
 μ 2. μ SBC μ μ 1 μ (μ (32%) μ
 2. μ μ μ μ 3- 5, μ SNH NPW
 μ μ μ μ , μ .
 « » , μ μ XLW SBC.

5.1. μ *Reeves et al. (2007)*

Model 1 (M1)	$Y_t = \mu$	$+ \varepsilon_t$
Model 2 (M2)	$Y_t = \mu + \beta_1 t$	$+ \varepsilon_t$
Model 3 (M3)	$Y_t = \mu + \Delta I(t > c)$	$+ \varepsilon_t$
Model 4 (M4)	$Y_t = \mu + \beta_1 t + \Delta I(t > c)$	$+ \varepsilon_t$
Model 5 (M5)	$Y_t = \mu + \beta_1 t + \Delta I(t > c) + \beta_2 t I(t > c)$	$+ \varepsilon_t$

9. Beaulieu et al. (2008):

8 μ μ μ μ
 μ . μ :

- SNHT (Alexandersson, 1986, Khaliq and Ouarda, 2007),
- MREG (multiple regression, Vincent, 1998),
- REG2 (two-phase regression, Easterling and Peterson, 1995, Lund and Reeves, 2002),
- BIVT (bivariate test, Maronna and Yohai, 1978, Potter, 1981),
- WILS (sequential Wilcoxon test, Karl and Williams, 1987, Lanzante, 1996, Ducre-Robitaille et al., 2003),
- STUS (sequential t-test, Gullett et al., 1990),
- JARU (Jaruskova's method, Jaruskova, 1996),
- BAYE (Bayesian approach, Rasmussen, 2001).

μ 10 μ μ μ μ
 μ , μ μ
 μ . μ 10 μ
 μ μ μ
 μ *Easterling and Peterson (1992).*
 μ μ μ μ μ μ μ
 (μ μ , μ , μ 1

test, Shapiro-Wilk normality

$$z_i = \rho z_{i-1} + e_i \quad (5.8)$$

0.02. 1089 mm, 142 mm

10000, 5000, 60, 5000, 100

Beaulieu *et al.* (2008)

- 50000 (25000, 60, 25000)
- 30000 (15000, 2)
- 10000
- 10000

(

5.9,

$$w_i = \rho w_{i-1} + e_i \quad (5.9)$$

5.10.

$$w_i = z_i + w_i \quad (5.10)$$

z_i w_i i i w 0.7 $()$ 0.55 $300km$ 0.55 $300km$ $Quebec$ 60 100 (4.4) 5% $BAYE (21.2\%)$ 5% $BAYE$ 60 100 $JARU$ $SNHT$ $MREG$ $BAYE$ $STUS$ $REG2$ 17 11 7 $:$

10. Domonkos (2008):

17 11 7 $:$

- tta (t-test, Ducré-Robitaille *et al.*, 2003)
- ttb (t-test, Kyselý and Domonkos, 2006)
- Buia (Buishand-test, maximum of the absolute values of accumulated anomalies, Buishand, 1982)
- Buib (Buishand-test, difference between maximum and minimum values of accumulated anomalies, Buishand, 1982)
- SNHa (Standard Normal Homogeneity Test for shifts only, Alexandersson, 1986)
- WRS (Wilcoxon Rank Sum test, Karl and Williams, 1987)
- MLR (Multiple Linear Regression, Vincent, 1998)
- Baya (Bayesian test, Ducré-Robitaille *et al.*, 2003, with serial correlation analysis, Sneyers, 1999)
- Bayb (Bayesian test, Ducré-Robitaille *et al.*, 2003, with penalised maximum likelihood method for calculating number of change-points, Caussinus and Lyazrhi, 1997, Mestre, 2004)
- Pett (Pettitt-test, Pettitt, 1979)
- M-K (Mann-Kendall test, Aesawy and Hasanean, 1998)
- Mest (method of Mestre, Mestre, 2004)
- Mesb (method of Mestre with parameterised minimum unit-length)
- SNHT (Standard Normal Homogeneity Test for shifts and trends, Alexandersson and Moberg, 1997)
- East (Easterling-Peterson test, Easterling and Peterson, 1995)
- MASH, (Multiple Analysis of Series for Homogenisation, Szentimrey, 1999)
- MASb, (Multiple Analysis of Series for Homogenisation with parameterised minimum unitlength)

Homogeneity Test-	μ	(MLR	μ	Standard Normal
SNHT)	μ	μ	μ	(μ)
	(μ).	μ	μ	μ
	μ	.		
		(μ , μ , μ)	μ	μ
μ	μ	215	μ	μ
.		μ	μ	112
		μ	μ	98-100
		μ		.
μ	μ		μ	
	0.4.			
	μ		μ	
μ	,	μ	μ	μ
μ	,)	μ)	μ	,) μ
μ	,)	$\mu\mu$,) μ	μ , .
	μ	μ) μ	
μ	μ	.		

μ μ μ 100 .
 μ μ μ μ 4 .
 μ μ μ μ .
 μ μ μ μ μ μ μ μ μ .
 17 μ μ μ μ μ μ μ μ μ μ .
 μ : μ μ μ μ μ .
 • μ μ μ μ μ μ μ μ .
 • (u), μ μ (c). μ μ μ μ .
 μ μ μ μ μ μ 1000.
 μ μ μ μ μ μ .
 μ μ μ , . . .
 μ μ μ μ μ μ Domonkos (2008) Mesb,
 MASb, Mest, Bayb, MLR, MASH SNHa. μ μ μ μ μ .
 μ μ μ μ μ μ μ μ μ μ μ .
 μ (μ μ μ μ μ μ μ μ).
 μ μ μ μ μ μ μ μ .
 μ μ μ μ μ SNHT (Alexandersson, 1986),
 μ μ μ μ μ Domonkos (2008) μ μ .
 μ μ μ μ μ μ μ μ .

11. Domonkos (2011a):

μ μ μ μ μ μ μ μ μ μ μ μ μ μ μ μ μ μ μ 7 μ μ μ μ μ μ μ μ μ μ . 7 μ :

- MLR (Multiple Linear Regression, Vincent, 1998),
- PMT (Penalised Maximal t test, Wang *et al.*, 2007),
- SNH (Standard Normal Homogeneity Test for shifts only, Alexandersson, 1986),
- SNHT (Standard Normal Homogeneity Test for shifts and trends, Alexandersson and Moberg, 1997),
- tts (t test, Ducre-Robitaille et al., 2003),
- C-M (Caussinus and Mestre, 2004),
- MASH (Szentimrey, 1999).

μ SNH (SNH1 SNH2):

1 μ 0.4. μ Domonkos
(2011a) μ μ μ μ
 μ μ A μ 1 μ μ B-D
 μ μ E-F μ
(μ , μ).
 μ μ μ μ μ 4
 μ μ μ $\mu\mu$ μ , μ ,
 μ 5.12.

$$F_r = \frac{S_F}{S_R + S_F} \tag{5.12}$$

S_R μ μ S_F μ μ μ .
(2011a) μ μ μ μ Domonkos
 μ μ , μ μ , μ μ tts
 μ .
 μ μ μ μ μ μ μ
 μ (A), μ D μ B, C E μ μ 5-
 μ 20% , μ 5%. μ F (30-45%) .
 μ μ F $\mu\mu$
 μ , μ μ μ μ .
A-C μ , μ PMT SNH μ D-F μ C-M
MASH. μ μ μ 8 μ .
E F μ , μ μ
 μ μ C-M MASH.
 μ SNH2 μ SNH1 SNH2,
 μ SNH1, μ μ μ
 μ μ μ μ (μ $\mu\mu$ μ)
 μ μ μ) .

12. Venema *et al.* (2012):

μ μ μ μ μ μ μ
 μ μ μ μ μ . 25
 μ μ μ 5.2.

PMFred (Wang, 2008)

(surrogate),

5.2. Venema et al. (2012).

Method	Comparison		Detection		References
	Comparison	Time step	Search	Criterion	
MASH	Multiple references	Annual, parallel monthly	Exhaustive	Statistical test (MLR)	Szentimrey (2007, 2008)
PRODIGE	Pairwise, human synthesis	Annual, parallel monthly	DP	Penalized Likelihood	Caussinus and Mestre (2004)
USHCN	Pairwise, automatic synthesis	Serial monthly	HBS	Statistical test (MLR)	Menne et al. (2009)
AnClim	Reference series	Annual, parallel monthly	HBS, moving window	Statistical test	Štepanek et al. (2009)
Craddock	Pairwise, human synthesis	Serial monthly	Visual	Visual	Craddock (1979); Brunetti et al. (2006)
RhtestV2	Reference series or absolute	Serial monthly	Stepwise	Statistical test (modified Fisher)	Wang (2008)
SNHT	Reference series	Annual	HBS	Statistical test (MLR)	Alexandersson and Moberg (1997)
Climatol	Reference series	Parallel monthly	HBS, moving window	Statistical test	Guijarro (2011)
ACMANT	Reference series	Annual, joint seasonal	DP	Penalized Likelihood	Domonkos et al. (2011)

DP=Dynamic programming (optimization method); HBS=(semi-)hierarchic binary splitting; MLR=Maximum Likelihood Ratio test.

Iterative Amplitude Adjusted Fourier Transform Algorithm (IAAFT), Venema et al. (2006b). Fourier

100 (1900 1999). 5, 9 15

100 (mirroring)

Craddock, MASH, PRODIGE, ACMANT, PMFred, MASH, 5%, 10%, 15%, SNHT.

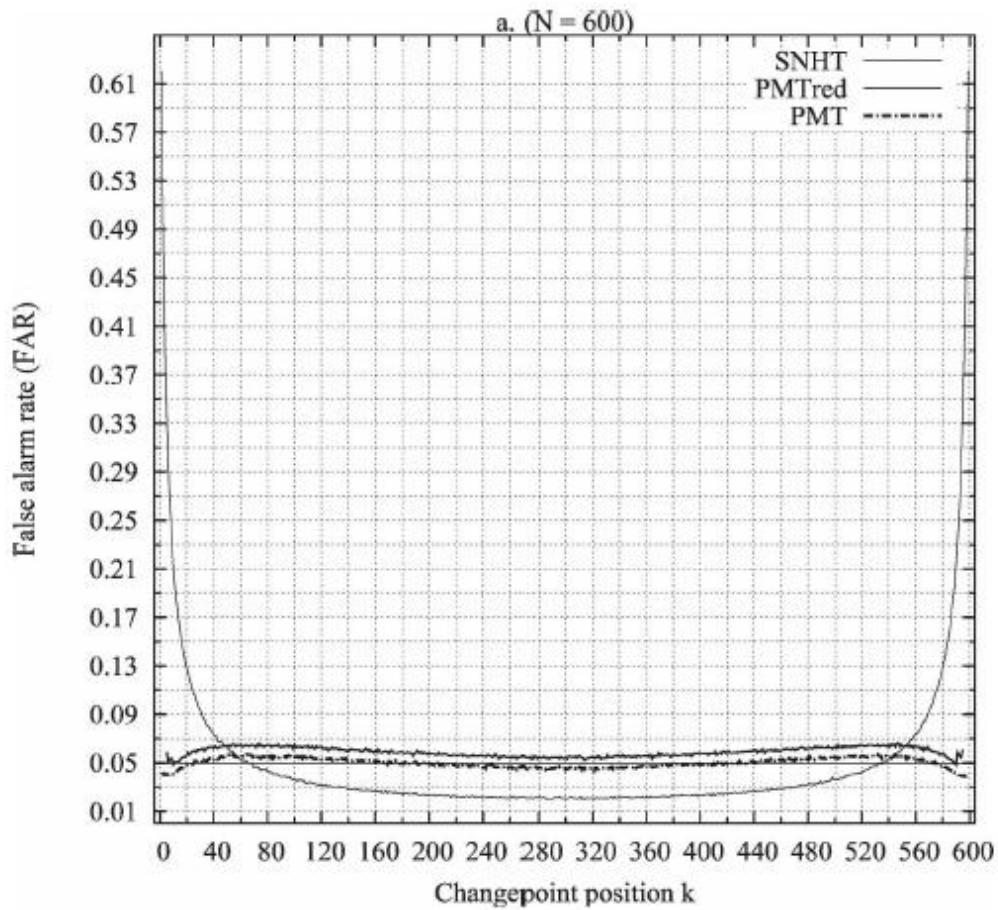
5.4.2.

1. Wang (2008)

PMT, PMF, PMTred, PMFred, SNHT, $(N - 2) \times (N_{\min} + 1) \times 10000$, 600, (50), 1 (AR(1)). N_{\min}

$\mu = 600$. 1
 $\mu = 0.1925$.

alarm rate - FAR) μ (False
 μ (μ 5.1). FAR μ μ
 μ μ μ μ



μ 5.1. μ μ (FAR) μ PMTred
 μ μ μ PMT SNHT. : Wang (2008).

μ μ μ 5.1 PMTred PMT μ μ μ
 μ μ μ FAR μ μ μ k μ
 μ μ PMTred μ PMT, μ
 μ μ μ SNHT μ μ μ
 μ μ μ , μ μ μ
 μ μ μ FAR. μ 5.1
 μ μ 60 μ (μ 5)
 μ (FAR>5%).

1 (AR(1) $\mu = 0.1366$, $\sigma = 0.1200$), PMT, PMTred, Wang (2008) $\mu = 0.01$, (5%).

10000 μ , $\mu = 0$, $\mu = 600$, 2) $\mu = -0.1925$, $\mu = 600$, FAR. μ

PMTred, PMFred, $\mu = -0.200, -0.150, -0.100, -0.050, 0, 0.025, \dots, 0.0950$. μ , 43 μ , 10000000

AR(1), FAR, $\mu\mu$

2. Rust et al. (2008)

Causinus and Mestre (2004), CM.

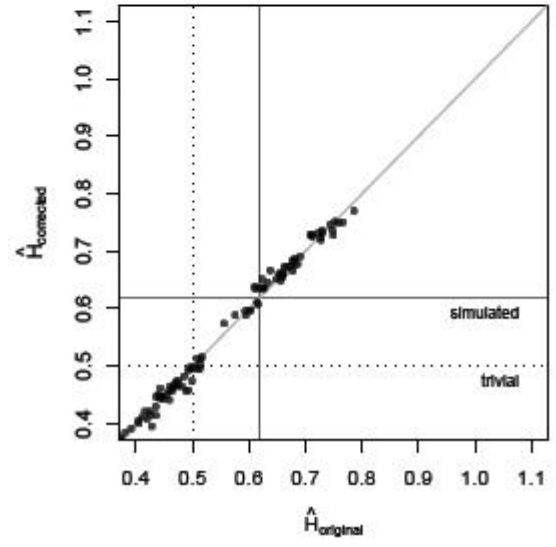
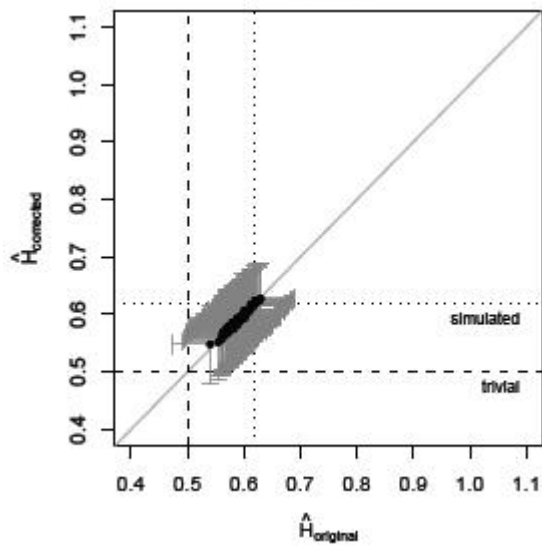
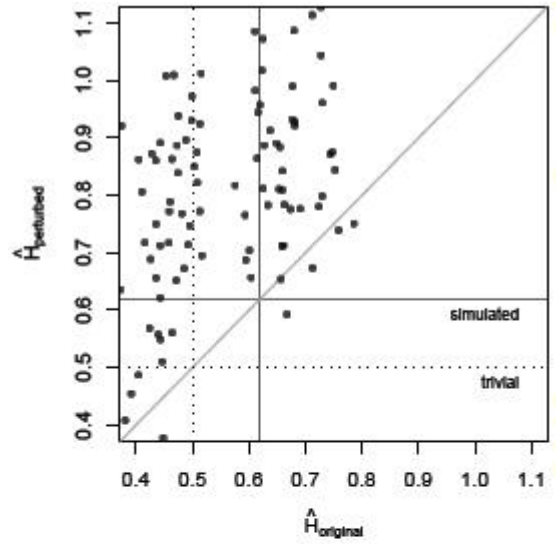
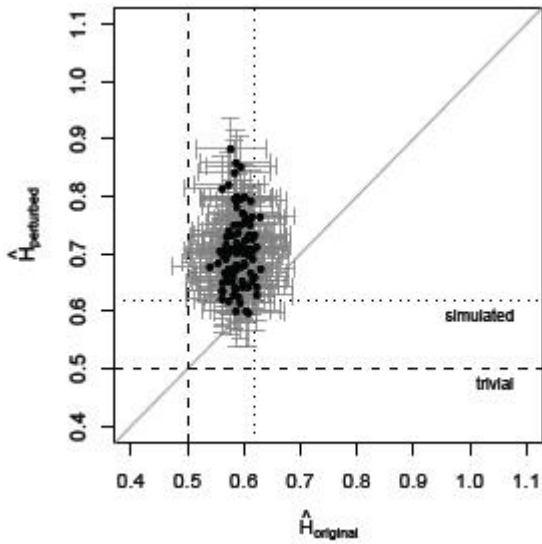
μ_t, μ_{t-1}, \dots (anomalies, μ_t).

Alexandersson (1986). Hurst CM μ .

24 μ (surface air temperature). Causinus and Mestre (2004). 19 μ .

Hurst μ .

- μ FARIMA[p; d; q], (μ , μ FARIMA[1; d; 0] FARIMA[1; d; 1]),



μ 5.2. μ *Hurst*
 μ *Rust et al. (2008)*, μ
FARIMA[1; d; 0] μ *DFA* μ : μ
Hurst μ μ μ μ μ μ
 μ μ μ μ μ μ μ *Hurst*
 μ μ μ μ μ μ μ μ *Hurst*
 . : *Rust et al. (2008)* μ .

5.5. - μ μ
 μ μ 5.4.1 5.4.2 μ
 μ μ μ μ μ μ μ μ
 μ μ μ μ μ μ μ μ

5.5.1.

(Buishand, 1982, Lubes-Niel *et al.*, 1998, Beaulieu *et al.*, 2008).
 Venema *et al.* (2012)
 2008
 100
 60 100 Beaulieu *et al.* (2008)
 60 100
 Rust *et al.* (2008). Rust *et al.* (2008)
 Caussin and Mestre (2004) Hurst (
 FARIMA[1,d,0])
 =0.62.
 AR(1), (Venema *et al.* (2012)
 Lubes-Niel *et al.* (1998)
 Rust *et al.* (2008),
 Conrad and Pollak (1962) ()
 (Karl and Williams, 1987).
 0.45-0.9.
 $\mu_{XY}=0.8$ (Easterling and Peterson, 1995, Ducré-Robitaille *et al.*, 2003, DeGaetano, 2006).

5.5.2.

(Alexandersson, 1986), (Reeves *et al.*, 2007).

(Potter (1981) Bayesian tests (Perreault *et al.*, 1999, 2000, Ducré-Robitaille et al., 2003).

95%, 0.95. 5%,

(Domonkos, 2011a, Venema *et al.*, 2012)

MASH, Craddock, PRODIGE, ACMANT Caussinus and Mestre (2004).

5.4.1 5.4.2 Vincent (1998), (AR(1)) Easterling and Peterson (1995), Menne and Williams (2005).

Domonkos (2011a),
 Venema *et al.*
 (2012), MASH, 5%.

SNHT. MASH.

- (cutting algorithm),
- μ - (semihierarchic algorithm),

Domonkos (2011a).

Beaulieu *et al.* (2008).
 Wang (2008), SNHT 5%.

5.6.

X_t Y_t

$$Z_t = X_t - Y_t \tag{5.14}$$

$$Y_t = rX_t + \sqrt{1-r^2}V_t \tag{5.15}$$

$$Cov[X_t, X_{t-1}] = Cov[Y_t, Y_{t-1}] = \alpha_t \tag{5.16}$$

$$Var[X_t] = Var[Y_t] = \alpha_0 = 1 \tag{5.17}$$

$$\dots_t = \frac{\alpha_t}{\alpha_0} = \alpha_t \tag{5.18}$$

$$E[X_t Y_{t-1}] = E[rX_t X_{t-1}] + E[X_t \sqrt{1-r^2} V_{t-1}] = \alpha_t \tag{5.19}$$

$$\begin{aligned} E[Z_t Z_{t-1}] &= Cov[Z_t, Z_{t-1}] = E[(X_t - Y_t)(X_{t-1} - Y_{t-1})] = \\ &= E[X_t X_{t-1}] + E[Y_t Y_{t-1}] - E[X_t Y_{t-1}] - E[X_{t-1} Y_t] = \\ &= 2\alpha_t - 2r\alpha_t = 2(1-r)\alpha_t \end{aligned} \tag{5.20}$$

$$Var[Z_t] = 2(1-r)\alpha_0 \tag{5.21}$$

$$\rho_t = \frac{Cov[Z_t, Z_{t-1}]}{\sqrt{Var[Z_t]Var[Z_{t-1}]}} = \frac{2(1-r)\alpha_t}{2(1-r)\alpha_0} = \frac{\alpha_t}{\alpha_0} = \dots_t \tag{5.22}$$

(1962), Conrad and Pollak (1962), $Cov[X_t, Y_t] = 0$, $Var[X_t] = Var[Y_t]$

6.1. μ μ μ

	μ	
μ	0 (°C)	1 (°C)
	1000 (mm)	300 (mm)

μ μ μ : μ μ

_____ μ _____ :

1. Hurst,
2. μ μ ,
3. μ .

_____ μ _____ :

1. ,
 2. μ μ μ μ μ .
- μ μ μ μ μ μ

μ μ 5.1.4.1 5.1.4.5.

6.3.1. Hurst

μ μ μ [0.5, 1),
 μ , 4.2.1. μ
 μ =0.5 μ =0.9 μ 0.05, 9
 μ .
 : = {0.50, 0.55, ..., 0.90}.

6.3.2.

μ μ μ μ μ μ μ μ .
 μ μ μ μ μ μ μ .

Moberg and Alexandersson (1997), SNHT

$$\frac{dT}{dt}, \quad (X) \quad (Y) \quad (3.1).$$

$$\rho_{XY} := \frac{Cov[X, Y]}{\sqrt{Var[X]Var[Y]}} = \frac{\dagger_{XY}}{\dagger_X \dagger_Y} \quad (-1 \leq \rho_{XY} \leq 1) \quad (6.1)$$

Easterling and Peterson (1995) Beaulieu *et al.* (2008). 0.5, 0.1, 0.95.

0.9. 0.95. 6 : $\rho_{XY} = \{0.5, 0.6, 0.7, 0.8, 0.9, 0.95\}$.

6.3.3

3, 9 (6.2).

6.2.

$\mu \quad \mu \quad \mu$

$\mu \quad \mu$	μ
1/1	1 $\mu \quad \mu$
3/7	7 $\mu \quad \mu \quad 3$
3/1	1 « » 3 $\mu \quad \mu$
2/2	2 $\mu \quad \mu \quad 2$
3/3	3 $\mu \quad \mu \quad 3$
3/10	10 $\mu \quad \mu \quad 3$
3/20	20 $\mu \quad \mu \quad 3$
3/7-50 years*	7 $\mu \quad \mu \quad \mu$ $\mu \quad 50 \quad \mu \quad 100 \quad 3$,
3/7-j10*	7 $\mu \quad \mu$ $\mu \quad \mu \quad 10 \quad \mu \quad 3 \quad \mu \quad 5$,

* $\mu \quad \mu \quad \mu \quad 3/7 \quad \mu \quad \mu \quad ,$

$\mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu$
 $\mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu$
 $\mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu \quad :$

$\mu \quad 1/1 \quad \mu \quad \mu$
 $\mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu$
 ($\mu \quad \mu \quad \mu$) $\mu \quad \mu$
 $\mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu$
 $\mu \quad \mu \quad 3/7 \quad \mu \quad \mu$
 $\mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu$ PHA (Menne and Williams, 2009), μ
 $\mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu \quad \mu$, SNHT,
 $\mu \quad \mu \quad \mu \quad 7 \quad \mu$
 $\mu \quad \mu \quad \mu \quad \mu \quad \mu$

μ 3 (. 3.2.4). 4
 4 / 3/7.
 μ μ 3
 (3/1). μ
 μ (μ μ μ) μ
 μ ,
 μ (Searcy and Hardison, 1960).

μ 3 μ
 (3/7 3/1) (Ducré-
 Robitaille *et al.*, 2003, Vincent, 1998, Beaulieu *et al.*, 2008).

μ 3/10
 3/20.
 μ μ μ μ (μ
 , μ μ μ μ
 μ) μ μ μ
 3/7-50 years 3/7-j10.
 6.3.4 6.3.5.

6.3.4.

μ μ μ μ ,
 μ μ 100 μ μ
 . μ μ μ (. .
 Caussin and Mestre, 2004, Easterling and Peterson, 1995, Moberg and Alexandersson,
 1997 . .).

μ 100 μ μ 50 ,
 μ μ μ
 μ .

6.3.5.

μ μ μ μ μ
 μ μ μ μ μ
 μ μ μ μ μ
 . . μ μ 99 μ μ
 100 .

μ μ 5
 μ μ μ μ .
 μ μ (. . Easterling and Peterson, 1995),
 μ μ μ μ ,

6.4.1. μ μ μ $\mu\mu$

μ μ HK. μ , μ (Koutsoyiannis, 2002).

μ μ μ $\mu\mu$, μ μ Koutsoyiannis (2002). μ μ

μ 1000.

X_i μ μ μ , μ Hurst, AR(1), A_i, B_i, C_i ,

$$X_i = A_i + B_i + C_i \tag{6.2}$$

$$\mu \dots = 1.52(H - 0.5)^{1.32}, \tag{6.3}$$

$$\{ = 0.953 - 7.69(1 - H)^{3.85}, \tag{6.4}$$

$$< = \begin{cases} 0.932 + 0.087H, & H \leq 0.76 \\ 0.993 + 0.007H, & H > 0.76 \end{cases} \tag{6.5}$$

$$Var[A_i] = (1 - c_1 - c_2)\chi_0 \tag{6.6}$$

$$Var[B_i] = c_1\chi_0 \tag{6.7}$$

$$Var[C_i] = c_2\chi_0 \tag{6.8}$$

c_1 c_2 μ μ μ

$$\dots_j = (1 - c_1 - c_2)\dots^j + c_1\{^j + c_2<^j \tag{6.9}$$

μ μ μ c_1 c_2 μ μ : 1 100. ,

$$\begin{cases} \dots_1 = (1 - c_1 - c_2)\dots^1 + c_1\{^1 + c_2<^1 \\ \dots_{100} = (1 - c_1 - c_2)\dots^{100} + c_1\{^{100} + c_2<^{100} \end{cases} \tag{6.10}$$

6.11. μ j

$$\dots_j = (1/2)[(j+1)^{2H} + (j-1)^{2H}] - j^{2H}, \quad j > 0 \tag{6.11}$$

6.4.2.

$$Y = AX \tag{6.12}$$

$$E[Y, Y^T] = Cov[Y, Y^T] = A E[X, X^T] A^T = A I A^T = A A^T \tag{6.13}$$

$$Cov[Y, Y^T] = A A^T = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \tag{6.13}$$

$$Y = AX$$

$$YY^T = AXX^T A^T$$

$$E[X, X^T] = Cov[X, X^T] = A \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & \dots \\ \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & 1 \end{bmatrix} \tag{6.13}$$

$$E[Y, Y^T] = A E[X, X^T] A^T = A I A^T$$

$$Cov[Y, Y^T] = A A^T = \begin{bmatrix} 1 & \dots_{12} & \dots & \dots_{1\epsilon} \\ \dots_{21} & 1 & \dots & \dots \\ \dots & \dots & \dots & \dots_{\epsilon-1, \epsilon} \\ \dots_{\epsilon 1} & \dots & \dots_{\epsilon, \epsilon-1} & 1 \end{bmatrix}, \tag{6.14}$$

$$12 = 13 = \dots = 1 = 21 = 1 = \dots = ij \tag{6.15}$$

$$\rho_{XY} = 0.9$$

$$Cov[Y, Y^T] = AA^T = \begin{bmatrix} 1 & 0.9 & 0.9 & 0.9 \\ 0.9 & 1 & 0.9 & 0.9 \\ 0.9 & 0.9 & 1 & 0.9 \\ 0.9 & 0.9 & 0.9 & 1 \end{bmatrix}$$

Cholesky.

$$Y_{ref, final} = \frac{(Y_1 + Y_2 + Y_3)}{3} \quad (6.16)$$

6.17.

$$W = \frac{X + |Y|}{\}} \quad (6.17)$$

$$Cov[W, Y] = E \left[\frac{X' + |Y'|}{\}} Y' \right] = E \left[\frac{X'Y' + |Y'|^2}{\}} \right] = \frac{1}{\}} Var[Y] = \frac{1}{\}} \tau_y^2 \quad (6.18)$$

$$\dots = \frac{Cov[W, Y]}{\tau_w \cdot \tau_y} \quad \dots = \frac{1 \cdot \tau_y}{\}} \cdot \tau_w \quad (6.19)$$

$$\tau_w^2 = E \left[\left(\frac{X' + |Y'|}{\}} \right)^2 \right] = E \left[\frac{1}{\}^2 X'^2 + 2 \frac{1}{\}} X'Y' + \frac{1}{\}^2 Y'^2 \right] = \frac{1}{\}^2 \tau_x^2 + \frac{1}{\}^2 \tau_y^2 \quad (6.20)$$

6.19 μ

$$\dots^2 = \frac{|^2 \cdot \dagger_Y^2}{\}^2 \cdot \dagger_W^2} \quad \dagger_W^2 = \frac{|^2 \dagger_Y^2}{\}^2 \dots^2} \quad (6.21)$$

6.20 6.21 μ :

$$\frac{1}{\}^2 \dagger_X^2 + \frac{|^2}{\}^2 \dagger_Y^2 = \frac{|^2 \dagger_Y^2}{\}^2 \dots^2} \quad \dagger_X^2 + |^2 \dagger_Y^2 = \frac{|^2 \dagger_Y^2}{\dots^2} \quad \dots^2 \dagger_X^2 + \dots^2 |^2 \dagger_Y^2 = |^2 \dagger_Y^2$$

$$|^2 = \frac{\dots^2 \cdot \dagger_X^2}{\dagger_Y^2 - \dots^2 \cdot \dagger_Y^2} \quad (6.22)$$

μ μ $\dagger_W^2 = 1$ 6.20

$$\}^2 = \dagger_X^2 + |^2 \cdot \dagger_Y^2 = \frac{1}{1 - \dots^2} \dagger_X^2 \quad (6.23)$$

W

μ Hurst. H μ μ μ
 μ μ μ (LSV, least squares based on variance)
 Tyrallis and Koutsoyiannis (2011).

μ μ X, Y, μ W,
 $\mu\mu$ Hurst μ μ μ 15
 μ μ Hurst W, μ W

$$\begin{cases} H15 \leq H + 0.05 \\ H15 \geq H - 0.05 \end{cases} \quad (6.24)$$

μ , Hurst. W
 μ μ μ
 μ .

6.5 μ μ

6.5.1. μ μ SNHT

μ SNH μ (. 3.2.2).
 μ , μ
 μ μ , μ

SNH2) , Domonkos (2011a). (SNH1)

5.3. SNH1 (cutting algorithm).

Domonkos (2011a) 10

Domonkos (2011a) 5

SNH2 SNH1. SNH1.

SNH1 SNH2,

SNH1 Domonkos (2011a).

SNH ,

SNH 95%. 5%

5%.

test value T_{crit} Khaliq and Ouarda (2007),

SNH Monte Carlo 108

Alexandersson and Moberg (1997),
 Khaliq and Ouarda (2007)
 T_{crit}
 T_{crit} ()

6.5.2.

SNHT, μ
 XY = {0.50, 0.55, ..., 0.90} XY = {0.5, 0.6, 0.7, 0.8, 0.9, 0.95}.

- 1/1, 3/7, 2/2, 3/3, 3/10
3/20
 - 3/7-50 years 3/7-j10
 - 1 Hurst
- 10000
- SNHT 1/1, 1
- 3/7,
- μ μ , μ
- μ μ
- μ μ

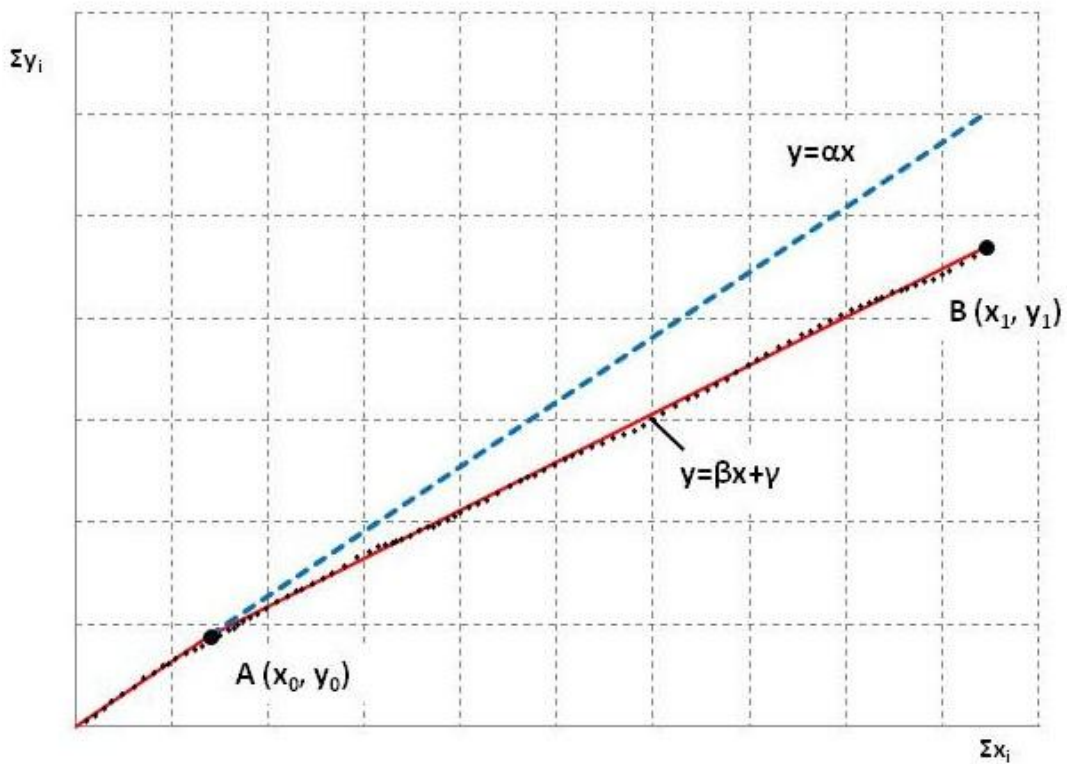
6.2. (x_0, y_0) , (x_1, y_1)

$y = r x$

(x_0, y_0) , (x_1, y_1)

$$y = r x \quad r = \frac{y_0}{x_0} \quad (6.25)$$

$$y = \frac{y_0}{x_0} x \quad (6.26)$$



6.2. (x_0, y_0) , (x_1, y_1)

$$\mu \quad y = Sx + X, \quad :$$

$$\begin{cases} y_1 = Sx_1 + X \\ y_0 = Sx_0 + X \end{cases}, \quad :$$

$$S = \frac{y_1 - y_0}{x_1 - x_0} \quad (6.27)$$

$$\mu \quad x_0, x_1, y_0, y_1 \quad \mu \quad :$$

$$X = \frac{y_1 - y_0}{x_1 - x_0} x + y_0 - \frac{y_1 - y_0}{x_1 - x_0} x_0 \quad (6.28)$$

$$\mu \quad \mu \quad (x_i, y_i) \quad \mu \quad \mu \quad .$$

$$\mu : \quad \sum (y_i - \hat{y}_i)^2 = \min$$

$$\mu \quad \mu \quad 6.26, 6.27$$

$$Q = \sum_{x \leq x_0} \left(y_i - \frac{y_0}{x_0} x_i \right)^2 + \sum_{x > x_0} \left(y_i - \frac{y_1 - y_0}{x_1 - x_0} x_i - y_0 + \frac{y_1 - y_0}{x_1 - x_0} x_0 \right)^2 \quad (6.29)$$

$$\mu \quad \mu \quad Q \quad \mu \quad , y_0 \quad y_1, \quad y_0 \quad y_1.$$

$$\mu : \quad \mu$$

$$\begin{cases} \frac{\partial Q}{\partial y_0} = 0 \\ \frac{\partial Q}{\partial y_1} = 0 \end{cases} \quad (6.30)$$

$$\mu \quad \mu \quad \mu \quad y_0 \quad y_1 \quad (6.31) \quad (6.32).$$

$$y_0 = \frac{\frac{D}{C} \frac{x_1 - x_0}{x_0} \sum_{x \leq x_0} x_i y_i - \frac{D}{C} \sum_{x > x_0} x_i y_i + \frac{D}{C} x_1 \sum_{x > x_0} y_i - \sum_{x > x_0} x_i y_i - x_0 \sum_{x > x_0} y_i}{mx_0 - \sum_{x > x_0} x_i + \frac{x_1 - x_0}{x_0^2} \frac{D}{C} \sum_{x \leq x_0} x_i^2 - \frac{D}{C} \sum_{x > x_0} x_i + \frac{D}{C} x_1 m} \quad (6.31)$$

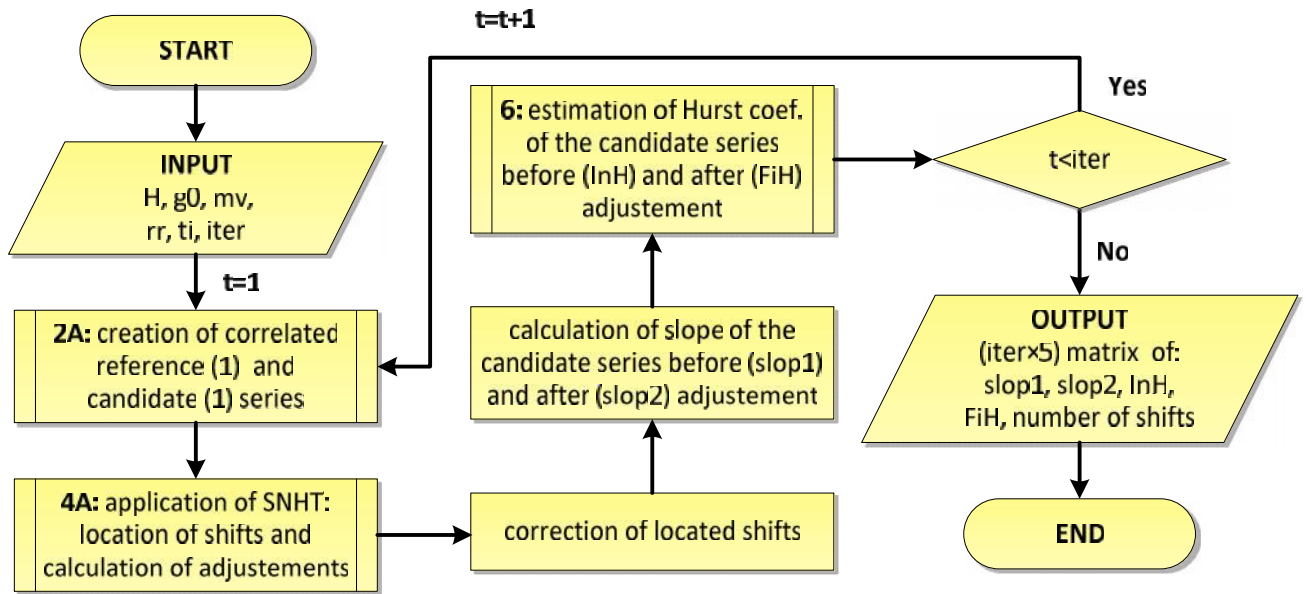
$$y_1 = y_0 + \frac{(x_1 - x_0)(mx_0 y_0 + \sum_{x > x_0} x_i y_i - x_0 \sum_{x > x_0} y_i - y_0 \sum_{x > x_0} x_i)}{D} \quad (6.32)$$

$$: \quad C = x_0 \sum_{x > x_0} x_i - mx_0 x_1 - \sum_{x > x_0} x_i^2 + x_1 \sum_{x > x_0} x_i \quad (6.33)$$

$$D = mx_0^2 + \sum_{x > x_0} x_i^2 - 2x_0 \sum_{x > x_0} x_i \quad (6.34)$$

) CheckSNHT_multiple:

μ μ SNHT μ μ μ
 μ μ μ μ Hurst μ
 μ μ μ SNHT.
 7.2. $\mu\mu$
 CheckSNHT_multiple μ 7.1.

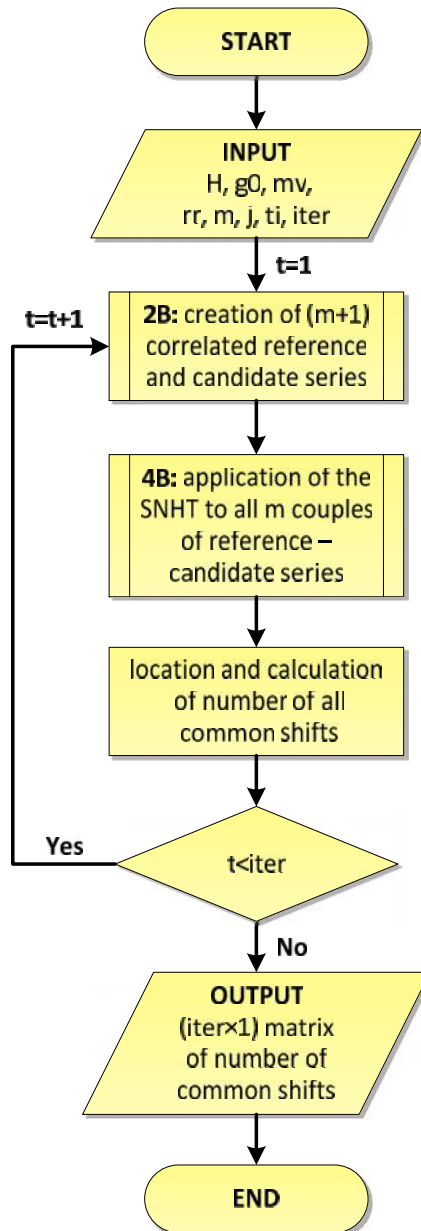


μ 7.1. $\mu\mu$ *CheckSNHT_multiple,* μ μ
 SNHT μ μ

CheckSNHT_multiple μ μ :
 H: μ Hurst
 g0: μ
 mv: μ
 rr: μ - μ
 ti: μ
 iter: μ μ Monte Carlo
 :
 slop1: $\mu\mu$ μ μ
 slop2: $\mu\mu$ μ μ μ
 InH: Hurst μ μ
 FiH: Hurst μ μ μ
 alm: μ μ

) CheckSNHT Mref:

μ μ SNHT μ μ
SNHT. μ μ μ μ
 $\mu\mu$ CheckSNHT_Mref μ 7.2.



μ 7.2. $\mu\mu$ *CheckSNHT_Mref,* μ μ μ μ
SNHT μ μ

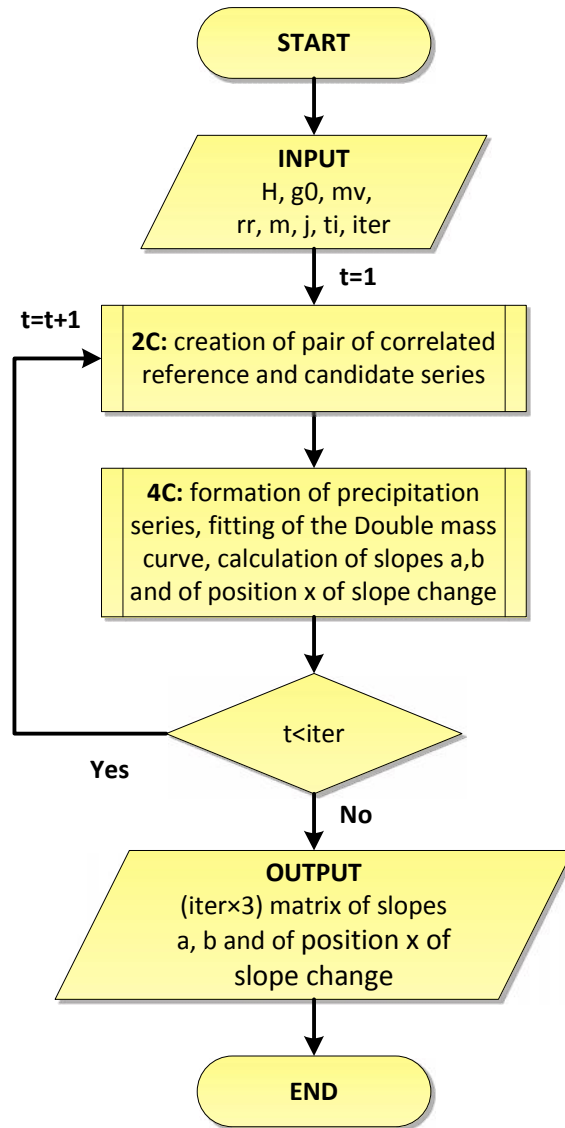
H: μ CheckSNHT_Mref μ Hurst μ :
 g0: μ
 mv: μ μ
 rr: μ - μ
 m: μ
 j: μ - μ μ μ
 ti: μ
 iter: μ μ Monte Carlo
 alm: μ μ μ , μ μ μ

) CheckDMass1:

μ μ μ μ μ
 μ μ μ μ μ . μ μ μ .

H: μ CheckDMass1 μ Hurst μ :
 g0: μ
 mv: μ μ
 rr: μ - μ
 m: μ
 ti: μ
 iter: μ μ Monte Carlo
 a: μ
 b: μ
 x:

μ
 $\mu\mu$ CheckDMass1 μ 7.3.



μ 7.3.

μμ

CheckDMass1,

μ μ

μ

μ

SNHT

7.2.

,

11

μ

μ

7.1,

Tyralis and

Koutsoyiannis (2011).

μ

7

μ

μ

μ

μ

.

μ

μ

μ

7.2

6,

μ

Hurst

o

Tyralis and Koutsoyiannis (2011).

7, μ μ SNHT μ

7.2. μ μ

		μ μ	
1	μ μ μ μ μ	1	hurst_3AR1
2	μ μ - μ μ	2A	series_ref_cand
		2	creation_series_ref_cand
		2C	series_ref_cand_DMass
3	μ μ μ μ μ	3A	SNHT_breaking_pos
		3B	piecewise_linear
		3C	DMC_mult_ref_cand
4	μ μ μ μ	4A	multiple_shifts2
		4B	multiple_shifts_mref2
5		5	SCor
6	μ Hurst	6	lsv
7	μ μ SNHT	7	Tcritical

1) hurst 3AR1

μ μ μ μ Hurst.

series_ref_cand μ μ :

H: μ Hurst

g0: μ

mv: μ μ

ti: μ

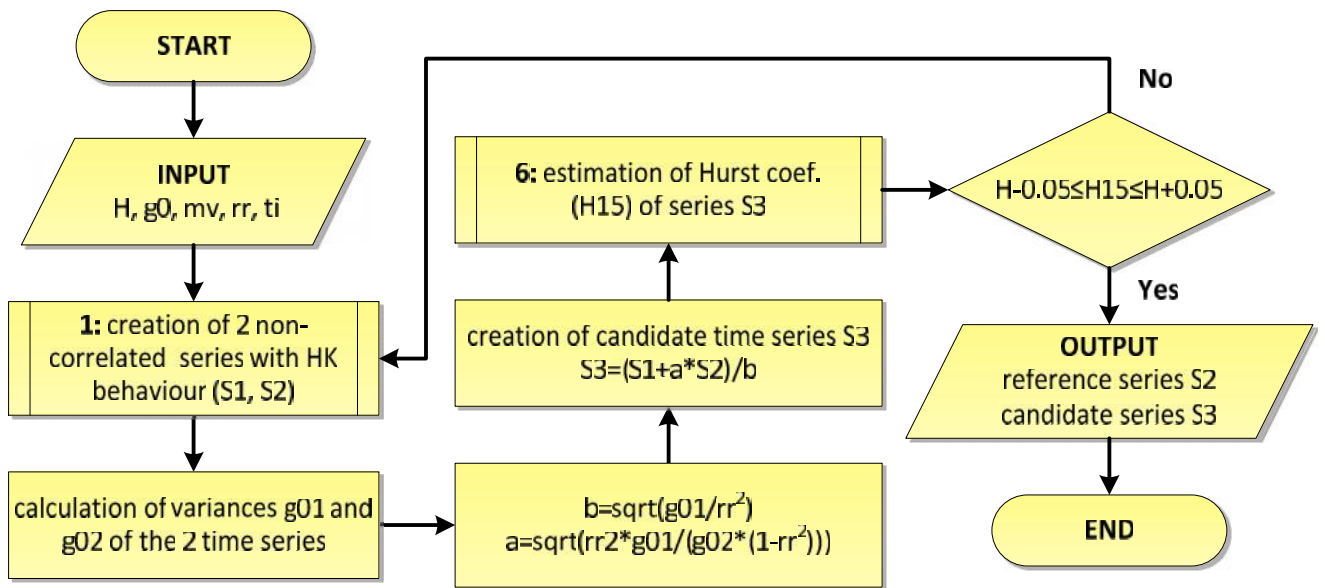
XT: μ μ Hurst

6.4.1,

2) series_ref_cand:

μ μ - μ , μ μ .

μ 7.4.

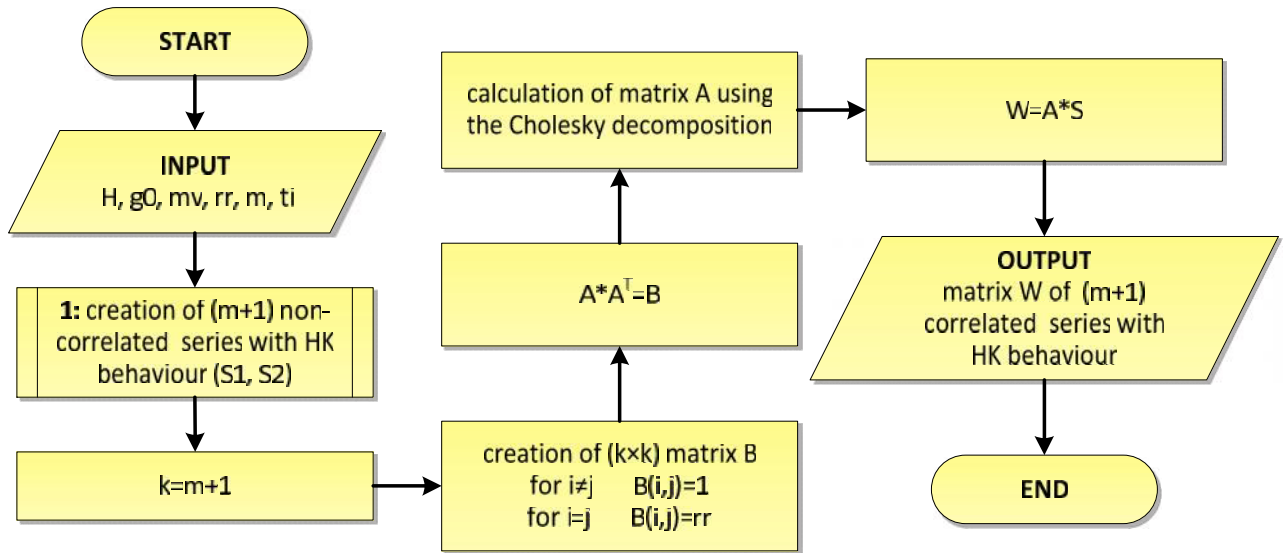


μ 7.4. $\mu\mu$ $series_ref_cand,$ μ
 μ - μ

H: μ $series_ref_cand$ μ μ $:$
 Hurst
 g0: μ
 mv: μ μ
 rr: μ - μ
 ti: μ
 S2:
 S3: μ

2) creation series ref cand:

μ μ $(m+1)$ μ μ , m
 1 μ .
 μ μ $:$
 H: μ Hurst
 g0: μ
 mv: μ μ
 rr: μ - μ
 m: μ
 ti: μ
 W: μ $(m+1)$ μ μ
 $\mu\mu$ $creation_series_ref_cand$ μ 7.5.



μ 7.5. $\mu\mu$ $creation_series_ref_cand,$ μ $(m+1)$
 μ μ $-$ μ

2C) series_ref_cand_DMass:

μ $-$ μ , μ μ .

() . μ

μ μ

$\mu\mu$ $series_ref_cand_DMass$ μ 7.6.

μ :

H: μ Hurst

g0: μ

mv: μ μ

rr: μ - μ

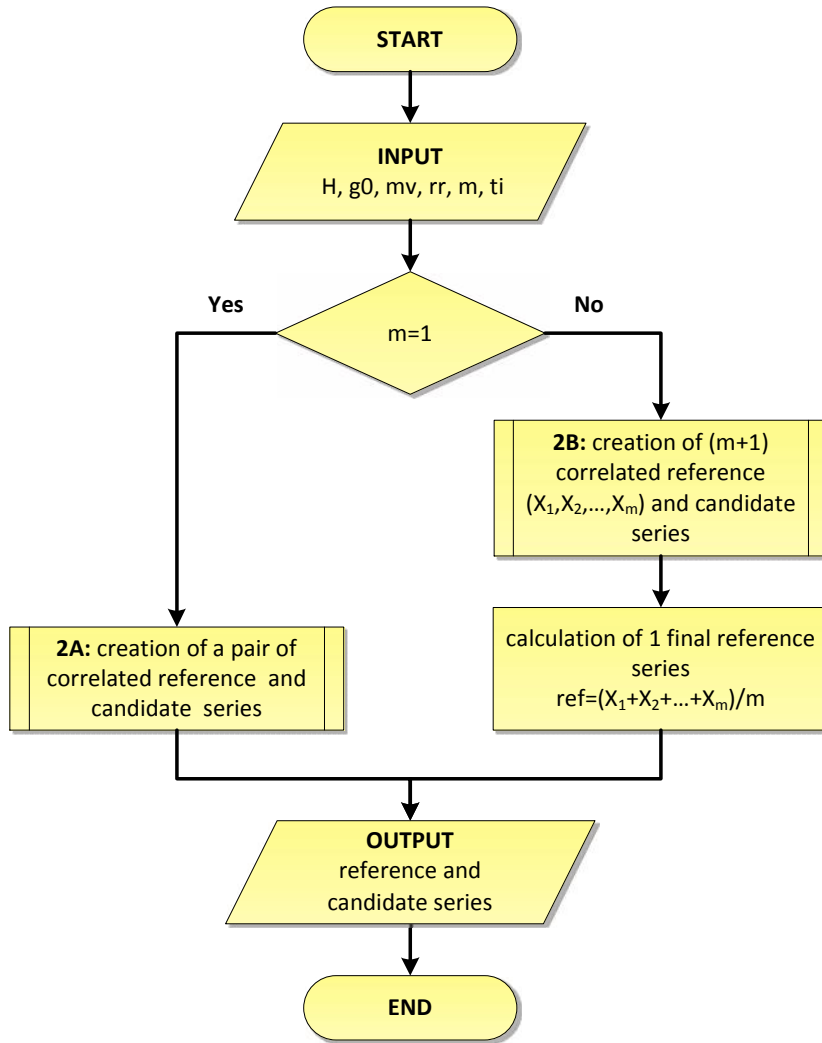
m: μ

ti: μ

:

ref:

Y: μ



μ 7.6. μ series_ref_cand_DMass, μ

3A) SNHT breaking pos:

μ μ μ μ μ SNHT for single μ .

μ μ 5 5 μ μ μ μ

μ μ μ (-999.9, -999.9).

μ μ 3.2.2 6.5.1,

μ μ μ :

X:

Y: μ μ μ μ

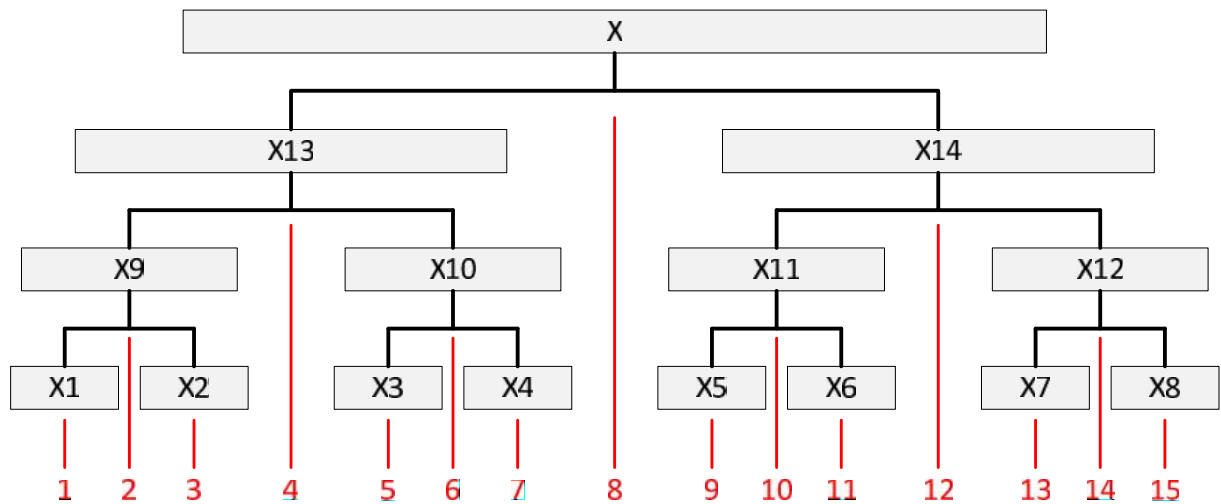
x0: μ μ μ μ μ

adj: μ μ

a: μ
 b: μ
 x: μ μ ()

4A) multiple shifts2:

μ SNHT μ
 μ μ μ μ μ μ
 μ μ μ μ μ μ
 μ 15 μ , μ μ ,
 μ 7.8. μ μ ,
 μ μ μ μ 8.



μ 7.8. μ μ
multiple_shifts2. SNHT μ μ μ μ μ μ
 μ μ μ μ 15 μ μ μ μ
 μ μ μ μ μ μ μ μ

multiple_shifts2_ μ μ :

X:
 Y: μ

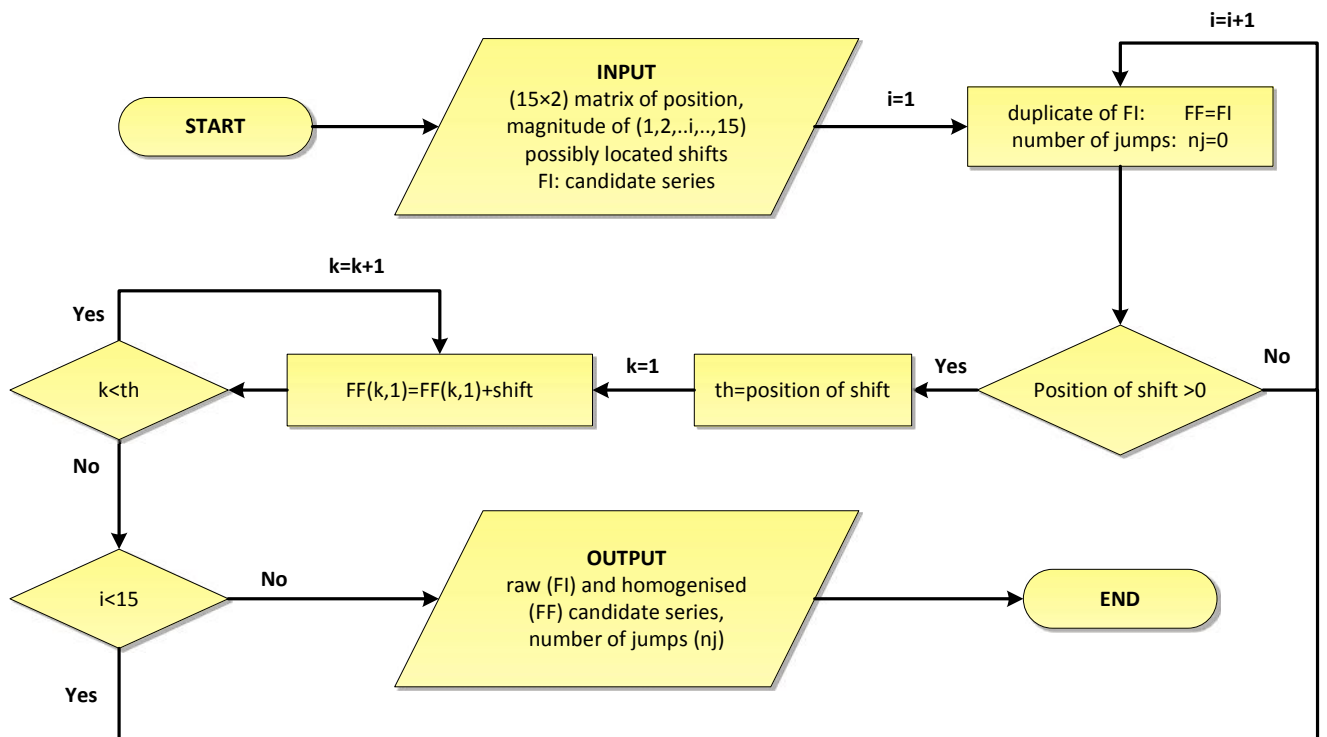
Uresult: μ (15 \times 2) μ 15 μ

4B) multiple_shifts_mref2:

multiple_shifts2
 Uresult 77,
 88,

5) SCor

7.9.



7.9. Scor, 15

SCor :
 InSeries: μ
 Uresult: μ (15x2) μ 15
 FI: μ μ
 FF: μ μ μ μ
 Njumps: μ μ

μ 5%. μ 17-45% (8.1), μ

8.2. μ μ 3/7
 μ μ SNHT μ μ μ 3 μ 7
 μ Hurst
 6.3.1 6.3.2, = {0.50,
 0.55, ..., 0.90} $x_{XY} = \{0.5, 0.6, 0.7, 0.8, 0.9, 0.95\}$. $9 \times 6 = 54$

μ μ μ 10000 μ
 54 8.2 μ 8.2.

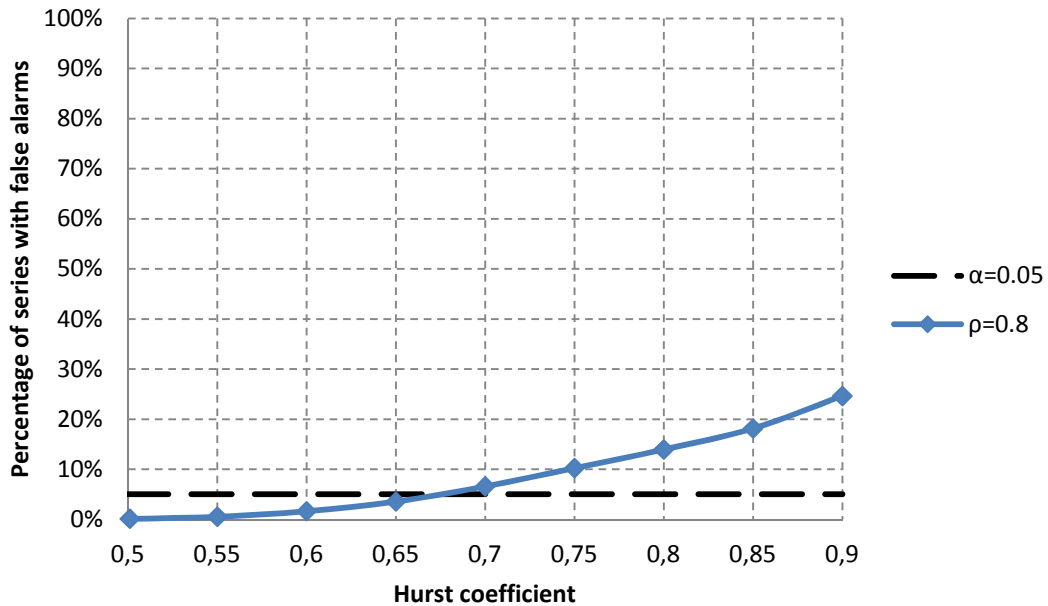
Hurst 8.2. μ μ μ 54 μ
 μ μ (μ 3/7).
 μ μ $x_{XY} = 0.8$, μ μ

\ H	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9
0.95	0.3%	1.4%	4.4%	9.1%	16.9%	28.0%	38.1%	48.9%	59.1%
0.9	0.4%	1.4%	4.0%	9.0%	16.9%	28.4%	38.3%	48.6%	59.8%
0.8	0.4%	1.2%	3.6%	8.7%	17.4%	27.8%	38.4%	49.2%	58.9%
0.7	0.3%	1.3%	3.8%	9.2%	17.7%	27.3%	38.6%	48.6%	59.8%
0.6	0.3%	1.4%	4.2%	9.3%	16.8%	28.1%	39.0%	49.1%	59.4%
0.5	0.4%	1.4%	3.7%	9.2%	17.1%	28.0%	39.3%	48.7%	60.1%

8.2, μ μ μ 1/1,
 μ μ . μ
 μ μ .
 μ , μ μ μ μ μ
 μ μ , μ μ , μ
 μ μ μ μ 0.8. μ ,
 6.6.3, μ μ
 μ μ (. . Ducré-
 Robitaille *et al.*, 2003, DeGaetano, 2006, Vincent, 1998, Easterling and Peterson, 1995).

8.3.
Hurst

\ H	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9
0.8	0.1%	0.5%	1.7%	3.6%	6.6%	10.3%	14.0%	18.2%	24.6%



8.3. $\rho = 0.8$.

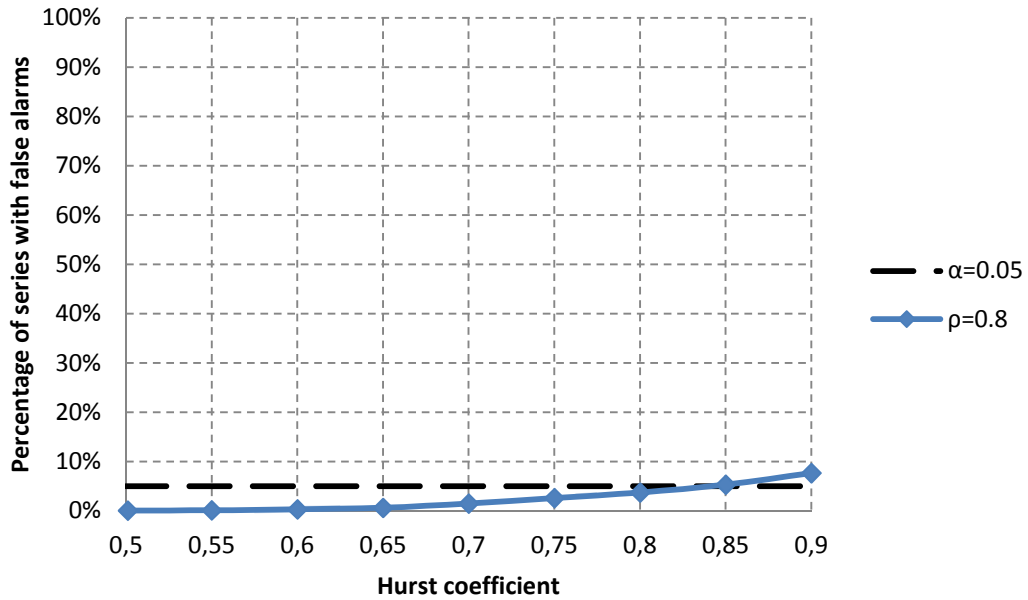
Hurst. $\mu = 0.5-0.7$. SNHT $\mu = 25\%$.
 $\mu = 0.6-0.7$, $\mu = 1.7-6.6\%$ (8.3).

8.4. $\mu = 3/3$

(8.2), SNHT $\mu = 0.8$.
 Hurst $\mu = \{0.50, 0.55, \dots, 0.90\}$.
 10000 $\mu = 8.4$.

8.4. Hurst coefficient H (0.5 to 0.9) and μ (0.05, 0.8). The percentage of series with false alarms is shown in the table below.

$H \backslash \mu$	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9
0.8	0.1%	0.1%	0.3%	0.6%	1.5%	2.6%	3.7%	5.3%	7.7%



8.4. SNHT test results for Hurst coefficients H (0.5 to 0.9) and μ (0.05, 0.8). The percentage of series with false alarms is shown in the table below.

SNHT test results for $\mu=0.05$ and $\mu=0.8$. The percentage of series with false alarms is shown in the table below.

H	$\mu=0.05$ (%)	$\mu=0.8$ (%)
0.5	0.1	0.1
0.55	0.1	0.1
0.6	0.3	0.1
0.65	0.6	0.1
0.7	1.5	0.3
0.75	2.6	0.6
0.8	3.7	1.5
0.85	5.3	2.6
0.9	7.7	5.3

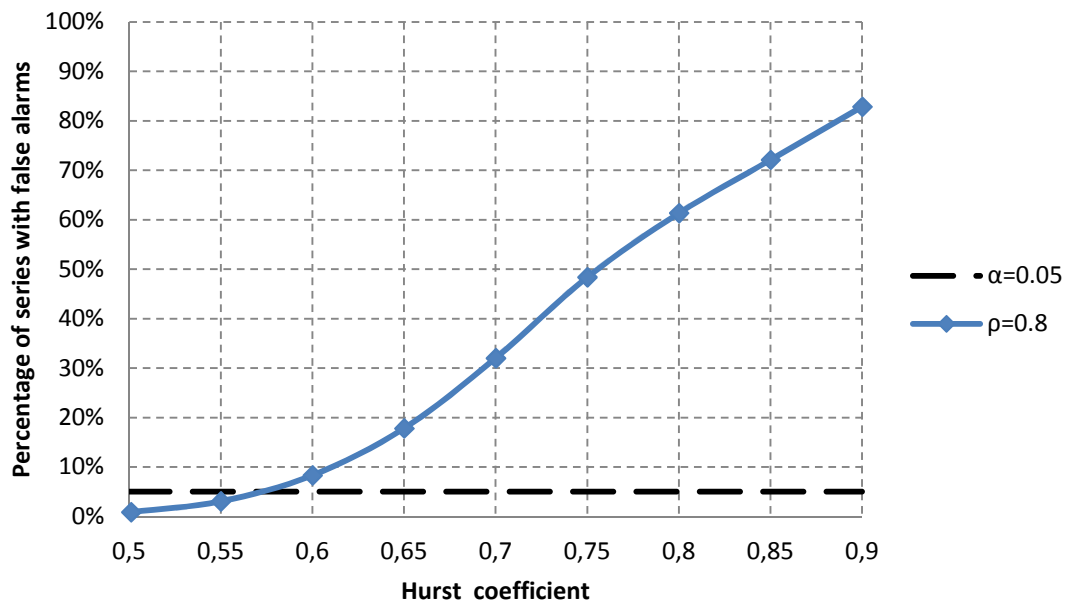
8.5. SNHT test results for Hurst coefficients H (0.5 to 0.9) and μ (0.05, 0.8). The percentage of series with false alarms is shown in the table below.

SNHT test results for $\mu=0.05$ and $\mu=0.8$. The percentage of series with false alarms is shown in the table below.

H	$\mu=0.05$ (%)	$\mu=0.8$ (%)
0.5	0.1	0.1
0.55	0.1	0.1
0.6	0.3	0.1
0.65	0.6	0.1
0.7	1.5	0.3
0.75	2.6	0.6
0.8	3.7	1.5
0.85	5.3	2.6
0.9	7.7	5.3

8.5.
Hurst

\ H	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9
0.8	0.9%	3.0%	8.3%	17.8%	32.0%	48.4%	61.3%	72.1%	82.8%



8.5.
Hurst

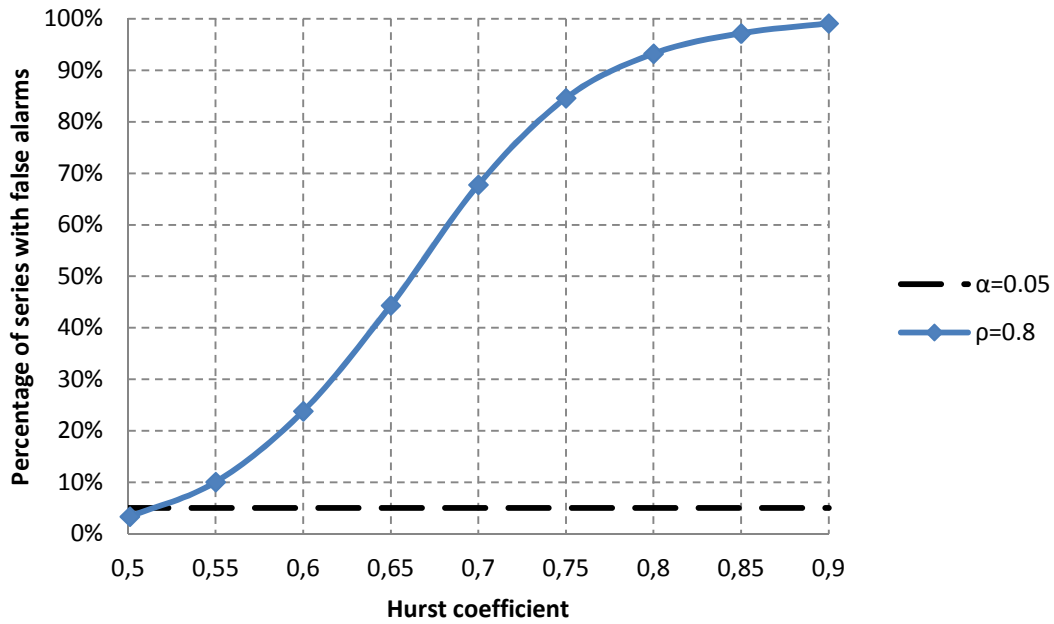
SNHT
 $\mu = 0.5-0.55$,
 $\mu > 0.55$ (μ) $\mu = 83%$ $\mu = 0.9$.
 $\mu = 0.6-0.7$,
: 8.3-32% (8.5).

8.6.
Hurst

(8.2),
20
Hurst $= \{0.50, 0.55, \dots, 0.90\}$.
3
10000
8.6
8.6.

8.6. Hurst

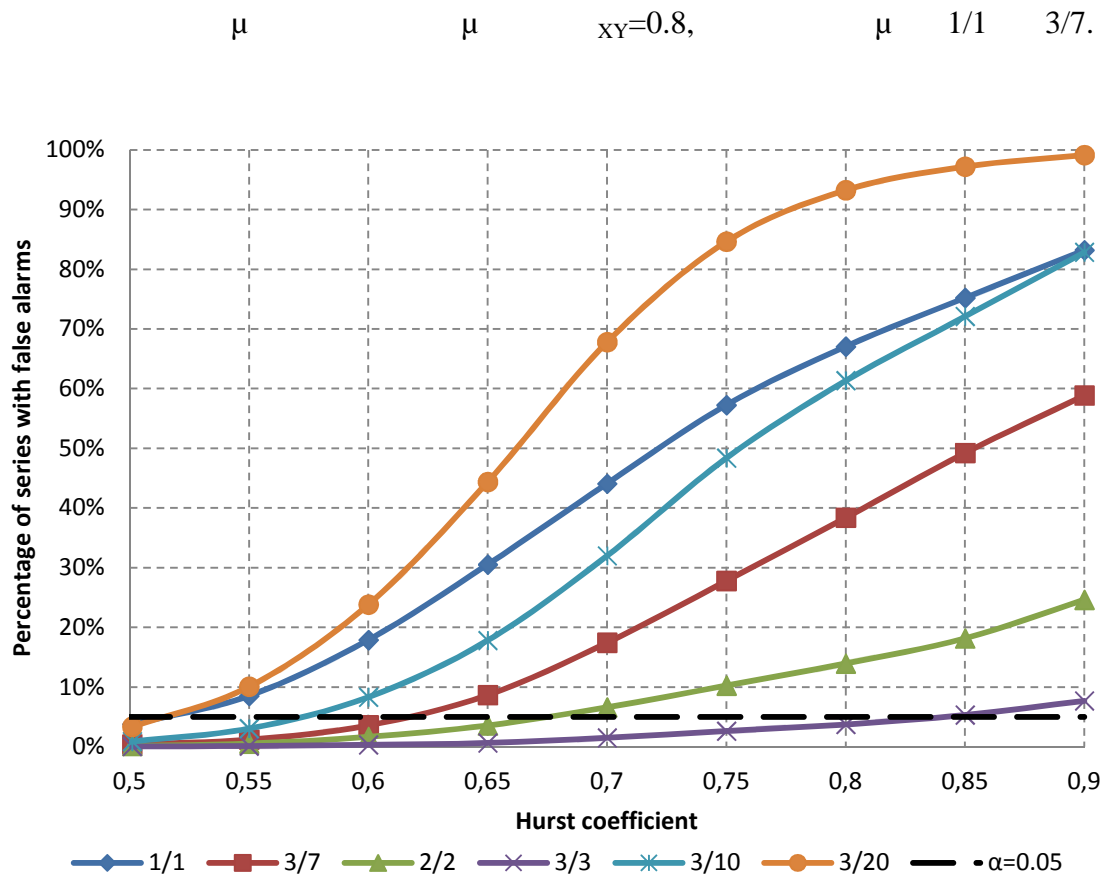
\ H	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9
0.8	3.4%	10.1%	23.8%	44.4%	67.8%	84.6%	93.2%	97.2%	99.1%



8.6. Hurst coefficient H is a measure of the long-term memory of a time series. For $H < 0.5$, the series is anti-persistent; for $H = 0.5$, it is a random walk; for $H > 0.5$, it is persistent. The SNHT test is used to detect non-stationarity in time series. The Hurst coefficient H is estimated from the time series. For $H = 0.9$, the percentage of series with false alarms is 99.1% (8.6).

8.7.

8.7. The Hurst coefficient H is a measure of the long-term memory of a time series. For $H < 0.5$, the series is anti-persistent; for $H = 0.5$, it is a random walk; for $H > 0.5$, it is persistent. The SNHT test is used to detect non-stationarity in time series. The Hurst coefficient H is estimated from the time series. For $H = 0.9$, the percentage of series with false alarms is 99.1% (8.6).



μ 8.7. μ 7.1-7.6. μ
 μ μ $\mu_{XY}=0.8.$ μ
 μ μ 3/3 (μ 5%. μ).

μ 8.7 Hurst μ
 μ μ SNHT. μ =0.5 (μ μ SNHT)
 μ μ (<5%), μ μ , , μ
 (=0.8-0.9) μ μ μ .
 μ 3/3, μ μ >8%. μ
 μ 2/2, μ μ <0.7, μ 3/7 μ
 <0.6. μ μ μ
 , 7 μ μ 3
 μ μ μ >5%
 μ μ (=0.5).
 μ (3/20) μ μ
 μ μ μ μ (=0.55), 100% >0.8.

SNHT
Hurst,

8.8.

(8.2), SNHT
Hurst = {0.50, 0.55, ..., 0.90}.

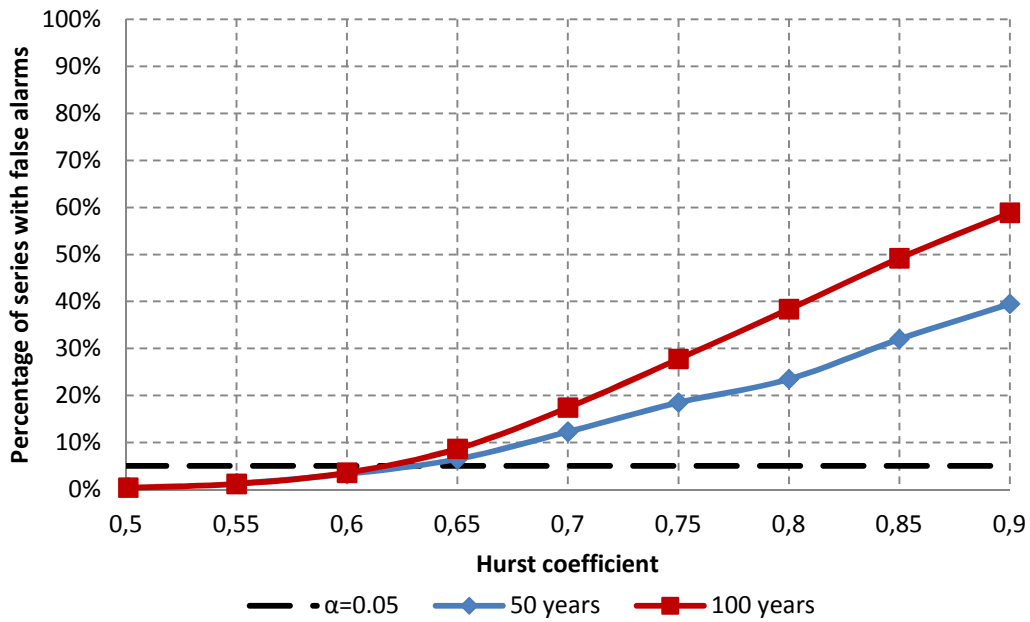
10000
8.7
8.8
3/7-50

8.7. 3/7, 9, 50, Hurst
0.8, 100

\ H	0.501	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9
0.8	0.5%	1.3%	3.3%	6.5%	12.3%	18.5%	23.5%	32.0%	39.5%

50 100 =0.5-0.6.
SNHT. >0.6
Hurst.

μ 50 SNHT μ , μ



μ 8.8.
 50 , 9
 (μ μ) μ
 μ 100 (μ 3/7 μ).
 μ Hurst μ μ μ 7.2 μ 0.8

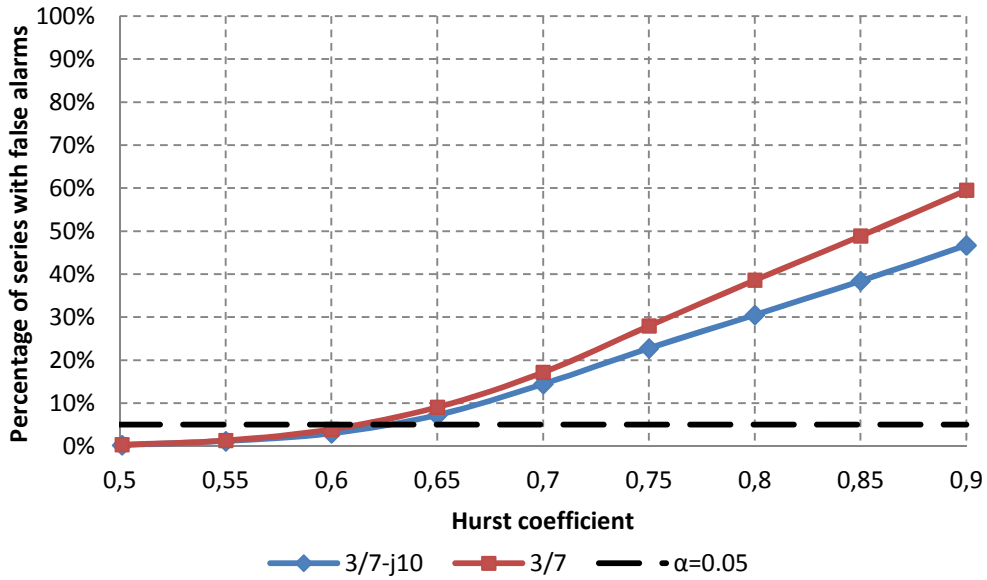
8.9.

μ μ
 (μ 8.2), μ μ SNHT μ μ 3
 7 , μ μ
 10 (5) μ μ
 Hurst = {0.50, 0.55, ..., 0.90}. μ 9
 μ μ μ 10000 μ μ 8.8 μ 8.9.
 μ 8.9 μ μ 3/7-j10
 μ μ μ 5).

8.8.

Hurst

\ H	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9
0.8	0.2%	1.2%	3.0%	7.2%	14.4%	22.7%	30.5%	38.4%	46.7%



8.9.

100 , 9
 10 (μ) μ Hurst, μ
 μ 8.2 μ 100 (μ) μ
 μ μ , μ
 μ .

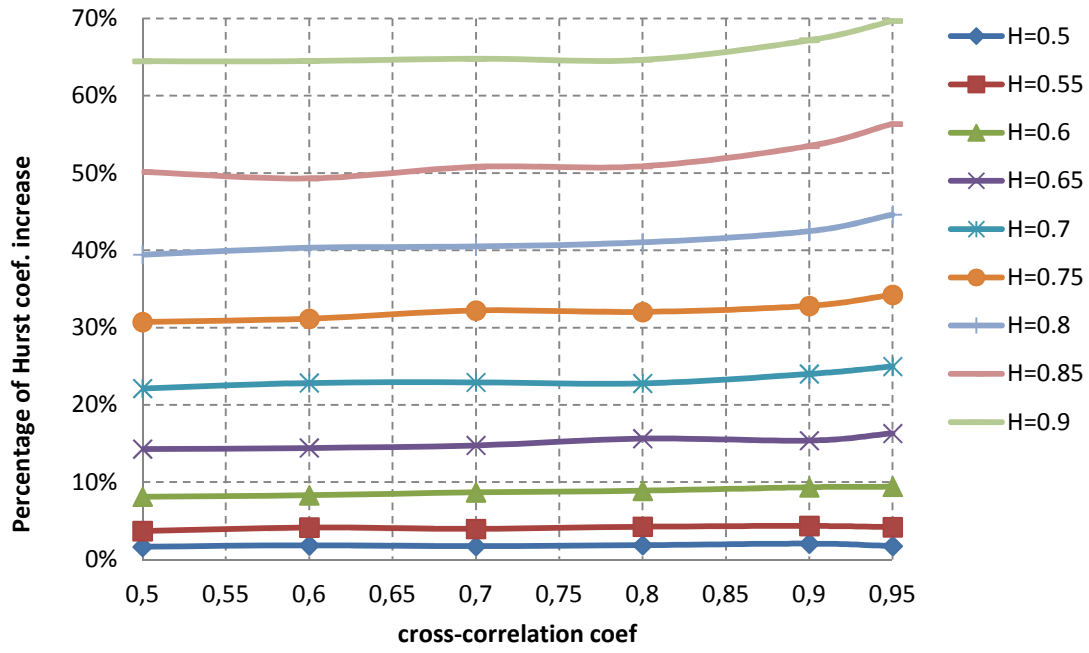
SNHT

(overdetection at the edges, . . . Wang, 2008).

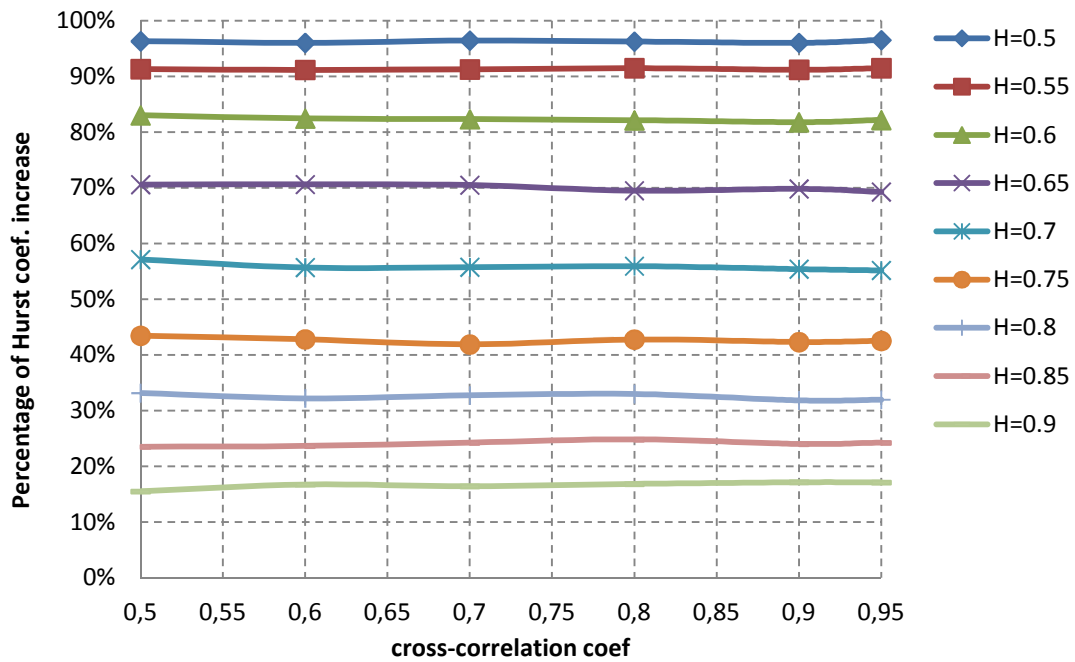
μ μ μ μ μ μ μ μ μ μ μ
 μ μ μ μ μ μ μ μ μ μ μ
 5 10 .
 μ μ SNHT μ
 μ μ , μ μ μ μ 8.9 μ
 μ μ .

8.10.

μ , 5.5.2, μ
 μ μ 1/1, SNHT. SNHT
 μ μ

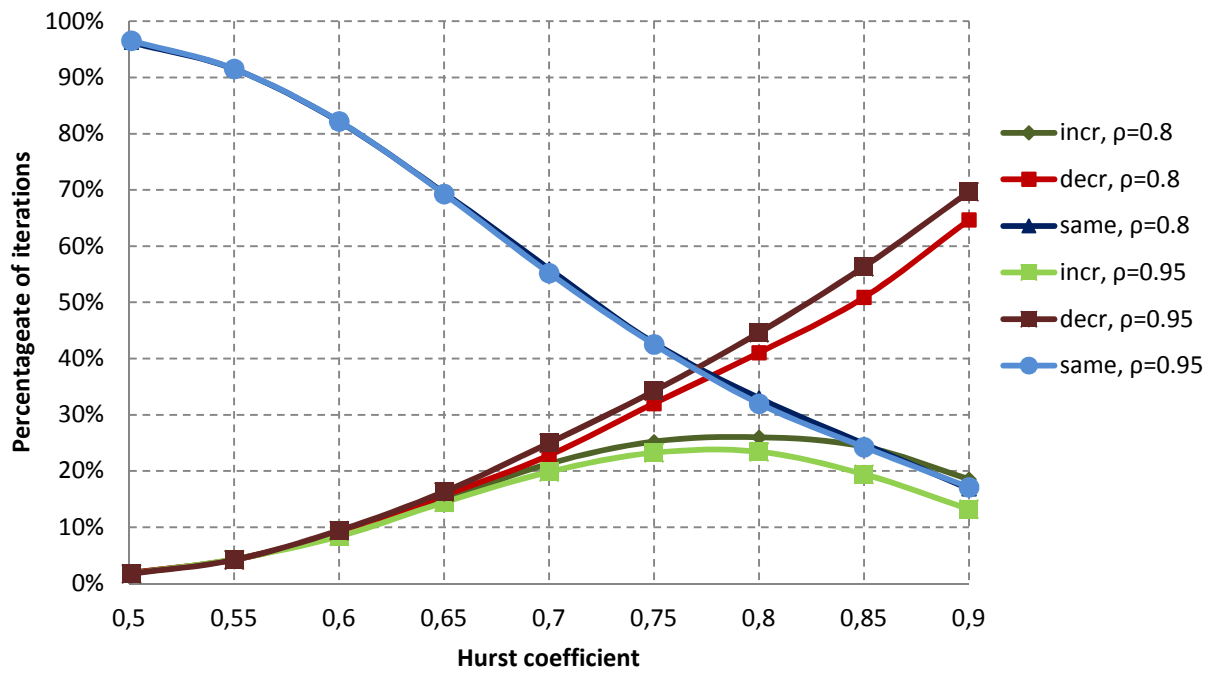


8.11. μ SNHT, μ Hurst μ



8.12. μ SNHT, μ Hurst μ

μ 8.13
 μ
 Hurst. μ
 μ
 μ 8.13
 μ
 :
 • $\rho_{XY}=0.8$, $\mu\mu$
 • $\rho_{XY}=0.95$, μ



μ 8.13. μ μ μ μ SNHT μ Hurst
 μ
 μ 8.14 μ μ :
 1. Hurst μ
 μ μ Hurst.
 2. μ μ $\mu\mu$ (<0.65)
 Hurst μ
 3. μ μ μ μ
 >0.65 urst μ μ μ
 μ

9.1.

μ μ μ μ μ μ μ μ μ μ

μ μ μ μ μ μ μ μ μ μ

μ μ μ μ μ μ μ μ μ μ

μ μ μ μ μ μ μ μ μ μ

μ μ μ μ μ μ μ μ μ μ

μ μ μ μ μ μ μ μ μ μ

μ μ μ μ μ μ μ μ μ μ

μ μ μ μ μ μ μ μ μ μ

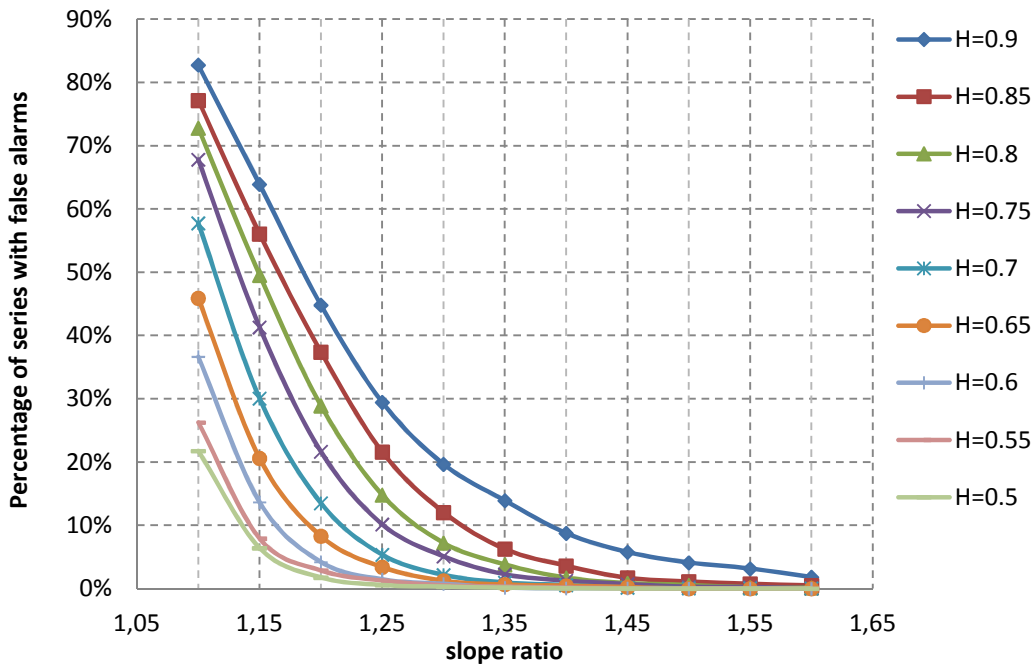
μ μ μ μ μ μ μ μ μ μ

μ μ μ μ μ μ μ μ μ μ

μ μ μ μ μ μ μ μ μ μ

μ μ μ μ μ μ μ μ μ μ

\ H	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9
1.10	21.8%	26.3%	36.7%	45.9%	57.8%	67.8%	72.8%	77.2%	82.8%
1.15	6.4%	7.9%	13.7%	20.6%	30.1%	41.3%	49.6%	56.1%	63.9%
1.20	1.8%	2.9%	4.2%	8.3%	13.5%	21.7%	28.9%	37.4%	44.8%
1.25	0.6%	1.3%	1.5%	3.4%	5.4%	10.2%	14.8%	21.6%	29.5%
1.30	0.3%	0.5%	0.8%	1.3%	2.2%	5.1%	7.2%	12.1%	19.7%
1.35	0.2%	0.3%	0.2%	0.7%	1.0%	2.3%	3.8%	6.3%	13.9%
1.40	0.1%	0.2%	0.1%	0.5%	0.6%	1.3%	1.7%	3.6%	8.8%
1.45	0.0%	0.1%	0.1%	0.3%	0.2%	0.7%	0.9%	1.7%	5.8%
1.50	0.0%	0.0%	0.0%	0.0%	0.1%	0.3%	0.6%	1.1%	4.1%
1.55	0.0%	0.0%	0.0%	0.0%	0.1%	0.3%	0.2%	0.8%	3.2%
1.60	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%	0.5%	1.8%



9.1.

μ μ μ μ μ μ μ μ μ μ

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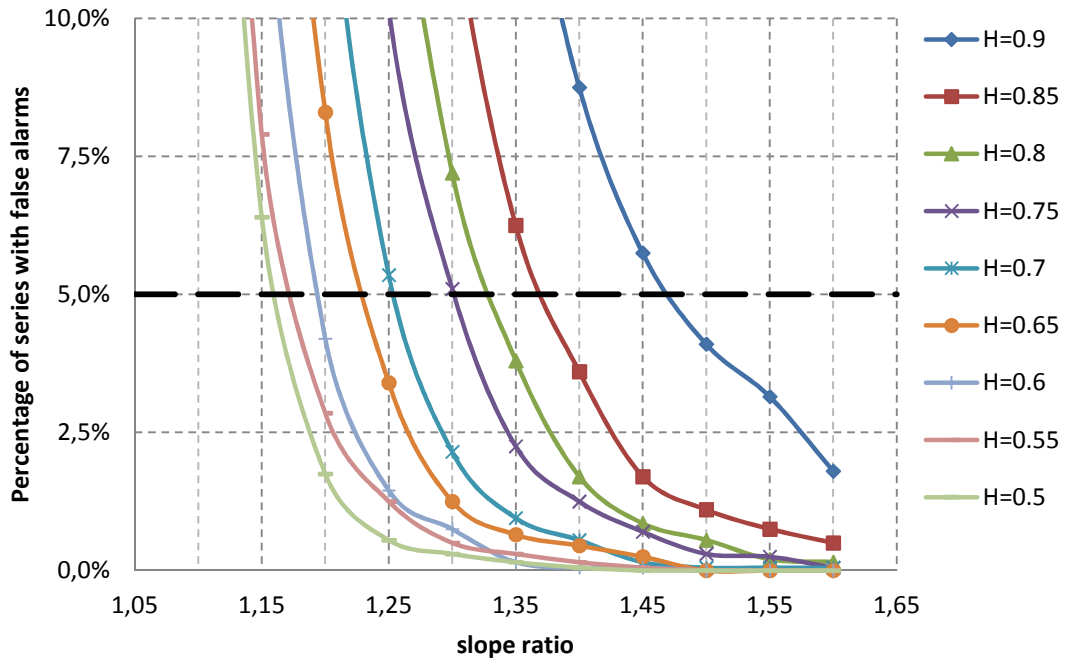
μ μ μ μ μ μ μ μ μ μ

μ μ μ μ μ μ μ μ μ μ

μ μ μ μ μ μ μ μ μ μ

μ μ μ μ μ μ μ μ μ μ

μ Hurst. =0.9, μ 1.5. μ μ Hurst μ 5%. μ



μ 9.2. μ μ 9.1. μ 5%. μ μ

9.2. μ μ 3/1 μ 0.8, μ μ (μ μ 1/1) μ Hurst μ = {0.50, 0.55, ..., 0.90}. μ « μ »

μ μ μ μ 2000 μ μ μ Hurst μ

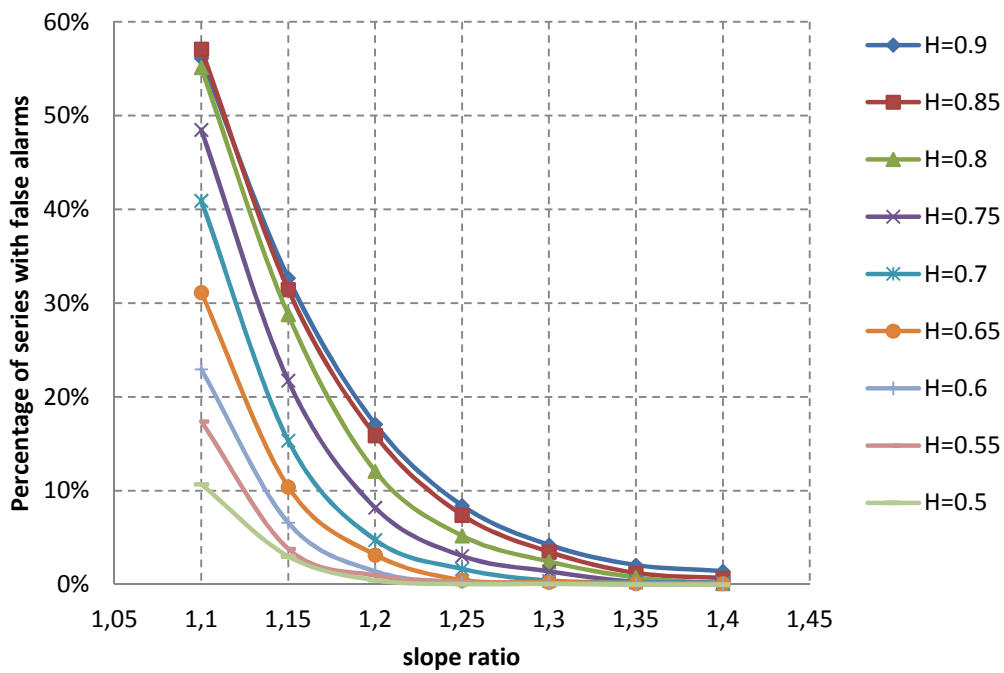
9.2 μ 9.3. μ 9.2 μ 1/1, μ :

- μ Hurst,
- μ μ .

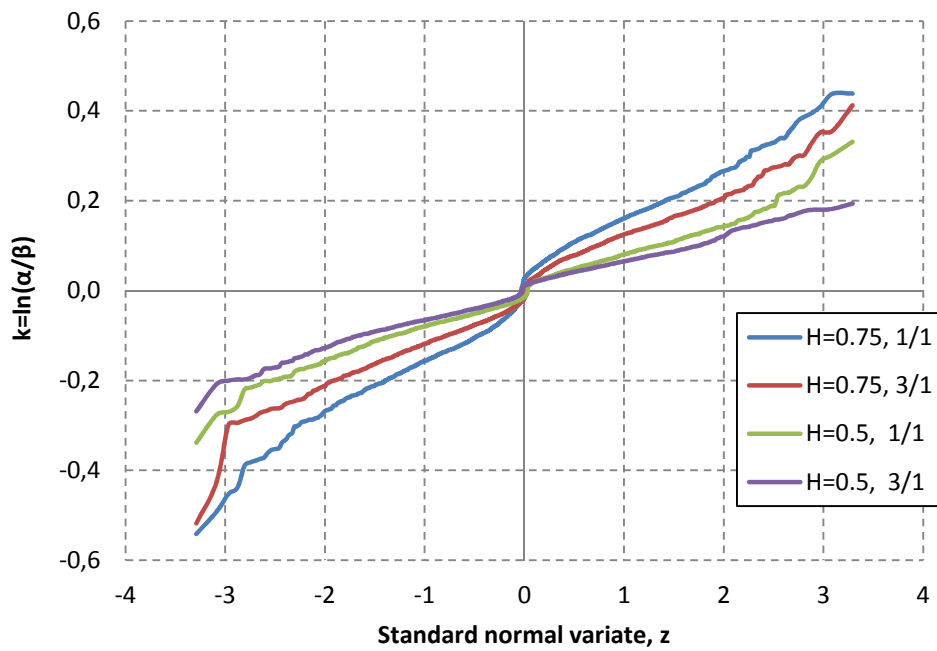
9.2).

9.2.

\ H	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9
1.10	10.7%	17.4%	23.0%	31.2%	41.0%	48.5%	55.2%	57.1%	56.2%
1.15	3.0%	3.8%	6.6%	10.4%	15.4%	21.8%	28.8%	31.5%	32.7%
1.20	0.5%	1.0%	1.4%	3.2%	4.8%	8.2%	12.1%	15.9%	17.1%
1.25	0.1%	0.3%	0.3%	0.5%	1.7%	3.1%	5.2%	7.4%	8.4%
1.30	0.1%	0.1%	0.2%	0.3%	0.4%	1.4%	2.5%	3.5%	4.2%
1.35	0.0%	0.0%	0.1%	0.1%	0.3%	0.3%	0.8%	1.2%	2.1%
1.40	0.0%	0.0%	0.1%	0.1%	0.2%	0.2%	0.1%	0.7%	1.4%



9.3.



$\mu = 9.5$, $H = 0.75$ ($\mu = 1/1$) $k = \ln(\alpha/\beta)$ $Hurst = 0.5$
 $\mu = 9.1$ $\mu = 9.2$ $\mu = 9.5$
 $\mu = 1/1$ $\mu = 3/1$ $\mu = 2000$
 Monte Carlo.

- $\mu = 9.1$ $\mu = 9.2$ $\mu = 9.5$
 - $\mu = 1/1$ $\mu = 3/1$ $\mu = 2000$
- $Hurst = 0.75$ $\mu = 1.35$ $\mu = -0.3$ $z = 0.3$
- $Hurst = 0.75$ $\mu = 3/1$ $\mu = 1.2$
- $Hurst = 0.5$ ($\mu = 1/1$) $\mu = 1.1$

10.

10.1. $\mu \quad \mu$

μ μ μ μ μ , μ μ μ , μ
 μ μ μ μ μ (2, 3).
 μ μ μ (4).

_____ μ μ :
 μ μ μ

1. μ μ μ μ μ μ SNHT
 μ μ , μ μ
 μ μ μ μ , PHA (. 3).

μ μ μ μ μ μ μ
 μ μ μ μ .

2. μ μ μ μ μ μ μ μ μ
 μ μ , μ μ μ μ

3. μ μ μ μ μ μ μ
 μ μ μ : μ $\mu \mu$.

4. μ μ μ μ μ μ μ μ
 μ μ μ μ μ ().
 μ μ , μ μ

μ (5.6).

μ μ μ SNHT μ μ μ
 μ μ μ , μ μ μ μ .
 μ μ μ μ , μ
Hurst . SNHT
 μ .

7. $H < 0.65$ $H > 0.65$, Hurst, Hurst.

1. To Hurst.
2. Hurst.
3. $< 5\%$ (1.3) 1.5.

10.2.

PHA (Menne and Williams, 2009), MASH (Szentimrey, 1999) Caussinus and Mestre (2004),

« »

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<http://www.ncdc.noaa.gov/oa/climate/research/ushcn/ushcn.html>

ΠΑΡΑΡΤΗΜΑ Α

μ SNHT for single shifts μ T_{crit} (test value) 0
 μ 10-100 .
 Khaliq and Ouarda (2007) T_{crit} μ μ n
 μ μ μ μ μ (μ μ
 μ μ μ μ μ μ μ μ .

n	Tcrit	n	Tcrit	n	Tcrit
10	5.637	41	8.1825	71	8.829
11	5.8525	42	8.214	72	8.844
12	6.068	43	8.2435	73	8.8585
13	6.235	44	8.273	74	8.873
14	6.402	45	8.302	75	8.8855
15	6.538	46	8.331	76	8.898
16	6.674	47	8.3565	77	8.912
17	6.7865	48	8.382	78	8.926
18	6.899	49	8.407	79	8.9385
19	6.994	50	8.432	80	8.951
20	7.089	51	8.456	81	8.9635
21	7.173	52	8.48	82	8.976
22	7.257	53	8.502	83	8.9885
23	7.3285	54	8.524	84	9.001
24	7.4	55	8.545	85	9.0135
25	7.4645	56	8.566	86	9.026
26	7.529	57	8.586	87	9.0365
27	7.586	58	8.606	88	9.047
28	7.643	59	8.6265	89	9.057
29	7.695	60	8.647	90	9.067
30	7.747	61	8.665	91	9.0785
31	7.794	62	8.683	92	9.09
32	7.841	63	8.7	93	9.1
33	7.8855	64	8.717	94	9.11
34	7.93	65	8.7345	95	9.1185
35	7.9695	66	8.752	96	9.127
36	8.009	67	8.768	97	9.137
37	8.045	68	8.784	98	9.147
38	8.081	69	8.799	99	9.157
39	8.116	70	8.814	100	9.167
40	8.151				

μ μ , μ μ SNHT
 μ $\mu\mu$
 7.

```

) _____ CheckSNHT multiple:
function apotelesma6=CheckSNHT_multiple(H,g0,mv,rr,ti,iter)

% description: location and correction of jumps identified by
% 1 reference series

% H: Hurst coefficient
% g0: variance
% mv: mean value
% rr: cross-correlation coefficient
% ti: length of time series
% iter: number of iterations

apotelesma6=zeros(iter,5);

for t=1:iter

% subroutine 2A - creation of correlated reference and candidate
series
apotelesma2=series_ref_cand(H,g0,mv,rr,ti);
X=apotelesma2(:,1);
Y=apotelesma2(:,2);

% subroutine 4A - application of the SNHT to all couples of
% reference-candidate series
Uresult=multiple_shifts2(X,Y);

% subroutine 5 - correction of located shifts
apotelesma4=Scor(apotelesma2,Uresult);

% calculation of initial and final slope slop1, slop2
x=zeros(length(apotelesma2),1);
x(:)=1:100;

y1=apotelesma4(:,1);
p1=polyfit(x,y1,1);
slop1=p1(1);

y2=apotelesma4(:,2);

```

```

p2=polyfit(x,y2,1);
slop2=p2(1);

% subroutine 6 - estimation of initial and final Hurst coef. InH,
FiH
x1=apotelesma4(:,1);
H1 = lsv(x1);

x2=apotelesma4(:,2);
H2 = lsv(x2);

alm=apotelesma4(1,3);

apotelesma6(t,:)=[slop1 slop2 H1 H2 alm];

end

end

```

) CheckSNHT Mref:

```

function apotelesmaMULT2=CheckSNHT_Mref(H,g0,mv,rr,m,j,ti,iter)

% description: location of common jumps identified by multiple
reference series

% H: Hurst coefficient
% g0: variance
% mv: mean value
% rr: cross-correlation coefficient
% m: number of reference series
% j: threshold - number of simultaneous jumps in order to be
considered significant
% ti: length of time series
% iter: number of iterations

apotelesmaMULT2=zeros(iter,1);

for t=1:iter

% subroutine 2B - creation of (m+1) correlated reference and
candidate series
Mref=creation_series_ref_cand(H,g0,mv,rr,m,ti);

k=m+1;

% extraction of reference series
X=zeros(ti,k-1);

for i=1:k-1
    X(:,i)=Mref(:,i);
end

% extraction of candidate series

```

```

Y1=Mref(:,k);

% subroutine 4B - application of the SNHT to all m couples of
reference-candidate series
pos=zeros(ti,m);

for l=1:m
    pos(:,l)=multiple_shifts_mref2(X(:,l),Y1,ti);
end

% identification of all common jumps
Njumps=0;
d=0;

n=0;
while n<ti-2;
    n=n+1;
    comj=pos(n:n+2,:);

    ncj1=length(find(comj==77));
    ncj2=length(find(comj==88));

    if ncj1>=j || ncj2>=j
        Njumps=Njumps+1;
        d=1;
    end
    if d==1
        n=n+2;
        d=0;
    end
end

apotelesmaMULT2(t,:)=Njumps;

end

end

)_____ CheckDMass1:
function DoubleMass1=CheckDMass1(H,g0,mv,rr,m,ti,iter)

% description: identification of the most significant inhomogeneity
of a
% precipitation time series using the Double mass curve and a single
% reference series or 1 final reference series calculated by
multiple
% initial reference series

% H: Hurst coefficient
% g0: variance
% mv: mean value
% rr: cross-correlation coefficient

```

```

% m:    number of reference series
% ti:   length of time series
% iter: number of iterations

DoubleMass1=zeros(iter,3);

for t=1:iter

    % subroutine 2C - production of reference and candidate series
    apotelesma2=series_ref_cand_DMass(H,g0,mv,rr,m,ti);

    X=apotelesma2(:,1);
    Y=apotelesma2(:,2);

    % subroutine 4C - formation of precipitation series,
    % double mass procedure, calculation of trends, position of
change
    DMC_mult=DMCmult_ref_cand(X,Y,ti);

    % trends before and after the most significant inhomogeneity,
position
    % of change
    a=DMC_mult(1,1);
    b=DMC_mult(1,2);
    x=DMC_mult(1,3);

    DoubleMass1(t,:)=[a b x];

end

end

```

1) hurst 3AR1

```

function apotelesma1=hurst_3AR1(H,g0,mv,ti)

% Subroutine 1: creation of time series with HK behaviour. Method
proposed by Koutsoyiannis (2002)

% calculation of autocorrelation coef. for lag 1 of the 3 AR(1)
series
r=1.52*(H-0.5)^1.32;
f=0.953-7.69*(1-H)^3.85;
if H<=0.76
    ksi=0.932+0.087*H;
else
    ksi=0.993+0.007*H;
end

```

```

% calculation of autocorrelation coef. for lag 1 and 100 of
fractional Gaussian noise
r1=0.5*((1+1)^(2*H)+(1-1)^(2*H))-1^(2*H);
r100=0.5*((100+1)^(2*H)+(100-1)^(2*H))-100^(2*H);

% calculation of parameters c1, c2
a11=f-r;
a12=ksi-r;
a21=f^100-r^100;
a22=ksi^100-r^100;

A=[a11 a12;a21 a22];

b1=r1-r;
b2=r100-r^100;

B=[b1;b2];

C=A\B;

% selection of the mean value of the 3 AR(1) series
m1=0;
m2=0;
m3=0;

% calculation of variances of the 3 AR(1) series
var1=(1-C(1,1)-C(2,1))*g0;
var2=C(1,1)*g0;
var3=C(2,1)*g0;

varv1=(1-r^2)*var1;
varv2=(1-f^2)*var2;
varv3=(1-ksi^2)*var3;

% creation 3 AR(1) series of length ti
W1=zeros(ti+1,1);
W1(1,1)=norminv(rand(1),m1,sqrt(var1));

W2=zeros(ti+1,1);
W2(1,1)=norminv(rand(1),m2,sqrt(var2));

W3=zeros(ti+1,1);
W3(1,1)=norminv(rand(1),m3,sqrt(var3));

XT=zeros(ti,1);

for i=2:ti+1
    W1(i,1)=W1(i-1,1)*r+norminv(rand(1),m1,sqrt(varv1));
    W2(i,1)=W2(i-1,1)*f+norminv(rand(1),m2,sqrt(varv2));
    W3(i,1)=W3(i-1,1)*ksi+norminv(rand(1),m3,sqrt(varv3));
end

% sum of the 3 independent AR(1) processes and of the selected mean
value

```



```

for i=1:ti
    XT(i,1)=W1(i+1,1)+W2(i+1,1)+W3(i+1,1)+mv;
end

apotelesma1=XT;

end

```

2) series ref cand:

```

function apotelesma2=series_ref_cand(H,g0,mv,rr,ti)

% Subroutine 2A: formation of a pair of correlated reference-
candidate
% series (S2,S3)

for i=1:100

% subroutine 1 - creation of 2 non-correlated time series S1 and S2
% with HK behaviour
S1=hurst_3AR1(H,g0,mv,ti);
S2=hurst_3AR1(H,g0,mv,ti);

% creation of the candidate series S3 with HK behaviour as a linear
% function of S1, S2
sS1=std(S1);
sS2=std(S2);

g01=sS1^2;
g02=sS2^2;

b=sqrt(g01/(1-rr^2));
a=sqrt(rr^2*g01/(g02*(1-rr^2)));

S3=(S1+a.*S2)./b;

% subroutine 6: estimation of Hurst coef. of series S3
x15=S3;
H15 = lsv(x15);

% examination if the Hurst coef. of the candidate series is out of
bounds
if H15<=H+0.05 && H15>=H-0.05, break, end;

end;

if H15>H+0.05 || H15<H-0.05, error('hurst coef. out of bounds');
end;

apotelesma2=[S2 S3];

end

```

2) creation series ref cand:

```

function Mref=creation_series_ref_cand(H,g0,mv,rr,m,ti)

% Subroutine 2B: creation of a matrix of (m+1) correlated series W
from a
% matrix of (m+1) non-correlated series S. Procedure: W=A*S

% creation of independent reference and candidate series - matrix S
k=m+1;

S=zeros(ti,k);

% subroutine 1 - creation of (m+1) series with HK behaviour
for i=1:k
    S(:,i)=hurst_3AR1(H,g0,mv,ti);
end

% creation of matrix A
B=zeros(k,k);
B(:)=rr;
for i=1:k
    for j=1:k
        if i==j, B(i,j)=1; end;
    end
end

A = chol(aa, 'lower');

% creation of correlated reference and candidate series - matrix W
W=zeros(ti,k);
R=zeros(ti,k);
D=zeros(ti,1);

for i=1:k
    for j=1:k
        R(:,j)=A(i,j).*S(:,j);
    end

    for l=1:ti
        D(l,1)=sum(R(l,:));
    end

    W(:,i)=D;
end

Mref=W;

end

```

2C) series_ref_cand_DMass:

```
function Series_Dmass=series_ref_cand_DMass(H,g0,mv,rr,m,ti)
```

```

% Subroutine 2C: creation of a pair of correlated reference and
candidate
% series

Ser=zeros(ti,2);

if m==1

    % subroutine 2A - creation of a pair of correlated reference and
    % candidate series
    Ser=series_ref_cand(H,g0,mv,rr,ti);

elseif m>=2

    % subroutine 2B - creation of a matrix of (m+1) correlated
series,
    % m reference and 1 candidate series
    Mref=creation_series_ref_cand(H,g0,mv,rr,m,ti);
    X=Mref(:,1:m);
    Y=Mref(:,m+1);
    ref=zeros(ti,1);

    % creation of 1 final reference series calculated by the m
initial
    % reference series
    for j=1:ti
        ref(j,1)=sum(X(j,:)/m);
    end

    Ser=[ref Y];
end

Series_Dmass=Ser;

end

```

3A) SNHT breaking pos:

```

function result_p=SNHT_breaking_pos(X,Y)

% Subroutine 3A: A candidate series (Y) is tested using the SNHT for
the
% existence of a statistically significant shift at a 95%
significance
% level, in comparison to a reference time series (X)

% creation of time series Q from the pair of reference-candidate
time
% series
Q0=Y-X+mean(X)-mean(Y);

% calculation of mean value, standard deviation and length of the
series Q
mQ0=mean(Q0);
sq=std(Q0);

```

```

n=length(Q0);

% examination if the length of the time series Q is adequate for the
test
if n<10
    apot=[-999.9,-999.9];

elseif n>=10

% normalisation of time series Q
Z0=(Q0-mQ0)./sq;

z1=zeros(n-1,1);
z2=zeros(n-1,1);
T0=zeros(n-1,1);

% Calculation of test value T0
for i=1:n-1
    z1(i,1)=mean(Z0(1:i));
    z2(i,1)=mean(Z0(i+1:end));

    T0(i,1)=i.*z1(i,1).^2+(n-i).*z2(i,1).^2;
end

% identification of maximum value of T0 (mT0)
mT0=max(T0);

% location of position x0 of mT0
x0=find(T0==mT0);

% subroutine 7: critical values of the SNHT for multiple series
lengths at
% a significance level 95%
Tcr=Tcritical(n);

% examination if mT0 is statistically significant and calculation of
% adjustment
if (mT0>Tcr)
    if (x0>5 && x0<(n-5))

        mq1=sq.*mean(Z0(1:x0))+mQ0;
        mq2=sq.*mean(Z0(x0+1:end))+mQ0;
        adj=mq2-mq1;

        apot=[x0,adj];
    else
        apot=[-999.9,-999.9];
    end
else

    apot=[-999.9,-999.9];

end

end

```

```

% Output: if the shift is significant, location x0 of maximum value,
% adjustment
result_p=apot;

```

```
end
```

3) piecewise linear:

```
function proc_pl=piecewise_linear(S1,S2,le)
```

```
% Subroutine 3B: Double mass curve procedure
```

```
k=length(S1);
```

```
% extraction of reference (X) and candidate (Y) series before and
after the
```

```
% possible inhomogeneity
```

```
X1=S1(1:le,1);
```

```
X2=S1(le+1:k,1);
```

```
Y1=S2(1:le,1);
```

```
Y2=S2(le+1:k,1);
```

```
m=k-le;
```

```
n=le;
```

```
x0=S1(le);
```

```
x1=S1(k);
```

```
Sx1=sum(S1(1:le));
```

```
Sx2=sum(S1(le+1:k));
```

```
Sy2=sum(S2(le+1:k));
```

```
Sx1_2=sum(X1.*X1);
```

```
Sx2_2=sum(X2.*X2);
```

```
xy1=sum(X1.*Y1);
```

```
xy2=sum(X2.*Y2);
```

```
% calculation of "help" values
```

```
cc=x0*Sx2-m*x0*x1-Sx2_2+x1*Sx2;
```

```
aa=m*x0^2+Sx2_2-2*x0*Sx2;
```

```
y01=aa*xy1*(x1-x0)/(x0*cc)-aa*xy2/cc+aa*x1*Sy2/cc-xy2+x0*Sy2;
```

```
y02=m*x0-Sx2+(x1-x0)*aa*Sx1_2/(x0^2*cc)-aa*Sx2/cc+aa*x1*m/cc;
```

```
y0=y01/y02;
```

```
y11=(x1-x0)*(m*x0*y0+xy2-x0*Sy2-y0*Sx2);
```

```
y1=y0+y11/aa;
```

```
a=y0/x0;
```

```
b=(y1-y0)/(x1-x0);
```

```
g=y0-((y1-y0).*x0)./(x1-x0);
```

```
% least squares
```

```

K1=a.*X1;
Res1=(Y1-K1).^2;
sR1=sum(Res1);

K2=b.*X2+g;
Res2=(Y2-K2).^2;
sR2=sum(Res2);

% sum of least squares
SRes=sR1+sR2;

proc_pl=[a b SRes];

end

```

3C) DMCmult_ref_cand:

```

function DMC_mult=DMCmult_ref_cand(X,Y,ti)

% Subroutine 3C: formation of precipitation series, fitting of the
Double mass curve

A=X*250+1000;
B=Y*250+1000;

% replacement of negative precipitation values with zero values

for g=1:ti
    if A(g)<0;A(g)=0;end
    if B(g)<0;B(g)=0;end
end

% formation of aggregated values with reverse order

Q1=flipud(A);
Q2=flipud(B);

S1=cumsum(Q1,1);
S2=cumsum(Q2,1);

% identification of the most significant trend change

kliseis=zeros(ti-9,3);

for i=1:ti-9
    kliseis(i,:)=piecewise_linear(S1,S2,i+4);
end

MR=min(kliseis(:,3));
loc=find(kliseis(:,3)==MR);

% output: slopes of the Double mass curve, position of inhomogeneity
a=kliseis(loc,1);
b=kliseis(loc,2);
x=ti-loc-4;

```

```
DMC_mult=[a b x];
```

```
end
```

4A) multiple shifts2:

```
function Uresult=multiple_shifts2(X,Y)
```

```
% Subroutine 4A: location of shifts identified by successive  
application  
% of the SNHT every time a shifts is located the time series is cut  
in the  
% place of the jump and both parts are retested
```

```
bpoint=zeros(15,2);  
bpoint(:, : )=-999.9;
```

```
bpoint(8,1:2)=SNHT_breaking_pos(X,Y);  
x8=bpoint(8,1);
```

```
% breaking the time series in two parts, 1/2 & 2/2, if a significant  
shift  
% is identified
```

```
if bpoint(8,1)>0
```

```
    % first part of the time series
```

```
    XN13=X(1:x8,1);  
    YN13=Y(1:x8,1);
```

```
    XN14=X(x8+1:end,1);  
    YN14=Y(x8+1:end,1);
```

```
    bpoint(4,1:2)=SNHT_breaking_pos(XN13,YN13);  
    x4=bpoint(4,1);
```

```
    if bpoint(4,1)>0
```

```
        % breaking the time series in two parts, 1/4 & 2/4, because  
of the % identification of a significant shift
```

```
        XN9=XN13(1:x4,1);  
        YN9=YN13(1:x4,1);
```

```
        XN10=XN13(x4+1:end,1);  
        YN10=YN13(x4+1:end,1);
```

```
        bpoint(2,1:2)=SNHT_breaking_pos(XN9,YN9);  
        x2=bpoint(2,1);
```

```

of the      % breaking the time series in two parts, 1/8 & 2/8, because
            % identification of a significant shift

            if bpoint(2,1)>0

                XN1=XN9(1:x2,1);
                YN1=YN9(1:x2,1);

                XN2=XN9(x2+1:end,1);
                YN2=YN9(x2+1:end,1);

                bpoint(1,1:2)=SNHT_breaking_pos(XN1,YN1);
                bpoint(3,1:2)=SNHT_breaking_pos(XN2,YN2);

                if bpoint(3,1)>0
                    bpoint(3,1)=bpoint(3,1)+x2;
                end

            end

            bpoint(6,1:2)=SNHT_breaking_pos(XN10,YN10);
            x6=bpoint(6,1);

            if bpoint(6,1)>0

                % breaking the time series in two parts, 3/8 & 4/8,
because of the % identification of a significant shift

                bpoint(6,1)=bpoint(6,1)+x4;

                XN3=XN10(1:x6,1);
                YN3=YN10(1:x6,1);

                XN4=XN10(x6+1:end,1);
                YN4=YN10(x6+1:end,1);

                bpoint(5,1:2)=SNHT_breaking_pos(XN3,YN3);
                if bpoint(5,1)>0
                    bpoint(5,1)=bpoint(5,1)+x4;
                end
                bpoint(7,1:2)=SNHT_breaking_pos(XN4,YN4);
                if bpoint(7,1)>0
                    bpoint(7,1)=bpoint(7,1)+bpoint(6,1);
                end

            end

        end

        % second part of the time series

        bpoint(12,1:2)=SNHT_breaking_pos(XN14,YN14);
        x12=bpoint(12,1);

```



```

if bpoint(12,1)>0
    % breaking the time series in two parts, 3/4 & 4/4, because
of the % identification of a significant shift
    bpoint(12,1)=bpoint(12,1)+x8;
    XN11=XN14(1:x12,1);
    YN11=YN14(1:x12,1);
    XN12=XN14(x12+1:end,1);
    YN12=YN14(x12+1:end,1);
    bpoint(10,1:2)=SNHT_breaking_pos(XN11,YN11);
    x10=bpoint(10,1);
    % breaking the time series in two parts, 5/8 & 6/8, because
of the % identification of a significant shift
    if bpoint(10,1)>0
        bpoint(10,1)=bpoint(10,1)+x8;
        XN5=XN11(1:x10,1);
        YN5=YN11(1:x10,1);
        XN6=XN11(x10+1:end,1);
        YN6=YN11(x10+1:end,1);
        bpoint(9,1:2)=SNHT_breaking_pos(XN5,YN5);
        if bpoint(9,1)>0
            bpoint(9,1)=bpoint(9,1)+x8;
        end
        bpoint(11,1:2)=SNHT_breaking_pos(XN6,YN6);
        if bpoint(11,1)>0
            bpoint(11,1)=bpoint(11,1)+bpoint(10,1);
        end
    end
    bpoint(14,1:2)=SNHT_breaking_pos(XN12,YN12);
    x14=bpoint(14,1);
    if bpoint(14,1)>0
        % breaking the time series in two parts, 7/8 & 8/8,
because of the % identification of a significant shift
        bpoint(14,1)=bpoint(14,1)+bpoint(12,1);
        XN7=XN12(1:x14,1);
        YN7=YN12(1:x14,1);

```

```

        XN8=XN12(x14+1:end,1);
        YN8=YN12(x14+1:end,1);

        bpoint(13,1:2)=SNHT_breaking_pos(XN7,YN7);
        if bpoint(13,1)>0
            bpoint(13,1)=bpoint(13,1)+bpoint(12,1);
        end
        bpoint(15,1:2)=SNHT_breaking_pos(XN8,YN8);
        if bpoint(15,1)>0
            bpoint(15,1)=bpoint(15,1)+bpoint(14,1);
        end

    end

end

end

Uresult=bpoint;

end

```

4B) multiple shifts mref2:

```

function MRshifts2=multiple_shifts_mref2(X,Y,ti)

% Subroutine 4B: location of shifts identified by successive
% application
% of the SNHT every time a shifts is located the time series is cut
% in the
% place of the jump and both parts are retested - attribution of
% shifts to
% the candidate series

bpoint=zeros(15,2);
bpoint(:,:)= -999.9;

bpoint(8,1:2)=SNHT_breaking_pos(X,Y);
x8=bpoint(8,1);

% breaking the time series in two parts, 1/2 & 2/2, if a significant
% shift
% is identified

if bpoint(8,1)>0

    % first part of the time series

    XN13=X(1:x8,1);
    YN13=Y(1:x8,1);

    XN14=X(x8+1:end,1);
    YN14=Y(x8+1:end,1);

```

```

bpoint(4,1:2)=SNHT_breaking_pos(XN13,YN13);
x4=bpoint(4,1);

if bpoint(4,1)>0

    % breaking the time series in two parts, 1/4 & 2/4, because
of the % identification of a significant shift

    XN9=XN13(1:x4,1);
    YN9=YN13(1:x4,1);

    XN10=XN13(x4+1:end,1);
    YN10=YN13(x4+1:end,1);

    bpoint(2,1:2)=SNHT_breaking_pos(XN9,YN9);
    x2=bpoint(2,1);

    % breaking the time series in two parts, 1/8 & 2/8, because
of the % identification of a significant shift

    if bpoint(2,1)>0

        XN1=XN9(1:x2,1);
        YN1=YN9(1:x2,1);

        XN2=XN9(x2+1:end,1);
        YN2=YN9(x2+1:end,1);

        bpoint(1,1:2)=SNHT_breaking_pos(XN1,YN1);
        bpoint(3,1:2)=SNHT_breaking_pos(XN2,YN2);

        if bpoint(3,1)>0
            bpoint(3,1)=bpoint(3,1)+x2;
        end

    end

    bpoint(6,1:2)=SNHT_breaking_pos(XN10,YN10);
    x6=bpoint(6,1);

    if bpoint(6,1)>0

        % breaking the time series in two parts, 3/8 & 4/8,
because of the % identification of a significant shift

        bpoint(6,1)=bpoint(6,1)+x4;

        XN3=XN10(1:x6,1);
        YN3=YN10(1:x6,1);

        XN4=XN10(x6+1:end,1);
        YN4=YN10(x6+1:end,1);

```

```

        bpoint(5,1:2)=SNHT_breaking_pos(XN3,YN3);
        if bpoint(5,1)>0
            bpoint(5,1)=bpoint(5,1)+x4;
        end
        bpoint(7,1:2)=SNHT_breaking_pos(XN4,YN4);
        if bpoint(7,1)>0
            bpoint(7,1)=bpoint(7,1)+bpoint(6,1);
        end

    end

end

% second part of the time series

bpoint(12,1:2)=SNHT_breaking_pos(XN14,YN14);
x12=bpoint(12,1);

if bpoint(12,1)>0

    % breaking the time series in two parts, 3/4 & 4/4, because
of the % identification of a significant shift

    bpoint(12,1)=bpoint(12,1)+x8;

    XN11=XN14(1:x12,1);
    YN11=YN14(1:x12,1);

    XN12=XN14(x12+1:end,1);
    YN12=YN14(x12+1:end,1);

    bpoint(10,1:2)=SNHT_breaking_pos(XN11,YN11);
    x10=bpoint(10,1);

    % breaking the time series in two parts, 5/8 & 6/8, because
of the % identification of a significant shift

    if bpoint(10,1)>0

        bpoint(10,1)=bpoint(10,1)+x8;

        XN5=XN11(1:x10,1);
        YN5=YN11(1:x10,1);

        XN6=XN11(x10+1:end,1);
        YN6=YN11(x10+1:end,1);

        bpoint(9,1:2)=SNHT_breaking_pos(XN5,YN5);
        if bpoint(9,1)>0
            bpoint(9,1)=bpoint(9,1)+x8;
        end
        bpoint(11,1:2)=SNHT_breaking_pos(XN6,YN6);

```

```

        if bpoint(11,1)>0
            bpoint(11,1)=bpoint(11,1)+bpoint(10,1);
        end

    end

    bpoint(14,1:2)=SNHT_breaking_pos(XN12,YN12);
    x14=bpoint(14,1);

    if bpoint(14,1)>0

        % breaking the time series in two parts, 7/8 & 8/8,
because of the
        % identification of a significant shift

        bpoint(14,1)=bpoint(14,1)+bpoint(12,1);

        XN7=XN12(1:x14,1);
        YN7=YN12(1:x14,1);

        XN8=XN12(x14+1:end,1);
        YN8=YN12(x14+1:end,1);

        bpoint(13,1:2)=SNHT_breaking_pos(XN7,YN7);
        if bpoint(13,1)>0
            bpoint(13,1)=bpoint(13,1)+bpoint(12,1);
        end
        bpoint(15,1:2)=SNHT_breaking_pos(XN8,YN8);
        if bpoint(15,1)>0
            bpoint(15,1)=bpoint(15,1)+bpoint(14,1);
        end

    end

end

end

end

% attribution of positive and negative shifts to the time series

shifts=zeros(ti,1);

for i=1:15
    if bpoint(i,1)>0
        p=bpoint(i,1);
        if bpoint(i,2)>0
            shifts(p)=77;
        elseif bpoint(i,2)<0
            shifts(p)=88;
        end
    end

end

end

MRshifts2=shifts;

```

end

5)_____SCor

```
function apotelesma4=Scor(InSeries,Uresult)

% Subroutine 5: correction of located shifts at candidate series

% extraction of candidate series
FI(:,1)=InSeries(:,2);
FF(:,1)=FI(:,1);

nj=0;

% correction of 1-15 possible shifts
for i=1:15
    if Uresult(i,1)>0
        th=Uresult(i,1);
        nj=nj+1;
        for k=1:th
            FF(k,1)=FF(k,1)+Uresult(i,2);
        end
    end
end

Njumps=zeros(length(FI),1);
Njumps(1,1)=nj;

% output: raw and adjusted candidate time series, number of
corrected
% shifts

apotelesma4=[FI FF Njumps];

end
```